New Uzbekistan University

Discrete Mathematics | MATH 221

Tutorial Week 11 | Pigeonhole Principle, Linear Recurrence Relations

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Applications of Recurrence Relations

1. A country uses as currency coins with values of 1 peso, 2 pesos, 5 pesos, and 10 pesos. Find a recurrence relation for the number of ways to pay a bill of n pesos if the order in which the coins are paid matters.
2. Find a recurrence relation for the number of bit strings of length n that contain three consecutive 0s. What are the initial conditions? How many bit strings of length seven contain three consecutive 0s?
3. Find a recurrence relation for the number of ways to climb *n* stairs if the person climbing the stairs can take one stair or two stairs at a time. What are the initial conditions? In how many ways can this person climb a flight of eight stairs?
4. Consider strings of length *n* of characters of 0-9 and find a recurrence relation for the number of such sequences of length *n* with an even number of 0s.

Pigeonhole principle

5. What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?
6. Suppose that every student in a discrete mathematics class of 25 students is a freshman, a sophomore, or a junior. a) Show that there are at least nine freshmen, at least nine sophomores, or at least nine juniors in the class.
7*. Show that if there are 101 people of different heights standing in a line, it is possible to find 11 people in the order they are standing in the line with heights that are either increasing or decreasing.
8. A computer network consists of six computers. Each computer is directly connected to zero or more of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers.

Linear Recurrence Relations

9. In how many ways can a $2 \times n$ rectangular checkerboard be tiled using 1×2 and 2×2 pieces?