New Uzbekistan University

Discrete Mathematics | MATH 221

Tutorial - Week 10 | Basic Counting Principles

Dr. Rustam Turdibaev

1. Consider a modified version of the binary search algorithm, where instead of splitting the array into two equal halves, we split it into three equal parts and recurse into the part containing the target. Assume n is always a power of 3.
(a) Write the recurrence relation for the worst-case runtime.
(b) Use Master Theorem to estimate the complexity.
(c) Compare the complexity of this algorithm to standard binary search.

2. Given an array A of size n, design a recursive algorithm to find the maximum element in A. Your algorithm should:
Divide the array into two halves. Recursively find the maximum in each half. Combine the results to determine the overall maximum.
(a) Write a recurrence relation for the time complexity $T(n)$ and estimate it using Master Theorem.
(b) Prove the correctness of the algorithm using strong induction.

1. An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?

2. A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each gender. How many different types of this shirt are made?

3. A palindrome is a string whose reversal is identical to the string. How many bit strings of length n are palindromes?

4. How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

5. In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if
a) the bride must be next to the groom?
b) the bride is not next to the groom?
c) the bride is positioned somewhere to the left of the groom?

6. How many bit strings of length seven either begin with two 0s or end with three 1s?

7. How many bit strings of length 10 contain either five consecutive 0s or five consecutive 1s?

8. Every student in a discrete mathematics class is either a computer science or a mathematics major or is a joint major in these two subjects. How many students are in the class if there are 38 computer science majors (including joint majors), 23 mathematics majors (including joint majors), and 7 joint majors?

9. How many ways are there to arrange the letters a, b, c, and d such that a is not followed immediately by b?

10. Determine the number of matches played in a single-elimination tournament with n players, where for each game between two players the winner goes on, but the loser is eliminated.