New Uzbekistan University

Discrete Mathematics | MATH 221

Tutorial Week 12 | Linear Recurrence Relations

Dr. Rustam Turdibaev

Linear Recurrence Relations with Constant Coefficients

1. In how many ways can a $2 \times n$ rectangular checkerboard be tiled using 1×2 and 2×2 pieces?
2. Solve these recurrence relations together with the initial conditions:
$a_n = -4a_{n-1} - 4a_{n-2} \quad \text{for} \quad n \geq 2, \quad a_0 = 0, \quad a_1 = 1$
3. Solve these recurrence relations together with the initial conditions:
$a_n = 4a_{n-2} \quad \text{for} \quad n \geq 2, \quad a_0 = 0, \quad a_1 = 4.$
4. Find the solution to the recurrence relation
$a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3} \quad \text{with} \quad a_0 = 7, \quad a_1 = -4, \quad a_2 = 8.$
5. Solve $a_n = 3a_{n-1} + 2^n$ with $a_0 = 1$.
6. Solve $a_n = 2a_{n-1} + 2^n$ with $a_0 = 2$.
7. Solve $a_n = 5a_{n-1} - 6a_{n-2} + 2^n + 3n$ with the initial condition $a_0 = 21/4$ and $a_1 = 1/4$.

8. What is the general form of the solutions of a linear homogeneous recurrence relation if its characteristic equation has the roots -1, -1, -1, 2, 2, 5, 5, 7?

9. What is the general form of the particular solution guaranteed to exist by Theorem 6 of the linear nonhomogeneous recurrence relation $a_n = 8a_{n-2} - 16a_{n-4} + F(n)$ if

a) $F(n) = n^3$

b) $F(n) = (-2)^n$

c) $F(n) = n^2 n$

d) $F(n) = n^2 4^n$

e) $F(n) = (n^2 - 2)(-2)^n$

f) $F(n) = n^4 2^n$

g) $F(n) = 2$

Other recurrence relations

1. Suppose that $f(n) = f(n/5) + 3n^2$ when n is a positive integer divisible by 5, and $f(1) = 4$. Find big-O estimate of $f(n)$.

2. Solve the recurrence relation $T(n) = nT^2(\dfrac{n}{2})$ with initial condition $T(1) = 6$.

(Hint: Consider the case $n = 2^k$ for some integer k.)

3*. Solve the recurrence relation $T(n) = nT^2(\frac{n}{2})$ with initial condition $T(1) = 6$.

4*. Suppose that the function f satisfies the recurrence relation

$$f(n) = 2f(\sqrt{n}) + 1$$

whenever n is a perfect square greater than 1 and f(2) = 1.

a) Find f(16).

b) Give a big-O estimate for f(n).