# Towards reproducibility in small-N treatment research in aphasiology: a tutorialy

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#### Abstract

Purpose: Small-N studies are the dominant study design supporting evidence-based treatment studies in communication sciences and disorders, and specifically in research on aphasia and related disorders. However, there is little guidance on conducting reproducible analysis of such studies, which has implications for scientific review, rigor, and replication.

Methods: This tutorial demonstrates how to implement reproducible analyses of small-N designs by reanalyzing data from Wambaugh et al. (2017), a single-case experimental design study with 20 individuals with post-stroke apraxia of speech and aphasia receiving Sound Production Treatment. A comparison and discussion of the strengths and weaknesses of small-N effect sizes is provided so that researchers can make informed decisions about how to best characterize treatment effects for their own work.

Results: Tutorial code demonstrates how to implement the following effect sizes: standardized mean difference, Proportion of Maximal Gain, Tau-U, and mixed-effects models at the individual and group level in the statistical language R. Data and code are publicly available as a resource for students, researchers, and clinicians.

Conclusion: This tutorial demonstrates how researchers in aphasia and related disorders can conduct reproducible analysis of small-N studies, the dominant intervention design in the field. We also demonstrate how properties of different approaches to statistical analysis can affect the interpretation and replication of small-N studies. This article may serve as a template for conducting reproducible analyses of the small-N designs common to aphasia and related disorders.

#### Setup

## Load packages and functions

```
library(here)
                       # for locating files
library(tidyverse)
                       # data wrangling
library(SingleCaseES)
                       # calculating SMD, Tau-U
library(lme4)
                       # frequentist mixed-effects models
library(emmeans)
                       # estimating effect sizes from lme4
library(brms)
                       # bayesian mixed-effects models
library(tidybayes)
                     # estimating effect sizes from brms
library(ggdist)
                       # Visualizing posterior distributions
# set a seed for reproducibility
set.seed(42)
```

#### Read in data

Note that the current setup uses RStudio R projects (https://support.rstudio.com/hc/en-us/articles/200526207-Using-RStudio-Projects). One of the features of R projects is that the working directory is automatically set to the project root (the folder with the .Rproj). A discussion of R projects can be found at https://www.tidyverse.org/blog/2017/12/workflow-vs-script/. In this case here("study-data") refers to the /study-data folder inside the project.

# Preview the data

```
head(df)
## # A tibble: 6 x 11
```

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##		participant	condition	${\tt phoneme}$	$\verb itemType $	phase	session	item	trials	spt2017
##		<chr></chr>	<chr></chr>	<chr></chr>	<chr></chr>	<chr></chr>	<dbl></dbl>	<chr></chr>	<dbl></dbl>	<chr></chr>
##	1	P1	blocked	pr	tx	${\tt baseline}$	1	pr-1	10	pre
##	2	P1	blocked	pr	tx	${\tt baseline}$	1	pr-12	10	pre
##	3	P1	blocked	pr	tx	${\tt baseline}$	1	pr-4	10	pre
##	4	P1	blocked	pr	tx	${\tt baseline}$	1	pr-15	10	pre
##	5	P1	blocked	pr	tx	${\tt baseline}$	1	pr-5	10	pre
##	6	P1	blocked	pr	tx	${\tt baseline}$	1	pr-7	10	pre
##	#	# with 2 more variables: response <dbl>. n baselines <dbl></dbl></dbl>								

Table 1
Data variables and their description

Variable	Description
participant	de-identified participant ID
condition	probe schedule (blocked or random)
phoneme	target_phoneme
itemType	item condition (treatment or generalization)
phase	treatment phase
session	session number from Wambaugh 2017
item	item identifier
trials	number of items in the list (per phoneme)
$\operatorname{spt2017}$	phase used to calcualte effect sizes in Wambaugh et al., 2017
response	accuracy of participant response
$n$ _baselines	Number of baseline sessions

### Case example: Participant 10

# Filter data for Participant 10

Starting from the entire dataset, filter for participant 10, treated items, and the blocked condition. Then to calculate session-level data (the number of correct responses per session), group by session, and use the summarize function to calculate the number of correct responses per session. The <code>group\_by</code> function also includes phase and <code>spt2017</code> because we want to keep these variables in the summary data frame, but their addition doesn't affect grouping. The .groups argument removes the grouping after summarize.

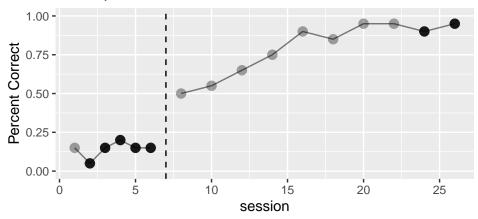
```
P10 <- df %>%
  filter(participant == "P10",
        itemType == "tx",
        condition == "blocked") %>%
  group_by(session, phase, spt2017) %>%
  summarize(sum_correct = sum(response), .groups = "drop")
```

#### Plot performance over time

Plotting data from participant (also Figure 1.). First, we select only the baseline and treatment phases (ignoring the washout and maintenance phases for the purpose of this paper). The we create a dummy variable reflecting whether or not the session was included in the SMD/PMG calculations. Finally, the {ggplot2} package. A recent primer on {ggplot2} for researchers unfamilar with R can be found here: https://doi.org/10.1177/25152459221074654

```
P10 %>%
  filter(phase == "baseline" | phase == "treatment") %>%
  mutate(Measure = factor(
    ifelse(!is.na(spt2017), "include", "exclude"),
           levels = c("exclude", "include"))) %>%
  ggplot(aes(x = session, y = sum\_correct/20, group = phase)) +
  geom_point(aes(alpha = Measure), size = 3) +
  geom line(alpha = 0.5) +
  geom_vline(aes(xintercept = 7), linetype = "dashed") +
  scale x continuous(breaks = seg(0,30,5)) +
  ylim(0, 1) +
  scale alpha discrete(range = c(0.35, 0.9)) +
  labs(title = "Participant 10, treated words, blocked condition",
       caption = "Dark circles represent data points used to calculate
       the within-case standardized mean difference in Wambaugh et al.,
       (2017)",
       y="Percent Correct") +
  guides(alpha = "none")
```

# Participant 10, treated words, blocked condition



Dark circles represent data points used to calculate the within–case standardized mean difference in Wambaugh et al., (2017)

#### Within-case standardized mean difference

There are any number of ways to calculate the within case standardized mean difference using R code. In this example, we have used the SMD() function from the established package

{SingleCaseES} by James Pustejovsky because it includes additional functions that may be of interest to researchers in aphasiology.

Note that the bias\_correct argument is set to FALSE to match what is typically done in aphasia research, though aphasia researchers may benefit from using the bias correction for small sample sizes as it can reduce procedural sensitivities of the within-case standardized mean difference.

Additionally, we do not show all information returned by the function, which also includes a 95% confidence interval, as it is not clear that this confidence interval applies to the the  $d_{\rm BR}$  modification of the original within-case standardized mean difference.

```
## [1] 14.33207
```

To calculate  $d_{\rm BR}$  for all participants and conditions in the Wambaugh et al, (2017) study, we created a custom function which can be found in the R/effect-size-functions.R file.

## Proportion of potential maximal gain

There is no R package that includes a function to calculate PMG to our knowledge. However, creating such a function is relatively straightforward. A function that calculates PMG similar to the SMD() function from the {SingleCaseES} package might take the following form, with an additional argument for the number of items treated (nitems). The function calculates the mean of the A phase and B phase, and then calculates and returns the PMG value from the same data as  $d_{\rm BR}$  above.

```
# the function is named PMG and takes 3 arguments:
# vectors of the a_data and b_data, and
# a single number indicating how many items were treated
PMG <- function(a_data, b_data, nitems){
   mean_a <- mean(a_data) # calculate mean of a_data
   mean_b <- mean(b_data) # calculate mean of b_data
   pmg <- (mean_b-mean_a)/(nitems-mean_a) # calculate PMG
   return(pmg) # return the PMG value.
}

PMG(a_data = A, b_data = B, nitems = 20)</pre>
```

```
## [1] 0.9127907
```

To calculate PMG for all participants and conditions in the Wambaugh et al., (2017) study,

we created a custom function which can be found in the R/effect-size-functions.R file.

#### Tau-U

The Tau-U family of effect sizes (and a number of other non-overlap measures) can be calculated using the {SingleCaseES} package. In this case, we use all data summarized in the P10 dataframe (and not just the data used to calculate  $d_{\rm BR}$ ).

First, we estimate the trend line during the baseline phase, which can be generated by creating a simple linear model using the lm() function. The model includes the number of correct responses as the dependent variable and the session number as the independent variable. The session coefficient reflects the slope during the baseline phase. The coef() function simple extracts the model coefficients.

```
P10 %>%
   filter(phase == "baseline") %>%
   lm(data = ., sum_correct~session) %>%
   coef()
```

```
## (Intercept) session
## 2.133333 0.200000
```

The Tau() and Tau\_U() functions take the same data structure as the SMD() and PMG() functions above. I

Using the conservative benchmark of 0.33 recommended by Lee and Cherney (2018), we would calculate Tau~A VS. B~ as the slope of the baseline phase is only 0.2. To calculate Tau-U~A VS. B~, we can use the Tau() function.

```
A = P10 %>% filter(phase == "baseline") %>% pull(sum_correct)
B = P10 %>% filter(phase == "treatment") %>% pull(sum_correct)
Tau(A_data = A, B_data = B)
```

```
## ES Est SE CI_lower CI_upper
## 1 Tau 1 0.02710291 1 1
```

However, if we had elected to correct for baseline trends and use Tau-U $\sim$ A VS. B - TREND A $\sim$ , we can use the similar Tau\_U() function.

```
Tau_U(A_data = A, B_data = B)
## ES Est
## 1 Tau-U 0.95
```

#### Mixed-effects model-based effect sizes

The mixed-effects model example for participant 10 uses item-level data, so we need to create a new dataframe for this model. The model formula is based on a structure from Huitema & McKean (2000). We recommend the reader read Huitema & McKean for a clear description and justification for this model structure.

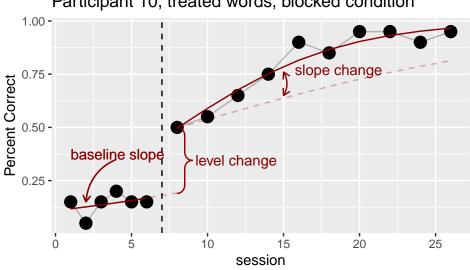
$$Y_t = \beta_0 + \beta_1 T_t + \beta_2 D_t + \beta_3 [T_t - (n_1 + 1)]D_t + \epsilon_t$$

After selecting data from participant 10, the coefficients are created by:

- setting baseline\_slope equal to the session variable
- level\_change is a dummy variable, 0 during baseline and 1 during treatment
- slope\_change is created by subtracting the number of baselines plus 2 from the baseline slope value, and then multiplying the result with the level\_change variable. Typically, if probing every session, the formula calls for subtracting the number of baselines plus 1. However, because Wambaugh et al., (2017) used intermittent probing schedules, and probed every other treatment session starting at the second, we need to add 2 to the number of baselines to ensure that the slope change variable starts at 0 on the first recorded treatment probe.

```
P10 <- df %>%
  filter(participant == "P10",
         condition == "blocked",
         itemType == "tx",
         phase == "baseline" | phase == "treatment") %>%
  mutate(baseline_slope = session,
         level_change = ifelse(phase == "baseline", 0, 1),
         slope_change = (baseline_slope - (6+2))*level_change,
         level_change = as.factor(level_change))
```

Figure 2. visualizes each parameter in this model structure. The code can be found in the .Rmd file, and is omitted from the pdf due to its length.



# Participant 10, treated words, blocked condition

The following shows how we arrived at the final model for P10

1. First, we fit the maximal random effects structure. However, the model did not converge.

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.192497 (tol = 0.002, component 1)
```

2. Second, we tried specifying a different optimizer, following recommendations that can be found at https://bbolker.github.io/mixedmodels-misc/glmmFAQ.html#convergence-warnings.

Because the model structure is pre-determined, we tried a different optimizer, which we have had more success with in past studies. This removed the convergence warning.

We can now examine the model summary:

```
summary(mod1)
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
     Approximation) [glmerMod]
##
## Family: binomial (logit)
## Formula: response ~ baseline_slope + level_change + slope_change + (1 +
##
       baseline_slope + level_change + slope_change | item)
      Data: P10
##
## Control: glmerControl(optimizer = "bobyqa")
##
##
        AIC
                 BIC
                       logLik deviance df.resid
               302.3 -110.8
##
      249.5
                                 221.5
                                            306
##
## Scaled residuals:
##
      Min
                1Q Median
                                30
                                       Max
## -2.2995 -0.2462 0.0202 0.1563 3.6534
##
## Random effects:
   Groups Name
                          Variance Std.Dev. Corr
##
           (Intercept)
##
   item
                         4.5354
                                   2.1296
##
           baseline_slope 0.3358
                                   0.5795
                                            -0.75
                                             0.34 -0.01
##
           level_change1 5.6514
                                   2.3773
```

```
##
           slope change
                           0.8646
                                    0.9298
                                              0.66 - 0.97 - 0.14
## Number of obs: 320, groups:
                                 item, 20
##
## Fixed effects:
##
                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                  -2.70900
                               1.33242
                                        -2.033
                                                   0.042 *
## baseline_slope
                   0.01743
                               0.33249
                                         0.052
                                                   0.958
                                         1.480
## level_change1
                   2.50785
                               1.69431
                                                   0.139
## slope_change
                                         0.989
                                                   0.323
                   0.39216
                               0.39651
## ---
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
## Correlation of Fixed Effects:
##
               (Intr) bsln_s lvl_c1
## baselin_slp -0.861
## level chng1 0.535 -0.723
## slope_chang 0.761 -0.916
```

We note that in this case, further reducing the random effects structure often returns a significant result for the level\_change parameter, demonstrating how our choice of random effect structure can influence the statistical significance of model parameters.

Calculating an overall effect size for this participant requires contrasting performance either at the end of treatment with and without the level change and slope change parameters, or contrasting performance at the end of treatment with performance at the end of baseline. The former option assumes that any baseline trend would have continued throughout the treatment phase in the absence of treatment, is typically more conservative.

While there is a small, empirical baseline slope in this data, it may be reasonable to consider that this slope is largely driven by lower performance on the second probe session, and that performance in baseline sessions 3-6 are stable, and therefore estimate the difference in performance from the end of baseline to the end of treatment. Criteria for such decisions should ideally be made a-priori if possible.

1. First, we generate the marginal means for each combination of baseline slope, level change, and slope change.

```
marginal_means
    baseline slope level change slope change emmean
                                                        SE df asymp.LCL asymp.UCL
##
                 7 0
                                            0 -2.5870 1.36 Inf
                                                                  -5.252
                                                                            0.0782
##
                26 0
                                            0 -2.2559 7.53 Inf
                                                                 -17.009
                                                                           12.4976
##
                 7 1
                                            0 -0.0792 1.20 Inf
                                                                  -2.427
                                                                            2.2684
                26 1
                                            0 0.2520 6.39 Inf
##
                                                                 -12.263
                                                                           12.7672
##
                 7 0
                                           19 4.8641 6.46 Inf
                                                                  -7.801
                                                                           17.5290
##
                26 0
                                           19 5.1952 3.06 Inf
                                                                  -0.802
                                                                           11.1928
                                           19 7.3719 7.35 Inf
                                                                  -7.041
##
                 7 1
                                                                           21.7846
                                           19 7.7030 2.49 Inf
##
                26 1
                                                                   2.819
                                                                           12.5873
##
## Results are given on the logit (not the response) scale.
## Confidence level used: 0.95
```

2. This returns a table of all possible comparisons, and we are only interested in contrasting the first row (beginning of treatment) with the last row (end of treatment). After selecting these two rows, we can then contrast their estimates.

```
# code to select first and last rows
# The 1 indicates that the row should be selected
A = c(1, 0, 0, 0, 0, 0, 0, 0)
B = c(0, 0, 0, 0, 0, 0, 0, 1)

# contrast the marginal means
# infer argument returns a confidence interval and p value if
# both are set to TRUE.
contrast(marginal_means,
    method = list("Unadjusted effect size" = B-A),
    infer = c(TRUE, TRUE))
```

We could also make the more conservative assumption that any baseline trend continues by choosing the second row where baseline slope is set to the last treatment session.

```
# code to select first and last rows

# The 1 indicates that the row should be selected

A = c(0, 1, 0, 0, 0, 0, 0, 0)

B = c(0, 0, 0, 0, 0, 0, 0, 1)
```

```
# contrast the marginal means
# infer argument returns a confidence interval and p value if
# both are set to TRUE.
contrast(marginal_means,
    method = list("Unadjusted effect size" = B-A),
    infer = c(TRUE, TRUE))
```

Notice that there is much greater uncertainty in this contrast, and as a result the p-value is no longer significant.

**Group-level model.** We can extend this individual model to all participants, still focusing on treated items in the blocked condition. First, we create a new dataframe that includes all participants and then repeat the model

Then we can start again with a relatively maximal random effect structures, noting that we could also include random slopes for items. However, it is unlikely that such a model structure could be supported by the data. In this case we have chosen to include the most theoretically important random effects (Matsucheck, 2018) that we expect to be supported by the data.

The model takes a little longer to run, but returns a convergence warning

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.0786858 (tol = 0.002, component 1)
```

Again, we change the optimizer.

Since the model appears to have converged, we can examine the model results summary (mod2)

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
    Approximation) [glmerMod]
## Family: binomial (logit)
## Formula: response ~ baseline slope + level_change + slope change + (1 +
##
      baseline_slope + level_change + slope_change | participant) +
##
       (1 | item)
     Data: df_glmm
##
## Control: glmerControl(optimizer = "bobyqa")
##
##
       AIC
                BIC
                      logLik deviance df.resid
##
    5652.2 5755.8 -2811.1
                               5622.2
                                          7385
##
## Scaled residuals:
      Min
               1Q Median
                               3Q
                                      Max
## -8.1144 -0.3551 -0.1630 0.3241 13.5510
##
## Random effects:
                              Variance Std.Dev. Corr
## Groups
               Name
## item
               (Intercept)
                             1.611099 1.26929
## participant (Intercept)
                              0.404805 0.63624
##
               baseline_slope 0.002648 0.05146
                                                0.41
##
               level_change1 2.390503 1.54613
                                                0.18 0.86
                              0.007304 0.08546 -0.78 -0.54 -0.52
               slope_change
## Number of obs: 7400, groups: item, 322; participant, 20
##
## Fixed effects:
                 Estimate Std. Error z value Pr(>|z|)
##
## (Intercept)
               -3.44259 0.20649 -16.672 < 2e-16 ***
## baseline_slope 0.07624
                             0.01883 4.050 5.13e-05 ***
## level change1 0.85952 0.39754 2.162 0.030611 *
## slope_change
                            0.02582 3.534 0.000409 ***
                  0.09124
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Correlation of Fixed Effects:
## (Intr) bsln_s lvl_c1
## baselin_slp -0.161
## level_chng1 0.096 0.310
## slope_chang -0.192 -0.666 -0.363
```

The summary table shows us there there are statistically reliable effects for all three parameters: a small but reliable trend at baseline, a fairly substantial level change on average, and an increase in slope from baseline that is slightly more than double the initial average trend. Additionally, performance at baseline is predicted to be low, just 3%. We also note that there there is much more variation in the level change parameter between participants relative to the baseline slope and slope change parameters. Finally, the correlation of fixed effects shows a positive association between individuals baseline trend and their level change, but a negative association between individuals baseline trend and slope change and level change.

We can calculate an overall effect size using the same approach as the individual model. In this case, we assume the median number of baseline sessions (11) and treatment sessions (20)

```
baseline_slope level_change slope_change
##
                                                emmean
                                                           SE
                                                               df asymp.LCL asymp.UCL
##
                11 0
                                             0 -2.6040 0.268 Inf
                                                                    -3.1289
                                                                              -2.07905
##
                31 0
                                             0 -1.0792 0.587 Inf
                                                                    -2.2293
                                                                               0.07091
                                             0 -1.7444 0.545 Inf
                                                                    -2.8119
##
                11 1
                                                                              -0.67695
                                             0 -0.2197 0.814 Inf
##
                31 1
                                                                    -1.8145
                                                                               1.37515
##
                11 0
                                            20 -0.7791 0.393 Inf
                                                                    -1.5503
                                                                              -0.00791
##
                31 0
                                            20
                                                0.7457 0.410 Inf
                                                                    -0.0587
                                                                               1.54996
##
                11 1
                                            20
                                                0.0804 0.480 Inf
                                                                    -0.8613
                                                                               1.02214
##
                31 1
                                            20
                                               1.6052 0.581 Inf
                                                                     0.4669
                                                                               2.74347
```

## Results are given on the logit (not the response) scale.
## Confidence level used: 0.95

Because the baseline trend, on average, was statistically reliable, we calculated an overall effect size assuming that it would have continued in the absense of treatment.

```
# code to select first and last rows
# The 1 indicates that the row should be selected
A = c(0, 1, 0, 0, 0, 0, 0, 0)
B = c(0, 0, 0, 0, 0, 0, 0, 1)
# contrast the marginal means
# infer argument returns a confidence interval and p value if
# both are set to TRUE.
contrast(marginal_means,
    method = list("Unadjusted effect size" = B-A),
    infer = c(TRUE, TRUE))
## contrast
                                       SE df asymp.LCL asymp.UCL z.ratio p.value
                           estimate
## Unadjusted effect size
                               2.68 0.525 Inf
                                                   1.66
                                                             3.71
                                                                    5.112 <.0001
##
```

This results in a statistically reliable group effect size of 2.7 logits. Given that the group model suggests a starting place of only around 3%, this indicates a gain of about 29 percentage points on average can be attributed to the level and slope changes. we can calculate this by running plogis(-3.44 + 2.68)-plogis(3.44). However, we're not aware of a straightforward method of estimating individual effect sizes and confidence intervals using the frequentist approach.

## Results are given on the log odds ratio (not the response) scale.

### Bayesian Mixed effects models

## Confidence level used: 0.95

Bayesian mixed-effects models can be used in the same fashion as model 2 above to obtain both group and individual effect size estimates. First, a group-level model is estimated.

```
),
# extra arguments, see rmd file
cores = 4,
file = "models/group_brm",
file_refit = "on_change")
```

We can preview the model results using summary() again. Notably, the model estimates are largely similar to the frequentist model.

summary(mod3)

```
## Family: bernoulli
     Links: mu = logit
##
## Formula: response ~ 0 + Intercept + baseline_slope + level_change + slope_change + (1 +
##
      Data: df_glmm (Number of observations: 7400)
     Draws: 4 chains, each with iter = 3000; warmup = 1000; thin = 1;
##
##
            total post-warmup draws = 8000
##
## Group-Level Effects:
## ~item (Number of levels: 322)
                 Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
                                         1.15
                                                  1.46 1.00
                                                                 2366
                                                                          4243
## sd(Intercept)
                     1.30
                               0.08
##
## ~participant (Number of levels: 20)
                                      Estimate Est.Error 1-95% CI u-95% CI Rhat
## sd(Intercept)
                                                    0.23
                                                              0.40
                                                                       1.28 1.00
                                          0.77
## sd(baseline slope)
                                                             0.03
                                                                       0.11 1.00
                                          0.06
                                                    0.02
## sd(level_change1)
                                          1.80
                                                    0.40
                                                             1.12
                                                                       2.70 1.00
## sd(slope_change)
                                          0.10
                                                    0.03
                                                             0.05
                                                                       0.16 1.00
## cor(Intercept,baseline_slope)
                                          0.21
                                                    0.33
                                                            -0.45
                                                                       0.79 1.00
## cor(Intercept,level_change1)
                                          0.15
                                                    0.28
                                                            -0.43
                                                                       0.65 1.00
## cor(baseline_slope,level_change1)
                                          0.36
                                                    0.32
                                                            -0.32
                                                                       0.87 1.00
## cor(Intercept,slope_change)
                                         -0.56
                                                    0.25
                                                            -0.93
                                                                       0.02 1.00
## cor(baseline_slope,slope_change)
                                         -0.37
                                                            -0.85
                                                                       0.33 1.00
                                                    0.31
## cor(level_change1,slope_change)
                                                            -0.72
                                                                       0.35 1.00
                                         -0.22
                                                    0.28
##
                                      Bulk_ESS Tail_ESS
## sd(Intercept)
                                                   3834
                                          2463
## sd(baseline_slope)
                                          2559
                                                   3731
## sd(level_change1)
                                          3090
                                                   4799
## sd(slope_change)
                                          1180
                                                   2898
## cor(Intercept,baseline_slope)
                                          1883
                                                   3701
## cor(Intercept,level_change1)
                                          1310
                                                   2675
## cor(baseline_slope,level_change1)
                                           915
                                                   1814
## cor(Intercept,slope_change)
                                          1053
                                                   2559
## cor(baseline_slope,slope_change)
                                           954
                                                   1958
```

```
## cor(level_change1,slope_change)
                                                   2921
                                         1657
##
## Population-Level Effects:
##
                  Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept
                     -3.41
                                0.24
                                        -3.89
                                                  -2.96 1.00
                                                                 2118
                                                                          3857
## baseline_slope
                      0.06
                                0.02
                                         0.02
                                                   0.11 1.00
                                                                 1309
                                                                          2171
## level_change1
                      0.90
                                0.45
                                        -0.01
                                                   1.78 1.00
                                                                 2587
                                                                          3970
## slope_change
                      0.11
                                0.03
                                         0.05
                                                   0.17 1.00
                                                                 1582
                                                                          2628
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk ESS
## and Tail ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
df contrasts <- mod3$data %>%
    group_by(level_change, participant) %>%
    mutate(last_session = max(baseline_slope)) %>%
    filter(baseline_slope == last_session) %>%
    select(-response) %>%
    distinct()
es_logit = df_contrasts %>%
    add_linpred_draws(mod3) %>%
    ungroup() %>%
    mutate(timepoint = ifelse(level_change == 0, "entry", "exit")) %>%
    select(timepoint, item, .draw, .linpred, participant) %>%
    pivot_wider(names_from = "timepoint", values from = .linpred) %>%
    mutate(ES = exit-entry) %>%
    group by(participant) %>%
    point_interval(ES)
```

Examine the results:

```
head(es_logit, 20)
```

```
## # A tibble: 20 x 7
     participant
                    ES .lower .upper .width .point .interval
##
##
     <chr>
                 <dbl> <dbl> <dbl> <chr> <chr>
## 1 P1
                  3.34 2.36
                               4.34
                                      0.95 median qi
   2 P10
##
                  6.35 5.13
                               7.78
                                      0.95 median qi
## 3 P11
                  4.99 3.69
                               6.52 0.95 median qi
## 4 P12
                  3.73 2.90
                               4.58
                                      0.95 median qi
## 5 P13
                  6.54 5.43
                               7.72
                                      0.95 median qi
## 6 P14
                  4.85 3.91
                               5.86
                                      0.95 median qi
                  3.39 2.13
## 7 P15
                               4.77
                                      0.95 median qi
## 8 P16
                  7.22 5.27
                               9.73
                                      0.95 median qi
## 9 P17
                  5.75 4.07
                               8.12
                                      0.95 median qi
```

##	10	P18	1.46	0.342	2.66	0.95	${\tt median}$	qi
##	11	P19	7.46	6.14	9.00	0.95	${\tt median}$	qi
##	12	P2	8.31	6.65	10.3	0.95	${\tt median}$	qi
##	13	P20	4.42	3.44	5.52	0.95	${\tt median}$	qi
##	14	P3	6.40	5.20	7.74	0.95	${\tt median}$	qi
##	15	P4	6.27	5.17	7.55	0.95	${\tt median}$	qi
##	16	P5	3.61	2.35	5.03	0.95	${\tt median}$	qi
##	17	P6	4.76	3.48	6.26	0.95	${\tt median}$	qi
##	18	P7	2.57	1.73	3.41	0.95	${\tt median}$	qi
##	19	P8	4.15	3.19	5.18	0.95	${\tt median}$	qi
##	20	P9	2.29	1.43	3.21	0.95	${\tt median}$	qi

These results can also be easily visualized

```
df_contrasts %>%
   add_linpred_draws(mod3) %>%
   ungroup() %>%
   mutate(timepoint = ifelse(level_change == 0, "entry", "exit")) %>%
   select(timepoint, item, .draw, .linpred, participant) %>%
   pivot_wider(names_from = "timepoint", values_from = .linpred) %>%
   mutate(ES = exit-entry) %>%
   ggplot(aes(x=ES, y = participant)) +
   ggdist::stat_halfeye() +
   geom_vline(aes(xintercept = 0), linetype = "dashed", color ="darkred")
```

