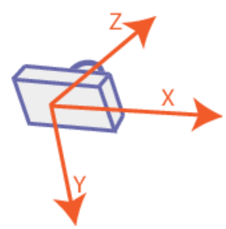
CP 313
Autonomous Navigation

Homework Assignment #2 Due: November 5, 2019

1. Consider two views of the high voltage department building provided at the following link: These images were taken by a GoPro camera whose intrinsics are given below.
$$\begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2.0095765446472751 14e+03 & 0 & 2.0e+03 \\ 0 & 1.9990083158359148 00e+03 & 1.5e+03 \\ 0 & 0 & 1 \end{bmatrix}$$
The distortion parameters are given by: $[k_1, k_2, p_1, p_2, k_3] = [-1.6245705992374331 19e-02, -5.0516206107488755 01e-02, 2.6181652464636758 46e-04, 4.5522549650748381 22e-03, 6.3469805548287711 04e-02]$. Estimate the fundamental matrix associated with the two views, and use this to compute the motion of the camera between the two views. The camera coordinate system is shown in the figure below.



2. Consider two views of the Entrepreneurship Centre Building following link: The second position of the camera was obtained by traslating by the amount $[0.24, 0, -1.8]^T$ and rotating it about the $y-$axis by $-19°$. Based on the above, compute the height and width of the monitor in the image. Use the same convention for the camera coordinate sytem as in Problem 1. Also, assume the camera intrinsics.

3. Given the translation $\mathbf{t}$ and rotation $\mathbf{R}$ between two poses of a camera, we know that the essential matrix can be written as $\mathbf{E} = [\mathbf{t_x}]\mathbf{R}$.

   (a) Show that the cross product operation $[t_x]$ can be written as $[t_x] = \mathbf{S Z R_{90} S^T}$, where $\mathbf{S}$ is an orthonormal basis, $\mathbf{Z}$ is a 3×3 identity matrix with the last diagonal element zeroed out, and $\mathbf{R_{90}}$ is a matrix that represents a $90°$ rotation about the first basis vector of $\mathbf{S}$.

   (b) Based on the result in (a), show how you would extract the translation vector $\mathbf{t}$ and the rotation matrix $\mathbf{R}$ out of $\mathbf{E}$.

4. Let $\{(\mathbf{x_i}, \mathbf{x'_i})\}$ be a set of matched feature points in two images, where $\mathbf{x'} = \mathbf{f}(\mathbf{x}; \mathbf{p})$. Here $\mathbf{f}(.)$ is a planar transformation parametrized by $\mathbf{p}$.

(a) Assume that the motion between the two views is small. Let $\Delta\mathbf{x} = \mathbf{x}' - \mathbf{x} = \mathbf{J}(\mathbf{x})\mathbf{p}$, where $\mathbf{J}$ is the Jacobian of the transformation $\mathbf{f}$. Derive an equation with $\mathbf{p}$ as the variable that can be solved for p.

(b) Using the result in (a) find the solution for the case where the transformation represents a simple translation.