Problem 1: Cows? Outside Ruminating Needlessly (28 Pts)

22

Circle **T** or **F** as appropriate for each of the following statements. Each correct action is worth 2 points. Unless otherwise stated, assume that n refers to the number of keys present in the structure.

1.1 (T) **F** The external nodes of a 4-way search trie have no children.

1.2 (T) (F) All nodes in an 9-way search tree have 9 children.

1.3 (T) **F** A 4-way search tree has keys in the leaves.

1.4 (T) **F** The root of a non-empty k-ary tree has level 0 and depth 0.

1.5 (T) **F** Every linked list containing keys is ordered.

1.6 (T) (F) The leaves of a 6-ary tree have up to 6 empty children.

1.7 (T) **F** A single `find_key` operation for a PR octree based on a 2^b by 2^b by 2^b grid is in $O(\log_4 n)$.

1.8 **T** (F) The shape of a binary search tree (BST) is independent of the order in which keys are inserted.

1.9 **T** (F) An MX quadtree with a grid of $2^v \times 2^v$ one by one blocks has up to v levels.

1.10 **T** (F) The amount of memory required by **PR octree** with a maximum of 2^d by 2^d by 2^d one by one by one cubes is in $O(\log_8(n))$.

1.11 **T** (F) An algorithm with execution time in $\Theta(\log n)$ must examine at least $(\alpha \log n)$ nodes, for some positive constant α.

1.12 **T** (F) If $T(n)$, the execution time of an algorithm, is in $\Theta(\log n)$, then there exist positive constants b, c, and d such that $b \leq c \leq d$ and $T(n) = c \log n$.

1.13 (T) **F** All nodes in a **k-d tree** of order 8 have 2 children.

1.14 (F) **F** Dr. Hugue's favorite beverage is Mountain Dew with tequila.

+16

Problem 2: Beagles Eagerly Emitting Train Sounds (20 pts)

Circle the letter of any statement below that you believe to be **True**. There is at least one **True** statement below; so, a page with nothing circled will be treated as blank.

The Assumptions–PM3 Quadtree

You should assume that the PM3 quadtrees are based on a $2^b \times 2^b$ grid, where b is allowed to grow to support asymptotic analysis. They represent graphs with distinct edges, where any 2 edges can intersect in at most one of the n vertexes in the graph. The PM3 partitions are closed on all boundaries, and each black node, or non-empty leaf, can contain at most one vertex.

The Questions–Circle True

✓ A. A PM3 quadtree is an object-space partitioned structure.

✓ B. Insertion of an edge into a PM3 quadtree requires the generation of at least one new partition.

✓ C. PM3 quadtrees are used because vertex search is in $O(n)$. ✗

✓ D. A PM3 quadtree leaf node must contain at least one vertex. ✗

✓ (E) A PM3 quadtree for the assumed graphs must consist of at least one gray node and two black nodes.

(F) A PM quadtree based on the $2^b \times 2^b$ grid above can be used to represent a polygonal map with 4^b distinct vertexes.

(G) A PM3 quadtree can represent an n-vertex graph with up to $n(n-1)/2$ undirected edges.

✓ (H) The number of nodes in a PM quadtree is in $O(n)$, where n is the number of vertexes which can be contained by the structure.

✓ (I) The number of black nodes in a PM quadtree is bounded above by $3g+1$, where g is the number of gray nodes.

✓ J. A PM3 quadtree without a vertex can contain at most one q-edge.

Problem 3 : Koalas Are Looking Energetic (22 pts)

For each term or phrase in the 2nd list, select an answer code from the first list that is the best description of the term. Codes may be used more than once, and there may be more than one acceptable answer. We will read and consider what you write if you go blank and have to 'write your own' answers. Each acceptable answer is worth 2 points.

Code	Answer
A	2d Array or matrix
E	k-way search triE
G	undirected acyclic connected Graph
K	K-ary tree
L	list
OTHER	explain for credit

2 1. Binary tree K

2 2. K-D tree of order 4 K

2 3. General tree G

2 4. AVL tree K

2 5. MX quadtree E

2 6. Point Region quadtree E

7. PM3 quadtree K

8. Non-empty graph L

2 9. Point OcTree K

2 10. Linked List L

2 11. Hash Table A

18

33

Problem 4: Playful Elks Attacking Spirals (34pts)

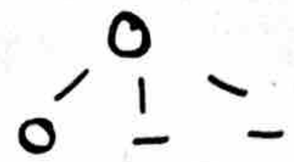
Some Definitions

1. **Full** k-ary tree: all children of internal nodes are non-empty.
2. **Complete** k-ary tree: full with all leaves at the same level. adding one more key will require an extra level.
3. **Left Complete** k-ary tree: not complete; leaves at two levels; complete through the level of the shallower leaves, and filled from left to right at the level of the deepest leaves.

4.1 (10) Draw a 3-ary tree having *no less than two* and *no more than three levels* that satisfies the properties associated with the definitions listed below. If no such structure exists, convince the grader that it is impossible. No credit for a blank answer

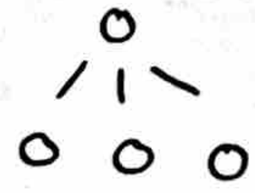
4.1.1 (2) neither full nor complete

+2



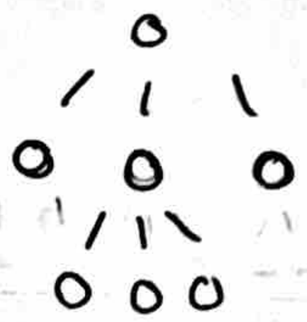
4.1.2 (2) full but not left complete.

+1



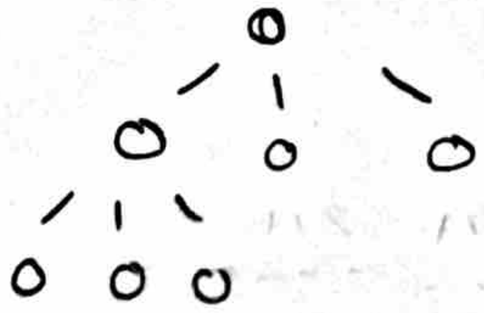
4.1.3 (2) full but not complete

+2



4.1.4 (2) left complete and full

+2

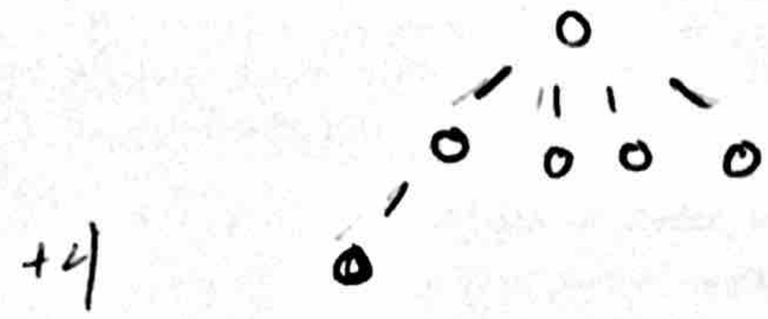


4.1.5 (2) complete but not full

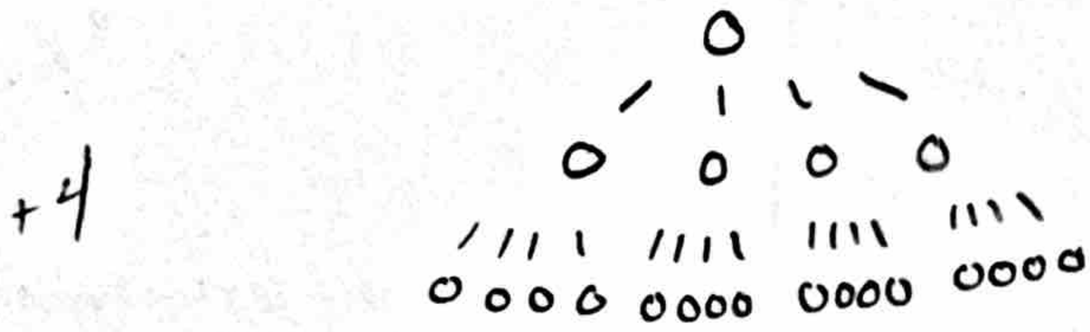
+2 Not possible, a complete tree has all nodes above the last level full.

4.3–4.6 (16) Sketch a **4-ary tree with 3 levels** which meets the requirements given in the problem. If no such structure exists, convince the grader. No credit for a blank answer

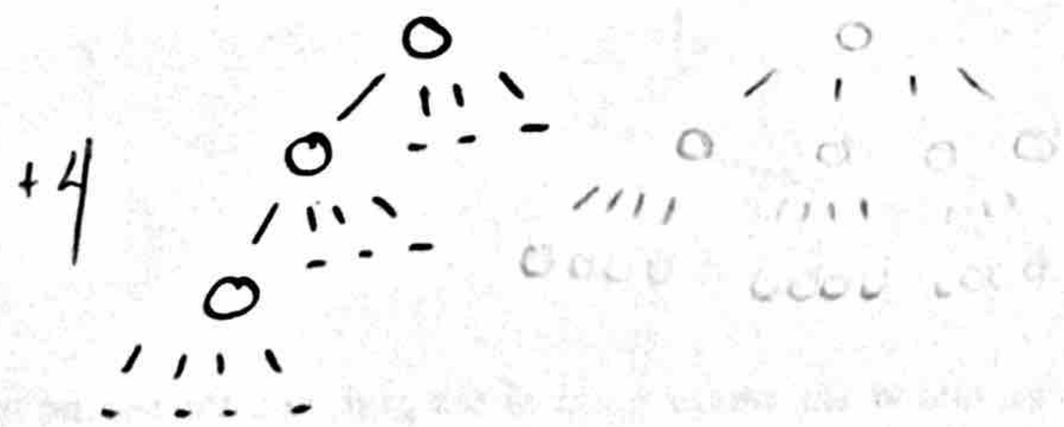
4.3 (4) left-complete having the fewest nodes



4.4 (4) full with all the leaves at the same level



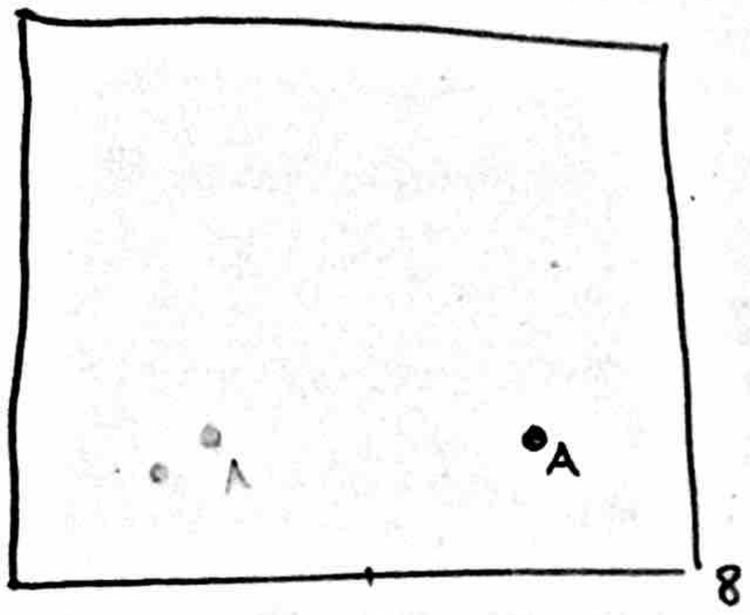
4.5 (4) the number of empty child pointers is minimal.



4.6 (4) the fewest internal nodes

4.7–4.8 (8) For 4 points per problem, draw a graph and the associated PR quadtree representation which satisfy the descriptions given below. Assume that the PR quadtree uses a grid of 2^3 by 2^3 one by one blocks. As in class, your gray nodes should look like:<>. Your black nodes should be circles. You must use the course convention for the correspondence between quadrants of the PR quadtree and the order of children of the gray nodes.
Do not draw unnecessary partitions or feel the need to generate coordinates for your vertices. Don't forget that the PR quadtree quadrants are closed on the left side and on the bottom.

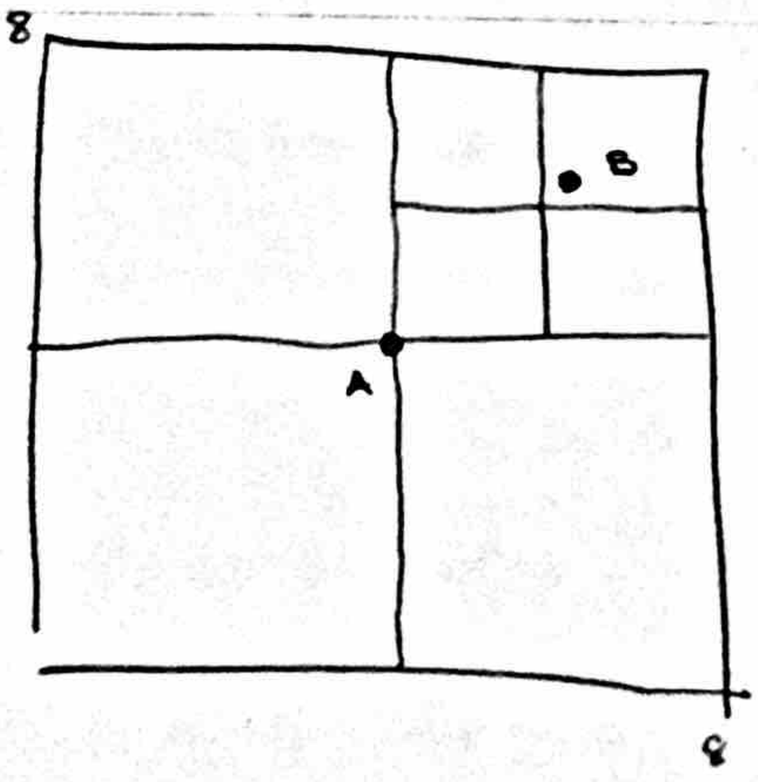
4.7 (4) The graph consists of exactly one vertex, located somewhere in the SE-most 2x2 block.



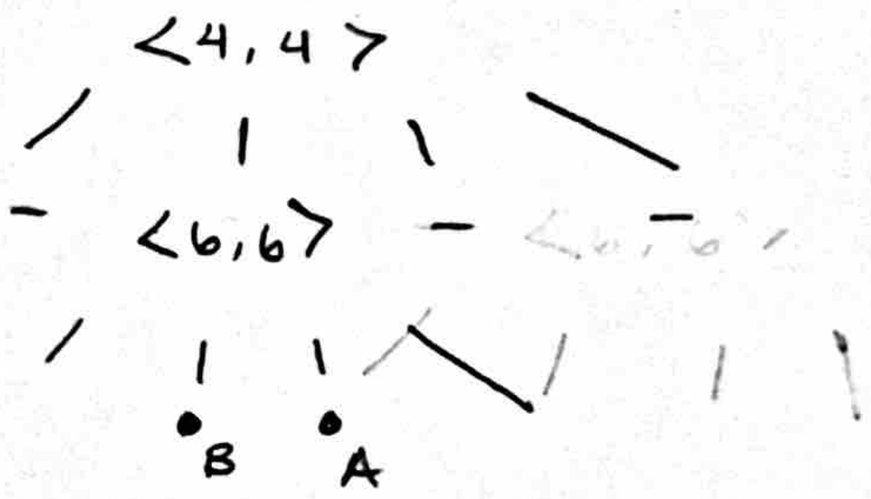
Tree

+4

4.8 (4) The graph consists two vertices, one at the center point of the grid, and the second somewhere in the NE-most 2x2 block but not on a boundary point of that block.



+4



Problem 5: Cougars And Brawny Badgers Ate, Growling Effortlessly (20+ pts)

Some of the following statements are True; others, woefully, are False. Your job is to circle **T** or **F** as appropriate. You will receive two points for each correct action. Zero points for a blank answer.
If you decide that a statement is False, you can disprove it using either a counter example or another type of proof for up to 2 more points. Merely explaining why it is False will earn one more point. Pictures often make excellent counter examples.
Unless otherwise stated, the letter n represents the number of nodes or keys in the structure, as appropriate for the question.

(T) **F** **5A.** All 8-ary trees with the leaves at the same level are full.[1].

(T) **F** **5B.** A PR quadtree with a maximum grid of $2^t \times 2^t$ one by one blocks will have to examine at least t nodes for a successful search.

T (F) **5C.** The order in which the n keys are inserted has no impact on the level of the deepest leaf in the resulting point quadtree.

[1] Go visit problem 4 for this definition

(T) F 5D. All ordered lists are sorted.

+1

(T) F 5E. The number of empty child pointers in a binary tree is one more than the number of nodes in it.

+2

(T) F 5F. A point octree with n keys has exactly $(7n + 1)$ empty child pointers

+2

(T) F 5G. An internal node in a k-ary tree has at least one non-empty child subtree.

+2

7

(T) F **5H.** Only one gray node is needed in a PR quadtree representing a graph with two vertices and no edges.

T F **5I.** All AVL trees are full 2-ary trees.

T F **5J.** A point region (PR) quadtree containing n black nodes has exactly $(n-1)/3$ gray nodes.