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Part I

Theory

A legal matter processed in IDEAL traverses four states:

- *Generation* (\mathcal{G}).
- *Representation* (\mathcal{R}).
- *Transformation* (\mathcal{T}).
- *Presentation* (\mathcal{P}).

We can understand IDEAL as a system of *plug-ins* which either generate, or commonly access and transform a unified representation of a legal matter into derivative states. Consequently, the system is a series of machines mapping $[\alpha_i \in \mathcal{G} \cup \mathcal{T} \cup \mathcal{P}] \rightarrow \mathcal{R}$, or $\mathcal{R} \rightarrow [\beta_i \in \mathcal{G} \cup \mathcal{T} \cup \mathcal{P}]$. We denote $\langle \alpha_i \rangle$ as a *generator* state which can generate \mathcal{R} , and β_i as a *producible* state, which can be produced by some action on \mathcal{R} .

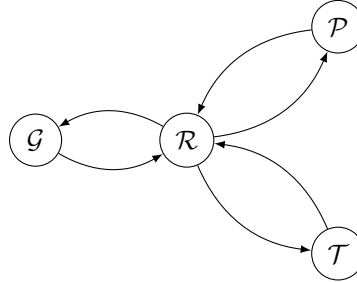


Figure 1: Visualising the interactions between the states.

1 Defining the Representation

Our goal is to define a mathematical structure for \mathcal{R} which encodes legal information and maximises the number of producible states. Given that the representation is driven by the encoded legal information, we begin with the idea that a lawyer is a mechanism for analysing the *existence* or *non-existence* of a legal relationship between *objects*.

Facts, Nodes and the NodeState. We describe a legal object as a **Node**, and define the admissible *factual* actions, attributes or character of the **Node** as the **NodeState**. The NodeState_n is an unordered

set of n **Fact** objects which are generated and edited by an oracle, \mathcal{F} , in combination with a **SourceOfLaw**:

$$\mathcal{F} : (\text{SourceOfLaw}, \{ \text{Fact}_0, \dots, \text{Fact}_n \}) \rightarrow \text{NodeState}_n \quad (1.1)$$

$$\mathcal{F} : (\text{Fact}, \text{NodeState}_n) \rightarrow \text{NodeState}_{n'} \quad (1.2)$$

$$\mathcal{F} : \text{NodeState}_n \rightarrow \text{NodeState}_{n''} \quad (1.3)$$

Sources of Law. A **SourceOfLaw** defines the conditions, attributes or characteristics which are required for the oracle to generate both a **NodeState**, as well as the **Role** (1.1) associated with a **NodeState**. Concretely, these are objects such as legislation or common law which have the inherent capacity to generate legal rights or obligations relevant to an object.

1.1 The Role Object

A **Role** defines the *legal personality* of a **Node** by capturing the attributes, actions, or characteristics attributable under law. A **Node** can be subject to multiple **Role** objects of arbitrary complexity, provided they are distinct under (1.6). The **Role** associated with a **Node** is generated by a pair (**NodeState**, **SourceOfLaw**) under the oracle function, \mathcal{F} , and is transformable under the **Consequence** of a **Link** (1.2):

$$\mathcal{F} : [(\text{NodeState}, \text{SourceOfLaw}) \rightarrow \text{Role}] \quad (1.4)$$

$$\text{Consequence} : [\text{Role} \rightarrow \text{Role}'] \quad (1.5)$$

Equivalence of Roles. We define an equivalence relation on a pair (**Role** _{i} , **Role** _{j}) by comparing their generating states, such that they are only pairwise distinct where the generative facts and law diverge:

$$[\text{Role}_i = \text{Role}_j] \iff [(\text{NodeState}_i \iff \text{NodeState}_j) \wedge (\text{SourceOfLaw}_i \iff \text{SourceOfLaw}_j)] \quad (1.6)$$

Role Extension. The **NodeState** of a **Node** can generate multiple **Role** objects *iff* the (**NodeState**, **SourceOfLaw**) pair are distinct under (1.6). A **Role** is *reducible* when a subset of the generative **NodeState** can produce another distinct **Role**:

$$[N := \{ \text{Fact}_0, \dots, \text{Fact}_n \}] \wedge [M := \{ \text{Fact}_0, \dots, \text{Fact}_m \}] : [M \subset N] \quad (1.7)$$

$$[\text{NodeState}_i = \mathcal{F}(N, \text{SourceOfLaw}_i)] : [\text{Role}_i = \mathcal{F}(\text{NodeState}_i, \text{SourceOfLaw}_i)] \quad (1.8)$$

$$[\text{NodeState}_j = \mathcal{F}(M, \text{SourceOfLaw}_j)] : [\text{Role}_j = \mathcal{F}(\text{NodeState}_j, \text{SourceOfLaw}_j)] \quad (1.9)$$

$$\implies [N \text{ is } \textit{reducible}] \quad (1.10)$$

A **Role** which is reducible is an *extension* of another **Role**, and the **Role** objects which are extended are called the *components of the extension*. The extended **Role** will automatically import any components

of the extension objects into its own definition. We denote an extension using subset notation, such that the following indicates Role_i is an extension of Role_j :

$$\text{Role}_j \subset \text{Role}_i \quad (1.11)$$

$$[\text{Role}_j \subset \text{Role}_i] : [\text{Role}_i \implies \text{Role}_j] \quad (1.12)$$

Given that an extension implies any components of the extension, a **Node** with multiple **Role** objects *may* replace any **Role** with an extension. We distinguish a **Role** extension from a **Role composition**, which is a set of distinct **Role** objects where there does not exist a **Role** in the composition which is an extension of any other **Role**:

$$[N := \{ \text{Role}_0, \dots, \text{Role}_n \}] : [\nexists (\text{Role}_i, \text{Role}_j) \in N : \text{Role}_i \subset \text{Role}_j] \quad (1.13)$$

1.2 Links

A **Link** is a directed, pairwise relationship between a source (Role_i) and a destination (Role_j), which has been generated with the assistance of a **SourceOfLaw**. Given a pair $(\text{Role}_i, \text{Role}_j)$ and an associated **SourceOfLaw**, the oracle may draw a $\text{Link}_{i \rightarrow j}$:

$$\mathcal{F} : (\text{SourceOfLaw}, \text{Role}_i, \text{Role}_j) \rightarrow \text{Link}_{i \rightarrow j} \quad (1.14)$$

Hooking a Link. The **Hook** and **Anchor** objects are a specialisation of the **Role** object which represent the directed, relational requirements of a **Link**, and are generated under a $(\text{SourceOfLaw}, \text{Role})$ pairing. The oracle defines a **Link** by consuming a $(\text{SourceOfLaw}, \text{Hook}, \text{Anchor})$ triple:

$$[\mathcal{F}(\text{SourceOfLaw}, \text{Role}_i) = \text{Hook}] \wedge [\mathcal{F}(\text{SourceOfLaw}, \text{Role}_j) = \text{Anchor}] \quad (1.15)$$

$$\implies \mathcal{F} : (\text{SourceOfLaw}, \text{Role}_i, \text{Role}_j) \rightarrow (\text{Hook}, \text{Anchor}) \quad (1.16)$$

$$\mathcal{F} : (\text{SourceOfLaw}, \text{Hook}, \text{Anchor}) \rightarrow \text{Link}_{i \rightarrow j} \quad (1.17)$$

1.3 Consequence

Given a **Link** to a relationship, the oracle, \mathcal{F} , generates a **Consequence** from a **SourceOfLaw**:

$$\text{Link}_{i \rightarrow j} \implies [\mathcal{F} : \text{SourceOfLaw} \rightarrow \text{Consequence}] \quad (1.18)$$

A **Node** may be modified under the **Consequence** of a **Link**, changing the **NodeState** or associated **Role**:

$$[\mathcal{F}(\text{Consequence}, \text{Node}) = \text{Node}'] \implies \{ [\text{NodeState} \rightarrow \text{NodeState}'] \vee [\text{Role}_i \rightarrow \text{Role}'_i] \} \quad (1.19)$$

The oracle subsequently *walks the consequence forward* to generate or decouple any **Hook** or **Anchor** objects which have been invalidated by a disturbance of the **Conditions** required by the **SourceOfLaw**.

1.4 Sources of Law

Given a **SourceOfLaw**, there exists a related set of **Conditions** which must be satisfied before the oracle function, \mathcal{F} , can evaluate any expression:

$$\text{SourceOfLaw} \implies \text{Conditions} \quad (1.20)$$

$$\mathcal{F} : \text{Conditions} \rightarrow \{ T, F \} \quad (1.21)$$

Application to Links. The **Hook** or **Anchor** required to draw a **Link** will have **Conditions** containing evaluations related to the **NodeState** of an object. The oracle, \mathcal{F} , will only generate a **Link** where the **Conditions** imposed by the **SourceOfLaw** are satisfied. Consequently, a **Link** will 'decouple' where a **Consequence** modifies the **NodeState** such that the evaluation of the **Conditions** of the relevant **SourceOfLaw** fail.

1.5 The Stage Controller

Time is encoded in \mathcal{R} as an ordered set of n distinct **Interval** objects related by a **Transition**:

$$\text{IntervalSet} := \{ \text{Interval}_0, \dots, \text{Interval}_n \} \quad (1.22)$$

$$\text{ForwardTransition} : \text{Interval}_i \rightarrow \text{Interval}_{i+1} \quad (1.23)$$

$$\text{BackwardTransition} : \text{Interval}_i \rightarrow \text{Interval}_{i-1} \quad (1.24)$$

The **BackwardTransition** and **ForwardTransition** are a specialisation of a **Transition** that form an identity map under function composition:

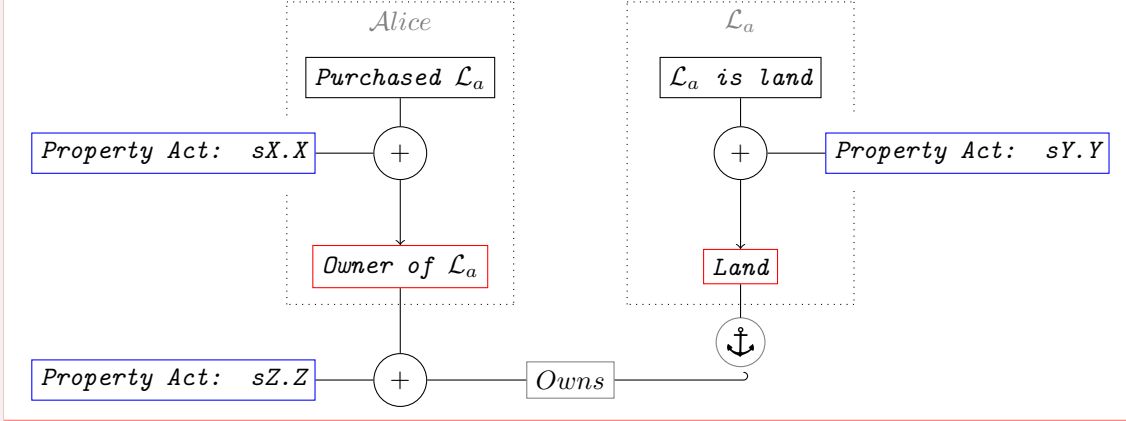
$$\text{BackwardTransition} \circ \text{ForwardTransition} : a \rightarrow a \quad (1.25)$$

The **TransitionSet** of an **Interval** is the minimum set of **ForwardTransition** and **BackwardTransition** objects required to reconstruct the **IntervalSet** under the oracle function, \mathcal{F} :

$$\mathcal{F} : (\text{TransitionSet}, \text{Interval}) \rightarrow \text{IntervalSet} \quad (1.26)$$

Example.

Consider a boundary dispute between *Alice* and *Bob* on two adjacent plots of land: \mathcal{L}_a and \mathcal{L}_b . In this case, *Alice* is the owner of \mathcal{L}_a , and *Bob* is the owner of \mathcal{L}_b . We are attempting to define whether it is permissible for *Alice* to build a structure by defining a boundary line delineating the properties, called **Bound**. The graphs below represents the information encoded in **Interval**₀.

Diagram 1.1 *Alice owns a parcel of land.*

The above diagram (1.1) reflects the following:

1. *Alice* and \mathcal{L}_a are both a Node.
2. [*Purchased \mathcal{L}_a*] and [*\mathcal{L}_a is land*] are both instances of a Fact.
3. (*Property Act: $sX.X$* , *Purchased \mathcal{L}_a*) generates the Role [*Owner of \mathcal{L}_a*].
4. (*Property Act: $sY.Y$* , *\mathcal{L}_a is land*) generates the Role [*Real Property*].
5. The (Hook, Anchor) pairing below generates a Link [*Owns*]:
 - (a) (*Property Act: $sZ.Z$* , *Owner of \mathcal{L}_a*) generates a Hook.
 - (b) [*Real Property*] generates an Anchor.

The Consequence of the Link [*Owns*] requires that any other Anchor on \mathcal{L}_a satisfies the additional Condition of having either a superior 'property' claim in relation to \mathcal{L}_a , or *Alice*'s consent.

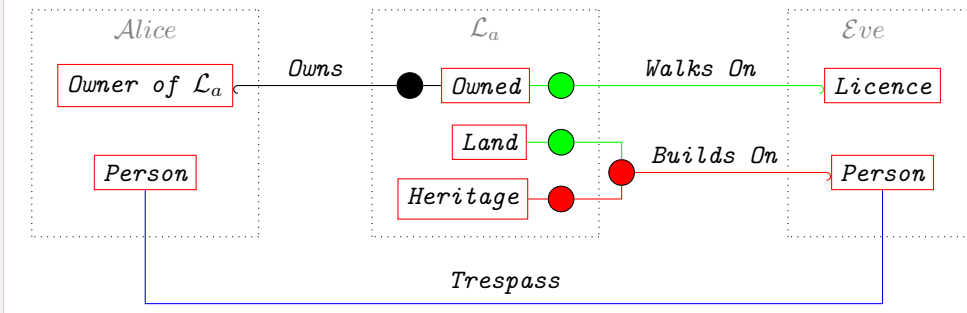
Diagram 1.2 *Can Eve walk on \mathcal{L}_a ?*

Diagram (1.2) reflects that:

1. \mathcal{L}_a has the NodeState [*Owned*].
2. The NodeState [*Owned*] is a Consequence under the Link [*Owns*].
3. The Condition of [*Owns*] requires that any other Anchor on \mathcal{L}_a has the Role [*Licence*]:
 - (a) *Eve* may walk on \mathcal{L}_a because:

- i. The [*Licence*] is a **Hook** which satisfies the **Conditions** for [*Walks On*].
 - ii. [*Owned*] is an **Anchor** which satisfies the **Conditions** for **Link** [*Walks On*].
- (b) *Eve* may **not** build on \mathcal{L}_a because:
- i. [*Person*] is a **Hook** which **does not** satisfy the **Conditions** for [*Builds On*].
 - ii. [*Land*] is an **Anchor** which satisfies the **Conditions** for [*Builds On*].
 - iii. [*Heritage*] is an **Anchor** which **does not** satisfy the **Conditions** for [*Builds On*].

Compliance and Deviation. A lawyer constructs a **Link(Consequence)** by analysing the **Role** of an object relative to a set of legal conditions. We define both compliance and deviation as:

$$[\exists \text{Link(Existence, Consequence)}] \wedge \begin{cases} [\text{Role} \implies \text{Link}(\dots)], & \text{Compliance} \\ [\text{Role} \not\implies \text{Link}(\dots)], & \text{Deviation} \end{cases} \quad (1.27)$$

Example. Continuing the previous example, we define compliance as Bob enforcing the terms of C against Alice, because Alice fulfils the relevant **Role**. Bob could not, however, enforce C against a third party, unless they also fulfilled a role which generated a **Link** to C.