90 POINTS **HOMEWORK 4** **DUE: 2/11/15**

1. (20 pts.) A reciprocating machine produces a unbalanced force with magnitude $F = 13\text{ N}$ at its operating speed of $\omega = 20\text{ rad/s}$. The machine and its support structure have the values $m = 75\text{ kg}$ and $k = 2500\text{ N/m}$.

   a. (10 pts.) Design a dynamic vibration absorber (i.e. choose values of m_a, k_a) such that the vibrations are completely canceled out. The maximum deflection of the absorber is constrained to $X_a = 0.002\text{ m}$.

   b. (10 pts.) What is the maximum operating range of this machine if it is required that $\left|\frac{X}{\delta_{st}}\right| \le 1$?

2. (20 pts.) A machine with an operating range of $1000\text{ rpm} \le \omega \le 2000\text{ rpm}$ has an undesirable resonance at $\omega = 1750\text{ rpm}$. When a tuned dynamic vibration absorber with mass $m_a = 5\text{ kg}$ is attached, resonances occur at $\omega = 1400\text{ rpm}$ and $\omega = 2190\text{ rpm}$.

   a. (10 pts.) Design a dynamic vibration absorber (i.e. choose values of m_a, k_a) such that $\left|\frac{X}{\delta_{st}}\right| \le 10\text{ dB}$ over the entire operating range. Make m_a as small as possible.

   b. (10 pts.) Plot the frequency response of the system without a dynamic vibration absorber and with the absorber designed in part (a) on the same graph, using rpm on the horizontal axis and dB on the vertical axis. Add a horizontal line on the graph at 10 dB to show that your design from part (a) satisfies the constraints.

3. (40 pts.) Consider the system shown in Figure 1. We wish to reduce the force transmissibility of the system such that it is below a certain value over all frequencies of the input ω.

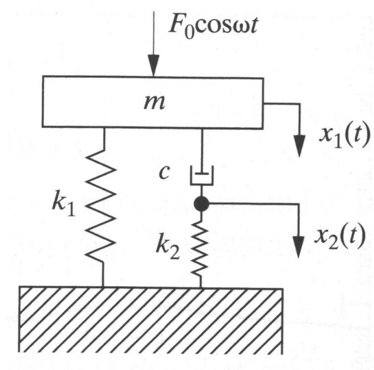

Figure 1

a. (20 pts.) Show that the force transmissibility (ratio of input force magnitude to force magnitude transmitted to the foundation) is given by

$$\left|\frac{F_t(i\omega)}{F(i\omega)}\right| = \frac{\sqrt{1+4(1+\gamma)^2\zeta^2 r^2}}{\sqrt{\left(1-r^2\right)^2+4\zeta^2 r^2\left(1+\gamma-r^2\gamma\right)^2}}$$

where

$$r = \frac{\omega}{\sqrt{\frac{k_1}{m}}}, \qquad \gamma = \frac{k_1}{k_2}, \qquad \zeta = \frac{c}{2\sqrt{mk_1}}$$

Note that x_2 is an independent degree of freedom since it can move independently of x_1.

b. (20 pts.) The peak frequency is given by $r_p = \sqrt{\frac{2(1+\gamma)}{1+2\gamma}}$ and the optimal damping ratio is given by $\zeta_{opt} = \frac{\sqrt{2(1+2\gamma)/\gamma}}{4(1+\gamma)}$. (You do not need to derive these formulas). Using these formulas, show that the peak magnitude of the force transmissibility is given by $\left|\frac{F_t(i\omega)}{F(i\omega)}\right|_{\max} = 1+2\gamma$.

4. (10 pts.) Consider again the system shown in Figure 1. Design the isolator (i.e. choose c and k_2) such that when a 70 N sinusoidal force is applied, no more than 100 N is transmitted to the ground, no matter what the frequency of input. You are given $m = 1000\text{ kg}$ and $k_1 = 40{,}000\text{ N/m}$.