120 POINTS **HOMEWORK 9** **DUE: 11/20/14**

1. (40 pts.) We wish to describe the torsional EOM of the shaft shown in Figure 1.

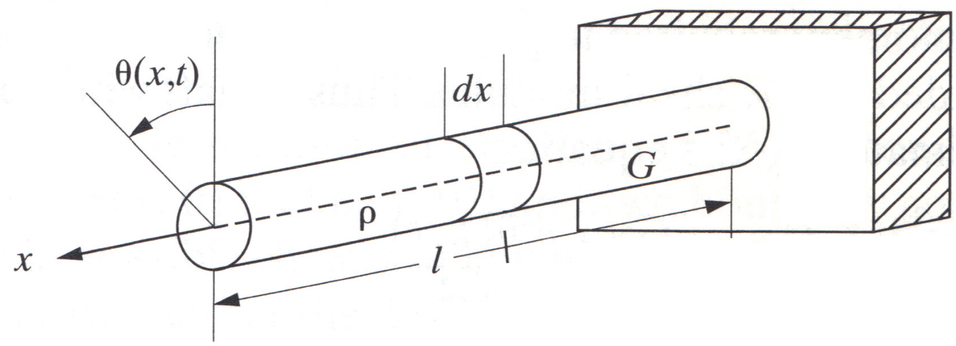

Figure 1

a. (20 pts.) Show, using strength of materials, that the internal twisting moment is given by

$$M(x,t) = GJ(x)\frac{\partial \theta(x,t)}{\partial x}$$

where G is the shear modulus, and $J(x) = \iint_{A(x)} r^2 dA$ is the polar moment of inertia of the cross section with area $A(x)$. You may assume radial symmetry in the shaft.

b. (20 pts.) Show that the EOM is given by

$$\frac{\partial}{\partial x}\left[GJ(x)\frac{\partial \theta(x,t)}{\partial x}\right] + \tau(x,t) = \rho J(x)\frac{\partial^2 \theta(x,t)}{\partial t^2}$$

where $\tau(x,t)$ is the external torque per unit length and ρ is the density.

2. (40 pts.) Consider the shaft shown in Figure 1, which is considered a fixed-free shaft. Assume that the shaft has a uniform cross section, so $J(x) = J$, a constant.

a. (20 pts.) Show that the natural frequencies are given by $\omega_k = \sqrt{\frac{G}{\rho}}\frac{(2k+1)\pi}{2l}$.

b. (20 pts.) Show that the mode shapes are given by $\phi_k(x) = c\sin\left[\frac{(2k+1)\pi x}{2l}\right]$.

3. (40 pts.) Consider the transverse vibrations of an Euler-Bernoulli beam with a uniform cross section. Suppose the beam is pinned at both ends.

a. (20 pts.) Show that the natural frequencies are given by $\omega_k = \sqrt{\frac{EI(k\pi)^4}{\rho A l^4}}$.

b. (20 pts.) Show that the mode shapes are given by $\phi_k(x) = c\sin(\beta_k x)$.