60 POINTS | **HOMEWORK 8** | **DUE: 3/25/15**

1. (30 pts.) Let $x(t)$ be a random process and $\dot{x}(t)$ be its time derivative. Please prove the following:

a. (10 pts.) $R_{\dot{x}\dot{x}}(t_1, t_2) = \frac{\partial^2}{\partial t_1 \partial t_2} R_{xx}(t_1, t_2)$. Hint: you can exchange the order of integration and differentiation since we assume $x(t)$ is bounded and continuous.

b. (10 pts.) If $x(t)$ is stationary, then $R_{\dot{x}\dot{x}}(\tau) = -\frac{\partial^2}{\partial \tau^2} R_{xx}(\tau)$.

c. (10 pts.) If $x(t)$ is stationary, then $S_{\dot{x}\dot{x}}(\omega) = \omega^2 S_{xx}(\omega)$. Hint: consider the rule for taking the Laplace transform of $\dot{x}(t)$ and modify it for the Fourier transform.

2. (30 pts.) A rotating shaft supported by bearings has the equation of motion

$$J\dot{\omega} + c\omega = T$$

where ω is the speed of the shaft, $J = 5\ \mathrm{kgm}^2$, c is the bearing damping constant, and T is the applied torque. Suppose T is a zero-mean, stationary white noise process with power spectral density $S_0 = 2\ (\mathrm{Nm})^2/(\mathrm{rad/s})$.

a. (10 pts.) Find the value of c such that, in steady state, $E[\omega^2(t)] = 1\,(\mathrm{rad/s})^2$.

b. (20 pts.) Run the simulation of this system multiple times (at least 20 times) and average $\omega^2(t)$ at each t of the simulation to get an estimate of $E[\omega^2(t)]$. Plot this estimate against the theoretical curve for $E[\omega^2(t)]$.