9.10 Vibration Isolation

Vibration isolation is a procedure by which the undesirable effects of vibration are reduced [9.22–9.24]. Basically, it involves the insertion of a resilient member (or isolator) between the vibrating mass (or equipment or payload) and the source of vibration so that a reduction in the dynamic response of the system is achieved under specified conditions of vibration excitation. An isolation system is said to be active or passive depending on whether or not external power is required for the isolator to perform its function. A passive isolator consists of a resilient member (stiffness) and an energy dissipator (damping). Examples of passive isolators include metal springs, cork, felt, pneumatic springs, and elastomer (rubber) springs. Figure 9.17 shows typical spring and pneumatic mounts that can be used as passive isolators, and Fig. 9.18 illustrates the use of passive isolators in the mounting of a high-speed punch press [9.25]. The optimal synthesis of vibration isolators is presented in references [9.26–9.30]. An active isolator is comprised of a servomechanism with a sensor, signal processor, and actuator.

FIGURE 9.17 (a) Undamped spring mount; (b) damped spring mount; (c) pneumatic rubber mount. (Courtesy of *Sound and Vibration*.)

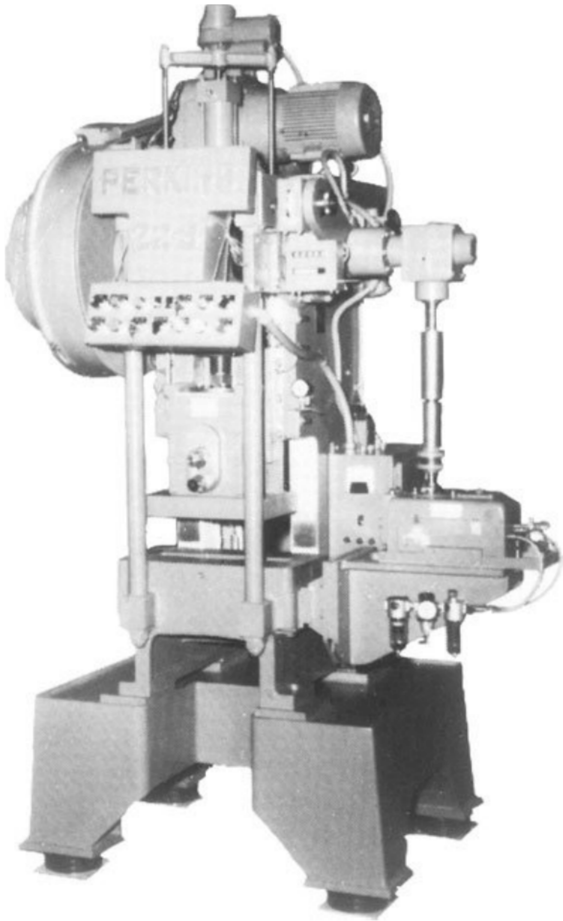

FIGURE 9.18 High-speed punch press mounted on pneumatic rubber mounts. (Courtesy of *Sound and Vibration*.)

Vibration isolation can be used in two types of situations. In the first type, the foundation or base of a vibrating machine is protected against large unbalanced forces. In the second type, the system is protected against the motion of its foundation or base.

The first type of isolation is used when a mass (or a machine) is subjected to a force or excitation. For example, in forging and stamping presses, large impulsive forces act on the object to be formed or stamped. These impacts are transmitted to the base or foundation of the forging or stamping machine, which can damage not only the base or foundation but also the surrounding or nearby structures and machines. They can also cause discomfort to operators of these machines. Similarly, in the case of reciprocating and rotating machines, the inherent unbalanced forces are transmitted to the base or foundation of the machine. In such cases, the force transmitted to the base, $F_t(t)$, varies harmonically, and the resulting stresses in the foundation bolts also vary harmonically, which might lead to fatigue

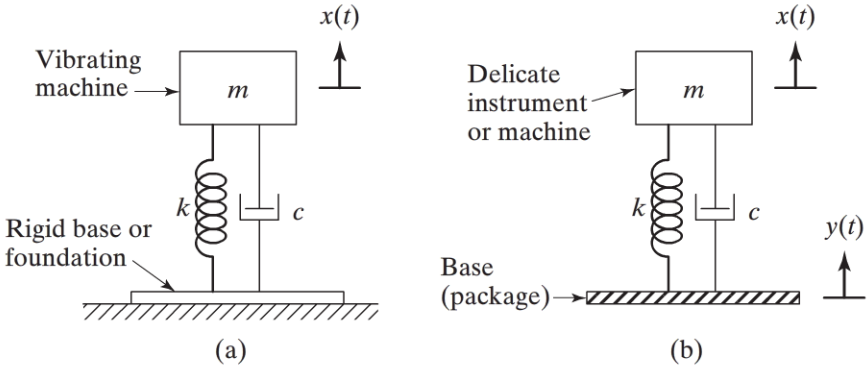

FIGURE 9.19 Vibration isolation.

failure. Even if the force transmitted is not harmonic, its magnitude is to be limited to safe permissible values. In these applications, we can insert an isolator (in the form of stiffness and/or damping) between the mass being subjected to force or excitation and the base or foundation to reduce the force transmitted to the base or foundation. This is called *force isolation.* In many applications, the isolator is also intended to reduce the vibratory motion of the mass under the applied force (as in the case of forging or stamping machines). Thus both force and displacement transmissibilities become important for this of isolators.

The second type of isolation is used when a mass to be protected against the motion or excitation of its base or foundation. When the base is subjected to vibration, the mass m will experience not only a displacement $x(t)$ but also a force $F_t(t)$. The displacement of the mass $x(t)$ is expected to be smaller than the displacement of the base $y(t)$. For example, a delicate instrument or equipment is to be protected from the motion of its container or package (as when the vehicle carrying the package experiences vibration while moving on a rough road). The force transmitted to the mass also needs to be reduced. For example, the package or container is to be designed properly to avoid transmission of large forces to the delicate instrument inside to avoid damage. The force experienced by the instrument or mass m (same as the force transmitted to mass m) is given by

$$F_t(t) = m\ddot{x}(t) = k\{x(t) - y(t)\} + c\{\dot{x}(t) - \dot{y}(t)\} \tag{9.87}$$

where $y(t)$ is the displacement of the base, $x(t) - y(t)$ is the relative displacement of the spring, and $\dot{x}(t) - \dot{y}(t)$ is the relative velocity of the damper. In such cases, we can insert an isolator (which provides stiffness and /or damping) between the base being subjected to force or excitation and the mass to reduce the motion and/or force transmitted to the mass. Thus both displacement isolation and force isolation become important in this case also.

It is to be noted that the effectiveness of an isolator depends on the nature of the force or excitation. For example, an isolator designed to reduce the force transmitted to the base or foundation due to impact forces of forging or stamping may not be effective if the disturbance is a harmonic unbalanced force. Similarly, an isolator designed to handle harmonic excitation at a particular frequency may not be effective for other frequencies or other types of excitation such as step-type excitation.

9.10.1 Vibration Isolation System with Rigid Foundation

Reduction of the Force Transmitted to Foundation. When a machine is bolted directly to a rigid foundation or floor, the foundation will be subjected to a harmonic load due to the unbalance in the machine in addition to the static load due to the weight of the machine. Hence an elastic or resilient member is placed between the machine and the rigid foundation to reduce the force transmitted to the foundation. The system can then be idealized as a single-degree-of-freedom system, as shown in Fig. 9.20(a). The resilient member is assumed to have both elasticity and damping and is modeled as a spring k and a dashpot c, as shown in Fig. 9.20(b). It is assumed that the operation of the machine gives rise to a harmonically varying force $F(t) = F_0 \cos \omega t$. The equation of motion of the machine (of mass m) is given by

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t \tag{9.88}$$

Since the transient solution dies out after some time, only the steady-state solution will be left. The steady-state solution of Eq. (9.88) is given by (see Eq. (3.25))

$$x(t) = X \cos(\omega t - \phi) \tag{9.89}$$

where

$$X = \frac{F_0}{[(k - m\omega^2)^2 + \omega^2 c^2]^{1/2}} \tag{9.90}$$

and

$$\phi = \tan^{-1}\left(\frac{\omega c}{k - m\omega^2}\right) \tag{9.91}$$

The force transmitted to the foundation through the spring and the dashpot, $F_t(t)$, is given by

$$F_t(t) = kx(t) + c\dot{x}(t) = kX \cos(\omega t - \phi) - c\omega X \sin(\omega t - \phi) \tag{9.92}$$

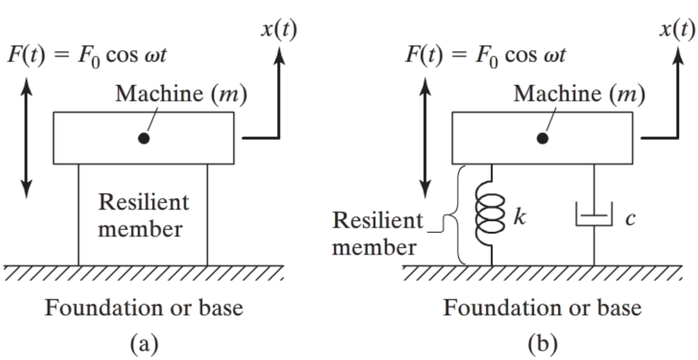

FIGURE 9.20 Machine and resilient member on rigid foundation.

The magnitude of the total transmitted force (F_T) is given by

$$F_T = [(kx)^2 + (c\dot{x})^2]^{1/2} = X\sqrt{k^2 + \omega^2 c^2}$$

$$= \frac{F_0(k^2 + \omega^2 c^2)^{1/2}}{[(k - m\omega^2)^2 + \omega^2 c^2]^{1/2}} \tag{9.93}$$

The transmissibility or transmission ratio of the isolator (T_f) is defined as the ratio of the magnitude of the force transmitted to that of the exciting force:

$$T_f = \frac{F_T}{F_0} = \left\{\frac{k^2 + \omega^2 c^2}{(k - m\omega^2)^2 + \omega^2 c^2}\right\}^{1/2}$$

$$= \left\{\frac{1 + (2\zeta r)^2}{[1 - r^2]^2 + (2\zeta r)^2}\right\}^{1/2} \tag{9.94}$$

where $r = \frac{\omega}{\omega_n}$ is the frequency ratio. The variation of T_f with the frequency ratio $r = \frac{\omega}{\omega_n}$ is shown in Fig. 9.21. In order to achieve isolation, the force transmitted to the foundation needs to be less than the excitation force. It can be seen, from Fig. 9.21, that the forcing frequency has to be greater than $\sqrt{2}$ times the natural frequency of the system in order to achieve isolation of vibration.

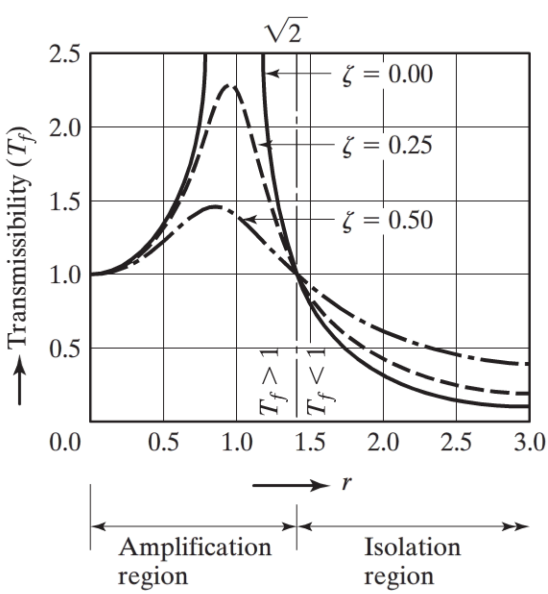

FIGURE 9.21 Variation of transmission ratio (T_f) with r.

For small values of damping ratio ζ and for frequency ratio $r > 1$, the force transmissibility, given by Eq. (9.94), can be approximated as

$$T_f = \frac{F_t}{F} \approx \frac{1}{r^2 - 1} \quad \text{or} \quad r^2 \approx \frac{1 + T_f}{T_f} \tag{9.95}$$

Notes

1. The magnitude of the force transmitted to the foundation can be reduced by decreasing the natural frequency of the system (ω_n).
2. The force transmitted to the foundation can also be reduced by decreasing the damping ratio. However, since vibration isolation requires $r > \sqrt{2}$, the machine should pass through resonance during start-up and stopping. Hence, some damping is essential to avoid infinitely large amplitudes at resonance.
3. Although damping reduces the amplitude of the mass (X) for all frequencies, it reduces the maximum force transmitted to the foundation (F_t) only if $r < \sqrt{2}$. Above that value, the addition of damping increases the force transmitted.
4. If the speed of the machine (forcing frequency) varies, we must compromise in choosing the amount of damping to minimize the force transmitted. The amount of damping should be sufficient to limit the amplitude X and the force transmitted F_t while passing through the resonance, but not so much to increase unnecessarily the force transmitted at the operating speed.

Reduction of the Vibratory Motion of the Mass. In many applications, the isolation is required to reduce the motion of the mass (machine) under the applied force. The displacement amplitude of the mass m due to the force $F(t)$, given by Eq. (9.90), can be expressed as:

$$T_d = \frac{X}{\delta_{st}} = \frac{kX}{F_0} = \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \tag{9.96}$$

where $\frac{X}{\delta_{st}}$ is called, in the present context, the *displacement transmissibility* or *amplitude ratio* and indicates the ratio of the amplitude of the mass, X, to the static deflection under the constant force F_0, $\delta_{st} = \frac{F_0}{k}$. The variation of the displacement transmissibility with the frequency ratio r for several values of the damping ratio ζ is shown in Fig. 9.22. The following observations can be made from Fig. 9.22:

1. The displacement transmissibility increases to a maximum value at (Eq. (3.33)):

$$r = \sqrt{1 - 2\zeta^2} \tag{9.97}$$

Equation (9.97) shows that, for small values of damping ratio ζ, the displacement transmissibility (or the amplitude of the mass) will be maximum at $r \approx 1$ or $\omega \approx \omega_n$.

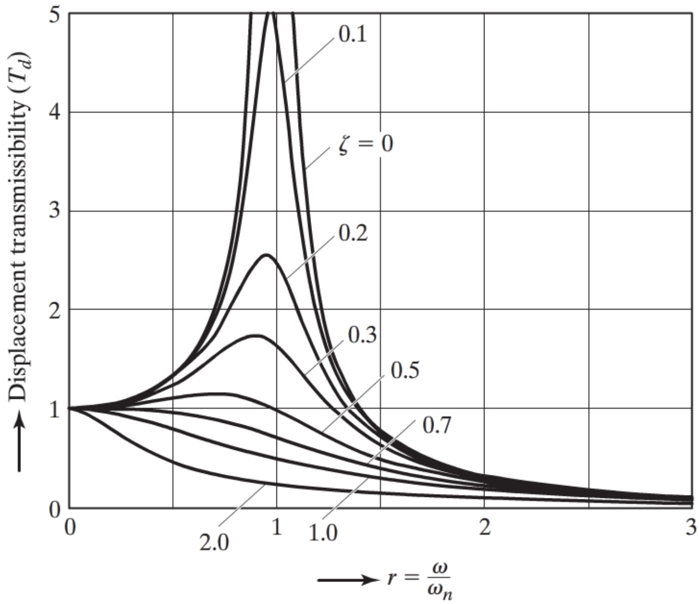

FIGURE 9.22 Variation of displacement transmissibility (T_d) with r.

Thus the value of $r \approx 1$ is to be avoided in practice. In most cases, the excitation frequency ω is fixed and hence we can avoid $r \approx 1$ by altering the value of the natural frequency $\omega_n = \sqrt{\frac{k}{m}}$ which can be accomplished by changing the value of either or both of m and k.

2. The amplitude of the mass, X, approaches zero as r increases to a large value. The reason is that at large values of r, the applied force $F(t)$ varies very rapidly and the inertia of the mass prevents it from following the fluctuating force.

EXAMPLE 9.4 Spring Support for Exhaust Fan

An exhaust fan, rotating at 1000 rpm, is to be supported by four springs, each having a stiffness of K. If only 10 percent of the unbalanced force of the fan is to be transmitted to the base, what should be the value of K? Assume the mass of the exhaust fan to be 40 kg.

Solution: Since the transmissibility has to be 0.1, we have, from Eq. (9.94),

$$0.1 = \left[\frac{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2} \right]^{1/2} \tag{E.1}$$

where the forcing frequency is given by

$$\omega = \frac{1000 \times 2\pi}{60} = 104.72 \text{ rad/s} \tag{E.2}$$

and the natural frequency of the system by

$$\omega_n = \left(\frac{k}{m}\right)^{1/2} = \left(\frac{4K}{40}\right)^{1/2} = \frac{\sqrt{K}}{3.1623} \tag{E.3}$$

By assuming the damping ratio to be $\zeta = 0$, we obtain from Eq. (E.1),

$$0.1 = \frac{\pm 1}{\left\{1 - \left(\frac{104.72 \times 3.1623}{\sqrt{K}}\right)^2\right\}} \tag{E.4}$$

To avoid imaginary values, we need to consider the negative sign on the right-hand side of Eq. (E.4). This leads to

$$\frac{331.1561}{\sqrt{K}} = 3.3166$$

or

$$K = 9969.6365 \text{ N/m}$$

■

EXAMPLE 9.5 Design of an Undamped Isolator

A 50-kg mass is subjected to the harmonic force $F(t) = 1000 \cos 120t$ N. Design an undamped isolator so that the force transmitted to the base does not exceed 5% of the applied force. Also, find the displacement amplitude of the mass of the system with isolation.

Solution: By setting the value of force transmissibility as 0.05 and using $\zeta = 0$, Eq. (9.95) gives

$$r^2 \approx \frac{1 + T_f}{T_f} = \frac{1 + 0.05}{0.05} = 21 \tag{E.1}$$

Using the definition of r, along with the values of $m = 50$ kg and $\omega = 120$ rad/s, Eq. (E.1) yields

$$r^2 = \frac{\omega^2}{\omega_n^2} = \frac{\omega^2 m}{k}$$

or

$$k = \frac{\omega^2 m}{r^2} = \frac{(120^2)(50)}{21} = 34.2857 \times 10^3 \text{ N/m} \tag{E.2}$$

The displacement amplitude of the mass of the system with isolation can be found from Eq. (9.96), with $\zeta = 0$:

$$X = \frac{F_0}{k} \frac{1}{(r^2 - 1)} = \frac{1000}{34.2857 \times 10^3} \frac{1}{(21 - 1)} = 1.4583 \times 10^{-3} \text{ m} \tag{E.3}$$

■

Design Chart for Isolation:

The force transmitted to the base or ground by a source of vibration (vibrating mass) is given by Eq. (9.94) and is shown in Fig. 9.21 as a graph between $T_f = F_T/F_0$ and $r = \omega/\omega_n$. As noted earlier, vibration isolation—reduction of the force transmitted to the ground—can be achieved for $r > \sqrt{2}$. In the region $r > \sqrt{2}$, low values of damping are desired for more effective isolation. For large values of r and low values of ζ, the term $(2\zeta r)^2$ becomes very small and can be neglected in Eq. (9.94) for simplicity. Thus Eq. (9.94) can be approximated as shown in Eq. (9.95) for $r > \sqrt{2}$ and ζ small.

By defining the natural frequency of vibration of the undamped system as

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta_{st}}} \tag{9.98}$$

and the exciting frequency ω as

$$\omega = \frac{2\pi N}{60} \tag{9.99}$$

where δ_{st} is the static deflection of the spring and N is the frequency in cycles per minute or revolutions per minute (rpm) of rotating machines such as electric motors and turbines, Eqs. (9.95) to (9.99) can be combined to obtain

$$r = \frac{\omega}{\omega_n} = \frac{2\pi N}{60} \sqrt{\frac{\delta_{st}}{g}} = \sqrt{\frac{2 - R}{1 - R}} \tag{9.100}$$

where $R = 1 - T_f$ is used to indicate the quality of the isolator and denotes the percent reduction achieved in the transmitted force. Equation (9.100) can be rewritten as

$$N = \frac{30}{\pi} \sqrt{\frac{g}{\delta_{st}} \left(\frac{2 - R}{1 - R} \right)} = 29.9092 \sqrt{\frac{2 - R}{\delta_{st}(1 - R)}} \tag{9.101}$$

Equation (9.101) can be used to generate a graph between $\log N$ and $\log \delta_{st}$ as a series of straight lines for different values of R, as shown in Fig. 9.23. This graph serves as a design chart for selecting a suitable spring for the isolation.

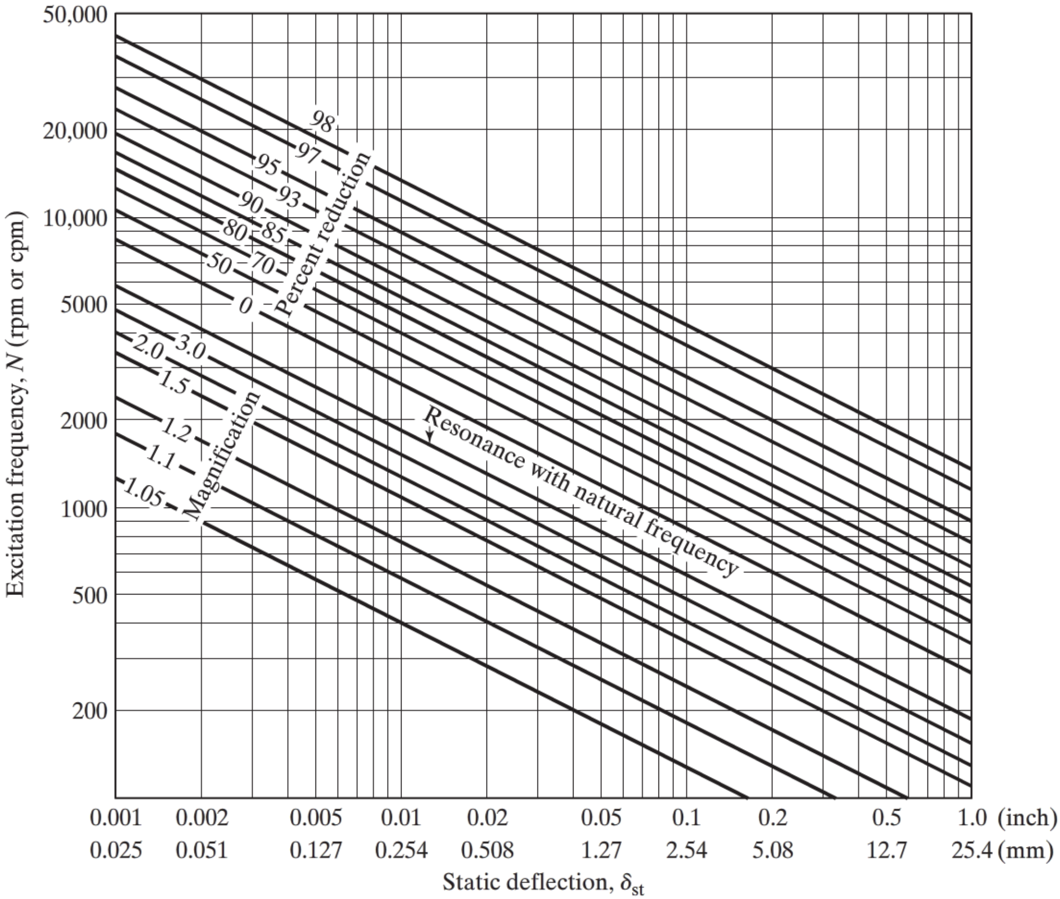

FIGURE 9.23 Isolation efficiency.

EXAMPLE 9.6

Isolator for Stereo Turntable

A stereo turntable, of mass 1 kg, generates an excitation force at a frequency of 3 Hz. If it is supported on a base through a rubber mount, determine the stiffness of the rubber mount to reduce the vibration transmitted to the base by 80 percent.

Solution: Using $N = 3 \times 60 = 180$ cpm and $R = 0.80$, Eq. (9.105) gives

$$180 = 29.9092\sqrt{\frac{2 - 0.80}{\delta_{st}(1 - 0.80)}}$$

or

$$\delta_{st} = 0.1657 \text{ m}$$

The static deflection of the rubber mount can be expressed, in terms of its stiffness (k), as

$$\delta_{st} = \frac{mg}{k}$$

which gives the stiffness of the rubber mount as

$$0.1657 = \frac{1(9.81)}{k} \quad \text{or} \quad k = 59.2179 \text{ N/m}$$

■

Isolation of systems with rotating unbalance:

A common source of forced harmonic force is imbalance in rotating machines such as turbines, centrifugal pumps, and turbogenerators. Imbalance in a rotating machine implies that the axis of rotation does not coincide with the center of mass of the whole system. Even a very small eccentricity can cause a large unbalanced force in high-speed machines such as turbines. A typical rotating system with an unbalance is shown in Fig. 9.24. Here the total mass of the system is assumed to be M and the unbalanced mass is considered as a point mass m located at the center of mass of the system (which has an eccentricity of e from the center of rotation) as shown in Fig. 9.24. If the unbalanced mass rotates at an angular velocity ω and the system is constrained to move in the vertical direction, the equation of motion of the system is given by

$$M\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t \equiv me\omega^2 \sin \omega t \tag{9.102}$$

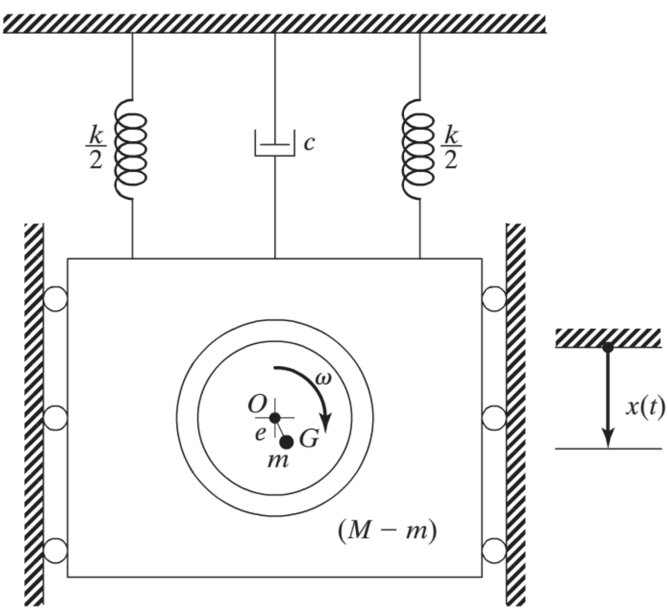

FIGURE 9.24 A system with rotating unbalance.

Using $F_0 = me\omega^2$, the force transmissibility of the system can be found from Eq. (9.88).

However, the presence of ω^2 in F_0 results in the following equation for the force transmissibility (T_f) due to rotating unbalance:

$$T_f = \frac{F_t}{F_0} = \frac{F_t}{me\omega^2} = \frac{F_t}{mer^2\omega_n^2}$$

or

$$\frac{F_t}{me\omega_n^2} = r^2\left\{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right\}^{\frac{1}{2}} \tag{9.103}$$

EXAMPLE 9.7 Centrifugal Pump with Rotating Unbalance—Rattle Space

A centrifugal pump, with a mass of 50 kg and rotational speed of 3000 rpm, is mounted in the middle of a simply supported beam of length 100 cm, width 20 cm, and thickness 0.5 cm. The damping ratio of the system (beam) can be assumed as $\zeta = 0.05$. The impeller (rotating part) of the pump has a mass of 5 kg with an eccentricity of 1 mm. If the maximum deflection of the beam is constrained to be less than the available rattle space[6] of 3 mm. Determine whether the support system of the pump is adequate.

Solution: The bending stiffness or spring constant of the simply supported beam is given by

$$k = \frac{48EI}{l^3}$$

where the moment of inertia of the beam cross section can be computed as

$$I = \frac{1}{12}wt^3 = \frac{(20)(0.05^3)}{12} = 0.208333\ \text{cm}^4 = 20.8333 \times 10^{-10}\ \text{m}^4$$

Using $E = 207 \times 10^9$ Pa, the spring constant of the beam can be found as

$$k = \frac{48(207 \times 10^9)(20.8333 \times 10^{-10})}{(1.0^3)} = 206{,}999.6688\ \text{N/m}$$

Using the density of steel as 7.85 gram/cm^3, the mass of the beam (m_b) can be determined as

$$m_b = 7.85(100)(20)(0.5) = 7850\ \text{gram} = 7.85\ \text{kg}$$

[6]The available clearance space that permits the system to undergo the induced deflection freely during vibration is called the *rattle space* or *clearance*. If the rattle space is too small to accommodate the deflection of the system, the system will undergo impacts (as it hits the surrounding or nearby surface or object) in each cycle of vibration.

The total mass of the system (M) is equal to the mass of the pump plus the effective mass of the beam at its center (equal to $\frac{17}{35}m_b$; see Problem 2.86):

$$M = m_{\text{pump}} + \frac{17}{35}m_b = 50 + \frac{17}{35}(7.85) = 53.8128 \text{ kg}$$

The natural frequency of the system is given by

$$\omega_n = \sqrt{\frac{k}{M}} = \sqrt{\frac{206999.6688}{53.8128}} = 62.0215 \text{ rad/s}$$

The impeller (rotor) speed of 3000 rpm gives $\omega = 2\pi(3000)/60 = 314.16$ rad/s. Thus the frequency ratio (r) becomes

$$r = \frac{\omega}{\omega_n} = \frac{314.16}{62.0215} = 5.0653; \qquad r^2 = 25.6577$$

The amplitude of the forcing function is

$$me\omega^2 = 5(10^{-3})(314.16^2) = 493.4825 \text{ N}$$

Using $\zeta = 0.05$, the steady-state amplitude of the pump can be found from Eq. (9.96), using $me\omega^2$ for F_0, as

$$X = \frac{me\omega^2}{k}\frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{493.4825}{206999.6688}\frac{1}{\sqrt{(1-25.6577)^2 + \{2(0.05)(5.0653)\}^2}}$$

$$= \frac{493.4825}{206999.6688}\frac{1}{24.6629} = 9.6662 \times 10^{-5}\, m$$

The static deflection of the beam under the weight of the pump can be determined as

$$\delta_{\text{pump}} = \frac{W_{\text{pump}}}{k} = \frac{(50)(9.81)}{206999.6688} = 236.9569 \times 10^{-5} \text{ m}$$

Thus the total deflection of the system is

$$\delta_{\text{total}} = X + \delta_{\text{pump}} = 9.662 \times 10^{-5} + 236.9569 \times 10^{-5} = 246.6231 \times 10^{-5} \text{ m} = 2.4662 \text{ mm}$$

This deflection is less than the rattle space of 3 mm. As such the support system of the pump is adequate. In case the value of δ_{total} exceeds the rattle space, we need to redesign (modify) the support system. This can be achieved by changing the spring constant (dimensions) of the beam and/or by introducing a damper.

■

9.10.2 Vibration Isolation System with Base Motion

In some applications, the base of the system is subjected to a vibratory motion. For example, the base or foundation of a machine such as a turbine in a power plant may be subjected to ground motion during an earthquake. In the absence of a suitably designed isolation system, the motion of the base transmitted to the mass (turbine) might cause damage and power failure. Similarly, a delicate instrument (mass) may have to be protected from a force or shock when the package containing the instrument is dropped from a height accidentally. Also, if the instrument is to be transported, the vehicle carrying it may experience vibration as it travels on a rough road with potholes. In this case, also, proper isolation is to be used to protect the instrument against excessive displacement or force transmitted from the base motion.

For a single-degree-of-freedom system with base excitation, shown in Fig. 9.19(b), the analysis was presented in Section 3.6. When the base of the system is subjected to a harmonic motion, $y(t) = Y \sin \omega t$, the equation of motion is given by Eq. (3.75):

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \tag{9.104}$$

where $z = x - y$ denotes the displacement of the mass relative to the base. If the base motion is harmonic, then the motion of the mass will also be harmonic. Hence the displacement transmissibility, $T_d = \frac{X}{Y}$, is given by Eq. (3.68):

$$T_d = \frac{X}{Y} = \left\{ \frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right\}^{1/2} \tag{9.105}$$

where X and Y denote the displacement amplitudes of the mass and the base, respectively, and the right-hand-side expression can be identified to be the same as that in Eq. (9.94). Note that Eq. (9.105) is also equal to the ratio of the maximum steady-state accelerations of the mass and the base. The variation of the displacement transmissibility with the frequency ratio (r) for different values of the damping ratio (ζ) is shown in Fig. 9.25. The following observations can be made from Fig. 9.25:

1. For an undamped system, the displacement transmissibility approaches infinity at resonance ($r = 1$). Thus the undamped isolator (stiffness) is to be designed to ensure that the natural frequency of the system (ω_n) is away from the excitation frequency (ω).
2. For a damped system, the displacement transmissibility (and hence the displacement amplitude) attains a maximum for frequency ratios close to 1. The maximum displacement amplitude of the mass can be larger than the amplitude of base motion—that is, the base motion can get amplified by a large factor.
3. The displacement transmissibility is close to 1 for small values of the frequency ratio (r) and is exactly equal to 1 at $r = \sqrt{2}$.
4. The displacement amplitude is larger than 1 for $r < \sqrt{2}$ and smaller than 1 for $r > \sqrt{2}$. Note that a smaller damping ratio corresponds to a larger T_d for $r < \sqrt{2}$ and a smaller T_d for $r > \sqrt{2}$. Thus, if the damping of the system cannot be altered, the natural frequency of the system (stiffness) can be changed to achieve a value of $r > \sqrt{2}$.

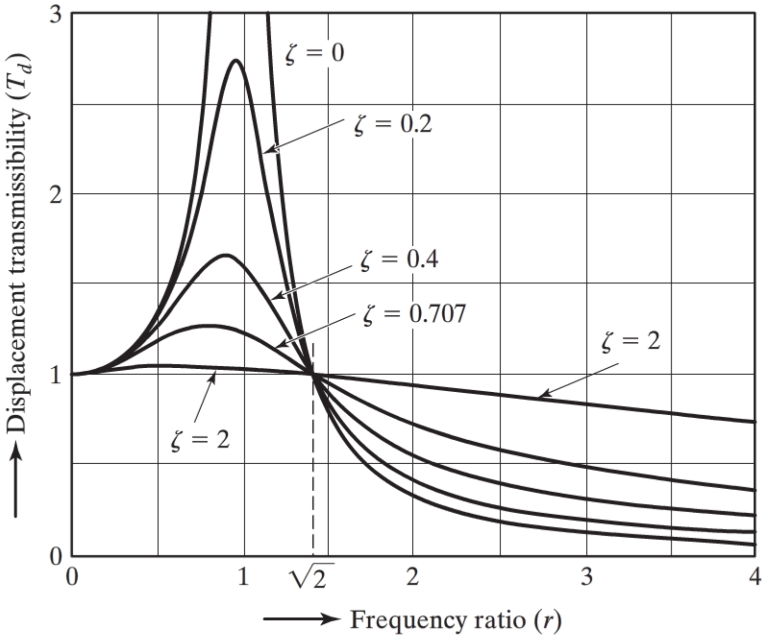

FIGURE 9.25 Variation of T_d with r (for base motion).

If F_t denotes the magnitude of the force transmitted to the mass by the spring and the damper, the force transmissibility (T_f) of the system is given by Eq. (3.74):

$$T_f = \frac{F_t}{kY} = r^2 \left\{ \frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right\}^{\frac{1}{2}} \tag{9.106}$$

where kY is used to make the force transmissibility dimensionless. Note that once the displacement transmissibility, T_d, or the displacement amplitude of the mass (X) is computed using Eq. (9.105), the force transmitted to the mass, F_t, can be determined using the relation

$$\frac{F_t}{kY} = r^2 \frac{X}{Y} \quad \text{or} \quad F_t = kr^2 X \tag{9.107}$$

The variation of the force transmissibility with the frequency ratio (r) for different values of the damping ratio (ζ) is shown in Fig. 9.26. The following observations can be made from Fig. 9.26:

1. The force transmissibility (T_f) will be 2 at the frequency ratio $r = \sqrt{2}$ for all values of the damping ratio (ζ).
2. For $r > \sqrt{2}$, a lower damping ratio corresponds to a lower value of force transmissibility.
3. For $r > \sqrt{2}$, for any specific value of the damping ratio, the force transmissibility increases with r. This behavior is opposite to that of displacement transmissibility.
4. The force transmissibility is close to zero at small values of the frequency ratio r and attains a maximum at values of r close to 1.

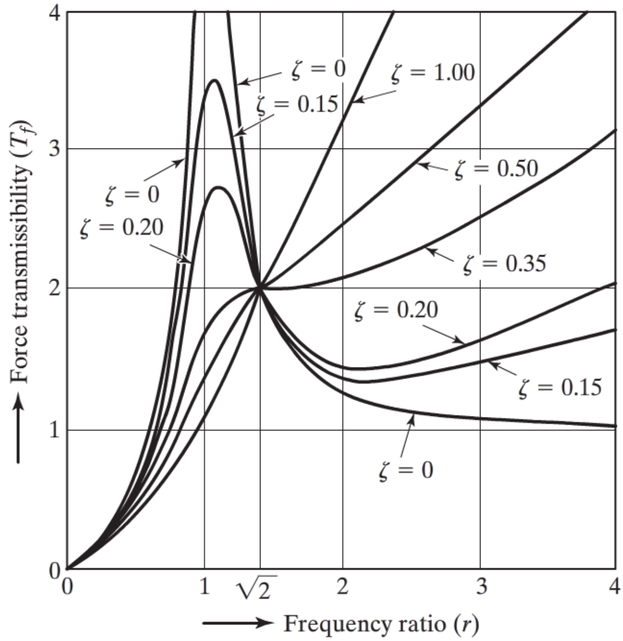

FIGURE 9.26 Variation of T_f with r (for base motion).

EXAMPLE 9.8 Isolation from Vibrating Base

A vibrating system is to be isolated from its vibrating base. Find the required damping ratio that must be achieved by the isolator to limit the displacement transmissibility to $T_d = 4$. Assume the system to have a single degree of freedom.

Solution: By setting $\omega = \omega_n$, Eq. (9.105) gives

$$T_d = \frac{\sqrt{1 + (2\zeta)^2}}{2\zeta}$$

or

$$\zeta = \frac{1}{2\sqrt{T_d^2 - 1}} = \frac{1}{2\sqrt{15}} = 0.1291$$

■

EXAMPLE 9.9 Design of Isolation for a Precision Machine with Base Motion

A precision machine used for the manufacture of integrated circuits, having a mass of 50 kg, is placed on a work bench (as base). The ground vibration transmitted by a nearby internal combustion engine causes the base (all four corners of the bench) to vibrate at a frequency of 1800 rpm. Helical springs, with a damping ratio of $\zeta = 0.01$ and a relationship of bilinear load (P) to deflection (x) given by

$$P = \begin{cases} 50{,}000x; & 0 \le x \le 8 \times 10^{-3} \\ 10^5 x - 4 \times 10^5; & 8 \times 10^{-3} \le x \le 13 \times 10^{-3} \end{cases} \tag{E.1}$$

(P in newtons and x in meters) are available for use as isolators at the four corners of the base. If no more than 10% of the vibration of the base is to be transmitted to the precision machine, determine a method of achieving the isolation.

Solution: Since the displacement transmissibility is required to be 0.1, Eq. (9.105), for $\zeta = 0.01$, gives

$$T_d = \frac{X}{Y} = 0.1 = \sqrt{\frac{1 + \{2(0.01)r\}^2}{(1 - r^2)^2 + \{2(0.01)r\}^2}} \tag{E.2}$$

The simplification of Eq. (E.2) yields a quadratic equation in r^2 as

$$r^4 - 2.0396r^2 - 99 = 0 \tag{E.3}$$

The solution of Eq. (E.3) gives

$$r^2 = 11.0218, -8.9822$$

which gives the positive value of r as 3.3199. Using the excitation frequency of

$$\omega = \frac{2\pi(1800)}{60} = 188.496 \text{ rad/s}$$

and the frequency ratio of $r = 3.3199$, the required natural frequency of the system can be determined as

$$r = 3.3199 = \frac{\omega}{\omega_n} = \frac{188.496}{\omega_n} \tag{E.4}$$

Equation (E.4) gives $\omega_n = 56.7776$ rad/s.

We assume that that one helical spring is installed at each corner of the base (under the four corners of the work bench). Because the expected deflection of the springs is unknown, the correct stiffness of the springs (out of the two possible values) is unknown. Hence we use the relation (see Eq. (2.28)):

$$\omega_n = \sqrt{\frac{g}{\delta_{st}}} \quad \text{or} \quad 56.7776 = \sqrt{\frac{9.81}{\delta_{st}}} \tag{E.5}$$

to find the static deflection of the system (δ_{st}) as

$$\delta_{st} = \frac{9.81}{56.7776^2} = 3.0431 \times 10^{-3}\ \text{m}$$

Since all the four springs experience δ_{st}, the static load acting on each spring can be found from Eq. (E.1) as

$$\text{P} = 50000(3.0431 \times 10^{-3}) = 152.155\ \text{N}$$

The total load on the four springs is $4 \times 152.155 = 608.62$ N. Because the weight of the machine is $50\ \text{g} = 50(9.81) = 490.5$ N, in order to achieve the total load of 608.62 N, we need to add a weight of $609.62 - 490.50 = 118.12$ N to the system. This weight, in the form of a rectangular steel plate, is to be attached at the bottom of the machine, so that the total vibrating mass becomes 62.0408 kg (with a weight of 608.62 N).

■

EXAMPLE 9.10 Isolation System for a System with Base Motion

A printed circuit board (PCB) made of fiber-reinforced plastic composite material is used for the computer control of an automobile engine. It is attached to the chassis of the computer, which is fixed to the frame of the automobile as shown in Fig. 9.27(a). The frame of the automobile and the chassis of the computer are subject to vibration at the engine speed of 3000 rpm. If it is required to achieve a displacement transmissibility of no more than 10% at the PCB, design a suitable isolation system between the chassis of the computer and the frame of the automobile. Assume that the chassis of the computer is rigid with a mass of 0.25 kg.

Data of PCB: Length (l): 25 cm, width (w): 20 cm, thickness (t): 0.3 cm, mass per unit surface area: 0.005 kg/cm^2, Young's modulus (E): 15×10^9 N/m^2, damping ratio: 0.01.

Solution: The PCB is assumed to be fixed to the chassis of the computer as a cantilever beam. Its mass (m_{PCB}) is given by $25 \times 20 \times 0.005 = 2.5$ kg. The equivalent mass at the free end of the cantilever is m_b is (see Example 2.9):

$$m_b = \frac{33}{140} m_{\text{PCB}} = \frac{33}{140}(2.5) = 0.5893\ \text{kg}$$

Using the moment of inertia of the cross section of the PCB

$$I = \frac{1}{12} wt^3 = \frac{1}{12}(0.20)(0.003)^3 = 45 \times 10^{-8}\ \text{m}^4$$

the stiffness of the PCB as a cantilever beam can be computed as

$$k_b = \frac{3EI}{l^3} = \frac{3(15 \times 10^9)(45 \times 10^{-8})}{(0.25)^3} = 1.296 \times 10^6\ \text{N/m}$$

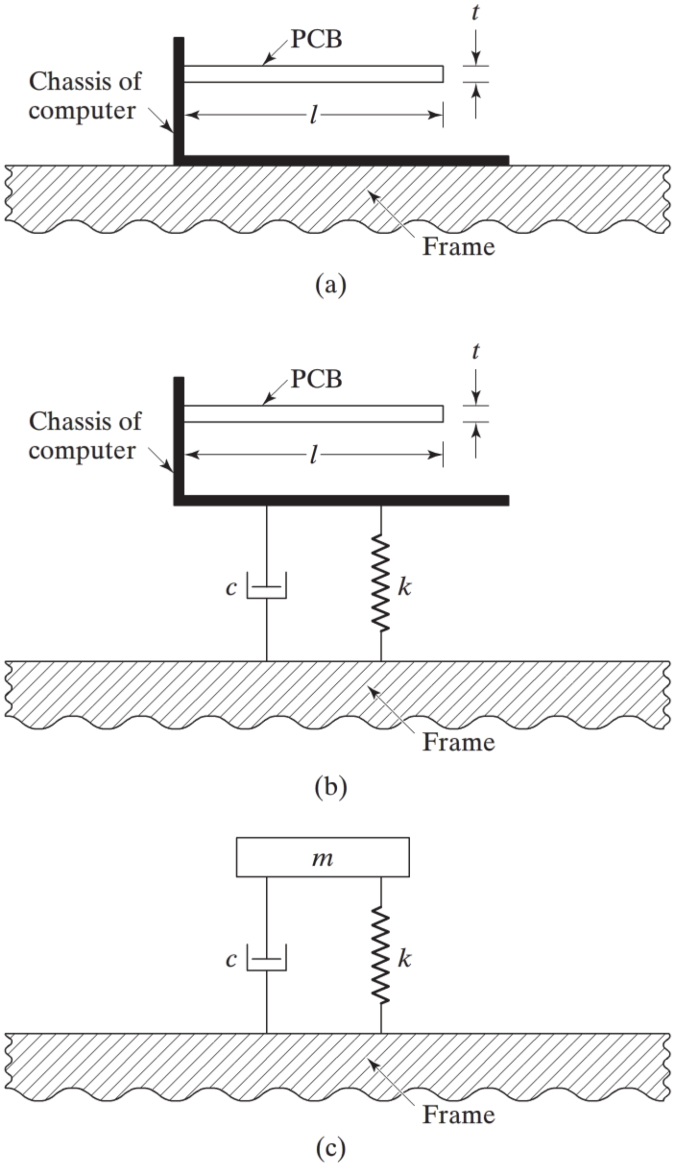

FIGURE 9.27

The natural frequency of the PCB is given by

$$\omega_n = \sqrt{\frac{k_b}{m_b}} = \sqrt{\frac{1.296 \times 10^6}{0.5893}} = 1482.99 \text{ rad/s}$$

The frequency of vibration of the base (chassis of the computer) is

$$\omega = \frac{2\pi(3000)}{60} = 312.66 \text{ rad/s}$$

The frequency ratio is given by

$$r = \frac{\omega}{\omega_n} = \frac{312.66}{1482.99} = 0.2108$$

Using the damping ratio $\zeta = 0.01$, the displacement transmissibility can be determined from Eq. (9.105):

$$T_d = \frac{X}{Y} = \left\{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{1 + [2(0.01)(0.2108)]^2}{(1 - 0.2108^2)^2 + [2(0.01)(0.2108)]^2}\right\}$$

$$= 1.0465 \tag{E.1}$$

This value of $T_d = 104.65\%$ exceeds the maximum permissible value of 10%. Hence we design an isolator (with stiffness k and damping constant c) between the chassis of the computer and the frame of the automobile as shown in Fig. 9.27(b). If we model the PCB with stiffness k_b and mass m_b as before, the addition of the isolator makes the problem a two-degree-of-freedom system. For simplicity, we model the cantilever beam (PCB) as a rigid mass with no elasticity. This leads to the single-degree-of-freedom system shown in Fig. 9.27(c), where the equivalent mass m is given by

$$m = m_{\text{PCB}} + m_{\text{chassis}} = 2.5 + 0.25 = 2.75 \text{ kg}$$

Assuming a damping ratio of 0.01, for the required displacement transmissibility of 10%, the frequency ratio r can be determined from the relation

$$T_d = 0.1 = \left\{\frac{1 + [2(0.01)r]^2}{(1 - r^2)^2 + [2(0.01)r]^2}\right\}^{\frac{1}{2}} \tag{E.2}$$

By squaring both sides of Eq. (E.2) and rearranging the terms, we obtain

$$r^4 - 2.0396r^2 - 99 = 0 \tag{E.3}$$

The positive root of Eq. (E.3) is $r^2 = 11.0218$ or $r = 3.3199$. The stiffness of the isolator is given by

$$k = \frac{m\omega^2}{r^2} = \frac{(2.75)(312.66^2)}{11.0218} = 24{,}390.7309 \text{ N/m}$$

The damping constant of the isolator can be computed as

$$c = 2\zeta\sqrt{mk} = 2(0.01)\sqrt{(2.75)(24390.7309)} = 5.1797 \text{ N-s/m}$$

■