50 POINTS **HOMEWORK 12** **DUE: 4/22/15**

1. (50 pts.) Consider the problem of designing a dynamic vibration absorber for an undamped system, as shown in Figure 1. Suppose that the spectral density of the input force is a band-limited white noise:

$$S_{ff}(\omega) = S_0 \frac{(\beta\omega)^2}{\left(\omega_c^2 - \omega^2\right)^2 + (\beta\omega)^2}$$

You are given the constants $m = 100\,\text{kg}$, $k = 20{,}000\,\text{N/m}$, $\omega_c = 16\,\text{rad/s}$, $\beta = 20\,\text{rad/s}$, and $S_0 = 1\,\text{N}^2/(\text{rad/s})$.

a. (10 pts.) Prove that $E\left[f^2(t)\right] = \pi\beta S_0$.

b. (10 pts.) By approximating the input force as white noise, find the optimal values of μ, p, ζ if $E\left[\xi^2(t)\right]_{\max} = 20$.

c. (20 pts.) Plot $S_{ff}(r)$, $S_{\xi\xi}(r)$ without the DVA, and $S_{\xi\xi}(r)$ with the DVA on the same plot.

d. (10 pts.) Using $S_{\xi\xi}(r)$ with the DVA, estimate $E\left[\xi^2(t)\right]$. How does this computed value compare with the design value of 20? Explain the difference.

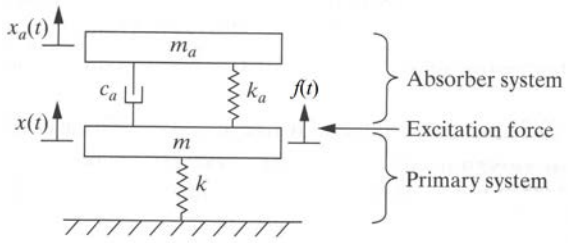

Figure 1