HOMEWORK 2 SOLUTIONS

1. a. To avoid oscillations in the response, the roots of the characteristic equation must be real. This is because, if the roots are complex, then you will get terms like $e^{-at}\cos(bt)$ or $e^{-at}\sin(bt)$ in the response.

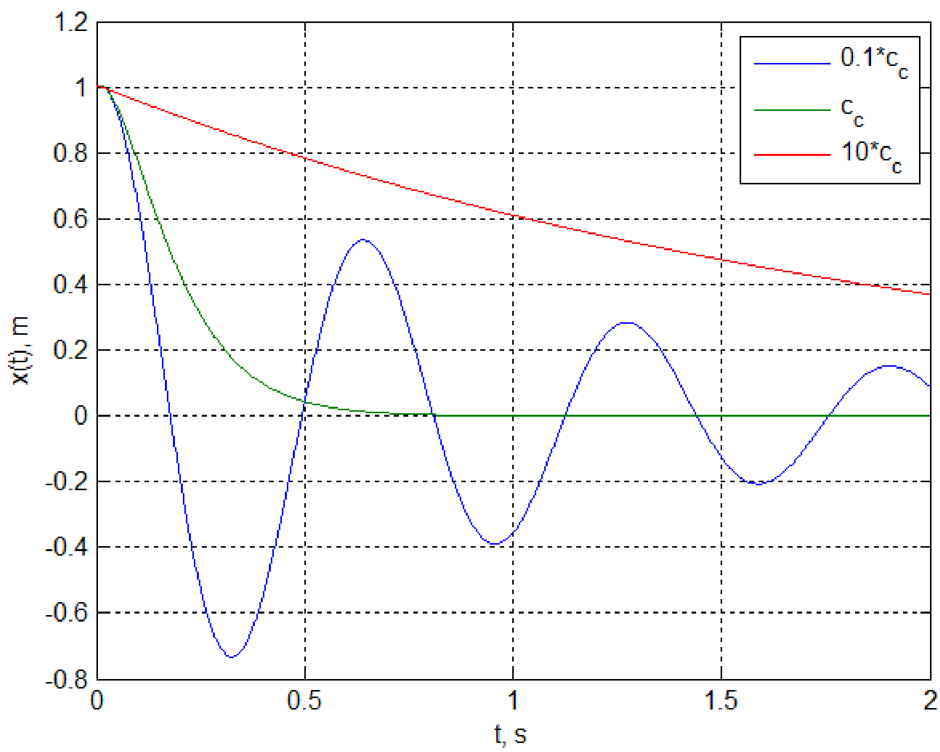
The transition from real to complex roots occurs when the discriminant is 0, i.e.

$$c = \sqrt{4mk} = \sqrt{4(50\text{ kg})(5000\text{ N/n})} = 1000\text{ N/(m/s)}$$

Of course, larger values of c will also have no oscillations in the response, but this is the minimum value.

b.

```
function hw2_1()

clear all;
close all;

m = 50;
k = 5000;
c = sqrt(4*m*k);
c = [0.1*c c 10*c];

t = linspace(0,2,1001);
x = zeros(length(t),length(c));
x0 = [1; 1];
for i = 1:length(c)
    [dum,y] = ode45(@loc_eom_fun,t,x0);
    x(:,i) = y(:,1);
end

    function xdot = loc_eom_fun(loc_t,loc_x)
        xdot = [loc_x(2); -c(i)/m*loc_x(2)-k/m*loc_x(1)];
    end

plot(t,x);
ylabel('x(t), m');
xlabel('t, s');
legend('0.1*c_c','c_c','10*c_c');
grid on;

end
```



2. Once the engine makes contact with the stopper, it becomes a simple spring-mass-damper system.

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$m\left(s^2 X(s) - sx_0 - \dot{x}_0\right) + c\left(sX(s) - x_0\right) + kX(s) = 0$$

Rearranging and substituting in the numerical values gives

$$X(s) = \frac{m\dot{x}_0}{ms^2 + cs + k} = \frac{20{,}000}{2{,}000s^2 + 20{,}000s + 40{,}000} = \frac{10}{s^2 + 10s + 20}$$

Checking the discriminant of the denominator polynomial yields $10^2 - 4 \cdot 20 = 20 > 0 \Rightarrow \text{real}$, so the fraction can be expanded, using `residue` for example, as

$$X(s) = \frac{-2.2361}{s + 7.2361} + \frac{2.2361}{s + 2.7639}$$

whose inverse Laplace transform is $x(t) = -2.2361e^{-7.2361t} + 2.2361e^{-2.7639}$.

a. To find the maximum compression, the velocity is set equal to 0:

$$\dot{x}(t) = 16.1806e^{-7.2361t} - 6.18036e^{-2.7639} = 0$$
$$e^{-4.4722t} = 0.3820$$
$$t = \frac{\ln 0.3820}{-4.4722} = 0.2125\ \text{s}$$

b. The maximum compression is then

$$\max x(t) = -2.2361e^{-7.2361 \cdot 0.2152} + 2.2361e^{-2.7639 \cdot 0.2152}$$
$$= 0.762\ \text{m}$$

3. We know that in general, the amplitude of motion decreases linearly when Coulomb friction is the sole damping force:

$$x_{i+1} = x_i - \frac{4\mu N}{k}$$

where, in this problem, the normal force is $N = \frac{\sqrt{3}}{2}mg$.

The motion stops when $\dot{x} = 0$ and $|x| \le \frac{\mu N}{k}$. In terms of half cycles n, this condition becomes

$$|x_0| - \frac{2\mu N}{k}n \le \frac{\mu N}{k}$$

Solving for μ gives

$$\mu \ge \frac{k|x_0|}{(2n+1)N} = \frac{2\omega_n^2|x_0|}{\sqrt{3}(2n+1)g} = 0.227$$

This gives us a lower bound for μ. An upper bound for μ can be found from the same equation assuming the mass stopped after 19 half cycles:

$$\mu \le \frac{k|x_0|}{[2(n-1)+1]N} = \frac{2\omega_n^2|x_0|}{\sqrt{3}[2(n-1)+1]g} = 0.238$$

Therefore, $0.227 \le \mu \le 0.238$.

4.

$$s_{1,2} = \frac{-12 \pm \sqrt{144-8k}}{4} = -3 \pm \frac{1}{4}\sqrt{144-8k}$$

$0 < k < 18$ => overdamped

$k = 18$ => critcally damped

$k > 18$ => underdamped