110 POINTS **HOMEWORK 1** **DUE: 1/21/15**

1. (40 pts.) Consider the system shown in Figure 1. You are given the following: $k = 5000\ \text{N/m}$, $c = 1000\ \text{N/(m/s)}$, $l = 1\,\text{m}$, $m = 10\,\text{kg}$, $M_0 = 100\ \text{N - m}$, $\omega = 1000\ \text{rpm}$. $M_0 \cos(\omega t)$ represents an externally applied moment about the hinge.

a. (10 pts.) Derive the transfer function of the system, where the externally applied moment is the input, and θ is the output.

b. (10 pts.) Calculate general response of the system $\theta(t)$ starting from rest.

c. (10 pts.) Calculate the steady-state response $\theta(t)$ as $t \to \infty$.

d. (10 pts.) Plot these two responses (part b and c) on the same graph. Choose the scales appropriately to see the convergence to steady-state clearly.

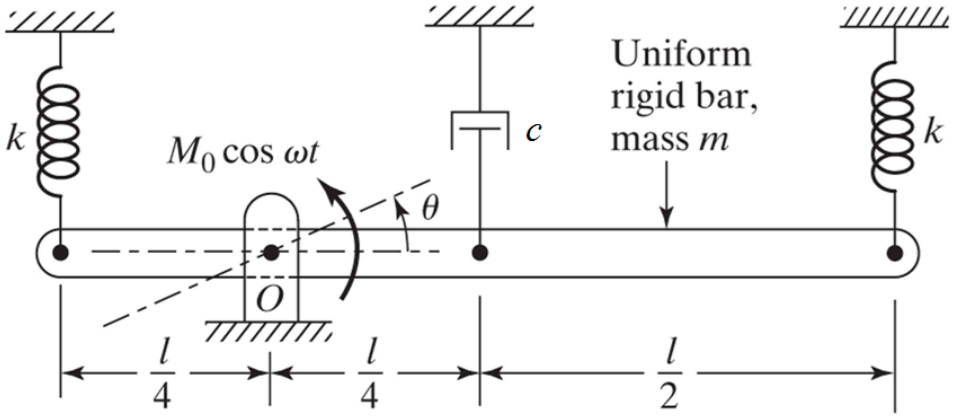

Figure 1

2. (20 pts.)

a. (10 pts.) Prove that the unit ramp response of a system is just the integral (assuming 0 initial conditions) of the unit step response. (Hint: this is easily shown using transfer functions.)

b. (10 pts.) Show how a state-space model of a system can be augmented to output its ramp response if the input is a unit step.

3. (20 pts.) Solve the following ODE using the Laplace Transform method:

$$\ddot{x} + 4\dot{x} + 40x = 10, \quad x_0 = \dot{x}_0 = 0$$

Please write out by hand all steps.

4. (30 pts.) The unit step response of a 2nd-order system is shown in Figure 2.

a. (10 pts.) Where are its poles located in the s-plane?

b. (10 pts.) What is the transfer function of this system?

c. (5 pts.) How many times more should its damping be increased over its current value to eliminate overshoot in the step response?

d. (5 pts.) If the conditions of part (c) are met, where would the poles be located in the s-plane?

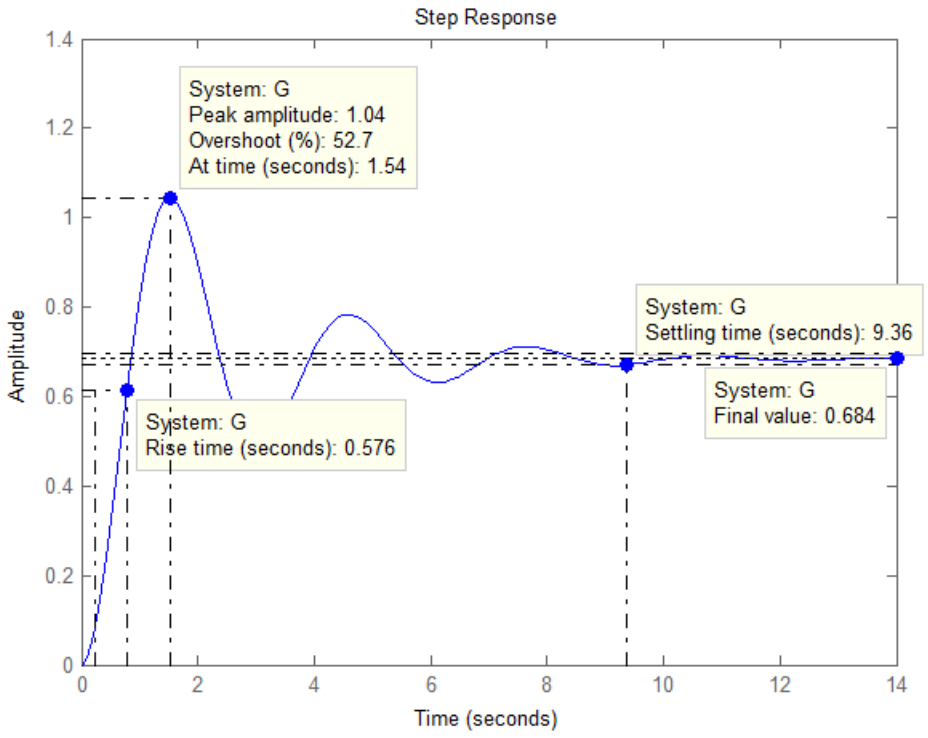

Figure 2