70 POINTS **HOMEWORK 6** **DUE: 2/25/15**

1. (30 pts.) Consider an undamped, pinned-pinned beam with constant properties $\rho A = 1\,\text{kg/m}$, $EI = 1\,\text{Nm}^2$, and $l = 1\,\text{m}$. A machine with an operating range of $35\,\text{rad/s} \le \omega \le 45\,\text{rad/s}$ applies a sinusoidal force to the beam at $x = l/10$. We wish to attach a tuned, damped dynamic vibration absorber such that $\left|\frac{W(l/4, i\omega)}{\delta_{st}(l/4)}\right| \le 25\,\text{dB}$ over the operating range without greatly modifying the other modes. Suppose the absorber has a damping ratio $\zeta_a = c_a/(2m_a\omega_a) = 0.05$.

a. (20 pts.) Find values of m_a, c_a, k_a to satisfy the design requirements. Make m_a as small as possible. Include plots to show the effects of adding the DVA. Add markers to show that your design meets the requirements.

b. (10 pts.) Calculate how much adding this DVA affects the first natural frequency of the beam. (Recall that a good approximation is to assume the DVA mass is a fixed mass on the beam at low frequencies.) Does this match the shift in frequency you see on your plot?

2. (40 pts.) Consider the 2-DOF system shown in Figure 1, where $m_1 = 1\,\text{kg}$, $m_2 = 0.1\,\text{kg}$, $k = 1\,\text{N/m}$, and $c = 0.004\,\text{N/(m/s)}$.

a. (10 pts.) Derive a formula for the transfer function $G(s) = \frac{X_1(s)}{F(s)}$ in terms of the following parameters: $\omega_2 = \sqrt{\frac{k}{m_2}}$, $\zeta = \frac{c}{2m_2\omega_2}$, and $\mu = \frac{m_2}{m_1}$. (Don't plug in the values.)

b. (10 pts.) Consider a lead compensator of the form $H(s) = K\frac{Ts+1}{\alpha Ts+1}$, where $T = 10$ and $\alpha = 0.004$. Prove that the closed-loop system has an infinite gain margin.

c. (10 pts.) Derive a formula for the transfer function $G(s) = \frac{X_2(s)}{F(s)}$ in terms of the same parameters. (Don't plug in the values.)

d. (10 pts.) Consider the same lead compensator for this transfer function. Compute the gain margin. What is the difference between this system and the one in parts (a,b)?

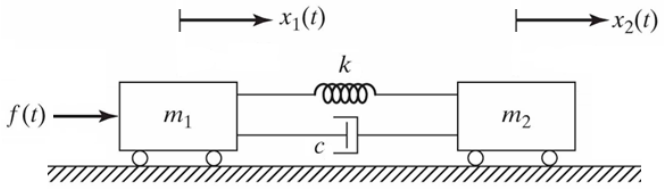

Figure 1