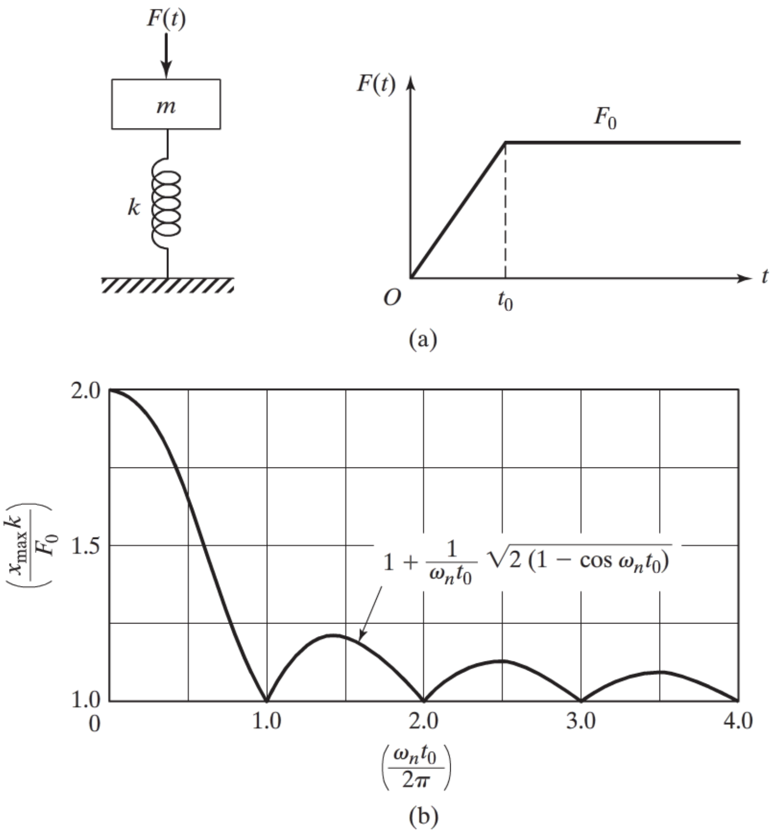

FIGURE 9.30 Shock load on electronic instrument.

The root of Eq. (E.4) gives the desired stiffness value as $k = 6.2615 \times 10^5$ N/m. The following MATLAB program can be used to find the root of Eq. (E.4):

```
>> x=1000:1:10000000;
>> f='(100/sqrt(x))*sqrt(2*(1-cos(0.02*sqrt(x))))-0.000002*x+1';
>> root=fzero(f,100000)

root =

 6.2615e+005

>>
```

■

9.10.6 Active Vibration Control

A vibration isolation system is called active if it uses external power to perform its function. It consists of a servomechanism with a sensor, signal processor, and an actuator, as shown schematically in Fig. 9.31 [9.31–9.33]. This system maintains a constant distance (l) between the vibrating mass and the reference plane. As the force $F(t)$ applied to the system (mass) varies, the distance l tends to vary. This change in l is sensed by the sensor and a signal, proportional to the magnitude of the excitation (or response) of the vibrating

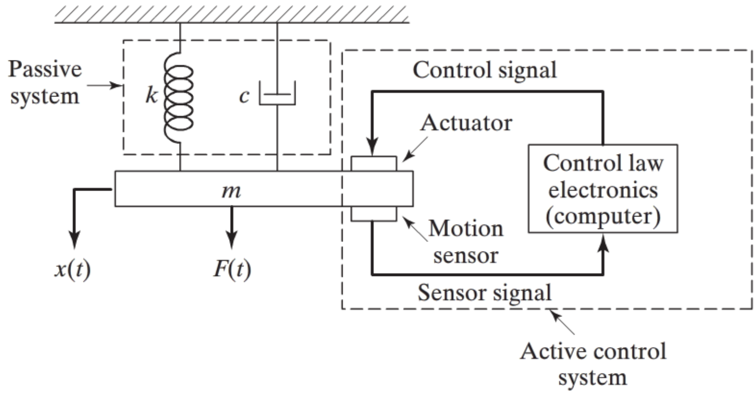

FIGURE 9.31 Active vibration isolation system.

body, is produced. The signal processor produces a command signal to the actuator based on the sensor signal it receives. The actuator develops a motion or force proportional to the command signal. The actuator motion or force will control the base displacement such that the distance l is maintained at the desired constant value.

Different types of sensors are available to create feedback signals based on the displacement, velocity, acceleration, jerk, or force. The signal processor may consist of a passive mechanism, such as a mechanical linkage, or an active electronic or fluidic network that can perform functions such as addition, integration, differentiation, attenuation, or amplification. The actuator may be a mechanical system such as a rack-and-pinion or ball screw mechanism, a fluidic system, or piezoelectric and electromagnetic force generating system. Depending on the types of sensor, signal processor, and actuator used, an active vibration control system can be called *electromechanical*, *electrofluidic*, *electromagnetic*, *piezoelectric*, or *fluidic*.

Analysis: Consider a single-degree-of-freedom system in which the mass m is subjected to an applied force $f(t)$ as shown in Fig. 9.31. If we use an active control system to control the vibration of the mass m, the actuator will be designed to exert a control force $f_c(t)$ so that the equation of motion of the system becomes

$$m\ddot{x} + c\dot{x} + kx = F(t) = f(t) + f_c(t) \tag{9.127}$$

Most commonly, the sensor (computer) measures the displacement x and the velocity $\dot{x}$ of the mass in real time (continuously). The computer computes the control force $f_c(t)$ necessary to control the motion and commands the actuator to exert the force $f_c(t)$ on the mass m.

Usually the computer is programmed to generate the control force proportional to the displacement $x(t)$ and the displacement derivative or velocity $\dot{x}(t)$ of the mass so that

$$f_c(t) = -g_p x - g_d \dot{x} \tag{9.128}$$

where g_p and g_d are constants whose values are to be determined and programmed into the computer by the designer. The constants g_p and g_d are known as control gains, with g_p denoting the proportional gain and g_d indicating the derivative or rate gain. The control algorithm in this case is known as the proportional and derivative (PD) control. By substituting Eq. (9.128) into Eq. (9.127), we obtain

$$m\ddot{x} + (c + g_d)\dot{x} + (k + g_p)x = f(t) \tag{9.129}$$

which shows that g_d acts like additional (or artificial) damping and g_p like additional (or artificial) stiffness. Equation (9.129), known as the closed-loop equation, can be solved to find the response characteristics of the system. For example, the new (effective) natural frequency is given by

$$\omega_n = \left(\frac{k + g_p}{m}\right)^{\frac{1}{2}} \tag{9.130}$$

and the new (effective) damping ratio by

$$\zeta = \frac{c + g_d}{2\sqrt{m(k + g_p)}} \tag{9.131}$$

The new (effective) time constant of the system, for $\zeta \leq 1$, is given by

$$\tau = \frac{2m}{c + g_d} \tag{9.132}$$

Thus the functioning of the active vibration control system can be described as follows: Given the values of m, c, and k, compute the control gains g_p and g_d to achieve the desired values of ω_n, ζ, or τ. In practice, the response of the system is continuously monitored, the computations are done, and the actuator is made to apply the control force f_c to the mass in real time so that the response of the system lies within the stated limits. Note that the gains g_p and g_d can be positive or negative depending on the measured and desired responses.

EXAMPLE 9.13 Vibration Control of a Precision Electronic System

It is proposed to control the vibration of a precision electronic system supported on an elastic pad (with no damping) by either a passive or an active method. The system has a mass of 15 kg and a natural frequency of 20 rad/s. It is estimated that the system requires a damping ratio of $\zeta = 0.85$ to control the vibration. Assume that the available dashpots can provide damping constants only in the range $0 \leq c \leq 400$ N-s/m.

Solution: First, we investigate the use of an available dashpot to control the vibration (passive control). From the known natural frequency of the system, we can find the stiffness of the elastic pad as

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{or} \quad k = m\omega_n^2 = 15(20)^2 = 6000 \text{ N/m} \tag{E.1}$$

The required damping ratio of the system gives the necessary damping constant (c) as

$$\zeta = \frac{c}{2\sqrt{km}} = 0.85 \quad \text{or} \quad c = 2\zeta\sqrt{km} = 2(0.85)\sqrt{6000(15)} = 510 \text{ N-s/m} \tag{E.2}$$

Since the available dashpots can provide damping constant values up to 400 N-s/m only, we cannot achieve the desired control using passive damping.

Thus we consider an active control system to create the required amount of damping into the system. Let the control force be of the form $f_c = -g_d\dot{x}$, so that the damping ratio, alternate form of Eq. (9.131), can be expressed (with $g_p = 0$):

$$2\zeta\omega_n = \frac{c + g_d}{m} \tag{E.3}$$

By adding the available dashpot, with a damping constant of 400 N-s/m, Eq. (E.3) can be rewritten as

$$400 + g_d = 2m\zeta\omega_n = 2(15)(0.85)(20) = 510 \text{ N-s/m}$$

or

$$g_d = 110 \text{ N-s/m}$$

This gives the value of the damping constant to be provided by the active control (also known as derivative gain) as $g_d = 110$ N-s/m.

■

EXAMPLE 9.14 Active Control of a System with Rotating Unbalance

A single-degree-of-freedom system consists of a mass (m) = 150 kg, damping constant (c) = 4000 N-s/m, and stiffness (k) = 6×10^6 N/m. The mass is subjected to a rotating unbalanced force given by $f(t) = 100 \sin 60\pi t$ N. The following observations can be made from the given data:

(i) The natural frequency of the system, $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{6(10^6)}{150}} = 200$ rad/s, is close to the frequency of the disturbance, $\omega = 60\pi = 188.4955$ rad/s.

(ii) The damping ratio of the system is small with a value of

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{4000}{2\sqrt{[6(10^6)(160)]}} = 0.06667$$

It is desired to change the natural frequency of the system to 100 rad/s and the damping ratio to 0.5. Because the values of k and c of the system cannot be altered, it is proposed to use an active control system. Determine the control gains required to achieve the desired values of ω_n and ζ. Also find the magnitude of the response and the actuator force of the system in the steady state.

Solution: When an active control system is used with control gains g_p and g_d, the natural frequency of the system can be expressed as

$$\omega_n = 100 = \sqrt{\frac{6(10^6) + g_p}{150}}$$

or

$$g_p = 150(10^4) - 6(10^6) = -4.5(10^6) \text{ N/m}$$

This implies that the stiffness of the system is to be reduced to 1.5×10^6 N/m. The new damping ratio of the system is given by

$$\zeta = 0.5 = \frac{c + g_d}{2\sqrt{km}} = \frac{4000 + g_d}{2\sqrt{[1.5(10^6)](150)}}$$

or

$$g_d = 15000 - 4000 = 11000 \text{ N-s/m}$$

This implies that the damping of the system is to be increased to 15000 N-s/m.

The equation of motion of the actively controlled system can be written as

$$m\ddot{x} + c\dot{x} + kx = f(t) = f_0 \sin \omega t \tag{E.1}$$

which, in this case, takes the form

$$150\ddot{x} + 15000\dot{x} + 1.5(10^6)x = f(t) = 100 \sin 60\pi t \tag{E.2}$$

From Eq. (E.1), the general transfer function of the system can be expressed as (see Section 3.12)

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k} \tag{E.3}$$

The magnitude of the steady-state response of the system corresponding to Eq. (E.3) is given by (see Section 3.13)

$$X = \frac{f_0}{\left[(k - m\omega^2)^2 + (c\omega)^2\right]^{\frac{1}{2}}} \tag{E.4}$$

In the present case, $f_0 = 100$ N, $m = 150$ kg, $c = 15000$ N-s/m, $k = 1.5 \times 10^6$ N-s/m, and $\omega = 188.4955$ rad/s. Thus Eq. (E.4) gives

$$X = \frac{150}{\left[\{1.5(10^6) - 150(188.4955)^2\}^2 + \{15000(188.4955)\}^2\right]^{\frac{1}{2}}}$$

$$= \frac{150}{4.7602(10^6)}$$

$$= 31.5113(10^{-6}) \text{ N}$$

The actuator (control) force, F_t, at steady state can be obtained from the relation

$$\frac{F_t(s)}{F(s)} = \frac{F_t(s)}{X(s)} \frac{X(s)}{F(s)} = \frac{k + cs}{ms^2 + cs + k} \tag{E.5}$$

as

$$\begin{aligned} F_t(i\omega) &= |4.5(10^6) - 11000 i\omega| X(i\omega) \\ &= |4.5(10^6) - 11000(188.4955)i|(31.5113(10^{-6})) \\ &= \sqrt{\{4.5(10^6)\}^2 + \{11000(188.4955)\}^2}\,(31.5113(10^{-6})) \\ &= 156.1289 \text{ N} \end{aligned}$$

■

9.11 Vibration Absorbers

The *vibration absorber*, also called *dynamic vibration absorber*, is a mechanical device used to reduce or eliminate unwanted vibration. It consists of another mass and stiffness attached to the main (or original) mass that needs to be protected from vibration. Thus the main mass and the attached absorber mass constitute a two-degree-of-freedom system, hence the vibration absorber will have two natural frequencies. The vibration absorber is commonly used in machinery that operates at constant speed, because the vibration absorber is tuned to one particular frequency and is effective only over a narrow band of frequencies. Common applications of the vibration absorber include reciprocating tools, such as sanders, saws, and compactors, and large reciprocating internal combustion engines which run at constant speed (for minimum fuel consumption). In these systems, the vibration absorber helps balance the reciprocating forces. Without a vibration absorber, the unbalanced reciprocating forces might make the device impossible to hold or control. Vibration absorbers are also used on high-voltage transmission lines. In