**80 POINTS** | **HOMEWORK 9** | **DUE: 4/1/15**

1. (40 pts.) A car driving over a bumpy road can be modeled as a single DOF system as shown in Figure 1. Suppose the bumpiness of the road can be modeled by zero-mean white noise, i.e. $S_{yy}(\omega) = S_0$.

a. (20 pts.) Calculate the steady-state mean square value of $x(t)$, i.e. $E[x^2]$, in terms of $S_0$, $m$, $c$, and $k$.

b. (20 pts.) Using the values $S_0 = 0.004\ \text{m}^2/(\text{rad/s})$, $m = 9000\ \text{N}$, $k = 45{,}000\ \text{N/m}$ and $\zeta = 0.2$, run the simulation of this system multiple times (at least 20 times) and average $x^2(t)$ at each $t$ of the simulation to get an estimate of $E[x^2(t)]$. Show that this estimate converges to the theoretical steady-state value of $E[x^2]$.

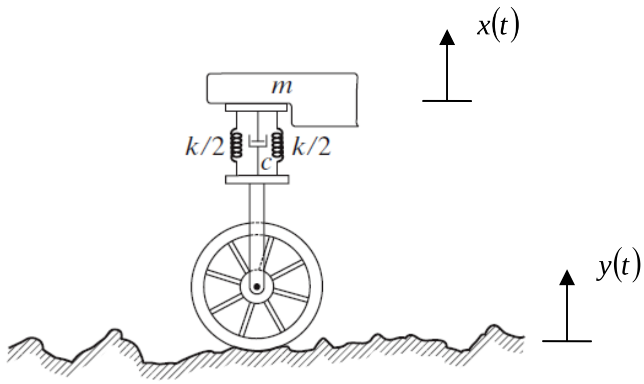


**Figure 1**

2. (40 pts.) The wing of an airplane subject to vertical gusts can be modeled as a spring-mass-damper system, as shown in Figure 2. Suppose that the mean-square deflection of the wing is $E\left[x^2(t)\right]=x_{ms}$ when subjected to a white noise gust with spectral density $S(\omega)=S_0$.

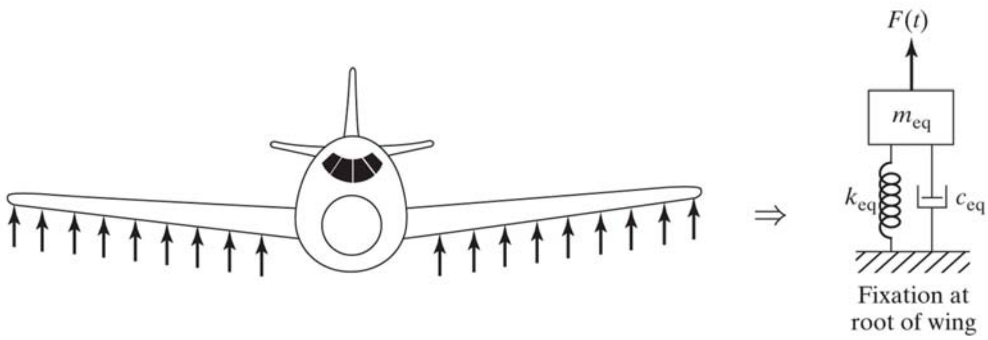


**Figure 2**

a. (20 pts.) Derive expressions for the equivalent system parameters $m_{eq}$, $c_{eq}$, and $k_{eq}$ in terms of $\omega_n$, $\omega_d$, $x_{ms}$, and $S_0$.

b. (20 pts.) The unit impulse response and the response to zero mean, unit variance white noise (i.e. $S_0=1$) are shown in Figure 3. Using these figures and your answer to part (a), find $m_{eq}$, $c_{eq}$, and $k_{eq}$. The one standard deviation envelope is shown in red.

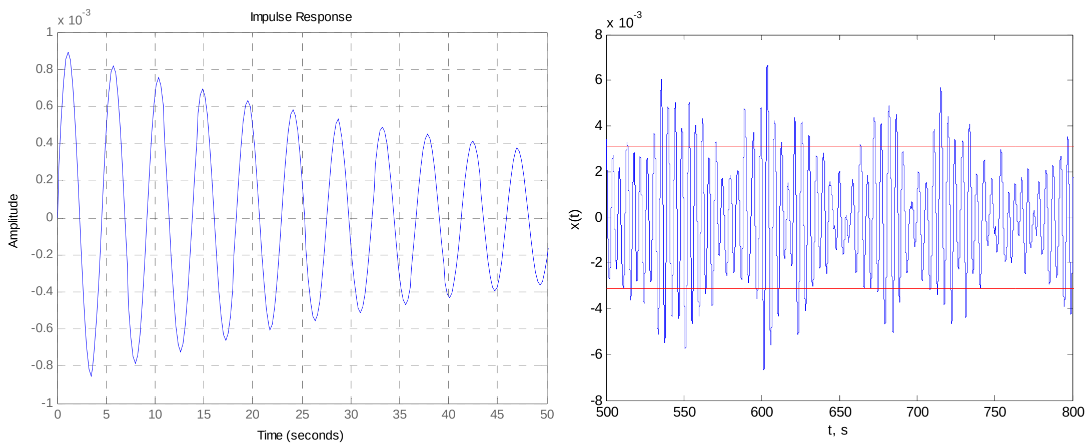


**Figure 2**