60 POINTS **HOMEWORK 11** **DUE: 4/15/15**

1. (60 pts.) Consider a spring-mass-damper system with natural frequency $f_n = 1\,\text{Hz}$ and $m = 5\,\text{kg}$. The system is subject to stationary, zero-mean white noise with $S_0 = 1\ \text{N}^2/(\text{rad/s})$. It is common to model the yield failure of a system in terms of its standard deviation, i.e. $U = k\sigma_x$, where k is a positive constant.

a. (30 pts.) Calculate, from repeated simulations, $E\left[\dot{n}_+(U)\right]$ and $E\left[\dot{n}_+^a(U)\right]$ as a function of k over the range $0 \le k \le 8$ and for several values of ζ. Compare these to plots of the theoretical values using the standard assumptions of stationary, narrow band, Gaussian processes. Similar to class, plot k on the horizontal axis. Are these comparisons better for higher or lower values of ζ, and why?

b. (30 pts.) For the same values of k and ζ, plot the distribution of t_f, the time to failure, using $E\left[\dot{n}_+(U)\right]$ and $E\left[\dot{n}_+^a(U)\right]$. Are the mean and standard deviations as expected? Are the expectations closer to the theoretical values for higher or lower values of ζ, and why?