75 POINTS **HOMEWORK 7** **DUE: 3/18/15**

1. (40 pts.) The mean and variance of a random process can only be approximated because only a finite number of samples N are available (see §5 of the handout "Review of random variables and expected values"). Consider 1000 samples of data from a Gaussian (normal) distribution with a 0 mean and 1 variance.

a. (20 pts.) For sample sets of size $N = 10,20,50,100$, calculate the mean of the means $E\left[\hat{\bar{x}}\right]$ and the variance of the means $\text{var}\left(\hat{\bar{x}}\right)$ of the sets. Plot the four means of the means on a graph, with N on the horizontal axis, along with the theoretical relationship $E\left[\hat{\bar{x}}\right] = \bar{x}$. Plot the four variances of the means on a separate graph, with N on the horizontal axis, along with the theoretical relationship $\text{var}\left(\hat{\bar{x}}\right) = \frac{\sigma_x^2}{N}$.

b. (20 pts.) For sample sets of size $N = 10,20,50,100$, calculate the mean of the variances $E\left[\hat{\sigma}_x^2\right]$ and the variance of the variances $\text{var}\left(\hat{\sigma}_x^2\right)$ of the sets. Plot the four means of the variances on a graph, with N on the horizontal axis, along with the theoretical relationship $E\left[\hat{\sigma}_x^2\right] = \sigma_x^2$. Plot the four variances of the variances on a separate graph, with N on the horizontal axis, along with the theoretical relationship $\text{var}\left(\hat{\sigma}_x^2\right) = \frac{1}{N}\left(\sigma_x^4 - \frac{N-3}{N-1}\left(\sigma_x^2\right)^2\right)$.

2. (15 pts.) Consider the following random process:

$$x(t) = A\cos(t) + B\sin(t)$$

where A and B are zero-mean, unit-variance, independent random variables. Find $\bar{x}(t)$ and $R_{xx}(t_1, t_2)$. Is $x(t)$ a stationary random process?

3. (10 pts.) Let $x(t)$ be a stationary random process. Prove the following:

$$P\left(\left|x(t+\tau) - x(t)\right| \geq a\right) \leq \frac{2}{a^2}\left[R_{xx}(0) - R_{xx}(\tau)\right] \text{ for all } a > 0$$

4. (10 pts.) Let $x(t)$ be a stationary random process. Using the fact that $R_{xx}(\tau) = R_{xx}(-\tau)$, prove that $S_{xx}(\omega) = S_{xx}(-\omega)$. Hence, both functions are <u>even</u>.