60 POINTS **HOMEWORK 10** **DUE: 4/8/15**

1. (30 pts.) In order to model a material specimen cracking and eventually breaking, a spring-mass-damper system with 4 springs and 4 dampers in parallel can be used. Each spring and damper represents one quarter of the width of the specimen. Each quarter has $k = 50{,}000\ \text{N/m}$, and $c = 0.1\ \text{N/(m/s)}$, and obeys the fatigue law $N = \frac{d}{S^b}$. Each quarter has $b = 0.25$ but different values of $d = 2500,\ 5000,\ 7500,\ \text{and}\ 10000$. The total mass of the system is $m = 0.2\ \text{kg}$. The system is subject to stationary, zero-mean white noise with $S_0 = 1\ \text{m}^2\text{/(rad/s)}$

a. (20 pts.) Calculate the time at which each quarter section of the specimen fails. Plot time on the x-axis and number of specimens failed on the y-axis. You may use the theoretical lifetime rather than having to simulate the system.

b. (10 pts.) Calculate the number of cycles of motion (assuming narrow band motion) at which each quarter section of the specimen fails. Plot number of cycles on the x-axis and number of specimens failed on the y-axis. How does this plot differ than the one in part a?

2. (30 pts.) If a random variable $x(t)$ is narrow-band, Gaussian, stationary, and zero mean, then the probability distribution of the peak magnitude random variable $S(t)$ is given by the Rayleigh distribution

$$p_S(s) = \frac{s}{\sigma_x^2} e^{-\frac{s^2}{2\sigma_x^2}}$$

One can also show that for a wide-band process, the probability distribution is Gaussian:

$$p_S(s) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{s^2}{2\sigma_x^2}}$$

One can vary the bandwidth of a system by varying its damping ratio.

Choose several values of damping ratio for a single DOF system, and plot the normalized histogram of the peak distribution along with curves representing the Rayleigh and Gaussian distributions. You should see that for low damping, the histogram approaches the Rayleigh distribution, and for high damping, the histogram approaches the Gaussian distribution.