80 POINTS **HOMEWORK 3** **DUE: 2/4/15**

1. (20 pts.) An instrument with mass $m = 20\text{ kg}$ is to be isolated from the floor with a spring and damper such that $\omega_n = 15\text{ rad/s}$ and $\zeta = 0.2$. The available springs have values of $k = 1000\text{ N/m}, 2000\text{ N/m}, 4000\text{ N/m}$ and the available dampers have values of $c = 10\text{ N/(m/s)}, 50\text{ N/(m/s)}, 100\text{ N/(m/s)}$. Only one spring and one damper may be used. Design a feedback control law $f_c = -K_V\dot{x} - K_P x$ such that $|f_c|$ is <u>minimized</u>.

2. (20 pts.) A spring-mass-damper system has the unit step response shown in Figure 1. Design a feedback control law using position, velocity, <u>and acceleration</u> such that the unit step response appears as in Figure 2.

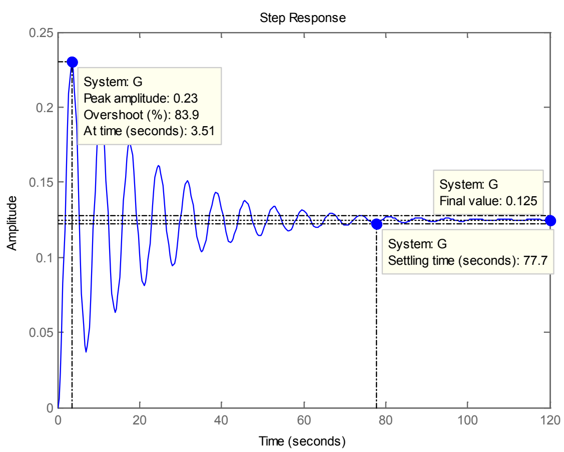

Figure 1

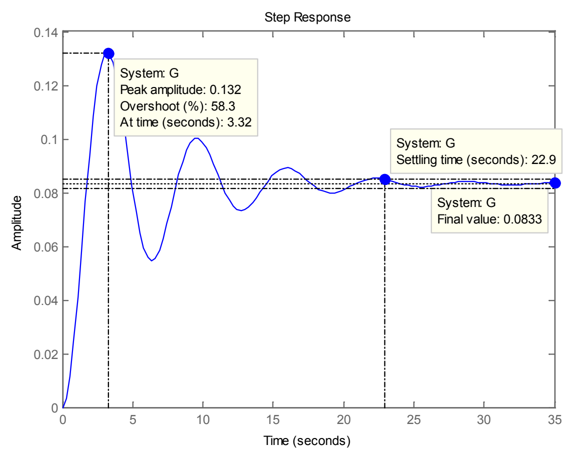

Figure 2

3. (40 pts.) Consider the active control problem discussed in class in which the following values were given: $m = 20\text{ kg}$, $c = 4000\text{ N/(m/s)}$, $k = 6\times10^6\text{ N/m}$, $f(t) = 100\sin(\omega t)$, and $\omega = 30\text{ Hz}$, and the control gains $K_P = -4.5\times10^6\text{ N/m}$ and $K_V = 11{,}000\text{ N/(m/s)}$ yielded $\omega_n = 100\text{ rad/s}$ and $\zeta = 0.5$. Suppose the actuator has its own dynamics; specifically, the actuator behaves as a first order system with a time constant T. Thus, the actuator has a transfer function $H(s) = \dfrac{1}{Ts+1}$.

a. (20 pts.) Using Simulink, simulate the closed-loop system with the actuator dynamics over several orders of magnitude of T. Compare your results to the open-loop and closed-loop performance of the system discussed in class.

b. (20 pts.) Plot the root locus of the closed-loop poles including the actuator dynamics. How do the actuator dynamics affect the shape of the root locus? Where is the actuator's pole in relation to the plant's poles? How does this explain the results from part (a)? Plot root loci for the same values of T considered in part (a).