50 POINTS | **HOMEWORK 4** | **DUE: 10/2/14**

1. (20 pts.) A spring-mass-damper system with properties $m = 1\,\text{kg}$, $k = 15{,}000\ \text{N/m}$, and $\zeta = 0.1$ is subjected to a periodic force, one period of which is shown in Figure 1. Find the steady-state response of the system. Hint: you can use Eq. 4.9–4.11 in the handout "General Periodic Forcing" to approximate the integrals for the Fourier Series coefficients.

2. (20 pts.) The torsional vibrations of a driven gear on a shaft, shown in Figure 2, are governed by the equation

$$J_0\ddot{\theta} + k_t\theta = M(t)$$

where $J_0 = 0.1\,\text{kg-m}^2$ is the moment of inertia of the gear, $k_t = 392{,}700\ \text{N-m/rad}$ is the torsional stiffness of the shaft, and $M(t)$ is the applied moment from the driving gear. Suppose the driving gear nominally supplies a constant moment of 1000 N-m at 1000 rpm, but one of the gear's 16 teeth is broken and does not contact the driven gear. Determine the resulting steady-state torsional vibration of the driven gear $\theta(t \to \infty)$.

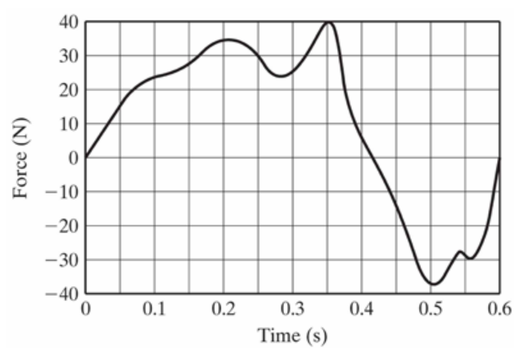

Figure 1

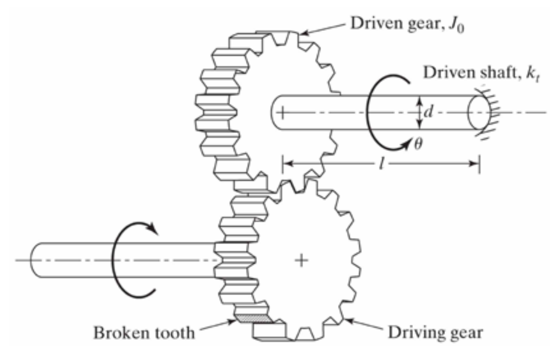

Figure 2

3. (10 pts.) Derive the EOMs in matrix-vector form for the system shown in Figure 3. Use x and θ as your generalized coordinates. Assume no slip between the rope and the pulley. Hint: When drawing the FBD of the pulley, the tension in the rope on the left is not the same as the tension in the rope on the right (otherwise the pulley would never rotate!).

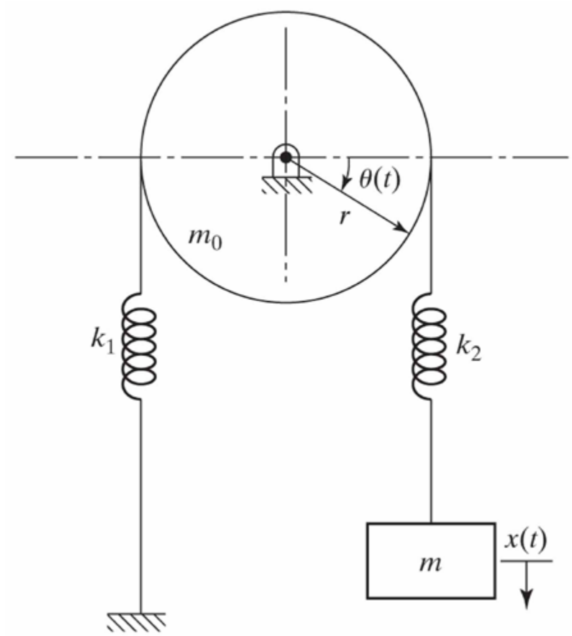

Figure 3