MAE 6254 Final Exam Problem 6

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**Paper:** Sliding-Mode Velocity Control of Mobile-Wheeled Inverted-Pendulum Systems

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This paper proposes a velocity-tracking controller for inverted pendulum systems. Specifically, the model they use is based on the Segway and other similar commercially available products, which is referred to as a mobile wheeled inverted pendulum (MWIP). This type of system is used as a mode of transportation (Segway platform) as well as a common robot model (warehouse/service robots and hobby projects). The authors propose a controller to stabilize the system in the presence of inclines and uncertainty in the robot model. For the Segway in particular this robustness to uncertainty is important, since it is often used by many different people in outdoor environments. Model uncertainty makes linearization difficult, because the equilibrium position isn't known exactly and so some approximations may not be valid.

Several assumptions underly the analysis presented in the paper. First, the model travels only in 1 dimension. A more accurate model would be able to move in at least two dimensions, although the assumption that the robot does not leave the ground is safe. However, since this type of system can turn in place, the assumption that it moves in only straight lines may be a reasonable approximation. Another assumption made during the controller design is that the estimation error in modeling uncertainty is bounded by a known function. This may be true based on empirical testing or further analysis, but there is no motivation or basis given for this assumption. Additionally, the fact that the proposed method tracks a velocity assumes that the input to the system is a velocity. Intuitively, the input to the system would be both position and velocity. However, this simplification may be reasonable since a demanded trajectory could be decomposed into constant velocity sections. During simulation, the desired trajectory is a constant, which is probably not a safe assumption over any extended period of time.

The proposed method is a Sliding Mode Controller. The authors show two separate controllers. the first is stable and robust to uncertainty but leaves a steady state error. The second controller is asymptotically stable, and removes any tracking error. Each controller is designed to push the system to a virtual sliding surface and then shown to be stable by the Lyapunov method. Then, the system is simulated in the presence of an inaccurate initial model estimate and an additional disturbance, and both controllers keep the system stable.

The state is given as tilt angle, forward velocity, and tilt rate. The parameters of the system are likely realistic for a vehicle the size of a Segway. The authors assume that the desired equilibrium is subject to a constraint relating the tilt angle and forward velocity are related. Since the desired velocity is an input to the system, this constraint determines desired tilt angle, which is always a constant. So, based on the single input of desired velocity the entire desired equilibrium is determined.

For the first controller, a linear combination of tracking errors defines the virtual sliding surface. The Lyapunov function V=s2/2 is implied, so V\_dot =s s\_dot. The input is designed by showing that s s\_dot \leq -k\*abs(s). Based on this equation, the equations of motion, and the bounds on estimation error, a control input is determined. The control input drives the system to the sliding surface, but the coefficients of that surface are not yet determined. Next stability of the surface is analyzed. This produces inequalities for the coefficients of the sliding surface which lead to a locally asymptotically stable equilibrium at the surface. To do this, the authors use a linearized system around a reduced state vector. This leads to asymptotic stability of the linearized system, and local exponential stability of the full system. The authors do not show that the system is driven to the surface in finite time. Because of this, the system does not go to the desired equilibrium, which is represented by the error variables on the surface all going to zero. Instead, there is some tracking error, which the authors show in the simulation section.

The second sliding mode controller is the main contribution of this work. An expanded state vector is used which includes an integral term and the velocity tracking error. The authors claim that similar to a PID controller, the integral term works to remove steady-state tracking error. The control input and stability of the surface are determined using the same method as in the first controller. Again, the stability analysis shows local exponential stability of the full system, but does not guarantee that the system is driven to the surface in finite time.

Next, both control methods are analyzed in simulation. To show that they are robust to the uncertain modeling error, the actual system simulated is significantly different than the nominal system for which the controller is designed. Additionally, measurement noise and frictional terms are treated as Gaussian variables with known mean and covariance. During the simulation, a small intermittent torque is applied to show that the controllers are robust to disturbance. The simulation is run for three different ground incline angles, which are also treated as a part of the uncertainty in the model. The addition of these disturbances and errors makes the simulation convincing for its application to a real system. However, only a constant velocity is used for each simulation. This is likely unrealistic except for short periods of time. A better assumption would be that the commanded velocity is always smooth, and that both acceleration and velocity are bounded. An interesting extension of this work would be to collect data from actual users of an MWIP system like the Segway, and use a realistic commanded velocity profile.

The results of the simulation are interesting, and not exactly what I expected. Both controllers are stable, and drive the system to a steady state in a relatively short amount of time despite the modeling errors. The first controller has some steady state error, which is dependent on the ground incline, but this steady state is achieved relatively quickly. The second controller produces no steady state error as discussed, but it produces some significant transient oscillations. These transients are also dependent on ground incline, with a downward slope producing the largest transients. Intuitively, this would make for an uncomfortable ride until these transients die out. Since the authors only simulate a constant commanded velocity, it is possible that these transients are also large for any changes in velocity. Although the theoretical analysis neglects to show that the system is driven to the sliding surface in finite time, the simulations show that the system does reach a steady state in finite time and remains stable in the presence of a disturbance.

The combination of stability analysis and simulation are a fairly strong validation of the proposed control strategy. The two most glaring issues are that the theoretical analysis only shows exponential stability, and the simulation only covers a constant demanded velocity. It appears that each side of the analysis covers the other's weakness; the theoretical analysis doesn't assume a constant velocity and the simulation shows that stability is achieved in finite time. Therefore, it seems that the controller is sufficiently validated despite these shortcomings.

The main contribution of the work is the design of a robust controller for an MWIP system. The authors claim that other attempts to solve this problem do not take into account the same uncertainty in the model and therefore in the equilibrium. Both proposed controllers are effective, although each has some drawbacks. Future work proposed by the authors is to test the controller on an actual system.