



# NAVAL POSTGRADUATE SCHOOL

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## THESIS

**ADAPTIVE CONTROL FOR FIXED WING AIRCRAFT**

by

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June 2017

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**ADAPTIVE CONTROL FOR FIXED WING AIRCRAFT**

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Submitted in partial fulfillment of the  
requirements for the degree of

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## ABSTRACT

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## List of Acronyms and Abbreviations

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<b>DoD</b>	Department of Defense
<b>NPS</b>	Naval Postgraduate School
<b>USN</b>	U.S. Navy
<b>SISO</b>	Single Input Single Output
<b>LTI</b>	Linear Time Invariant
<b>MAV</b>	Mirco Aerial Vehcile
<b>MIMO</b>	Multiple Input Multiple Output
<b>MRAC</b>	Model Reference Adaptive Control
<b>NED</b>	North East Down
<b>UAS</b>	unmanned aerial system
<b>IMU</b>	inertial measurement unit

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## Executive Summary

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This research will focus on.....

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## Acknowledgments

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# CHAPTER 1:

## Introduction and Literature Review

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This chapter outlines a brief history of adaptive control. The motivation for adaptive control applied to fixed wing aircraft can be seen in this history as well as practical applications for modern aerodynamics. Some limitations of adaptive control will be discussed and ultimately explain why the  $\mathcal{L}_1$  adaptive control architecture was chosen for this research. This will include the features specific to the  $\mathcal{L}_1$  approach which address some of these traditional limitations as well as a basic derivation as applied to a small fixed wing unmanned aerial system (UAS).

### **1.1 Adaptive Control History**

#### **1.1.1 Failures of Adaptive Control**

Failures in early adaptive control were largely impart due to a very naive understanding of robustness. As paralleled in the X-15's robustness issues, Brian Anderson concludes that "it is clear that the identification time scale needs to be faster than the plant variation time scale, else identification cannot keep up" [1].

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## CHAPTER 2:

# UAV Avionics Trends and Problem Formulation

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## CHAPTER 3:

### Adaptive Control Derivation

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This section will outline the various derivations for controllers used in this research. The controllers designed in this research was simplified assuming generic dynamics of the plant. For example, the coupling inherent in lateral fixed wing aerodynamics is assumed to be negligible. With this assumption, each controller is assumed to be Single Input Single Output (SISO) to reduce complexity.

### 3.1 Preliminaries

This thesis uses the following notation, nomenclature, and fundamental equations of motion for fixed wing rigid body aerodynamics.

#### 3.1.1 Kinematics

The following is the nomenclature that will be used to describe the kinematic equations. Euler angles for pitch ( $\theta$ ), roll ( $\phi$ ), and yaw ( $\psi$ ) will have the units of radians. The following Figure 3.1 illustrates the North East Down (NED) reference frame definitions used for body rotational rates about the  $x$  axis ( $p$ ),  $y$  axis ( $q$ ), and the  $z$  axis ( $r$ ) as well as the body velocities in the  $x$  axis ( $u$ ),  $y$  axis ( $v$ ), and the  $z$  axis ( $w$ ).

#### 3.1.2 First Order Model

The following nomenclature will be used to illustrate the modeling of first order systems where  $\dot{x}$  is the time derivative of the state,  $A$  is the Hurwitz matrix,  $B$  is the input matrix, and  $u$  is the input vector.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (3.1)$$

Often times in this thesis,  $B$  will be set equal to  $A$ . This is to ensure the DC gain is unity when desired.

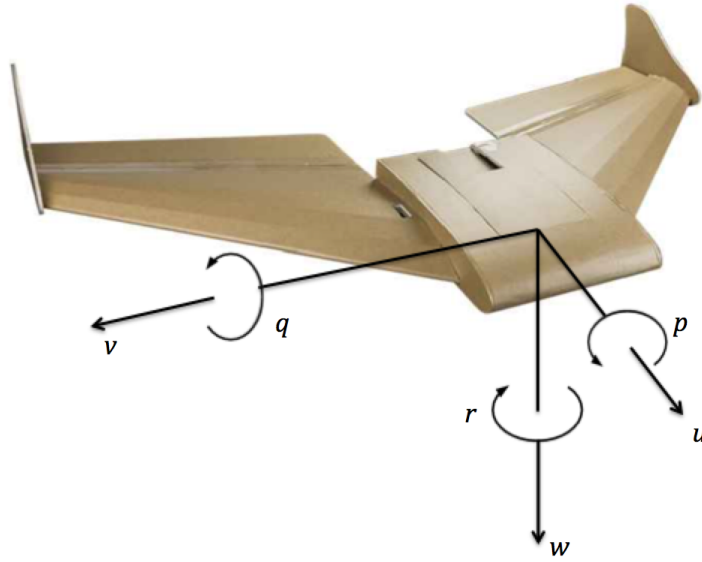


Figure 3.1. Reference frame - body rates and velocities

### 3.2 $\mathcal{L}_1$ Adaptive Control

The  $\mathcal{L}_1$  adaptive controller is a slight deviation from traditional Model Reference Adaptive Control (MRAC). MRAC control algorithms typically take two forms topologically; direct and indirect. They are similar approaches designed to model a Linear Time Invariant (LTI) system with unknown constant parameters. The desired outcome is to ensure the error between the plant (system) and the system model (state predictor) asymptotically approaches zero. These architectures can be seen below in Figures 3.2 and 3.3.

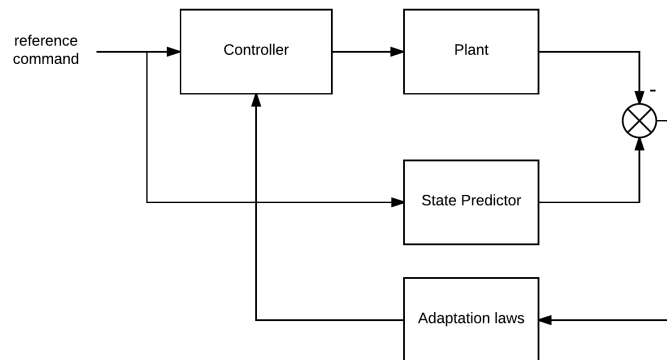


Figure 3.2. Direct MRAC architecture



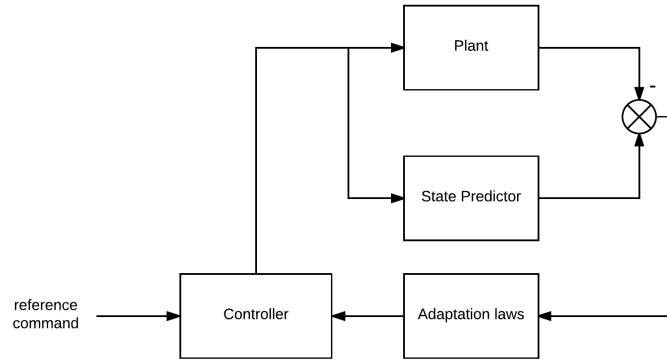


Figure 3.3. Indirect MRAC architecture

The  $\mathcal{L}_1$  adaptive control algorithm asserts that trying to estimate the plant uncertainties outside of the control actuators' bandwidth is overly ambitious. The system's actuator bandwidth is most commonly the system's limiting factor, and the estimator's robustness/stability could be in question if un-modeled high frequency content exists in the plant. The  $\mathcal{L}_1$  adaptive control constrains the objective function by using a low-pass filter (first or second order) to band the frequency response in order to meet robustness specifications. This low-pass filter should be tuned to a frequency response commensurate with the actuator's frequency response. When looking at examples of where to place the low-pass filter in the direct and indirect architectures, it becomes clear that the indirect architecture is the only candidate. Figures 3.4 and 3.5 illustrate the placement of the low-pass filter and its implication on the closed loop model.

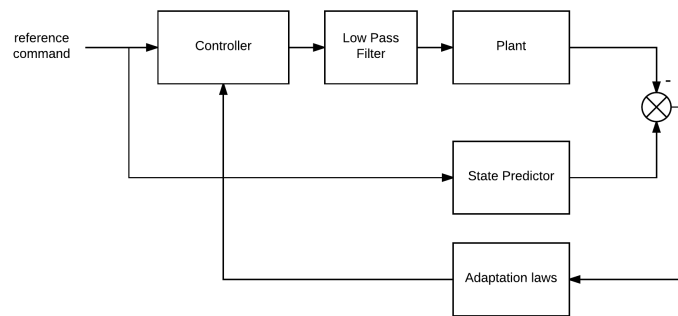


Figure 3.4. Direct MRAC architecture with low-pass filter

It can be seen that the low-pass filter in the direct architecture inherently changes the

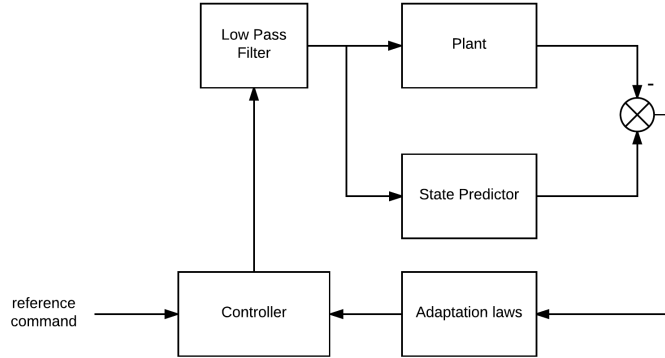


Figure 3.5. Indirect MRAC architecture with low-pass filter

structure of the model with the cascading of the low-pass filter and plant block diagrams. This change mathematically is not mirrored in the state predictor and therefore is not subtractable. However, in the indirect case, the structure of the model is kept intact and the low-pass filter is applied to both the plant and the state predictor. This ensures that the low-pass filter is subtractable when calculating the error state and the model's structure is kept intact.

In the primary literature for this research [2], the author often refers to the state predictor as the reference model or companion model for the direct and indirect architectures respectively. The reference model (direct architecture) intuitively maps the desired model response to the error feedback. In the indirect architecture case, the error state is a result of the companion model plus the low-pass filter. This subtle distinction is necessary because it must be accounted for when tuning the companion model with the included low-pass filter.

Many slight variations of the  $\mathcal{L}_1$  adaptive architectures have been derived for various use cases [2]. Some of the following forms were studied for viability in the fixed wing UAS use case:

- SISO with constant but unknown state parameters
- SISO with time variant and/or nonlinear unknown state parameters
- Multiple Input Multiple Output (MIMO) with constant but unknown state parameters
- MIMO with time variant and/or nonlinear unknown state parameters

MIMO control algorithms would potentially afford the controller more ability to cope with

system coupling if present. A fixed wing UAS would exhibit coupled behavior due to the coupling present in the aerodynamics but was not chosen due to the added architectural complexity. Unknown state parameters that are assumed to be constant or time invariant are considered matched uncertainty. Unknown state parameters that are non-constant (time variant) and/or exhibit non-linear behavior are considered unmatched uncertainty. The unmatched uncertainty architecture offers a more appealing solution for fixed wing use cases (asymmetric actuator failure, aerodynamic coefficients scaled by dynamic pressure, etc.), but adds a significant amount of complexity to the architecture. In summary, the SISO architecture with matched uncertainty was chosen for this research.

The SISO controller with matched uncertainty was chosen to control pitch rate ( $q$ ) and roll rate ( $p$ ) of the aircraft using two separate but parallel controllers. This meant that the controller could be generalized to a first principles physical point mass model similar to derivations found in rigid body equations of motion. In this implementation of the  $\mathcal{L}_1$  adaptive controller, the desired state  $x$  to be controlled was an individual body rate (e.g.,  $q$ ,  $p$ ).

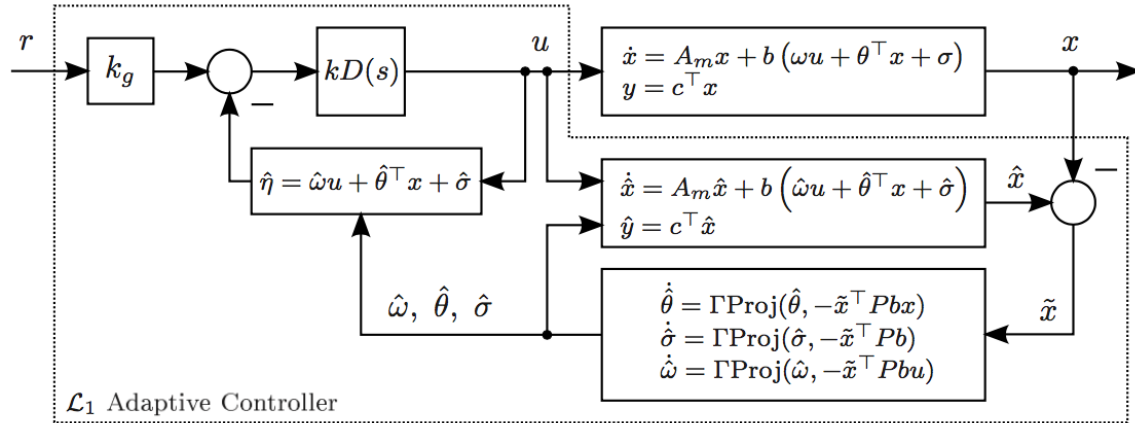


Figure 3.6.  $\mathcal{L}_1$  Architecture with Matched Uncertainty Block Diagram [2]

As seen in Figure 3.6, the generalized  $\mathcal{L}_1$  architecture in block diagram form and the following elements can be identified:

$k_g$  - feed forward input gain

$kD(s)$  - user described filter (second order low pass plus integrator)

$\hat{\eta}$  -  $\mathcal{L}_1$  controller state

$\dot{x}$  - first order differential equation of state model  
 $\hat{x}$  - state estimate  
 $\tilde{x}$  - state error  
 $u$  - reference objective  
 $A_m$  - Hurwitz matrix  
 $b$  - input matrix  
 $\hat{\omega}$  - unknown input gain coefficient  
 $\hat{\theta}$  - unknown constant state coefficient  
 $\hat{\sigma}$  - unknown disturbance estimate  
 $\Gamma$  - adaptation gain  
 $Pb$  - solution to the Lyapunov stability criterion

It should also be noted that the architecture presented in Figure 3.6 includes the use of a projection operator. The parameters for  $\hat{\omega}$ ,  $\hat{\theta}$ , and  $\hat{\sigma}$  are all projection based adaptation laws. This simply ensures that the adaptation stays bounded around the feasible region of parameter space. The Lyapunov stability proofs for this architecture rely on this method to guarantee stability [2] . More discussion on the specific application of this operator can be found in Appendix [???].

One of the main benefits of using the SISO architecture is that the solution to the Lyapunov stability criterion ( $Pb$ ) used in the projection based adaptation laws is greatly simplified.

In this case,  $Pb$  reduces to:

$$Pb = \frac{1}{2\omega_n} \quad (3.2)$$

where  $\omega_n$  is the natural frequency in rad/s for the companion model in discrete recursive form assuming DC gain of 1.

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## CHAPTER 4:

### Design of Experimental Platform

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The aircraft used for this research was the Flitetest Spear [?]. The Spear airframe was chosen for its endurance capability of greater than 45 minutes of flight time and its large capacity fuselage. The flying-wing architecture keeps the actuation requirement to a minimum of two servos by utilizing an elevon configuration.



Figure 4.1. Spear Airframe

The large blunt nose provides adequate space for two 2,200 mAh (12.6volts) lithium polymer batteries wired in parallel. The remaining cargo space was used for accommodating the Pixhawk autopilot.



Figure 4.2. Spear Cargo Capacity

This plane was constructed out of craft foam board. The plans were downloaded from flitetest.com [?] and converted to CorelDraw vector files for use in a laser cutter. These files

were then cut out of four sheets of foam board using the laser cutter. The wing halves were joined with standard box tape and hot-glue. This provided a cheap and rapid construction process which was achievable under four hours of build time.

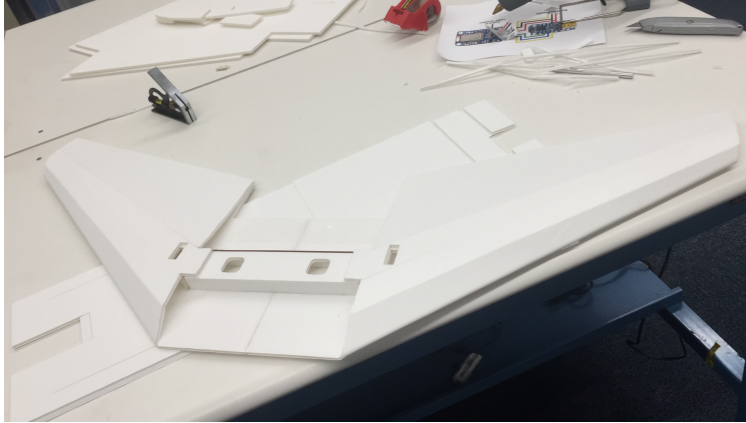


Figure 4.3. Spear Build Process

The aircraft specifications are as follows:

- Weight without battery: 1.45 lbs (658 g)
- Center of gravity: 3 – 3.5" (76 – 89 mm) in front of firewall
- Control surface throws: 16° deflection – Expo 30%
- Wingspan: 41 inches (1041 mm)
- Recommended motor: 425 sized 1200 kv minimum
- Recommended prop: 9 x 4.5 CW (reverse) prop
- Recommended ESC: 30 amp minimum
- Recommended Battery: (2) 2200 mAH 12.6 volt minimum
- Recommended Servos: (2) 9 gram servos

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## CHAPTER 5:

### Flight Testing and Performance Evaluation

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## CHAPTER 6:

### Recommendation

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## CHAPTER 7:

### Conclusion

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## List of References

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- [1] B. D. Anderson *et al.*, “Failures of adaptive control theory and their resolution,” *Communications in Information & Systems*, vol. 5, no. 1, pp. 1–20, 2005.
- [2] N. Hovakimyan and C. Cao, *L1 adaptive control theory: guaranteed robustness with fast adaptation*. Siam, 2010, vol. 21.

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