University of Zurich
Department of Banking and Finance

**Bachelor Thesis**

# Modeling Conditional Betas with Application in Asset Allocation

**Author**

Rino R. Beeli

Freilagerstrasse 92

8047 Zurich

rino.beeli@uzh.ch

15-709-371

**Supervisor**

Prof. Dr. Marc S. Paolella

Department of Banking and Finance

University of Zurich

January 26, 2019

## Abstract

Empirical evidence shows that market betas vary substantially over time, hence time-varying beta models are of interest in the field of financial modeling. This thesis reexamines the findings of Bali et al. (2017) of a positive link between the dynamic conditional beta and the cross section of daily stock returns. Their investment strategy takes a long position in stocks in the highest beta decile and a short position in stocks in the lowest beta decile, and produces average returns and alphas in the range of 0.60%–0.80% per month. We are able to replicate their findings based on the DCC-GARCH construct and show that the value-weighted High-Low difference portfolio yields even higher monthly excess returns and alphas in the range of 1.7%–1.9% on our sample data. Replacing DCC-GARCH with the so-called COMFORT model, which is statistically more advanced and accounts for major stylized facts of financial asset returns, does not increase performance, nor does it result in lower portfolio risk due to model estimation errors and a longer estimation window.

## Executive Summary

The beta measure was introduced by the renowned Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965). It is a cornerstone in asset pricing theory and still widely used in practice, despite earning a lot of critique because of its poor empirical performance. The measure is the sensitivity of the security returns to changes in the market portfolio returns. The economic intuition behind the CAPM is attractive and easily understood, but Fama and French (2004) showcase poor empirical performance of the model compared to the expectations of the theoretical framework.

More advanced estimation methods which account for heteroscedasticity, volatility-clustering and time-dependency exist, e.g. GARCH-based models. Bali, Engle, and Tang (2017) estimate stock betas using a time-varying conditional correlation model in conjunction with a GARCH model, also called DCC-GARCH, which was introduced by Engle (2002).

Bali et al. (2017) find a significant positive relation between conditional betas and the cross section of daily stock returns using the DCC-GARCH model for beta estimation. They estimate betas for all stocks in their sample for each day using a moving window. Based on the estimated betas, the cross-section of stock returns is predicted using beta deciles. They show that stocks in the lowest beta decile have significantly lower returns on the next trading day compared to stocks in the highest beta decile. The difference portfolio of Bali et al. (2017), which takes a long position in stocks in the highest beta decile and a short position in stocks in the lowest beta decile, produces average returns and alphas in the range of 0.60%–0.80% per month. The strategy was backtested using all U.S.-based common stocks trading on the NYSE, AMEX, and NASDAQ exchanges with a stock price of $5/share or more and with a market capitalization greater than $10 million. The sample period starts in July 1963 and ends in December 2013.

The replication of this finding is the first objective of this thesis. We are able to replicate their findings with our sample data based on the DCC-GARCH construct and show that the value-weighted High-Low beta portfolio yields even higher monthly excess returns and alphas in the range of 1.7%–1.9%. After incorporating the estimated transaction costs of 35 basis points, the strategy yields monthly excess returns and alphas in the range of 1.35%–1.55%, which is almost twice the monthly excess return Bali et al. (2017) achieved in their study. We use a more recent time period and a different sample to test the strategy. Our sample covers all S&P 500 constituents and covers the period from January 1996 to December 2013. Additionally, we used an exponential moving average on the covariance estimates, which increased performance and lowered the portfolio turnover rate, allowing the strategy to be implemented at lower costs.

The second objective of this thesis is to replace the DCC-GARCH construct used by Bali et al. (2017) with the so-called COMFORT model developed by Paolella and Polak (2015a). The more realistic, statistically advanced COMFORT model allows to model all major stylized facts of financial returns, including volatility clustering, dynamics in the dependency structure, asymmetry, and heavy tails. It supports various extensions which account for time-varying correlation dynamics and a hybrid GARCH-SV extension for modeling shocks across assets, which are an additional source of dynamics in the correlations. The COMFORT-DCC model used throughout this thesis uses a fat-tailed, multivariate asymmetric Laplace distribution (MALap) in conjunction with the Dynamic Conditional Correlation model of Engle and Sheppard (2001) for the correlation dynamics; the hybrid GARCH-SV extension is not employed. The superior capabilities of the COMFORT-DCC model should enable it to outperform the DCC-GARCH model in terms of more significant beta estimates and a significantly better performance of the investment strategy, but the empirical results show that neither does the performance increase, nor do we find lower portfolio risk.

There are various reasons why the COMFORT-DCC model does not outperform the DCC-GARCH construct in our setting. Covariance estimation errors impair our High-Low portfolio analysis, because reliable and consistent beta estimates are crucial for our study—any extreme values due to estimation errors directly bias our results. Our analysis shows that especially the highest beta values are prone to estimation errors and therefore worsen the performance of the High-Low portfolio.

Also, the COMFORT-DCC model was estimated each day with a window size of 1000 days, which is approximately four times bigger than the window size used by Bali et al. (2017). The DCC-GARCH model with a window size of 1000 days performs still better than the COMFORT-DCC model, leading to the conclusion that the window size alone is not the deciding factor for the worse performance. Furthermore, the bivariate estimation method introduces an additional degree of freedom compared to a full-sample estimation approach for the mixing factor, because the model assumes that the mixing factor estimates used in COMFORT-DCC are the same for all asset returns. This additional degree of freedom likely introduces less precise/more random COMFORT-DCC model estimates.

The replication findings show promising results regarding the investment strategy used by Bali et al. (2017), it outperforms the market and has higher risk-adjusted returns. Further studies of the COMFORT-DCC model with more reliable model estimates, different model extensions, and different estimation approaches are required in order to outperform the DCC-GARCH construct in our setting. In case of success, the resulting investment strategy would be highly interesting to investors.

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# 1 Introduction

The beta measure was introduced by the renowned Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965). It is a cornerstone in asset pricing theory and still widely used in practice, despite earning a lot of critique because of its poor empirical performance. The measure is the sensitivity of the security returns to changes in the market portfolio returns. The traditional estimation method uses ordinary least squares (OLS) for estimating a linear regression of security excess returns against market excess returns, resulting in an unconditional beta coefficient. The economic intuition behind the CAPM is attractive and easily understood, but Fama and French (2004) showcase poor empirical performance of the model compared to the expectations of the theoretical framework.

More advanced estimation methods which account for heteroscedasticity, volatility-clustering and time-dependency exist, e.g. GARCH-based models. Bali et al. (2017) estimate stock betas using a time-varying conditional correlation model in conjunction with a GARCH model, also called DCC-GARCH, which was introduced by Engle (2002). It is supposed to have a significant positive relation to the cross section of daily stock returns. A market neutral investment strategy taking a long position in stocks of the highest conditional beta decile and a short position in stocks of the lowest conditional beta decile is supposed to yield monthly excess returns and alphas in the range of 0.60% to 0.80%.

Paolella and Polak (2015a) developed the more realistic, statistically advanced COMFORT model, which allows to model all major stylized facts of financial returns, including volatility clustering, dynamics in the dependency structure, asymmetry, and heavy tails. It supports various extensions which account for time-varying correlation dynamics and a hybrid GARCH-SV extension for modeling shocks across assets, which are an additional source of dynamics in the correlations.

This thesis replicates in a first step the empirical study conducted by Bali et al. (2017). We use their Gaussian DCC-GARCH construct with daily stock returns data and focus on Long-Short strategy returns for comparison. In a second step, the DCC-GARCH construct is replaced with the COMFORT model. The COMFORT-DCC model used throughout this thesis uses a fat-tailed, multivariate asymmetric Laplace distribution (MALap) in conjunction with the Dynamic Conditional Correlation model of Engle and Sheppard (2001) for the correlation dynamics; the hybrid GARCH-SV extension is not employed. The ability of the COMFORT-DCC model to capture more stylized facts of financial asset returns compared to the DCC-GARCH model gives us the hope and expectation of more significant beta estimates and a significantly better performance of the investment strategy, namely lower risk with equal and/or higher returns.

We estimate stock betas for all S&P 500 index constituents for each day using a moving window. The sample data covers the period from January 1996 to December 2013. Based on the estimated betas, we predict the cross-section of stock returns and analyze the findings based on decile portfolios. Stocks in the lowest beta decile are expected to have significantly lower returns on the next trading day compared to stocks in the highest beta decile. This hypothesis is tested using a High-Low difference portfolio consisting of a long position in the highest-beta decile and a short position in the lowest-beta decile. Moreover, these portfolios serve as a vehicle to compare the performance of the DCC-GARCH and COMFORT-DCC model, which is the second objective of this thesis.

The remainder of this thesis is structured as follows. Section 2 provides general definitions for unconditional and conditional beta measures. Section 3 presents the methodology used to conduct this study. Section 4 states model specifications for the CAPM, DCC-GARCH and COMFORT-DCC models, including remarks on the estimation procedure. Section 5 presents the findings of our empirical study—it shows the results of our replication of Bali et al. (2017), and it compares the performance of DCC-GARCH with COMFORT-DCC. Section 6 concludes the findings and provides possible explanations and context thereof. The appendix contains useful statistics and figures of our empirical findings; model estimation runtimes for DCC-GARCH and COMFORT-DCC are provided.

## 2 Definitions

### 2.1 Beta Measure

The beta measure ($\beta$) creates a linear relationship between the expected returns of a stock and the expected market portfolio returns. The resulting measure is the sensitivity of the stock returns to changes in market portfolio returns. The beta of the market is 1 by definition. If a stock moves on average less than the market typically does, the stock's beta will be smaller than 1. On the other hand, if a stock moves more than the market typically does, the stock's beta will be greater than 1. In other words, the higher the beta, the higher the volatility of the corresponding stock.

### 2.2 Unconditional Beta

The unconditional CAPM beta creates a linear relationship of the form

$$\mathrm{E}[R_i] - \mathrm{E}[R_f] = \beta_i(\mathrm{E}[R_m] - \mathrm{E}[R_f]), \tag{1}$$

where $\mathrm{E}[R_i]$ denotes the expected return of the asset $i$, $\mathrm{E}[R_f]$ – the expected risk-free rate with a beta of zero by definition, and $\mathrm{E}[R_m]$ – the expected market portfolio return. Unconditional in this context means that the CAPM beta does not vary within the estimation period, it is constant.

### 2.3 Conditional Beta

The assumption of a firm having a constant beta over the estimation period seems implausible because its cash flow risks and exposure to market risks are likely to vary over time. A conditional beta measure incorporates the information available at each given day $t$ and should therefore yield more reliable and significant results compared to the unconditional (CAPM) beta:

$$\mathrm{E}[R_{i,t+1} - R_{f,t+1} \mid \Omega_t] = \mathrm{E}[\beta_{i,t+1} \mid \Omega_t] \cdot \mathrm{E}[R_{m,t+1} - R_{f,t+1} \mid \Omega_t] \tag{2}$$

where $\Omega_t$ denotes the information set available at time $t$ about future returns and betas, $\mathrm{E}[R_{i,t+1} - R_{f,t+1} \mid \Omega_t]$ and $\mathrm{E}[R_{m,t+1} - R_{f,t+1} \mid \Omega_t]$ are the expected excess returns of the risky asset $i$ and the market portfolio at time $t+1$, conditional on the information set at time $t$. $\mathrm{E}[\beta_{i,t+1} \mid \Omega_t]$ is the expected conditional beta measure of asset $i$ at time $t$, conditional on the information set available at $t$, given by

$$\mathrm{E}[\beta_{i,t+1} \mid \Omega_t] = \frac{\mathrm{Cov}[R_{i,t+1} - R_{f,t+1}, R_{m,t+1} - R_{f,t+1} \mid \Omega_t]}{\mathrm{Var}[R_{m,t+1} - R_{f,t+1} \mid \Omega_t]}. \tag{3}$$

# 3 Methodology

## 3.1 Data Sample

The sample dataset includes all constituents of the S&P 500 stock market index. The time period used by Bali et al. (2017) ranges from July 1963 to December 2013, but we use a shorter time period ranging from January 1996 to December 2013 because of constraints in computation time. The WRDS API data query is provided in Appendix A.4.1. The period covers bear as well as bull markets and the financial crisis of 2008, which is especially important for testing the model's ability to incorporate shocks into (co-)variance predictions. In accordance with Bali et al. (2017), daily stock returns including dividends of all S&P 500 constituents were obtained from the Center for Research in Security Prices (CRSP) database. The daily market returns and risk-free rates used for excess-returns calculation were extracted from Kenneth French's Data Library (French (2018)).

## 3.2 Returns Calculation

A return is the change in the total value of a security over some period of time per unit of initial investment. $R_t$ is the return for a sale on day $t$ at the last tradable price $P_t$. It is based on a purchase at the most recent time previous to $t$ when the security had a valid price $P_{t-1}$. The return incorporates all incurred adjustments relevant for the value of the security, e.g. stock dividends and other price adjustments like stock splits. We use returns provided by CRSP, which are given by

$$R_t = \frac{P_t \cdot f(t) + d(t)}{P_{t-1}} - 1, \tag{4}$$

where $P_t$ denotes the current price of the stock, $P_{t-1}$ – the previous time period price of the stock, $f(t)$ represents a price adjustment factor at time $t$, and $d(t)$ denotes cash adjustments at time $t$. It is important to realize that calculating returns based on prices only without accounting for necessary adjustments like dividends or stock splits can cause jumps in the price time series, which ultimately induces a bias into further calculations.

If a company gets delisted on day $t$, we incorporate the delisting return $R_t^{\text{del}}$ into its last trading day return $R_t$ via a compounded return calculation given by

$$R_t = (1 + R_t)(1 + R_t^{\text{del}}) - 1. \tag{5}$$

Conforming to Bali et al. (2017), excess returns are used for model estimation. The excess return $Z_t$ at day $t$ for a stock is defined as the difference between absolute returns $R_t$ and the risk-free rate $R_{f,t}$ (Bali, Engle, and Murray (2016)):

$$Z_t = R_t - R_{f,t}. \tag{6}$$

Daily risk-free rates were extracted from Kenneth R. French's Data Library (French (2018)).

### 3.2.1 Avoiding Delisting Bias in CRSP Data

Following Bali et al. (2017) and Shumway (1997), delisting returns $R_t^{\text{del}}$ of all delisted S&P 500 constituents within our CRSP sample are adjusted in order to avoid survivorship bias and to obtain realistic backtest results. According to Shumway (1997), many CRSP delisting returns are too low or even missing, leading to returns higher than actually attainable. Investors cannot reliably anticipate the delisting of a company, therefore delisting returns need to be taken into account (Bali et al. (2016)).

If provided by CRSP, their delisting return is used. If no delisting return is provided, the following rules apply:

1. A delisting return of $R_t^{\text{del}} = -30\%$ applies to the following delisting codes: 500 (reason unavailable), 520 (went to over-the-counter), 551-573, 580 (various reasons), 574 (bankruptcy), or 584 (does not meet exchange financial guidelines).

2. For all codes not mentioned above, the delisting return is $R_t^{\text{del}} = -100\%$ (total loss).

Applying this ruleset to the S&P 500 sample from January 1996 to December 2013 yields 8 corrections which result in a $-30\%$ delisting return and no corrections for the $-100\%$ (total loss) delisting return case.

### 3.2.2 Simple Returns vs. Logarithmic Returns

Logarithmic returns are commonly used by practitioners in finance because of attractive properties like time additivity, log-normality, stable distribution, approximate raw-log equality, etc., but these advantages are less pronounced in shorter time periods, e.g. daily- or hourly time spans (Hudson and Gregoriou (2015)); we use daily returns throughout this thesis. In contrast to log-returns, simple returns possess the linear additivity property across portfolio components.

The paper of Bali et al. (2017) uses the equity beta in the context of the Capital Asset Pricing Model (see section 2.2), which creates a linear relationship between excess returns and risk. The derivation of the CAPM is based on expected portfolio returns formed as the weighted average of asset returns, implying linear additivity of portfolio components. Log-returns are unsuitable in this context because they are not linearly additive and, furthermore, beta estimates based on log-returns systematically differ from those calculated using simple returns (Hudson and Gregoriou (2015)). In conclusion, we choose simple returns over log-returns for our analysis because it's the appropriate measure in the CAPM context.

### 3.2.3 Optimization Convergence Issues

Practical tests highlight the importance of scaling returns data used for model estimation. The optimization algorithms used for DCC-GARCH and COMFORT-DCC model estimation run into convergence issues without scaling. Using percentage excess returns $Z_t^*$ instead of simple excess returns $Z_t$ circumvents this issue, which is a consequence of limited precision floating point data types used by numerical optimization algorithms. We scale the simple excess returns $Z_t$ by a factor of 100 to obtain percentage returns, given by

$$Z_t^* = 100 \cdot Z_t. \tag{7}$$

This does not affect our beta-estimate $\hat{\beta}_i$, which is equivalent to the percentage-returns beta-estimate $\hat{\beta}_i^*$ due to basic variance properties:

$$\hat{\beta}_i^* = \frac{\mathrm{Cov}(100Z_m, 100Z_i)}{\mathrm{Var}(100Z_m)} = \frac{100^2\mathrm{Cov}(Z_m, Z_i)}{100^2\mathrm{Var}(Z_m)} = \frac{\mathrm{Cov}(Z_m, Z_i)}{\mathrm{Var}(Z_m)} \equiv \hat{\beta}_i. \tag{8}$$

## 3.3 Portfolio Returns Calculation

This section defines how the average returns of equal-weighted, value-weighted and High-Low portfolios are calculated, and corresponds to Bali et al. (2016).

### 3.3.1 Equal-Weighted Portfolio Returns

The average equal-weighted portfolio return $R_{pf,d}^{\mathrm{eq}}$ at day $d$ is simply the arithmetic mean of all returns of the portfolio components:

$$R_{pf,d}^{\mathrm{eq}} = \frac{1}{N}\sum_{i=1}^{N} R_{i,d}, \tag{9}$$

where $N$ denotes the number of portfolio components, and $R_{i,d}$ denotes the return of the portfolio component $i$ at day $d$.

### 3.3.2 Value-Weighted Portfolio Returns

The average value-weighted portfolio return $R_{pf,d}^{\mathrm{vw}}$ at day $d$ uses the market capitalization of the firm as weighting:

$$R_{pf,d}^{\mathrm{vw}} = \frac{\sum_{i=1}^{N} V_{i,d-1} \cdot R_{i,d}}{\sum_{i=1}^{N} V_{i,d-1}}, \tag{10}$$

where $N$ denotes the number of portfolio components, $R_{i,d}$ denotes the return of the portfolio component $i$ at day $d$, and $V_{i,d-1}$ denotes the market capitalization of firm $i$ at day $d-1$.

### 3.3.3 High-Low Beta Portfolio Returns

The High-Low beta portfolio return $R_{pf,d}^{\text{hl}}$ at day $d$ is the equal-weighted average return of going long in the highest-beta decile portfolio and short in the lowest-beta decile portfolio:

$$R_{pf,d}^{\text{hl}} = R_{pf,d}^{\text{highest}} - R_{pf,d}^{\text{lowest}}, \tag{11}$$

where $R_{pf,d}^{\text{highest}}$ denotes the average return of the highest-beta decile portfolio, and $R_{pf,d}^{\text{lowest}}$ denotes the average return of the lowest-beta portfolio. Note that the decile portfolios can either be equal- or value-weighted.

## 3.4 Covariance Smoothing

Our findings for the DCC-GARCH and COMFORT-DCC models are based on smoothed (co-)variances in order to reduce the effects of noise and numerical optimization errors (e.g. local optima). We use an Exponentially Weighted Moving Average (EWMA) to smooth (co-)variance estimates, given by

$$S_t = \begin{cases} Y_1, & t = 1 \\ \alpha Y_t + (1-\alpha) S_{t-1}, & t > 1 \end{cases} \tag{12}$$

where $\alpha = 2/(n+1)$ denotes the degree of weighting decrease for decreasing $t$, $Y_t$ denotes the estimated covariance at day $t$ and $n$ denotes the window size (look-back time steps). The use of an EWMA does not introduce a look-ahead bias, the value calculated at day $t$ only uses data within the sample window ranging from day $t - n + 1$ to day $t$.

We choose $n$ so that it maximizes risk-adjusted High-Low portfolio returns for the DCC-GARCH and COMFORT-DCC model. The Sharpe ratio is used as maximization criteria, given by

$$SR = \frac{\mathrm{E}[R_{pf} - R_f]}{\sqrt{\mathrm{Var}[R_{pf} - R_f]}}. \tag{13}$$

Maximizing the Sharpe ratio yields a window size of $n = 7$.

# 4 Model Specifications

## 4.1 CAPM

### 4.1.1 Definition

Considering a time series linear regression, the CAPM model takes the form given by

$$Z_{i,t} = \alpha_i + \beta_i \; Z_{m,t} + \epsilon_{i,t}, \tag{14}$$

where $Z_{i,t}$ denotes the excess return of stock $i$ at day $t$, $\alpha_i$ denotes the constant linear regression intercept, $\beta_i$ – the unconditional beta measure of stock $i$, $Z_{m,t}$ – the market excess return, and $\epsilon_{i,t}$ is the error term.

### 4.1.2 Estimation

For every day, we estimate the beta for each stock using a moving window approach with the past 252 trading days' data by regressing stock excess returns onto market excess returns.

Ordinary least squares (OLS) method is used for estimating the beta coefficient. According to the Gauss-Markov theorem, the OLS estimator is the best linear unbiased estimator given the OLS assumptions are fulfilled, which we assume to hold because of the stated CAPM model definition.

## 4.2 DCC-GARCH

Bali et al. (2017) use the Dynamic Conditional Correlation (DCC) model introduced by Engle and Sheppard (2001) to estimate the covariance between each stock and the market portfolio returns for their main findings. It belongs to the class of models with conditional variances and conditional correlations, hence time-varying. The idea behind DCC is that the covariance matrix can be decomposed into conditional standard deviations and conditional correlations. DCC uses a two-step estimation procedure. In the first step, the conditional variance for each asset is estimated via a univariate GARCH model. In the second step, the residuals from the univariate models are used to estimate the time-varying correlation matrix. This approach is computationally more efficient compared to multivariate GARCH approaches, because the number of parameters in the correlation process subject to estimation is independent of the number of series to be correlated. Thus, potentially large correlation matrices can be estimated, making the DCC-GARCH model highly attractive to practitioners in finance.

### 4.2.1 Definition

The following definitions closely follow Bali et al. (2017). The stationary excess return time series are given by

$$R_{i,d+1} - R_{f,d+1} = \alpha_0^i + \epsilon_{i,d+1} = \alpha_0^i + \sigma_{i,d+1} \cdot u_{i,d+1} \tag{15}$$

$$R_{m,d+1} - R_{f,d+1} = \alpha_0^m + \epsilon_{m,d+1} = \alpha_0^m + \sigma_{m,d+1} \cdot u_{m,d+1}, \tag{16}$$

where $R_{i,d+1} - R_{f,d+1}$ and $R_{m,d+1} - R_{f,d+1}$ denote the excess return at day $(d+1)$ of stock $i$ and the market portfolio, respectively, and $u_{i,d+1} = \epsilon_{i,d+1}/\sigma_{i,d+1}$ and $u_{m,d+1} = \epsilon_{m,d+1}/\sigma_{m,d+1}$ denote the standardized residuals for stock $i$ and the market portfolio, respectively.

The variances $\sigma^2_{i,d+1}$ and $\sigma^2_{m,d+1}$ of the excess return time series are modeled using univariate GARCH(1,1) specifications and equal the squared error terms $\epsilon^2_{i,d+1}$ and $\epsilon^2_{m,d+1}$, because their mean is zero:

$$\mathrm{E}[\epsilon^2_{i,d+1} \mid \Omega_d] \equiv \sigma^2_{i,d+1} = \beta_0^i + \beta_1^i \sigma^2_{i,d} u^2_{i,d} + \beta_2^i \sigma^2_{i,d} \tag{17}$$

$$\mathrm{E}[\epsilon^2_{m,d+1} \mid \Omega_d] \equiv \sigma^2_{m,d+1} = \beta_0^m + \beta_1^m \sigma^2_{m,d} u^2_{m,d} + \beta_2^m \sigma^2_{m,d}, \tag{18}$$

where $\sigma^2_{i,d+1}$ denotes the expected conditional variance of stock $i$ at day $d$, and $\sigma^2_{m,d+1}$ denotes the expected conditional variance of the market portfolio at day $d$.

The Dynamic Conditional Correlation model uses Pearson's correlation coefficient, conditioned on the information set $\Omega_d$ available at day $d$ (Engle (2002)). Thus, the covariance $\sigma^2_{im,d+1}$ between stock $i$ and the market portfolio excess returns is given by

$$\mathrm{E}[\epsilon_{i,d+1}\epsilon_{m,d+1} \mid \Omega_d] \equiv \sigma_{im,d+1} = \rho_{im,d+1} \cdot \sigma_{i,d+1} \cdot \sigma_{m,d+1}, \tag{19}$$

with

$$\begin{aligned}
\rho_{im,d+1} &= \frac{q_{im,d+1}}{\sqrt{q_{ii,d+1} \cdot q_{mm,d+1}}} \\
q_{im,d+1} &= \overline{\rho}_{im} + a_1(u_{i,d} \cdot u_{m,d} - \overline{\rho}_{im}) + a_2(q_{im,d} - \overline{\rho}_{im}) \\
q_{ii,d+1} &= (1 - a_1 - a_2) + a_1 \cdot u_{i,d}^2 + a_2 \cdot q_{ii,d} \\
q_{mm,d+1} &= (1 - a_1 - a_2) + a_1 \cdot u_{m,d}^2 + a_2 \cdot q_{mm,d},
\end{aligned} \tag{20}$$

where $\rho_{im,d+1}$ denotes the expected conditional correlation, $\overline{\rho}_{im}$ denotes the unconditional correlation, $q_{im,d+1}$ denotes the covariance, $q_{ii,d+1}$ and $q_{mm,d+1}$ denote the variance of stock $i$ and the market portfolio, respectively, and $a_1$ and $a_2$ are GARCH(1,1) model parameters of the correlation process.

#### 4.2.2 DCC Beta

The DCC beta, analogously to the unconditional CAPM beta, is given by the ratio of the variables defined in (19) and (18):

$$\beta_{i,d+1}^{\text{DCC}} = \frac{\sigma_{im,d+1}}{\sigma_{m,d+1}^2}. \tag{21}$$

#### 4.2.3 Estimation

Bali et al. (2017) use a moving window approach to estimate the parameters of the DCC-GARCH model. For every day within the sample period, they estimate the model parameters based on the past 252 trading days' data and forecast the 1-step ahead covariance matrix, but they do not specify how the variance of the market and the covariance of each stock and the market are obtained. An estimation of the whole window of size 252x500 (252 days, 500 index components) is not feasible without the use of special estimation methods like shrinkage estimation, because the covariance matrix becomes singular, i.e. it cannot be inverted to retrieve the precision matrix ($n < p$).

Consequently, we choose a bivariate estimation, i.e. we estimate the covariance matrix between each stock and the market portfolio with a window size of 252 days.

Bali et al. (2017) use correlation targeting to ease optimization convergence with the parameters $a_1$ and $a_2$, and assume time-varying correlations mean reverting to the unconditional sample correlation $\overline{\rho}_{im}$, which requires $a_1 + a_2 < 1$ to hold.[1]

In order to compare the DCC-GARCH and COMFORT-DCC models better, we also estimate the DCC-GARCH model using a window size of 1000 days.

## 4.3 COMFORT-DCC

The COMFORT-DCC model allows to model various stylized facts of financial returns, including volatility clustering, dynamics in the dependency structure, and the asymmetry and heavy tails of financial returns distributions. The hybrid GARCH-SV extension allows to model shocks across assets, which are an additional source of dynamics in the correlations.

### 4.3.1 Definition

A definition of the COMFORT model, which allows the dependency matrix to vary over time (DCC), is described in Paolella and Polak (2015b). The COMFORT-DCC setting in this thesis uses the fat-tailed, multivariate asymmetric Laplace distribution (MALap) in conjunction with the DCC model of Engle and Sheppard (2001) for the correlation dynamics. The multivariate asymmetric Laplace (MALap) distribution is a limiting case of the multivariate generalized hyperbolic (MGHyp) distribution, see McNeil, Frey, and Embrechts (2015) and Kotz, Kozubowski, and Podgorski (2001). We do not employ the hybrid GARCH-SV extension, since it would not provide a notable benefit in our bivariate estimation setting.

### 4.3.2 Estimation

The EM-algorithm used for estimating our COMFORT-DCC model is described in the Appendix 7.1 of Paolella and Polak (2015b).

[1] Maximum Likelihood Estimation of the DCC-GARCH model is described in Appendix I of *Bali etal.*(2017).

# 5 Empirical Results

In this section, we examine the cross-sectional relation between unconditional beta, dynamic conditional beta (DCC), COMFORT-DCC beta (CDCC) and daily stock returns; detailed statistics and performance measures are provided. We compare our results with the results of Bali et al. (2017) and evaluate if the COMFORT-DCC model outperforms the DCC-GARCH model in terms of higher portfolio returns and lower portfolio risk.

## 5.1 Univariate Portfolio Analysis

A Univariate Portfolio Analysis approach as described in Bali et al. (2016) is used to analyze the empirical data. For each day, we estimate the CAPM, DCC-GARCH and COMFORT-DCC betas of each S&P 500 index constituent using daily excess returns data. Afterwards, we assign each stock to a decile based on the sorted beta measure, where decile 1 contains the stocks with the lowest betas and decile 10 contains the stocks with the highest betas. All deciles hold the same number of stocks. We expect that stocks in the lowest beta decile have significantly lower returns on the next trading day compared to stocks in the highest beta decile. This hypothesis is tested using a High-Low beta portfolio consisting of a long position in the highest-beta decile and a short position in the lowest-beta decile.

We present results for equal-weighted as well as value-weighted portfolios for the sample period from January 1996 to December 2013.

### 5.1.1 Portfolio Analysis based on Equal-Weighted Deciles

Table 1 reports performance figures and average betas for decile portfolios calculated using the CAPM, DCC-GARCH and COMFORT-DCC models. Interestingly, and in contrast to the paper of Bali et al. (2017), looking at the Fama-French 5 factor alpha value and the High-Low portfolio return value, CAPM does not perform substantially worse than DCC-GARCH or COMFORT-DCC using equal-weighted portfolios.

**Table 1:** Univariate equal-weighted portfolios of all S&P 500 stocks sorted by beta

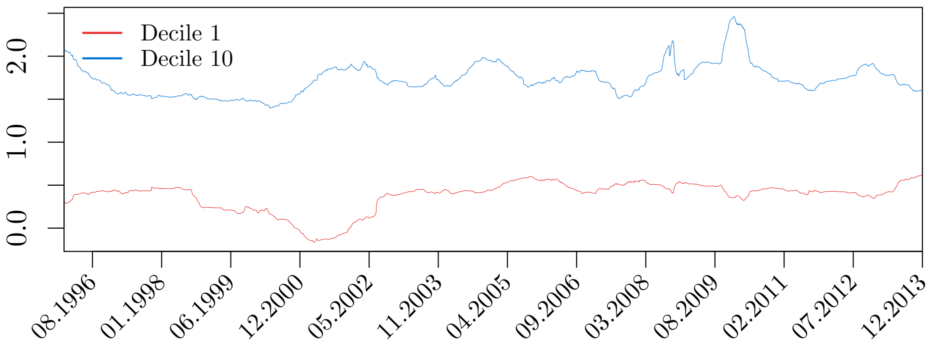
For each day, all stocks of our S&P 500 sample within the period from January 1996 to December 2013 are sorted into univariate decile portfolios. The CAPM-252 and DCC-252 columns were estimated using a moving window of size 252 days, and DCC-1000 and CDCC-1000 were estimated using the past 1000 days. The column *RET* reports the equal-weighted average excess return of the respective portfolio, the column $\beta$ reports the average equal-weighted beta within each decile. The row "High−Low" reports the average excess return taking an equal-weighted long position in the highest beta decile and a short position in the lowest beta decile. FF5 $\alpha$ is the constant coefficient of a Fama-French 5 factor regression (Fama and French (2015)). CAPM $\alpha$ is the coefficient of a CAPM regression. The last row *SR* reports the Sharpe ratio of the High-Low portfolio. All returns are reported as monthly excess returns in percentage terms, assuming 21 trading days in a month. Newey-West $t$-statistics are reported in parentheses.

| | CAPM-252 | | DCC-252 | | DCC-1000 | | CDCC-1000 | |
|---|---|---|---|---|---|---|---|---|
| Decile | *RET* | $\beta$ | *RET* | $\beta$ | *RET* | $\beta$ | *RET* | $\beta$ |
| 1 (Low) | 0.62 | 0.38 | 0.49 | 0.32 | 0.48 | 0.34 | 0.42 | 0.38 |
| 2 | 0.69 | 0.53 | 0.72 | 0.50 | 0.57 | 0.49 | 0.64 | 0.53 |
| 3 | 0.73 | 0.64 | 0.66 | 0.60 | 0.86 | 0.60 | 0.77 | 0.64 |
| 4 | 0.92 | 0.74 | 0.85 | 0.70 | 0.85 | 0.69 | 0.97 | 0.73 |
| 5 | 0.72 | 0.83 | 1.10 | 0.79 | 0.83 | 0.78 | 0.89 | 0.82 |
| 6 | 0.81 | 0.92 | 0.69 | 0.89 | 0.93 | 0.88 | 0.86 | 0.92 |
| 7 | 0.94 | 1.02 | 0.91 | 0.99 | 0.90 | 0.98 | 0.90 | 1.03 |
| 8 | 1.25 | 1.14 | 1.21 | 1.12 | 0.92 | 1.11 | 1.08 | 1.15 |
| 9 | 1.27 | 1.30 | 1.27 | 1.31 | 1.31 | 1.29 | 1.16 | 1.34 |
| 10 (High) | 1.15 | 1.73 | 1.20 | 1.80 | 1.25 | 1.75 | 1.20 | 2.07 |
| High−Low | 0.53 | | 0.71 | | 0.77 | | 0.78 | |
| | (0.85) | | (1.24) | | (1.28) | | (1.28) | |
| FF5 $\alpha$ | 0.52 | | 0.62 | | 0.57 | | 0.62 | |
| | (1.36) | | (1.62) | | (1.53) | | (1.63) | |
| CAPM $\alpha$ | -0.27 | | -0.08 | | -0.01 | | -0.02 | |
| | (-0.68) | | (-0.20) | | (-0.03) | | (-0.05) | |
| SR | 0.11 | | 0.18 | | 0.20 | | 0.20 | |

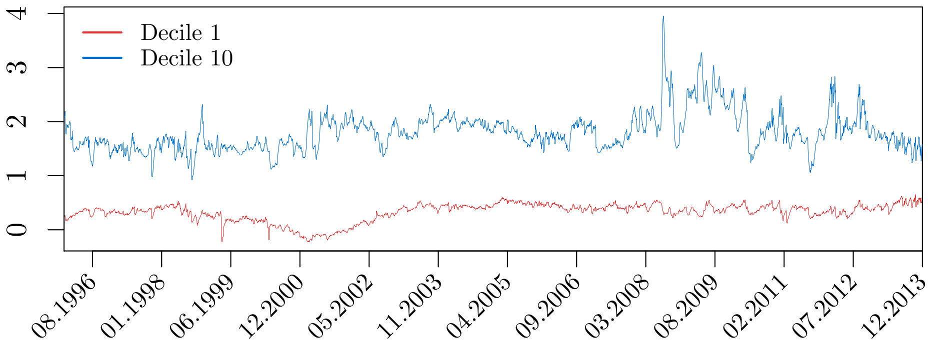
Comparing DCC-GARCH and COMFORT-DCC performance, we see very similar results—the High-Low portfolio returns, alpha values and Sharpe ratios are almost identical. The only noteworthy difference is the relatively high average beta value in decile 10 of the COMFORT-DCC model. Using equal-weighted portfolios, COMFORT-DCC does not outperform the DCC-GARCH model, and both do not significantly outperform CAPM either. A very high correlation value of 0.96 among all High-Low portfolio returns indicates no significant difference in predicting one-day ahead stock returns.

Furthermore, Bali et al. (2017) report a monotonically increasing average return for their DCC-GARCH decile portfolios, and our returns data shows a very similar pattern. All models generate higher average returns in the highest beta decile compared to the lowest beta decile. This evidence supports the hypothesis of a link between the dynamic conditional beta and the cross section of daily stock returns, although the corresponding $t$-statistics are not significant like in the findings of Bali et al. (2017) because of a much smaller sample size.

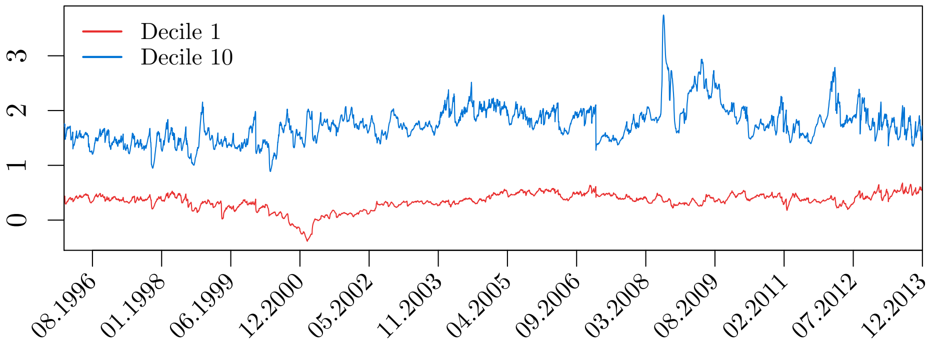
Figure 1 shows the beta dynamics of decile 1 and decile 10 for the CAPM, DCC-GARCH and COMFORT-DCC models over the sample period. As expected, the CAPM beta shows a smooth curve compared to the DCC and CDCC betas, because it does not vary within the estimation period. On the other hand, DCC and CDCC beta show more movements due to their time-varying nature. As mentioned before, the relatively high average beta value in decile 10 of the COMFORT-DCC model manifests in relatively high beta values during the global financial crisis and the European debt crisis period. Looking at the covariance estimates of COMFORT-DCC, convergence issues seem to be the cause of these high beta values. Smoothing the covariance matrix does not help in this case, because the estimation errors are too large, and stronger smoothing would impair the time-varying estimation advantage compared to CAPM.



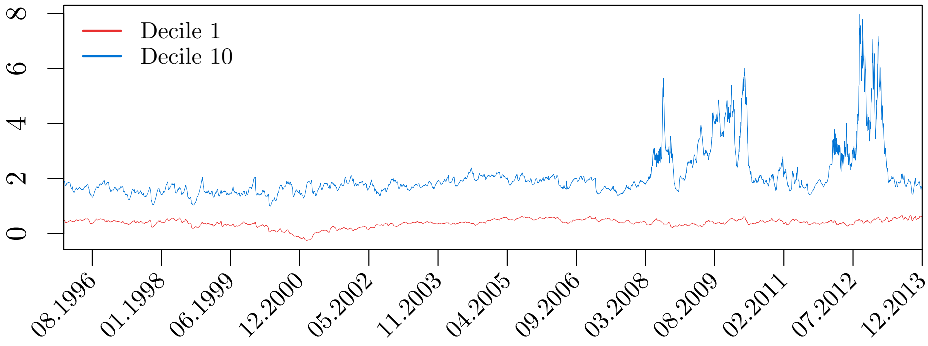

(a) Average CAPM-252 beta of deciles 1 and 10




(b) Average DCC-252 beta of deciles 1 and 10




(c) Average DCC-1000 beta of deciles 1 and 10




(d) Average CDCC-1000 beta of deciles 1 and 10

**Figure 1:** Average equal-weighted CAPM, DCC and CDCC betas of deciles 1 and 10

**Table 2:** Beta standard deviations within each decile

This table presents the standard deviations of the average betas within each decile measured over the whole sample period for the different beta estimation methods.

| Decile | $\sigma(\beta_{\text{CAPM-252}})$ | $\sigma(\beta_{\text{DCC-252}})$ | $\sigma(\beta_{\text{DCC-1000}})$ | $\sigma(\beta_{\text{CDCC-1000}})$ |
|---|---|---|---|---|
| 1 | 0.17 | 0.16 | 0.15 | 0.15 |
| 2 | 0.16 | 0.16 | 0.15 | 0.15 |
| 3 | 0.16 | 0.16 | 0.14 | 0.15 |
| 4 | 0.15 | 0.16 | 0.14 | 0.15 |
| 5 | 0.15 | 0.17 | 0.15 | 0.16 |
| 6 | 0.15 | 0.18 | 0.17 | 0.17 |
| 7 | 0.16 | 0.20 | 0.18 | 0.19 |
| 8 | 0.17 | 0.22 | 0.20 | 0.21 |
| 9 | 0.16 | 0.24 | 0.22 | 0.23 |
| 10 | 0.18 | 0.35 | 0.33 | 0.89 |

Table 2 further highlights the beta estimation problem in decile 10 for COMFORT-DCC. The standard deviation of the average beta within decile 10 is more than twice the DCC-GARCH counterpart. The bad covariance estimates resulting in wrong betas cannot be filtered out without introducing a look-ahead bias, therefore the COMFORT-DCC model optimization processes need to be improved, but this is outside the scope of this thesis. Assuming covariance estimation errors being the cause of the high beta standard deviation in decile 10, conclusions for the High-Low portfolio of COMFORT-DCC should be drawn with the knowledge of estimation errors influencing the results, because the portfolio is based on a long position in decile 10.

Clear evidence of estimation errors and/or optimization convergence issues for COMFORT-DCC are provided in Appendix A.3.

**Table 3:** Firm characteristics and risk attributes of COMFORT-DCC portfolios

For each day, all stocks of our S&P 500 sample within the period from January 1996 to December 2013 are sorted into univariate decile portfolios based on the COMFORT-DCC beta. The column *RET* reports the equal-weighted average excess return of the respective portfolio and the column $\beta$ reports the average equal-weighted beta within each decile. *SIZE* reports the average market capitalization in \$1M units, *ILLIQ* Amihud's average illiquidity measure (Amihud (2002)), *TURN* the average number of stocks changed each day proportional to the number of stocks in the respective decile, and the last column reports the average market share by aggregated capitalization of a decile relative to all stocks.

| Decile | *RET* | $\beta$ | *SIZE* | *ILLIQ* | *TURN* (%) | Market share (%) |
|---|---|---|---|---|---|---|
| 1 (Low) | 0.42 | 0.38 | 31,076 | 0.056 | 7.1 | 10.1 |
| 2 | 0.64 | 0.53 | 29,933 | 0.062 | 14.6 | 9.8 |
| 3 | 0.77 | 0.64 | 29,680 | 0.066 | 18.2 | 9.7 |
| 4 | 0.97 | 0.73 | 30,606 | 0.069 | 20.5 | 10.0 |
| 5 | 0.89 | 0.82 | 28,475 | 0.070 | 21.5 | 9.5 |
| 6 | 0.86 | 0.92 | 27,283 | 0.071 | 21.2 | 9.5 |
| 7 | 0.90 | 1.03 | 26,277 | 0.067 | 20.2 | 9.4 |
| 8 | 1.08 | 1.15 | 25,663 | 0.061 | 17.9 | 9.5 |
| 9 | 1.16 | 1.34 | 27,680 | 0.057 | 13.6 | 10.6 |
| 10 (High) | 1.20 | 2.07 | 31,034 | 0.058 | 7.1 | 12.0 |

Table 3 presents average firm characteristics and risk attributes of all COMFORT-DCC decile portfolios. The turnover (*TURN*) measures how many stocks of a decile portfolio change each day, measured in percentage. For deciles 1 and 10, the turnovers are the lowest with 7.1% of the stocks changing each day, indicating that extreme beta values usually stay extreme for a certain time compared to the middle deciles. Moreover, Amihud's illiquidity measure (*ILLIQ*) is the lowest for the extreme deciles indicating that those stocks are traded more often than the middle-decile stocks. Interestingly, the average firm size does not seem to correlate with beta—the market shares and average market capitalizations (*SIZE*) are roughly the same. This does not reflect the findings of Bali et al. (2017), who report that high DCC-beta deciles with higher average returns consist of stocks with larger market capitalizations compared to lower decile averages. The corresponding table for the DCC-GARCH model can be found in the Appendix 6. This finding can be explained by Bali et al. (2017) using a much bigger stock sample than ours.

### 5.1.2 Portfolio Analysis based on Value-Weighted Deciles

In this section, we conduct the same analysis as in the previous section, with the only difference being in the way average returns and average betas are calculated. Instead of using equal-weighted means for portfolio returns and betas, we use market capitalization as weighting to calculate the mean as described in Section 3.3. The findings for the firm characteristics as presented in 3 do not change; the variables other than average return ($RET$) and average beta ($\beta$) are still equal-weighted.

**Table 4:** Univariate value-weighted portfolios of all S&P 500 stocks sorted by beta

For each day, all stocks of our S&P 500 sample within the period from January 1996 to December 2013 are sorted into univariate decile portfolios. The CAPM-252 and DCC-252 columns were estimated using a moving window of size 252 days, and DCC-1000 and CDCC-1000 were estimated using the past 1000 days. The column *RET* reports the value-weighted average excess return of the respective portfolio, the column $\beta$ reports the average value-weighted beta within each decile. The row "High−Low" reports the average excess return taking an equal-weighted long position in the highest beta decile and a short position in the lowest beta decile. FF5 $\alpha$ is the constant coefficient of a Fama-French 5 factor regression (Fama and French (2015)). CAPM $\alpha$ is the coefficient of a CAPM regression. The last row *SR* reports the Sharpe ratio of the High-Low portfolio. All returns are reported as monthly excess returns in percentage terms, assuming 21 trading days in a month. Newey-West $t$-statistics are reported in parentheses.

| | CAPM-252 | | DCC-252 | | DCC-1000 | | CDCC-1000 | |
|---|---|---|---|---|---|---|---|---|
| Decile | *RET* | $\beta$ | *RET* | $\beta$ | *RET* | $\beta$ | *RET* | $\beta$ |
| 1 (Low) | 0.84 | 0.39 | 0.61 | 0.33 | 0.66 | 0.35 | 0.54 | 0.39 |
| 2 | 0.79 | 0.53 | 0.86 | 0.50 | 0.73 | 0.49 | 0.92 | 0.53 |
| 3 | 1.10 | 0.64 | 0.98 | 0.60 | 1.04 | 0.59 | 0.88 | 0.64 |
| 4 | 1.17 | 0.74 | 1.21 | 0.70 | 1.11 | 0.69 | 1.17 | 0.73 |
| 5 | 1.03 | 0.83 | 1.42 | 0.79 | 1.22 | 0.78 | 1.03 | 0.82 |
| 6 | 1.12 | 0.92 | 1.09 | 0.89 | 1.35 | 0.88 | 1.33 | 0.92 |
| 7 | 1.15 | 1.02 | 1.39 | 0.99 | 1.32 | 0.98 | 1.32 | 1.03 |
| 8 | 1.47 | 1.14 | 1.38 | 1.12 | 1.06 | 1.11 | 1.39 | 1.15 |
| 9 | 1.35 | 1.31 | 1.24 | 1.31 | 1.72 | 1.29 | 1.34 | 1.34 |
| 10 (High) | 2.03 | 1.70 | 2.37 | 1.75 | 2.10 | 1.71 | 1.85 | 2.26 |
| High−Low | 1.18 | | 1.75 | | 1.43 | | 1.30 | |
| | (1.89) | | (2.88) | | (2.32) | | (2.13) | |
| FF5 $\alpha$ | 1.32 | | 1.84 | | 1.38 | | 1.30 | |
| | (2.84) | | (3.54) | | (2.82) | | (2.72) | |
| CAPM $\alpha$ | 0.37 | | 0.96 | | 0.65 | | 0.51 | |
| | (0.77) | | (1.89) | | (1.30) | | (1.05) | |
| SR | 0.32 | | 0.51 | | 0.41 | | 0.37 | |

Comparing the results based on value-weighted excess returns in Table 4 to equal-weighted excess returns as shown in Table 1, we see significantly better performance and higher Sharpe ratios for all High-Low portfolios. The DCC-252 portfolio in particular shows a significant improvement of average excess returns increasing from

0.71% per month (Newey-West $t$-statistic of 1.24) to 1.75% (Newey-West $t$-statistic of 2.88) and the Sharpe ratio increasing from 0.18 to 0.51. The performance of the High-Low CDCC-1000 portfolio also increases, but it neither outperforms the DCC-252, nor the DCC-1000 model. It has lower excess returns of 1.3% per month, compared to 1.75% of DCC-252 and 1.43% of DCC-1000. The Sharpe ratio of 0.37 is also substantially smaller than the 0.51 of DCC-252 and 0.41 of DCC-1000.

To further analyze the performance of value-weighted High-Low portfolios, we plot the cumulative excess returns in Figure 2. The total excess returns of the market portfolio were added to the figure for the purposes of comparison.

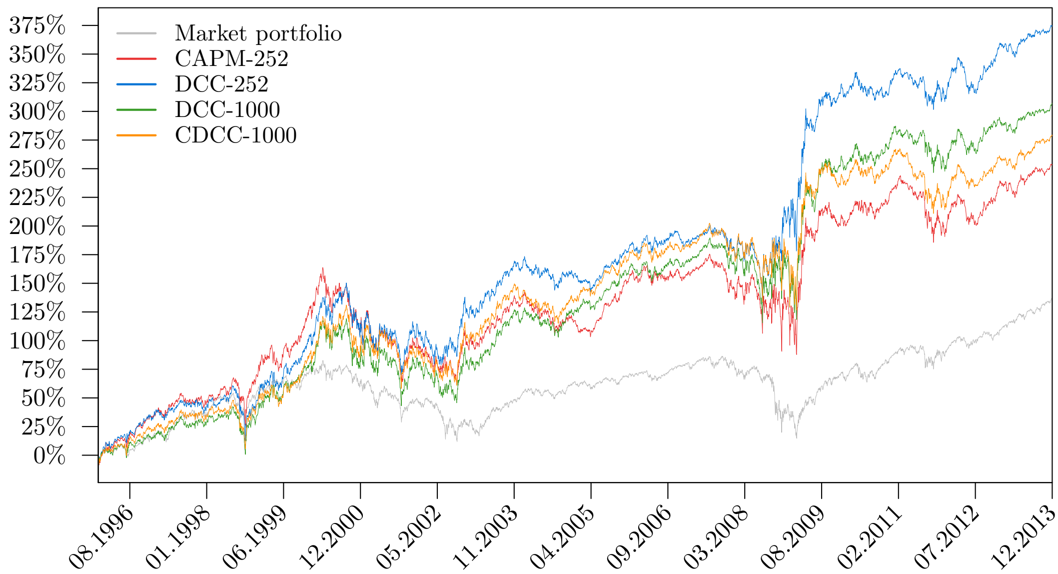



**Figure 2:** Cumulative value-weighted excess returns of CAPM, DCC and CDCC High-Low beta portfolios

One can easily see that the DCC-252 model is superior to the others in terms of performance and risk. It reports an annualized Sharpe ratio of 0.51, CAPM-252 reports 0.32, DCC-1000 reports 0.41 and CDCC-1000 reports only 0.37. For comparison, the market portfolio has an annualized Sharpe ratio of 0.25 over the same estimation period ranging from January 1996 to December 2013. This means that DCC-252 enables the investor to achieve more performance at lower risk compared to the market portfolio, visible in the substantially higher Sharpe ratio. The main performance driver of the DCC-252 High-Low portfolio is the strong recovery period after the financial crisis of 2008—before that, all three portfolios performed similarly. The superior performance of the value-weighted High-Low portfolio during the recovery period after periods of market stress seem particularly interesting to investors, allowing them to recover the previously experienced losses and to outperform the market at the same time.

# 6 Conclusion

Our empirical findings for the DCC-GARCH construct confirm the findings of Bali et al. (2017). The High-Low portfolio, used to test the hypothesis of a link between the dynamic conditional beta and the cross section of daily stock returns, yields positive excess returns and the decile portfolio returns show an increasing pattern, although not monotonically. The value-weighted High-Low portfolio return of 1.75% per month without accounting for transaction costs is statistically highly significant (Newey-West $t$-statistic of 2.88), the corresponding Fama-French alpha of 1.84% per month is highly significant as well (Newey-West $t$-statistic of 3.54). Bali et al. (2017) report for the value-weighted High-Low portfolio a monthly return of 0.91% (Newey-West $t$-statistic of 2.03) without accounting for transaction costs, the corresponding alpha is 1.24% per month (Newey-West $t$-statistic of 3.80). Using the monthly transaction costs estimate of 35 basis points yields a value-weighted monthly excess return of 1.4% for our High-Low portfolio, and 0.56% for the portfolio of Bali et al. (2017). The illiquidity measures are almost identical, but our portfolio turnover is lower because of covariance smoothing, making it cheaper to implement the strategy compared to Bali et al. (2017). Turnovers for the highest and lowest decile portfolios are identical in our study—Bali et al. (2017) report high turnover in the highest-beta decile and low turnover in the lowest-beta decile. Furthermore, they report that the average market capitalization of firms in the lowest beta decile is four times smaller compared to the highest decile, but the corresponding values in our study are the same. These differences can be explained by the different data sample and time frames used—Bali et al. (2017) use a substantially bigger stock universe and a longer time frame. Our sample goes back to January 1996, whereas theirs goes as far back as July 1963. They use all U.S.-based common stocks trading on the NYSE, AMEX, and NASDAQ exchanges with a stock price of $5/share or more and with a market capitalization greater than $10 million, whereas we use the S&P 500 index with 500 constituents.

Moreover, the replication of the paper of Bali et al. (2017) was subject to interpretation, because they do not explicitly specify precisely how they estimate the covariances of stock excess returns and market excess returns. There exist multiple possible approaches. Our approach was to estimate the covariance in a bivariate way. For each stock of the S&P 500 index, the covariance matrix of stock excess returns and market excess returns was estimated bivariately, which then was used to calculate the respective beta. A possible alternative is a multivariate estimation approach by using many stock excess return time series in conjunction with the market excess return time series to estimate the covariance matrix. This could be done in one step by the estimation of the full covariance matrix, or in multiple steps by taking a subsample of all index constituents.

There are various reasons why the COMFORT-DCC model (CDCC-1000) does not outperform the DCC-GARCH constructs (DCC-252 and DCC-1000) in our setting.

First, covariance estimation errors, especially for the highest beta decile stocks, impair our High-Low portfolio analysis. The empirical findings provide evidence of model optimization problems, see Appendix A.3. They are also apparent by looking at the average beta of the highest beta decile, which is substantially greater than the average beta of decile 9 and the average beta of decile 10 for the DCC-GARCH model. Same holds true for the standard deviation of the average beta in decile 10 of COMFORT-DCC—it is substantially greater than the ones of DCC-GARCH. Reliable and consistent beta estimates are crucial for our study, because we rely on the extreme cases (highest and lowest betas) to predict the one-step-ahead cross section of stock returns; any extreme values due to estimation errors directly bias our results.

Second, the COMFORT-DCC model was estimated each day with a window size of 1000 days, which is approximately four times bigger than the one of the DCC-252 model. The window size for COMFORT-DCC was chosen that way to ease parameter convergence and reduce estimation errors. The DCC-GARCH model with a window size of 1000 days performs worse than the 252 days counterpart, but still better than the COMFORT-DCC model.

Third, the bivariate estimation method likely results in different estimates for the parameters of the Generalized Inverse Gaussian (GIG) distribution of the mixing factor $G_t$ (see Paolella and Polak (2015b)), although the model assumes that the mixing factor estimates are the same for all asset returns at time $t$. This additional degree of freedom likely introduces less precise/more random COMFORT-DCC model estimates.

These findings let us conclude that reliable beta estimates combined with a shorter estimation window are more important than having a model accounting for asymmetry, fat-tails and other stylized facts of financial returns, because accounting for these comes at the cost of less reliable estimates and tend to require bigger samples for estimation. The estimation problems the COMFORT-DCC model faces do not necessarily imply that the underlying model is less suitable for the task presented—since it accounts for major stylized facts of asset returns, which the DCC-GARCH does not—but it indicates that the estimation process is more intricate. This leaves us with an inconclusive answer whether COMFORT-DCC outperforms the DCC-GARCH model.

The replication findings show promising results regarding the investment strategy used by Bali et al. (2017), it outperforms the market and has higher risk-adjusted returns. Further studies of the COMFORT-DCC model with more reliable model estimates are required. A multivariate estimation approach should be considered in order to estimate the GIG parameters of the univariate mixing factor over all assets. The GARCH-SV extension as additional source of correlation dynamics, and other extensions are other options to potentially increase the forecasting accuracy. As the results of DCC-252 and DCC-1000 show, a shorter estimation window yields a higher risk-adjusted and absolute performance, and should therefore be considered. Another interesting finding is the strong performance of DCC-252 after periods of market stress, where it outperforms the market portfolio by a big margin. This effect seems to be particularly interesting in combination with other strategies, therefore making it attractive for further studies.

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# A Appendix

## A.1 Supplemental Figures for Equal-Weighted Empirical Analysis

### A.1.1 Portfolio and Firm Characteristics

**Table 5:** Characteristics of equal-weighted CAPM portfolios with a window size of 252 days

| Decile | *RET* | $\beta$ | *SIZE* | *ILLIQ* | *TURN* (%) | Market share (%) |
|---|---|---|---|---|---|---|
| 1 (Low) | 0.62 | 0.38 | 29,600 | 0.055 | 3.2 | 9.5 |
| 2 | 0.69 | 0.53 | 30,741 | 0.059 | 4.7 | 9.8 |
| 3 | 0.73 | 0.64 | 31,019 | 0.063 | 5.4 | 10.0 |
| 4 | 0.92 | 0.74 | 30,391 | 0.066 | 6.1 | 9.7 |
| 5 | 0.72 | 0.83 | 30,735 | 0.067 | 6.3 | 9.9 |
| 6 | 0.81 | 0.92 | 28,791 | 0.071 | 6.1 | 9.7 |
| 7 | 0.94 | 1.02 | 26,156 | 0.068 | 5.7 | 9.0 |
| 8 | 1.25 | 1.14 | 23,827 | 0.062 | 5.4 | 8.6 |
| 9 | 1.27 | 1.30 | 31,160 | 0.053 | 4.7 | 11.7 |
| 10 (High) | 1.15 | 1.73 | 30,935 | 0.049 | 3.7 | 12.0 |

**Table 6:** Characteristics of equal-weighted DCC-GARCH portfolios with a window size of 252 days

| Decile | *RET* | $\beta$ | *SIZE* | *ILLIQ* | *TURN* (%) | Market share (%) |
|---|---|---|---|---|---|---|
| 1 (Low) | 0.49 | 0.32 | 29,525 | 0.056 | 9.2 | 9.5 |
| 2 | 0.72 | 0.50 | 30,120 | 0.060 | 18.4 | 9.6 |
| 3 | 0.66 | 0.60 | 31,060 | 0.063 | 22.0 | 10.1 |
| 4 | 0.85 | 0.70 | 30,973 | 0.065 | 24.0 | 10.0 |
| 5 | 1.10 | 0.79 | 30,318 | 0.066 | 24.9 | 9.9 |
| 6 | 0.69 | 0.89 | 28,536 | 0.066 | 24.5 | 9.6 |
| 7 | 0.91 | 0.99 | 27,085 | 0.064 | 23.3 | 9.3 |
| 8 | 1.21 | 1.12 | 25,867 | 0.062 | 20.3 | 9.3 |
| 9 | 1.27 | 1.31 | 29,802 | 0.056 | 15.5 | 11.3 |
| 10 (High) | 1.20 | 1.80 | 30,023 | 0.055 | 8.1 | 11.5 |

**Table 7:** Characteristics of equal-weighted DCC-GARCH portfolios with a window size of 1000 days

| Decile | *RET* | $\beta$ | *SIZE* | *ILLIQ* | *TURN* (%) | Market share (%) |
|---|---|---|---|---|---|---|
| 1 (Low) | 0.48 | 0.34 | 30,443 | 0.055 | 7.6 | 10.0 |
| 2 | 0.57 | 0.49 | 30,163 | 0.062 | 15.2 | 9.8 |
| 3 | 0.86 | 0.60 | 29,580 | 0.067 | 18.3 | 9.7 |
| 4 | 0.85 | 0.69 | 29,890 | 0.069 | 20.2 | 9.9 |
| 5 | 0.83 | 0.78 | 28,368 | 0.070 | 21.1 | 9.5 |
| 6 | 0.93 | 0.88 | 27,837 | 0.068 | 20.8 | 9.7 |
| 7 | 0.90 | 0.98 | 27,557 | 0.064 | 19.8 | 9.6 |
| 8 | 0.92 | 1.11 | 26,724 | 0.061 | 17.5 | 9.8 |
| 9 | 1.31 | 1.29 | 28,213 | 0.059 | 13.3 | 10.9 |
| 10 (High) | 1.25 | 1.75 | 28,008 | 0.062 | 7.0 | 11.1 |

**Table 8:** Characteristics of equal-weighted COMFORT-DCC portfolios with a window size of 1000 days

| Decile | *RET* | $\beta$ | *SIZE* | *ILLIQ* | *TURN* (%) | Market share (%) |
|---|---|---|---|---|---|---|
| 1 (Low) | 0.42 | 0.38 | 31,076 | 0.056 | 7.1 | 10.1 |
| 2 | 0.64 | 0.53 | 29,933 | 0.062 | 14.6 | 9.8 |
| 3 | 0.77 | 0.64 | 29,680 | 0.066 | 18.2 | 9.7 |
| 4 | 0.97 | 0.73 | 30,606 | 0.069 | 20.5 | 10.0 |
| 5 | 0.89 | 0.82 | 28,475 | 0.070 | 21.5 | 9.5 |
| 6 | 0.86 | 0.92 | 27,283 | 0.071 | 21.2 | 9.5 |
| 7 | 0.90 | 1.03 | 26,277 | 0.067 | 20.2 | 9.4 |
| 8 | 1.08 | 1.15 | 25,663 | 0.061 | 17.9 | 9.5 |
| 9 | 1.16 | 1.34 | 27,680 | 0.057 | 13.6 | 10.6 |
| 10 (High) | 1.20 | 2.07 | 31,034 | 0.058 | 7.1 | 12.0 |

### A.1.2 Average Equal-Weighted Excess Returns per Decile

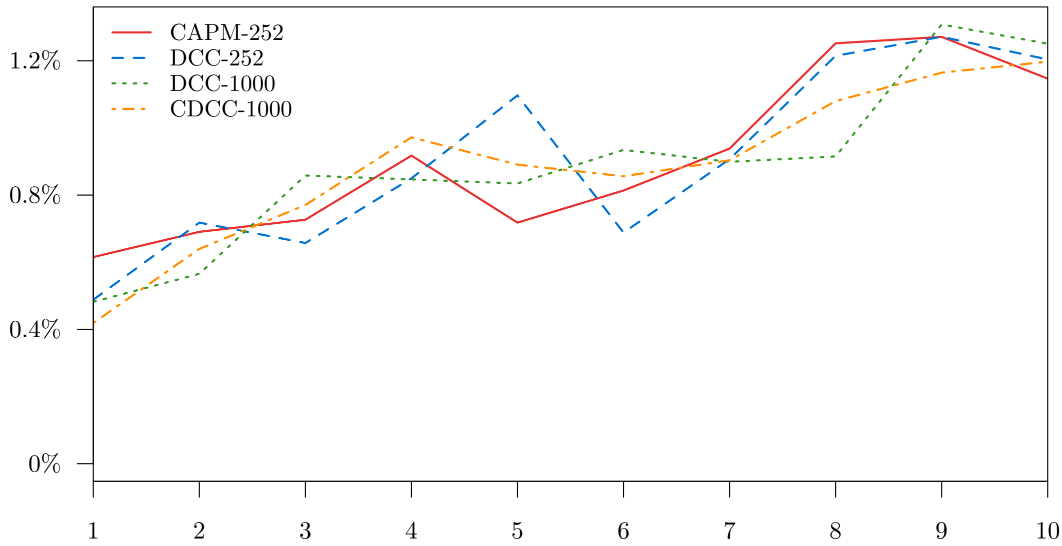


**Figure 3:** Average equal-weighted excess returns per decile for CAPM-, DCC and CDCC beta portfolios

### A.1.3 Equal-Weighted High-Low Portfolio Performance

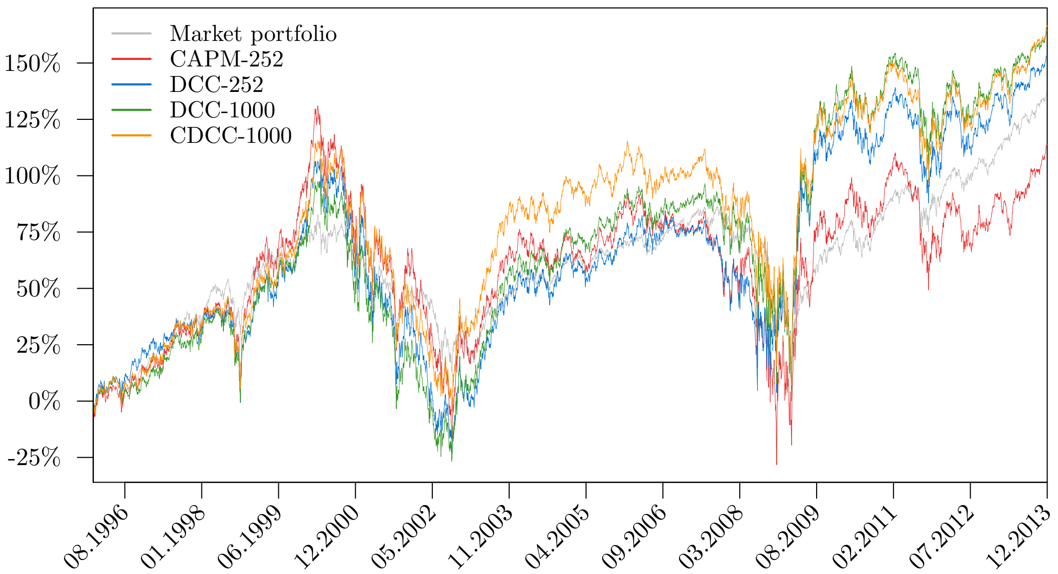


**Figure 4:** Cumulative equal-weighted excess returns of CAPM, DCC and CDCC High-Low beta portfolios

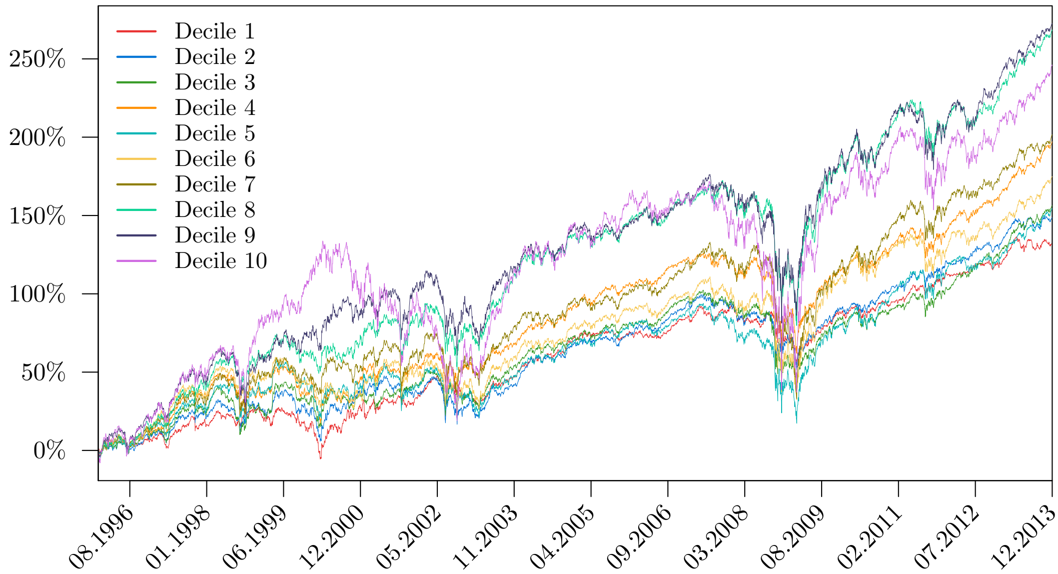


**Figure 5:** Cumulative equal-weighted excess returns of CAPM beta decile portfolios with a window size of 252 days

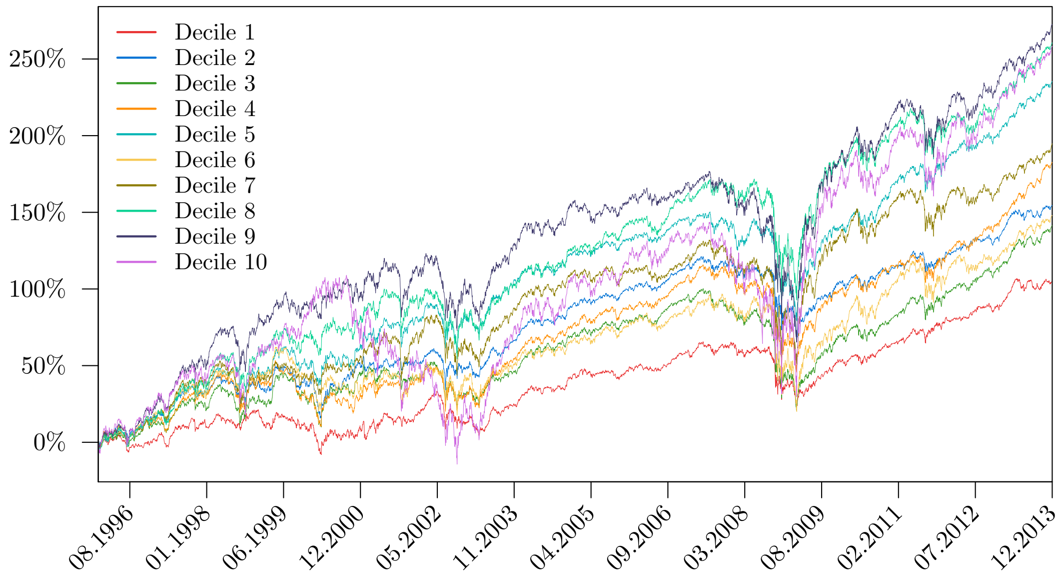


**Figure 6:** Cumulative equal-weighted excess returns of DCC-GARCH beta decile portfolios with a window size of 252 days

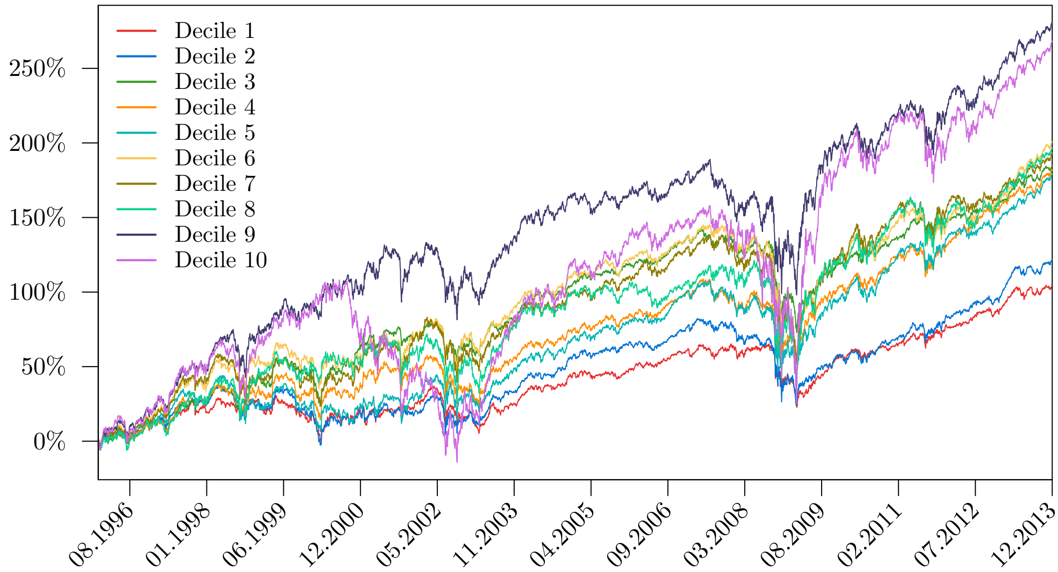


**Figure 7:** Cumulative equal-weighted excess returns of DCC-GARCH beta decile portfolios with a window size of 1000 days

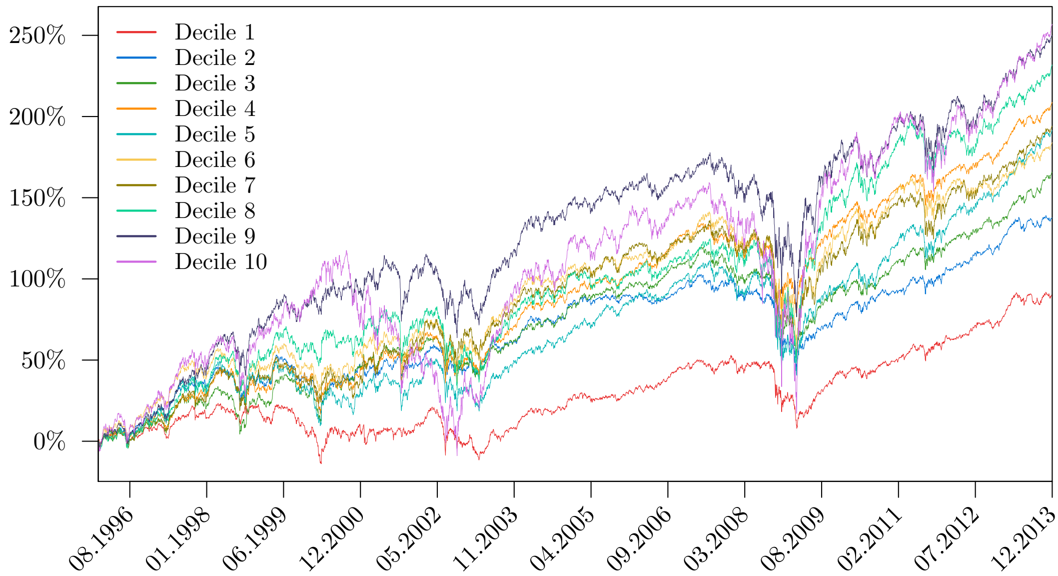


**Figure 8:** Cumulative equal-weighted excess returns of COMFORT-DCC beta decile portfolios with a window size of 1000 days

## A.2 Supplemental Figures for Value-Weighted Empirical Analysis

### A.2.1 Portfolio and Firm Characteristics

**Table 9:** Characteristics of value-weighted CAPM portfolios with a window size of 252 days

| Decile | *RET* | $\beta$ | *SIZE* | *ILLIQ* | *TURN* (%) | Market share (%) |
|---|---|---|---|---|---|---|
| 1 (Low) | 0.84 | 0.39 | 29,600 | 0.055 | 3.2 | 9.5 |
| 2 | 0.79 | 0.53 | 30,741 | 0.059 | 4.7 | 9.8 |
| 3 | 1.10 | 0.64 | 31,019 | 0.063 | 5.4 | 10.0 |
| 4 | 1.17 | 0.74 | 30,391 | 0.066 | 6.1 | 9.7 |
| 5 | 1.03 | 0.83 | 30,735 | 0.067 | 6.3 | 9.9 |
| 6 | 1.12 | 0.92 | 28,791 | 0.071 | 6.1 | 9.7 |
| 7 | 1.15 | 1.02 | 26,156 | 0.068 | 5.7 | 9.0 |
| 8 | 1.47 | 1.14 | 23,827 | 0.062 | 5.4 | 8.6 |
| 9 | 1.35 | 1.31 | 31,160 | 0.053 | 4.7 | 11.7 |
| 10 (High) | 2.03 | 1.70 | 30,935 | 0.049 | 3.7 | 12.0 |

**Table 10:** Characteristics of value-weighted DCC-GARCH portfolios with a window size of 252 days

| Decile | *RET* | $\beta$ | *SIZE* | *ILLIQ* | *TURN* (%) | Market share (%) |
|---|---|---|---|---|---|---|
| 1 (Low) | 0.61 | 0.33 | 29,525 | 0.056 | 9.2 | 9.5 |
| 2 | 0.86 | 0.50 | 30,120 | 0.060 | 18.4 | 9.6 |
| 3 | 0.98 | 0.60 | 31,060 | 0.063 | 22.0 | 10.1 |
| 4 | 1.21 | 0.70 | 30,973 | 0.065 | 24.0 | 10.0 |
| 5 | 1.42 | 0.79 | 30,318 | 0.066 | 24.9 | 9.9 |
| 6 | 1.09 | 0.89 | 28,536 | 0.066 | 24.5 | 9.6 |
| 7 | 1.39 | 0.99 | 27,085 | 0.064 | 23.3 | 9.3 |
| 8 | 1.38 | 1.12 | 25,867 | 0.062 | 20.3 | 9.3 |
| 9 | 1.24 | 1.31 | 29,802 | 0.056 | 15.5 | 11.3 |
| 10 (High) | 2.37 | 1.75 | 30,023 | 0.055 | 8.1 | 11.5 |

**Table 11:** Characteristics of value-weighted DCC-GARCH portfolios with a window size of 1000 days

| Decile | *RET* | $\beta$ | *SIZE* | *ILLIQ* | *TURN* (%) | Market share (%) |
|---|---|---|---|---|---|---|
| 1 (Low) | 0.66 | 0.35 | 30,443 | 0.055 | 7.6 | 10.0 |
| 2 | 0.73 | 0.49 | 30,163 | 0.062 | 15.2 | 9.8 |
| 3 | 1.04 | 0.59 | 29,580 | 0.067 | 18.3 | 9.7 |
| 4 | 1.11 | 0.69 | 29,890 | 0.069 | 20.2 | 9.9 |
| 5 | 1.22 | 0.78 | 28,368 | 0.070 | 21.1 | 9.5 |
| 6 | 1.35 | 0.88 | 27,837 | 0.068 | 20.8 | 9.7 |
| 7 | 1.32 | 0.98 | 27,557 | 0.064 | 19.8 | 9.6 |
| 8 | 1.06 | 1.11 | 26,724 | 0.061 | 17.5 | 9.8 |
| 9 | 1.72 | 1.29 | 28,213 | 0.059 | 13.3 | 10.9 |
| 10 (High) | 2.10 | 1.71 | 28,008 | 0.062 | 7.0 | 11.1 |

**Table 12:** Characteristics of value-weighted COMFORT-DCC portfolios with a window size of 1000 days

| Decile | *RET* | $\beta$ | *SIZE* | *ILLIQ* | *TURN* (%) | Market share (%) |
|---|---|---|---|---|---|---|
| 1 (Low) | 0.54 | 0.39 | 31,076 | 0.056 | 7.1 | 10.1 |
| 2 | 0.92 | 0.53 | 29,933 | 0.062 | 14.6 | 9.8 |
| 3 | 0.88 | 0.64 | 29,680 | 0.066 | 18.2 | 9.7 |
| 4 | 1.17 | 0.73 | 30,606 | 0.069 | 20.5 | 10.0 |
| 5 | 1.03 | 0.82 | 28,475 | 0.070 | 21.5 | 9.5 |
| 6 | 1.33 | 0.92 | 27,283 | 0.071 | 21.2 | 9.5 |
| 7 | 1.32 | 1.03 | 26,277 | 0.067 | 20.2 | 9.4 |
| 8 | 1.39 | 1.15 | 25,663 | 0.061 | 17.9 | 9.5 |
| 9 | 1.34 | 1.34 | 27,680 | 0.057 | 13.6 | 10.6 |
| 10 (High) | 1.85 | 2.26 | 31,034 | 0.058 | 7.1 | 12.0 |

### A.2.2 Average Value-Weighted Excess Returns per Decile

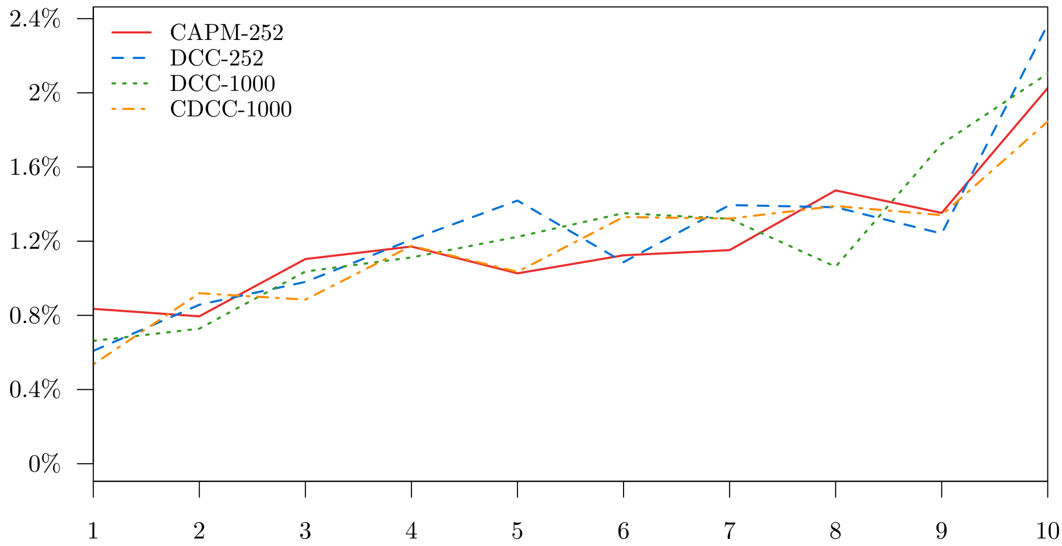


**Figure 9:** Average value-weighted excess returns per decile for CAPM-, DCC and CDCC beta portfolios

### A.2.3 Value-Weighted High-Low Portfolio Performance

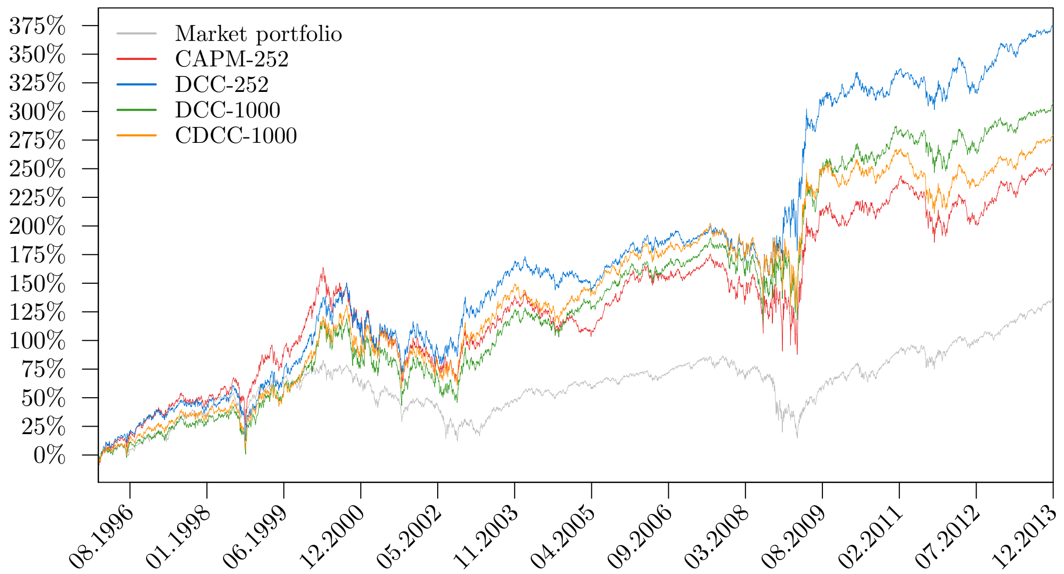


**Figure 10:** Cumulative value-weighted excess returns of CAPM, DCC and CDCC High-Low beta portfolios

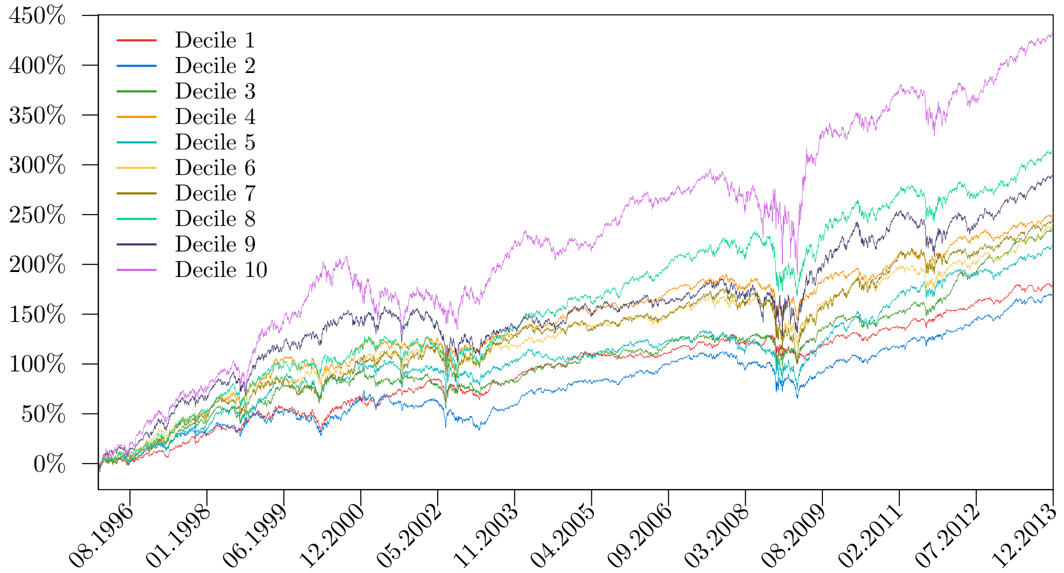


**Figure 11:** Cumulative value-weighted excess returns of CAPM beta decile portfolios with a window size of 252 days

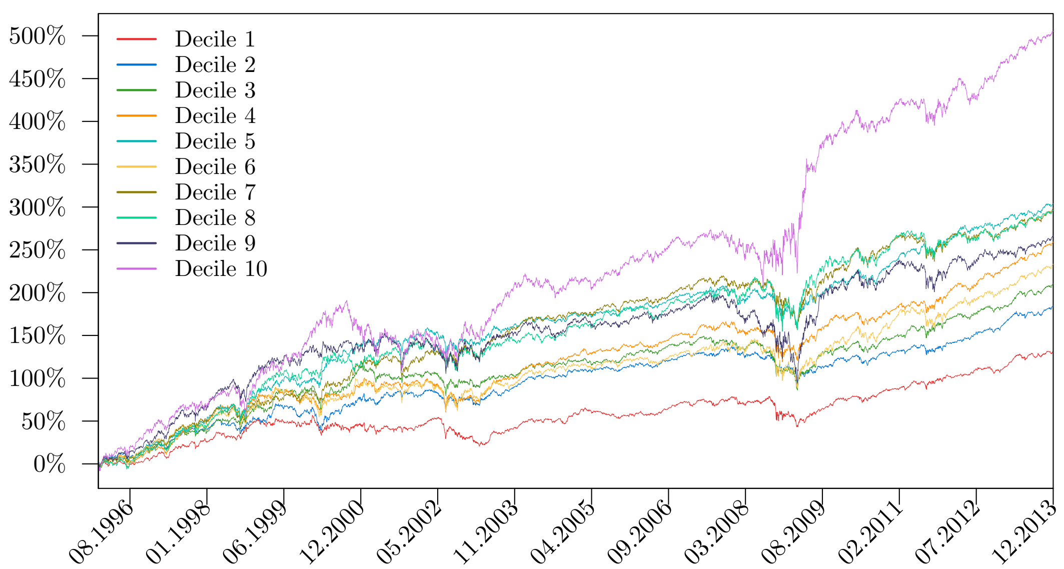


**Figure 12:** Cumulative value-weighted excess returns of DCC-GARCH beta decile portfolios with a window size of 252 days

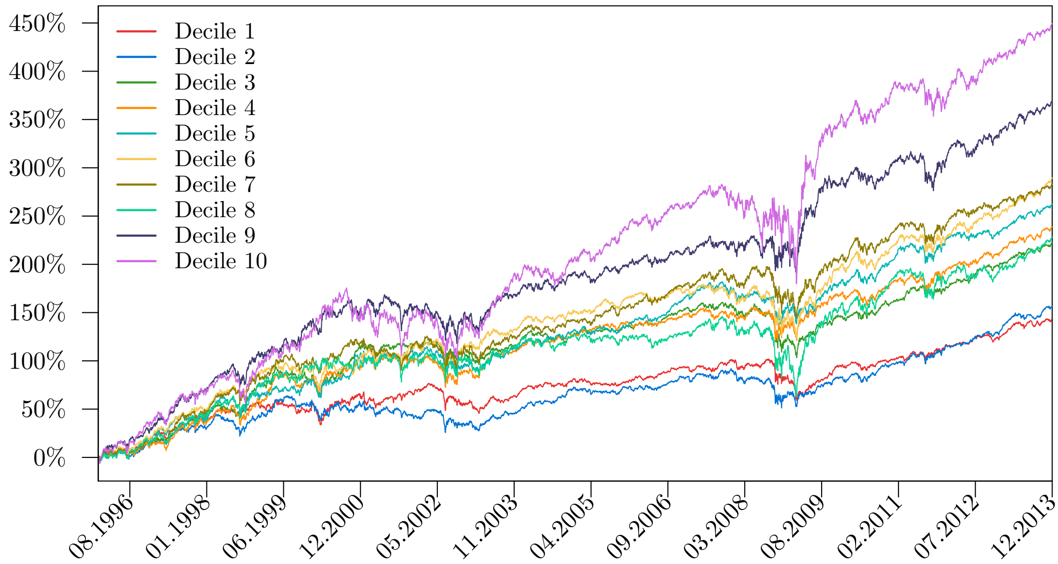


**Figure 13:** Cumulative value-weighted excess returns of DCC-GARCH beta decile portfolios with a window size of 1000 days

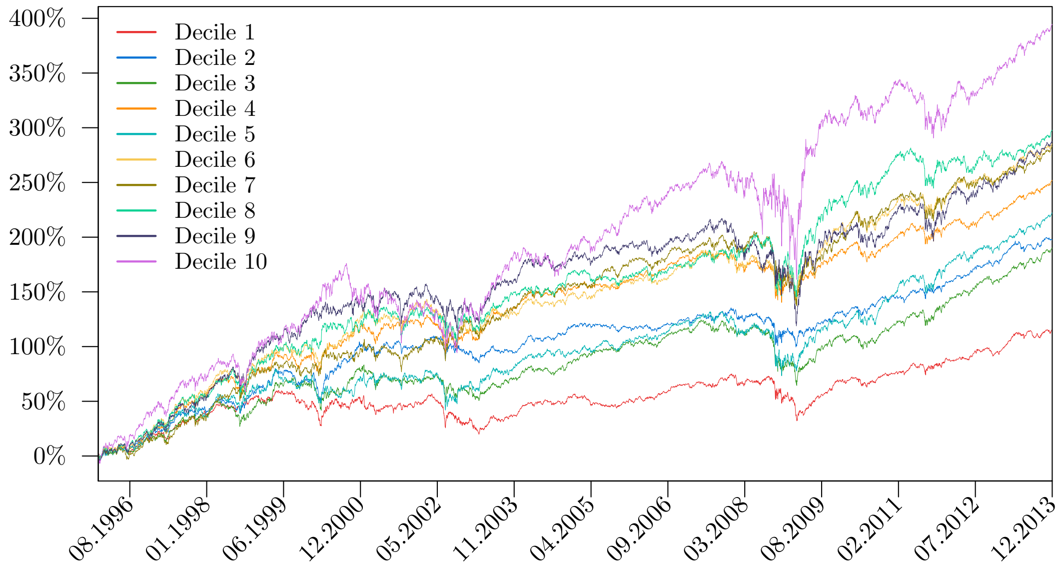


**Figure 14:** Cumulative value-weighted excess returns of COMFORT-DCC beta decile portfolios with a window size of 1000 days

## A.3 Erroneous COMFORT-DCC Covariance Estimates Compared to DCC-GARCH Estimates

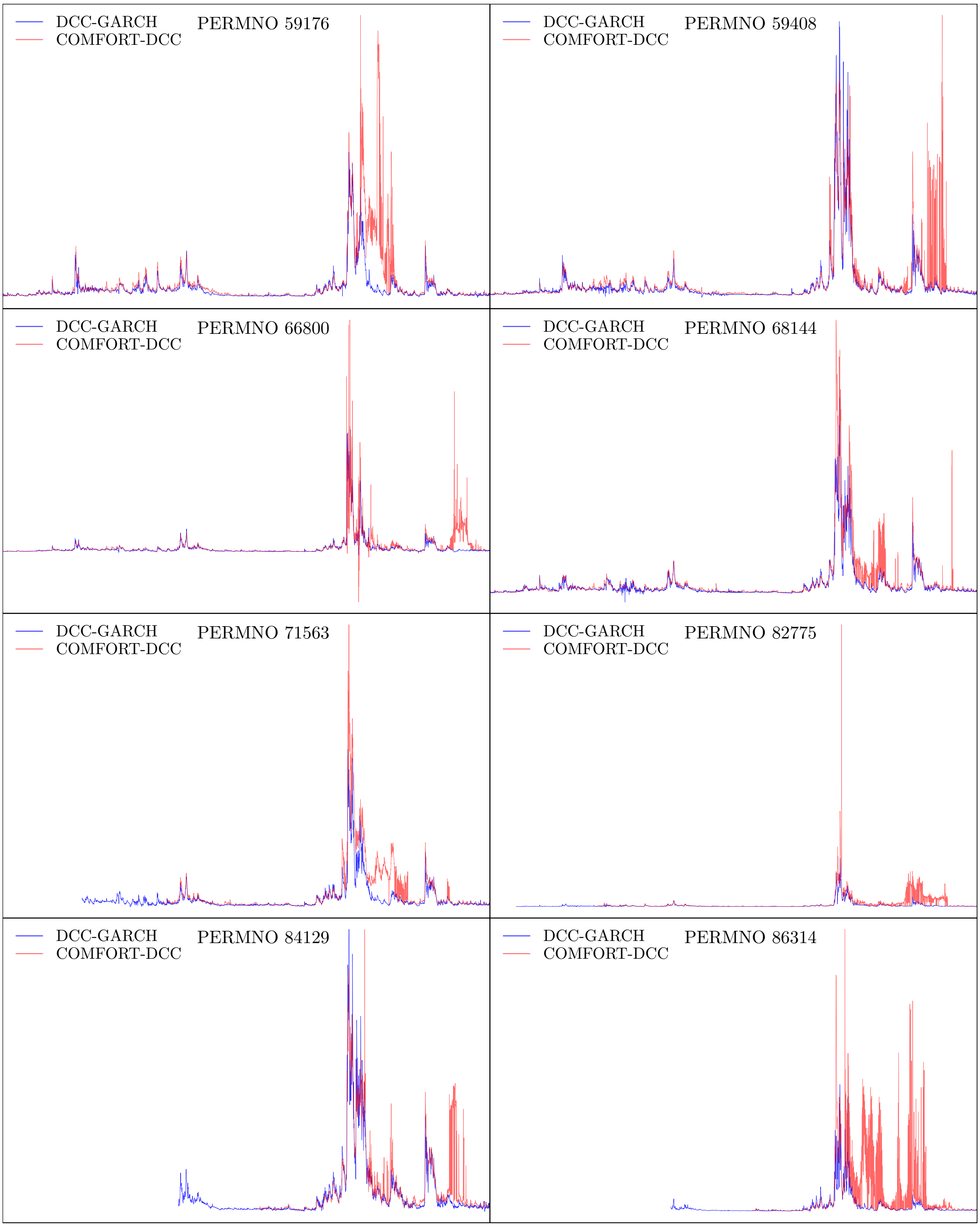


**Figure 15:** Erroneous COMFORT-DCC covariance estimates compared to DCC-GARCH of various CRSP stocks identified by PERMNO

## A.4 Codes

### A.4.1 S&P 500 Constituents WRDS Data Query

Figure 16 presents the SQL query used to retrieve CRSP pricing data and firm characteristics of all S&P 500 constituents via the Wharton Research Data Services (WRDS) API (The Wharton School (2018)) for the sample period from January 1996 to December 2013.

```
select b.permno, b.date, b.ret, d.dlret, d.dlstcd,
           (ABS(b.prc) * (b.shrout * 1000)) as cap,
           GREATEST(b.vol, 0) as vol,
           ABS(b.prc) as prc, (b.shrout * 1000) as shrout
       from crsp.dsp500list a
       join crsp.dsf b on b.permno=a.permno
  left join crsp.dse d on d.permno=b.permno and
                          d.date=b.date and
                          d.dlstcd is not null
      where b.date >= a.start and b.date <= a.ending
        and b.date >= '1996-01-01' and b.date <= '2013-12-31'
   order by b.date
```

**Figure 16:** WRDS SQL query used to load all S&P 500 index constituents for the sample period from January 1996 to December 2013

### A.4.2 COMFORT-DCC Matlab setType.m Model Configuration

Figure 17 presents the model configuration used for COMFORT-DCC estimation. It configures a multivariate, asymmetric Laplace distribution (MALap), no GARCH-SV dynamics and a Dynamic Conditional Correlation (DCC) model. Moreover, EM-algorithm is used for model estimation.

```
function [type] = setType(Nassets,winsize)
    type.model.FREECOMFORT = 0;
    type.model.IID = 0;
    type.model.estimation = 'EM';
    type.model.mixGARCH_SV = 0;
    type.model.distribution = 'MALap';
    type.model.Corrmodel = 'DCC';
    type.model.GARCHtype = 'GARCH';
    type.model.for = 0;
    [type] = LowerLevelParametersAndPrint(type,Nassets,winsize);
end
```

**Figure 17:** COMFORT-DCC Matlab setType.m model configuration

### A.5 Model Optimization Runtimes

All model optimization runtimes were measured using a single computation thread. Table 13 lists the specification of the machine used to run the calculations.

| | |
|---|---|
| Type | Personal Computer |
| CPU | Intel® Core™ i7-5820K, 3.97 GHz |
| Cores | 6 physical cores, 12 logical processors |
| Memory | DDR4, 2448 MHz, 16 GB |
| Operating System | Windows 10 Pro, 64 bit, build 1803 |
| Matlab | R2017b |

**Table 13:** Specification of machine running model estimations

#### A.5.1 Runtimes of DCC-GARCH Optimization with a Window Size of 252 Days

Table 14 reports summary statistics measured in seconds for the estimation of a single window with size 252 and two assets for the DCC-GARCH model. The statistics were derived using 542 samples and report an average runtime of approximately 1.691 seconds per window, resulting in an expected runtime of 7 minutes to estimate the covariance of two assets over a whole year, assuming 252 trading days in a year.

**Table 14:** Model optimization runtimes in seconds for two assets and an estimation window of 252 days using the DCC-GARCH model

| N | Mean | St. Dev. | Min | Pctl(25) | Pctl(75) | Max |
|---|---|---|---|---|---|---|
| 542 | 1.691 | 0.152 | 0.949 | 1.653 | 1.765 | 2.010 |

### A.5.2 Runtimes of DCC-GARCH Optimization with a Window Size of 1000 Days

Table 15 reports summary statistics measured in seconds for the estimation of a single window with size 1000 and two assets for the DCC-GARCH model. The statistics were derived using 578 samples and report an average runtime of approximately 4.487 seconds per window, resulting in an expected runtime of 19 minutes to estimate the covariance of two assets over a whole year, assuming 252 trading days in a year.

**Table 15:** Model optimization runtimes in seconds for two assets and an estimation window of 1000 days using the DCC-GARCH model

| N | Mean | St. Dev. | Min | Pctl(25) | Pctl(75) | Max |
|---|---|---|---|---|---|---|
| 578 | 4.487 | 0.473 | 3.746 | 4.072 | 4.934 | 5.830 |

### A.5.3 Runtimes of COMFORT-DCC Optimization with a Window Size of 1000 Days

Table 16 reports summary statistics measured in seconds for the estimation of a single window with size 1000 and two assets for the COMFORT-DCC model. The statistics were derived using 9329 samples and report an average runtime of approximately 31 seconds per window, resulting in an expected runtime of 2 hours and 10 minutes to estimate the covariance of two assets over a whole year, assuming 252 trading days in a year.

**Table 16:** Model optimization runtimes in seconds for two assets and an estimation window of 1000 days using the COMFORT-DCC model

| N | Mean | St. Dev. | Min | Pctl(25) | Pctl(75) | Max |
|---|---|---|---|---|---|---|
| 9329 | 30.876 | 4.882 | 17 | 28 | 33 | 69 |

## Statutory Declaration

I, Rino Beeli, hereby declare that my thesis with title

*Modeling Conditional Betas with Application in Asset Allocation*

has been composed by myself autonomously and that no means other than those declared were used. In every single case, I have marked parts that were taken out of published or unpublished work, either verbatim or in a paraphrased manner, as such through a quotation.

This thesis has not been handed in or published before in the same or similar form.

Zurich, January 26, 2019 ......................................