Learning Opportunity 3-2

3-2-1 This exercise will explore basic properties of nilpotent matrices. See the section on Nilpotent Matrices in SCLA for help.

1. The matrix Abelow is nilpotent. Provide the necessary computation and explain how your computation shows the matrix is nilpotent.
2. What is the **index** of A?
3. Compute the dimensions of the kernels of powers of T, *only for the powers for which this computation is interesting*. "Extra" powers will decrease your marks.

$$A = \begin{bmatrix} 9 & 14 & 18 & -3 & -18 & 16 & -32 & 10 & -93 & -146 & -57 & -16 \\ -23 & -51 & -82 & 43 & 90 & -38 & 167 & -14 & 432 & 590 & 172 & 96 \\ 25 & 45 & 77 & -22 & -65 & 49 & -133 & 27 & -368 & -527 & -205 & -108 \\ 9 & 9 & 19 & 9 & 2 & 24 & -13 & 27 & -52 & -105 & -78 & -37 \\ 15 & 41 & 81 & -46 & -86 & 25 & -166 & -8 & -427 & -538 & -162 & -131 \\ 31 & 74 & 125 & -70 & -140 & 49 & -257 & 4 & -665 & -884 & -256 & -155 \\ 12 & 19 & 33 & -5 & -23 & 25 & -52 & 17 & -150 & -222 & -98 & -52 \\ -26 & -61 & -107 & 56 & 115 & -43 & 215 & -2 & 562 & 743 & 229 & 145 \\ 9 & 24 & 40 & -26 & -50 & 12 & -87 & -7 & -224 & -289 & -77 & -47 \\ -15 & -35 & -62 & 32 & 66 & -25 & 124 & -1 & 325 & 429 & 134 & 86 \\ 21 & 44 & 77 & -33 & -75 & 38 & -146 & 12 & -389 & -529 & -180 & -108 \\ -5 & -11 & -18 & 9 & 20 & -8 & 36 & 0 & 96 & 129 & 41 & 22 \end{bmatrix}$$

```
A: matrix(QQ, [[9, 14, 18, -3, -18, 16, -32, 10, -93, -146, -57, -16] [-23, -51, -82, 43, 90, -38,
[25, 45, 77, -22, -65, 49, -133, 27, -368, -527, -205, -108] [9, 9, 19, 9, 2, 24, -13, 27, -52, -105,
[15, 41, 81, -46, -86, 25, -166, -8, -427, -538, -162, -131] [31, 74, 125, -70, -140, 49, -257, 4, -6
[12, 19, 33, -5, -23, 25, -52, 17, -150, -222, -98, -52] [-26, -61, -107, 56, 115, -43, 215, -2, 562,
[9, 24, 40, -26, -50, 12, -87, -7, -224, -289, -77, -47] [-15, -35, -62, 32, 66, -25, 124, -1, 325, 4
[21, 44, 77, -33, -75, 38, -146, 12, -389, -529, -180, -108] [-5, -11, -18, 9, 20, -8, 36, 0, 96, 129
```

Full marks for the verification of nilpotentcy, the index, and the dimensions of the kernels.

3-2-2 Challenge: Lights Out!

1. Learn how to play the "Lights Out!" game at `http://www.logicgamesonline.com/lightsout/`.
2. Write a Python routine which accepts an initial state of a game as input (in a matrix) and returns the coefficient matrix of a system of equations that may be used to solve the game. Do not just create the matrix "by hand." Your routine should be general enough to accept a square matrix of any size (not just 5×5).
3. Entries of your matrix should come from the field `Integers(2)` rather than `QQ` or `QQbar`. Use `1` for a light being on and `0` for a light being off.
4. Use your coefficient matrix to solve a particular nontrivial instance of the game.

Notes:

1. The hardest part of this exercise is to not confuse the table of on/off lights with the coefficient matrix you will build. When the table is `3\times 3`, the coefficient matrix is `9\times 9`.

2. Start with $n = 3$, but generalize your code to work for any n, the website gives you puzzles with just $n = 5$.

3. Theory+computation question: other than solutions which have you clicking the same cell many times (an even number of times), how many different ways are there to solve a given 5×5 puzzle?

4. Super-challenge: design an unsolvable puzzle.