Learning Opportunity 2-4

2-4-1 This is a Sage exercise that builds a pair of complementary orthogonal projectors.

1. Create two non-trivial vectors, each with six integer entries. Choose the entries so that (a) the vectors are orthogonal, and (b) the length of each vector is an integer, (c) there are not too many zeros in each vector.
2. Let Q be a matrix over the rationals whose columns are your orthogonal vectors, but scaled by their length so that they are now a set of two orthonormal vectors.
3. Create $P = QQ^*$ and $C = I_6 - P$.
4. Use Sage to check that P and C are projector matrices.
5. Compute the rank of P and of C. Is this what you would expect for complementary projectors?
6. For each projector, compute the column space (i.e. its range). These column spaces (ranges) are vector spaces, but you can use Sage's `.basis_matrix()` command to get the basis vectors as rows of a matrix. Perform a matrix computation with the basis matrices from these two projector's ranges that will clearly demonstrate that the ranges are orthogonal vector spaces.
7. Create a random vector, v, with six entries from the rationals. Find vectors v_1 and v_2, from the two orthogonal vector spaces of the previous part, so that

$$v = v_1 + v_2$$

Full marks for two orthogonal vectors, the matrices P and Q, and a correct check on the two orthogonal vector spaces and a decomposition of a random vector.

2-4-2 This exercise will utilize the least-squares curve fitting results.

The Buzby bird in Botswana had very low populations at the start of the last decade. Because of its importance to tourism, a government effort was launched to restore the health of the Buzby population. In the late 1990's the number surviving in Chobe National Park was estimated at about 50. Part of the effort to restore the population included daily monitoring and reporting.

Efforts began January 1, 2000 and within about a year, populations began to increase. However in the middle of 2005, despite continued efforts, populations began to decline again. By January 1, 2010, populations were lower than when the program had began.

Data file includes two numbers on each line. The first is time, measured in years, starting at January 1, 2002, and ending at December 31, 2009. There is a line for each day of the year (ignore the possibility of leap years). The next number on that line is an estimate of the population within Chobe National Park, formed using standard population ecology techniques.

The following Sage code will *help get you started* with reading in the data, but there is more work to do.

```
f = open(DATA+'buzby_populations.txt')
lines = f.readlines()
for x in lines:
    map(RDF, x.split())
```

1. Model the Buzby population, as a function of time, with a polynomial function.
2. Use least-squares techniques to find the best-fitting polynomial.
3. In an effort to understand the cause of the decline, use your function to provide the best estimate of the date (day, month, year) when the decline began.

4. When might the Buzby bird be extinct in Chobe National Park?

5. Prepare a graph describing the changes in the Buzby population, with data points and model, that your boss can include in her presentation to the Botswana National Park Commission that has been funding this project. You might begin this with a Sage plot and add to it with some other graphics tool you know.

6. To add your graphic back into the notebook, use the "Data" combo box at the top of the worksheet to store it back with your worksheet. Suppose you named the edited file `buzby-plot.png` and saved it into your worksheet by that name. Then open up the TinyMCE editor (shift-click on a blue bar), click on the tree icon to insert an image, and then where it asks for the "Image URL" type in just the name of the file, here `buzby-plot.png`, and it will be visible in your worksheet (making it much easier for the tutors to see!).

Full marks for a reasonable model, an approximating polynomial and a highly informative and persuasive graphic.