Image Compression using Haar wavelet transformation

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January 22, 2015

Introduction

- Image compression is the process of minimizing the number of bytes of a graphics file.
- It also reduces the time required for images to be sent over the internet or downloaded from Web pages.

$5.4kb$ $4.5kb$

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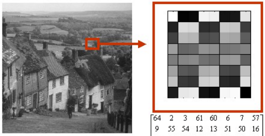
Image compression

- Convert the image in the matrix form:
  In the computer image is stored in the form of array of pixels. These pixel values are ranging from zero (for black) to 225 (for white).
- To transform a matrix representing of the image using **Haar Wavelet Transformation.**



Images are comprised of pixels represented by numbers

$$\begin{bmatrix} 64 & 2 & 3 & 61 & 60 & 6 & 7 & 57 \\ 9 & 55 & 54 & 12 & 13 & 51 & 50 & 16 \\ 17 & 47 & 46 & 20 & 21 & 43 & 42 & 24 \\ 40 & 26 & 27 & 37 & 36 & 30 & 31 & 33 \\ 32 & 34 & 35 & 29 & 28 & 38 & 39 & 25 \\ 41 & 23 & 22 & 44 & 45 & 19 & 18 & 48 \\ 49 & 15 & 14 & 52 & 53 & 11 & 10 & 56 \\ 8 & 58 & 59 & 5 & 4 & 62 & 63 & 1 \end{bmatrix}$$

Haar Wavelet Transformation

- Introduced by Hungarian mathematician Alfred Haar .

Results in Mathematics, Vol. 8 (1985)
0378-6218/85/020194-03$01.50 + 0.20/0

Alfred Haar (1885–1933)

Alfred Haar was born in Budapest one hundred years ago, on October 11th, 1885. During his relatively short life he became a professor of mathematics of world wide fame, member of the Hungarian Academy of Sciences, one of the founders of the internationally renowned Szeged school of mathematics and its journal Acta Scientiarum Mathematicarum.

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Haar Wavelet Transformation

Consider a pixel matrix A representing 8x8 images.

$$A = \begin{bmatrix} 88 & 88 & 89 & 90 & 92 & 94 & 96 & 97 \\ 90 & 90 & 91 & 92 & 93 & 95 & 97 & 97 \\ 92 & 92 & 93 & 94 & 95 & 96 & 97 & 97 \\ 93 & 93 & 94 & 95 & 96 & 96 & 96 & 96 \\ 92 & 93 & 95 & 96 & 96 & 96 & 96 & 95 \\ 92 & 93 & 95 & 96 & 96 & 96 & 96 & 95 \\ 92 & 94 & 96 & 98 & 99 & 99 & 98 & 97 \\ 94 & 96 & 99 & 101 & 103 & 103 & 102 & 101 \\ 95 & 97 & 101 & 104 & 106 & 106 & 105 & 105 \end{bmatrix}$$

We will concentrate for the first row:

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Let $v = (88, 88, 89, 90, 92, 94, 96, 97)$

Since the data string has length $2^3 = 8$, there are three steps in the transform process

1st Step: Group all of the columns in the pairs:

$$(88, 88), (89, 90), (92, 94), (96, 97)$$

2nd Step:Averaging and Subtracting.

$$T_1 = (88, 89.5, 93, 96.5, \mathbf{0}, -\mathbf{0.5}, -\mathbf{1}, -\mathbf{0.5})$$

The first 4 entries are called **approximation coefficient** and the last 4 are called **detail coefficients**. **3rd Step**:Apply the 1st and the 2nd step for the first four entries.

$$T_2 = (88.75, 84.75, \text{-}\mathbf{0.75}, -\mathbf{1.75}, \mathbf{0}, -\mathbf{0.5}, -\mathbf{1}, -\mathbf{0.5})$$

Last step: Apply the 1st and the 2nd step for the first 2 entries

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$$T_3 = (91.75, -\mathbf{3}, -\mathbf{0.75}, -\mathbf{1.75}, \mathbf{0}, -\mathbf{0.5}, -\mathbf{1}, -\mathbf{0.5})$$

Haar Wavelet Transformation Matrix

If we let

$$H_1 = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

Haar Wavelet Transformation Matrix

then AH_1 is equal to the first step above applied to all of the rows of A.In particular,

$$AH_1 = \begin{bmatrix} 88 & 89.5 & 93 & 96.5 & 0 & -0.5 & -1 & -0.5 \\ 90 & 91.5 & 94 & 97 & 0 & -0.5 & -1 & 0 \\ 92 & 93.5 & 95.5 & 97 & 0 & -0.5 & -1 & 0 \\ 93 & 94.5 & 96 & 96 & 0 & -0.5 & 0 & 0 \\ 92.5 & 95.5 & 96 & 95.5 & -0.5 & -0.5 & 0 & 0.5 \\ 93 & 97 & 99 & 97.5 & -1 & -1 & 0 & 0.5 \\ 95 & 100 & 103 & 101.5 & -1 & -1 & 0 & 0.5 \\ 96 & 102.5 & 106 & 105 & -1 & -1.5 & 0 & 0 \end{bmatrix}$$

Similary,by defining H_2 and H_3 by:

$$H_2 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad H_3 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$AH_1H_2 = \begin{bmatrix} 88.75 & 94.75 & -0.75 & -1.5 & 0 & -0.5 & -1 & -0.5 \\ 90.75 & 95.5 & -0.75 & -1.5 & 0 & -0.5 & -1 & 0 \\ 92.75 & 96.25 & -0.75 & -0.75 & 0 & -0.5 & 0 & 0 \\ 93.75 & 96 & -0.75 & 0 & 0 & -0.5 & 0 & 0 \\ 94 & 95.75 & -1.5 & 0.25 & -0.5 & -0.5 & 0 & 0.5 \\ 95 & 98.25 & -2 & 0.75 & -1 & -1 & 0 & 0.5 \\ 97.5 & 102.25 & -2.5 & 0.75 & -1 & -1 & 0 & 0.5 \\ 99.25 & 105.5 & -3.25 & 0.5 & -1 & -1.5 & 0 & 0 \end{bmatrix}$$

$$AH_1H_2H_3 = \begin{bmatrix} 91.75 & -3 & -0.75 & 0 & -1.75 & 0 & -10 & -0.5 \\ 93.13 & -2.4 & -0.8 & -1.5 & 0 & -0.5 & -10 & 0 \\ 94.5 & -1.75 & 0 & -0.75 & -0.75 & -0.5 & -0.5 & 0 \\ 94.9 & -1.1 & -0.75 & 0 & 0 & -0.5 & 0 & 0 \\ 94.9 & -0.9 & -1.5 & -0.3 & -0.5 & -0.5 & 0 & 0.5 \\ 96.6 & -1.6 & -20 & 0.75 & -10 & -1 & 0 & 0.5 \\ 99.9 & -2.4 & -2.5 & 0.75 & -1 & -1 & 0 & 0.5 \\ 102.4 & -3.1 & -3.25 & 0.5 & -1 & -0.5 & 0 & 0 \end{bmatrix}$$

Let $H = H_1H_2H_3$

To apply the procedure to the columns,we just multiply A on the left by H^T

$$B = H^T AH = \begin{bmatrix} 96 & -2.0 & -1.5 & -0.2 & -0.4 & -0.8 & -0.3 & 0.1 \\ -2.4 & -0.0 & 0.8 & -0.8 & 0.5 & 0.3 & -0.3 & 0.25 \\ -1.13 & -0.63 & 0 & -0.63 & 0 & 0 & 0 & -0.38 \\ -2.69 & 0.75 & 0.56 & -0.06 & 0.13 & 0.25 & 0 & 0.125 \\ -0.69 & -0.31 & 0 & -0.13 & 0 & 0 & 0 & 0 \\ -0.19 & -0.32 & 0 & -0.38 & 0 & 0 & -0.25 & 0 \\ -0.88 & 0.38 & 0.25 & -0.25 & 0.25 & 0.25 & 0 & 0 \\ -1.25 & 0.38 & 0.38 & 0.13 & 0 & 0.25 & 0 & 0.25 \end{bmatrix}$$

- H is an invertible and orthogonal matrix.Then

$$B = H^T AH \Longrightarrow A = (H^T)^{-1} BH^{-1} = HBH^T$$

- B represent the compressed image of A(in the sense that it is sparse).This compression is lossless.

Lossy Compression

-It reduce image quality.
Let $\epsilon > 0$. If we set all the the entries of $H^T AH$ with absolute value at most ϵ to zero.Then this matrix has more 0 values and represents a more compressed image.
if $a < \epsilon$,set a=0 ,then the result matrix becomes sparse matrix.

If we choose $\epsilon = 0.25$,then we end up with the matrix which is close to the original matrix.

$$\begin{bmatrix} 96 & -2.0 & -1.5 & 0.0 & -0.4 & -0.8 & -0.3 & 0.0 \\ -2.4 & 0.0 & 0.8 & -0.8 & 0.5 & 0.3 & -0.3 & 0 \\ -1.13 & -0.63 & 0.0 & -0.63 & 0.0 & 0.0 & 0.0 & -0.38 \\ -2.69 & 0.75 & 0.56 & -0.06 & 0.0 & 0.0 & 0.0 & 0 \\ -0.69 & -0.31 & 0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -0.32 & 0 & -0.38 & 0 & 0 & 0.0 & 0 \\ -0.88 & 0.38 & 0.0 & 0.0 & 0.0 & 0.0 & 0 & 0 \\ -1.25 & 0.38 & 0.38 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

conclusion

- Haar wavelet transformation is computationally efficient and effective algorithm for image compression.
- It is faster than the other Image compression such that Discrete Fourier Transform(DFT)
- It preserve length and angle of the image.

The End

Thank You