Learning Opportunity 1-2

AIMS-ZA Advanced Linear Algebra with Sage

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1-2-1 For the data points below, (x_i, y_i), $1 \leq i \leq 10$ find an interpolating polynomial through the points by employing a VanderMonde matrix and solving the appropriate linear system.

```
(0.62, 0.014491456), (-1.18, -0.78141223),
(-1.90, 0.24276501), (-1.28, -0.83824589),
(1.13, 1.675439), (-0.80, -0.078602254),
(-0.98, -0.39967253), (-2.00, 2.0010134),
(-1.84, 0.027698042), (0.70000, 0.017342488)
```

Hint: `ymin=` and `ymax=` can help to control the scale of your plot.

Full marks for the correct polynomial and a plot of the data and the polynomial together on one set of axes.

1-2-2 This exercise asks you to solve the linear system with coefficent matrix A and vector of constants b via an LU decomposition.

$$A = \begin{bmatrix} -1 & -3 & -3 & -1 & -3 & 8 & 1 & -4 \\ 1 & 2 & 2 & 1 & 2 & -6 & 0 & 5 \\ -2 & -3 & -2 & -2 & -4 & 5 & 6 & -7 \\ 0 & 1 & 1 & 1 & 0 & -6 & 8 & -1 \\ 0 & -3 & -4 & 1 & -2 & 9 & -1 & 0 \\ 0 & 4 & 4 & 0 & 3 & -9 & 0 & -4 \\ -1 & 0 & 0 & 1 & -1 & -2 & 7 & -6 \\ 0 & -1 & -2 & 0 & -1 & 8 & -8 & 4 \end{bmatrix} \qquad b = \begin{bmatrix} -56 \\ 47 \\ -107 \\ -43 \\ -15 \\ 9 \\ -76 \\ 50 \end{bmatrix}$$

```
A:
[[-1, -3, -3, -1, -3, 8, 1, -4], [1, 2, 2, 1, 2, -6, 0, 5],
[-2, -3, -2, -2, -4, 5, 6, -7], [0, 1, 1, 1, 0, -6, 8, -1],
[0, -3, -4, 1, -2, 9, -1, 0], [0, 4, 4, 0, 3, -9, 0, -4],
[-1, 0, 0, 1, -1, -2, 7, -6], [0, -1, -2, 0, -1, 8, -8, 4]]
b: [-56, 47, -107, -43, -15, 9, -76, 50]
```

1. Use Sage to create an LU decomposition of A *without* providing any options for the pivoting strategy. Note carefully that you will get back a permutation matrix (P) as part of the output/decomposition. You will need to decide how this affects the subsequent steps in ways different than what was shown in lecture.
2. Use Sage to solve the two (simple) linear systems with L and U as coefficient matrices. Notice that because of the triangular form of the matrices, these solutions could be obtained easily "by hand" with backsolving.
3. Form the eventual solution, x, to the system and exhibit the correctness of your result by showing that $Ax = b$ with a single Sage statement that returns `True`.

Full marks for two intermediate solutions from the backsolving steps, and a solution with the requested check.