Learning Opportunity 3-4

AIMS-ZA Advanced Linear Algebra with Sage

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3-4-1 This exercise will familiarize you with rational canonical form. Use the matrix A below, the PDF will not contain the whole matrix, so see the worksheet version.

$$
A = \begin{bmatrix}
21928482449510 & 100556052911902 & -235111658224485 & 791805265072981 & -747346360744229 & 20518\cdots \\
18593117807143 & 85261282016259 & -199350716079749 & 671370137416051 & -633673516943483 & 17397\cdots \\
9541025670851 & 43751676875271 & -102296469754808 & 344512406841077 & -325168452723126 & 89277\cdots \\
11981840525378 & 54944366722082 & -128466269509679 & 432646637487153 & -408354052353997 & 11211\cdots \\
13890299302220 & 63695865240618 & -148928283117783 & 501558278193267 & -473396388070839 & 12997\cdots \\
42733887106725 & 195962078604439 & -458181955960303 & 1543057828528120 & -1456417002502565 & 39986\cdots \\
-36923220033687 & -169316470732667 & 395881449495023 & -1333243183895978 & 1258383195666016 & -34549\cdots \\
26240580250078 & 120329766476084 & -281344882068409 & 947508773157291 & -894307305120467 & 24553\cdots \\
7456492586158 & 34192765673804 & -79946632543678 & 269242984509188 & -254125317588676 & 69771\cdots \\
17130046946785 & 78552171182291 & -183664042997585 & 618540813083438 & -583810497381790 & 16028\cdots \\
-15926876561498 & -73034868377120 & 170763949571782 & -575096084315419 & 542805134822621 & -14903\cdots \\
4533960669100 & 20791096830075 & -48611986862015 & 163714669713237 & -154522292379380 & 42425\cdots \\
-27426428759333 & -125767636843412 & 294059250590751 & -990328019069197 & 934722302270697 & -25663\cdots \\
-2550696499339 & -11696567078026 & 27347922582011 & -92101893610926 & 86930482844520 & -2386\cdots \\
-25192369217877 & -115523051644602 & 270106228493684 & -909659416395748 & 858583144246368 & -23573\cdots \\
1368040820684 & 6273338163029 & -14667788867193 & 49397943735413 & -46624309303937 & 12801\cdots \\
12629721457624 & 57915313182335 & -135412684888143 & 456040663322286 & -430434531656063 & 11817\cdots \\
18480027346571 & 84742690524076 & -198138190545960 & 667286614029947 & -629819279625326 & 17292\cdots \\
-3659403350178 & -16780694436574 & 39235201823717 & -132135683029093 & 124716426158252 & -3424\cdots \\
-3409823506352 & -15636211831968 & 36559268826466 & -123123718583815 & 116210472027320 & -3190\cdots \\
-3986522613241 & -18280744241446 & 42742490147002 & -143947471631730 & 135864995946996 & -3730\cdots
\end{bmatrix}
$$

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[-3986522613241, -18280744241446, 42742490147002, -143947471631730, 135864995946996, -373026875614163, -1
```

1. Compute the eigenvalues of B and note their algebraic multiplicities. Verify that the matrix is deficient.

2. Compute the characteristic polynomial of B, and then its factored form.

3. Verify the Cayley-Hamilton Theorem.

4. From the characteristic polynomial, compute the determinant. Check with Sage.

5. Compute the minimum polynomial of B, and then its factored form.

6. Check that B satisfies its minimum polynomial.

7. From the minimum polynomial, determine the dimensions of the generalized eigenspaces of B. Check with Sage.

8. Compute the rational canonical form of B.

9. From the rational canonical form, compute the characteristic polynomial.

10. From the rational canonical form, compute the determinant.

11. From the rational canonical form, compute the minimum polynomial.

12. From the rational canonical form, compute the dimensions of the generalized eigenspaces.

3-4-2 Study the "Heat Distribution" problem at http://aix1.uottawa.ca/~jkhoury/temp.htm.

1. Form a new problem in Sage, even larger than the 6×6 grid in the last example given in the problem description.

2. Construct sensible boundary conditions by hand, or with Sage's random vector constructor.

3. Solve the problem you have constructed.