Learning Opportunity 1-1

AIMS-ZA Advanced Linear Algebra with Sage

Robert A. Beezer

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1-1-1 Consider the square matrix A below. Compute the following items using Sage and answer the associated questions for this matrix. (Note: you will likely need to use Sage's help facilities, such as tab-completion to determine which methods to use.)

$$A = \begin{bmatrix} 0 & -1 & 0 & 2 \\ 1 & 4 & -1 & -7 \\ 2 & 4 & -1 & -6 \\ 0 & 2 & 1 & -3 \end{bmatrix}$$

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[[0, -1, 0, 2], [1, 4, -1, -7], [2, 4, -1, -6], [0, 2, 1, -3]]
```

1. Row-reduce A. What important property of the matrix is implied by this result?
2. Use a single Sage command to determine if this matrix is singular or not.
3. What do you expect for the null space of the matrix and why? Compute the null space with Sage and verify your suspicion. (Read the documentation for the Sage command `.right_kernel()`.)
4. Use Sage to solve the system of equations that has A as its coefficient matrix and the vector of constants $b = (2, -1, 7, -2)$. What is the size of the solution set, and how do you know this size before even using Sage? Can you find the relevant theorem in FCLA?
5. How would the solution set change if we used a different vector for b? (The *size* of the set, and the *contents* of the set, might both change.)

Full marks for five answers, including careful explanations where requested.

1-1-2 This exercise uses theorems and techniques from Sections FS and DM of FCLA, as applied to the matrix T.

$$T = \begin{bmatrix} 1 & 2 & 0 & 4 & -1 \\ -2 & -4 & 0 & -8 & 2 \\ -1 & -2 & 2 & -8 & 7 \end{bmatrix}$$

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[[1, 2, 0, 4, -1], [-2, -4, 0, -8, 2], [-1, -2, 2, -8, 7]]
```

1. Determine by hand the row operations necessary to convert T into its reduced row-echelon form, a matrix we call B. You do not need to submit this part of your work. Hint: follow the algorithm in FCLA at the start of the proof of Theorem REMEF.
2. For each row operation you found, form the associated elementary matrix, using Sage's `matrix.elementary()` constructor.
3. Use Sage to form the product of these elementary matrices as the matrix J, so that the matrix product JT equals B. Hint: be sure to use the proper order when you use matrix multiplication to form your product.
4. Compare your answer to the previous part to the J matrix produced by Sage's `.extended_echelon_form()` method for T. Can you explain the discrepancy?
5. Find a matrix S that provides the matrix decomposition of T as $T = SB$.

Full marks for five answers, including explanations where requested.