Learning Opportunity 2-5

AIMS-ZA Advanced Linear Algebra with Sage

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2-5-1 This exercise will utilize the least-squares curve fitting results.

The Buzby bird in Botswana had very low populations at the start of the last decade. Because of its importance to tourism, a government effort was launched to restore the health of the Buzby population. In the late 1990's the number surviving in Chobe National Park was estimated at about 50. Part of the effort to restore the population included daily monitoring and reporting.

Efforts began January 1, 2000 and within about a year, populations began to increase. However in the middle of 2005, despite continued efforts, populations began to decline again. By January 1, 2010, populations were lower than when the program had began.

Data file included here has two numbers on each line. The first is time, measured in years, with time zero being January 1, 2002. The next number on that line is an estimate of the population within Chobe National Park, formed using standard population ecology techniques.

The following Sage code will *help get you started* with reading in the data, but there is more work to do. You will need to be sure `buzby-populations.txt` is located in the same directory as your notebook.

```
f = open('buzby-populations.txt')
lines = f.readlines()
for x in lines:
    map(RDF, x.split())
```

1. Model the Buzby population, as a function of time, with a polynomial function.
2. Use least-squares techniques to find the best-fitting polynomial.
3. In an effort to understand the cause of the decline, use your function to provide the best estimate of the date (day, month, year) when the decline began.
4. When might the Buzby bird be extinct in Chobe National Park?
5. Prepare a graph describing the changes in the Buzby population, with data points and model, that your boss can include in her presentation to the Botswana National Park Commission that has been funding this project. You might begin this with a Sage plot and add to it, using some other graphics tool you know.
6. Add your graphic into your Jupyter notebook and make sure it gets communicated to the tutors when you submit this learning opportunity.

Full marks for a reasonable model, an approximating polynomial, and a highly informative and persuasive graphic.

2-5-2 This exercise will analyze the 10×10 matrix A below:

$$A = \begin{bmatrix} -11 & -22 & 24 & -50 & -14 & -24 & -24 & -11 & 27 & 28 \\ 6 & 17 & -12 & 29 & 5 & 16 & 16 & 7 & -14 & -16 \\ -18 & 2 & 26 & -44 & -37 & -14 & -14 & -20 & 31 & 35 \\ -21 & 36 & 15 & -6 & -45 & 11 & 11 & -20 & 15 & 25 \\ 1 & 3 & 3 & -3 & -4 & 0 & 0 & 1 & 3 & 2 \\ 6 & -14 & -18 & 21 & 22 & 0 & 2 & 6 & -18 & -15 \\ 3 & -51 & 30 & -75 & 7 & -42 & -44 & 0 & 34 & 25 \\ 8 & 16 & -26 & 50 & 17 & 22 & 22 & 8 & -29 & -26 \\ -7 & 24 & 4 & 8 & -18 & 11 & 11 & -5 & 1 & 6 \\ -3 & -3 & 1 & -3 & -2 & -2 & -2 & -3 & 1 & 5 \end{bmatrix}$$

```
[[-11, -22, 24, -50, -14, -24, -24, -11, 27, 28],
[6, 17, -12, 29, 5, 16, 16, 7, -14, -16],
[-18, 2, 26, -44, -37, -14, -14, -20, 31, 35],
[-21, 36, 15, -6, -45, 11, 11, -20, 15, 25],
[1, 3, 3, -3, -4, 0, 0, 1, 3, 2],
[6, -14, -18, 21, 22, 0, 2, 6, -18, -15],
[3, -51, 30, -75, 7, -42, -44, 0, 34, 25],
[8, 16, -26, 50, 17, 22, 22, 8, -29, -26],
[-7, 24, 4, 8, -18, 11, 11, -5, 1, 6],
[-3, -3, 1, -3, -2, -2, -2, -3, 1, 5]]
```

1. Use Sage and theorems from FCLA to verify that A is deficient.
2. Compute the generalized eigenspace of A for each eigenvalue using the definition and Sage's `.right_kernel()` matrix method. (Generalized eigenspaces do not appear to be implemented.)
3. Describe a decomposition of $\mathbb{C}^{10}$ into invariant subspaces relative to the linear transformation T given by $T(x) = Ax$.
4. Construct a random vector, v, from $\mathbb{C}^{10}$. Express v as a sum of generalized eigenvectors of A. Provide computational proof that each of your generalized eigenvectors really is a generalized eigenvector and also really provide a decomposition of v.

Full marks for a convincing deficiency check, efficient computation of the generalized eigenspaces, and a decomposition of a random vector which checks that the decomposition is correct.