AIMS DMG Sage Demonstration 06
AIMS 2013-14: Designs, Matroids and Graphs

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1 Linear Algebra over $GF(7)$, Sage Style

Chalkboard example, reprised. Note how we set the field of scalars.

```
F = FiniteField(7)
A = matrix(F, 3, 3, [[1, 0, 1], [5, 1, 3], [2, 6, 5]])
b = vector(F, [2, 5, 1])
A, b
```

Augment and row-reduce for solution.

```
X
```

Check solution

```
X
```

Grab columns (`.columns()`), verify linear combination.

```
X
```

Coefficient matrix is nonsingular, so full rank, unique solutions, etc.

```
X
```

Make the relevant vector space (`V`), so we can test a set of vectors for linear independence. Columns of coefficient matrix are linearly independent (empty list indicates no relations of linear dependence):

```
V = F^3
V.linear_dependence(A.columns())
```

Try eigenvalues directly, then via definitions and characteristic polynomial.

```
X
```

2 Linear Algebra over $GF(17)$

Larger example. Rectangular coefficient matrix.

```
G = FiniteField(17)
B = matrix(G, [
[1, 15, 0, 0, 1, 3, 0, 0],
[1, 16, 0, 0, 0, 1, 1, 14],
[15, 7, 1, 2, 14, 11, 16, 15],
[1, 15, 16, 16, 0, 15, 1, 0],
[1, 2, 0, 0, 15, 0, 1, 11],
[2, 16, 0, 1, 0, 1, 1, 12],
[14, 6, 0, 16, 14, 12, 2, 10]
])
B
```

```
c = vector(G, [5, 2, 3, 11, 9, 0, 0])
```

```
B.solve_right(c)
```

```
d = vector(G, [5, 2, 3, 11, 9, 0, 1])
```

```
B.solve_right(d)
```

How do we explain this different behavior?

```
X
```

c and d, inside and outside column space? By hand, first.

```
X
```

c and d, inside and outside column space? With Sage membership, second.

```
X
```

Infinitely many solutions for c, find them with null space.

```
X
```

```
X
```

3 Eigen-stuff over a Finite Field

```
E = matrix(G, [
[8, 10, 15, 1, 7, 2, 15, 8, 13, 6],
[4, 16, 5, 11, 14, 15, 3, 9, 14, 12],
[0, 5, 10, 4, 15, 1, 7, 10, 3, 11],
[15, 14, 6, 5, 16, 1, 10, 16, 6, 9],
[1, 16, 5, 13, 0, 16, 7, 9, 11, 16],
[13, 9, 13, 9, 13, 15, 1, 16, 6, 9],
[4, 0, 0, 0, 16, 1, 16, 13, 0, 0],
[0, 4, 0, 0, 13, 16, 13, 8, 16, 0],
[0, 0, 4, 0, 4, 13, 12, 5, 8, 9],
[0, 0, 0, 4, 13, 5, 8, 12, 4, 11]
```

```
])
```

Characteristic polynomial, factored.

```
X
```

`.eigenvalues()`, `.eigenspaces_right()`, `.eigenmatrix_right()`

```
X
```

Perform similarity check with output of eigenmatrix method.

```
X
```