These exercises are to be done by hand and solutions to these problems should be typed up in LaTeX.

Exercise 1. Consider the following graph G.

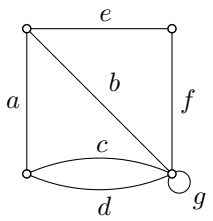

Figure 1: The graph G.

(i) What is the rank of $M(G)$, the cycle matroid on G?

(ii) List all the bases in $M(G)$.

Defn: A set of elements of a matroid is a *dependent set* if it is not independent.

(iii) Find three dependent sets of this matroid of three different sizes.

Defn: A *circuit* is a minimal dependent set. (Minimal means that the removal of any element will yield an independent set)

(iv) List all the circuits in $M(G)$.

(v) How is a circuit in the matroid represented in the graph? That is, the circuits in the matroid correspond to what in the graph?

(vi) Do you have any circuits with two elements? Any with one?

Defn: A circuit with only one element is called a *loop* in the matroid. Elements in a two-element circuit are called *parallel elements*.

(vii) Draw a geometric representation of $M(G)$.

Note: In the geometric representation, parallel elements are drawn by placing two points adjacent to each other (with no line in between). A loop is listed off to the side (it does not get a point!).

Exercise 2. A matroid has ground set $E = \{a, b, c\}$ with bases $\{\{a, b\}, \{a, c\}\}$. Draw a graph representing this matroid. Daw a second graph also representing this matroid that is different from the first and explain how the graphs differ in a structural way.

Recall: A basis is a maximal independent set. This is a set that is not dependent, and is represented by a spanning tree or spanning forest of the graph.

Exercise 3. A matroid M has ground set $E = \{a, b, c, d\}$ with bases
$\{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$.

(i) Find the circuits of this matroid.

(ii) Deduce from the circuits that there is no graph representing this matroid. (Explain in full sentences your reasoning).

Exercise 4. Let M_1 and M_2 be the rank-3 matroids shown in Figure 2.

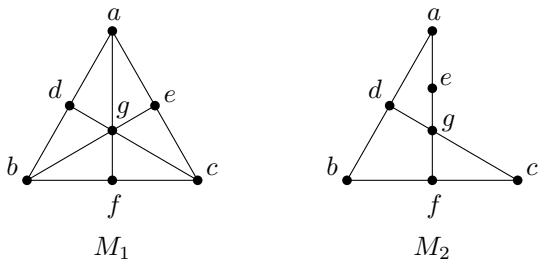

Figure 2: Rank-3 matroids M_1 and M_2.

(i) Give an example of a subset that is a basis in one of M_1 and M_2, but a circuit in the other.

(ii) Do you think it is possible that these matroids are isomorphic? Why or why not?

(iii) For each of M_1 and M_2, give examples of the following:

  (a) A basis other than $\{a, b, c\}$.

  (b) A four-element circuit.

  (c) A three-element circuit.

(iv) What is the rank of each matroid?