Euler's Formula

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Let G be a plane graph with n vertices, e edges, and f faces.

Euler's formula. (1750) If G is connected, then $n - e + f = 2$.

Corollary 1. If G has k components, then $n - e + f = k + 1$.

Corollary 2. All planar embeddings of a planar graph have the same number of faces.

Corollary 3. If G is a simple planar graph with $n \geq 3$, then $e \leq 3n - 6$.

Corollary 4. K_5 is not planar.

Corollary 5. If G is a simple planar graph, then $\delta \leq 5$.

Corollary 6. If G is a simple triangle-free planar graph with $n \geq 3$, then $e \leq 2n - 4$.

Corollary 7. $K_{3,3}$ is not planar.

Corollary 8. There are only 5 Platonic solids.