Physics Connected

Learn like you think — an interconnected view of algebra-based physics

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Credit for MathBookXML / PreTeXt format:
Robert A. Beezer

Compiled: 15:22:12 (-04:00), October 2, 2017

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Edition: Annual Edition 2017

Website: TMC Physics

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Preface

This text is intended for a one or two-semester undergraduate course in introductory algebra-based physics.

The purpose of creating this book is to make better use of the technology that electronic texts allow for without losing the functionality of a print book. While this text should be comparable to any other print text, when this is provided in the online format it will provide links back and forth between early and later topics. Linking from later material to earlier material will allow students to refresh their memory of what was previously discussed. Linking from earlier material to later material will inspire students to look ahead to how that topic will be used in more interesting scenarios.

Having these links will allow for some other interesting features that can be placed in the back of the book and accessed through links. Examples of this might be:

1. "Dig Deeper" where some of the more tedious and some of the more interesting aspects can be investigated. For example in 1 on the equations of motion, one might see how these equations are direct applications of calculus for those students who happen to have taken that course (which is common for biology and pre-medical students).
2. "Every Equation Tells a Story" which discusses how the description-in-English and the description-with-math interrelate to build intuition in both directions.
3. "Examples", with the difference from a traditional textbook being that students can interact with the example as: "If you have this question, then go here. If you have that question, then go there."
4. "In the 'Real World"' where students see how the concept lives in the messy real world and why physicists simplify or ignore complicating aspects.
5. "Connections", which might take one of three forms:
   (a) *Looking Back* which links back to earlier material to address the question "Where have I seen this before?"
   (b) *Looking Ahead* which links ahead to later material to address the question "When will I ever use this?"
   (c) *In Detail* which links to popular or complex topics to address the question "Why is this interesting?"

The goal of the book is to encourage curiosity in the reader. Since there is an expectation that students will explore the material on their own, advanced topics will explicitly note where the reader can look for supporting material and basic topics will be motivated with links to more advanced topics. To help maintain the interest of the reader, recurring characters will be featured in the examples. These characters will live a storyline and interact with each other. It is possible to read the examples as a separate storyline for the N interacting characters.

I am choosing the approach described above based on the assumption that students will prefer to develop their knowledge by building a world-view that connects to their current understanding, their interests, and their world-view. Providing the cross-referencing links without distracting students with all of the information at once will enable them to explore the information. Writing the text in a narrative style that helps students see the explanations for the world they live in will encourage them to explore "what happens when I do this" in their real life. Fostering this spirit of exploration will enable the instructors to bring their own active-learning techniques into the classroom.

This textbook is in several Parts: *Part I* is for the preliminaries, including descriptions of science in general, physics in particular, and the use of math. *Part II* is intended to introduce three fundamental and powerful concepts. These concepts are motion, force, and energy. I have found that if a student can understand these ideas sufficiently well, then they can quickly pick up any other idea that we introduce, even if the idea seems initially unfamiliar. *Part III* develops the ideas in Part II by introducing momentum, circular motion, rotational motion, torque, and the Newtonian theory of gravitation. *Parts IV and V* are oscillations and thermodynamics. With the traditional organization of the two-semester introductory physics, these parts can be covered in either order and can be chosen to be put in either semester. *Part VI* covers electricity, magnetism, light, and optics. This is traditionally the meat of the second semester. *Part VII* touches on the topics that are usually referred to as "modern physics". The goal with including these chapters is to provide some inspiration for what some students see as the tedium of the standard material. These chapters will be linked to throughout the book as examples of how the traditional material supports the material that may be in the news and is more talked about in popular science. The last final part, *Part VIII*, holds the answers to the interactive examples mentioned above, the bulk of the adventures the reader can investigate in order to test their understanding of the material, and the story lines of each of the characters in the text.

> ***A note about viewing the PDF online:*** If you are viewing this as a PDF set to view "single page," then the links will take you to the top of the relevant page, rather than to the specific topic. If, on the other hand, you are viewing this in "continuous view" then you should go directly to the location of interest. If you are viewing this in "two-page" mode (whether continuous or not), it might not be immediately obvious to which page (left or right) you have jumped. Most of the PDF viewers I have encountered allow you to follow links and to return to your previous location. On most PCs, the way to return to your previous location is by holding the `ctrl` key and pressing the [←] (`Left-Arrow`) button. There are a few PDF viewers that do not allow you to "go back" to the location you linked from. Whether or not you have that capability, I have placed "return links" in the margins so that you can get back to the place from which you linked.

Contents

Part I

Prerequisites

The chapters in this Part are the preliminaries, including descriptions of science in general, physics in particular, and the use of math.

Chapter 1

The Story of Science

Once upon a time somebody saw the world around them and thought something equivalent to "well, that's an interesting pattern…" and predictions were born. Every human and many animals build their own world of expectations such as: objects will fall down, food will arrive at mealtime, or certain people will smile at me. Scientists study the patterns in the world around us and do so in a fairly specific way. Novelists, sociologists, historians, and cartoonists also look at the world around us in a very particular way. The story of humanity is a story about observing the world around us.

Scientists, in general, observe patterns through careful, detailed measurements …

Physicists, in particular, consider the patterns in the physical world around us.

Some patterns that you might experience help us take very different experiences and group them together. For example, there are ways in which dropping your keys and throwing a dog toy are very similar. They both fall, even thought the fall along rather different paths. There are also patterns that you experience that might look very similar but can be treated very differently. For example, the path of baseball pitch (See Investigation 2.2.1.) is very different for a fast ball compared to a slider, a curve ball, or a knuckleball.

images/i-BoonePark.jpg

Figure 1.0.1: Life is full of examples of physics all around us.

1.1 Careful, Detailed Observation

[Discussion of “casual observer” as intuition versus “scientific observing” and mathematical modelling]

Paragraph referenced by Clarification of Newton’s laws 3.2.1

[Discussion of common student comment: “in physics class it is this way, but in *real life* it is that way.”]

1.2 Theory versus Law

Section referenced by Section 3.2

List of examples

Chapter 2

Seeing Physics

What you will find in this book is a series of chapters that, on the surface, feel like a list of isolated topics. Each chapter will have examples that focus your attention on examples of that specific concept. However, the really interesting aspect of physics is that these descriptions of the world around us come together in different ways to explain complex systems that might feel unrelated. For example, the thermodynamics of making your refrigerator work on Earth comes from the same theories of thermodynamics that help us understand the heat flow of the sun. Furthermore, in order to understand the sun, we also need to understand the gravitational interaction, which also describes how baseballs fly through the air.

This chapter will introduce a set of quick-overview explanations of phenomena to indicate how different ideas tie together in some complex systems. The point is specifically to over-simplify complex ideas in order to "get the idea". You will also be pointed to the various chapters that go into the details of the relevant physics where you can learn more. Then, at the end of the book in Chapter 1, we will revisit each of these ideas and go into the description in more depth assuming you have understood each of the relevant chapters, with reference back to the sections that provide the basis of our understanding.

Warning 2.0.1. Since this particular chapter is intended to be background introduction, rather than a place to study details, none of the links to other sections here will have return links in the rest of the text. So, if you intend to use this as a jumping off point, you might want to create a bookmark here so that you can return after you read the details in other sections.

2.1 The Flame Challenge and Other Brief Descriptions

The point of this section is to whet your whistle, to provide an appetizer before the main meal, to inspire your curiosity. This section will have very brief descriptions, inspired by The Flame Challenge. Since that is the inspiration, we will start by referencing it explicitly. After that you will find some brief explanations for complex ideas.

2.1.1 The Flame Challenge

- The Flame Challenge
- Useful? How Alan Alda Makes Science Understandable
  - 2012: What is a flame?
  - 2013: What is time?
  - 2014: What is color?
  - 2015: What is sleep?
  - 2016: What is sound?
  - 2017: What is energy?

2.1.2 The Forming of Matter in the Universe

In the early ages of the universe, which is an entirely different story that could be told, there were a ridiculously large number of particles created and drifting around. There were a variety of types (Subsection 2.3.4), some being positively charged (Section 1.1), some negatively charged, and some were neutral; but the larger ones tended to gradually decay (Subsection 2.3.5) into smaller ones. The smaller of the positively-charged baryons (Section 2.3), which we call protons, and the smallest of the negatively-charged leptons (Section 2.3), which we call electrons, also tended to stick together because of their electrical charges (Section 1.1), forming hydrogen atoms. You may note that as this happens, sometimes the more ambitious of the particles form larger clumps of two protons and two neutrons, making helium atoms that are held together by the strong nuclear force ([need ref])

2.1.3 Things in the Sky

Example 2.1.1 (*The Sun*)

The bright, shiny sun, which keeps us all alive, is a nice example of a rather complex system that allows us to verify our various theories of the world around us. As an over-simplification of the process, we can consider the existence of a star in three phases: the ignition (some have said "birth") of a star, the shining (some would say "life") of the star, and the snuffing ("death"?) of the star.

2.1.4 Things on the ground

Example 2.1.2 (*Hot Tea and Iced Tea*)

Example referenced by *Section 1.2*

On 28 April, 2017, CBC Broadcasting published a Quirks and Quarks episode discussing why hot water sounds different from cold water when they are poured. Spoiler Alert: It is due to surface tension, size of droplets when heated, and auditory perception.

2.1.5 Kitchen Appliances

Example 2.1.3 (*Oven*)

...

Example 2.1.4 (*Refrigerator*)

...

Example 2.1.5 (*Microwave*)

...

Example 2.1.6 (*Television*)

...

2.1.6 Automobile

Example 2.1.7 (*Coolant and Antifreeze*)

...

Example 2.1.8 (*Tires*)

...

Example 2.1.9 (*Torque*)

...

2.1.7 Cool Ideas

Example 2.1.10 (*Black Holes*)

...

Example 2.1.11 (*Quantum Mechanics*)

...

Example 2.1.12 (*Relativity*)

...

Example 2.1.13 (*String Theory*)

...

Example 2.1.14 (*Fusion*)

On 28 April, 2017, CBC Broadcasting published a Quirks and Quarks *episode discussing a documentary compares the massive scale ITER approach to fusion with the much smaller approach by a Canadian company.* I don't think I want to use this, but it might be helpful to listen again to the nice summary of fusion. *Maybe get some resources on "state of the art".*

2.2 Effective Theory

All of our explanations are approximations. This section will describe some physics in the world around us in one or two paragraphs with links to the sections in the book that provide the detailed understanding of that piece which connects to the mathematics and the underlying foundation. Each topic will also link to a more detailed discussion at the end of the book with a longer conversation that gets into more nitty-gritty details which assume you have learned the details from the book. In short, this section looks forward to what is possible to understand and that chapter looks back at how you do understand. Each of these topics will also be accompanied by a five-minute podcast describing the topic.

The term "effective theory" is used in physics to describe a wide-reaching phenomenon which can be approximated by a simpler theory in a smaller circumstance. So, for example, Einstein's theory of general relativity as a complex description of the gravitational interaction. It would be unwieldy and impractical to use that to describe our day-to-day interactions with the gravitational interaction. On the other hand, Newton's theory of the gravitational interaction is a special case of Einstein's general theory of relativity that works perfectly well so long as you behave yourself and do not try to travel at a significant fraction of the speed of light. We can say that Newton's theory of gravity is an effective theory for Einstein's theory of gravity that accounts for acceleration at low speeds. Likewise, Einstein's special theory of relativity is an effective theory of the general theory of relativity. The special theory is relevant when you do not allow for acceleration, but do allow for faster speeds. Once you reach beyond the limitations of the effective theory, the description "breaks down".

For example, see the discussion on the acceleration due to gravity being "locally constant".

List of examples

(Continued on next page)

Example 2.1.7.10	Black Holes
Example 2.1.7.11	Quantum Mechanics
Example 2.1.7.12	Relativity
Example 2.1.7.13	String Theory
Example 2.1.7.14	Fusion

Chapter 3

Why so much math?

Mathematics is its own language. It is the language of patterns. Humans are very adept at tracking patterns. Physics is the study of patterns in the physical world. It turns out that the language of physics provides a natural and concise mechanism for expressing patterns in a uniquely precise manner.

3.1 Every equation tells a story

Equations allow us to connect physical reality to very specific predictions. For example, the equation for thermal conductivity, (1.3.1) in Subsection 1.3.1, allows Abdul to predict the time it takes for his oven to warm up to a specific temperature because $\frac{Q}{\Delta t} = \kappa A \frac{\Delta T}{\Delta x}$ says that the rate at which energy flows depends on how well air allows energy to flow, the size of the oven, and the amount the temperature needs to change across the height of the oven as follows:

$\frac{Q}{\Delta t}$	$=$	κ	A	$\frac{\Delta T}{\Delta x}$
the rate at which energy flows	depends on	how well air allows energy to flow,	the size of the oven,	and the amount the temperature needs to change / across the height of the oven

Table 3.1.1: An example of how the math describes the "story-of" a physical situation.

We will see this particular story in more detail with Example 1.3.1 when Abdul prepares to bake some bread for his friends. Some of the more important equations are listed below. By jumping between these narratives, you can get a better sense of how to think about physics in general.

Equation	Location
$\vec{F}_{\text{net}} = m\vec{a}$	Translation 3.2.8

Table 3.1.2: Locations of the "Story of" various equations

3.2 The Metric System

Section referenced by Subsection 4.1.1

The International System of Units (SI) [3.5.1] was adopted in 1960 at the eleventh meeting of the International Bureau of Weights and Measures (BIPM). [1]

In 1901 at the third meeting of the BIPM, it was declared that

1. The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram;
2. The word "weight" denotes a quantity of the same nature as a "force": the weight of a body is the product of its mass and the acceleration due to gravity; in particular, the standard weight of a body is the product of its mass and the standard acceleration due to gravity;
3. The value adopted in the International Service of Weights and Measures for the standard acceleration due to gravity is 980.665 $^{cm}/_{s^2}$, value already stated in the laws of some countries.

Other related decisions were made at the following meetings:

1. The 11th meeting (1960) redefined the meter in terms of wavelengths of light.
2. The 13th meeting (1967) redefined the second in terms of the frequency of radiation from ^{133}Cs.
3. The 17th meeting (1983) redefined the meter in terms of the speed of light and seconds.
4. The 24th (2011) and 25th (2014) meeting discussed redefining the kilogram in terms of the Planck constant, with an expectation that it will be redefined at the 26th meeting (Nov, 2018). See the references in Subsection 3.2.3 for more information.

Note Handbook 44, page B-6 talks about SI.

Note Handbook 44 webpage still links to the 2016 pdf instead of the the 2017 pdf even though it says it was updated in 2017.

There is also a special publication from NIST that summarizes the use and conversation between units in the SI.

3.2.1 Units Quantify Dimensions

.

3.2.2 Conversion from English Units

Subsection referenced by Section 4.2, Exercise 1.3.1

Note internet search comments in Subsection 4.1.1 regarding the "conversion" of kilograms-to-pounds, with special attention to (See significant digits 4.2.).

3.2.3 Fundamental Units versus Derived Units

Subsection referenced by Subsubsection 3.2.2.1, Subsection 4.1.1

Note conversation in Subsubsection 3.2.2.1 about the Newton.

See the 2012 article from SciTechDaily.com and the NIST explanation about redefining the kilogram in terms of the Planck constant at the 26th meeting (Nov, 2018) of BIPM.

Paragraph referenced by historical discussion on the definition of the kilogram

This paragraph should describe a fundamental unit (as opposed to a derived unit).

This paragraph should describe a derived unit (as opposed to a fundamental unit).

[1] In French this organization is the Bureau International des poids et mesures, so the acronym is BIPM.

3.3 A graph is worth a thousand pictures

3.3.1 Coordinate Systems

Discussion of the choice of origin (possible reference to zero-value of the potential energy)

Discussion of the choice of the positive-direction (possible reference to falling objects and using positive-up versus positive-down)

Paragraph referenced by Subsection 1.2.3 Subsection 1.6.1 Clarification of Newton's laws 3.2.1

Definition of a reference frame

- (different locations) The view from the roof versus from the ground
- (different speeds) The view from the sidewalk versus from a moving car (See also Subsection 1.6.1.)
- (different types of motion) The view from a park bench versus from a merry-go-round. (See also Section 2.4.)

3.3.2 The Vocabulary of Graphs

[Quick review of parameters and variables of $y = mx + b$ and $y = ax^2 + bx + c$.]

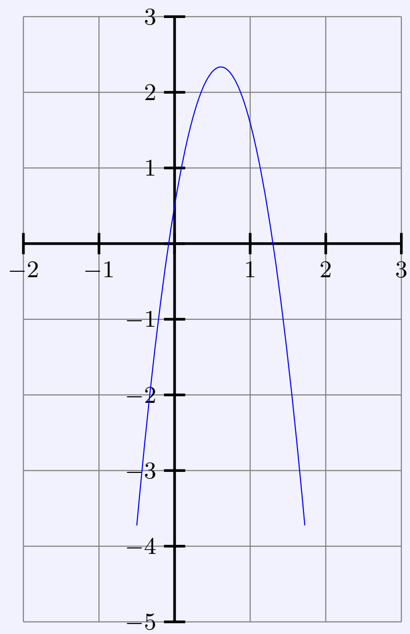


Figure 3.3.1: A random parabola

3.4 Trigonometry and Vectors

3.4.1 Trigonometry

.

3.4.2 Vectors

Subsection referenced by Subsubsection 3.2.2.2

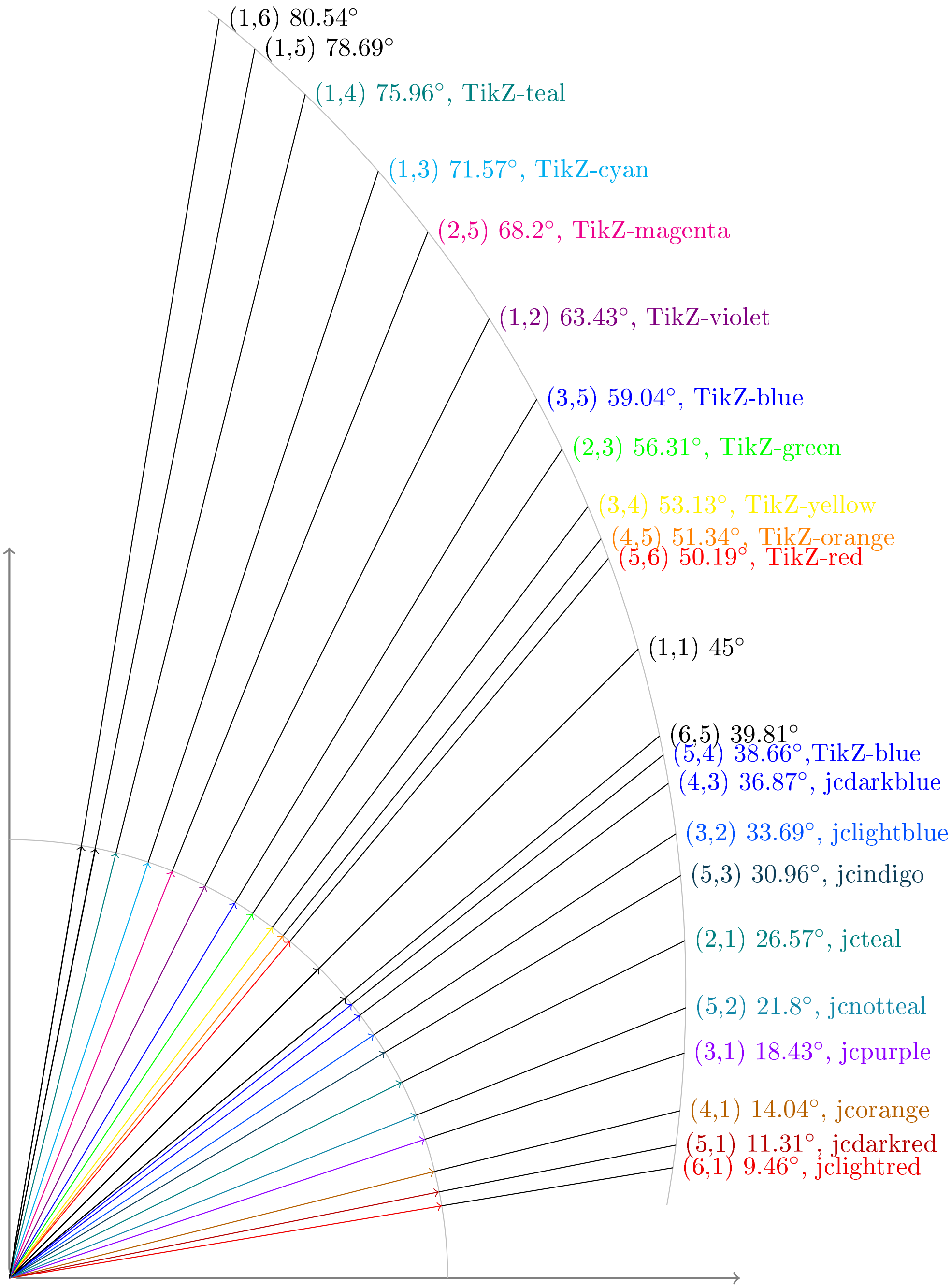


Figure 3.4.1: LaTeX lines and vectors. This will be deleted, but is here for reference.

3.4.2.1 Scalar Quantities versus Vector Quantities

definition of scalar

Paragraph referenced by Discussion of the direction of forces

definition of magnitude

Paragraph referenced by Discussion of the direction of forces

definition of direction (of a vector); how it gets expressed as an angle measured from a given reference point (usually the $+x$-axis).

Paragraph referenced by Discussion of the direction of forces

definition of vector

3.4.2.2 Vector Equations

Section referenced by the ballistic freefall $F = ma$

... If $\vec{A} = 3\vec{B}$, then this is true for each component.

$$A_x = 3B_x \tag{3.4.1}$$
$$A_y = 3B_y \tag{3.4.2}$$
$$A_z = 3B_z \tag{3.4.3}$$

This can also be written in two different ways: $A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = 3\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = (3B_x)\hat{i} + (3B_y)\hat{j} + (3B_z)\hat{k}$

Connection 3.4.2 (*Looking Ahead*)

> *This will be useful when we are discussing ballistics (2-dimensional motion), Newton's second law (combining multiple forces pushing on an object), 2-dimensional collisions, and the calculation of electrical fields.*

3.4.2.3 Multiplication, but Not Division

[define dot product]

[define cross-product]

Can do magnitude-equations like $F = ma$ or $m = F/a$. But for vector equations, while you can do $\vec{F} = m\vec{a}$, you cannot do something like $m = \dfrac{3\hat{i} + 4\hat{j}}{2\hat{i} - 5\hat{j}}$; but, in that case, you can use the magnitudes as follows $m = \frac{\sqrt{(3)^2+(4)^2}}{\sqrt{(2)^2+(-5)^2}} = \frac{\sqrt{25}}{\sqrt{29}} = \sqrt{\frac{25}{29}} = 0.928$

Paragraph referenced by Example 3.2.21

3.5 References and Further Reading

[1] *International System of Units (SI)*, Wikipedia (Visited June, 2017), https://en.wikipedia.org/wiki/International_System_of_Units.

[2] *General Conference on Weights and Measures*, Wikipedia (Visited June, 2017), https://en.wikipedia.org/wiki/General_Conference_on_Weights_and_Measures.

List of examples

Chapter 4

Estimating and Uncertainty

4.1 Precision and Accuracy

In this section, we will consider the benefits of being precise both in measurements and in our language. Sometimes people confuse the words precise and accurate, but they mean different things. It may help to remember that the opposite of precise is vague. Being precise makes it easier to determine if a statement is accurate. If we already know the answer, then we can know if a result is accurate. However, the exciting aspect of science is to study that which we do not already know. In this case, gauging accuracy can be tricky. If we are do not already know an answer, then we can try to be consistent within our accepted precision.

Since physics has its roots in the natural philosophy of the ancient Greeks and developed mathematically with Galileo and Newton, it has been around long enough for the technical language to both evolve (Newton used the word "action" for what we refer to as "force") and to be absorbed into everyday (colloquial) language. Words like force and energy have taken on broader meanings in English. In this text, we will try to be precise with the language. Hopefully we can avoid using the dismissive phrase, "Oh, you *know* what I *mean*."

One example of not being careful with the language comes when people use the term "massive" to mean "big." The word massive actually means "has a large amount of material" whereas big means "takes up a large amount of space" (which might be replaced by the word "voluminous" rather than "massive"). These are related by the density but it is possible to be massive and not voluminous (see, for example, the discussion of black holes). While it is *technically* inaccurate to use massive to mean big, "we" know what "we" mean.

Paragraph referenced by Subsection 4.1.1

4.2 Significant Figures

Section referenced by Subsection 3.2.2 Subsection 4.1.1

Note the comments in Subsection 4.1.1 regarding an internet search on the "conversion" of kilograms-to-pounds.

A short Google™ search by the author found that the conversation rate between pounds and kilograms was $1\,\text{kg} = 2.2046226218\,\text{lbs}$. Several sites go on to list about 10 decimal places for all of the conversions. First, you should recall our discussion about significant digits. Second you should note that the unit of pounds is a measure of force (how much the Earth pulls on you), whereas the unit of kilogram is a measure of mass (how much "stuff" there is). These are related in proportion to the strength of the gravitational field, which varies in the third digit (on the order of about 1%) around the globe. Some sites indicate that they are shortening their conversion factor to 3 digits for convenience, but this is not an issue of convenience, it is an issue of precision.

4.3 Scientific Notation

.

4.4 Effective Theories

Section referenced by non-inertial reference frames, air resistance Subsection 1.6.2, air resistance Subsection 2.2.1, Newton's first law, Newton's second law, Section 4.1, fundamental forces, Section 4.5.

Life is complicated. One mechanism that scientists in general and physicists in particular use to simplify their descriptions of the world around us is to build an effective theory. These are not intended to be true (accurate) to as many decimal places as can be calculated, but rather are intended to be good enough. In this context, good enough is most likely to mean something like: true to a reasonable number of decimal places.

A colloquial example of this is when you wear a smile to give the impression of happiness even if you are not in the mood. Most of your casual interactions will be the same as when you are in a good mood, but your friends who know you better will recognize the small discrepancies.

A technical example of this is that Newton's theory of gravity is very precise as long as none of the objects being described are travelling "close" to the speed of light. How close counts as close depends on the level of precision the measurement needs to be. If any of the objects are moving close to the speed of light, then we need Einstein's general theory of relativity. One way to describe this is that Newton's theory is a special case of Einstein's Theory. Another way is describe it is that Newton's theory is an effective theory for Einstein's theory, effective when the speed is low. It is possible for us to measure the difference between Newton's theory and Einstein's theory, but it is often not worth the effort of using the more complex theory in the cases where the simpler one will do; it is effectively true (rather than actually true).

Another technical example is that Einstein's special theory of relativity is a special case of the general theory of relativity. The aspect that makes it a special case is that the special theory only considers motion without acceleration.

One final case that should be mentioned up front is to notice that humans experience the Earth *as if* it were stationary.

Sometimes we imagine objects to be "massless" or "frictionless". In these instances we *usually* mean that the mass (or amount of friction) is small enough to not impact the significant digits of our calculation. In those cases, if we consider a more-precise measurement/computation, then the mass (or amount of friction) would impact the significant digits. (See comments in Example 4.5.16.)

List of examples

Part II

Introducing Motion, Force, and Energy

The chapters in this Part are intended to introduce three fundamental and powerful concepts. These concepts are motion, force, and energy. I have found that if a student can understand these ideas sufficiently well, then they can quickly pick up any other idea that we introduce, even if the idea seems initially unfamiliar.

The trio of topics in this part of the book are fundamental and powerful concepts[1]. These are fundamental in that most other topics in physics are built upon them. They are powerful in that if they are well-understood, then one is empowered to use them to understand and develop an intuition for nearly any other topic that is experienced. It has been my experience that with these topics, students can jump into a surprisingly wide variety of other, significantly more esoteric, topics and develop a reasonable grasp of the key concepts. Furthermore, the development of understanding of these ideas introduce the language and thought processes of being a professional physicist such that it nicely bridges the language barrier that might otherwise exist due to the jargon of physics.

[1]The idea of Fundamental and Powerful Concepts (F&PC) is taken from Dr. Gerald Nosich, *Learning to Think Things Through*, Prentice Hall, 2012.

Chapter 1

One-Dimensional Motion

1.1 How Physicists Use the Words (Notation)

- Position = where is it? Also discuss location as a vector and giving directions as defining a coordinate system (locate a common origin and unit-vector, then give a series of magnitudes and directions).
  - This chapter will distinguish location versus distance.
  - This chapter will distinguish distance traveled versus displacement.
- Velocity = which way did it go? Is its position changing?
  - This chapter will distinguish speed and velocity.
  - Introduce the language of “at rest”.
- Acceleration = Is its velocity changing?
  - This chapter will clarify acceleration, deceleration, and changing direction.
  - This chapter will distinguish distance traveled versus displacement.

1.2 Connecting the Concepts- distance equals rate times time

1.2.1 Position

Identifying the position requires identifying a common known position (which we could call “the origin”), a distance from that known location (which we could call “a magnitude”), and a direction from the origin in which to travel such a distance. The common example that identifies the location as “I am in my room” references “your room” as the common, known origin. If the author of this text were to tell you that he was in his room, then your next obvious question is: “OK, but where is your room?”

Position can be seen to be a vector when you describe a meeting place or destination to a friend who has never been to that location: “Well, you know where the bookstore is, right?” (establishes a common origin). “OK, so, if you face the sports gear shop. . . ” (sets the coordinate axis and defines the position direction) “. . . turn left and walk a block” (defines the magnitude and the direction).

1.2.2 Speed versus Velocity

When you are not moving, physicists will describe you as being “at rest”. When you drive to the store, your car “starts from rest” and then travels some distance in some time. When you arrive at the store, your car “ends at rest” when you arrive at your destination.

Paragraph referenced by Discussion of Newton’s First Law

When you are moving. . .

To be moving, you must be moving in a particular direction.

1.2.3 Adding Velocities

Comment on inertial reference frames.

1.3 Extending the Concepts: Changing How You Move

Section referenced by Clarification of Newton's laws 3.2.1 $F = ma$

The $F = ma$ reference to this section is about the statement that the direction of the acceleration does not determine the direction *of the motion*, but rather determines the direction *of the change* in motion. (adding sideways motion does not necessarily mean removing forwards motion.) Do we need to move that reference?

1.3.1 Moving versus Speeding Up

Paragraph referenced by Clarification of Newton's laws 3.2.2 Newton's First Law

Description of "moving" as *moving at constant velocity*.

Paragraph referenced by Section 3.1, Clarification of Newton's laws 3.2.1

The technical term **acceleration** means *changing the velocity*, which refers to *either* **speeding up** (colloquially "acceleration") *or* **slowing down** (colloquially "deceleration") *or* **changing the direction** (colloquially "turning").

Discussion of Exercise 1.3.1 and Exercise 1.3.2.

Exercise 1.3.1 (*How far will you go?*)

Exercise referenced by *Subsection 1.3.1*

You and your friend, Beth, are driving along at 55.0 mph *and run out of gas* 2.25 mi *from a gas station. You leave the car in gear and find that after* $t_1 = 1.00$ min, *you are travelling* $v_1 = 30$ mph. *Will you make it to the gas station?*

Hint. *The first thing we should do is notice what information is given to us and make sure that everything is in consistent units. I will convert everything to SI units.*

$$v_i = 55.0\ {}^{\text{mi}}\!/\!_{\text{hr}} \times \left(\frac{1609\,\text{ft}}{1.0000\,\text{mi}}\right)_{4\,\text{sig}} \times \left(\frac{1\,\text{hr}}{3600\,\text{s}}\right)_{\text{exact}} = \mathbf{24.5}8\ {}^{\text{m}}\!/\!_{\text{s}}$$

$$\Delta x = 2.25\,\text{mi} \times \left(\frac{1609\,\text{ft}}{1.0000\,\text{mi}}\right)_{4\,\text{sig}} = \mathbf{362}0.3\,\text{m} = \mathbf{3.62}0 \times 10^3\,\text{m}$$

Solution. *[This example is not done, but the work will result in the following numbers: With* t_1 *and* v_1*, you can find* $a = -1500\ {}^{\text{mi}}\!/\!_{\text{hr}^2}$*. From that you can find, for* $v_f = 0\ {}^{\text{m}}\!/\!_{\text{s}}$*, that* $t = 2.2$ min *and* $\Delta x = \mathbf{1.00}8$ mi*.]*

You do not make it to the gas station.

Exercise 1.3.2 (*How fast should you start?*)

Exercise referenced by *Subsection 1.3.1*

You and your friend, Beth, are driving along at 55.0 mph *and run out of gas* 2.25 mi *from a gas station. You put the car in neutral because you know that the car will slow down with an acceleration of* $a = 500\ {}^{\text{mi}}\!/\!_{\text{hr}^2}$*. With what speed should you be going when you put your car into neutral in order to coast to a stop at the gas station?*

Hint. *The first thing we should do is notice what information is given to us and make sure that everything is in consistent units. I will convert everything to SI units.*

$$v_i = 55.0\ ^{\mathrm{mi}}\!/_{\mathrm{hr}} \times \left(\frac{1609\,\mathrm{ft}}{1.0000\,\mathrm{mi}}\right)_{4\,\mathrm{sig}} \times \left(\frac{1\,\mathrm{hr}}{3600\,\mathrm{s}}\right)_{\mathrm{exact}} = \mathbf{24.5}8\ ^{\mathrm{m}}\!/_{\mathrm{s}}$$

$$\Delta x = 2.25\,\mathrm{mi} \times \left(\frac{1609\,\mathrm{ft}}{1.0000\,\mathrm{mi}}\right)_{4\,\mathrm{sig}} = \mathbf{362}0.3\,\mathrm{m} = \mathbf{3.62}0 \times 10^3\,\mathrm{m}$$

Solution. *[This example is not done, but the work will result in the following numbers: With $a = -500\ ^{\mathrm{mi}}\!/_{\mathrm{hr}^2}$, you can find, for $v_f = 0\ ^{\mathrm{m}}\!/_{\mathrm{s}}$, that $t = 6.6\,\mathrm{min}$ and $\Delta x = 3.025\,\mathrm{mi}$. You clearly make it to the gas station. You can also find that for $\Delta x = 2.25\,\mathrm{mi}$, $t = \mathbf{3.25}9\,\mathrm{min}$ and $v_f = \mathbf{27.8}388\ ^{\mathrm{mi}}\!/_{\mathrm{hr}}$.]*

So, if you start at $55.0\ ^{\mathrm{mi}}\!/_{\mathrm{hr}} - 27.8\ ^{\mathrm{mi}}\!/_{\mathrm{hr}} = \mathbf{27.1}6\ ^{\mathrm{mi}}\!/_{\mathrm{hr}}$ you should make it exactly.

1.4 Connecting the English to the Math

Section referenced by Example 3.2.18

The equations of constant acceleration can be summarized as

$$\begin{aligned} x_f &= x_i + v_i t + \frac{1}{2} a t^2 \\ v_f &= v_i + at \\ v_f^2 &= v_i^2 + 2a\,\Delta x \end{aligned}$$

Paragraph referenced by Exercise 1.6.1

1.5 Examples

1.5.1 Freefall

Section referenced by Exercise 1.6.1

Since acceleration is the change in velocity (magnitude and/or direction), it is possible to select your own rate of change while driving your car. However, that acceleration is difficult to measure directly. Your speedometer measures the speed and you have to compute your acceleration based on how quickly your speed changes. It turns out that there is a convenient way to start from the acceleration and compute the expected velocity: Drop a ball or your keys. To convince yourself that objects do, in fact, accelerate when they fall, we can consider dropping items. One of the complications during such an experiment will be discussed in Subsection 1.6.2. If we drop a sheet of paper, air resistance causes an obvious effect (fluttering). For this section, I will assume that the mass-to-surface-area ratio is large enough that we can effectively ignore the air resistance.

Paragraph referenced by Discussion of the description of physics

The patterns that you see when you drop objects is that objects fall faster than humans are used to paying attention to. This Investigation shows you how you can pay close attention to the patterns that result from observing falling objects. You should go do those experiments before reading further. Go ahead. I'll wait.

Investigation 1.5.1 (*The motion of dropped objects*)

Because Carl is a pitcher on the local baseball team, he decides to drop a ball and watch what happens. You and Diane decide to join him. Diane provides a few other objects that can also be dropped: a tennis *ball, a hammer, a small Wonder Woman toy, and a broken cell phone. Some of these are dropped at the same time.*

Diane notices that ***it is important to release the objects at exactly the same time.***

Carl notices that ***it is important to have the bottoms of the objects line up (rather than the tops of the objects) so that if they travel at the same speed, then they hit at the same time.***

(a) *Drop any two objects at the same time from the same height.*

(i) *Are there any objects that always hit first or last?*

Solution. *As long as you are careful about releasing at the same time, you should not see any object consistently land first or consistently land last. It is true that a piece of paper will consistently land last, but this is because of the air resistance that we previously agreed to avoid. If you crumple the paper into a tight ball (yes, it has to be a tight ball), then this will minimize the effect of air resistance and you might still be able to make the comparisons. It is possible that some of the objects you are dropping (such as those in Solution 5.5.1.1.a.ii.1) have a shape that makes air resistance relevant.*

(ii) *If so, what are the properties of those objects?*

Solution. *As long as you are careful about releasing at the same time, it is unlikely that you will find anything consistently falling faster or slower than the others. If you do notice a pattern, then the likely culprit is that air resistance is having an effect. If you have something flat, like a computer (!) or a book that is falling more slowly than something else, like a hammer, then try dropping the flat object in different orientations to see if that affects the air resistance. If you have something somewhat cylindrical, like a wine bottle (!) or a pencil that is falling more quickly than something else, like a hammer, then try dropping the cylindrical object in different orientations to see if that affects the air resistance. Remember that we are trying to eliminate differences due to air resistance so that we can study the effect of the gravitational force.* The effect you should notice is that so long as air resistance does not affect one object differently than the other, all objects fall at the same rate.

(b) *Drop one of these objects from about eye-level*

(i) *Observe the speed of the object as it falls.*

(ii) *Is the object moving at a constant speed?*

Solution. *When you drop something from eye-level, it takes less than a half-second for it to hit the ground. Due to the limited need to gauge speed, it is very difficult for most humans to distinguish constant speed from accelerated motion in this small of a time interval. Athletes can often tell is an object is moving fast or slow, but even then it is difficult to gauge acceleration. Practice measuring the time-of-flight by counting out loud: "one-one-thousand... two-one-thousand...". For this fall, you will likely only get to "one-one-thou".*

(c) *Climb a tall ladder, drop the ball from at least eight-feet high*[1]

(i) *Observe the time it takes the object to pass four rungs near the top of the ladder and compare it to the time it takes the object to pass four rungs near the bottom of the ladder*

(ii) *Is one set of four-rungs a shorter time or are they the same amount of time?*

Solution. *Measure the time-of-flight by counting out loud: "one-one-thousand... two-one-thousand...". For the four rungs near the top, you will likely only get to "one-one-thou". For the four rungs near the bottom, you will likely only get to "one-wa". Since those two distances are the same, it should be clear that the object is going faster at the bottom of the ladder.* Objects speed up (accelerate) while they fall.

You and your friends should get together to see if you can come up with a way to measure the acceleration due to the gravitational force.

Return to: freefall, the force of gravity

You did do them, right? You're not just reading ahead? Really? OK. Doing that experiment will help you see (1) that everything falls at the same rate and (2) that objects accelerate as they fall.It turns out that, ignoring the effect of air resistance, all objects fall with the same acceleration (due to the gravity). As discussed in Subsection 4.1.2, this varies slightly round the world, but only slightly. (See Table 4.1.14 for some representative samples.)

Definition 1.5.2. In this book, "being in freefall" will mean moving only under the influence of gravity and, when near the surface of the Earth, having an acceleration consistent with Convention 4.1.12.

We will start to discuss the reason for this in Section 4.1 and then get into more detail in Chapter 6. For now, Exercise 1.5.3 shows the type of experiment that can allow you to calculate the acceleration due to gravity.

Exercise 1.5.3 (*How quickly does it fall?*)

Exercise referenced by *freefall, discussion of falling objects.*

Your friend, Carl, is a baseball player and is curious to learn about the rate that baseballs fly through the air. You get on a 12 ft ladder and he lays on the ground below you aiming his radar gun (which measures speed) upwards. Each rung is 1.0 ft apart and his gun is at the first rung. When you drop the ball three rungs above the gun, he measures the final velocity to be 4.24 $^{\mathrm{m}}\!/\!_{\mathrm{s}}$. When you drop the ball six rungs above the gun, he measures the final velocity to be 6.00 $^{\mathrm{m}}\!/\!_{\mathrm{s}}$. When you drop the ball eleven rungs above the gun, he measures the final velocity to be 8.11 $^{\mathrm{m}}\!/\!_{\mathrm{s}}$. Find the acceleration of the ball in each case.

Hint. *The first thing we should do is notice what information is given to us and make sure that everything is in consistent units. I will convert everything to metric.*

$$\begin{aligned} 3\,\text{rungs} &= 3.00\,\text{ft} \times \left(\frac{0.3048\,\text{m}}{1.00000\,\text{ft}}\right) = 0.\mathbf{9144}\,\text{m} \\ 6\,\text{rungs} &= 6.00\,\text{ft} \times \left(\frac{0.3048\,\text{m}}{1.00000\,\text{ft}}\right) = \mathbf{1.829}\,\text{m} \\ 11\,\text{rungs} &= 11.00\,\text{ft} \times \left(\frac{0.3048\,\text{m}}{1.00000\,\text{ft}}\right) = \mathbf{3.353}\,\text{m} \end{aligned}$$

Solution. *To find the acceleration in each case, we can solve $v_f^2 = v_i^2 + 2a\,\Delta x$ for the acceleration:*

$$\begin{aligned} a_3 &= \frac{(4.23\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2 - (0\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2}{2(0.\mathbf{9144}\,\text{m})} &= \mathbf{9.831}\ ^{\mathrm{m}}\!/\!_{\mathrm{s}^2} \\ a_6 &= \frac{(6.00\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2 - (0\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2}{2(\mathbf{1.829}\,\text{m})} &= \mathbf{9.841}\ ^{\mathrm{m}}\!/\!_{\mathrm{s}^2} \\ a_1 1 &= \frac{(8.11\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2 - (0\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2}{2(\mathbf{3.353}\,\text{m})} &= \mathbf{9.80}8\ ^{\mathrm{m}}\!/\!_{\mathrm{s}^2} \end{aligned}$$

Notice that these have some variation due to the rounding. It turns out that the variation in the value of acceleration depends on the composition of the earth in your location as well as your altitude above sea-level. That will be discussed in detail in Chapter 6, for simplicity we will assume (by Convention 4.1.12) that all objects accelerate at the rate of 9.81 $^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$ when they are solely under the influence of gravity.

It turns out that you can also see this acceleration when you throw an object straight up into the air.

1.6 Complications

1.6.1 Non-Inertial Accelerated Reference Frames

Subsection referenced by Reference Frames, Subsection 3.2.1.

[Discuss non-rotating linearly accelerating reference frames. See also Section 2.4 for a discussion on rotating reference frames.]

[Comment on the Earth as essentially stationary? See Section 4.4 on effective theories.]

1.6.2 Air Resistance

Subsection referenced by freefall, Answer 7.3.0.2.5, Section 4.1

Terminal velocity…

When do we include air resistance and when can we ignore it? …

[Comment on air resistance being a small effect in some cases? See Section 4.4 on effective theories.]

1.6.3 Multi-Step Solutions

Exercise 1.6.1 (*Carl hits the ceiling!*)

Exercise referenced by *Example 4.3.8, Example 4.7.1*

Carl gets bored one day in physics class[1] *and tossed a baseball($m_b = 0.145\,\mathrm{kg}$) at the ceiling… a little too hard. The initial velocity is $v_i = +5.00\ \mathrm{m/s}\,\hat{j}$ and it leaves his hand $1.00\,\mathrm{m}$ below the ceiling. The ball hits the ceiling and when it returns to his hand, it is travelling $\vec{v}_f = -4.73\ \mathrm{m/s}\,\hat{j}$, slower than he expected.*

(a) *Assuming that the ball is in contact with the ceiling for $\Delta t = 0.142\,\mathrm{s}$, find the acceleration of the ball during the collision. Solution 5.6.3.1.1*

(b) *On the other hand, if the ceiling had not been there, then how high would the ball have gone and how fast would it have been going when it returned to Carl's hand? Solution 5.6.3.1.2*

Exercise Not Done *part b*

Hint. *During **the first stage**, the ball is accelerating upwards and Carl is interacting with the ball. We are not going to consider this part of the motion at all because we are given the velocity that ends this stage (and begins the next stage).*

Hint. ***The second stage*** *of the motion is while the ball moves from Carl's hand up to the ceiling. During this stage only the gravitational force is acting on the ball. Since it has left Carl's hand, he is not interacting with it. Since it has not yet hit the ceiling, the ceiling is not interacting with it. We can therefore use the equations of constant acceleration to describe the motion. During this portion of the motion we know that the velocity at the bottom is $\vec{v}_{\mathrm{bot}} = +5.00\ \mathrm{m/s}\,\hat{j}$, that it travels $\Delta\vec{x} = +1.00\,\mathrm{m}\,\hat{j}$, and that (because it is in freefall) it is accelerating at $\vec{a}_g = -9.81\ \mathrm{m/s^2}\,\hat{j}$.*

We can find the time of flight (not useful) and the velocity when the ball reaches the ceiling:

$$v_{\mathrm{top}} = \sqrt{v_{\mathrm{bot}}^2 + 2a\,\Delta x}$$
$$v_{\mathrm{top}} = \sqrt{(+5.00\ \mathrm{m/s})^2 + 2(-9.81\ \mathrm{m/s^2})(+1.00\,\mathrm{m})}$$
$$v_{\mathrm{top}} = +\mathbf{2.31}9\ \mathrm{m/s}$$

Note: When you take the square root, you have to decide if you should take the positive sign or the negative sign. In this case, the ball is still moving upwards, so we choose the positive sign.

Hint. ***The third stage*** *is while it is interacting with the ceiling. In order to find the acceleration during this motion, we need to know the velocity immediately before hitting the ceiling (which we just found) and the velocity just after it finishes hitting the ceiling (which we have not yet found). We will come back to this step.*

Hint. ***The fourth stage***, *like the second, is while the ball moves from the ceiling down to Carl's hand. During this portion of the motion we know that the velocity at the bottom (final) is* $\vec{v}_{\text{bot}} = -1.67\,{}^{\text{m}}\!/\!_{\text{s}}\hat{\jmath}$, *that it travels* $\Delta\vec{x} = -1.00\,\text{m}\hat{\jmath}$, *and that (because it is in freefall) it is accelerating at* $\vec{a}_g = -9.81\,{}^{\text{m}}\!/\!_{\text{s}^2}\hat{\jmath}$. *We can find the time of flight (not useful) and the velocity when the ball leaves the ceiling (initial), solving* $v_{\text{bot}}^2 = v_{\text{top}}^2 + 2a\,\Delta x$ *for* v_{top}:

$$v_{\text{top}} = \sqrt{v_{\text{bot}}^2 - 2a\,\Delta x}$$
$$v_{\text{top}} = \sqrt{(-1.67\,{}^{\text{m}}\!/\!_{\text{s}})^2 - 2(-9.81\,{}^{\text{m}}\!/\!_{\text{s}^2})(+1.00\,\text{m})}$$
$$v_{\text{top}} = -\mathbf{4.73}4\,{}^{\text{m}}\!/\!_{\text{s}}$$

Note: When you take the square root, you have to decide if you should take the positive sign or the negative sign. In this case, the ball is now moving downwards, so we choose the negative sign.

Hint. *During* ***the fifth stage***, *the ball is accelerating upwards while moving downwards and so Carl is stopping the ball. We are not going to consider this part of the motion at all.*

Hint. *The acceleration is only* constant *during* each *stage. The acceleration is different from one stage to another.* ***This*** *is the observation that tells us that* we must consider the stages separately*.*

Solution. *In order to solve Question 5.6.3.1.a for the acceleration, we need to recognize that*

1. *there are five stages to the motion of the baseball: the throw (Hint 5.6.3.1.1), the ball moving from Carl's hand up to but not yet hitting the ceiling (Hint 5.6.3.1.2), the ball hitting the ceiling (Hint 5.6.3.1.3), the ball falling from the ceiling down to but not yet touching Carl's hand (Hint 5.6.3.1.4), and the catching of the ball (Hint 5.6.3.1.5), and*

2. *the equations of constant acceleration* assume that the acceleration is constant *(Hint 5.6.3.1.6).*

The acceleration is $a = \dfrac{v_f - v_i}{\Delta t}$, *but the story of this equation says that since the acceleration is only during the interaction with the ceiling, then the velocities in this equation are just-before the ball hits and just-after the ball hits (not the very beginning velocity and not the very final velocity). Similarly, the* Δt *in this equation is only the time during which it was interacting with the ceiling, not the entire flight.*

Now that we have the velocities immediately before (from Hint 5.6.3.1.2) and after (from Hint 5.6.3.1.4) the collision with the ceiling, we can find the acceleration: $a = \frac{v_f - v_i}{\Delta t} = \frac{(-\mathbf{4.73}4\,{}^{\text{m}}\!/\!_{\text{s}})-(+\mathbf{2.31}9\,{}^{\text{m}}\!/\!_{\text{s}})}{(0.142\,\text{s})} = -\mathbf{28}.09\,{}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\jmath}$ *Notice that the acceleration is negative because the ball went from going up to going down.*

Solution. *To solve Question 5.6.3.1.b, we can just consider from after-thrown to before-caught (both given in the question). During this motion, assuming there is no ceiling, the entire motion is in freefall, so we can use* $v_f^2 = v_i^2 + 2a\,\Delta x$ *and solve for* Δx. *However, we only want to consider from the lowest point to the highest point, not all the way back to Carl's hand.*

List of examples

Chapter 2

Two-Dimensional Motion

2.1 Components of Motion

2.1.1 Cross-wind

Bikes and canoes

2.1.2 Ballistic Freefall

Subsection referenced by Subsubsection 3.4.2.2

Discussion about throwing a ball...

Paragraph referenced by Discussion of the description of physics

For 2-dimensional motion, we will use vector equations to describe the relationships. When we write $\vec{v}_f = \vec{v}_i + \vec{a}t$, we mean that this relationship holds for the x-components and separately for the y-components:

$$v_{fx} = v_{ix} + a_x t \qquad v_{fy} = v_{iy} + a_y t$$

Paragraph referenced by Discussion of $F = ma$

2.2 Complications

2.2.1 Air Resistance

Terminal velocity... non-parabolic paths ...

Words to indicate what is of interest in the IRL...

Investigation 2.2.1 (*Baseball pitches are not usually parabolic*)

Carl is a pitcher on the local baseball team. He throws a fast ball, a slider, a curve ball, and a knuckleball.

(a) *Go to a baseball game on a calm day. Sit near third base.*

(i) *Watch the path of fly balls to left field. Are they parabolic?*

Solution. *If you watch them carefully, you will notice that long fly balls are not parabolic. It turns out that the air resistance is fairly complicated, but in the case of baseballs, the part that is relevant is that air resistance is strong when the ball is moving faster and weak when the ball is moving slower. (This is different than the surface friction you will see in Section 4.4.) The effect of this is that the ball (usually) looks like it travels up into the air on a fairly straight* *path with a slight bend, which would produce a very wide parabola. As it slows, the horizontal motion decreases, which tightens the parabola. By the time the ball gets to its highest point, it is often travelling fairly slowly and has mostly all vertical motion by the time it drops into the outfielder's glove.*

(ii) *The path of pitch towards home plate. Are they parabolic?*

Solution. *The way a pitch travels is highly dependant on the way the pitcher releases the ball. As the ball rolls out of the pitcher's hand, a spin is (usually) given to the ball and this spin interacts with the air to modify the direction that the air presses on the ball during the flight. This will slightly affect the flight of the ball during the time it takes for the ball to get from the pitcher's mound to home plate. In addition, fast balls have less time for the gravitational force to pull the ball down, so they will curve downwards less than a slower pitch. This makes following the path of the ball somewhat difficult, but with some practice and careful attention, you should be able to see it. All balls will drop somewhat, but the effect of the air resistance is exactly the mechanism for making a pitch unpredictable, so it is unlikely that you see the ball drop in a clean parabolic path.*

(b) *Go to a baseball game. Sit up high behind home plate. Watch the path of the baseball for various pitches.*

(i) *Do they all fly straight over the plate?*

Solution. *The way a pitch travels is highly dependant on the way the pitcher releases the ball. As the ball rolls out of the pitcher's hand, a spin is (usually) given to the ball and this spin interacts with the air to modify the direction that the air presses on the ball during the flight. In many cases, this will affect the flight of the ball (especially to the right or to the left) during the time it takes for the ball to get from the pitcher's mound to home plate. If you watch from behind home plate, this sideways motion should be fairly clear.*

Air resistance has an effect on most objects that move through the air. The faster an object moves, the bigger the effect. Spinning objects also feel an effect. Objects that have a somewhat large cross sectional area and a somewhat small mass are also affected, but this depends on the actual area and the actual mass.

Return to: the description of physics, 2.2.1

[Comment on air resistance being a small effect in some cases? See Section 4.4 on effective theories.]

List of examples

Chapter 3

Force

3.1 How Physicists Use the Words (Notation)

Section referenced by Discussion of heat as a verb

The technical term **force** refers to the general idea of pushing or pulling. In the same way that physicists use the technical word acceleration differently than the general population uses the colloquial word, force has a specific meaning. We will use *force as a noun* (not as a verb) referring to the act of pushing or pulling and having specific relationships that will be outlined in this chapter.

Connection 3.1.1 (*Looking Back*)

> *Forces are necessarily vectors, because pushing on something intrinsically involves both an amount and a direction.*

You will use this property to show that multiple people pushing in the same direction increases the effect, whereas multiple people pushing in opposite directions reduces the effect. One might say that people who push an object in opposite directions work[1] against each other. Because the force is a vector, whenever you are answering a question about a force, you should always expect to give the strength of the force (the magnitude) and the direction of the force (relative to some specific axis, usually the positive x-axis).

Insight 3.1.2

> *You can't have a push or pull without* both *a thing that pushes or pulls* and *a thing that is pushed or pulled. Forces are* necessarily *an interaction between two objects.*

Clarification 3.1.3 (*Push or Pull*)

> *By now you may have noticed that it is tedious to keep saying "pushed or pulled," so we will only say "push" even when we are including the possibility of "pushing or pulling".*

Sometimes we care about the thing doing the pushing or pulling, sometimes we don't. We always care about the thing being pushed or pulled. We will *distinguish these objects* by referring to the object that is pushing as the object "causing the force" or "exerting the force" and by referring to the object that is being pushed as the object "feeling the force". We will *distinguish these forces* as follows:

Convention referenced by Example 3.3.4

Convention 3.1.4

> *Let's imagine that Beth gives Abdul a good-natured shove in the arm. The following are useful descriptions and are different ways of describing the same action.*
>
> - *Beth exerted a force* ***on*** *Abdul.*
> - *Abdul felt a force* ***from*** *Beth.*

[1] After you study Section 5.2, this play on words will be hilarious!

- *There was a force* **on** *Abdul* **by** *Beth.*

The notation for this will be $F_{A,B}$ where the first subscript is the person who felt the force (who the force is "on") and the second subscript is who exerted the force (who the force is "by"). In those instances when we only care about who is feeling the force and not who is exerting the force, we might just use one subscript F_A.

In some cases, there may be two forces acting on one person (or object). In that case, it will be obvious who is feeling the force and we will use the subscript to distinguish which force it is in addition to *who feels the force, such as F_{A1} and F_{A2},* or rather than *who feels the force, such as F_1 and F_2. This will be more relevant when we discuss in Chapter 4 the types of forces that might be applied.*

Touchstone Recall the distinction between theories and laws.

Connection 3.1.5 (*Looking Ahead*)

Looking ahead to Newton's Laws (Section 2), you should be ready to notice that the first law (Subsection 3.2.1) is about objects that are not feeling a force, the second law (Subsection 3.2.2) is about a specific object that is feeling a force, and the third law (Subsection 3.2.3) is about the interaction between the two objects. In all three of these, we care about the object feeling the force. It is only in the third law that we care about the object exerting the force.

3.2 Connecting the Concepts: Newton's Laws

Section referenced by Discussion of how to describe forces

Touchstone Recall the distinction between Theory versus Law.

Newton's Laws describe our observations about three questions:

1. What happens to an object when I *don't* push on it?
2. What happens *to an object* when I do push on it?
3. What happens *to me* when I push on an object?

The answers to these questions have precise, concise, technical language and the point of the next three subsections is to translate that into (modern) English, into math, and into intuition. The statement of these laws has slightly different versions in different texts to emphasize different points. We will state them as follows:

1. When viewed from an inertial reference frame, an object with no forces acting on it will maintain its velocity, which may be zero.
2. When viewed from an inertial reference frame, the vector-sum of all forces acting on an object will cause that object to accelerate in proportion to its mass: $\vec{F}_{\text{net}} = m\vec{a}$.
3. For every force acting (the "action") on one object by an other object, there is an equal-in-magnitude reaction-force acting on the other object in the opposite direction.

There are a few terms that should be clarified in these laws.

Clarification 3.2.1 (*Inertial Reference Frame*)

Being in an inertial reference frame essentially means being in a place in which you do not have to hold on in order to maintain your position.

If you are accelerating then you are not in an inertial reference frame, but rather are in a non-inertial reference frame. In this case, you will misinterpret the forces acting. This will be discussed in more detail *in Section 2.4 when we discuss the surface of the Earth as a non-inertial rotating reference frame. The fact that the Earth spins will be relevant for global-sized systems, such as the atmosphere and the ocean (discussed in Subsection 2.4.1 with the Coriolis effect).*

For human-sized interactions, the effect of the Earth spinning is small enough that for most of what we casually observe, we can safely pretend that the Earth is stationary and that we are actually at rest while sitting on the curb watching the world go by. This is so true that our human brains already interpret everything around us as though it were an inertial reference frame. This psychological perspective is exactly the feature that both allows us to make fairly reliable predictions about the world around us and causes us to make incorrect judgements when we encounter human-sized non-inertial reference frames. That is to say, as long as we don't measure our world too closely, we are viewing it from an essentially inertial reference frame. This point is so implicit, that many books do not even include[1] the portion of the statement referring to the reference frame.

Touchstone Recall effective theories.

Clarification 3.2.2 (*Objects in motion*)

Sometimes Newton's first law is written[2] to include the phrase "an object in motion", which I will be careful to link directly to velocity, as was done above. However, it technically should reference the momentum, which is discussed in Chapter 1.

Clarification 3.2.3 (*Action\Reaction*)

The way Newton's third law is often written[3] (and referred to) includes the words "action" and "reaction". Newton was referring to forces with these words and to keep it clear in our discussion, we will use the word force, with the occasional clarification of the action-force or the reaction-force.

3.2.1 Translating Newton's First Law: The Law of Inertia

Subsection referenced by Discussion of how to describe forces

Newton's First Law

When viewed from an inertial reference frame, an object with no forces acting on it will maintain its velocity, which may be zero.

Touchstone You might also recall the discussion in Subsection 1.6.1.

Let's take this apart and connect it to your daily experiences.

As indicated in Clarification 3.2.1, it is usually safe to assume you are in an inertial reference frame for your daily experience. So, at this point we won't worry about the words before the comma.

DISCUSS at rest no force. and the intuition that goes with force not causing motion.

The rest of this statement is often written a little differently (and less concisely) as "an object at rest remains at rest unless acted on by an external force and an object in motion remains in motion unless acted on by an external force." Since being "at rest" is a statement about the velocity($\vec{v} = 0$) and being "in motion" is also a statement about the velocity, each of these statements can be understood as saying that

Insight 3.2.4

Forces are those things that cause a change in the velocity.

In other words, Newton's first law says that without a force, the velocity will not change.

Connection 3.2.5 (*Looking Ahead*)

In the discussion of equilibrium (Subsubsection 3.2.2.4), we will note that this is often extended to say that without a net force (as opposed to "without a force" as explained in Subsubsection 3.2.2.2) the velocity

> *will not change, but that is a special case of Newton's second law (Subsection 3.2.2).*

3.2.1.1 Inertia

This law is often called the "law of inertia". The concept of inertia can be described as *the tendency of an object to maintain its velocity*. This is describing how the object behaves when you don't do anything to it. The inertia is not a quantity that physicists calculate, but physicists do refer to objects as having a lot of inertia, usually to indicate that it will take a large force to change the object's motion, or as having a small amount of inertia, usually to indicate that it should be relatively easy to change the object's motion. However, the inertia does not actually refer to the force needed. Instead, the inertia most often refers to the "inertial mass" of an object, which shows up in the second law.

Insight 3.2.6

> *Inertia is not a force.*

Sometimes when physicists are not being careful with their language, they will appear to use the word inertia interchangeably with the term momentum, which we will discuss in more detail in Subsection 1.1.1.

3.2.1.2 How the Laws Work Together

You should notice that Newton's First Law is about what happens when you are *not pushing* on the object, which is to say, the tendency of an object to maintain its own motion without a force acting on it; this is the inertia of the object. On the other hand, Newton's Second Law is about what happens *to the object* when you *do push* on an it. This is what we will consider next. After that, Newton's third law will describe what happens *to the thing pushing* rather than just to the thing being pushed. Subsubsection 5 of Subsection 3.2.2 will explore these ideas further.

3.2.2 Translating Newton's Second Law: The Equation Law

Subsection referenced by Subsubsection 3.4.2.2, how to describe forces, Newton's first law, discussion of falling objects

Newton's Second Law

When viewed from an inertial reference frame, the vector-sum of all forces acting on an object will cause that object to accelerate in proportion to its mass: $\vec{F}_{\text{net}} = m\vec{a}$.

Touchstone Effective theories

Let's take this apart and connect it to your daily experiences. As with Newton's First Law, the non-inertial rotating reference frame of the surface of the Earth is a small enough effect that, as long as we don't measure our world too closely, we can pretend that we are viewing it essentially from an inertial reference frame.

Touchstone Vector equations

Insight 3.2.7 ($F = ma$)

> *For this law, it is often sufficient to write down the equation and know that the words are there for back-up. While most people have no trouble remembering $F = ma$, it is important to pay attention to two aspects:*
>
> - *This is a vector-equation, which means that*
>   - *the equation is true for each component separately (recall Subsubsection 3.4.2.2), and*
>   - *the direction of $\vec{F}_{\text{net}}$ is the same as the direction of $\vec{a}$ (which, of course, might be different than the direction of the velocity - recall Paragraph).*

- *The force in this equation is the* net force, *which means that we must consider* all *forces that are acting on this object and only those forces that are acting on this object.*

Translation 3.2.8 (*The Story of $\vec{F}_{\text{net}} = m\vec{a}$*)

Referenced by *Exercise 3.2.16*

This equation is all about what happens to a specific object, m. If the object, m, is accelerating in a particular direction, $\vec{a}$, then it is because the combination of forces, $\vec{F}_{\text{net}}$, do not entirely cancel each other out. This also can be expressed as: if the combination of forces, $\vec{F}_{\text{net}}$, do not entirely cancel each other out, then our friend m must be accelerating, $\vec{a}$, in a particular direction. Furthermore the resulting direction of the net force determines the direction of the acceleration. Connecting the English and the math:

$\vec{F}_{\text{net}}$	$=$	m	$\vec{a}$
the combination of all forces acting on m	*causes*	*that object*	*to change its velocity*

Connection 3.2.9 (*Looking Back*)

You should recall, that the direction of the acceleration does not determine the direction of the motion, *but rather determines the direction* of the change *in motion.*

Connection 3.2.10 (*Looking Ahead*)

That idea will be important when we discuss how a tension acts as a centripetal force, the relationship between velocity and acceleration in a spring that oscillates, and objects that are propelled through either a gravitational or an electrical field.

3.2.2.1 Units of Force

Referenced by 3.2.3

Recall that the fundamental units of the SI-system are meters, kilograms, and seconds (MKS). With our relationship connecting force to mass (kg) and acceleration ($\mathrm{m/s^2}$), we can see that the units of force are $\mathrm{kg{\cdot}m/s^2}$. This quantity is so common that we would like to have a shorthand for it. Furthermore, Sir Isaac Newton did such ground-breaking work on the concept, that it was decided in 1948[4] to name the unit the Newton, such that

Definition 3.2.11. The **unit of Newton** is defined to be the amount of force necessary to accelerate one kilogram by 1 meter-per-second-squared: $1\,\mathrm{N} = 1\,\mathrm{kg{\cdot}m/s^2}$

3.2.2.2 Calculating the Net Force

Subsubsection referenced by Discussion of Newton's first law

The word "net" that goes with force is here to indicate the total, which is useful to think of as "everything collected with the net."[5] The intention here is that wherever there are multiple forces acting on a single object, we must combine them as vectors as follows:

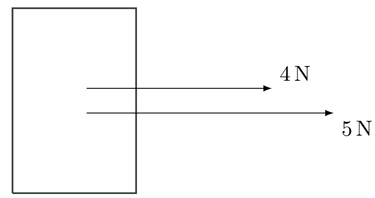
[4] According to: International Bureau of Weights and Measures (1977), The international system of units (330-331) (3rd ed.), U.S. Dept. of Commerce, National Bureau of Standards, p. 17, which refers to the 7th resolution (Mar, 2017) of the 9th CGPM (Mar, 2017).

[5] Although according to etymonline.com (Mar, 2017), it is actually from the Old French *net* for "neat" or "clean", having the sense of trim and elegant.

Example 3.2.12 (*Net Force, Vector-Add Forces in the Same-Direction*)

Example referenced by *Example 3.2.18*, *Equilibrium*, *Example 3.2.28*, *Example 3.2.25*, *discussion about 3.2.28*

If there is a 5.0 N force to the right and a 4.0 N force to the right, then the net force is 9.0 N to the right.



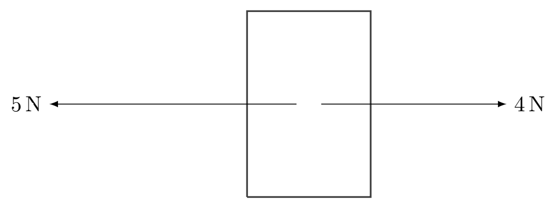
Solution.

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = \left(5.0\,\text{N}\hat{\imath}\right) + \left(4.0\,\text{N}\hat{\imath}\right) = +9.0\,\text{N}\hat{\imath}$$

Example 3.2.13 (*Net Force, Vector-Add Forces in the Opposite-Direction*)

Example referenced by *Example 3.2.21, Equilibrium*

If there is a 5.0 N force to the left and a 4.0 N force to the right, then the net force is 1.0 N to the left.



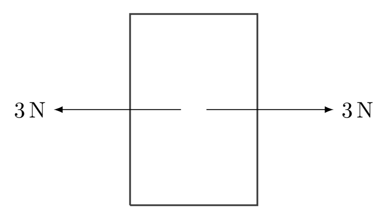
Solution.

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = \left(-5.0\,\text{N}\hat{\imath}\right) + \left(4.0\,\text{N}\hat{\imath}\right) = -1.0\,\text{N}\hat{\imath}$$

Example 3.2.14 (*Net Force, Vector-Add Equal-Magnitude Opposite-Direction Forces*)

Example referenced by *Equilibrium, discussion about 3.2.14*

If there is a 3.0 N force to the right and a 3.0 N force to the left, then the net force is 0.0 N.



In this case, the object is said to be "in equilibrium."

Solution.

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = \left(3.0\,\text{N}\hat{\imath}\right) + \left(-3.0\,\text{N}\hat{\imath}\right) = 0.0\,\text{N}\hat{\imath}$$

Connection 3.2.15 (*Looking Ahead*)

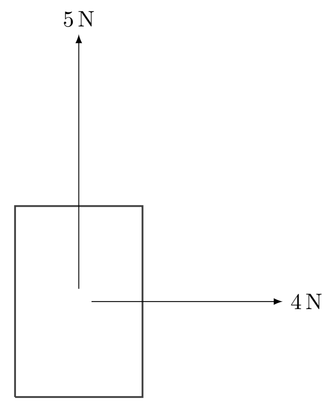
The images included in these examples will eventually be referred to as "free-body diagrams," but for now, you can just consider them images of the forces acting on the bodies.

Next, we should do a couple of examples that show the math for situations with forces in two dimensions. The first, Exercise 3.2.16, has one force in the x-direction and another in the y-direction. The second, Exercise 3.2.17, has one force in the x-direction and the other in the second quadrant.

Exercise 3.2.16 (*An object is pushed by perpendicular forces*)

Exercise referenced by the discussion of the net force, Exercise 3.2.17

A 2.0 kg mass is being pushed north with 5.0 N and east with 4.0 N. What is the net force?



Solution. *Since we have multiple forces acting on a mass to cause an acceleration, it should be clear (recall the story) that we need to use Newton's second law and find the net force in order to compute the acceleration. We will, as usual, start with a free-body diagram (at right).*

This example is made easier because the forces happen to be at right angles and so finding their x and y components is not difficult. By adding the x-components and separately adding the y-components, we have found the components of the net force.

	x-comp	y-comp
F_1	0 N	+5 N
F_2	+4 N	0 N
F_{net}	+4 N	+5 N

From there, we can easily find the magnitude and direction of the net force.

$$\text{Magnitude:} \quad F_{\text{net}} = \sqrt{(+4\,\text{N})^2 + (+5\,\text{N})^2} = \mathbf{6.40}\,\text{N}$$

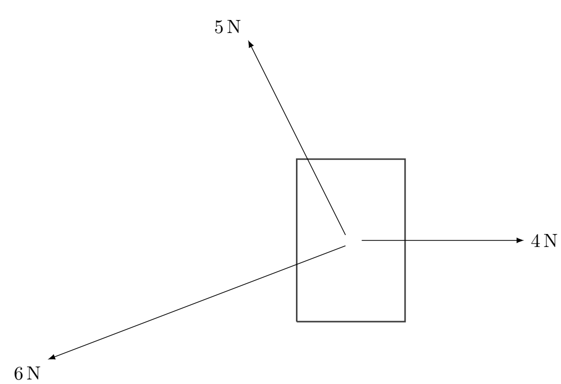
$$\text{Direction:} \quad \theta = \tan^{-1}\left(\frac{+5\,\text{N}}{+4\,\text{N}}\right) = \mathbf{51.3}\,^\circ\text{N of E}$$

(The direction can be stated as $\theta = 51^\circ$N of E or as $\phi = 39^\circ$E of N.) Exercise 3.2.19 will use this calculation to find the acceleration.

Exercise 3.2.17 (*Three forces act on an object*)

Exercise referenced by the net force

A 2.0 kg mass is being pushed northwest with 5.0 N at an angle 63.4°N of W, southwest with 6.0 N at an angle of 21.8°S of W, and east with 4.0 N. What is the net force?



Solution. *This follows the same logic as Exercise 3.2.16, which I will not restate here.*

This example is slightly harder because the forces have to be split into their x and y components. By adding the x-components and separately adding the y-components, we have found the components of the *net force.*

	x-comp	*y-comp*
F_1	$-(5.0\,\text{N})\cos(63.4°) = -\mathbf{2.24}\,\text{N}$	$+(5.0\,\text{N})\sin(63.4°) = +\mathbf{4.47}\,\text{N}$
F_2	$-(6.0\,\text{N})\cos(21.8°) = -\mathbf{5.57}\,\text{N}$	$-(6.0\,\text{N})\sin(21.8°) = -\mathbf{2.23}\,\text{N}$
F_3	$+4\,\text{N}$	$0\,\text{N}$
F_{net}	$-\mathbf{3.81}\,\text{N}$	$+\mathbf{2.24}\,\text{N}$

From there, we can easily find the magnitude and direction of the net force.

$$\text{Magnitude:}\quad F_{\text{net}} = \sqrt{(-\mathbf{3.81}\,\text{N})^2 + (+\mathbf{2.24}\,\text{N})^2} = \mathbf{4.42}\,\text{N}$$

$$\text{Direction:}\quad \theta = \tan^{-1}\left(\frac{+\mathbf{2.24}\,\text{N}}{-\mathbf{3.81}\,\text{N}}\right) = \mathbf{30}.5\,°\text{N of W}$$

(The direction can be stated as $\theta = 31°$N of W or as $\phi = 60°$W of N.) *Exercise 3.2.20 will use this calculation to find the acceleration.*

3.2.2.3 Using the Net Force to Calculate Other Quantities

Generally, the point of finding the net force is that it causes an object to change its velocity. Let's also consider a few simple examples of this calculation.

Example 3.2.18 (*The acceleration of a box feeling a net force*)

> **Example referenced by** *finding m from $F = ma$, Example 3.2.28, Example 3.2.25, discussion about 3.2.28, Example 4.1.7, Answer 8.1.2.7.1, Example 4.3.2*

If the forces in Example 3.2.12 are applied to an object with mass 2.0 kg, *then it will accelerate at the rate of*

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{+(9.0\,\text{N})\hat{\imath}}{2.0\,\text{kg}} = 4.5\ {}^{\text{N}}\!/\!_{\text{kg}}\,\hat{\imath} = 4.5\ {}^{\text{kg}\cdot\text{m}}\!/\!_{\text{s}^2\cdot\text{kg}}\,\hat{\imath} = 4.5\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\imath}$$

which (recall Section 1.4), after acting for 1.6 s *on an object originally at rest, would result in a final speed of*

$$v_f = (0\ {}^{\text{m}}\!/\!_{\text{s}}) + (+4.5\ {}^{\text{m}}\!/\!_{\text{s}^2})(1.6\,\text{s}) = 7.2\ {}^{\text{m}}\!/\!_{\text{s}}$$

We can do this same kind of procedure for the case when forces are in two dimensions.

Exercise 3.2.19 (*Accelerating a box from $\vec{F}_{\text{net}}$*)

A 2.0 kg mass is being pushed north with 5.0 N and east with 4.0 N. What is the acceleration?

Hint. *Exercise 3.2.16 already found the net force to be $\vec{F}_{\text{net}} = 4.0\,\text{N}\hat{\imath} + 5.0\,\text{N}\hat{\jmath}$ which is $F_{\text{net}} = 6.4\,\text{N}$ at $51°$ N of E. (What remains is to find the acceleration.)*

Hint. *It is possible to find the (vector) acceleration from the (vector) net force by using* either *the x- and y-components,* or *by using the magnitude and direction.*

Solution. ***components:*** *Given the mass and the components of the net force, we can calculate the acceleration.*

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{4.0\,\text{N}\hat{\imath} + 5.0\,\text{N}\hat{\jmath}}{2.0\,\text{kg}} = 2.0\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\imath} + 2.5\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\jmath}$$

You can then use the components of the acceleration to find the magnitude and direction of the acceleration.

$$\begin{aligned} \textit{Magnitude:}\quad a &= \sqrt{(+2.0\ {}^{\text{m}}\!/\!_{\text{s}^2})^2 + (+2.5\ {}^{\text{m}}\!/\!_{\text{s}^2})^2} &&= \mathbf{3.2}0\,\text{N} \\ \textit{Direction:}\quad \theta &= \tan^{-1}\left(\frac{+2.5\ {}^{\text{m}}\!/\!_{\text{s}^2}}{+2.0\ {}^{\text{m}}\!/\!_{\text{s}^2}}\right) &&= \mathbf{51}\,°\textit{N of E} \end{aligned}$$

Solution. ***magnitude and direction:*** *Given the mass and the magnitude and direction of the net force, we can calculate the acceleration.* $a = \frac{6.4\,\mathrm{N}}{2.0\,\mathrm{kg}} = 3.2\,\mathrm{m/s^2}$ *and know that the direction of the acceleration is the same as the acceleration of the net force:* $51°$ *N of E.*

Exercise 3.2.20 (*Accelerating a box pushed by three forces*)

A $2.0\,\mathrm{kg}$ *mass is being pushed northwest with* $5.0\,\mathrm{N}$ *at an angle* $63.4°$*N of W, southwest with* $6.0\,\mathrm{N}$ *at an angle of* $21.8°$*S of W, and east with* $4.0\,\mathrm{N}$*. What is the acceleration?*

Hint. *Exercise 3.2.17 already found the net force to be* $\vec{F}_{\text{net}} = 3.8\,\mathrm{N}\hat{\imath} + 2.2\,\mathrm{N}\hat{\jmath}$ *which is* $F_{\text{net}} = 4.4\,\mathrm{N}$ *at* $31°$ *N of W. (What remains is to find the acceleration.)*

Hint. *It is possible to find the (vector) acceleration from the (vector) net force by using* either *the* x*- and* y*-components,* or *by using the magnitude and direction.*

Solution. *Given the mass and the components of the net force, we can calculate the acceleration.*

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{3.8\,\mathrm{N}\hat{\imath} + 2.2\,\mathrm{N}\hat{\jmath}}{2.0\,\mathrm{kg}} = 1.9\,\mathrm{m/s^2}\,\hat{\imath} + 1.1\,\mathrm{m/s^2}\,\hat{\jmath}$$

You can then use the components of the acceleration to find the magnitude and direction of the acceleration.

$$\text{Magnitude:}\quad a = \sqrt{(+1.9\,\mathrm{m/s^2})^2 + (+1.1\,\mathrm{m/s^2})^2} = \mathbf{2.2}\,\mathrm{m/s^2}$$
$$\text{Direction:}\quad \theta = \tan^{-1}\left(\frac{+1.1\,\mathrm{m/s^2}}{+1.9\,\mathrm{m/s^2}}\right) = \mathbf{31}\,°\text{N of W}$$

Solution. *Given the mass and the magnitude of the net force we can find the magnitude of the acceleration* $a = \frac{4.4\,\mathrm{N}}{2.0\,\mathrm{kg}} = 2.2\,\mathrm{m/s^2}$ *and know that the direction of the acceleration is the same as the acceleration of the net force:* $31°$ *N of W.*

In Example 3.2.18, we used the forces to find the acceleration. It is also possible to use the forces to find the mass of an object, as follows:

Example 3.2.21 (*Finding the mass of a box from its force and acceleration*)

If the forces in Example 3.2.13 are applied to an object with unknown mass and produce an acceleration of $3.2\,\mathrm{m/s^2}$*, then what is the mass of the object?*

Solution. *Naively, one might consider* $m = \frac{\vec{F}_{\text{net}}}{\vec{a}}$*, but it does not make mathematical sense to divide vectors. In this case, you must consider the magnitudes of force and acceleration, knowing that their directions are the same. (We are* not *"cancelling" the directions.)*

$$m = \frac{F_{\text{net}}}{a} = \frac{9.0\,\mathrm{N}}{3.2\,\mathrm{m/s^2}} = \mathbf{2.8}1\,\mathrm{N{\cdot}s^2/m} = 2.8\,\mathrm{kg{\cdot}m{\cdot}s^2/s^2{\cdot}m} = 2.8\,\mathrm{kg}$$

Connection 3.2.22 (*Looking Ahead*)

Yet another example of using this equation can be seen in many bathrooms. The scale that people stand on uses a spring (introduced in Subsection 4.3.1 and discussed in detail in Section 4.6) to adjust the force provided until your acceleration is zero (placing you in equilibrium) and then tells you the force it needed to balance your weight.

It will be easier to visualize these ideas when we introduce the tool of a free-body diagram in Subsubsection 3.2.3.1.

3.2.2.4 Equilibrium

Subsubsection referenced by Example 3.2.14, Newton's first law, discussion of falling objects

This word can be traced back to Latin and Old English with the prefix *equi-* for *equal* and the root *libra* referring to a *pair of scales, as in a balance*, such as those depicted in images of the astronomical constellation Libra. When the scales are equal, they are in equilibrium. Since the second law asks us to calculate the sum of the forces acting on an object, one of the primary questions is to determine if those forces balance each other. In Example 3.2.12 and Example 3.2.13, the forces are not balanced, the object "is not in equilibrium", and it will be accelerated in a particular direction. In the Example 3.2.14, the forces are balanced, the object "is in equilibrium", and it will *not change* its velocity (in accord with the first law).

Definition 3.2.23. An object in **equilibrium** has $\vec{F}_{\text{net}} = 0\,\text{N}$ and $\vec{a} = 0$.

3.2.2.5 How the Laws Work Together

Referenced by Subsubsection 3.2.1.2

When forces act on an object, Newton's second law applies, so we usually start with the second law. If those forces combine to give a net force of zero, such that the object is in equilibrium, then Newton's first law applies. If we also care about the person or thing pushing, then the third law also applies.

To better understand how the first and second laws work together, Investigation 3.3.2 provides some activities that you can do or consider in order to think about the patterns you can see when you are or aren't pushing on objects. Exploration 3.2.24 will help you think through some of the consequences of the first and second law. When you are ready to solve some problems, you can jump to Section 3.3, but some of those examples will also reference Newton's third law.

Exploration 3.2.24 (*Out of gas*)

Exploration referenced by *Subsubsection 3.2.2.5*

On a long road trip with your friend Beth, your car starts to sputter as it runs out of gas shortly before arriving in a new town. You see a sign for a gas station in the distance and have to decide what to do. You and Beth can think of three options.

Plan A *Pull over, park the car, walk to the gas station, buy a gas can, fill it up, carry it back to the car, and drive on! If you follow this plan, then read Answer 1.*

Plan B *Leave the car in drive, continue holding the gas-pedal down until there is absolutely no gas, and hope against all hope that you get the car to the gas station so that nobody needs to carry a heavy gas can. If you follow this plan, then read Answer 2.*

Plan C *Speed up to just over the speed limit, put the car in neutral, turn on your blinking hazard-lights, coast as far as you can possibly coast, and hope against all hope that you get the car to the gas station so that nobody needs to carry a heavy gas can. If you follow this plan, then read Answer 3.*

Answer. *Just as planned, you pull over and park the car. Beth suggests one of you stays with the car, probably because she has physics homework to do. If you decide to separate, read Answer 5. If you decide to journey together, read Answer 7.*

Answer. *As soon as you decide to do this, the gas runs out. Thinking you can make it to the gas station, you take your foot off of the gas pedal. You slow down fairly quickly and get nervous that you might get rear-ended. You turn on the hazard-lights. After about a minute you are travelling* 30 mph *and you pass the time by working out Exercise 1. People are honking at you as they try to pass. Beth turns to you and asks you why you are going so slow. If you start a discussion about Newton's First Law, then go to Answer 4. If you get embarrassed and decide to pull over, then read Answer 6.*

Answer. *You speed up to* 60 mph *before the gas runs out and then you quickly pop the car into neutral. You slow down gradually and, in an effort to not get rear-ended, you cleverly turn on the hazard-lights. After about a minute you are travelling* 52 mph *and you pass the time by working out Exercise 1.3.2). After* 2 min*, you are travelling* 43 mph *and people are getting impatient as they try to pass. After* 2.79 min*, you triumphantly coast into the gas station at a comfortable speed of* 36.7 mph*. Beth is so happy, she buys* *you a full tank of gas and the two of you start a discussion about Newton's First Law while pumping the gas. Please read Answer 8.*

Answer. *After some discussion, you and Beth realize that when the car is in drive, the transmission (the part of the car that converts how-fast-the-engine-spins to how-fast-the-axel-and-wheels-turn) is connected to the axel, which means that the rolling wheels are trying to turn the engine parts as well as the wheels themselves. The engine parts have grease and oil, but still take a lot of energy to turn. This causes friction, which dissipates energy and, more importantly, exerts a backwards force on the spinning wheels. Your car is not being described by Newton's First Law, which requires there to be no force applied. Instead your car is being described by Newton's Second Law and the force is changing the velocity to cause you to go slower. It only takes the car* 2.2 min *to stop and you still have to walk to the gas station. Beth laments "If only there were a way to reduce the force on the axel..." If it occurs to you to speculate about putting the car in neutral when the gas ran out, then imagine reading Answer 3. If you stop talking and walk to the gas station, then read Answer 10.*

Answer. *You leave Beth in the car and walk the 45 minutes to the gas station. You buy a gas can, fill it up, and start to carry it back to the car. It is very heavy and you notice vultures circling overhead. You hope you survive this. It might have been a better idea to bring Beth with you to share the burden. You stumble once, and then again. You swear to be more cautious about estimating your gas consumption. After walking for what seems like hours and stumbling back to the car, you find Beth very excited. She declares that she has invented a time machine so you can go back to the* adventure *and start over to learn something about Newton's First Law!*

Answer. *You pull over and park the car. Beth suggests one of you stays with the car, probably because she has physics homework to do. If you decide to separate, read Answer 9. If you decide to journey together, read Answer 7.*

Answer. *Everything goes as planned. You drive off into the sunset sadly ignorant of the physics you might have learned.* ***The end!***

Answer. *During the discussion, you and Beth realize that the rolling wheels and the spinning axel are still connected to the not-spinning frame of the car. While this causes less friction than if the car were in drive, there is still some friction, which dissipates energy and, more importantly, exerts a backwards force on the spinning wheels. Your car is not being described by Newton's First Law, which requires there to be no force applied. Instead your car is being described by Newton's Second Law and the force is changing the velocity to cause you to go slower. You finish getting gas and drive on to many happy adventures.* ***The end!***

Answer. *You leave Beth in the car and walk the 31 minutes to the gas station. You buy a gas can, fill it up, and start to carry it back to the car. It is very heavy and you notice vultures circling overhead. You hope you survive this. It might have been a better idea to bring Beth with you to share the burden. You stumble once, and then again. You swear to be more cautious about estimating your gas consumption. After walking for what seems like hours and stumbling back to the car, you find Beth very excited. She declares that she has invented a time machine so you can go back to the* adventure *and start over to learn something about Newton's First Law!*

Answer. *You and Beth happily walk the 25 minutes to the gas station, discussing and working out physics problems the whole way. You buy a gas can, fill it up, and share the burden of carrying a heavy gas can. You return to the car, add gas, and drive on to many happy adventures.* ***The end!***

3.2.3 Translating Newton's Third Law: Action & Reaction

Subsection referenced by how to describe forces, Example 3.3.4, Example 3.3.7

Newton's Third Law

For every force acting *on* one object *by* an *other object*, there is an equal-in-magnitude reaction-force acting *on* the *other object* in the opposite direction.

This law is often shortened to "For every action, there is an equal and opposite reaction." The statement given above is meant to emphasize several points:

- These "actions" are specifically forces.
- Forces are an interaction in which the acting force is *on one object* by another and necessitates that there is a reaction force on the other object by the one. That is to say, an object cannot feel a force without also exerting a force back on the other object.

Another way to say this is that all forces come in action/reaction pairs that necessarily have equal magnitude and opposite direction and necessarily act on different objects.

Foreshadow This law is the force-version of the statement of the conservation of momentum.

Let's take this apart and connect it to your daily experiences. Students of physics will often see the terms action and reaction and connect it to the way humans react to the actions of their friends. However, this implies a causal[6] response that is not true for Newton's forces. That is to say, this is not a "revenge law" whereby if you push on me, then I will choose to push you back. Instead, it is expressing that forces are intrinsically interactions between a pair of objects. When you push on me, I am – independent of my choosing to do so – necessarily pushing back on you. But, you might say, "if that were true, then why am I able to sneak up on you and push you over without falling over myself?" Well, you can think about how that works by reading Exploration 3.3.3. After we introduce the tool of a free-body diagram in the next subsection, you can also explore this idea by comparing Example 3.3.4 to Example 3.3.7.

3.2.3.1 The Free-Body Diagram (FBD)

Referenced by Example 3.2.14, uses of $F = ma$

In the more interesting situations where there are several forces acting, it can be easy to lose track of what is pushing whom where. In order to better organize our information and direct our attention, we can make use of **free-body diagrams**. The basic idea is to make a diagram for each individual object that we care about in a given situation, and *free* from the overall picture. This allows us to identify the forces acting on a single object (relevant for Newton's second law) and more easily pair them with third-law pairs that act on different objects.

To see how this works, the next example will build on the previous examples to help us consider not only the (2nd law) forces on the object, but also the (3rd law) forces on the people doing the pushing and pulling.

Example 3.2.25 (*The reaction forces on people pushing a box*)

Let's reconsider Example 3.2.12 and imagine that Abdul stands to the left of the object and exerts a force to the right (pushes) with magnitude $\vec{F}_1 = +5.0\,\mathrm{N}\hat{\imath}$ and that Beth stands to the right of the object and exerts a force to the right (pulls) with magnitude $\vec{F}_2 = +4.0\,\mathrm{N}\hat{\imath}$, as drawn in Figure 3.2.26. Both of these forces are on the object and, by Newton's second law cause it to accelerate (as described in Example 3.2.18). Newton's third law tells us about the interaction between *objects and, from this, we can deduce the force on Abdul and Beth.*[7]

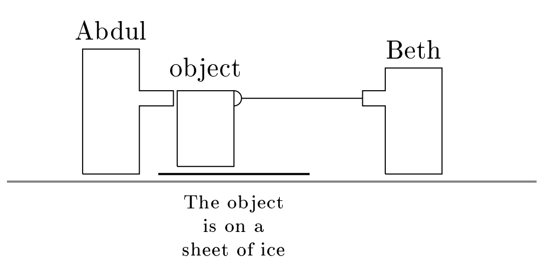
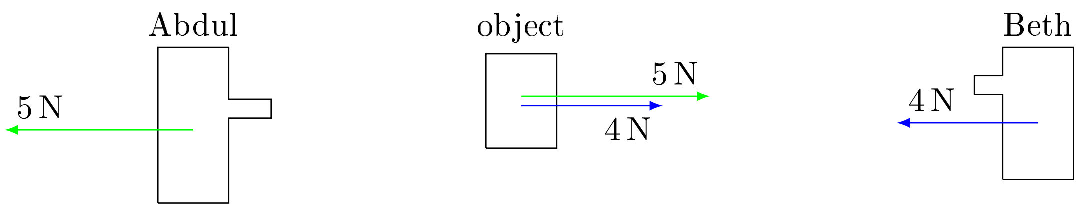


Figure 3.2.26: Abdul and Beth push on a box

[6] Please note that "causal" and "casual" are different words.

Now we will draw a free-body diagram for each individual. Notice that each free-body *diagram is on its own, free from the rest of the picture.*



Because Abdul pushes the object to the right, Newton's third law tells us Abdul feels a force to the left *on* him *by* the object.

For the object (in the middle), Example 3.2.12 showed that $F_{\text{net}} = 9.0\,\text{N}$.

Because Beth pulls on the object to the right, Newton's third law tells us Beth feels a force to the left *on* her *by* the object.

Figure 3.2.27: These are the free fbody diagrams for Figure 25.

It turns out that this is a little oversimplified. When we get to Section 4.1 and *Section 4.3, we will see that we have to include a downwards force of gravity and an upwards support force. This will be explained in Example 4.3.2.*

Return to: 3.3.4, 3.3.7, rope-tension

As indicated above, the free-body diagram also helps us visualize the third-law (action/reaction) force pairs. By evaluating the colored forces in Figure 3.2.27, we can see that the green forces form an action-reaction pair and separately the blue forces form an action-reaction pair. Notice the following:

1. The third law action-reaction pairs are located of different objects and cannot be added. (You cannot add blue to blue nor green to green.)

2. The second law forces on the object should be added to find that specific object's acceleration. (You *should* add the blue force to the green force on the middle object.)

None of these three are in equilibrium, because they each have a net force. Example 3.2.18 calculates the acceleration of the object. Example 3.2.28 calculates the acceleration of the people.

Example 3.2.28 (*The acceleration of people pushing a box*)

Referenced by *3.2.25*

In the same way that Example 3.2.18 was able to use Newton's second law to compute the acceleration of an object after Example 3.2.12 found the net force, we can use the forces found in Example 3.2.25 to compute the acceleration of the people pushing the box. Recall that Figure 3.2.26 shows how Abdul and Beth exert forces on the box (and, from Newton's third law, how the box exerts force on the people).

1. *Since Abdul exerts a force of* $5.0\,\text{N}$ *to the right* $(+\hat{\imath})$ *on the object, the* third law *reminds us that the object exerts a force of* $5.0\,\text{N}$ *to the left* $(-\hat{\imath})$ *on Abdul. Since Abdul has a mass of* $85.0\,\text{kg}$*, the* second law *reminds us that he is accelerated at the rate of*

$$\vec{a}_1 = \frac{-5.0\,\text{N}\hat{\imath}}{85.0\,\text{kg}} = -0.0\mathbf{588}\,{}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\imath} = -5.9 \times 10^{-2}\,{}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\imath}$$

which is to the left with a small enough value that it is easy for him to brace against. Even if he doesn't brace, if he starts from rest, he will only be moving

$$v_{1f} = (0\,{}^{\text{m}}\!/\!_{\text{s}}) + (-0.0\mathbf{588}\,{}^{\text{m}}\!/\!_{\text{s}^2})(1.6\,\text{s}) = -0.0\mathbf{941}\,{}^{\text{m}}\!/\!_{\text{s}},$$

which is much slower than the object.

2. *Since Beth exerts a force of* 4.0 N *to the right on the object, the* third law *reminds us that the object exerts a force of* 4.0 N *to the left on Beth. Since Beth has a mass of* 75.0 kg, *the* second law *reminds us that she is accelerated at the rate of*

$$\vec{a}_2 = \frac{-4.0\,\mathrm{N}\hat{\imath}}{75.0\,\mathrm{kg}} = -0.0\mathbf{533}\ \mathrm{m/s^2}\,\hat{\imath} = -5.3 \times 10^{-2}\ \mathrm{m/s^2}$$

which is also to the left with a small enough value that it is easy for her to brace against. Even if she doesn't brace, if she starts from rest, she will only be moving

$$v_{2f} = (0\ \mathrm{m/s}) + (-0.0\mathbf{533}\ \mathrm{m/s^2})(1.6\,\mathrm{s}) = -0.0\mathbf{853}\ \mathrm{m/s},$$

which is also much slower than the object and different from Abdul.

It might be interesting to note that, since everybody and thing has a different mass, the accelerations are all different. This would also be true if the forces were all the same.

If we now do the same thing with the Example 3.2.14, but let Carl be the person on the left pushing to the right and Diane be the person on the right pushing to the left, then we see that while the green forces form an action-reaction pair showing the third-law interaction between Carl and the object and while the blue forces form an action-reaction pair showing the third-law interaction between Diane and the object, it is the combination of the blue and the green forces, which only act on the object itself, that coincidentally cancel to leave the object in (second law) equilibrium. In these images, Carl and Diane are *not* in equilibrium.

The point of Examples 3.2.25–3.2.28 in comparison to Examples 3.2.12 (which told us about the forces on the object) and 3.2.18 (which told us about how the object moved) is to highlight the different roles of Newton's second and third laws. The earlier examples were relevant to the second law and only affected the object itself. To say this more specifically, the forces within one free-body diagram are described by Newton's second law. They do get added together to form the net-force (which is to say that we can add the object's green force to the object's blue force). They are able to cancel each other if they *happen to* be equal in magnitude and opposite in direction. Finally, they will determine how that specific object accelerates. On the other hand, Newton's third law describes any specific pair of forces that interact between free-body diagrams (each colored pair); they *will necessarily be* equal in magnitude and opposite in direction, but they cannot be canceled because they cannot be added because they are on different objects.

3.3 Examples

Referenced by Subsubsection 3.2.2.5

Next, we can consider a simple interactive example that is intended to help you think about how you know a force is acting.

Exploration 3.3.1 (*Drop a Book*)

You hold a book a little above your desk. When you let go, it falls and then hits your desk.

1. *While you are holding it, it has no acceleration. Are there forces acting on it? Select one: "Yes" (Answer 1) or "No" (Answer 2).*
2. *While you are holding it, is it in equilibrium? Select one: "Yes" (Answer 3) or "No" (Answer 4).*
3. *After you let go and while the book falls, it accelerates downwards. Are there forces acting on it? Select one: "Yes" (Answer 5) or "No" (Answer 5).*
4. *While it is hitting the desk, is it accelerating? Select one: "Yes" (Answer 6) or "No" (Answer 7).*
5. *After it has landed and is sitting on the desk, is it in equilibrium? Select one: "Yes" (Answer 8) or "No" (Answer 9).*
6. *After it has landed and is sitting on the desk, how many forces are acting on it? Select one: "Zero" (Answer 11), "One" (Answer 12), or "Two" (Answer 13).*

Answer. *There are forces acting on it. You should be able to tell this because you are exerting one of the forces. While it is true that there are forces on it, it is also true that there is no* net force. *If you are exerting an upward force on the book, can you guess (Answer 10) what the downward force is?*

Answer. *It is true that while you hold the book, there is no* net force, *but that does not mean that there is no force acting. If there were no forces on the book, then your hand would not need to be there. In fact, if you remove the force your hand provides, then the book falls. This shows that there is an upward force (by your hand on the book) and a downward force (of gravity by the Earth on the book).*

Answer. *It is in equilibrium. When the acceleration is zero, then the net force must be zero and those properties are what define equilibrium.*

Answer. *The definition of equilibrium is that the forces balance. The result of this is that the net force must be zero and the acceleration is then zero. You can tell this is true because the velocity is* not changing. *It is not important that the velocity is zero, what is important is that the velocity* stays zero. *While you hold it, the book is in equilibrium.*

Answer. *Both "Yes" and "No" bring you to this answer. Yes, there is* a force *on the book while it falls (the force of gravity), but no, there are not force**s** (plural). There is only one force. "But, wait!" you say, "What about the force of air resistance?" Aha! You are correct; there is a force of air resistance, but in this case, it is negligible and we will not consider it. Please read Subsection 1.6.2* for *more information about deciding when to use or ignore this phenomenon.*

Answer. *The book is accelerating. The velocity* is *changing from "moving downwards" to "stopped". The book is not in equilibrium.*

Answer. *While it is hitting the desk, the velocity is changing from "moving downwards" to "stopped". Since the velocity is changing,* the book is accelerating. *Since it is accelerating, the book is not in equilibrium.*

Answer. *It is in equilibrium. The book is at rest and* continues to be *at rest on the desk. There are forces acting, but they cancel each other, resulting in no net force.*

Answer. *After it has landed, the book stops moving. Once the book comes to rest on the desk, it* continues to stay at rest. *This says that the velocity is not changing, so the book is not accelerating. That means that the book is in equilibrium. There are forces acting, but they cancel each other, resulting in no net force.*

Answer. *It is the force of gravity.*

Answer. *Recall the situation when you were holding the book. Gravity is still pulling the book down and the desk is holding the book up. There are two forces acting on the book while it is at rest on the desk.*

Answer. *If there were only one force on the book, it could not be a balanced force, so the book could not be in equilibrium and the book would be accelerating. The book is not accelerating, so there are either two forces (Answer 13) or no forces (Answer 11).*

Answer. *There are two forces acting on the book while it is at rest on the desk. Similar to the situation when you were holding the book, gravity is pulling the book down and the desk is holding the book up.*

Next, we can consider pushing an object across the floor in Investigation 3.3.2 to get a different sense of observations we can make that help us recognize patterns that are due to forces we might not have thought to look for.

Investigation 3.3.2 (*Pushing an Object Across the Floor*)

(a) *Push a chair across a carpet floor. Notice that when you stop pushing, it stops moving. Based on this, do you think force causes motion?*

Solution. *If we refer to "motion" as describing the velocity, then no. Force causes a change in velocity. When you stop pushing, the chair stops because there is a force from the carpet acting to* *oppose the force you apply while you push the chair.*

(b) *Push a chair across a tile floor. Notice that when you stop pushing, it probably*[1] *stops moving. Based on this, do you think force causes motion?*

Solution. *This is essentially the same as Task a, but the carpet exerts more force than the tile. In either case, force causes a* change *in velocity. You are trying to speed the chair up and the floor is trying to slow the chair down. (Both are trying to change the velocity, but cancel to result in a constant velocity.) When you stop pushing, the chair stops moving because there is a force from the tile acting to oppose the force you apply while you push the chair; when you let go, this force slows the chair until the chair stops and then the force stops acting. (See Section 4.4* for more details.*)*

(c) *Push a chair* with wheels *across a tile floor, with some strength, then let it go.*

(i) *What happens when you stop pushing?*

Solution. *For a chair with wheels being pushed across a tile floor, when you stop pushing it probably continues to move across the floor for at least a short distance.*

(ii) *If force causes motion, why does the chair move after you stop touching it?*

Solution. *The chair continues to move for the same reason that the chair without wheels and the chair on carpet* all *continued to move when you let go. The reason is that this is* how all objects behave; they maintain their velocity when allowed to act without interference. *(This is why Newton's first law says what it does.) Because the chair with wheels has much less friction there is a smaller force trying to interfere with the motion and so it continues to move for a noticeable distance. The other chairs slowed to a stop almost immediately. The wheel-less chair on tile might have continued for a short distance if it was moving fast enough that it required a long enough time to change its velocity to zero.*

(d) *Push a chair* with wheels *across a tile floor, change your behavior after you let it go.*

(i) *Do your actions when you are not touching the chair have* any *impact on the chair?*

Solution. *No. But if it does not matter what you do after you let go of the chair, then why do coaches (in basketball free-throws, tennis serves and swings, baseball pitches, and all manner of arm and leg propulsion) tell you to pay attention to your "follow through"?*

Select one: "They have been fooled; follow-through doesn't matter" (Solution 2) or "they are right; follow-through does matter!" (Solution 3).

Solution. *They haven't been fooled, but follow-through matters in a different way. What does matter is not literally how you move* after *the release, but rather how you move* before *you release the ball. By paying attention to your follow-through, you are also changing the way you move before you release or impact the ball. You want a smooth flow throughout the motion and a sloppy follow-through often implies a sloppy initiation of the motion.*

Solution. *What does matter is not literally how you move* after *the release, but rather how you move* before *you release the ball. By paying attention to your follow-through, you are also changing the way you move before you release or impact the ball. You want a smooth flow throughout the motion and a sloppy follow-through often implies a sloppy initiation of the motion.*

(ii) *Is it possible that there is a "residual effect" that you have on the chair after letting it go?*

Solution. *No. When you throw a ball very high into the air, you can dance a jig or do any manner of things and it will obviously not affect the ball. The force you exerted on the chair goes away the instant you stop touching the chair. It is, however, true that your force gave the chair some velocity (actually momentum) and Newton's first law (inertia) says that the chair would prefer to keep that velocity. Unfortunately, the friction with the ground slows it down. The careful way to describe the situation is that your force gave the chair some velocity (actually momentum) and its characteristic inertia made it difficult for the frictional force to slow it down rapidly.*

(e) *Newton's First Law says that if you give the chair a velocity, it should keep that velocity. Repeat the first three suggestions and correlate the interaction-with-the-ground to the motion-after-you-push-and-release. Is there a force that the chair feels after you release it?*

Solution. *The force that the chair feels after you release it is friction. For the carpet, there is a lot of friction and the chair slows down very quickly (essentially instantaneously). For the wheel-less chair on the tile floor, the chair slows rapidly although it may leave your hand. The wheels provide the least amount of friction and that chair goes the furthest. You may note that the friction slowing the chair-with-wheels is primarily between the rolling wheel and its axel (where it connects to the non-rolling chair leg) rather than between the wheel and the floor (although the friction between the wheel and the floor also plays a role). This is discussed in more detail in Section 4.4.*

(f) *Newton's Second Law says that a net force will change the velocity. Push a chair gently across the floor. A constant force (balanced by the force of friction) will move at a constant speed. What if there were no friction?*

Solution. *If there were no friction, then you could start the chair and it would move on its own at a constant speed; you wouldn't need to continue pushing to keep it moving. On the other hand, if you did continue to push, then the chair would continue to speed up and you would have to run faster and faster to keep up with it. On the other hand, if the chair were not experiencing friction, then you probably wouldn't either and you couldn't get enough traction to keep up with the chair, so it would sail away almost immediately, being then described by Newton's first law!*

(g) *Newton's Second Law says that a net force will change the velocity. Push a chair forcefully across the floor. A constant force (stronger than the force of friction) will accelerate the object away from your push. Can you list surfaces that are essentially frictionless?*

Hint. *You should include surfaces that are very easy to push objects across.*

Hint. *Ice is an obvious (?) choice. You might have experience with the table-top "air hockey" game. You can look-up "mag-lev trains".*

Hint. *You should also think about how smooth or rough the surfaces you list are.*

Notice in each case that you are not the only thing interacting with the chair. The floor is also interacting with the chair. The floor exerts a force of friction on the chair. So, when you interpret how your force causes the chair to move, you must also account for the interaction with the floor in your expectations. We can minimize the effect of friction, by modifying the floor surface. If you have ever driven on ice and felt out of control, you might have begun to develop your Newtonian intuition.

Return to: 3.2.2.5, section 3.3 reference to Theorem 3.3.2

Building on that, it is useful to also consider how human beings behave when they are pushing or getting pushed. Because people have *intention* in their actions, we subconsciously balance ourselves and we don't always recognize that we are doing it. Exploration 3.3.3 provides an interactive storyline that starts to show some of the patterns that can lead to a recognition of how we balance ourselves.

Exploration 3.3.3 (*The Town Bully*)

Exploration referenced by *the discussion of action-reaction forces, section 3.3 reference to Theorem 3.3.3, Example 3.3.4, Example 3.3.7*

Zambert is the town bully. One day, he spies a biology student, Carl, minding his own business studying an interesting ecological phenomenon. At the same time, you are standing across the street chatting with your friend Diane, who happens to be taking a psychology class. Diane has been quite fascinated lately with watching the way others interact and points out the way Zambert is menacingly approaching the unsuspecting Carl. You both predict that Zambert is going to push Carl over. Diane is mesmerized by *the psychological effects and you, having just learned about Newton's laws, are excited to see if this action does indeed produce a reaction.*

1. *If you watch the way Carl is standing before, during, and after Zambert pushes him, then read Answer 1. (See also Answer 8.)*
2. *If you watch the way Zambert is standing before, during, and after he pushes Carl, then read Answer 2. (See also Answer 8.)*
3. *If, on the other hand, you shout a warning to Carl and a criticism to Zambert, trying to keep the incident from becoming violent, then read Answer 3.*

Answer.

> ***Foreshadow*** *The physics of why an object (or person) rotates when they fall over is discussed with torque.*

As Carl gets pushed, you notice that he was not aware of the pending doom. He is standing casually with his feet set to support his own weight, but not to brace him against the sideways force. When he gets pushed from the side, his feet stay in place and his torso topples, rotating him about his center of mass as he falls to the ground. Diane points to her phone and says, "I recorded the whole thing!" If you respond, "Awesome! Can I watch the part about how Zambert acts?", please read Answer 2. If you respond, "Awesome! Let's show the psychology and physics faculty our cool video!", please read Answer 6. If you respond, "Yeah, we probably should have intervened before this happened instead of just watching. Let's go talk to Campus Security.", please read Answer 7.

Answer. *As Zambert pushes, you notice that because he was being intentional, he put one foot behind him to brace his body during the push. He leans into the push and stays standing. You are intrigued. If you decide to do a follow-up experiment by pushing Diane over without bracing yourself, then read Answer 4. If you decide to exercise self-restraint, then read Answer 5.*

Answer. *Being the thoughtful and considerate person you are, you rush over and startle Carl out of his reverie. Zambert is quite angry and now focuses his attention on you! He rushes towards you and shoves as hard as he can. You go flying backwards and land on your tailbone while he just stands there laughing. Carl and Diane both rush over to help you while Zambert wanders off. Surprisingly, Carl has an icepack, which helps. If you go speak to your faculty members about this, please read Answer 6. If you decide to talk to Campus Security, please read Answer 7.*

Answer. *You turn and push Diane over. Like Carl, she did not expect it and was not braced, so she falls over. Similarly, you decided not to brace yourself and in pushing Diane, you fall over backwards! Diane did not have to choose to push on you. The act of you deciding to push her necessarily and simultaneously produces a force on you, equal in magnitude and opposite in direction. Unfortunately, Diane doesn't think this was a useful exercise and shouts "I have the whole thing on video!" and storms off to Campus Security. You are arrested for assault, miss your physics class for a couple of weeks and ultimately fail all of your classes. I certainly hope this was all happening in your head and not in real life! You learned something about physics, but at what cost to your humanity?* ***The end!***

Answer. *Diane points to her phone and says, "I recorded the whole thing!" If you respond, "Awesome! Can I watch the part about how Carlacts?", please read Answer 1. If you respond, "Awesome! Let's show the psychology and physics faculty our cool video!", please read Answer 7.3.0.4.6. If you respond, "Yeah, we probably should have intervened before this happened instead of just watching. Let's go talk to Campus Security", please read Answer 7.*

Answer. *The psychology faculty member speaks to you both about how to be good citizens and about the psychological effects of bullies both on the bully and on the recipient. If you decide to learn more about this, please read Psychology Today. The physics faculty member points out that when one person pushes another, the person being pushed does not brace himself, whereas the person doing the pushing does. Furthermore one might imagine what would happen if you did not brace yourself when you pushed each other, such as in Answer 4. You are asked to review both Example 3.3.4 and Example 3.3.7 before* *the next exam. On your way out the door, you hear a voice suggest "... and you might want to talk to Campus Security about the incident..."*

Answer. *You speak with Campus Security about the incident and Zambert gets taken in for assault. The Dean thanks you for being brave enough to speak up.* ***The end!***

Answer. *You feel guilty for letting Zambert push Carl down despite your amazing score on the next physics test. It wasn't worth it.* ***The end!***

(To better understand Newton's third law, you should compare Example 3.3.4 to Example 3.3.7.)

Example 3.3.4 (*Zambert intentionally braces when pushing Carl*)

Example referenced by action-reaction, Example 3.3.4

Zambert, the town bully (with $m_Z = 95.0\,\mathrm{kg}$), decides to vent his frustration on Carl for all the times that Carl makes Zambert look bad in class. While Carl ($m_C = 90.0\,\mathrm{kg}$) has his back turned, Zambert walks up, leans in, and shoves Carl with a force of $\vec{F}_{C,Z} = 215\,\mathrm{N}\hat{\imath}$. How does Zambert accelerate during this exchange?

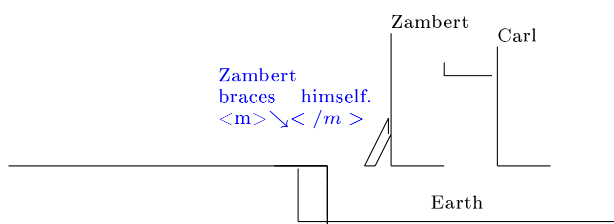


Figure 3.3.5: This image shows Zambert pushing Carl in order to see the reaction forces.

Solution. ***What do we know?*** *As usual, it is convenient to start with the picture to help decide on the appropriate coordinate system. We know m_Z, which is useful for relating $F_{Z,\mathrm{net}}$ to a_Z. We know m_C, which is useful for relating $F_{C,\mathrm{net}}$ to a_C. (This is not asked for, but is asked in homework problem Exercise 7.4.3.1.) We know $F_{C,Z}$, how hard Zambert pushes on Carl. We also know that Carl is not bracing himself (because he "has his back turned") so he only feels one force, and that Zambert is bracing himself (because the problem states that he "leans in") so he exerts multiple forces.*

What do we want to know? *We want to know about the forces acting on Zambert, in order to find $F_{Z,\mathrm{net}}$ and therefore a_Z.*

How are what-we-know and what-we-want related? *First, since Zambert exerts a force on Carl, Newton's third law tells us that Zambert feels a force of $F_{Z,C} = -215\,\mathrm{N}\hat{\imath}$.*

Second, because Zambert knew he was going to feel this reaction force, he compensates by bracing himself. This means he chooses to exert a force of $215\,\mathrm{N}$ on the Earth in the $-\hat{\imath}$ direction, probably by putting one leg behind himself and pushing the ground backwards with his foot. Newton's third law then tells us that Zambert feels a force of $F_{Z,C} = +215\,\mathrm{N}\hat{\imath}$ from the ground.

Free-Body Diagrams *We are told of the force on Carl. We are told that Zambert braces himself, which tells us the force on the Earth. Newton's third law then helps us recognize the forces on Zambert. (Recall the "on-by" notation.)*

<!— *Zambert is pushed by Carl to the left. Zambert is pushed to the right by the Earth.* –>

<!– *Carl feels Zambert push to the right.* –>

<!– Earth feels a force by Zambert to the left. –>

Figure 3.3.6: This image shows Zambert pushing Carl in order to see the reaction forces.

Concepts to Consider: *Newton's third law guarantees that the action-reaction force pairs, such as $F_{Z,C}$ and $F_{C,Z}$ or $F_{Z,E}$ and $F_{E,Z}$, are equal and opposite. There is no such guarantee on $F_{Z,C}$ and $F_{Z,E}$. These are equal because Zambert chose to make $F_{C,Z}$ and $F_{E,Z}$ equal. He pushed on the two others in equal amounts so that the reaction forces that act on him will balance for Newton's second law so that his acceleration would be zero.*

After using Newton's third law to find the forces on Zambert, we can use Newton's second law to find his acceleration:

$$a_Z = \frac{F_{Z,\text{net}}}{m_Z} = \frac{\left[\vec{F}_{Z,C} + \vec{F}_{Z,E}\right]}{95.0\,\text{kg}} = \frac{\left[\left(-215\,\text{N}\hat{\imath}\right) + \left(+215\,\text{N}\hat{\imath}\right)\right]}{95.0\,\text{kg}} = 0\ \text{m/s}^2$$

Aside about Example 33: This example only considers the left-right forces that act in order to make a point about our intuition regarding forces we intend to apply. Please consider how Example 4.3.2 updates Example 3.2.25 to make yourself aware of the other forces that are acting here, but are being ignored.

Example 3.3.7 (*Diane does not brace herself when pushing Carl.*)

Example referenced by *the discussion of action-reaction forces*, *Example 3.3.4*

In the lab room one day, while waiting for the instructor, Diane (who has a mass of $m_D = 80.0\,\text{kg}$) decides to try a physics experiment to test Newton's third law. She politely asks her lab partner, Carl ($m_C = 90.0\,\text{kg}$), to turn his back while she squares her feet underneath herself and pushes with a force of $\vec{F}_{C,D} = 215\,\text{N}\hat{\imath}$. Despite the experience of Example 3.3.4 (as told in Exploration 3.3.3), Carl reluctantly agrees. How does Diane accelerate during this exchange?

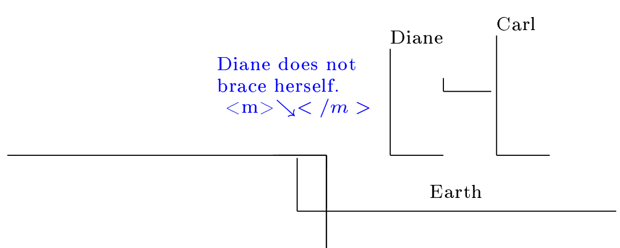


Figure 3.3.8: This image shows Diane pushing Carl in order to see the reaction forces.

Solution. ***What do we know?*** *As usual, it is convenient to start with the picture to help decide on the appropriate coordinate system. We know m_D, which is useful for relating $F_{D,\text{net}}$ to a_D. We know m_C, which is useful for relating $F_{C,\text{net}}$ to a_C. (This is not asked for, but is asked in homework problem Exercise 7.4.3.1.) We know $F_{C,D}$, how hard Diane pushes on Carl. We also know that neither person is bracing for the push. So, both Carl and Diane each only feel one force.*

What do we want to know? *We want to know about the forces acting on Diane, in order to find $F_{D,\text{net}}$ and therefore a_D.*

How are what-we-know and what-we-want related? *First, since Diane exerts a force on Carl, Newton's third law tells us that Diane feels a force of $F_{D,C} = -215\,\text{N}\hat{\imath}$.*

Second, unlike Zambert in Example 3.3.4, Diane chooses not to exert a force on the Earth in the $-\hat{\imath}$ direction.

Free-Body Diagrams: *We again draw free-body diagrams:*

Concepts to Consider: *Newton's third law guarantees that the action-reaction force pairs, $F_{D,C}$ and $F_{C,D}$, are equal and opposite. Because these forces are not on the same person, we cannot add these* *forces. Newton's second law will then indicate how each person accelerates.*

After using Newton's third law to find the forces on Diane, we can use Newton's second law to find her acceleration:

$$a_D = \frac{F_{D,\text{net}}}{m_D} = \frac{\left[\vec{F}_{D,C}\right]}{80.0\,\text{kg}} = \frac{\left[\left(-215\,\text{N}\hat{\imath}\right)\right]}{80.0\,\text{kg}} = -\mathbf{2.68}75\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\imath}$$

Aside about Example 36: This example only considers the left-right forces that act in order to make a point about our intuition regarding forces we intend to apply. Please consider how Example 4.3.2 updates Example 3.2.25 to make yourself aware of the other forces that are acting here, but are being ignored.

3.4 Summary and Homework

3.4.1 Summary of Concepts and Equations

This chapter introduced the way physicists describe forces. The concept of force encodes how objects interact. After reading this chapter, you should be comfortable responding to the following questions or comments. Unlike the other links in this book, if you follow the links in this summary section, there is no link to return to this page. (This is on purpose to encourage you to answer these points without following these links.)

- State Newton's Laws. (Answer)
- How is the unit of Newton related to the fundamental units of the SI system? (Answer)
- How do you know when a system is in equilibrium? (Answer)
- You should know how to draw a free-body diagram. (Example)

3.4.2 Conceptual Questions

1. In order to climb a tree, you reach up and grab a branch and pull. Most people refer to this as "pulling yourself up." In terms of Newton's third law, describe what is happening in more technical terms.

2. Some cars have a "cruise-control" feature that keeps your speed constant as you drive down the highway. (a) If you are driving due north with the cruise-control on, are you in equilibrium? (b) If, instead, you have the cruise-control set while you are following the road around a gradual curve of the road as it follows the shore of a lake, then are you in equilibrium? (c) In both cases, how can you tell if you are in equilibrium?

3.4.3 Problems

1. If Zambert, with $m_Z = 95.0\,\text{kg}$, braces himself (so that he does not accelerate) and pushes Carl ($m_C = 90.0\,\text{kg}$) with a force of $\vec{F}_{C,Z} = 215\,\text{N}\hat{\imath}$, find the following:

(a) What is the acceleration of Carl? (Solution 1)
(b) What net force does Zambert feel? (Solution 2)
(c) If Zambert braces himself against the Earth, then what must that bracing force be? (Solution 3)
(d) What are the individual forces that Zambert feels? (Solution 4)
(e) What is the acceleration of the Earth? (Solution 5)
(f) Which of Newton's laws allows you to answer each of these questions?

Solution. $\vec{a}_C = \dfrac{215\,\text{N}\hat{\imath}}{90.0\,\text{kg}} = \mathbf{2.38}9\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\imath}.$

Solution. $F_{Z,\text{net}} = 0\,\text{N}.$

Solution. $\vec{F}_{E,Z} = -215\,\text{N}\hat{\imath}.$

Solution. $F_{Z,C} = -215\,\text{N}\hat{\imath}$ and $F_{Z,E} = 215\,\text{N}\hat{\imath}.$

Solution. $\vec{a}_E = \dfrac{-215\,\text{N}\hat{\imath}}{5.97 \times 10^{24}\,\text{kg}} = -\mathbf{3.60}1 \times 10^{-23}\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\imath}.$

2. If you apply a force of 4.65 N to a mass of 2.18 kg, then how much will it accelerate?

3. How much force must you apply to cause a mass of 80.0 kg to accelerate at $a = 0.795\ ^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$?

4. You arrive home to find a box that came in the mail. You find that you have to exert 54.3 N to cause it to accelerate $a = 1.25\ ^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$. (a) What is its mass? (b) Is that a heavy box or a light box? (c) Is it likely that this box would fit in a mailbox?

5. Your 2538 kg car has run out of gas. So you ask your friend, Beth who has a mass of 75.0 kg, to put it in neutral, sit inside, and steer while you push. If you apply enough force to cause a net forward force of magnitude 37.5 N, how much time will it take for the car to move faster than you can walk? Assume your walking speed is $3.0\ ^{\mathrm{mi}}\!/\!_{\mathrm{hr}}$. How far will the car have travelled in that time?

6. Find the components of the net force on a large crate if three forces are applied: $\vec{F}_1 = -3.0\,\mathrm{N}\hat{\imath} + 2.5\,\mathrm{N}\hat{\jmath}$, $\vec{F}_2 = -6.25\,\mathrm{N}\hat{\jmath}$, and $\vec{F}_3 = 4.5\,\mathrm{N}\hat{\imath} + 1.63\,\mathrm{N}\hat{\jmath}$.

7. Find the components of the net force on a large crate if three forces are applied: $F_1 = 3.61\,\mathrm{N}$ at 71.6° north of east, $F_2 = 4.61\,\mathrm{N}$ due west, and $F_3 = 8.13\,\mathrm{N}$ at 21.8° south of east.

8. Find the magnitude and direction of the net force on a large crate if three forces are applied: $\vec{F}_1 = 4.25\,\mathrm{N}\hat{\imath} - 4.66\,\mathrm{N}\hat{\jmath}$, $\vec{F}_2 = -2.65\,\mathrm{N}\hat{\jmath}$, and $\vec{F}_3 = -5.4\,\mathrm{N}\hat{\imath} + 2.93\,\mathrm{N}\hat{\jmath}$.

9. Find the magnitude and direction of the net force on a large crate if three forces are applied: $F_1 = 2.65\,\mathrm{N}$ at 26.6° north of west, $F_2 = 2.22\,\mathrm{N}$ at 56.31° south of west, and $F_3 = 7.12\,\mathrm{N}$ at 28.4° north of east.

List of examples

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(Revised September 1, 2017)

Chapter 4

The Many Types of Force

Chapter referenced by Discussion of subscript notation of forces

There are many mechanisms for imparting force on an object. Perhaps the most obvious is that objects fall towards the Earth. We call this force "gravity" and we will discuss it first. It turns out to be somewhat more complicated because it turns out that the force that helps us fall when we trip is the same force that holds the planets in their orbits. The complicated general theory will be discussed later, but for now Section 1 will describe the phenomenon at the surface of the Earth where we usually experience it.

After a brief introduction to the four fundamental forces in Section 2, we will consider the normal force, tension (such as in ropes), friction, springs, and end with the catch-all additional category of applied forces. Section 8 will wrap it up by using examples including multiple types of force to compute the net force, which connects back to Newton's Laws in the previous chapter.

4.1 Gravity at the Surface of the Earth

Section referenced by freefall, Example 3.2.25

Perhaps the force that is the most obvious to humanity is the one that helps us fall when we stumble: the gravitational force. This is one of the fundamental forces discussed in Section 4.2. In addition, the details about how the planets, moon, and the sun experience this force will be discussed in Chapter 6. For now, we can consider how this interaction manifests itself on our daily lives. In this section, we will start with how objects move when the gravitational force is the only force acting. Subsections 4.1.1 and 6.1.1 will clarify some subtleties and then we'll jump into the examples in Subsection 2.

We can investigate what happens when the gravitational force is the only force acting on an object by holding it in the air and dropping it. One of the complications during such an experiment was discussed in Subsection 1.6.2. If we drop a sheet of paper, there is air resistance in addition to the gravitational force. For this section, I will assume that the mass-to-surface-area ratio is large enough that we can effectively ignore the air resistance.

Touchstone Recall effective theories.

Since objects fall faster than humans are used to paying attention to, the patterns are difficult to see. Investigation 1.5.1 shows you how you can pay close attention to the patterns that result from observing falling objects. You should go do those experiments before reading further. Go ahead. I'll wait.

You did do them, right? You're not just reading ahead? Really? OK. Doing that experiment will help you see (1) that everything falls at the same rate and (2) that objects accelerate as they fall. This first point is a bit less intuitive and will be discussed further in Subsection 6.1.1. This second point should be exactly what you expect, when you consider Newton's second Law: If there is only one force (the gravitational force), then the object cannot be in equilibrium and it must be accelerating. (You should notice that this is the language of the story of Newton's second law.)

Touchstone the on-by notation

In order to evaluate this further, let's consider a specific object, like a baseball. Our baseball has a mass of $m_b = 0.145\,\text{kg}$. If the only force acting *on* the ball is the gravitational force *by* the Earth, then the net force is the gravitational force: $\vec{F}_{\text{net}} = \vec{F}_{bEg}$. Here the subscripts are b (because the force is on the $b$all), E (because the force is exerted by the Earth), and g (because it is a $g$ravitational force). Since the acceleration is due to the gravitational force, I will use either a_g (usually when the object is in freefall and therefore accelerating at this rate) or g (usually when the object is not actually accelerating at that rate). With this notation, Newton's second law becomes:

$$\vec{F}_{bEg} = m_b \vec{a}_g$$

At this point, we know the mass, but we don't know the force or the acceleration. However, we have conveniently already done the experiment (recall Exercise 1.5.3) that will tell us the acceleration is $a_g = 9.81\,{}^{\text{m}}\!/\!_{\text{s}^2}$ downwards. (Recall that "downwards" is the direction of the vector, which can be expressed as $-\hat{\jmath}$.) If we know the mass and the acceleration, then we can compute the force.

Example 4.1.1 (*Calculate the weight of a ball in freefall*)

If a baseball with mass $m_b = 0.145\,\text{kg}$ *is dropped (allowed to fall freely) so that it accelerates at* $a_g = 9.81\,{}^{\text{m}}\!/\!_{\text{s}^2}$ *downwards, then while it falls it feels the gravitational force:*

$$\vec{F}_g = m\vec{g} = (0.145\,\text{kg})[-(9.81\,{}^{\text{m}}\!/\!_{\text{s}^2})\,\hat{\jmath}] = -\mathbf{1.42}24\,\text{N}\hat{\jmath} = -1.42\,\text{N}\hat{\jmath}.$$

This is the force of the gravitational force on the baseball. Although we computed the force while the ball was falling, the gravitational force does not magically vanish when the ball is sitting on the floor. So, we can say that (as long as the ball is close to the surface of the Earth, as noted in Chapter 15) the force always has this value. Rather than continuing to say "the force of gravity" we call this force the weight.

Definition 4.1.2. The **weight** of an object is computed as its mass times the acceleration due to gravity, even when the object is not actually accelerating at that rate: $\mathbf{F_g} \equiv \mathbf{mg}$.

4.1.1 Weight versus Mass

Referenced by Subsection 3.2.2, Section 4.2

Since all objects have the same acceleration due to gravity at the surface of the Earth, the weight of an object and the mass of an object are very closely correlated, but they are not the same quantity. This tends to cause some confusion when the discussion is not explicitly technical. Recall the discussion about being precise in our language, Section 4.1. One complication for people in the United States is that there are two definitions of the pound; one is a unit of mass[1] and the other is a unit of force. Since the pound-force[2] is defined as the standard unit of mass times the standard unit for the acceleration due to gravity, as discussed in Section 3.2, the conversion directly from pound-force to Newtons will *not* match the longer, but more appropriate, conversion from pound-mass to kilogram that gets multiplied by the local acceleration due to gravity (as opposed to the standard g) into Newtons. It may also be useful to review the comments about unit-conversion in the section on significant digits.

In the discussion about being precise, we distinguished "massive" (the amount) from "voluminous" (the size). Now that we understand Newton's second law, we can distinguish "massive" (an amount of material causing a difficulty in making accelerate) from "weighty" (a strength needed to lift). The concept that goes with

Clarification 4.1.3

mass is the amount of material,

whereas, the concept that goes with

[1] There are also multiple versions of the pound-mass. You can find these explained on the internet, but most of these are considered obsolete. The one I will use is the "avoirdupois-pound", which is defined in the NIST publication Handbook 44, page C-19, as exactly 453.59237 g.

[2] There is also a unit of force called the kilogram-force.

Touchstone Recall Definition 2

Clarification 4.1.4

weight is how strongly the gravitational force pulls on the object.

Having mass affects both the inertia (ease of moving) and the weight (force of gravity). Having weight expresses the gravitational force due to whichever large object (moon, planet, sun, etc.) you happen to be on or near. Noticing that the SI-unit (Section 3.2) is different for different types of quantities, such as a kilogram (a fundamental unit) for mass and a Newton (a derived unit) for weight, may help you remember that these are different kinds of quantities.

The interesting aspect of this relationship is that while having more mass makes an object harder to move (the same force produces less acceleration for more massive objects), when objects fall under the influence of the gravitational force, they accelerate at the *same* rate. This reveals that the gravitational force must be stronger for more massive objects *by the exact amount* needed to compensate for that larger mass. This is called the equivalence principle and is discussed in Subsection 6.1.1.

4.1.2 Calculating the weight

When calculating the forces acting on a person or an object, we will often need to account for the force of gravity, while other forces may also be at work. As mentioned above, the weight is found by multiplying the mass times the local acceleration due to gravity, even if the object is not actually accelerating at that rate. Chapter 6 will clarify why it is true[3], but for now please note that the acceleration due to gravity is (1) different according to where we are and also (2) the same for all objects at that location.

Because of the peculiarities in the definition of pound it will be useful to build some intuition about masses in terms of kilograms and Newtons. Table 5 lists the mass of some common objects and, using the standard value for g, their corresponding weights. Exercise 8.9.2.1 asks you to estimate the mass of some other common objects. Exercise 8.9.2.2 asks you to think of common objects with a specified mass.

Object	pounds	mass (kg)	weight (N)
apple	0.33	0.15	1.5
lean, healthy cat	10	4.6	45
medium-sized dog	44	20	196
human	200	91	890
horse	1000	362	3.56×10^3
large pick-up truck	4000	1.81×10^3	1.78×10^4

Table 4.1.5: The list of objects is intended to give a sense of scale so that the reader can better estimate the value of the mass of an object. You might notice that (except for the apple) each of these is between 4 and 4.5 times heavier than the previous object. Note that these are rough estimates; for example, while the author weighs about 200 lbs this is not typical, nor average.

Now let's do some calculations...

Example 4.1.6 (*Calculate the mass from the weight*)

Abdul notices that he needs to exert $F = 1.5\,\mathrm{N}$ *to support the apple listed in* Table 5. *He then drops it and notices its acceleration of* $9.81\ \mathrm{m/s^2}$. *He computes the mass to be*

$$m = \frac{F_g}{a_a} = \frac{1.5\,\mathrm{N}}{9.81\ \mathrm{m/s^2}} = \frac{1.5\ \mathrm{kg\cdot m/s^2}}{9.81\ \mathrm{m/s^2}} = 0.\mathbf{153}\,\mathrm{kg}$$

(If you know the weight, you can compute the mass, even if the mass is not actually in freefall.)

[3] The short answer is that the altitude (distance from the surface of the Earth) and local geology affect the strength of the gravitational field. Since the Earth is slightly oblate (bulges at the equator), the altitude at different latitudes corresponds to a different distance from the center of the Earth. In addition, while the spin of the Earth does not affect the strength of the gravitational field, it does affect how objects accelerate. The GRACE project has measured the variations across the globe.

Example 4.1.7 (*Calculate weight from the mass*)

Referenced by Example 4.3.2, Example 4.5.6

Abdul, who knows his own mass (85.0 kg), then imagines dropping himself (!) from a (small) height. While he falls, he recognizes the gravitational force on him, which is computed to be

$$\vec{F}_g = m\vec{g} = (85.0\,\mathrm{kg})[-(9.81\,\mathrm{m/s^2})\,\hat{\jmath}] = -\mathbf{833}.85\,\mathrm{N}\hat{\jmath} = -834\,\mathrm{N}\hat{\jmath}$$

Since he is in freefall and there is only one force is acting on him, the net force is easy to compute: $\vec{F}_{\text{net}} = -834\,\mathrm{N}$*. However, if you know the mass something, you can compute the weight even if that object is not in freefall. You should repeat this calculation for the mass in Example 3.2.18.Since the object in Example 3.2.18 has a mass of* 2.0 kg*, we can find the weight by*

$$\vec{F}_g = m\vec{g} = (2.0\,\mathrm{kg})[-(9.81\,\mathrm{m/s^2})\,\hat{\jmath}] = -\mathbf{19}.62\,\mathrm{N}\hat{\jmath} = -20\,\mathrm{N}\hat{\jmath}.$$

Insight 4.1.8 (*ma versus mg*)

- F_{net} $(= ma)$ *is always related to the actual acceleration of the object.*
- F_g $(= mg)$ *is always related to the local acceleration due to gravity.*

You should also note that

Insight 4.1.9

the actual acceleration, (a), is only equal to the local acceleration due to gravity, (g), if the object is in freefall (by Definition 1.5.2).

Example 4.1.10 (*Deducing the existence of forces using Newton's second law*)

Referenced by Example 4.3.2, Example 4.5.6

If Beth is not falling, but rather standing safely on the floor, then the gravitational force is still acting. It can be computed as

$$\vec{F}_g = m\vec{g} = (75.0\,\mathrm{kg})[-(9.81\,\mathrm{m/s^2})\,\hat{\jmath}] = -\mathbf{735}.75\,\mathrm{N}\hat{\jmath} = -736\,\mathrm{N}\hat{\jmath}$$

However, since we can see that her acceleration is zero, the $\vec{F}_{\text{net}}$ *must be zero. The only way that can happen, though is if there is another force acting upwards on Beth. What could possibly be pushing up on her? Answer 1. Whatever it is pushing up on her, it is supplying a support force, which can be calculated since* $\vec{F}_{\text{net}} = \vec{F}_g + \vec{F}_{\text{support}}$ *and we can solve for*

$$\vec{F}_{\text{support}} = \vec{F}_{\text{net}} - \vec{F}_g = m\,(0\,\mathrm{m/s^2}) - \left[-(\mathbf{735}.8\,\mathrm{N})\hat{\jmath}\right] = +736\,\mathrm{N}\hat{\jmath}$$

Because it is in the direction opposite to $\vec{F}_g$*, it is upwards* $(+\hat{\jmath})$*.*

Can you identify why the support force is equal in magnitude and opposite in direction to the gravitational force? Select one: Newton's second law or Newton's third law.I hope you guessed the floor. That is the only thing pushing up on Beth. One useful way to think about it is that the floor is the thing keeping her from falling. The direction of this force is normal *(perpendicular) to the horizontal floor, so it is in the vertical direction. This will be discussed in more detail in Section 4.3.You can tell that it is Newton's second law* ($\vec{F}_{\text{net}} = m\vec{a}$) *because the forces we are considering are acting on the same object. In this case, the gravitational force is caused by the Earth and the normal force is caused by the floor by they are both felt by Beth. These forces happen to be equal and opposite because she happens to be in equilibrium. She does not have to be in equilibrium, such as when she jumps, in which case the forces would not be equal and might not be opposite.If it were Newton's third law, then the two forces we were discussing would be acting on different objects and would be unrelated to the fact that the object (in this case, Beth) is in* *equilibrium. The gravitational force and the normal force in this case are both acting on Beth, so although they happen to be equal and opposite, this is not due to Newton's third law.*

You should, however, note that the force that is reaction-paired to the gravitational force on Beth by the Earth is a gravitational force on the Earth by Beth. Similarly, the reaction-paired force to the normal force on Beth by the floor is a normal force on the floor by Beth. (Please note the "on" and "by" in each case.)

As was mentioned earlier, the value of the acceleration due to gravity also varies across the surface, although this is less than about a percent or so (see Table 14). Nonetheless, this means that your weight can change even when your mass remains the same.

Example 4.1.11 (*Weight can vary even if mass does not*)

While talking to your friend Beth, you learn that her parents, Erik and Frances, grew up in Norway, visited Puerto Rico, and climbed Mount Everest before settling in the United States. Using Table 14, compute Erik's weight at each location, assuming his mass is 95.0 kg*: Norway, Puerto Rico, and Mt. Everest.*[Norway] $F_g = mg = (95.0\,\mathrm{kg})(9.825\,\mathrm{m/s^2}) = \mathbf{933}.4\,\mathrm{N}$.[Puerto Rico] $F_g = mg = (95.0\,\mathrm{kg})(9.782\,\mathrm{m/s^2}) = \mathbf{929}.3\,\mathrm{N}$.[Mount Everest] $F_g = mg = (95.0\,\mathrm{kg})(9.763\,\mathrm{m/s^2}) = \mathbf{927}.5\,\mathrm{N}$.

The precision of g Because the variation is small, throughout this text when we are considering situations "at the surface of the Earth", we will assume that

Convention 4.1.12

the acceleration due to gravity is $9.81\,\mathrm{m/s^2}$ *to three significant figures.*

This statement is making an approximation that allows us to compute values to a useful precision. You should note from Table 14([4]) that the value of g does vary around the world, but that your distance from the center of the Earth has to change *significantly* for the value to change by a noticeable amount. The consequence of this is that whenever we consider motion that stays within a vertical range of tens-of-meters, we can consider the acceleration due to gravity to be constant. Sometimes this is referred to as being "locally constant". This is an example of a useful approximation that is essentially true, as discussed in Section 4.4.

Example 4.1.13 (*The acceleration due to gravity is only "locally constant"*)

Consider the values for the acceleration due to gravity at various locations around the Earth. Look for a pattern in the values as the latitude increases. You might notice the values for Mount Everest and Denver; Can you explain any peculiarity?Because the Earth was spinning as it cooled (forming the crust), it formed an oblate spheroid.[5] *Since the strength of the gravitational interaction depends (among other things) on how far you are from the center (slightly weaker further away), the acceleration due to gravity is smaller when you are at smaller latitudes (closer to the equator).In addition to being an oblate spheroid (Footnote 5), the Earth has mountains and valleys. Since the strength of the gravitational interaction depends (among other things) on how far you are from the center (slightly weaker further away), the acceleration due to gravity is smaller when you are at at high altitudes, such as Denver, CO and Mount Everest.*

[4]While both the latitude-longitude and the local value of g were found using the WolframAlpha® computational knowledge engine, these g values do not necessarily correspond to these coordinates. The g values are based on a theoretical model of the Earth.

Location	latitude	longitude	local $g(\mathrm{m/s^2})$
San Juan, Puerto Rico	18°26′24″ N	66°7′48″ W	9.782 $\mathrm{m/s^2}$
Brownsville, TX	26°1′6″ N	97°27′14″ W	9.788 $\mathrm{m/s^2}$
Mount Everest (Napal/China border)	27°59′17″ N	86°55′31″ E	9.763 $\mathrm{m/s^2}$
Cincinnati, OH	39°8′24″ N	84°30′23″ W	9.801 $\mathrm{m/s^2}$
Denver, CO	39°45′43″ N	104°52′50″ W	9.798 $\mathrm{m/s^2}$
Paris, France	48°51′36″ N	2°20′24″ E	9.813 $\mathrm{m/s^2}$
Oslo, Norway	59°54′36″ N	10°45′ E	9.825 $\mathrm{m/s^2}$
Anchorage, AK	61°10′39″ N	149°16′28″ E	9.826 $\mathrm{m/s^2}$

Table 4.1.14: Comparison of g at a few places on Earth. (See Footnote 8.1.2.4.) Example 13 considers some patterns in this table.

4.2 Fundamental Forces

The previous section describes our (macroscopic) experience of the gravitational interaction when standing on the surface of the Earth. This is essentially the same across the surface, but does change with altitude and the difference can be measured on mountain tops and in caves. In fact, one can use the differences from one location to another to predict where we might find a a pocket of oil.

In later sections, we will consider this and other interactions that depend on the physical properties, such as mass and charge. All particles with the property of mass (which we will start to call gravitational charge) will interact according to the gravitational force; however, this description is better described by the mathematics in Chapter 6. All particles with the property of electrical charge will interact according to the electrical force. The basic theory will be discussed in Chapter 1. A more complicated version that incorporates quantum mechanics is called quantum electrodynamics (QED) and this will be touched on in Subsection 2.3.2. Particles like protons and neutrons (hadrons) are actually made up of other particles (quarks) that are held together by an interaction that is sometimes called the strong nuclear force (Subsection 2.2.2) and is described by the theory of quantum chromodynamics (QCD); this will be touched on in Subsection 2.3.3. Finally, in Subsection 2.2.3 another fundamental force, called the weak nuclear force, will be discussed.

For the most part, these theories describe the interaction between microscopic particles, so we will not discuss them in detail here. However, the gravitational interaction is exception in a variety of ways. In particular, the gravitational interaction does affect macroscopic objects. These fundamental forces have a particular description that allows us to pretend (recall effective theories) that they are action-at-a-distance interactions. All other forces (introduced next) will require physical contact in order to exert the force.

4.3 Normal Force

Section referenced by 3.2.25, Answer 8.1.2.10.1, 4.5

The word "normal" originates with the idea of conformity to the pattern. While in everyday life this the typical state of being, the origins actually refer to a carpenter's square, which put corners into a right angle. In math and physics, the word is used to mean perpendicular.

Insight 4.3.1

> *In the context of forces, the normal force is the force that a surface exerts to keep objects from passing through them. The direction of this force is always in the outward direction, normal (perpendicular) to the surface.*

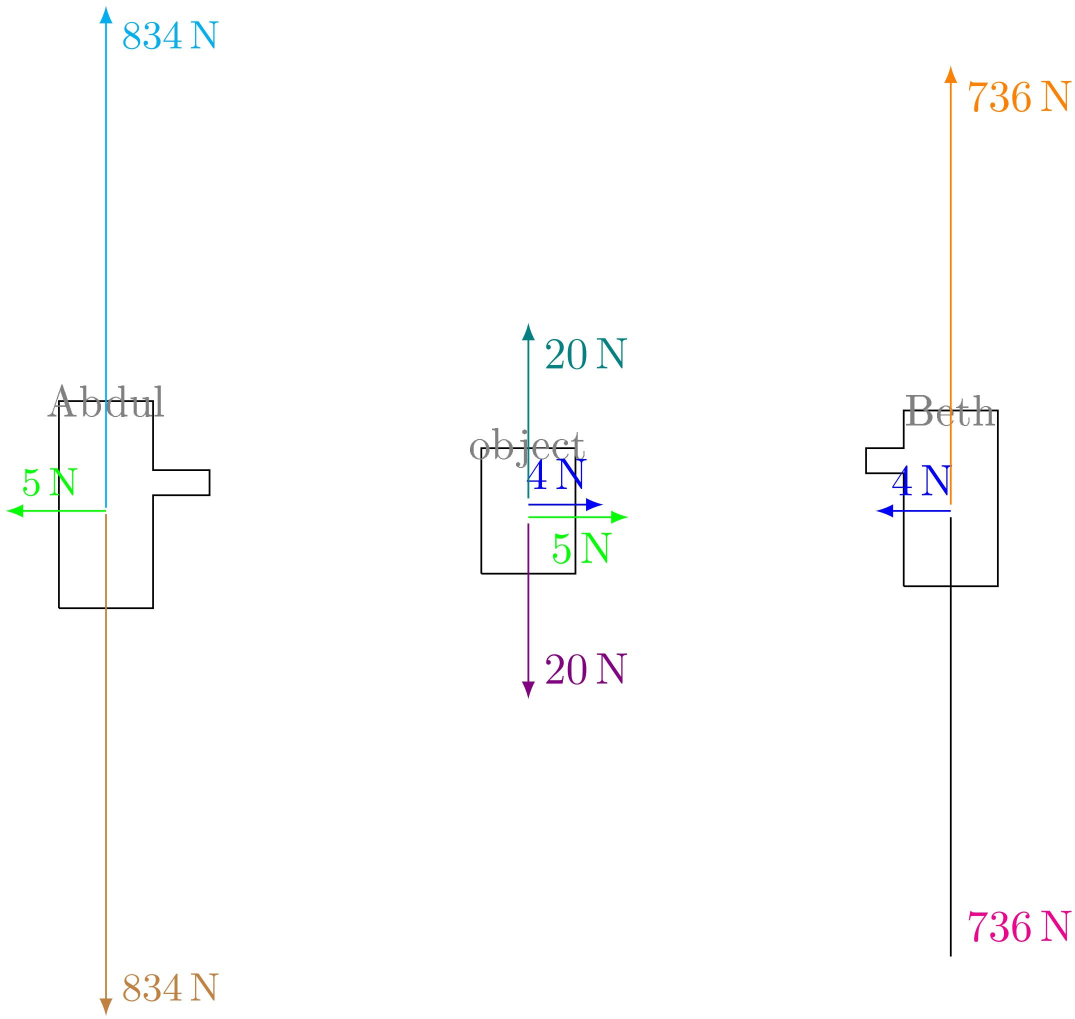
Let's consider some specific situations... In Example 4.1.10, Beth felt the downwards gravitational force even while she was standing on the ground. We noticed that she was not falling (and so not accelerating). Colloquially, we say that the ground is supporting Beth. This support force is keeping Beth from passing through the floor; this is a normal force. The normal force from the floor is acting upwards, which is normal (perpendicular) to the surface of the floor. Example 4.3.2 updates the free-body diagrams of Example 3.2.25 to show how the gravitational and normal forces impact that calculation.

Example 4.3.2 (*People pushing a box also feel the gravitational and the normal forces*)

Let's start from a familiar example so that we focus our attention on the new *aspect. Starting from Example 3.2.25, we can observe that each of the three bodies also has a downwards gravitational force. This is analogous to the calculation in Example 4.1.7, which was only for Abdul; but you can calculate the weight for the mass in Example 3.2.18 and Beth's weight was computed in Example 4.1.10. In addition to the downward gravitational force (the weight), Newton's second law and the fact that nothing is accelerating up or down together tells us that there must also be a normal force on each body. This is analogous to the calculation in Example 4.1.10, which was only for Beth; but you can deduce it for the object and for Abdul.*

Figure 4.3.3: Abdul and Beth push on a box

Now, as in Figure 3.2.27, we will draw a free-body diagram for each individual separately. However, this time we will use the information calculated in Example 4.1.7 and Example 4.1.10 to include the gravitational force (the weight) and the normal force. I will use the "on-by" notation to distinguish the forces.



Even with the vertical forces, Abdul still has a $\vec{F}_{\text{net}} = -5.0\,\text{N}\hat{\imath}$.

Even with the vertical forces, the object still has a $\vec{F}_{\text{net}} = +9.0\,\text{N}\hat{\imath}$.

Even with the vertical forces, Beth still has a $\vec{F}_{\text{net}} = -4.0\,\text{N}\hat{\imath}$.

Figure 4.3.4: This figure modifies Example 3.2.25 by allowing for the gravitational and the normal forces.

The normal forces and the gravitational forces are all caused by each object interacting with the Earth. So, while we can use Newton's second law to deduce the value of each normal force, the normal and gravitational forces are not third-law pairs. In order to see the third-law pairs, please review Example 4.3.12

Return to: 3.3.4, 3.3.7, 4.3, rope-tension, 4.5.6

Let's consider some other specific situations... If you decide to lean against a wall, the wall will provide a normal force that pushes horizontally, keeping you from moving through the wall.

Example 4.3.5 (*Ladders push on the wall and on the floor*)

Carl leans a $22.7\,\text{kg}$ *ladder against a wall at an angle of* $75.5°$, *consistent with* OSHA *standard* 1926.1053(a)(1)(ii), *so that about* $^1\!/_8$ *of the weight is leaning into the wall.*

1. *Find the magnitude and direction of the normal force exerted* by the wall *on the ladder. (Answer 1)*
2. *Find the magnitude and direction of the normal force exerted* by the floor *on the ladder. (Answer 2)*

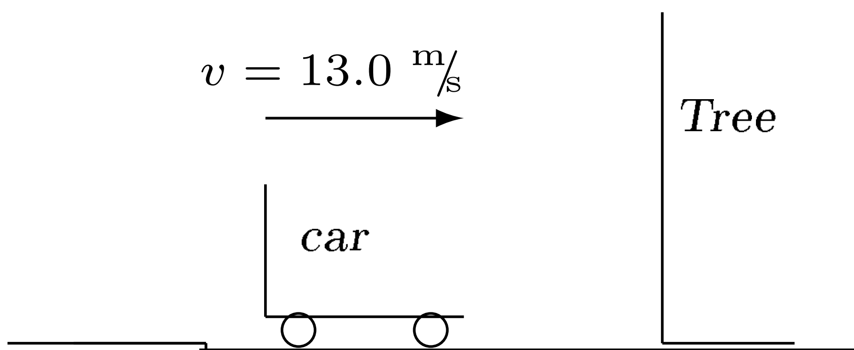
Note: Example 4.2.1 *goes into the full details of how one calculates the necessary values.Since the weight is* $F_g = mg = (22.7\,\text{kg})(9.81\,{}^{\text{m}}\!/_{\text{s}^2}) = \mathbf{222}.69\,\text{N}$, *an eighth of this is* $\mathbf{27.8}36\,\text{N}$. *This force is pressing* *into the wall (horizontally, which I will choose as the* $+\hat{\imath}$ *direction). By* Newton's third law *if the ladder presses into the wall with* $\mathbf{27}.836\,\mathrm{N}$ *in the* $+\hat{\imath}$ *direction (this is also a normal force), then the wall pushes the ladder with a normal force of* $\mathbf{27}.836\,\mathrm{N}$ *in the* $-\hat{\imath}$ *direction.* Notice that this is normal (perpendicular) to the surface of the wall.*Since the full weight of the ladder,* $F_g = \mathbf{222}.69\,\mathrm{N}$, *is still pressing downwards* $(-\hat{\jmath})$ *into the floor (as a normal force),* Newton's third law *says that the floor pushes the ladder upwards* $(+\hat{\jmath})$ *with a normal force of* $\mathbf{222}.69\,\mathrm{N}$. Notice that this is normal (perpendicular) to the surface of the floor.

If you lose control of your car and run into a tree, the tree also provides a normal force pushing the car away from the tree; this normal force will stop the car.

Example 4.3.6 (*The normal force stops a crashing car*)

Zambert is driving home after a late night of studying at the library. He is kind of tired and drifts off during the drive. While traveling $\vec{v}_i = 13.0\ \mathrm{m/s}\,\hat{\imath}$, *Zambert runs into a tree, bringing his car* ($m = 2.1 \times 10^3\,\mathrm{kg}$) *to a halt in* $\Delta t = 0.243\,\mathrm{s}$. *(Zambert remains unharmed because he was awake enough to wear his seatbelt and had a car with a functioning airbag. Whew.) Find the normal force by the tree on the car.*

To be clear about what is happening, I will draw the picture.



Solution. *In order to find the force, we will first need to find the acceleration.*

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{(0\ \mathrm{m/s}) - (13.0\ \mathrm{m/s}\,\hat{\imath})}{0.243\,\mathrm{s}} = -\mathbf{53}.49\ \mathrm{m/s^2}\,\hat{\imath}$$

That the acceleration is in the direction opposite the velocity corresponds to the object slowing down. Now we can find the ***net force*** *from Newton's second law:*

$$\vec{F}_{\text{net}} = m\vec{a} = (2.1 \times 10^3\,\mathrm{kg})(-\mathbf{53}.49\ \mathrm{m/s^2}\,\hat{\imath}) = -\mathbf{1}.12 \times 10^5\,\mathrm{N}\hat{\imath}$$

There are three forces acting on the car, as can be seen in analogy with the free-body diagrams of Example 4.3.2. *So, we can draw a free-body diagram here as well.*

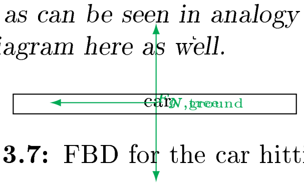


Figure 4.3.7: FBD for the car hitting the tree

The gravitational force on the car is

$$\vec{F}_g = m\vec{g} = (2.1 \times 10^3\,\mathrm{kg})(-9.81\ \mathrm{m/s^2}\,\hat{\jmath}) = -\mathbf{2}.06 \times 10^4\,\mathrm{N}\hat{\jmath}$$

Since this in the vertical direction and the net force is in the horizontal direction, there must be an upwards normal force from the ground

$$F_{N,\text{ground}} = \mathbf{2}.06 \times 10^4\,\mathrm{N}\hat{\jmath}.$$

This is normal (perpendicular) to the surface of the ground. *The remaining horizontal force is the normal force from the tree,*

$$F_{N,\text{tree}} = -\mathbf{1.12} \times 10^5\,\text{N}\hat{\imath}.$$

This is normal (perpendicular) to the surface of the tree.

(Notice that Example 4.3.6 also shows why it is not always necessary to consider the vertical forces when we "know" that they cancel.) If you throw a ball at the ceiling, the ceiling will provide a normal force downwards, keeping the ball from moving through the surface.

Example 4.3.8 (*The normal force acts to reflect objects off a surface (ceiling)*)

Carl recalls that one time he got bored one day in physics class (what?!?) and tossed a baseball ($m_b = 0.145\,\text{kg}$) at the ceiling... a little too hard ... as recounted in Exercise 1.6.1. The acceleration during that collision with the ceiling was $\vec{a} = -\mathbf{28.09}\,{}^{\text{m}}\!/\!_{\text{s}^2}\hat{\jmath}$. Find the normal force by the ceiling on the ball.

Solution. *There are five stages to the motion: (a) throwing, (b) falling up, (c) hitting the ceiling, (d) falling down, and (e) catching. We can show the forces involved in each (although we only care about the forces during (c) hitting the ceiling).*

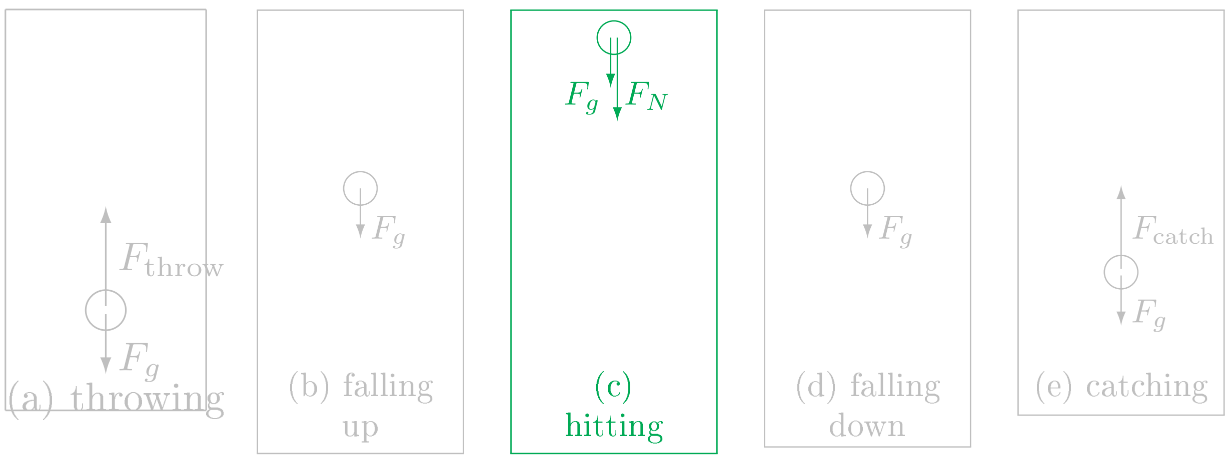


Figure 4.3.9: When a ball is thrown there are five distinct stages because the forces on the ball change and we can, at this point, only manage to describe a situation in which the forces do not change.

In this particular problem, we are only concerned with step (c) when the ball hits the ceiling, because that is the only part where the normal force acts. Example 4.7.1 *will describe what happens during steps (a) and (e).*

During step (c), we have the actual acceleration, which tells us about the net force. We will also need to know the weight of the baseball, because gravity is still acting during the collision.

$$\begin{aligned}
\vec{F}_N + \vec{F}_g = \vec{F}_{\text{net}} &= m\vec{a} \\
\vec{F}_N + m\vec{g} &= m\vec{a} \\
\vec{F}_N &= m\vec{a} - m\vec{g} \\
\vec{F}_N &= \left[(0.145\,\text{kg})(-\mathbf{28.09}\,{}^{\text{m}}\!/\!_{\text{s}^2}\hat{\jmath})\right] - \left[(0.145\,\text{kg})(-9.81\,{}^{\text{m}}\!/\!_{\text{s}^2}\hat{\jmath})\right] \\
\vec{F}_N &= \left[-\mathbf{4.07}3\,\text{N}\hat{\jmath}\right] - \left[-\mathbf{1.42}2\,\text{N}\hat{\jmath}\right] = -\mathbf{2.65}1\,\text{N}\hat{\jmath}
\end{aligned}$$

You can see that the downward normal force ($\mathbf{2.65}1\,\text{N}$) *combined with the downward gravitational force* ($\mathbf{1.42}2\,\text{N}$) *together create the downward net force* ($\mathbf{4.07}3\,\text{N}$).

If you make a "bank shot" with either a basketball off the backboard or a pool ball[1] off the bumper, then the

[1]Resources for specifications and a PDF version. These provide: weight (5.5 oz = 0.**155**92 kg and 6.0 oz = 0.**170**097 kg cue),

surface provides a normal force that is perpendicular to the surface, in this case redirecting the ball rather than stopping it. Unfortunately, the actual mechanism is somewhat more complicated than we are ready for; these are considered a little bit in the Investigation 1.

diameter (2.250 ± 0.005 in $=$ **5.71**5 cm $\pm$ 0.0127 cm), rail height (63.5% of the ball height, $=$ **3.62**9 cm), and dimension limits on the cue stick: $L_{\mathrm{min}} = 40.00$ in $= 1.016$ m, $m_{\mathrm{max}} = 25.0$ oz $=$ 0.**708**75 kg, and tip-width $w_{\mathrm{max}} = 1.4$ cm. You might also consider the information and calculations at Dr. Dave's site, which gives slow (1 mph), medium (3 mph), and fast (7 mph); coefficient of friction ball-to-ball $\mu = 0.06$; and ball-ball collision times as 250 μs-300 μs.

Investigation 4.3.10 (*Pool balls and bumpers / cushions*)

Diane is relaxing with the local physics club, playing pool. She shoots a bank-shot and the ricochet reminds all of you about the normal force from the bumper on the ball.

(a) *Find a billiards table. Notice the felt, the bumpers (cushion), and the dimensions of the table. Does the ball roll as far on felt as it does on hardwood? (Answer 1) How soft is the bumper? (Answer 2)*

Answer. *First, you should not roll a pool ball across just any floor; there is felt on the pool table for a reason. However, if you have a clean, smooth surface and are able to reproduce your rolling speed, you will find that the pool ball rolls further on the stiff, nonyielding surface than it will on the felt. The reason for this is beyond the scope of this textbook, but you can read more from "Sliding and rolling: the physics of a rolling ball," J. Hierrezuelo and C. Carnero, Physics Education, Volume 30, Number 3* (unofficially at *this PDF*).

Answer. *The cushion (sometimes called a bumper) is pretty still to the touch, but it is made of a springy rubber that allows the balls to bounce reasonably well. The* document *indicates that you should be able to firmly strike a ball at some angle to the far wall and have it bounce around the table four to four-and-a-half times. If the bumpers were perfectly* elastic*, then the normal force would be normal to the resting surface; but since the bumper has some flexibility, when the ball hits the bumper with a glancing blow, then bumper bends inwards and the normal force is directed in a way that depends on the shape of the dent.*

(b) *Find a set of pool balls. Compare the solid-colored balls, the striped balls, and the cue ball. Are there differences in size of weight?*

Answer. *The* specifications *show that there is no difference between the solids and stripes, but the cue ball weighs 9% more that the other balls* (6.0 oz *versus* 5.5 oz)*. The colored balls and the cue ball are otherwise identical.*

(c) *Hit the cue-ball off of a bumper in the manner intended for* testing cushions*. Compare the angle it leaves the bumper (reflected angle) match the angle at which it came in (incident angle). Does the spin of the ball matter?*

Answer. *Because the bumper is covered in felt, it has a small grip on the ball. Because the bumper has some give to it, it dents in when hit and provide more surface area, which increases the grip. Both of these mean that the spin of the ball gets transferred to the pool table somewhat and change the way a spinning ball exits from the bumper collision.*

(d) *Place a pool ball against the bumper and ricochet the cue ball off the pool ball instead of the bumper itself. Notice how the pool ball reacts. Why does the pool ball jump off the bumper?*

Does the pool ball move along the wall?

Where did you hit the pool ball?

Answer. *[Answer]*

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: friction, elastic versus inelastic collisions, rotational motion, *and* angular momentum.

Return to: the discussion of pool

4.3.1 Bathroom Scales Measure the Normal Force

Referenced by Discussion of uses of $F = ma$

To get a good sense of what how the normal force works, it helps to consider the way a bathroom scale works. Consider the concepts presented in the Investigation 4.3.11.

Investigation 4.3.11 (*Playing with a scale.*)

While speaking to your friend, Beth about her recent accomplishment of losing 45 N, you mention that your scale always gives a different number than the one in the doctor's office. You suggest she gets on your scale to verify the calibration. Beth currently has a mass of 75.0 kg.

(a) *Imagine losing 45 N. Compare this to your weight. Is this a lot of weight to lose?*

Solution. *Since Beth weighs* $(75.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 736\,\mathrm{N}$, $45\,\mathrm{N}$ *is about 6% of her weight. This is fairly substantial. You should compute how much 6% of your weight is and convert that to kilograms and Newtons.*

(b) *Place your toe on the scale while Beth weighs herself. This increases the value the scale reads. Does Beth weigh more?*

Solution. *When one person stands on the scale, the scale provides just enough of an upwards normal force to keep that person in equilibrium. In that case, the upwards force is balancing the weight of the person. This gives the impression that the scale is telling you your weight; however, when you press down or help support whomever is standing on the scale, the scale adjusts the amount it must provide. The scale is not trying to tell you your weight. Rather the scale is trying to create equilibrium by balancing whatever force(s) are pressing into it. Your weight is determined by the gravitational force and does not change when you press harder or lighter onto the scale.*

(c) *With your hands, press down on Beth's shoulders while she stands on the scale. Control the value read by the scale (HINT: Task b). Increase the reading by 20 N, 30 N, etc. Does Beth's weight change? (Solution 8.3.1.2.b.1) Are you adding weight to the scale? (Solution 8.3.1.2.c.1)*

Solution. *When you press down on Beth's shoulders, you are not adding weight. Weight has a specific definition: it is specifically the value that the gravitational force pulls on any object. Pushing the person does not change their weight; it does, however, change the amount that they press into the Earth. That is to say, it increases their downwards normal force, but not their weight.*

(d) *Have Beth lean on a nearby table or counter while she stands on the scale. Control the value read by the scale (HINT: Task c). Decrease the reading by 20 N, 30 N, etc. Does Beth's weight change? (Solution 8.3.1.2.b.1)*

(e) *Hold the scale against the wall and press into it. Control the value read by the scale (HINT: Task d). Increase the reading by 20 N, 30 N, etc. What is the scale measuring?*

Solution. *Since your weight is a force pulling downwards, having the scale on the wall shows that the scale cannot be balancing weight. Since you are pushing into the wall, you are exerting a normal force into the scale and the scale is exerting a normal force back at you. Both of these forces are horizontal (assuming the wall is plumb).*

Another way to think about this is: If you can control the value read by the scale (such as against the wall) while at the same time not changing your actual mass, the scale cannot literally be measuring the weight of the object on the scale.

(f) *Imagine placing a scale on a ramp that can be laid flat or raised to any angle up to a vertical (making it a wall). Imagine standing on the scale on the ramp while it is lifted from horizontal (like a floor) to vertical (like a wall). Does the scale always read the same value while it is raised to different angles?*

Solution. *When the scale is on the flat, horizontal floor, it balances your full weight. When the scale is on the vertical wall it does not carry any of your weight. At any angle in between those values, it carries some fraction of your weight while friction keeps you from sliding down the ramp. It will turn out that since the cosine function behaves in just the right way, we can use the cosine to find the component of the weight that the normal force from the scale has to support.*

Return to: Subsection 4.3.1

Some digital scales are inconvenient for understanding how they work because they don't display the value until it has come to something close to equilibrium. If you have access to an analog scale, then you can watch the value change as it settles down and it might be easier to build your intuition.

As you consider the values that you read on the scale, you should consider what happens if you jump off of or land upon a scale. *Note that actually doing this can decalibrate your scale, if not break it entirely. Scales are not meant to be handled this way.* While you are jumping from your scale, it must provide not only the force necessary to support your weight, but also the upwards force require to accelerate you upwards. While you are landing on the scale, it musty provide not only the force necessary to support your weight, but also the upwards force necessary to decelerate you.

Bathroom scales use leverage (i.e., torque) and a spring-system to balance the force pressing into them. The mechanism can be seen at How Stuff Works.

4.3.2 The normal force and the third law

the third law

Example 4.3.12 (*Using the second and third laws to find all forces acting*)

See Example 4.3.2 and add the FBD for the Earth.

4.4 Frictional Force

Section referenced by Task 3.3.2.b, Task 3.3.2.d.ii, Task 3.3.2.e, Solution 6.2.1.1.a.i.1

4.4.1 sliding friction

4.4.1.1 static friction

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4.4.1.2 kinetic friction

.

4.4.2 rolling friction

Investigation 4.4.1 (*Rolling pool balls and friction*)

Diane is relaxing with the local physics club, playing pool. She hits the cue ball and counts the number of walls she can hit in one shot.

(a) *Hit the cue-ball off of a bumper in the manner intended for testing cushions. Compare the strength of the hit to the distance travelled.*

How much is the total distance affected by the number of bumpers hit? (Answer 1) Does it matter if you shoot along the length of the table versus the width of the table? (Answer 2) Why does friction slow the ball down instead of just make it turn $v = \omega r$ (no slip) (Answer 3)

Answer. *[Answer]*

Answer. *[Answer]*

Answer. *[Answer]*

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: normal force, elastic versus inelastic collisions, rotational motion, and angular momentum.

Return to: pool

4.4.3 Air Resistance

.

4.5 Tension

Section referenced by Discussion of $F = ma$

Insight 4.5.1

Where the normal force is appropriate for pushing against surfaces, ***tension*** *is the pulling force that is transmitted through materials such as cable, chain, or rope.*

Tension is closely related to the compression force experienced by support beams. One can simplistically think of tension as pulling and compression as pushing on the intermediate object that transmits force between the objects at either end.[1] When engineers design the skeleton of bridges and buildings, one of the primary considerations is the tension and compression of the steel beams. You can build your intuition by considering the Investigation 4.5.2.

[1]It doesn't usually make sense to talk about the compression of a rope or chain.

Investigation 4.5.2 (*Pull my finger.*)

We talk about tension and stress in our daily lives. This is an analogy to the physical version of tension, stress, and strain. While stress and strain come from the the concept of tightening, tension comes from the concept of stretching.

(a) *Sit on a swing. Notice the tightness of the support ropes/chains. How tight are the supports when the swing is empty? When a small child is in the swing? When a full-sized adult is in the swing?*

Answer. *To make this comparison, let's consider a swing that is supported by chains. If you are sitting in the swing and take hold of the chains at about shoulder height, you should be able to shake them in (towards your chest) and out (away from you, towards your neighbor swings). You can do this same motion while standing next to the swing. If you do this when the swing is empty, it is very easy to do this. If you ask a series of successively larger people to sit in the swing, you will notice that it gets progressively more difficult to extend them very far. The chains are increasing in tension; they are pulled more taut. Your ability to move the chain in this way is exactly analogous to the way a bow draws across a violin or the way your fingers pluck a guitar, as described in Subsection 3.1.1.*

(b) *Install a fan or light fixture that hangs from the ceiling. You don't want the fan to be supported by the electrical wires, but rather by the metal shaft. How is the fan supported?*

Answer. *You might also consider Answer 2, which discusses the case of hanging a light fixture from the ceiling. If you have ever installed a fan in your house, then you will notice that you have to support the fan while the wires are connected. Usually the fan has a shaft that connects to the ceiling at one end and the fan at the other and provides a mechanism for supporting the fan while you manage the wires, which pass through the shaft. Since the fan houses the motor, it is usually reasonably heavy. The nice property of use a metal shaft to support the fan is that it doesn't stretch or wiggle like a chain might. The difficulty in this example is that it is more difficulty to notice the tension in the shaft. If you are the person hanging the fan, then one thing you might be able to notice is that if you flick the metal with a finger when it is not supporting anything, it will have a slightly different "ting" than when it is supporting the fan.*

Answer. *If you have ever installed a chandelier in your house, then you will notice that the light has to be supported between the joists of the ceiling. There will be an electrical box with a screw to which you will attach the support for the chain that holds the chandelier. The wires will run through the support chain. The heavier the chandelier, the tauter the chain, much as described in Answer 1. This tension is much easier to see than the tension in the shaft of the fan.*

(c) *Pull on a doorknob. Imagine replacing the knobs (inside and outside) with large knots on a rope that runs through the hole the doorknob used to occupy. What if the doorknob were replaced with a rope, knotted on either side of the door?*

Answer. *[Answer]*

(d) *Take a dog for a walk on a leash. Try to pay attention to Newton's second and third law when the dog changes its level of enthusiasm for pulling on the leash. If the dog pulls very hard on the leash and you balance that force without allowing the dog to move away from you, then describe the way the force connects you to the dog.*

Answer. *[Answer]*

Return to: Section 4.5

When considering the tension in the rope, the context is generally that the rope is connecting two objects that are trying to pull on each other. It is convenient to recognize that each object only "sees" the rope, not the object at the far side. This can be seen in a couple of contexts.

We will start with the simplistic approximation of ropes that only transmit the force. As your understanding improves, we will add some examples where the tension in the rope also affects the rope itself. In that more complicated situation, the tension will change across the rope and the rope may stretch. Since ropes and cables are twisted strands while chains are links, ropes and cables can also introduce a torsion that tends not to occur in chains.

Foreshadow ...

4.5.1 Tension as a Support Force

Ropes and chains (and beams) can use tension to support (from above) dangling objects.

Example 4.5.3 (*Tension supports hanging objects*)

Diane hangs her purse ($m = 1.36\,\mathrm{kg}$) on a hook. How much tension is in the shoulder strap to keep it from falling?Since the strap supports the purse, what does the strap support against?If we assume the purse is hanging peacefully, then it is not accelerating. Is there an equation that relates forces to accelerations?

Solution. *The strap connects the hook to the purse. We can consider the interaction between the hook and the strap or between the purse and the strap. We will consider the latter since we don't know anything about the hook.*

Considering the forces on the purse, we know that there is a downwards gravitational force of $F_g = (1.36\,\mathrm{kg})(9.81\,{}^{\mathrm{m}}\!/_{\mathrm{s}^2}) = \mathbf{13.3}4\,\mathrm{N}$ and that the net force must zero (because the purse is not accelerating). So, the strap must provide an upwards (tension) force.

$$\begin{aligned} \vec{F}_T + \vec{F}_g &= m\cancelto{0}{\vec{a}} \\ \vec{F}_T + (-\mathbf{13.3}4\,\mathrm{N}\hat{\jmath}) &= 0\,\mathrm{N} \\ \vec{F}_T &= +\mathbf{13.3}4\,\mathrm{N}\hat{\jmath} \end{aligned}$$

This is the upwards force that the strap applies to the purse.

Note: *The tension strap is doing two jobs: It is pulling up on the purse (as indicated above)* and *it is pulling down on the hook.*

The important thing to take away from Example 4.5.3 is not that we can compute the value (although that is, of course, a useful skill), but rather that

Insight 4.5.4

the tension is conveying the force between the two objects.

In the same way that the normal force on a scale does not measure your weight, but rather the amount you press into the scale, the tension passes force on to the attached object. The hook doesn't feel the weight of the purse, but does feel the tension required to support the purse.

In an upcoming section, we will consider what happens when multiple masses are hung from the rope.

4.5.1.1 How Physicists Use the Words (Vocabulary)

You can probably think of several examples of objects dangling: a purse on a hook, a flag on a pole, a shop sign attached to a post, a pendulum,a swing set, etc. Since these are all similar in some ways (although different in other ways), *we can treat all of them as a mass at the end of a rope*. Typically, because we do not want to deal with the complications that come from sagging supports, we will use the approximation of an "*immovable support*." This will be indicated by hashing the surface.

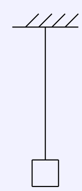


Figure 4.5.5: [Add caption]

4.5.2 Tension as Dragging Force

We can also consider the tension in a rope used to drag an object across the floor. You may recall that in Example 3.2.25 (and the updated version, Example 4.3.2) Beth pulled a box across a sheet of ice. It is possible that Beth was grabbing the object itself, but it is more likely that she was pulling on a rope that was attached to the object. In that case, the tension in the rope was 4.0 N. This tension is what pulled Beth leftwards *and* what pulled the object rightwards.

We can further update this by considering the case where Beth pulls the rope up at an angle. In that case, some of the tension is used to drag the box and some is used to reduce the normal force. In Example 4.5.6, we will have Abdul continue to push with 5.0 N horizontally and have Beth pull with 4.0 N at a 14° angle above the horizontal. You should note that since the tension on the object is pulling up, helping the normal force, this allows the normal force on the object (what a scale would read) to be a little smaller. You should also note that since the tension on Beth is pulling down, counter-acting the normal force, this requires the normal force on Beth (what a scale would read) to be a little larger.

Example 4.5.6 (*People pushing a box at an angle*)

Again, we can start by drawing a picture of the situation. The description is the same as it was for People pushing a box also feel the gravitational and the normal forces except that Beth pulls at a slight angle upwards. We will again need the gravitational force for Abdul (Example 4.1.7) and Beth (Example 4.1.10). As before, since nothing is accelerating up or down together, there must also be a normal force on each body.

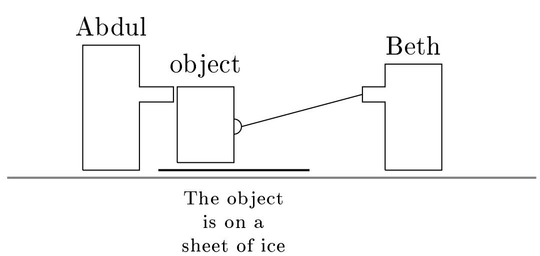


Figure 4.5.7: Abdul and Beth push on a box. Beth pulls at an angle.

Now, as in Example 4.3.2, we will draw a free-body diagram for each individual separately. However, *this time we will put the tension of the rope at the appropriate angle. We will need to do a small calculation to find the value of the normal forces.*

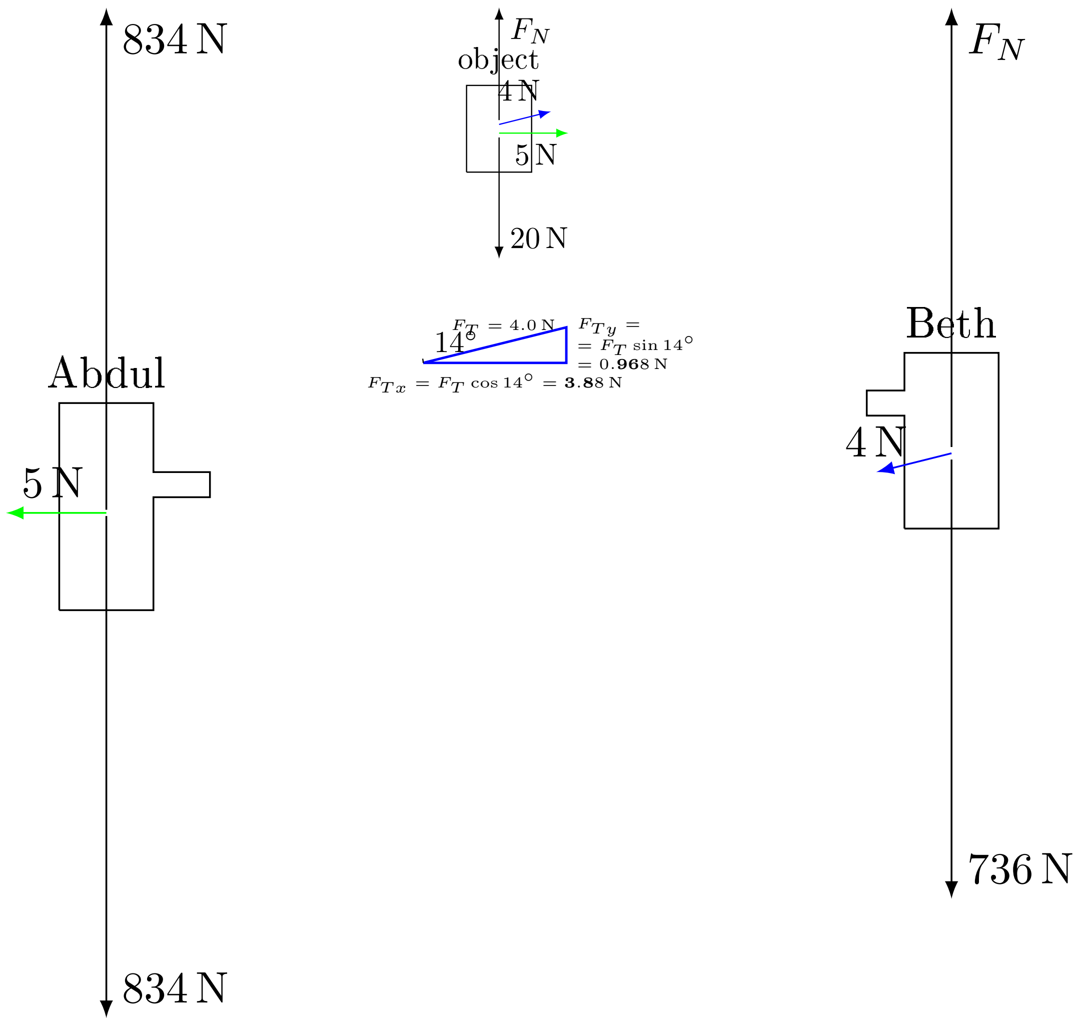


Figure 4.5.8: The forces on Abdul have not changed. ... The forces on the object *have* changed. ... The forces on Beth *have* changed.

For the object*: Since the y-component of the net force is zero, we can find the normal force to be* $F_N = -[(-20\,\text{N})+(+0.\mathbf{96}8\,\text{N})] = 19\,\text{N}$. *The x-component of the net force is* $F_{\text{net},x} = (5.0\,\text{N})+(\mathbf{3.8}8\,\text{N}) = \mathbf{8.8}8\,\text{N}$.

For Beth*: Since the y-component of the net force is zero, we can find the normal force to be* $F_N = -[(-736\,\text{N}) + (-0.\mathbf{96}8\,\text{N}) = 737\,\text{N}$. *The x-component of the net force is* $F_{\text{net},x} = (-\mathbf{3.8}8\,\text{N})$.

Return to: rope-tension

4.5.3 Pulleys

While the flexibility of ropes makes them inconvenient for pushing, their flexibility makes them *very useful* for changing the direction of the pull. The mechanism for changing the direction is the pulley. Furthermore, by allowing us to change the direction of the pull, we are also able to double, triple, or further improve the strength of the pull. The term for this is **the mechanical advantage** of a pulley-system.

First we will consider three simple cases of redirecting the force. In each of these cases, I will assume that the pulley and rope have no mass and that there is no friction in the turning of the pulley (assume it is trivially easy to spin). If we do not make this assumption, then the problem gets significantly more complicated.

Example 4.5.9 (*The tension in a rope over a pulley*)

Abdul decides to hold a box that weighs 20 N *using a pulley-system. What is the tension in the rope?*

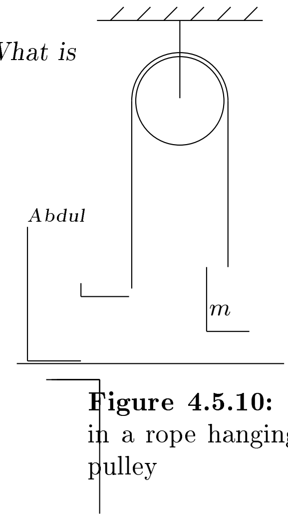


Figure 4.5.10: Tension in a rope hanging over a pulley

Solution. *Since the mass is in equilibrium, the net force is zero and the tension must balance the weight. This tells us that the tension in the rope is* 20 N.

If the pulley were difficult to turn (had friction) that stickiness could help support the mass and the tension on Abdul's side might be less than 20 N;[2] *but since we assumed the pulley to be frictionless, Abdul must provide the full* 20 N *of tension to the rope.*

The interesting aspect is that Abdul must pull *down* in order to produce the *upward* tension on the box. This means that both Abdul and the mass are pulling down. Since the rope is draped over the pulley, the pulley feels 40 N downwards, 20 N from the tension supporting the mass and 20 N from Abdul who is creating the tension that supports the mass. This means that the second rope that is connecting the pulley to the ceiling must be supporting the full 40 N in order to keep the pulley in equilibrium.

4.5.4 Interesting Complications

4.5.4.1 What is the net force on the rope itself?

The answer to this depends on how complicated you want the answer to be (recall the discussion about effective theories in Section 4.4). Some reasonable answers are:

- If the rope is static (whether massive or massless), then the net force on the rope must be zero even while it maintains the tension.
- If the rope is accelerating (and massive), the net force on the rope while it transfers the forces between the objects at each end is whatever is necessary to produce the acceleration $\vec{F}_{\text{net}} = m_{\text{rope}}\vec{a}_{\text{rope}}$. In English, you have to drag the rope as well as the sled to which it is attached.
- If we assume that the mass of the rope is small enough (**insignificant**) then whether it is in equilibrium or accelerating, it does not require a net force and it merely passes its tension through to the object at the other end.

On the other hand, if the rope has mass, then the answer is a different interesting complication, which you can read about in Subsubsection 4.5.4.3. In this case, the tension through the rope changes from one end to the other unless it is not supporting its own weight against gravity (which, unlike other forces, pulls on each portion of the rope) and is static (has no net force).

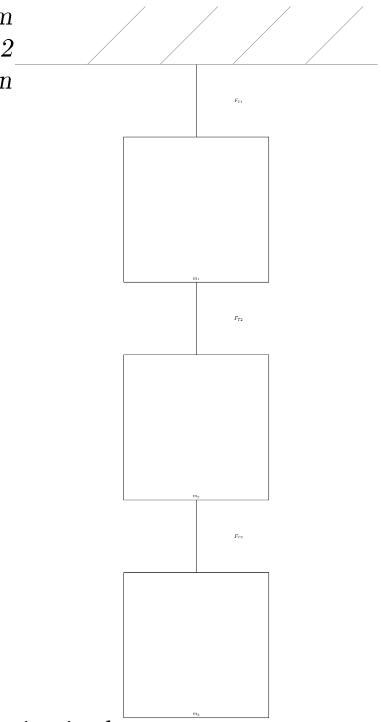
4.5.4.2 Multiple Masses

Referenced by Subsection 4.5.1

Now that we have a few examples of tension under our belts, we can consider some more interesting examples. Example 10 considers the case of hanging multiple masses, which extends the ideas of Subsection 1.

Example 4.5.11 (*Tension between masses hung in a chain*)

While preparing to hang some ornament on a tree, you chain them from a hook on the wall. You hang ornament 3 ($m_3 = 30\,\mathrm{g}$) below ornament 2 ($m_2 = 20\,\mathrm{g}$), which is below ornament 1 ($m_1 = 10\,\mathrm{g}$). What is the tension in each subsequent string?



The tension in the bottom rope only supports the mass beneath it.The tension in the upper ropes supports the mass beneath it and *whatever it has to carry.*

Solution. *The first thing we should do is notice what information is given to us and make sure that everything is in consistent units. I will convert everything to SI units and find the weight of each mass:*

$$
\begin{aligned}
F_{g1} = m_1 g &= (0.010\,\mathrm{kg})(9.81\ \mathrm{m/s^2}) = 0.0\mathbf{98}1\,\mathrm{N} \\
F_{g2} = m_2 g &= (0.020\,\mathrm{kg})(9.81\ \mathrm{m/s^2}) = 0.\mathbf{196}\,\mathrm{N} \\
F_{g3} = m_3 g &= (0.030\,\mathrm{kg})(9.81\ \mathrm{m/s^2}) = 0.\mathbf{29}4\,\mathrm{N}
\end{aligned}
$$

The free-body diagrams show that the bottom mass is the easiest to manage. We should begin there.

Starting at the bottom *of the picture, we can use Newton's second law to find the tension* F_{T3}.

$$\begin{aligned}
\vec{F}_{T3} + \vec{F}_{g3} &= m_3 \cancelto{0}{\vec{a}_3} \\
\vec{F}_{T3} &= -\vec{F}_{g3} \\
\vec{F}_{T3} &= -(-0.\mathbf{29}4\,\mathrm{N}\,\hat{\jmath}) \\
\vec{F}_{T3} &= 0.\mathbf{29}4\,\mathrm{N}(+\hat{\jmath})
\end{aligned}$$

Since $\vec{F}_{g3}$ *is down, we find that* F_{T3} *is upwards (as expected).*
When we then jump to m_2, *we note that the tension* F_{T3} *pulls* m_2 *downwards (even though it pulls* m_3 *upwards), so we have to change the sign. Once we do this, though, we can find* F_{T2}.

$$\begin{aligned}
\vec{F}_{T2} + \vec{F}_{T3} + \vec{F}_{g2} &= m_2 \cancelto{0}{\vec{a}_2} \\
\vec{F}_{T2} &= -\vec{F}_{T3} - \vec{F}_{g2} \\
\vec{F}_{T2} &= -(-0.\mathbf{29}4\,\mathrm{N}\,\hat{\jmath}) - (-0.\mathbf{19}6\,\mathrm{N}\,\hat{\jmath}) \\
\vec{F}_{T2} &= 0.\mathbf{49}0\,\mathrm{N}(+\hat{\jmath})
\end{aligned}$$

Since both $\vec{F}_{g2}$ *and* $\vec{F}_{T3}$ *are down, we find that* F_{T2} *is upwards (as expected). You should also note that* m_2 *does not experience the* weight *of* m_3, *but rather the* tension *support-ing that weight. This is more obvious when we consider* m_1.
Now we jump to the top *of the picture with* m_1 *and note that the tension* F_{T2} *pulls* m_1 *down-wards (even though it pulls* m_2 *upwards), so we have to change the sign. Once we do this, though, we can find* F_{T1}.

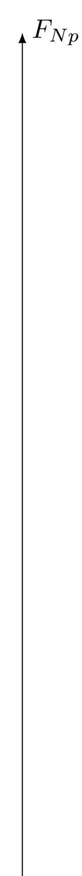
$$\begin{aligned}
\vec{F}_{T1} + \vec{F}_{T2} + \vec{F}_{g1} &= m_1 \cancelto{0}{\vec{a}_1} \\
\vec{F}_{T1} &= -\vec{F}_{T2} - \vec{F}_{g1} \\
\vec{F}_{T1} &= -(-0.\mathbf{49}0\,\mathrm{N}\,\hat{\jmath}) - (-0.0\mathbf{98}1\,\mathrm{N}\,\hat{\jmath}) \\
\vec{F}_{T1} &= 0.\mathbf{58}8\,\mathrm{N}(+\hat{\jmath})
\end{aligned}$$

Since both $\vec{F}_{g1}$ *and* $\vec{F}_{T2}$ *are down, we find that* F_{T1} *is upwards (as expected). You should also note that* m_1 *does not experience the* weight *of the other masses, but rather expe-riences the* tension *supporting that weight.*
You might also note that F_{T1} *pulls the ceiling downwards with a force of* $0.\mathbf{58}8\,\mathrm{N}(-\hat{\jmath})$.

Example 4.5.12 considers the case of dragging multiple masses, which extends the ideas of Subsection 2. Since we have not yet introduced friction, we will assume this is a frictionless surface. We will update this example in Section 4.4 with Example 4.5.13.

Example 4.5.12 (*Tension in a caravan*)

When Beth was a child in Norway, she was pulled through the woods in a sled by her parent, Frances. Beth's sled ($m_B = 50.0\,\mathrm{kg}$) *was also tied to a sled on which her dog sat. The dog's sled* ($m_d = 30.0\,\mathrm{kg}$) *was then connected to a sled with provisions for the day* ($m_p = 10.0\,\mathrm{kg}$)*. As they start their journey, the* *entire system is accelerated at* $a = 0.215\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$.


F_{Np}

Solution. *The first thing we should do is draw the free-body diagrams. These are given in Answer 1. If there were friction, then we would*

You should note that these examples are essentially expressing the same idea in two different contexts.

We can now update Example 4.5.12.

Example 4.5.13 (*Tension in a caravan with friction*)

While pulling a sled on which your son sits, your son's sled is tied to a sled on which your dog sits. Your dog's sled is then connected to a sled with provisions for the day. What is the tension in each subsequent string?

Solution. *The first thing we should do is notice what information is given to us and make sure that everything is in consistent units. I will convert everything to SI units.*

4.5.4.3 Ropes with Mass

If a rope has mass, then it can be thought of as a series of tiny pieces of mass hung by tiny massless strings.

4.5.4.4 Atwood's Machine

Note https://en.wikipedia.org/wiki/Atwood_machine. "invented in 1784 by the English mathematician George Atwood as a laboratory experiment to verify the mechanical laws of motion with constant acceleration. Atwood's machine is a common classroom demonstration used to illustrate principles of classical mechanics."

Example 4.5.14 (*The acceleration of masses on Atwood's Machine*)

The two crates in the figure (p. 114) hang over a pulley (in what is called an "Atwood's machine"). I will select $m_1 = 35\,\text{kg}$ (because it looks smaller) and $m_2 = 85\,\text{kg}$ (because it looks bigger). We will assume that the pulley is massless and frictionless (so that the tension is the same throughout the rope). Find the acceleration and the time it takes m_2 to accelerate down for the $12\,\text{m}$ to the floor.

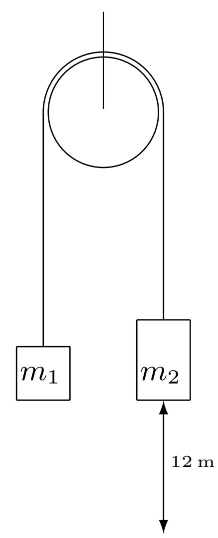


Figure 4.5.15: Masses hung from a rope draped over Atwood's Machine

Solution. *The easy way to do this is to say that m_1 pulls down on the left with $F_{g1} = (35\,\text{kg})(9.81\,{}^{\text{m}}\!/\!_{\text{s}^2}) = \mathbf{34}3.4\,\text{N}$ and m_2 pulls down on the right with $F_{g2} = (85\,\text{kg})(9.81\,{}^{\text{m}}\!/\!_{\text{s}^2}) = \mathbf{83}3.5\,\text{N}$ for a difference of $F_{net} = \mathbf{490}\,\text{N}$ down to the right. Since this has to move both m_1 and m_2, the acceleration is*

$$a = \frac{F_{\text{net}}}{m_1 + m_2} = \frac{\mathbf{490}\,\text{N}}{(35\,\text{kg}) + (85\,\text{kg})} = \frac{\mathbf{490}\,\text{N}}{\mathbf{120}\,\text{kg}} = \mathbf{4.0}87\,{}^{\text{m}}\!/\!_{\text{s}^2}$$

This acceleration then causes m_2 to drop and the time it takes is found from the equation that include *distance and time,*

$$y_f = y_i + v_i\,t + \frac{1}{2}a\,t^2$$
$$(0\,\text{m}) = (12\,\text{m}) + (0\ {}^{\text{m}}\!/\!_{\text{s}})\,t + \frac{1}{2}(-\mathbf{4.09}\ {}^{\text{m}}\!/\!_{\text{s}^2})\,t^2$$

which we can solve for time:

$$t = \sqrt{\frac{-(12\,\text{m})}{\frac{1}{2}(-\mathbf{4.09}\ {}^{\text{m}}\!/\!_{\text{s}^2})}} = \sqrt{\mathbf{5.87}\,\text{s}^2} = \mathbf{2.4}2\,\text{s}$$

Example 13 computes the acceleration, but does so in a way that avoids the question of the tension in the rope. Based on that example, it is possible to then deduce the tension by considering the net force on either of the masses.

Example 4.5.16 (*Atwood's tension from the acceleration*)

Given the acceleration from Example 13, find the tension in the rope.

Solution. *Since that example used the two masses as a single system, it did not reference the "internal force" of tension. By considering the two masses as separate objects, the tension is no longer "internal" to the system. This allows us to compute the tension with Newton's second law using one or the other mass. Here we do both to verifyt hat the result is the same.*

For the lighter mass (on the left),

$$\begin{aligned} F_{\text{net}} &= m_1 a_1 \\ &= (35\,\text{kg})(\mathbf{4.09}\ {}^{\text{m}}\!/\!_{\text{s}^2}) \\ &= \mathbf{143}\,\text{N} \end{aligned}$$

With the net force and the gravitational force (weight), we can find the tension:

$$\begin{aligned} \vec{F}_{\text{net}} &= \vec{F}_g + \vec{F}_T \\ \vec{F}_T &= \vec{F}_{\text{net}} - \vec{F}_g \\ \vec{F}_T &= (+\mathbf{143}\,\text{N}\hat{j}) - (-\mathbf{343}\,\text{N}\hat{j}) \\ \vec{F}_T &= +\mathbf{486}\,\text{N}\hat{j} \end{aligned}$$

For the heavier mass (on the right),

$$\begin{aligned} F_{\text{net}} &= m_1 a_1 \\ &= (85\,\text{kg})(\mathbf{4.09}\ {}^{\text{m}}\!/\!_{\text{s}^2}) \\ &= \mathbf{348}\,\text{N} \end{aligned}$$

With the net force and the gravitational force (weight), we can find the tension:

$$\begin{aligned} \vec{F}_{\text{net}} &= \vec{F}_g + \vec{F}_T \\ \vec{F}_T &= \vec{F}_{\text{net}} - \vec{F}_g \\ \vec{F}_T &= (-\mathbf{348}\,\text{N}\hat{j}) - (-\mathbf{833}\,\text{N}\hat{j}) \\ \vec{F}_T &= +\mathbf{485}\,\text{N}\hat{j} \end{aligned}$$

These are consistent to within the precision of our calculation. You should notice that

1. *the tension is larger than the weight of the lighter mass and therefore pulls it upwards,*
2. *the tension is smaller than the weight of the heavier mass and therefore keeps it from falling as fast as it would in freefall.*
3. *The tension is the same on both sides of the pulley specifically because we assumed that it did not take any effort to turn the pulley. This is usually expressed with three specific (and independent) assumptions:*
   (a) *The rope has an insignificant amount of mass – the rope is "massless". (This means that its mass is too small to impact the significant digits.)*
   (b) *The pulley has an insignificant amount of mass – the pulley is "massless". (This means that its inertial mass[3] is too small to impact the significant digits.)*
   (c) *The pulley has an insignificant amount of friction – the pulley is "frictionless". (This means that any force that might resist turning the pulley is too small to impact the significant digits.)*

The examples above are straightforward mathematically, but seem to cause some conceptual problems with students. Because of this, we can consider the same problem in a way that seems to be easier conceptually, although it is a little more involved mathematically.

Example 4.5.17 (*Computing the acceleration and tension for Atwood's machine*)

Given the description from Example 13, we can calculate the tension and acceleration.

Solution. *Figure 14 allows us to draw the free-body-diagrams from which we can write down Newton's second law for each individual object.*

For these equations, I am explicitly putting the sign in by hand to indicate the direction (positive is up and negative is down). Doing this, the value of F_T, F_g, and a will be positive because they are the magnitudes.
The equation for m_1 is

$$\vec{F}_{g1} + \vec{F}_{T1} = m_1\vec{a}_1$$
$$(-F_{g1}) + (+F_T) = m_1(+a)$$
$$(-\mathbf{343}\,\mathrm{N}) + (+F_T) = (35\,\mathrm{kg})(+a)$$

The equation for m_2 is

$$\vec{F}_{g2} + \vec{F}_{T2} = m_1\vec{a}_2$$
$$(-F_{g2}) + (+F_T) = m_1(-a)$$
$$(-\mathbf{833}\,\mathrm{N}) + (+F_T) = (85\,\mathrm{kg})(+a)$$

In these equations, I have noticed that the tensions F_{T1} and F_{T2} have to have the same magnitude, even though they are different forces. I am using F_T to indicate that magnitude. Similarly, the accelerations a_1 and a_2 have to have the same magnitude (because the masses are connected) even though they have opposite directions. I am using a to indicate that magnitude. In each case, the sign indicates the different directions.

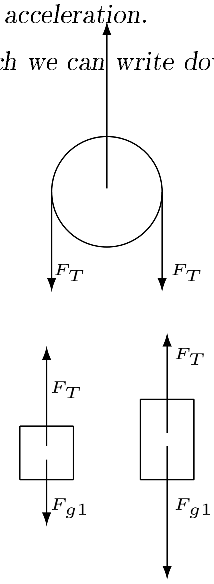


Figure 4.5.18: The free-body diagrams for the Atwood's machine system

These equations can be combined as two-equations-with-two-unknowns and solved in the familiar ways. It is useful to be aware that you can do these independently of each other. So, if you need the acceleration, you can solve Item 1 and if you need the tension, you can solve Item 2.

1. *If we solve one equation for F_T and plug it into the other, then we get the following equation. (Since the coefficient of F_T is the same in these equations, if we subtract the second equation from the first, then we also get the following equation.)*

   Computing the acceleration:

$$(-\mathbf{343}\,\mathrm{N}) - (-\mathbf{833}\,\mathrm{N}) = [(35\,\mathrm{kg}) + (85\,\mathrm{kg})]\,(a)$$
$$a = \frac{\mathbf{490}\,\mathrm{N}}{(35\,\mathrm{kg}) + (85\,\mathrm{kg})} = \mathbf{4.0}87\ \mathrm{m/s^2}$$

2. *If we solve the first equation for a and plug it into the second, then we get the following equation.*

   Computing the tension:

$$(-\mathbf{833}\,\mathrm{N}) + (F_T) = -(85\,\mathrm{kg})\left[\frac{(-\mathbf{343}\,\mathrm{N}) + (F_T)}{(35\,\mathrm{kg})}\right]$$
$$F_T = \frac{-(35\,\mathrm{kg})(-\mathbf{833}\,\mathrm{N}) - (85\,\mathrm{kg})(-\mathbf{343}\,\mathrm{N})}{[(35\,\mathrm{kg}) + (85\,\mathrm{kg})]} = \mathbf{486}\,\mathrm{N}$$

It is useful to verify the following: The tension is not enough to support m_2, so it falls:

$$\vec{a}_2 = \frac{\vec{F}_{\text{net}}}{m_2} = \frac{(+\mathbf{486}\,\text{N}) + (-\mathbf{833}\,\text{N})}{(85\,\text{kg})} = -\mathbf{4.09}\ {}^{\text{m}}\!/_{\text{s}^2}.$$

The tension is also more than enough to lift m_1, so it rises:

$$\vec{a}_2 = \frac{\vec{F}_{\text{net}}}{m_2} = \frac{(+\mathbf{486}\,\text{N}) + (-\mathbf{343}\,\text{N})}{(35\,\text{kg})} = +\mathbf{4.09}\ {}^{\text{m}}\!/_{\text{s}^2}.$$

4.5.4.5 Surface Tension

As a final note, surface tension is something else entirely. It is mentioned here only because it has the word "tension" in the name.

See Example 2.1.2 for a comment on the contribution to hot versus cold spoon noises.

4.6 Spring Force

Referenced by $F = ma$, uses of $F = ma$, Subsection 4.3.1

Initial definition of elastic (reference Paragraph

4.7 Applied Force

The term "an applied force" is used to describe any force applied by any object when there isn't really a formula to find it. So this is kind of a "any other force" category. I will use this type of force to describe forces exerted by people. We have seen some examples where a person throws an object. We can now revisit those examples and consider the force exerted (applied) by the person who threw the object.

Example 4.7.1 (*Applying a force to throw a ball*)

Carl recalls that one time he got bored one day in physics class (what?!?) and tossed a baseball ($m_b = 0.145\,\mathrm{kg}$) at the ceiling... a little too hard ... as recounted in Exercise 1.6.1. Recall that Example 4.3.8 found the normal force by the ceiling on the ball. Please now find the force Carl applied while throwing and catching the ball assuming that the throw took $0.200\,\mathrm{s}$ to gain the speed of $5.00\,\mathrm{m/s}$ and the catch took $0.250\,\mathrm{s}$ to slow the ball from $4.73\,\mathrm{m/s}$ to rest.

Solution.

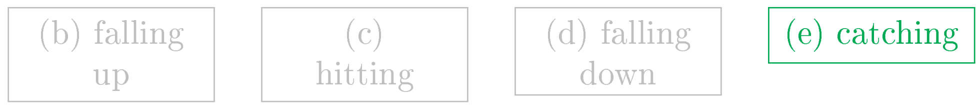


Figure 4.7.2: When a ball is thrown there are five distinct stages because the forces on the ball change and we can, at this point, only manage to describe a situation in which the forces do not change.

In this particular problem, we are only concerned with steps (a) and (e) because that's where Carl throws and catches the ball. In each case, we need the acceleration:

$$\vec{a}_{\text{throw}} = \frac{(+5.00\,\mathrm{m/s}\,\hat{j}) - (0\,\mathrm{m/s}\,\hat{j})}{0.200\,\mathrm{s}} = +\mathbf{25.00}\,\mathrm{m/s^2}\,\hat{j}$$

$$\vec{a}_{\text{catch}} = \frac{(0\,\mathrm{m/s}\,\hat{j}) - (-4.73\,\mathrm{m/s}\,\hat{j})}{0.250\,\mathrm{s}} = +\mathbf{18.92}\,\mathrm{m/s^2}\,\hat{j}$$

During each step, we have the actual acceleration, which tells us about the net force. We will also need to know the weight of the baseball $F_g = \mathbf{1.42}2\,\mathrm{N}$*, because gravity is still acting during the collision. Let's consider the throwing part first.*

$$\begin{aligned} \vec{F}_N + \vec{F}_g = \vec{F}_{\text{net}} &= m\vec{a} \\ \vec{F}_A &= m\vec{a} - \vec{F}_g \\ \vec{F}_A &= \left[(0.145\,\mathrm{kg})(+\mathbf{25.00}\ {}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2}\hat{\jmath})\right] - \left[-\mathbf{1.42}2\,\mathrm{N}\hat{\jmath}\right] \\ \vec{F}_A &= \left[+\mathbf{3.62}5\,\mathrm{N}\hat{\jmath}\right] - \left[-\mathbf{1.42}2\,\mathrm{N}\hat{\jmath}\right] \ = \ +\mathbf{5.047}\,\mathrm{N}\hat{\jmath} \end{aligned}$$

You can see that the upward applied force ($\mathbf{5.047}\,\mathrm{N}$) *has to be large enough so that when it is combined with the downward gravitational force* ($\mathbf{1.42}2\,\mathrm{N}$) *they can together result in the necessary (but smaller) upward net force* ($\mathbf{3.62}5\,\mathrm{N}$) *to get it going upwards.*

For the catching part, the ball is moving downwards and needs to be stopped, so the catching applied force must be upwards.

$$\begin{aligned} \vec{F}_A + \vec{F}_g = \vec{F}_{\text{net}} &= m\vec{a} \\ \vec{F}_A &= m\vec{a} - \vec{F}_g \\ \vec{F}_A &= \left[(0.145\,\mathrm{kg})(+\mathbf{18.92}\ {}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2}\hat{\jmath})\right] - \left[-\mathbf{1.42}2\,\mathrm{N}\hat{\jmath}\right] \\ \vec{F}_A &= \left[+\mathbf{2.74}3\,\mathrm{N}\hat{\jmath}\right] - \left[-\mathbf{1.42}2\,\mathrm{N}\hat{\jmath}\right] \ = \ +\mathbf{4.16}5\,\mathrm{N}\hat{\jmath} \end{aligned}$$

You can see that the upward applied force ($\mathbf{4.16}5\,\mathrm{N}$) *has to be large enough so that when it is combined with the downward gravitational force* ($\mathbf{1.42}2\,\mathrm{N}$) *they can together result in the necessary upward net force* ($\mathbf{2.74}3\,\mathrm{N}$) *to stop it from continuing downwards.*

4.8 Putting it Together, F_{net}

4.8.1 Translational Equilibrium

blah blah blah Translational equilibrium: $F_{\text{net}} = m\not{a}$. blah blah blah

4.8.2 Static Equilibrium

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4.8.3 Dynamic Equilibrium

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4.9 Summary and Homework

4.9.1 Summary of Concepts and Equations

...

4.9.2 Conceptual Questions

1. Estimate, preferably without using the internet, the mass of the following: (a) a four-door sedan, (b) dishwasher, (c) a pair of glasses, (d) a cell phone. You should be able to estimate to within one significant digit.

2. List at least one object, preferably without using the internet, that has the following mass: (a) 2500 kg (b) 41 kg, (c) 3 kg, (d) 50 g.

4.9.3 Problems

List of examples

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(Revised September 1, 2017)

Chapter 5

Energy and the Transfer of Energy

Chapter referenced by Discussion of heat as a verb

Energy is a noun; objects can *have* energy.

Work is a verb; doing work is the process of *exchanging* energy.

Referenced by Discussions of force as a noun and heat as a verb

5.1 Objects Can Have Energy

.

5.2 A Force Can Transfer Energy

Referenced by Discussion of the direction of forces

5.3 Dissipating Energy

pool balls on cushion/bumper

5.4 Conserving Energy

Investigation 5.4.1 (*1-D elastic collisions of pool balls and inelastic collisions off the bumper*)

Diane is relaxing with the local physics club, playing pool. She hits the cue ball and counts the number of walls she can hit in one shot.

(a) *collide some pool balls and notice where they hit each other. Can you determine the angle at which they roll away?*

Answer. 90°

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: normal force, elastic versus inelastic collisions, rotational motion, and angular momentum.

Return to: pool

5.4.1 Gravitational Potential Energy

Referenced by Section 6.2

See also Section 6.2.

5.4.2 Spring Potential Energy

.

5.4.3 Conservative Forces in General

.

Part III

Interesting Uses of Motion, Force, and Energy

The chapters in this Part develop the ideas in Part II by introducing momentum, circular motion, rotational motion, torque, and the Newtonian theory of gravitation.

Chapter 1

Momentum: A Better Way to Describe Force

Chapter referenced by Clarification of Newton's laws 3.2.2 Subsection 3.2.3, Task 3.3.2.d.ii

Paragraph referenced by Subsubsection 3.2.1.1

Define momentum

$$\vec{p} = m\vec{v}.$$

Useful to include? The Physics of Bullets Vs. Wonder Woman's Bracelets

1.1 Revising Newton's First and Second Laws

1.1.1 Inertia and Momentum

Referenced by Subsubsection 3.2.1.1

Recall Subsubsection 3.2.1.1.

Probably define elastic (Paragraph) and inelastic collisions

1.2 Revising Newton's Third Law: Conservation of Momentum

Referenced by Subsection 3.2.3

1.3 Two-Dimensional Collisions

Referenced by Subsubsection 3.4.2.2

pool balls? What about rolling?

Investigation 1.3.1 (*2-D collisions of pool balls*)

Diane is relaxing with the local physics club, playing pool. She hits the cue ball and counts the number of walls she can hit in one shot.

(a) *collide some pool balls and notice where they hit each other. Can you determine the angle at which they roll away?*

Answer. 90°

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: normal force, elastic versus inelastic collisions, rotational motion, and angular momentum.

Return to: pool

Chapter 2

Rotational Motion

2.1 The Equations of Rotational Motion

Investigation 2.1.1 (*Rolling pool balls*)

Diane is relaxing with the local physics club, playing pool. She hits the cue ball and counts the number of walls she can hit in one shot.

(a) *Roll a striped ball along the table. Use the stripe to notice the rate of rotation. How does the rotation compare to the translation?*

Answer. *[Answer]*

(b) *Roll a striped ball along the table. Notice the distance the ball travels. Why does friction slow the ball down instead of just make it turn $v = \omega r$ (no slip)*

Answer. *[Answer]*

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: normal force, elastic versus inelastic collisions, rotational motion, and angular momentum.

Return to: pool

2.2 Moment of Inertia

In this video of eggs in space from 14 Oct, 2011 created by Science Friday, we can watch how the momentum of inertia affects the way fresh eggs spin differently than hard-boiled eggs.

2.3 Angular Momentum

Investigation 2.3.1 (*Rolling pool balls*)

Diane is relaxing with the local physics club, playing pool. She hits the cue ball and counts the number of walls she can hit in one shot.

(a) *Roll a striped ball along the table. Use the stripe to notice the rate of rotation. How does the rotation compare to the translation?*

Answer. *[Answer]*

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: normal force, elastic versus inelastic collisions, rotational motion, and angular momentum.

Return to: *pool*

2.4 Non-inertial Rotational Reference Frames

Section referenced by Subsection 1.6.1, Clarification of Newton's laws 3.2.1

Paragraph referenced by Subsection 3.2.2

Because the Earth rotates, we are actually in a non-inertial reference frame. In fact, we can prove that the Earth rotates by observing the effects, such as the Coriolis effect, that in our non-inertial frame seem to require unexplainable forces but which, in a non-rotating frame, follow the expected laws of physics.

2.4.1 The Coriolis Effect

Subsection referenced by Clarification of Newton's laws 3.2.1

Paragraph referenced by Non-inertial Rotational Reference Frames

weather . . . AND . . . In her podcast, *Spacepod*[1] Dr. Carrie Nugent interviews Dr. Andy Thompson about "underwater flying objects" that investigate the ocean. He notes that ocean waters, because they are such a large-scale system, can see the effect of the rotation of the Earth.

2.4.2 The Foucault Pendulum

See youtube video by Sixty Symbols.

[1] Nugent, Carrie (Producer, Host). *Spacepod* [Audio podcast], episode 89 (19 May, 2017). Retrieved from http://spacepod.libsyn.com/ on 9 Apr. 2017.

Chapter 3

Circular Motion and Centripetal Force

3.1 Circular Motion

.

3.2 Centripetal Force

Referenced by Discussion of $F = ma$

Chapter 4

Torque and the $F = ma$ of Rotations

Chapter referenced by Answer 7.3.0.4.1

4.1 Leverage

.

Referenced by Subsection 4.3.1

4.2 Putting it all together, τ_{net}

4.2.1 Rotational Equilibrium

blah blah blah. Rotational equilibrium: $\tau_{\text{net}} = I\cancelto{0}{\alpha}$. blah blah blah

4.2.2 Static (Rotational) Equilibrium

.

4.2.3 Dynamic (Rotational) Equilibrium

Example 4.2.1 (*Carluses a ladder*)

Example referenced by *Example 4.3.5, Paragraph*

Carl leans a 22.7 kg ladder against a wall at an angle of 75.5°, consistent with OSHA standard 1926.1053(a)(1)(ii). The coefficient of friction between the ladder and the floor is $\mu_f = 0.31$. The coefficient of friction between the ladder and the wall is $\mu_w = 0.19$. Use the rotational and translational equilibrium to determine if the ladder slides.Since the full weight of the ladder, $F_g = \mathbf{222}.69\,\text{N}$, is still pressing downwards into the floor (as a normal force), it is tempting to say that Newton's third law implies that the floor pushes the ladder upwards with a normal force of $\mathbf{222}.69\,\text{N}$ but this would not account for the frictional force on the wall, F_{fw}. If there were no friction between the ladder and the wall, then we could deduce F_{Nf}, but at this point, we cannot.If we consider $\mu_w \to 0$, then $F_{fw} = 0\,\text{N}$, $\vec{F}_{Nf} = -\vec{F}_g = \mathbf{222}.7\,\text{N}\hat{\jmath}$, and $\vec{F}_{Nw} = -\vec{F}_{ff} = \mathbf{28}.79\,\text{N}\hat{\imath}$. In this case, μ_f could be as small as $0.\mathbf{129}3$ and still hold the ladder in place, unless Carl climbs the ladder, in which case see Answer 13.2.3.1.3.If we consider $\mu_w \to 0$ with Carl ($m = 90.0\,\text{kg}$) at the third-rung-from-the-top of the ladder, ($1.53\,\text{m}$ up the ladder), then $F_{fw} = 0\,\text{N}$, $\vec{F}_{Nf} = \mathbf{1105}.6\,\text{N}\hat{\jmath}$, and $\vec{F}_{Nw} = -\vec{F}_{ff} = \mathbf{171}.97\,\text{N}\hat{\imath}$. In this case, μ_f could be as small as $0.\mathbf{155}5$ and still hold the ladder in place.

Solution. *Since we are asked to distinguish between two cases that cannot both be true, we should assume one (the easier one to calculate is that the ladder does not slip) and then verify that the result is consistent with that assumption.*

What do we know? *We know that the floor has a normal force (F_{Nf}) upwards and a frictional force (F_{ff}) to the left. We know that the wall has a normal force (F_{Nw}) to the right and a frictional force (F_{fw}) up (keeping the ladder from sliding down). We know the weight is* $F_g = mg = (22.7\,\text{kg})(9.81\ {}^{\text{m}}\!/_{\text{s}^2}) = \mathbf{222}.69\,\text{N}$.

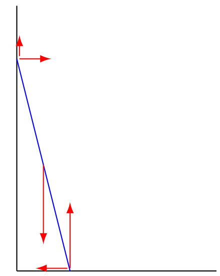

Figure 4.2.2: Diagram of the ladder against the wall

What do we want to know? *We want to know about the the magnitudes of both normal forces and both frictional forces. Can we easily deduce the magnitude of F_{Nf}? Answer 13.2.3.1.1.*

How are what-we-know and what-we-want related? *The forces acting on any body are related by static translational equilibrium*

$$\begin{aligned} x: &\quad 0 = \cancel{0}F_{gx} + \cancelto{0}{F_{Nfx}}0 + F_{ffx} + F_{Nwx} + \cancelto{0}{F_{fwx}} \\ y: &\quad 0 = F_{gy} + F_{Nfy} + \cancelto{0}{F_{ffy}} + \cancelto{0}{F_{Nwy}} + F_{fwy} \end{aligned}$$

and static rotational equilibrium, assuming the pivot point is at the ground, and using the relationship $F_f = \mu F_N$, *we find*

$$\begin{aligned} 0 &= \tau_g + \cancelto{0}{\tau_{Nf}} + \cancelto{0}{\tau_{ff}} + \tau_{Nw} + \tau_{fw} \\ 0 &= \left[F_g \frac{l}{2} \sin 14.5^\circ\right] + \left[-F_{Nw} l \sin(75.5^\circ)\right] + \left[-F_{fw} l \sin(14.5^\circ)\right] \\ F_{Nw} &= \left[F_g \frac{l}{2} \sin 14.5^\circ\right] / \left[l \sin(75.5^\circ) + \mu_w l \sin(14.5^\circ)\right] \end{aligned}$$

Concepts to consider: *First, the length of the ladder cancels from the expression; what matters is the angle at which it is propped.*

Second, every force value will be linearly dependent on the mass of the ladder. So once we solve this problem, we can easily scale the answers to any mass.

Third, the friction with the wall is, by far, the smallest effect and it might be interesting to approximate all of this with $\mu_w = 0$. You can check your calculation against Answer 13.2.3.1.2.

Solution to the example: *When we worry about significant figures,*

$$\begin{aligned} F_{Nw} &= \frac{[(\mathbf{222}.7\,\text{N})(1/2)(0.\mathbf{2504})]}{[(0.\mathbf{9682}) + (0.19)(0.\mathbf{2504})]} &= \frac{[(\mathbf{27}.88\,\text{N})]}{[(0.\mathbf{9682}) + (0.0\mathbf{476})]} \\ F_{Nw} &= \frac{[(\mathbf{27}.88\,\text{N})]}{[(\mathbf{1.0157})]} &= \mathbf{27}.44\,\text{N} \\ F_{fw,\max} &= (0.19)(\mathbf{27}.44\,\text{N}) &= \mathbf{5}.215\,\text{N} \\ F_{Nf} &= F_g - F_{fw} = (\mathbf{222}.7\,\text{N}) - (\mathbf{5}.215\,\text{N}) &= \mathbf{217}.5\,\text{N} \\ F_{ff,\max} &= (0.31)(\mathbf{217}.5\,\text{N}) &= \mathbf{672}.4\,\text{N} \end{aligned}$$

Since $F_{ff} > F_{Nw}$, the friction is sufficient to hold the ladder in place, as assumed.

Note about Example 1: Since F_{ff} only needs to be $\mathbf{27}.\mathbf{4}4\,\text{N}$ to hold the ladder in place, it is possible for the ladder to not slide on a floor that only has $\mu_{\min} = (\mathbf{27}.\mathbf{4}4\,\text{N})/(\mathbf{217}.5\,\text{N}) = 0.\mathbf{126}2$; but that would not allow a person to climb the ladder.

Homework related to Example 1: Homework problem Exercise 13.4.3.1 asks you to determine if the ladder slides when Carl climbs to different locations on the ladder.

4.3 Torsion

Referenced by Section 4.5

4.4 Summary and Homework

4.4.1 Summary of Concepts and Equations

...

4.4.2 Conceptual Questions

4.4.3 Problems

1. Carl leans a 22.7 kg ladder against a wall at an angle of 75.5°, consistent with OSHA standard 1926.1053(a)(1)(ii). The coefficient of friction between the ladder and the floor is $\mu_f = 0.31$. The coefficient of friction between the ladder and the wall is $\mu_w = 0.19$. Use the rotational and translational equilibrium to determine if the ladder slides when Carl (90.0 kg) climbs to ...

(a) the third-rung from the top of the ladder, so that he is 1.53 m from the bottom of the ladder. (Solution 1)

- (You might also consider Answer 13.2.3.1.3 for the case of $\mu_w = 0$.)

(b) the third-rung from the bottom of the ladder, so that he is 0.914 m from the bottom of the ladder. (Solution 2)

Solution. The normal force from the wall is the only force to the right. The frictional force from the floor is the only force to the left. Using the calculations in Example 4.2.1, we notice that

$$\begin{aligned} F_{Nw} &= \mathbf{163}.9\,\text{N} \\ F_{fw,\max} &= (0.19)(\mathbf{163}.9\,\text{N}) \;=\; \mathbf{31}.14\,\text{N} \\ F_{Nf} &= \mathbf{1074}.4\,\text{N} \\ F_{ff,\max} &= \mathbf{333}.0\,\text{N} > \mathbf{163}.9\,\text{N} \end{aligned}$$

The frictional force with the floor can support a normal force from the wall of 333 N, but the normal force is only 164 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{152}56$.

Solution. The normal force from the wall is the only force to the right. The frictional force from the floor is the only force to the left. Using the calculations in Example 4.2.1, we notice that

$$\begin{aligned} F_{Nw} &= \mathbf{108}.97\,\text{N} \\ F_{fw,\max} &= (0.19)(\mathbf{108}.97\,\text{N}) \;=\; \mathbf{20}.70\,\text{N} \\ F_{Nf} &= \mathbf{1084}.9\,\text{N} \\ F_{ff,\max} &= \mathbf{336}.3\,\text{N} > \mathbf{108}.97\,\text{N} \end{aligned}$$

The frictional force with the floor can support a normal force from the wall of 336 N, but the normal force is only 109 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{100}45$

If, hypothetically, $\mu_w = 0$, then

$$F_{Nw} = \mathbf{114}.3\,\text{N}$$

$$\begin{aligned} F_{fw,\max} &= 0\,\mathrm{N} \\ F_{Nf} &= \mathbf{1105}.6\,\mathrm{N} \\ F_{ff,\max} &= \mathbf{342}.7\,\mathrm{N} > \mathbf{114}.3\,\mathrm{N} \end{aligned}$$

The frictional force with the floor can support a normal force from the wall of 336 N, but the normal force is only 109 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{103}4$

Chapter 5

Energy of Rotating Objects

5.1 Rotational Kinetic Energy

pool balls

Chapter 6

The Gravitational Force on a Large Scale

Chapter referenced by freefall, fundamental forces

6.1 Gravitational Force and Field

Section referenced by Discussion of $F = ma$

The value of the acceleration due to gravity varies according to the mass and size of any celestial body. This means that, as was seen in Example 4.1.11, your weight can change even when your mass remains the same.

Example 6.1.1 (*Weight is not mass*)

In conversation with a visiting alien, Xerxes, you find that Xerxes has been to the moon and several planets both within and outside of our solar system. In addition to the Earth, Xerxes has visited our moon, Mars, Pluto, and Planet X. Using Table 2, compute Xerxes's weight are each location, assuming Xerxes's mass is 62.5 kg.

Solution.

1. *[Earth]* $F_g = (62.5\,\text{kg})\left[\frac{GM_E}{R_E^2}\right] = (62.5\,\text{kg})(9.825\,{}^{\text{m}}\!/\!_{\text{s}^2}) \;=\; \mathbf{933}.4\,\text{N}$

2. *[moon]* $F_g = (62.5\,\text{kg})\left[\frac{GM_m}{R_m^2}\right] = (62.5\,\text{kg})(9.782\,{}^{\text{m}}\!/\!_{\text{s}^2}) \;=\; \mathbf{929}.3\,\text{N}$

3. *[Mars]* $F_g = (62.5\,\text{kg})\left[\frac{GM_M}{R_M^2}\right] = (62.5\,\text{kg})(9.763\,{}^{\text{m}}\!/\!_{\text{s}^2}) \;=\; \mathbf{927}.5\,\text{N}$

4. *[Pluto]* $F_g = (62.5\,\text{kg})\left[\frac{GM_P}{R_P^2}\right] = (62.5\,\text{kg})(9.763\,{}^{\text{m}}\!/\!_{\text{s}^2}) \;=\; \mathbf{927}.5\,\text{N}$

5. *[Planet X]* $F_g = (62.5\,\text{kg})\left[\frac{GM_X}{R_X^2}\right] = (62.5\,\text{kg})(9.763\,{}^{\text{m}}\!/\!_{\text{s}^2}) \;=\; \mathbf{927}.5\,\text{N}$

Planet	Mass (kg)	Mean Radius (m)	$g(\,{}^{\text{m}}\!/\!_{\text{s}^2})$

Table 6.1.2: Properties of various celestial bodies.

6.1.1 Inertial Mass versus Gravitational Mass

Referenced by Subsection 4.1.1

6.2 Gravitational Potential Energy

Referenced by Subsection 5.4.1

Recall Subsection 5.4.1

6.3 Making Connections

Referenced by Section 1.3

$$\vec{F} = m\vec{g}$$

$$F = G\frac{m_1 m_2}{R^2} \leftrightarrow g = G\frac{m}{R^2}$$

$$\Delta\text{PE} = -\vec{F}\cdot\Delta\vec{x} \updownarrow \qquad \updownarrow \qquad \text{[for later]}$$

$$\text{PE} = G\frac{m_1 m_2}{R} \leftrightarrow \text{[for later]}$$

$$\text{[for later]}$$

(Look ahead to the parallel with the electrical interaction in Section 1.3.)

6.4 Orbits

.

Part IV

Making Waves

The chapters in this Part are oscillations and thermodynamics. With the traditional organization of the two-semester introductory physics, these parts can be covered in either order and can be chosen to be put in either semester.

Chapter 1

Fluids

1.1 Density

Referenced by Subsection 4.1.1

1.2 Surface Tension

Referenced by Discussion of surface tension

Chapter 2

Oscillations

2.1 Oscillating Springs

Referenced by Discussion of $F = ma$

2.2 Oscillating Pendulums

.

2.3 Other Examples of Oscillations

On 13 April, 2017, CBC Broadcasting published a *Quirks and Quarks* episode discussing how we can find solutions to health issues caused by swaying office towers and vibrating floors.

Chapter 3

Sound

3.1 TBD

3.1.1 Musical Instruments

Subsection referenced by Answer 8.5.0.4.a.1

Part V

Is It Hot in Here?

The chapters in this Part are oscillations and thermodynamics. With the traditional organization of the two-semester introductory physics, these parts can be covered in either order and can be chosen to be put in either semester.

Chapter 1

The flow of thermal energy

Energy is a noun; objects can *have* energy.

Heat is a verb; heating is a process of *exchanging* energy. Recall our discussions of force and work.

Referenced by Discussions of force as a noun and work as a verb

1.1 Specific Heat Capacity

Heating (positive Q) can warm (positive ΔT) a material.

$$Q = mc\,\Delta T \tag{1.1.1}$$

but (1.2.1) (as one example) shows that it is possible to heat (positive Q) a material without warming it (constant T). When we get to Section 2.1 we will see other examples of "isothermal processes" that have a non-zero Q (heat the system or heat the surroundings) without warming or cooling the system.

1.2 Latent Heat

Referenced by Discussion of Paragraph heating versus warming

Heating might also change the phase of a material.

$$Q = \pm mL \tag{1.2.1}$$

1.3 The Flow of Thermal Energy

1.3.1 Thermal Conductivity

Referenced by Section 3.1

$$\frac{Q}{\Delta t} = \kappa A \frac{\Delta T}{\Delta x} \tag{1.3.1}$$

Example 1.3.1 (*Abdulwarms his oven*)

Abdul decides to bake some bread for the dinner party at Beth's house, but he is on a tight schedule. In order to set his schedule, he needs to know how long it will take his oven to warm up.

Solution.

Referenced by Section 3.1

1.3.2 Convection

.

1.3.3 Radiation

.

Chapter 2

Ideal Gas Law

2.1 P-V Diagrams

Referenced by Discussion of heating versus warming

Part VI

Let There Be Light!

The chapters in this Part cover electricity, magnetism, light, and optics. This is traditionally the meat of the second semester.

Chapter 1

The Electrical Interaction

Chapter referenced by Discussion of fundamental forces

1.1 Electrical Charge

.

1.2 The Big Picture

1.2.1 Electric Forces and Fields

Subsection referenced by Subsubsection 3.4.2.2, $F = ma$

pst-electricfield

1.2.1.1 Examples

.

1.2.2 Electric Forces, Fields, and Potential Energy

.

1.2.3 Electric Fields, Potential Energy, and Potential

.

1.3 Making Connections

Referenced by Section 6.3

$$\vec{F} = q\vec{E}$$

$$\begin{array}{rcl} & F = k\dfrac{q_1 q_2}{r^2} & \leftrightarrow E = k\dfrac{q}{r^2} & \\ \Delta\text{PE} = -\vec{F}\cdot\Delta\vec{x} & \updownarrow & \updownarrow & \Delta V = -\vec{E}\cdot\Delta\vec{x} \\ & \text{PE} = k\dfrac{q_1 q_2}{r} & \leftrightarrow V = k\dfrac{q}{r} & \end{array}$$

$$\Delta\text{PE} = q\Delta V$$

(Recall the parallel with the gravitational interaction in Section 6.3.)

Chapter 2

Electricity

.

Chapter 3

The Magnetic Interaction

pst-magneticfield

Chapter 4

"Magnicity?"

.

Chapter 5

Light

.

Chapter 6

Optics

.

Part VII

What Have You Done for Me Lately?

The chapters in this Part touch on the topics that are usually referred to as “modern physics”. The goal with including these chapters is to provide some inspiration for what some students see as the tedium of the standard material. These chapters will be linked to throughout the book as examples of how the traditional material supports the material that may be in the news and is more talked about in popular science.

Chapter 1

Relativity

.

Chapter 2

Quantum Mechanics

2.1 Atomic Physics

2.1.1 The Periodic Table and Quantum Numbers

.

2.2 Nuclear Physics

2.2.1 Nuclear Decay

.

2.2.2 The Strong Nuclear Force

Referenced by Discussion of fundamental forces

2.2.3 The Weak Nuclear Force

Referenced by Discussion of fundamental forces

2.3 Particle Physics

2.3.1 Field Theory

.

2.3.2 Quantum Electrodynamics

Referenced by Discussion of fundamental forces

2.3.3 Quantum Chromodynamics

Referenced by Discussion of fundamental forces

2.3.4 The Standard Model

.

2.3.5 Particle Decay

.

Chapter 3

Condensed Matter

.

Chapter 4

Astronomy

.

Chapter 5

Cosmology

.

Part VIII

Supplements

This final part holds the answers to the interactive examples mentioned above, the bulk of the adventures the reader can investigate in order to test their understanding of the material, and the story lines of each of the characters in the text.

Chapter 1

Deeper Dive

1.1 [Need title]

This is where I will put the fuller explanations.

1.1.1 The Sun

The bright, shiny sun, which keeps us all alive, is a nice example of a rather complex system that allows us to verify our various theories of the world around us. We can consider the existence of a star in three phases: the birth of a star, the life of the star, and the death of the star.

1.1.1.1 The Birth of a Star

.

1.1.1.2 The Life of a Star

.

1.1.1.3 The Death of a Star

.

1.1.2 Kitchen Appliances

1.1.2.1 Oven

.

1.1.2.2 Refrigerator

.

1.1.2.3 Microwave

.

1.1.2.4 Television

.

1.1.3 Automobile

1.1.3.1 Coolant and Antifreeze

.

1.1.3.2 Tires

.

1.1.3.3 Torque

.

1.1.4 Cool Ideas

1.1.4.1 Black Holes

Referenced by Subsection 4.1.1

On 7 April, 2017, CBC Broadcasting published a *Quirks and Quarks* episode discussing how we can turn our planet into a giant telescope to get a photo of a black hole. The results should be available by the early 2018.

1.1.4.2 Quantum Mechanics

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1.1.4.3 Relativity

.

1.1.4.4 String Theory

.

Chapter 2

Podcasts and Videos

2.1 Podcasts

Spacepod with Carrie Nugent
Science Friday with Ira Flatow

2.2 Videos

Physics Footnotes
Sixty Symbols

2.3 Websites

The Flame Challenge

Chapter 3

Characters

This textbook has five characters who follow you throughout the book. They appear in the examples and some homework problems. They also remember previous experiences. I need to adjust the examples in Chapter 7 such that the people pushing boxes are helping the reader rearrange furniture.

The index lists the pages that the characters appear. The point of this chapter is to highlight some of the primary adventures of the characters according to their own perspectives. *None of the links in this chapter will be given a corresponding return link.* This chapter is for me to track relationships and will likely go away when the book is ready for publication

I can, at the header of the code, define the name, gender, mass, and dimensions of each individual.

3.1 Abdul

- In Section 3.1, Beth gives Abdul a good-natured shove in the arm in order to get the language clarified and begin the conversation about the on-by notation.
- In Example 3.2.28 Abdul helps Beth...
  - (in the current version) push an object to make it accelerate and feel a reaction force causing him to accelerate backwards.
  - (in the future version) will help the reader move into or out of their residence hall by pushing on heavier furniture.
  - [NOTE:] This is all drawn in Example 3.2.25, which is updated in Example 4.3.2.
- In Example 4.1.7, Abdul falls from a small height. (maybe he is jumping off a short ledge while taking a short-cut to class?)
- In Example 1.3.1, Abdul decides to bake some bread for a party at Beth's house, measuring the time it takes to warm his oven.

3.2 Beth

- Beth is a passenger in the reader's car in Exercise 1.3.1 when the reader runs out of gas and coasts to a stop.
- Beth is a passenger in the reader's car in Exercise 1.3.2 and speculates about how fast to go before putting the car in neutral to coast to a stop.
- Beth joins the reader on a road trip in Exploration 3.2.24 and runs out of gas. This results in multiple possible adventures:
  - Answer 7.2.2.1.1, which leads to either an end at Answer 7.2.2.1.5 or an end at Answer 7.2.2.1.7.

  - Answer 7.2.2.1.2, which leads to either Answer 7.2.2.1.4 (choose Answer 7.2.2.1.3 or end with Answer 7.2.2.1.10) or Answer 7.2.2.1.6 (choose Answer 7.2.2.1.7 or end at Answer 7.2.2.1.9)
  - Answer 7.2.2.1.3, which leads to an end at Answer 7.2.2.1.8.

- In Section 3.1, Beth gives Abdul a good-natured shove in the arm in order to get the language clarified and begin the conversation about the on-by notation.
- In Example 3.2.28, Beth helps Abdul…
  - (in the current version) pull an object to make it accelerate and feel a reaction force causing her to accelerate backwards.
  - (in the future version) will help the reader move into or out of their residence hall by pushing on heavier furniture.
  - [NOTE:] This is all drawn in Example 3.2.25, which is updated in Example 4.3.2.
- In Example 4.1.10, Beth has a normal force supporting her. (This touches Answer 8.1.2.10.1, Answer 8.1.2.10.2, and Answer 8.1.2.10.3.)
- At some point, Beth has a party, because in Example 1.3.1, Abdul decides to bake some bread for a party at Beth's house.
- According to Example 4.1.11, Beth's parents are and Frances. They have lived in Norway, where Beth grew up, and visited both Puerto Rico and Mount Everest.

3.3 Carl

.

3.4 Diane

.

3.5 Erik

According to Example 4.1.11, Erik is the father of Beth. They have lived in Norway, where Beth grew up, and visited both Puerto Rico and Mount Everest.

3.6 Frances

According to Example 4.1.11, Frances is the mother of Beth. They have lived in Norway, where Beth grew up, and visited both Puerto Rico and Mount Everest.

3.7 Xerxes

.

3.8 Zambert

.

Chapter 4

Holder

This chapter holds a few empty-format structures that I have used once and expect to use again.

This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes.

Touchstone Aside with title.

This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes.

This is a biographical, title should be automatically changed to “Touchstone”.

This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes. This is filler to space the margin notes.

This is a historical, title should NOT be automatically changed. Perhaps use this for “Foreshadow”?

This is filler to space the margin notes. This is filler to space the margin notes.

4.1 Centering Assemblages (Important)

To solve Question 5.6.3.1.b, we can just consider from after-thrown to before-caught (both given in the question). During this motion, assuming there is no ceiling, the entire motion is in freefall, so we can use $v_f^2 = v_i^2 + 2a\,\Delta x$ and solve for Δx. However, we only want to consider from the lowest point to the highest point, not all the way back to Carl's hand.

THIS NEEDS TO BE FINISHED

THIS NEEDS TO BE FINISHED

1THIS NEEDS TO BE FINISHED 2

1THIS NEEDS TO BE FINISHED 2

1 THIS NEEDS TO BE FINISHED 2

1 THIS NEEDS TO BE FINISHED 2

THIS NEEDS TO BE FINISHED

THIS NEEDS TO BE FINISHED

THIS NEEDS TO BE FINISHED

THISNEEDSTOBEFINISHED

THIS NEEDS TO BE FINISHED

More attempts

Carl notices that it is important to have the objects line up at the bottom so that if they travel at the same speed, then they hit at the same time. (Using p seems to make extra blank space above.)

Carl notices that it is important to have the objects line up at the bottom so that if they travel at the same speed, then they hit at the same time. (Using `sidebysidep` seems to make *even more* blank space above.)

Carl notices that it is important to have the objects line up at the bottom so that if they travel at the same speed, then they hit at the same time. (Using `sidebyside tabular col width="100%"` forces a smaller "tabular-sized" font.)

Carl notices that it is important to have the objects line up at the bottom so that if they travel at the same speed, then they hit at the same time. (Using `tabular col width="100%"` forces a smaller "tabular-sized" font; apparently we *must* use either `p` or `sidebyside` but both still give the awkward additional space above.)

Carl notices that it is important to have the objects line up at the bottom so that if they travel at the same speed, then th

4.2 Stolen from sample-article

An ampersand is used, in two ways, to describe positioning several equations per line, organized in columns. We suggest in ⟨⟨subsection-reserved-characters⟩⟩ that the pre-defined LaTeX macro `\amp` is the safest way to specify these. The second, fourth, sixth, ... ampersands separate columns, and the spacing between columns will be provided automatically. The first, third, fifth, ... ampersands are alignment points for the equations in each column. Typically this is placed just prior to a binary operator, such as an equal sign (`\amp =`), or for a column of explanations or commentary, just prior to the `\text{}` macro. Note that this scenario suggests always having an odd number of amersands in each `mrow`. In the example below, alignment is on the equals sign in the first two columns, and provides left-justification to the explanations in the third column. N.B.: the use below of the `\text{}` macro does not include mathematics within its argument. Doing so may yield unpredictable results depending on your choice of delimiters for the mathematics (and using an `m` tag will be ineffective).

$$\begin{aligned} \frac{dx}{dt} &= x^2 - 4x - y + 4 & \frac{dy}{dt} &= x^3 - y & &x, y \text{ version}\\ \frac{dw}{dt} &= z^3 - w & \frac{dz}{dt} &= z^2 - 4z - w + 4 & &z, w \text{ version} \end{aligned}$$

PreTeXt will automatically detect the presence or absence of ampersands, but by defining macros for entire aligned equations, you can effectively hide the ampersands. So the `@alignment` attribute can override automatic detection. We use a simple LaTeX macro to demonstrate setting `alignment='align'` to override the use of a `gather` environment and use a `align` environment instead. (example deleted, has macro)

The AMSmath LaTeX package's `alignat` environment is a third variant of alignment. It never happens automatically, you need to ask for it with `alignment="alignat"`. It is very similar to `align` but adds no space between the equation columns. So you can leave it that way, or you can add your own "extra" space to suit. Here is a previous example with no inter-column space:

$$\begin{alignedat}{3} \frac{dx}{dt} &= x^2 - 4x - y + 4 & \frac{dy}{dt} &= x^3 - y & \qquad &x, y \text{ version}\\ \frac{dw}{dt} &= z^3 - w & \frac{dz}{dt} &= z^2 - 4z - w + 4 & &z, w \text{ version.} \end{alignedat}$$

This modified example has a middle row with three columns, while the other rows have just one column, as a test of our routines for determining the `mrow` with the greatest number of ampersands (and how many there are),

$$\begin{aligned}
\frac{dw}{dt} &= z^3 - w\\
\frac{dx}{dt} &= x^2 - 4x - y + 4 \frac{dy}{dt} = x^3 - yx, y \text{ third column}\\
\frac{dw}{dt} &= z^3 - w.
\end{aligned}$$

Final example demonstrates that ampersands in other objects (matrices here) can wreak havoc with computing the number of columns. So we provide yet another attribute to override automatic detection, `alignat-columns`. This is the number of *columns* not the number of *ampersands*. Generally, for c columns, there will be $2c - 1$ ampersands.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

One caveat: if your number of ampersands is even (see advice above about using an odd number) behavior should still be correct, as in next example.

If you want super-precise control over alignment of the terms of a system of equations (linear or not) you can use the `alignat` option to advantage by not including any extra space. This example is modified slightly from a post by Alex Jordan:

$$\begin{alignedat}{4}
2x &+{} & y &+{} & 3z &={} & 10\\
x & & &+{} & z &={} & 6\\
x &+{} & 3y &+{} & 2z &={} & 13.
\end{alignedat}$$

Beautiful.

Check for references to biographical Touchstone , historical Historical Aside , [cannot use reference in <todo>], and touchstone-aside Aside twice Touchstone.

Part IX

Notation

The following table defines the notation used in this book. Page numbers or references refer to the first appearance of each symbol.

Symbol	Description	Page
acceleration	refers to the general idea of changing your motion (velocity), meaning *either* **speeding up** (colloquially "acceleration") *or* **slowing down** (colloquially "deceleration") *or* **changing the direction** (colloquially "turning")	25
acceleration	refers to the general idea of changing your motion (velocity), meaning *either* **speeding up** (colloquially "acceleration") *or* **slowing down** (colloquially "deceleration") *or* **changing the direction** (colloquially "turning")	26
freefall	"being in freefall" will mean moving only under the influence of gravity	29
force	refers to the general idea of pushing or pulling; every force is enacted **by** one person or object and is acted **on** another.	35
$F_{\text{on,by,type}}$	Use of subscripts on force labels: If it is obvious which object feels the force, the the subscript will only indicate *which force*. If it is not obvious which object feels the force, the the subscript will also indicate which object the force is acted *on*. Sometimes we need to clarify who exerts the force as well as who feels the force. In this case, the subscripts indicate who the force is *on*, who the force is *by*, and which force we are referring to: $F_{\text{on,by,type}}$.	36
$1\,\text{N} = 1\ ^{\text{kg}\cdot\text{m}}/_{\text{s}^2}$	Definition of the unit Newton	39
equilibrium	An object in equilibrium has $\vec{F}_{\text{net}} = 0\,\text{N}$ and $\vec{a} = 0$.	46
weight	$\mathbf{F_g \equiv mg}$	60

Part X

Solutions to Selected Exercises

7.4.3 Problems

Add more variety of problems.**1.** If Zambert, with $m_Z = 95.0\,\text{kg}$, braces himself (so that he does not accelerate) and pushes Carl ($m_C = 90.0\,\text{kg}$) with a force of $\vec{F}_{C,Z} = 215\,\text{N}\hat{\imath}$, find the following:

(a) What is the acceleration of Carl? (Solution 1)

(b) What net force does Zambert feel? (Solution 2)

(c) If Zambert braces himself against the Earth, then what must that bracing force be? (Solution 3)

(d) What are the individual forces that Zambert feels? (Solution 4)

(e) What is the acceleration of the Earth? (Solution 5)

(f) Which of Newton's laws allows you to answer each of these questions?

$$\vec{a}_C = \frac{215\,\text{N}\hat{\imath}}{90.0\,\text{kg}} = \mathbf{2.38}9\ \text{m/s}^2\,\hat{\imath}.$$

$F_{Z,\text{net}} = 0\,\text{N}.$

$\vec{F}_{E,Z} = -215\,\text{N}\hat{\imath}.$

$F_{Z,C} = -215\,\text{N}\hat{\imath}$ and $F_{Z,E} = 215\,\text{N}\hat{\imath}.$

$$\vec{a}_E = \frac{-215\,\text{N}\hat{\imath}}{5.97\times10^{24}\,\text{kg}} = -\mathbf{3.60}1\times10^{-23}\ \text{m/s}^2\,\hat{\imath}.$$

13.4.3 Problems

Add more problems.**1.** Carl leans a 22.7 kg ladder against a wall at an angle of 75.5°, consistent with OSHA standard 1926.1053(a)(1)(ii). The coefficient of friction between the ladder and the floor is $\mu_f = 0.31$. The coefficient of friction between the ladder and the wall is $\mu_w = 0.19$. Use the rotational and translational equilibrium to determine if the ladder slides when Carl (90.0 kg) climbs to . . .

(a) the third-rung from the top of the ladder, so that he is 1.53 m from the bottom of the ladder. (Solution 1)

- (You might also consider Answer 13.2.3.1.3 for the case of $\mu_w = 0$.)

(b) the third-rung from the bottom of the ladder, so that he is 0.914 m from the bottom of the ladder. (Solution 2)

The normal force from the wall is the only force to the right. The frictional force from the floor is the only force to the left. Using the calculations in Example 4.2.1, we notice that

$$\begin{aligned} F_{Nw} &= \mathbf{163}.9\,\text{N} \\ F_{fw,\max} &= (0.19)(\mathbf{163}.9\,\text{N}) = \mathbf{31}.14\,\text{N} \\ F_{Nf} &= \mathbf{1074}.4\,\text{N} \\ F_{ff,\max} &= \mathbf{333}.0\,\text{N} > \mathbf{163}.9\,\text{N} \end{aligned}$$

The frictional force with the floor can support a normal force from the wall of 333 N, but the normal force is only 164 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{152}56$.

The normal force from the wall is the only force to the right. The frictional force from the floor is the only force to the left. Using the calculations in Example 4.2.1, we notice that

$$\begin{aligned} F_{Nw} &= \mathbf{108}.97\,\text{N} \\ F_{fw,\max} &= (0.19)(\mathbf{108}.97\,\text{N}) = \mathbf{20}.70\,\text{N} \\ F_{Nf} &= \mathbf{1084}.9\,\text{N} \\ F_{ff,\max} &= \mathbf{336}.3\,\text{N} > \mathbf{108}.97\,\text{N} \end{aligned}$$

The frictional force with the floor can support a normal force from the wall of 336 N, but the normal force is only 109 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{100}45$

If, hypothetically, $\mu_w = 0$, then

$$\begin{aligned} F_{Nw} &= \mathbf{114}.3\,\mathrm{N} \\ F_{fw,\max} &= 0\,\mathrm{N} \\ F_{Nf} &= \mathbf{1105}.6\,\mathrm{N} \\ F_{ff,\max} &= \mathbf{342}.7\,\mathrm{N} > \mathbf{114}.3\,\mathrm{N} \end{aligned}$$

The frictional force with the floor can support a normal force from the wall of 336 N, but the normal force is only 109 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{103}4$

Part XI

List of Features

.1 List of Investigations

(Continued on next page)

.2 List of Tables

.3 List of Figures

(Continued on next page)

.4 List of Asides

(Continued on next page)

Aside Paragraph referenced by

Section 3.3 A graph is worth a thousand pictures

Subsection 3.3.1 Coordinate Systems

Aside Paragraph referenced by

Section 3.4 Trigonometry and Vectors

Subsection 3.4.2 Vectors

Aside Subsection referenced by
Aside Paragraph referenced by
Aside Paragraph referenced by
Aside Paragraph referenced by
Aside Section referenced by
Aside Paragraph referenced by

Chapter 4 Estimating and Uncertainty

Section 4.1 Precision and Accuracy

Aside Paragraph referenced by

Section 4.2 Significant Figures

Aside Section referenced by

Section 4.4 Effective Theories

Aside Section referenced by

Chapter 5 One-Dimensional Motion

Section 5.2 Connecting the Concepts- distance equals rate times time

Subsection 5.2.2 Speed versus Velocity

Aside Paragraph referenced by

Section 5.3 Extending the Concepts: Changing How You Move

Aside Section referenced by

Subsection 5.3.1 Moving versus Speeding Up

Aside Paragraph referenced by
Aside Paragraph referenced by

(Continued on next page)

(Continued on next page)

Section 7.2 Connecting the Concepts: Newton's Laws

Aside	Section referenced by
Touchstone	Touchstone
Aside	Touchstone

Subsection 7.2.1 Translating Newton's First Law: The Law of Inertia

Aside	Subsection referenced by
Aside	Touchstone

Subsection 7.2.2 Translating Newton's Second Law: The Equation Law

Aside	Subsection referenced by
Aside	Touchstone
Aside	Touchstone
Aside	Referenced by
Aside	Referenced by
Aside	Subsubsection referenced by
Aside	Example referenced by
Aside	Example referenced by
Aside	Example referenced by
Aside	Exercise referenced by
Aside	Exercise referenced by
Aside	Example referenced by
Aside	Subsubsection referenced by
Aside	Referenced by
Aside	Exploration referenced by

Subsection 7.2.3 Translating Newton's Third Law: Action & Reaction

Aside	Subsection referenced by
Aside	Foreshadow
Aside	Referenced by
Aside	Referenced by

Section 7.3 Examples

Aside	Referenced by
Aside	Exploration referenced by
Aside	Foreshadow
Aside	Example referenced by
Aside	Example referenced by

Chapter 8 The Many Types of Force

Aside	Chapter referenced by

Section 8.1 Gravity at the Surface of the Earth

Aside	Section referenced by
Touchstone	Touchstone
Touchstone	Touchstone

(Continued on next page)

(Continued on next page)

Aside	Chapter referenced by
Aside	Paragraph referenced by

Section 10.1 Revising Newton's First and Second Laws

Subsection 10.1.1 Inertia and Momentum

Aside	Referenced by

Section 10.2 Revising Newton's Third Law: Conservation of Momentum

Aside	Referenced by

Section 10.3 Two-Dimensional Collisions

Aside	Referenced by

Chapter 11 Rotational Motion

Section 11.4 Non-inertial Rotational Reference Frames

Aside	Section referenced by
Aside	Paragraph referenced by

Subsection 11.4.1 The Coriolis Effect

Aside	Subsection referenced by
Aside	Paragraph referenced by

Chapter 12 Circular Motion and Centripetal Force

Section 12.2 Centripetal Force

Aside	Referenced by

Chapter 13 Torque and the $F = ma$ of Rotations

Aside	Chapter referenced by

Section 13.1 Leverage

Aside	Referenced by

Section 13.2 Putting it all together, τ_{net}

Subsection 13.2.3 Dynamic (Rotational) Equilibrium

Aside	Example referenced by

Section 13.3 Torsion

(Continued on next page)

(Continued on next page)

Section 19.2 Latent Heat

Aside Referenced by

Section 19.3 The Flow of Thermal Energy

Subsection 19.3.1 Thermal Conductivity

Aside Referenced by

Aside Referenced by

Chapter 20 Ideal Gas Law

Section 20.1 P-V Diagrams

Aside Referenced by

Chapter 21 The Electrical Interaction

Aside Chapter referenced by

Section 21.2 The Big Picture

Subsection 21.2.1 Electric Forces and Fields

Aside Subsection referenced by

Section 21.3 Making Connections

Aside Referenced by

Chapter 28 Quantum Mechanics

Section 28.2 Nuclear Physics

Subsection 28.2.2 The Strong Nuclear Force

Aside Referenced by

Subsection 28.2.3 The Weak Nuclear Force

Aside Referenced by

Section 28.3 Particle Physics

Subsection 28.3.2 Quantum Electrodynamics

Aside Referenced by

(Continued on next page)

Subsection 28.3.3 Quantum Chromodynamics

Aside Referenced by

Chapter 32 Deeper Dive

Section 32.1 [Need title]

Subsection 32.1.4 Cool Ideas

Aside Referenced by

Chapter 35 Holder

Aside Touchstone

Touchstone

Historical Aside

.5 To-Do List

To Do storyboard the characters and how they develop.[NUM]

To Do Decide how many characters.[NUM]

To Do (em)Book Layout:(/em) Here we should add information about Adventures, Examples, Equation-Stories, and IRLs..[N

Chapter 1 The Story of Science

To Do Add description of science..[NUM]

To Do Add description of physics.[NUM]

Section 1.1 Careful, Detailed Observation

To Do Consider the ”casual to the obvious observer” joke.[NUM]

Chapter 2 Seeing Physics

To Do This chapter should mirror (xref ref="c-revisted" text="type-global" /).[NUM]

Section 2.1 The Flame Challenge and Other Brief Descriptions

Subsection 2.1.2 The Forming of Matter in the Universe

To Do Stopped mid-stream. This is a good place to jump back in when I am stuck someplace else..[NUM]

Section 2.2 Effective Theory

To Do Should this be here or in (xref ref="s-effective2" text="type-global" /)?.[NUM]

Chapter 3 Why so much math?

Section 3.1 Every equation tells a story

To Do I would love for this to be a mouse-over in the equation.[NUM]
To Do Consider "chunking" the "story" with colors to indicate the pieces..[NUM]
To Do Is there a way to include something in the LaTeXversion but not in the HTML version? (such as **??**).[NUM]
To Do Is there a way to include something in the LaTeXversion but not in the HTML version? (such as 39).[NUM]
To Do Decide if should use the "public version" or the "me version" (which uses)..[NUM]

Section 3.2 The Metric System

Subsection 3.2.2 Conversion from English Units

To Do rephrase this. I moved that discussion to (xref ref="s-sigfig" text="type-global" /)..[NUM]

Chapter 4 Estimating and Uncertainty

Section 4.2 Significant Figures

To Do Remove this sentence and make the next paragraph sensible. It makes more sense here than in (xref ref="ss-weight
To Do Refocus this paragraph as an example about significant digits..[NUM]
To Do Add link: variation in g.[NUM]

Section 4.4 Effective Theories

To Do Should this be here or in (xref ref="s-effective1" text="type-global" /)?.[NUM]
To Do Decide if this should be filled out more or if it should reference the variety of places where the text fills out these ki

Chapter 5 One-Dimensional Motion

Section 5.1 How Physicists Use the Words (Notation)

To Do Decide if the [notation] goes here (xref ref="intro-acc" text="type-global" /) or in (xref ref="d-acc" text="type-gl

Section 5.2 Connecting the Concepts- distance equals rate times time

Subsection 5.2.2 Speed versus Velocity

To Do Discussion of speed as ${}^{\Delta x}/_{\Delta t}$.[NUM]
To Do Discussion of velocity as a vector.[NUM]

Section 5.3 Extending the Concepts: Changing How You Move

Subsection 5.3.1 Moving versus Speeding Up

To Do Decide if the [notation] goes (xref ref="d-acc" text="type-global" /) or in (xref ref="intro-acc" text="type-global"
To Do Is there a way to include something in the LaTeXversion but not in the HTML version? (such as 26 and 26).[NUM]

Section 5.5 Examples

Subsection 5.5.1 Freefall

To Do Maybe add another exercise?.[NUM]

Section 5.6 Complications

Subsection 5.6.3 Multi-Step Solutions

To Do finish (xref ref="ex-ceiling" text="type-global" /). Maybe make it two examples, instead of one?.[NUM]

Chapter 7 Force

Section 7.2 Connecting the Concepts: Newton's Laws

To Do check the implications of this footnote..[NUM]

Subsection 7.2.2 Translating Newton's Second Law: The Equation Law

To Do maybe note the MKS-to-SI transition. maybe leave that in (xref ref="ss-units" text="type-global" /)..[NUM]
To Do Maybe these should be sidebyside with the figure above the example?.[NUM]
To Do Make this object a desk so that we can have Abdul and Beth helping you rearrange your room in your residence hall..
To Do As before, make this object a desk so that we can have Abdul and Beth helping you rearrange your room in your resi
To Do Merge (xref ref="ex-2Dfa" text="type-global" /) and (xref ref="ex-2Dfa2" text="type-global" /). Also reference the
To Do I like "answer" better, but it doesn't fit. "hint" fits, but I don't like it. I can't figure out how to extend that line. sid

Subsection 7.2.3 Translating Newton's Third Law: Action & Reaction

To Do As before, make this object a desk so that we can have Abdul and Beth helping you rearrange your room in your resi
To Do Figure out how to color text..[NUM]
To Do As before, make this object a desk so that we can have Abdul and Beth helping you rearrange your room in your resi
To Do Make an exercise that calculates the acceleration of Carl and Diane..[NUM]

Section 7.4 Summary and Homework

To Do Add more conceptual questions.[NUM]
To Do Add more variety of problems..[NUM]

Chapter 8 The Many Types of Force

To Do add reference to the general theory.[NUM]

Section 8.1 Gravity at the Surface of the Earth

Subsection 8.1.1 Weight versus Mass

To Do Update (xref ref="s-SI-MKS" text="type-hybrid" /) with this information..[NUM]

Subsection 8.1.2 Calculating the weight

To Do Gather values of g at various locations. Wiki has a list, but need to find the source. Wolfram has numbers, but the

Section 8.3 Normal Force

To Do Do we need to repeat the example here? no?.[NUM]
To Do (xref ref="irl-poolcushion" text="type-hybrid" /) should be moved to a section that has more about friction and a
To Do This answer is getting too complex for the section it is in. I need to move the IRL before I finish considering how t

Subsection 8.3.1 Bathroom Scales Measure the Normal Force

To Do link equilibrium?.[NUM]
To Do link the gravitational force?.[NUM]
To Do link the gravitational force.[NUM]
To Do link to the section on friction and ramps.[NUM]
To Do link to the trig section.[NUM]
To Do link "can use" to the section on ramps.[NUM]

Section 8.5 Tension

To Do add a link to (and the section itself) to a section on the modulus and stress/strain..[NUM]
To Do Maybe add an IRL about a house settling and the compression forces. Loading a pick-up truck and watching the b
To Do Still need to update the (xref ref="irl-tension" text="type-global"/)..[NUM]
To Do Is this sufficiently noticeable?.[NUM]
To Do maybe add links about the tension chaning in the rope.[NUM]
To Do maybe add links to the stretch of the rope.[NUM]

Subsection 8.5.1 Tension as a Support Force

To Do Add (insert)a description of the(/insert) (stale)an(/stale) image of an immovable surface to that section.[NUM]

Subsection 8.5.3 Pulleys

To Do add a reference to the section (problem?) where this is considered..[NUM]

Subsection 8.5.4 Interesting Complications

To Do YOU ARE HERE. Finish this example.[NUM]
To Do imported a homework problem from Giordano. Need to modify it to fit my purposes..[NUM]

Section 8.9 Summary and Homework

Subsection 8.9.3 Problems

To Do Add more problems..[NUM]

Chapter 13 Torque and the $F = ma$ of Rotations

Section 13.4 Summary and Homework

Subsection 13.4.2 Conceptual Questions

To Do Add conceptual problems..[NUM]
To Do Add more problems..[NUM]
To Do Make this a different homework problem..[NUM]

Chapter 15 The Gravitational Force on a Large Scale

Section 15.1 Gravitational Force and Field

To Do Reference a table of g on other planets and compute the weight of a space craft at each planet..[NUM]

Chapter 32 Deeper Dive

To Do This chapter should mirror (xref ref="c-physics" text="type-global" /)..[NUM]

Section 32.1 [Need title]

Subsection 32.1.4 Cool Ideas

To Do Follow-up in 2018 to find the results..[NUM]

Chapter 34 Characters

To Do The index will recognize the people in two different formats. One is by my name for them, which is Xerxes(where X is
To Do (url href="http://malveyauthor.com/")Madeline Alvey(/url), the author of (url href="http://escapepod.org/2017/03

Chapter 35 Holder

To Do Consider adding comment about Newton not including "inertial reference frame" to his laws..[NUM]

Index

This article was authored in PreTeXt.