LinearRegressionGradientDecentOneVariable

July 11, 2022

[61]:
```
# Import
import math, copy
import numpy as np
import matplotlib.pyplot as plt
```

0.1 Cost function

In linear regression with one variable, the cost function is a measure on how well our model is predicting the target values.

The equation for the cost function with one variable is:

$$J(w,b) = \frac{1}{2m} \sum_{i=0}^{m-1} (f_{w,b}(x^{(i)}) - y^{(i)})^2 \tag{1}$$

where

$$f_{w,b}(x^{(i)}) = wx^{(i)} + b \tag{2}$$

- $f_{w,b}(x^{(i)})$ is the prediction for sample i using parameters w, b.
- $(f_{w,b}(x^{(i)}) - y^{(i)})^2$ is the squared difference between the target value and the prediction.
- These differences are summed over all the m samples and divided by $2m$ to produce the cost, $J(w,b)$.
- Note that the summation ranges from 0 to m-1 (compatible with python).

Here is the code to compute the cost function:

[62]:
```
# Cost function calculator
def compute_cost(x, y, w, b):
    """
    Computes the cost function for linear regression
    Args:
      x (ndarray (m,)): features, m data sets
      y (ndarray (m,)): target values
      w,b (scalar)    : linear model parameters
    Returns
      total_cost (scalar): The total cost
     """
```

```
    # initialize
    m = x.shape[0]
    cost = 0

    # loop over number of data sets
    for i in range(m):
        f_wb = w * x[i] + b
        cost += (f_wb - y[i])**2
    # total cost
    total_cost = 1 / (2 * m) * cost

    return total_cost
```

0.2 Gradient of the cost function

From the cost function equation, the gradient of the cost function with respect to w and b is given as follows:

$$\frac{\partial J(w,b)}{\partial w} = \frac{1}{m} \sum_{i=0}^{m-1} (f_{w,b}(x^{(i)}) - y^{(i)})x^{(i)} \tag{4}$$

$$\frac{\partial J(w,b)}{\partial b} = \frac{1}{m} \sum_{i=0}^{m-1} (f_{w,b}(x^{(i)}) - y^{(i)}) \tag{5}$$

$$\tag{1}$$

- Note that all partial derivatives are computed simultaniously.

Here is the code to compute the gradient of the cost function:

[63]:
```
# Cost function gradient calculator
def compute_cost_gradient(x, y, w, b):
    """
    Computes the gradient of the cost function based on the linear regression␣
 ↪model
    Args:
      x (ndarray (m,)): features, m data sets
      y (ndarray (m,)): target values
      w,b (scalar)    : model parameters
    Returns
      dj_dw (scalar): The gradient of the cost function w.r.t. the parameters w
      dj_db (scalar): The gradient of the cost function w.r.t. the parameter b ␣
 ↪
     """

    # Initialize
    m = x.shape[0]
```

```
    dj_dw = 0
    dj_db = 0

    # loop over number of data sets
    for i in range(m):
        f_wb = w * x[i] + b
        dj_dw_i = (f_wb - y[i]) * x[i]
        dj_db_i = (f_wb - y[i])
        dj_dw += dj_dw_i
        dj_db += dj_db_i
    dj_dw = dj_dw / m
    dj_db = dj_db / m

    return dj_dw, dj_db
```

0.3 Gradient descent

In linear regression with one variable, the linear model that predicts $f_{w,b}(x^{(i)})$ is given as:

$$f_{w,b}(x^{(i)}) = wx^{(i)} + b \tag{1}$$

In linear regression, we utilize input training data to fit the parameters w,b by minimizing a measure of the error between our predictions $f_{w,b}(x^{(i)})$ and the actual data $y^{(i)}$. The measure is called the *cost*, $J(w,b)$. In training we measure the cost over all of our training samples $x^{(i)}, y^{(i)}$ by:

$$J(w,b) = \frac{1}{2m} \sum_{i=0}^{m-1} (f_{w,b}(x^{(i)}) - y^{(i)})^2 \tag{2}$$

Using the *gradient descent* method, parameters w, b are updated simultaneously as follows:

$$\begin{aligned} w &= w - \alpha \frac{\partial J(w,b)}{\partial w} \\ b &= b - \alpha \frac{\partial J(w,b)}{\partial b} \end{aligned} \tag{3}$$

where α is the learing rate.

- We repeat the update precedure until the convergence criteria are met.
- Note that when the gradient is negative, w or b is decreased and vice-versa.

Here is the code for gradient decent method with one variable:

[64]:
```
# Gradient decent method
def gradient_descent(x, y, w_in, b_in, alpha, num_iters, rel_err):
    """
    Performs gradient descent to fit w,b. Updates w,b by taking
    num_iters gradient steps with learning rate alpha
```

```
Args:
  x (ndarray (m,))  : features, m examples
  y (ndarray (m,))  : target values
  w_in,b_in (scalar): initial values of model parameters
  alpha (float)     : learning rate
  num_iters (int)   : number of iterations to run gradient descent
  rel_err(float)    : relative error in the gradient decent

Returns:
  w (scalar)      : Updated value of parameter after running gradient␣
↪descent
  b (scalar)      : Updated value of parameter after running gradient␣
↪descent
  J_history (List): History of cost values
  p_history (list): History of parameters [w,b]
  """

# Initialize
w = copy.deepcopy(w_in) # avoid modifying global w_in

# An array to store cost J and w's at each iteration primarily for graphing␣
↪later
J_history = []
p_history = []
b         = b_in
w         = w_in

# Loop over number of iterations
i = 0
while i < num_iters:

    # Calculate the gradient and update the parameters using␣
↪gradient_function
    dj_dw, dj_db = compute_cost_gradient(x, y, w , b)

    # Update Parameters using equation for the gradient decent
    b = b - alpha * dj_db
    w = w - alpha * dj_dw

    # Save cost J at each iteration
    if i < 100000: # prevent resource exhaustion
        J_history.append(compute_cost(x, y, w , b))
        p_history.append([w,b])
    # Relative difference
    rel_diff = 1.0
    if i > 0:
```

```
            rel_diff = abs(J_history[i]-J_history[i-1])/J_history[i]
        # Print cost every at intervals 10 times or as many iterations if < 10
        if i% math.ceil(num_iters/20) == 0:
            print(f"Itr {i:4}: Cost {J_history[-1]:0.2e} ",
                  f"dj_dw: {dj_dw: 0.3e}, dj_db: {dj_db: 0.3e}  ",
                  f"w: {w: 0.3e}, b:{b: 0.5e}, rel_err: {rel_diff: 0.5e}")

        # Check the convergence and update
        i += 1
        if rel_diff < rel_err:
            return w, b, J_history, p_history

    # return w and J,w history for graphing
    return w, b, J_history, p_history
```

0.4 Plot cost function

[65]:
```
def plot(J_hist):
    # plot cost versus iteration
    fig, (ax1, ax2) = plt.subplots(1, 2, constrained_layout=True,␣
 ↪figsize=(12,4))
    ax1.plot(J_hist[:100])
    ax2.plot(1000 + np.arange(len(J_hist[1000:])), J_hist[1000:])
    ax1.set_title("Cost vs. iteration(start)");  ax2.set_title("Cost vs.␣
 ↪iteration (end)")
    ax1.set_ylabel('Cost')            ;  ax2.set_ylabel('Cost')
    ax1.set_xlabel('iteration step')  ;  ax2.set_xlabel('iteration step')
    plt.show()
```

0.5 Run

[66]:
```
def main():
    # initialize parameters
    w_init = 0
    b_init = 0

    # some gradient descent settings
    num_iters = 10000
    alpha     = 1.0e-2
    rel_err   = 1.0e-3

    # Load our data set
    x_train = np.array([1.0, 2.0])   #features
    y_train = np.array([300.0, 500.0])   #target value

    # run gradient descent
```

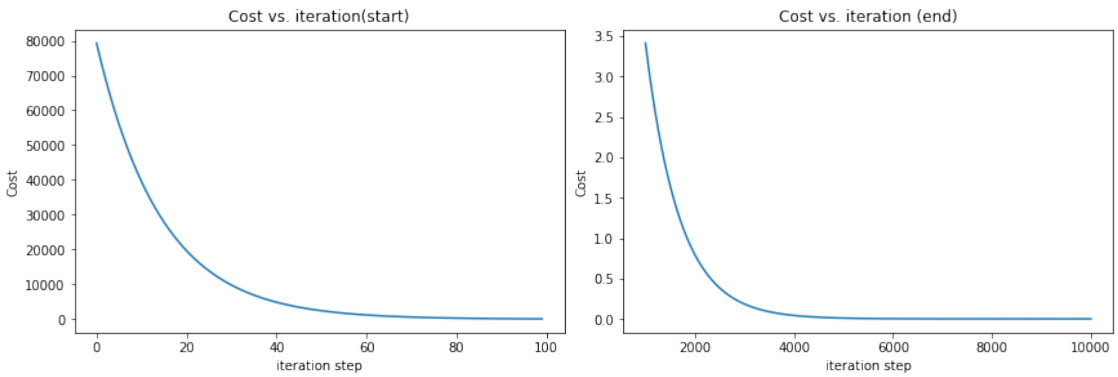
```python
    w_final, b_final, J_hist, p_hist = gradient_descent(x_train ,y_train,␣
↪w_init, b_init, alpha, num_iters, rel_err)
    print(f"(J, w, b) found by gradient descent: ({J_hist[-1]:0.7f}, {w_final:8.
↪4f}, {b_final:8.4f})")

    # plot
    plot(J_hist)

if __name__ == '__main__':
    main()
```

```
Itr    0: Cost 7.93e+04  dj_dw: -6.500e+02, dj_db: -4.000e+02   w:  6.500e+00,
b: 4.00000e+00, rel_err:  1.00000e+00
Itr  500: Cost 7.08e+00  dj_dw: -5.347e-01, dj_db:  8.651e-01   w:  1.927e+02,
b: 1.11851e+02, rel_err:  1.46058e-03
Itr 1000: Cost 3.41e+00  dj_dw: -3.712e-01, dj_db:  6.007e-01   w:  1.949e+02,
b: 1.08228e+02, rel_err:  1.46058e-03
Itr 1500: Cost 1.64e+00  dj_dw: -2.577e-01, dj_db:  4.170e-01   w:  1.965e+02,
b: 1.05713e+02, rel_err:  1.46058e-03
Itr 2000: Cost 7.93e-01  dj_dw: -1.789e-01, dj_db:  2.895e-01   w:  1.975e+02,
b: 1.03966e+02, rel_err:  1.46058e-03
Itr 2500: Cost 3.82e-01  dj_dw: -1.242e-01, dj_db:  2.010e-01   w:  1.983e+02,
b: 1.02754e+02, rel_err:  1.46058e-03
Itr 3000: Cost 1.84e-01  dj_dw: -8.625e-02, dj_db:  1.396e-01   w:  1.988e+02,
b: 1.01912e+02, rel_err:  1.46058e-03
Itr 3500: Cost 8.88e-02  dj_dw: -5.989e-02, dj_db:  9.690e-02   w:  1.992e+02,
b: 1.01327e+02, rel_err:  1.46058e-03
Itr 4000: Cost 4.28e-02  dj_dw: -4.158e-02, dj_db:  6.727e-02   w:  1.994e+02,
b: 1.00922e+02, rel_err:  1.46058e-03
Itr 4500: Cost 2.06e-02  dj_dw: -2.887e-02, dj_db:  4.671e-02   w:  1.996e+02,
b: 1.00640e+02, rel_err:  1.46058e-03
Itr 5000: Cost 9.95e-03  dj_dw: -2.004e-02, dj_db:  3.243e-02   w:  1.997e+02,
b: 1.00444e+02, rel_err:  1.46058e-03
Itr 5500: Cost 4.79e-03  dj_dw: -1.391e-02, dj_db:  2.251e-02   w:  1.998e+02,
b: 1.00308e+02, rel_err:  1.46058e-03
Itr 6000: Cost 2.31e-03  dj_dw: -9.660e-03, dj_db:  1.563e-02   w:  1.999e+02,
b: 1.00214e+02, rel_err:  1.46058e-03
Itr 6500: Cost 1.11e-03  dj_dw: -6.707e-03, dj_db:  1.085e-02   w:  1.999e+02,
b: 1.00149e+02, rel_err:  1.46058e-03
Itr 7000: Cost 5.37e-04  dj_dw: -4.657e-03, dj_db:  7.535e-03   w:  1.999e+02,
b: 1.00103e+02, rel_err:  1.46058e-03
Itr 7500: Cost 2.59e-04  dj_dw: -3.233e-03, dj_db:  5.231e-03   w:  2.000e+02,
b: 1.00072e+02, rel_err:  1.46058e-03
Itr 8000: Cost 1.25e-04  dj_dw: -2.245e-03, dj_db:  3.632e-03   w:  2.000e+02,
b: 1.00050e+02, rel_err:  1.46058e-03
Itr 8500: Cost 6.01e-05  dj_dw: -1.558e-03, dj_db:  2.522e-03   w:  2.000e+02,
```

```
b: 1.00035e+02, rel_err:  1.46058e-03
Itr 9000: Cost 2.90e-05  dj_dw: -1.082e-03, dj_db:  1.751e-03   w:  2.000e+02,
b: 1.00024e+02, rel_err:  1.46058e-03
Itr 9500: Cost 1.40e-05  dj_dw: -7.512e-04, dj_db:  1.215e-03   w:  2.000e+02,
b: 1.00017e+02, rel_err:  1.46058e-03
(J, w, b) found by gradient descent: (0.0000067, 199.9929, 100.0116)
```



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