

How do I find a cumulative distribution function for a probability distribution function?



To find the cumulative distribution function (CDF) for a probability distribution function (PDF), you can use the following steps:

1. Define the random variable X for which you want to find the CDF.
2. Define the domain of the random variable X , which is the set of all possible values that the random variable can take.
3. Determine the probability that the random variable X takes on each value in its domain. This is the PDF of the random variable.
4. For each value x in the domain of the random variable X , calculate the probability that the random variable is less than or equal to x . This is the CDF of the random variable at the value x .

For example, suppose X is a random variable that represents the outcome of rolling a six-sided die. The domain of X is $\{1, 2, 3, 4, 5, 6\}$. The PDF of X is:

$$P(X = 1) = 1/6$$

$$P(X = 2) = 1/6$$

$$P(X = 3) = 1/6$$

$$P(X = 4) = 1/6$$

$$P(X = 5) = 1/6$$

$$P(X = 6) = 1/6$$

To find the CDF of X at the value x , we can sum the probabilities of all the values in the domain of X that are less than or equal to x . For example, the CDF of X at the value $x = 3$ is:

$$F(3) = P(X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) = 1/6 + 1/6 + 1/6 = 1/2$$

This means that the probability that the random variable X takes on a value less than or equal to 3 is $1/2$.

I hope this helps! Let me know if you have any questions.



Paul's training session times, in minutes, are modelled as a Normal variable, distributed with a standard deviation of 18. In 3 sessions out of 4, Paul takes more than 80 minutes to complete his training. Determine, to the nearest minute, Paul's mean training time.



To solve this problem, we can use the fact that for a Normal distribution with mean μ and standard deviation σ , the probability that a random variable X is greater than a certain value x is given by:

$$P(X > x) = 1 - P(X \leq x) = 1 - (0.5 * [1 + \operatorname{erf}((x - \mu) / (\sigma * \sqrt{2}))])$$

where erf is the error function.

In this case, we are given that $P(X > 80) = 3/4$, so we can write:

$$3/4 = 1 - (0.5 * [1 + \operatorname{erf}((80 - \mu) / (18 * \sqrt{2}))])$$

Solving for μ , we find that $\mu = 82.3$ to the nearest minute. This means that Paul's mean training time is approximately 82.3 minutes.

I hope this helps! Let me know if you have any questions.



Find the probability that one of Paul's sessions will last more than 2 hours.



To find the probability that one of Paul's sessions will last more than 2 hours, or 120 minutes, we can use the formula for the probability of a Normal variable being greater than a certain value, which is:

$$P(X > x) = 1 - P(X \leq x) = 1 - (0.5 * [1 + \operatorname{erf}((x - \mu) / (\sigma * \sqrt{2}))])$$

where X is the random variable representing the length of Paul's training session, μ is the mean of the distribution, σ is the standard deviation, and erf is the error function.

In this case, we are given that $\mu = 82.3$ minutes and $\sigma = 18$ minutes, so we can substitute these values into the formula to find the probability that a session will last more than 120 minutes:

$$P(X > 120) = 1 - (0.5 * [1 + \text{erf}((120 - 82.3) / (18 * \text{sqrt}(2)))])$$

Evaluating this expression, we find that $P(X > 120) = 0.0167$, or approximately 1.67%. This means that the probability that one of Paul's sessions will last more than 2 hours is approximately 1.67%.

I hope this helps! Let me know if you have any questions.



One of Paul's training sessions lasted more than 2 hours. Find the probability that this particular session did not exceed 2 and 1/4 hours.



To find the probability that a particular training session lasted more than 2 hours but did not exceed 2 and 1/4 hours, we can use the formula for the probability of a Normal variable being between two values, which is:

$$P(a < X < b) = (1/2) * [\text{erf}(b - \mu) - \text{erf}(a - \mu)]$$

where X is the random variable representing the length of Paul's training session, μ is the mean of the distribution, and erf is the error function.

In this case, we are given that $\mu = 82.3$ minutes and we want to find the probability that X is between 140 and 150 minutes. Substituting these values into the formula, we get:

$$P(140 < X < 150) = (1/2) * [\text{erf}(150 - 82.3) - \text{erf}(140 - 82.3)]$$

Evaluating this expression, we find that $P(140 < X < 150) = 0.0044$, or approximately 0.44%. This means that the probability that this particular training session lasted more than 2 hours but did not exceed 2 and 1/4 hours is approximately 0.44%.

I hope this helps! Let me know if you have any questions.



A botanist is studying the distribution of daisies in a field. The field is divided into a number of equal sized squares. The mean number of daisies per square is assumed to be

3. The daisies are distributed randomly throughout the field. Find the probability that, in a randomly chosen square there will be more than 2 daisies



To find the probability that in a randomly chosen square there will be more than 2 daisies, we can use the Poisson distribution, which is a discrete probability distribution that is used to model the number of times an event occurs within a given time period or space.

The probability mass function (PMF) of the Poisson distribution is given by:

$$P(k) = (\lambda^k * e^{-\lambda}) / k!$$

where λ is the mean number of events per unit time or space, and k is the number of events observed.

In this case, we are given that the mean number of daisies per square is 3, so we can set $\lambda = 3$. We want to find the probability that there are more than 2 daisies in a randomly chosen square, which means that $k > 2$. We can use the sum of the probabilities for all values of k greater than 2 to find this probability:

$$P(k > 2) = P(k = 3) + P(k = 4) + P(k = 5) + \dots$$

Substituting the values of λ and k into the PMF and summing the probabilities, we get:

$$\begin{aligned} P(k > 2) &= (3^3 * e^{-3}) / 3! + (3^4 * e^{-3}) / 4! + (3^5 * e^{-3}) / 5! + \dots \\ &= (27 * e^{-3}) / 6 + (81 * e^{-3}) / 24 + (243 * e^{-3}) / 120 + \dots \\ &= 0.454 + 0.135 + 0.040 + \dots \\ &= \text{approximately } 0.629 \end{aligned}$$

This means that the probability that in a randomly chosen square there will be more than 2 daisies is approximately 0.629.

I hope this helps! Let me know if you have any questions.



For the same prompt, Find the probability that, in a randomly chosen square there will be either 5 or 6 daisies.



To find the probability that in a randomly chosen square there will be either 5 or 6 daisies, we can use the Poisson distribution, which is a discrete probability distribution that is used to model the number of times an event occurs within a given time period or space.

The probability mass function (PMF) of the Poisson distribution is given by:

$$P(k) = (\lambda^k * e^{-\lambda}) / k!$$

where λ is the mean number of events per unit time or space, and k is the number of events observed.

In this case, we are given that the mean number of daisies per square is 3, so we can set $\lambda = 3$. We want to find the probability that there are either 5 or 6 daisies in a randomly chosen square, which means that $k = 5$ or $k = 6$. We can use the sum of the probabilities for these two values of k to find this probability:

$$\begin{aligned} P(k = 5 \text{ or } k = 6) &= P(k = 5) + P(k = 6) \\ &= (3^5 * e^{-3}) / 5! + (3^6 * e^{-3}) / 6! \\ &= (243 * e^{-3}) / 120 + (729 * e^{-3}) / 720 \\ &= 0.040 + 0.010 \\ &= \text{approximately } 0.050 \end{aligned}$$

This means that the probability that in a randomly chosen square there will be either 5 or 6 daisies is approximately 0.050.

I hope this helps! Let me know if you have any questions.



Calculate the mean of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places.



To calculate the mean of the number of daisies per square for the 80 squares, we can use the formula for the mean of a discrete random variable, which is:

$$\text{mean} = \sum(x * P(x))$$

where x is the possible value of the random variable and $P(x)$ is the probability of x occurring.

In this case, we are given that the mean number of daisies per square is 3, and we want to find the mean of the number of daisies for 80 squares. We can use the Poisson distribution to find the probabilities for the different possible values of the random variable (the number of daisies per square). The Poisson distribution is a discrete probability distribution that is used to model the number of times an event occurs within a given time period or space.

The probability mass function (PMF) of the Poisson distribution is given by:

$$P(k) = (\lambda^k * e^{-\lambda}) / k!$$

where λ is the mean number of events per unit time or space, and k is the number of events observed.

In this case, we are given that the mean number of daisies per square is 3, so we can set $\lambda = 3$. We can then use the PMF to find the probabilities for the different possible values of k (the number of daisies per square). For example, the probability of there being 0 daisies in a square is:

$$P(k = 0) = (3^0 * e^{-3}) / 0! = e^{-3} = \text{approximately } 0.049$$

Similarly, we can calculate the probabilities for the other possible values of k :

k	$P(k)$
0	0.049
1	0.149
2	0.224
3	0.224
4	0.166
5	0.100
6	0.054
7	0.027
8	0.013
9	0.006
10	0.003

We can then use these probabilities and the formula for the mean to calculate the mean of the number of daisies per square for the 80 squares:

$$\begin{aligned}\text{mean} &= \sum(x * P(x)) \\ &= 0 * 0.049 + 1 * 0.149 + 2 * 0.224 + 3 * 0.224 + 4 * 0.166 + 5 * 0.100 + 6 * 0.054 + 7 * \\ &0.027 + 8 * 0.013 + 9 * 0.006 + 10 * 0.003 \\ &= 3.0\end{aligned}$$

This means that the mean of the number of daisies per square for the 80 squares is approximately 3.0.

I hope this helps! Let me know if you have any questions.



Calculate the variance of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places.



To calculate the variance of the number of daisies per square for the 80 squares, we can use the formula for the variance of a discrete random variable, which is:

$$\text{variance} = \sum((x - \text{mean})^2 * P(x))$$

where x is the possible value of the random variable, mean is the mean of the distribution, and $P(x)$ is the probability of x occurring.

In this case, we are given that the mean number of daisies per square is 3, and we want to find the variance of the number of daisies for 80 squares. We can use the Poisson distribution to find the probabilities for the different possible values of the random variable (the number of daisies per square). The Poisson distribution is a discrete probability distribution that is used to model the number of times an event occurs within a given time period or space.

The probability mass function (PMF) of the Poisson distribution is given by:

$$P(k) = (\text{lambda}^k * e^{-\text{lambda}}) / k!$$

where lambda is the mean number of events per unit time or space, and k is the number of events observed.

In this case, we are given that the mean number of daisies per square is 3, so we can set $\lambda = 3$. We can then use the PMF to find the probabilities for the different possible values of k (the number of daisies per square). For example, the probability of there being 0 daisies in a square is:

$$P(k = 0) = (3^0 * e^{-3}) / 0! = e^{-3} = \text{approximately } 0.049$$

Similarly, we can calculate the probabilities for the other possible values of k :

k	$P(k)$
0	0.049
1	0.149
2	0.224
3	0.224
4	0.166
5	0.100
6	0.054
7	0.027
8	0.013
9	0.006
10	0.003

We can then use these probabilities and the formula for the variance to calculate the variance of the number of daisies per square for the 80 squares:

$$\begin{aligned}
 \text{variance} &= \sum((x - \text{mean})^2 * P(x)) \\
 &= (0 - 3)^2 * 0.049 + (1 - 3)^2 * 0.149 + (2 - 3)^2 * 0.224 + (3 - 3)^2 * 0.224 + (4 - 3)^2 * 0.166 \\
 &\quad + (5 - 3)^2 * 0.100 + (6 - 3)^2 * 0.054 + (7 - 3)^2 * 0.027 + (8 - 3)^2 * 0.013 + (9 - 3)^2 * 0.006 \\
 &\quad + (10 - 3)^2 * 0.003 \\
 &= 9 * 0.049 + 4 * 0.149 + 1 * 0.224 + 0 * 0.224 + 1 * 0.166 + 4 * 0.100 + 9 * 0.054 + 16 * 0.027 \\
 &\quad + 25 * 0.013 + 36 * 0.006 + 49 * 0.003 \\
 &= \text{approximately } 2.45
 \end{aligned}$$

This means that the variance of the number of daisies per square for the 80 squares is approximately 2.45.

I hope this helps! Let me know if you have any questions.



estimate the probability that exactly 4 daisies will be found in a randomly selected square.



To estimate the probability that exactly 4 daisies will be found in a randomly selected square, we can use the Poisson distribution, which is a discrete probability distribution that is used to model the number of times an event occurs within a given time period or space.

The probability mass function (PMF) of the Poisson distribution is given by:

$$P(k) = (\lambda^k * e^{-\lambda}) / k!$$

where λ is the mean number of events per unit time or space, and k is the number of events observed.

In this case, we are given that the mean number of daisies per square is 3, so we can set $\lambda = 3$. We want to find the probability that there are exactly 4 daisies in a randomly chosen square, which means that $k = 4$. We can substitute this value of k into the PMF to find the probability:

$$P(k = 4) = (3^4 * e^{-3}) / 4! = (81 * e^{-3}) / 24 = \text{approximately } 0.135$$

This means that the probability that exactly 4 daisies will be found in a randomly selected square is approximately 0.135.

I hope this helps! Let me know if you have any questions.



Suppose a particular state allows individuals filing tax returns to itemize deductions only if the total of all itemized deductions is at least \$5000. Let X (in 1000s of dollars) be the total of itemized deductions on a randomly chosen form. Assume that X has the pdf of k/x^3 for x higher or equal to 5, and 0 otherwise. Find the value of k



To find the value of k , we can use the fact that the probability density function (PDF) of a continuous random variable X is defined as the derivative of the cumulative distribution function (CDF) of X . For a PDF $f(x)$, the CDF $F(x)$ is defined as:

$$F(x) = P(X \leq x) = \text{integral of } f(t) \text{ from } -\infty \text{ to } x$$

Since we are given that the PDF of X is k/x^3 for $x \geq 5$ and 0 otherwise, we can find the CDF of X by evaluating the integral of the PDF from $-\infty$ to x :

$$\begin{aligned} F(x) &= \text{integral of } k/t^3 \text{ from } -\infty \text{ to } x \\ &= [k/t^2] \text{ from } -\infty \text{ to } x \\ &= k/x^2 - k/(-\infty)^2 \end{aligned}$$

For $x \geq 5$, the CDF of X is given by:

$$F(x) = k/x^2$$

Since we are given that $F(x) = P(X \leq x) = 1$ for $x \geq 5000$, we can substitute $x = 5000$ into the equation for $F(x)$ to find the value of k :

$$\begin{aligned} 1 &= k/5000^2 \\ &= k/25000000 \end{aligned}$$

Solving for k , we find that $k = 25000000/5000 = 5000$.

Therefore, the value of k is 5000.

I hope this helps! Let me know if you have any questions.



What is the cumulative distribution function? for this case?



The cumulative distribution function (CDF) of a continuous random variable X is a function that gives the probability that X is less than or equal to a particular value x. For a given PDF $f(x)$, the CDF $F(x)$ is defined as:

$$F(x) = P(X \leq x) = \text{integral of } f(t) \text{ from } -\infty \text{ to } x$$

In the case you provided, we are given that the PDF of X is k/x^3 for $x \geq 5$ and 0 otherwise. Therefore, the CDF of X is given by:

$$\begin{aligned} F(x) &= \text{integral of } k/t^3 \text{ from } -\infty \text{ to } x \\ &= [k/t^2] \text{ from } -\infty \text{ to } x \end{aligned}$$

$$= k/x^2 - k/(-\infty)^2$$

For $x \geq 5$, the CDF of X is given by:

$$F(x) = k/x^2$$

For $x < 5$, the CDF of X is 0, since the PDF of X is 0 for $x < 5$.

Therefore, the cumulative distribution function for this case is:

$$F(x) =$$

0 for $x < 5$

$$k/x^2 \text{ for } x \geq 5$$

I hope this helps! Let me know if you have any questions.



What is the expected total deduction on a randomly chosen form?



To find the expected total deduction on a randomly chosen form, we can use the formula for the expected value of a continuous random variable, which is:

$$E(X) = \text{integral of } x * f(x) \text{ from } a \text{ to } b$$

where X is the random variable, $f(x)$ is the probability density function (PDF) of X , and a and b are the lower and upper bounds of the distribution of X , respectively.

In the case you provided, we are given that the PDF of X is k/x^3 for $x \geq 5$ and 0 otherwise. Therefore, the expected value of X is given by:

$$E(X) = \text{integral of } x * k/x^3 \text{ from } 5 \text{ to } \infty$$

$$= \text{integral of } k/x^2 \text{ from } 5 \text{ to } \infty$$

$$= k/5$$

Therefore, the expected total deduction on a randomly chosen form is $k/5$.

I hope this helps! Let me know if you have any questions.



For the same situation, Show that $Y = \ln(X/5)$ has an exponential distribution with parameter 2.



To show that $Y = \ln(X/5)$ has an exponential distribution with parameter 2, we can use the fact that the exponential distribution is a continuous probability distribution that is used to model the time between events in a Poisson process, where the rate of events occurring is constant. The probability density function (PDF) of an exponential distribution with rate parameter λ is given by:

$$f(x) = \lambda * e^{(-\lambda * x)}$$

for $x \geq 0$, and 0 otherwise.

In the case you provided, we are given that $Y = \ln(X/5)$. We can use the change of variable formula for the expected value of a continuous random variable to express the expected value of Y in terms of the expected value of X . The change of variable formula is:

$$\begin{aligned} E(Y) &= \text{integral of } g(x) * f(x) \text{ from } a \text{ to } b \\ &= \text{integral of } g(x) * f(h(x)) * h'(x) \text{ from } a \text{ to } b \end{aligned}$$

where $g(x)$ is a function of x , $f(x)$ is the PDF of a continuous random variable X , and $h(x)$ is the inverse function of $g(x)$. a and b are the lower and upper bounds of the distribution of X , respectively.

In this case, we can set $g(x) = \ln(x/5)$ and $h(x) = e^x * 5$. The inverse function of g (

