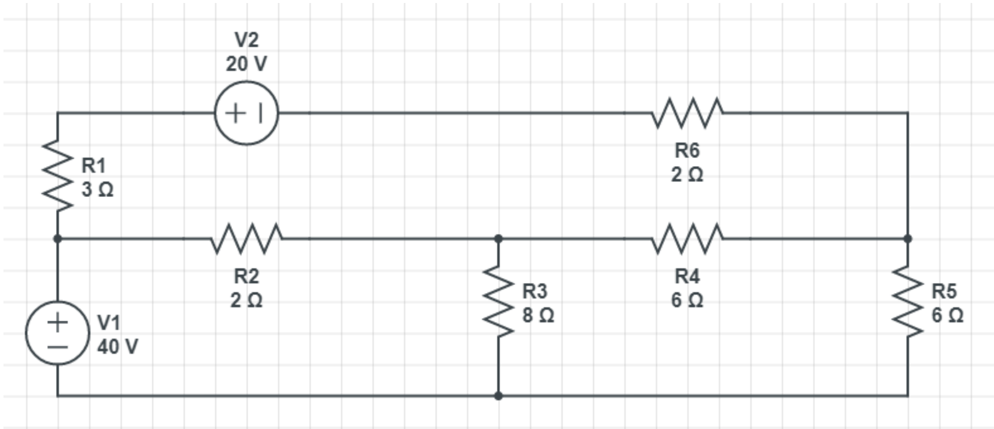
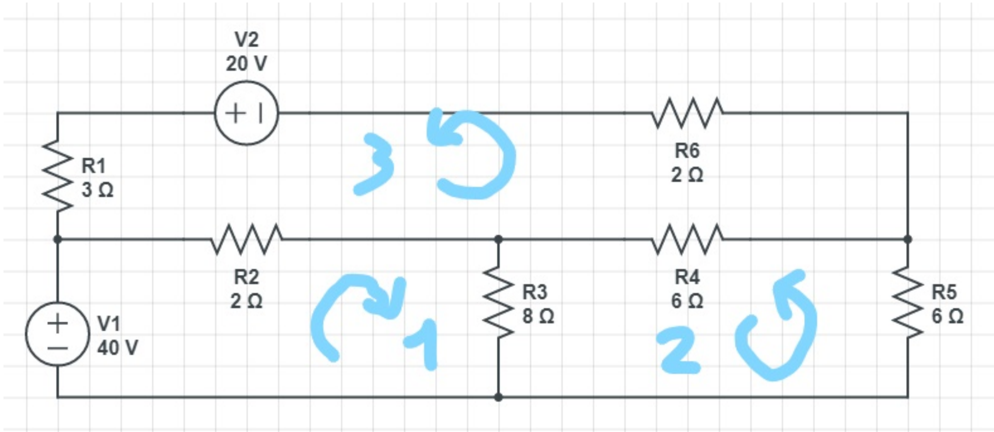
Problems

Problem 1

Find the currents going through R2 and R4 in the following circuit



Solution



Mesh equations:

$$(2I_1 + 2I_3) + (8I_1 + 8I_2) = 40$$
$$(8I_2 + 8I_1) + (6I_2) + (6I_2 - 6I_3) = 0$$
$$(2I_3 + 2I_1) + (6I_3 - 6I_2) + (2I_3) + (3I_3) = 20$$

Hence,

$$10I_1 + 8I_2 + 2I_3 = 40$$
$$8I_1 + 20I_2 - 6I_3 = 0$$
$$2I_1 - 6I_2 + 13I_3 = 20$$

which leads to,

$$I_1 = 6,34\text{A}, I_2 = -2,75\text{A}, I_3 = -0,71\text{A}$$

Finally

$$I_{R2} = I_1 + I_3 = 5,63\text{A}$$
$$I_{R4} = I_3 - I_2 = 2,04\text{A}$$

In [2]:
```
from numpy.linalg import solve
```

In [5]:
```
A=np.matrix('10,8,2;8,20,-6;2,-6,13')
b=np.matrix('40;0;20')
x=solve(A,b)
print(x)
```

```
[[ 6.33802817]
 [-2.74647887]
 [-0.70422535]]
```

In [2]:
```
A=np.matrix('10,-8,-2;-8,20,-6;-2,-6,13')
b=np.matrix('40;0;-20')
x=solve(A,b)
print(x)
```

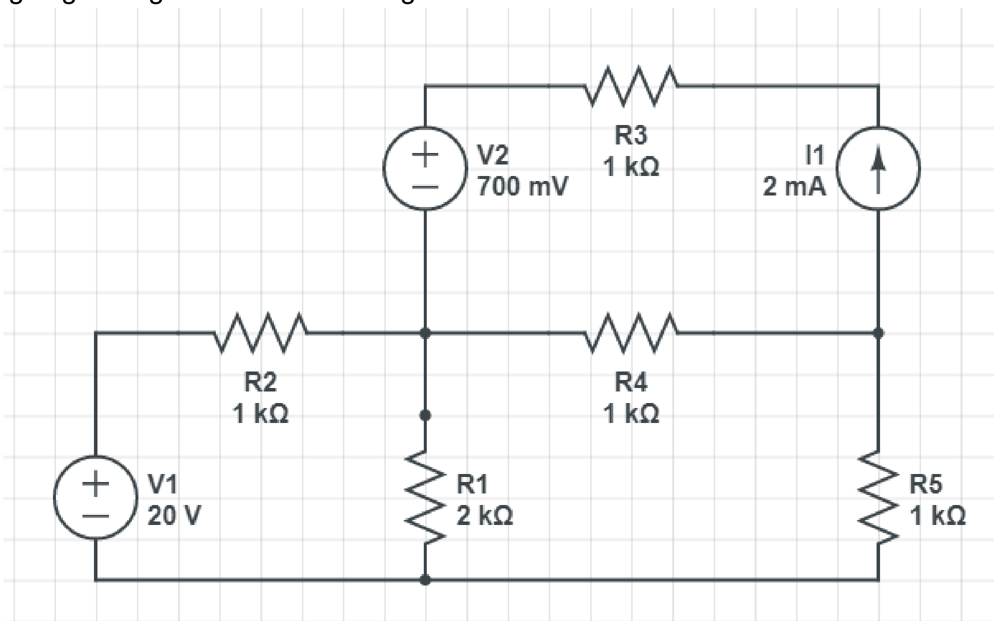
```
[[6.33802817]
 [2.74647887]
 [0.70422535]]
```

```
In [4]: (6.33802817-0.70422535)*2+(6.33802817-2.74647887)*8
```

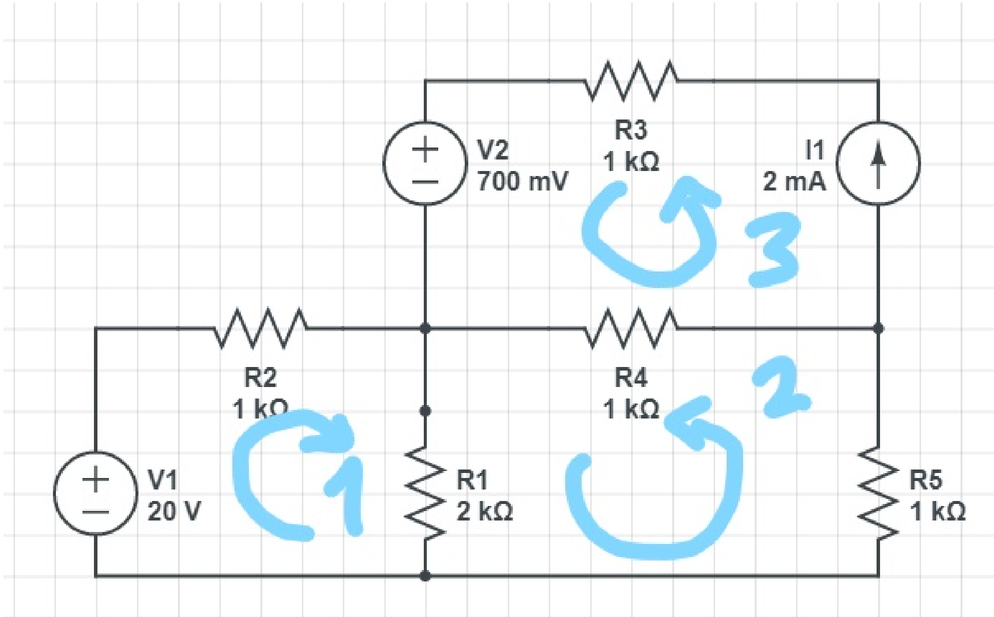
```
Out[4]: 40.00000004
```

Problem 2

Find the current going through R4 in the following circuit



Solution



Mesh equations

$$(1I_1) + (2I_1 + 2I_2) = 20$$
$$(2I_2 + 2I_1) + (1I_2) + (1I_2 - 1I_3) = 0$$
$$I_3 = 2mA$$

Then,

$$3I_1 + 2I_2 = 20$$
$$2I_1 + 4I_2 - I_3 = 0$$
$$I_3 = 2$$

The solution to the system of equations is,

$$I_1 = 9,5\mathrm{mA}, I_2 = -4,25\mathrm{mA}, I_3 = 2\mathrm{mA}$$

Then

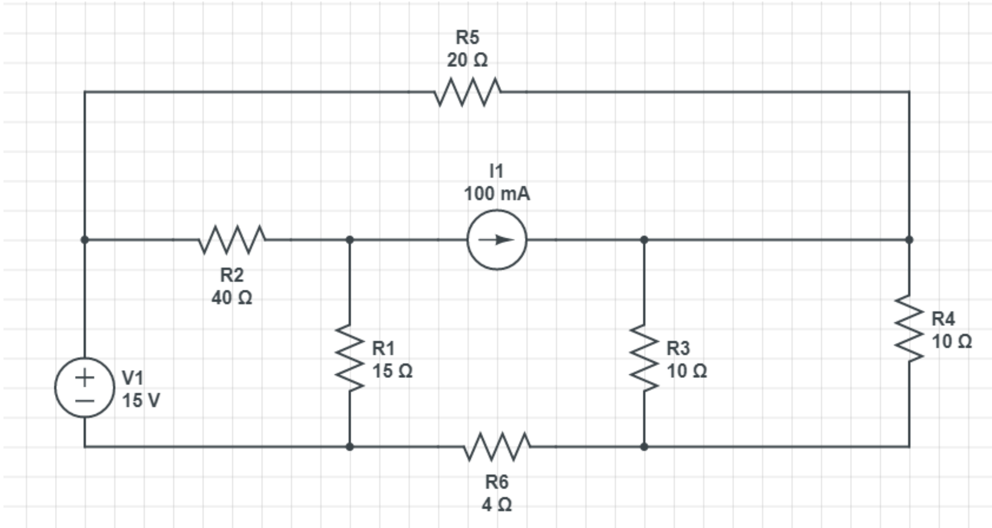
$$I_{R4} = I_3 - I_2 = 6,25\mathrm{mA}$$

In [7]:
```
A=np.matrix('3,2,0;2,4,-1;0,0,1')
b=np.matrix('20;0;2')
x=solve(A,b)
print(x)
```

```
[[ 9.5 ]
 [-4.25]
 [ 2.  ]]
```

Problem 3

Find the current and voltage drop in through R4 in the following circuit. How many nodes are there?



Solution

Let's apply mesh analysis for the three lower meshes and the outer loop, assuming they are all clockwise. The equations are:

$$I_1(40+15)+I_2(-15)=15$$
$$I_2=0.1$$
$$I_2(-10)+I_3(20)+I_4(10)=0$$
$$I_2(4)+I_3(10)+I_4(20+10+4)=15$$

In [3]:
```python
A=np.matrix('55,-15,0,0;0,1,0,0;0,-10,20,10;0,4,10,34')
b=np.matrix('15;0.1;0;15')
x=solve(A,b)
print(x)
```

```
[[ 0.3       ]
 [ 0.1       ]
 [-0.19310345]
 [ 0.4862069 ]]
```

$$I_{R4}=I_3+I_4=0.29A$$

$$\Delta V_{R4}=I_{R4}R_4=0.29\times 10=2.9V$$

Alternative solution: Let's combine R3 and R4 as they are in parallel, so $R_{eq} = (R_3 R_4)/(R_3 + R_4) = 100/20 = 5\Omega$. It is not required, but we simplify the analysis as we now require 3 equations instead of 4. Now there are only 3 meshes.Using the outer loop instead of mesh 3

$$I_1(40+15) + I_2(-15) + I_3(0) = 15$$
$$I_2 = 0.1$$
$$I_2(5+4) + I_3(20+5+4) = 15$$

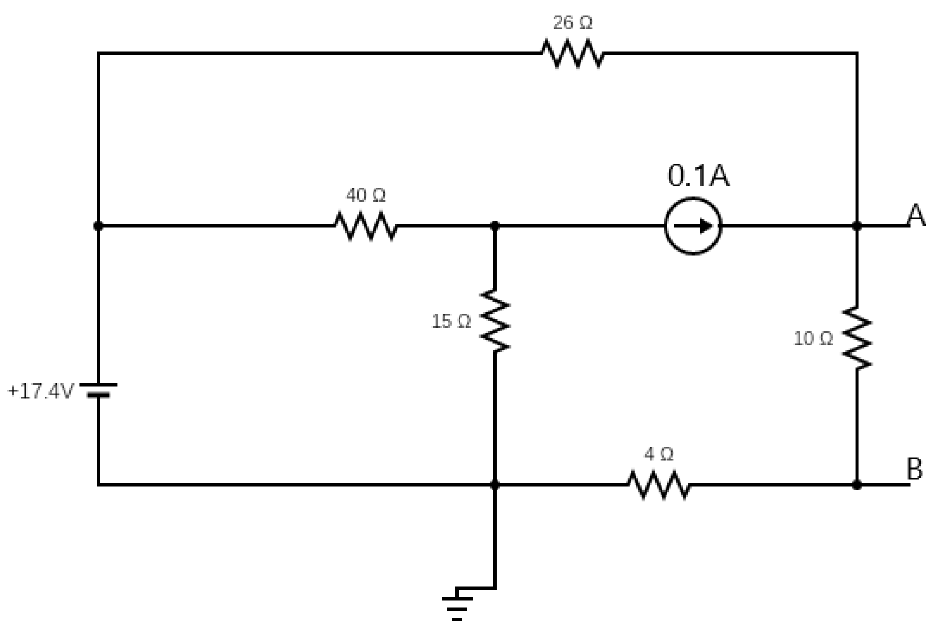
In [13]:
```
A=np.matrix('55,-15,-40;0,1,0;0,9,29')
b=np.matrix('15;0.1;15')
x=solve(A,b)
print(x)
```

```
[[0.65360502]
 [0.1       ]
 [0.4862069 ]]
```

The net current is going though R_4 is then $I_{R_4} = (I_2 + I_3)R_{eq}/R_4 = 0.29A$, which is the same as our previous result.

Problem 4

Find the Thevenin's equivalent between A and B



Solution

To calculate V_{th}, let us choose the the two lower meshes and the outer loop, then

$$55I_1 - 15I_2 = 17,4$$
$$I_2 = 0,1$$
$$14I_2 + 40I_3 = 17,4$$

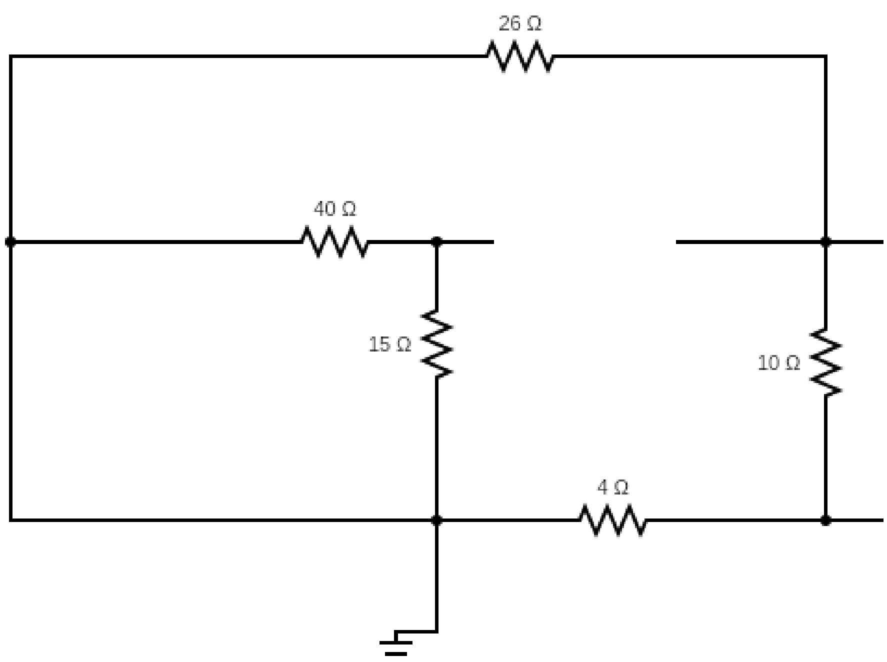
Then

$$I_1 = 0.34 \text{ A}; I_2 = 0.1 \text{ A}; I_3 = 0.4 \text{ A}$$

Hence

$$V_{th} = 10\Omega \cdot (0.4 + 0.1)\text{A} = 5V$$

Now R_{th}:



There is a short circuit for the 40Ω and 15Ω resistors, so they do not contribute. Everything comes down to two resistors in parallel with values 10Ω and $(25+4)\Omega$

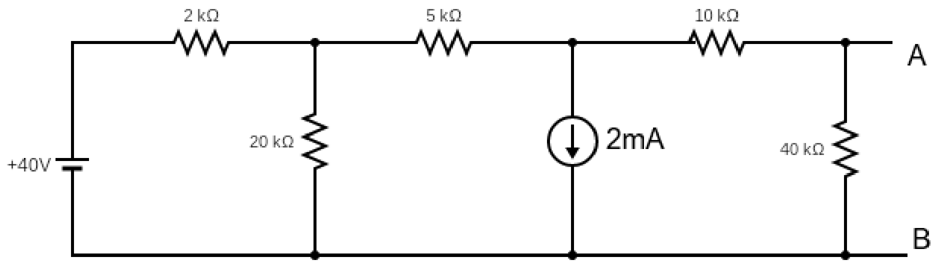
$$R_{th} = \frac{10\Omega \cdot (26+4)\Omega}{10\Omega + (26+4)\Omega} = 7,5\Omega$$

In [26]:
```
A=np.matrix('55,-15,0;0,1,0;0,14,40')
b=np.matrix('17.4;0.1;17.4')
x=solve(A,b)
print(x)
```

```
[[0.34363636]
 [0.1       ]
 [0.4       ]]
```

Problem 5

Find the Thevenin's equivalent between A and B



Answer

In [11]:
```
A=np.matrix('22,-20,-20;0,1,0;-20,25,75')
b=np.matrix('40;2;0')
x=solve(A,b)
print(x)
```

```
[[4. ]
 [2. ]
 [0.4]]
```

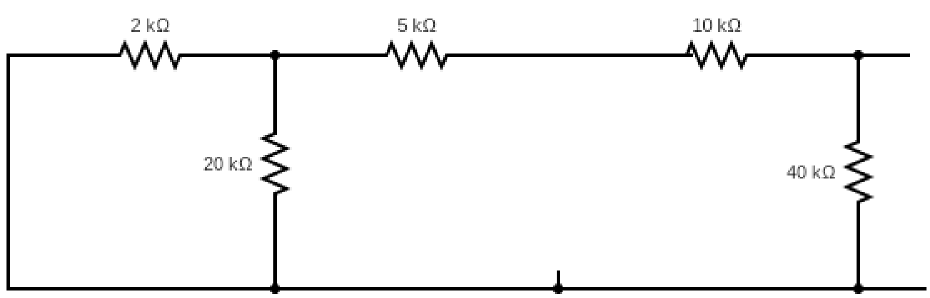
$$I_1 = 4\text{mA}; I_2 = 2\text{mA}; I_3 = 0.4\text{mA}$$

Hence

$$V_{th} = 40k\Omega \cdot I_3 = 16V$$

In [12]:
```
40*0.4
```

Out[12]:
```
16.0
```



$R_{th} = 11.84k\Omega$

In [11]:

```
parallel1=20*2/(20+2)
series1=parallel1+10+5
parallel2=(series1)*40/(series1+40)
print(parallel2)
```

```
11.84
```