Lectures 35&36 – Two-Way ANOVA with Interactions

Example 1: The next two examples are based on marketing studies. A new apple juice product was entering the marketplace. It had three distinct advantages relative to existing apple juices. First, it was not a concentrate and was therefore considered to be of higher “quality” than many similar products. Second, as one of the first juices packaged in cartons, it was cheaper than competing products. Third, partly because of the packaging, it was more convenient. The director of marketing for the company would like to know which advantage should be emphasized in advertisements. The director would also like to know whether local television or newspapers are better for sales.

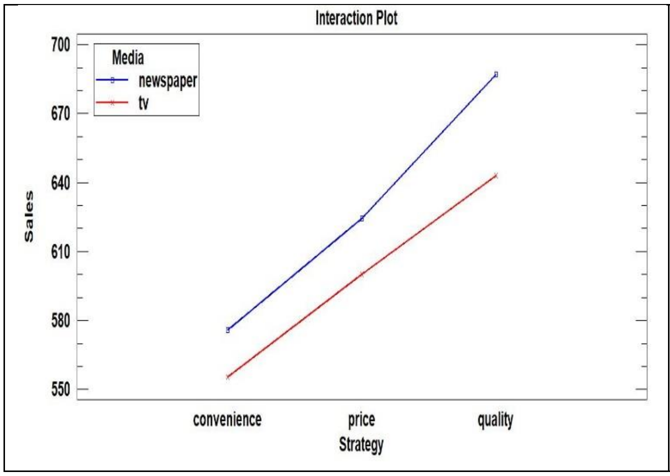
Consequently, six cities with similar demographics are chosen, and a different combination of *Marketing Strategy* and *Media* is tried in each. The unit sales of apple juice for the ten weeks immediately following the start of the ad campaigns are recorded for each city in the file *Apple Juice*. The two-way table below describes the city assignments for the six possible combinations of levels for the two factors. Below the assignment table is the ANOVA Table for interactions.

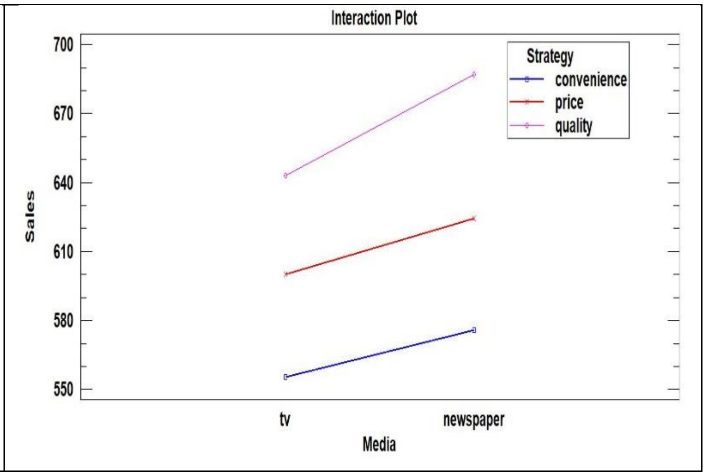
	Convenience	Quality	Price
Local Television	City 1	City 3	City 5
Newspaper	City 2	City 4	City 6

Analysis of Variance for Sales - Type III Sums of Squares

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*
MAIN EFFECTS					
A:Strategy	98838.6	2	49419.3	5.33	0.0077
B:Media	13172.0	1	13172.0	1.42	0.2387
INTERACTIONS					
AB	1609.63	2	804.817	0.09	0.9171
RESIDUAL	501137.	54	9280.31		
TOTAL (CORRECTED)	614757.	59			

Interactions are not significant to the model (*P*-value equals 0.9171), a fact which is reinforced by looking at the *Interaction Plot* under *Graphical Options*. Note that the two curves are almost parallel, a sign that interactions are not significant.



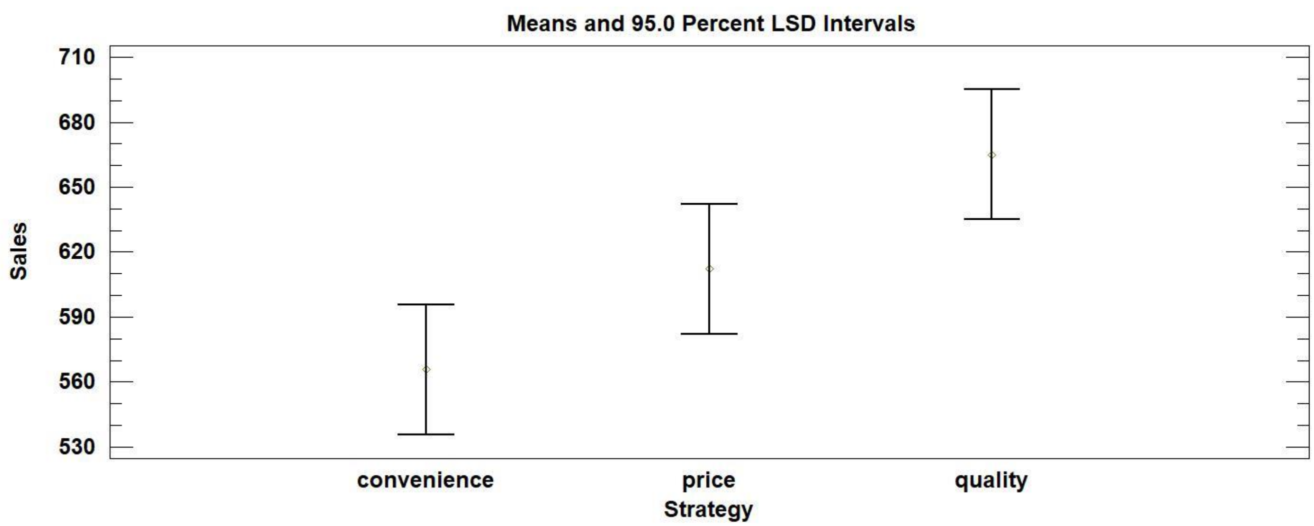


Parallel curves in the Interaction Plots reflect the absence of interaction between *Strategy* and *Media*

Removing interactions, we obtain the ANOVA Table below, from which we conclude that the marketing strategy is significant, but the media used may not be. Since only marketing strategy appears to affect sales, we'll restrict ourselves to the means plot for the factor Strategy below. Only the difference in mean sales when emphasizing quality versus emphasizing convenience is statistically significant at the 5% level of significance.

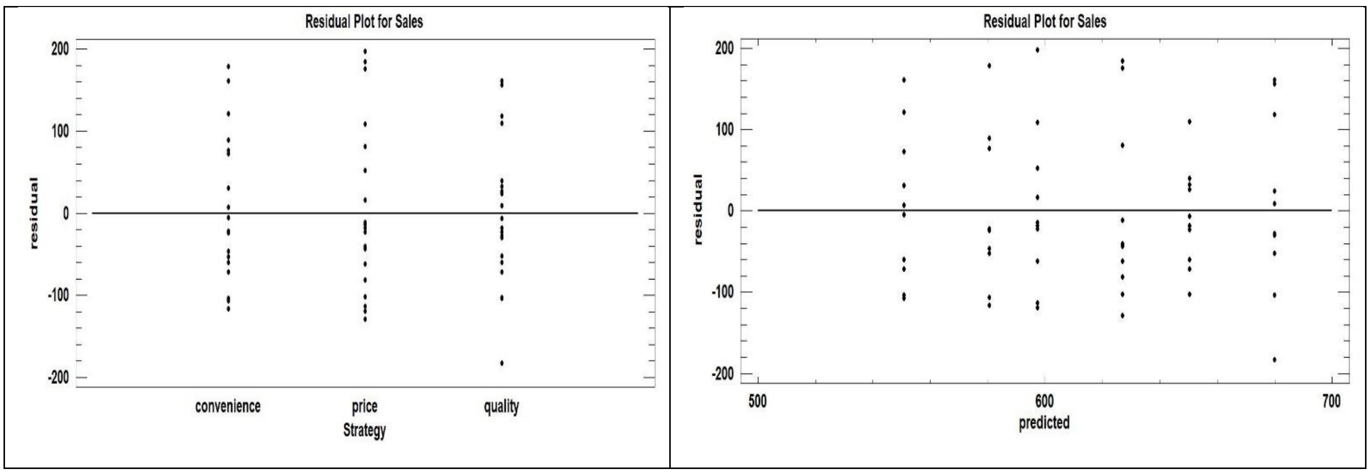
Analysis of Variance for Sales - Type III Sums of Squares

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*
MAIN EFFECTS					
A:Strategy	98838.6	2	49419.3	5.50	0.0066
B:Media	13172.0	1	13172.0	1.47	0.2309
RESIDUAL	502746.	56	8977.61		
TOTAL (CORRECTED)	614757.	59			

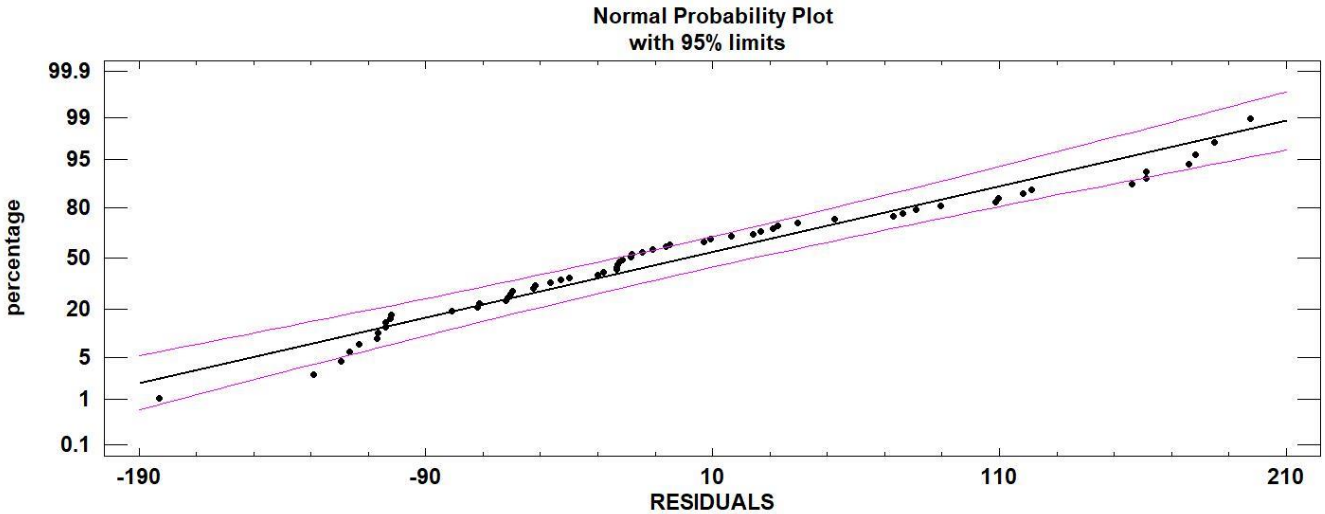


Notice that although there is no clear winner, we are unlikely to emphasize convenience in a marketing campaign.

Below are some of the graphs used to verify assumptions about the error. The residuals plotted against marketing strategy and against predicted unit sales exhibit no heteroscedasticity.



The normal probability plot of residuals shown below doesn't show a systematic departure from the fitted line, together with the accompanying standardized skewness and kurtosis values, won't make us reject the assumption that errors are normally distributed.



Stnd. skewness	1.63233
Stnd. kurtosis	-0.775092

The StatAdvisor: The standardized skewness and standardized kurtosis are within the range expected for data from a normal distribution.

Next, we move on to the case in two-way ANOVA where significant interactions exist between the factors.

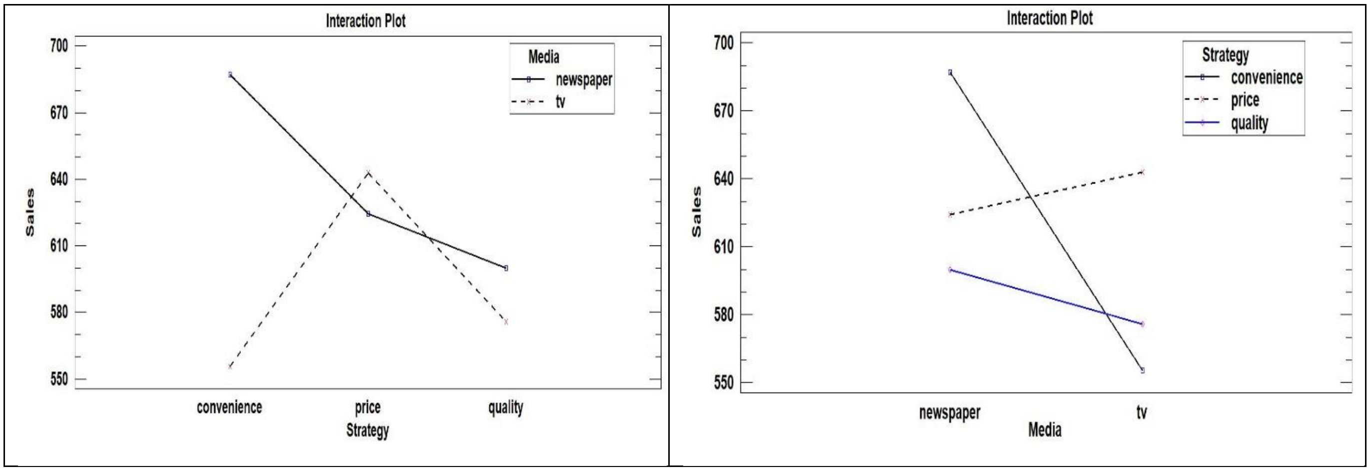
Example 2**:** This is just the apple juice problem revisited (see file *Apple Juice – Interaction*). By a judicious rearrangement of sales figures, I've created a marketing study in which interactions are significant. (See the two-way table below for the new assignments.) The comparison of the interaction plots for this example and example 1 should help to clarify the role of interactions in the interpretation of ANOVA output. The small *P*-value of 0**.**0474 for the hypothesis test of interactions implies that certain combinations of marketing strategy and media are important to sales.

	Convenience	Quality	Price
Local Television	City 1	City 2	City 3
Newspaper	City 4	City 5	City 6

Analysis of Variance for Sales - Type III Sums of Squares

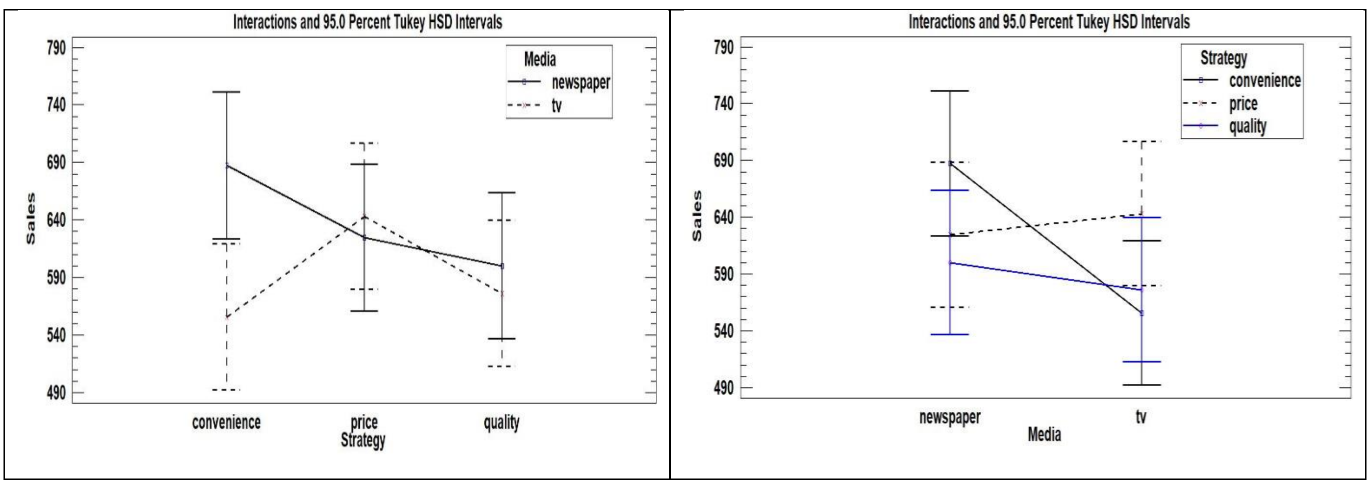
Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*
MAIN EFFECTS					
A:Strategy	22393.6	2	11196.8	1.21	0.3072
B:Media	31327.4	1	31327.4	3.38	0.0717
INTERACTIONS					
AB	59899.3	2	29949.6	3.23	0.0474
RESIDUAL	501137.	54	9280.31		
TOTAL (CORRECTED)	614757.	59			

Looking at the interaction plots, notice that emphasizing convenience lead to both the lowest and highest mean sales, depending upon whether local television or newspapers were used. Thus, it wouldn't make sense to talk about the effect of emphasizing convenience without consideration of the media used, i.e., we should only interpret levels of the two factors taken together (the combinations). Therefore, we will not investigate the means plots for Strategy and Media (although we will discuss them later). From the interaction plots, it appears that the most effective campaign would emphasize convenience in newspapers. The least effective combination is to emphasize convenience on local television. (Note: Since the interaction plot doesn't display confidence intervals for the six possible combinations, we cannot attach a particular significance level to our conclusions as we could with the means plots.)



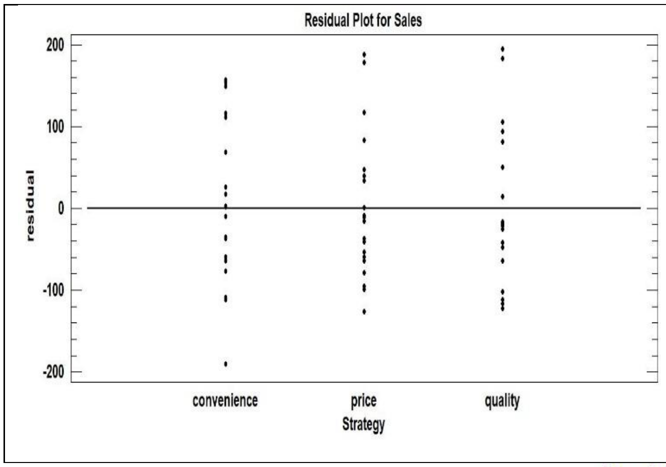
Both Interaction Plots exhibit crossing characteristic of Factor Interaction

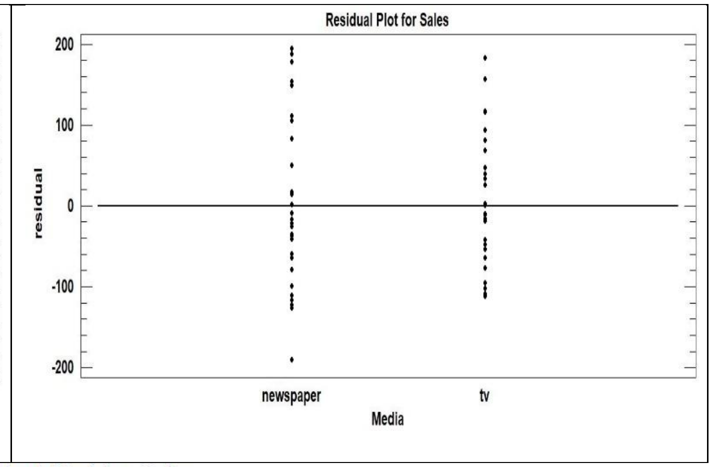
Below, I've added 95% Tukey HSD intervals to the interaction plots. You'll notice there is a lot of overlap in the intervals, making it difficult to attach a significance level to conclusions. An exception are the intervals for convenience conditioned on media, where newspapers are the clear winner. I used HSD intervals to control the significance level for all $r = 2 \times 3 = 6$ confidence intervals. I experimented with increasing α, but many intervals continued to overlap. This doesn't mean you can't make a decision based on the results, but you'll have to live with more uncertainty than you might like.

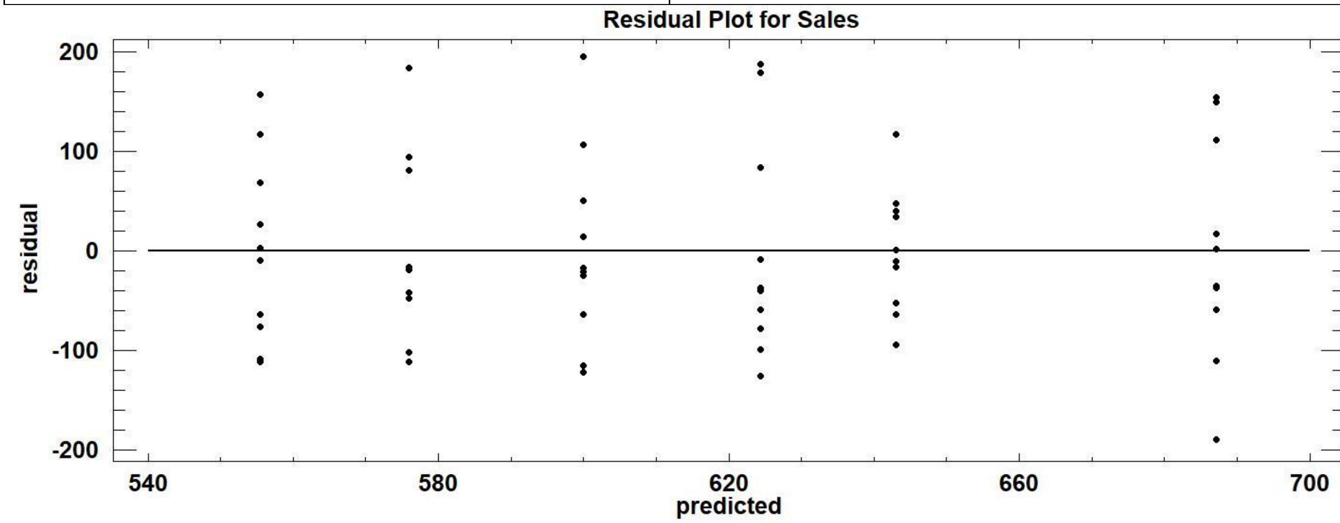


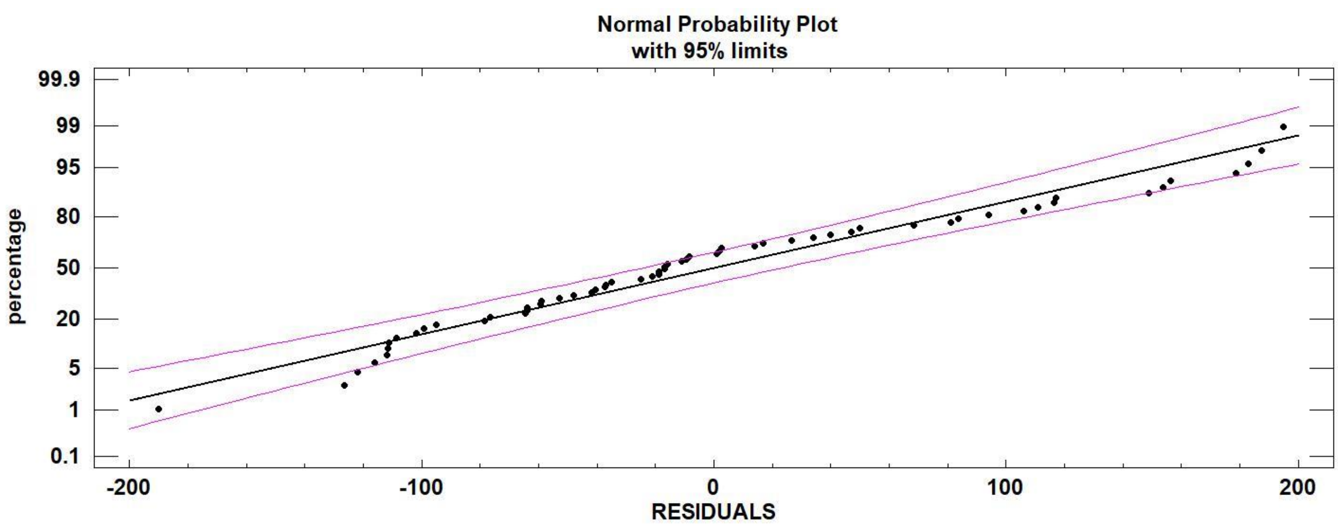
Few of the Differences are Significant in the 95% HSD Intervals

Speaking of confidence intervals, to have much confidence in them we should check assumptions. Below I've reproduced residual plots versus both factors and the predicted unit sales. All support the equal variance assumption. The normal probability plot and standardized skewness and kurtosis scores are reasonable for normally distributed error.





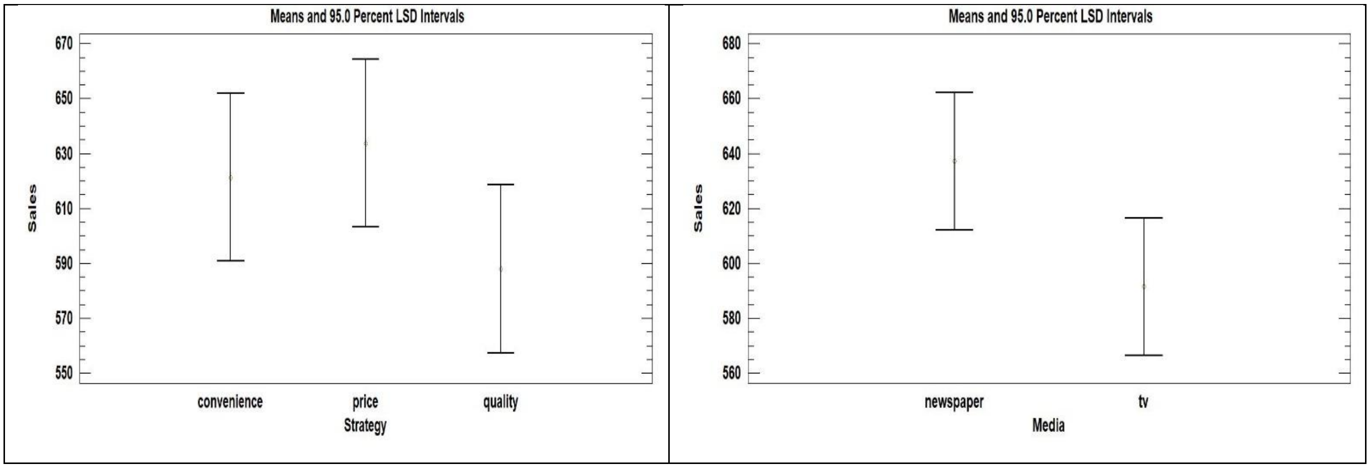




Stnd. skewness	1.53437
Stnd. kurtosis	-0.733494

The StatAdvisor: The standardized skewness and standardized kurtosis are within the range expected for data from a normal distribution.

Usually, there wouldn't be much follow-up with the means plots for the factors if interaction was deemed significant, but I've included them below. Notice that there are no clear winners at the 5% level, which is consistent with the *P*-values for the *Main Effects* in the ANOVA table.



A 5% level test would fail to reject the hypothesis of equal Factor Level means for either factor

The Two-Way **ANOVA** Table With Interaction

Notation and terminology for a two-way ANOVA with interaction is displayed in the table below.

- Factor A has a Levels, and Factor B has b Levels.
- A Treatment is a cross of one level of Factor A with another level of Factor B.
- In a Complete Factorial Design, there will be $a \times b$ treatments. In the coliform example, there are nine treatments (such as ocean-west and bay-east).
- In a Balanced Design, such as the apple juice marketing example, all treatment sample sizes are the same, n_s .
- The total number of observations in a balanced design is $n = n_s ab$.

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*
MAIN EFFECTS				
A: (Factor A)	*SSA*	*a* - 1	*SSA*/(*a* – 1)	*MSA*/*MSE*
B: (Factor B)	*SSB*	*b* - 1	*SSB*/(b – 1)	*MSB*/*MSE*
INTERACTIONS				
AB	*SSAB*	(*a* – 1)(*b* – 1)	*SSAB*/(*a* – 1)(*b* – 1)	*MSAB*/*MSE*
RESIDUAL (Error)	*SSE*	$ab(n_s - 1)$	$SSE/ab(n_s - 1)$	
TOTAL	*SST*	*n* - 1		

Below is the ANOVA table for the apple juice marketing example, with $a = 3$, $b = 3$, and $n_s = 2$. Note:

- *SSA* + *SSB* + SSAB + *SSE* = *SST*
- The degrees of freedom for the Main Effects, Interactions, and the Error sum to the total degrees of freedom.

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INTERACTIONS					
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RESIDUAL	501137.	54	9280.31		
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Finally, a comparison of the ANOVA table above for the interaction model to the table below with interactions removed reveals the following:

- *SSA*, *SSB*, and *SST* are the same for both models, as are their degrees of freedom.
- The error sum of squares in the Main Effects model decomposes into interaction and error sum of squares in the Interaction model. The degrees of freedom for error also decomposes when interactions are added to the model.
- The *F-Ratios* for the main effects change when interactions are added because they are based on a different mean square error.

Analysis of Variance for Sales - Type III Sums of Squares

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*
MAIN EFFECTS					
A:Strategy	22393.6	2	11196.8	1.12	0.3342
B:Media	31327.4	1	31327.4	3.13	0.0825
RESIDUAL	561036.	56	10018.5		
TOTAL (CORRECTED)	614757.	59			