Part 1

Main constraints

- each client is visited (served) once
- all providers start at 14HS (depot) and return to it
- providers must return to 14HS by hour l.
- service times of clients must be respected
  - → provider can arrive earlier, or at, but no later than the start time of a client's service.

Data

- l : end of work (all providers must be back to 14HS by hour l)
- $N = \{2, \dots, N+1\}$ clients (N of them)
  - s_i : service start time
  - d_i : service duration
- $M = \{1, \dots, M\}$ providers (M of them)
  - f_i : hiring cost (one-off)
  - w_i : earliest available starting hour.
- $N' = \{1, 2, \dots, N, N+1\}$ = all locations

  (1 ↑ 14HS; $2, \dots, N$ underbrace: N clients)
- t_{ij} : travel time between location $i, j \in N'$.

Variables

- $x_{ijk} = 1$ if provider k moves from location i to j, $i, j \in N'$.

Objective (1)

$$\min \sum_{k \in M} \sum_{j \in N} f_k \cdot x_{1jk}$$

($\sum$ hiring cost × provider leaves 14HS)

Constraints — routing constraints

(2)
$$\sum_{i \in N'} \sum_{k \in M} x_{ijk} = 1 \quad , j \in N$$
(clients are served once, by one vehicle)

(3)
$$\sum_{i \in N'} x_{ipk} = \sum_{j \in N'} x_{pjk} \quad , k \in M, p \in N$$
(route continuity)

(4)
$$\sum_{j \in N} x_{1jk} \leq 1, \quad k \in M$$
(not all providers need to work)

(5)
$$\sum_{i \in N} x_{i1k} \leq 1, \quad k \in M$$

(6)
$$x_{iik} = 0 \quad , i \in N', k \in M$$
(no self-loop)

(7)
$$\sum_{j \in N} \sum_{k \in M} x_{1jk} \geq 1$$
(at least 1 provider must work)

→ (8)
$$\sum_{i \in N} \sum_{j \in N} x_{ijk} \leq L \cdot \sum_{j \in N} x_{1jk} \quad , k \in M$$
(all providers must start at 14HS)

(9)
$$\sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - 1 \quad , S \subseteq N, k \in M$$
(subtour elimination for each provider)

time constraints

(10) $x_{ijK} \cdot (w_K + t_{ij}) \leq s_j$, $K \in M, j \in N$ (provider must not arrive later than start time of first client)

(11) $x_{ijK} \cdot (s_i + t_{ij} + d_i) \leq s_j$, $i \in N, j \in N, K \in M$ (service times respected)

(12) $x_{i1K} (s_i + t_{i1} + d_i) \leq l$, $i \in N, K \in M$ (come back to HHs by hour l)

(13) $x_{ijK} \in \{0,1\}$, $i,j \in N', K \in M$ (integrality constraints)

→ Example 1: (5 providers, 6 clients)

Providers	Routes
1	$1 \to 2 \to 1$
2	$1 \to 7 \to 1$
3	$1 \to 6 \to 1$
4	$1 \to 3 \to 4 \to 5 \to 1$

$z^* = 45$. $(= \sum_{i=1}^{4} f_i)$