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Home health care problem

An extended multiple Traveling Salesman Problem

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Abstract This paper deals with the routing problem of health care staff in a home health care problem. Given a list of patients needing several cares, the problem is to assign cares to care workers. Some cares have to be performed by several persons and some cares cannot be performed with others. If a patient needs several cares, he may want to be treated by the same person. Moreover, some skills constraints and time windows have to be satisfied. We show that this problem is equivalent to a multiple traveling salesman problem with time windows (mTSPWT) with some specific constraints. For solving this problem, we propose an integer linear program with some technical improvements.

1 Introduction

The Home Health Care (HHC) demand is growing rapidly in France as well as in several other countries [19]. It includes services such as nursing, physical therapy, speech therapy, medical social services, medical visits, house cleaning, home life aides, old people assistance, etc. This development is mainly due to economic factors [1], congestion of hospitals, preferences of the patients [21], ageing of the population [2]. The HHC aims to reducing the number of patients in hospitals, to improving the quality of care, and minimizing the costs of health [17]. Many papers deal with the HHC and its advantages (see [10] and [23]).

A new problem of organization appears: how is it possible to coordinate care workers while ensuring good quality of care. In this paper, we are only interested in the routing problem of health care staff, subject to some coordination problems in the residences of patients. Some cares may need simultaneously several care workers, for example a nurse and a doctor, or two care assistants to help to get up an impotent person. Conversely, some care workers cannot work simultaneously, for example a nurse and a physical therapist. It is better if patients always see the same care workers (same nurse,

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same doctor, etc.) as long as possible. We show that this problem is equivalent to a multiple traveling salesman problem with time windows (mTSPWT) and some special constraints. We propose an integer linear programming formulation (ILP) with some cuts for solving it.

2 Literature review

Some articles related to home health care workers routing problem already exist in the literature. In [5], the authors study the scheduling and routing of home health care nursing in Alabama and develop a spatial decision support system. They build a heuristic to construct the routes of each nurse, taking unavailabilities constraints into account. In a paper of V. De Angelis [9], the main problem is not a problem of routing but of allocating resources (doctors, nurses, social assistance and caring for patients) within a given budget, in order to maximize the number of patients delivered. To solve this problem, the author formulates a linear program model. Another linear integer model is developed in [7]. This model aims to solve the human resource short term planning in order to satisfy each patient on time. T. Fahle, in [13], studied the problem of vehicle routing and staff rostering aspects in home health care. He described a model data of generic home health care problem software, and presented a method to solve this problem. In our study, we are not interested in the rostering problem. In [8], the problem of routing home health care is also studied by considering two types of nursing, part time and full time, with a different cost per hour. They define a mixed integer program taking into account the lunch pauses and they present a basic heuristic with the objective of minimizing the total cost. A similar problem is tackled in [6] with nurses having different skills. Here, the objective is not only to minimize the total cost but a weighted sum of the total travel time plus a sum of several penalties like the violation of time windows or of patients preferences. The heuristic developed by the authors for solving this problem is divided into two parts: (1) build a set of patients to serve for each nurse and (2) find an optimal sequencing for each set of patients. A very similar problem is studied in [11] in a Swedish environment, but the objective is to minimize the travel time and the waiting time of patients. The authors solve this problem using a set partitioning model with two types of variables (some for assigning a staff member to a schedule, some for assigning a staff member to a visit with a vehicle). A matching approach is used iteratively for finding a solution. They described also the development of a decision support system called LAPS CARE to eliminate the manual planning of home care unit assignments. In another paper[12], the same authors presented and discussed some results and experiences from two local government organizations and from the Laps Care. They conclude on an improvement of operational efficiency and of the quality of home care for elderly citizens. In [3], the same problem is solved by a Particle Swarm Optimization algorithm (population-based evolutionary algorithm with a local search algorithm). The first algorithm explores the solution space globally whereas the local search explores the neighborhood of a solution, using improvement procedures like swap or insertion. Finally, one of the most recent articles dealing with the problem of routing home health care people is [4]. The authors take the most possible constraints into account but they consider only one type of care workers and do not take cares exclusions into account. They formulate an integer linear model and compare two commercial solvers (Cplex and Lingo). However, the computation time is quite important even on small instances.

3 Problem presentation

We are interested in the routing problem of the care staff. Several doctors, nurses, physical therapists, care assistants, housekeepers, etc., are available to treat some patients at home, but under restrictions on working hours or locations. The patients to be treated are dispersed geographically and require possibly several cares. A care is characterized by a duration, an earliest and a latest starting times and one needed category of care workers (a nurse or a doctor or etc.) but a care can also be pre-assigned to a specific person, for example because he is the family doctor or the nurse of the patient. A care can also require several care workers. In this case, we consider that there are as many cares as the number of care workers needed and that these cares must begin at the same time or with known delays. Conversely, other cares can be in disjunction: they cannot take place simultaneously. For example, a speech therapist and a doctor cannot work together, either the doctor comes first or it is the speech therapist. Some cares have to be performed by the same person (not necessary pre-assigned), for example if a nurse has to come at the morning and at the afternoon, the patient may prefer that the same nurse makes the two cares, whoever is the nurse.

We consider a set $M = \{1, ..., m\}$ of care workers, c_k the skill of person k (doctor, nurse, etc.). We have to assign to each person an ordered list of cares to perform. Each person k must leave and return to its domicile (or office) D_k, with D the set of domiciles. A person works only during its shift noted by a time window $[e_{D_k}, l_{D_k}]$ which respectively represents the earliest date and latest date of work of person k on its domicile D_k. In addition, a person k may require one or more breaks during his service. We will see that this break may be seen like a fictitious pre-assigned care.

There is a set of patients to treat at home possibly several times by perhaps several people. We define a service by only one care to one patient performed by one person. A patient requiring a treatment involving three care workers will be represented by three synchronized services. We define a set $S = \{1, ..., n\}$ of services. A service i is characterized by a required skill q_i, a time window $[e_i, l_i]$ during which the service to perform must begin and a duration of treatment p_i.The travel time between two locations of services i and j is denoted by $d_{i,j}$. We note $\Omega = \{\omega_1, ..., \omega_r\}$ the set of treatments requiring more than one care worker. Each element ω_t is in fact a set of synchronized services, the synchronization between two services i and j is given by a delay $\delta_{i,j}$: service i must begin $\delta_{i,j}$ time units after service j. We note $\Phi = \{\phi_1 ... \phi_o\}$ the set of set of services which must not be performed simultaneously, and $\Gamma = \{\gamma_1 ... \gamma_s\}$ the set of set of services which must be performed by the same person. If a service i is pre-assigned we note π_i the number of the pre-assigned people, if i is not pre-affected, π_i is set to 0. We can therefore represent a pause by a pre-assigned service to the person concerned.

This problem is clearly similar to a traveling salesman problem with time windows and some special constraints. The TSP [14] has been often studied in the literature. This type of problem with a fixed number of traveling salesman is called m-TSP. It is a generalization of the classical TSP. In [22], the author enumerates the possible extensions of this problem (time windows, fixed number of traveling salesmen, maximum or minimum number of vertices to visit, etc.), its practical applications, some formulations in linear programming and some heuristic or exact methods to solve these problems. Considering the time windows version, check if an instance of m-TSP with time windows is feasible is NP-complete [18]. One of the first ILP formulation for TSP was made by [16]. This formulation has been extended to several traveling salesmen by

[20], the authors introduce also a branch-and-bound algorithm to solve the m-salesman traveling problem by using linear relaxations. In [15], the authors propose extending these linear models by integrating a minimum and maximum number of vertices to visit for each salesman in the case of multiple depots.

The HHC problem is thus equivalent to a m-TSP with some special constraints on a complete directed graph noted $G = \{V; A\}$ with: $V = S \cup D$ the set of vertices (one vertex for each service and for each depot), and A the set of arcs, the length of an arc between vertices i and j being the sum of the travel time $d_{i,j}$ between i and j the duration p_i of service i, if $i \in S$. Finally, we note $cost_{i,j}^{k}$ the cost of passage of the person on the arc (i, j). The goal is to find the routes of each person such as each vertex is visited by exactly one person and the constraints are satisfied. The objective is to minimize the total cost of the routes followed by each care worker.

4 ILP

4.1 Linear programming model

We propose now an exact resolution of this problem by using an integer linear programming formulation (ILP).

This formulation uses the three following types of variables:

$$\forall i, j \in V \ (i \neq j), \forall k \in M : \quad x_{i,j}^{k} = \begin{cases} 1 \text{ if person } k \text{ performs service } i \text{ and then } j \\ 0 \text{ otherwise} \end{cases}$$

$$\forall \phi_h \in \Phi, \forall i, j \in \phi_h \ (i \neq j) : \quad y_{i,j} = \begin{cases} 1 \text{ if service } i \text{ is performed before } j \\ 0 \text{ otherwise} \end{cases}$$

$$\forall i \in V : \quad t_i \in [e_i, l_i] : \text{starting date of service } i$$

The first type of variables is used to define the routes of care workers. Thanks to the time windows we can set some variables to the value 0 (if $e_i + p_i + d_{i,j} > l_j$ then $\forall k \in M : \ x_{i,j}^{k} = 0$). The second type is used to ensure the disjunctions between services which cannot be performed at the same time. The last type of variables is used for the satisfaction of all time windows.

The problem can be defined by 11 sets of constraints:

$$\forall j \in S: \quad \sum_{\forall k \in M} \sum_{\forall i \in V \setminus j} x^k_{i,j} = 1 \tag{1}$$

$$\forall k \in M, \forall i \in V: \quad \sum_{\forall j \in V \setminus i} x^k_{j,i} = \sum_{\forall j \in V \setminus i} x^k_{i,j} \tag{2}$$

$$\forall k \in M: \quad \sum_{\forall i \in V \setminus D_k} x^k_{D_k,i} \leq 1 \tag{3}$$

$$\forall k \in M, \forall i \in S: \quad x^k_{D_k,i}(e_{D_k} + d_{D_k,i}) \leq t_i \tag{4}$$

$$\forall i,j \in S \ i \neq j: \quad t_i + d_{i,j} + p_i \leq t_j + HV(1 - \sum_{\forall k \in M} x^k_{i,j}) \tag{5}$$

$$\forall k \in M, \forall i \in S: \quad t_i + d_{i,D_k} + p_i \leq x^k_{i,D_k}(l_{D_k} - L) + L \tag{6}$$

$$\forall \omega_h \in \Omega, \forall i,j \in \omega_h \ (i \neq j): \quad t_i = t_j + \delta_{i,j} \tag{7}$$

$$\forall \phi_h \in \Phi, \forall i,j \in \phi_h \ (i \neq j): \quad y_{i,j} = 1 - y_{j,i} \tag{8}$$

$$\forall \phi_h \in \Phi, \forall i,j \in \phi_h \ (i \neq j): \quad t_i + p_i \leq t_j + HV(1 - y_{i,j}) \tag{9}$$

$$\forall \gamma_h \in \Gamma, \forall i,j \in \gamma_h \ (i \neq j), \forall k \in M: \quad \sum_{\forall l \in V \setminus i} x^k_{l,i} = \sum_{\forall l \in V \setminus j} x^k_{l,j} \tag{10}$$

$$\forall i \in S/\pi_i \neq 0: \quad \sum_{\forall l \in V \setminus i} x^{\pi_i}_{l,i} = 1 \tag{11}$$

Constraints (1) impose that each service is performed by exactly one care worker. Constraints (2) ensure the continuity of routes: when a person enters in a vertex, he has to exit. Constraints (3) impose that each care worker will exit from its domicile only once (remarks: $\forall k, k' \in M, k \neq k', \forall i \in V: \ x^{k'}_{D_k,i} = x^{k'}_{i,D_k} = 0$). Constraints (4) define the earliest starting time of work of the care worker k if he performs service i. Constraints (5) formulate the travel times between two vertices (HV is a notation to indicate a value presumably infinite). The constraints (6) impose to each care worker k to come back to its domicile before its date of end of work (L is a constant value equal to $max_{k \in M}(l_{D_k})$, the greatest date of end of work). Constraints (7) link the synchronized services, considering the time lags $\delta_{i,j}$. Constraints (8) and (9) deal with the services that are in disjunction. Constraints (8) impose that service i is performed before service j, or the contrary. If service i is performed before j then the constraints (9) ensure that the treatment of j does not begin before the end of treatment of i. However, it is unnecessary to add this constraint if $l_i + p_i < e_j$ or $l_j + p_j < e_i$. Constraints (10)) express that some services must be performed by the same care worker. Constraints (11) assign all care workers. The competence of the services he performed is ensured by the following pre-processing:

$$if \ q_i \neq c_k \ then \ \forall j \in V, (i \neq j): \ x^k_{i,j} = x^k_{j,i} = 0$$

The objective function of the ILP is to minimize the total travelling cost:

$$\min \sum_{\forall k \in M} \sum_{\forall i \in V} \sum_{\forall j \in V \setminus i} cost^k_{i,j} x^k_{i,j} \tag{12}$$

4.2 Technical improvements

In order to improve the resolution time, many techniques have been tested. After several experiments, the following techniques are used.

First, some cuts can be added according to the time windows. Two services with two overlapping time windows require two different care providers. This type of cut can be formulated as follows:

$$
\begin{aligned}
&\forall i, j \in S \; i \neq j : \\
&\qquad if \; e_i + p_i + d_{i,j} > l_j \; and \; e_j + p_j + d_{j,i} > l_i \; then \; \forall k \in M : \\
&\qquad \sum_{\forall l \in V \setminus i} x_{l,i}^k + \sum_{\forall l \in V \setminus j} x_{l,j}^k \leq 1
\end{aligned}
$$

In the same way, the cares which require several people are split into several services that cannot be assigned to the same person. We present here the cuts concerning the care requiring 2 and 3 people, but it can be easily extended to more people.

$$
\begin{aligned}
&\forall \omega_h \in \Omega : \\
&\qquad if \; \|\omega_h\| = 2 \; then \; \forall i, j \in \omega_h \; (i \neq j), \; \forall k \in M : \\
&\qquad \sum_{\forall l \in V \setminus i} x_{l,i}^k + \sum_{\forall l \in V \setminus j} x_{l,j}^k \leq 1 \\
&\qquad if \; \|\omega_h\| = 3 \; then \; \forall i, j, o \in \omega_h \; (i \neq j \neq o), \; \forall k \in M : \\
&\qquad \sum_{\forall l \in V \setminus i} x_{l,i}^k + \sum_{\forall l \in V \setminus j} x_{l,j}^k + \sum_{\forall l \in V \setminus o} x_{l,o}^k \leq 1
\end{aligned}
$$

Finally, to avoid the exploration of identical solutions, we added a term in the objective function. Indeed, each route of two identical care workers (same domicile, same skill, same work time and same cost) can be exchanged without changing the value of the objective function. As there are often identical care workers in real instances, many solutions with the same cost are explored. In order to improve the exploration significantly, we define a new objective function with two criteria to optimize in a lexicographical way. The first criterion is the previously defined objective function. The second criterion is a sort of corrective term that helps avoiding identical solutions, it tends to assign the longest routes to care workers with the smallest numbers. We thus define the new following objective function.

$$
min \; \alpha \times \sum_{\forall k \in M} \sum_{\forall i \in V} \sum_{\forall j \in V \setminus i} cost_{i,j}^k x_{i,j}^k + \sum_{\forall k \in M} k \sum_{\forall i \in V} \sum_{\forall j \in V \setminus i} d_{i,j} x_{i,j}^k
$$

The term α is supposed to be greater than the maximal value of the corrective term, so that the lexicographic optimization is guaranteed.

5 Computational experiments

The ILP has been tested on random generated instances with the solver Cplex. We have considered three types of instances with 1, 2 and 3 skills and with 10, 20, 30 and 40 services. For each type of instance, we have generated 200 difficult instances. Duration of cares belongs to the interval [10 minutes, 1 hour]. The length of time windows is between half an hour and three hours. On average 20% of services are synchronized ones, 20% of services are in exclusion, and 20% of services must be perform by the same care worker. The number of care workers is calculated such that they are occupied on average 70% of the day to obtain difficult instances and sometimes infeasible. All care workers have the same associated cost ($cost^k_{i,j} = d_{i,j}$). The various locations are randomly generated on a 100×100 square and the travel time $d_{i,j}$ is set to the Euclidean distance between these two locations. The experiments have been conducted on a Pentium IV at 2.8 GHz with 512 Mo of RAM. The time of resolution was limited to 10 minutes only. Table 1 presents the average time to solve an instance, the standard deviation of time resolution, the percentage of instances solved for each type of instance, and the GAP of Cplex for unsolved instances. The Gap is defined as $(f(ILP) - f(LP))/f(LP)$ where f(ILP) is the best integer solution found by Cplex and f(LP) is the optimal linear solution. We compare in this table two ILP: with and without technical improvements. We achieved these results after trying to set the parameters of CPLEX. The resolution settings of CPLEX were not changed except the level of probing at the highest level.

Table 1 Computational experiments

Number of services	Number of skills	Without Technical improvements				With Technical improvements			
		CPU(s)	σ	τ(%)	GAP(%)	CPU (s)	σ	τ(%)	GAP(%)
10	3	0,03	0,02	100,0	-	0,03	0,02	100,0	-
10	2	0,06	0,07	100,0	-	0,05	0,06	100,0	-
10	1	0,45	0,81	100,0	-	0,42	0,58	100,0	-
20	3	1,14	3,9	100,0	-	1,65	7,14	100,0	-
20	2	10,1	55,2	100,0	-	8,8	40,5	100,0	-
20	1	220,6	253,6	58	18,2	181,7	229,2	58	16,1
30	3	41,0	106,1	90,0	9,1	46,8	132,3	89,5	9,7
30	2	144,1	200,1	63,5	9,8	136,2	184,6	66,5	10,0
40	3	167,9	229,1	52,0	7,0	107,4	142,8	57,0	5,8

The number of skills is strongly connected to the time of resolution because the number of possible solution increases when the number of skills decreases. Lastly, the time of resolution, as well as the standard deviation, is decreased thanks to the technical improvements but also the number of unsolved instances and the GAP.

6 Conclusion

We have presented a routing problem in a very general context of Home Health Care. It involves care workers with various skills and patients with various needs. The patients must be visited according to times windows. This problem can be assimilated to a multi-traveling salesman problem with time windows in which some cities must be visited by the same traveler or by a specific traveler. Some cities cannot be visited by the same traveler and some cities must be visited simultaneously or with a known delay. The proposed integer linear program is not able to deal with instances of real size but it appears in the tests that it gives rapidly a good solution which is promising to apply technique like column generation.

References

1. Afrite A., Com-Ruelle L., Or Z., Renaud T.: L'hospitalisation à domicile, une alternative économique pour les soins de suite et de réadaptation. Question d'économie de la santé, n119, (2007).
2. Ahlner-Elmqvist M., Jordhoy M.S., Bjordal K., Jannert M., Kaasa S.: Characteristics and Quality of Life of Patients Who Choose Home Care at the End of Life. Journal of Pain and Symptom Management, Volume 36, Issue 3, (2008).
3. Akjiratikarl C., Yenradee P., Drake P.R.: PSO-based algorithm for home care worker scheduling in the UK. Computers & Industrial Engineering 53, 559-583, (2007).
4. Ben Bachouch R., Fakhfakh M., Guinet A., Hajri-Gabouj S.: Planification de la tournée des infirmiers dans une structure de soins à domicile. Gestion et Ingénierie des SystèmEs Hospitaliers, 4ème conférence GISEH EPFL-Lausanne, (2008).
5. Begur S.V., Miller D.M., Weaver J.R.: An integrated spatial Decision Support System for scheduling and routing home health care nurses. Institute of operations research and the management science, 35-48, (1997).
6. Bertels S., Fahle T.: A hybrid setup for a hybrid scenario: combining heuristics for the home health care problem Computers & Operations Research 33, 2866-2890, (2006).
7. Borsani V., Matta A., Beschi G., Sommaruga F.: A home care scheduling model for human resources. Service Systems and Service Management, International Conference, 449-454, (2006).
8. Cheng E., Rich J. L.: A home care routing and scheduling problem. Technical Report TR98-04, Department of Computational And Applied Mathematics, Rice University (1998).
9. De Angelis V.: Planning Home Assistance for AIDS Patients in the City of Rome, Italy, Institute of operations research and the management science 28, 75-83 (1998).
10. Duke M.: Home Care. International Encyclopedia of Public Health. Academic Press, Oxford, 455-460, (2008).
11. Eveborn P., Flisberg P., Ronnqvist M.: LAPS CARE-an operational system for staff planning of home care. European Journal of Operational Research 171, 962-976, (2006).
12. Eveborn P., Ronnqvist M., Einarsdottir H., Eklund M., Liden K., Almroth M.: Operations Research Improves Quality and Efficiency in Home Care. Interfaces, Vol. 39, No. 1, 18-34, (2009).
13. Torsten Fahle: Production and Transportation Planning Modeling Report. Report, (2001).
14. Gutin G., Abraham P.: The Traveling Salesman Problem and Its Variation. Punne, Springer Verla, (2002).
15. Kara I., Tolga B.: Integer linear programming formulations of multiple salesman problems and its variations. European Journal of Operational Research, 174, 1449-1458, (2006).
16. Miller C.E., Tucker A.W., Zemlin R.A.: Integer programming formulation of travelling salesman problems. J. Assoc. Comput. Mach., 7, 326-329, (1960).
17. Raffy N.: L'hospitalisation à domicile : une alternative économique = Home care : an economic alternative. Concours médical ISSN 0010-5309 Coden Comeao. Vol. 116, n13, (1994).
18. Savelsbergh M.W.P.: Local search in routing problems with time windows. Ann. Oper. Res., 4, 285-305, (1985).
19. Shepperd S. and Iliffe S.: Hospital at home versus inpatient hospital care. The Cochrane Database of Systematic Reviews (2005).
20. Svetska J.A., Huckfeldt V.E.: Computational experience with an m-salesman travelling salesman algorithm. Management Sci., 19, 790-799, (1973).
21. Sentilhes-Monkam A.: L'hospitalisation à domicile et la prise en charge de la fin de vie : le point de vue des patients et de leurs proches. Santé publique - 3 (Vol. 18), ISSN 0995-3914, (2006).
22. Tolga B.: The multiple traveling salesman problem: an overview of formulations and solution procedures. Omega, 34, 209-219, (2006).
23. Van den Berg B., Hassink W.H.J.: Cash benefits in long-term home care. Health PolicyIn Press, (2008).