01 19 sim check real

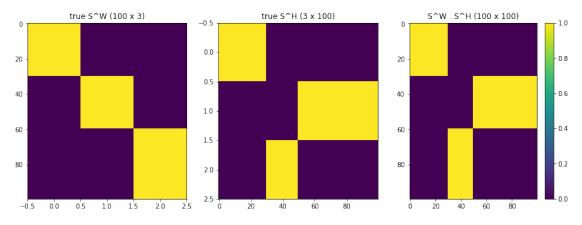
January 21, 2021

```
[1]: import dssvi_fn
     import time
     import numpy as np
     import matplotlib.pyplot as plt
     import random
     import pandas as pd
[2]: true_pi_W = [0.3, 0.3, 0.4]
     true_pi_H = [0.3, 0.5, 0.2]
[3]: def number K(obj):
         len_good_k = [len(obj.good_k_list[i]) for i in range(len(obj.good_k_list))]
         plt.figure()
         plt.plot(len_good_k)
         plt.xticks(np.arange(0, 19+1, 1.0))
         plt.title("number of K's across iterations")
         plt.show()
     def log_likelihoods(obj, burn_in, max_iter):
         log_ll_list = [obj.log_ll[i,burn_in] for i in range(max_iter)]
         plt.figure()
         plt.plot(log_ll_list)
         plt.xticks(np.arange(0, len(log_ll_list),1))
         plt.title("log 11")
         plt.show()
     def K_and_log_ll(obj, burn_in, max_iter):
         len_good k = [len(obj.good k_list[i]) for i in range(len(obj.good k_list))]
         log_ll_list = [obj.log_ll[i,burn_in] for i in range(max_iter)]
         fig, ax = plt.subplots(1, 1)
         ax.plot(len_good_k, 'r', label="number of K")
         ax.set_yticks(np.arange(0, max(len_good_k)+1))
         ax.set_xticks(np.arange(0, len(log_ll_list), 1.0))
         ax2 = ax.twinx()
```

```
ax2.plot(log_ll_list, label="log likelihood")
         ax2.ticklabel_format(useOffset=False, style='plain')
         plt.draw()
         fig.legend()
         print(log_ll_list)
         return
     def plot_pi(obj, true_pi_W=true_pi_W, true_pi_H=true_pi_H, label=True):
         fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(10, 5), sharey=True)
         labels = np.arange(0, obj.n_components,1)
         for i in range(obj.n_components):
                 axes[0].plot(obj.Epi_W_list[:,i], label="pi_" + str(labels[i]))
         axes[0].title.set_text("Value of Pi_W's across iterations")
         for j in range(len(true_pi_W)):
             axes[0].axhline(y=true_pi_W[j], color='grey', linestyle='dashed')
         if label:
             axes[0].legend()
         for i in range(obj.n_components):
             axes[1].plot(obj.Epi_H_list[:,i], label="pi_" + str(labels[i]))
         axes[1].title.set_text("Value of Pi_H's across iterations")
         for j in range(len(true_pi_H)):
             axes[1].axhline(y=true_pi_H[j], color='grey', linestyle='dashed')
         if label:
             axes[1].legend()
         return
[4]: S W = np.vstack((np.hstack((np.ones(30), np.zeros(70))),
                     np.hstack((np.zeros(30), np.ones(30), np.zeros(40))),
                      np.hstack((np.zeros(60), np.ones(40))))).T
     S_H = np.vstack((np.hstack((np.ones(30), np.zeros(70))),
                      np.hstack((np.zeros(50), np.ones(50))),
                     np.hstack((np.zeros(30), np.ones(20), np.zeros(50)))))
[5]: print("true pi_W:", true_pi_W)
     print("true pi_H:", true_pi_H)
    true pi_W: [0.3, 0.3, 0.4]
    true pi_H: [0.3, 0.5, 0.2]
[6]: fig, axes = plt.subplots(ncols=3, figsize=(15,5))
     ax1, ax2, ax3 = axes
```

```
im1, im2, im3 = ax1.imshow(S_W, aspect="auto", interpolation='none'), ax2.

→imshow(S_H, aspect="auto", interpolation='none')\
, ax3.imshow(S_W.dot(S_H), aspect="auto", interpolation="none")
ax1.title.set_text('true S^W (100 x 3)')
ax2.title.set_text('true S^H (3 x 100)')
ax3.title.set_text('S^W . S^H (100 x 100)')
plt.colorbar(im3, ax=ax3)
plt.show()
```



```
[7]: np.random.seed(10)

cluster1 = np.random.gamma(20, 1, 50)
cluster2 = np.random.gamma(10, 1, 50)
cluster3 = np.random.gamma(5, 1, 50)
```

```
[8]: W_S_W = np.zeros((S_W.shape))
W_S_W[:30, 0] = S_W[:30, 0] * cluster1[:30]
W_S_W[30:60, 1] = S_W[30:60, 1] * cluster2[:30]
W_S_W[60:, 2] = S_W[60:, 2] * cluster3[:40]
```

```
[9]: H_S_H = np.zeros((S_H.shape))

H_S_H[0, :30] = S_H[0, :30] * cluster1[:30]

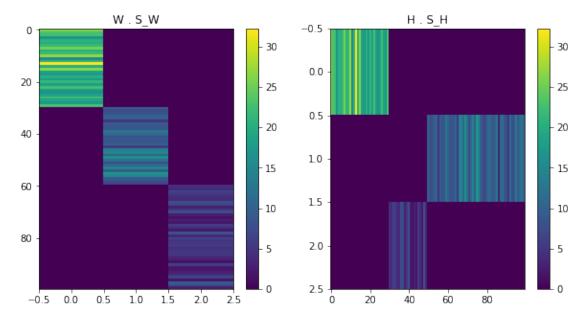
H_S_H[1, 50:] = S_H[1, 50:] * cluster2[:50]

H_S_H[2, 30:50] = S_H[2, 30:50] * cluster3[:20]
```

```
fig, axes = plt.subplots(ncols=2, figsize=(10,5))
ax1, ax2 = axes
im1, im2 = ax1.imshow(W_S_W, aspect="auto", interpolation='none'), ax2.

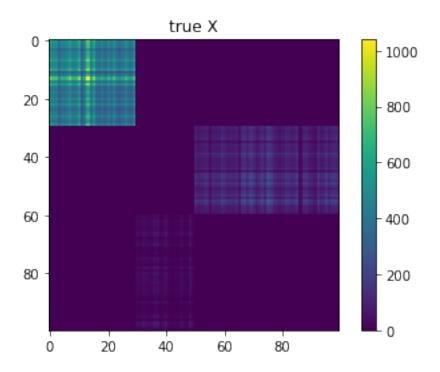
→imshow(H_S_H, aspect="auto", interpolation='none')
ax1.title.set_text('W . S_W')
```

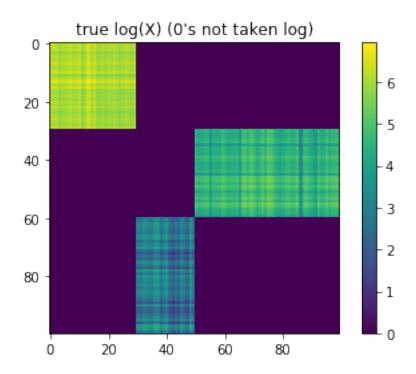
```
ax2.title.set_text('H . S_H')
plt.colorbar(im1, ax=ax1)
plt.colorbar(im2, ax=ax2)
plt.show()
```



```
[11]: X = np.dot(W_S_W, H_S_H)
X = pd.DataFrame.from_records(X)

[12]: plt.imshow(X, interpolation='none')
plt.colorbar()
plt.title("true X")
plt.show()
```





0.1 Run 1: K=3, burn=30, max_iter=20

```
[14]: dssvi_run = dssvi_fn.SSMF_BP_NMF(n_components=3, burn_in=30, random_state=10, u 

→verbose=True, max_iter=20, 

a=1, b=1, c=5, d=5, a0_W=1, b0_W=1, a0_H=1, u 

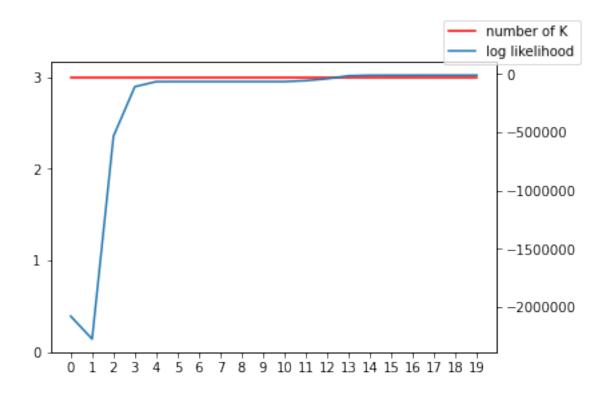
→b0_H=1)
```

```
[15]: start_t = time.time()
dssvi_run.fit(X)
t = time.time() - start_t
```

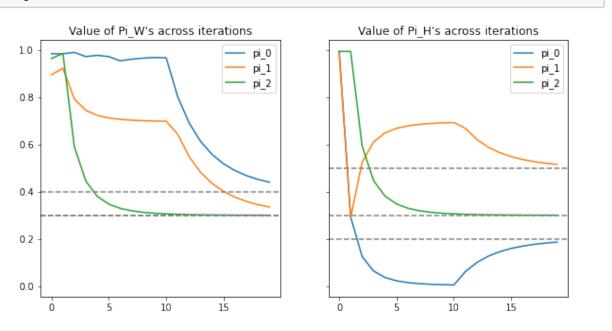
```
SSMF-A iteration 0
                         good K:3:
        Gibbs burn-in: 30
SSMF-A iteration 1
                         good K:3:
        Gibbs burn-in: 30
SSMF-A iteration 2
                         good K:3:
        Gibbs burn-in: 30
SSMF-A iteration 3
                         good K:3:
        Gibbs burn-in: 30
SSMF-A iteration 4
                         good K:3:
        Gibbs burn-in: 30
SSMF-A iteration 5
                         good K:3:
        Gibbs burn-in: 30
SSMF-A iteration 6
                        good K:3:
```

```
good K:3:
              SSMF-A iteration 7
                                    Gibbs burn-in: 30
              SSMF-A iteration 8
                                                                                  good K:3:
                                    Gibbs burn-in: 30
              SSMF-A iteration 9
                                                                                 good K:3:
                                    Gibbs burn-in: 30
              SSMF-A iteration 10
                                                                                  good K:3:
                                    Gibbs burn-in: 30
              SSMF-A iteration 11
                                                                                 good K:3:
                                     Gibbs burn-in: 30
              SSMF-A iteration 12
                                                                                  good K:3:
                                     Gibbs burn-in: 30
                                                                                 good K:3:
              SSMF-A iteration 13
                                     Gibbs burn-in: 30
              SSMF-A iteration 14
                                                                                 good K:3:
                                     Gibbs burn-in: 30
              SSMF-A iteration 15
                                                                                  good K:3:
                                    Gibbs burn-in: 30
              SSMF-A iteration 16
                                                                                 good K:3:
                                    Gibbs burn-in: 30
              SSMF-A iteration 17
                                                                                  good K:3:
                                    Gibbs burn-in: 30
                                                                                 good K:3:
              SSMF-A iteration 18
                                    Gibbs burn-in: 30
              SSMF-A iteration 19
                                                                                  good K:3:
                                    Gibbs burn-in: 30
[16]: t
[16]: 20.361778497695923
[17]: K_and_log_ll(dssvi_run, 30, 20) # prints log-likelihoods calculated each_
                   \rightarrow iteration
               [-2079981.0108942022, -2275746.4001533287, -534322.3292252664,
              -109227.16724647142, -65870.11504878564, -65419.76439393273, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.44634637383, -65419.4463463738, -65419.44634637383, -65419.4463463738, -65419.4463463738, -65419.4463463, -65419.4463463, -65419.4463463, -65419.4463463, -65419.4463463, -65419.4463463, -65419.4463463, -65419.4463463, -65419.446346, -65419.446346, -65419.446346, -65419.446346, -65419.44646, -65419.44646, -65419.44646, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.4464, -65419.446, -65419.446, -65419.446, -65419.446, -65419.446, -65419.446, -65419.4460, -65419.446, -65419.446, -65419.446, -65419.446, -65419.446, -65419.446, -65419.446, -65419
              -65400.33376564072, -65408.89049433645, -65384.36166670424, -65408.79461889039,
              -57453.73543446944, -41382.67636371251, -15505.17615876438, -11282.329502832557,
              -10800.613318588363, -10802.833776747699, -10705.740700997861,
              -10721.737012364805, -10701.361428483533]
```

Gibbs burn-in: 30



[18]: plot_pi(dssvi_run)



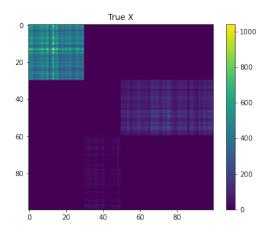
```
[19]: print("Expected pi_H at last iteration:", dssvi_run.Epi_W_list[-1]) print("Expected pi_W at last iteration:",dssvi_run.Epi_H_list[-1])
```

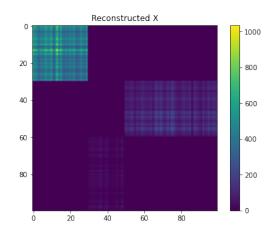
Expected pi_H at last iteration: [0.4405041 0.33534436 0.30076966] Expected pi_W at last iteration: [0.18713403 0.51566943 0.30077525]

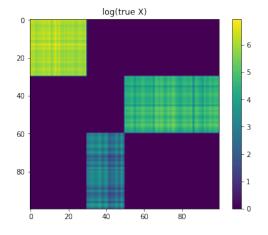
[20]: W_post_mean, H_post_mean, X_post_mean = dssvi_fn.get_post_means(dssvi_run)

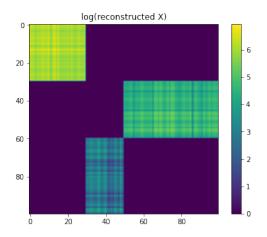
dssvi_fn.draw_two_matrices(X, X_post_mean, 1, 1, first='True X', □

⇒second='Reconstructed X')

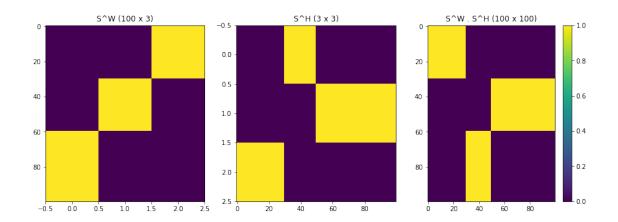








[21]: dssvi_fn.draw_S_H_S_W(dssvi_run)



1 Run 2: K=4

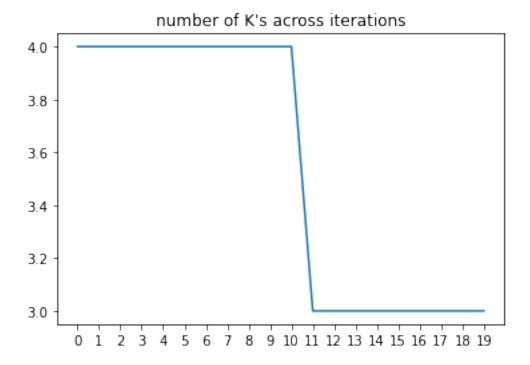
SSMF-A iteration 10

```
[22]: dssvi_run2 = dssvi_fn.SSMF_BP_NMF(n_components=4, burn_in=30, random_state=10,__
       →verbose=True, max_iter=20, cutoff=1e-2,
                                     a=1, b=1, c=5, d=5, a0_W=1, b0_W=1, a0_H=1,
       \rightarrowb0_H=1)
      start_t = time.time()
      dssvi_run2.fit(X)
      t2 = time.time() - start_t
     SSMF-A iteration 0
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 1
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 2
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 3
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 4
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 5
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 6
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 7
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 8
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 9
                              good K:4:
             Gibbs burn-in: 30
```

good K:4:

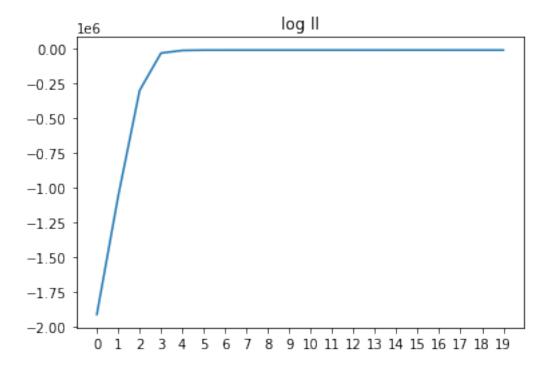
```
Gibbs burn-in: 30
     SSMF-A iteration 11
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 12
                              good K:3:
             Gibbs burn-in: 30
     SSMF-A iteration 13
                              good K:3:
             Gibbs burn-in: 30
     SSMF-A iteration 14
                              good K:3:
             Gibbs burn-in: 30
     SSMF-A iteration 15
                              good K:3:
             Gibbs burn-in: 30
     SSMF-A iteration 16
                              good K:3:
             Gibbs burn-in: 30
                              good K:3:
     SSMF-A iteration 17
             Gibbs burn-in: 30
     SSMF-A iteration 18
                              good K:3:
             Gibbs burn-in: 30
     SSMF-A iteration 19
                              good K:3:
             Gibbs burn-in: 30
[23]:
     t2
[23]: 26.059136152267456
[24]: len_good_k = [len(dssvi_run2.good_k_list[i]) for i in range(len(dssvi_run2.

    good_k_list))]
[25]: number_K(dssvi_run2)
```



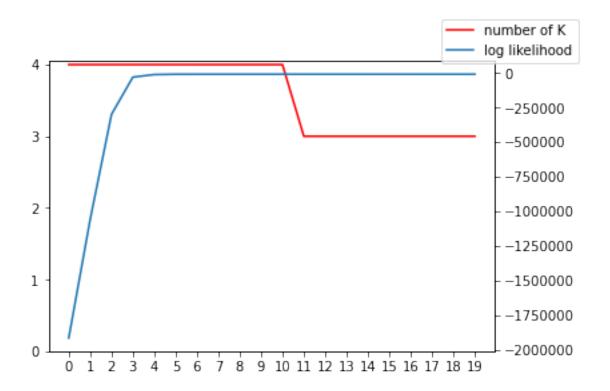
```
[26]: log_ll_list = [dssvi_run2.log_ll[i,30] for i in range(20)]
      log_ll_list
[26]: [-1913504.4145709837,
       -1060306.2990473085,
       -300630.0575652924,
       -31884.094783652807,
       -13147.570473712245,
       -10885.443479517977,
       -10779.035640741415,
       -10748.924756921937,
       -10758.015331446373,
       -10736.064428468133,
       -10752.302893094035,
       -10757.288597135994,
       -10736.797865532026,
       -10696.833403467792,
       -10699.062364684285,
       -10681.167884783144,
       -10702.315960650525,
       -10713.083941572258,
       -10687.371802909598,
       -10698.325910001935]
```

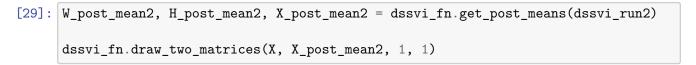
[27]: log_likelihoods(dssvi_run2,30,20)

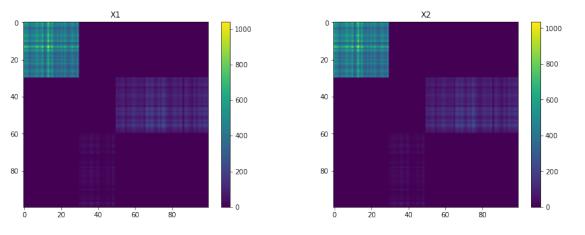


[28]: K_and_log_ll(dssvi_run2, 30, 20)

```
 \begin{array}{l} [-1913504.4145709837, \ -1060306.2990473085, \ -300630.0575652924, \\ -31884.094783652807, \ -13147.570473712245, \ -10885.443479517977, \\ -10779.035640741415, \ -10748.924756921937, \ -10758.015331446373, \\ -10736.064428468133, \ -10752.302893094035, \ -10757.288597135994, \\ -10736.797865532026, \ -10696.833403467792, \ -10699.062364684285, \\ -10681.167884783144, \ -10702.315960650525, \ -10713.083941572258, \\ -10687.371802909598, \ -10698.325910001935] \end{array}
```

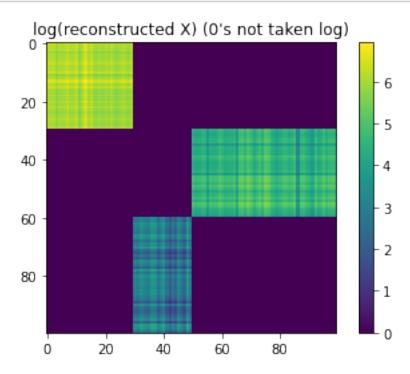


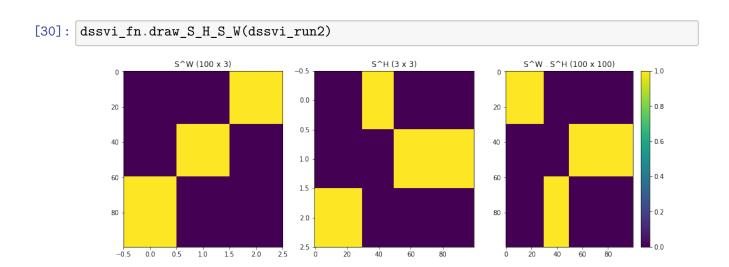




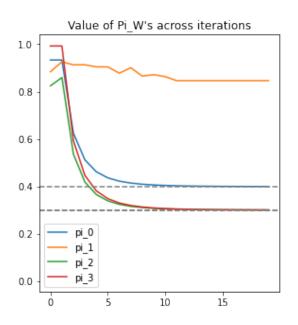
```
[111]: X_post_mean3 = pd.DataFrame.from_records(X_post_mean3)
plt.imshow(np.log(X_post_mean3.replace(0, np.nan)).replace(np.nan, 0),
interpolation='none')
plt.colorbar()
```

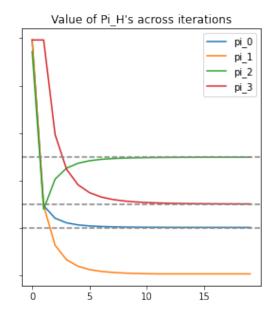
plt.title("log(reconstructed X) (0's not taken log)")
plt.show()





[34]: plot_pi(dssvi_run2)





1.1 Run 3: K = 5, burn = 30, max_iter = 20

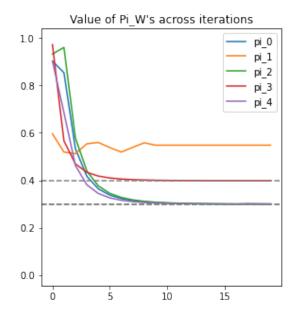
```
SSMF-A iteration 0
                        good K:5:
        Gibbs burn-in: 30
SSMF-A iteration 1
                         good K:5:
        Gibbs burn-in: 30
                         good K:5:
SSMF-A iteration 2
        Gibbs burn-in: 30
SSMF-A iteration 3
                         good K:5:
        Gibbs burn-in: 30
SSMF-A iteration 4
                         good K:5:
        Gibbs burn-in: 30
SSMF-A iteration 5
                        good K:5:
        Gibbs burn-in: 30
SSMF-A iteration 6
                         good K:5:
        Gibbs burn-in: 30
SSMF-A iteration 7
                        good K:5:
        Gibbs burn-in: 30
```

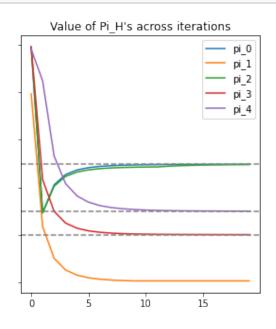
good K:5: SSMF-A iteration 8 Gibbs burn-in: 30 SSMF-A iteration 9 good K:5: Gibbs burn-in: 30 SSMF-A iteration 10 good K:4: Gibbs burn-in: 30 SSMF-A iteration 11 good K:4: Gibbs burn-in: 30 SSMF-A iteration 12 good K:4: Gibbs burn-in: 30 SSMF-A iteration 13 good K:4: Gibbs burn-in: 30 good K:4: SSMF-A iteration 14 Gibbs burn-in: 30 SSMF-A iteration 15 good K:4: Gibbs burn-in: 30 SSMF-A iteration 16 good K:4: Gibbs burn-in: 30 SSMF-A iteration 17 good K:4: Gibbs burn-in: 30 good K:4: SSMF-A iteration 18 Gibbs burn-in: 30 SSMF-A iteration 19 good K:4: Gibbs burn-in: 30

[36]: t3

[36]: 32.74858236312866

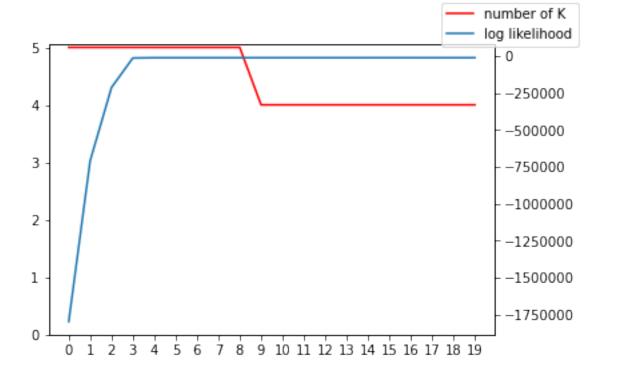
[37]: plot_pi(dssvi_run3)





```
[38]: K_and_log_ll(dssvi_run3, 30, 20)

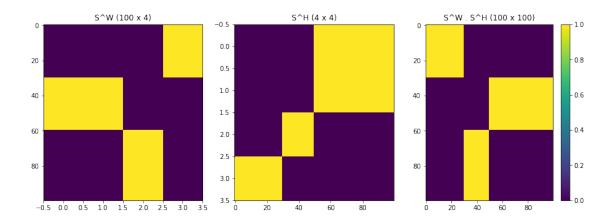
[-1797170.1395828, -709588.1554463771, -212713.4314165143, -12831.373081263053, -10968.045902495382, -10962.448775230177, -10947.996328305637, -10962.679968664757, -10947.014537005683, -10928.610385788288, -10872.785039239217, -10849.888706785952, -10892.748695469763, -10880.619157745672, -10862.926080445483, -10852.250646327779, -10875.606121177365, -10864.134086813257, -10871.814495368222, -10855.519718424905]
```

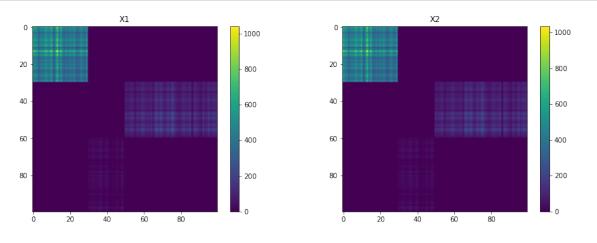


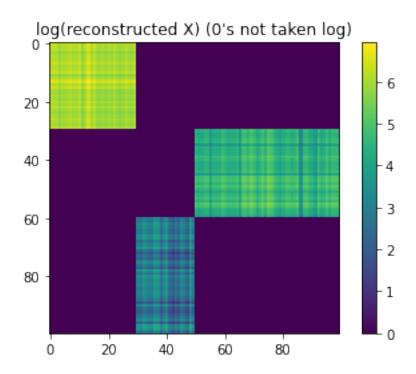
```
[39]: dssvi_run3.good_k

[39]: array([0, 2, 3, 4])

[40]: dssvi_fn.draw_S_H_S_W(dssvi_run3)
```





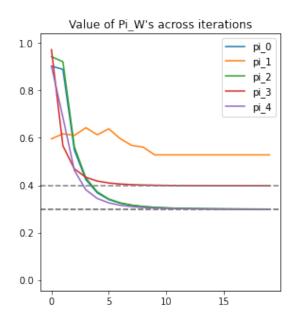


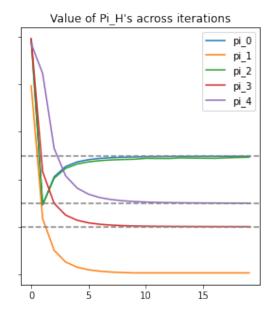
1.2 Run 4: K=5, burn = 60, max_iter = 20

SSMF-A iteration 0 good K:5: Gibbs burn-in: 60 SSMF-A iteration 1 good K:5: Gibbs burn-in: 60 SSMF-A iteration 2 good K:5: Gibbs burn-in: 60 good K:5: SSMF-A iteration 3 Gibbs burn-in: 60 SSMF-A iteration 4 good K:5: Gibbs burn-in: 60 SSMF-A iteration 5 good K:5: Gibbs burn-in: 60 SSMF-A iteration 6 good K:5:

```
SSMF-A iteration 7
                              good K:5:
             Gibbs burn-in: 60
     SSMF-A iteration 8
                              good K:5:
             Gibbs burn-in: 60
     SSMF-A iteration 9
                              good K:5:
             Gibbs burn-in: 60
     SSMF-A iteration 10
                              good K:4:
             Gibbs burn-in: 60
                              good K:4:
     SSMF-A iteration 11
             Gibbs burn-in: 60
     SSMF-A iteration 12
                              good K:4:
             Gibbs burn-in: 60
                              good K:4:
     SSMF-A iteration 13
             Gibbs burn-in: 60
     SSMF-A iteration 14
                              good K:4:
             Gibbs burn-in: 60
     SSMF-A iteration 15
                              good K:4:
             Gibbs burn-in: 60
     SSMF-A iteration 16
                              good K:4:
             Gibbs burn-in: 60
     SSMF-A iteration 17
                              good K:4:
             Gibbs burn-in: 60
     SSMF-A iteration 18
                              good K:4:
             Gibbs burn-in: 60
     SSMF-A iteration 19
                              good K:4:
             Gibbs burn-in: 60
[42]: t4
[42]: 62.98468327522278
[43]: plot_pi(dssvi_run4)
```

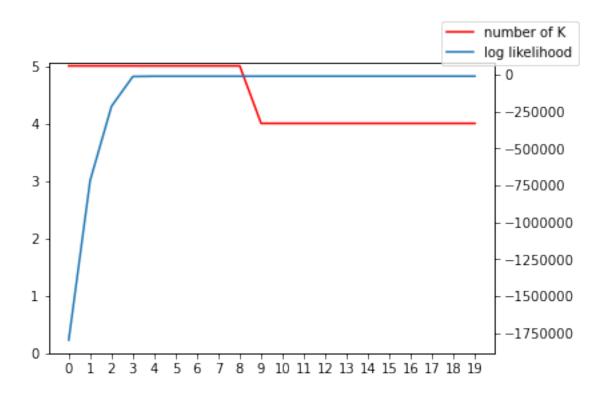
Gibbs burn-in: 60

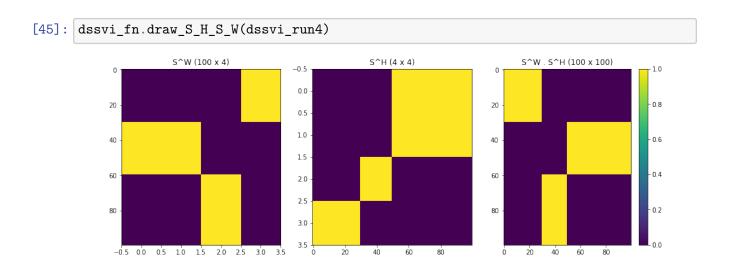




[44]: K_and_log_ll(dssvi_run4, 60, 20)

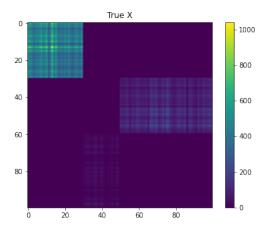
```
[-1797166.9513130942, -717833.9876844536, -214455.96168394867, -12865.889841415663, -10972.07384328549, -10959.990177463762, -10934.140250650435, -10961.448664493044, -10951.723469448112, -10927.279445537011, -10858.509310124206, -10880.469968200301, -10875.079817722904, -10864.363761174263, -10875.329584897392, -10869.411389907764, -10860.758394402508, -10850.895544754685, -10863.865127983596, -10872.569803331315]
```

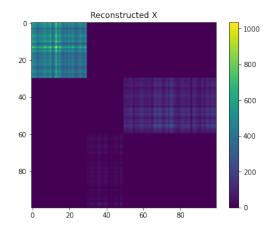


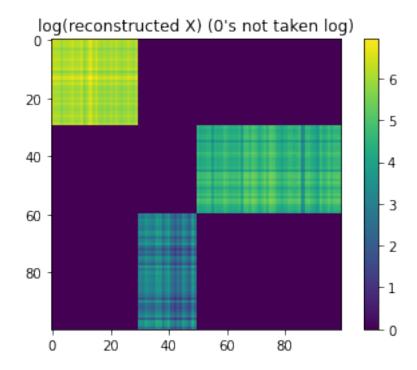


[46]: W_post_mean4, H_post_mean4, X_post_mean4 = dssvi_fn.get_post_means(dssvi_run4)

dssvi_fn.draw_two_matrices(X, X_post_mean4, 1, 1, 'True X', 'Reconstructed X')







1.3 Run: K=5, burn=30, max iter = 50

```
[47]: dssvi_run_ = dssvi_fn.SSMF_BP_NMF(n_components=5, burn_in=30, random_state=10,__
       →verbose=True, max_iter=50, cutoff=1e-2,
                                     a=1, b=1, c=5, d=5, a0_W=1, b0_W=1, a0_H=1,
       \rightarrowb0_H=1)
      start_t = time.time()
      dssvi_run_.fit(X)
      t_ = time.time() - start_t
     SSMF-A iteration 0
                              good K:5:
             Gibbs burn-in: 30
     SSMF-A iteration 1
                              good K:5:
             Gibbs burn-in: 30
                              good K:5:
     SSMF-A iteration 2
             Gibbs burn-in: 30
     SSMF-A iteration 3
                              good K:5:
             Gibbs burn-in: 30
     SSMF-A iteration 4
                              good K:5:
             Gibbs burn-in: 30
     SSMF-A iteration 5
                              good K:5:
             Gibbs burn-in: 30
                              good K:5:
     SSMF-A iteration 6
             Gibbs burn-in: 30
     SSMF-A iteration 7
                              good K:5:
             Gibbs burn-in: 30
     SSMF-A iteration 8
                              good K:5:
             Gibbs burn-in: 30
     SSMF-A iteration 9
                              good K:5:
             Gibbs burn-in: 30
     SSMF-A iteration 10
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 11
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 12
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 13
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 14
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 15
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 16
                              good K:4:
             Gibbs burn-in: 30
                              good K:4:
     SSMF-A iteration 17
             Gibbs burn-in: 30
```

good K:4:

SSMF-A iteration 18

Gibbs burn-in: 30

SSMF-A iteration 19 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 20 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 21 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 22 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 23 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 24 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 25 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 26 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 27 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 28 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 29 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 30 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 31 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 32 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 33 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 34 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 35 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 36 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 37 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 38 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 39 good K:4:

Gibbs burn-in: 30

SSMF-A iteration 40 good K:4:

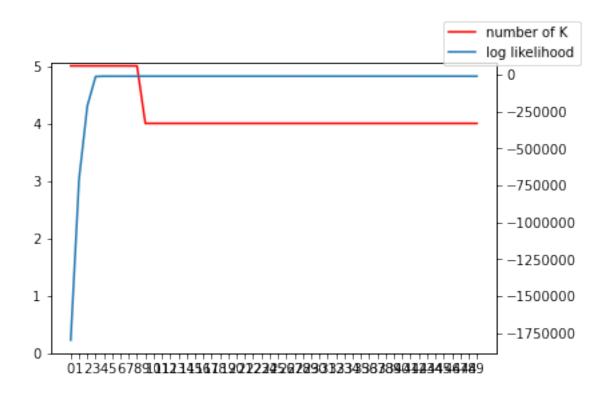
Gibbs burn-in: 30

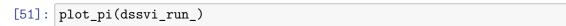
SSMF-A iteration 41 good K:4:

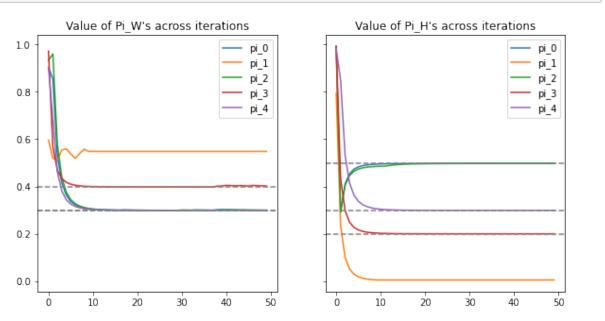
Gibbs burn-in: 30

SSMF-A iteration 42 good K:4:

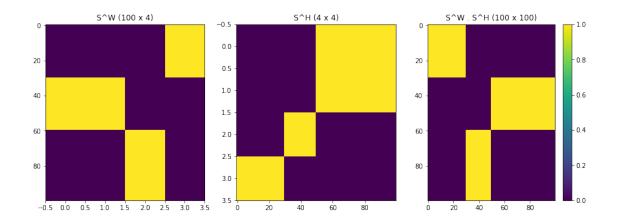
```
Gibbs burn-in: 30
     SSMF-A iteration 43
                             good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 44
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 45
                             good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 46
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 47
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 48
                              good K:4:
             Gibbs burn-in: 30
     SSMF-A iteration 49
                              good K:4:
             Gibbs burn-in: 30
[48]: t_
[48]: 81.53423643112183
[49]:
      dssvi_run_.good_k
[49]: array([0, 2, 3, 4])
[50]: K_and_log_ll(dssvi_run_, 30, 50)
     [-1797170.1395828, -709588.1554463771, -212713.4314165143, -12831.373081263053,
     -10968.045902495382, -10962.448775230177, -10947.996328305637,
     -10962.679968664757, -10947.014537005683, -10928.610385788288,
     -10872.785039239217, -10849.888706785952, -10892.748695469763,
     -10880.619157745672, -10862.926080445483, -10852.250646327779,
     -10875.606121177365, -10864.134086813257, -10871.814495368222,
     -10855.519718424905, -10866.214641650935, -10893.195570026415,
     -10855.714953733795, -10846.366448580626, -10878.167985067266,
     -10868.961471791697, -10874.354023701071, -10842.433238564143,
     -10853.497133010604, -10844.945969412644, -10863.167566257547,
     -10867.060528724538, -10858.14490249956, -10864.807749336413,
     -10865.534386667689, -10844.591896166407, -10853.603798417265,
     -10856.485440650773, -10861.315615835529, -10848.579858054363,
     -10840.506523955288, -10859.72207194283, -10862.987291944894,
     -10862.38461631021, -10852.52577604607, -10852.38535432219, -10870.134185578512,
     -10840.109184383817, -10867.894149553153, -10855.310666037865]
```





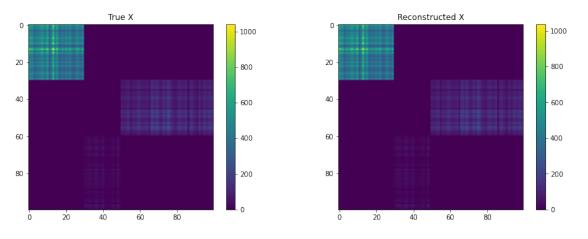


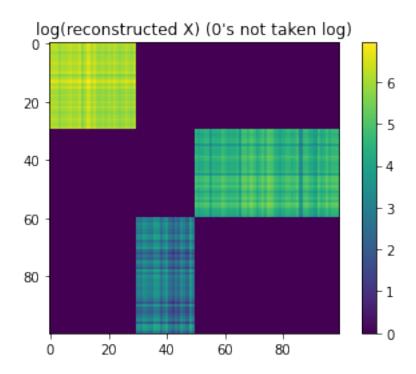
[52]: dssvi_fn.draw_S_H_S_W(dssvi_run_)



```
[53]: W_post_mean_, H_post_mean_, X_post_mean_ = dssvi_fn.get_post_means(dssvi_run_)

dssvi_fn.draw_two_matrices(X, X_post_mean_, 1, 1, 'True X', 'Reconstructed X')
```





1.4 Run 5: K = 10, burn = 50, max_iter = 50

SSMF-A iteration 0 good K:10: Gibbs burn-in: 50 SSMF-A iteration 1 good K:10: Gibbs burn-in: 50 SSMF-A iteration 2 good K:10: Gibbs burn-in: 50 good K:10: SSMF-A iteration 3 Gibbs burn-in: 50 SSMF-A iteration 4 good K:10: Gibbs burn-in: 50 SSMF-A iteration 5 good K:10: Gibbs burn-in: 50 SSMF-A iteration 6 good K:10: Gibbs burn-in: 50

SSMF-A iteration 7 good K:10:

Gibbs burn-in: 50

SSMF-A iteration 8 good K:10:

Gibbs burn-in: 50

SSMF-A iteration 9 good K:10:

Gibbs burn-in: 50

SSMF-A iteration 10 good K:9:

Gibbs burn-in: 50

SSMF-A iteration 11 good K:9:

Gibbs burn-in: 50

SSMF-A iteration 12 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 13 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 14 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 15 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 16 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 17 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 18 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 19 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 20 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 21 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 22 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 23 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 24 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 25 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 26 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 27 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 28 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 29 good K:8:

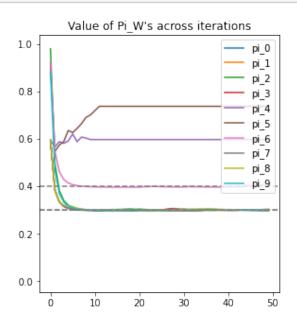
Gibbs burn-in: 50

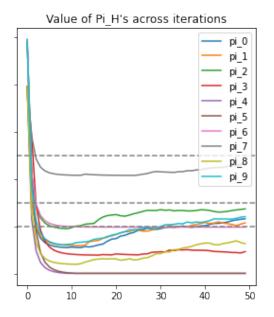
SSMF-A iteration 30 good K:8:

```
Gibbs burn-in: 50
     SSMF-A iteration 31
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 32
                              good K:8:
             Gibbs burn-in: 50
                              good K:8:
     SSMF-A iteration 33
             Gibbs burn-in: 50
     SSMF-A iteration 34
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 35
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 36
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 37
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 38
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 39
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 40
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 41
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 42
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 43
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 44
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 45
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 46
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 47
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 48
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 49
                              good K:8:
             Gibbs burn-in: 50
[55]: t5
[55]: 223.45148873329163
[56]: dssvi_run5.good_k
```

[56]: array([0, 1, 2, 3, 6, 7, 8, 9])

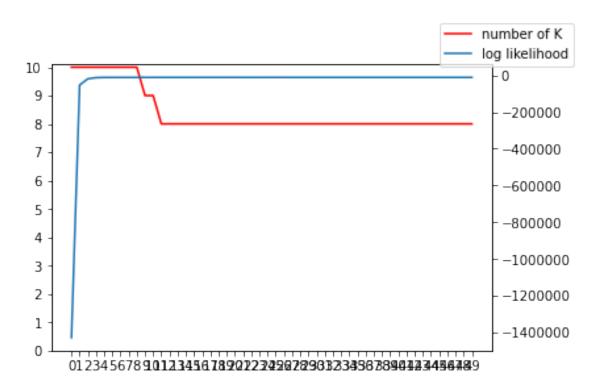
[57]: plot_pi(dssvi_run5)



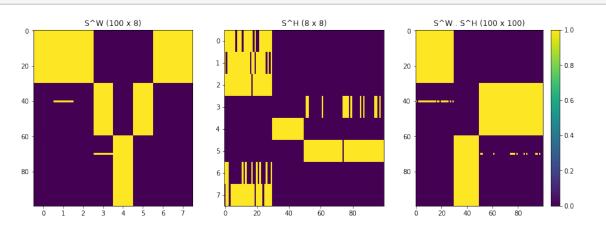


[58]: K_and_log_ll(dssvi_run5, 50, 50)

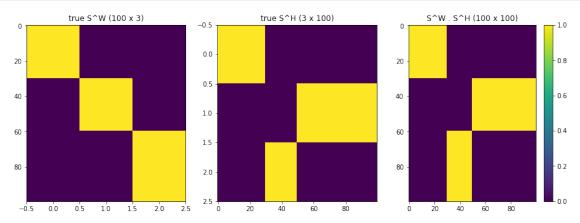
```
[-1428623.348077554, -53031.83471908225, -19198.723430477883,
-12788.87613836939, -11697.026787077579, -11530.849329025134,
-11511.09804520131, -11503.255640889867, -11491.81880147754,
-11453.806019340376, -11405.728445901428, -11370.564419818353,
-11301.08882555038, -11329.03183257621, -11318.638928420492,
-11296.106238867682, -11294.003753802033, -11291.391832244437,
-11324.995428876127, \ -11288.387514567268, \ -11315.54371319688,
-11302.31928793309, -11306.910073063968, -11302.05374535164,
-11318.386275184588, -11290.526006477421, -11301.187881227497,
-11298.936160315236, -11324.417216965016, -11311.939832264623,
-11323.996498222708, -11293.711125179194, -11330.979559336593,
-11320.256768545863, -11317.698204982316, -11333.763065221106,
-11313.440890369675, -11303.460174034832, -11307.1447839247,
-11295.963069746957, -11308.20617274677, -11295.109304815485,
-11296.722059190177, -11295.12129359343, -11312.820359421417,
-11315.86276548556, -11298.87744636344, -11308.994648898704, -11325.62098792596,
-11302.633815554937]
```



[59]: dssvi_fn.draw_S_H_S_W(dssvi_run5)

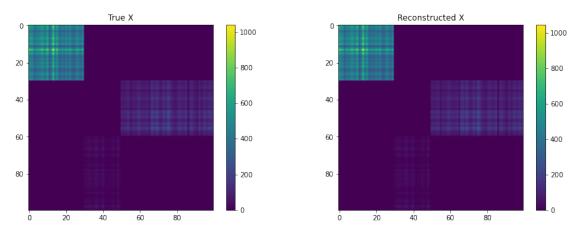


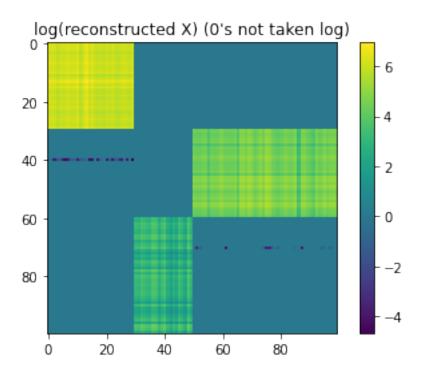
```
ax3.title.set_text('S^W . S^H (100 x 100)')
plt.colorbar(im3, ax=ax3)
plt.show()
```



[61]: W_post_mean5, H_post_mean5, X_post_mean5 = dssvi_fn.get_post_means(dssvi_run5)

dssvi_fn.draw_two_matrices(X, X_post_mean5, 1, 1, 'True X', 'Reconstructed X')





1.5 Run 7: K=10, burn = 100, max_iter= 50

```
dssvi_run7 = dssvi_fn.SSMF_BP_NMF(n_components=10, burn_in=100, u

→random_state=10, verbose=True, max_iter=50, cutoff=1e-2,

a=1, b=1, c=5, d=5, a0_W=1, b0_W=1, a0_H=1, u

→b0_H=1)

start_t = time.time()

dssvi_run7.fit(X)

t7 = time.time() - start_t
```

SSMF-A iteration 0 good K:10:

Gibbs burn-in: 100

SSMF-A iteration 1 good K:10:

Gibbs burn-in: 100

SSMF-A iteration 2 good K:10:

Gibbs burn-in: 100

SSMF-A iteration 3 good K:10:

Gibbs burn-in: 100

SSMF-A iteration 4 good K:10:

Gibbs burn-in: 100

SSMF-A iteration 5 good K:10:

Gibbs burn-in: 100

SSMF-A iteration 6 good K:10:

SSMF-A iteration 7 good K:10:

Gibbs burn-in: 100

SSMF-A iteration 8 good K:10:

Gibbs burn-in: 100

SSMF-A iteration 9 good K:9:

Gibbs burn-in: 100

SSMF-A iteration 10 good K:9:

Gibbs burn-in: 100

SSMF-A iteration 11 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 12 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 13 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 14 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 15 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 16 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 17 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 18 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 19 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 20 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 21 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 22 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 23 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 24 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 25 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 26 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 27 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 28 good K:8:

Gibbs burn-in: 100

SSMF-A iteration 29 good K:8:

Gibbs burn-in: 100

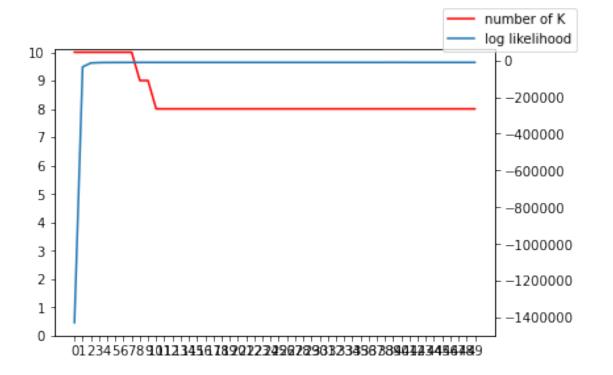
SSMF-A iteration 30 good K:8:

```
Gibbs burn-in: 100
     SSMF-A iteration 31
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 32
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 33
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 34
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 35
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 36
                              good K:8:
             Gibbs burn-in: 100
                              good K:8:
     SSMF-A iteration 37
             Gibbs burn-in: 100
     SSMF-A iteration 38
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 39
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 40
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 41
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 42
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 43
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 44
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 45
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 46
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 47
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 48
                              good K:8:
             Gibbs burn-in: 100
     SSMF-A iteration 49
                              good K:8:
             Gibbs burn-in: 100
[63]: t7
[63]: 445.0749204158783
[64]: dssvi_run7.good_k
```

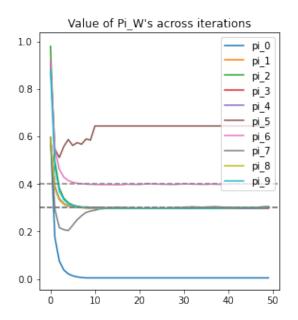
[64]: array([1, 2, 3, 4, 6, 7, 8, 9])

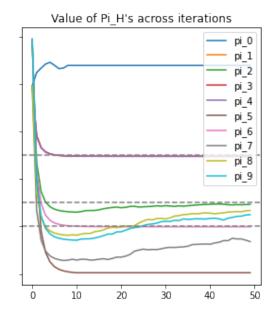
[65]: K_and_log_ll(dssvi_run7, 100, 50)

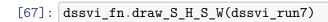
```
[-1428623.348077554, -35593.311805709185, -14850.747504378356,
-12652.842142464362, -12054.977683904614, -11925.904131050891,
-11771.255609677799, -11591.223668898583, -11565.725233412684,
-11479.102735075921, -11482.829235583205, -11398.213509626843,
-11411.779442024817, -11396.882587696815, -11393.93074268351,
-11391.216308236717, -11394.266708766434, -11394.706675078685,
-11375.098341717508, -11370.458996604539, -11370.02300389421,
-11385.937226798565, -11385.836236494659, -11379.657480935124,
-11397.377072248393, -11379.19693845303, -11373.586306973993,
-11361.056734656251, -11432.252693231894, -11397.175446075988,
-11370.172058956183, -11379.873066126138, -11392.774879360284,
-11386.207702613488, -11407.926401127688, -11387.803491412966,
-11378.385168560375, -11371.569038577381, -11374.180634012768,
-11353.231490276432, -11376.022839825971, -11387.992930869015,
-11371.461708827499, -11401.880478187808, -11378.60789138459,
-11402.941226841169, -11368.580923989888, -11360.495365137447,
-11380.295761202851, -11371.564351569963]
```

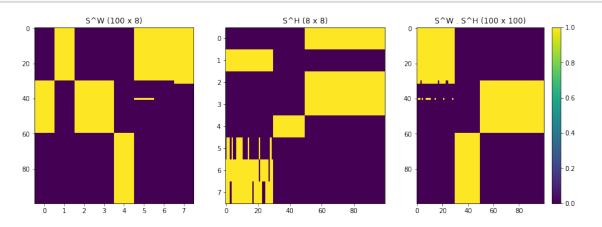


[66]: plot_pi(dssvi_run7)



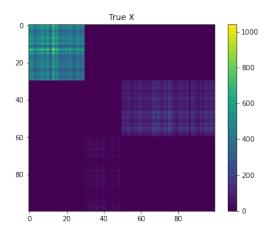


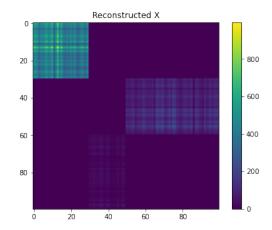


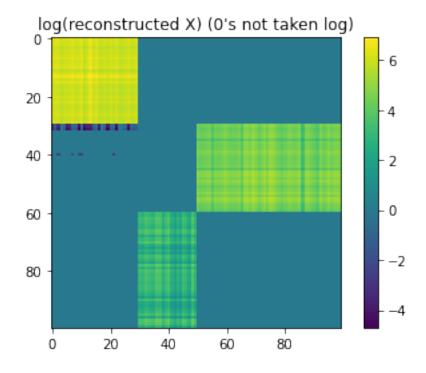


[68]: W_post_mean7, H_post_mean7, X_post_mean7 = dssvi_fn.get_post_means(dssvi_run7)

dssvi_fn.draw_two_matrices(X, X_post_mean7, 1, 1, 'True X', 'Reconstructed X')







1.6 Run 8: K = 10, burn = 50, max iter = 100

```
[69]: dssvi_run8 = dssvi_fn.SSMF_BP_NMF(n_components=10, burn_in=50, random_state=10,
       →verbose=True, max_iter=100, cutoff=1e-2,
                                     a=1, b=1, c=5, d=5, a0_W=1, b0_W=1, a0_H=1,
       \rightarrowb0_H=1)
      start_t = time.time()
      dssvi_run8.fit(X)
      t8 = time.time() - start_t
     SSMF-A iteration 0
                              good K:10:
             Gibbs burn-in: 50
     SSMF-A iteration 1
                              good K:10:
             Gibbs burn-in: 50
     SSMF-A iteration 2
                              good K:10:
             Gibbs burn-in: 50
     SSMF-A iteration 3
                              good K:10:
             Gibbs burn-in: 50
     SSMF-A iteration 4
                              good K:10:
             Gibbs burn-in: 50
     SSMF-A iteration 5
                              good K:10:
             Gibbs burn-in: 50
                              good K:10:
     SSMF-A iteration 6
             Gibbs burn-in: 50
     SSMF-A iteration 7
                              good K:10:
             Gibbs burn-in: 50
     SSMF-A iteration 8
                              good K:10:
             Gibbs burn-in: 50
     SSMF-A iteration 9
                              good K:10:
             Gibbs burn-in: 50
     SSMF-A iteration 10
                              good K:9:
             Gibbs burn-in: 50
     SSMF-A iteration 11
                              good K:9:
             Gibbs burn-in: 50
     SSMF-A iteration 12
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 13
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 14
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 15
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 16
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 17
                              good K:8:
             Gibbs burn-in: 50
```

good K:8:

SSMF-A iteration 18

SSMF-A iteration 19 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 20 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 21 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 22 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 23 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 24 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 25 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 26 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 27 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 28 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 29 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 30 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 31 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 32 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 33 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 34 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 35 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 36 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 37 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 38 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 39 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 40 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 41 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 42 good K:8:

SSMF-A iteration 43 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 44 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 45 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 46 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 47 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 48 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 49 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 50 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 51 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 52 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 53 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 54 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 55 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 56 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 57 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 58 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 59 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 60 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 61 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 62 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 63 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 64 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 65 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 66 good K:8:

SSMF-A iteration 67 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 68 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 69 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 70 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 71 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 72 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 73 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 74 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 75 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 76 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 77 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 78 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 79 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 80 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 81 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 82 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 83 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 84 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 85 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 86 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 87 good K:8:

Gibbs burn-in: 50

SSMF-A iteration 88 good K:8:

Gibbs burn-in: 50

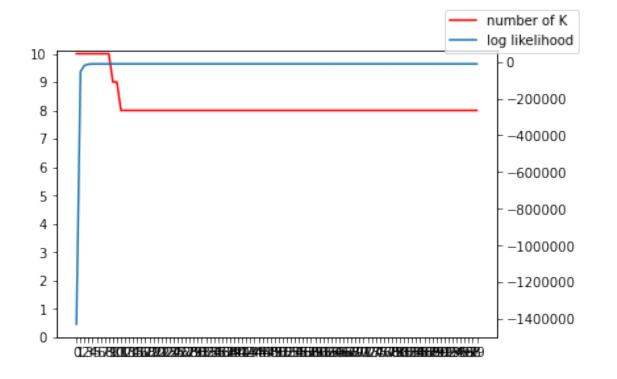
SSMF-A iteration 89 good K:8:

Gibbs burn-in: 50

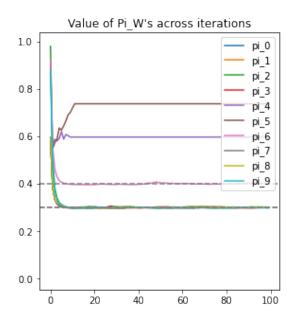
SSMF-A iteration 90 good K:8:

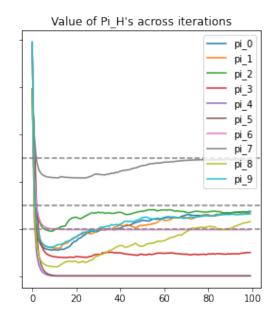
```
Gibbs burn-in: 50
     SSMF-A iteration 91
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 92
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 93
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 94
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 95
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 96
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 97
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 98
                              good K:8:
             Gibbs burn-in: 50
     SSMF-A iteration 99
                              good K:8:
             Gibbs burn-in: 50
[70]:
     t8
[70]: 452.51365661621094
      dssvi_run8.good_k
[71]: array([0, 1, 2, 3, 6, 7, 8, 9])
[72]: K_and_log_ll(dssvi_run8, 50, 100)
     [-1428623.348077554, -53031.83471908225, -19198.723430477883,
     -12788.87613836939, -11697.026787077579, -11530.849329025134,
     -11511.09804520131, -11503.255640889867, -11491.81880147754,
     -11453.806019340376, -11405.728445901428, -11370.564419818353,
     -11301.08882555038, -11329.03183257621, -11318.638928420492,
     -11296.106238867682, -11294.003753802033, -11291.391832244437,
     -11324.995428876127, -11288.387514567268, -11315.54371319688,
     -11302.31928793309, -11306.910073063968, -11302.05374535164,
     -11318.386275184588, -11290.526006477421, -11301.187881227497,
     -11298.936160315236, -11324.417216965016, -11311.939832264623,
     -11323.996498222708, -11293.711125179194, -11330.979559336593,
     -11320.256768545863, -11317.698204982316, -11333.763065221106,
     -11313.440890369675, -11303.460174034832, -11307.1447839247,
     -11295.963069746957, -11308.20617274677, -11295.109304815485,
     -11296.722059190177, -11295.12129359343, -11312.820359421417,
     -11315.86276548556, -11298.87744636344, -11308.994648898704, -11325.62098792596,
     -11302.633815554937, -11306.78988978581, -11336.43800362196,
     -11310.839599599121, -11324.966130597604, -11292.002660640826,
```

```
-11297.494731036475, -11317.9830365652, -11315.796546778387, -11318.39340625538, -11319.632363375884, -11296.30135785685, -11316.031939296718, -11315.12535807364, -11340.076411530456, -11300.902664593104, -11314.881102024918, -11315.970332096262, -11297.875686563362, -11328.355579362673, -11329.293623372454, -11312.66332612067, -11322.54616771053, -11279.745445973145, -11307.674290941823, -11307.039152380326, -11329.764864587838, -11307.309480680944, -11348.104761040886, -11332.914387176355, -11327.528693693004, -11322.6263166285, -11326.933217333906, -11323.535998608775, -11306.382799244384, -11318.719324883701, -11331.613765653256, -11325.79338107605, -11319.251808180514, -11322.020785070346, -11340.878762835608, -11307.13335319969, -11326.029238555837, -11325.204476722682, -11319.08116889412, -11329.919428812856, -11347.693889285802, -11318.172297269633, -11338.283881958014, -11313.735559909854, -11326.838886785474]
```

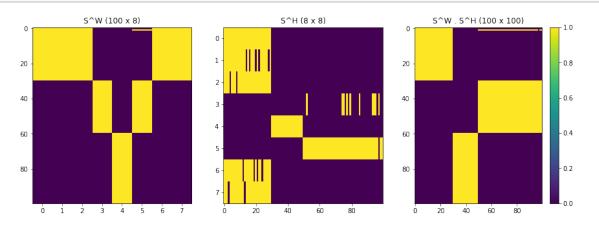


[73]: plot_pi(dssvi_run8)



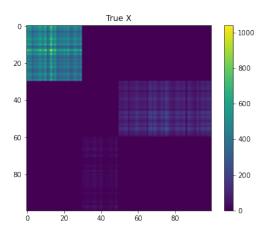


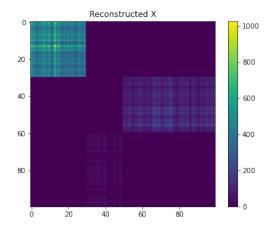


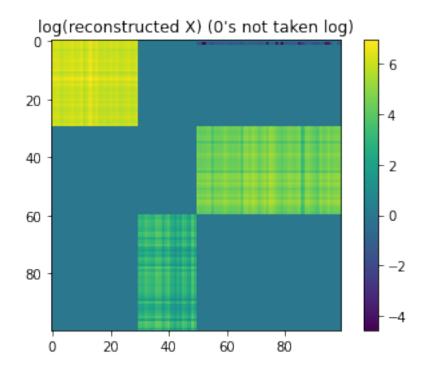


[75]: W_post_mean8, H_post_mean8, X_post_mean8 = dssvi_fn.get_post_means(dssvi_run8)

dssvi_fn.draw_two_matrices(X, X_post_mean8, 1, 1, 'True X', 'Reconstructed X')







1.7 K = 20, burn = 50, max iter = 50

```
[76]: dssvi_run6 = dssvi_fn.SSMF_BP_NMF(n_components=20, burn_in=50, random_state=10,_
       →verbose=True, max_iter=50, cutoff=1e-2,
                                     a=1, b=1, c=5, d=5, a0_W=1, b0_W=1, a0_H=1,
       \rightarrowb0_H=1)
      start_t = time.time()
      dssvi_run6.fit(X)
      t6 = time.time() - start_t
     SSMF-A iteration 0
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 1
                              good K:20:
              Gibbs burn-in: 50
     SSMF-A iteration 2
                              good K:20:
              Gibbs burn-in: 50
     SSMF-A iteration 3
                              good K:20:
              Gibbs burn-in: 50
     SSMF-A iteration 4
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 5
                              good K:20:
             Gibbs burn-in: 50
                              good K:20:
     SSMF-A iteration 6
             Gibbs burn-in: 50
     SSMF-A iteration 7
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 8
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 9
                              good K:20:
              Gibbs burn-in: 50
     SSMF-A iteration 10
                              good K:17:
              Gibbs burn-in: 50
     SSMF-A iteration 11
                              good K:17:
             Gibbs burn-in: 50
     SSMF-A iteration 12
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 13
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 14
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 15
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 16
                              good K:15:
              Gibbs burn-in: 50
                              good K:15:
     SSMF-A iteration 17
              Gibbs burn-in: 50
```

good K:15:

SSMF-A iteration 18

SSMF-A iteration 19 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 20 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 21 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 22 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 23 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 24 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 25 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 26 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 27 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 28 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 29 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 30 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 31 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 32 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 33 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 34 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 35 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 36 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 37 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 38 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 39 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 40 good K:15:

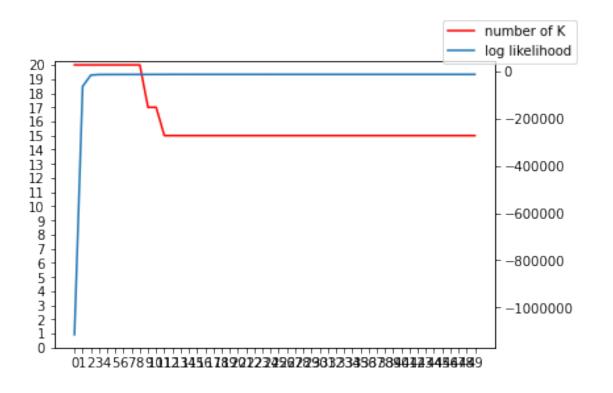
Gibbs burn-in: 50

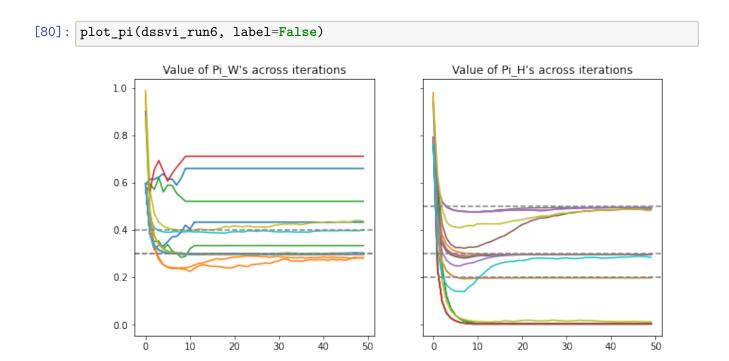
SSMF-A iteration 41 good K:15:

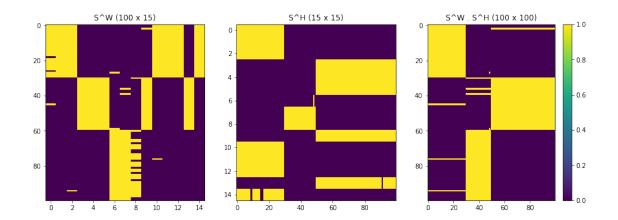
Gibbs burn-in: 50

SSMF-A iteration 42 good K:15:

```
Gibbs burn-in: 50
     SSMF-A iteration 43
                             good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 44
                             good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 45
                             good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 46
                             good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 47
                             good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 48
                             good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 49
                             good K:15:
             Gibbs burn-in: 50
[77]: t6
[77]: 391.66060400009155
[78]:
      dssvi_run6.good_k
[78]: array([ 1, 3, 4, 5, 6, 7, 8, 9, 11, 14, 15, 16, 17, 18, 19])
[79]: K_and_log_ll(dssvi_run6, 50, 50)
     [-1114876.983751481, -63197.26355503041, -15386.45227999154, -12990.07755311271,
     -12836.226983680413, -12768.617533565775, -12707.783622397616,
     -12656.943615197826, -12643.5813264265, -12602.008223958504,
     -12448.324821651076, -12446.88428554658, -12315.085690322803,
     -12294.164714027605, -12327.724468389424, -12303.214053331765,
     -12295.567742845175, -12299.978776378353, -12289.356620827155,
     -12312.50118637966, -12280.056399041365, -12287.413835926314,
     -12277.288395718342, -12301.381604461842, -12325.70086017958,
     -12279.988583307402, -12278.729843663825, -12261.987380222683,
     -12288.53852317285, -12296.427130938679, -12285.844609085672,
     -12289.773939479204, -12288.135154014093, -12277.747591475792,
     -12260.721277932555, -12282.315135804598, -12262.55127232295,
     -12306.669956743219, -12269.31351562593, -12269.100587798812,
     -12259.716077931515, -12271.625418264877, -12288.1766629993,
     -12268.776276136585, -12268.158455240895, -12260.565588129277,
     -12290.867095388665, -12266.575671119715, -12269.577241578409,
     -12255.941911533206]
```

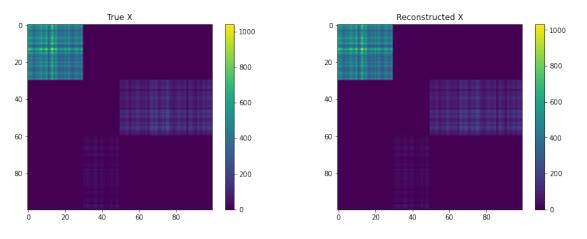


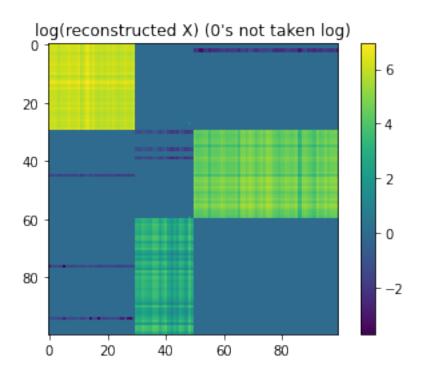




[82]: W_post_mean6, H_post_mean6, X_post_mean6 = dssvi_fn.get_post_means(dssvi_run6)

dssvi_fn.draw_two_matrices(X, X_post_mean6, 1, 1, 'True X', 'Reconstructed X')





1.8 K = 20, burn=100, max_iter=50

SSMF-A iteration 0 good K:20:

Gibbs burn-in: 100

SSMF-A iteration 1 good K:20:

Gibbs burn-in: 100

SSMF-A iteration 2 good K:20:

Gibbs burn-in: 100

SSMF-A iteration 3 good K:20:

Gibbs burn-in: 100

SSMF-A iteration 4 good K:20:

Gibbs burn-in: 100

SSMF-A iteration 5 good K:20:

Gibbs burn-in: 100

SSMF-A iteration 6 good K:20:

SSMF-A iteration 7 good K:20:

Gibbs burn-in: 100

SSMF-A iteration 8 good K:20:

Gibbs burn-in: 100

SSMF-A iteration 9 good K:15:

Gibbs burn-in: 100

SSMF-A iteration 10 good K:15:

Gibbs burn-in: 100

SSMF-A iteration 11 good K:15:

Gibbs burn-in: 100

SSMF-A iteration 12 good K:14:

Gibbs burn-in: 100

SSMF-A iteration 13 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 14 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 15 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 16 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 17 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 18 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 19 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 20 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 21 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 22 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 23 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 24 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 25 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 26 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 27 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 28 good K:13:

Gibbs burn-in: 100

SSMF-A iteration 29 good K:13:

Gibbs burn-in: 100

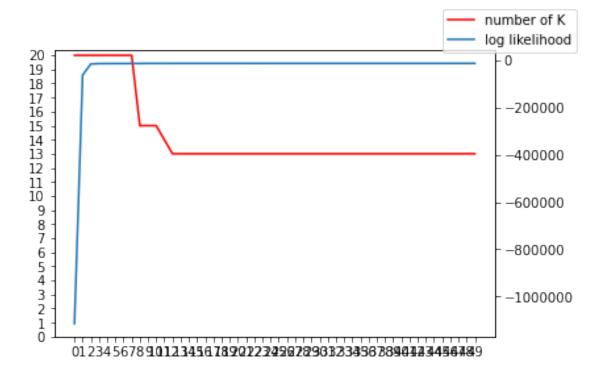
SSMF-A iteration 30 good K:13:

```
Gibbs burn-in: 100
     SSMF-A iteration 31
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 32
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 33
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 34
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 35
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 36
                              good K:13:
             Gibbs burn-in: 100
                              good K:13:
     SSMF-A iteration 37
             Gibbs burn-in: 100
     SSMF-A iteration 38
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 39
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 40
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 41
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 42
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 43
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 44
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 45
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 46
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 47
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 48
                              good K:13:
             Gibbs burn-in: 100
     SSMF-A iteration 49
                              good K:13:
             Gibbs burn-in: 100
[84]: t9
[84]: 720.0963819026947
[85]: dssvi_run9.good_k
```

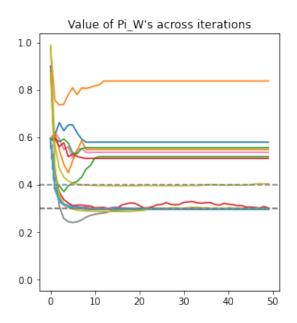
[85]: array([0, 3, 4, 5, 7, 8, 9, 14, 15, 16, 17, 18, 19])

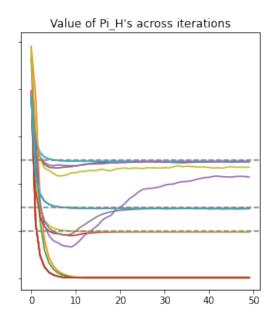
[87]: K_and_log_ll(dssvi_run9, 100, 50)

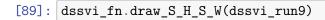
```
[-1114876.1586504786, -61376.539459310596, -14523.931474156747,
-13088.599662141522, -12837.094250935086, -12742.174107859264,
-12730.44000962798, -12641.738564237628, -12609.79955635812,
-12234.487017706242, -12225.48787916084, -12204.661028837452,
-12135.301711192424, -12064.071752770727, -12054.769562602054,
-12070.201163157133, -12059.634925911962, -12061.179053790573,
-12052.806238966734, -12084.636310715272, -12051.238073497494,
-12052.454158038163, -12074.682701688082, -12068.278836004229,
-12046.292629854963, -12061.500683491255, -12059.880397675384,
-12029.049859574763, -12059.173610508144, -12060.37483831762,
-12033.251904209963, -12083.675775277856, -12054.784898299706,
-12069.598515561234, -12028.238699699668, -12053.164607350287,
-12043.307353296668, -12051.48682968192, -12055.324405037554,
-12028.383664329527, -12076.626465125353, -12049.712221658021,
-12025.71809174355, -12052.776138636624, -12026.558453544087,
-12039.914095155316, -12037.054248738585, -12017.44521621172,
-12053.83882374272, -12064.373753643269]
```

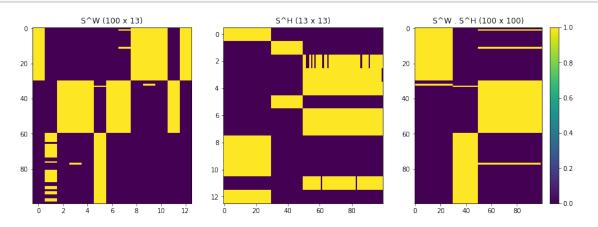


[88]: plot_pi(dssvi_run9, label=False)



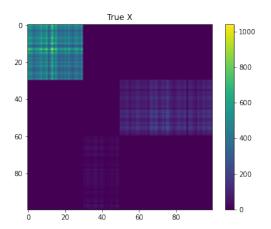


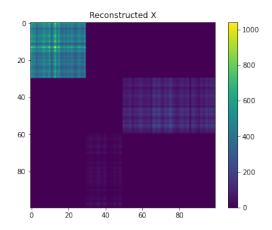


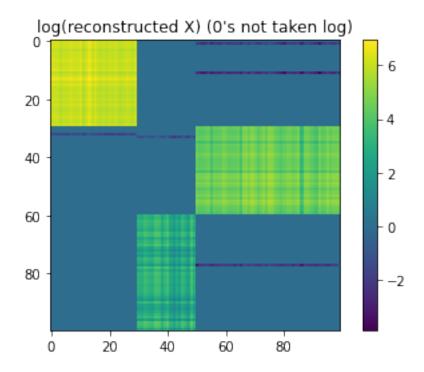


[90]: W_post_mean9, H_post_mean9, X_post_mean9 = dssvi_fn.get_post_means(dssvi_run9)

dssvi_fn.draw_two_matrices(X, X_post_mean9, 1, 1, 'True X', 'Reconstructed X')







1.9 K = 20, burn=50, max_iter = 100

```
[91]: dssvi_run10 = dssvi_fn.SSMF_BP_NMF(n_components=20, burn_in=50,
       →random_state=10, verbose=True, max_iter=100, cutoff=1e-2,
                                     a=1, b=1, c=5, d=5, a0_W=1, b0_W=1, a0_H=1,
       \rightarrowb0_H=1)
      start_t = time.time()
      dssvi_run10.fit(X)
      t10 = time.time() - start_t
     SSMF-A iteration 0
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 1
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 2
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 3
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 4
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 5
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 6
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 7
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 8
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 9
                              good K:20:
             Gibbs burn-in: 50
     SSMF-A iteration 10
                              good K:17:
             Gibbs burn-in: 50
     SSMF-A iteration 11
                              good K:17:
             Gibbs burn-in: 50
     SSMF-A iteration 12
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 13
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 14
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 15
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 16
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 17
                              good K:15:
             Gibbs burn-in: 50
```

good K:15:

SSMF-A iteration 18

SSMF-A iteration 19 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 20 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 21 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 22 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 23 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 24 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 25 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 26 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 27 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 28 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 29 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 30 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 31 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 32 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 33 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 34 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 35 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 36 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 37 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 38 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 39 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 40 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 41 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 42 good K:15:

SSMF-A iteration 43 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 44 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 45 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 46 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 47 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 48 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 49 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 50 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 51 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 52 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 53 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 54 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 55 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 56 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 57 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 58 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 59 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 60 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 61 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 62 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 63 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 64 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 65 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 66 good K:15:

SSMF-A iteration 67 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 68 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 69 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 70 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 71 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 72 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 73 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 74 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 75 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 76 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 77 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 78 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 79 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 80 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 81 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 82 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 83 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 84 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 85 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 86 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 87 good K:15:

Gibbs burn-in: 50

SSMF-A iteration 88 good K:15:

Gibbs burn-in: 50

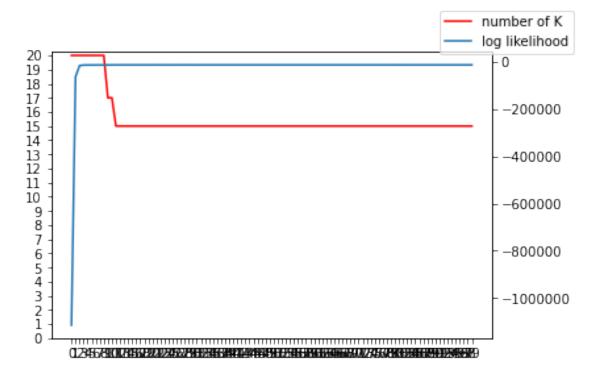
SSMF-A iteration 89 good K:15:

Gibbs burn-in: 50

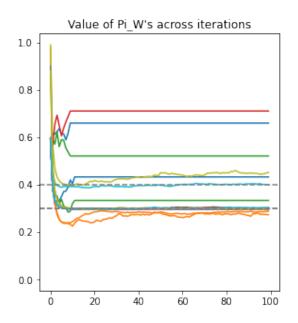
SSMF-A iteration 90 good K:15:

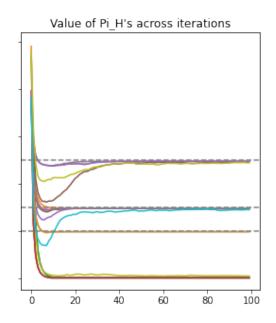
```
Gibbs burn-in: 50
     SSMF-A iteration 91
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 92
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 93
                             good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 94
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 95
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 96
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 97
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 98
                              good K:15:
             Gibbs burn-in: 50
     SSMF-A iteration 99
                              good K:15:
             Gibbs burn-in: 50
[92]:
     t10
[92]: 758.6015729904175
[93]:
      dssvi_run10.good_k
[93]: array([ 1, 3, 4, 5, 6, 7, 8, 9, 11, 14, 15, 16, 17, 18, 19])
[94]: K_and_log_ll(dssvi_run10, 50, 100)
     [-1114876.983751481, -63197.26355503041, -15386.45227999154, -12990.07755311271,
     -12836.226983680413, -12768.617533565775, -12707.783622397616,
     -12656.943615197826, -12643.5813264265, -12602.008223958504,
     -12448.324821651076, -12446.88428554658, -12315.085690322803,
     -12294.164714027605, -12327.724468389424, -12303.214053331765,
     -12295.567742845175, -12299.978776378353, -12289.356620827155,
     -12312.50118637966, -12280.056399041365, -12287.413835926314,
     -12277.288395718342, -12301.381604461842, -12325.70086017958,
     -12279.988583307402, -12278.729843663825, -12261.987380222683,
     -12288.53852317285, -12296.427130938679, -12285.844609085672,
     -12289.773939479204, -12288.135154014093, -12277.747591475792,
     -12260.721277932555, -12282.315135804598, -12262.55127232295,
     -12306.669956743219, -12269.31351562593, -12269.100587798812,
     -12259.716077931515, -12271.625418264877, -12288.1766629993,
     -12268.776276136585, -12268.158455240895, -12260.565588129277,
     -12290.867095388665, -12266.575671119715, -12269.577241578409,
     -12255.941911533206, -12268.303206320841, -12269.384646445507,
     -12247.349246106034, -12262.88723343311, -12251.175131279826,
```

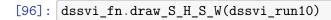
```
-12276.80399166911, -12251.277955675316, -12246.702261713262, -12250.812613063445, -12254.195483332673, -12264.590357206887, -12270.371031556435, -12253.410071260429, -12246.416391657667, -12264.906593085658, -12278.515680771356, -12280.207070536913, -12250.051098328648, -12267.247603519896, -12265.374203257252, -12252.396747923996, -12277.126825736534, -12267.619517724914, -12256.771549100786, -12269.581539283092, -12259.645858745955, -12290.665155322211, -12253.452623884627, -12301.567128144363, -12247.088678965183, -12230.96626050696, -12242.25358184581, -12257.68089293858, -12245.362650621448, -12243.150432715713, -12284.92910843033, -12276.166624015686, -12276.086153008895, -12273.672056228277, -12258.840977316095, -12252.764606000217, -12269.212923500054, -12242.634101225482, -12242.17864735025, -12262.49249877956, -12275.43125759339, -12265.447611165977, -12268.485086509343, -12263.318907763234, -12239.153749787049]
```

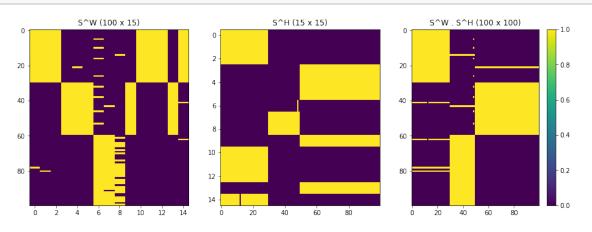


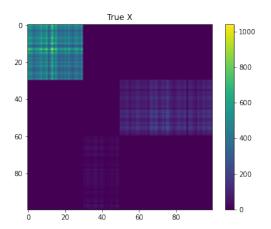
```
[95]: plot_pi(dssvi_run10, label=False)
```

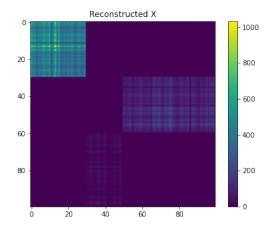


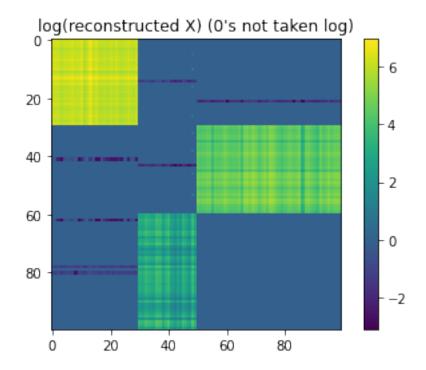












$2 K = 100, burn=200, max_iter=200$