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Singular Value Decomposition (SVD)

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Matrix Transpose:

$$\begin{bmatrix} 1 & 2 \end{bmatrix}^{T} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{T} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}^{T} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

OPTIONAL: R code

```
help(matrix)
A<-matrix(c(1,2),nrow=1,ncol=2,byrow=TRUE)
A
t(A)
B<-matrix(c(1,2,3,4),nrow=2,ncol=2,byrow=TRUE)
B
t(B)
C<matrix(c(1,2,3,4,5,6),nrow=3,ncol=2,byrow=TRUE)
C
t(C)
```

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Matrix Multiplication:

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1p} \\ B_{21} & B_{22} & \cdots & B_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ B_{m1} & B_{m2} & \cdots & B_{mp} \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} (AB)_{11} & (AB)_{12} & \cdots & (AB)_{1p} \\ (AB)_{21} & (AB)_{22} & \cdots & (AB)_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ (AB)_{n1} & (AB)_{n2} & \cdots & (AB)_{np} \end{pmatrix}$$

$$(AB)_{ij} = \sum_{k=1}^{m} A_{ik} B_{kj}.$$

$$A_{3x2} \cdot B_{2x4} = C_{3x4} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix} =$$

$$= \begin{pmatrix} 1\cdot1+2\cdot5 & 1\cdot2+2\cdot6 & 1\cdot3+2\cdot7 & 1\cdot4+2\cdot8 \\ 3\cdot1+4\cdot5 & 3\cdot2+4\cdot6 & 3\cdot3+4\cdot7 & 3\cdot4+4\cdot8 \\ 5\cdot1+6\cdot5 & 5\cdot2+6\cdot6 & 5\cdot3+6\cdot7 & 5\cdot4+6\cdot8 \end{pmatrix} = \begin{pmatrix} 11 & 14 & 17 & 20 \\ 23 & 30 & 37 & 44 \\ 35 & 46 & 57 & 68 \end{pmatrix}$$

OPTIONAL R Code

```
A<-
matrix(c(1,2,3,4,5,6),nrow=3,ncol=2,byrow=
TRUE)
A
B<-
matrix(c(1,2,3,4,5,6,7,8),nrow=2,ncol=4,byro
w=TRUE)
B
C<-A%*%B
D<-t(B)%*%t(A) ## note, B%*%A is not
possible;
```

how does D look like?

$AB \neq BA$

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Matrix Inverse:

If, *A B I*,identity matrix, Then, B = A^{-1}

Identity matrix

$$\begin{bmatrix} 1\,0 \ldots 0\,0 \\ 0\,1 \ldots 0\,0 \\ \ldots\ldots\ldots\ldots \\ 0\,0 \ldots 1\,0 \\ 0\,0 \ldots 0\,1 \end{bmatrix}$$

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OPTIONAL R Code

How to create n×n

Identity matrix?

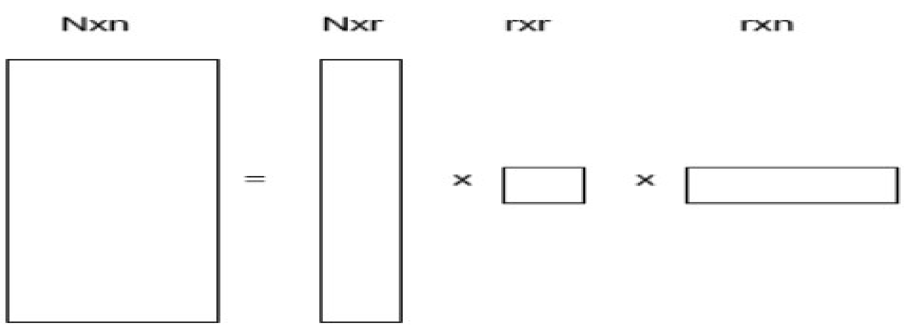
help(diag)

A<-diag(5)

find inverse of a matrix

solve(A)

$X = U\Sigma V^T$



U and V are orthonormal matrices

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OPTIONAL R Code

$$\overset{\mathbf{X}}{\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix}} = \overset{\mathbf{U}}{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix}} \times \overset{\mathbf{\Sigma}}{\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2.24 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} \times \overset{\mathbf{V}^T}{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0.45 & 0 & 0 & 0 & 0.89 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}}$$

```
>M=matrix(c(1,0,0,0,0,0,0,4,0,3,0,0,0,0,0,0,2,0,0,0),nrow=4,ncol= 5)
> X=svd(M)
> X$u
> X$d
> X$v
> X$u%*%diag(X$d)%*%t(X$v)
```

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Applications of SVD in image Processing

– Closest rank-k approximation for a matrix - X

$$X^k = \sum_{i=1}^{k} U_i \sum_i V_i^T$$

– Each term in the summation expression above is called principal image

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Original matrix (X)					Original size
1	0	0	0	2	4*5=20 bytes
0	0	3	0	0	
0	0	0	0	0	
0	4	0	0	0	

X						U					Σ					V^T				
1	0	0	0	**2**		0	0	1	0		4	0	0	0		0	1	0	0	0
0	0	**3**	0	0		0	1	0	0		0	3	0	0		0	0	1	0	0
0	0	0	0	0	=	0	0	0	-1	×	0	0	2.24	0	×	0.45	0	0	0	0.89
0	**4**	0	0	0		1	0	0	0		0	0	0	0		0	0	0	1	0

k=1																	
0	×	4	×	0	1	0	0	0	=	0	0	0	0	0	Compressed size
0										0	0	0	0	0	4*1+1+1*5=10 bytes
0										0	0	0	0	0	
1										0	4	0	0	0	

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k=2																	
0	0	×	4	0	×	0	1	0	0	0	=	0	0	0	0	0	Compressed size
0	1		0	3		0	0	1	0	0		0	0	3	0	0	4*2+2+2*5=20 bytes
0	0											0	0	0	0	0	
1	0											0	4	0	0	0	

k=3																				
0	0	1	×	4	0	0	×	0	1	0	0	0	=	1	0	0	0	2	Compressed size
0	1	0		0	3	0		0	0	1	0	0		0	0	3	0	0	4*3+3+3*5=30 bytes
0	0	0		0	0	2.24		0.45	0	0	0	0.89		0	0	0	0	0	
1	0	0												0	4	0	0	0	

k=4																							
0	0	1	0		4	0	0	0		0	1	0	0	0	=	1	0	0	0	2	Compressed size
0	1	0	0	×	0	3	0	0	×	0	0	1	0	0		0	0	3	0	0	4*4+4+4*5=40 bytes
0	0	0	-1		0	0	2.24	0		0.45	0	0	0	0.89		0	0	0	0	0	
1	0	0	0		0	0	0	0		0	0	0	1	0		0	4	0	0	0	

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The image compression example in http://journal.batard.info/post/2009/04/08/svdfun-profit

- Original size = 384*384 bytes = 147,456 bytes
- k=1: 384*1+1+1*384=769 bytes
- k=10: 384*10+10+10*384=7,690 bytes
- k=20: 384*20+20+20*384=15,380 bytes
- k=50: 384*50+50+50*384=38,450 bytes
- k=100: 384*100+100+100*384=76,900

bytes

- k=200: 384*200+200+200*384=153,800 bytes