Opportunities and challenges for municipal-scale biogas production from farming in southern Brazil

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***Abstract*—Waste-to-energy technologies like anaerobic digestion allow for the conversion of animal manure to production of biogas, a clean alternative fuel with wide applications in cooking and transportation. Animal manure is a substrate common to livestock farmers in southern Brazil, opening up the region to stakeholder interest in biogas projects. Multidisciplinary optimization performed at a municipal scale of small farms in the city of Curitiba in Paraná, Brazil found that the value of biogas is tempered by the cost of its construction and operation, and significant economic incentives are necessary to support future biogas projects to offset the financial toll on small farmers.**

***Keywords*—biogas, anaerobic digestion, multidisciplinary optimization**

I. Introduction

Though waste-to-energy technologies exist as established solutions for end-of-life waste treatment and energy generation for heat, power, and transportation, these solutions have largely found applications in high-income, urban areas. In rural areas, untreated and uncollected organic waste from animals on small farms is mostly dumped out in the open or washed into water channels, such as streams and rivers, causing local air and groundwater pollution. Consequently, this pollution also leads to significant harm to public health and a great loss of untapped potential in energy savings.

With the use of waste-to-energy technologies, this issue can be addressed to a great extent. Untreated organic waste can be collected and converted into consumable energy sources, such as methane and fertilizer, which can be used on farm or sold for additional revenue for farmers. This conversion allows farmers to not only reduce pollution, but also generate financial benefits to farms. Studies estimate that the waste-to-biogas potential contained within animal waste across small farms in Brazil could replace the country's total need for diesel and gasoline imports and supplement up to 44% of the total diesel demand [1].

While the outlook is promising, technical and social challenges within plant design and project implementation complicate the framework necessary for project success and longevity. Creating a sustainable bio-animal waste-to-energy system requires significant improvement and innovations particularly in two key areas. First, the technical efficiency of the waste-to-energy system must be high in order to generate enough units of gas from the amount of feedstock collected. Next, the financial sustainability of the system installation and operations is essential, which includes the financial implications of maintenance and operation costs, the price of gas sold to buyers, the incentives available for biogas producers, and payments for environmental services.

In this project, we explored an optimal system for decentralized biogas production through multidisciplinary modeling as a case study centered around municipalities in the Santa Catarina and Paraná regions of southern Brazil to balance energy capability with stakeholder satisfaction.

II. Background

A. Industry Growth

The beginning of 2020 saw more than 400 biogas plants in operation in Brazil, an increase of 40% compared to the previous year. The Brazilian Biogas Association believes it has been a successful year for the sector, which expanded its number of plants with large-scale projects underway totaling around R$700 million of investments [2]. Most of these plants are small-scale, processing up to 300,000 m^3 of biogas per year, and have been constructed and launched within the past five years since 2016. The biogas produced by these plants finds applications for electricity in the agroindustry [3]. While some regions support this production through crops like sugarcane and cassava, in south Brazil the prominent agroindustry farms produce livestock such as cattle, swine, and chicken. In this region, biogas plants serve additional opportunity to reduce the environmental impact of animal manure which, when improperly disposed of, can damage both public health and other renewable energy generation practices like hydropower operations which make up 12.4% of Brazil's internal energy supply [4].

B. Operations

Manure-based biogas plants works as follows: in a farm or ranch setup, manure is collected from a dairy, feedlot, piggery, or chicken farm and is transported to a digester facility to undergo anerobic digestion. Our model handles transportation via a single truck that travels to various farms in the surrounding area to collect manure. This truck drops the manure into a half-buried concrete pit with an impermeable cover. The biogas is temporarily collected under the cover before being transported to a storage tank, where it can later be retrieved. These operations need to be monitored daily to ensure the process is and the gas pipes are working safely. From this process, we capture biogas and generate organic fertilizer after 30 to 60 days of digestion in a digester of basic configuration as in Fig. 1.

III. Methods

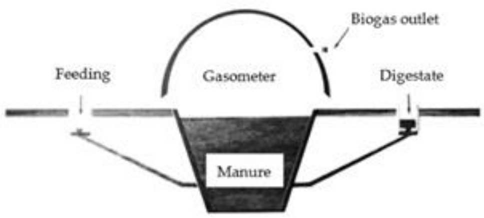

Fig. 1. Basic structure of an anerobic digestion unit.

A multidisciplinary design optimization (MDO) model was designed and built in Python[1] to simulate the operations of an anaerobic digestion system best able to meet the needs of localized farming operations, including farms that surround a specific municipality. The model design is described here.

A. Problem Statement

The purpose of the model is to maximize income and carbon captured from animal manure in Paraná in Brazil by generating biogas and other products at a municipal level using known biochemical treatment methods.

B. Objectives

The main objectives of the model are to

- maximize the net present value (NPV) of the system in Brazilian reals, R$, and
- maximize the global warming potential (GWP) captured in m^3 of carbon dioxide equivalents, or CO_2e, as defined by the US EPA [5].

These objectives are represented quantitatively in (1), where λ is the weight from zero to one on the objectives for optimization.

$$J(x) = \lambda * \max(NPV) + (1 - \lambda) * \max(GWP) \qquad (1)$$

C. System Model Design

A block diagram of our system was developed to capture the flow of the main processes and sub-systems, or modules, within the overall model. This diagram is shown in Fig. 2 to highlight the communication and dependencies between respective transportation, digester, biogas, fertilizer, and cost & profits modules. The high-level, first-order design structure matrix (DSM) constructed to identify respective dependencies between these modules is shown in Fig. 3.

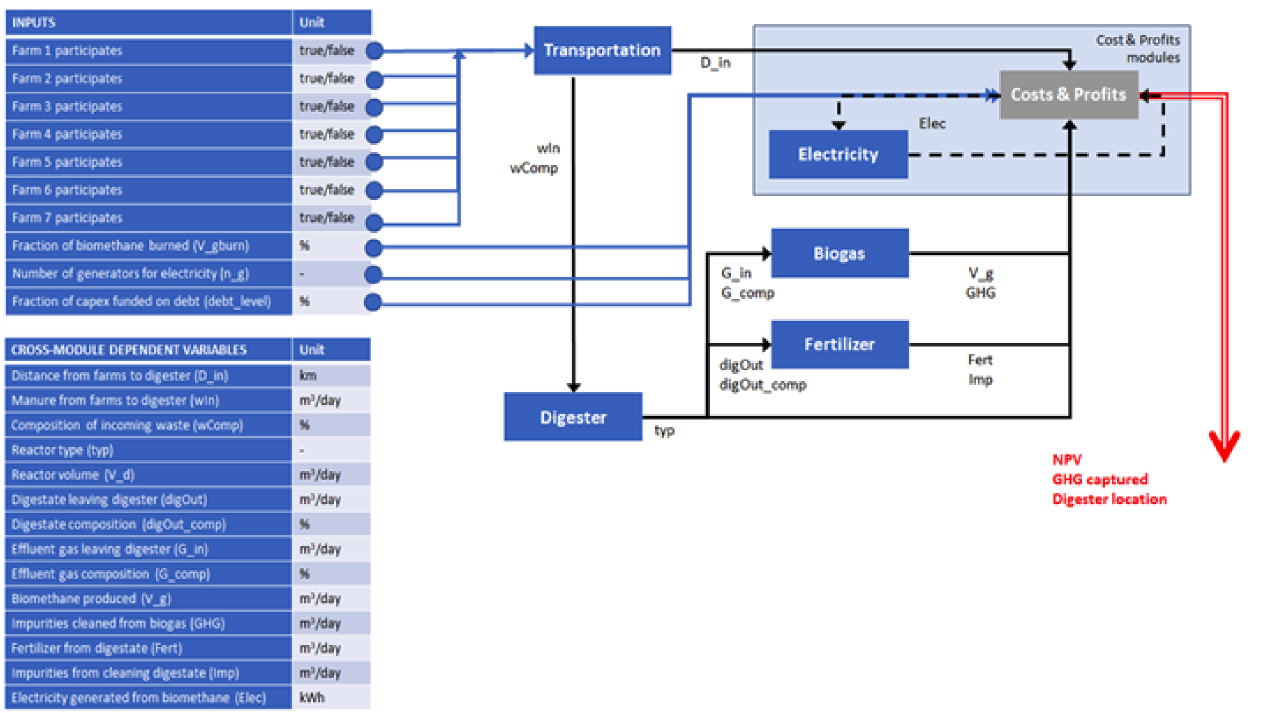

Fig. 2. Block diagram of a localized digester system.

[1] Tool can be viewed at https://tinyurl.com/MDObiogas.

Modules	Inputs	Cost & Profits	Transport	Digester	Fertilizer	Biogas	Electricity	Out puts
Cost & Profits	D_in, Elec, V_g, GHG, Fert, Imp	X	D_in	X	Fert, Imp	V_g	V_gburn	NPV
Transportation	D_in, manure_in	X		wIn, wComp				
Digester	wIn, wComp	X			digOut, digComp	G_in, G_comp		
Fertilizer	digOut, digOut_comp	X						
Biogas	G_in, G_comp	X						GWP
Electricity	V_g, n_g	X						

Fig. 3. Design Structure Matrix (DSM) of the main system.

The block diagram and DSM were further decomposed into a second-order composition as a master variable list. This master variable list contains all the design variables, parameters and constants used within the model. The approach ensured the standardization of names, definitions, and metrics between respective modules, and enabled effective interface management between the different modules. For example, the Cost & Profits module references the parameter *P_g* as the prices of gas sold in units of R$/kg.

Table 1 shows the 11 design variables, or inputs to the system, along with selected parameters. These variables can be changed to model and optimize expected system performance. The following section describes the contribution each module has on the overall system as well as their immediate objectives and constraints. A detailed variable list of the variables, parameters, and constants used across all modules can be found in Appendix A.

TABLE I. DESIGN VARIABLES AND SAMPLE PARAMETERS

Variable	**Inputs to System**		
	Description	***Unit***	***Range***
Design Variables			
Vgburn	Percent of gas produced per annum (pa) that is also consumed by the digester.	Percent	Real [0%, 100%]
n_g	Number of generators used to generate electricity.	Integer	Integer ">=1"
debtlevel	Percent of total capex required funded with debt.	Percent	Real [0%, 100%]
Vcng	Percentage of biogas converted into compressed natural gas (CNG) .	Percent	Real [0%, 100%]
F1 – F7	Whether a farm within the system (7 farms) are delivering their manure into the digester. There are 7 design variables for 7 different farms. A "1" indicates "Yes", the farm is transporting and delivering its manure into the digester and a "0" indicates "No", it does not contribute to the digester system (i.e. does not deliver manure to the digester).	Boolean	True/False
Sample Parameters			
rxVCap	Percent increase in digester size above feedstock volume; fixed/optimal value to 30%.	Percent	Real [10%, 100%]
P_g	Price of biogas sold; fixed/optimal value to 3.05.	R$/kg	Real [0, 10]
P_l	Price of fertilizer sold; fixed/optimal value to 3.	R$/kg	Real [0, 10]
E_priceS	Price of electrical energy sold; fixed/optimal value to 0.	R$/kWh	Real [0, 10]
E_c	Energy consumed inside the farm; fixed/optimal value to 121.2.	kWh/year	Real [100, 140]
E_priceB	Price of buying electrical energy; fixed/optimal value to 0.59.	R$/kWh	Real [0, 10]
C_rskm	Cost of transport including diesel and fixed cost; fixed/optimal value to 3.	R$/km	Real [0, 10]

1) Transportation

The objective of this nested optimization module is to minimize the total distance traveled to dispense manure across all the manure-contributing farms based on their GPS coordinates to the digester. In doing so, the transportation module also optimizes the location of the digester and the route traveled by the manure transportation truck. It utilizes a combination of first principles and heuristic techniques to optimize the travel order of the truck to different farms. The seven farms directly referenced in this project are located in Curitiba, Paraná of Brazil.

Unique constraints relevant to this module include

- At least one farm must dispense manure to the digester.
- The transport truck begins and ends at digester location every day.
- The truck capacity is <= 18 m^3.
- The digester is located at the farm closest to the weighted average GPS coordinate of all farms that dispense manure.

Initially, the transportation module used simulated annealing to minimize the total distance traveled. The tuning parameters used to influence the convergence speed and performance included the initial temperature *x0*, the temperature increment *L*, and the equilibrium condition *T_min*. The Python library we selected did not allow us to specify a cooling schedule, which is typically a choice between linear or exponential patterns.

Fig. 4 is an example of the optimal route and digester location optimized for all 7 farms being active. The optimal route is [3, 1, 5, 0, 2, 3, 4, 3, 4, 6, 3] with the digester located at Farm 3. The total daily distance traveled is 147.78 km, and the truck delivers 60 m^3 manure each day. Once the truck reaches capacity, it returns to the digester to dispose of its load.

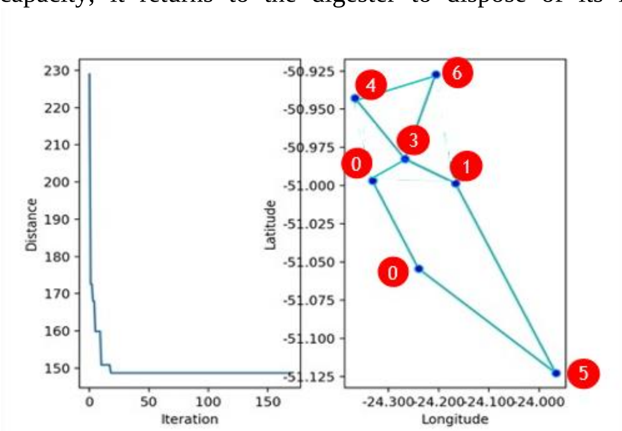

Fig. 4. Results from simulated annealing algorithm for route optimization.

Depth First Search (DFS) was the next heuristic analytical technique attempted for this module. It is an algorithm that physically tests and steps through different paths within a "maze" to determine the optimal route. Because the results were favorable both in performance and consistency and allowed a reduction of computational runtime, DFS was selected for the final version of the model available in the online biogas tool. This is the algorithm currently used by the biogas tool.

2) Digester

The digester module predicts the amount of gas produced from the manure collected from farms. The moisture content of the incoming waste determines the choice of digester between a covered lagoon and upflow system [6], and the volumetric flow at the inlet informs the digester size. Other chemical and physical properties of the waste were used to determine its reactivity by cattle, swine, and chicken manure [7,8]. Previous iterations modeled the digester operation from first principles calculations using a set of coupled ordinary differential equations (ODE) to represent the path of reactants and products through a continuously-stirred reactor [9,10], but to improve biogas yields this framework was replaced with equations fit to empirical data that predict the biogas and fertilizer production directly [11].

Constraints relevant to the digester are that the reactor capacity must be between 10% and 100% greater than the volume required to contain the waste feedstock. This restriction is to account for potential volume expansion in feedstock with rising ambient temperatures or with increases in the quantity of manure produced. In the coupled ODE setting, constraints expanded to included safety limits around operating temperature, mass balances around incoming and effluent streams, and confirmation that the final composition of methane in the effluent gas steam is within ranges expected from literature. These constraints were no longer necessary in the switch to the current version of the module as the use of empirical functions removes the need for a stepwise mass balance, and feedback from our sponsors allow the assumption that the digester operates at ambient temperatures.

3) Biogas

The biogas module maximizes the amount of methane (CH_4), carbon dioxide (CO_2), nitrogen oxide (NOx) and sulfur oxides (SOx) that are generated and captured by the system based on the amount of biogas produced within the digester module. The results of this module inform the second objective of the overall model, which is to maximize captured GWP as in (2) where the coefficients to each gaseous component is its equivalence to CO_2 over 100 years as reported in the Emission Factor Database managed by Intergovernmental Panel on Climate Change (IPCC) [12].

$$max\ GWP = CO_2 + 32\ CH_4 + 282\ NO_x + 282\ SO_x \quad (2)$$

Constraints relevant to the biogas module include the sum of the percentage of biogas consumed by the system and the percent exiting the system being less than or equal to 100%.

The biogas module also governs the amount of biomethane generated from the incoming untreated biogas, which is a mix of different gases such as methane, carbon dioxide, nitrogen oxide, sulfur oxides, and so on. The amount of biomethane produced is dependent on the amount of untreated biogas, the composition rate of methane, which is generally 0.6, and the purity rate of methane, which is regulated by the government.

For our project, we used the purity rate in Brazil which is 0.965. Below is the equation we used to calculate the amount of biomethane produced from the untreated biogas.

$$V_{biomethane} = G_{in} * \frac{CH_4\ recovery\ rate\ from\ biogas}{CH_4\ purity\ rate} \quad (3)$$

4) Fertilizer

The fertilizer module converts the solid and liquid residue remaining in the digester after biogas extraction into fertilizer for crops. The conversion rate of digestate into biofertilizer ranges from 70% to 90% depending on the technology we use. For our project, we used 90% as it is the rate that can be obtained through one of the digestate processing techniques indicated in literature [13]. With the conversion rate set to 90%, the amount of biofertilizer can be obtained simply by multiplying the amount of extracted digestate by 0.9.

5) Cost & Profits

The final module, cost & profits, includes calculations around the costs incurred throughout the preceding modules and the profits available from conversion of biogas to compressed natural gas (CNG) or electricity. The detailed calculations pertaining to this module are available in Appendix C.

The objective of the cost & profits module is to maximize the future free cash flow of the biogas project as per its net present value (NPV), and the decisions within the module are an assortment of empirical models and first principles equations. The objective is

$$Max\ NPV = r - i - C_m - C_t + C_e + f_s - C_{prod} \quad (4)$$

subject to

$$0 \le Debt_{level} \le Debt_{maxLevel} \quad (5)$$
$$n_g = 1, 2, 3, \ldots \quad (6)$$
$$0 \le V_{gburn} \le 1 \quad (7)$$
$$0 \le V_{cng} \le 1 \quad (8)$$
$$V_{gburn} + V_{cng} = 1 \quad (9)$$

Here, NPV is defined by revenue r, investment i, cost of maintenance C_m, cost of transport C_t, cost of savings of electrical energy C_e, fertilizer f_s, and cost of production C_{prod}.

The overall system was optimized with the Genetic Algorithm method (NSGA-II) from the pymoo Python library[2]. We chose to use genetic algorithms (GA) as we were interested in exploring how “natural selection” and evolution could assist us in obtaining better results through improving fitness. This approach was deemed appropriate for a number of reasons:

- First, our problem uses discrete Boolean and integer design variables and is discontinuous, which restricts our use of techniques that make use of derivatives. When the digester module used coupled ordinary differential equations (ODEs) in particular, we found symbolic expressions to be poorly suited for the numerical integration methods employed.
- Second, GA exposes us to selection, crossover, insertion, and mutation techniques that can be used to test whether a local or global optimum has been reached through random changes within the design vector, or “chromosomes”. We found this probabilistic nature of the transition rules to provide a “get out of jail free” card in testing local optima for convergence.
- Third, the limitation of GA in being unable to specify constraints assisted us in modeling our objective function such that constraints were embedded within modules instead.

IV. Sensitivity Analysis

We conducted a sensitivity analysis against 11 design variables and the seven selected parameters shown in Table 1. This analysis helped us gain a comprehensive understanding on the effects of those variables and parameters on our primary objective function. In this report, we will discuss the analysis results on two of the design variables which we believe have meaningful impacts on our objective. The optimal solution used in the analysis, x^*, was obtained through GA optimization.

Fig 5 below shows plots of the NPV, on the y axis, versus design variables V_{gburn} and *debt_level*, on the x axis. Please note that, as our objective is to maximize NPV but the standard optimization form is to minimize, the values shown are actually negated. Therefore, analyses on the burned gas per annum and total capex funded with debt resulted in a decrease in NPV as shown, as the potential values of most design variables tended to either increase the cost of operations or decrease the amount of biomethane available for sale. Both analyses show linear trends as well, as V_{gburn} and *debt_level* correspond to either integer values from one to three or fractions ranging from zero to one, and are used linearly in cost calculations.

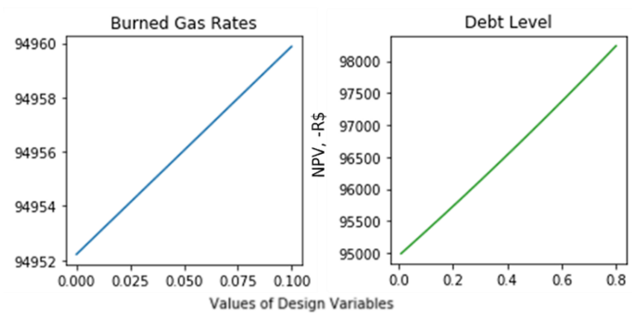

Fig. 5. Sensitivity of NPV to burned gas rate per annum and debt level.

In order to have a clear comparison on the overall impact of these design variables, we normalized the output values of each variable and plotted them in the same vector space. Here, too, the y-axis is NPV and the x axis is a scaled change in design variables as calculated by (10) and (11). For the y axis, we used the maximum absolute scaling method, which allows us to rescale each value between zero and one by dividing every

[2] The pymoo library is available at https://pymoo.org/.

observation by the maximum absolute value. For the x axis, we used the min-max normalization method to bring all the values of different units into a common vector space between zero and one.

$$y_{scaled} = \frac{y}{\max(|y|)} \quad (10)$$

$$x_{norm} = \frac{x - x_{min}}{x_{max} - x_{min}} \quad (11)$$

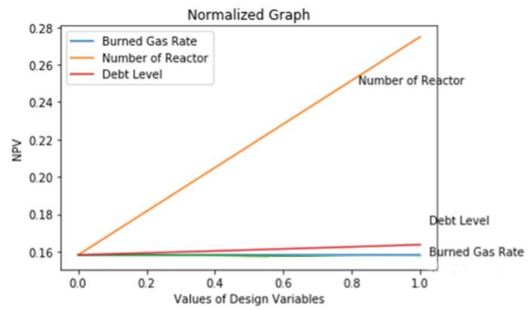

Fig. 6. Normalized sensitivity of NPV to non-Boolean design variables.

Fig. 6 is the resulting normalized graph, and shows the sensitivity of our non-Boolean design variables, including the burned gas rate per annum, the level of debt secured for capex funding, and the number of generators used. The number of generators has the greatest normalized impact on the NPV, and we believe this is because the construction, operation, and maintenance of each generator adds a significant cost to the system.

The sensitivity analysis helped us gain insights on how the output of our optimal solution behaves based on the changes in the design variables and parameters. We were able to identify the main drivers to the significant changes in NPV, and these insights helped us in prioritizing our model's needs for improvement.

V. Design of Experiments and Model Validation

A parameter study was created to evaluate the changes on the respective tuning parameters within the GA. The results from the simulations, including the GA model tuning parameters, are provided in Fig. 7. The respective tuning parameter changes did not materially impact the model results.

Factor	Base	Experiment 1	Experiment 2	Experiment 3	Experiment 4	Experiment 5	Experiment 6	Experiment 7	Experiment 8	Best design
max_num_iteration	500	100	500	500	500	500	500	500	500	500
population_size	100	100	500	100	100	100	100	100	100	500
mutation_probability	0,5	0,5	0,5	0,8	0,2	0,5	0,5	0,5	0,5	0,2
elit_ratio	0,01	0,01	0,01	0,01	0,01	0,1	0,01	0,01	0,01	0,1
crossover_probability	0,2	0,2	0,2	0,2	0,2	0,2	0,5	0,2	0,2	0,5
parents_portion	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,5	0,3	0,5
crossover_type	uniform	uniform	uniform	uniform	uniform	uniform	uniform	uniform	uniform	uniform
max_iteration_without_improv	200	100	200	200	200	200	200	200	300	300
DESIGN VECTOR										
VgBurn%	0,00165	0,00887	0,00040	0,03458	0,00031	0,00033	0,00056	0,00230	0,00203	0,00006
ng	1	1	1	1	1	1	1	1	1	1
debt_level	0,00189	0,00653	0,00041	0,00211	0,00008	0,00030	0,00161	0,00164	0,00024	0,00010
V_cng	0,00011	0,00057	0,00045	0,00026	0,00113	0,00009	0,00020	0,00039	0,00051	0,00011
F1	1	1	1	1	1	1	1	1	1	1
F2	0	0	0	0	0	0	0	0	0	0
F3	0	0	0	0	0	0	0	0	0	0
F4	0	0	0	0	0	0	0	0	0	0
F5	0	0	0	0	0	0	0	0	0	0
F6	0	0	0	0	0	0	0	0	0	0
F7	0	0	0	0	0	0	0	0	0	0
NPV	(97 706,22)	(97 733,14)	(97 699,08)	(97 758,69)	(97 697,67)	(97 698,54)	(97 703,84)	(97 706,07)	(97 700,40)	(97 697,47)
Difference from Base		(26,92)	7,14	(52,47)	8,55	7,68	2,38	0,15	5,82	8,75
Ranking		7	4	8	2	3	5	6	5	1

Fig. 7. Parameter study of GA tuning parameters.

For model validation, we tested the outputs of our modules against hand-calculated results. These tests are captured in an automated test script to ensure model accuracy and a sample are shown in Table 2.

TABLE II. Model Validation

Test	Example of Model Validations		
	Inputs	***Expected Output***	***Results***
Distance traveled by truck	Farm GPS	Less than 250 km/day Greater than or equal to 0km/day	Pass (147.78 km)
Total volume delivered	wIn by farm	60.22	Pass
Biogas production	G_in, G_comp	12.4612	Pass
Biofertilizer production	digOut	114.6413	Pass
GHG released	kilos, wComp	[0.0022, 0.0007, 1.096e-06, 0]	Pass
GHG captured	G_in, G_comp	[0.2492, 0.1427, 1.6895e-05, 0.0084]	Pass

To validate and verify our biogas production, we compared the outputs of the digester module to experimental data from our project sponsors and from literature. These data predicted the biomethane potential for anaerobic digestion to be 210 milliliters CH_4 per gram volatile solids from cow manure, and 323 milliliters CH_4 per gram volatile solids from swine manure [1]. Our model predicts values of 3.12 and 3.78 milliliters CH_4 per gram volatile solids from cow and swine manure, respectively, reflecting the trend that swine manure has a higher potential for biomethane conversion than cow manure. While these values are lower than those found in literature, they show improvements of over 400% and 200% biomethane yield, respectively, over the low values obtained from the first principles iteration of the digester module using coupled ODEs.

VI. Results And Discussion

A. Single-objective Optimization

Our design variables consisted of the volume percent of methane burned to produce electrical energy, V_{gburn}, the debt level for the financing of the digester system *debt_level*, the number of generators used for the generation of electricity, n_g, and the volume percent of gas upgraded to compressed natural gas, V_{cng}, to be used in the transportation of the manure. Additionally, we have seven Boolean inputs, *farm_i*, to determine whether a farm was active or not, "active" meaning the farm was transporting and dispensing their manure into the digester for processing. The model does not contain explicit loops for design variables. It begins with the transportation module where the distance, manure composition and weight are calculated, then presented as inputs to the digester module to calculate the digester size and the production of methane and fertilizer. Next, the biogas module calculates the greenhouse gases captured, and finally, these values are transferred to the cost & profits module where the model estimates cash flows and returns the system NPV.

The design variable vector is composed of seven Booleans, one positive integer, and three real variables. As mentioned previously, we found it difficult to calculate the gradient of this discontinuous system so we used a heuristic method for optimization. Nonetheless, using finite differences we were able to estimate the second order derivative of the real design variables, from which it was possible to conclude that the Hessian is not ill-conditioned. Our primary objective function is to the maximize net present value for the farmer. This objective was chosen despite the increasing public interest for cleaner energy sources because the project should be feasible as a financial decision to be attractive to the majority of farms and consequently widely adopted.

The genetic algorithm convergence history is shown in Fig. 8 and the run time is approximately 13.5 seconds. The optimal values generated by the model indicate that one generator should be used and all of the methane produced should be burned for electricity. Due to the random nature of GA, it is difficult to obtain results exactly equal to 1 or 0, the bounds of our system, but through increasing the number of iterations the GA population it became clear that the algorithm converges near these bounds, yielding an NPV of -R$97,706.22. The parameters used in the genetic algorithm were:

- population size = 100,
- iterations = 100,
- mutation rate = 2%,
- crossover = 20%, and
- number of offspring = 10

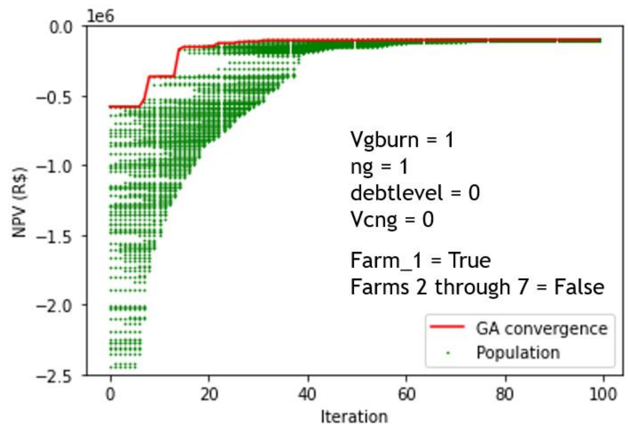

Fig. 8. GA convergence history

After several runs with varying parameters, the algorithm always converged to the same result, so our final parameter set is of a small population size and iterations. The negative NPV suggests that, based solely in the finances of the project, this biogas system should not be implemented. This result is consistent with current small farms in Paraná and Santa Catarina in Brazil that is not profitable.

Analyzing the optimal solution brings up further points of interest. First, the only active is farm 1, which, not coincidentally, is the smallest farm and produces lowest daily amount of manure. This preference suggests that the operations cost, linearly proportional to the size of the digester, is higher than the revenue generated by the system. Next, V_{cng} being zero is expected as the CNG produced is expected to go into the transport of the manure. However, with only a single farm active, there is no need to transport manure between farms. A single generator is enough to produce electricity from all of the methane produced, so n_g and V_{gburn} are equal to one. Lastly, and somewhat counterintuitively, the debt level for our optimal design is zero, which means that the system is fully funded by equity. The cost of debt is usually smaller than the cost equity, but requires interest to be paid periodically. Failure to pay could result in the debtholder taking over the company and the equity, so avoiding debt from the beginning helps farmers avoid delinquency. To this end, the system generates negative cash flows over the years of operation, and the actions the optimizer can take to move cash flows closer to zero are limited to:

- deactivating farms, within constraints,
- maximizing electricity generation,
- minimizing the number of generators, and
- increasing the weighted average cost of capital (WACC)

Equation (12) exemplifies what the optimizer is trying to achieve, where n is the year of operation and CF_n is the cash flow corresponding to that year. As the WACC approaches infinity, NPV converges to zero.

$$NPV = \sum_{n=1}^{10} \frac{CF_n}{(1+WACC)^n} \quad (12)$$

B. Multi-objective Optimization

Biogas is an alternative clean energy source. To show its environmental impacts, we added a second objective metric of maximizing the global warming potential captured in equivalent CO_2 over 100 years. The manure feedstock emits CO_2, CH_4, and NOx during processing in the digester. The estimated CO_2e from these emissions is calculated within the model, and the resulting tradespace is shown in Fig. 9. This plot was generated using the NSGA-II algorithm from the pymoo Python library. The Pareto front is shown by the red line and each blue point represents one possible design variable that was simulated in the iterations of the NSGA-II algorithm. It takes 67 seconds to run and plot these points in the tradespace, and we have shifted the axes to better visualize the area surrounding the utopia point. Of the three points highlighted in the Pareto front, point 1 was already discussed in the single-objective optimization section as the scenario where only one farm is active and all of the methane is burned for electricity.

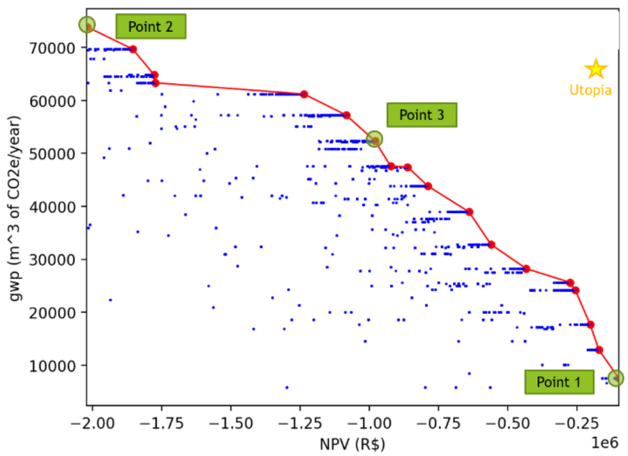

Fig. 9. Multi-objective tradespace and Pareto front.

Point 2 in Fig. 9 reflects when the system is optimized solely for the yearly GWP captured. Here all farms are active and dispense their manure into the digester. Point 3 is in the middle ground where both objectives are active, the farmer's loss is approximately R$1 million, and the GWP captured is 52,000 m^3 of CO_2e per year. The values of the design variables for each point is shown in Table 3, where active farms are "true" and inactive farms are "false".

TABLE III. TRADESPACE PARETO POINT SAMPLES

Variable/Result	Sample Points		
	Point 1	***Point 2***	***Point 3***
Vgburn	1	0.56	0.92
n_g	1	3	3
debt_level	0	0	0
Vcng	0	0.42	0
Farm_1	TRUE	TRUE	TRUE
Farm_2	FALSE	TRUE	FALSE
Farm_3	FALSE	TRUE	TRUE
Farm_4	FALSE	TRUE	FALSE
Farm_5	FALSE	TRUE	FALSE
Farm_6	FALSE	TRUE	TRUE
Farm_7	FALSE	TRUE	TRUE
NPV	R$(97,706)	R$(980,000)	R$(2,000,000)
GWP/year	7600	52,000	74,000

C. Feasibility Considerations

One way to make the system financially feasible for the farmer is through government incentives in the form of tax breaks or monetary compensation for the GWP captured. For the three scenarios above, farmers would need incentives of R$1.28, R$1.88 and R$2.70 per m^3 of CO_2e captured for points 1, 2 and 3 respectively. These incentives can be justified by other economic impacts, including reduced land rehabilitation costs, reduced healthcare costs related to the inhalation of toxic gases, reduced odors from livestock manure, and increased crop yields by applying organic fertilizer among other benefits difficult to quantify economically.

D. Model Performance Improvements

We implemented various changes to our model over the course of this project and found some decisions to have dramatic effects on the runtime. Our initial model ran for approximately five hours and contained both the genetic algorithm for the larger system and the simulated annealing algorithm for the transportation module. Key factors in reducing runtime lay in the transportation and digester modules.

First, pre-optimizing the nested transportation module by generating a data set of all possible directions and storing it in a data table to be re-used reduced runtime to 22.32 minutes. By restricting the optimal route calculation to this data table, we avoided having the module recalculate the next step in the route for each run (significant computation time saved).

Next, replacing the coupled ODEs of the digester model with a simpler surrogate model from empirical data further reduced runtime to 13.52 seconds. Reducing the model complexity from multiple numerical integration to heuristics removed the bottleneck of processing speed at the digester module.

Finally, the time our model takes to process changes in a single design variable is 0.03 seconds.

VII. Conclusions and Future Work

This work demonstrates that the construction and operation of a biogas plant at the municipal level is not financially viable as a standalone entity to process animal waste on small farms in southern Brazil. We identify a need for economic incentives in the form of subsidies or energy credits to supplement immediate project costs and reward users for the environmental benefits realizable from anaerobic digestion over traditional farming practices.

Barring these incentives, our final recommendation is for small farms in our region of study to construct and operate individual digesters under a decentralized framework. These results complement the current state of the biogas industry in that, while waste-to-energy technologies present a sustainable pathway to reducing waste through its repurposing and added value, this conversion has yet to achieve cost parity with existing methods of waste disposal and processing.

Future work in this space includes improving the accuracy of the digester module to assess whether potential increases in biogas yield raise the total revenue enough to justify the corresponding increase in model runtime, expanding the waste collection system to include more farms and more manure and potentially reduce the marginal costs of digester construction, and quantifying the economic benefit of other factors impacted by biogas production such as improved crop yields from nutrient-rich soil and reduced dependence on antibiotics for livestock from decreased air and water pollution. These supplementary economic benefits to be realized can support a case for government subsidies or other economic incentives.

Major learnings from this course include the idea of problem formulation as a dynamic, not static, process of iterative decision-making given industry background from our sponsors and practical knowledge from our own problem-solving strategies. We found the variable master list to be invaluable in module integration and interface management, as knowing what variables were called and what units they were reported in for the various modules was essential throughout the entire model. We also found the ability to significantly reduce runtime from hours to seconds through reducing model complexity to be incredibly useful as it allowed for swift turnarounds in finding and resolving problems that arose through model validation.

Additionally, we learned that developing and deploying an interactive web app with a user-friendly frontend significantly empowers stakeholder project planning and sparks curiosity in those new to the field. The online tool we created allows interested parties to run their own customized scenarios and draw their own conclusions independently of the primary modeler.

Acknowledgment

Throughout the duration of this project, we have received a great deal of support and guidance. It would not have been possible for us to achieve this much without the kind support and help of many others. We would first like to thank our teaching team, Dr. Maha Haji, Prof. Olivier de Weck, Johannes Norheim, and Elwyn Sirieys, whose expertise and guidance were invaluable in concretizing the foundation of knowledge for conducting the project.

We would also like to express our sincere gratitude to our sponsors, Dr. Afreen Siddiqi, Janaina Pasqual, and Felipe Souza Marques. We want to thank all their valuable insights and patient support given to further improve our project.

In addition, we would like to thank each of our team members for their patience and dedication in completing the project successfully.

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APPENDIX A: MASTER VARIABLE LIST (2ND-LEVEL DECOMPOSITION)

Symbol	Units	Size	type variable	Description	type	Modules Resp.	Used in	cross module?
a_d	R$/m^3	1	float	fit line aV_d+b to estimate digester investment cost	Parameter	Cost	Cost	FALSE
b_d	R$	1	float	fit line aV_d+b to estimate digester investment cost	Parameter	Cost	Cost	FALSE
p_f	R$/L	1	float	Price of fuel sold	Constant	Cost	Cost	FALSE
p_g	R$/kg	1	float	Price of gas sold	Parameter	Cost	Cost	FALSE
p_l	R$/kg	1	float	Price of Fertilizer sold	Parameter	Cost	Cost	FALSE
e_priceS	R$/kWh	1	float	Price of electrical energy sold	Parameter	Cost	Cost	FALSE
p_bf	R$/kg	1	float	Market price of Fertilizer	Parameter	Cost	Cost	FALSE
f_used	kg/ha	1	float	Fertilizer used inside the farm	Parameter	Cost	Cost	FALSE
e_c	kWh/year	1	float	Energy consumed inside the farm	Parameter	Cost	Cost	FALSE
e_priceB	R$/kWh	1	float	Price of buying electrical energy	Parameter	Cost	Cost	FALSE
L	years	1	integer	life time of digester	Parameter	Cost	Cost	FALSE
k_e	-	1	float	expected return of agriculture in Brazil	Parameter	Cost	Cost	FALSE
k_d	-	1	float	Interest paid in debt	Parameter	Cost	Cost	FALSE
g_d	R$/unit	1	float	Cost of an energy generator using biogas	Parameter	Cost	Cost	FALSE
g_m	R$/year	1	float	Maintenance cost of generator	Parameter	Cost	Cost	FALSE
i	R$	1	float	Investment	Dependent variable	Cost	Cost	FALSE
r	R$	1	float	Revenue	Dependent variable	Cost	Cost	FALSE
c_m	R$	1	float	Maintenance cost	Dependent variable	Cost	Cost	FALSE
c_t	R$	1	float	Transportation cost	Dependent variable	Cost	Cost	FALSE
c_e	R$	1	float	Electrical energy cost savings	Dependent variable	Cost	Cost	FALSE
c_p	R$	1	float	Pollution avoided cost savings	Dependent variable	Cost	Cost	FALSE
f_s	R$	1	float	Cost of fertilizer saved	Dependent variable	Cost	Cost	FALSE
e_s	kWh	1	float	Energy sold to energy company	Dependent variable	Cost	Cost	FALSE
c_f	R$	1	float	Direct cost of fuel sold	Parameter	Cost	Cost	FALSE
c_g	R$/m^3	1	float	Direct cost of gas sold	Dependent variable	Cost	Cost	FALSE
c_l	R$	1	float	Direct cost of fertilizer sold	Dependent variable	Cost	Cost	FALSE
w_l	kg	1	float	Weight of fertilizer sold	Dependent variable	Cost	Cost	FALSE
i_m	R$/year	1	float	Yealy cost of maintenance	Dependent variable	Cost	Cost	FALSE
D	km/year	1	float	Total distance covered by the trucks	Dependent variable	Cost	Cost	FALSE
V_d	m^3	1	float	Digester Volume	Parameter	Digester	Cost	TRUE
D_in	km/truck	1	float	av. distance from farm to digester	Design Variable	Transportation	Cost	TRUE
V_year	m^3/year	1	float	Annual volume of manure	Design Variable	Cost	Cost	FALSE
V_day, wln	m^3/day	1	float	Daily volume of manure	Design Variable	Transportation	Digester	TRUE
e_p	kWh/year	1	float	Annual electrical energy production	h(x)	Cost	Cost	FALSE
V_f	L/year	1	float	Annual fuel production	h(x)	Cost	Cost	FALSE
V_g	m^3/year	1	float	Annual biomethane gas production	Parameter	Biogas	Cost	TRUE

Fert	kg/year	1	float	Annual biofertilizer production	Parameter	Biogas	Cost	TRUE
W_a	kg/year	1	float	Annual weight of waste/manure processed	Dependent variable	Digester	Cost, Biogas	TRUE
typ	-	1	(0,1)	Type of digester (0=lagoon, 1=upflow)	Design Variable	Digester	Cost	TRUE
Farm1_lat	degrees	1	float	latitude degrees where Farm1 is located	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm1_lon	degrees	1	float	longitude degree where Farm1 is located	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm2_lat	degrees	1	float	latitude degrees where Farm2 is located	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm2_lon	degrees	1	float	longitude degree where Farm2 is located	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm3_lat	degrees	1	float	latitude degrees where Farm3 is located	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm3_lon	degrees	1	float	longitude degree where Farm3 is located	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm4_lat	degrees	1	float	latitude degrees where Farm4 is located	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm4_lon	degrees	1	float	longitude degree where Farm4 is located	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm5_lat	degrees	1	float	latitude degrees where Farm5 is located	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm5_lon	degrees	1	float	longitude degree where Farm5 is located	Design Variable	Transport ation	Transp ortatio n	FALSE
Digest_lon	radians	1	float	longitude radian of the optimal location for the digester	Dependent variable	Transport ation	Transp ortatio n	FALSE
Digest_lat	radians	1	float	latitude radian of the optimal location for the digester	Dependent variable	Transport ation	Transp ortatio n	FALSE
Farm 1 manure	m3/day	1	float	Manure per day supplied by Farm 1	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm 2 manure	m3/day	1	float	Manure per day supplied by Farm 2	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm 3 manure	m3/day	1	float	Manure per day supplied by Farm 3	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm 4 manure	m3/day	1	float	Manure per day supplied by Farm 4	Design Variable	Transport ation	Transp ortatio n	FALSE
Farm 5 manure	m3/day	1	float	Manure per day supplied by Farm 5	Design Variable	Transport ation	Transp ortatio n	FALSE
distance_t otal, distance	km/day	1	float	Total distance travelled by all trucks to digester	Design Variable	Transport ation	Cost	TRUE
wIn	m3/day	1	float	Total volume of all manure delivered per day	Dependent variable	Transport ation	Digeste r, Cost	TRUE
kilos	kg/day	1	float	Total kilograms of all manure delivered per day	Dependent variable	Transport ation	Transp ortatio n	FALSE

wComp	%	1	float	Weighted Average Percentage of Total Solids contained within the Volume manure delivered	Design Variable	Transportation	Digester	TRUE
manure_comp	%	3	list	Weighted Average Percentage of the manure volumes broken down per Cattle-Pigs-Chickens	Dependent variable	Transportation	Digester	TRUE
c_rskm	R$/km	1	float	Cost of transport including diesel and fixed cost	Parameter	Cost	Cost	FALSE
T_dig	degC	1	float	Reactor operating temperature	Design Variable	Digester	Digester	FALSE
G_in	m^3/day	1	float	Rate of effluent gas produced in reactor	Parameter	Digester	Biogas	TRUE
G_comp	%	4	[float,]	Fraction of CH4, CO2, NOX, SOX in effluent	Parameter	Digester	Biogas	TRUE
digOut	m3/day	1	float	Volumetric flow rate of digestate leaving reactor (PM)	Parameter	Digester	Biogas	TRUE
digOut_comp	%	3	[float,]	Composition in % of digestate leaving reactor [PM,NOX,SOX,Inert,Water]	Parameter	Digester	Biogas	TRUE
man_TS	%	3	float	Manure total solids % by animal type	Constant	Digester	Digester	FALSE
man_VS	%	3	float	Manure volatile solids % by animal type	Constant	Digester	Digester	FALSE
cng	days/year	1	integer	Working days per year	Constant	Cost	Cost	FALSE
working_hours	hours/day	1	integer	Working hours per day	Constant		Digester, Cost	TRUE
n_g	units	1	Natural	Number of generators	Design Variable	Cost	Cost	FALSE
CH4_comp	%		float	Rate of CH4 composition in biogas	Constant	Biogas	Biogas	FALSE
CH4_pur	%		float	Rate of CH4 purity in biomethane	Constant	Biogas	Biogas	FALSE
V_bm	scm/day		float	Biomethane produced from biogas	Dependent variable	Biogas	Cost	TRUE
kilos	kg/day	1	float	Total kilograms of all manure processed per day	Dependent variable	Transportation	Transportation	FALSE
vs_r	%		float	Rate of 'kilos' (manure) that is potentially converted to biogas	Dependent variable	Biogas	Biogas	FALSE
vs	kg/day		float	Total amount of volatile solid	Dependent variable	Biogas	Biogas	FALSE
PDY	kg/day		float	Potential Biofertilizer Yield produced from the manure AD process	Dependent variable	Biogas	Cost	TRUE
farmer_npv	R$	1	float	Farmer NPV	g(x)	Cost	Output	TRUE
g_eff	%	1	float	Generator efficiency	Parameter	Cost	Cost	FALSE
e_densitygas	kWh/m^3	1	float	Energy density of biomethane	Constant	Cost	Cost	FALSE
g	m/s^2	1	float	Gravity	Constant	Cost	Cost	FALSE
h_water	m	1	float	Height of water below ground	Parameter	Cost	Cost	FALSE
eff_pump	%	1	float	Pump efficiency	Parameter	Cost	Cost	FALSE
g_power	kW	1	float	Generator Power	Parameter	Cost	Cost	FALSE
V_gburn	m^3/year	1	float	Burned gas	Design Variable	Cost	Cost	FALSE
debt_level	%	1	float	Debt level of the project	Design Variable	Cost	Cost	FALSE
max_debt	%	1	float	max debt level acceptable	Parameter	Cost	Cost	FALSE
rxVCap	%	1	float	% larger than feedstock to size	Parameter	Digester	Digester	FALSE
tax	%	1	floar	Tax on profits. To the purpose of getting a tax shield	Parameter	Cost	Cost	FALSE
V_cng_p	%	1	float	Percentage of V_g used to convert to CNG	Design Variable	Cost	Cost	FALSE
V_cng	m^3/year	1	float	Volume of CNG produced	h(x)	Cost	Cost	FALSE

c_t_fixed	R$/km	1	float	Cost of transport excluding fuel	Parameter	Cost	Cost	FALSE
c_t_diesel	R$/km	1	float	Cost of diesel in transport	Parameter	Cost	Cost	FALSE
c_t_CNG	R$/km	1	float	Cost of CNG in transport	Parameter	Cost	Cost	FALSE
D_diesel	km/year	1	float	Total distance covered by the trucks using diesel	Parameter	Cost	Cost	FALSE
D_cng	km/year	1	float	Total distance covered by the trucks using CNG	Parameter	Cost	Cost	FALSE
T_m3_km_cng	m^3/km	1	float	Truck volume of CNG per km	Parameter	Transportation	Cost	TRUE
T_L_km_diesel	L/km	1	float	Truck diesel L per km	Parameter	Transportation	Cost	TRUE
P_diesel	R$/L	1	float	Price of diesel per liter	Parameter	Cost	Cost	FALSE
C_upgrade_cng	R$/m^3	1	float	Cost to upgrade biogas to CNG per m^3	Parameter	Cost	Cost	FALSE
C_V_gas	R$/m^3	1	float	Cost to produce biogas per m^3	Parameter	Cost	Cost	FALSE

APPENDIX B: MODEL CONSTRAINTS

Module	Type	Description	Verified by
Transportation	Inequality	Daily distance from farm to digester	Must be less than 40 km and greater than or equal to 0km
	Inequality	Volume of manure supplied from farm	Must be greater than 0.1 m^3 per day
	Equality	Manure composition	Composition of cattle-pig-chicken manure must equal 1
	Inequality	Manure composition	Each individual contribution of cattle, pig or chicken must be within the range of 0% and 100%
	Inequality	Solids percentage	Must be greater than 0% and less than 15%
	Inequality	Number of active farms	Number of farms dispensing manure into digester must be greater than 1
	Equality	Manure transport truck start and end	Manure transport truck must begin and end at digester every day
	Inequality	Manure truck capacity	Must be less than or equal to $18m^3$
Digester	Inequality	Digester size above minimum capacity	Digester size falls within 10% and 100% larger than necessary
Biogas	Inequality	The purity of biomethane after the upgrading process	The methane concentration rate must be over 96.5%
	Inequality	Greenhouse gas captured	The carbon dioxide recovery rate to be over 90%
Cost	As shown in appendix C		

APPENDIX C: COST MODULE

The final module of the model is the cost module, this is where the future free cash flows are calculated to estimate the net present value (NPV) of the project. This module uses a mix of empirical models and first order principles. The NPV is defined by the equation below:

$$NPV(r, i, C_m, C_t, C_e, f_s) = r - i - C_m - C_t + C_e + f_s - C_{prod}$$

For the system, NPV is defined by revenue (r), minus investment (i), minus cost of maintenance (C_m), minus cost of transport, plus cost savings of electrical energy (C_e) and fertilizer (f_s), minus the cost of production (C_{prod}). Firstly, it is calculated the weighted average cost (WACC) of capital which will be used as the discount rate.

$$WACC = Debt_{level}\left(1 - tax_{profit}\right)k_d + (1 - Debt_{level})k_e$$

The total yearly distance travel to transport the manure from the farms to the digester, and its decomposition, the distances traveled using CNG and diesel are calculated as follows:

$$D = distance_{travelded} * working_{days}$$

$$\mathrm{D_{cng}} = \frac{\mathrm{V_{cng}}V_g}{T_{m3kmcng}}$$

$$\mathrm{D_{diesel}} = \mathrm{D} - \mathrm{D_{cng}}$$

The cost decomposition for the transport is shown below:

$$\mathrm{C_{t_{diesel}}} = \mathrm{T_{Lkmdiesel}}\mathrm{P_{diesel}}$$

$$\mathrm{C_{t_{fixed}}} = C_{R\$km} - \mathrm{C_{t_{diesel}}}$$

$$\mathrm{C_{cng}} = \mathrm{C_{upgrade_{cng}}}T_{m3kmcng}$$

Finally, it is possible calculate the total cost of transport:

$$\mathrm{C}_t = \sum_{i=1}^{L}\left(C_{R\$km}D_{diesel} + \left(C_{tFixed} + C_{cng}\right)D_{cng}\right)\frac{1}{(1 + WACC)^i}$$

The investment cost shown below, has two main components, the linear relationship between digester volume ($a_dV_d + b_d$) and cost and the investment needed to buy "n" generators ($n_g g_d$).

$$i = a_dV_d + b_d + n_g g_d$$

Similarly, the yearly maintenance cost (i_m), is composed of a percentage ($i_{mainCost}$) of the digester cost plus a cost the maintain the generators (g_m). Finally, it is calculated the life cycle present value of the maintenance cost ($\mathrm{C_m}$).

$$i_m = n_g g_m + i_{mainCost}(a_dV_d + b_d)$$

$$\mathrm{C_m} = \sum_{i=1}^{L} i_m \frac{1}{(1 + WACC)^i}$$

The electrical energy produced by the system is given by the volume of gas burned ($V_{gBurn}V_g$) times the energy density of the gas generated in the digester ($e_{densitygas}$), times the generator efficiency (g_{eff}).

$$e_p = V_{gBurn}V_g e_{densitygas} g_{eff}$$

The energy produced must be smaller than the capacity of the generators to produce electricity running:

$$e_p \le n_g g_{power} working_{days} * working_{hours} g_{eff}$$

The savings in purchasing electrical energy is defined by the smallest of the energy consumed in the farm (e_c), and the energy produced (e_p), times the cost to buy energy from the utility company (e_{PriceB}) and then it is calculated its present value.

$$C_e = \sum_{i=1}^{L} \left(\min\left(e_c, e_p\right) e_{PriceB}\right)\frac{1}{(1 + WACC)^i}$$

The same logic is used for the savings in fertilizer. Where f_{used} is the amount of fertilizer used in the farm, f_p is the amount of fertilizer produced and P_{bf} is the cost to purchase fertilizer.

$$f_s = \sum_{i=1}^{L} \left(\min\left(f_{used}, f_p\right) P_{bf}\right) \frac{1}{(1+WACC)^i}$$

The excess fertilizer available for sale (W_L) is defined by the fertilizer produced (f_p) minus consumed (f_{used}), or zero, whichever is greater.

$$W_L = \max(f_p - f_{used}, 0)$$

Similarly, the excess electricity available for sale (e_s) is defined by the electricity produced (e_p) minus consumed (e_c), or zero, whichever is greater.

$$e_s = \max(e_p - e_c, 0)$$

Finally, it is calculated the revenue generated by the system. The surplus energy (e_s) is sold at the selling price of electricity (e_{priceS}), the biogas that was not used to generate electricity (V_{gburn}) nor used in the transport of manure (V_{cng}) is sold at the biogas price (P_g) and the excess fertilizer (W_L) is sold at the selling price of fertilizer (P_L).

$$r = \sum_{i=1}^{L} \frac{1}{(1+WACC)^i} \left(e_s e_{priceS} + V_{gi}\left(1 - V_{gburn} - V_{cng}\right) P_g + W_L P_L\right)$$

The value to be calculated is the cost of production of the biogas ($\mathrm{C_{prod}}$) by multiplying the volume of biogas produced (V_g) and the cost of producing it (C_{Vgas}).

$$\mathrm{C_{prod}} = \sum_{i=1}^{L} V_g C_{Vgas} \frac{1}{(1+WACC)^i}$$

The design variables are subject to the following constraints and bounds:

$$0 \le Debt_{level} \le \mathrm{Debt_{maxLevel}}$$
$$\mathrm{n_g} = 1,2,3,..$$
$$0 \le \mathrm{V_{gburn}} \le 1$$
$$0 \le \mathrm{V_{cng}} \le 1$$
$$V_{gburn} + V_{cng} \le 1$$

A. Cost Variables:

1) Design Variables:

$V_{gBurn} = Volume\ of\ biogas\ used\ in\ the\ production\ of\ electrical\ energy$ (%)
$n_g = Number\ of\ generators$
$Debt_{level} = Debt\ level\ for\ the\ project$
$V_{cng} = Volume\ of\ biogas\ used\ in\ the\ production\ of\ biodiesel$ (%)

2) Calculated within the module

$i = investment\ (R\$)$
$r = revenue\ (R\$)$
$\mathrm{C}_m = maintenance\ cost\ (R\$)$ [1]
$\mathrm{C}_t = transportation\ cost\ (R\$)$
$\mathrm{C}_e = energy\ cost\ savings (R\$)$
$f_s = Cost\ of\ fertilizer\ saved (R\$)$
$\mathrm{C_{prod}} = \text{Cost of producing biogas} \left(\frac{\mathrm{R\$}}{\mathrm{m^3}}\right)$
WACC = Weighted Average Cost of Capital (%)
$D = Total\ distance\ covered\ by\ the\ trucks\ \left(\frac{km}{year}\right)$
$\mathrm{D_{cng}} = Distance\ traveled\ using\ CNG\ (km)$

$\mathrm{D_{diesel}} = \mathit{Distance\ traveled\ using\ Diesel}\ (km)$

$\mathrm{C_{t_{fixed}}} = \text{Fixed cost portion of transport}\ \left(\frac{\text{R\$}}{\text{km}}\right)$

$\mathrm{C_{t_{diesel}}} = \text{Diesel cost of transport}\ \left(\frac{\text{R\$}}{\text{km}}\right)$

$\mathrm{C_{cng}} = \text{CNG cost of transport}\ \left(\frac{\text{R\$}}{\text{km}}\right)$

$i_m = Total\ yearly\ cost\ of\ maintetance$ [1]

$e_p = Energy\ produced\ inside\ the\ system\ \left(\frac{kWh}{year}\right)$

$W_L = Weight\ of fertilizer\ sold\ (kg)$

$e_s = Energy\ sold\ to\ energy\ company\ (kWh)$

Inputs from other modules:

$V_d = Digestor\ Volume\ (m^3)$

$distance_{travelded} = Daily\ distance\ traveled\ \left(\frac{km}{day}\right)$

$f_p = Fertilizer\ produced\ \left(\frac{kg}{year}\right)$

$V_g = Volume\ of gas\ \ produced\ (m^3/year)$

3) *Constants and Parameters*

$tax_{profit} = Corporate\ Taxes\ on\ profit\ (\%)$

$k_d = Cost\ of\ debt\ (interest)(\%) = 4\%$

$k_e = Cost\ of\ equity\ (\%) =\ 8\%$ [2]

$working_{days} = 365\frac{days}{year}$

$T_{m3kmcng} = \frac{1\mathrm{m}^3}{2.56\mathrm{km}}$ Truck consumption of CNG per km

$a_d, b_d = fit\ line\ to\ digestor\ volume = \sim(127R\$m^{-3}, 16500R\$\ )$ → fit line from Felipe's data adjusted for inflation

$g_d = Cost\ of\ an\ energy\ generator\ using\ biogas = R\$25000.00\ (36kVa)$ [1]

$P_{diesel} = Price\ of\ diesel\frac{3.3R\$}{L}$

$C_{R\$km} = \frac{3R\$}{km}\ Cost\ of\ truck\ transport$

$\mathrm{C_{upgrade_{cng}}} = Cost\ of\ upgrading\ biogas\ to\ CNG\frac{0.75R\$}{m^3}$

$g_m = g_d * 10\%\ Maintenance\ cost\ of\ generator$ [1]

$i_{mainCost} = Biodigester\ maintencance\ cost\ as\ percentage\ of\ investment\ (15\%)$

$e_{densitygas} = energy\ density\ of\ biogas\ \frac{20MJ}{m^3}$

$g_{eff} = Generator\ efficiency = 0.42\%$

$g_{power} = 36kW\ generator\ power$ [1]

$working_{hours} = \frac{8h}{day}$

$e_c = Energy\ consumed\ inside\ the\ farm = \frac{121.2MWh}{year}$ [1]

$e_{PriceB} = Price\ of\ buying\ energy\ from\ energy\ company = \sim R\$0.59(kWh)^{-1}$ [3]

$f_{used} = Fertilizer\ used\ inside\ the\ farm = 80kg$

$P_{bf} = Market\ price\ of\ fertilizer = R\$14.5kg^{-1}$ [4]

$e_{priceS} = Price\ paid\ by\ the\ energy\ company =\ 0.35R\$(kWh)^{-1}$

$P_g = Price\ of\ gas\ sold =\ R\$3.05kg^{-1}$ = [5]
$P_L = Price\ of\ fertilizer\ sold =\ R\$3.00kg^{-1}$ → Estimating 1 fifth of cost to buy
$L = life\ time\ of\ biodigestor \rightarrow 10\ years$
$\mathrm{Debt}_{\mathrm{maxLevel}} =$ Maximum debt level allowed for the project (%)

B. Parameter sources:

[1] BALDASSIN JUNIOR, Ricardo, CORTEZ, Luís Augusto Barbosa, JORDAN, Rodrigo Aparecido *et al*. Consumo de energia elétrica de um laticínio tipo "A" e estudo de racionalização dp uso de energia elétrica nos processos de resfriamento de leite e aquecimento de água: um estudo de caso. In *Anais do 5º Encontro de Energia no Meio Rural*, 2004, Campinas (SP) [online]. 2004

[2] SOUZA, Samuel N. Melegari de, PEREIRA, William Caldart and PAVAN, André Aparecido. Custo da eletricidade gerada em conjunto motor gerador utilizando biogás da suinocultura. In *Procedings of the 5th Encontro de Energia no Meio Rural*, 2004, Campinas (SP) [online]. 2004

[3] https://www.copel.com/hpcopel/root/nivel2.jsp?endereco=%2Fhpcopel%2Froot%2Fpagcopel2.nsf%2Fdocs%2F23BF37E67261209C03257488005939EB

[4] https://www.google.com/shopping/product/7535229380165716913?q=fertilizante&rlz=1C1SQJL_pt-BRBR785BR785&sxsrf=ALeKk01PwDkJQPMuJlfrg3CtQswN2TyP3w:1616266873646&biw=1366&bih=568&prds=epd:234859120399697191,paur:ClkAsKraX5Bm5kj2kqKeliv-cQ7WNKK0UCAj8xLeA7DyEYjPAkowF-8Z6kYNWsONJCxhWgmXHhqtAT8s65UBL0rGXn_Ae2G6R74J_4Ejf9V-9SlBngtjre67wRIZAFPVH72NaCCWb6mMcr0epuxOjuape_s2Dg,prmr:1&sa=X&ved=0ahUKEwjigoCrx7_vAhXqHbkGHUYhAS0Q8wIIwgM

[5] https://economia.uol.com.br/noticias/estadao-conteudo/2021/03/01/petrobras-reajusta-glp-em-valor-equivalente-a-r-190-por-13-quilos.htm#:~:text=Petrobras%20reajusta%20GLP%20em%20valor,1%2C90%20por%2013%20quilos&text=A%20Petrobras%20informou%20hoje%20que,%24%2039%2C69%20nas%20refinarias.

C. Cost Module Validation

We created an automated script to compare hand calculations (expected values) to the values returned by all the functions in the module. The list is shown below:

Cost Module Validation				
Test #	expected results	results	name	function result
0	0.04416	TRUE	['WACC']	0.04416
1	0.04416	TRUE	['WACC']	0.04416
2	0.08	TRUE	['WACC']	0.08
3	0.05312	TRUE	['WACC']	0.05312
4	0.04448	TRUE	['WACC']	0.04448
5	0.0956	TRUE	['WACC']	0.0956
6	8.26	TRUE	['npv']	8.264463
7	10	TRUE	['npv']	10
8	10	TRUE	['npv']	10
9	15.88	TRUE	['npv']	15.87664
10	81.1	TRUE	['total_npv']	81.10896
11	33.55	TRUE	['total_npv']	33.55041
12	3650	TRUE	['V_year']	3650
13	1825	TRUE	['V_year']	1825
14	3650	TRUE	['D']	3650
15	7300	TRUE	['D']	7300
16	4959.85	TRUE	['c_t']	4959.851
17	5229.656	TRUE	['c_t']	5229.657
18	1.073613	TRUE	['C_prod']	1.073613
19	1.297743	TRUE	['C_prod']	1.297743
20	1.6	TRUE	['C_prod']	1.6
21	107.3613	TRUE	['C_prod']	107.3613
22	-107.361	TRUE	['C_prod']	-107.361
23	72139	TRUE	['i']	72139.33
24	78311	TRUE	['i']	78311.56
25	45049	TRUE	['i']	45050.22
26	67515	TRUE	['i']	67515.47
27	9570.96	TRUE	['i_m']	9570.899
28	9246.76	TRUE	['i_m']	9246.734
29	5507.35	TRUE	['i_m']	5507.533
30	7627.25	TRUE	['i_m']	7627.321
31	64221.92	TRUE	['c_m']	64221.51
32	62046.51	TRUE	['c_m']	62046.34
33	44669.54	TRUE	['c_m']	44671.03
34	61863.83	TRUE	['c_m']	61864.41
35	1006.5	TRUE	['c_e']	1006.512
36	1677.52	TRUE	['c_e']	1677.52
37	20130.24	TRUE	['c_e']	20130.24
38	33550.4	TRUE	['c_e']	33550.41
39	608.3	TRUE	['c_e']	608.3172

Cost Module Validation				
Test #	expected results	results	name	function result
40	1013.9	TRUE	['c_e']	1013.862
41	13084.6	TRUE	['f_s']	13084.66
42	13084.6	TRUE	['f_s']	13084.66
43	8723.1	TRUE	['f_s']	8723.106
44	1744.62	TRUE	['f_s']	1744.621
45	0	TRUE	['f_s']	0
46	1	TRUE	['w_l']	1
47	0	TRUE	['w_l']	0
48	30.24	TRUE	['e_p']	30.24
49	15815	TRUE	['e_p']	15815.52
50	10.24	TRUE	['e_s']	10.24
51	5815	TRUE	['e_s']	5815.52
52	0	TRUE	['e_s']	0
53	44.18	TRUE	['r']	44.17918
54	13697.62	TRUE	['r']	13698.85