Task 2:

Imagine there is a robotic arm, e.g. Universal Robots UR10, with a distance sensor attached to its end effector. Let assume that the robotic arm base link is at the origin of a coordinate system. Describe how to calculate distance from the point that was measured by the sensor, to the robotic arm base link.

Answer:

In order to calculate the required distance we would need to know the position of the measured point. Assuming that we know our robot's parameters and configuration (vector q), we could calculate the forward kinematics to obtain the position and orientation of the end effector and then apply the measured distance accordingly, that is in the direction the sensor is facing. In terms of DH parameters, we could do that last step by considering the measurement as an additional link and assigning it an appropriate transformation matrix.

For example, if the sensor's measurement axis was aligned with the axis of the end effector, that matrix would describe translation only with the d_m parameter being the distance measured (plus possibly the thickness of the sensor itself). Taking the UR10 arm parameters from the Universal Robotics website*, the table of DH parameters would be as follows:

Kinematics	θ_i [rad]	a_i [m]	d_i [m]	α_i [rad]
Joint 1	q_1	0	0.1273	π/2
Joint 2	q_2	-0.612	0	0
Joint 3	q_3	-0.5723	0	0
Joint 4	q_4	0	0.1639	π/2
Joint 5	q_5	0	0.1157	-π/2
Joint 6	q_6	0	0.0922	0
Sensor / Joint 7	0	0	d_m	0

*https://www.universal-robots.com/articles/ur/parameters-for-calculations-of-kinematics-and-dynamics/

Based on the table the transformation matrices should be calculated:

$$^{n-1}T_n = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} & R & & T \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

To get the position of the measured point, we should read the vector T (as described above) of the following matrix:

$T_7^0 = T_1^0 T_2^1 ... T_7^6$

Then, calculate the euclidean distance of the obtained vector with respect to the base coordinates, usually (0, 0, 0).