Image Segmentation Using Local GMM in a Variational Framework

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by Yumin Cho

Introduction

Liu, J., Zhang, H. Image Segmentation Using a Local GMM in a Variational Framework. *J Math Imaging Vis* **46,** 161–176. 2013.

Image Segmentation Using a Local GMM in a Variational Framework

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Published online: 28 August 2012

Abstract In this paper, we propose a new variational framework to solve the Gaussian mixture model (GMM) based methods for image segmentation by employing the convex relaxation approach. After relaxing the indicator function in GMM, flexible spatial regularization can be adopted and efficient segmentation can be achieved. To demonstrate the superiority of the proposed framework, the global, local intensity information and the spatial smoothness are integrated into a new model, and it can work well on images with inhomogeneous intensity and noise. Compared to classical GMM, numerical experiments have demonstrated that our algorithm can achieve promising segmentation performance for images degraded by intensity inhomogeneity and noise.

Keywords Image segmentation · Gaussian mixture model · Regularization · Variational method · Inhomogeneous intensity

1 Introduction

Image segmentation is to partition an image into a finite number of regions or objects, which has long been an important and common problem in computer vision. Numerous segmentation methods have been proposed in the literature, and among them, we would like to mention the finite mixture model, which is a flexible and powerful probabilistic modeling tool to solve the clustering problem.

Gaussian mixture model is a standard tool (e.g. [1, 2]), which has been widely used (e.g. [3–5]). Usually, the parameters in the finite mixture model can be efficiently estimated by the well-known expectation maximization (EM) algorithm [6]. Once the parameters are determined, we can get the segmentation results in terms of the distribution parameters such as the means of the image intensity. A drawback of standard GMM is that it lacks in spatial dependency prior and is always sensitive to noise. One solution is to include a Markov Random Field (MRF) (e.g. [7, 8]) in GMM. However, solving MRF with EM algorithm may get stuck in a local minimum [9]. On the other hand, it has been shown that imposing a constraint on the boundary length of segments together with a convex relaxation technology can achieve a global minimizer [10, 11].

Intensity inhomogeneities occurs in many natural and medical images, which is usually caused by smoothing, spatially varying illumination, etc. Although it is not a big issue for visual inspection, it could dramatically affect the performance of intensity-based image segmentation. The literature is rich for image segmentation in the presence of inhomogeneous intensity. One straightforward strategy is to remove the inhomogeneity prior to the segmentation by a preprocessing step [12]. However, intermediate information captured during image segmentation may help inhomogeneity correction and subsequently segmentation process [5, 13–15]. In [5, 13, 14], the authors employ the logarithmic additive model to address the spatial inhomogeneity. However, the EM algorithm proposed in [5, 13] is short of spatial regularization. Besides, the additional regularized term for the bias function β in [13, 14] is computationally time-consuming.

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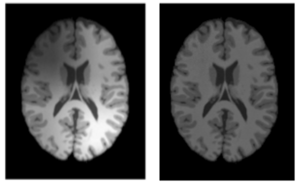
How to handle the intensity inhomogeneity in segmentation problem?

Intensity inhomogeneity



- Usually caused by smoothing, spatially varying illumination, etc.
- Affect the performance of intensity-based image segmentation

What about removing the inhomogeneity prior?

However, the intermediate information captured during image segmentation may help inhomogeneity correction and subsequently segmentation process

Integrate GMM and level set method

What's better than GMM?

Not require the extra constraints on the bias function

What's better than level set method?

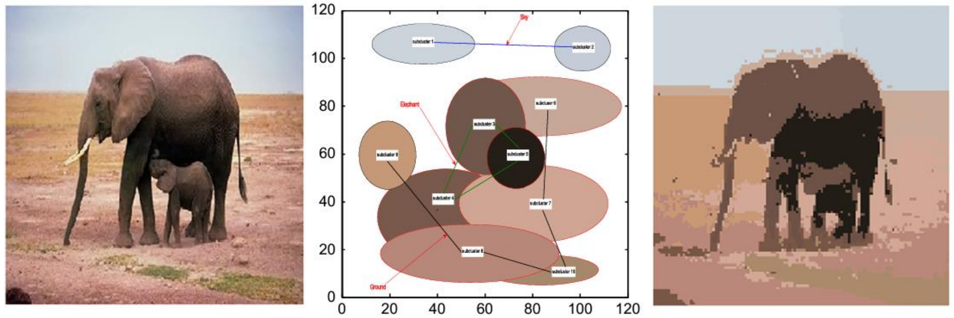
Easily deal with multi-clusters and control multiple boundary curves of different classes

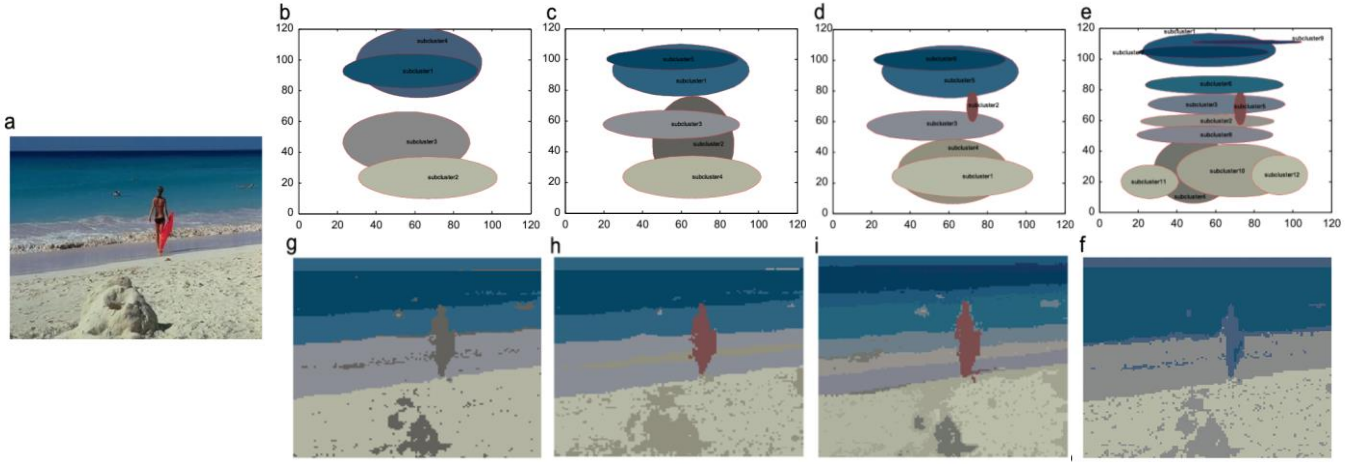
Image Segmentation Method

GMM-based model

Lacks in spatial dependency prior

Always sensitive to noise

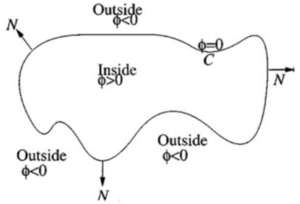




Level set method

Time-consuming

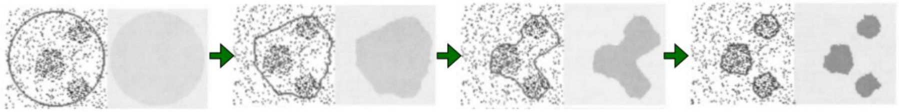
Inconvenient for multiphase segmentation



$$\phi(x(t), t + \nabla t) = \underbrace{\phi(x(t), t)}_{\text{initial } \phi} + \nabla t \underbrace{F}_{\text{energy function}} \|\nabla \phi\|$$

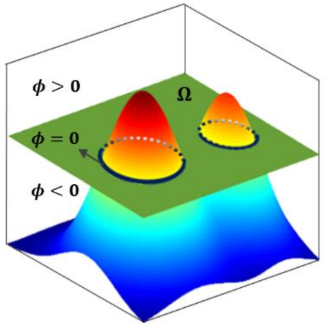
ϕ: distance function

F: edge-based / region-based / ...



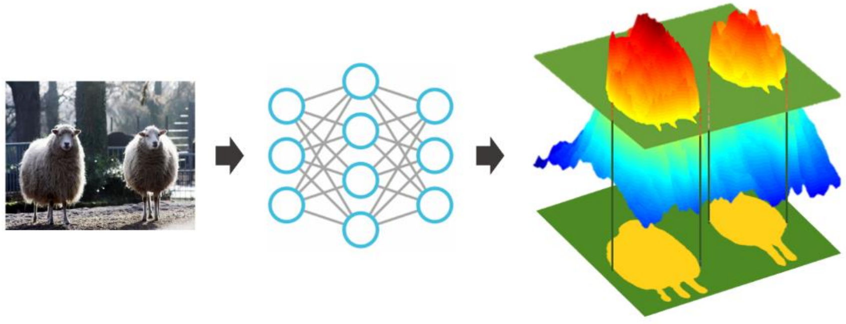
Initialize ϕ^0

solve the PDE in ϕ from F, obtain ϕ^{n+1}



$$\frac{\partial \phi_1}{\partial t} = \delta_\epsilon(\phi_1)\left\{\lambda \operatorname{div}\left(\frac{\nabla \phi_1}{|\nabla \phi_1|}\right) - \left[\left((u - c_{11})^2 - (u - c_{01})^2\right) H(\phi_2) + \left((u - c_{10})^2 - (u - c_{00})^2\right)(1 - H(\phi_2))\right]\right\}$$

[Level-set method]



Input image without labels

Unfolded level set evolution via CNN

n-th level function = n-th softmax channel

[Proposed neural network with level-set loss]

EM Algorithm in GMM

Gaussian Mixture Model

Let $f: \Omega \subset \mathbb{R}^2 \mapsto \mathbb{R}^M$ is a gray image and the pixel intensity $f(x)$ can be regarded as some realization of a random variable $\mathfrak{Z}$ with probability density function below

$$p_{\mathfrak{Z}}(z) = \sum_{k=1}^{K} \gamma_k p(z; c_k, \sigma_k^2).$$


(a) Gray natural image

the parameters $\Theta = \{\gamma_1, \cdots, \gamma_K, c_1, \cdots, c_K, \sigma_1^2, \cdots, \sigma_K^2\}$ in GMM can be efficiently estimated through MLE using EM algorithm

$$\mathcal{L}(\Theta) = \int_\Omega \log \sum_{k=1}^{K} \frac{\gamma_k}{\sqrt{2\pi}\sigma_k} \exp\left\{-\frac{[f(x) - c_k]^2}{2\sigma_k^2}\right\} \mathrm{d}x.$$

EM Algorithm

Iterates over below 2 steps

E-step

Update the expected value of $\mathcal{L}(\Theta)$ w.r.t the conditional distribution and the parameter set

$$P(\mathfrak{K} = k | \mathfrak{Z} = z; \Theta^\nu) = \frac{P(\mathfrak{K} = k; \Theta^\nu) P(\mathfrak{Z} = z | \mathfrak{K} = k; \Theta^\nu)}{P(\mathfrak{Z} = z; \Theta^\nu)}$$

$$q_k^\nu(x) := P(\mathfrak{K} = k | \mathfrak{Z} = f(x); \Theta^\nu) = \frac{\gamma_k^\nu p(f(x); c_k^\nu, (\sigma_k^2)^\nu)}{\sum_{i=1}^{K} \gamma_i^\nu p(f(x); c_i^\nu, (\sigma_i^2)^\nu)}$$

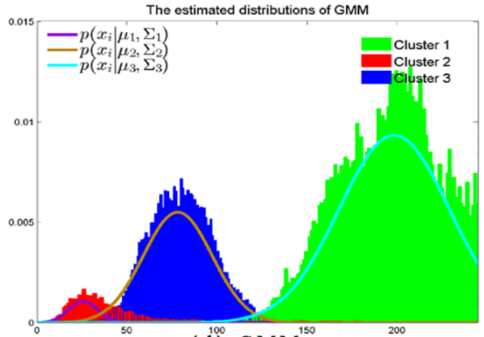
M-step

Update the parameters of each single Gaussian distribution once the conditional probability is obtained

$$\gamma_k^{\nu+1} = \frac{\int_\Omega q_k^\nu(x) \mathrm{d}x}{\int_\Omega 1 \mathrm{d}x},$$

$$c_k^{\nu+1} = \frac{\int_\Omega q_k^\nu(x) f(x) \mathrm{d}x}{\int_\Omega q_k^\nu(x) \mathrm{d}x},$$

$$(\sigma_k^2)^{\nu+1} = \frac{\int_\Omega q_k^\nu(x) [f(x) - c_k^{\nu+1}]^2 \mathrm{d}x}{\int_\Omega q_k^\nu(x) \mathrm{d}x}$$



(d) GMM

Proposed Model

Solving GMM with the Variational Method

Problem

Since the logarithm and summation operations are noncommutative, optimizing log-likelihood is not that easy

$$\mathcal{L}(\Theta) = \int_{\Omega} \log \sum_{k=1}^{K} \boxed{\frac{\gamma_k}{\sqrt{2\pi}\sigma_k}} \exp\left\{ -\boxed{\frac{[f(x) - c_k]^2}{2\sigma_k^2}} \right\} \mathrm{d}x.$$

Solution

1. Add a **convex relaxation** condition

$$\varDelta = \{\mathbf{v} |\ 0 < v_k < 1, \quad \textstyle\sum_{k=1}^{K} v_k = 1\}$$

$$\mathbf{v} = (v_1, \cdots, v_K) \qquad \mathbf{u}(x) = (u_1(x), u_2(x), \cdots, u_K(x))$$

$\varDelta$ is a convex relaxation of non-convex binary vector space $\{\mathbf{v}|\ v_k \in \{0,1\}, \quad \sum_{k=1}^{K} v_k = 1\}$

Based on the ***Lemma 1***, then the minimization problem becomes

$$\begin{aligned} \Theta^* &= \arg\max_{\Theta}\ \mathcal{L}(\Theta) = \arg\min_{\Theta}\ -\mathcal{L}(\Theta) \\ &= \arg\min_{\Theta} \left\{ \int_{\Omega} \min_{\mathbf{u}(x)\in\varDelta} \left\{ \sum_{k=1}^{K} \left[\boxed{\mathcal{B}_k(x)} - \log \boxed{\mathcal{A}_k(x)} \right] u_k(x) + \sum_{k=1}^{K} u_k(x) \log u_k(x) \right\} \mathrm{d}x \right\} \end{aligned}$$

Lemma 1 *[Commutativity of Log-sum operations] Given a function $\mathcal{A}_k(x) > 0$, for any function $\mathcal{B}_k(x) > 0$, we have*

$$-\log \sum_{k=1}^{K} \mathcal{A}_k(x) \exp\left[-\mathcal{B}_k(x)\right] = \min_{\mathbf{u}(x)\in\varDelta} \left\{ \begin{array}{l} \sum_{k=1}^{K} \left[\mathcal{B}_k(x) - \log \mathcal{A}_k(x)\right] u_k(x) \\ \quad + \sum_{k=1}^{K} u_k(x) \log u_k(x) \end{array} \right\}.$$

Proposed Model

2. Introduce a **functional** $\tilde{\mathcal{E}}(\Theta, \mathbf{u})$ **with two variables**

$$\begin{aligned}
&\tilde{\mathcal{E}}(\Theta, \mathbf{u}) \\
&= \int_\Omega \sum_{k=1}^K \left[\mathcal{B}_k(x) - \log \mathcal{A}_k(x)\right] u_k(x)\mathrm{d}x + \int_\Omega \sum_{k=1}^K u_k(x) \log u_k(x)\mathrm{d}x \\
&= \int_\Omega \sum_{k=1}^K \left[\frac{|f(x) - c_k|^2}{2\sigma_k^2} - \log \frac{\gamma_k}{\sqrt{2\pi}\sigma_k}\right] u_k(x)\mathrm{d}x + \int_\Omega \sum_{k=1}^K u_k(x) \log u_k(x)\mathrm{d}x.
\end{aligned}$$

Proposition 1 *The sequence Θ^ν produced by iteration scheme (4) satisfies*

$$-\mathcal{L}(\Theta^{\nu+1}) \leqslant -\mathcal{L}(\Theta^\nu).$$

3. Compute the minimizer of $\tilde{\mathcal{E}}(\Theta, \mathbf{u})$ via the following alternating algorithm:

$$\begin{cases}
\mathbf{u}^{\nu+1} = \underset{\mathbf{u} \in \Delta}{\arg\min} \ \ \tilde{\mathcal{E}}(\Theta^\nu, \mathbf{u}), \\
\Theta^{\nu+1} = \underset{\Theta}{\arg\min} \ \ \tilde{\mathcal{E}}(\Theta, \mathbf{u}^{\nu+1}).
\end{cases}$$

Instead of using $-\mathcal{L}(\Theta)$, minimize the constructed cost functional $\tilde{\mathcal{E}}(\Theta, \mathbf{u})$ which contains a convex relaxation term

∴ We solve GMM-based model using variational method with a convex relaxation and alternating minimization

☺ It may help us get a **global minimizer** if the constructed cost functional is convex

☺ The **spatial smoothness priority** can also be easily handled

Proposed Model

Model Assumption

The intensity inhomogeneity problem can be mathematically modeled by

observed data

: intensity of the observed image

$$\boxed{f(x)} = \boxed{\beta(x)}\boxed{g(x)}$$

ground truth image

: can be well segmented by GMM (**Assumption 1**)

bias field

: nonnegative and smoothly varying (**Assumption 2**)

Let $f(x)$ is a realization of a random variable $\mathfrak{z}$

Based on the ***Assumption 1*** and above model,

Proposition 2 *The PDF of* $\mathfrak{z}$ *has the expression*

$$p_{\mathfrak{z}}(z) = \sum_{k=1}^{K} \frac{\gamma_k}{\sqrt{2\pi}\sigma_k \beta(x)} \exp\left\{-\frac{[z - c_k\beta(x)]^2}{2\sigma_k^2\beta^2(x)}\right\}$$

cf.) classical GMM

$$p_{\mathfrak{z}}(z) = \sum_{k=1}^{K} \gamma_k p(z; c_k, \sigma_k^2).$$

Proposed Model

Data Term

Based on the ***Assumption 2*** and the ***Proposition 2***,

the local cost functional $\mathcal{E}_y$ in O_y (the neighborhood centered at y)

$$\mathcal{E}_y(\Theta) := -\mathcal{L}(\Theta) = -\int_{O_y} \log\left\{\sum_{k=1}^{K} \frac{\gamma_k}{\sqrt{2\pi}\sigma_k\beta(y)} \exp\left\{-\frac{[f(x)-c_k\beta(y)]^2}{2\sigma_k^2\beta^2(y)}\right\}\right\} \mathrm{d}x,$$

Add some weights for each pixel using Gaussian kernel and expand to the whole image domain Ω

$$\mathcal{E}_y(\Theta) = -\int_{O_y} G_\sigma(y-x) \log\left\{\sum_{k=1}^{K} \frac{\gamma_k}{\sqrt{2\pi}\sigma_k\beta(y)} \exp\left\{-\frac{[f(x)-c_k\beta(y)]^2}{2\sigma_k^2\beta^2(y)}\right\}\right\} \mathrm{d}x, \longrightarrow \mathcal{E}_y(\Theta) = -\int_{\Omega} G_\sigma(y-x) \log\left\{\sum_{k=1}^{K} \frac{\gamma_k}{\sqrt{2\pi}\sigma_k\beta(y)} \exp\left\{-\frac{[f(x)-c_k\beta(y)]^2}{2\sigma_k^2\beta^2(y)}\right\}\right\} \mathrm{d}x.$$

Finally, we get the total cost functional $\mathcal{E}(\Theta)$ and the segmentation problem $\Theta^* = \arg\min_{\Theta} \ \mathcal{E}(\Theta)$

$$\begin{aligned}\mathcal{E}(\Theta) &= \int_\Omega \mathcal{E}_y(\Theta)\mathrm{d}y \\ &= -\int_\Omega\int_\Omega G_\sigma(y-x) \log\left\{\sum_{k=1}^{K} \frac{\gamma_k}{\sqrt{2\pi}\sigma_k\beta(y)} \exp\left\{-\frac{[f(x)-c_k\beta(y)]^2}{2\sigma_k^2\beta^2(y)}\right\}\right\} \mathrm{d}x\mathrm{d}y.\end{aligned}$$

☹ However, it's difficult to directly optimize $\mathcal{E}(\Theta)$ due to the existence of the log-sum function

Based on the ***Lemma 1*** and the ***Proposition 1***, the final data term would be

$$\begin{aligned}\tilde{\mathcal{E}}(\Theta, \mathbf{u}) = &\frac{1}{2}\int_\Omega\int_\Omega \sum_{k=1}^{K} G_\sigma(y-x)\frac{\left[\frac{f(x)}{\beta(y)} - c_k\right]^2}{\sigma_k^2} u_k(x)\mathrm{d}x\mathrm{d}y - \int_\Omega \sum_{k=1}^{K} u_k(x)\log\gamma_k \mathrm{d}x \\ &+\frac{1}{2}\int_\Omega \sum_{k=1}^{K} u_k(x)\log\sigma_k^2\mathrm{d}x + \int_\Omega \log\beta(y)\mathrm{d}y + \int_\Omega \sum_{k=1}^{K} u_k(x)\log u_k(x)\mathrm{d}x.\end{aligned}$$

cf.) Data term for color images

$$\begin{aligned}&\tilde{\mathcal{E}}(\Theta, \mathbf{u}) = \\ &\frac{1}{2}\int_\Omega\int_\Omega \sum_{k=1}^{K} u_k(x)G_\sigma(y-x)\left[\beta^{-1}(y)\cdot\mathbf{f}(x) - \mathbf{c}_k\right]^{\mathrm{T}} \Sigma_k^{-1}\left[\beta^{-1}(y)\cdot\mathbf{f}(x) - \mathbf{c}_k\right]\mathrm{d}x\mathrm{d}y \\ &-\int_\Omega \sum_{k=1}^{K} u_k(x)\log\gamma_k\mathrm{d}x + \frac{1}{2}\int_\Omega\sum_{k=1}^{K} u_k(x)\log|\Sigma_k|\mathrm{d}x + 3\int_\Omega \log\beta(y)\mathrm{d}y \\ &+\int_\Omega \sum_{k=1}^{K} u_k(x)\log u_k(x)\mathrm{d}x,\end{aligned}$$

Proposed Model

Spatial Smoothness Term

Let mixture ratio be the number of clusters K and the variance is constant,

then minimizing the first term of $\tilde{\mathcal{E}}(\Theta, \mathbf{u})$ with binary constraint $u_k(x) \in \{0,1\}, \sum_{k=1}^{K} u_k(x) = 1$ is equivalent to the K-means clustering.

$$\tilde{\mathcal{E}}(\Theta, \mathbf{u}) = \frac{1}{2}\int_\Omega \int_\Omega \sum_{k=1}^{K} G_\sigma(y-x)\frac{\left[\frac{f(x)}{\beta(y)} - c_k\right]^2}{\sigma_k^2} u_k(x)\mathrm{d}x\mathrm{d}y - \int_\Omega \sum_{k=1}^{K} u_k(x)\log\gamma_k \mathrm{d}x$$
$$+\frac{1}{2}\int_\Omega \sum_{k=1}^{K} u_k(x)\log\sigma_k^2 \mathrm{d}x + \int_\Omega \log\beta(y)\mathrm{d}y + \int_\Omega \sum_{k=1}^{K} u_k(x)\log u_k(x)\mathrm{d}x.$$

From this point of view, EM algorithm for GMM is an entropy regularization of K-means clustering with a convex relaxation condition.

☹ However, such smoothness is not enough especially in the presence of intensity inhomogeneity and noise.

Since TV regularization is non-differentiable,

we introduce another **regularization term** which can still guarantee the solution of **u** is close-formed.

$$\mathcal{R}(\mathbf{u}) = \int_\Omega \sum_{k=1}^{K} u_k(x)\boxed{e(x)}\boxed{\int_{N(x;\omega)} [1-u_k(y)]\,\mathrm{d}y}\,\mathrm{d}x,$$

penalization term

: when the pixel with its neighbors belong to different clusters,

penalize s.t **the length of cluster boundaries** is short when minimizin $\mathcal{R}(\mathbf{u})$

edge detector function $e = \frac{1}{1+|\nabla G * f|}$

: prevent the regularization term from dramatically destroying the image edges

Proposed Model

The Segmentation Cost Functional

Combining the data term and regularization term, finally we get our new model as follows,

$$(\Theta^*, \mathbf{u}^*) = \underset{\Theta, \mathbf{u} \in \Delta}{\arg\min} \;\; \mathcal{J}(\Theta, \mathbf{u}) := \tilde{\mathcal{E}}(\Theta, \mathbf{u}) + \lambda \mathcal{R}(\mathbf{u})$$

$$= \frac{1}{2} \int_\Omega \int_\Omega \sum_{k=1}^{K} G_\sigma(y-x) \frac{\left[\frac{f(x)}{\beta(y)} - c_k\right]^2}{\sigma_k^2} u_k(x) \mathrm{d}x \mathrm{d}y - \int_\Omega \sum_{k=1}^{K} u_k(x) \log \gamma_k \mathrm{d}x$$

$$+ \frac{1}{2} \int_\Omega \sum_{k=1}^{K} u_k(x) \log \sigma_k^2 \mathrm{d}x + \int_\Omega \log \beta(y) \mathrm{d}y + \int_\Omega \sum_{k=1}^{K} u_k(x) \log u_k(x) \mathrm{d}x + \lambda \int_\Omega \sum_{k=1}^{K} u_k(x) e(x) \int_{N(x;\omega)} [1 - u_k(y)] \, \mathrm{d}y \mathrm{d}x$$

$$\Theta = \{\gamma_1, \cdots, \gamma_K, c_1, \cdots, c_K, \sigma_1^2, \cdots, \sigma_K^2\} \bigcup_y \{\beta(y)\}.$$

classical GMM

☹ Sensitive to noise

☹ Lacks in spatial smoothness constraint

☹ Global data term

☹ Hard to handle intensity inhomogeneity

Proposed Model

☺ Penalty term makes it robust to noise

☺ Integrates the local bias function information, the global intensity and edges information

-> Works well on images with intensity inhomogeneity

Proposed Model

Algorithm

Choose an initial guess value $\Theta^0, \mathbf{u}^0$, *set* $\nu = 0$ and $\mathcal{J}_{old} = +\infty$

Step 1. Update by $\mathbf{u}^{\nu+1} = \underset{\mathbf{u}\in\Delta}{\arg\min}\ \mathcal{J}(\Theta^\nu, \mathbf{u}),$

: It plays a similar role of the E-step in the original EM algorithm

-> **regularized** E-step (**RE-step**)

Step 2. Update parameter set by $\Theta^{\nu+1} = \underset{\Theta}{\arg\min}\ \mathcal{J}(\Theta, \mathbf{u}^{\nu+1})$.

$$
\begin{cases}
\gamma_k^{\nu+1} = \dfrac{\displaystyle\int_\Omega u_k^{\nu+1}(x)\mathrm{d}x}{\displaystyle\int_\Omega 1\mathrm{d}x}, \\
c_k^{\nu+1} = \dfrac{\displaystyle\int_\Omega u_k^{\nu+1}(x)f(x)\int_\Omega G_\sigma(y-x)\frac{1}{\beta^\nu(y)}\mathrm{d}y\mathrm{d}x}{\displaystyle\int_\Omega u_k^{\nu+1}(x)\mathrm{d}x}, \\
(\sigma_k^2)^{\nu+1} = \dfrac{\displaystyle\int_\Omega u_k^{\nu+1}(x)\int_\Omega G_\sigma(y-x)\left[\frac{f(x)}{\beta^\nu(y)} - c_k^{\nu+1}\right]^2\mathrm{d}y\mathrm{d}x}{\displaystyle\int_\Omega u_k^{\nu+1}(x)\mathrm{d}x}, \\
\beta^{\nu+1}(y) = \dfrac{-s^{\nu+1}(y) + \sqrt{[s^{\nu+1}(y)]^2 + 4t^{\nu+1}(y)}}{2}.
\end{cases}
$$

-> **M-step**

Step 3. Calculate new functional value and end the update if $||\mathcal{J}_{old} - \mathcal{J}_{new}||^2 < \varepsilon||\mathcal{J}_{old}||^2$ o.w return to **Step 1.**

cf.) classical GMM

E-step

Update the expected value of $\mathcal{L}(\Theta)$ w.r.t the conditional distribution and the parameter set

$$P(\mathfrak{K} = k|\mathfrak{Z} = z;\Theta^\nu) = \frac{P(\mathfrak{K} = k;\Theta^\nu)P(\mathfrak{Z} = z|\mathfrak{K} = k;\Theta^\nu)}{P(\mathfrak{Z} = z;\Theta^\nu)}$$

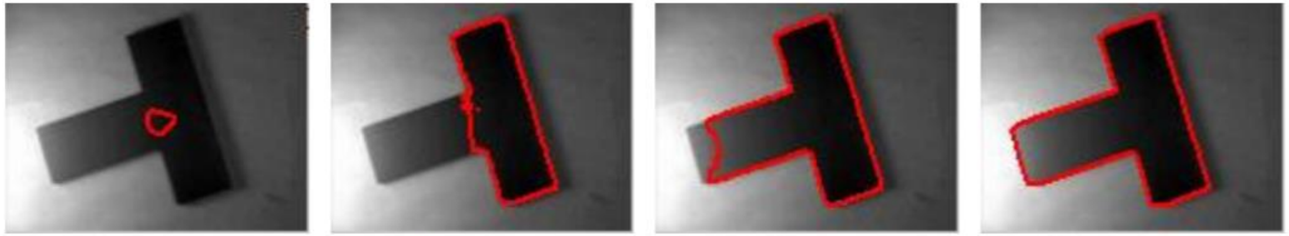
$$q_k^\nu(x) := P(\mathfrak{K} = k|\mathfrak{Z} = f(x);\Theta^\nu) = \frac{\gamma_k^\nu p(f(x);c_k^\nu,(\sigma_k^2)^\nu)}{\sum_{i=1}^K \gamma_i^\nu p(f(x);c_i^\nu,(\sigma_i^2)^\nu)}$$

M-step

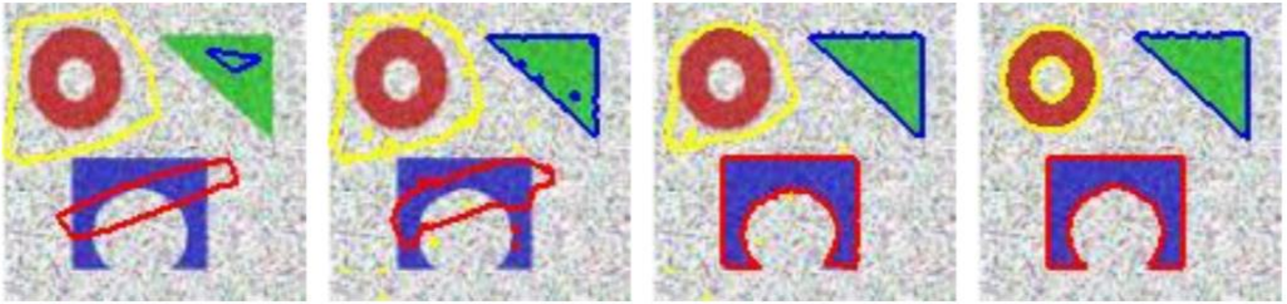
Update the parameters of each single Gaussian distribution once the conditional probability is obtained

$$
\gamma_k^{\nu+1} = \frac{\int_\Omega q_k^\nu(x)\mathrm{d}x}{\int_\Omega 1\mathrm{d}x},
\quad
c_k^{\nu+1} = \frac{\int_\Omega q_k^\nu(x)f(x)\mathrm{d}x}{\int_\Omega q_k^\nu(x)\mathrm{d}x},
\quad
(\sigma_k^2)^{\nu+1} = \frac{\int_\Omega q_k^\nu(x)[f(x)-c_k^{\nu+1}]^2\mathrm{d}x}{\int_\Omega q_k^\nu(x)\mathrm{d}x}
$$

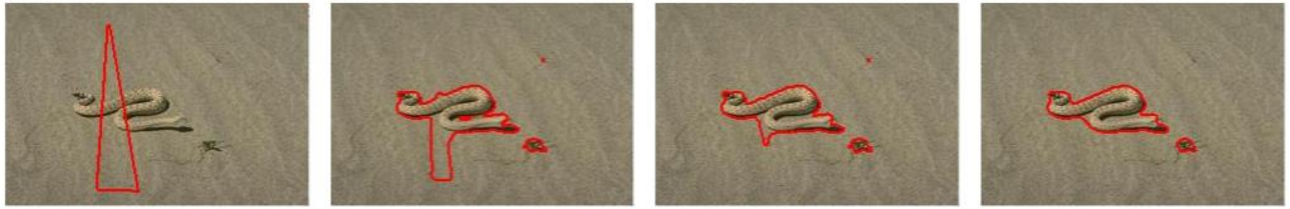
Experiments



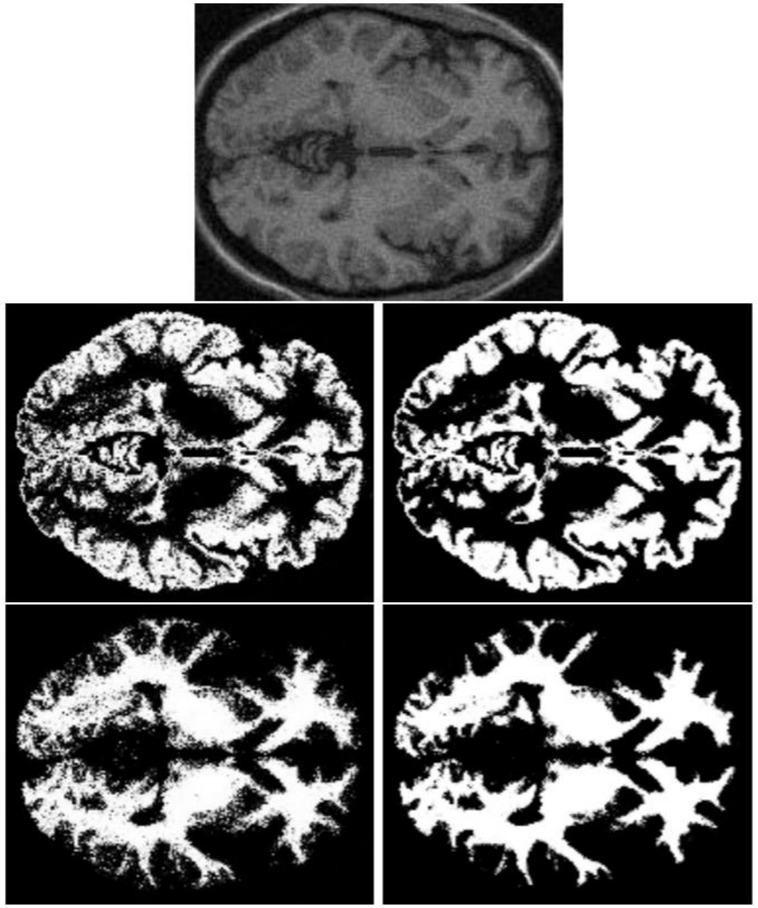
(a) Initial, $\nu = 0$ (b) $\nu = 10$ (c) $\nu = 20$ (d) Final, $\nu = 25$



(e) Initial, $\nu = 0$ (f) $\nu = 2$ (g) $\nu = 5$ (h) Final, $\nu = 10$



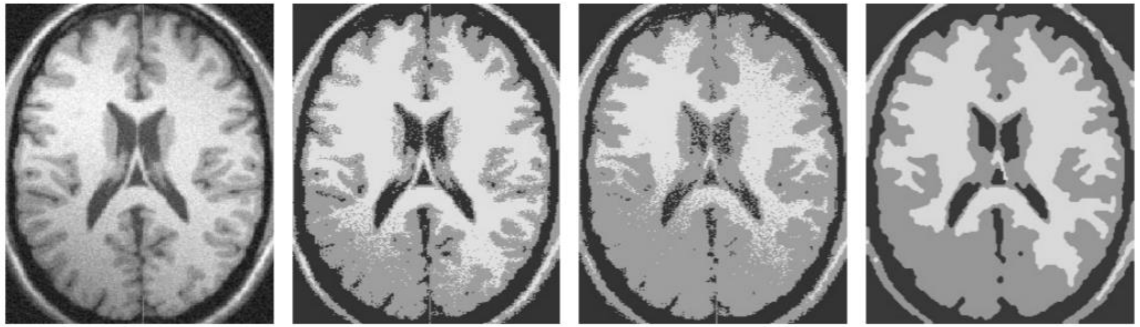
(i) Initial, $\nu = 0$ (j) $\nu = 20$ (k) $\nu = 35$ (l) Final, $\nu = 42$



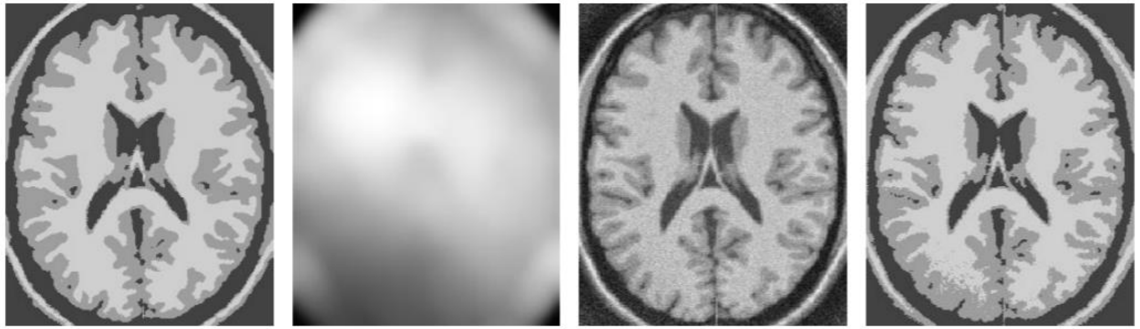
Experiments

Noise levels	SA			
σ	K-means	EM	REM	α-expansion
10	1.0000	1.0000	1.0000	1.0000
20	0.9775	0.9818	0.9992	**0.9998**
30	0.8801	0.9114	**0.9984**	0.9972
40	0.7704	0.8416	**0.9972**	0.9954
50	0.6753	0.7890	**0.9947**	0.9933
60	0.6046	0.7465	**0.9922**	0.9902

Fig. 2 Some results for the regularized GMM model. First column, the original images; Second column, clusters with GMM model by EM algorithm; Third column, results with model (14); Fourth column, classes with graph cut method α-expansion [39]. From top to bottom, the numbers of clusters are 2,4,5, respectively.



(a) Original (b) Initial (K-means) (c) EM (d) Proposed regularized EM



(e) Proposed LGMM (f) Estimated β (g) Estimated g, i.e. $\frac{f}{\beta}$ (h) Ahmed et. al. [14]

Conclusion

Contributions

1. Robust to noise with non-uniform illumination

It corrects the intensity or color by introducing a bias function β which doesn't need additional regularization

2. Formulate a new variational framework for regularized GMM

We reinterpret EM algorithm with a deterministic method (not random) in terms of a property of log-sum function.

So, one can easily get the connections among the EM algorithm, fuzzy clustering, and level set method for image segmentation.

References

- **Image Segmentation Using Local GMM**

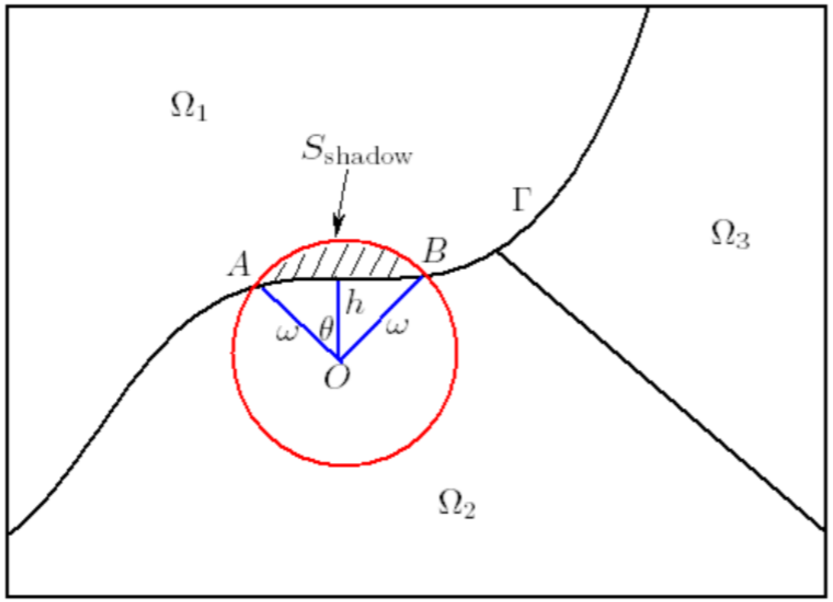
https://link.springer.com/article/10.1007/s10851-012-0376-5

- **Image segmentation using spectral clustering of Gaussian mixture models**

https://www.sciencedirect.com/science/article/abs/pii/S0925231214006298

- **A Rough Set Bounded Spatially Constrained Asymmetric Gaussian Mixture Model for Image Segmentation**

https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0168449


Ω_1
S_{shadow}
Γ
Ω_3
A
B
h
ω
θ
ω
O
Ω_2