Population forecasting and the importance of being uncertain

Rob J Hyndman

MONASH University

2007 Knibbs Lecture

Sir George Handley Knibbs (1858–1929)

- President of the Institute of Surveyors, 1892–1901
- President of the British Astronomical Society (NSW), 1897–1898
- President of the Royal Society of NSW, 1898–1899
- Lecturer in Geodesy, Astronomy and Hydraulics, Uni of Sydney, 1889–1905
- Director of Technical Education, NSW, 1905
- Professor of Physics, Uni of Sydney, 1905.
- Commonwealth Statistician, 1906–1921
- Director of Institute of Science and Industry, 1921–1926

Outline

Outline

What is it?

What is it?

Clay model of sheep's liver

Used by Babylonian forecasters approximately 600 B.C.

Now in British Museum.

Delphic oracle

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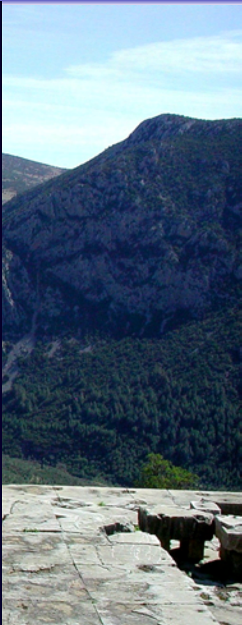

Delphic oracle

Vagrant forecasters

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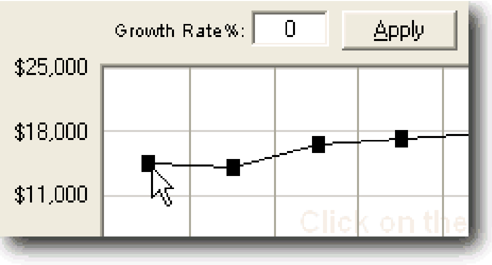
Punishment: a fine or three months' imprisonment with hard labour.

Outline

Standard business practice today

Graphic Forecaster

Create forecasts visually with a "drag and drop" graphic forecaster. The Graphic Forecaster is a simple and powerful tool to streamline the forecasting process. You can change your sales and expenses estimates by simply clicking your mouse button to move the line on your forecast chart or apply a specific growth rate to the whole year. Build forecasts using visual common sense.



View More Detail

Standard business practice today

Budget Maestro by Centage

Budget Maestro by Centage

[Click here for a free demo](#) FREE!

Application: Business Intelligence and Analytics

Price Range: Solutions start at $5K

A huge advance over spreadsheet-based systems, Budget Maestro is a complete solution for budgeting, forecasting, what-if scenario planning, reporting and analysis. Budget Maestro takes the pain out of the budgeting process (no tedious data entry and formula verification) while providing you a tool to more accurately analyze and measure business performance and profitability. Budget Maestro's capabilities include:

Budgeting and Forecasting: Budget Maestro utilizes database technology for real-time data collection and reporting. A common interface for all users fosters collaboration and increases the accuracy of data entry. There are no formulas or macros to create, no tedious re-keying of data and no mystery links to chase down and fix. Budget Maestro's built-in "financial intelligence and business rules" builds the formulas for you ensuring 100% accuracy.

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Is this any better than a sheep’s liver or hallucinogens?

ABS population projections

The Australian Bureau of Statistics provide population "projections".

"The projections are not intended as predictions or forecasts, but are illustrations of growth and change in the population that would occur if assumptions made about future demographic trends were to prevail over the projection period.
While the assumptions are formulated on the basis of an assessment of past demographic trends, both in Australia and overseas, there is no certainty that any of the assumptions will be realised. In addition, no assessment has been made of changes in non-demographic conditions."

ABS 3222.0 - Population Projections, Australia, 2004 to 2101

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- Based on assumed mortality, fertility and migration rates
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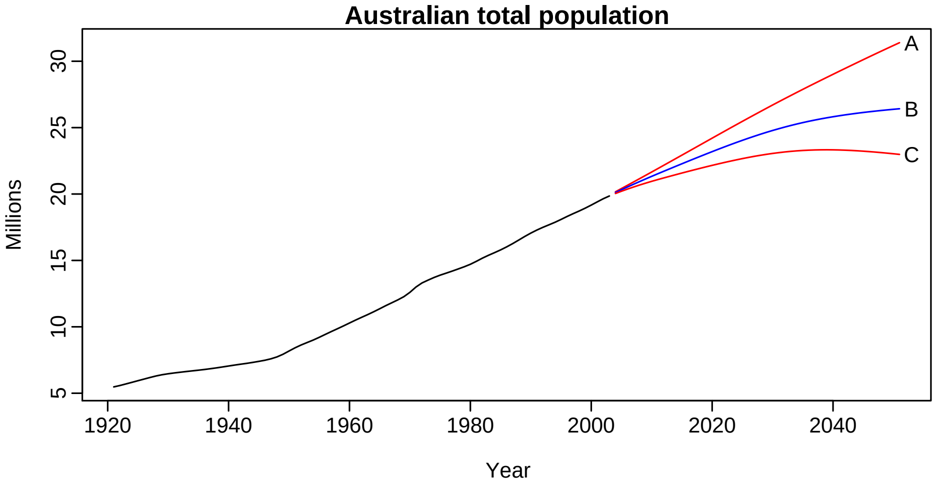
- Most users use the “Medium” projection, but it is unrelated to the mean, median or mode of the future distribution.

ABS population projections



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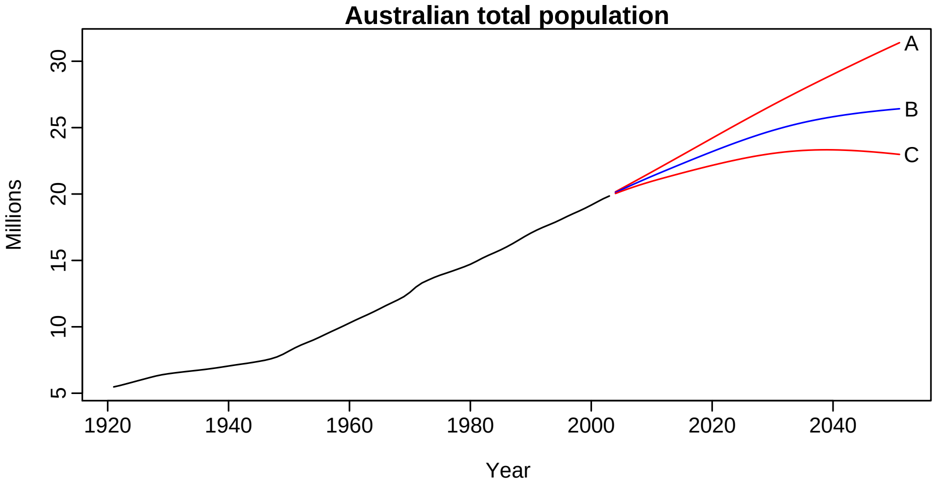

What do these projections mean?

Advantages of stochastic models

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Exponential smoothing

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"Unfortunately, exponential smoothing methods do not allow easy calculation of prediction intervals."

Makridakis, Wheelwright and Hyndman, p.177.
(Wiley, 3rd ed., 1998)

Exponential smoothing

Since 2002. . .

- a general class of state space models proposed underlying all the common exponential smoothing methods.
- analytical results for prediction intervals.
- likelihood calculation for estimation.
- AIC for model selection.
- an algorithm for automatic forecasting using the new class of models.
- new results on the admissible parameter space.

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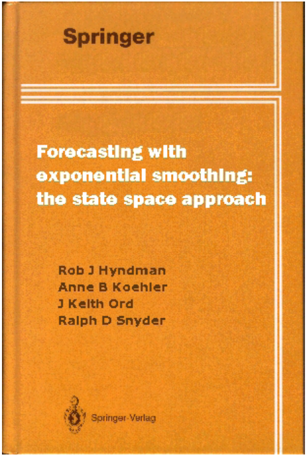
Implementation

This methodology is available as

- An R package (**forecast**) on CRAN
- An Excel add-in (**PhiCast**)

Exponential smoothing



New book due out in 2008

Forecasting with exponential smoothing: the state space approach

Rob J. Hyndman
Anne B. Koehler
J. Keith Ord
Ralph D. Snyder

Published by Springer

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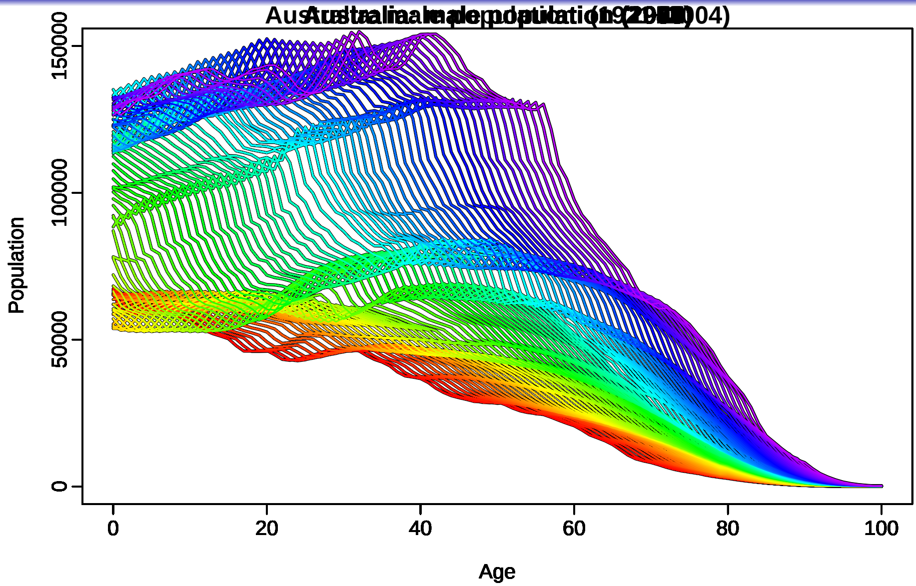
Annual age-specific population



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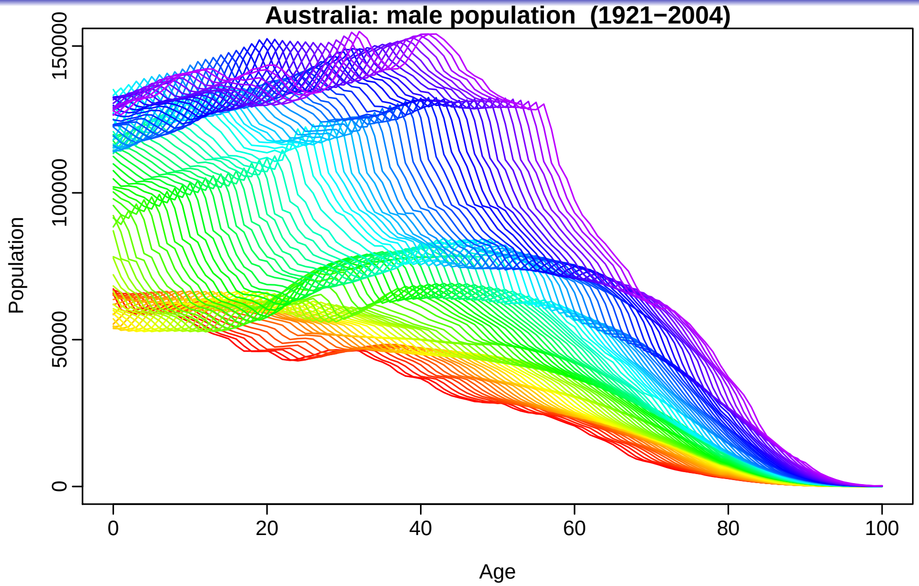

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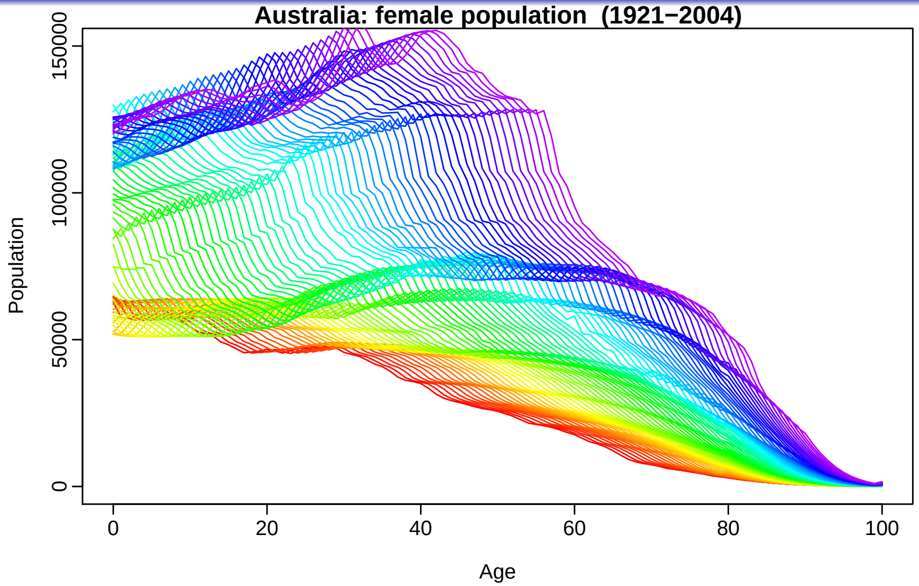

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- Combine the results to get **age-specific stochastic population forecasts**.

Demographic growth-balance equation

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\begin{aligned}
P_{t+1}(x+1) &= P_t(x) - D_t(x,x+1) + G_t(x,x+1) \\
P_{t+1}(0) &= B_t \qquad - D_t(B,0) \qquad + G_t(B,0)
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$x = 0, 1, 2, \dots.$

$P_t(x) =$ population of age x at 1 January, year t

$B_t =$ births in calendar year t

$D_t(x,x+1) =$ deaths in calendar year t of persons aged x at the beginning of year t

$D_t(B,0) =$ deaths in calendar year t of persons born in year t

$G_t(x,x+1) =$ net migrants in calendar year t of persons aged x at the beginning of year t

$G_t(B,0) =$ net migrants of infants born in calendar year t

The available data

The following data are available:

$P_t(x) =$ **population** of age x at 1 January, year t

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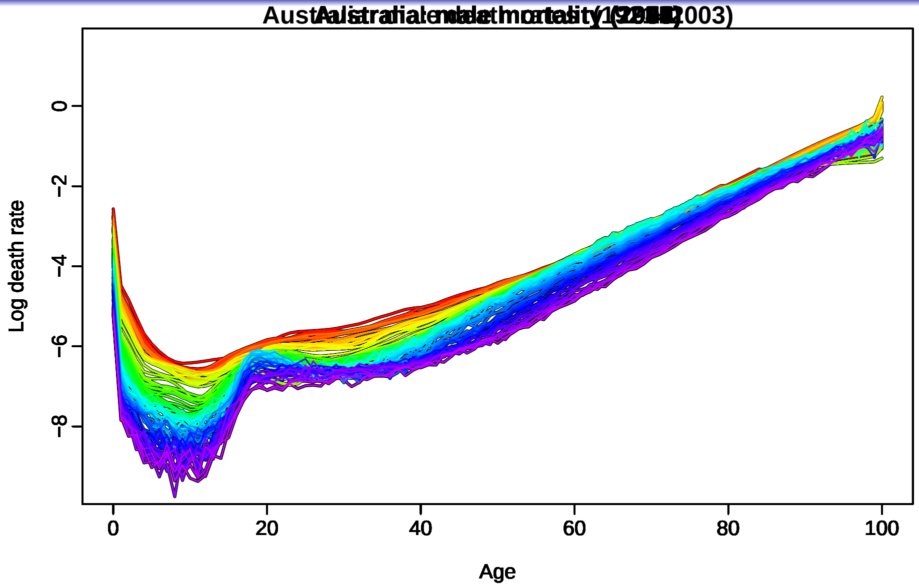
- $D_t(x, x+1)$ and $D_t(B, 0)$.

Mortality rates



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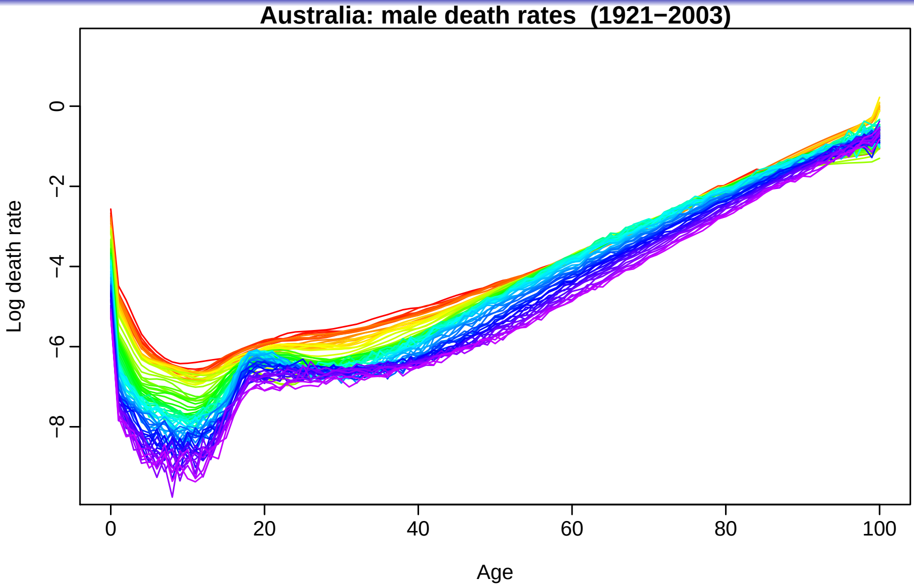

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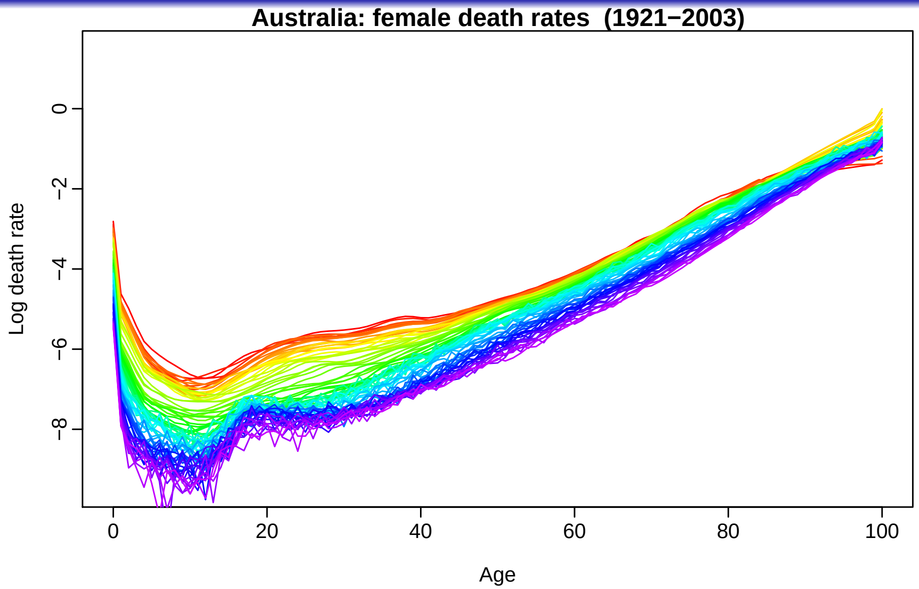
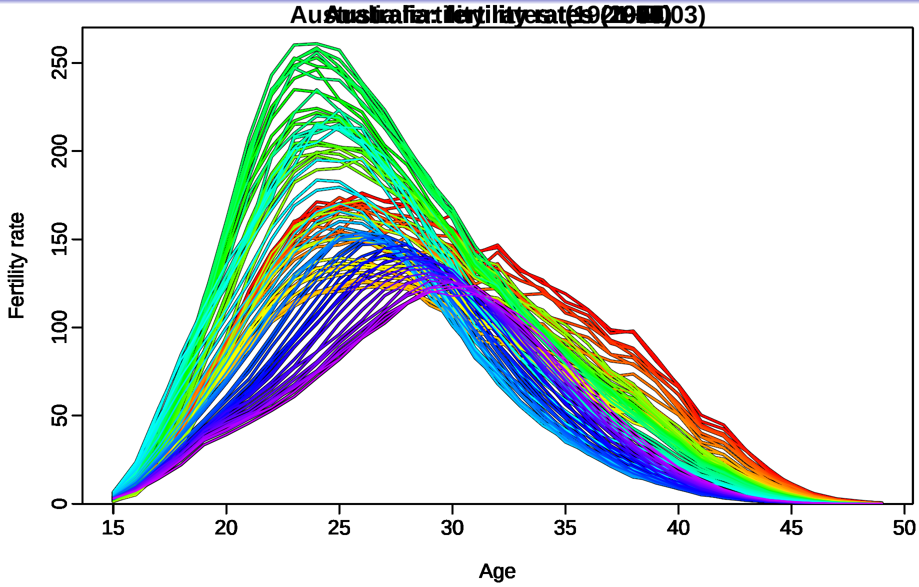


Fertility rates



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Note: "net migration" numbers also include **errors** associated with all estimates. i.e., a "residual".

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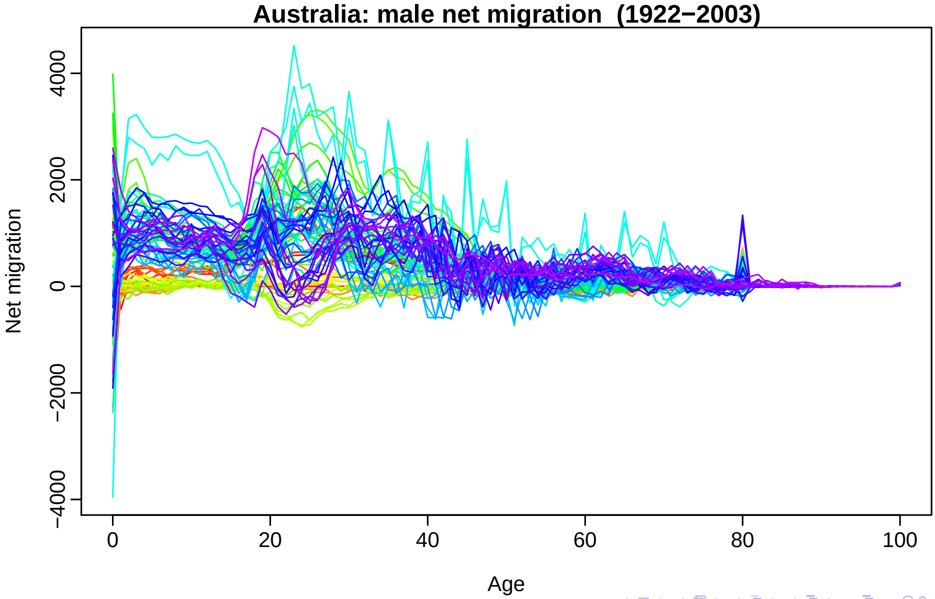

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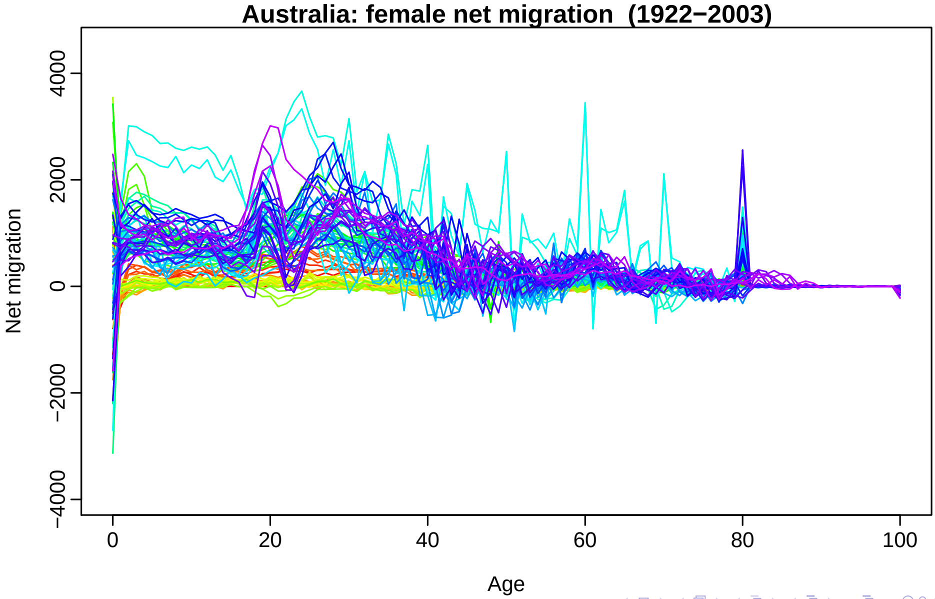

Some notation

Let $y_t(x_i)$ be the observed data (mortality, fertility or net migration) in period t at age x_i, $i = 1, \dots, p$, $t = 1, \dots, n$.

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- $s_t(x)$ and $\sigma_t(x)$ are smooth functions of x.
- We need to estimate $s_t(x)$ from the data for $x_1 < x < x_p$.
- We want to forecast **whole curve** $y_t(x)$ for $t = n + 1, \dots, n + h$.

Functional time series model

$$\begin{aligned} y_t(x) &= s_t(x) + \sigma_t(x)\varepsilon_{t,x} \\ s_t(x) &= \mu(x) + \sum_{k=1}^{K} \beta_{t,k}\,\phi_k(x) + e_t(x) \end{aligned}$$

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5. Put it all together to get forecasts of $y_t(x)$.

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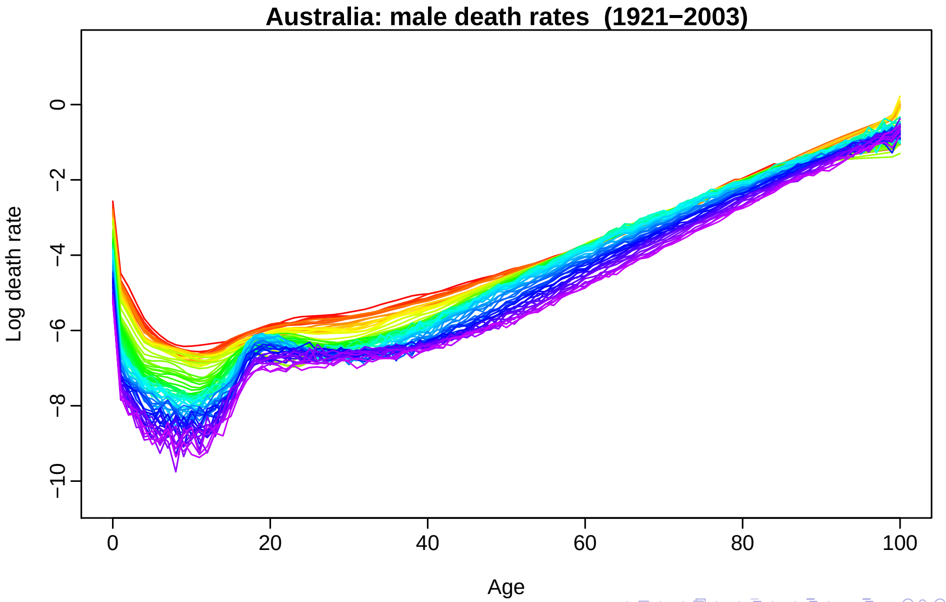

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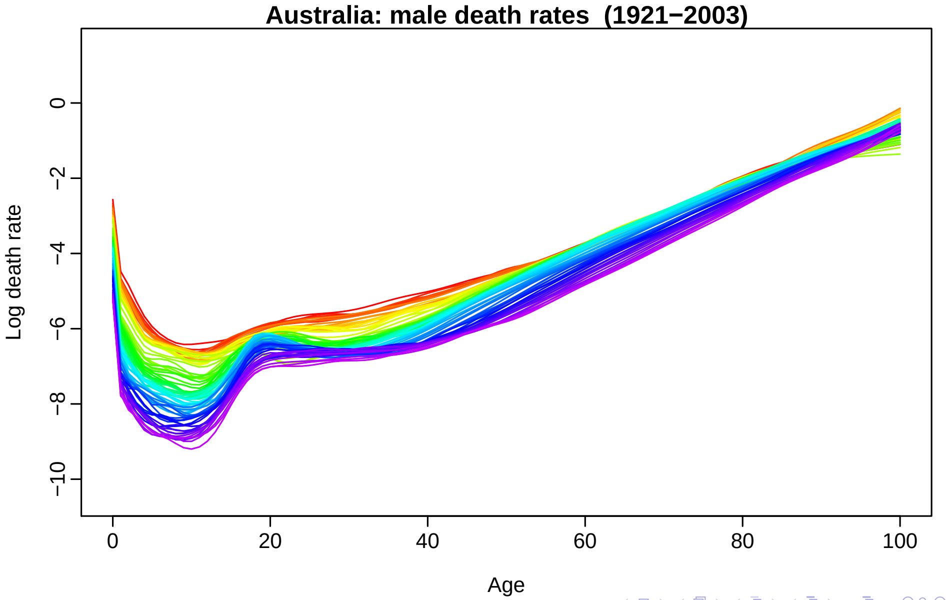

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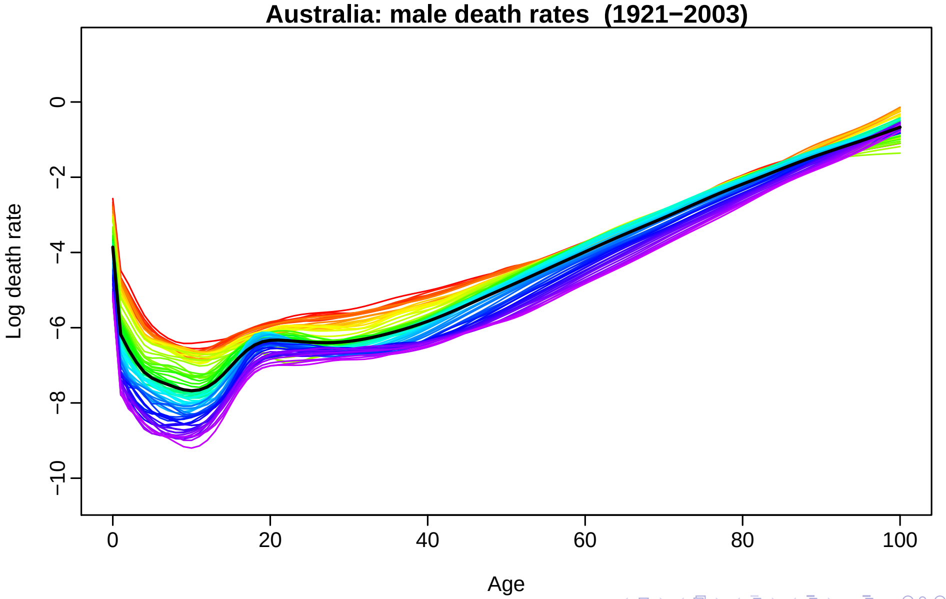

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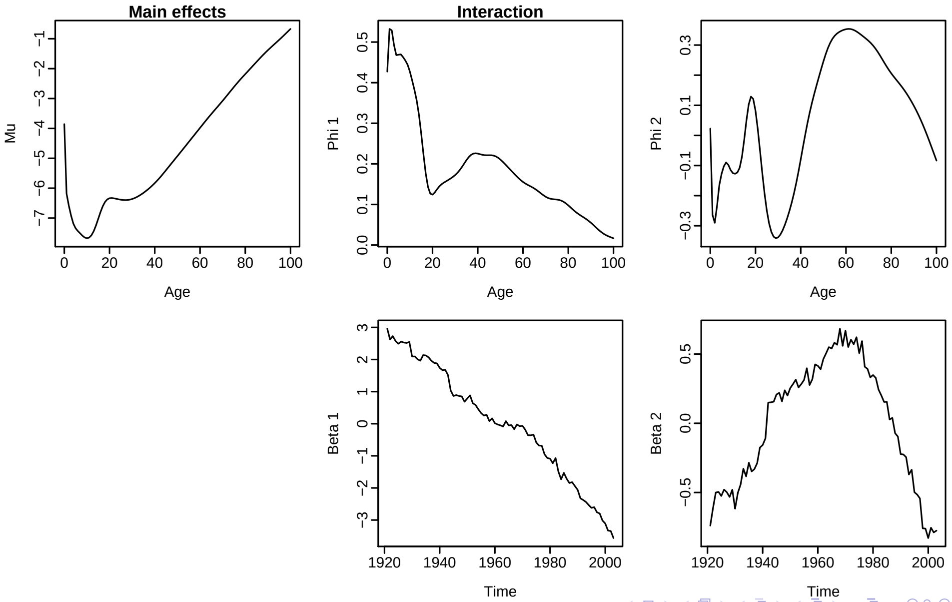
5. Put it all together to get forecasts of $y_t(x)$.

Functional time series model



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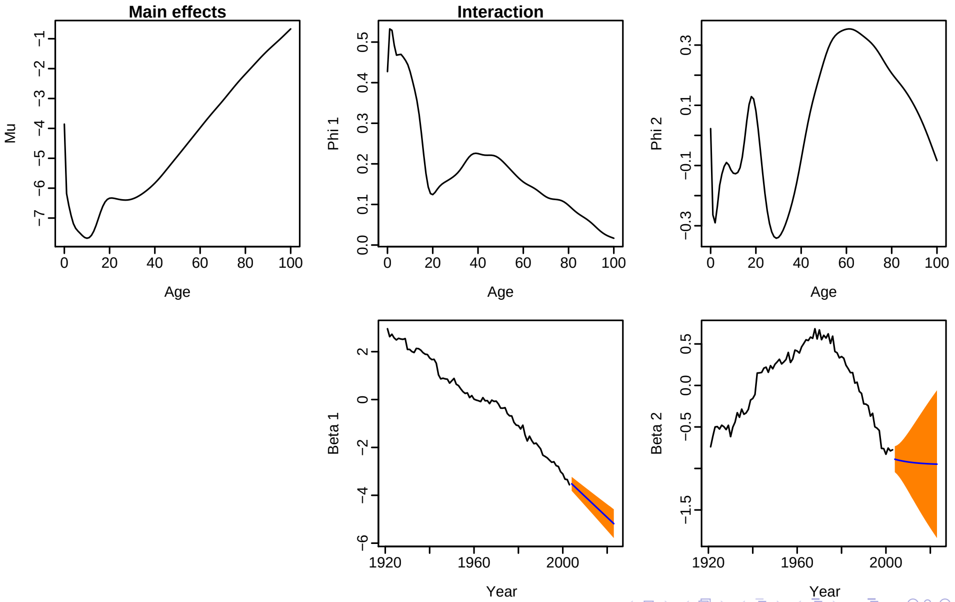

Functional time series model

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\begin{aligned}
y_t(x) &= s_t(x) + \sigma_t(x)\varepsilon_{t,x} \\
s_t(x) &= \mu(x) + \sum_{k=1}^{K} \beta_{t,k}\,\phi_k(x) + e_t(x)
\end{aligned}
$$

where $\varepsilon_{t,x} \overset{\text{iid}}{\sim} \mathrm{N}(0,1)$ and $e_t(x) \overset{\text{iid}}{\sim} \mathrm{N}(0, v(x))$.

1. Estimate smooth functions $s_t(x)$ using nonparametric regression.
2. Estimate $\mu(x)$ as mean $s_t(x)$ across years.
3. Estimate $\beta_{t,k}$ and $\phi_k(x)$ using functional principal components.
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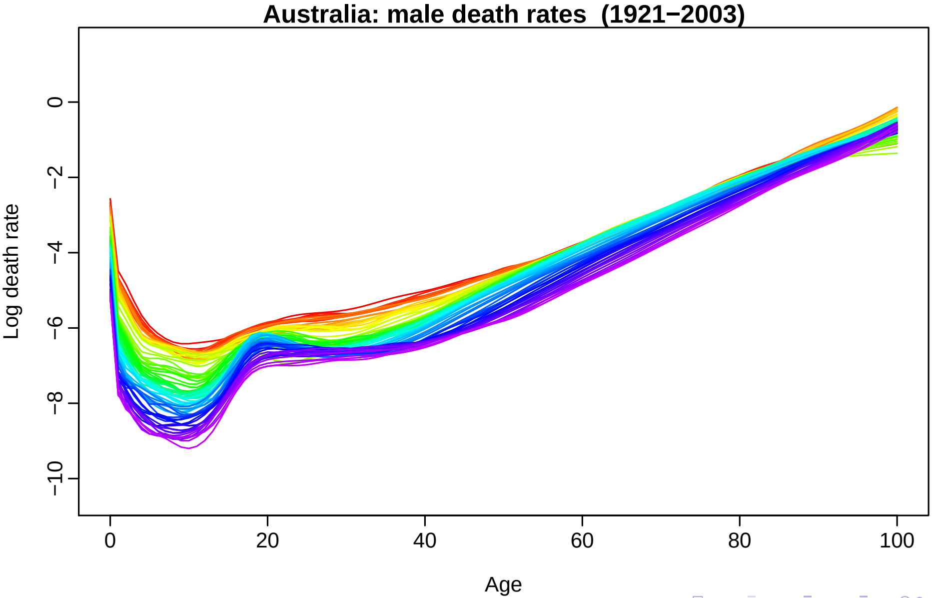

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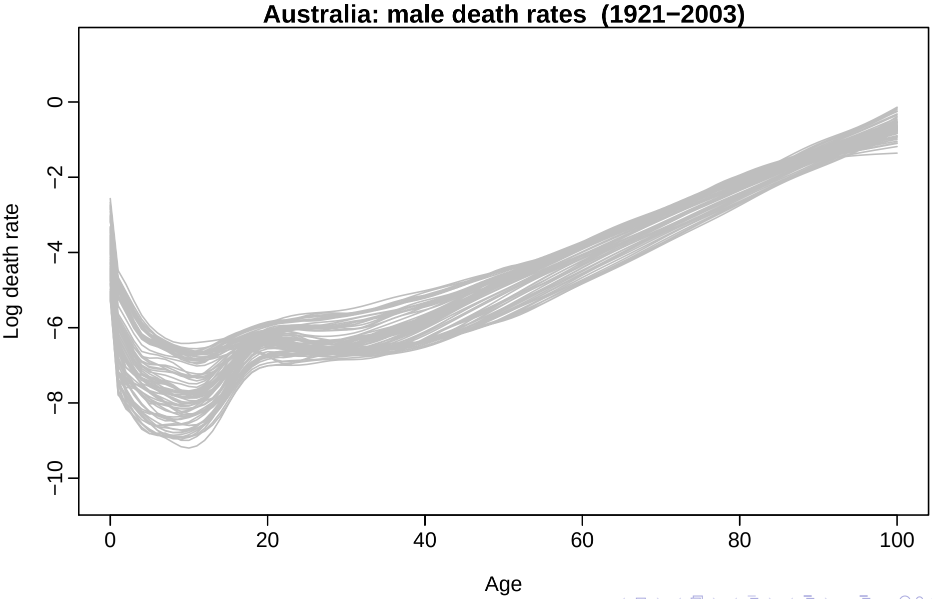

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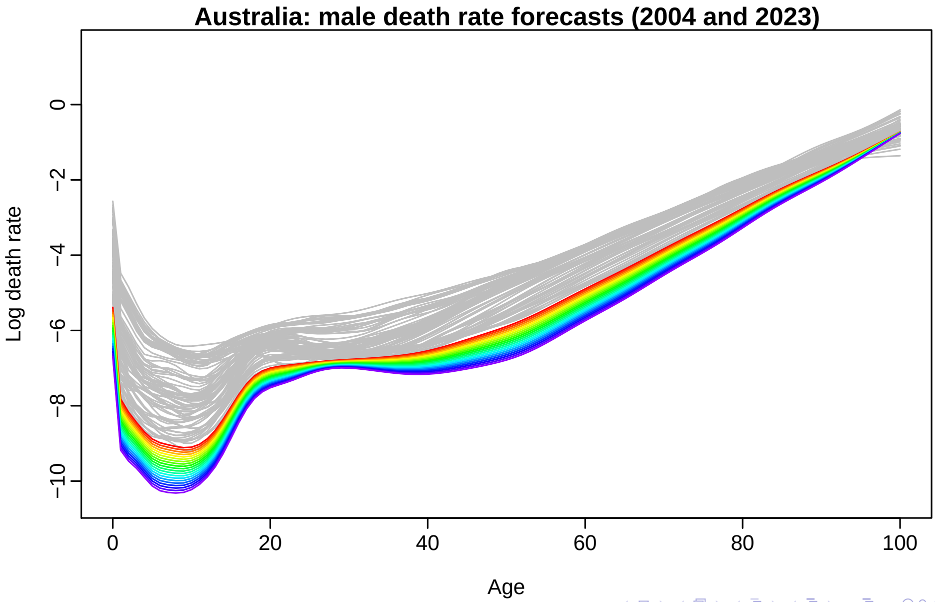

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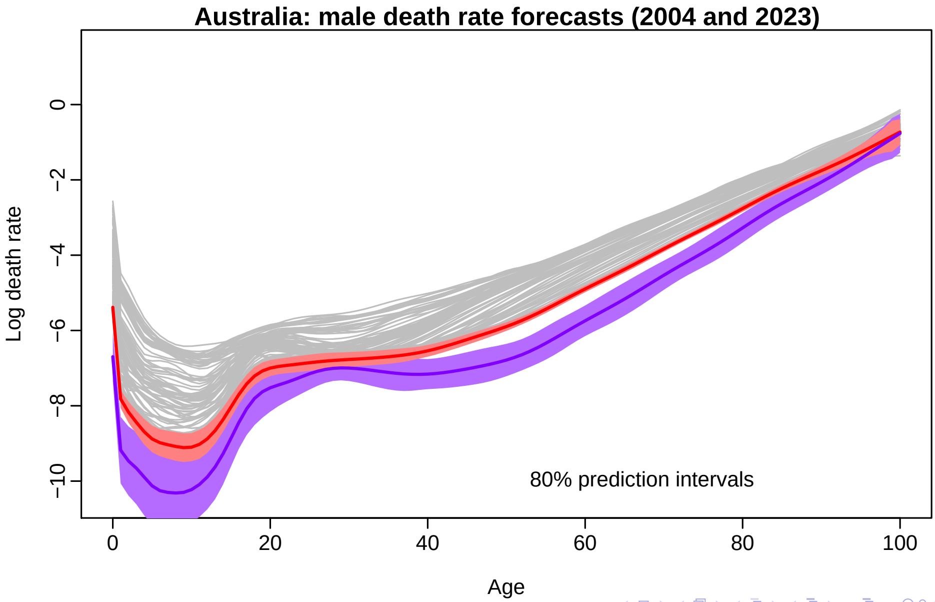
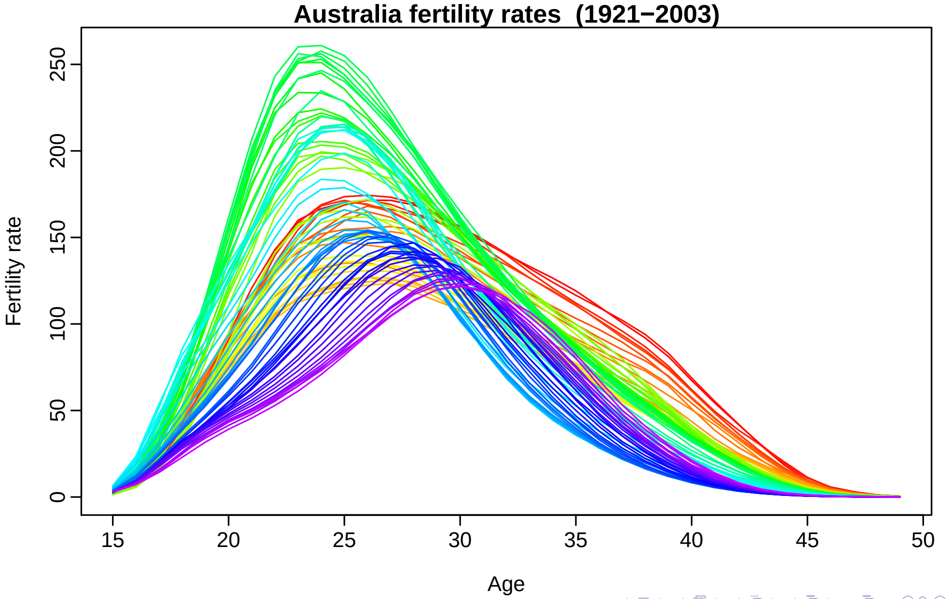
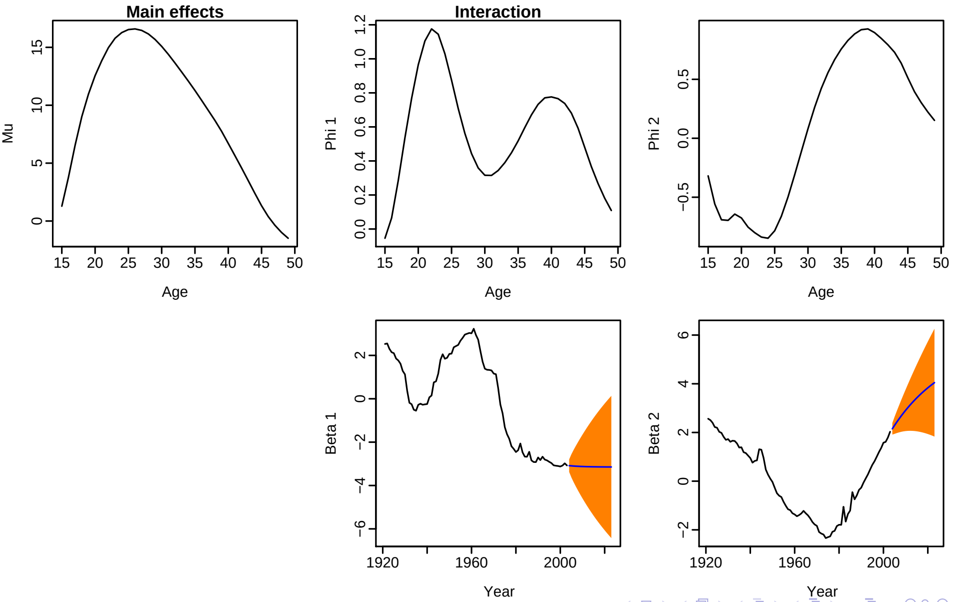


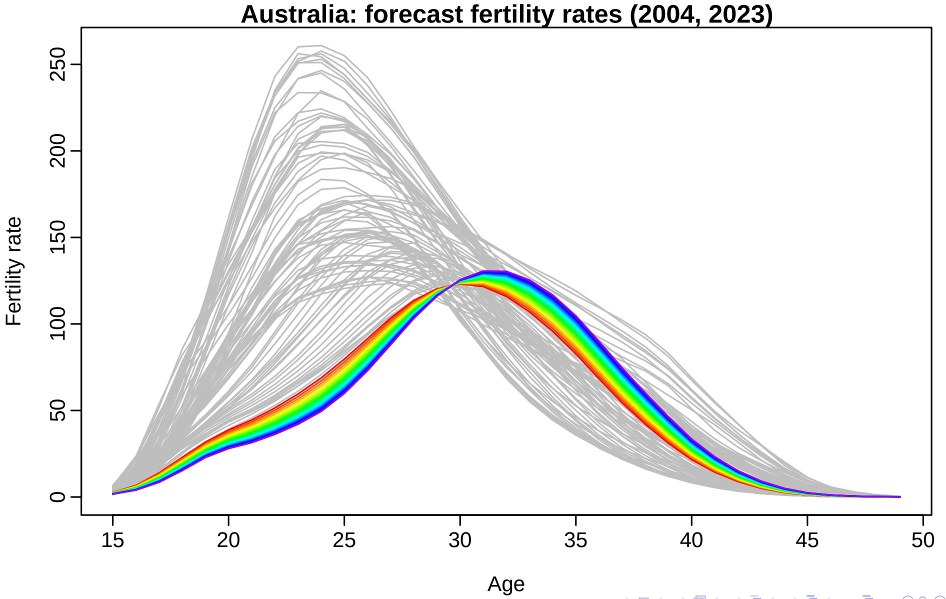
Fertility


Australia fertility rates (1921−2003)
Fertility rate
Age
0
50
100
150
200
250
15
20
25
30
35
40
45
50

Fertility


Main effects
Mu
Age
Interaction
Phi 1
Age
Phi 2
Age
Beta 1
Year
Beta 2
Year

Fertility


Australia: forecast fertility rates (2004, 2023)
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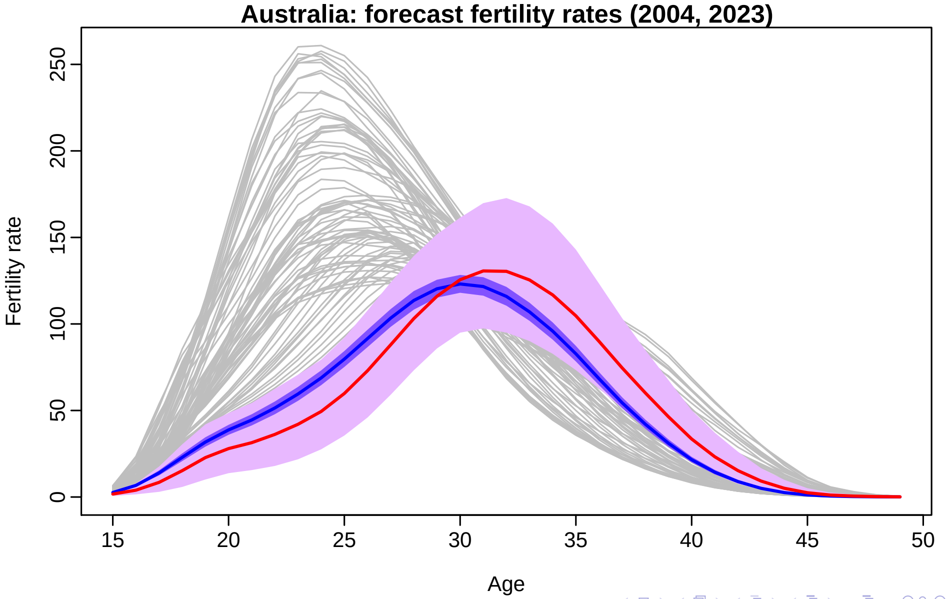
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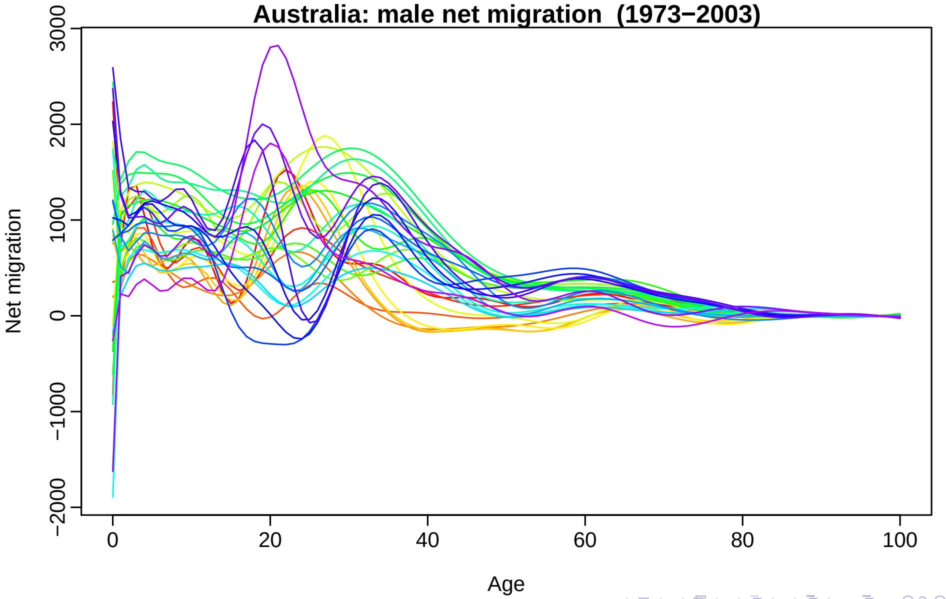
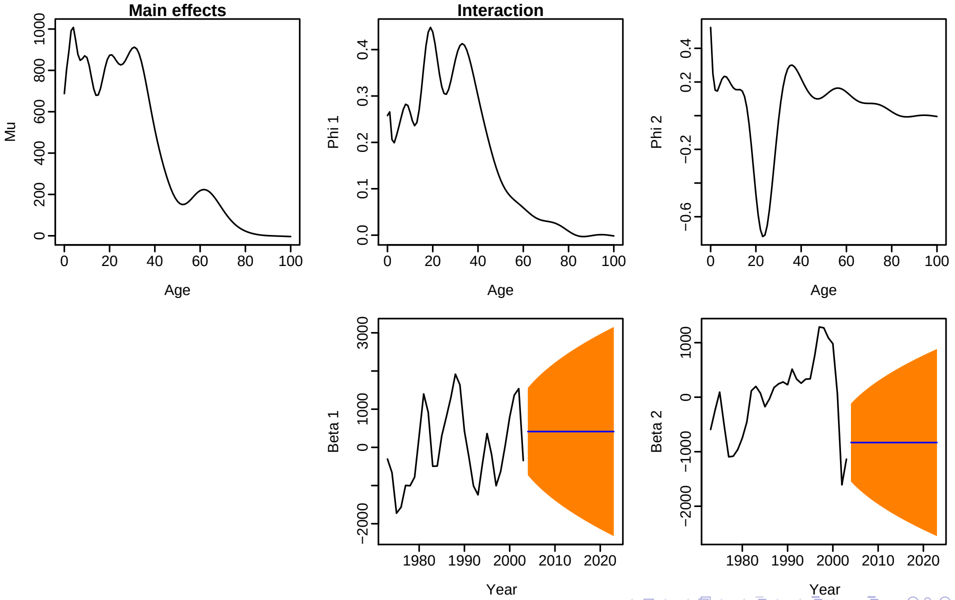
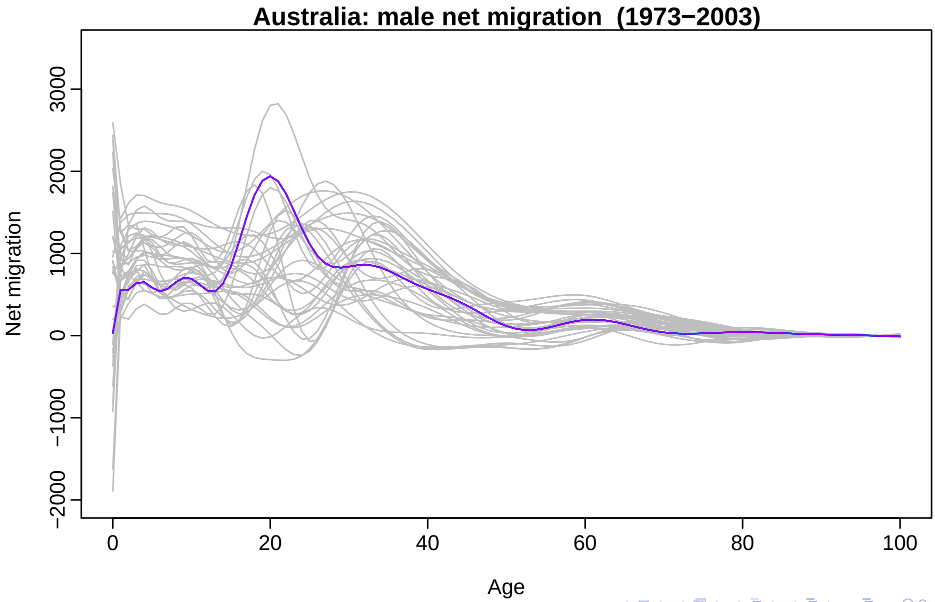
Migration: male



Migration: male



Migration: male



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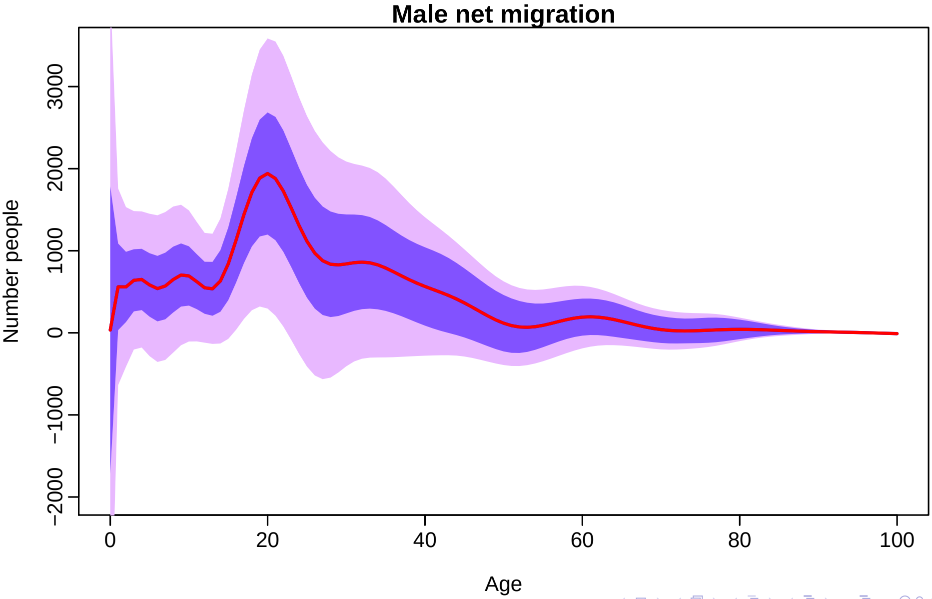

Simulation

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- For each of $m_t^F(x)$, $m_t^M(x)$, $f_t(x)$, $G_t^F(x, x+1)$, and $G_t^M(x, x+1)$:

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- Use simulated rates to generate $B_t(x)$, $D_t^F(x, x+1)$, $D_t^M(x, x+1)$ for $t = n+1, \dots, n+h$, assuming deaths and births are Poisson.

Simulation

Demographic growth-balance equation used to get population sample paths.

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\begin{aligned}
P_{t+1}(x+1) &= P_t(x) - D_t(x,x+1) + G_t(x,x+1) \\
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$x = 0, 1, 2, \ldots.$

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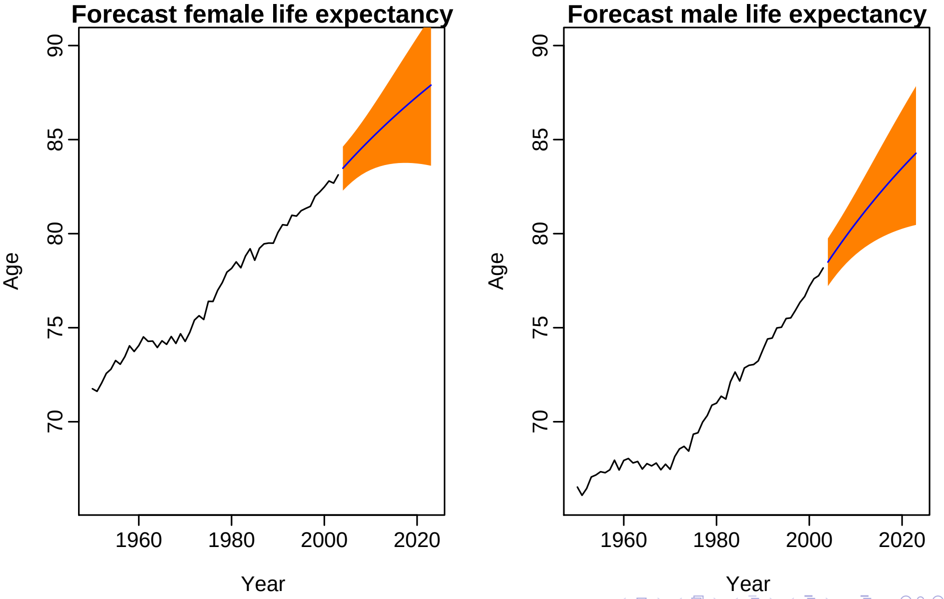
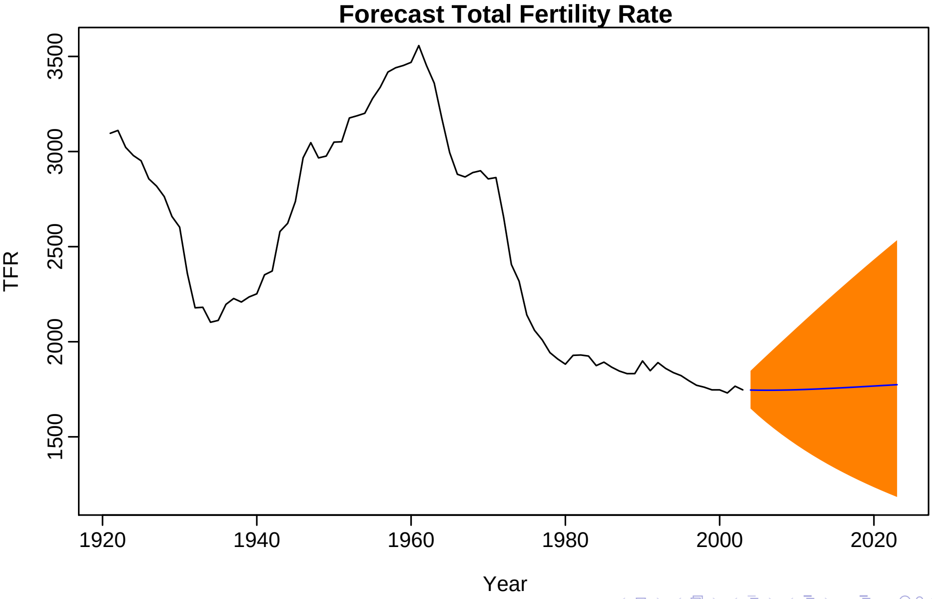
- This allows the computation of the empirical forecast distribution of any demographic quantity that is based on births, deaths and population numbers.

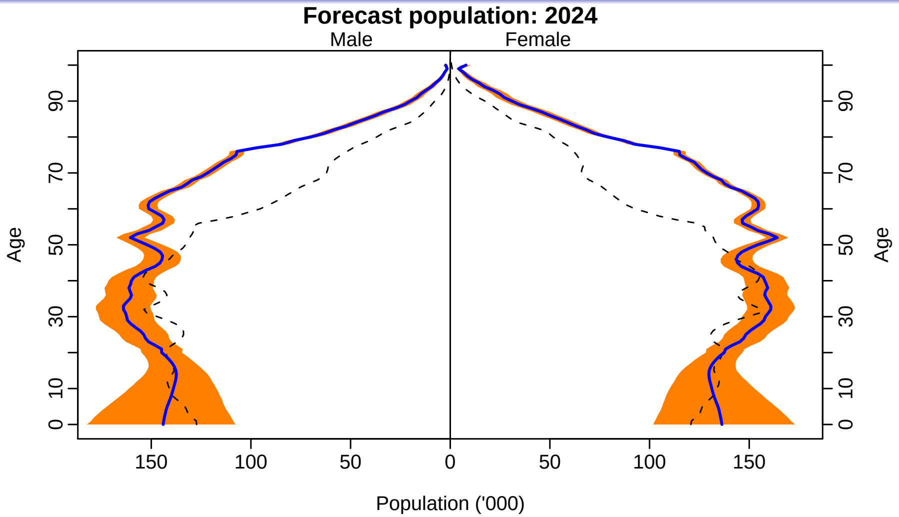
Forecasts of life expectancy at age 0



Forecasts of TFR

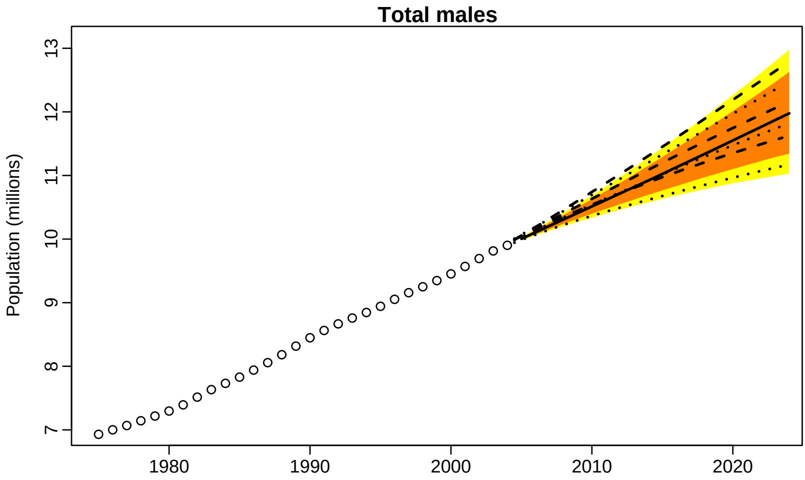


Population forecasts



Forecast population pyramid for 2024, along with 80% prediction intervals. Dashed: actual population pyramid for 2004.

Population forecasts



Twenty-year forecasts of total population along with 80% and 95% prediction intervals. Dashed: ABS (2006) projections. Dotted: ABS (2003) projections.

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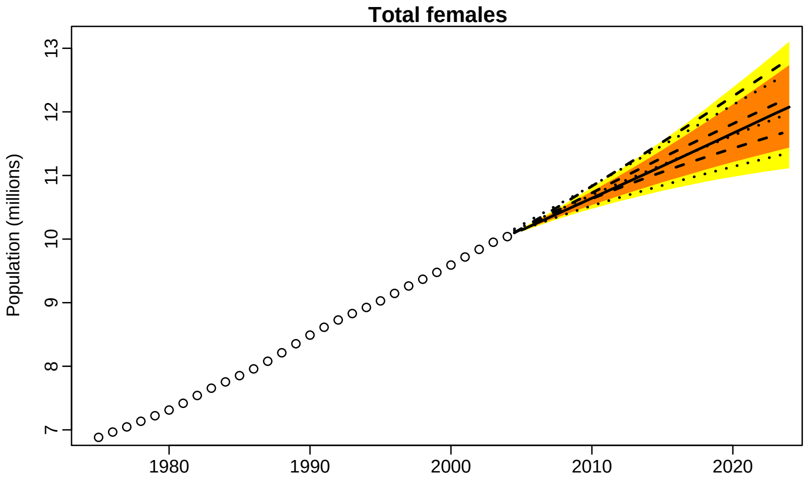

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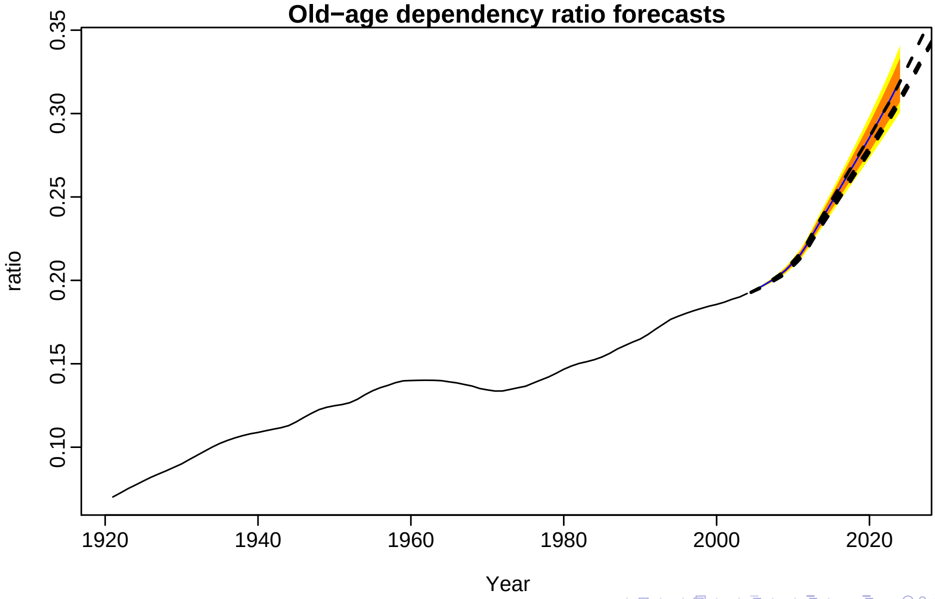
Old-age dependency ratio



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Extensions

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Software and papers:

- Booth (2004) On the importance of being uncertain: forecasting population futures for Australia. *People and Place*, **12**(2), 1–11.
- Hyndman and Booth (2007) Stochastic population forecasts using functional data models for mortality, fertility and migration. **www.robhyndman.info**
- Hyndman, Booth, Tickle and Maindonald (2006) demography **R package** v0.97. **www.robhyndman.info/Rlibrary/demography**

Outline

Conclusions

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Slides available from
http://www.robhyndman.info/