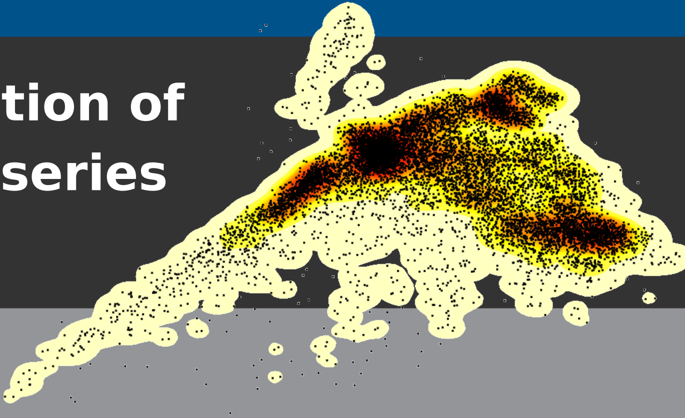
MONASH BUSINESS SCHOOL

Visualisation of big time series data



Rob J Hyndman

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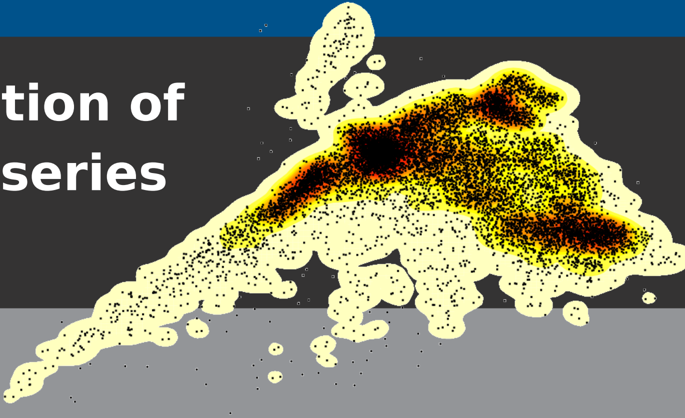

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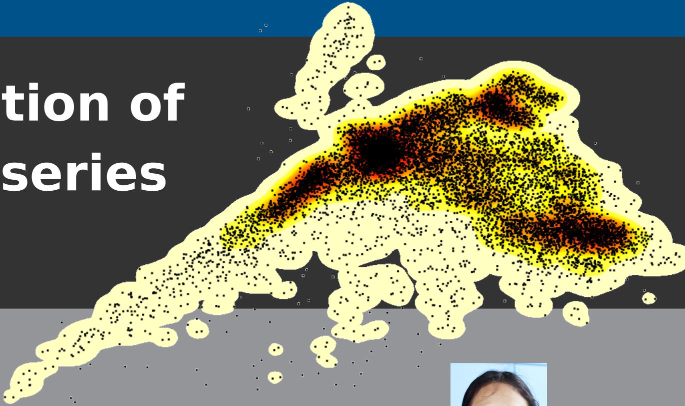

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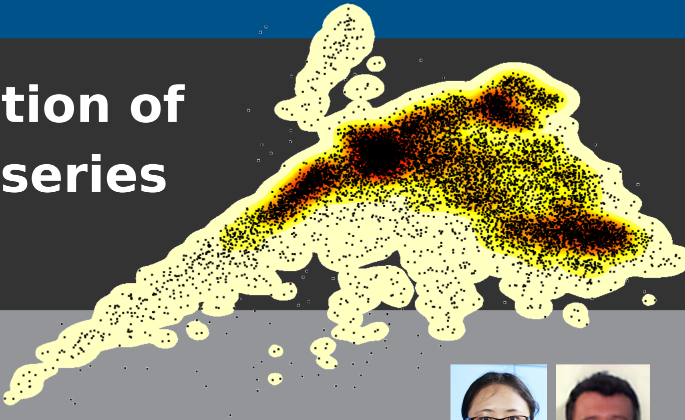

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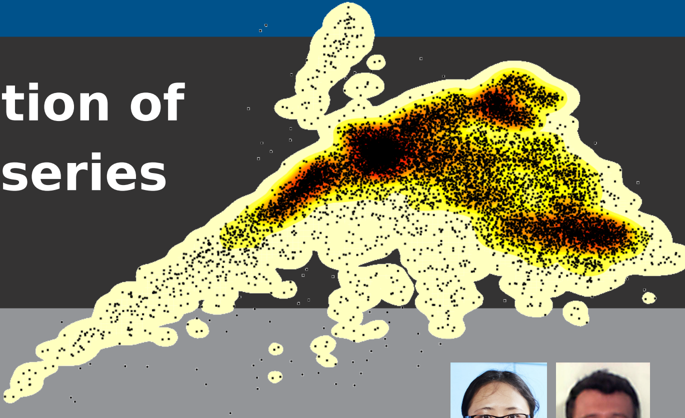

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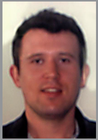

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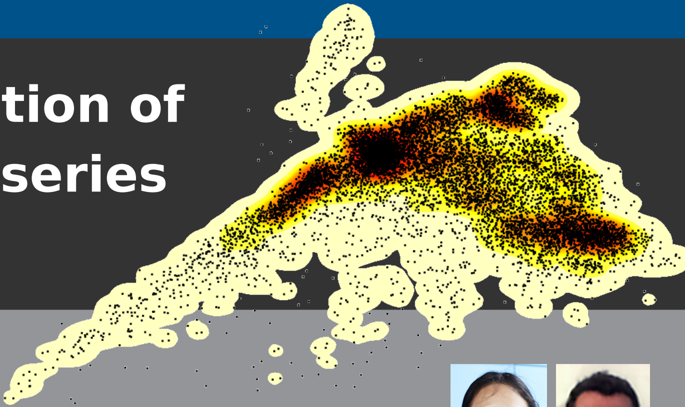

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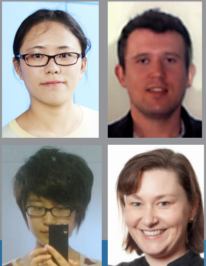

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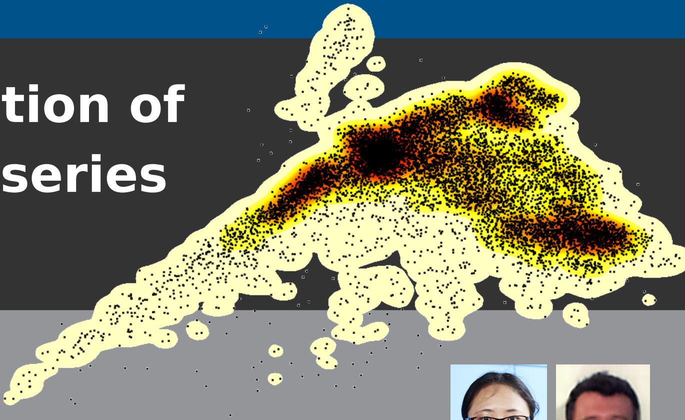

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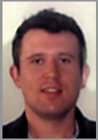
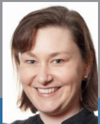
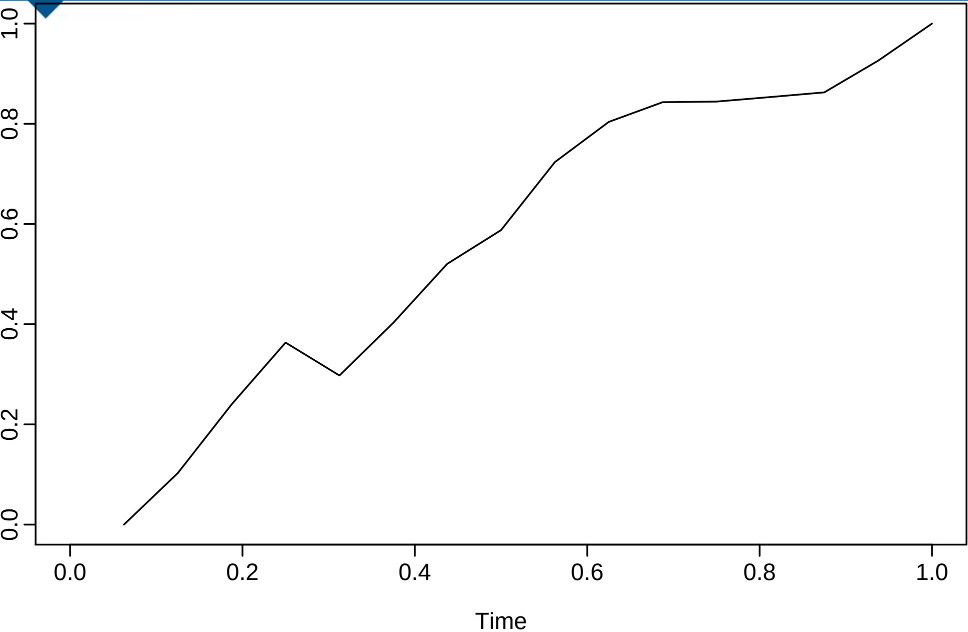




Outline

1. **The problem**
2. Australian tourism demand
3. M3 competition data
4. Yahoo web traffic
5. What next?

How to plot lots of time series?



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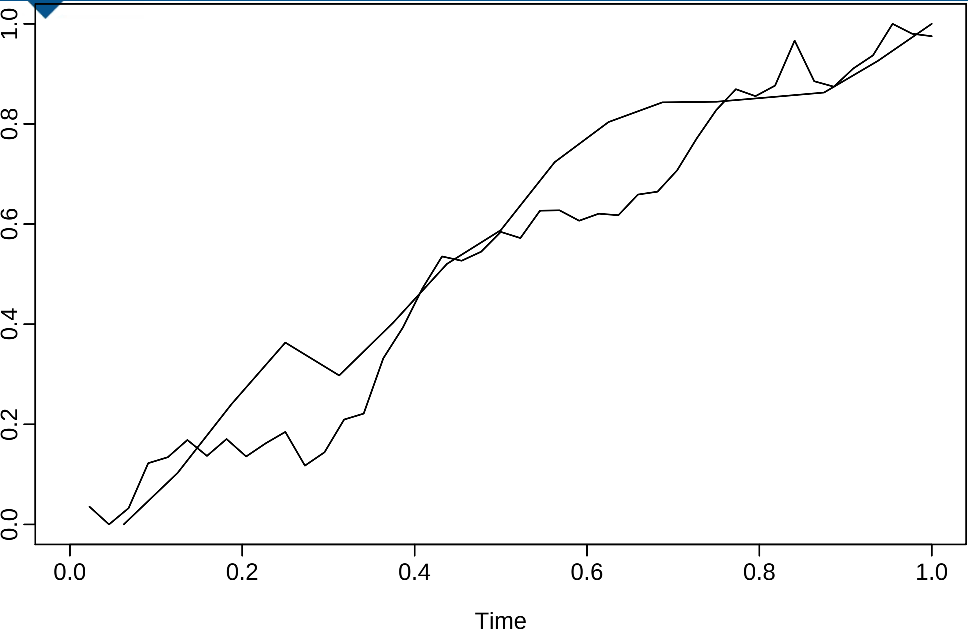

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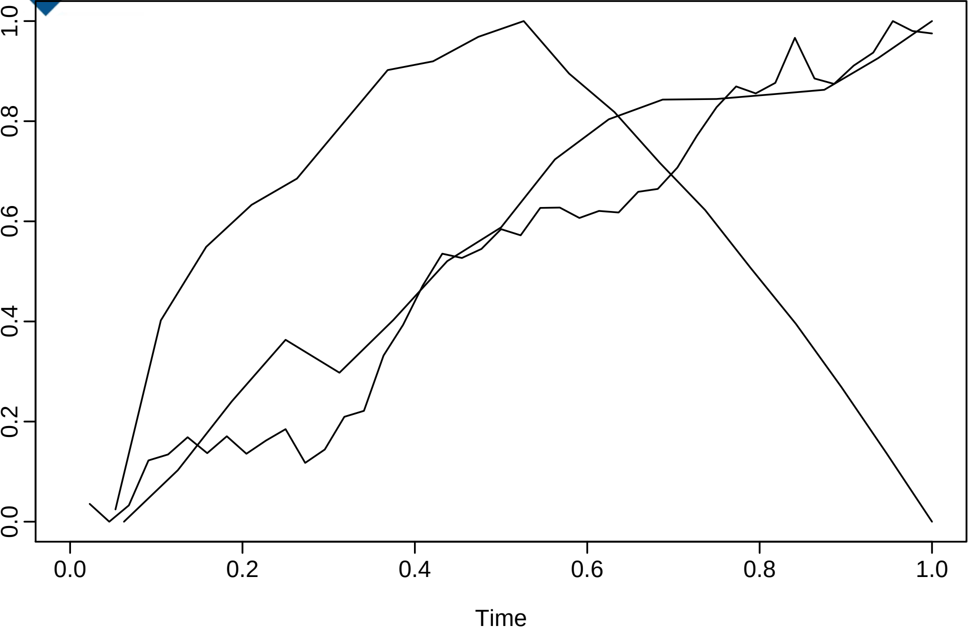

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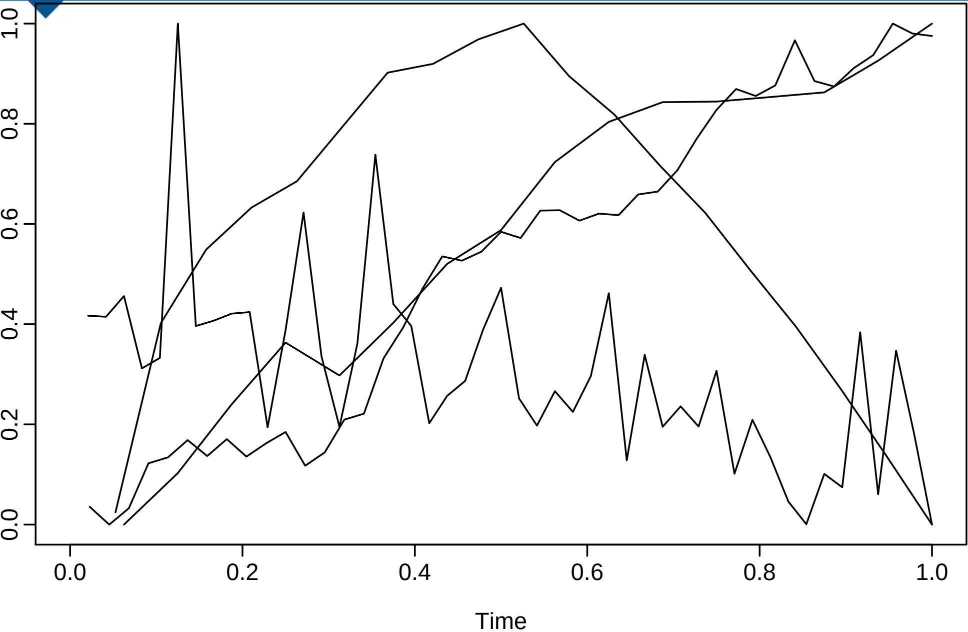

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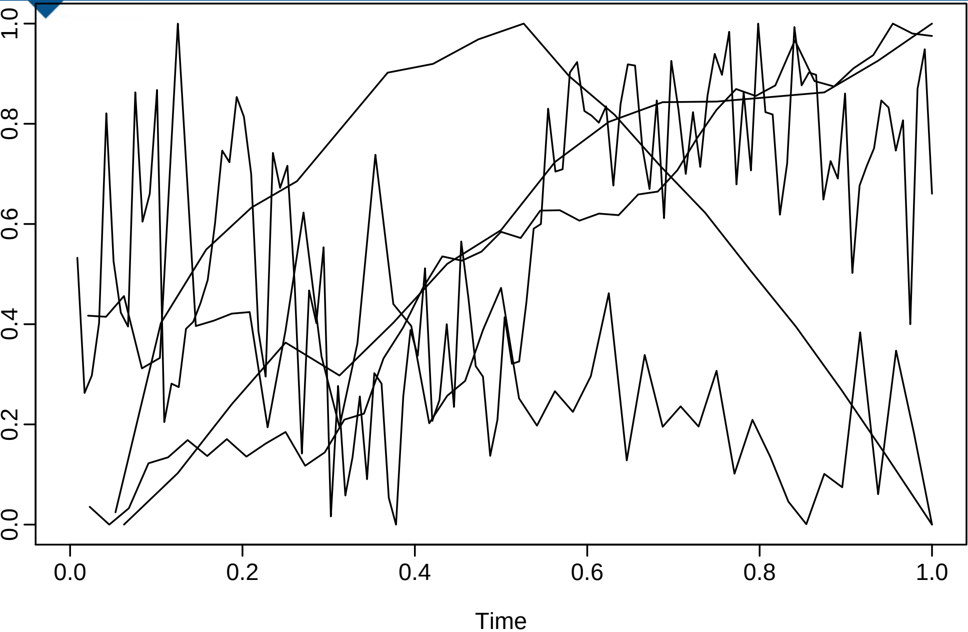

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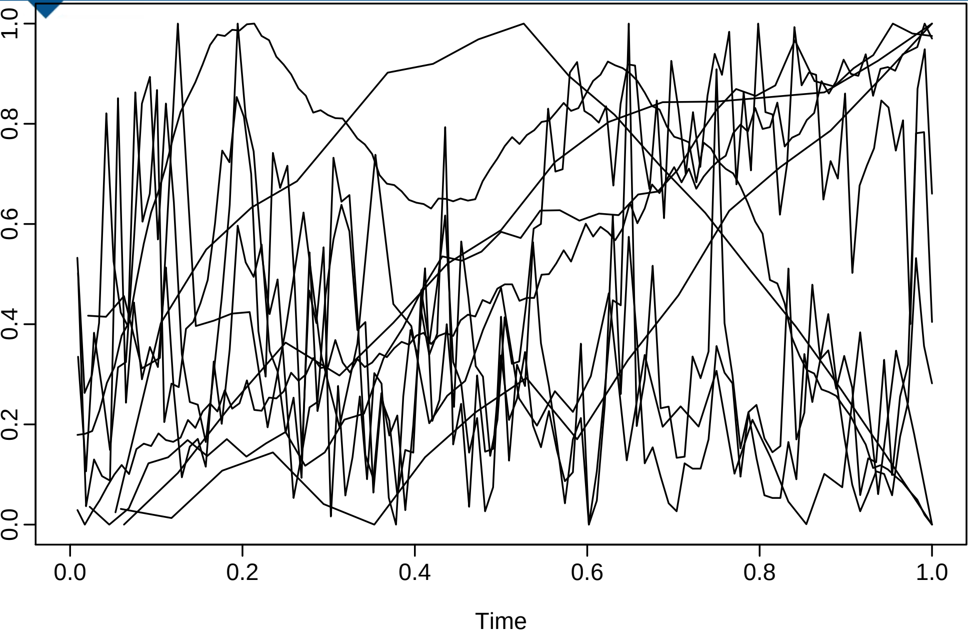

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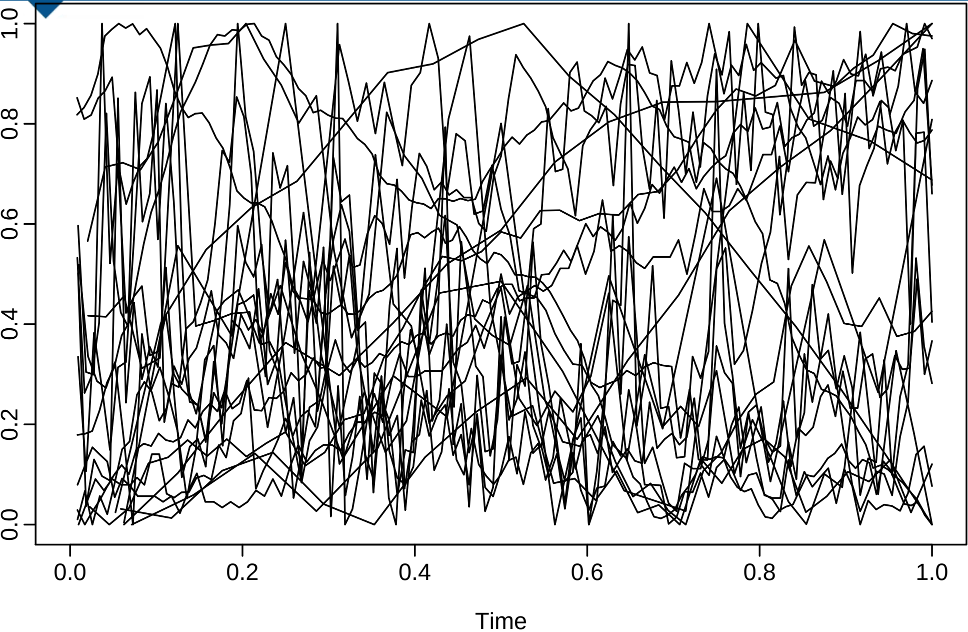

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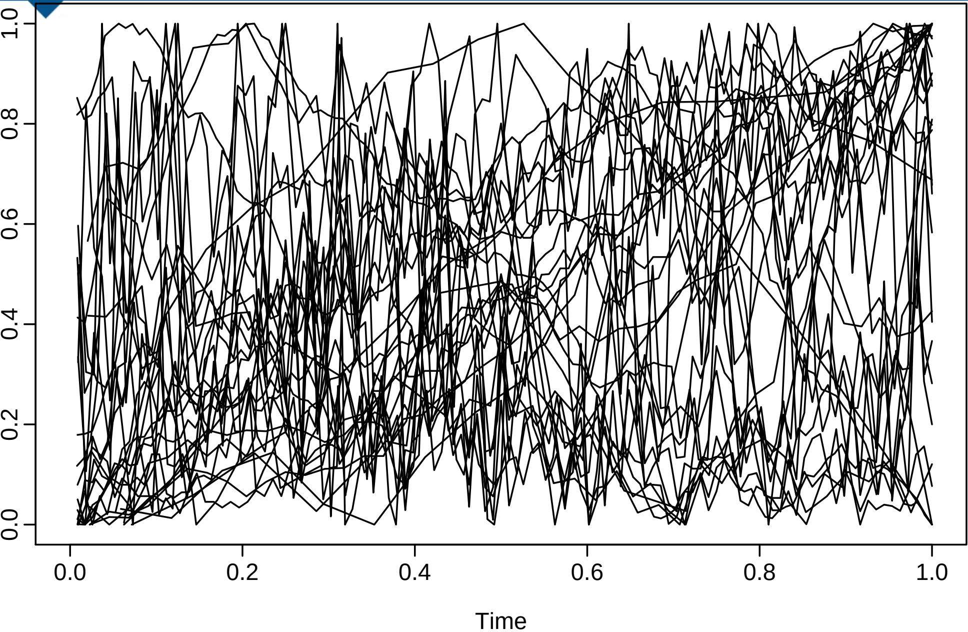

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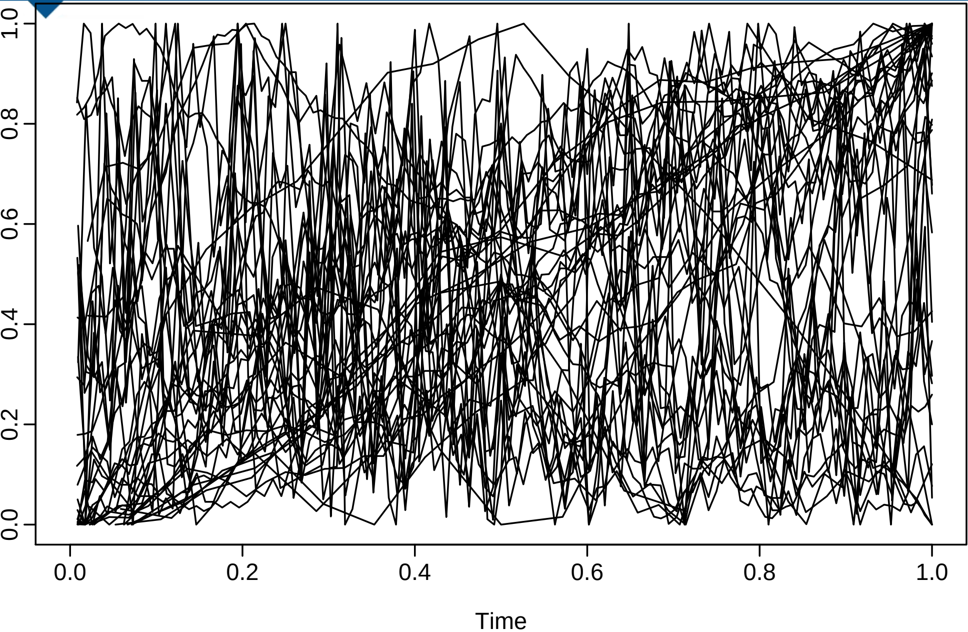

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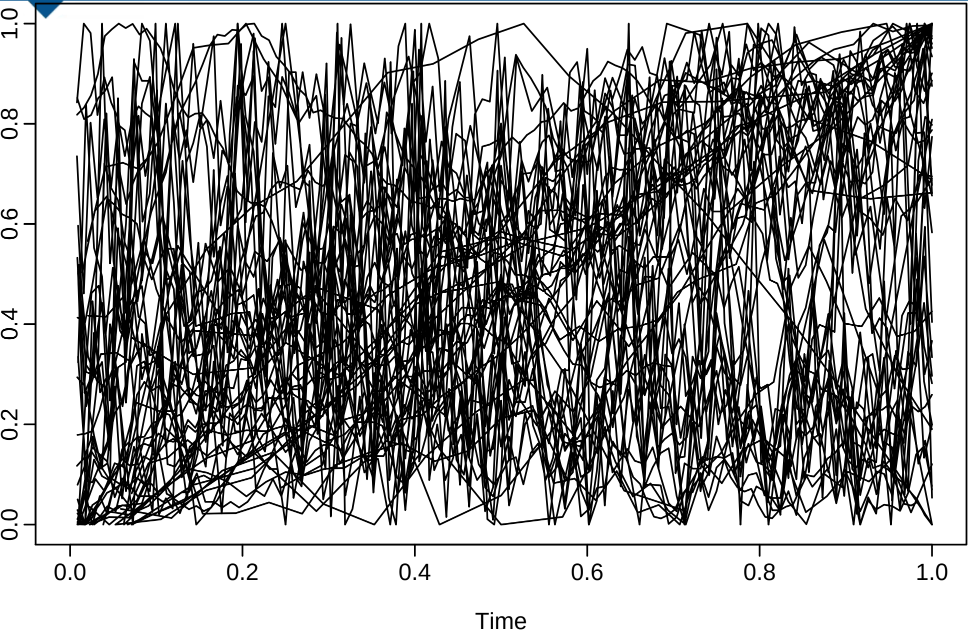

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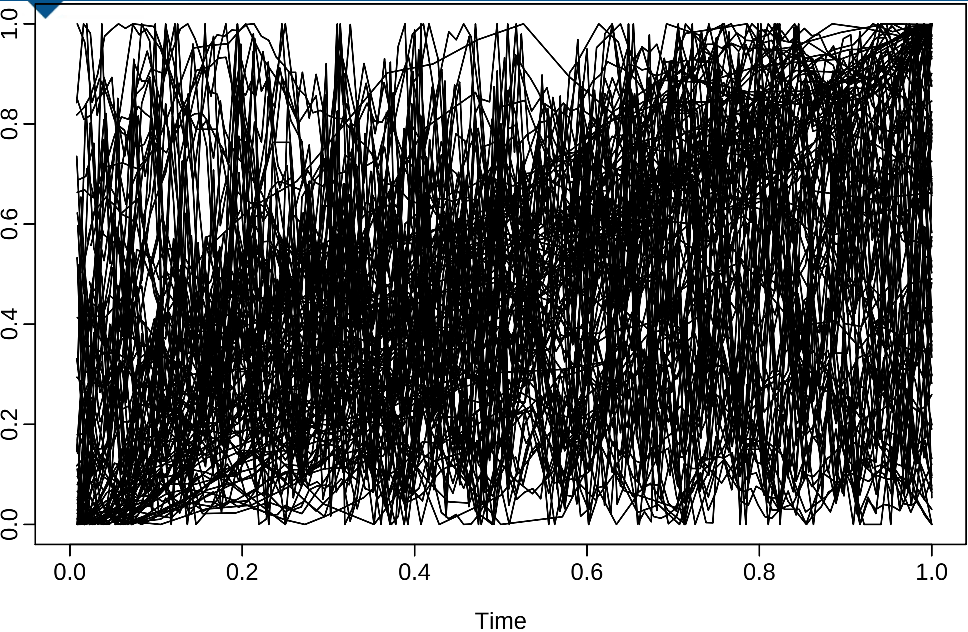

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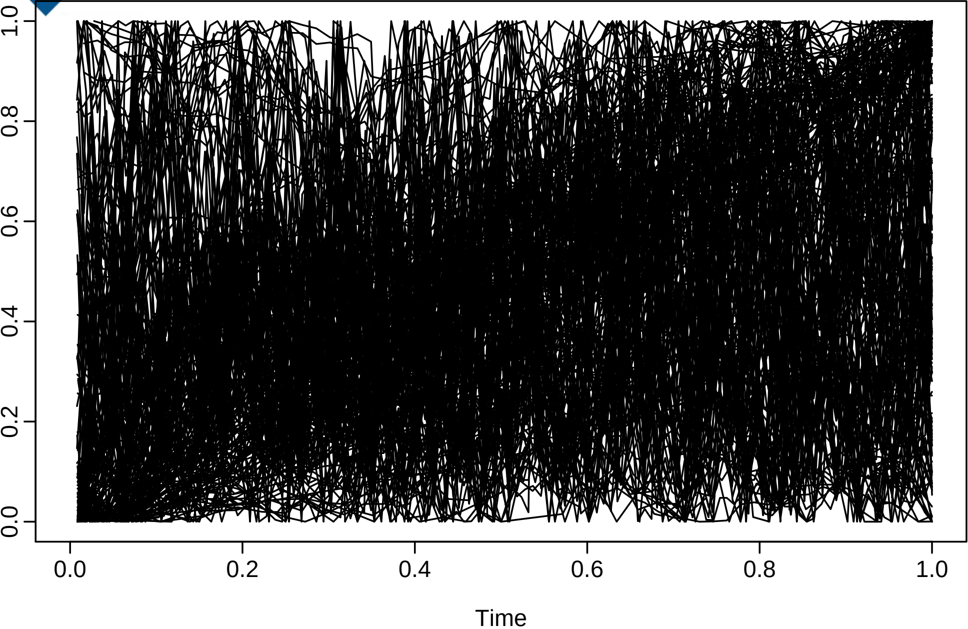

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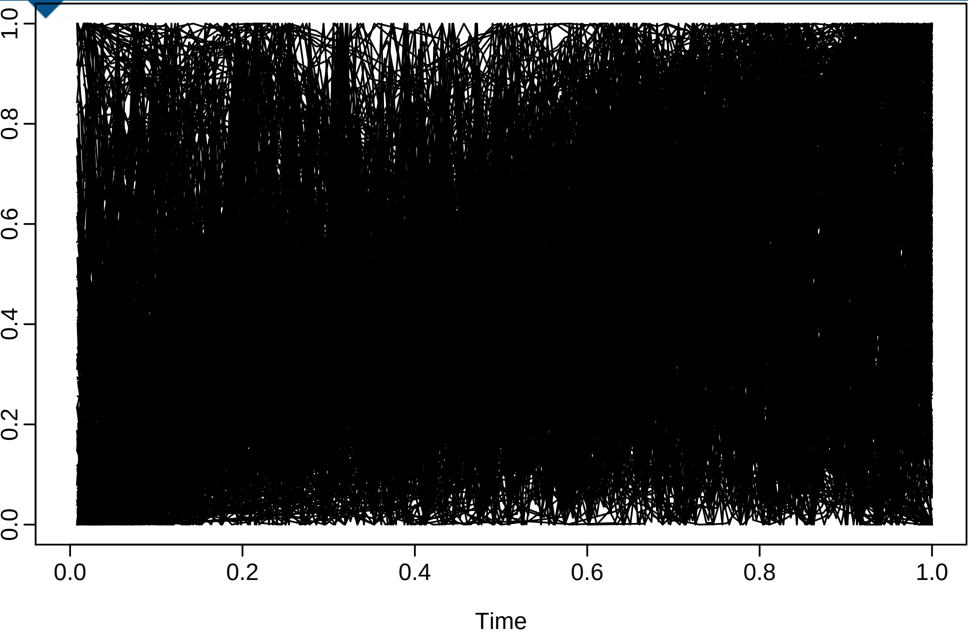

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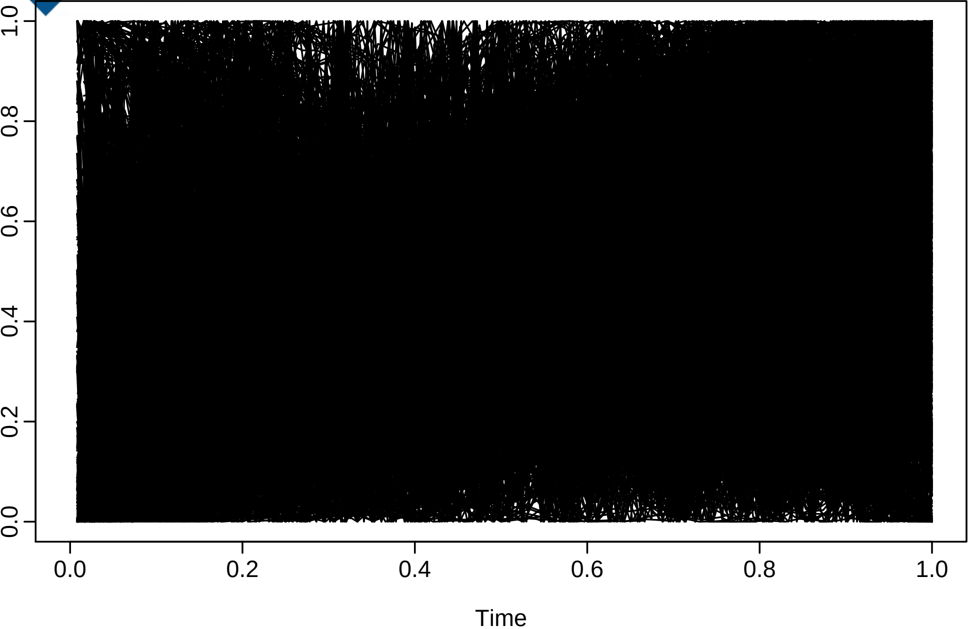

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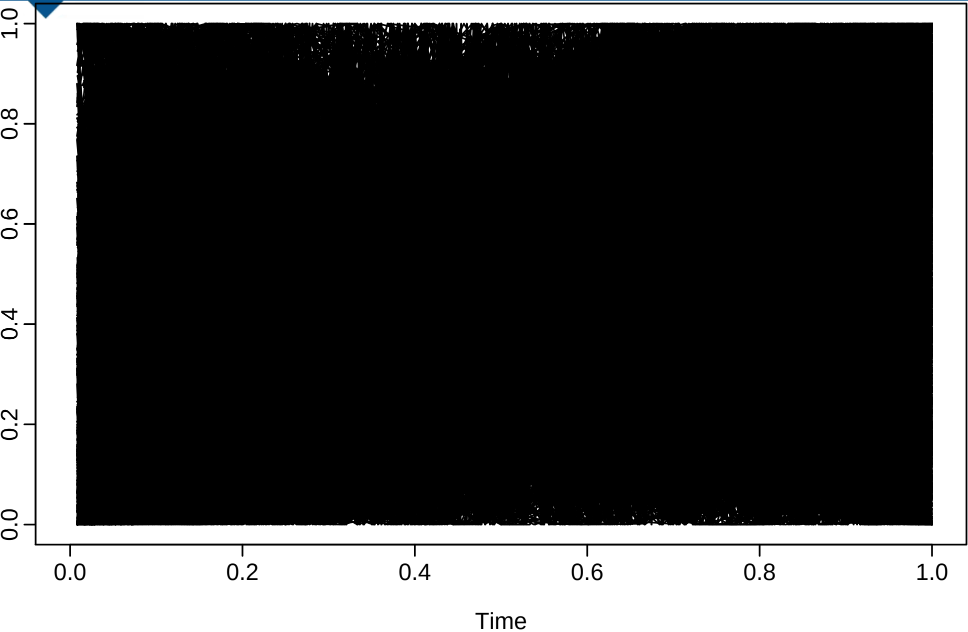

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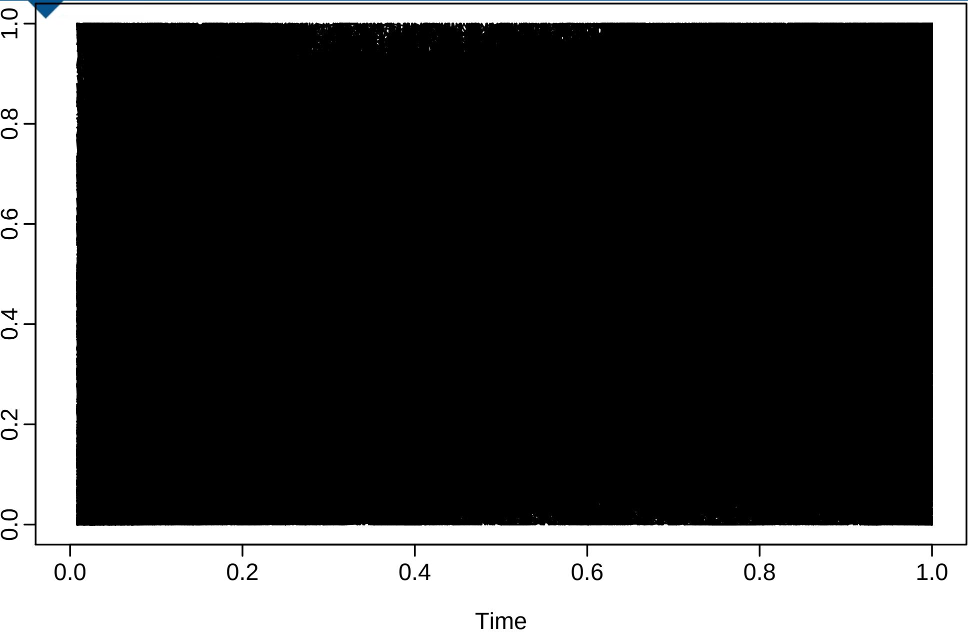

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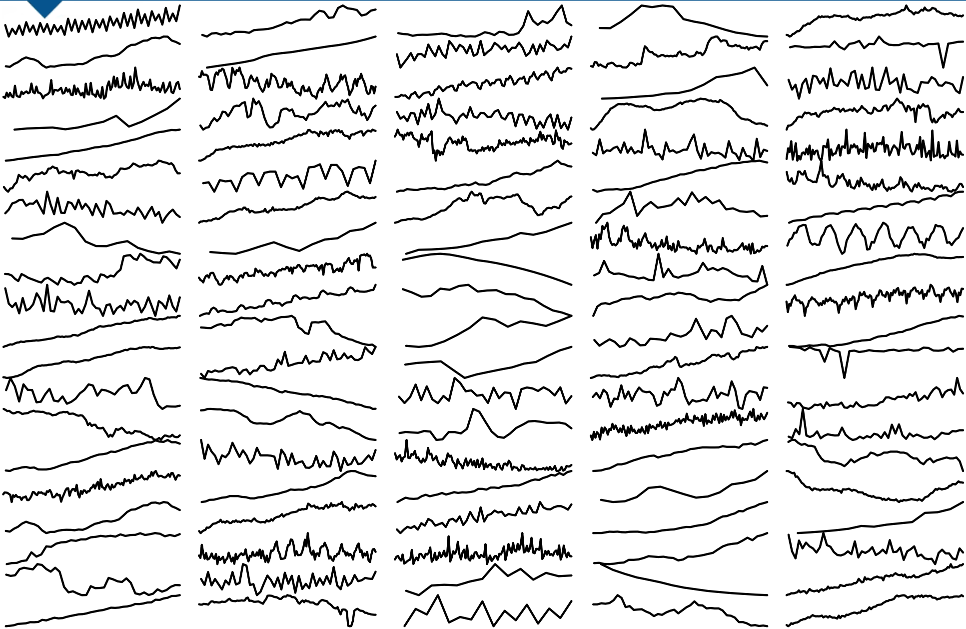

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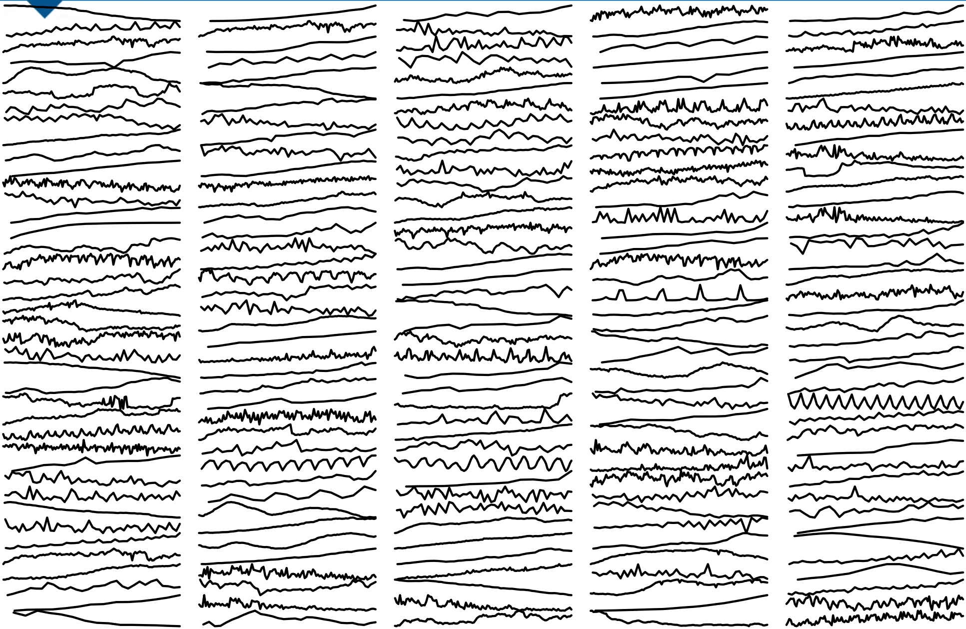

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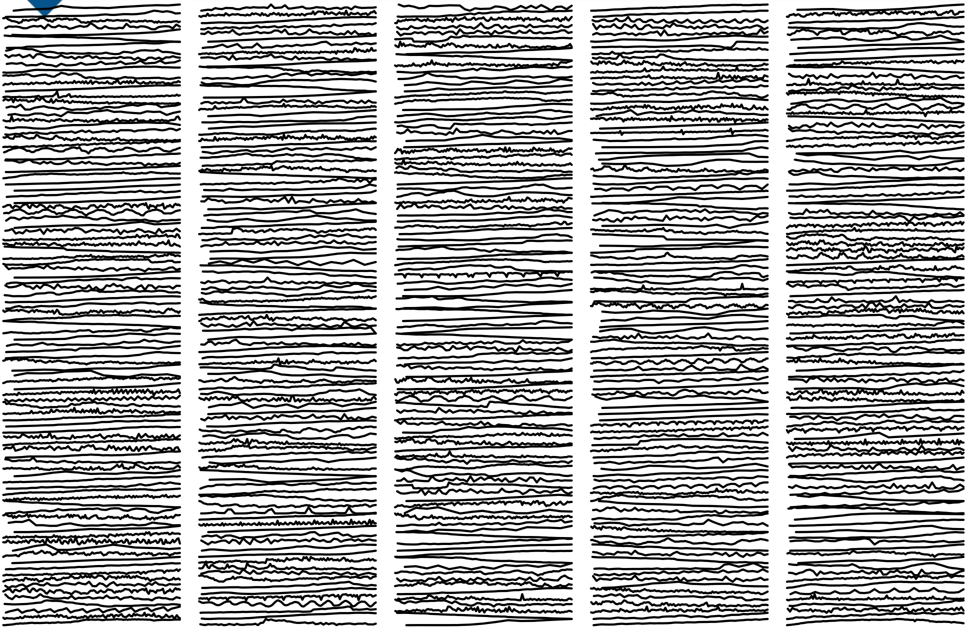

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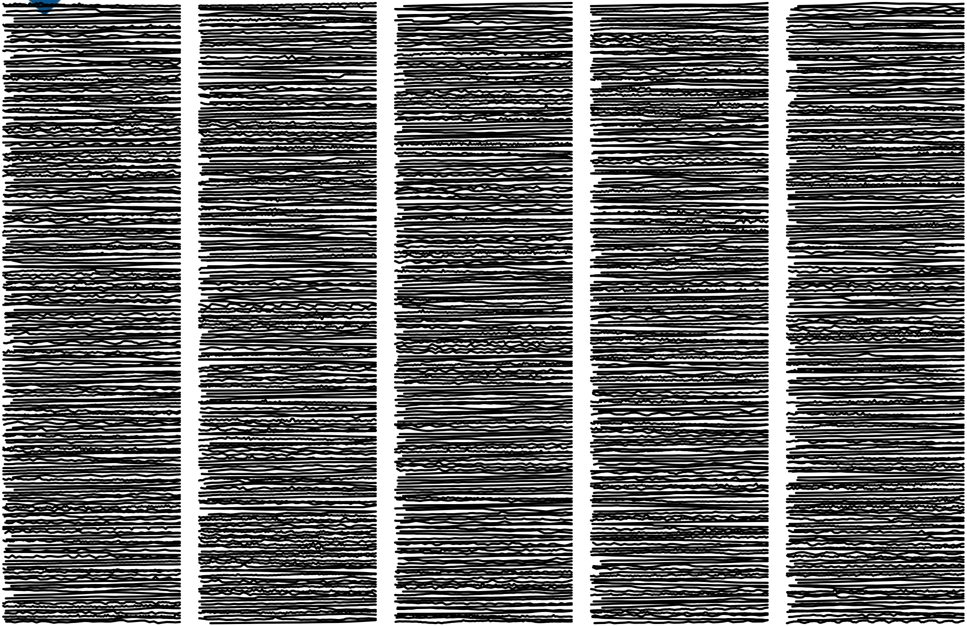

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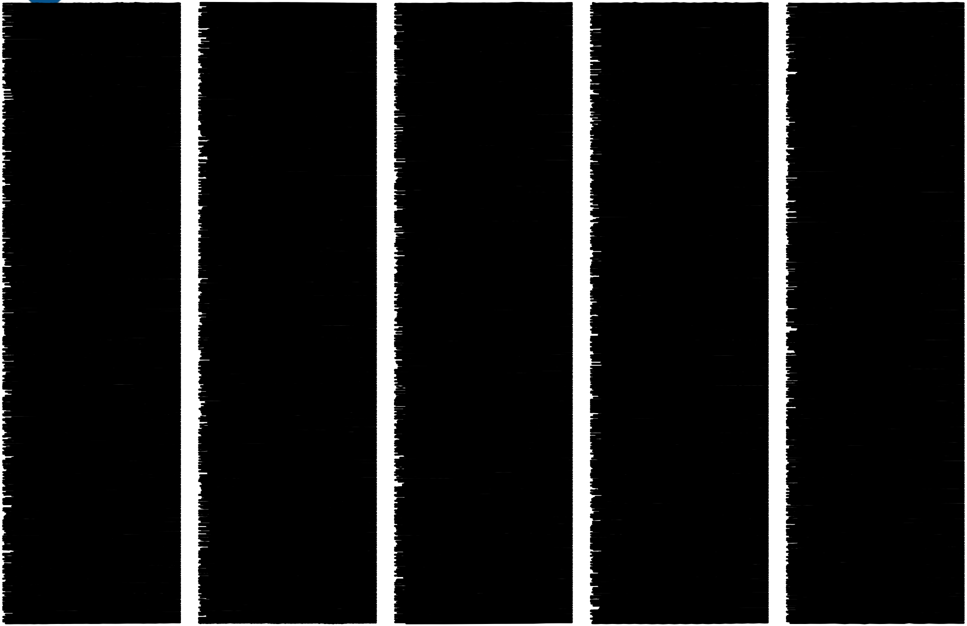
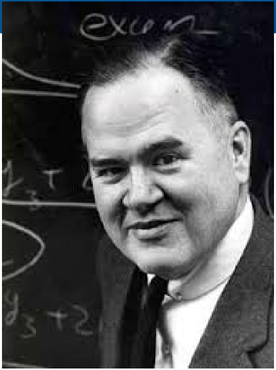


Key idea

Cognostics

Computer-produced diagnostics (Tukey and Tukey, 1985).



John W Tukey

Examples for time series

- lag correlation
- size and direction of trend
- strength of seasonality
- timing of peak seasonality
- spectral entropy

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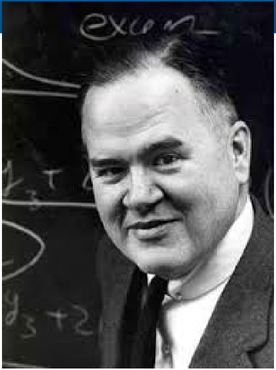

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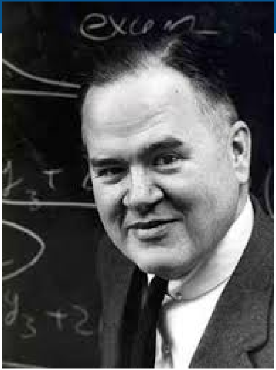

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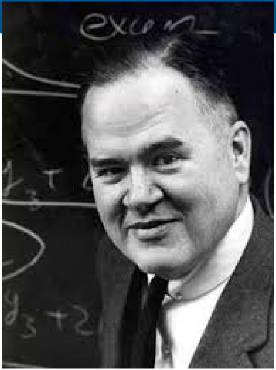

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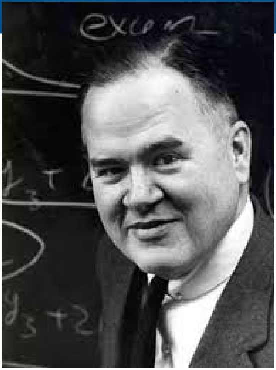

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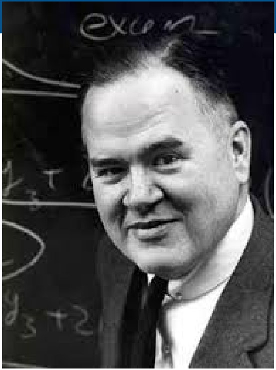

John W Tukey

Examples for time series

- lag correlation
- size and direction of trend
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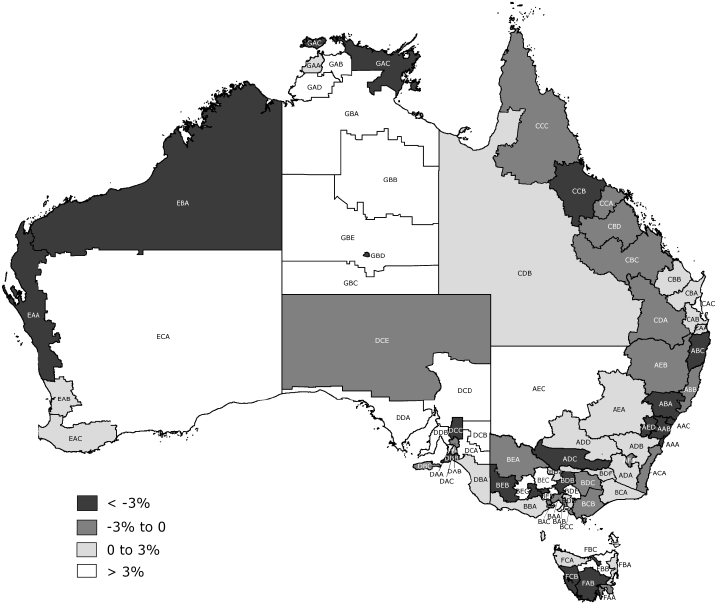
- spectral entropy

Called "features" or "characteristics" in the machine learning literature.

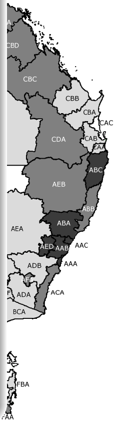
Outline

Australian tourism demand

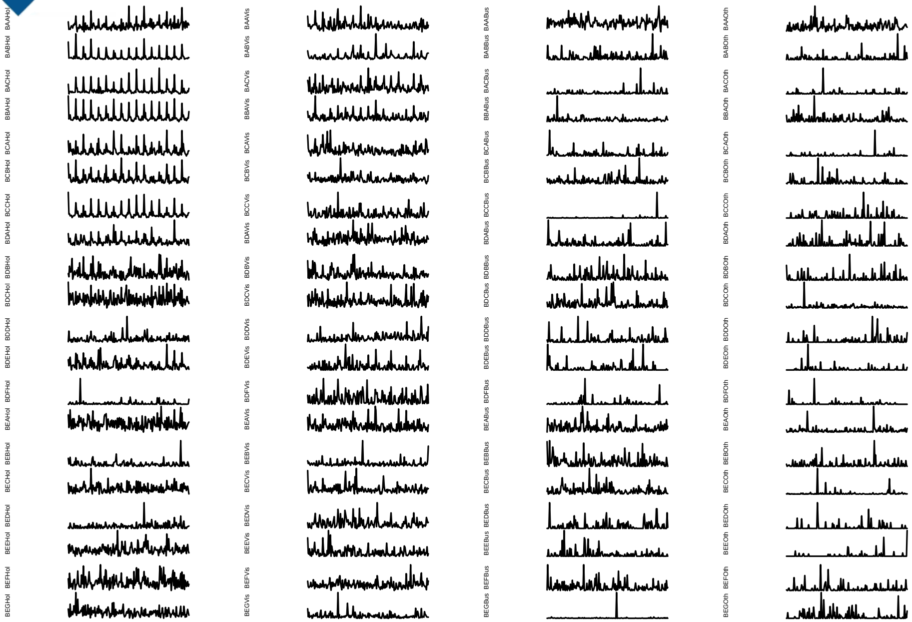


Australian tourism demand

- Quarterly data on visitor night from 1998:Q1 – 2013:Q4
- From: *National Visitor Survey*, based on annual interviews of 120,000 Australians aged 15+, collected by Tourism Research Australia.
- Split by 7 states, 27 zones and 76 regions (a geographical hierarchy)
- Also split by purpose of travel
  - Holiday
  - Visiting friends and relatives (VFR)
  - Business
  - Other
- 304 disaggregated series



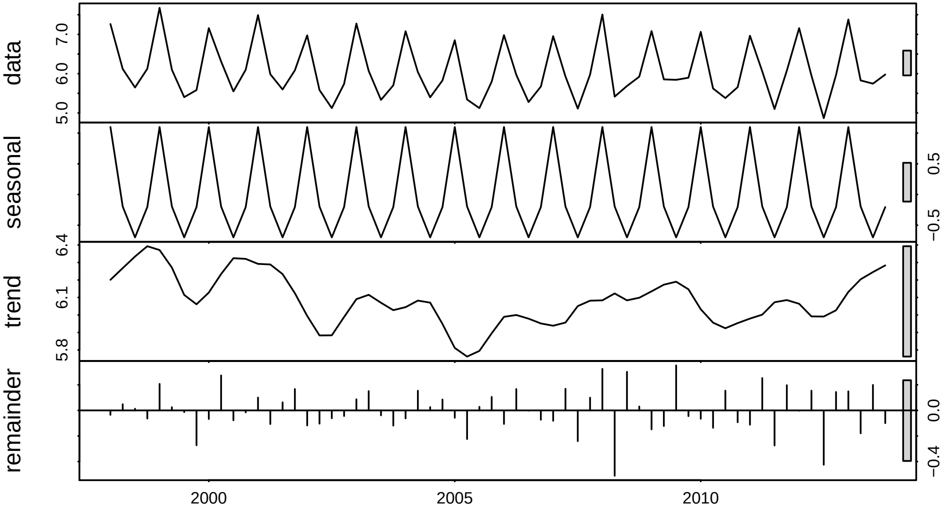
Domestic tourism demand: Victoria



An STL decomposition

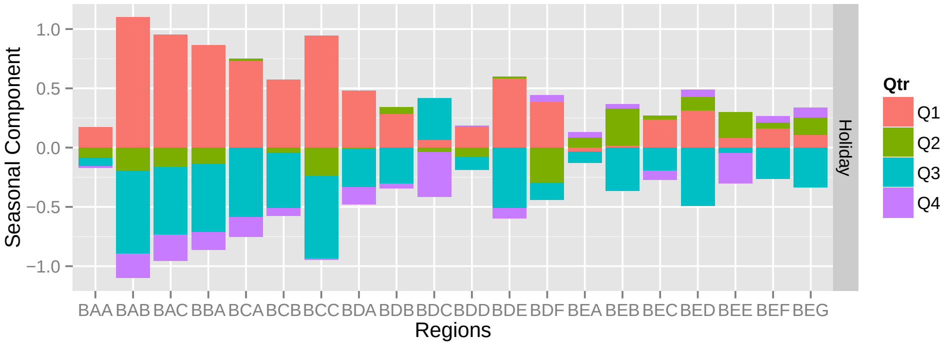
Tourism demand for holidays in Peninsula

$Y_t = S_t + T_t + R_t$ $\quad$ S_t is periodic with mean 0

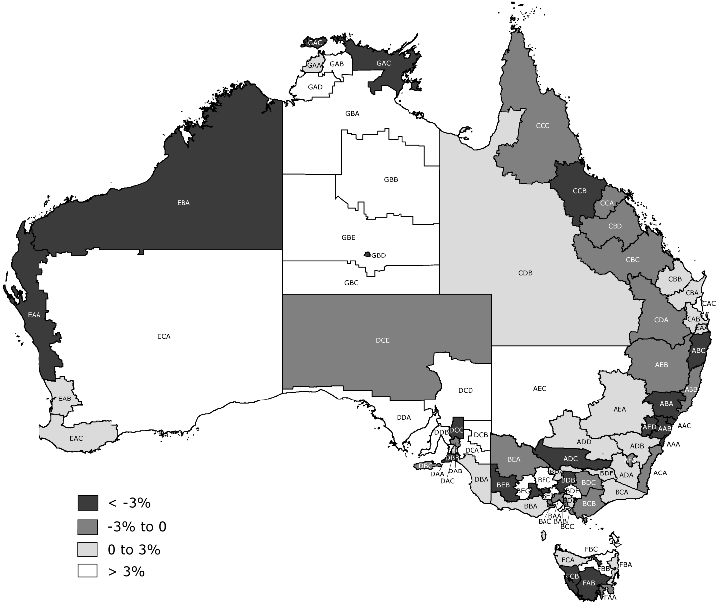


Seasonal stacked bar chart

- Place positive values above the origin while negative values below the origin
- Map the bar length to the magnitude
- Encode quarters by colours



Seasonal stacked bar chart: VIC



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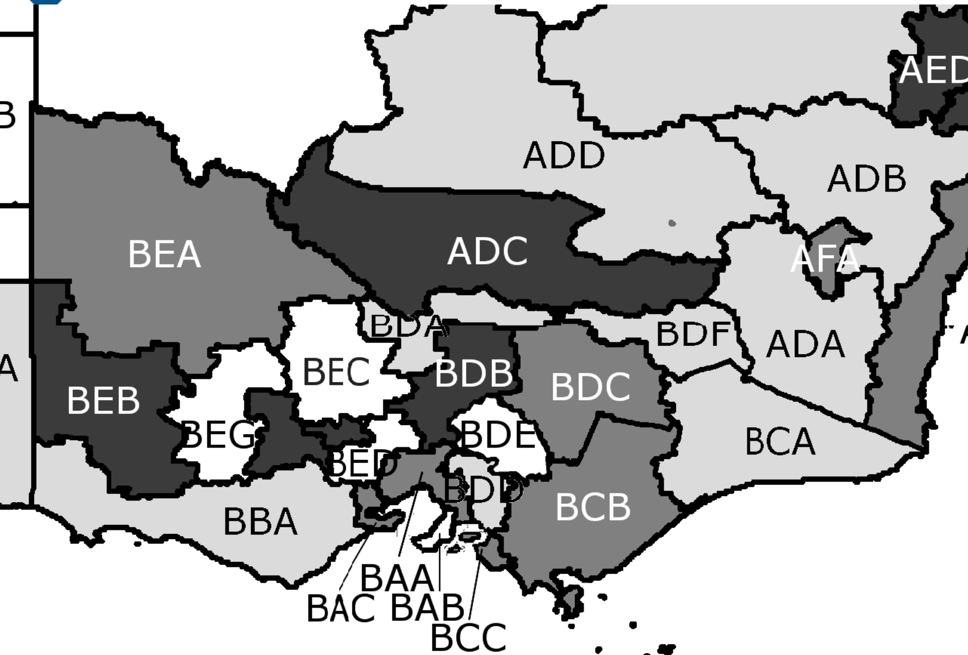

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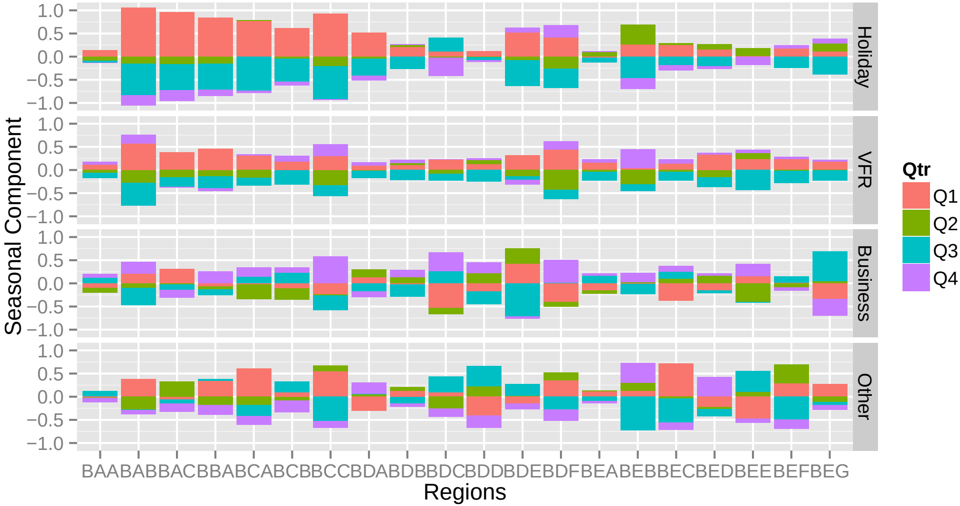


Trend analysis

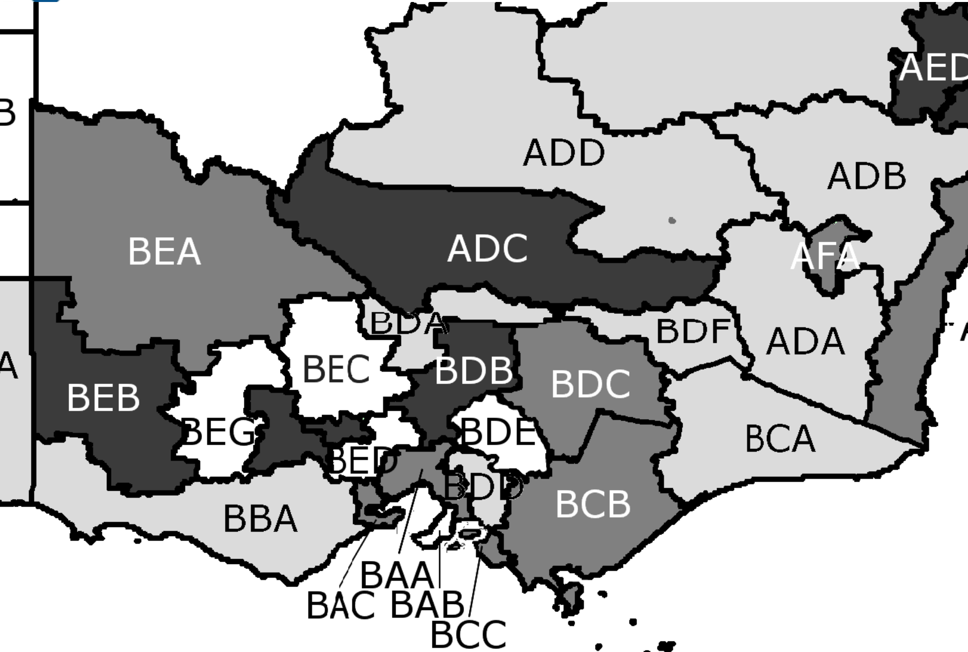
- **Linearity:** the long-term direction and strength of trend.
- **Curvature:** the “changing direction” of trend.
- Estimate by regression:

$$T_t = \hat{\beta}_0 + \hat{\beta}_1\phi_1(t) + \hat{\beta}_2\phi_2(t) + e_t$$

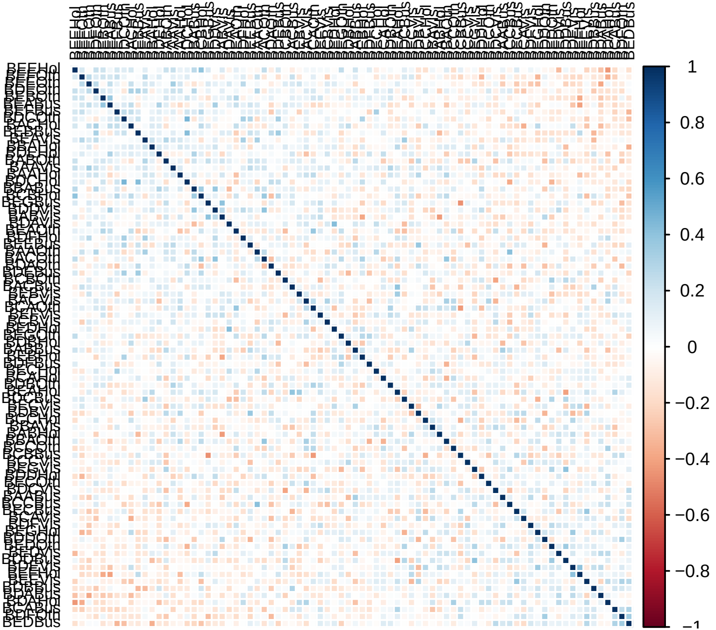
  where $\phi_k(t)$ is a kth-degree **orthogonal polynomial** in time t.
- To separate the linearity ($\hat{\beta}_1$) and curvature ($\hat{\beta}_2$).

Trend analysis

Trend analysis



Corrgram of remainder



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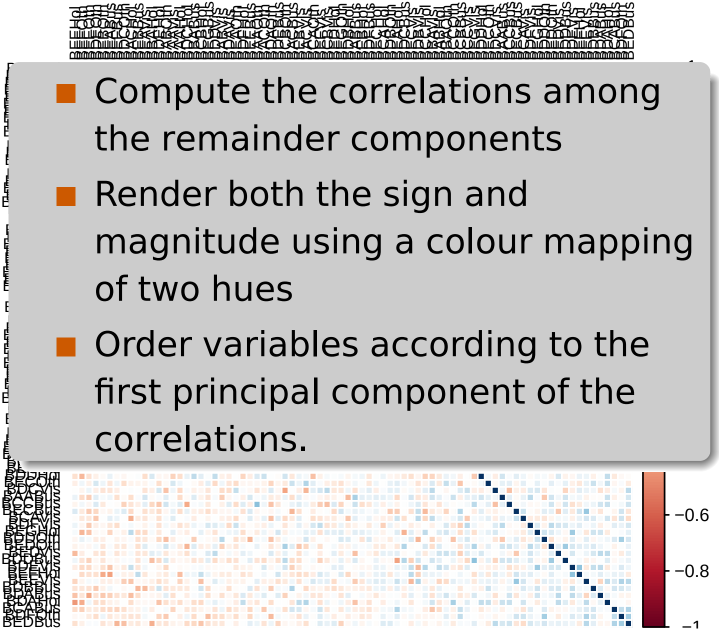
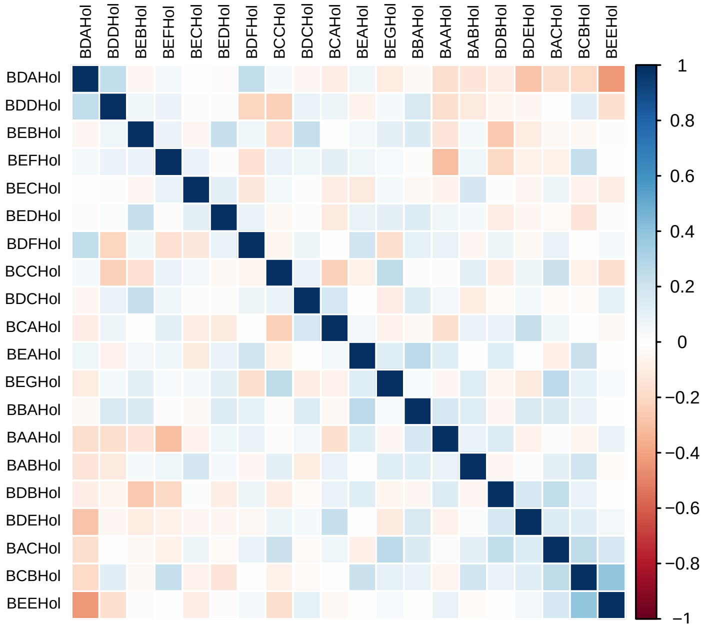
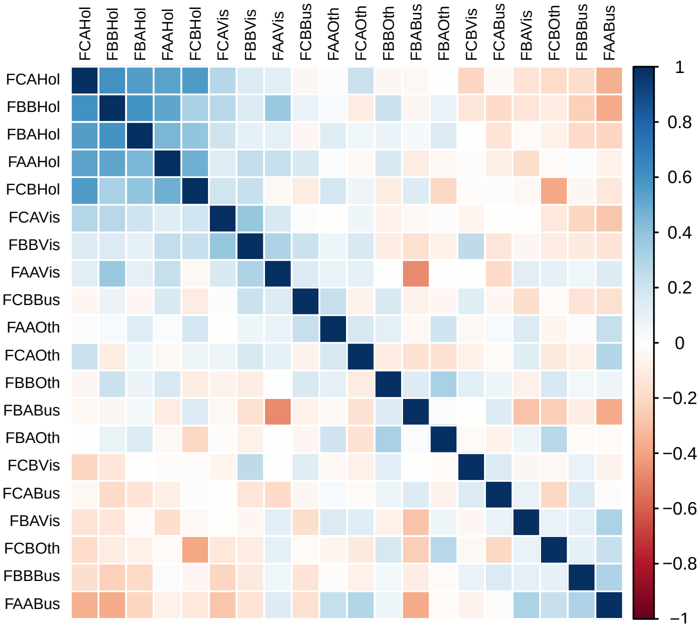


- Compute the correlations among the remainder components
- Render both the sign and magnitude using a colour mapping of two hues
- Order variables according to the first principal component of the correlations.

Corrgram of remainder



Corrgram of remainder: TAS

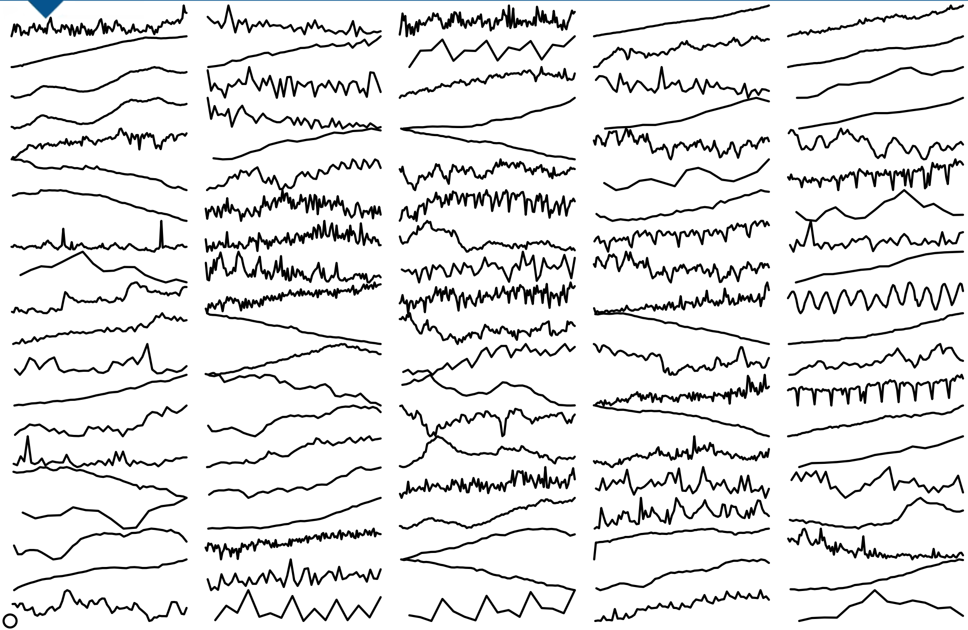


Outline

M3 forecasting competition

- 3003 series
- All data from business, demography, finance and economics.
- Series length between 14 and 126.
- Either non-seasonal, monthly or quarterly.
- All time series positive.

M3 forecasting competition



Candidate features

STL decomposition

$$Y_t = S_t + T_t + R_t$$

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- Seasonal period
- Strength of seasonality: $1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - T_t)}$
- Strength of trend: $1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - S_t)}$
- Spectral entropy: $H = -\int_{-\pi}^{\pi} f_y(\lambda) \log f_y(\lambda) d\lambda$, where $f_y(\lambda)$ is spectral density of Y_t. Low values of H suggest a time series that is easier to forecast (more signal).
- Autocorrelations: $r_1, r_2, r_3, \dots$
- Optimal Box-Cox transformation parameter λ

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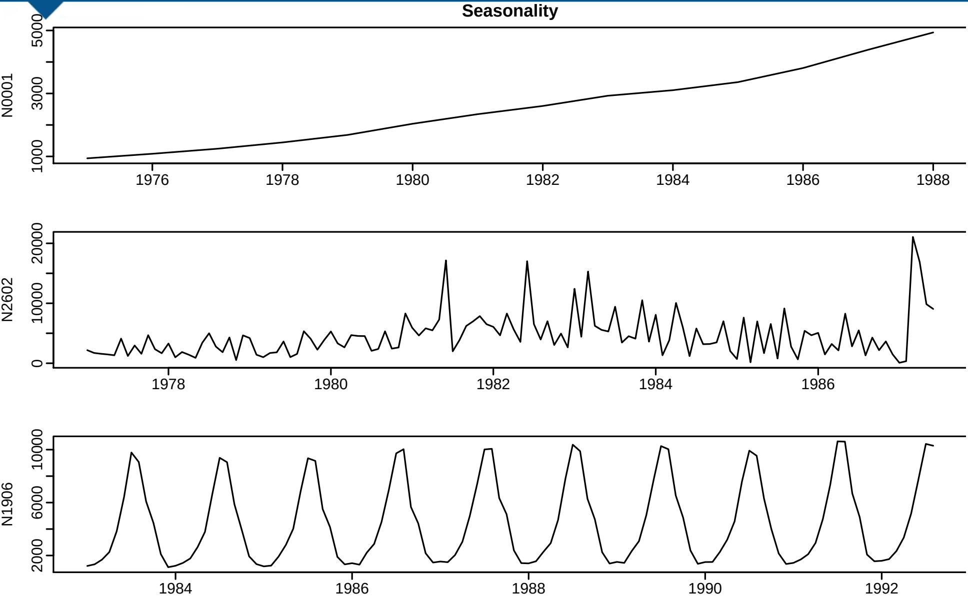

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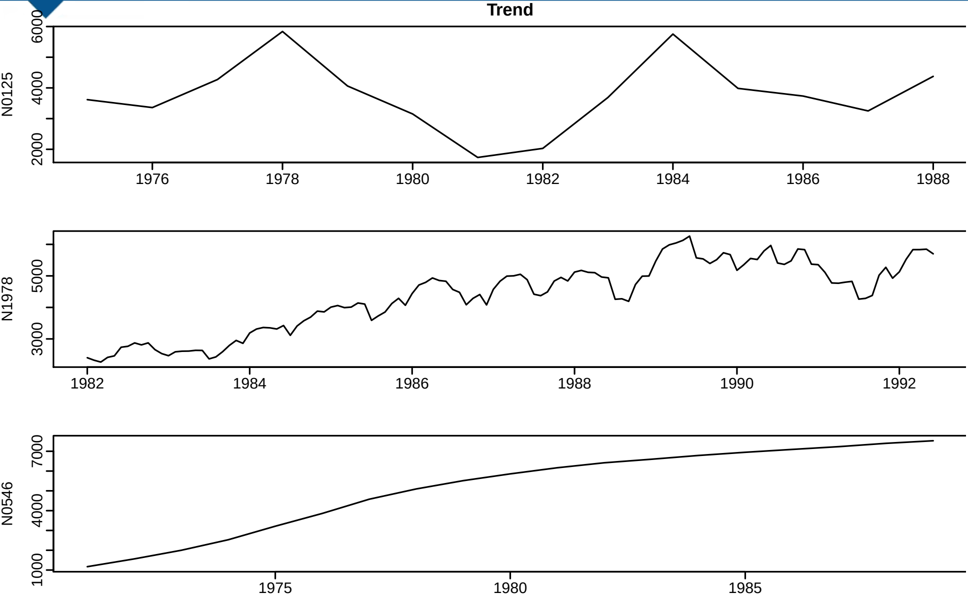

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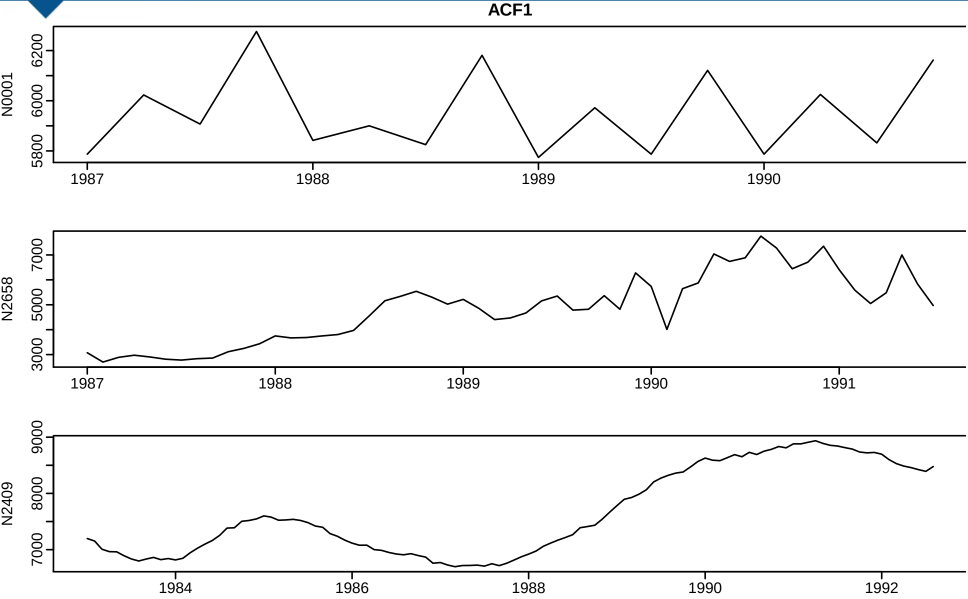

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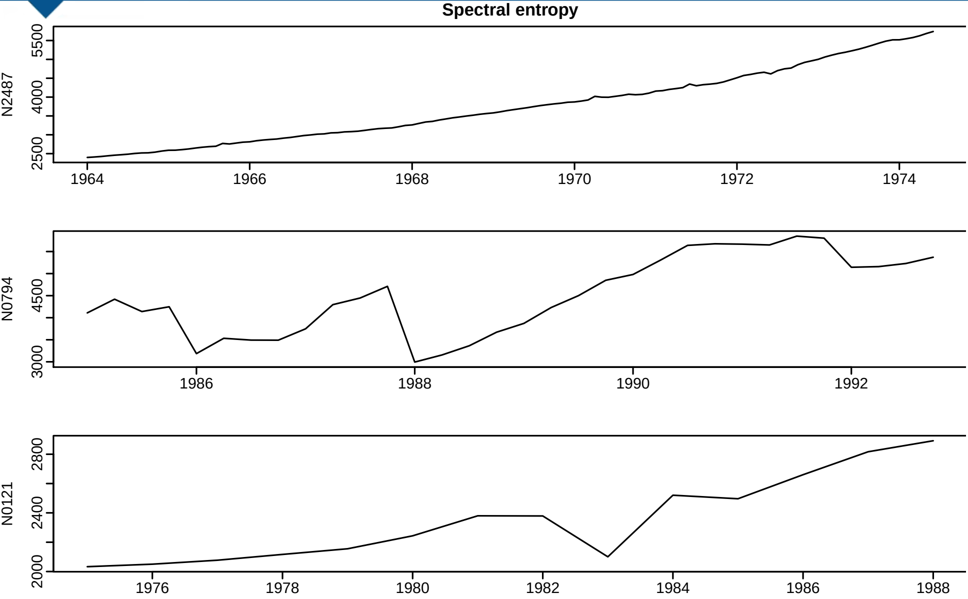

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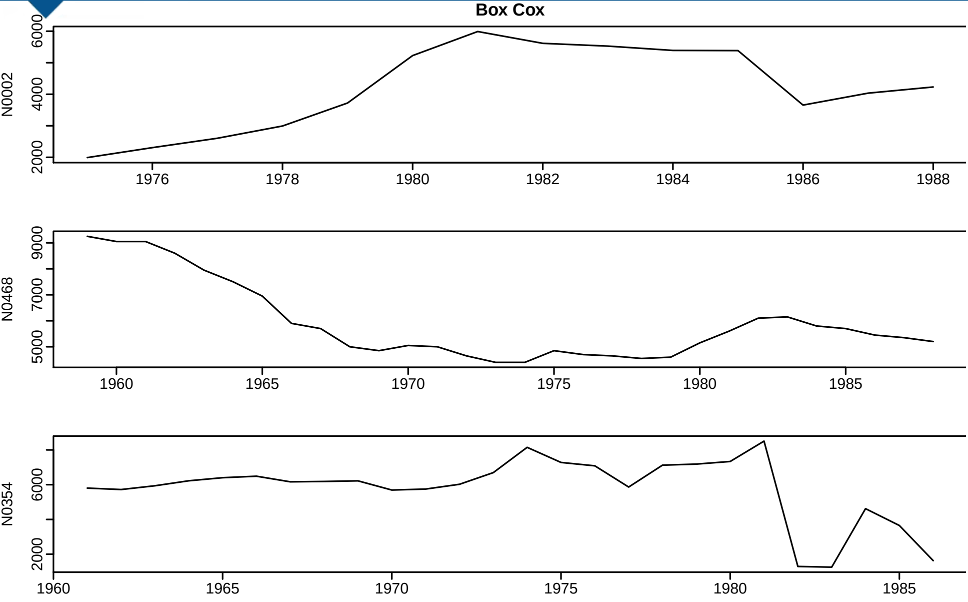

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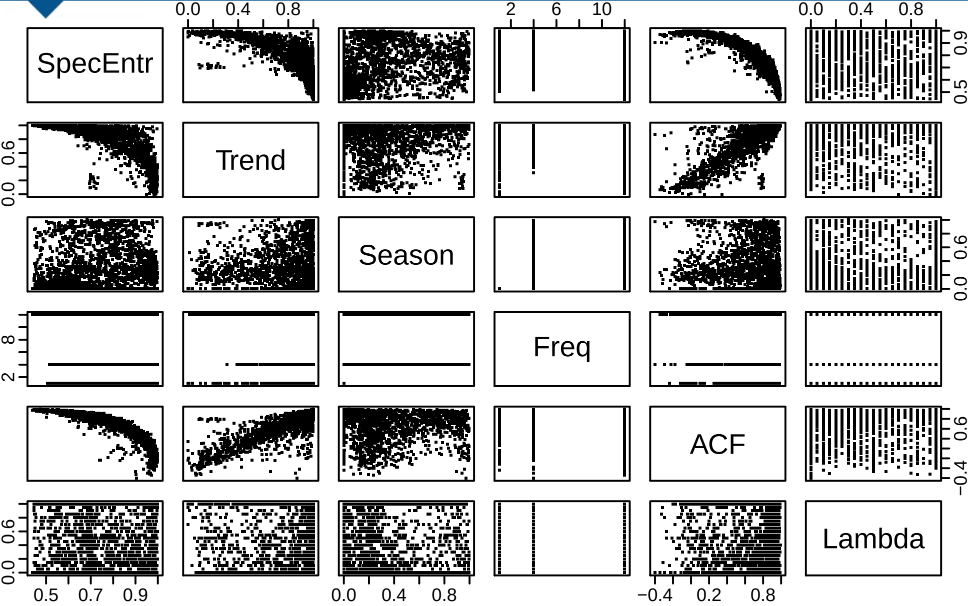
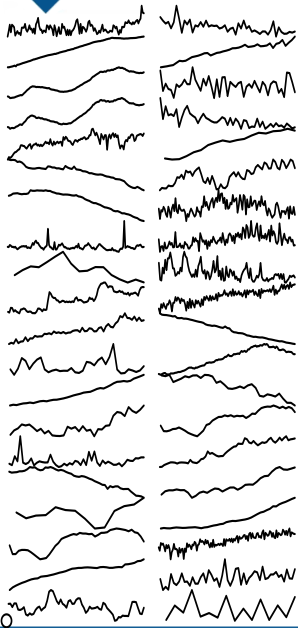


Dimension reduction for time series



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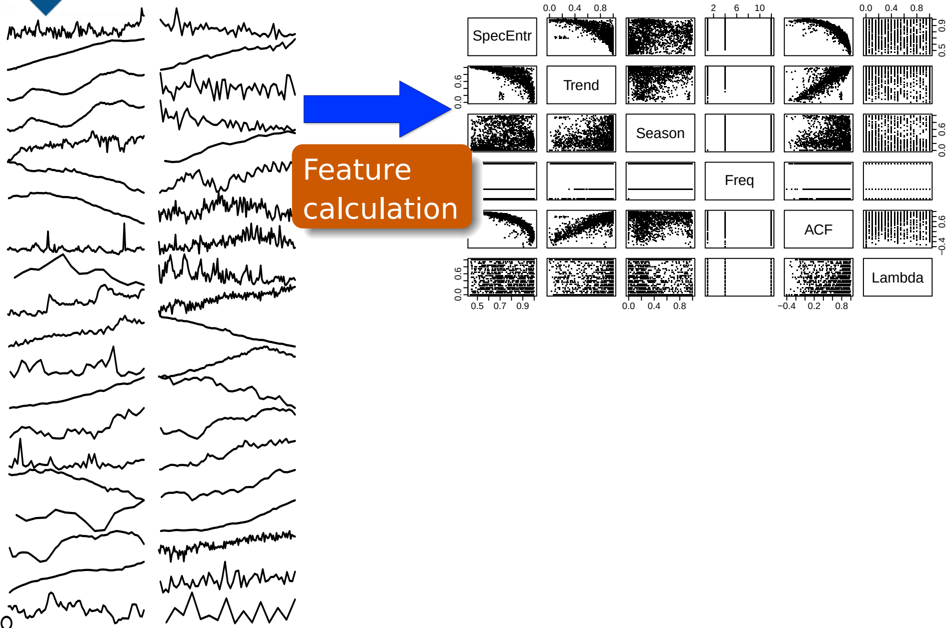

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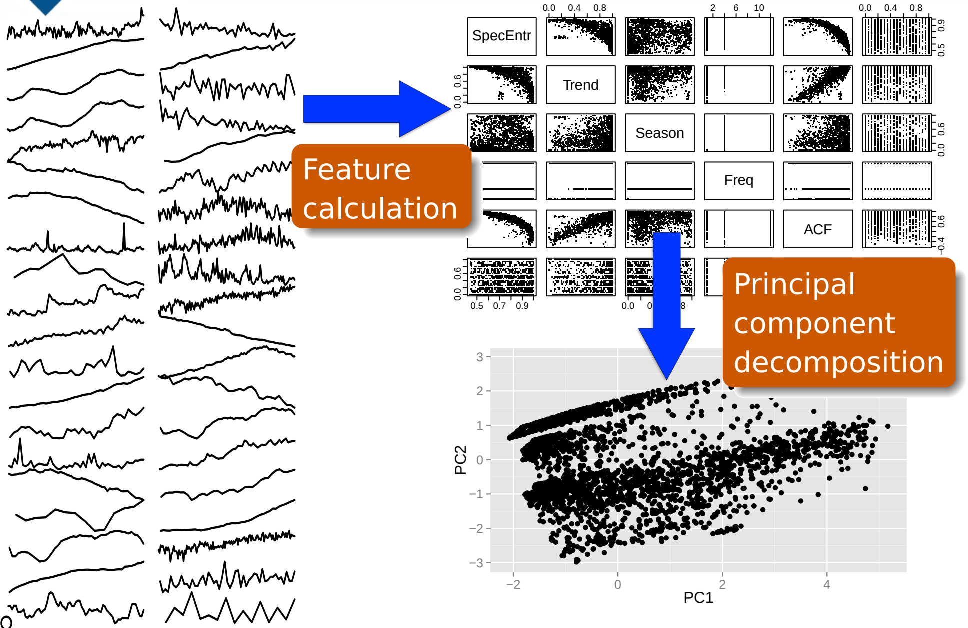
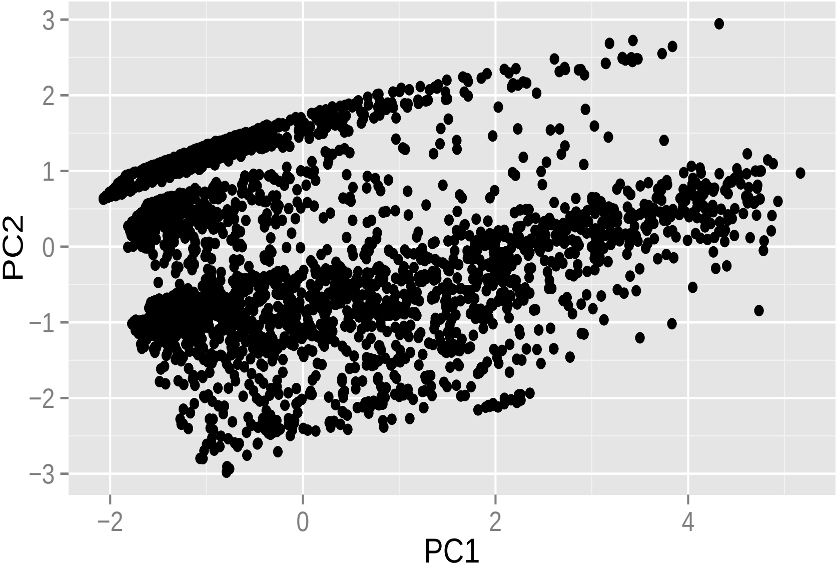


Feature space of M3 data

First two PCs explain 68% of variation.



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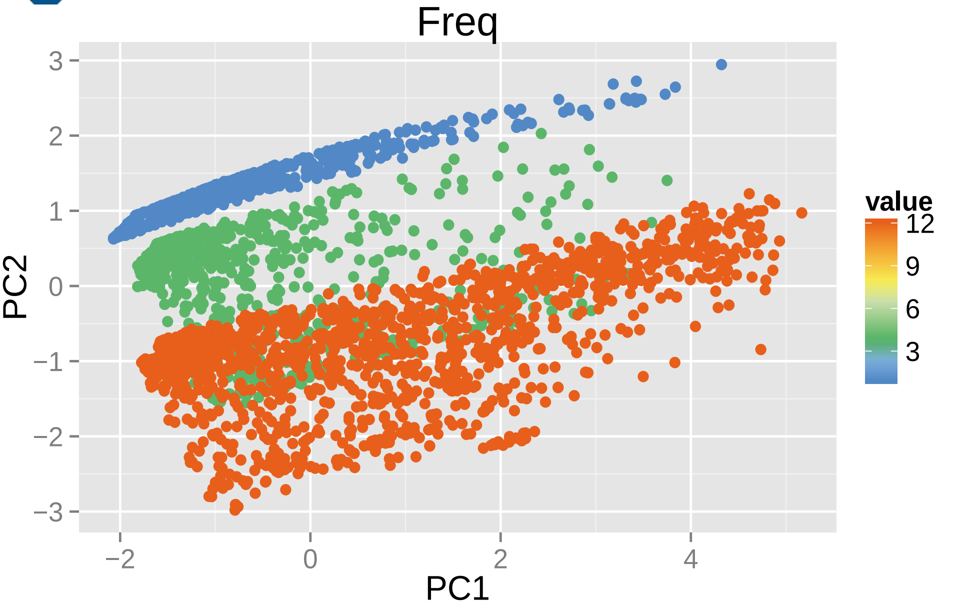

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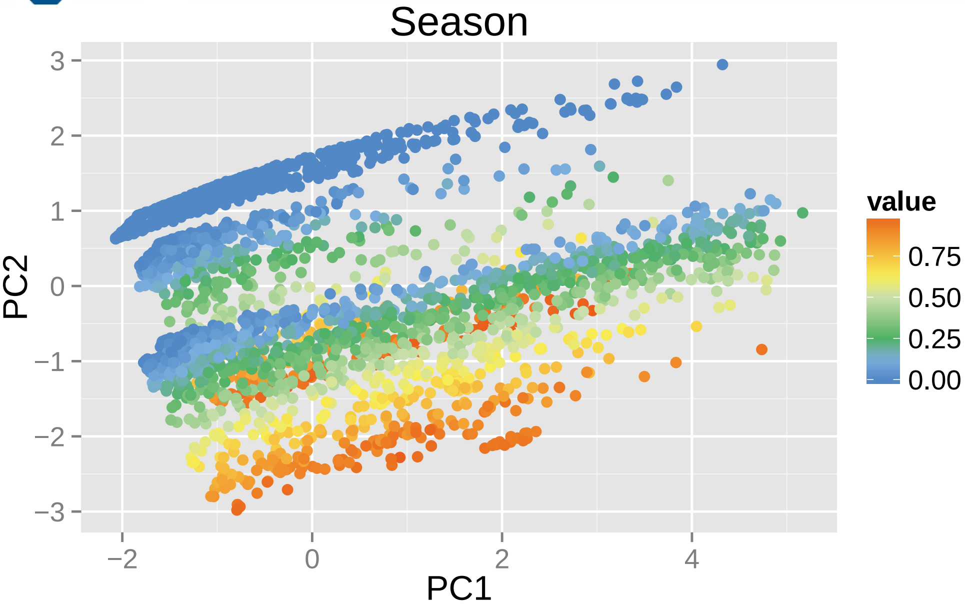

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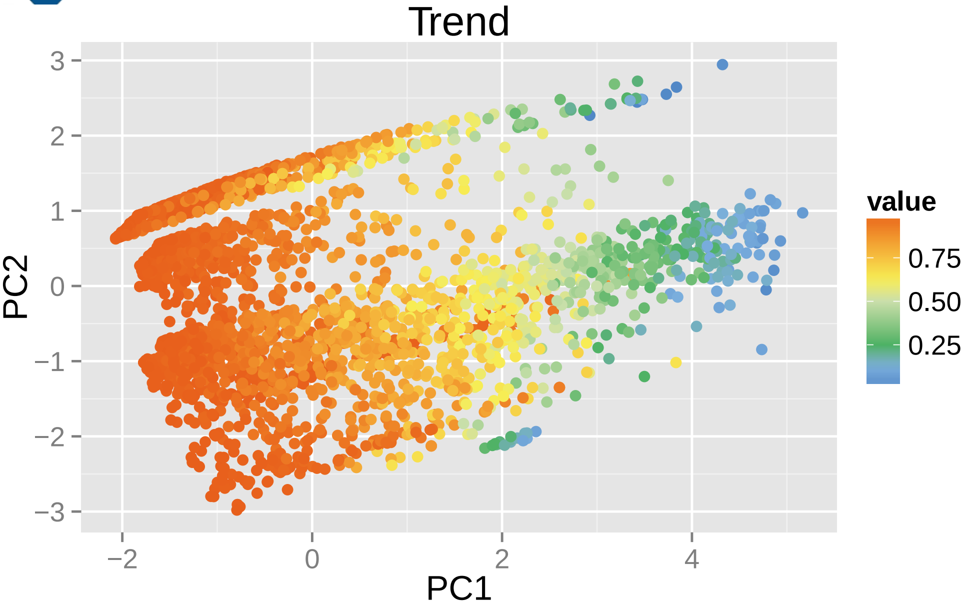

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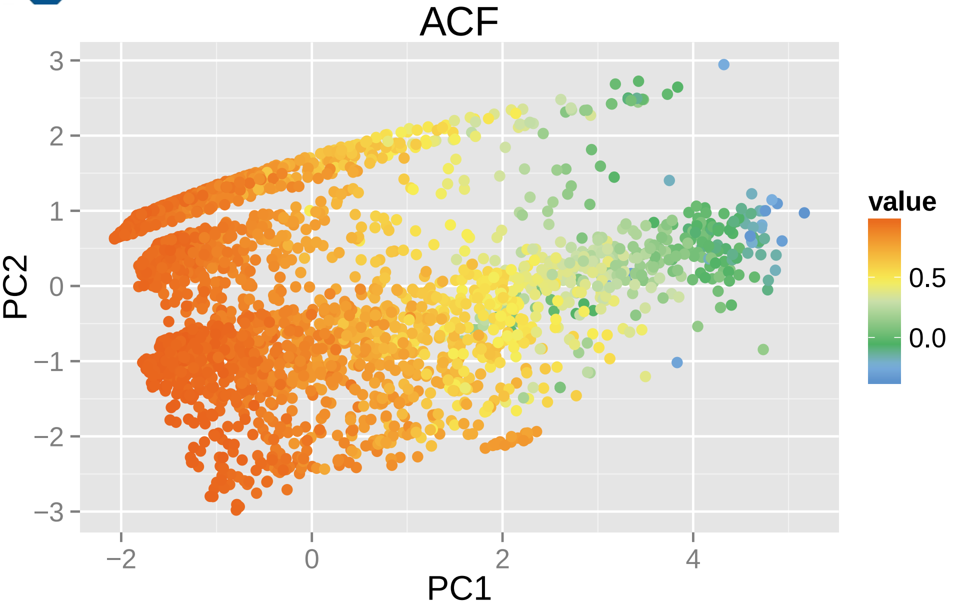

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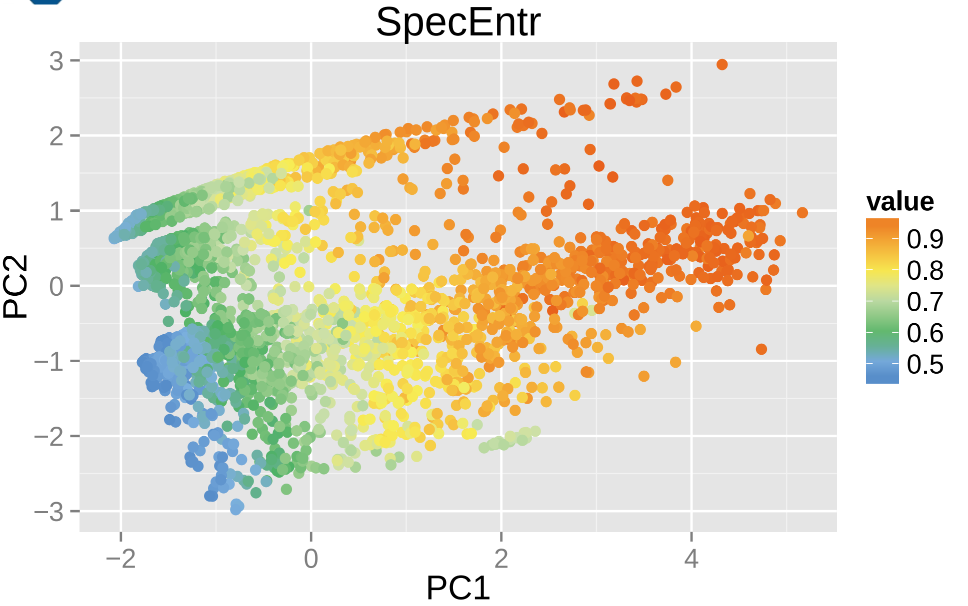

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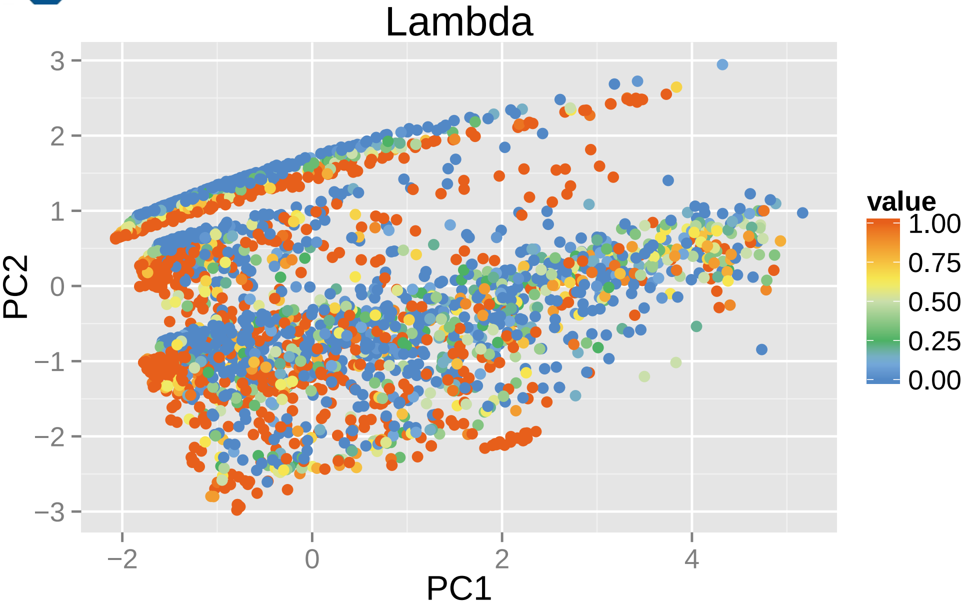

Predictability

Three general forecasting methods:

Theta method	Best overall in 2000 M3 competition
ETS	Exponential smoothing state space models
STL-AR	AR model applied to seasonally adjusted series from STL, and seasonal component forecast using the seasonal naive method.

Compute minimum MASE from all three methods

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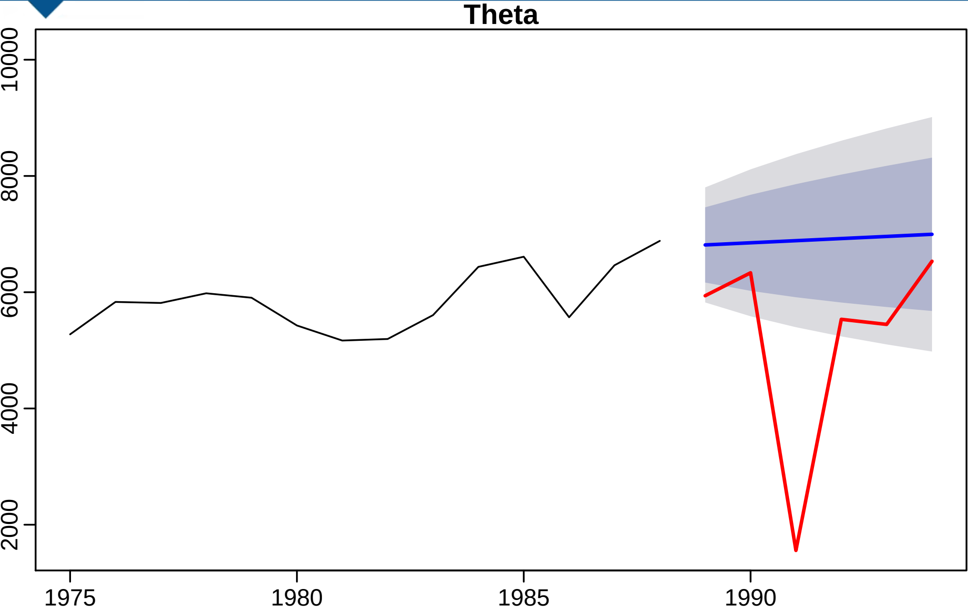

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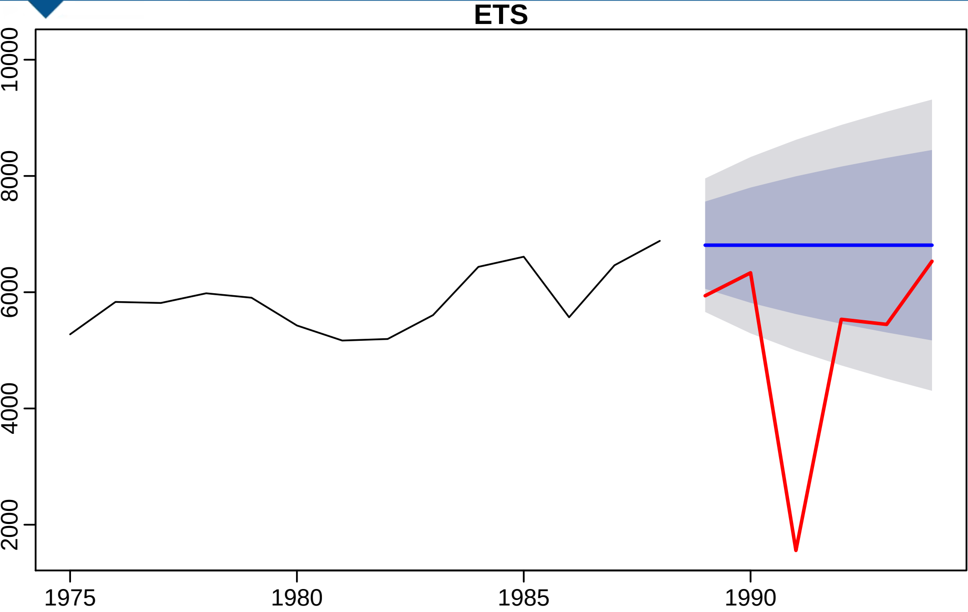

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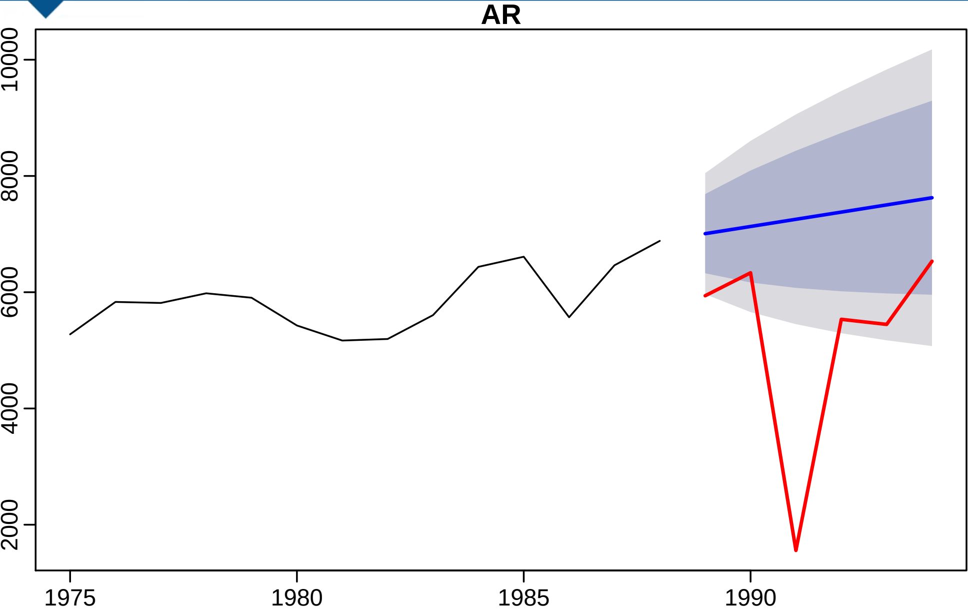

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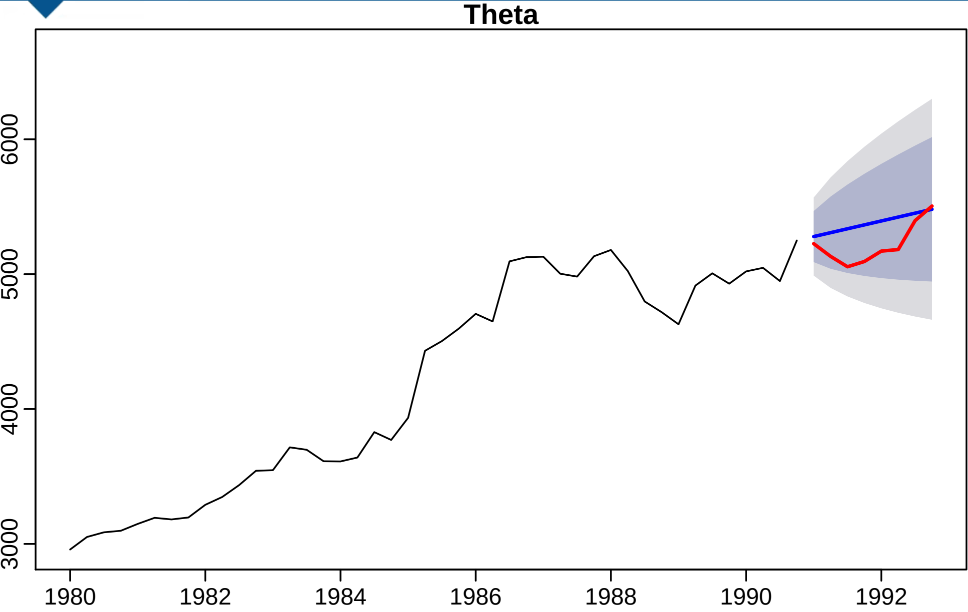

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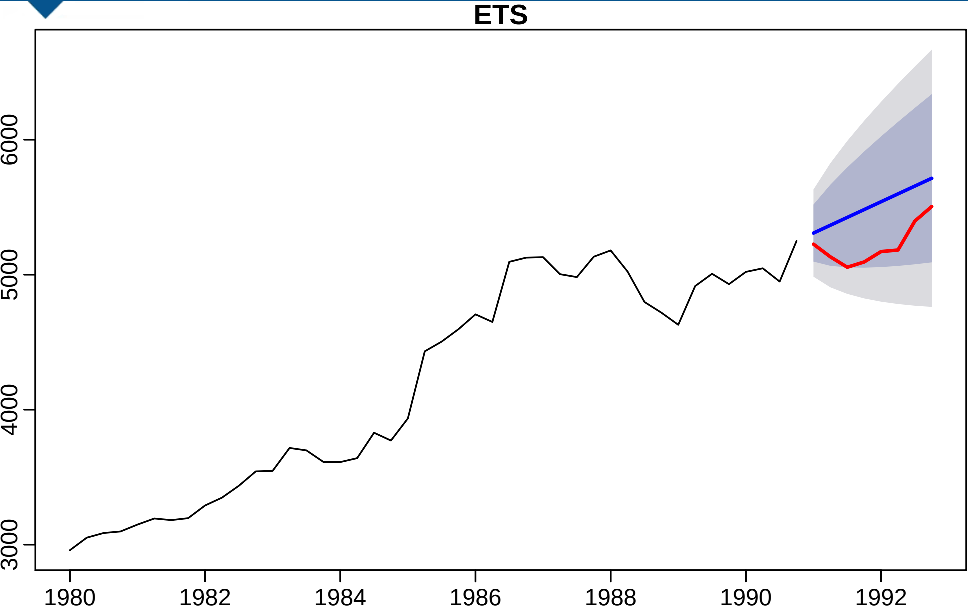


Predictability

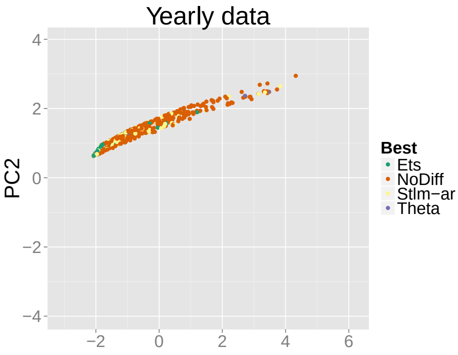
Predictability

Medium
MASE values

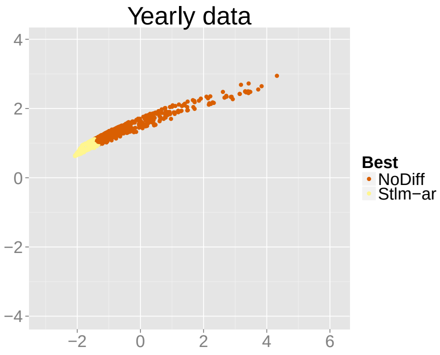
Predictability

High
MASE values

Predictability

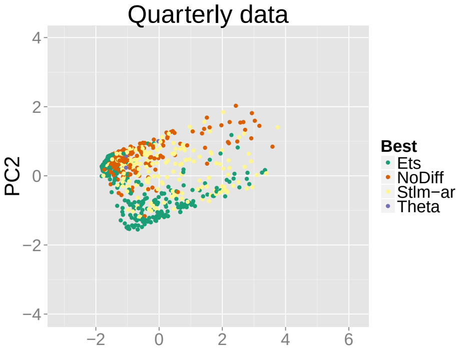


Actual



SVM prediction

Predictability



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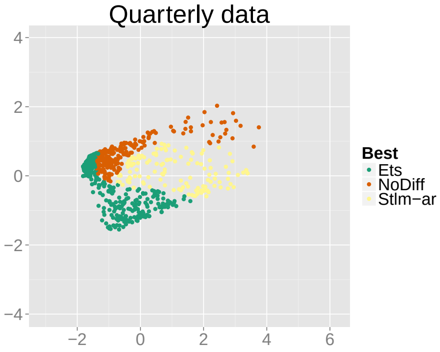

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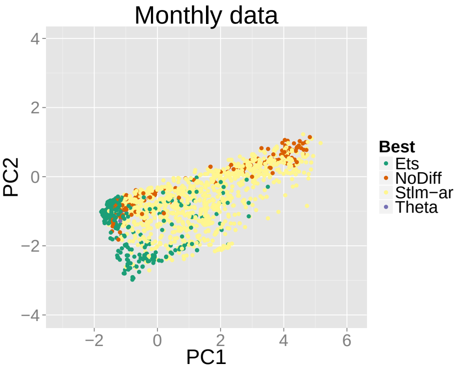

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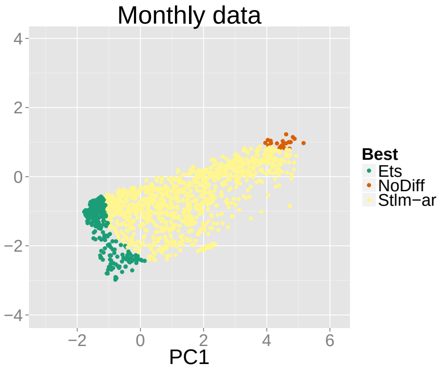

SVM prediction

Generating new time series

We can use the feature space to:

- Generate new time series with similar features to existing series
- Generate new time series where there are "holes" in the feature space.

- Let $\{\mathrm{PC}_1, \mathrm{PC}_2, \ldots, \mathrm{PC}_n\}$ be a "population" of time series of specified length and period.
- Genetic algorithm uses a process of selection, crossover and mutation to evolve the population towards a target point T_i.
- Optimize: Fitness $(\mathrm{PC}_j) = -\sqrt{(|\mathrm{PC}_j - T_i|^2)}$.
- Initial population random with some series in neighbourhood of T_i.

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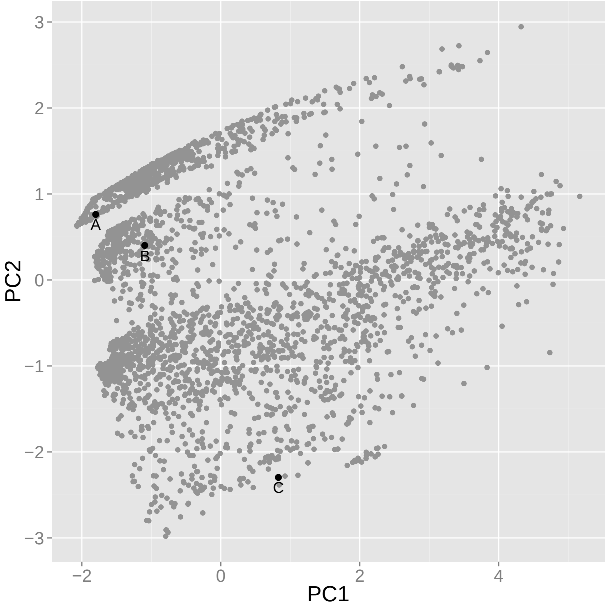

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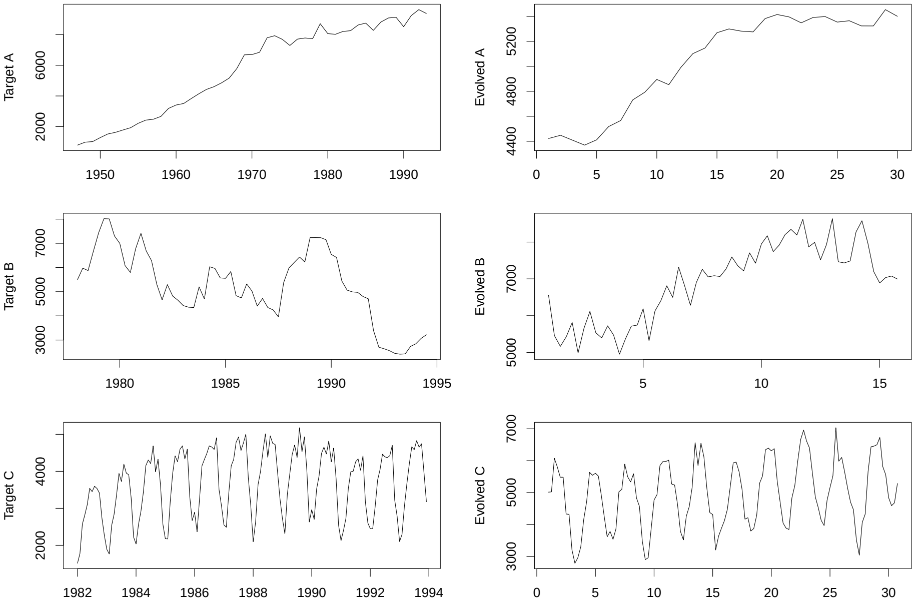

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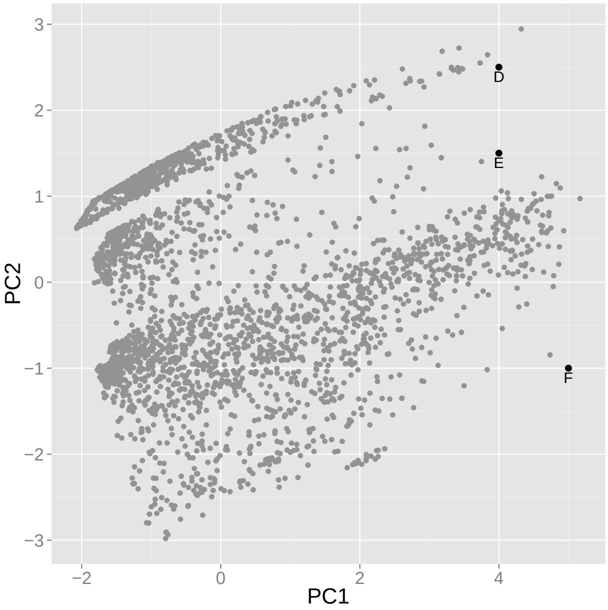

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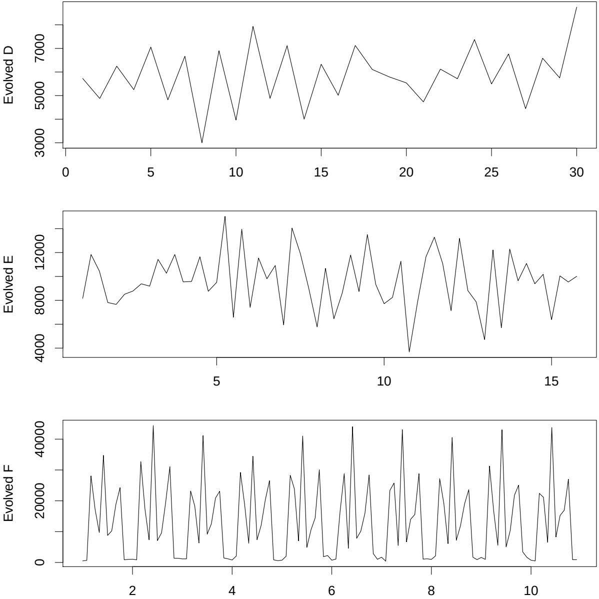

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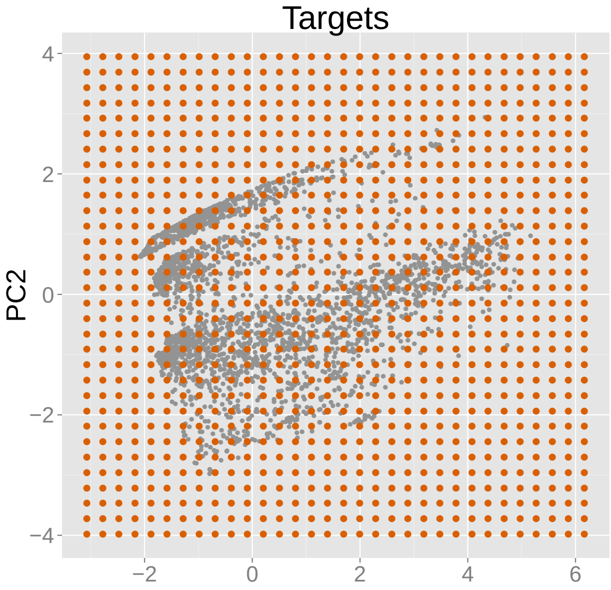

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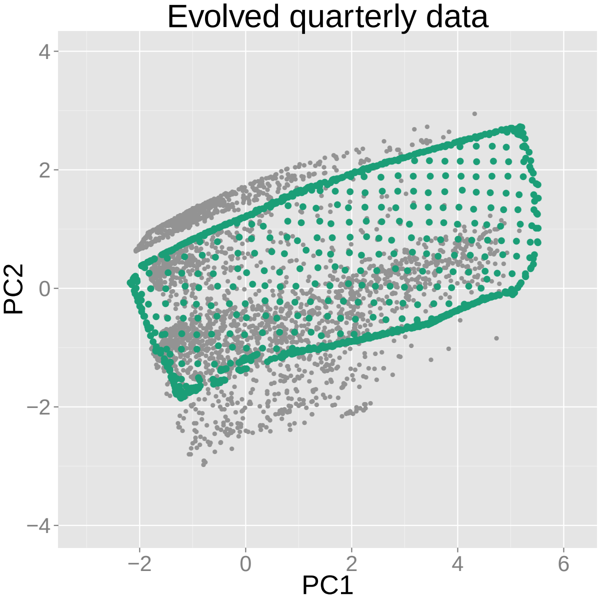

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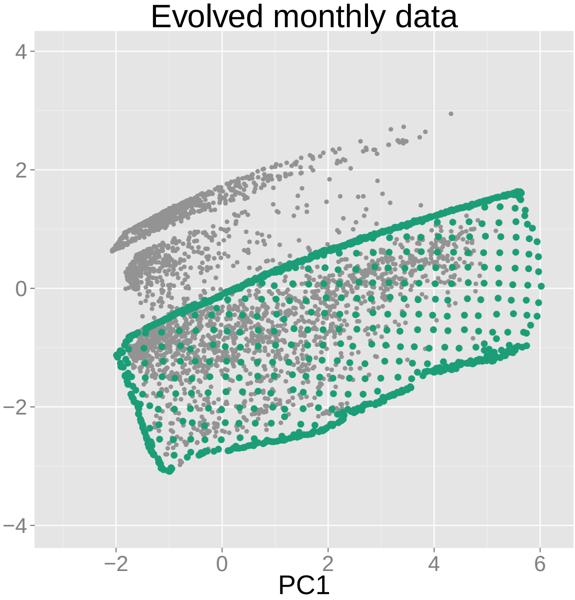

Questions raised

- Can SVM be used to create a forecast selection routine to give better forecasts?
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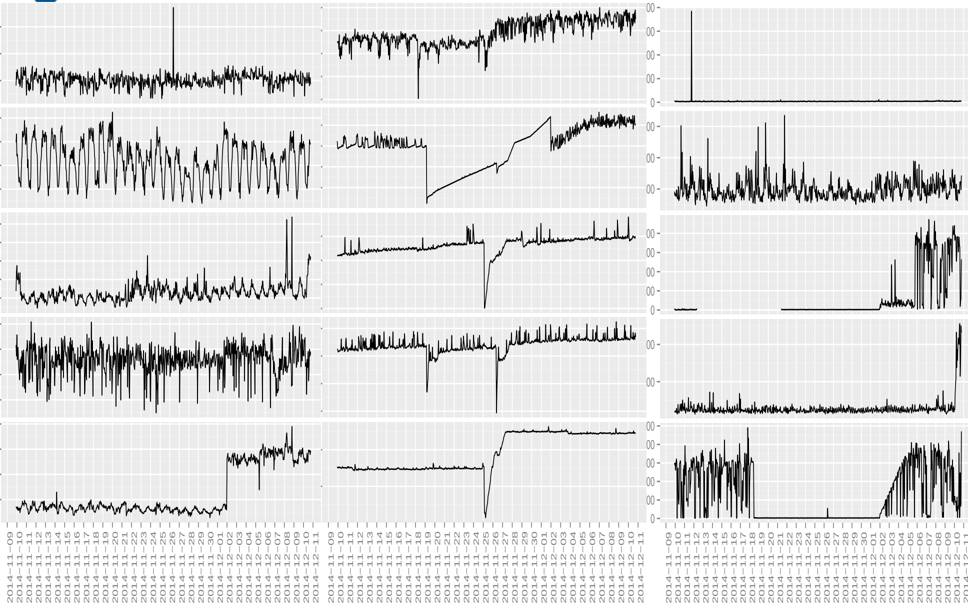
Outline

Yahoo web-traffic

- Tens of thousands of time series collected at one-hour intervals over one month.
- Consisting of several server metrics (e.g. CPU usage and paging views) from many server farms globally.
- Aim: find unusual (anomalous) time series.

Yahoo web-traffic



Feature space

- **ACF1**: first order autocorrelation = Corr(Y_t, Y_{t-1})
- Strength of **trend** and **seasonality** based on STL
- Trend **linearity** and **curvature**
- Size of seasonal **peak** and **trough**
- Spectral **entropy**
- **Lumpiness**: variance of block variances (block size 24).
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- **ACF1**: first order autocorrelation = $\text{Corr}(Y_t, Y_{t-1})$
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- **Lumpiness**: variance of block variances (block size 24).
- **Spikiness**: variances of leave-one-out variances of STL remainders.
- **Level shift**: Maximum difference in trimmed means of consecutive moving windows of size 24.
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- **Flat spots**: Discretize sample space into 10 equal-sized intervals. Find max run length in any interval.
- Number of **crossing points** of mean line.
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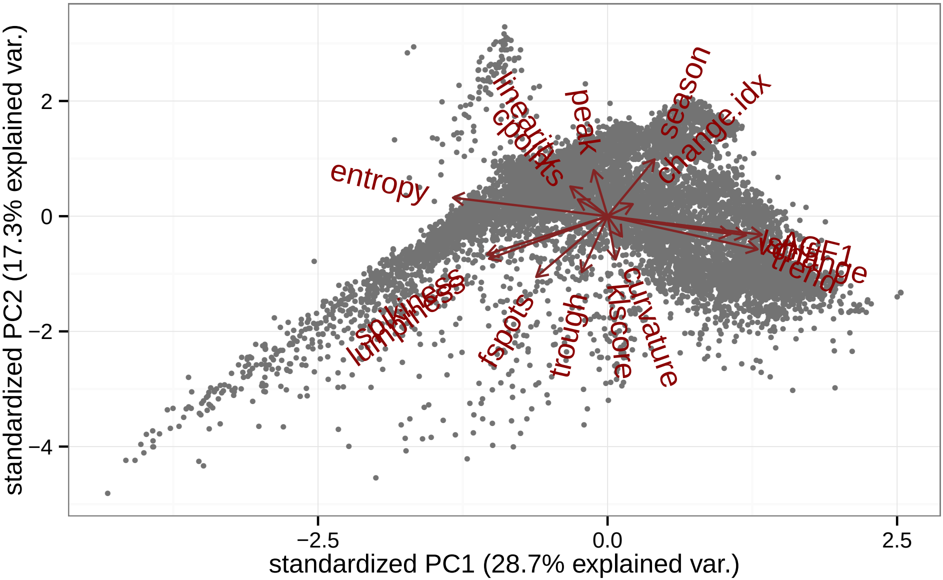
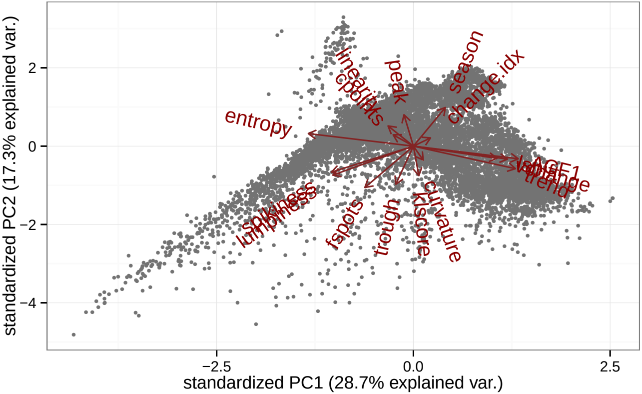
Principal component analysis

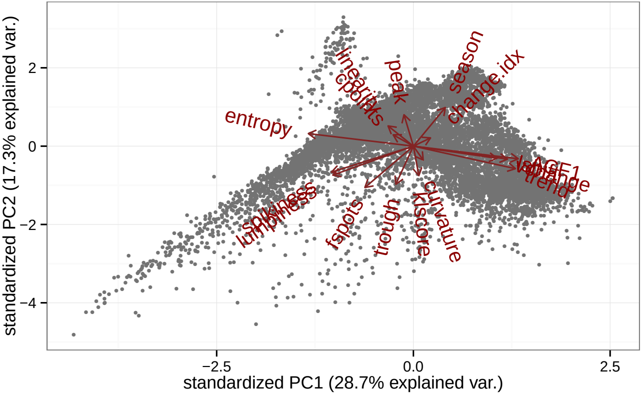




We need a measure of the “anomalousness” of a time series.

1. Rank points based on their local density.
2. Rank points based on whether they are within α-convex hulls of different radius.

What is “anomalous”



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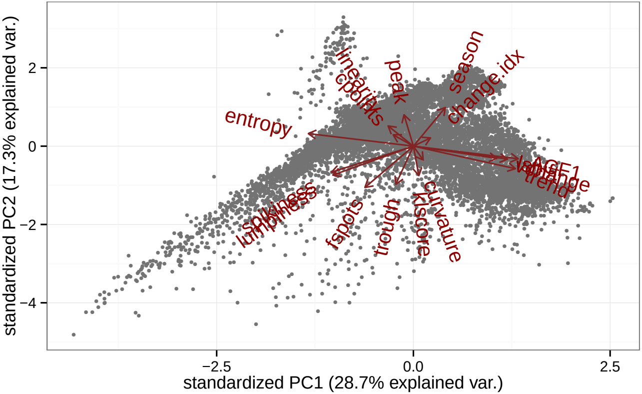

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$$\hat{f}(\mathbf{x};\mathbf{H}) = \frac{1}{n}\sum_{i=1}^{n} K_{\mathbf{H}}(\mathbf{x} - \mathbf{X}_i)$$

- $\mathbf{X}_i \in$ a bivariate random sample $\{\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n\}$
- $K_{\mathbf{H}}(\mathbf{x})$ is the standard normal kernel function
- $\mathbf{H}$ estimated by minimizing the sum of AMISE
- Rank points based on $\hat{f}$ values in 2d PCA space.

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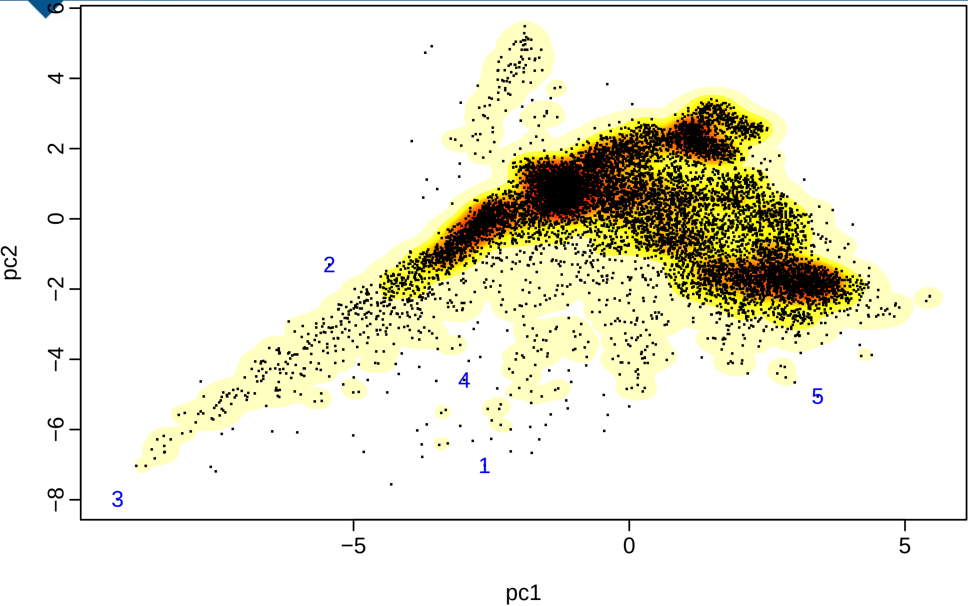

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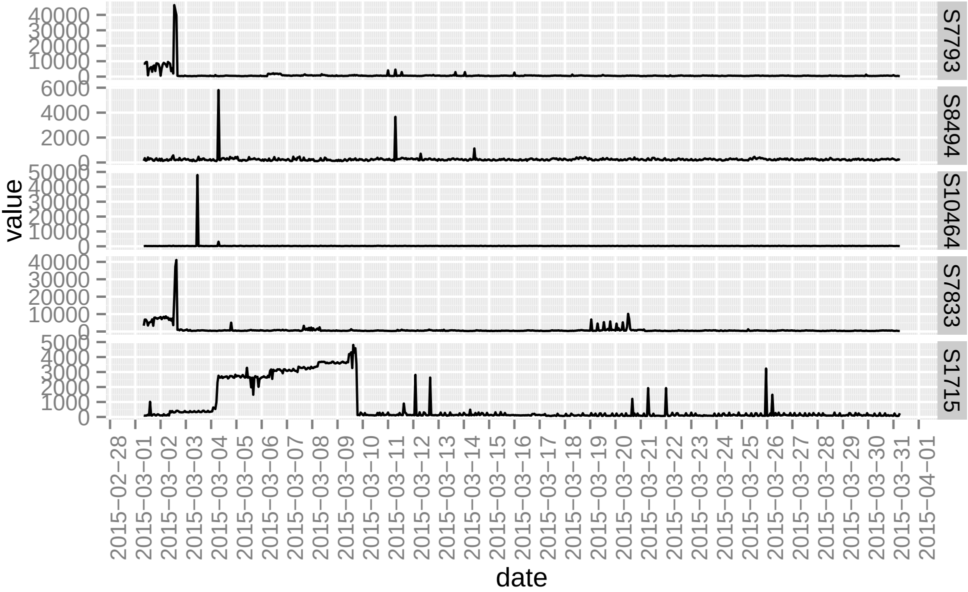
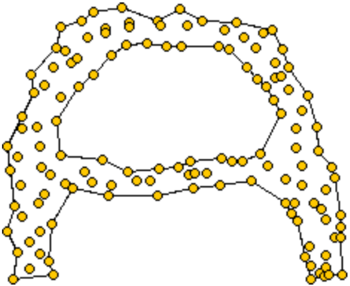
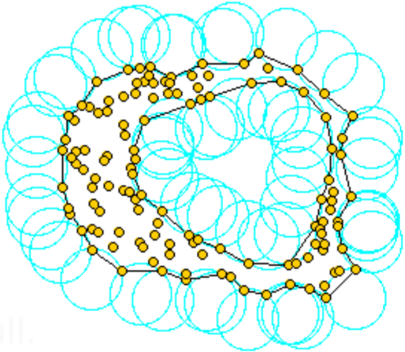


α-convex hulls

The space generated by point pairs that can be touched by an empty disc of radius α.





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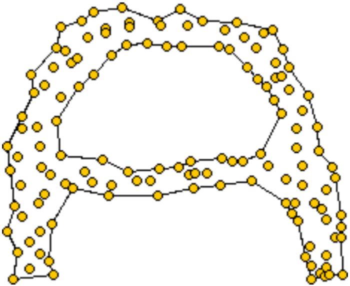

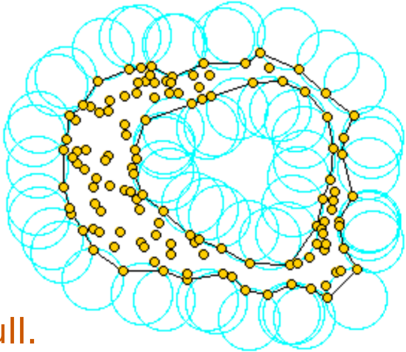

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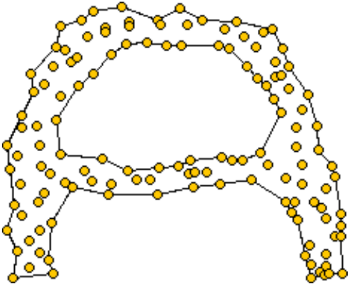

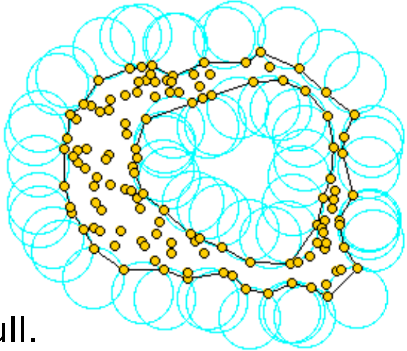

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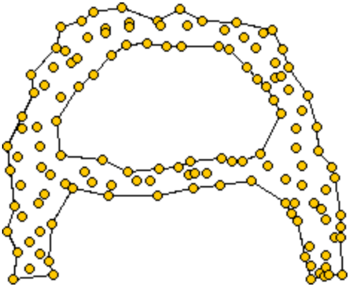

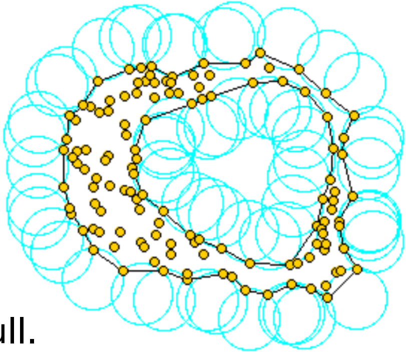

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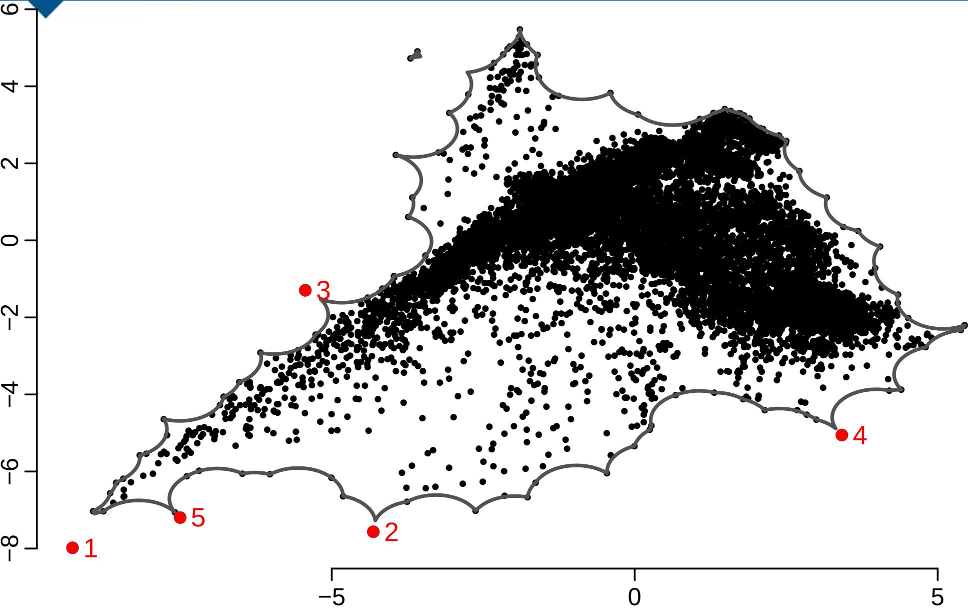

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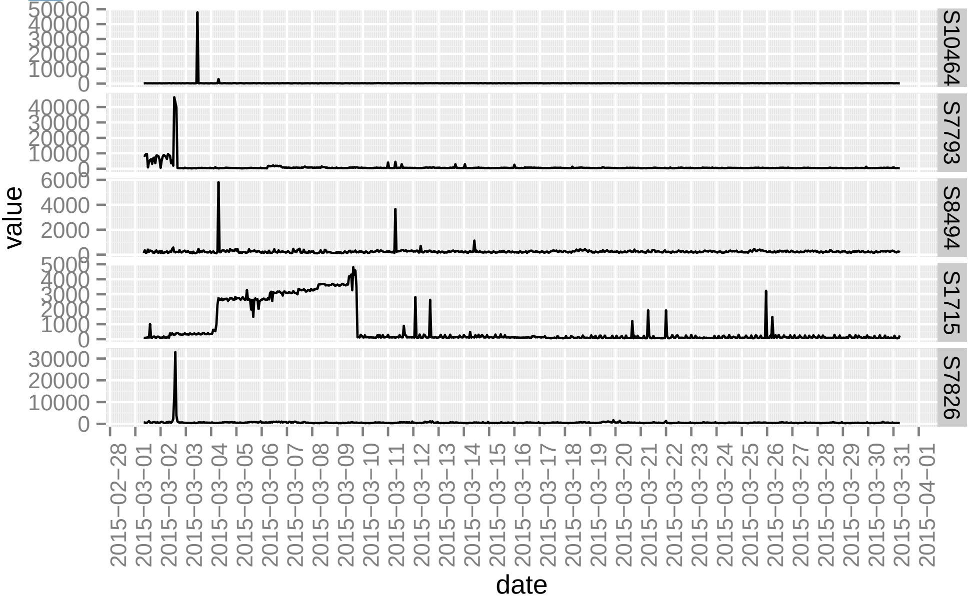
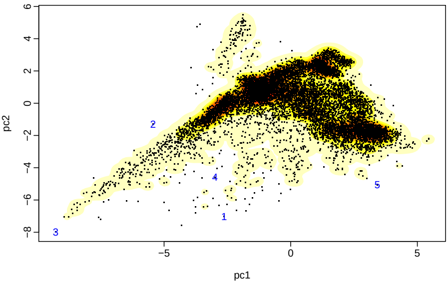


HDR versus α-convex hull

HDR boxplot



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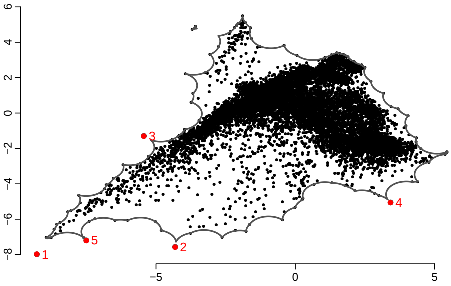
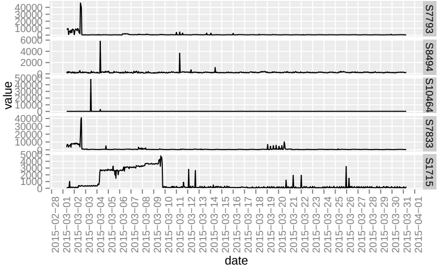


Top 5 anomalous time series

HDR



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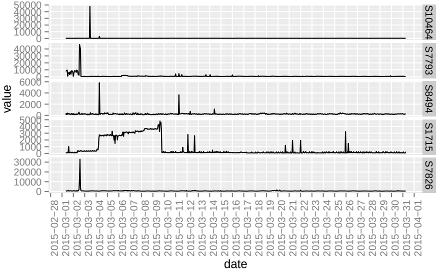

Outline

What next?

- Develop a more comprehensive set of features that are reliable measures and fast to compute. e.g., for finance data.
- Consider other dimension reduction methods and more than 2 dimensions.
- Develop dynamic and interactive visualization tools.
- Make methods available in an R package.

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Further information

- Papers and R packages: **robjhyndman.com**
- Blog: **robjhyndman.com/hyndsight**
- Code: **github.com/robjhyndman**
- Email: **Rob.Hyndman@monash.edu**