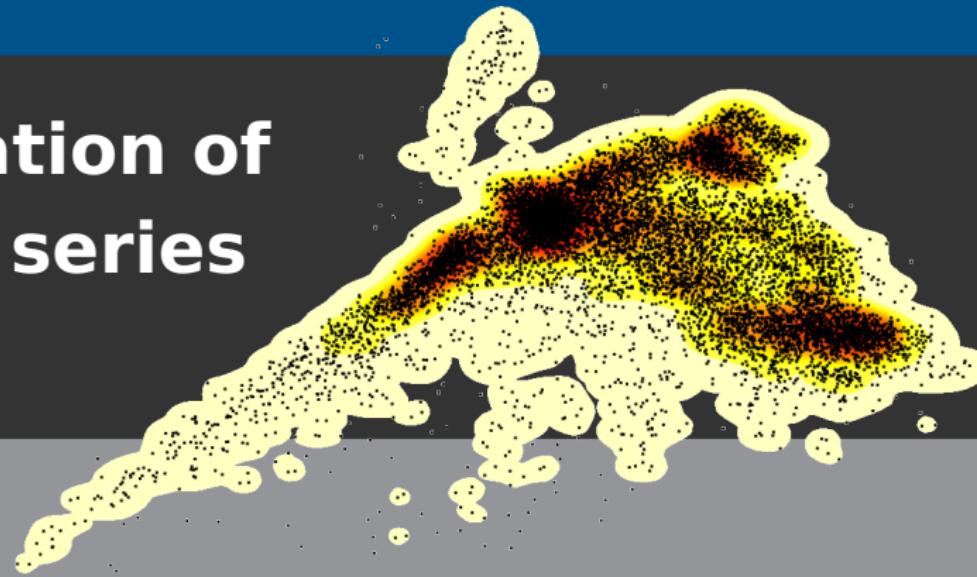




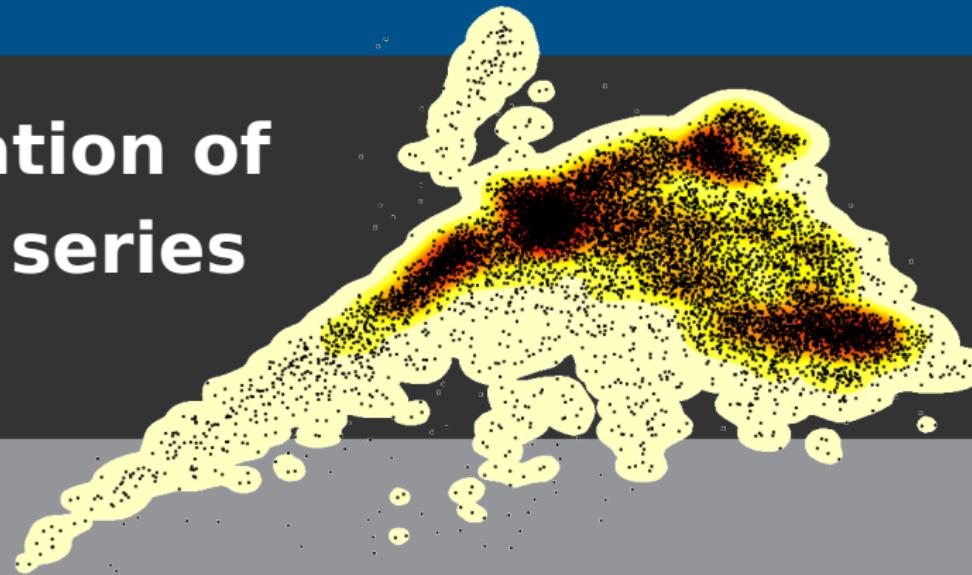
Visualisation of big time series data



Rob J Hyndman



Visualisation of big time series data

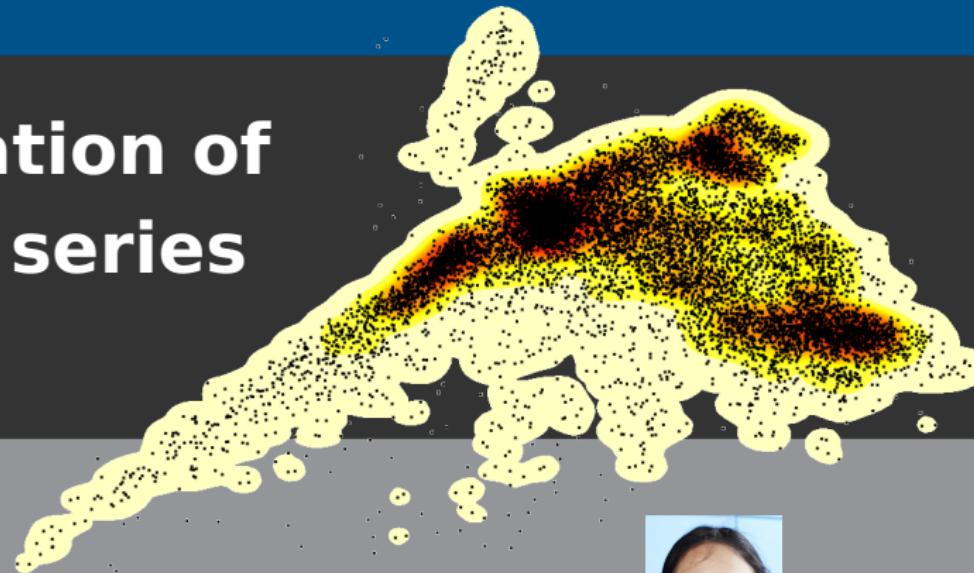


Rob J Hyndman

with Earo Wang, Nikolay Laptev
Yanfei Kang, Kate Smith-Miles



Visualisation of big time series data

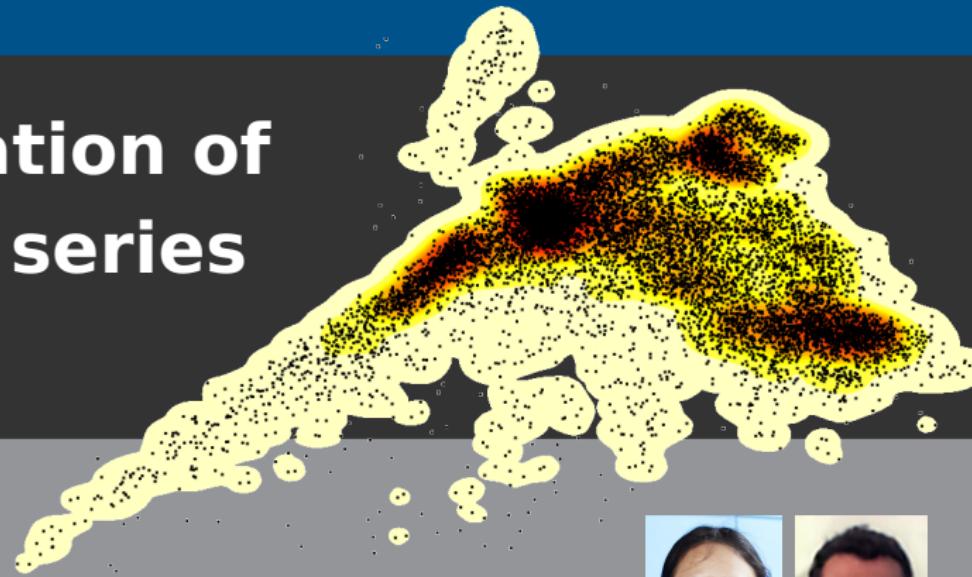


Rob J Hyndman

with **Earo Wang**, Nikolay Laptev
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Visualisation of big time series data



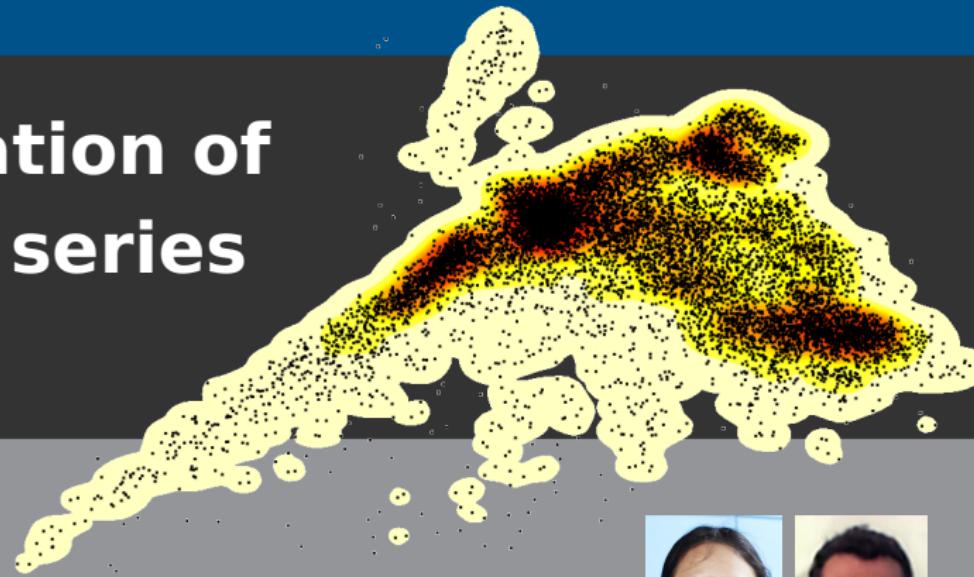
Rob J Hyndman

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Yanfei Kang, Kate Smith-Miles



Visualisation of big time series data



Rob J Hyndman

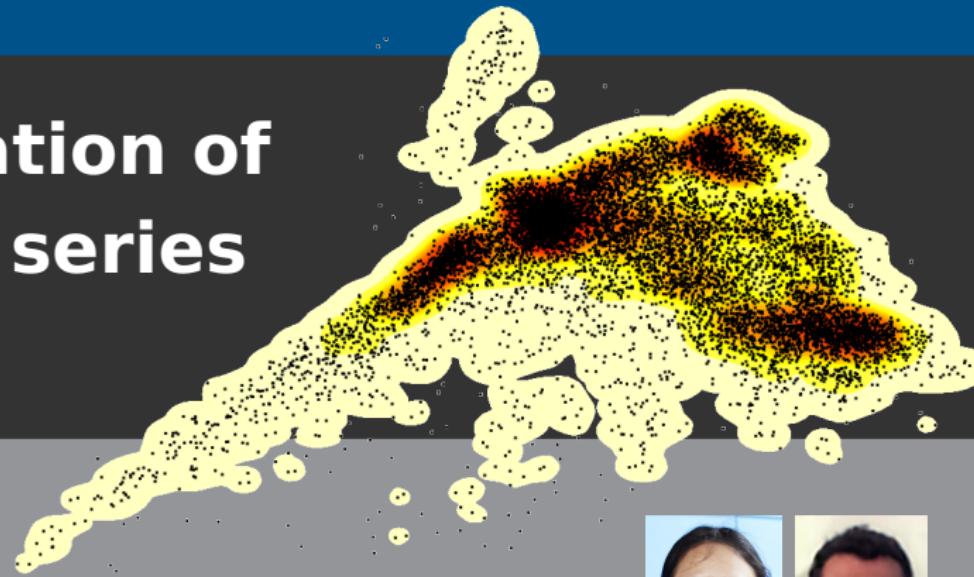
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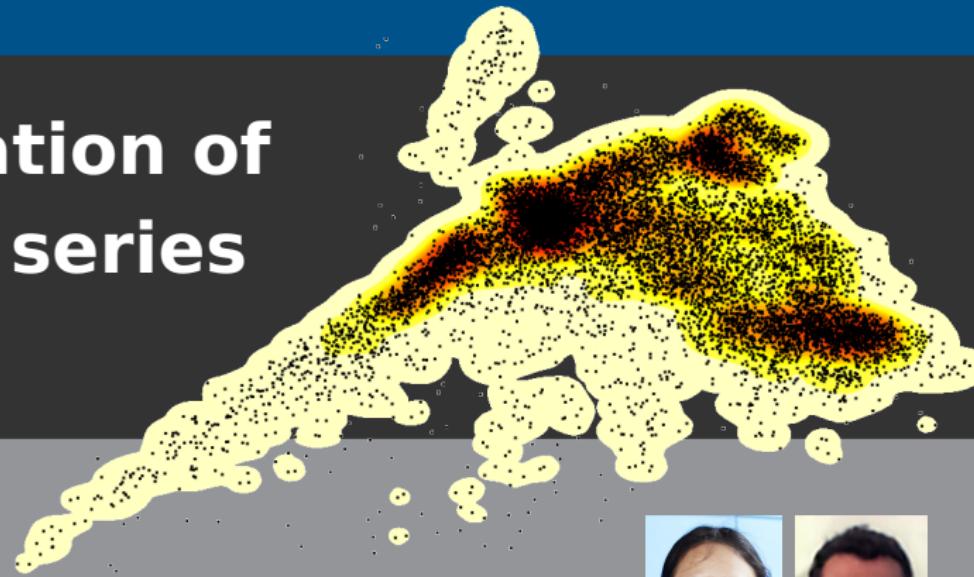
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MONASH BUSINESS SCHOOL

Visualisation of big time series data



Rob J Hyndman

with Earo Wang, Nikolay Laptev
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Outline

1 The problem

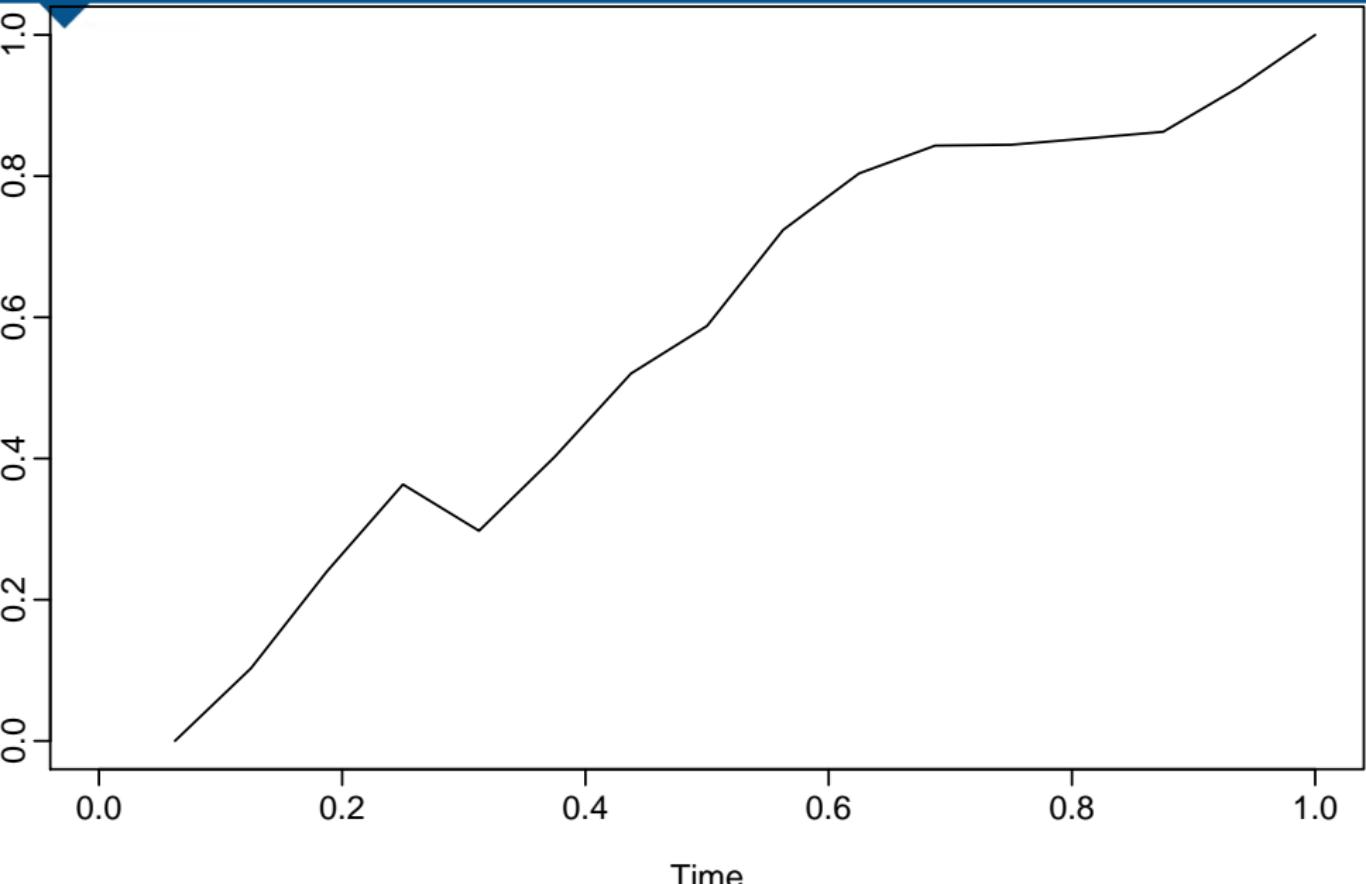
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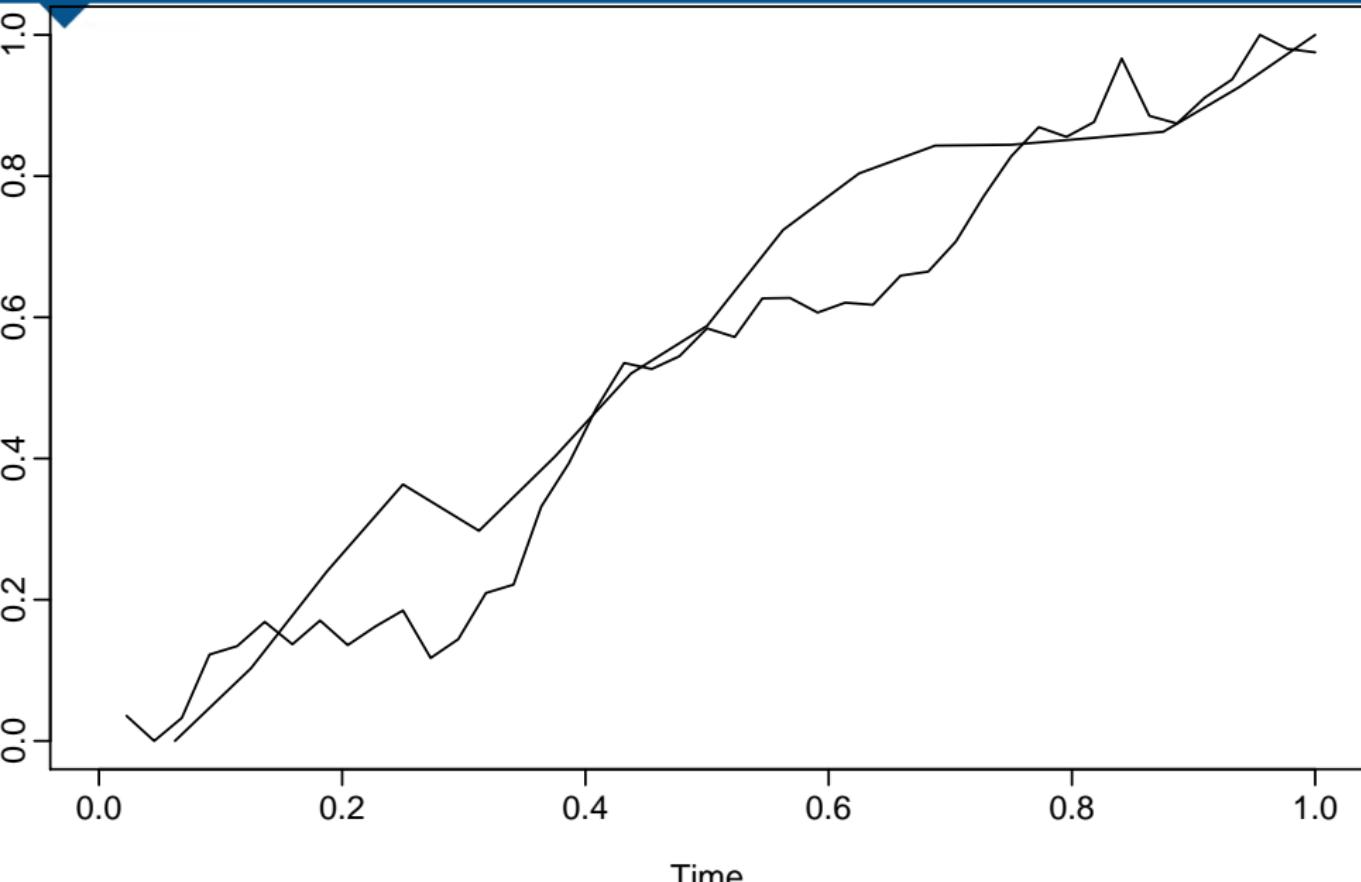
4 Yahoo web traffic

5 What next?

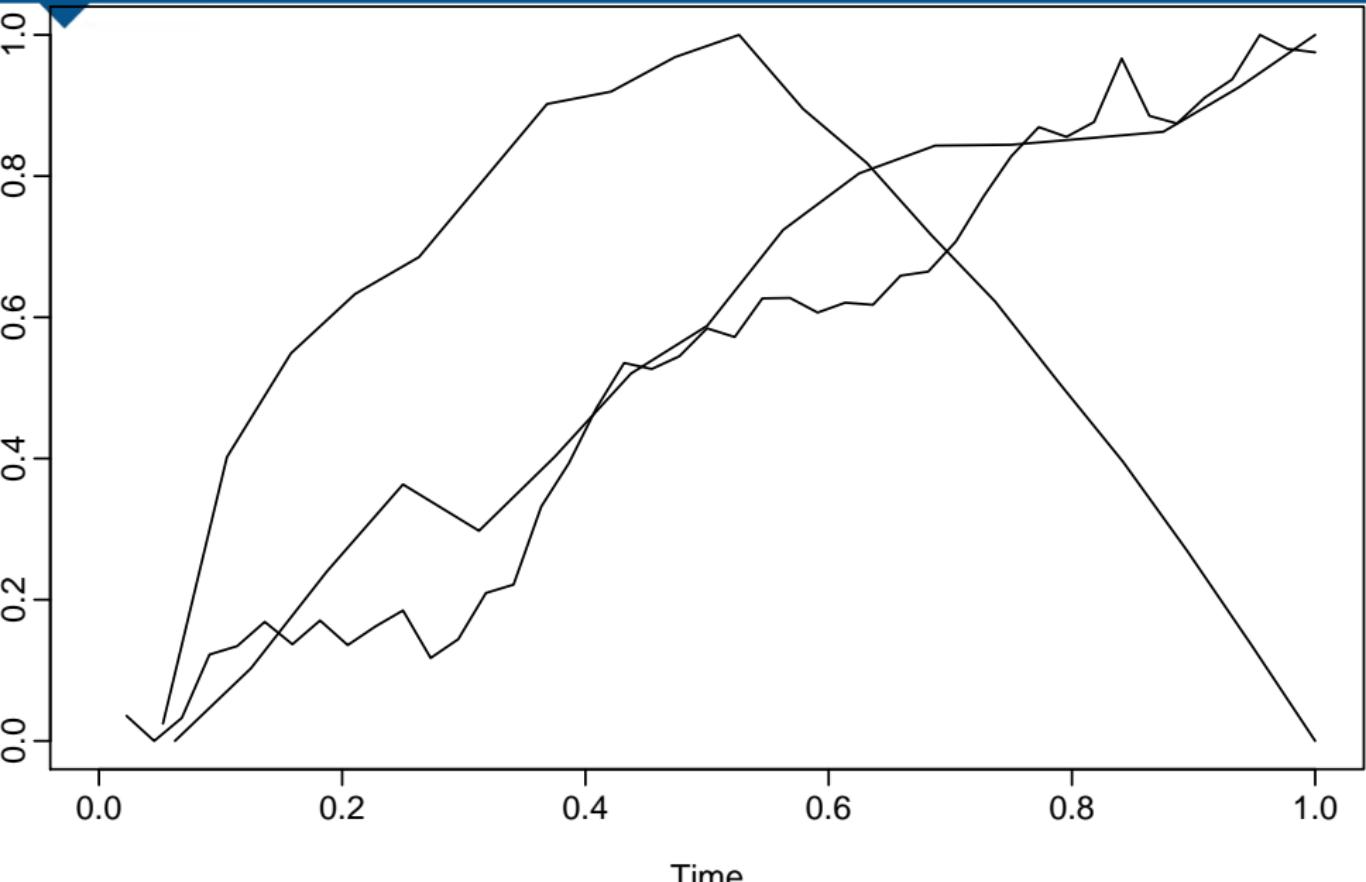
How to plot lots of time series?



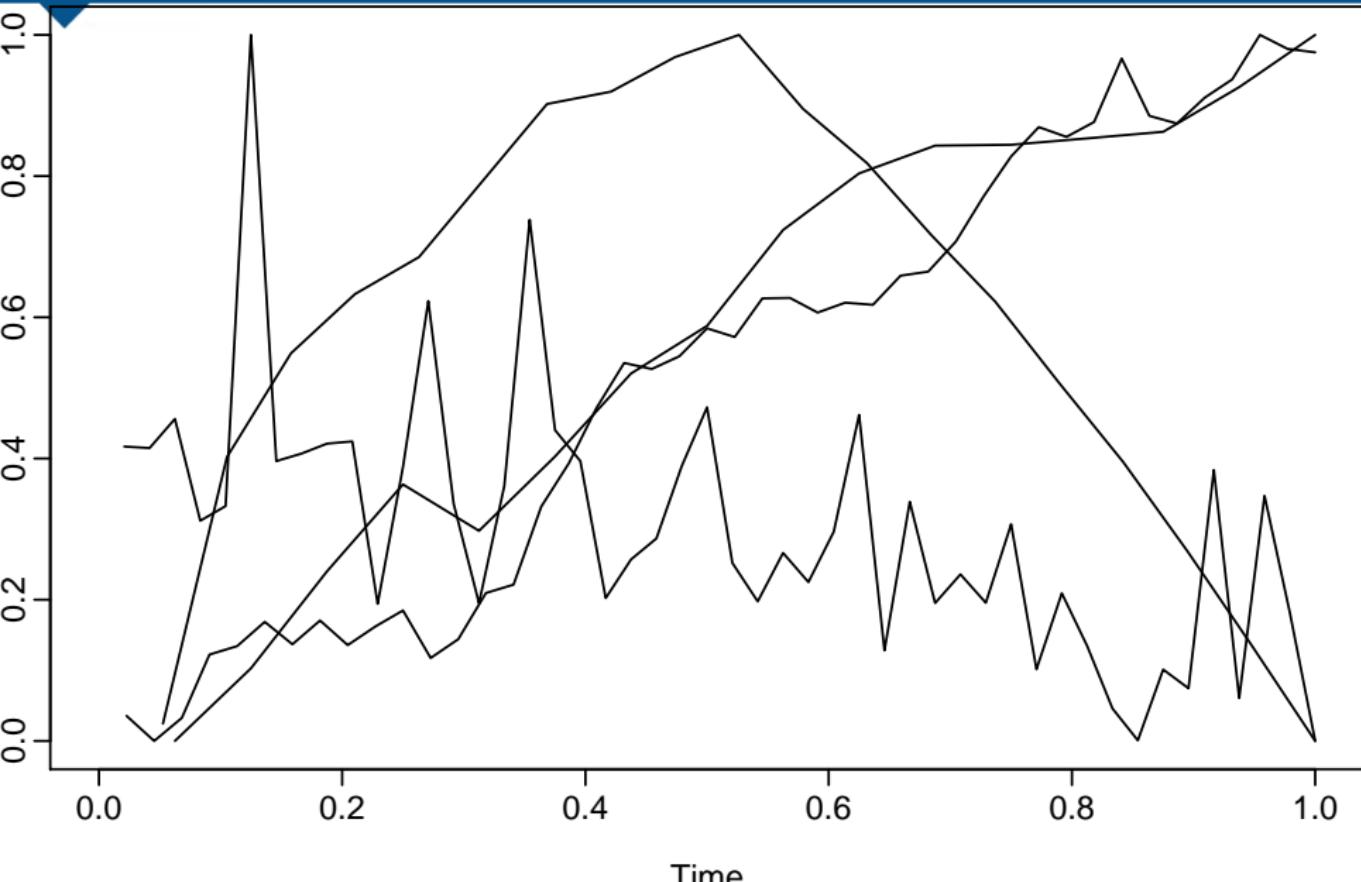
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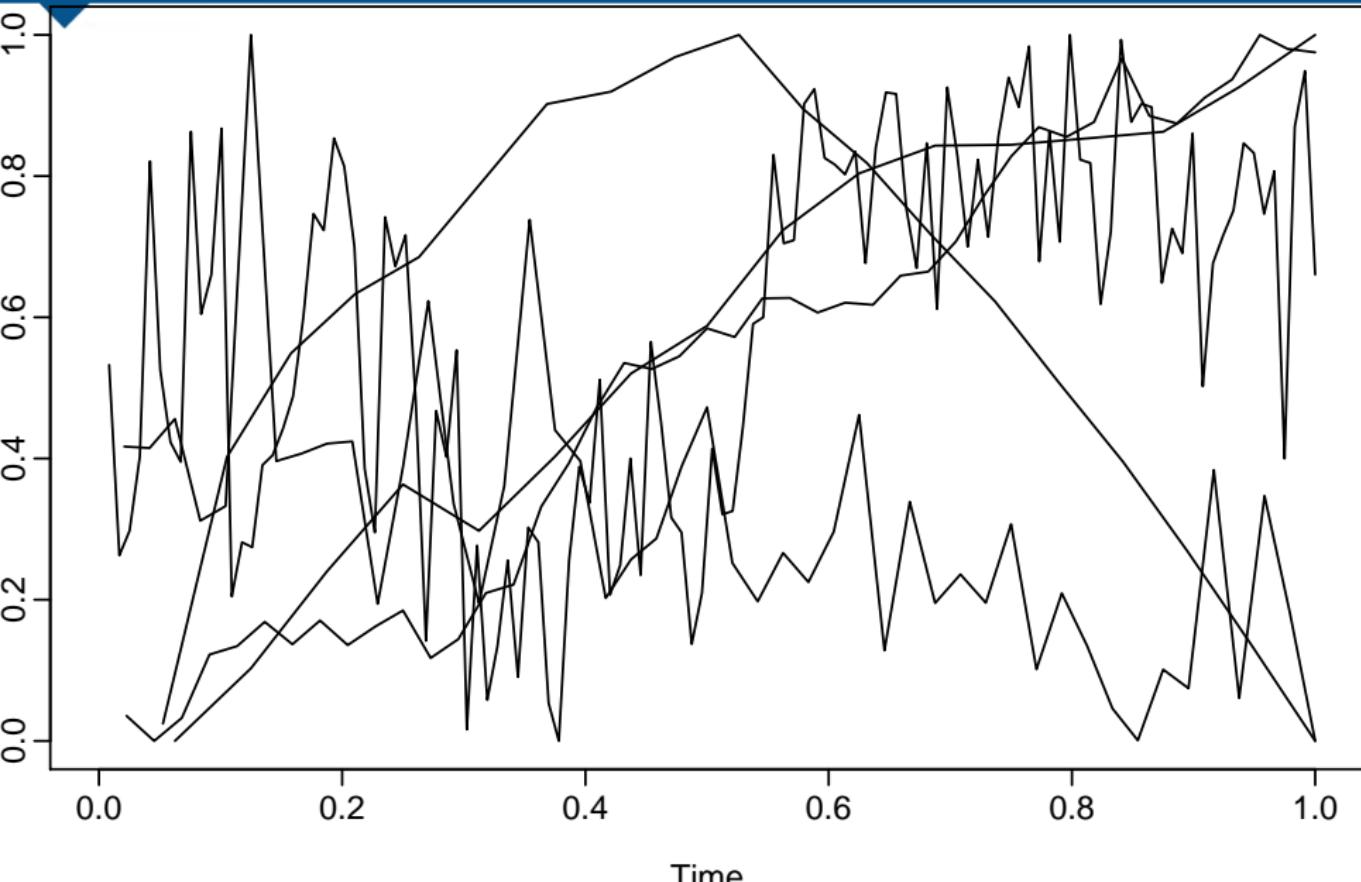
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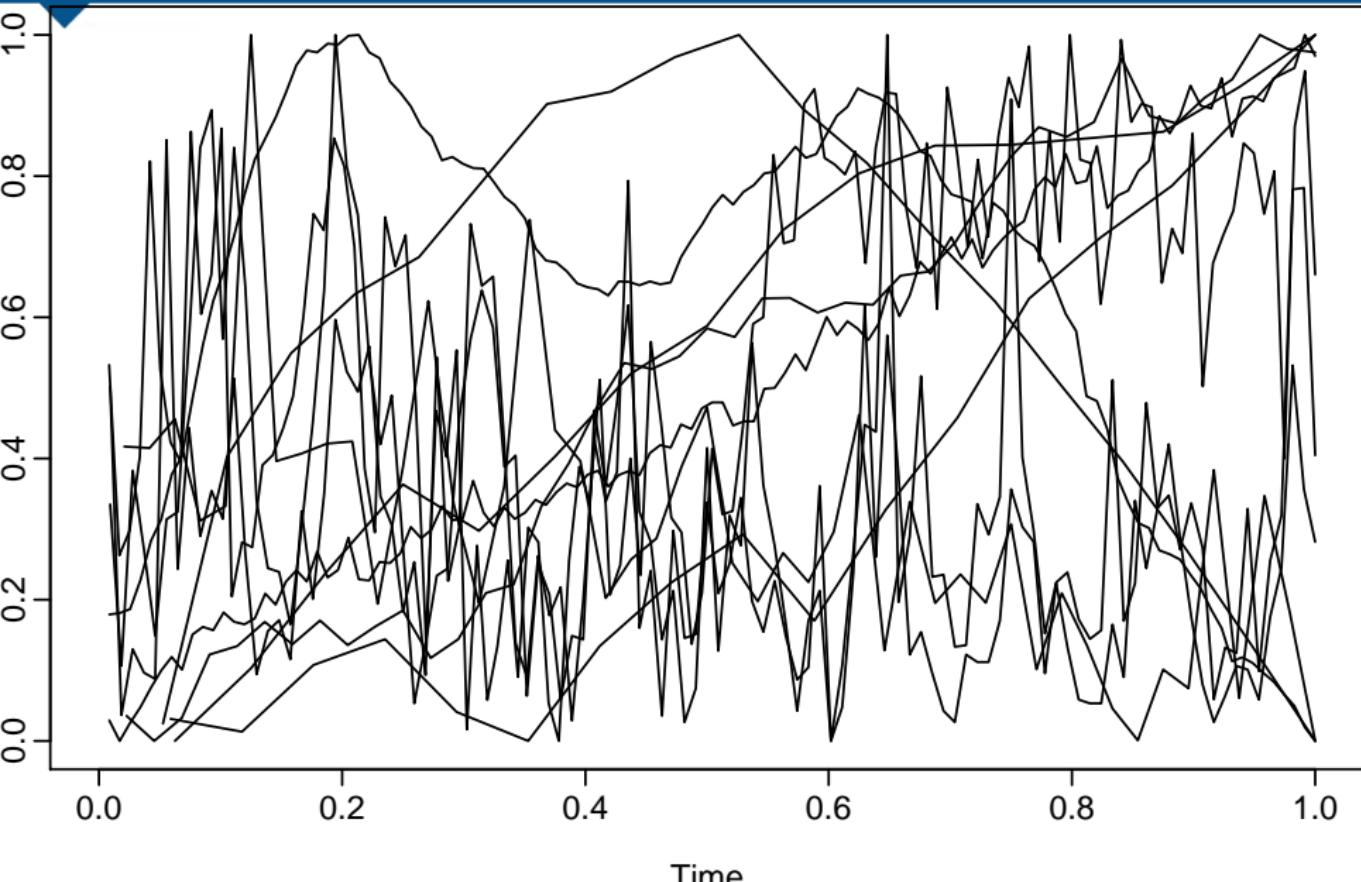
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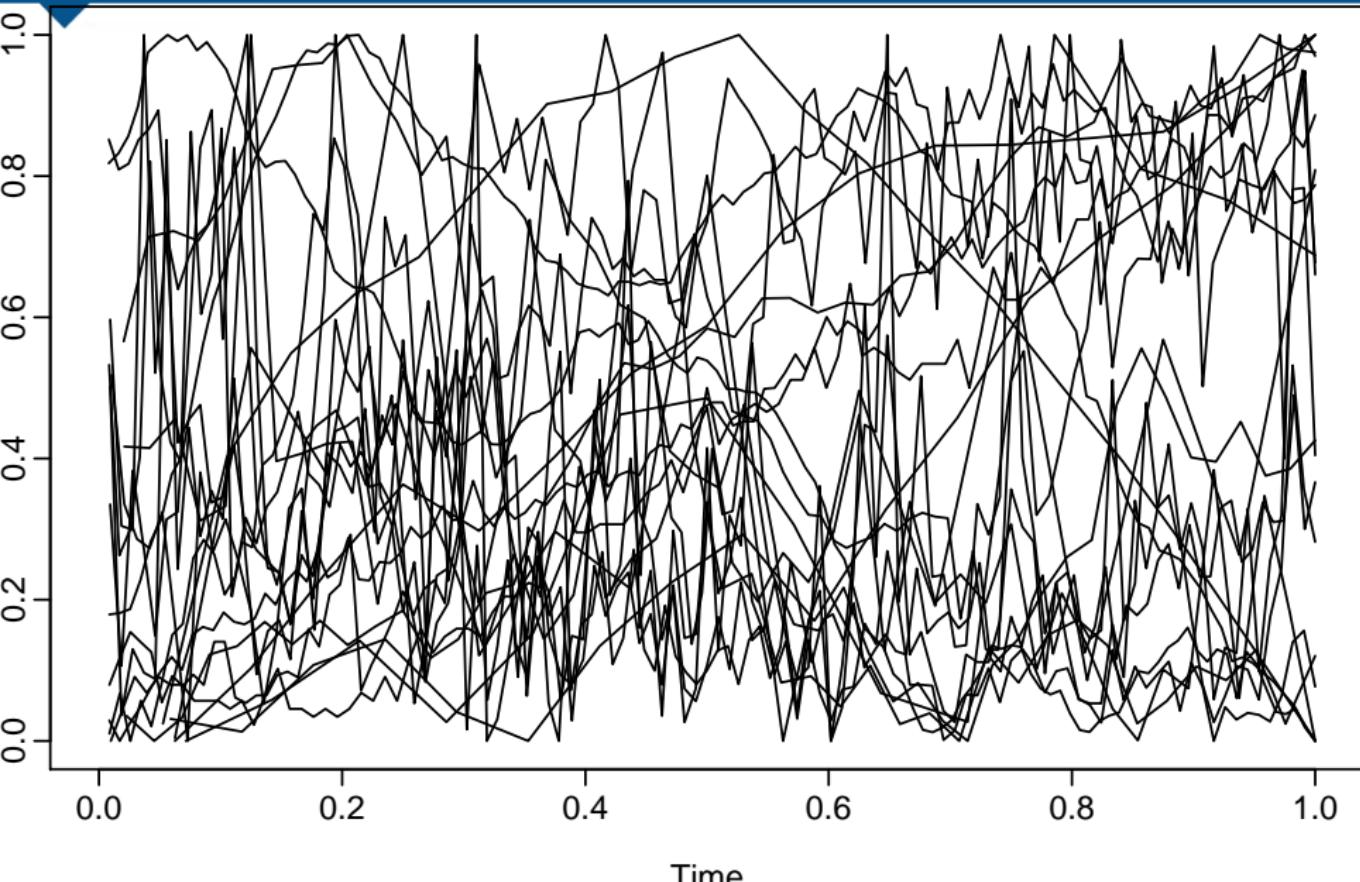
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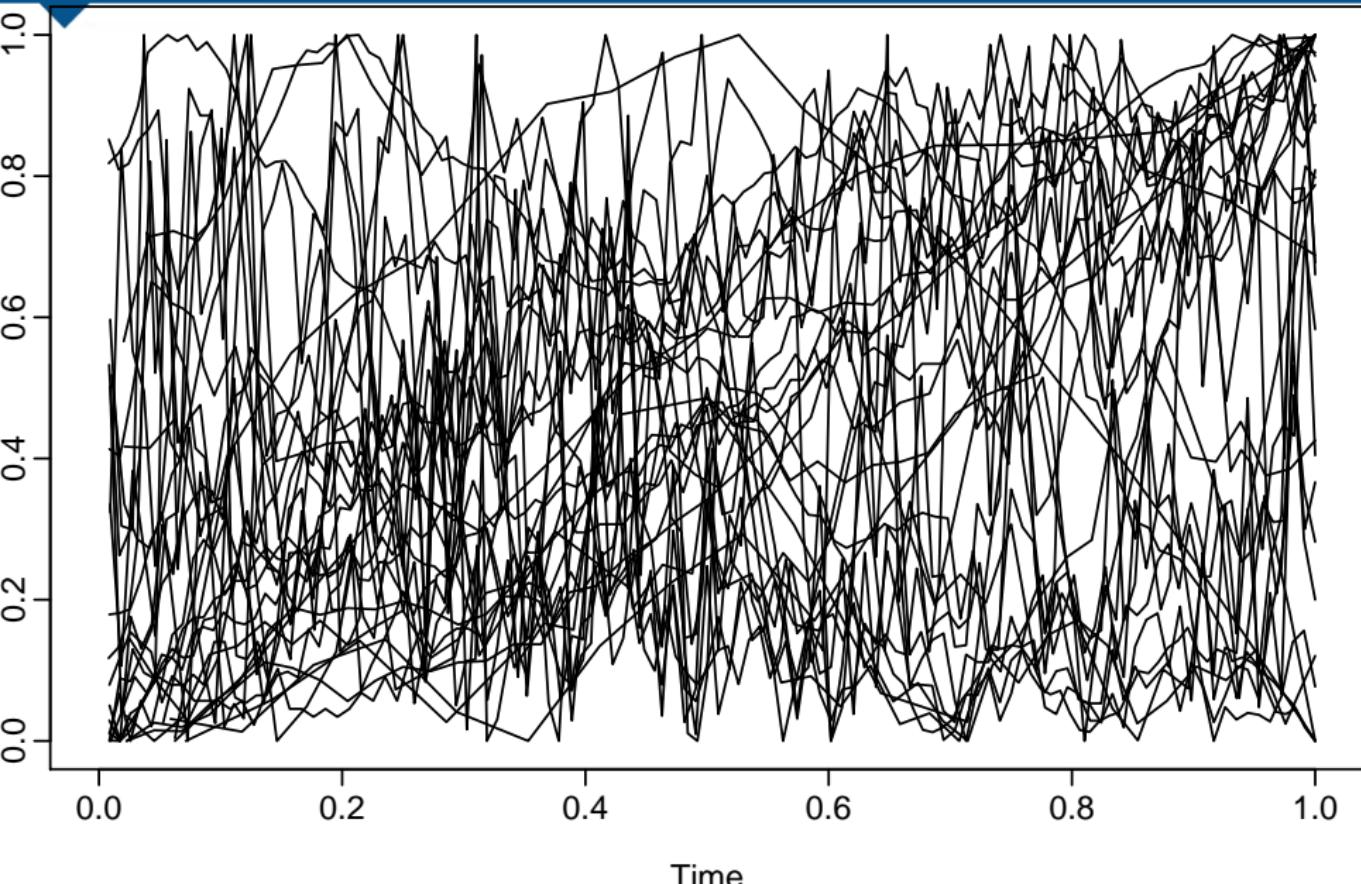
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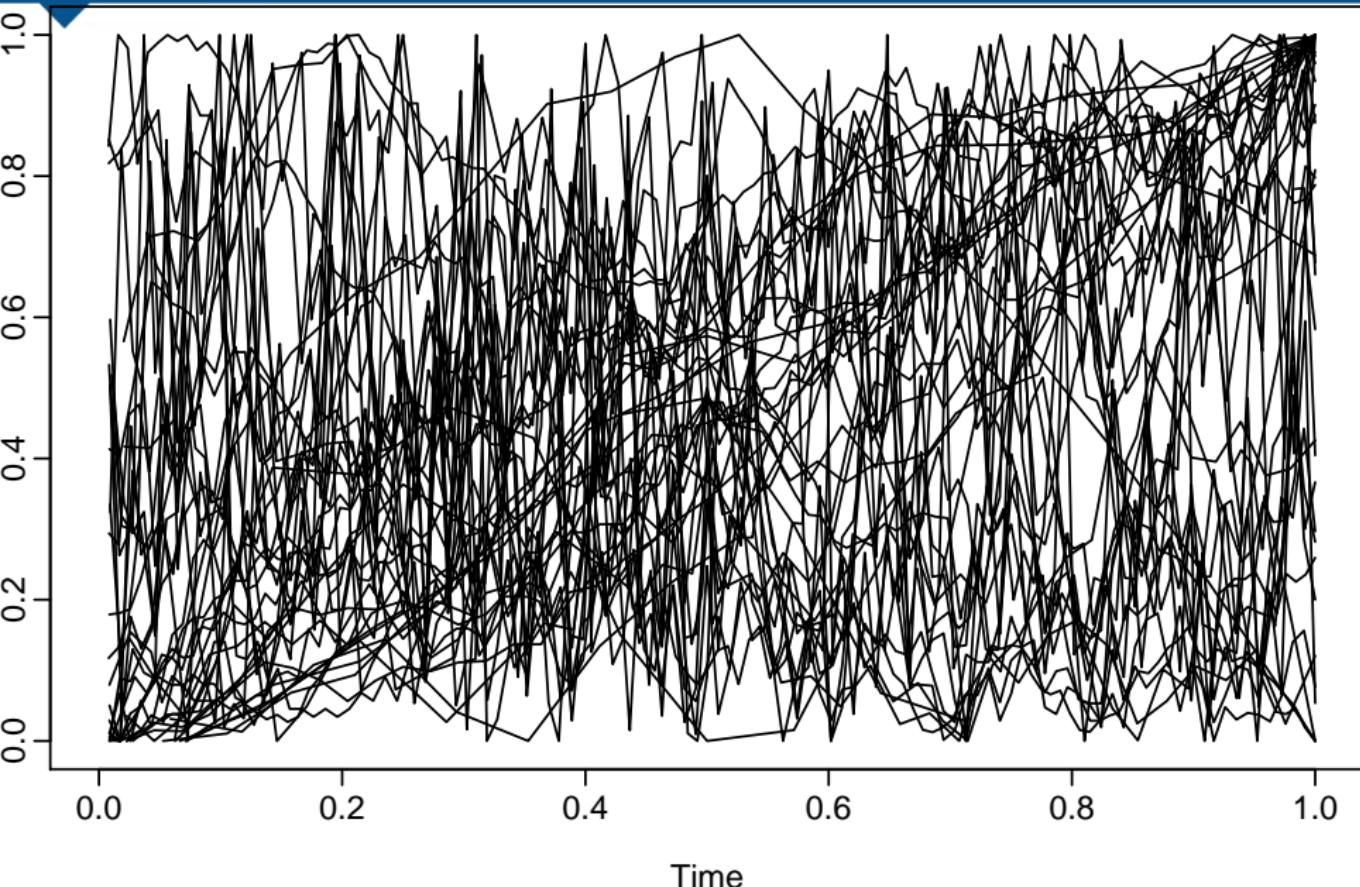
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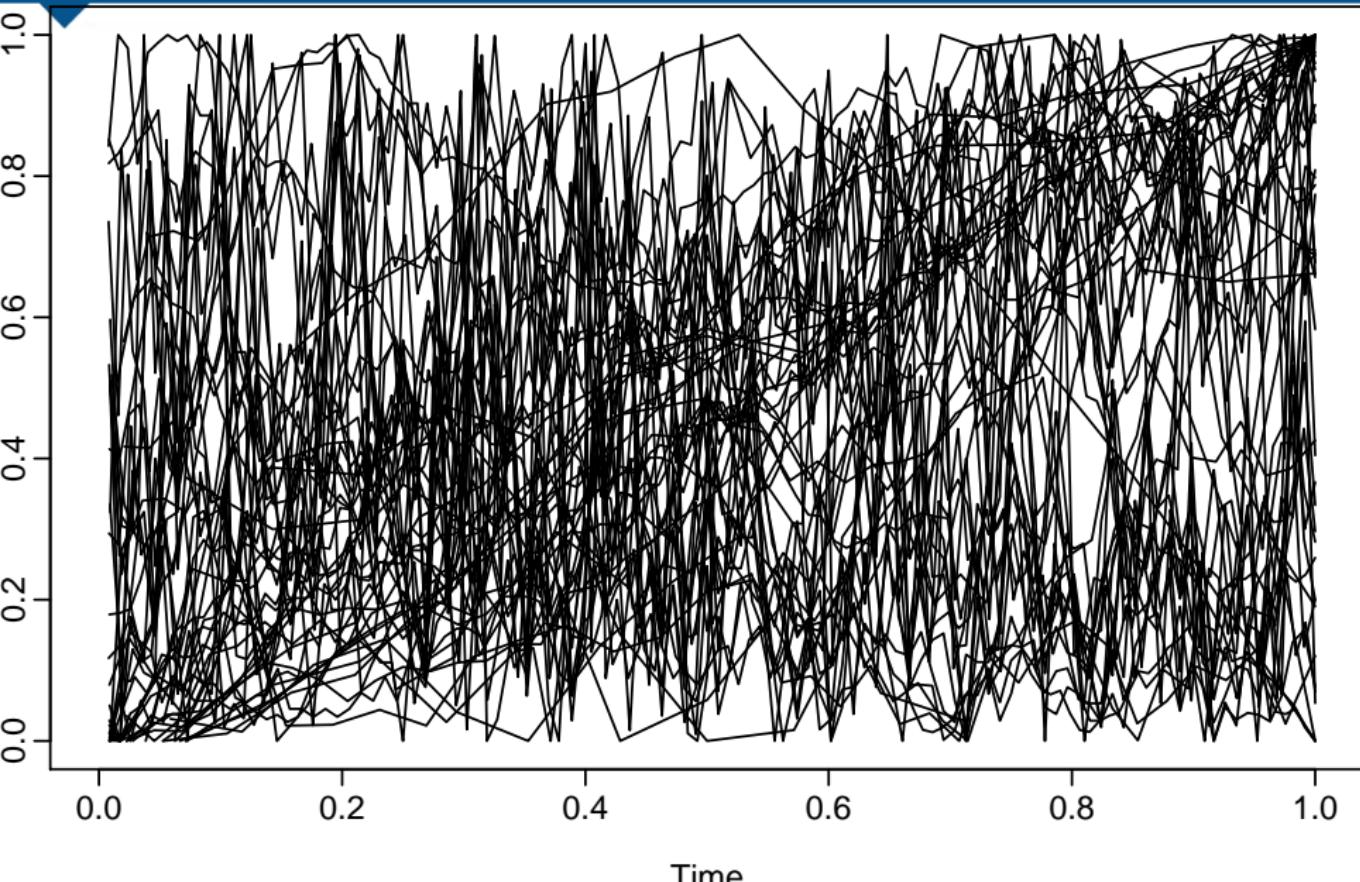
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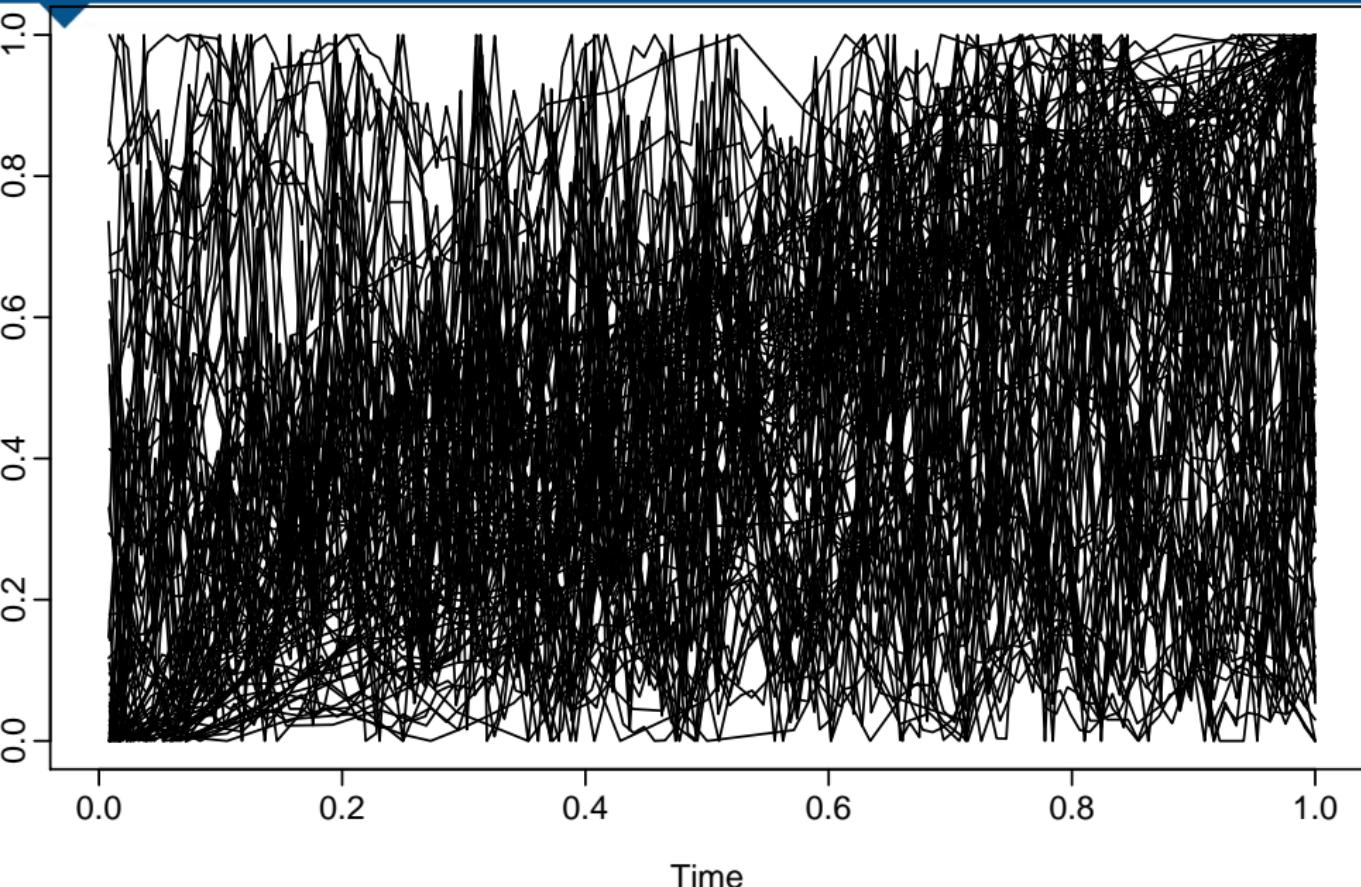
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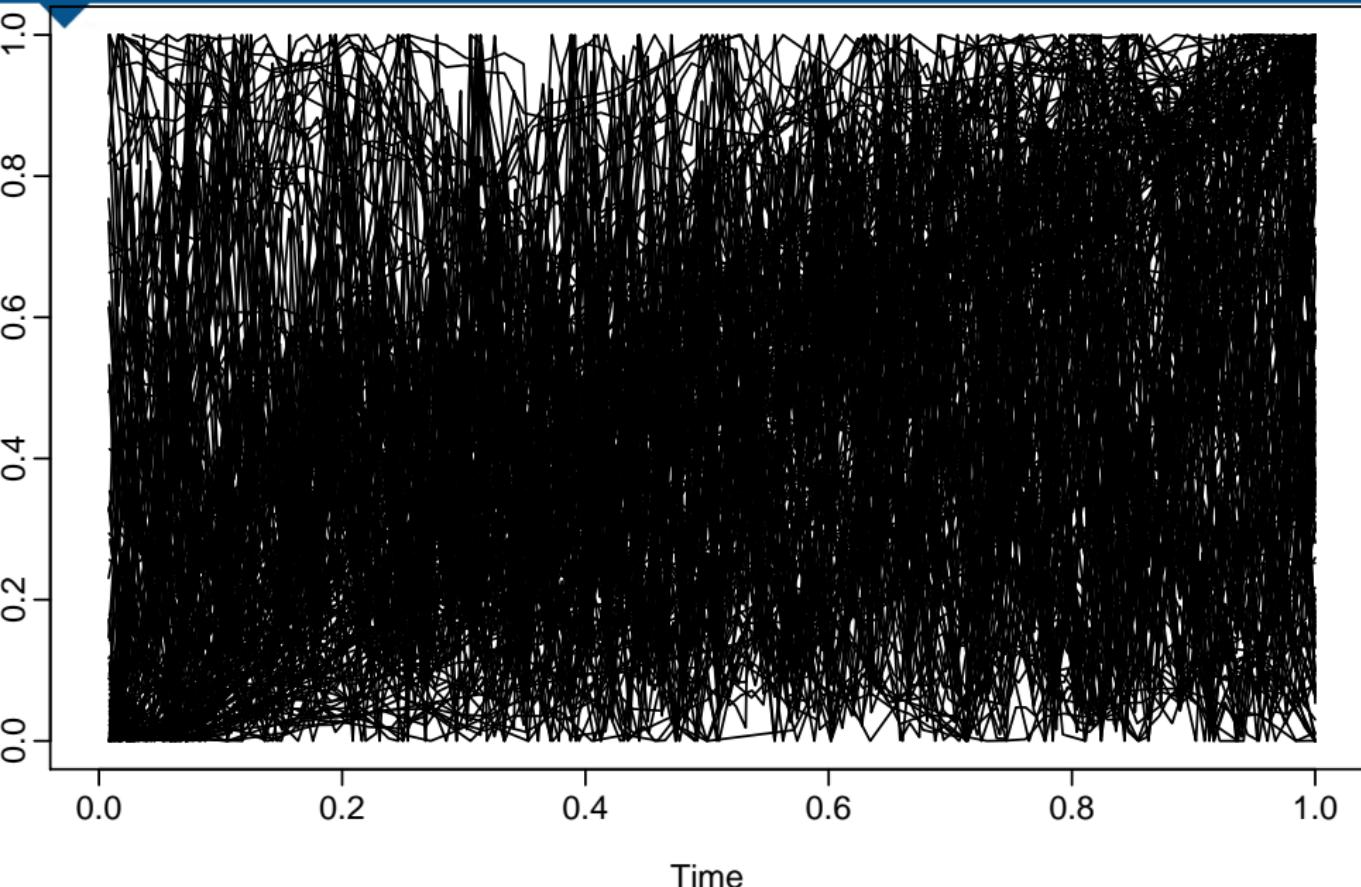
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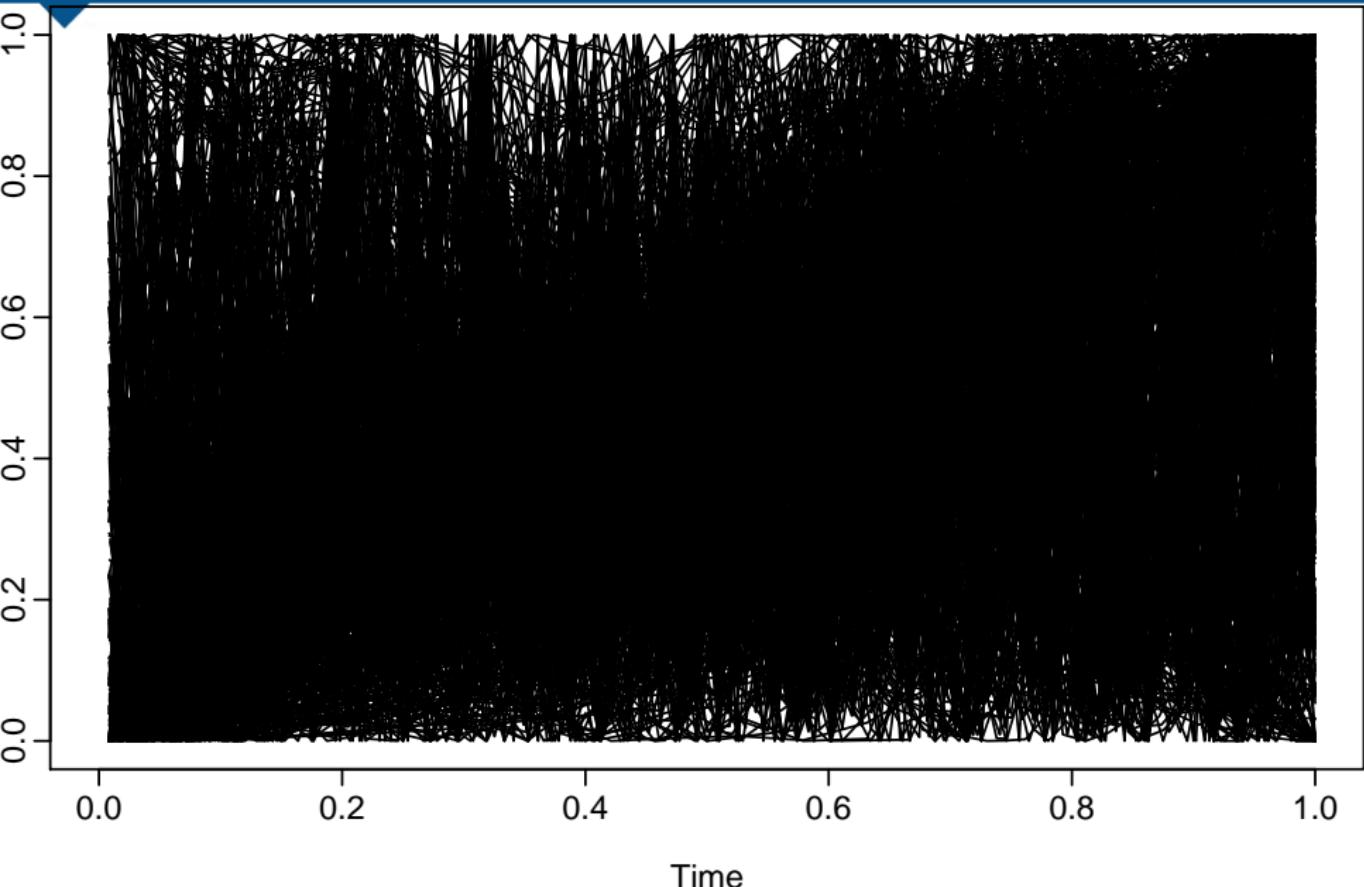
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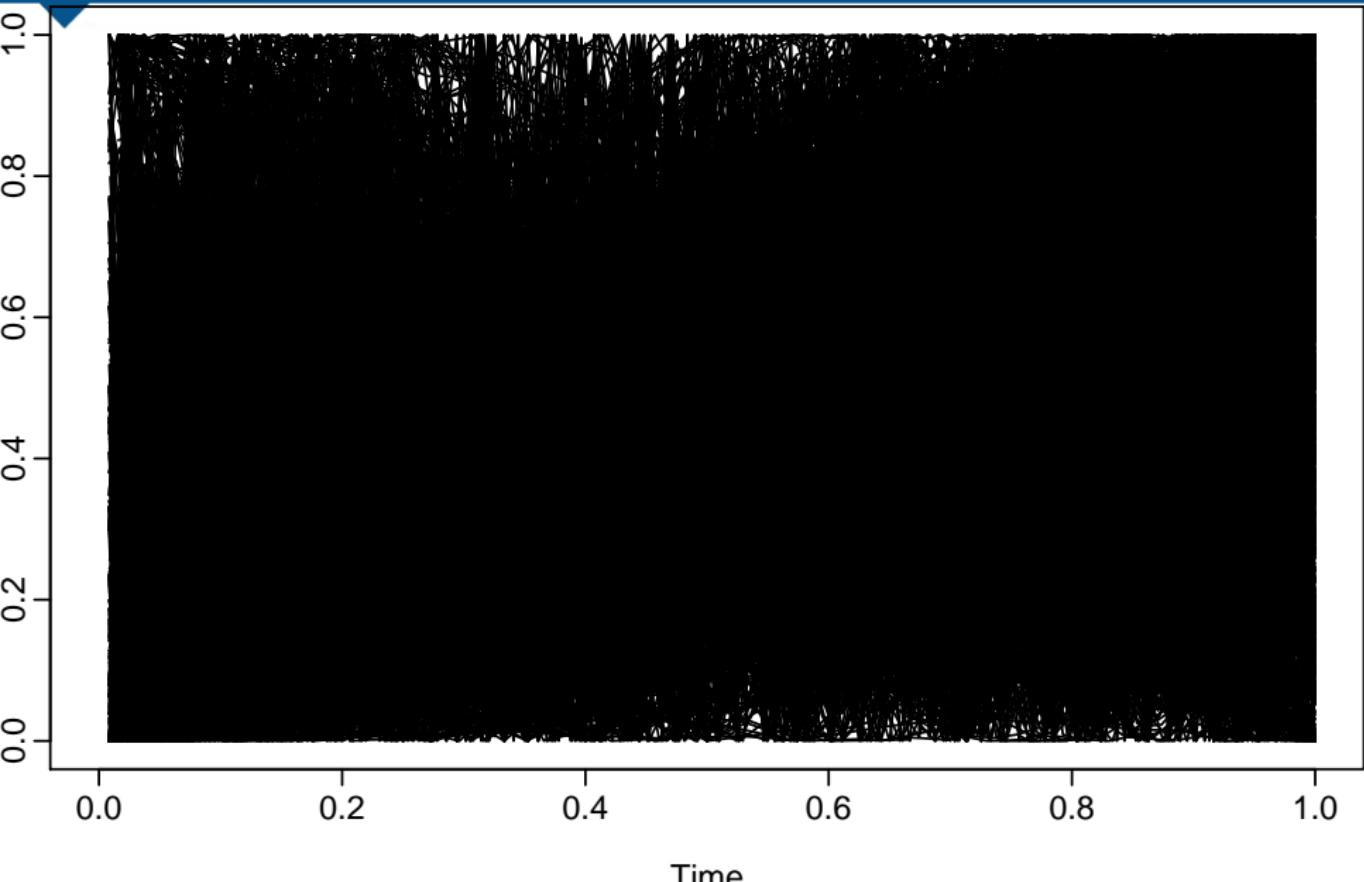
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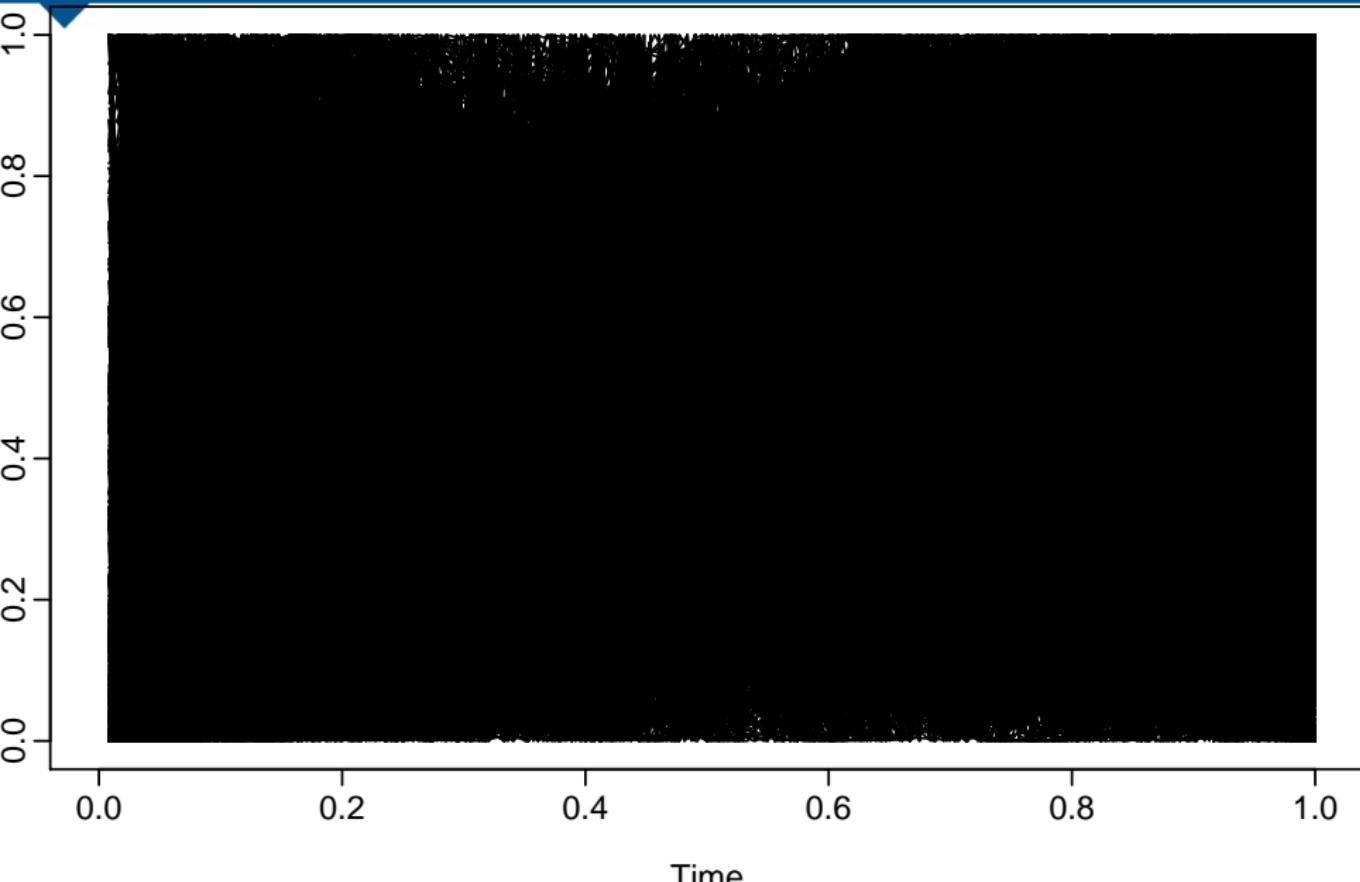
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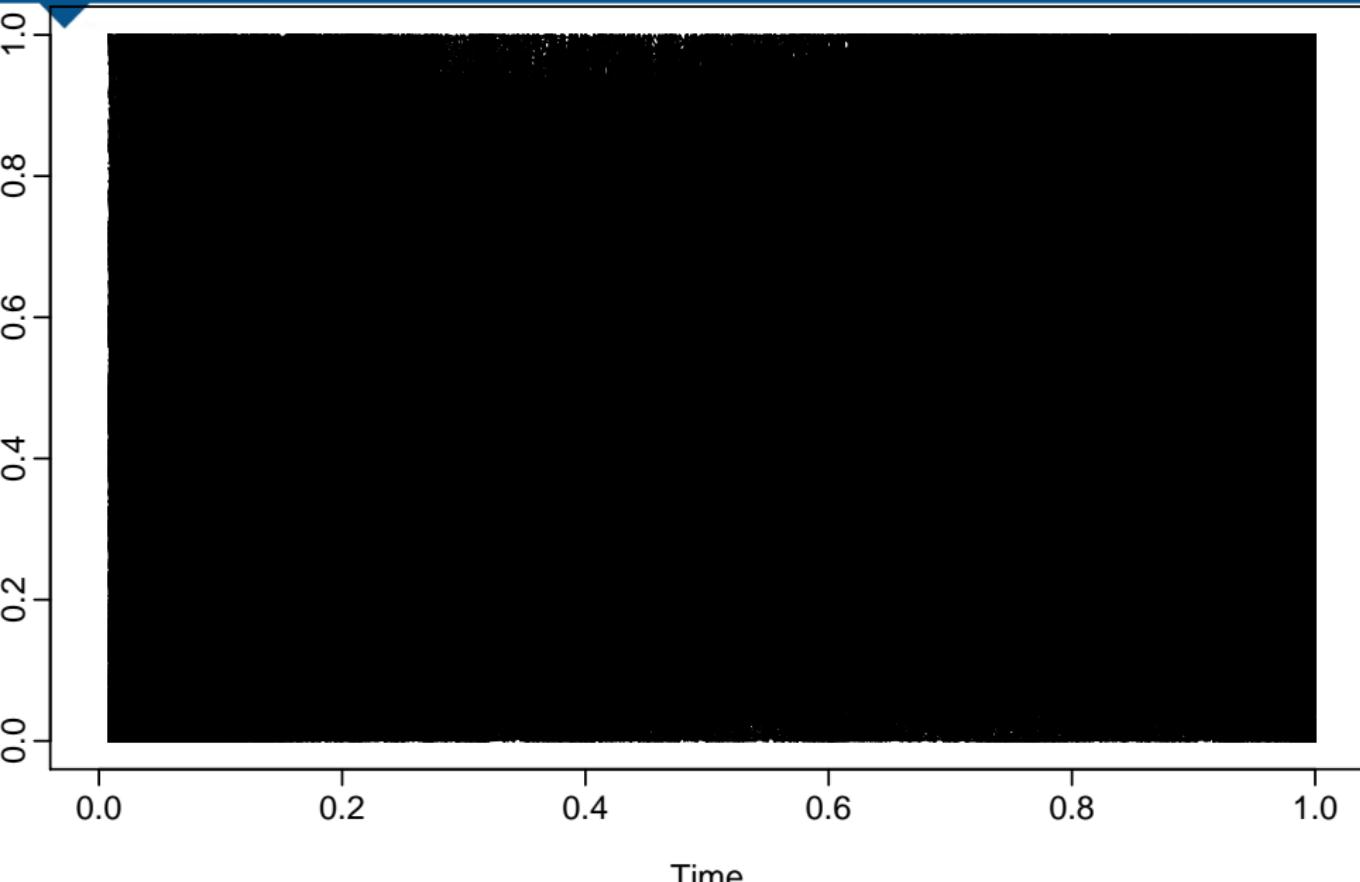
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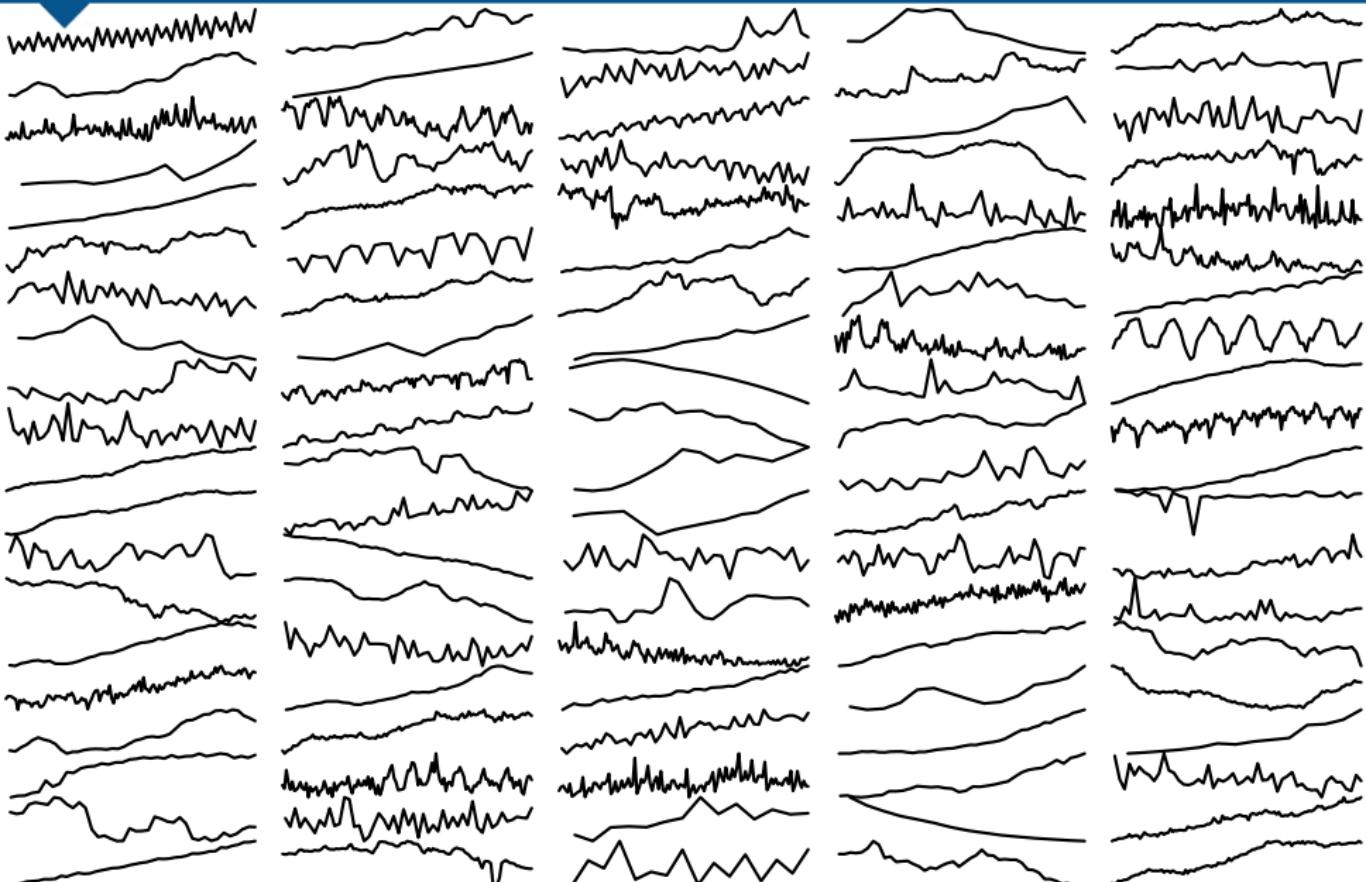
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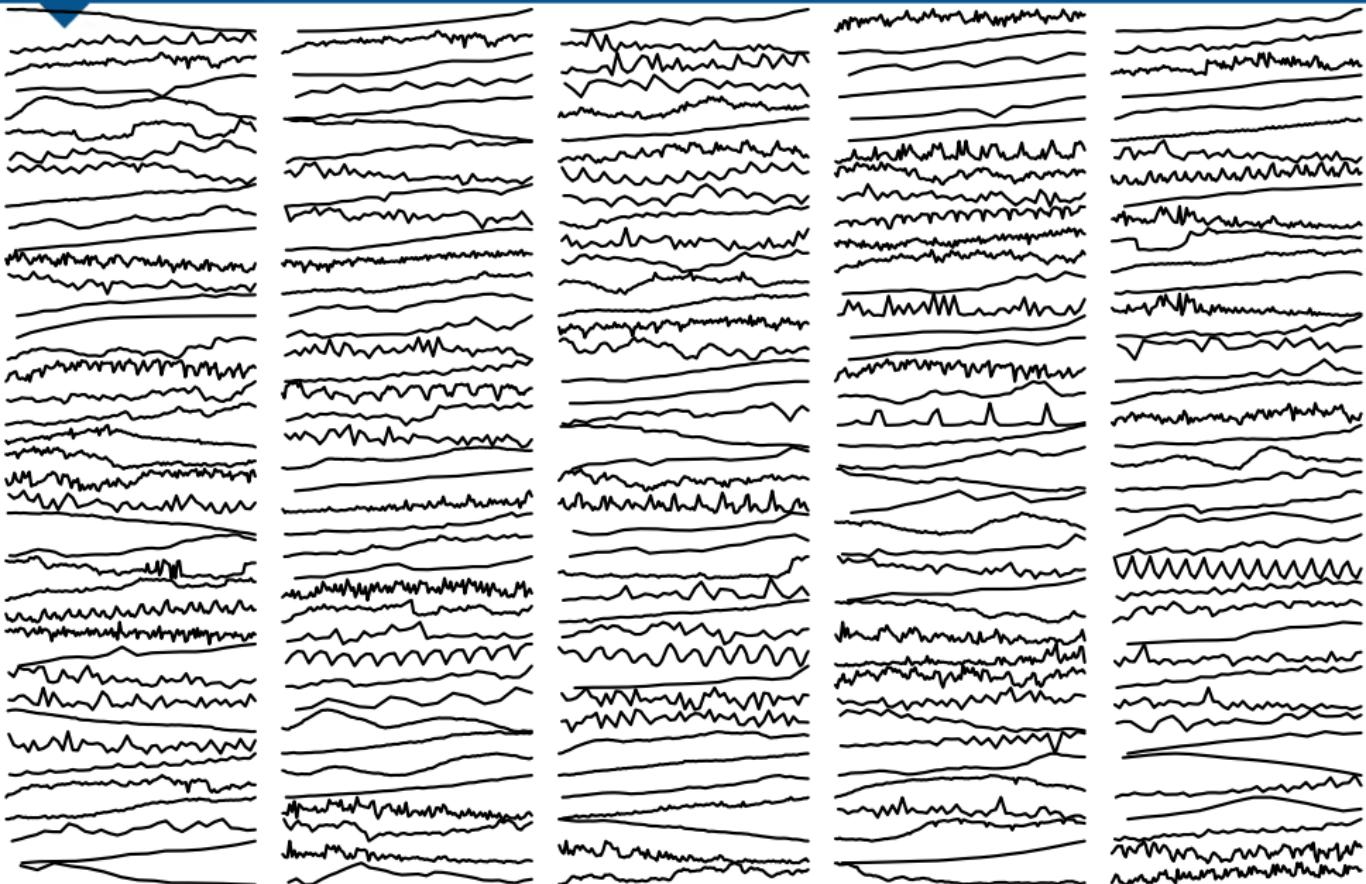
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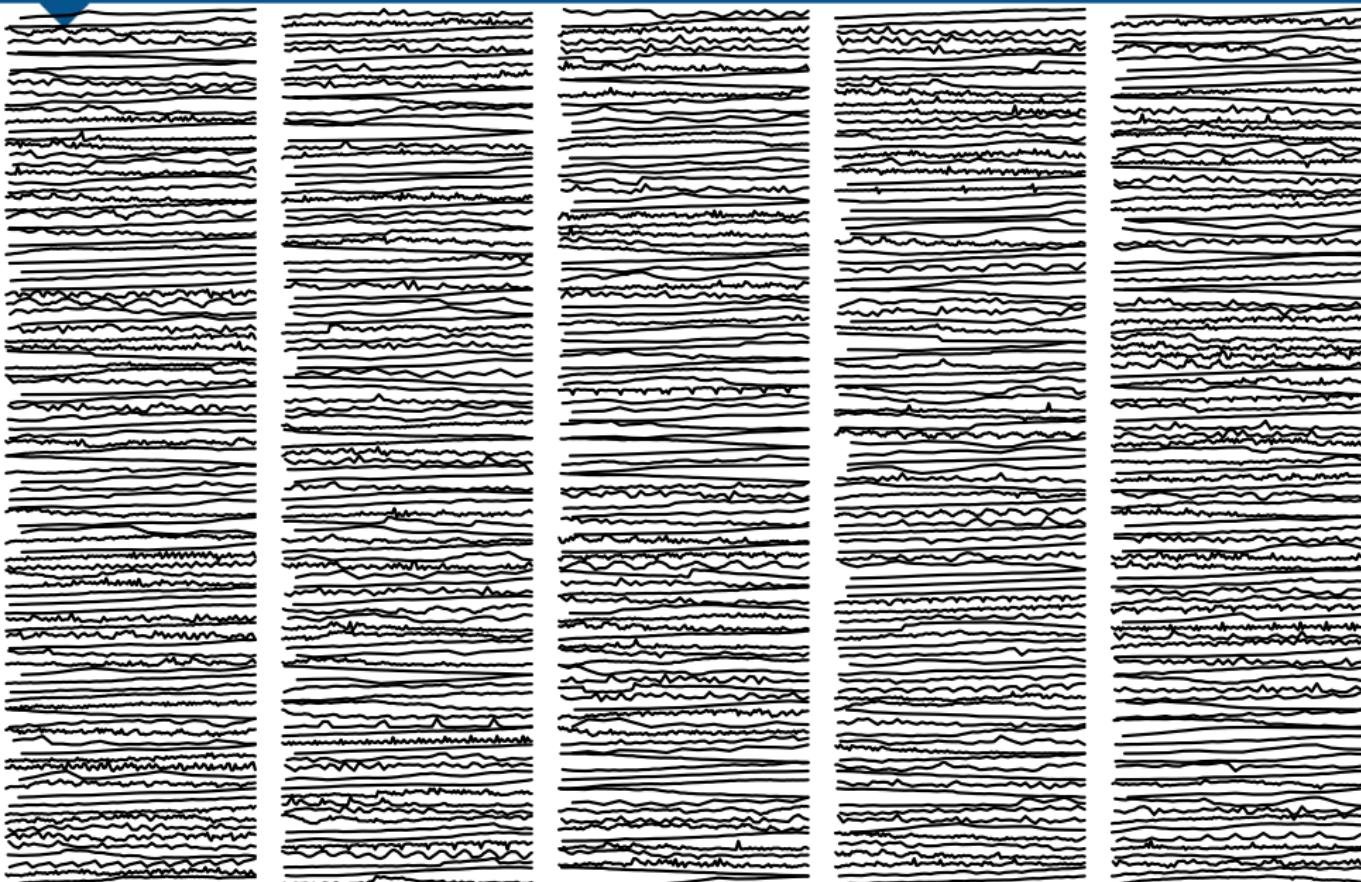
How to plot lots of time series?



Visualisation of big time series data

The problem

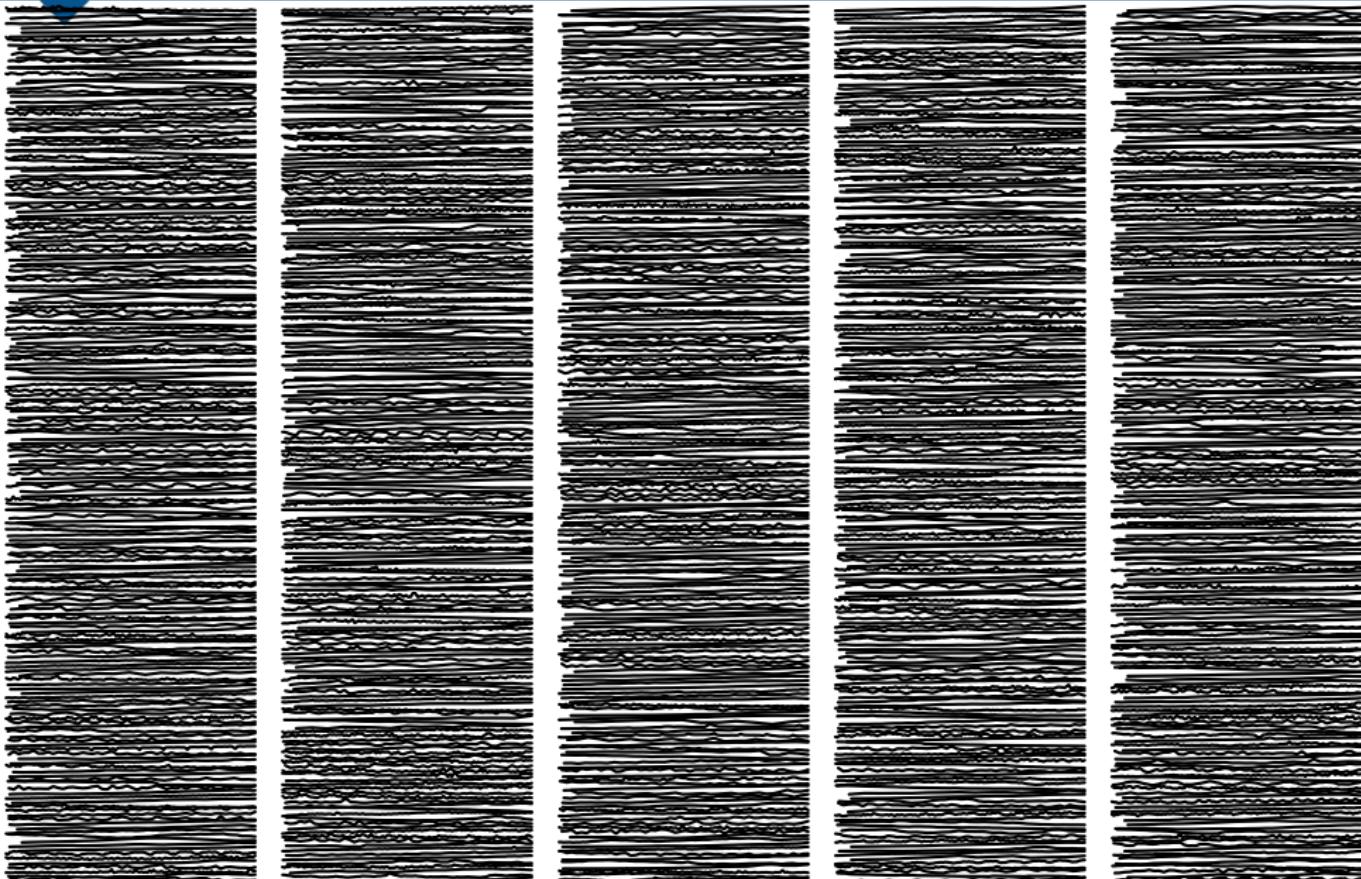
How to plot lots of time series?



Visualisation of big time series data

The problem

How to plot lots of time series?



Visualisation of big time series data

The problem

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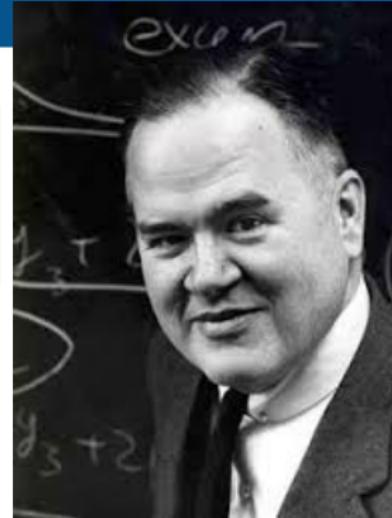
Key idea

Cognostics

Computer-produced diagnostics
(Tukey and Tukey, 1985).

Examples for time series

- lag correlation
- size and direction of trend
- strength of seasonality
- short-term fluctuations
- long-term fluctuations



John W Tukey

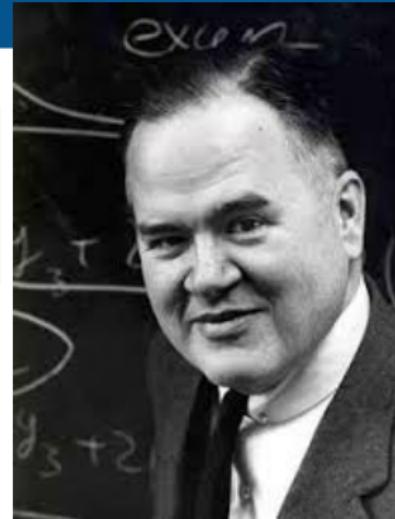
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Examples for time series

- lag correlation
- size and direction of trend
- strength of seasonality
- timing of peak seasonality
- spectral entropy



John W Tukey

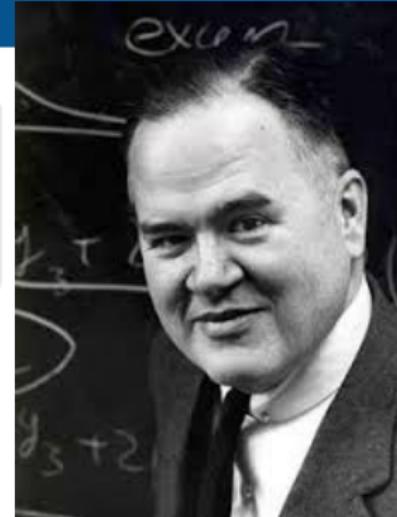
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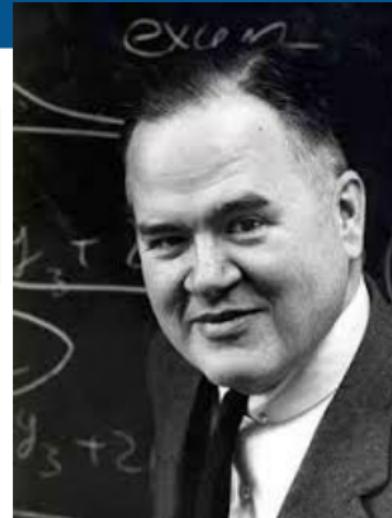
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John W Tukey

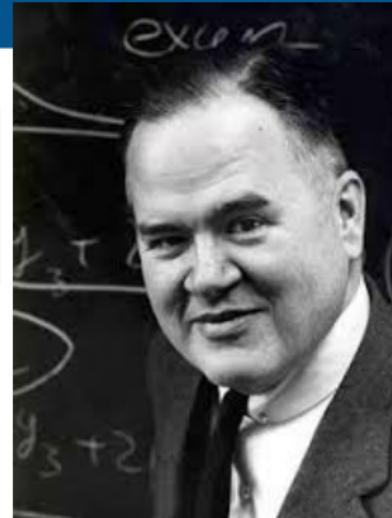
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John W Tukey

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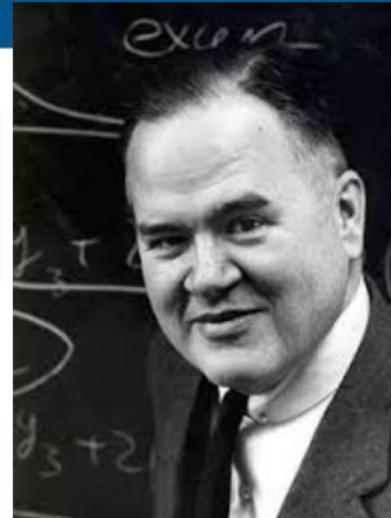
Cognostics

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Examples for time series

- lag correlation
- size and direction of trend
- strength of seasonality
- timing of peak seasonality
- spectral entropy

Called “features” or “characteristics” in the machine learning literature.



John W Tukey

Outline

1 The problem

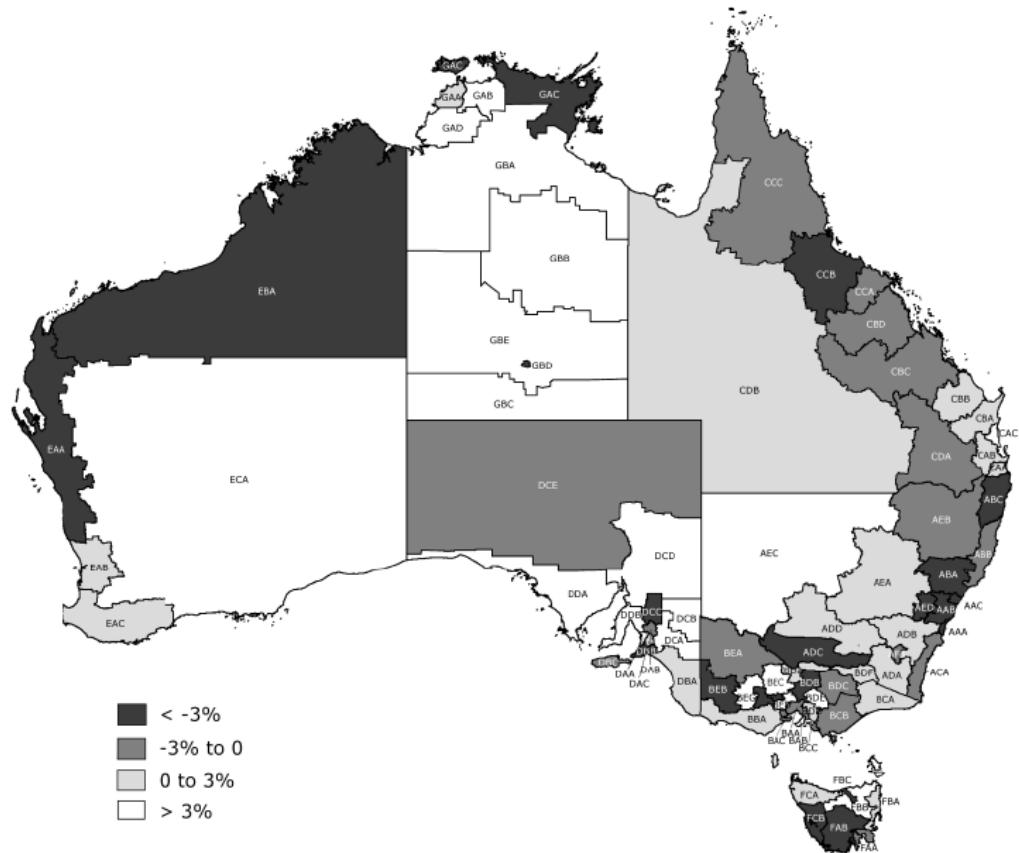
2 Australian tourism demand

3 M3 competition data

4 Yahoo web traffic

5 What next?

Australian tourism demand

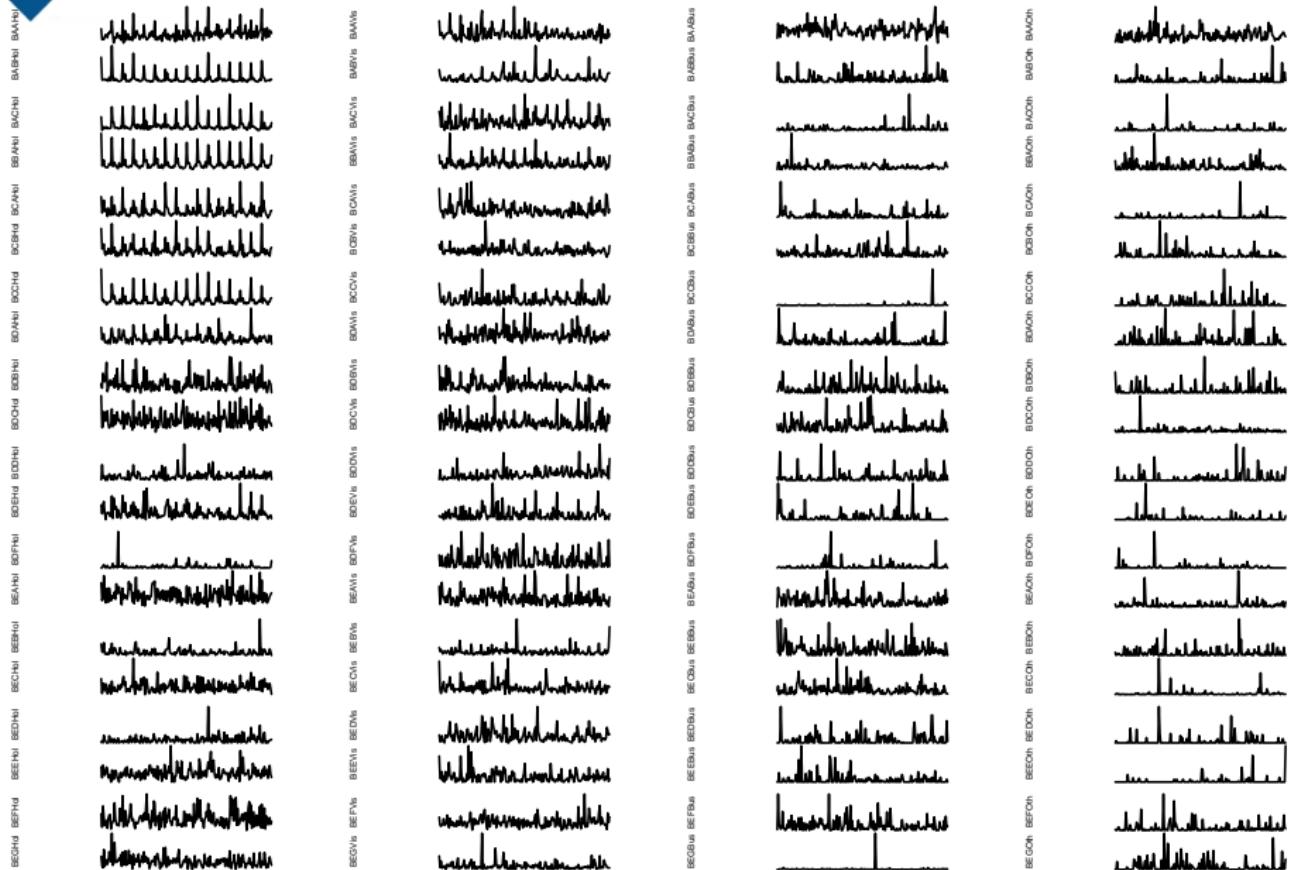


Australian tourism demand

- Quarterly data on visitor night from 1998:Q1 – 2013:Q4
- From: *National Visitor Survey*, based on annual interviews of 120,000 Australians aged 15+, collected by Tourism Research Australia.
- Split by 7 states, 27 zones and 76 regions (a geographical hierarchy)
- Also split by purpose of travel
 - Holiday
 - Visiting friends and relatives (VFR)
 - Business
 - Other
- 304 disaggregated series



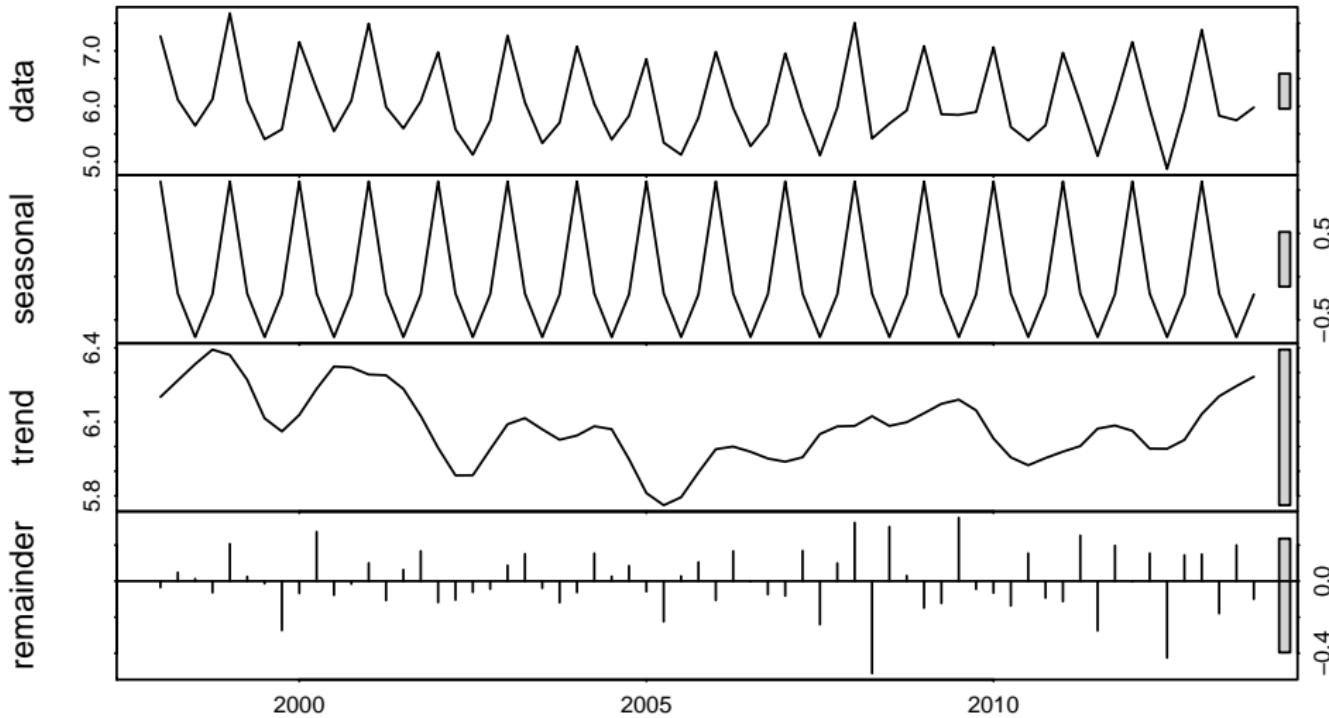
Domestic tourism demand: Victoria



An STL decomposition

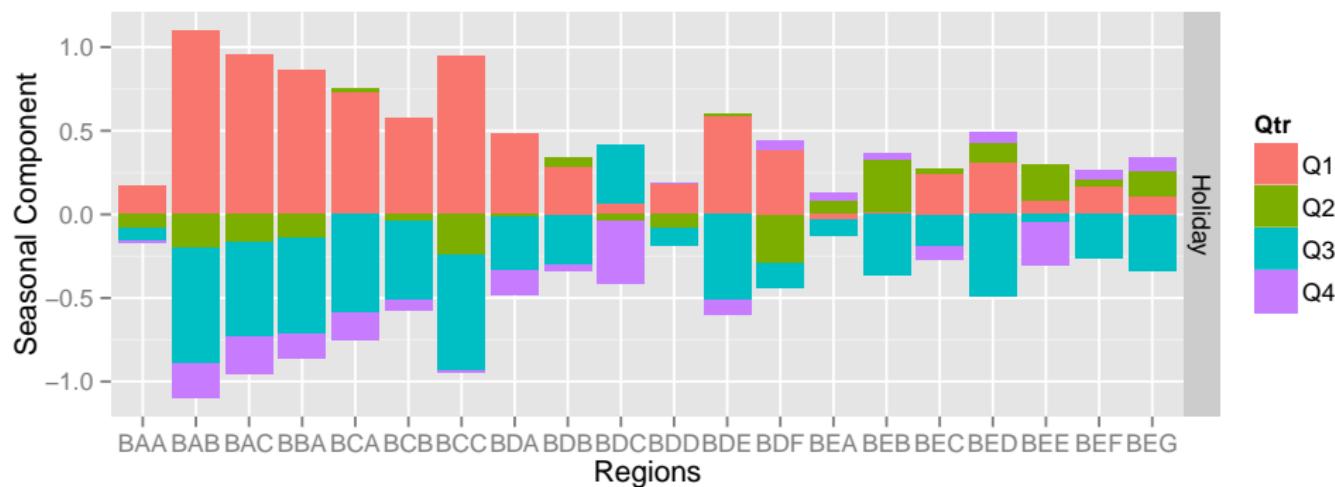
Tourism demand for holidays in Peninsula

$$Y_t = S_t + T_t + R_t \quad S_t \text{ is periodic with mean 0}$$

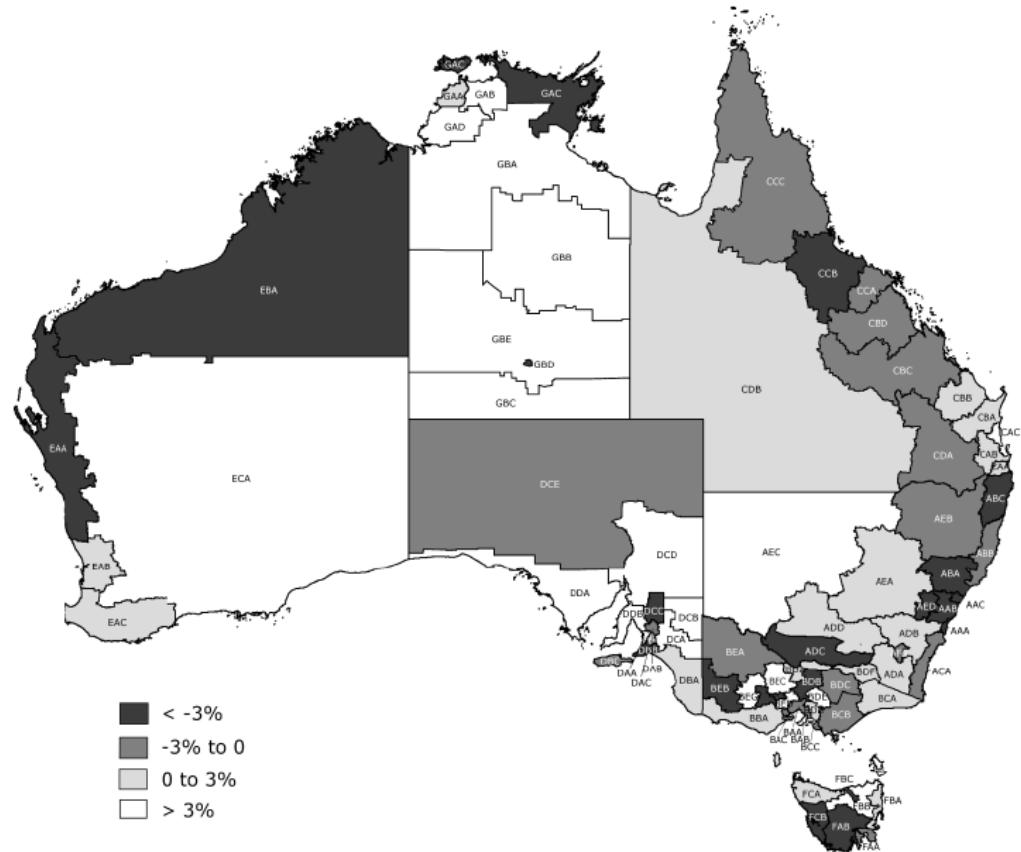


Seasonal stacked bar chart

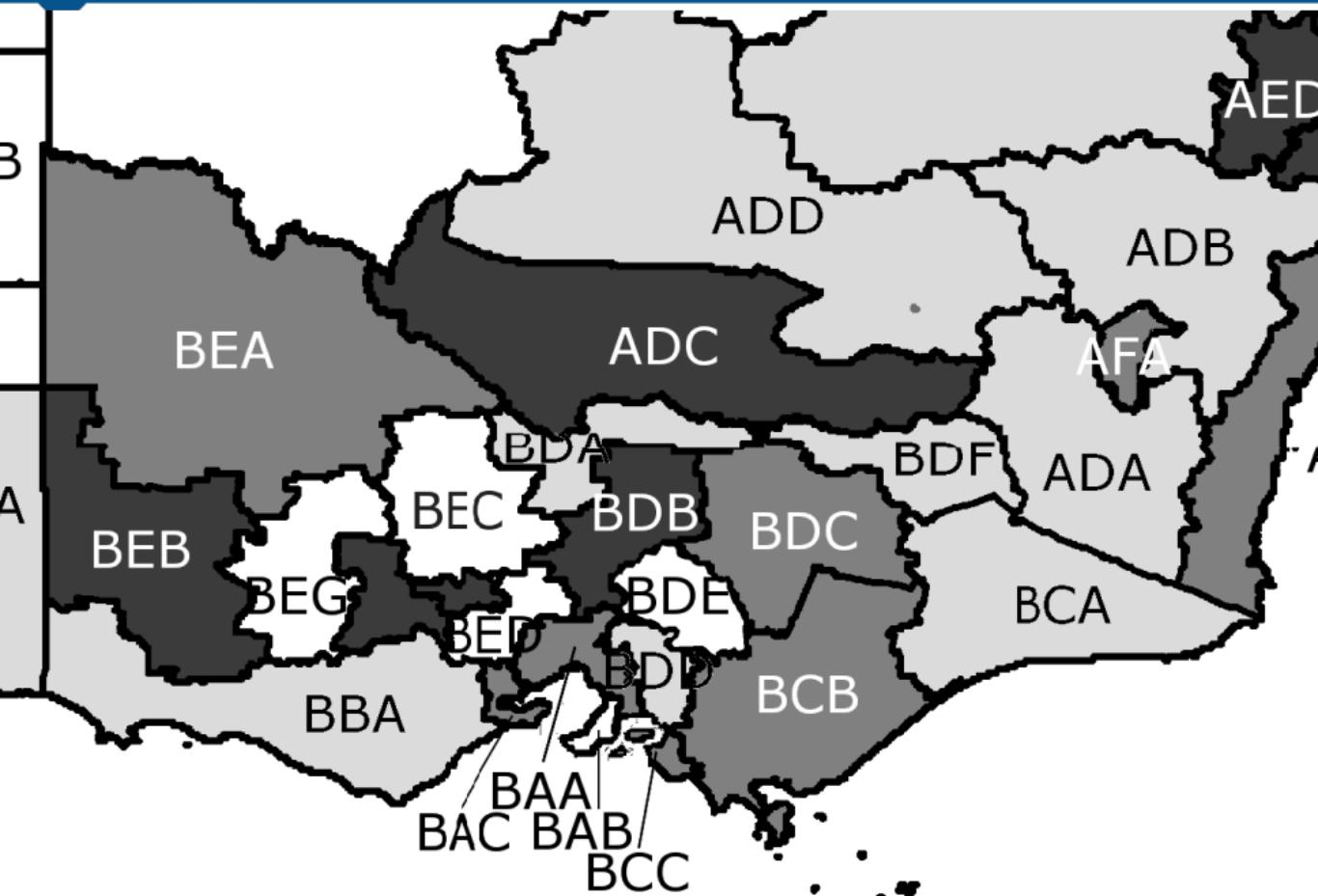
- Place positive values above the origin while negative values below the origin
- Map the bar length to the magnitude
- Encode quarters by colours



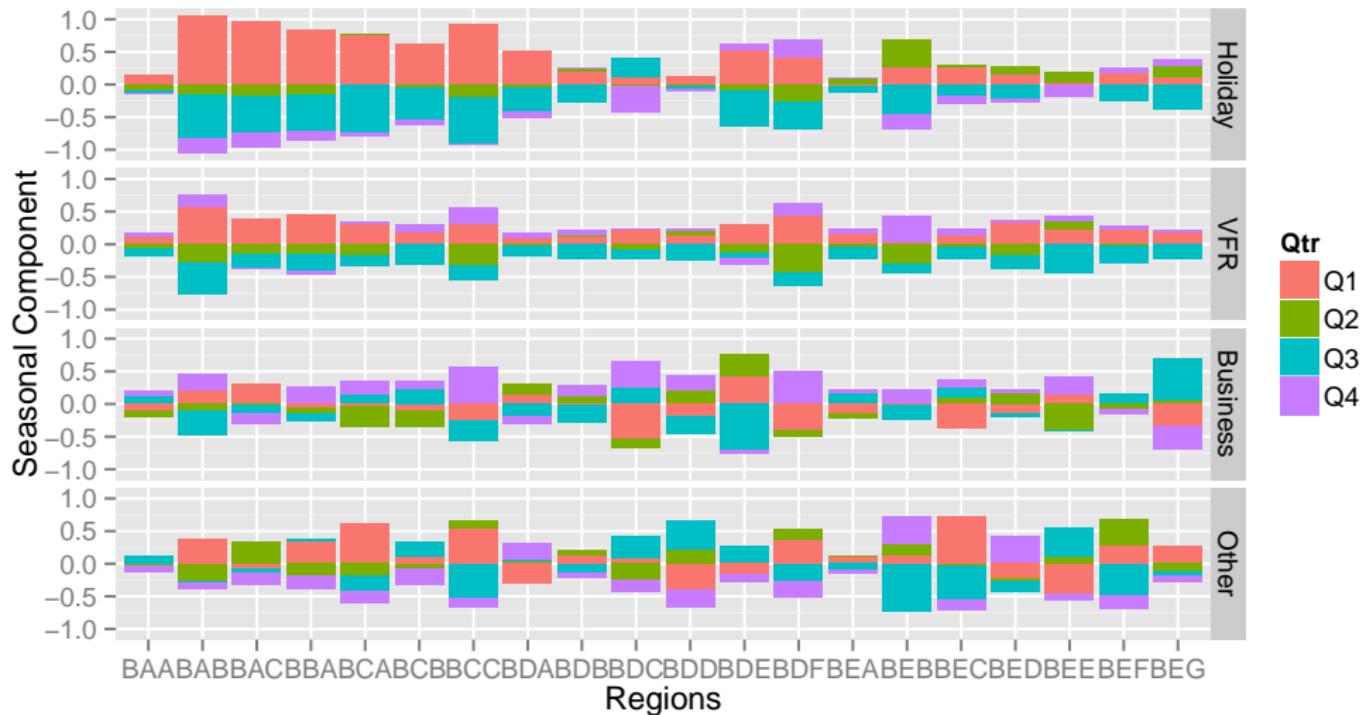
Seasonal stacked bar chart: VIC



Seasonal stacked bar chart: VIC



Seasonal stacked bar chart: VIC



Trend analysis

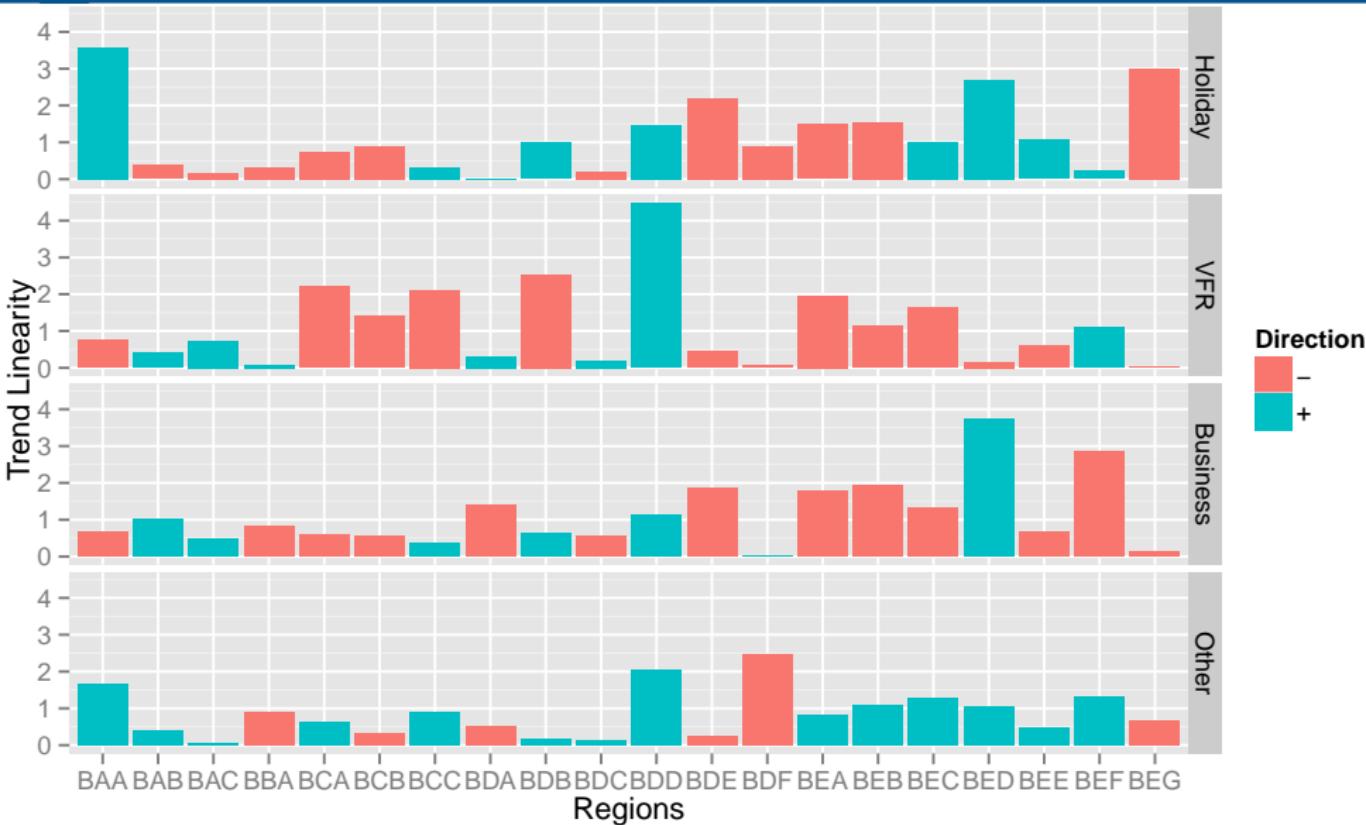
- **Linearity:** the long-term direction and strength of trend.
- **Curvature:** the “changing direction” of trend.
- Estimate by regression:

$$T_t = \hat{\beta}_0 + \hat{\beta}_1 \phi_1(t) + \hat{\beta}_2 \phi_2(t) + e_t$$

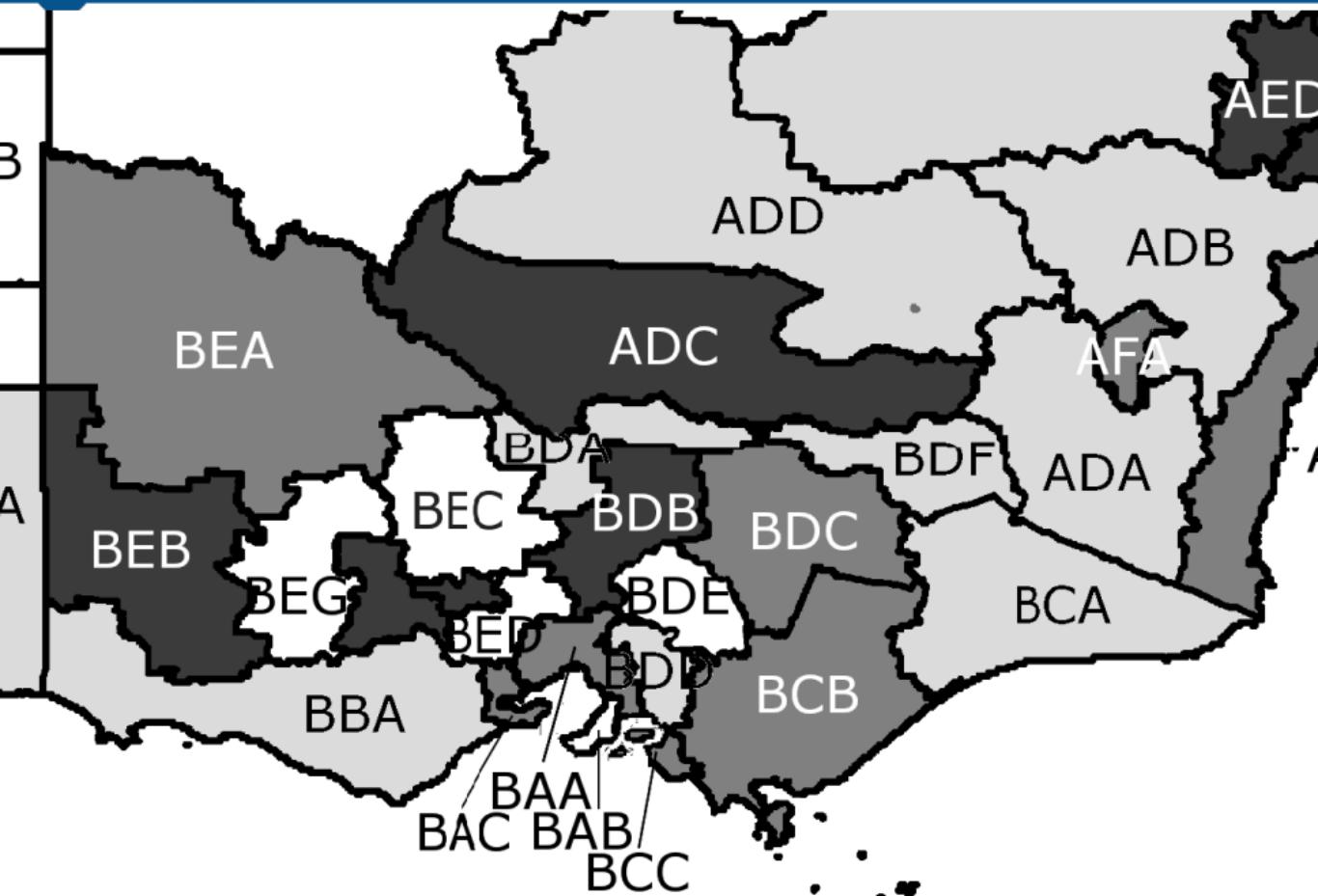
where $\phi_k(t)$ is a k th-degree **orthogonal polynomial** in time t .

- To separate the linearity ($\hat{\beta}_1$) and curvature ($\hat{\beta}_2$).

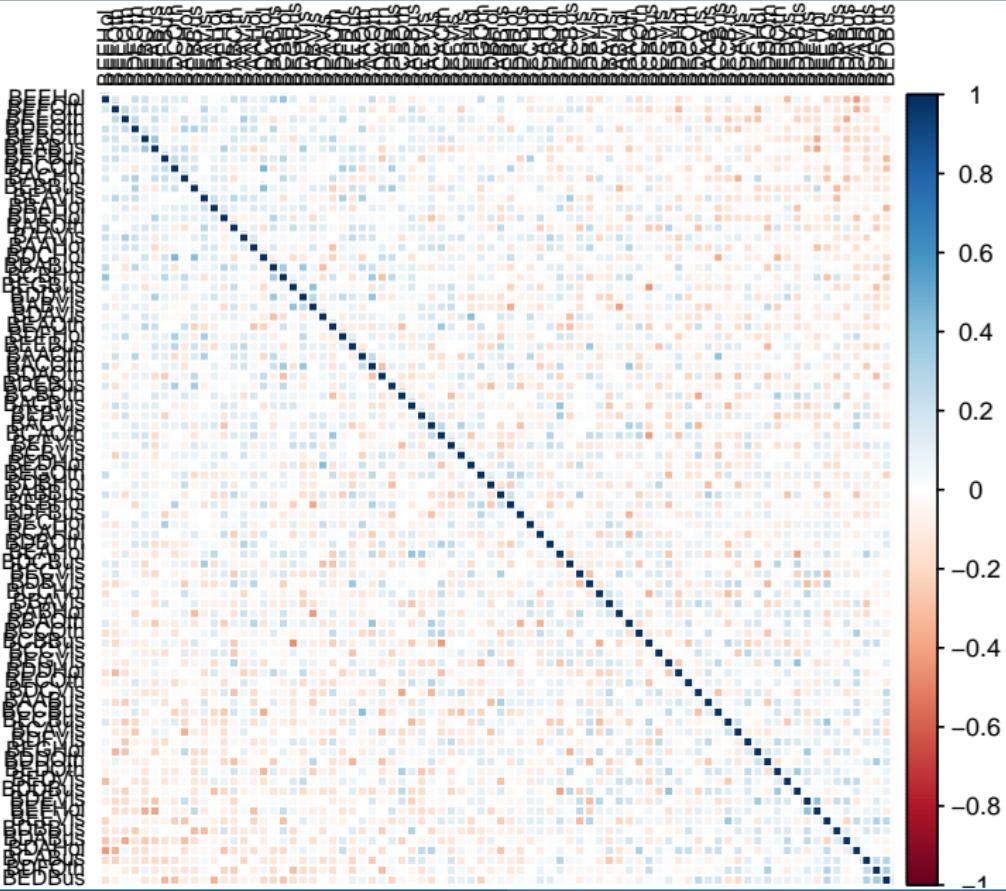
Trend analysis



Trend analysis



Corrrgram of remainder

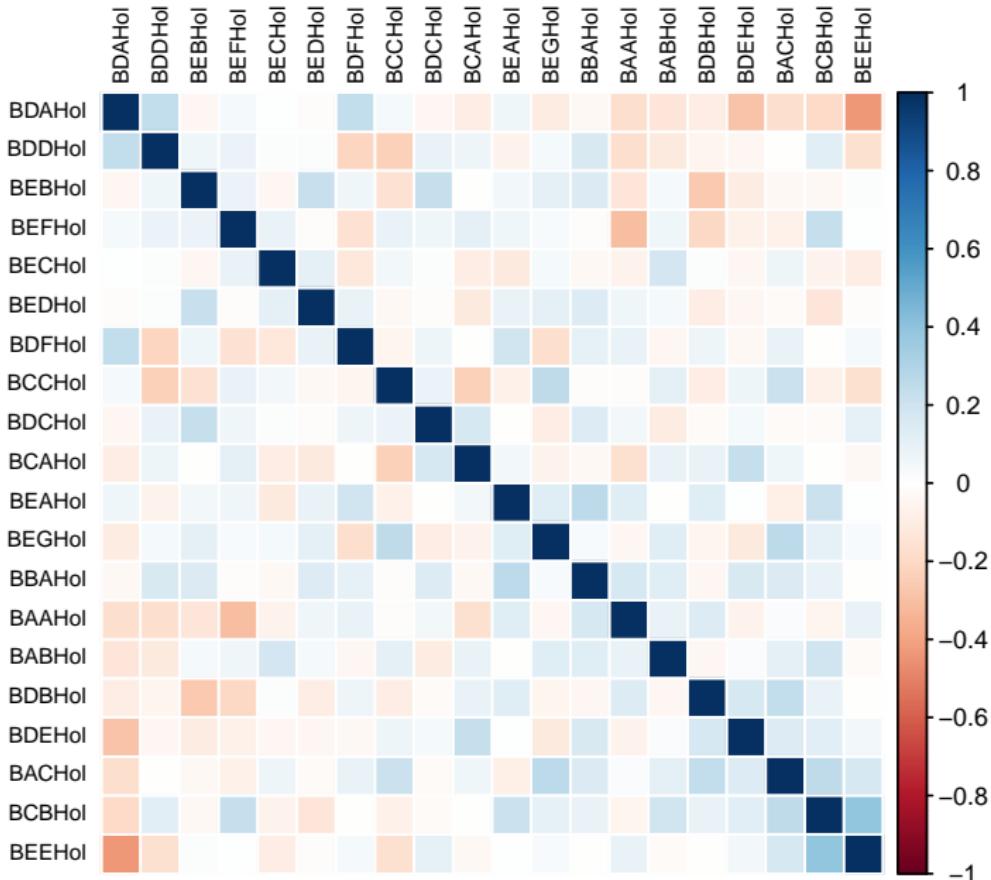


Corrgram of remainder

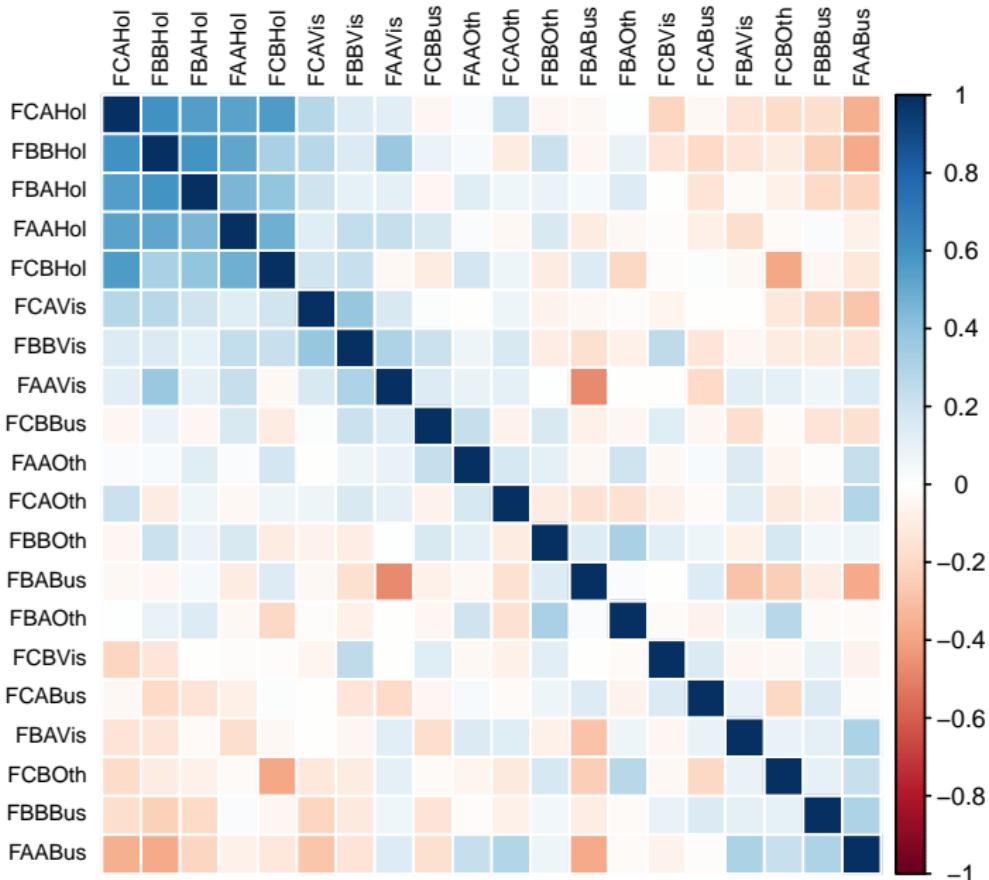
- Compute the correlations among the remainder components
 - Render both the sign and magnitude using a colour mapping of two hues
 - Order variables according to the first principal component of the correlations.



Corrgram of remainder



Corrgram of remainder: TAS



Outline

1 The problem

2 Australian tourism demand

3 M3 competition data

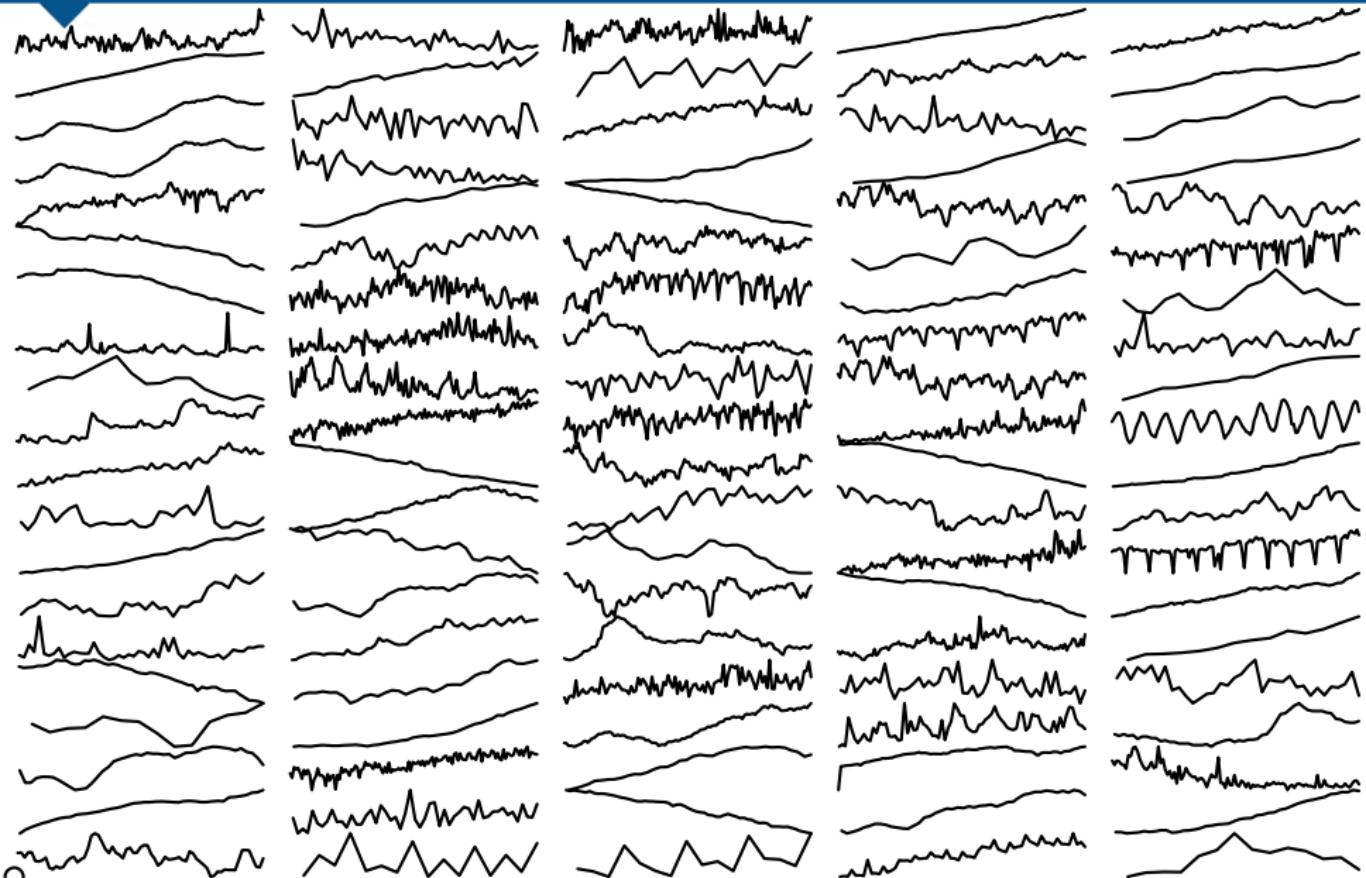
4 Yahoo web traffic

5 What next?

M3 forecasting competition

- 3003 series
- All data from business, demography, finance and economics.
- Series length between 14 and 126.
- Either non-seasonal, monthly or quarterly.
- All time series positive.

M3 forecasting competition



Candidate features

STL decomposition

$$Y_t = S_t + T_t + R_t$$

- Seasonal period
 - Strength of seasonality: $1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - T_t)}$
 - Strength of trend: $1 - \frac{\text{Var}(T_t)}{\text{Var}(Y_t - R_t)}$
- Strength of seasonal component = $\sqrt{\frac{\sum_{t=1}^T (Y_t - \hat{Y}_t)^2}{\sum_{t=1}^T (\hat{Y}_t - \bar{Y})^2}}$
- Strength of trend component = $\sqrt{\frac{\sum_{t=1}^T (Y_t - \hat{Y}_t)^2}{\sum_{t=1}^T (\hat{Y}_t - \bar{Y})^2}}$
- Strength of residual component = $\sqrt{\frac{\sum_{t=1}^T (Y_t - \hat{Y}_t)^2}{\sum_{t=1}^T (Y_t - \bar{Y})^2}}$

Candidate features

STL decomposition

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- Spectral entropy: $H = - \int_{-\pi}^{\pi} f_y(\lambda) \log f_y(\lambda) d\lambda$,
where $f_y(\lambda)$ is spectral density of Y_t .
Low values of H suggest a time series that is easier to forecast (more signal).
- Autocorrelations: r_1, r_2, r_3, \dots
- Optimal Box-Cox transformation parameter λ

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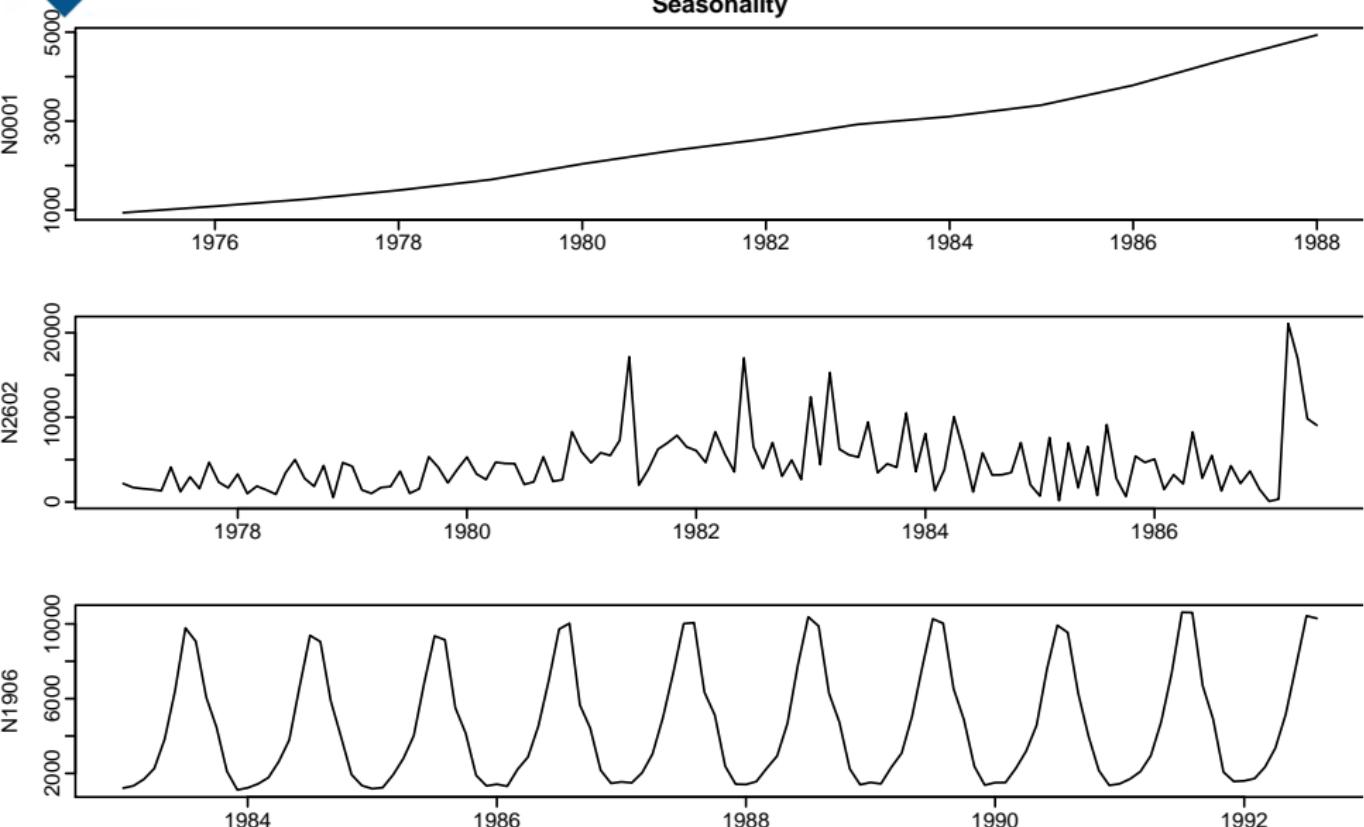
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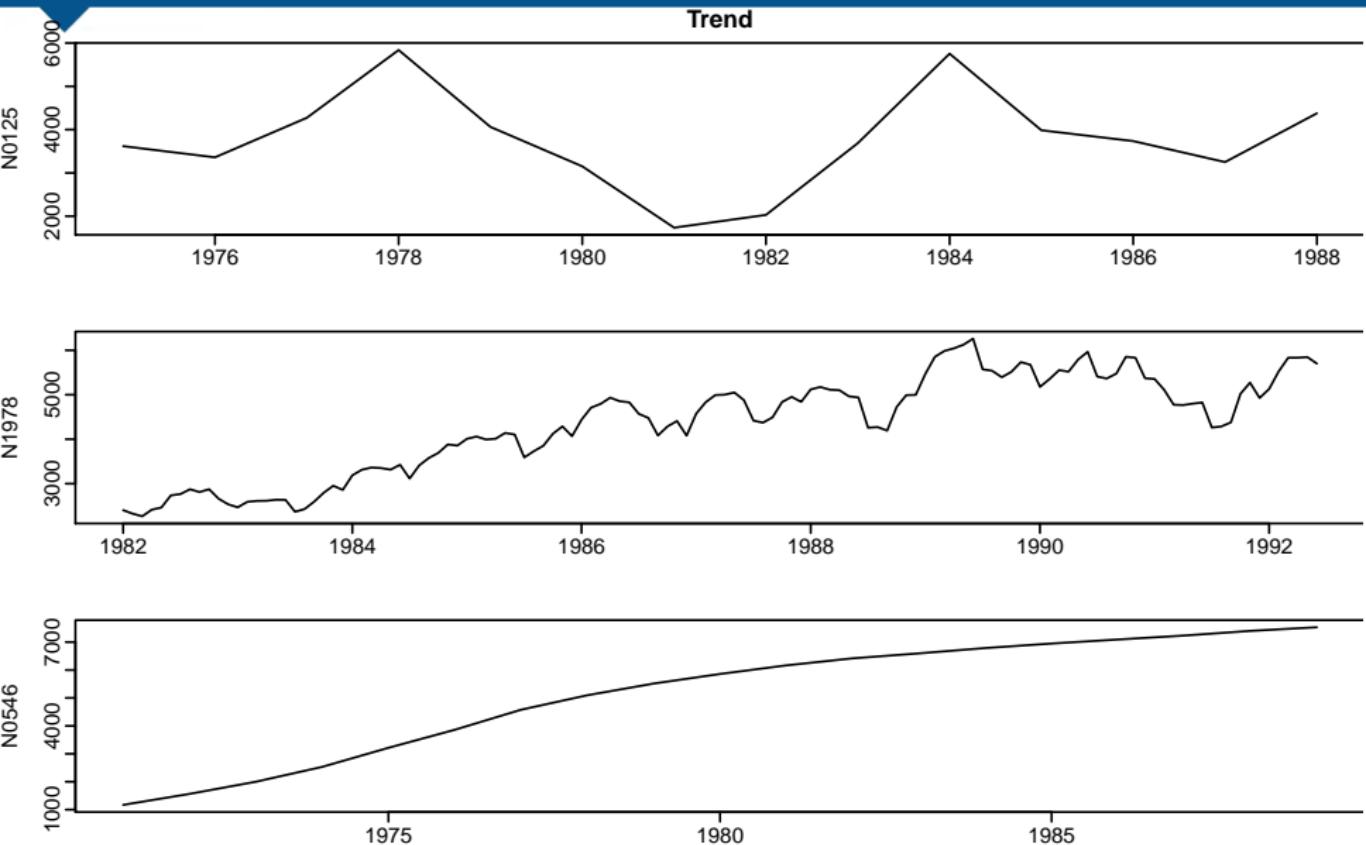
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Candidate features

Seasonality

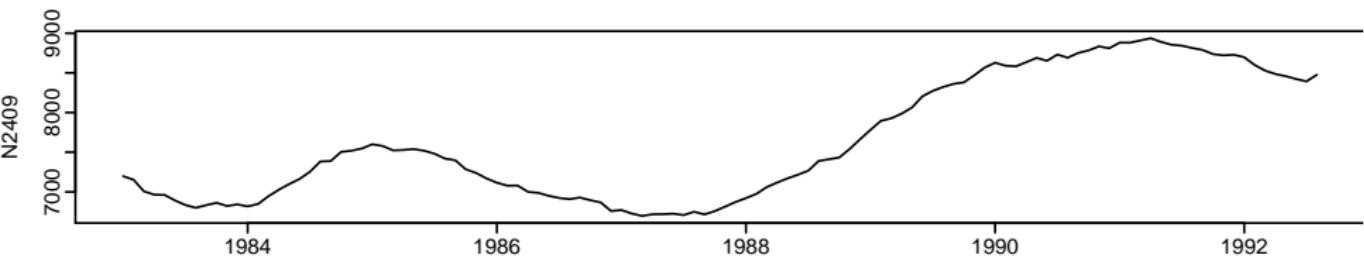
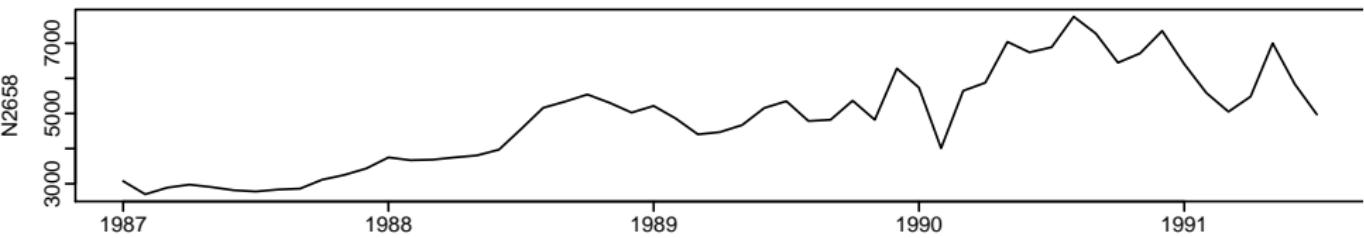
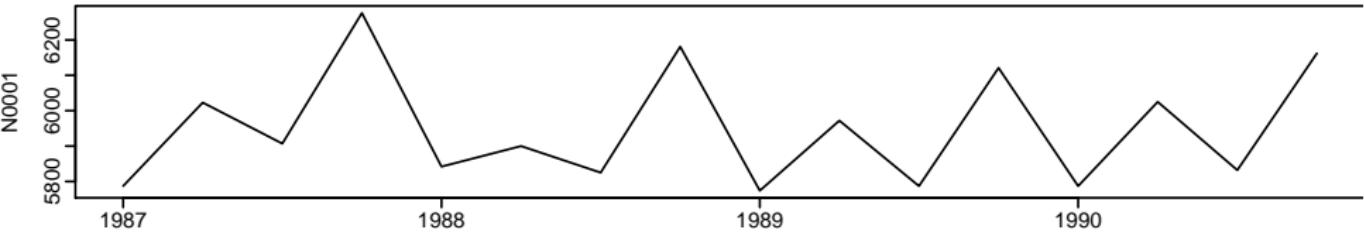


Candidate features



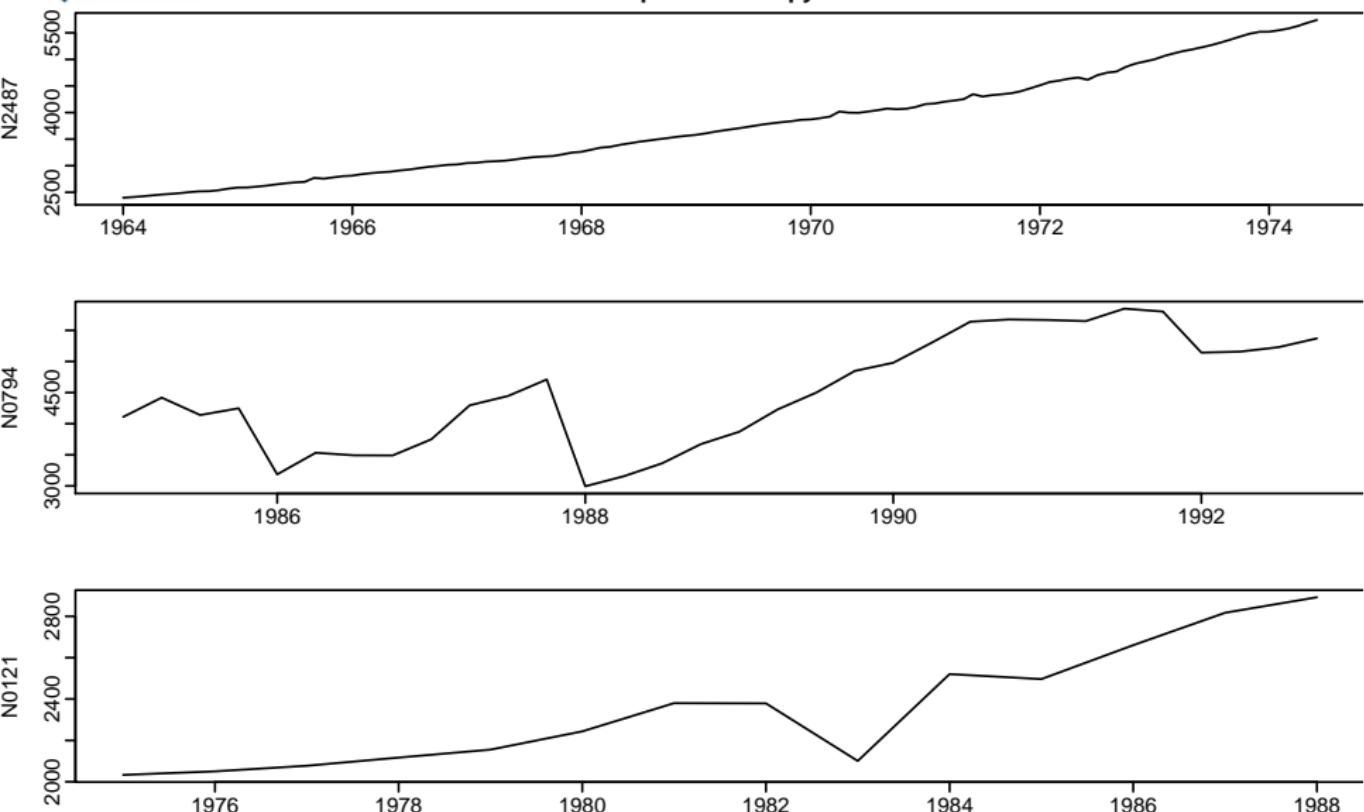
Candidate features

ACF1



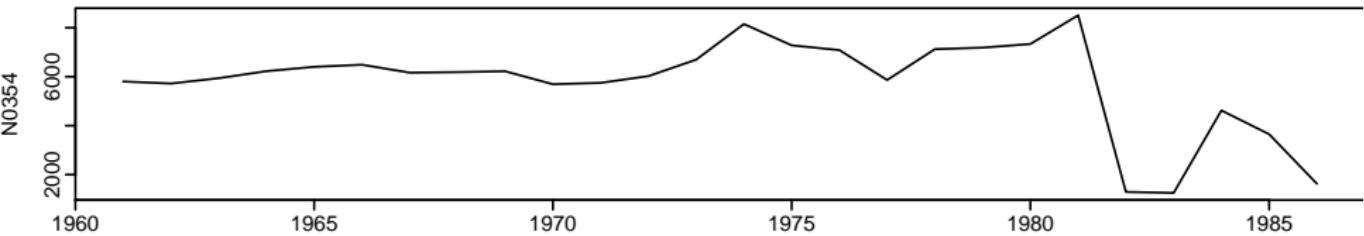
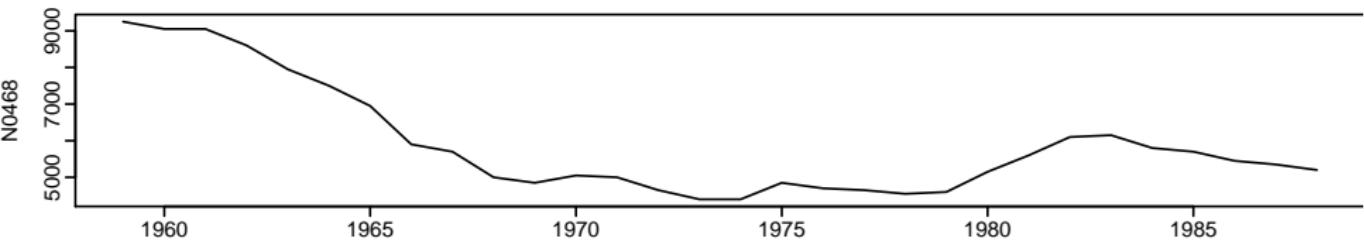
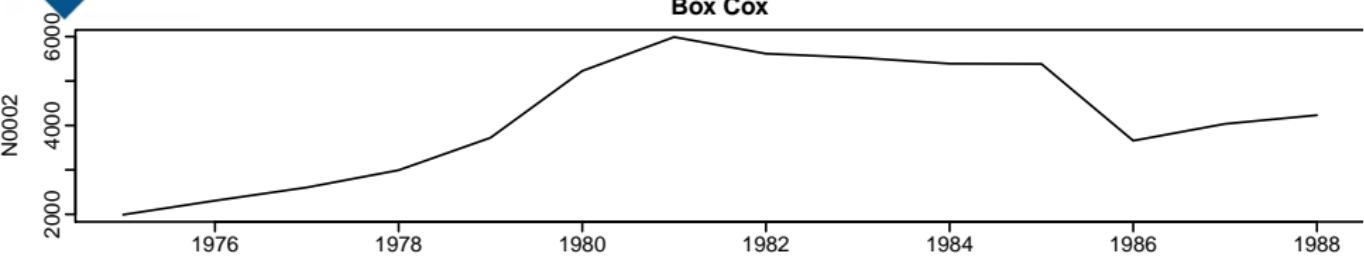
Candidate features

Spectral entropy

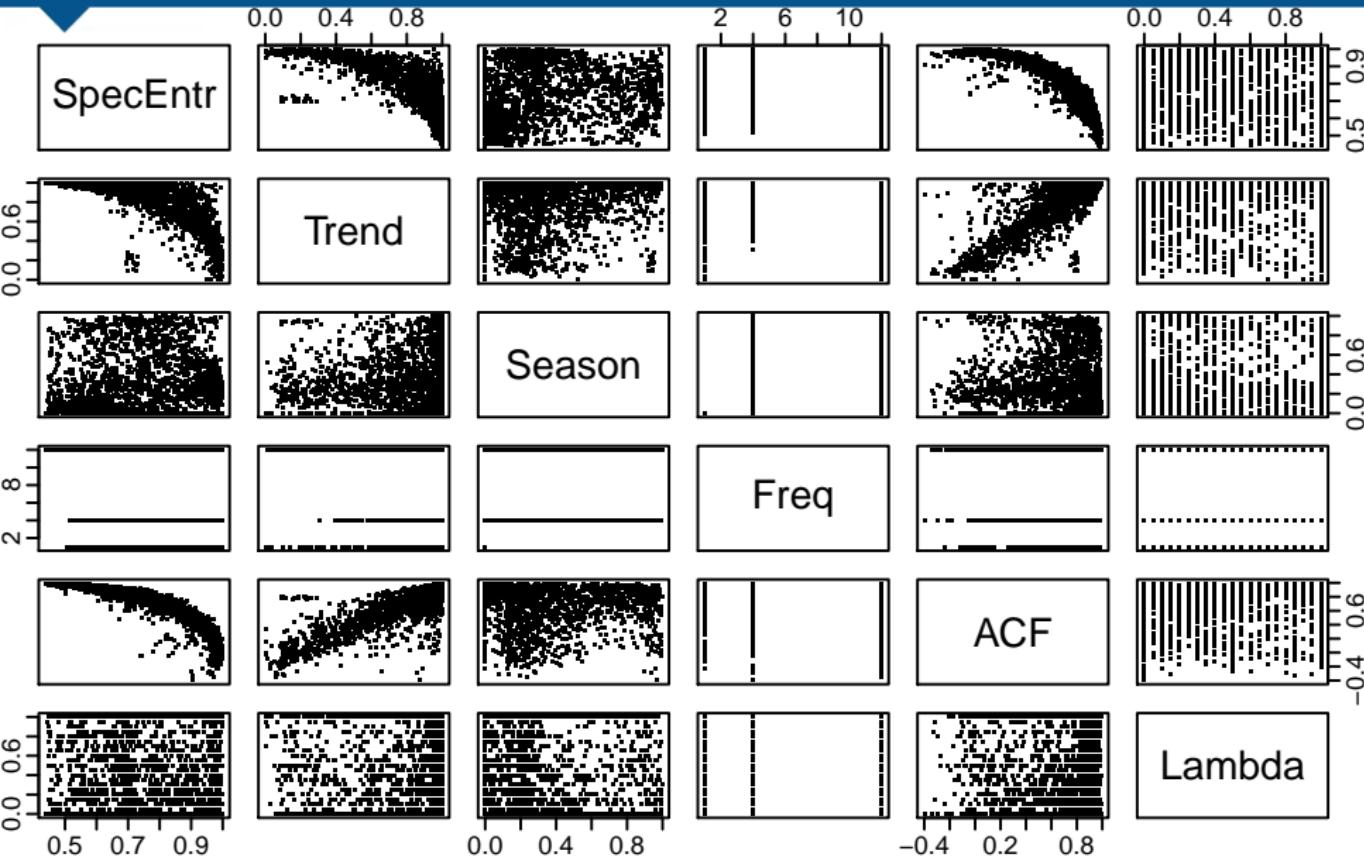


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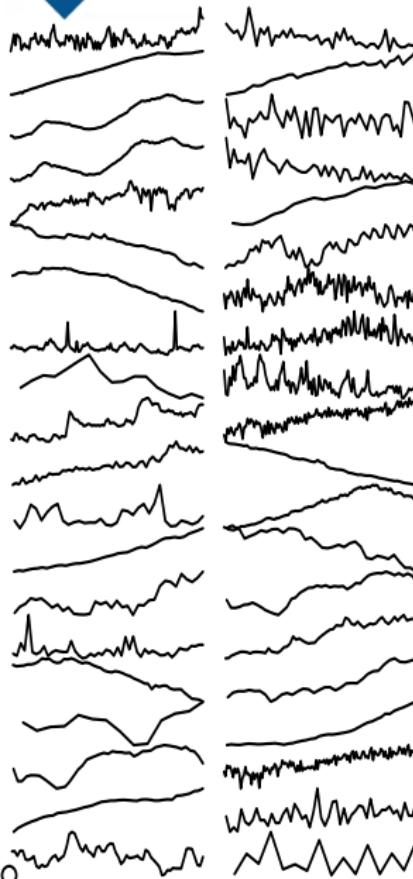
Box Cox



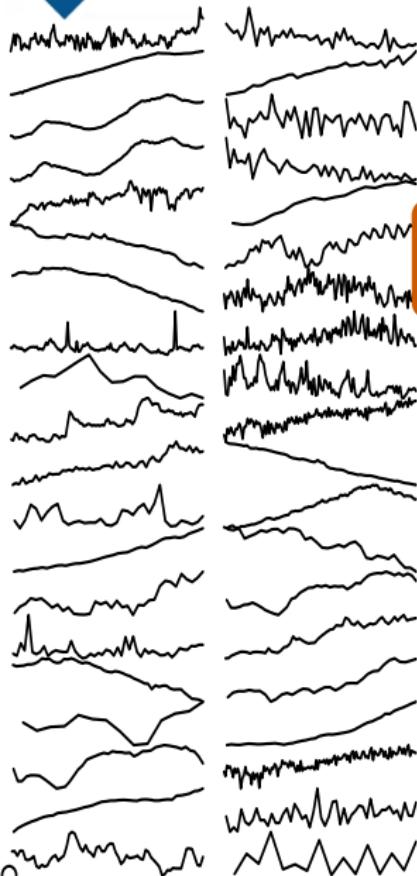
Candidate features



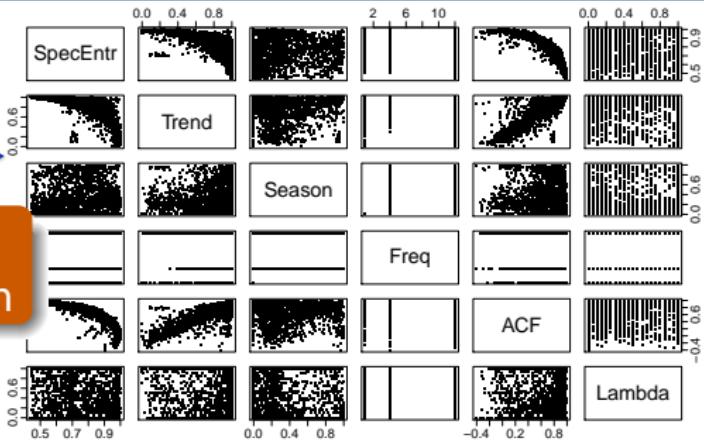
Dimension reduction for time series



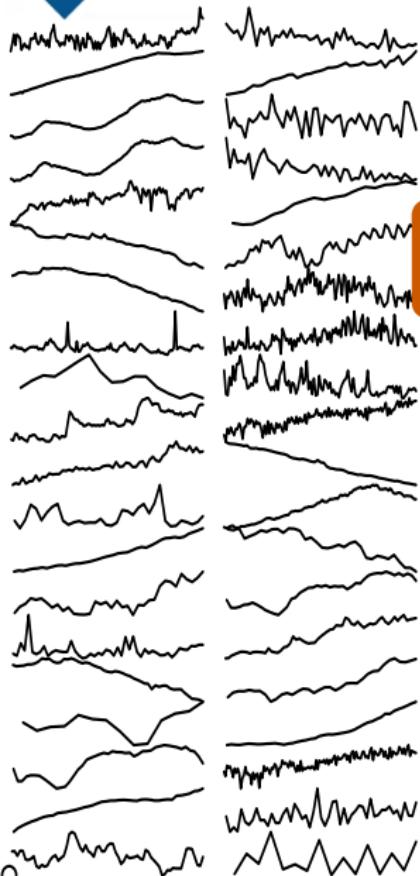
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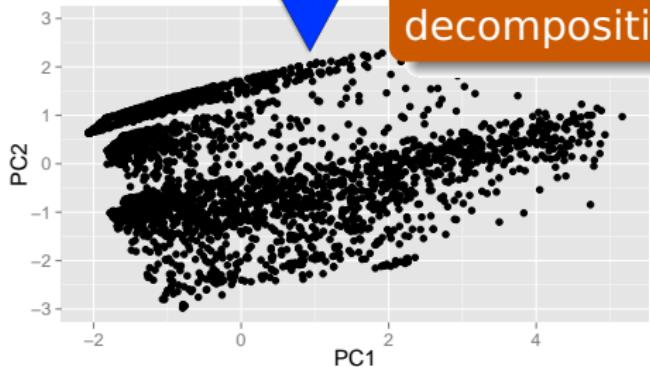
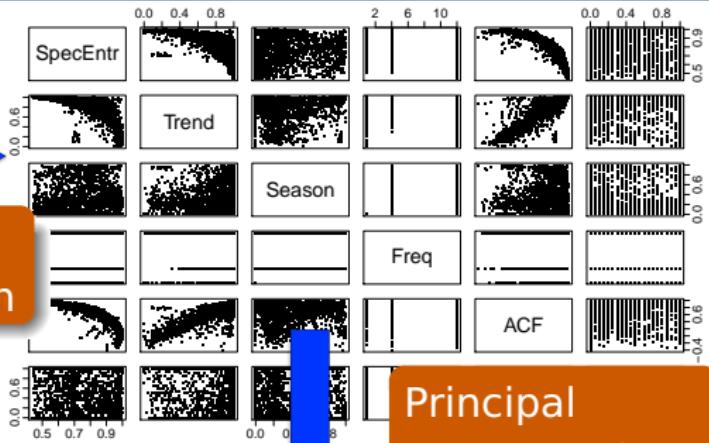
Feature
calculation



Dimension reduction for time series



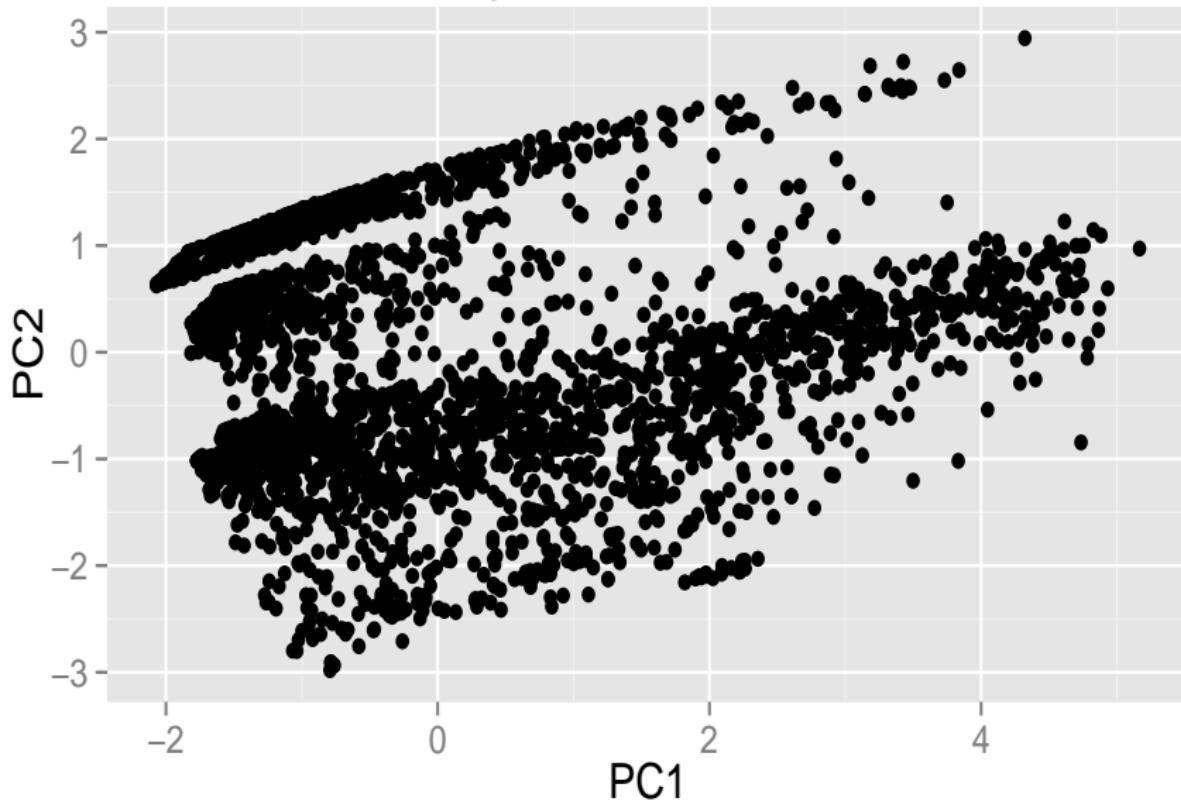
Feature
calculation



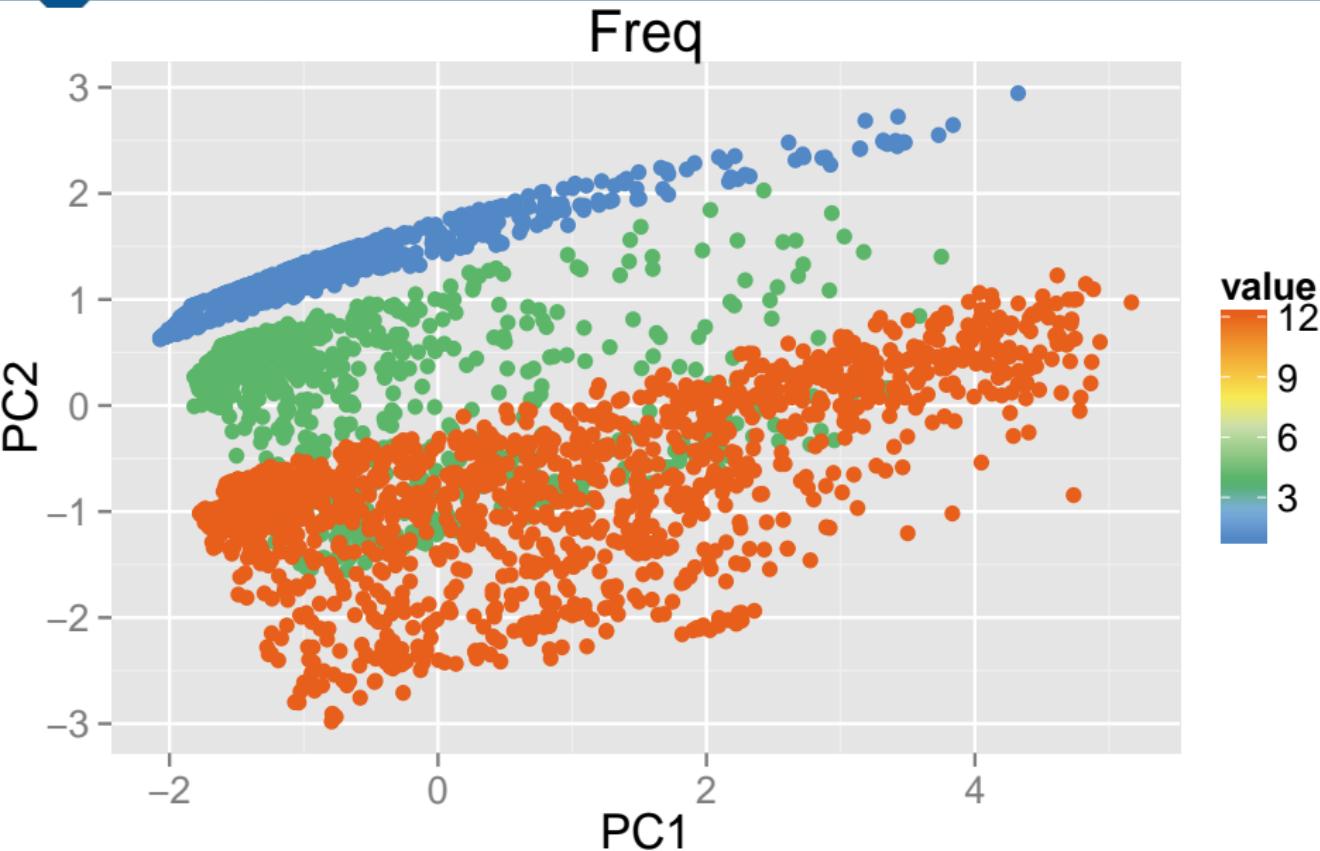
Principal
component
decomposition

Feature space of M3 data

First two PCs explain 68% of variation.

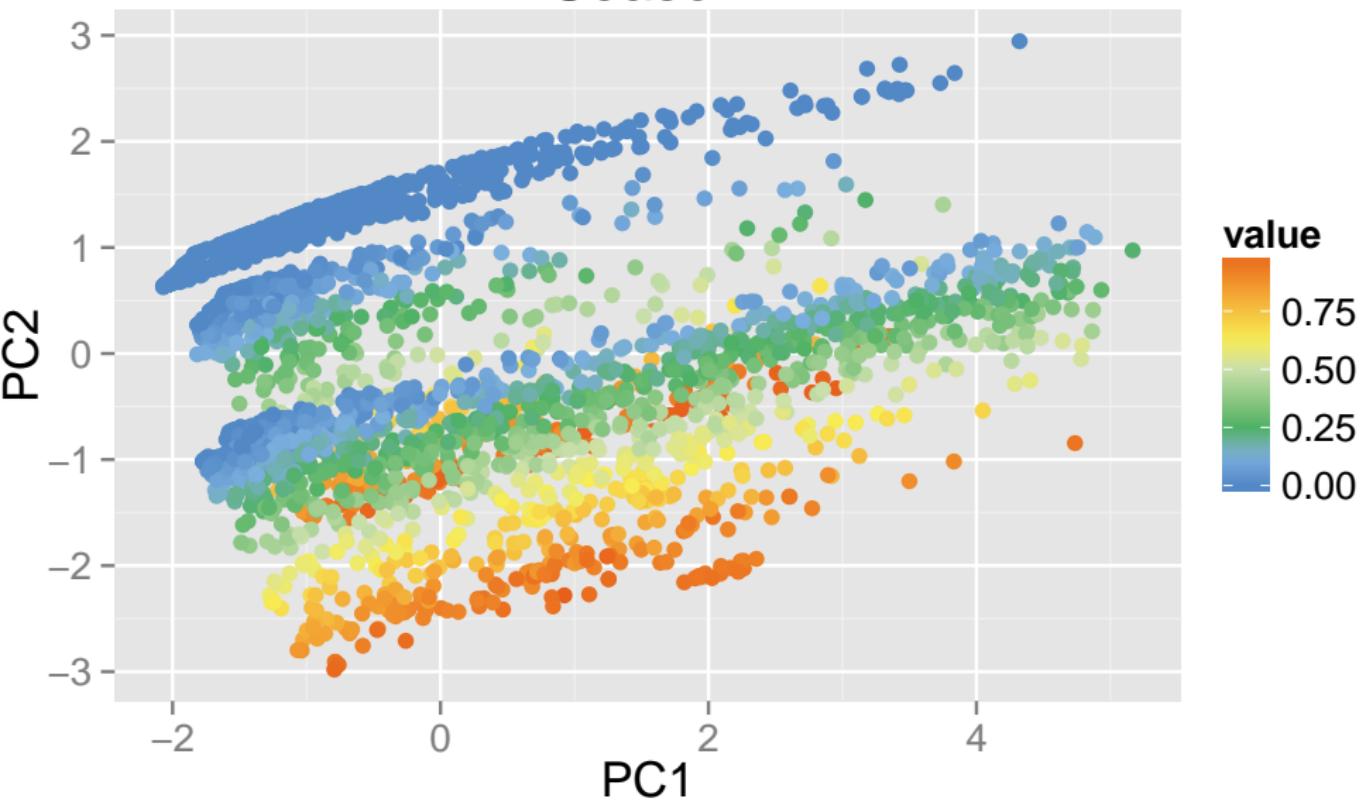


Feature space of M3 data



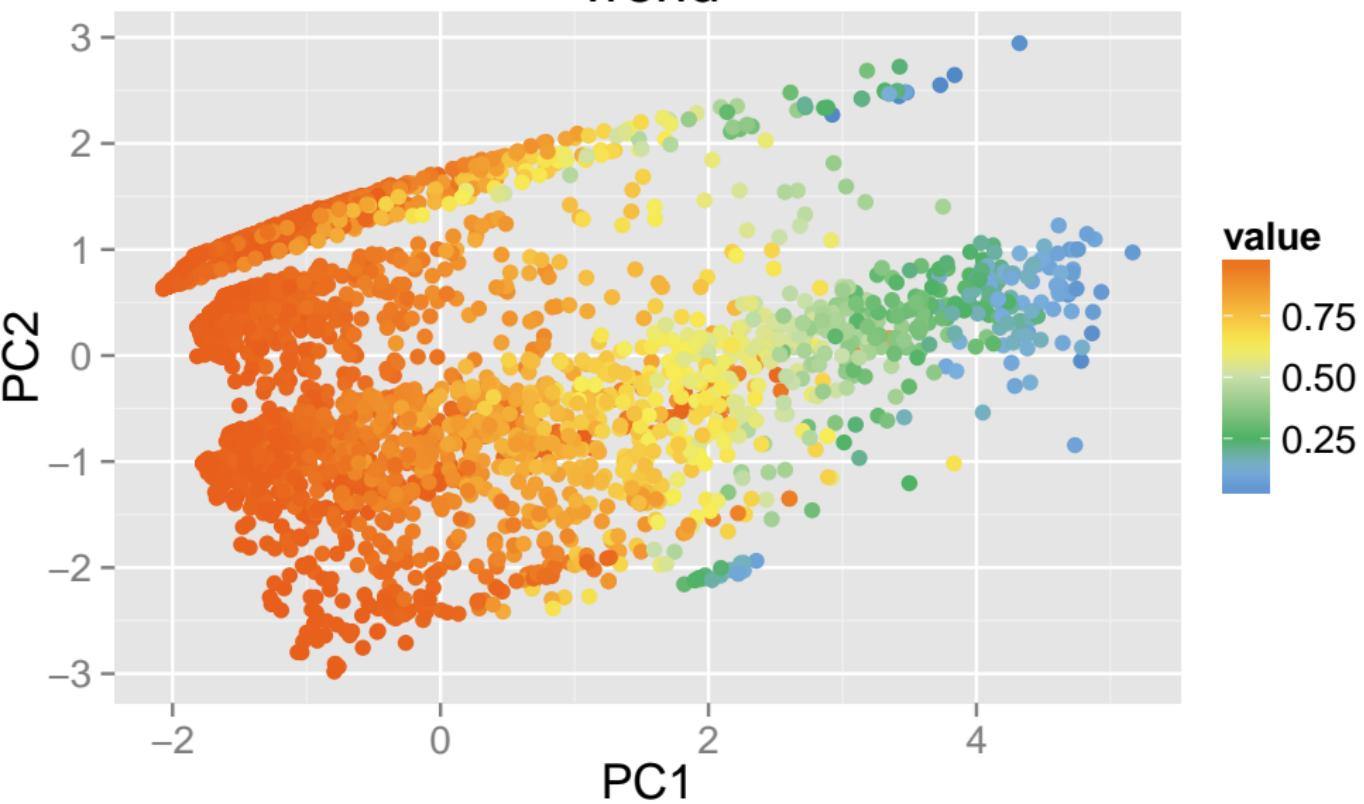
Feature space of M3 data

Season



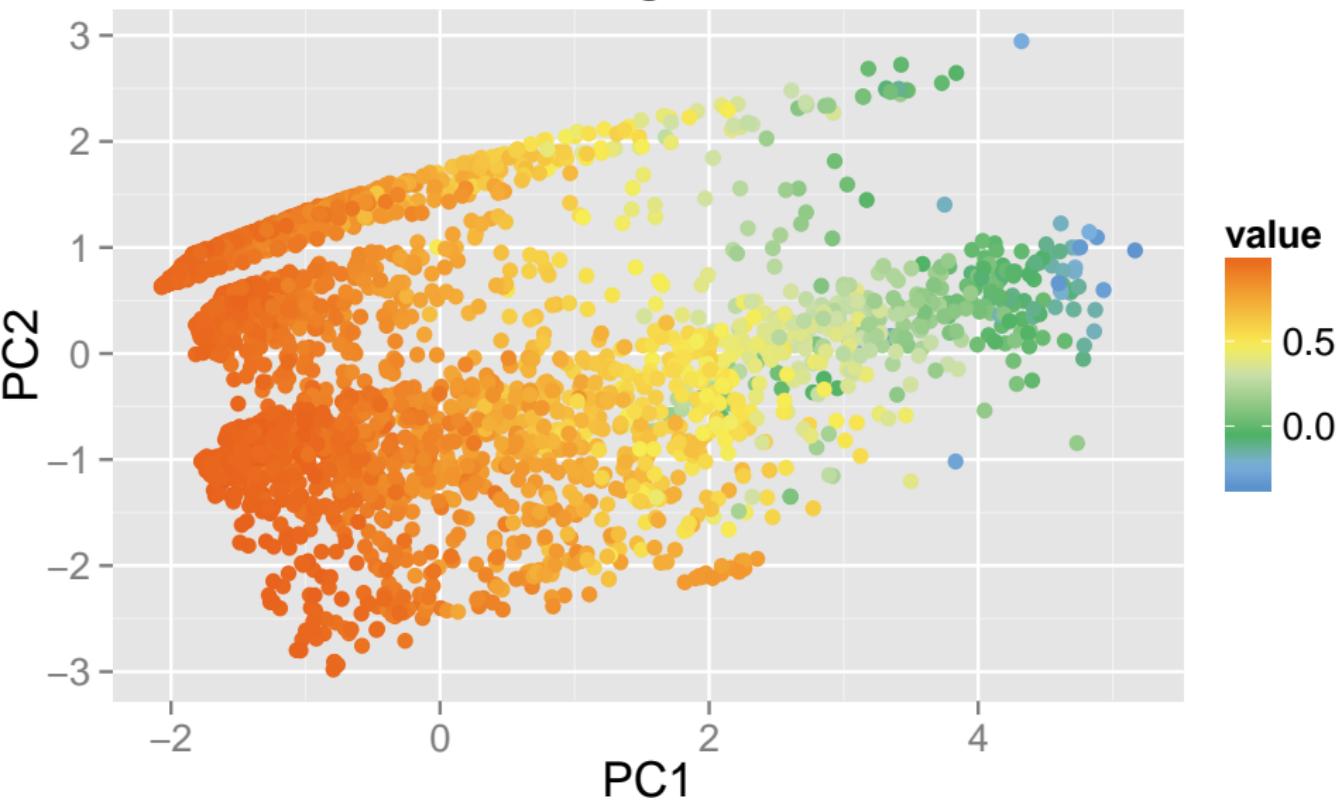
Feature space of M3 data

Trend



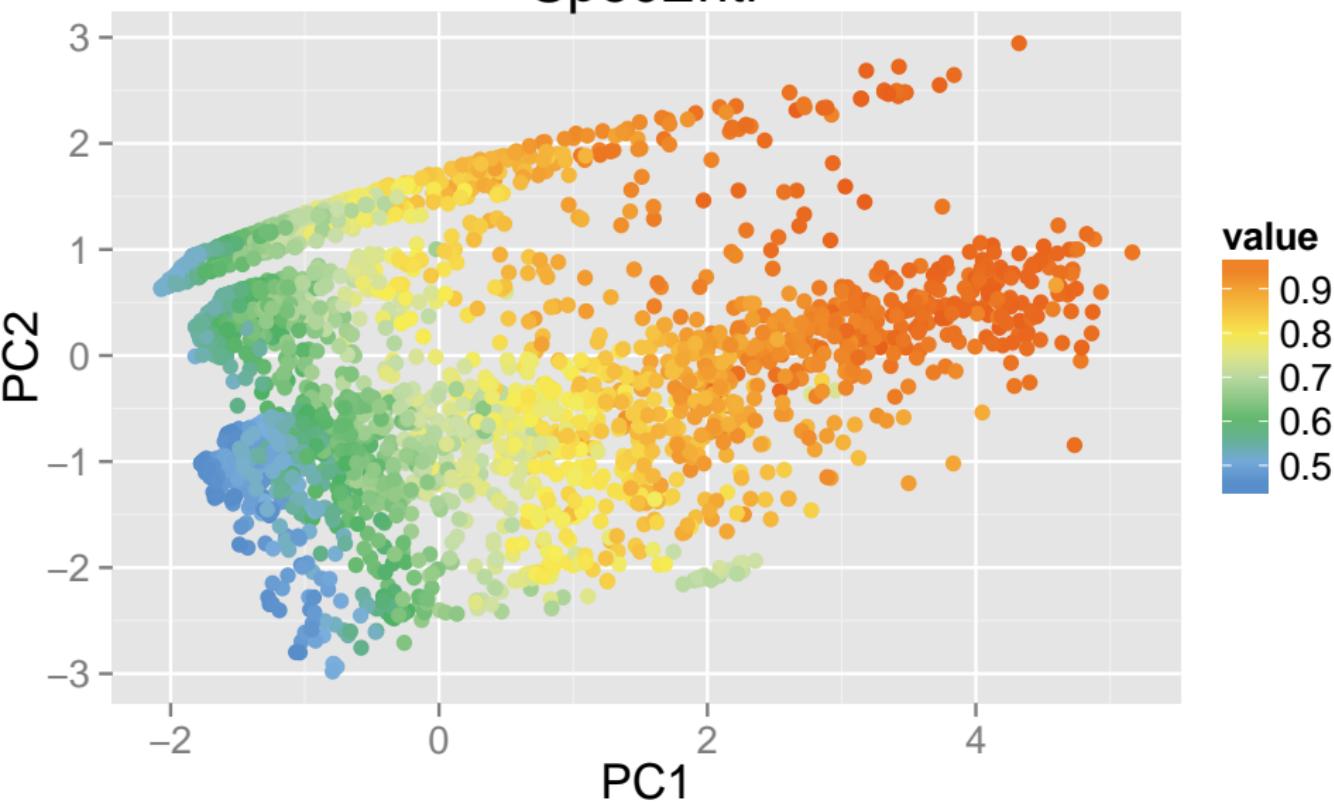
Feature space of M3 data

ACF



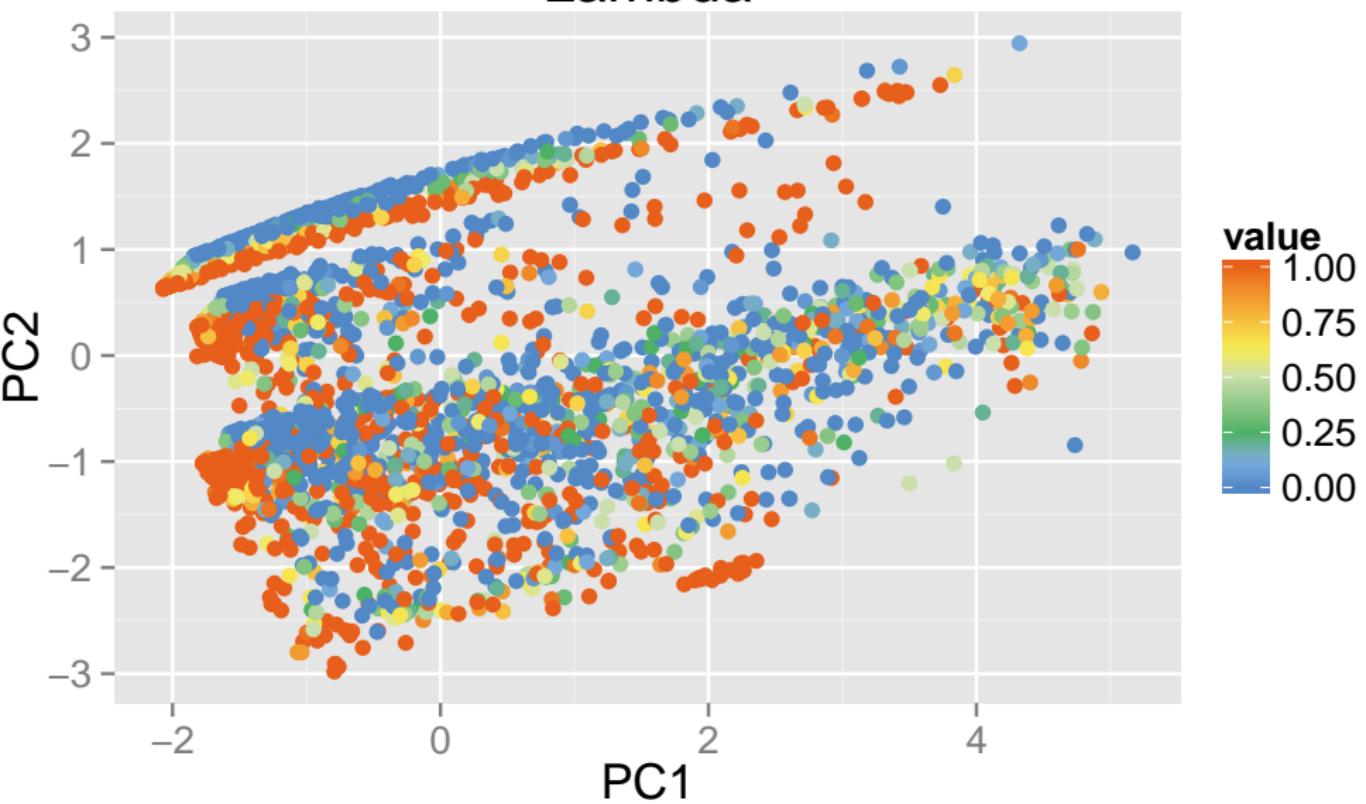
Feature space of M3 data

SpecEntr



Feature space of M3 data

Lambda



Predictability

Three general forecasting methods:

- | | |
|---------------------|---|
| Theta method | Best overall in 2000 M3 competition |
| ETS | Exponential smoothing state space models |
| STL-AR | AR model applied to seasonally adjusted series from STL, and seasonal component forecast using the seasonal naive method. |

Compute minimum MASE from all three methods

Predictability

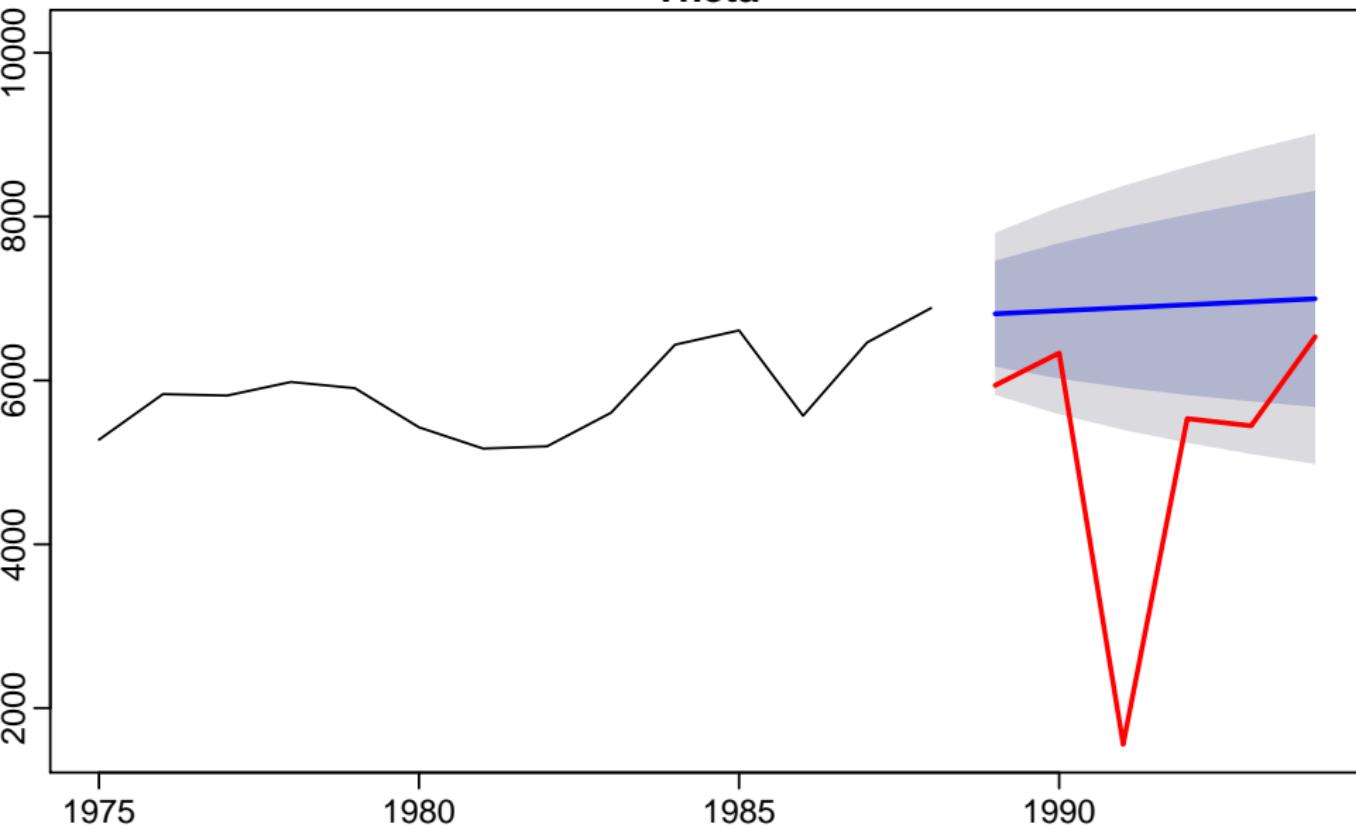
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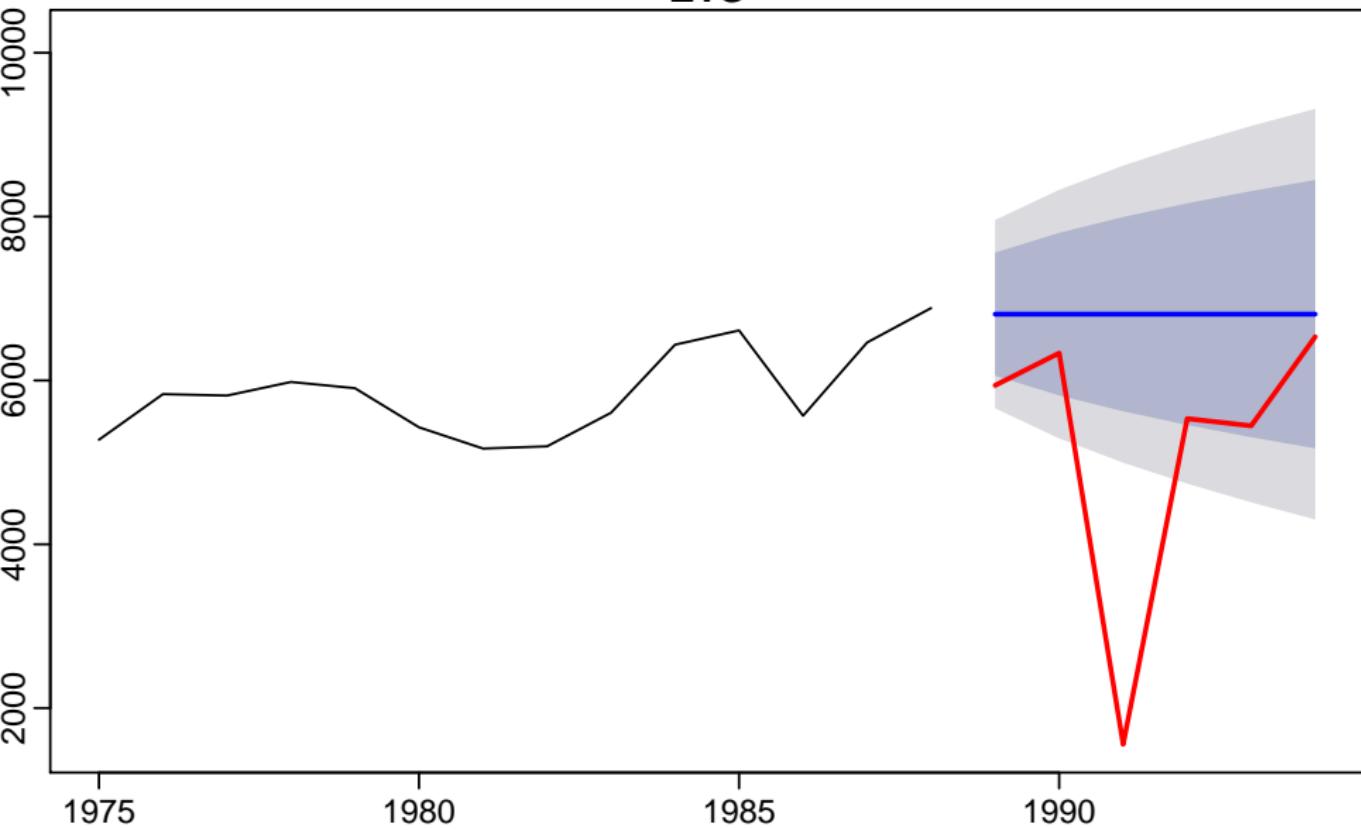
Predictability

Theta



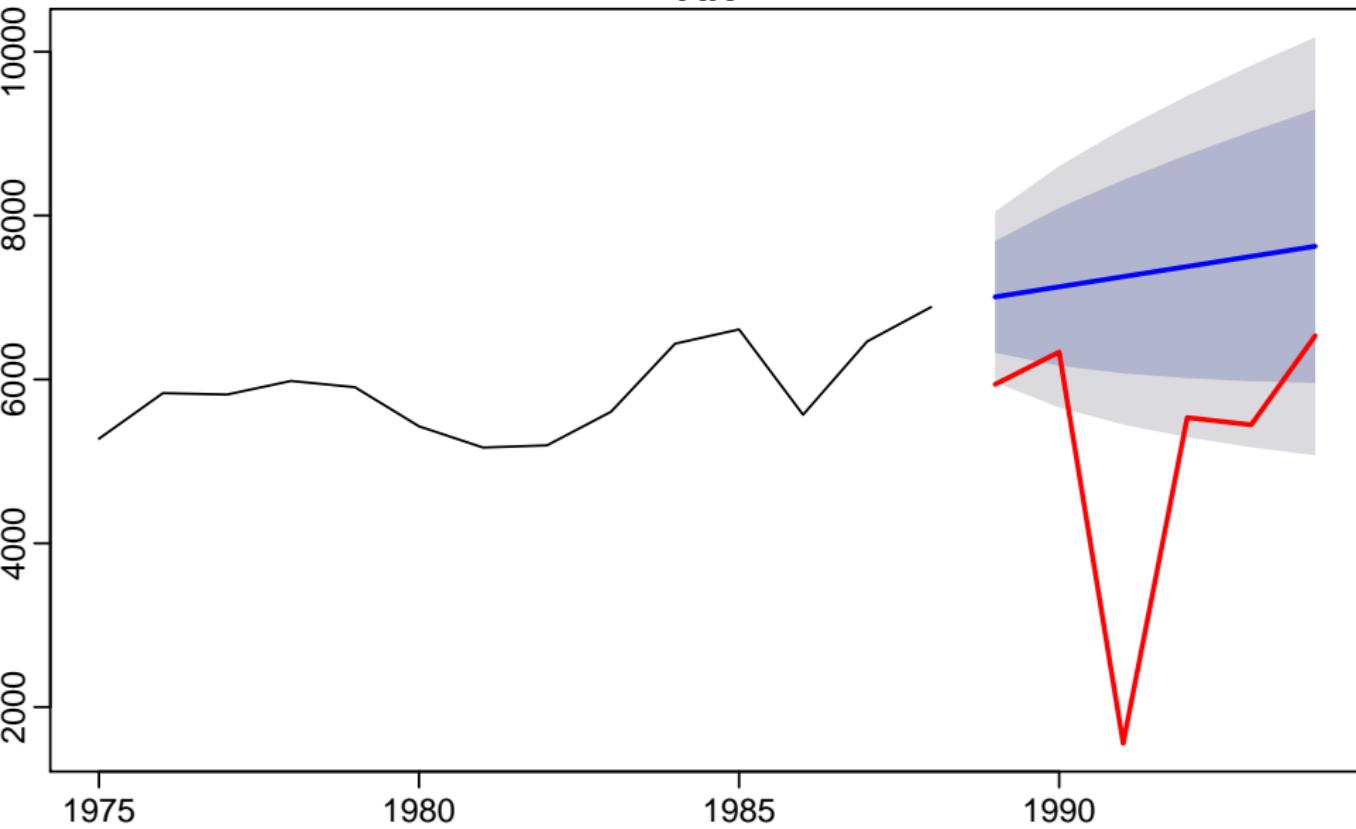
Predictability

ETS



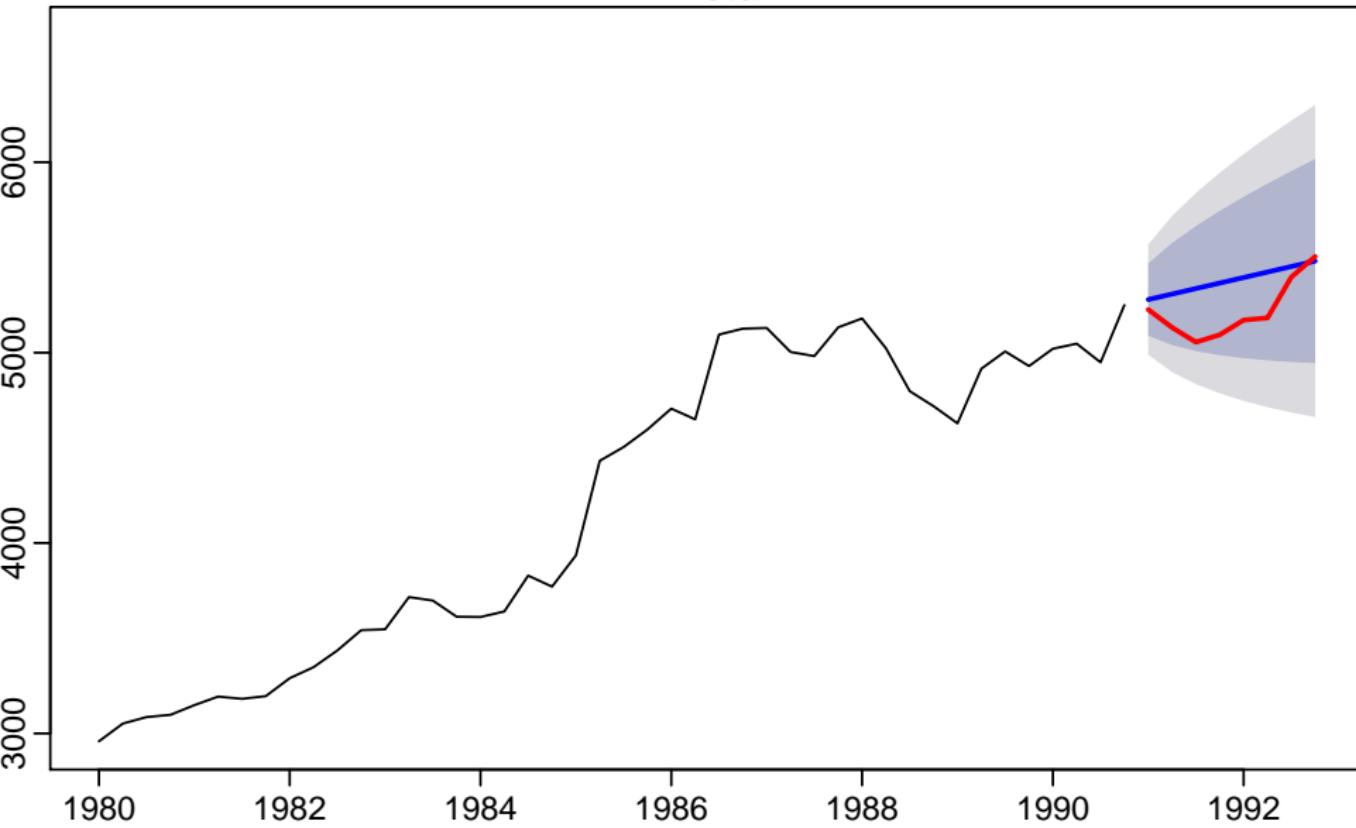
Predictability

AR



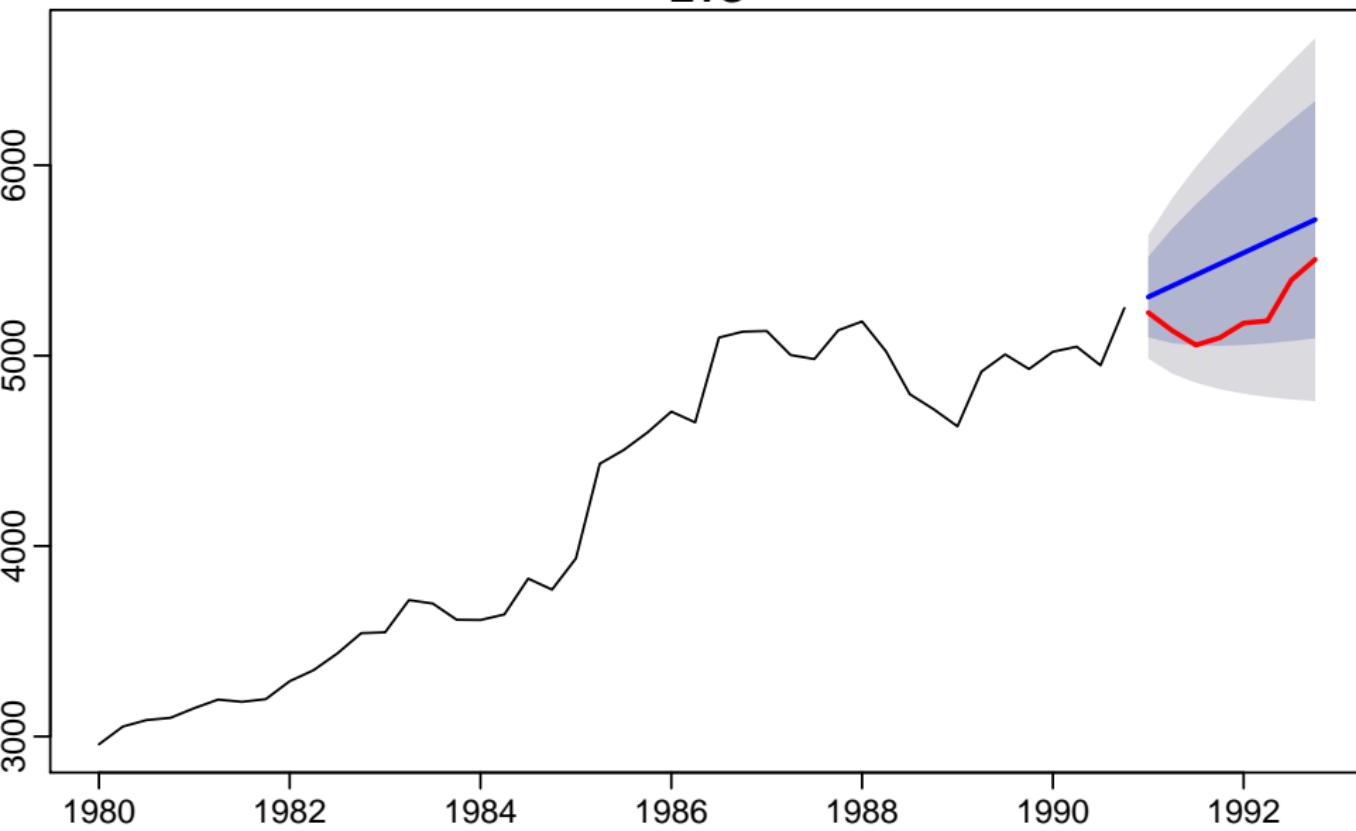
Predictability

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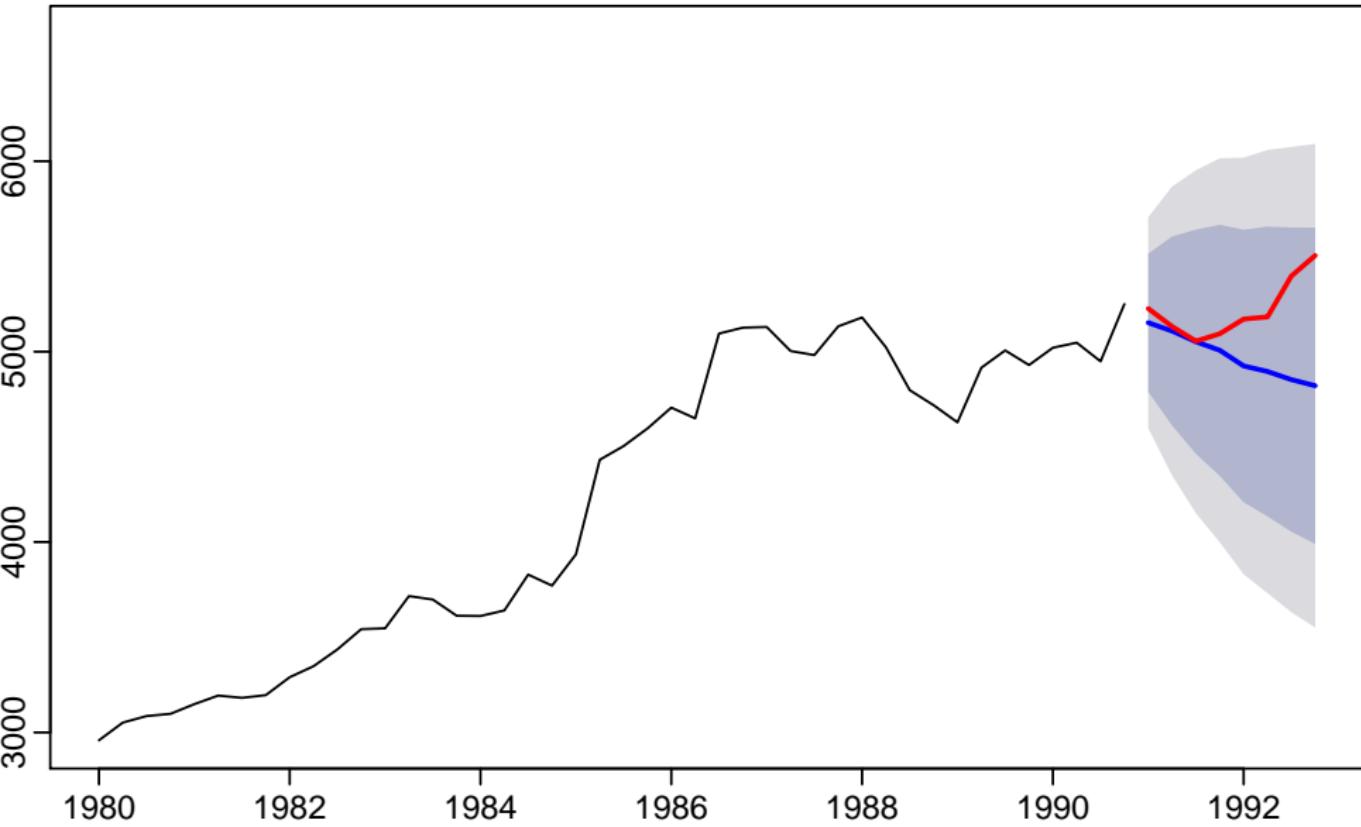
Predictability

ETS



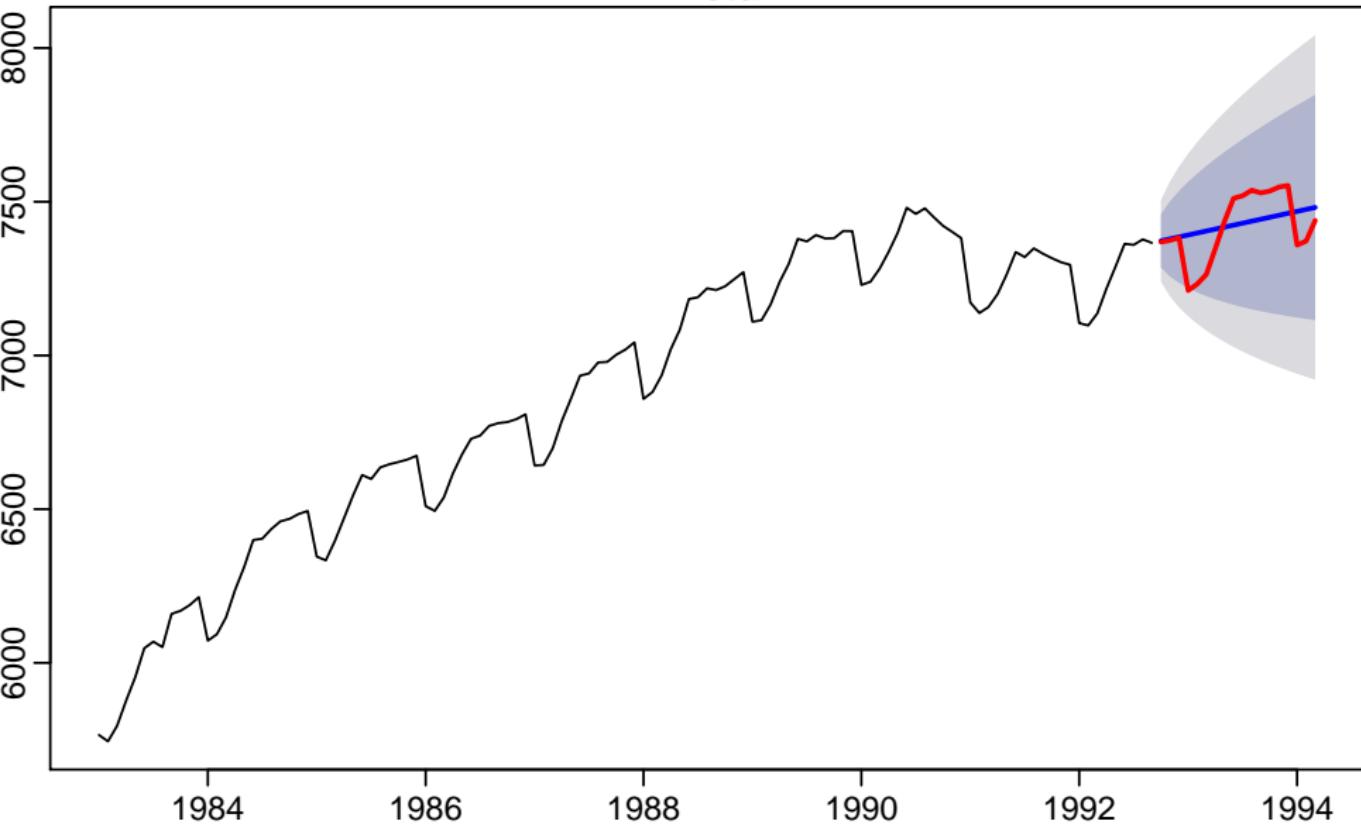
Predictability

STL-AR



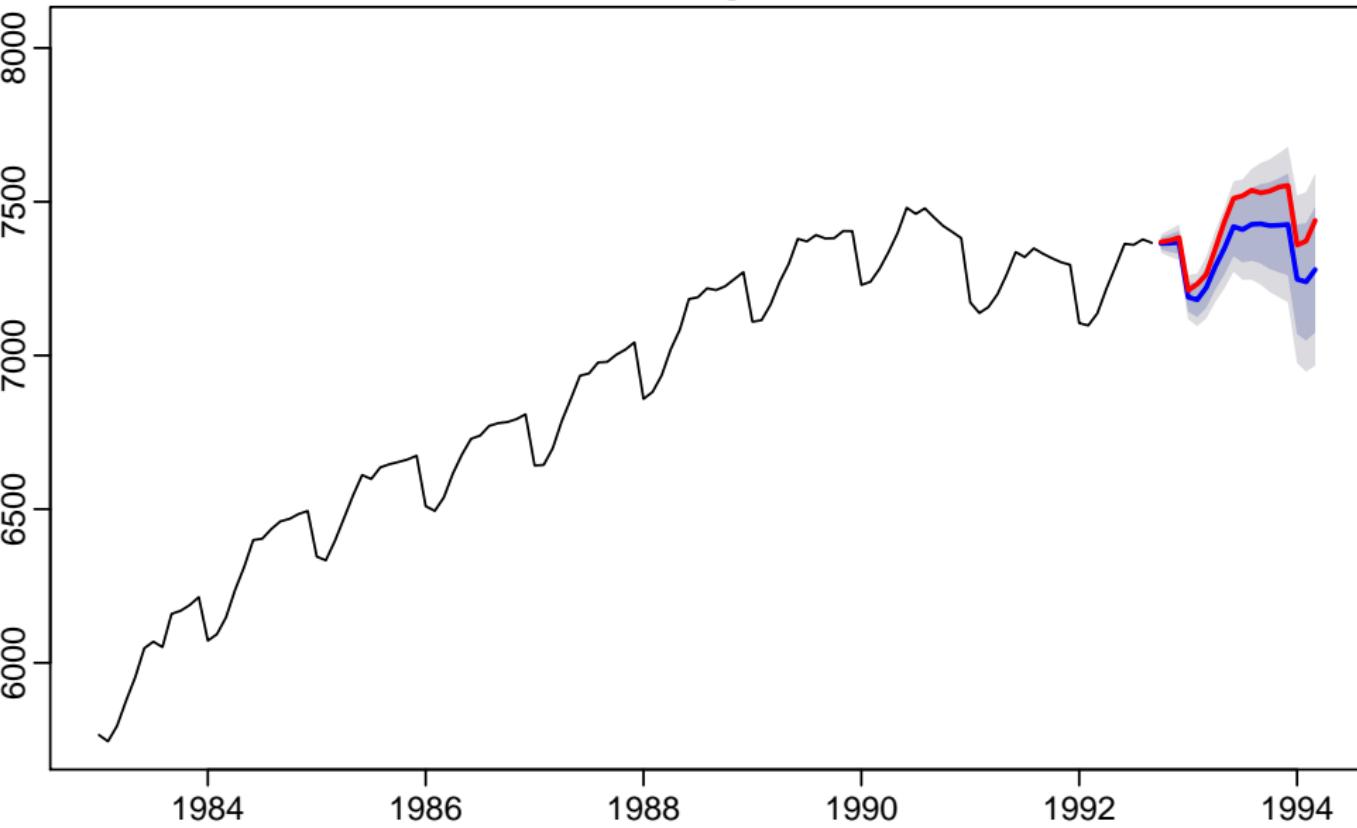
Predictability

Theta



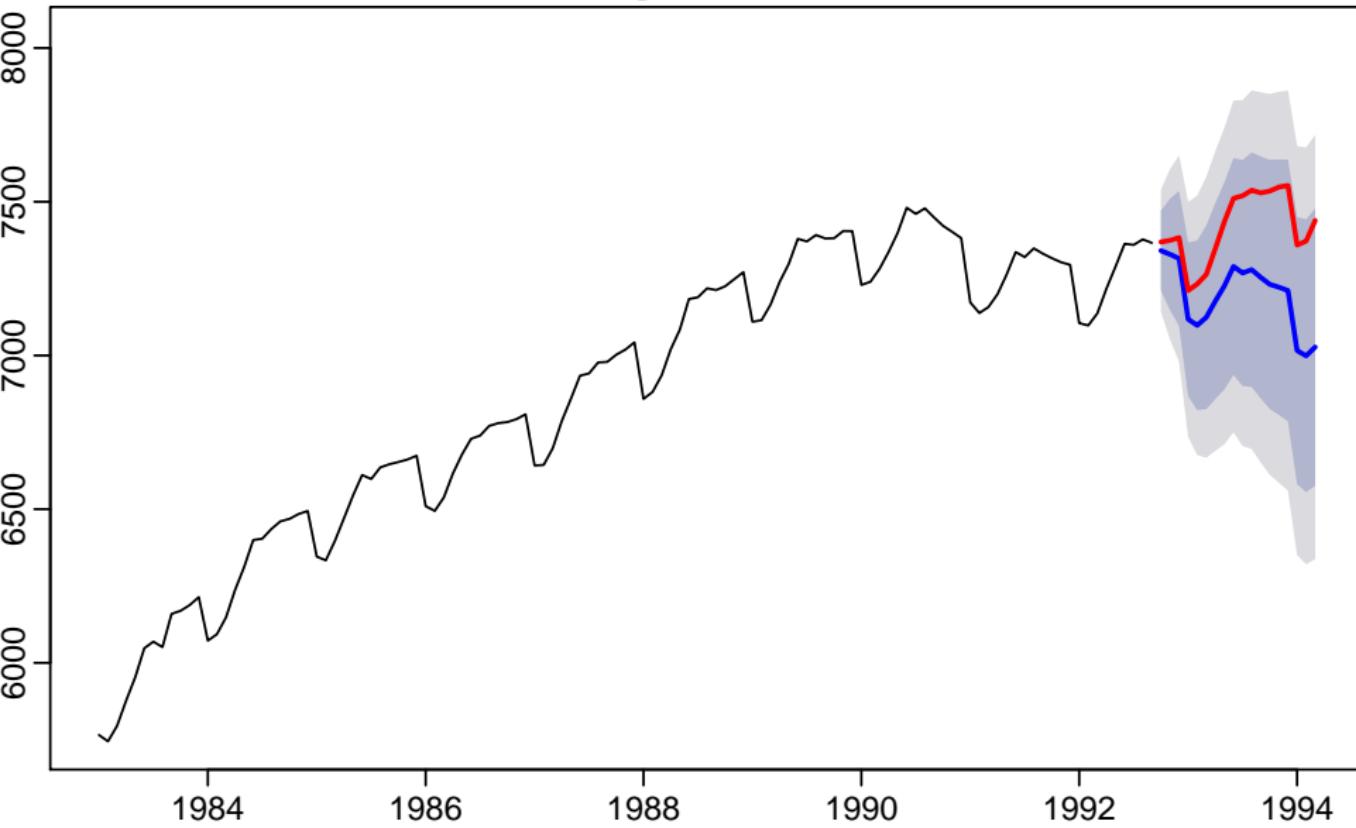
Predictability

ETS



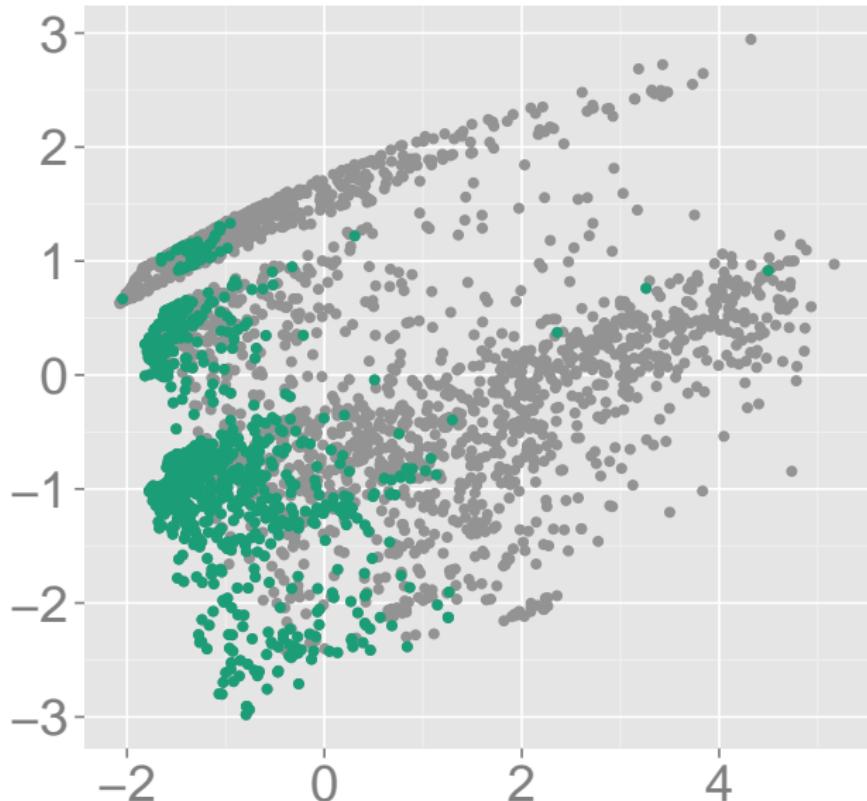
Predictability

STL-AR



Predictability

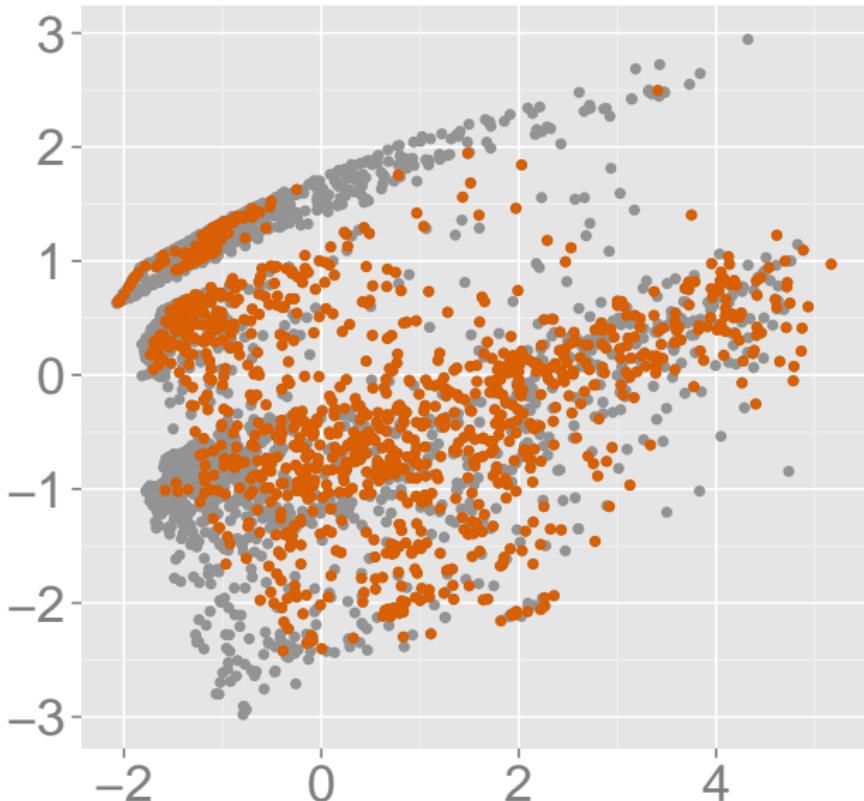
Low



Low
MASE values

Predictability

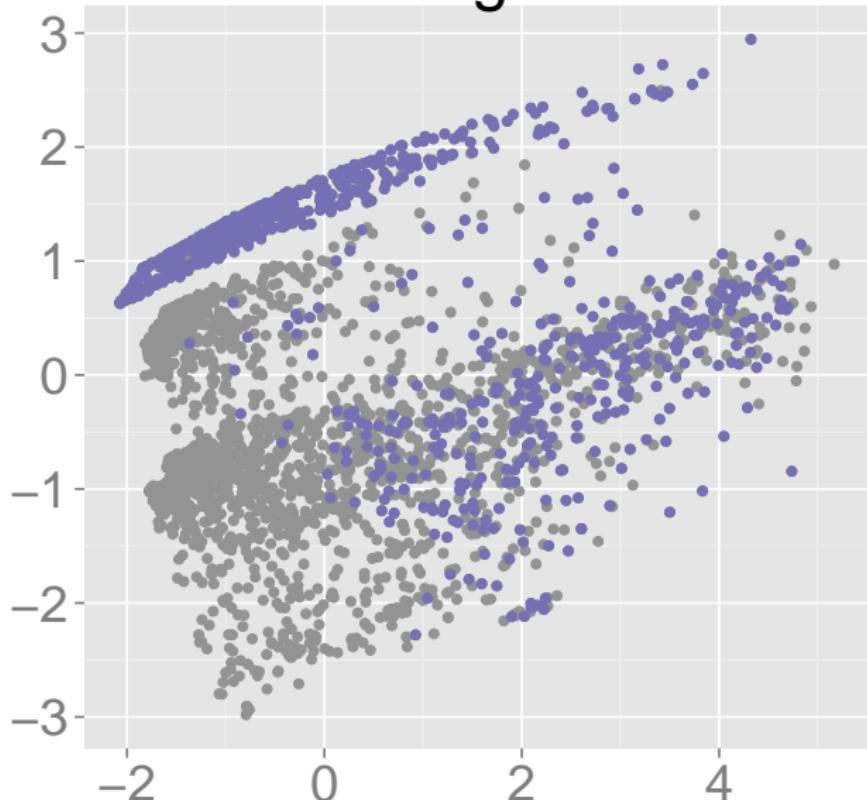
Middle



Medium
MASE values

Predictability

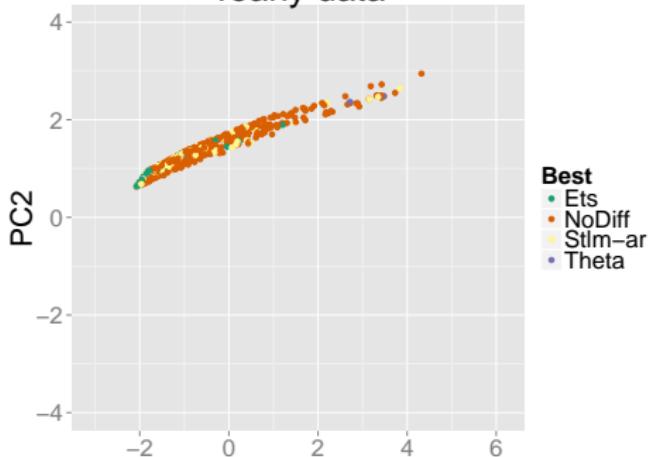
High



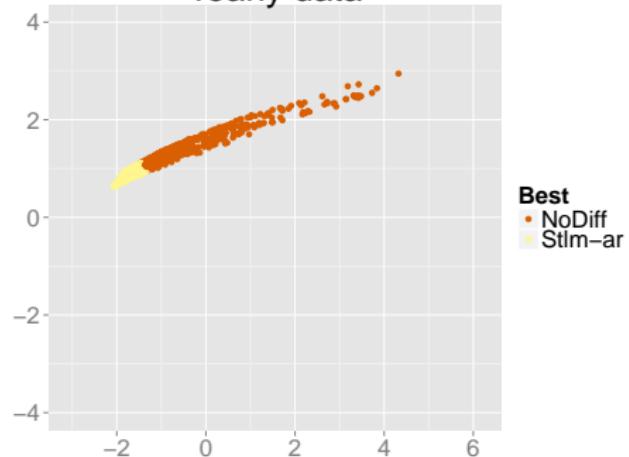
High
MASE values

Predictability

Yearly data

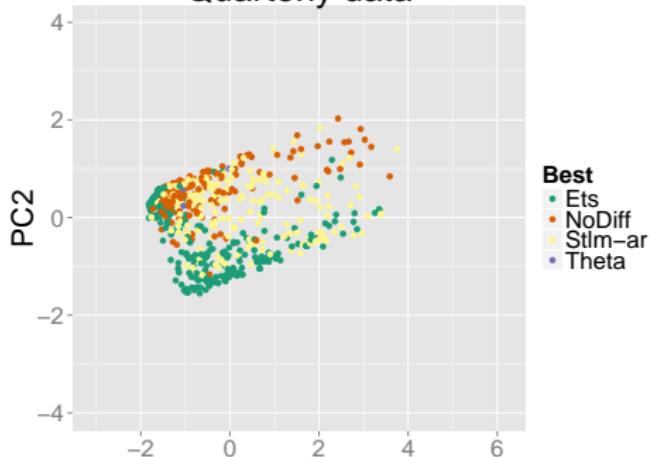


Yearly data

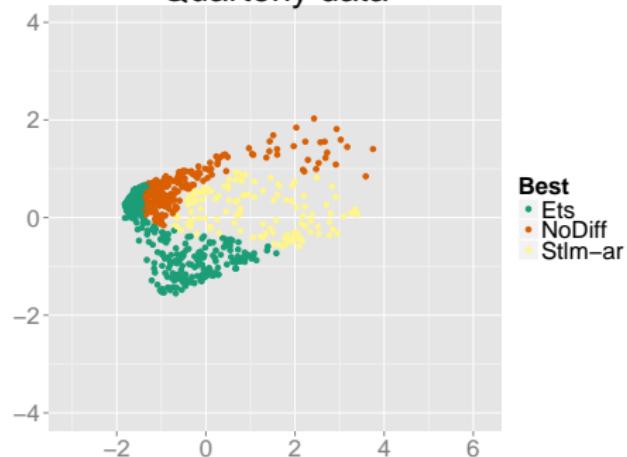


Predictability

Quarterly data

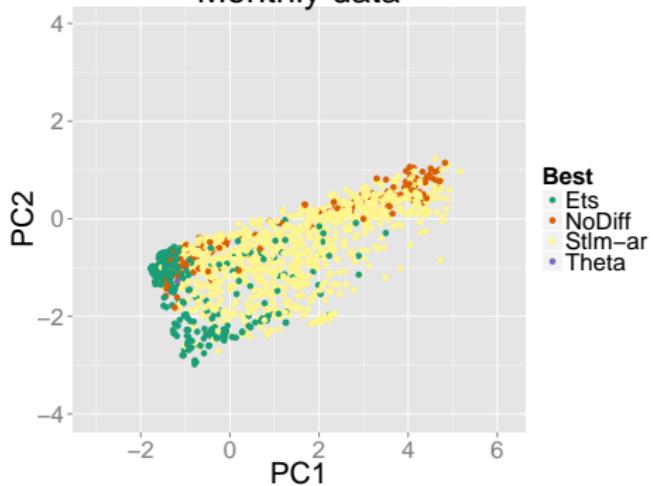


Quarterly data



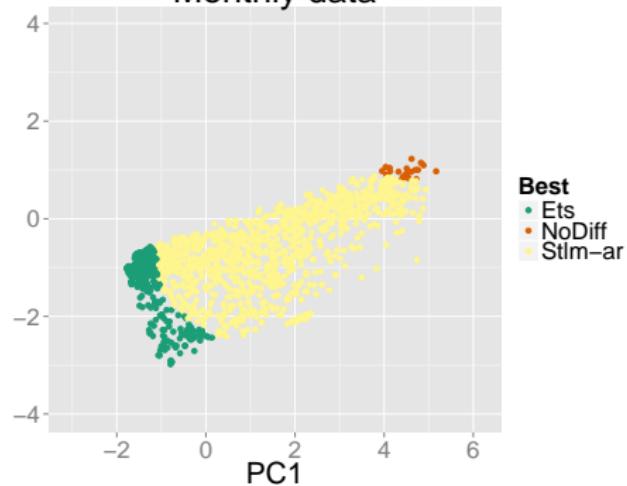
Predictability

Monthly data



Actual

Monthly data



SVM prediction

Generating new time series

We can use the feature space to:

- ▶ Generate new time series with similar features to existing series
- ▶ Generate new time series where there are “holes” in the feature space.

- ▶ Let $\{PC_1, PC_2, \dots, PC_n\}$ be a “population” of time series of specified length and period.
- ▶ Genetic algorithm uses a process of selection, crossover and mutation to evolve the population towards a target point T_t .
Optimize: $\text{Fitness}(PC) = -\sqrt{(T_t - PC)^2}$

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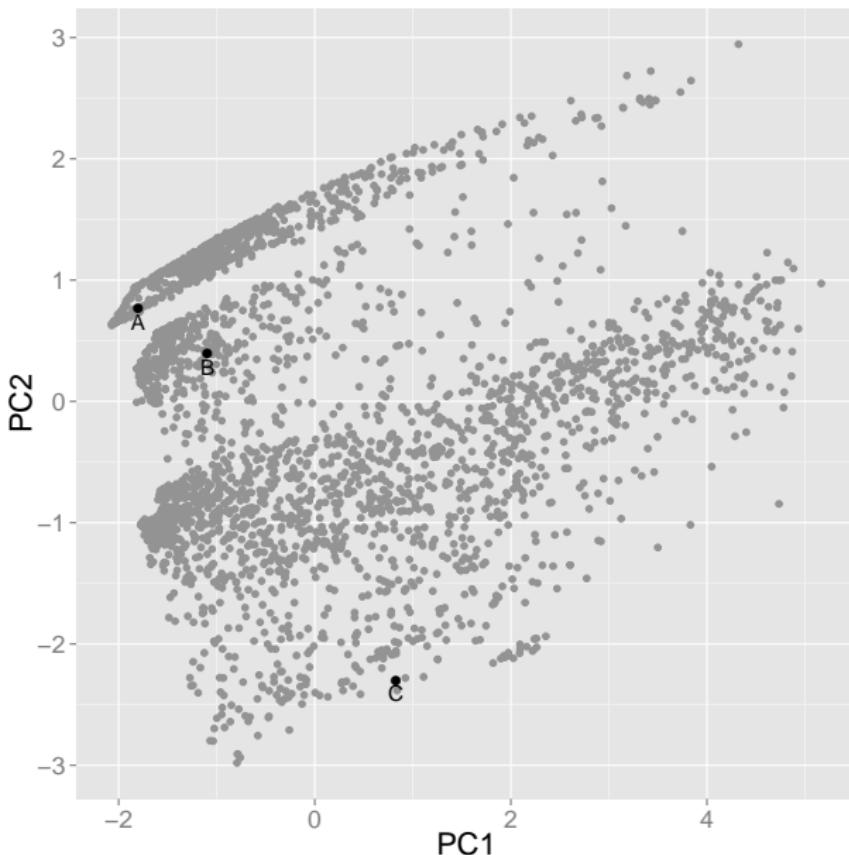
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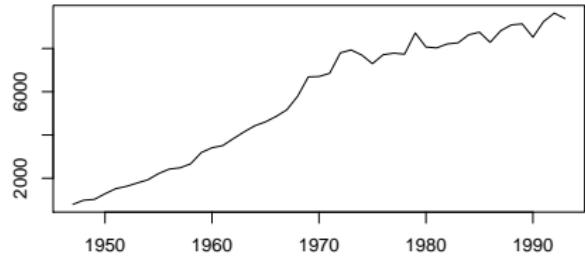
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Evolving new time series



Evolving new time series

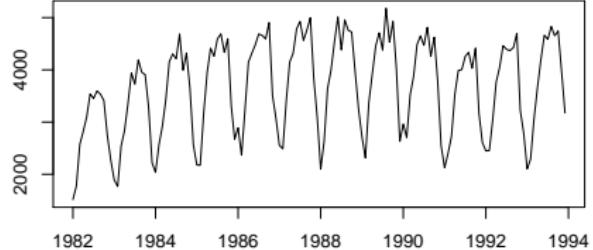
Target A



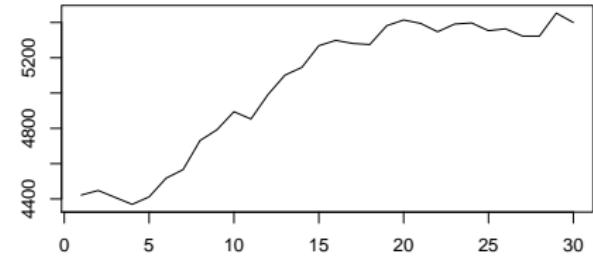
Target B



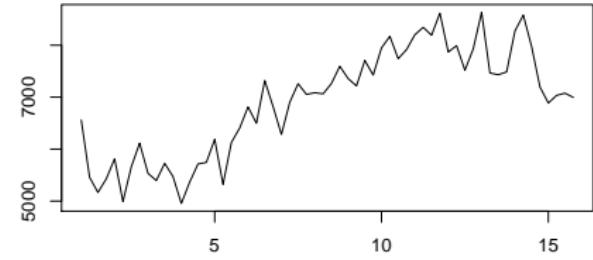
Target C



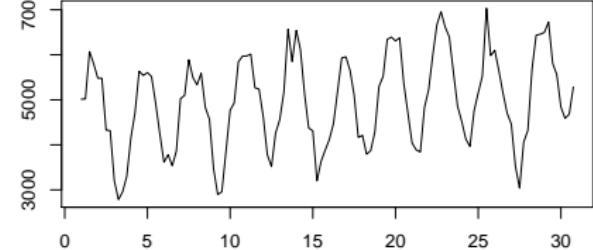
Evolved A



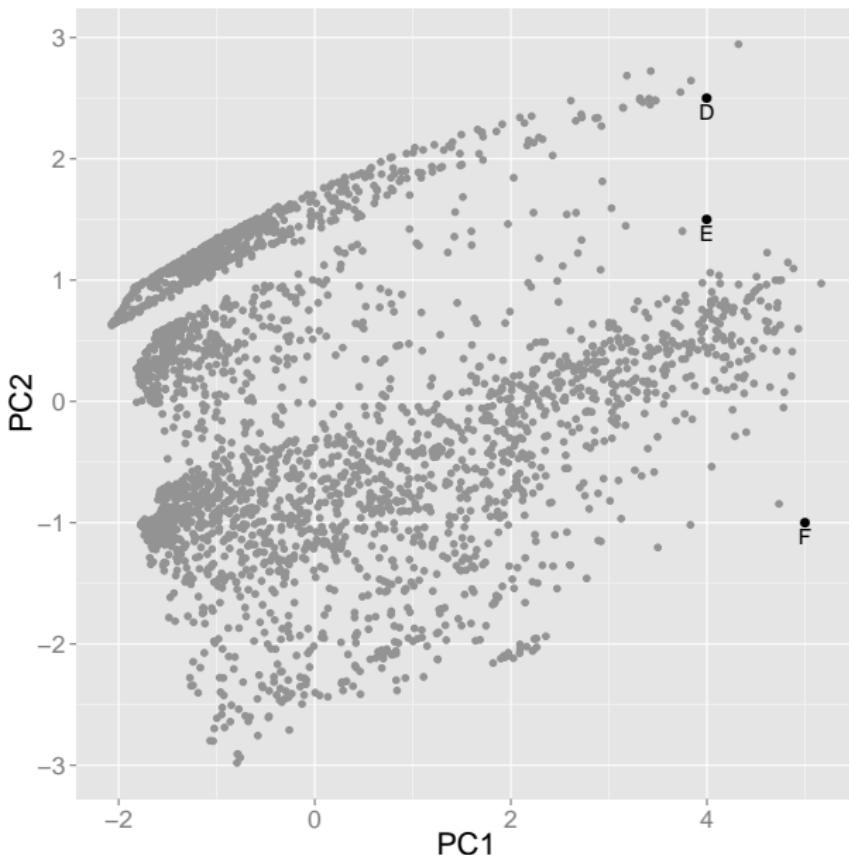
Evolved B



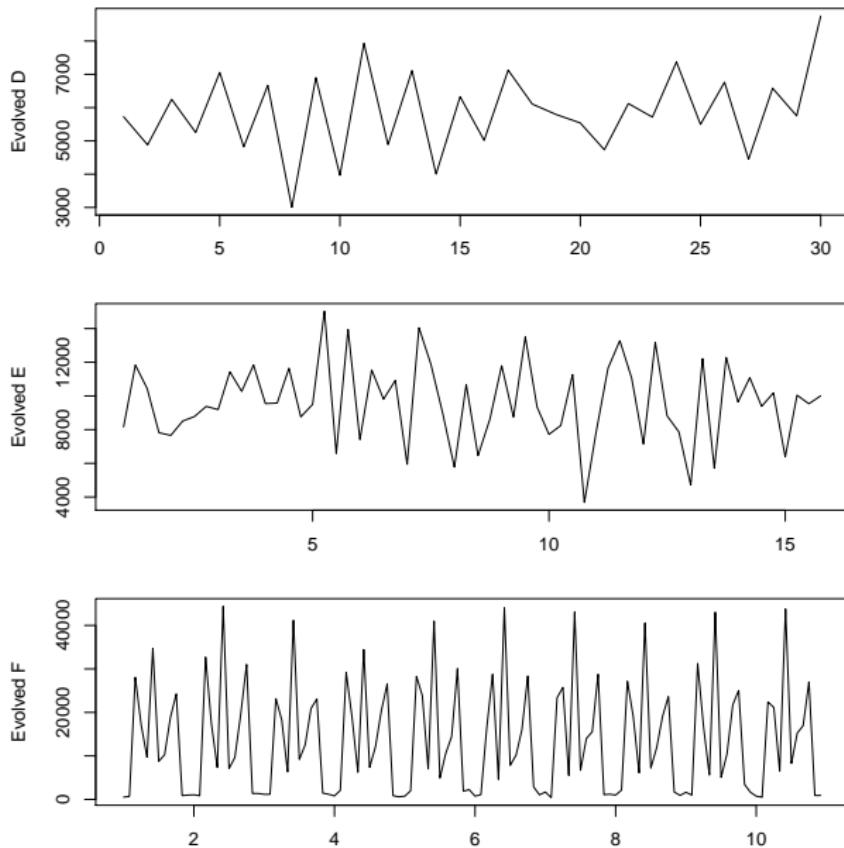
Evolved C



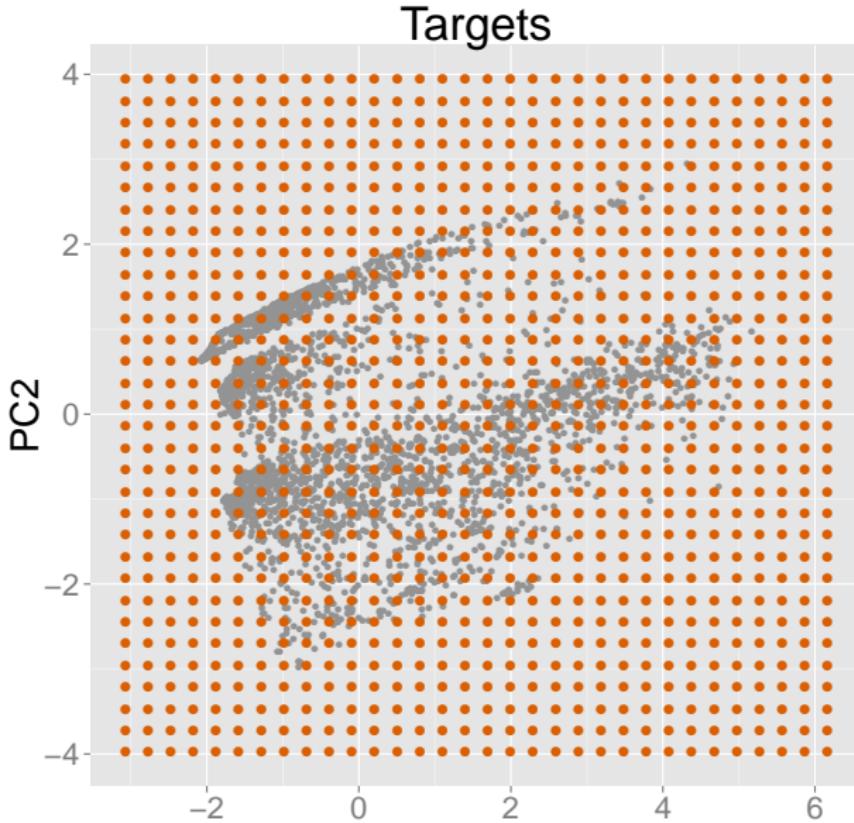
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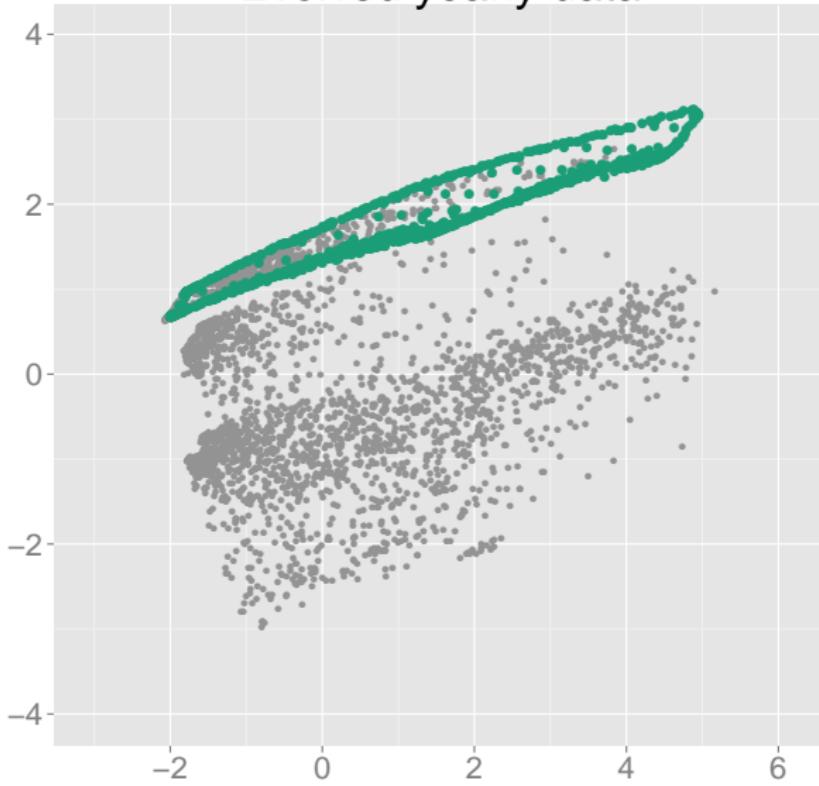


Evolving new time series

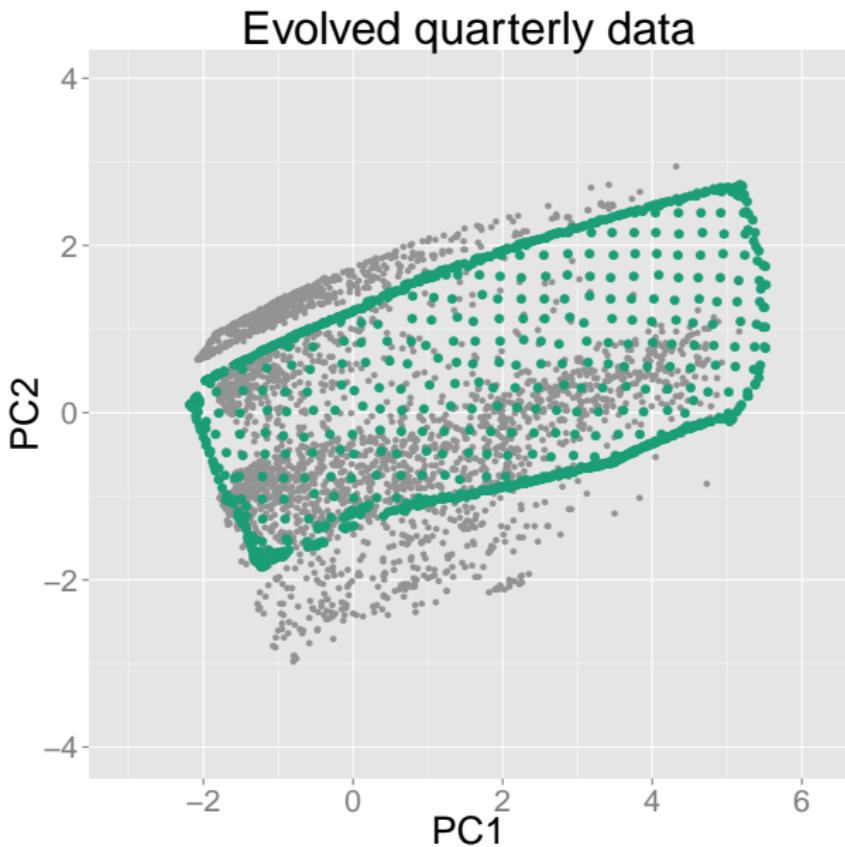


Evolving new time series

Evolved yearly data

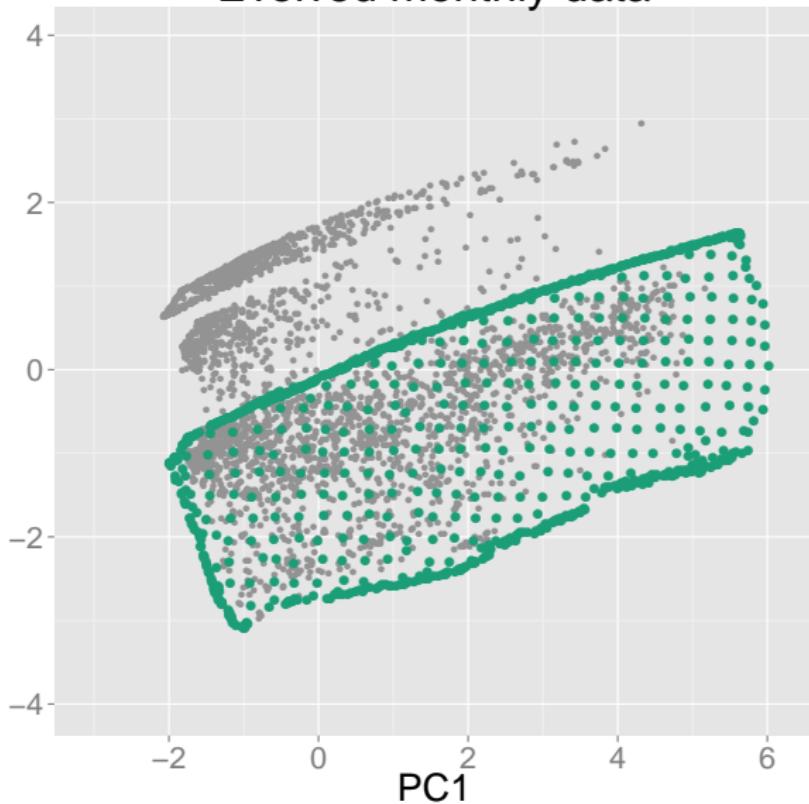


Evolving new time series



Evolving new time series

Evolved monthly data



Questions raised

- Can SVM be used to create a forecast selection routine to give better forecasts?
- How much do M3 conclusions depend on the particular set of time series involved?
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Outline

1 The problem

2 Australian tourism demand

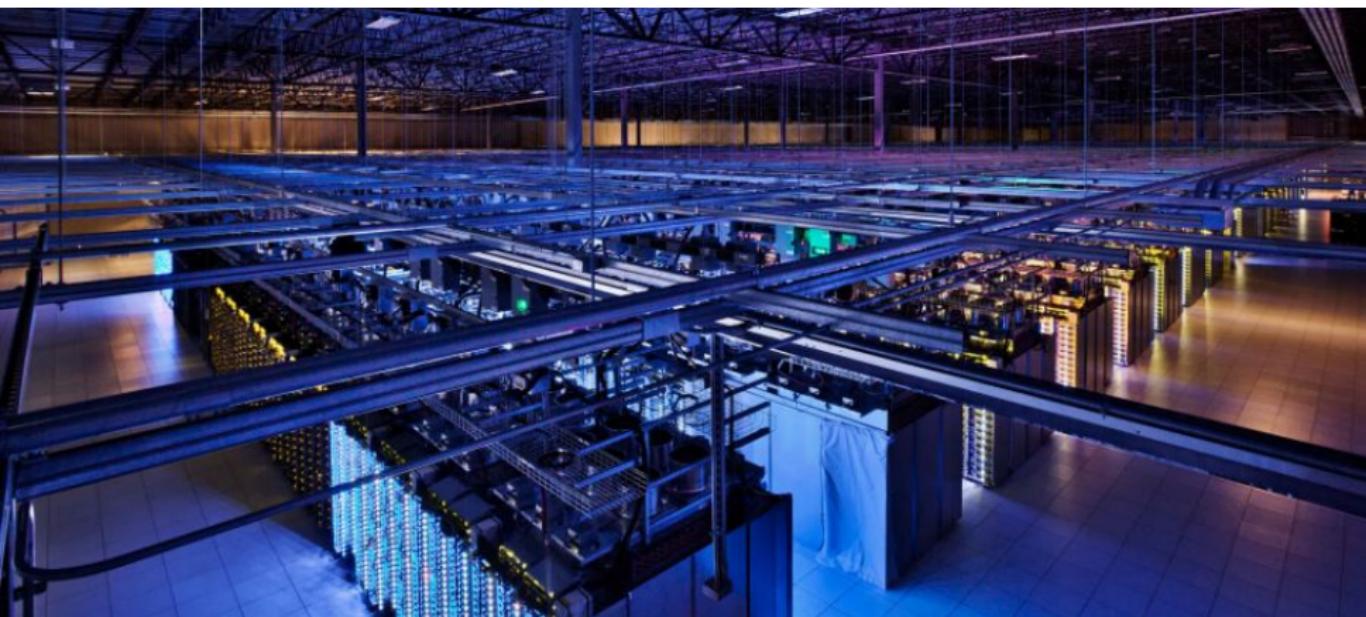
3 M3 competition data

4 Yahoo web traffic

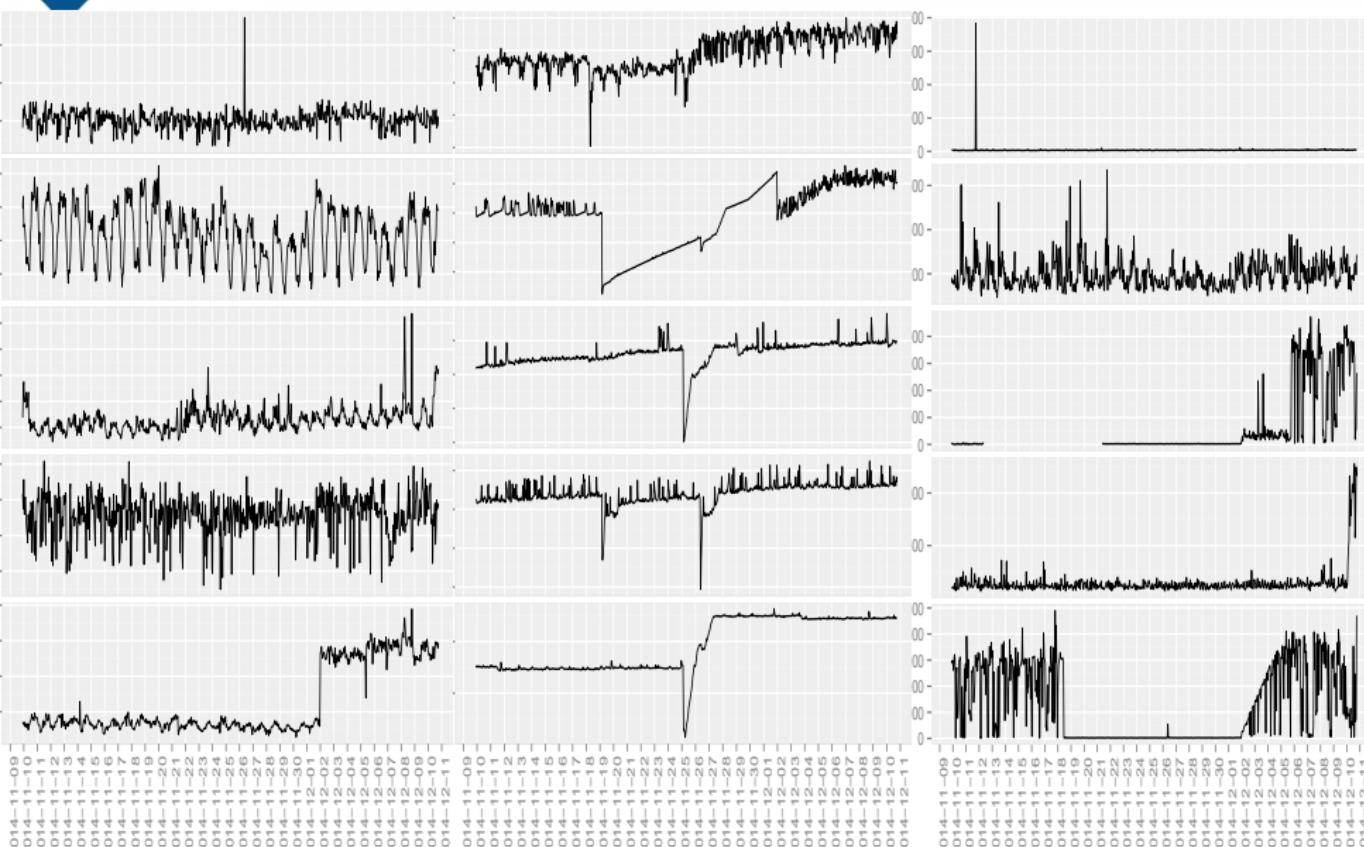
5 What next?

Yahoo web-traffic

- Tens of thousands of time series collected at one-hour intervals over one month.
- Consisting of several server metrics (e.g. CPU usage and paging views) from many server farms globally.
- Aim: find unusual (anomalous) time series.



Yahoo web-traffic



Feature space

- **ACF1:** first order autocorrelation = $\text{Corr}(Y_t, Y_{t-1})$
- Strength of **trend** and **seasonality** based on STL
- Trend **linearity** and **curvature**
- Size of seasonal **peak** and **trough**
- Spectral entropy
- Lumpiness: variance of block variances (block size 24).
- Spikiness: variances of leave-one-out variances of STL remainders.
- Level shift: Maximum difference in trimmed means of consecutive moving windows of size 24.
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- Flat spots: Discretize sample space into 10 equal-sized intervals.
Find max run length in any interval.
- Number of **crossing points** of mean line.
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- **ACF1:** first order autocorrelation = $\text{Corr}(Y_t, Y_{t-1})$
- Strength of **trend** and **seasonality** based on STL
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- Spectral **entropy**
- **Lumpiness:** variance of block variances (block size 24).
- **Spikiness:** variances of leave-one-out variances of STL remainders.
- **Level shift:** Maximum difference in trimmed means of consecutive moving windows of size 24.
- **Variance change:** Max difference in variances of consecutive moving windows of size 24.
- **Flat spots:** Discretize sample space into 10 equal-sized intervals.
Find max run length in any interval.
- Number of **crossing points** of mean line.
- **Kullback-Leibler score:** Maximum of
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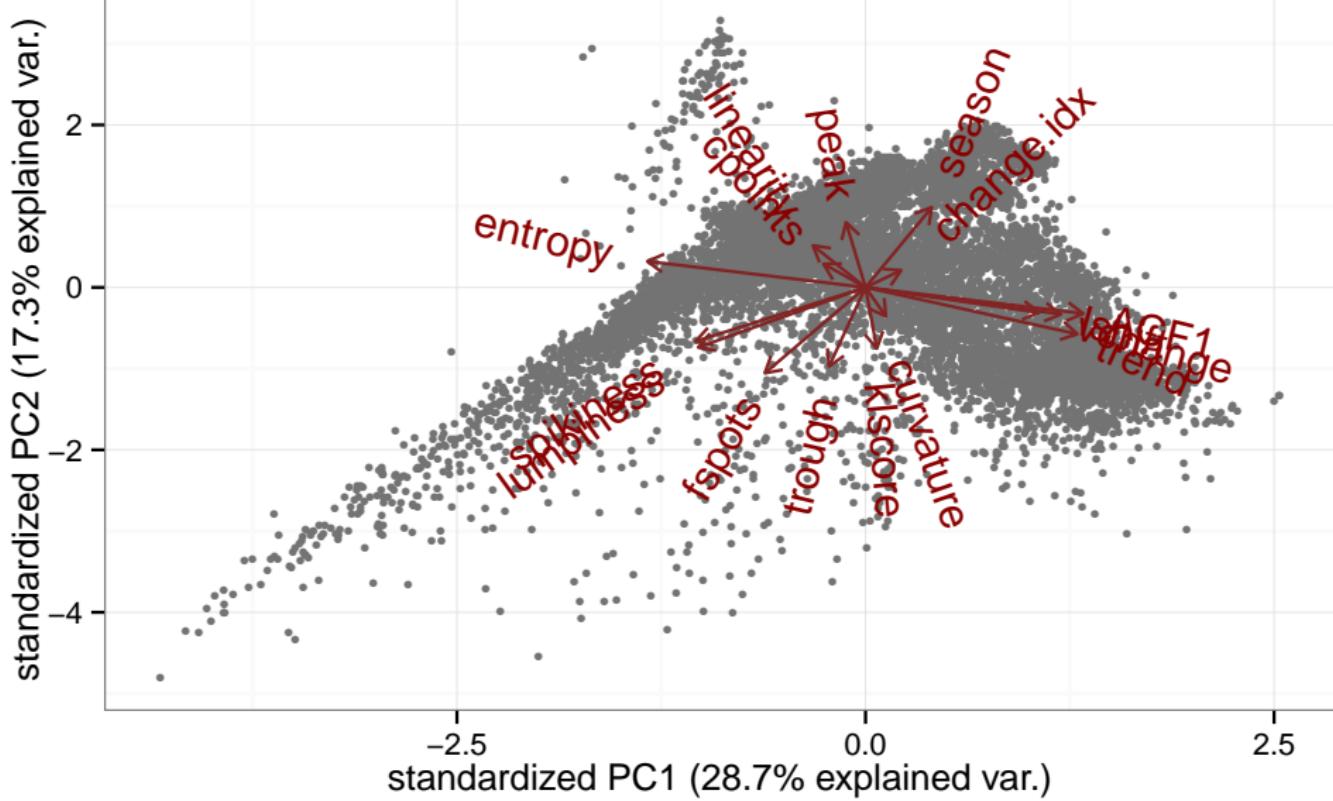
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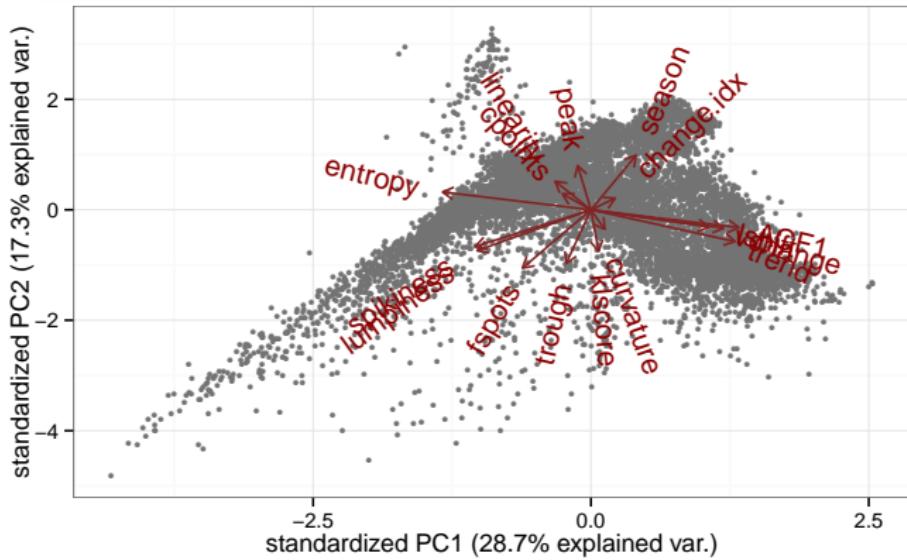
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Principal component analysis



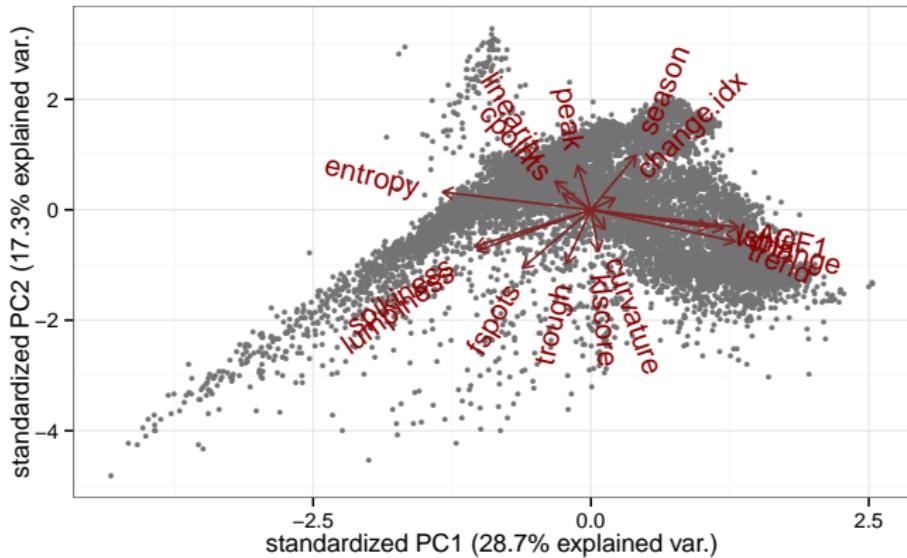
What is “anomalous”



We need a measure of the “anomalousness” of a time series.

- Rank points based on their local density.
- Rank points based on whether they are within α -Convex hulls of different radius.

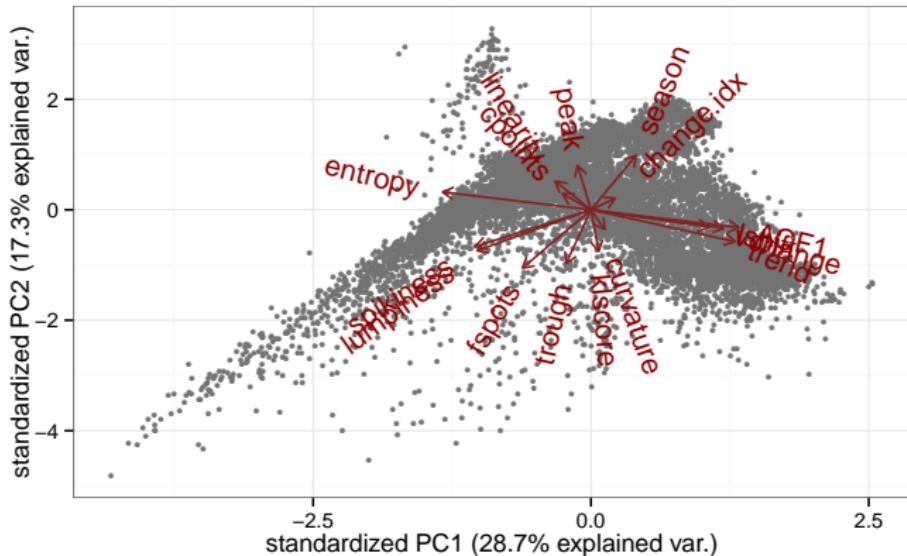
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Bivariate kernel density

$$\hat{f}(\mathbf{x}; \mathbf{H}) = \frac{1}{n} \sum_{i=1}^n K_{\mathbf{H}}(\mathbf{x} - \mathbf{x}_i)$$

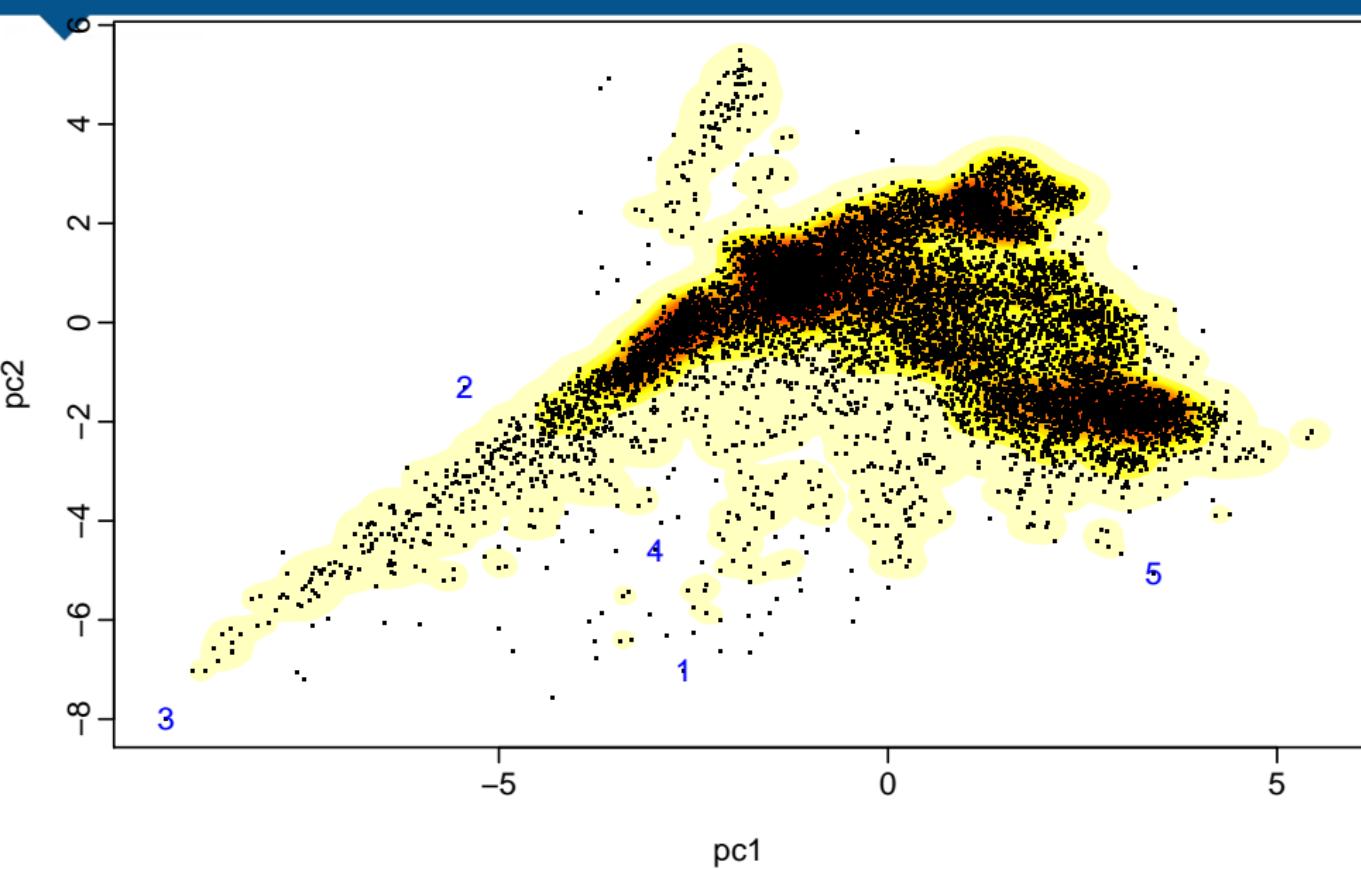
- $\mathbf{X}_i \in$ a bivariate random sample $\{\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_n\}$
- $K_{\mathbf{H}}(\mathbf{x})$ is the standard normal kernel function
- \mathbf{H} estimated by minimizing the sum of AMISE
- Rank points based on \hat{f} values in 2d PCA space.

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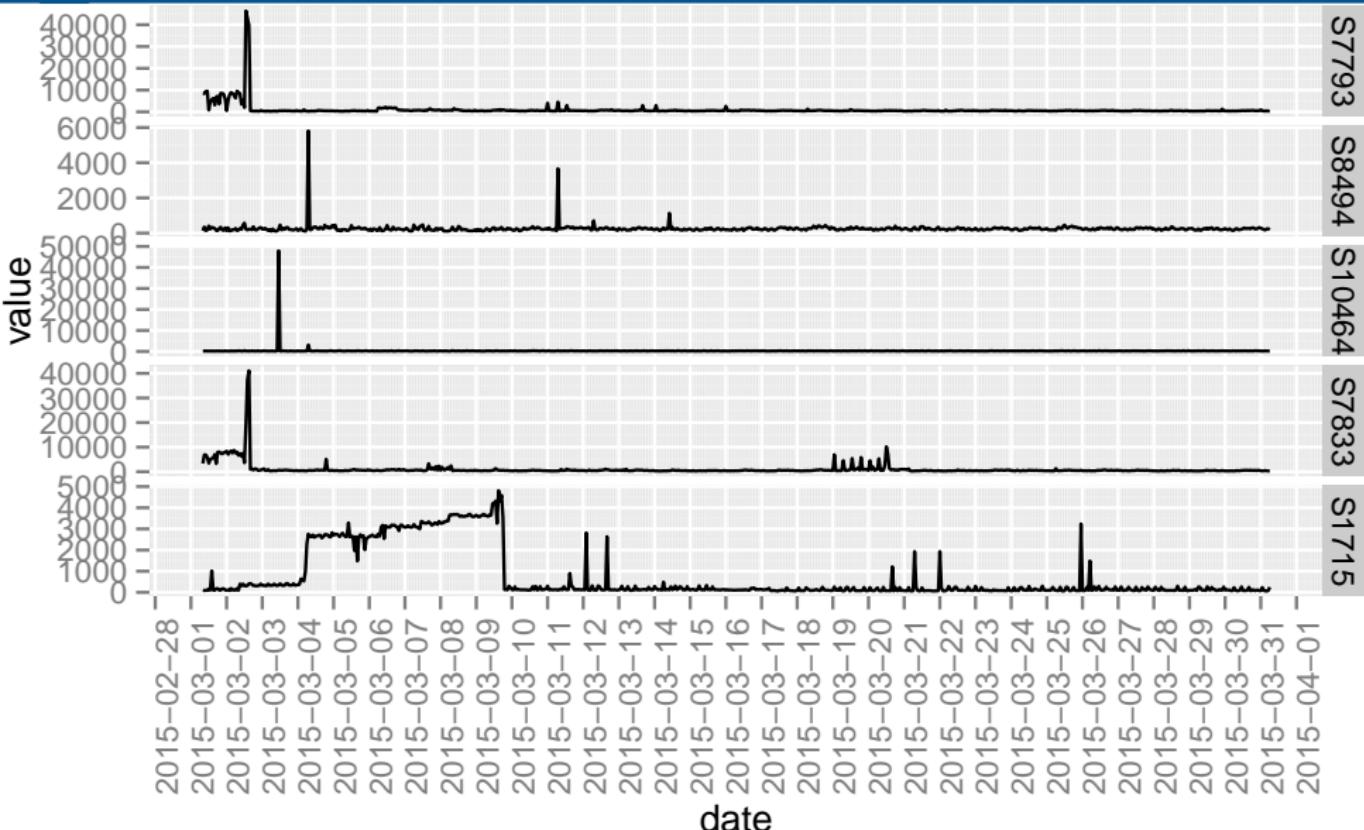
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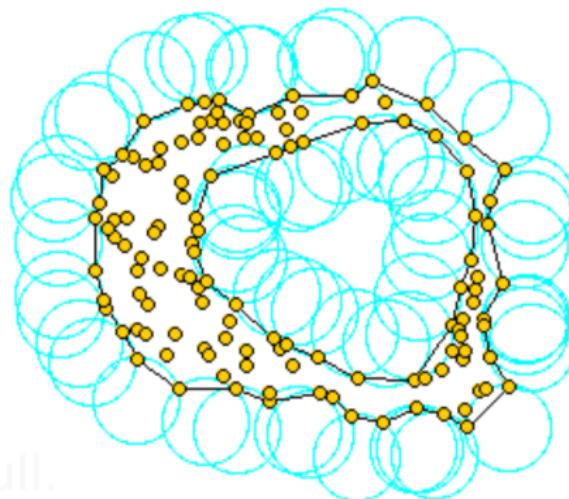
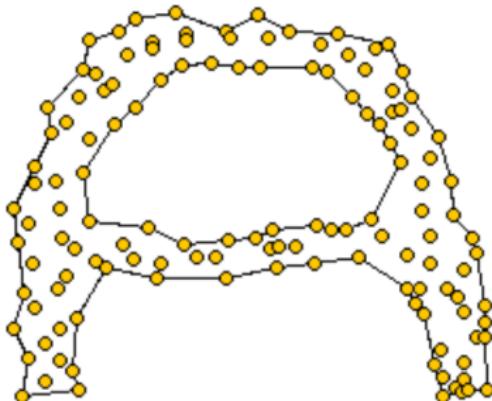


Bivariate density ranking



α -convex hulls

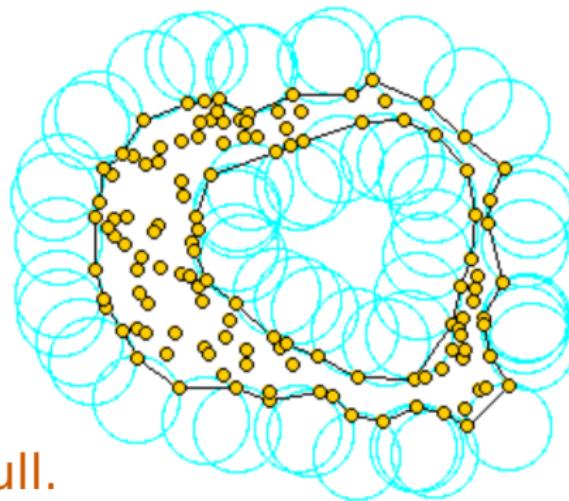
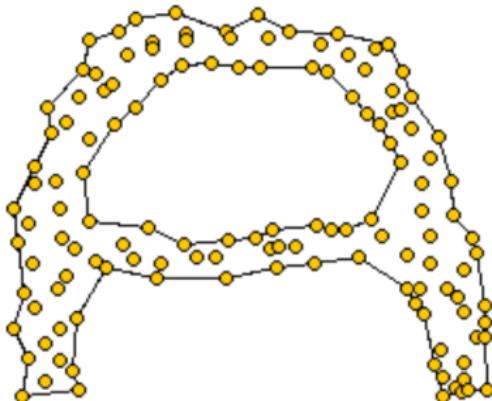
The space generated by point pairs that can be touched by an empty disc of radius α .



- $\alpha \rightarrow \infty$ gives a convex hull.
- Points can become isolated when α is small.
 - We rank points based on the value of α when they are connected to their

α -convex hulls

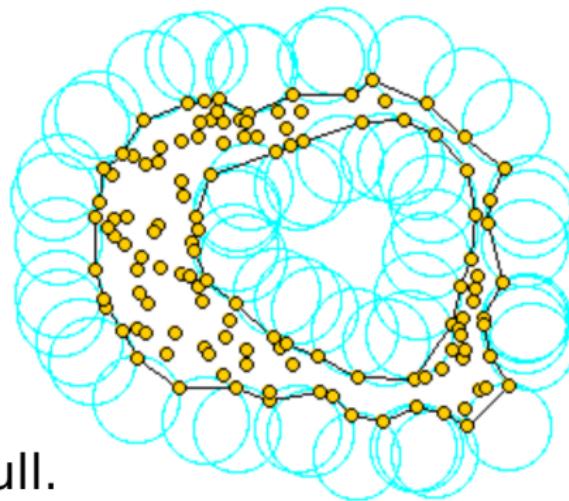
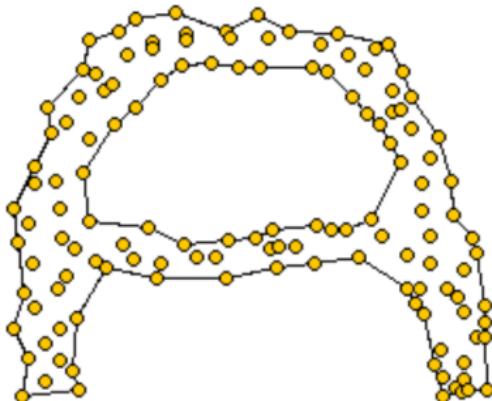
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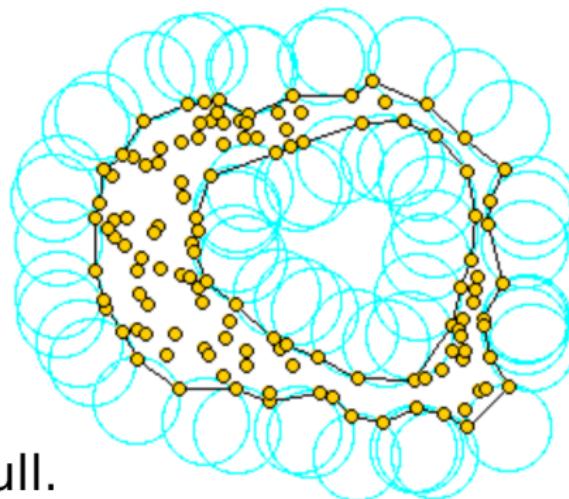
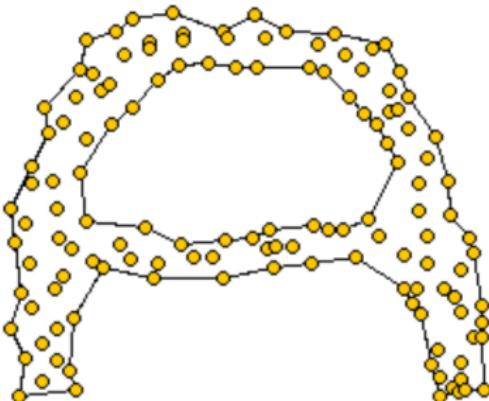
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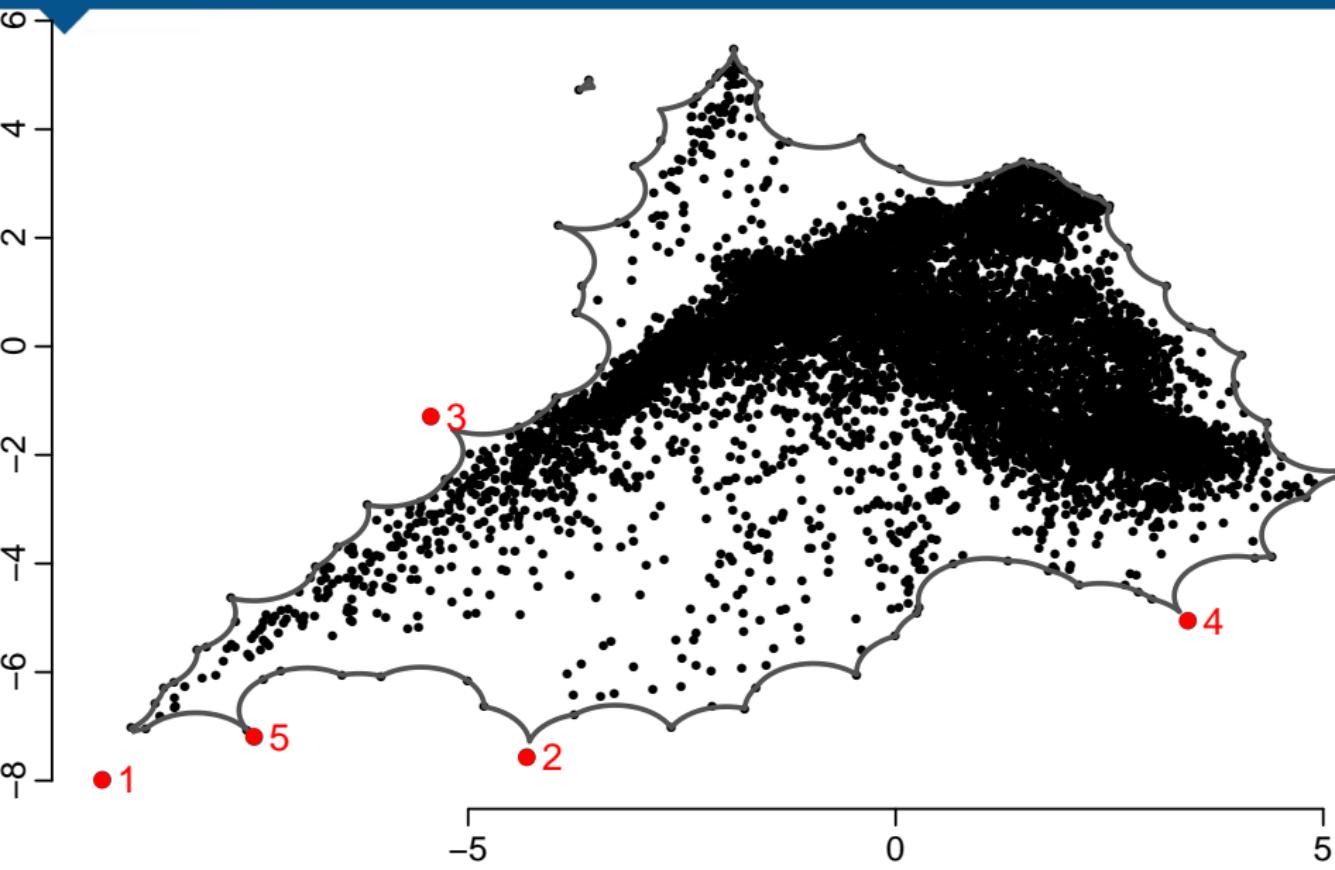
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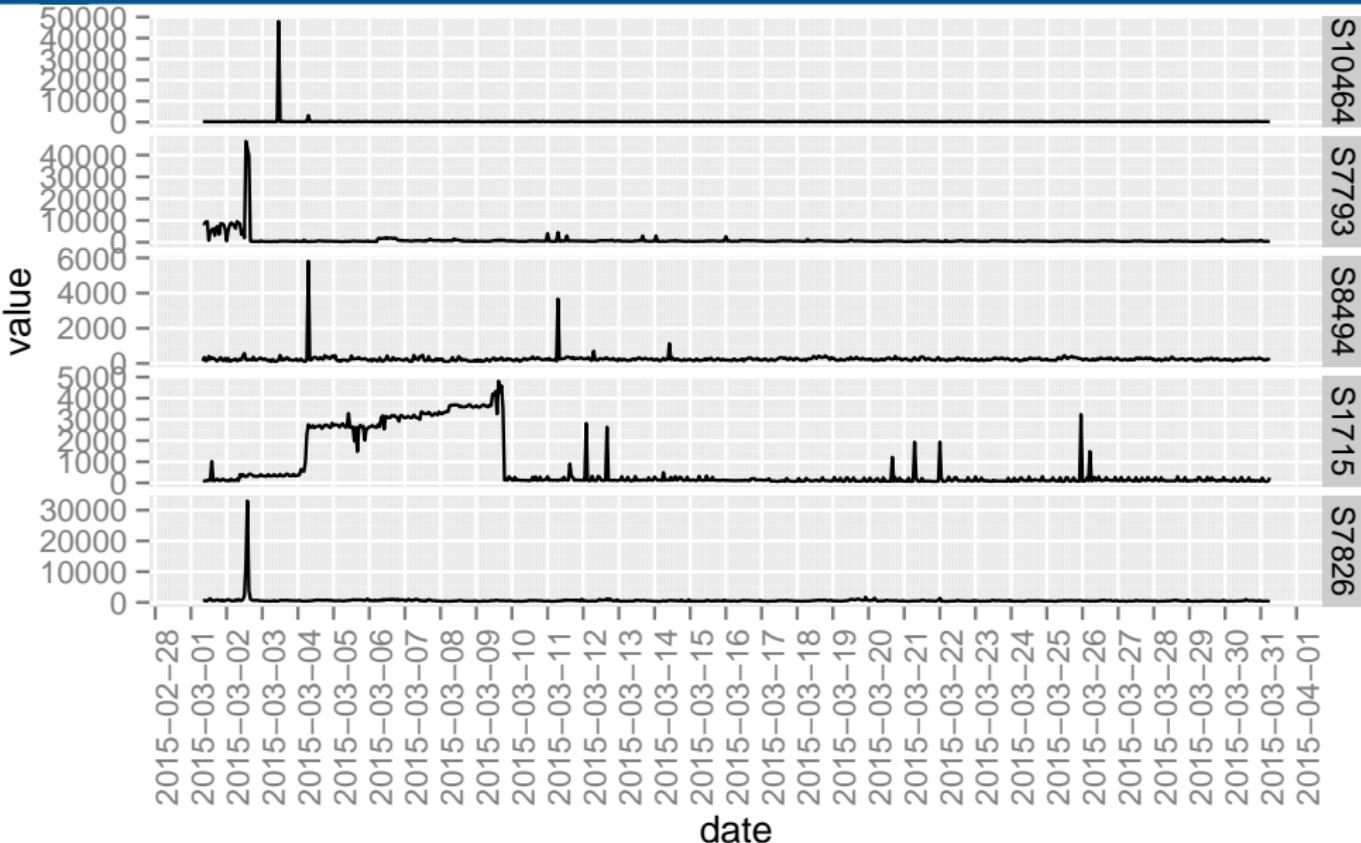
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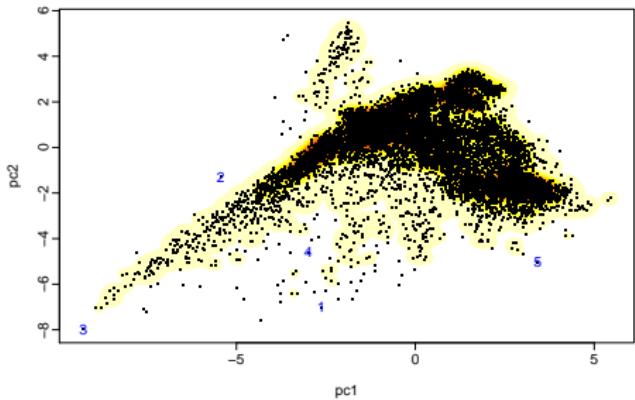


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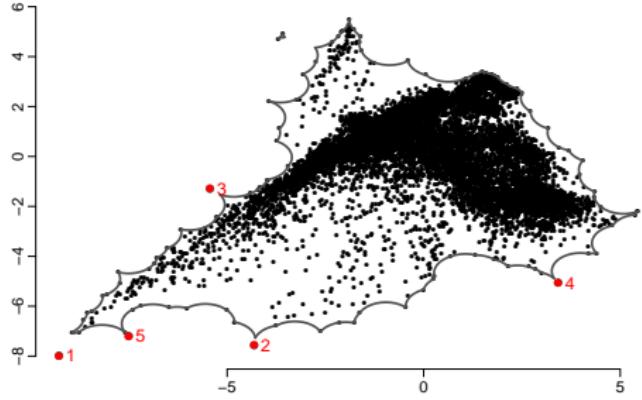


HDR versus α -convex hull

HDR boxplot

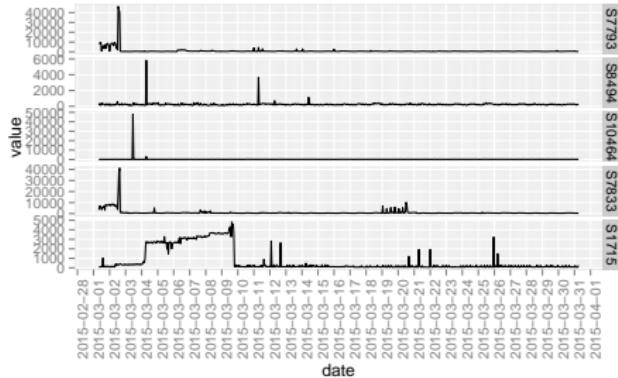


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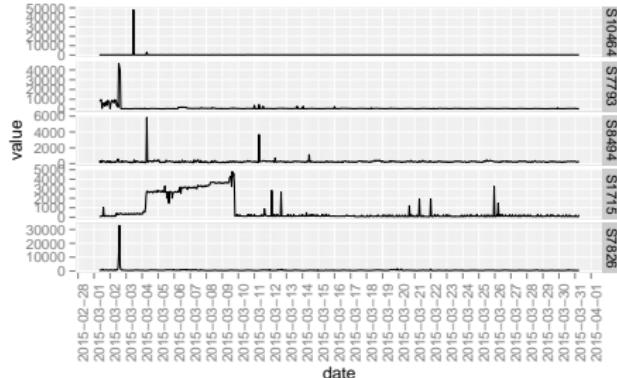


Top 5 anomalous time series

HDR



α -convex hull



Outline

1 The problem

2 Australian tourism demand

3 M3 competition data

4 Yahoo web traffic

5 What next?

What next?

- Develop a more comprehensive set of features that are reliable measures and fast to compute.
e.g., for finance data.
- Consider other dimension reduction methods and more than 2 dimensions.
- Develop dynamic and interactive visualization tools.
- Make methods available in an R package.

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http://www.r-project.org/doc/R-intro.html

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Further information

- ▶ Papers and R packages: robjhyndman.com
- ▶ Blog: robjhyndman.com/hyndsight
- ▶ Code: github.com/robjhyndman
- ▶ Email: Rob.Hyndman@monash.edu