CSC/ILIAS
Decision Systems

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Ongoing research

- Bipolar characteristic representations
- Outranking digraphs
- Multiple Criteria Decision Aid (MCDA)
- The RUBIS best choice method
- MCDA web services
- Universal MCDA - Modeling Language (XMCDA)

Bipolar characterizations

Examples

- Bachet's encoding of integers
- Net plurality characterization in voting theory
- Semiotics and logic computation
- Outranking concept
- Other

Bachet's encoding of integers

			–	+
-4	-1	-1	3 1	0
-3	-1	0	3	0
-2	-1	1	3	1
-1		-1	1	0
0		0	0	0
1		1	0	1
2	1	-1	1	3
3	1	0	0	3
4	1	1	0	1 3

Bachet encoding of integers

Bachet's theorem (1624)

Base 3 {-1,0,1}-*encoding is the unique* **n** *digits representation of integers from*

$$-(3^{n+1} - 1)/2 \text{ to } (3^{n+1} - 1)/2$$

which uses the smallest possible number of weights for balancing.

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Net plurality in voting

n members of a Committee have to elect their president. There is a unique candidate *P*.

voter #	1	2	3	4	5 ...	n
in favour	1		1	0	1 ...	1
against		1		0		
net result:	+1	-1	+1	0	+1...	+1 = *X*

X in [-*n*, *n*]: If *X* > 0, *P* becomes president elect

Condorcet's pairwise voting procedure

3 candidates {a,b,c} compete for the presidency of a Committee. The *11* members vote in turn on the proposition that :

" candidate ***x*** is preferred to candidate ***y*** " :

voting results

	a	***b***	***c***
a	-	+3	-5
b	-3	-	+7
c	+5	-7	-

Majority margins tables:

Condorcet (1785), Arrow & Raynaud (1986), Debord (1986), Lamboray(2007): prudent ranking rules

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Truth assessment by balancing reasons

“ Take the sum of all the feelings of belief which could be produced separately by all the arguments pro, subtract from that the similar sum for arguments con, and the remainder is the feeling of belief which ought to have the whole ”

C S Peirce, 1878

Valued epistemic logic

Let ***p*** be a proposition. To validate its truthfulness we use a set of logical (pragmatic) criteria:

criteria #	significance	value	weight
1	w_1	true	$+w_1$
2	w_2	true	$+w_2$
3	w_3	false	$-w_3$
4	w_4	-	0
...	...	...	...
			-------- +
			$= X$

If ($X > 0$), ***p*** is more true than false

If ($X < 0$), ***p*** is more false than true

If ($X = 0$), ***p*** is indeterminate

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Outranking concept

We consider a set choice decision problem where a decision maker tries to make the best choice from a given set of potential alternatives on the base of a set of performance measuring criteria.

An alternative " ***x* is said to outrank alternative *y*** " iff

1) there exists a significant majority of criteria who confirm that alternative ***x*** is to be considered as at least as worth as alternative ***y*** ;

2) no criterion opens to doubt the validity of the previous confirmation by revealing an otherwise serious counter-performance of alternative x compared to y.

(Roy 1968, Roy & Bouyssou 1993)

Outranking concept

- Examples of outranking situations
- Example of bipolar-valued outranking digraphs

Is a_{10} globally at least as good as a_7?

Outranking thresholds: weak preference ($\geqslant 10$), preference ($\geqslant 20$), veto ($\leqslant -80$).

criterion	w_i	a_{10}	a_7	$\Delta_i(10,7)$	balance	veto ?
g_1	7	98	18	80	+7	no
g_2	7	48	79	-31	-7	no
g_3	5	42	78	-36	-5	no
g_4	5	5	35	-30	-5	no
g_5	5	44	53	-9	+5	no
g_6	4	53	54	-1	+4	no
g_7	1	99	0	99	+1	no
				total balance	0	

Is a_7 globally at least as good as a_{10}?

Outranking thresholds: weak preference ($\geqslant 10$), preference ($\geqslant 20$), veto ($\leqslant -80$).

criterion	w_i	a_7	a_{10}	$\Delta_i(10,7)$	balance	veto ?
g_1	7	18	98	-80	-7	yes
g_2	7	79	48	+31	+7	no
g_3	5	78	42	+36	+5	no
g_4	5	35	5	+30	+5	no
g_5	5	53	44	+9	+5	no
g_6	4	54	53	+1	+4	no
g_7	1	0	99	-99	-1	yes
				total balance	+18-34	

We observe a veto situation on criteria g_1 and g_7.

a_7 is clearly not globally at least as good as a_{10}? !

Bipolar characterizations

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Other bipolar characterizations

- Statistics
  - Correlation coefficient (Galton 1888), Kendall's tau, the rank correlation coefficient (1942), Bayesian decision theory ?
- Psychometrics
  - The reciprocal matrix: $R_{ij} = 1/R_{ji}$ transformed to: $g_{ij} = \ln(R_{ij})$ (Saaty 1977)
- Social Choice
  - Comparison functions on (weak) tournaments (Dutta&Laslier 1998, Subiza&Peris 2003)
- Game theory
  - two-player zero-sum games (Von Neumann 1944), Nim games (Grundy 1939, Berge 1958)
- Computational logic
  - Bipolar Prolog (Bisdorff 2000)

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General characteristics of outranking digraphs

- Apart from evident reflexivity, any digraph may in fact result from a positive cut of the valuation (Bouyssou 1996)
- Convex combination of homogeneous semiorders (no hope to get ever a finite characterization of the outranking digraph)
- The same outranking digraph can be produced by many different performance tableaux. Two performance tableaux are called isometric if they produce the same normalized [-1,1]-valued outranking digraph.
- The rank of an outranking digraph is the minimal dimension of its set of isometric performance tableaux.

The maximal rank of the outranking digraph

- The maximal rank $r(2)$ of an outranking digraph of order n is 3
- The maximal rank $r(3)$ of an outranking digraph of order 3 is ≥ 5 ?
- The maximal rank $r(n)$ of an outranking digraph of order $n > 2$ is limited by Bachet's minimal encoding result:

$$r(n) \geq \frac{\log(3) + \log\left(\frac{n^2}{2} - \frac{n}{2} - 1\right)}{\log(3)}$$

Random outranking digraphs

- Monte Carlo generation of random instances
  - Reference case: 20 alternatives evaluated on 13 equi-significant criteria with 10% indifference, 20% preference and 80% veto thresholds.
  - Various generators for performances on each criteria: mode parametrized triangular - , truncated normal - , and uniform distributions.
- Statistical results
  - High density (70% - 80% fill rate)
  - More or less elliptic distribution of nodes with sometimes two or more ordered strongly connected components
  - Very short diameters, concentrated neighborhoods, etc

(Bisdorff & Roy 2008)

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Research community

- Scientific homeland
  - French Multiple Criteria Decision Aid School (CNRS-Lamsade) promoting the outranking approach (Roy, Bouyssou, ao)
- The COST Action IC0602 *Algorithmic Decision Theory*
  - AI preference handling - MCDA marriage !
  - Chair: Alexis Tsoukias (CNRS Paris-Dauphine),Vice-chair: Ulle Endriss (Univ. Amsterdam)
  - Associated US lab: DIMACS (Rutgers, F. Roberts)
- Decision-Deck Project
  - Founding partners:
    ULB (B), FPMs (B), Paris-Dauphine (F), KarmicSoft (F), INESC (PT), UL (L)

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Decision problematique

- Choice
  - Choose the single best, or the k best alternatives
- Ranking
  - Completely rank all alternatives without ties (linear ordering) or with ties (weak ordering)
- Rating (sorting)
  - Allocating to predefined (standard) ordered categories
- Clustering
  - Gathering similar objects

The Rubis best choice method

Pragmatic progressive choice recommendations (CR)

1) Non retainment for well motivated reasons
   CR is an outranking choice
2) Minimal size
3) Efficient, informative and stable
   CR is a chordless odd outranking circuit
4) Effectively better
   CR is strictly more outranking than outranked
5) Maximally credible
   CR is of maximal credibility

(Bisdorff, Meyer&Roubens 2007)

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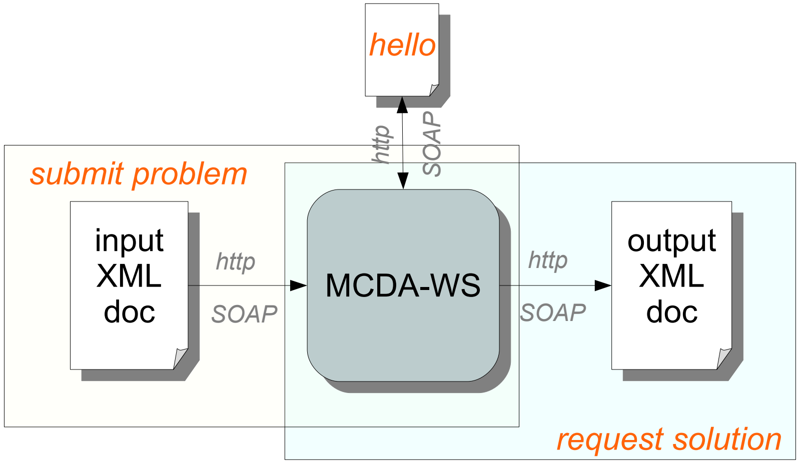
Asynchronous MCDA-WS



http://ernst-schroeder.uni.lu/d3/

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Thanks for your attention