

Decision aid protocols incorporating inverse decision analysis

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MULTICRITERIA DECISION AID

- · A definition of multicriteria problems
- Outranking methods
- Condorcet robustness denotation
- Computing the Condorcet denotation according to the decision maker
- · Mathematical modeling of the decision maker's preferences
- · Going towards an inquiring protocol
- Demos



Some examples of multicriteria problems

- Selecting a restaurant
 - Taking into account of the distance, the price, the quality, the atmosphere,...
- Buying a new car
 - · Studying its consumption, its price, its aesthetics, its color, the
 - equipments,...

 "An Aston Martin is much attractive than a Clio, but also much more expensive..."
- · Selecting a candidate for a role
 - Studying his CV, his motivation, his social skills, his salary expectations,...
- · Almost any decision in your life!

The "American" way

- · Aggregation of all the criteria in a unique criterion
- · Each alternative is associated with a score (a global value)
- Ex: Weighted sum, weighted mean, Choquet integral....

The "European" way

- · Based on pairwise comparisons of alternatives
- · Outranking methods and graph theory



Inverse analysis

- · Help a decision maker to identify all his preferences
 - By inquiring directly some method parameters
 - By asking him to explicit some partial a priori preferential information (obvious)
- Characterize the quality of a solution, in terms of robustness, satisfaction of all the process actors,...

- An alternative a outranks an alternative b when a significant majority of criteria validates the fact that a is performing at least as good as b and there is no criterion where b seriously outperforms a IRovI
- ELECTRE and Rubis methods are examples of outranking methods



Condorcet robustness denotation

Criteria weights preorder:

- Two sets of criteria weights are preorder-compatible if they have the same preorder on their criteria weights.
- Ex : $W = \{3,7,5,1,7\}$ and $W' = \{4,6,5,2,6\}$ have the same preorder $\{w_2,w_5\} \succ \{w_3\} \succ \{w_1\} \succ \{w_4\}$

Robustness of an outranking :

- We analyze the impact on an outranking statement with respect to some little modifications on the weights
- A solution is said to be robust if some "little" weights variations don't affect it
 - When the outranking statement stays the same for all sets of weights having the same preorder than the original one
 - . When modifying the criteria weights on some intervals



Condorcet denotation

We introduce the following denotation [Bisdorff 2004] :

- 3 if all criteria unanimously warrant the outranking
 - situation between x and y;
 2 if a significant majority of criteria warrants the
 outranking situation between x and y for all
 set of weights having the same preorder than W;
 if a significant majority of criteria warrants this
 outranking situation for W but not for all
 sets of weights having the same preorder than W;
 0 if W raises an indetermination;
 - -1 ...
 - -2 ...
 - -3 ...

Condorcet robustness denotation

Γ		g ₁	g ₂	g ₃
Γ	W	3.0	1.5	2.0
Γ	а	10	4	8
	b	5	6	4
	С	7	2	3
	d	5	7	2

$$3.0 + 1.5 + 2.0 = 6.5$$

$$(a, b): 3+0+2=5 > \frac{6.5}{2}$$

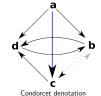


Outranking digraph

Condorcet robustness denotation

	g ₁	g ₂	g ₃		
W	3.0	1.5	2.0		
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С	7	2	3		
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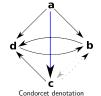
Cond	а	b	С	d
а	3	2	3	2
b	-2	3	1	2
С	-3	-1	3	2
d	-2	2	-2	3



40 P 45 P 45 P 45 P 900



	Cond	а	b	С	d
	а	3	2	3	2
I	b	-2	3	-1	2
ĺ	С	-3	1	3	2
ĺ	d	-2	2	-2	3



Condorcet robustness denotation

Condorcet robustness denotation

- · The robustness improves the strength of an outranking :
 - It is easier for a decision-maker to validate a preorder on the criteria weights
 - \bullet The comparison of alternatives with a Condorcet value equal to 2 or -2 are much more robust
- The denotation helps the decision-maker to focus on sensitive outrankings.

Robust inverse analysis

$Our\ objective$

Disaggregation of the Condorcet robustness denotation

Help the decision maker to explicit all his preferences in a robust manner via an interactive questioning protocol.

Our tools: The Condorcet robustness denotation and mathematical programming.

Starting from a complete or partial Condorcet robustness denotation, a performance table and discrimination thresholds on the criteria, we determine a set of compatible criteria weights.

⇒ Mixed integer linear programming, containing a set of generic constraints for each pair of alternatives, according to its Condorcet denotation.



Improvement of the model

Taking into account of further preferences of the decision maker :

- · Partial preorder on the weights of some criteria
- · Partial preorder on the weights of some criteria sets
- Some criteria sets which validate or invalidate the outranking relation
- Some numerical values for the weights of some criteria
- · Some intervals for the weights of some criteria

Each preferential information is modeled by a linear constraint

Web-service implementation

- Written in Python
- Also accessible via the diviz platform (cf demo)
- Mixed integer linear programing resolution
- Criteria weights must be integers (we can normalize it at the end if needed)





$Purpose\ of\ the\ sequel$

How to obtain this preferential information from a decision maker?

Preference, indifference, incomparability

- a is strictly prefered to b when a outranks b and b doesn't outrank a
- a and b are indifferent when a outranks b and b outranks a.
- a and b are incomparable when no one outranks the other one (at least one veto must be raised).

Proposal of a questioning method

Preference, indifference, incomparability

When a decision maker says "I prefer a to b" :

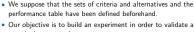
- We force the outranking of a over b, as robustly as possible
 - By adding strong "+1" constraints
 - By adding soft "+2" constraints
- ullet We force that b does not outrank a, as robustly as possible

Preference, indifference, incomparability

The particular context of our work

When a decision maker says "a and b are indifferent":

 We force the outranking in both directions, as robustly as possible



- questioning protocol.

 The construction of the criteria and the alternatives is not
- The construction of the criteria and the alternatives is not compatible with such an experiment.



The main steps of a multicriteria decision aid process (non exhaustive and in disorder)

- · Define the aim of the decision aid
- · Identify the potential alternatives
- Pre-select the alternatives
- · Identify the different points of views
- · Construct the criteria (regroup points of views, definition of the scales, ...)
- · Evaluate the alternatives on the points of views
- Collect a priori preferential information
- Present the data to the decision maker
- · Determine the various discrimination thresholds
- · Inquire interactively the decision maker

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Pre-selection of the alternatives

A priori preferential information

In the case of the choice problematique

- Each alternative must be acceptable by the decision maker (each one might be a potential candidate)
- For each criterion, the alternatives should be satisfactory (above or below a given value) (Danger: Some interesting alternatives might be left out this way!)
- Ne marche que dans le cas de la recherche du meilleur compromis!

Before presenting the data to the decision maker, determine, in an open way, the decision maker's a priori preferences on the criteria:

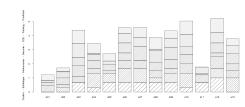
- · According to you, which are the most "important" criteria?
- · The less important ones?
- Are there some criteria which you sense as more important than other ones?
- .

40 × 48 × 48 × 48 × 40 ×

Data visualization

- Show the decision maker a set of descriptive statistics for a better understanding of the problem
 - Visualization, for each of the criteria, of the distribution of the values
 - Various representations of the performance table (data grouped by criteria or alternatives)
 - Present the weaknesses and the strengths of each of the alternatives
 - Graphical representation of the dominance relation
- Allow the decision maker to filter the performance table (remove alternatives or criteria) to better understand the problem.

Danger of the representation of the data "grouped by" alternatives



Danger of the representation of the data "grouped by" alternatives

First analysis of the data

- · Possible mix-up between valued methods and outranking ones
- · "Compensation" between the criteria
- By default, no taking into account of the weights of the criteria
- If the weights are considered, possible mix-up between compensation weights and significance weights

- On the basis of the previous visualization, we can determine some (couples of) alternatives or (couples of) criteria of particular interest
 - The well / badly performing alternatives
 - · Discriminating / non discriminating criteria
 - ...



Inquiring the decision maker

Macroscopic analysis:

- We ask the decision maker if he can "tag" some alternatives as "very good", "good", "less good" :
 - · Allows to focus on subsets of alternatives
 - We can "suppose" that the alternatives of "very good" are all preferred to those of "good", ...
 - Possibility to affect alternatives to more than one category in case of hesitation

Macroscopic analysis :

- We question the decision maker on "close" but differing alternatives:
 - Are there some pairs of alternatives where you prefer one over the other one?

Inquiring the decision maker

- Is there an alternative which is better than all the other ones?
- · We question the decision maker on conflicting alternatives

Goals : adjust the weights and thresholds!

Microscopic analysis:

- We present, to the decision maker, some couples of alternatives, and the associated elementary outranking vector
 - · Between two alternatives, is one preferred to the other one?
 - Is the criteria only in favor of the first alternative more important than those only in favor of the second alternative?

Goals: adjust the weights and thresholds!



Present the data to the decision maker

- · We shouldn't present the outranking digraph
 - . It is not intuitive for non-experts
 - · People often think/see transitivity
- We present a 2-dimensional table instead of the outranking digraph, with the following semantic:
 - Tab(a,b) = P if aPb
 - Tab(a,b) = I if alb
 - Tab(a,b) = . if bPa (⇒ aPb)
 - Tab(a,b) = ? when a and b are incomparable

- After each answer from the decision maker, we solve the mixed integer linear program, adding new constraints associated to the new statements
- Taking into account the robustness
 - · The decision maker's preferences must be as robust as possible



Conflicts resolution

- Take into account the fact that a decision maker might give some contradictory information
 - Better trust the last affirmations, as we can imagine that a
 decision maker, by a constructive effect, becomes much more
 confident with his preferences during the process.
 - · We need to make some tests

Future works Thanks a lot

- · Make some tests with "real" decision maker
 - · Define different questionning protocols
 - Have better knowledge about the real behavior of a decision maker
 - · Deal with indecisions, imprecisions
 - Deal with indecisions, imprecision
 Deal with incompatible answers
- · Improve the data visualization tools, add some new ones

http://www.decision-deck.org/ws/wsd-weightsFromCondorcetAndPreferences.html

