On polarising outranking relations with large performance differences

Raymond Bisdorff
University of Luxembourg, FSTC/CSC/ILIAS
Decision Systems Group
raymond.bisdorff@uni.lu

URPDM submission, *Rev* : 129, December 8, 2010

Abstract

We introduce a bipolarly extended veto principle – a positive, as well as negative, large performance differences polarisation – which allows us to extend the definition of the classical outranking relation in such a way that the identity between its asymmetric part and its bipolar codual relation is preserved.

Keywords: Multiple criteria decision aid; Outranking approach; Veto principle; Bipolar characteristic valuation.

Contents

1 Introduction

Recently, Pirlot and Bouyssou [8] have reported that a strict (asymmetric) outranking relation defined similarly to the classic outranking [9] is in general not identical to its codual relation, that is the converse of its negation. Indeed, from value-based orderings, we are used to think that a decision alternative x is considered strictly better than a decision alternative y, when it is not true that y is at least as good as x. Thereby, we genuinely expect the 'strictly better' relation to be asymmetric. This will however only be the case if the corresponding 'at least as good as' relation is complete, a fact which is generally not verified when dealing with classic outranking relations. This hiatus is problematic as the asymmetric part of an outranking relation is commonly identified as being in fact its codual relation.

In this paper we explore this problem in the context of our bipolar-valued credibility calculus [1, 2, 5]. Logical characteristic functions may here denote the empirical validation or not of a preferential statement with the help of three states : more true than false, more false than true, or logically indeterminate. Most important to notice is that in this bipolar setting the logical negation operation may no longer be identified with a standard set complementing. Contrary to classical logic, affirmation, as well as refutation of a preferential statement have, indeed here, both to be based on explicit, not necessarily complementary, empirical arguments.

In a first section, following the hint of Pirlot and Bouyssou [8], we illustrate formally this unsound hiatus between the asymmetric part and the codual in the case of the classic outranking concept [3]. In a second we introduce a bipolarly extended large performance difference (LPD) principle which allows us to extend the definition of the classical outranking concept in such a way that the identity between its asymmetric part and its codual is indeed given.

2 The classic outranking concept

Let $A = \{x, y, z, \ldots\}$ be a finite set of potential decision alternatives and let $F = \{1, \ldots, n\}$ be a coherent, i.e. finite, exhaustive, cohesive and non redundant, family of $n > 1$ criteria [9]. On each criterion $i \in F$, the alternatives are evaluated on a real performance scale $[0; M_i]$ supporting coherent indifference q_i and preference p_i discrimination thresholds such that $0 \leqslant q_i < p_i \leqslant M_i$ [9]. The performance of alternative x on criterion i is denoted x_i.

2.1 Overall *'at least as good'* relation

In order to characterize, between any two alternatives x and y, a marginal '*at least as good as*' situation [2, 3], we associate with each criterion i a double threshold ordering $\geqslant_i$ whose bipolar characteristic representation $r(\geqslant_i)$ is defined as follows:

$$r(x \geqslant_i y) = \begin{cases} 1 & , \text{ if } x_i + q_i \geqslant y_i \\ -1 & , \text{ if } x_i + p_i \leqslant y_i \\ 0 & , \text{ otherwise.} \end{cases}$$

Furthermore, we associate with each criterion $i \in F$ a rational *significance weight* w_i, which represents the contribution of criterion i to the *overall warrant or not* of the '*at least as good as*' preference situation between all pairs of alternatives. Let W denote the set of relative significance weights associated with F such that

$$W = \{w_i : i \in F\}, \text{ with } 0 < w_i < 1 \text{ and } \sum_{i \in F} w_i = 1.$$

Definition 2.1. *The bipolar-valued characteristic representation r of the* overall *'at least as good as' relation, denoted $\geqslant$, aggregating all the partial* at *least as good as situations $\geqslant_i$ for $i \in F$, is defined as follows*[1] *:*

$$r(x \geqslant y) \;=\; \sum_{i \in F} \big[w_i \cdot r(x \geqslant_i y) \big], \tag{1}$$

For each criterion $i \in F$, we can similarly characterize a marginal 'better than' situation between any two alternatives x and y of A. We use therefore a double threshold asymmetric ordering $>_i$ whose bipolar numerical representation $r(>_i)$ is defined as follows:

$$r(x >_i y) := \begin{cases} \;\;1 & , \text{ if } x_i - p_i \;\geqslant\; y_i \\ -1 & , \text{ if } x_i - q_i \;\leqslant\; y_i \\ \;\;0 & , \text{ otherwise.} \end{cases}$$

Again, the overall '*better than*' relation is characterized by:

$$r(x > y) \;:=\; \sum_{i \in F} \big[w_i \cdot r(x >_i y) \big], \tag{2}$$

Lemma 2.1
The asymmetric part $\gneqq$, i.e. $(x \geqslant y)$ and $(y \not\geqslant x)$, of the overall "at least as good" *relation $\geqslant$ on A is identical to the overall* "better than" *relation $>$ on A.*

Proof. Indeed, for each (x, y) in $A \times A$:

$$r(\neg(y \geqslant_i x)) = \begin{cases} -1 & , \text{ if } y_i + q_i \;\geqslant\; x_i \\ \;\;1 & , \text{ if } y_i + p_i \;\leqslant\; x_i \\ \;\;0 & , \text{ otherwise} \end{cases} \;.$$

Lemma 2.1 leads immediately to following proposition:

Proposition 2.2
The overall 'better than' *relation $>$ on A is the codual, i.e. the converse of the negation, of the overall* 'at least as good as' *relation $\geqslant$ on A.*

Proof. Following from Lemma 2.1, Formula (1) gives the same result for $(\neg \geqslant)^{-1}$ than Formula (2) gives for $>$. □

2.2 The classical veto principle

In order to characterize a marginal *veto* situation [9] between any two alternatives x and y of A, we may associate with each criterion i's performance scale $[0; M_i]$ a *veto* (v_i) discrimination threshold such that $p_i < v_i \leqslant M_i + \epsilon$.

We may thus define on each criterion $i \in F$ a threshold ordering, denoted $\ll_i$, which represents a '*seriously less performing than* situation on criterion i' and, whose bipolar characterisation $r(\ll_i)$ is given as follows:

$$r(x \ll_i y) := \begin{cases} \;\;1 & , \text{ if } x_i + v_i \;\leqslant\; y_i \\ -1 & , \text{ otherwise} \end{cases} \;.$$

[1] With $(r+1)/2$ we obtain the classical concordance index as used in the ELECTRE methods [9].

It is worthwhile noticing here that, in case $v_i = M_i + \epsilon$, the criterion i does not support any veto principle.

Definition 2.2. *The characteristic representation r of a (global)* 'veto' *situation '$(x \ll y)$' is hence defined by the overall disjunction of marginal* 'seriously less performing than' *situations:*

$$r(x \ll y) \;:=\; r\big(\bigvee_{i \in F} (x \ll_i y)\big) \;:=\; \max_{i \in F} \big[r(x \ll_i y) \big]\,. \tag{3}$$

With Definitions 2.1 and 2.2, we are ready now for defining the classical *outranking* relation.

2.3 The classical outranking relation

Definition 2.3. *An alternative x* outranks *an alternative y, denoted $x \succcurlyeq y$, when*

1. *a* significant majority *of criteria validates the fact that x is performing at least as good as y, i.e. $x \geqslant y$;*
2. *and there is no veto raised against this validation, i.e. $x \not\ll y$.*

The corresponding characteristic representation gives:

$$r(x \succcurlyeq y) \;:=\; r\big((x \geqslant y) \,\wedge\, (x \not\ll y) \big) \;=\; \min\big(r(x \geqslant y), r(x \not\ll y) \big) \tag{4}$$

Proposition 2.3 (Pirlot and Bouyssou [8])
Let $\succcurlyeq$ be a classic outranking relation.

1. *The asymmetric part $\gneqq$ of the classic outranking relation $\succcurlyeq$, i.e. $x \succcurlyeq y$ and $y \not\succcurlyeq x$, is in general not identical to its codual relation $(\not\succcurlyeq)^{-1}$.*
2. *The absence of any veto situation $\ll_i$, for i in F, is a sufficient and necessary condition for making $\gneqq$ identical to $(\not\succcurlyeq)^{-1}$.*

Proof.

(1) $r(x \gneqq y) = \min\big(r(x \succcurlyeq y), r(y \not\succcurlyeq x)\big) \;\leqslant\; r(y \not\succcurlyeq x) = r\big[\neg\big((y \geqslant x) \wedge (y \not\ll x)\big)\big] = r\big[\,(y \not\geqslant x) \vee \neg(y \not\ll x)\big] = r\big[\,(x > y) \vee (y \ll x)\big]$; the strict inequality appearing when $r(y \ll x) = 1$ and $r(x \succcurlyeq y) < 1$.

(2) $v_i = M_i + \epsilon$ for all $i \in F$ implies that $r(x \succcurlyeq y) = r(x \geqslant y)$ and the claimed identity follows from Proposition 2.2. Conversely, observing $\gneqq$ identical to $(\not\succcurlyeq)^{-1}$ implies for all $r(x \succcurlyeq y) < 1$ that $\min\big(r(x \succcurlyeq y), r(y \not\succcurlyeq x)\big) = r(y \not\succcurlyeq x)$. Hence $r(x \ll y)$ must necessarily admit the value -1 in all these cases. □

As recently reported by Pirlot and Bouyssou [8], this hiatus between the asymmetric part of the classical outranking relation and its codual raises a serious concern with respect to the logical soundness of Definition 2.3. Indeed, without this property, no sound strict preference statement can for instance be deduced from the asymmetric part of a given outranking relation. From the genuine point of view of the preferential semantics of the classical outranking concept, only the absence of any veto mechanism can guarantee this commonly expected property of converse outranking statements. But, rejecting the whole veto principle, is vanishing the very interest of Roy's original bipolar outranking concept itself [10].

In the next section we therefore introduce and extended bipolar veto principle.

3 Bi-polarising with large performance differences

From Proposition 2.3 we get the hint that the veto principle is in fact the concept that has to put into a bipolar epistemic setting in order to overcome the previous hiatus and recover the full power of the original outranking concept.

3.1 Taking into account large performance differences

Our bipolar-valued logical characterisation, with its potential indeterminate value, will allow us defining a veto situation which corresponds in fact to a dual counter-veto situation. On each criterion i, let v_i ($p_i < v_i \leqslant M_i + \epsilon$) denote a *large negative performance difference* (LPD) that appears, in the eyes of the decision maker, hardly compensable with positive performance differences potentially observed on other criteria. We denote $x \lll_i y$ such a marginal '*seriously worse performing than*' situation on criterion i, when the counter-performance of x with respect to y is larger or equal to v_i. Its bipolar characteristic representation r is defined as follows:

$$r(x \lll_i y) := \begin{cases} \;\;1 & , \text{ if } x_i + v_i \;\leqslant\; y_i \\ -1 & , \text{ if } x_i - v_i \;\geqslant\; y_i \\ \;\;0 & , \text{ otherwise} \end{cases} . \tag{5}$$

The dual situation, denoted $\ggg_i$, represents the corresponding converse '*seriously better performing than*' situation. Following from the semantics of the (strict) negation in our bipolar credibility calculus, the above defined $\lll_i$ and the $\ggg_i$ situations define on A two binary relations that are the codual one of the other. Indeed, from Formula (5) it is readily noticed that $r\big(\neg(x \lll_i y)\big) = -r(x \lll_i y) = r(x \ggg_i y)$. In case $v_i = M_i + \epsilon$, the criterion i will show neither $\lll_i$, nor $\ggg_i$ situations.

The bipolar characteristic representation of a global 'veto' situation $\lll$ (respectively its dual 'counter-veto' situation $\ggg$) is now given by the disjunctive aggregation of all marginal $\lll_i$ situations:

$$r(x \lll y) \;:=\; \bigoplus_{i \in F} \big[r(x \lll_i y)\big] \tag{6}$$

where $\oplus$ represents the bipolar sharpening operator, also called *'epistemic disjunction'* [6, 7], and defined as follows: $r \oplus r'$ equals $\max(r, r')$ if $r \geqslant 0$ and $r' \geqslant 0$; $\min(r, r')$ if $r \leqslant 0$ and $r' \leqslant 0$, and 0 otherwise.

We may thus observe that $r(x \lll y) = 1$ iff there exists a criterion i in F such that $r(x \lll_i y) = 1$, and there does not exist any criterion j in F such that $r(x \ggg_i y) = 1$. Or, conversely, $r(x \ggg y) = 1$ iff there exists a criterion i in F such that $r(x \ggg_i y) = 1$, and there does not exist any criterion j in F such that $r(x \lll_i y) = 1$.

Lemma 3.1
The codual $(\not\lll)^{-1}$ *of the global* 'seriously worse performing than' *relation* $\lll$ *on* A *is identical to the global* 'seriously better performing than' *relation* $\ggg$ *on* A.

Proof. It is sufficient to recall that the marginal situations $\lll_i$ and $\ggg_i$ are codual one to the other and to notice that the $\oplus$ operator used in Formula (6) is auto-dual. □

3.2 The bipolar outranking concept

Let x and y be two decision alternatives. From a bipolar point of view, we say that:

1. '*x outranks y*', denoted $x \succsim y$, if a significant majority of criteria validate a global outranking situation between x and y and no serious counter-performance is observed on a discordant criterion,

2. '*x does not outrank y*', denoted $x \not\succsim y$, if a significant minority only of criteria validate a global outranking situation between x and y and no seriously better performance is observed on a concordant criterion,

3. the statement '*x outranks y*' can *neither be validated nor invalidated* if we conjointly observe some seriously worse *as well as* some seriously better performances.

We say that In terms of our bipolar numeric representation r we obtain the following formal definition:

Definition 3.1 (Outranking with bipolar veto).

$$r(x \succsim y) := \begin{cases} 0, \quad \textit{if} \quad \exists\, i, j \in F : \big[r(x \lll_i y) \,=\, 1\big] \,\wedge\, \big[r(x \ggg_j y) \,=\, 1\big]; \\ \big[r(x \geqslant y) \,\oplus\, r(x \not\lll y)\big], \quad \textit{otherwise.} \end{cases} \tag{7}$$

If, on the one hand, $v_i = M_i + \epsilon$ for all $i \in F$, i.e. in the absence of any vetoes, $r(x \not\lll y) \,=\, 0$ and we recover the previous case where $r(x \succsim y) \,=\, r(x \succcurlyeq y) \,=\, r(x \geqslant y)$. If, on the other hand, we observe conjointly seriously better and worse performances on some criteria i and j, the validation or invalidation of the outranking statement '$x \succsim y$' gets doubtful. Consequently, its bipolar characteristic value is put to 0, i.e. the indeterminate case.

However, if we observe solely some seriously better performing situations ($r(x \ggg y) \,=\, 1$), coupled with a significant majority of validating criteria – $r(x \geqslant y) \geqslant 0$–, the outranking situation appears certainly validated: $r(x \succsim y) \,=\, 1$. Or, we may solely observe some seriously worse performing situations – $r(x \lll y) \,=\, 1$ –, coupled with a significant minority only of validating criteria – $r(x \geqslant y) \leqslant 0$ –, the outranking situation appears certainly invalidated: $r(x \succsim y) \,=\, -1$.

In all the other cases, i.e. – a minority only of validating criteria, but, some seriously better performance observed otherwise, or, a majority of validating criteria, but, some seriously worse performance observed otherwise –, the outranking statement, due to the potentially non compensable large performances we observe, may neither be validated, nor, invalidated anymore: consequently $r(x \succsim y) \,=\, 0$.

Let us now show that, with the bipolarly extended veto principle, the codual $(\not\succsim)^{-1}$ of the bipolar outranking relation $\succsim$ is indeed equal to its asymmetric part $\succnsim$.

Proposition 3.2
If $\succsim$ is a bipolar outranking relation, $\succnsim$ its corresponding asymmetric part, and $(\not\succsim)^{-1}$ its codual, then:

$$r(x \succnsim y) \;=\; r\big[(x \not\succsim y)^{-1}\big] \quad \forall (x, y) \in A^2. \tag{8}$$

Proof.

$$\begin{aligned} r\big[(x \not\succsim y)^{-1}\big] \;&=\; -\big\{\big[r(y \geqslant x) \,\oplus\, r(y \not\lll x)\big]\big\} && \text{(the negation of Formula (7)),} \\ &=\; \big[r(y \not\geqslant x) \,\oplus\, r(y \lll x)\big] && \text{(by Lemma 3.1),} \\ &=\; \big[r(x > y) \,\oplus\, r(x \not\lll y)\big] && \text{(by Proposition 2.2 and Lemma 3.1).} \end{aligned}$$

3.3 Numerical illustration

Let us consider performance evaluations (see Table 1) of five potential decision alternatives a_1, a_2, ..., a_5 we may observe on a coherent family of five criteria. All criteria: g_1, g_2, ..., g_5 , with given

criteria	w_i	a_1	a_2	a_3	a_4	a_5
g1	4/13	17.3	25.5	76.1	32.7	0.8
g2	1/13	7.3	5.7	53.0	99.7	91.3
g3	4/13	90.3	79.8	93.0	48.2	11.4
g4	2/13	13.2	94.1	48.9	49.9	19.6
g5	2/13	42.4	16.1	64.8	78.2	81.1

Table 1: Random performance tableau

significance weights w_i (see Table 1, column 2), admit a real performance scale running from 0.0 (worst level) to 100.0 (best level) supporting three discrimination thresholds: *indifference* ($\pm$10.0), *preference* (20.0) and, *seriously better or worse performing* ($\pm$60.0). The resulting bipolar-valued characterization of the overall '*at least as good as*' relation $\geqslant$ (see Definition 1) is shown below in Table 2 as intger multiples of 1/13. Below each characteristics, which may thus take any integer value between -13 to $+13$, we mention within brackets the number of large, positive ($+n$) and/or negative ($-n$) performance differences ($\geqslant$ 60.0), we observe in the pairwise comparisons of the marginal performances on each criterion.

$r(x \geqslant y) \cdot \frac{1}{13}$	a_1	a_2	a_3	a_4	a_5
a_1	–	$+\mathbf{9}$ (-1)	-5	-5 (-1)	$+7$ $(-\mathbf{1}, +\mathbf{1})$
a_2	$+\mathbf{5}$ $(+1)$	–	-5	$+7$ (-2)	$+7$ $(+\mathbf{2}, -\mathbf{2})$
a_3	$+13$	$+9$	–	$+9$	$+9$ $(+2)$
a_4	$+5$ $(+1)$	$+1$ $(+2)$	-3	–	$+13$
a_5	$+1$ $(+1, -1)$	-7 $(+2, -2)$	-7 (-2)	-7	–

Table 2: $r(x \geqslant y)$ characteristic values with, in brackets below, the number of positive, respectively negative, large performance differences

Let us first notice that, due to the potential indifference and indeterminate part of the $\geqslant$ relation, the $r(\geqslant)$ characteristic values are generally not symmetric: $r(a_1 \geqslant a_2) = +9/13$ for instance, whereas $r(a_2 \geqslant a_1) = +5/13$. However, $\lll$ and $\ggg$ being codual one to another (see Lemma 3.1), $r(a_1 \lll a_2) = -1$ implies $r(a_2 \ggg a_1) = +1$. Notice that the pairwise comparison between a_1 and a_5 (see Table 2 row a_1 right end position) shows conjointly a positive and a negative large performance difference. The comparison between a_2 and a_5 shows even, both, two positive and two negative large performance differences.

In Table 3, we show the eventual polarisation of the $\geqslant$ characteristics that results from taking into account all negative, as well as positive, large performance differences. We indicate in brackets

$r(x \succsim y)$	a_1	a_2	a_3	a_4	a_5
a_1	–	**0.0** (−**1.0**)	−.38	−1.0	0.0 (−1.0)
a_2	**+1.0** (+.38)	–	−.38	0.0	0.0 (−1.0)
a_3	**+1.0**	**+.69**	–	**+.69**	**+1.0** (+.69)
a_4	**+1.0** (+.38)	**+1.0** (+.08)	−.23	–	**+1.0**
a_5	0.0 (−1.0)	0.0 (−1.0)	−1.0	−.54	–

Table 3: Large performance differences polarised $x \succsim y$ characteristics with classical outranking in brackets when different

the corresponding classical veto polarisation $r(x \succcurlyeq y)$, when it is different from the actual bipolar $r(x \succsim y)$ one.

Let us reconsider the comparison between a_1 and a_2 (see row a_1 in Table 3). Whereas a highly significant majority of nearly 85% [2], all criteria, except g_4 validate the '*at least as good as*' statement. But, on criterion g_4, a_1 shows a very serious counter-performance of −80.9 when compared to a_2, a performance difference much larger than the already potentially non compensable considered difference threshold of 60.0. Following the classical veto principle, we would therefore certainly invalidate this outranking. But, as we have seen in Proposition 2.3, this certain invalidation breaks in fact the identity between the asymmetric part and the codual of the global outranking relation. In the bipolar setting therefore, this contradictory observations, a significant majority validating coupled with a serious counter-performance, will put instead doubt on the validation, *as well as on the invalidation*, of the outranking situation: $r(a_1 \succsim a_2) = 0.0$).

An even more unbalanced situation may be observed when comparing the performances of alternative a_1 with those of alternative a_5 (see Table 1). Here, all criteria, except g_2, with a significance of 77%, validate the outranking situation. Furthermore, a seriously better performance of +79.8 on criterion g_3 even reenforces this validation. However, on g_2, the counter-performance is again much larger (−84.1) than the non-compensable difference threshold of 60.0. Again, the classical outranking approach would certainly invalidate this outranking, whereas in the bipolar approach, we preferably put to doubt any validation as well as invalidation of this kind of outranking statement. We definitely can't know in an ordinal outranking approach what the aggregation of a very large positive and a very large negative performance may give as preferential result. It is therefore not indicated to validate this statemnt. At the same time it is as well not indicated, for the same reasons, to invalidate it. In the absence of compensable performance measures on each criteria, and to be prudent in our preferential constructions, we may only retain here all determined preferential judgments.

Let us, finally, consider those pairwise comparisons where a significant majority of criteria indeed validate an outranking, and where we observe, furthermore, some seriously better performances. In Table 2, we may notice, for instance, the comparisons between a_2 and a_1, between a_3 and a_5, and the comparisons between a_4 and a_1 or a_2. In all these cases, we may for certain validate these outrankings, a fact, not taken into account at all in the classical definition of the outranking concept.

[2] The conversion from the bipolar characteristic to simple majority percentage is computed as follows: $(r(\geqslant) + 1.0)/2.0$, i.e. that is a positive shift of one and a rescaling to the unity. For instance $r(a_1 \geqslant a_2) = 9/13$ and we obtain $(9 + 13)/26 = 11/13 = 0.8461$.

4 Conclusion

In this paper we have introduced a new bipolar veto and counter-veto principle which allows us to construct an extended global outranking relation that guarantees the formal identity of the corresponding strict outranking relation, – its asymmetric part, with its associated codual relation.

In the classical outranking definition, the conjointly invalidation of the outranking statement and its converse, models in fact an incomparability situation that captures the difficulty to compensate excellent performances with serious counter-performances. As formally shown, this apprfoach introduces. however, a hiatus between the asymmetric part and the codual of this outranking relation. To overcome this logical unsoundness, we rather rely in our characterisation of the actual outranking situation via the neutral value of the bipolar characteristic calculus for expressing our doubts concerning the effective compensation of such contrasted performances.

As a consequence of this new conceptual design, we preserve, on the one hand, all the original and interesting concordance versus discordance semantics of the classical outranking concept. On the other hand, we do not loose anymore the sound semantics of the logical negation of the actual outranking statement. Thus appeqars a much more prudent and robust outranking concept, that allows, in practice, to take coherently into account large non-compensable performance differences which may put to serious doubt otherwise significant majorities of concordant or discordant criteria.

Enhanced and simpler operational semantics for selectiong the best choice [3], or quick multiple criteria sorting, become potentially available. Also, inverse multiple criteria decision analysis, where preferential model parameters like criteria weights are inferred from indirect observation of global preferences [4], may now effictively tackle large non compensable performances differences and thereby use all the expressive power of the genuine outranking approach.

References

[1] R. Bisdorff. Logical foundation of fuzzy preferential systems with application to the electre decision aid methods. *Computers & Operations Research*, 27:673–687, 2000. 2

[2] R. Bisdorff. Logical foundation of multicriteria preference aggregation. In Bouyssou D et al., editor, *Aiding Decisions with Multiple Criteria*, pages 379–403. Kluwer Academic Publishers, 2002. 2

[3] R. Bisdorff, P. Meyer, and M. Roubens. Rubis: a bipolar-valued outranking method for the best choice decision problem. *4OR: A Quarterly Journal of Operations Research*, 6(2):143 – 165, june 2008. 2, 9

[4] R. Bisdorff, P. Meyer, and Th. Veneziano. Inverse analysis from a condorcet robustness denotation of valued outranking relations. In F. Rossi and A. Tsoukiàs, editors, *Algorithmic Decision Theory*, volume 5783 of *LNAI*, pages 180–191. Springer, 2009. 9

[5] R. Bisdorff, M. Pirlot, and M. Roubens. Choices and kernels from bipolar valued digraphs. *European Journal of Operational Research*, 175:155–170, 2006. 2

[6] Raymond Bisdorff. On computing fuzzy kernels from l-valued simple graphs. In *5th European Congress on Intelligent Techniques and Soft Computing EUFIT'97*, volume 1, pages 97–103, 1997. 5

[7] Michel Grabisch Jean-Luc Marichal, Radko Mesiar, and Endre Pap. *Aggregation Functions*. Encyclopedia of Mathematics and Its Application. Cambridge University Press, 2009. 5

[8] Marc Pirlot and Denis Bouyssou. Analysing the correspondence between strict and non-strict outranking relations. In Erwin Pesch and Gerhard Woeginger, editors, *23rd European Conference on Operational Research: Book of Abstracts*, Bonn, July 2009. 1, 2, 4

[9] B. Roy and D. Bouyssou. *Aide Multicritère à la Décision : Méthodes et Cas*. Economica, Paris, 1993. 1, 2, 3

[10] Bernard Roy. The outranking approach and the foudations of the ELECTRE methods. *Theory and Decision*, 31(1):49–73, 1991. 4