

The "American" way

- Aggregation of all the criteria in a unique criterion
- Each alternative is associated with a score (a global value)
- Ex : Weighted sum, weighted mean, Choquet integral, ...

The “European” way

- Based on pairwise comparisons of alternatives
- Outranking methods and graph theory

- An alternative a outranks an alternative b when a significant majority of criteria validates the fact that a is performing at least as good as b and there is no criterion where b seriously outperforms a [Roy]
- ELECTRE and Rubis methods are examples of outranking methods

Inverse analysis

- Help a decision maker to identify all his preferences
 - By inquiring directly some method parameters
or
 - By asking him to explicit some partial a priori preferential information (obvious)
- Characterize the quality of a solution, in terms of robustness, satisfaction of all the process actors....

CONDORCET ROBUSTNESS DENOTATION

Criteria weights preorder :

- Two sets of criteria weights are preorder-compatible if they have the same preorder on their criteria weights.
- Ex : $W = \{3, 7, 5, 1, 7\}$ and $W' = \{4, 6, 5, 2, 6\}$ have the same preorder $\{w_2, w_5\} \succ \{w_3\} \succ \{w_1\} \succ \{w_4\}$

Condorcet denotation

We introduce the following denotation [Bisdorff 2004] :

$$Cond_w(x, y) = \begin{cases} 3 & \text{if all criteria *unanimously warrant* the outranking situation between } x \text{ and } y; \\ 2 & \text{if a *significant majority* of criteria *warrants* the outranking situation between } x \text{ and } y \text{ for all set of weights having the same preorder than } W; \\ 1 & \text{if a significant majority of criteria *warrants* this outranking situation for } W \text{ but not for all sets of weights having the same preorder than } W; \\ 0 & \text{if } W \text{ raises an indetermination;} \\ -1 & \dots \\ -2 & \dots \\ -3 & \dots \end{cases}$$

Robustness of an outranking :

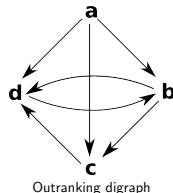
- We analyze the impact on an outranking statement with respect to some little modifications on the weights
- A solution is said to be **robust** if some "little" weights variations don't affect it
 - When the outranking statement stays the same for all sets of weights having the same preorder than the original one
 - When modifying the criteria weights on some intervals

Condorcet robustness denotation

	g ₁	g ₂	g ₃
W	3.0	1.5	2.0
a	10	4	8
b	5	6	4
c	7	2	3
d	5	7	2

$$3.0 + 1.5 + 2.0 = 6.5$$

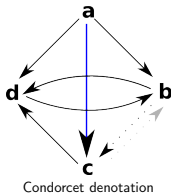
$$(a, b) : 3 + 0 + 2 = 5 > \frac{6.5}{2}$$



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d	5	7	2

Cond	a	b	c	d
a	3	2	3	2
b	-2	3	1	2
c	-3	-1	3	2
d	-2	2	-2	3

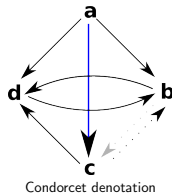


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b	-2	3	-1	2
c	-3	1	3	2
d	-2	2	-2	3



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Condorcet robustness denotation

- The robustness improves the strength of an outranking :
 - It is easier for a decision-maker to validate a preorder on the criteria weights
 - The comparison of alternatives with a Condorcet value equal to 2 or -2 are much more robust
- The denotation helps the decision-maker to focus on sensitive outrankings.

ROBUST INVERSE ANALYSIS

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Our objective

Help the decision maker to explicit all his preferences in a robust manner via an interactive questioning protocol.

Our tools : The Condorcet robustness denotation and mathematical programming.

Disaggregation of the Condorcet robustness denotation

Starting from a complete or partial Condorcet robustness denotation, a performance table and discrimination thresholds on the criteria, we determine a set of compatible criteria weights.

⇒ Mixed integer linear programming, containing a set of generic constraints for each pair of alternatives, according to its Condorcet denotation.

Improvement of the model

Taking into account of further preferences of the decision maker :

- Partial preorder on the weights of some criteria
- Partial preorder on the weights of some criteria sets
- Some criteria sets which validate or invalidate the outranking relation
- Some numerical values for the weights of some criteria
- Some intervals for the weights of some criteria

Each preferential information is modeled by a linear constraint

Web-service implementation

- Written in Python
- Also accessible via the diviz platform (cf demo)
- Mixed integer linear programming resolution
- Criteria weights must be integers (we can normalize it at the end if needed)

Purpose of the sequel

How to obtain this preferential information from a decision maker?

PROPOSAL OF A QUESTIONING METHOD

Preference, indifference, incomparability

- a is **strictly preferred** to b when a outranks b and b doesn't outrank a .
- a and b are **indifferent** when a outranks b and b outranks a .
- a and b are **incomparable** when no one outranks the other one (at least one veto must be raised).

Preference, indifference, incomparability

When a decision maker says "I prefer a to b " :

- We force the outranking of a over b , as robustly as possible
 - By adding strong "+1" constraints
 - By adding soft "+2" constraints
- We force that b does not outrank a , as robustly as possible

When a decision maker says " a and b are indifferent" :

- We force the outranking in both directions, as robustly as possible

- We suppose that the sets of criteria and alternatives and the performance table have been defined beforehand.
- Our objective is to build an experiment in order to validate a questioning protocol.
- The construction of the criteria and the alternatives is not compatible with such an experiment.

The main steps of a multicriteria decision aid process (non exhaustive and in disorder)

- Define the aim of the decision aid
- Identify the potential alternatives
- Pre-select the alternatives
- Identify the different points of views
- Construct the criteria (regroup points of views, definition of the scales, ...)
- Evaluate the alternatives on the points of views
- Collect a priori preferential information
- Present the data to the decision maker
- Determine the various discrimination thresholds
- Inquire interactively the decision maker

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In the case of the choice problematic

- Each alternative must be acceptable by the decision maker (each one might be a potential candidate)
- For each criterion, the alternatives should be satisfactory (above or below a given value) (**Danger** : Some interesting alternatives might be left out this way !)
- Ne marche que dans le cas de la recherche du meilleur compromis !

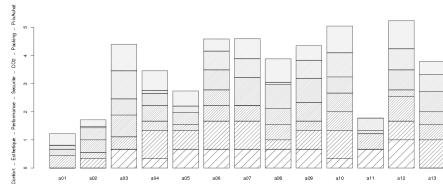
Data visualization

- Show the decision maker a set of descriptive statistics for a better understanding of the problem
 - Visualization, for each of the criteria, of the distribution of the values
 - Various representations of the performance table (data grouped by criteria or alternatives)
 - Present the weaknesses and the strengths of each of the alternatives
 - Graphical representation of the dominance relation
- Allow the decision maker to filter the performance table (remove alternatives or criteria) to better understand the problem.

Before presenting the data to the decision maker, determine, in an open way, the decision maker's a priori preferences on the criteria :

- According to you, which are the most "important" criteria ?
- The less important ones ?
- Are there some criteria which you sense as more important than other ones ?
- ...

Danger of the representation of the data "grouped by" alternatives



Microscopic analysis :

- We present, to the decision maker, some couples of alternatives, and the associated elementary outranking vector
 - Between two alternatives, is one preferred to the other one ?
 - Is the criteria only in favor of the first alternative more important than those only in favor of the second alternative ?

Goals : adjust the weights and thresholds !

- After each answer from the decision maker, we solve the mixed integer linear program, adding new constraints associated to the new statements
- Taking into account the robustness
 - The decision maker's preferences must be as robust as possible

Present the data to the decision maker

- We shouldn't present the outranking digraph
 - It is not intuitive for non-experts
 - People often think/see transitivity
- We present a 2-dimensional table instead of the outranking digraph, with the following semantic :
 - $\text{Tab}(a,b) = P$ if aPb
 - $\text{Tab}(a,b) = I$ if aIb
 - $\text{Tab}(a,b) = .$ if $bPa \Rightarrow a\bar{P}b$
 - $\text{Tab}(a,b) = ?$ when a and b are incomparable

Conflicts resolution

- Take into account the fact that a decision maker might give some contradictory information
 - Better trust the last affirmations, as we can imagine that a decision maker, by a constructive effect, becomes much more confident with his preferences during the process.
 - We need to make some tests

- Make some tests with "real" decision maker
 - Define different questioning protocols
 - Have better knowledge about the real behavior of a decision maker
 - Deal with indecisions, imprecisions
 - Deal with incompatible answers
- Improve the data visualization tools, add some new ones

<http://www.decision-deck.org/ws/wsd-weightsFromCondorcetAndPreferences.html>