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5. All mathematics must be typewritten and special symbols must be identified properly. The letter symbols used should be defined where they first appear, in figures, tables, or text, and arranged alphabetically in an appendix at the end of the paper titled Appendix.—Notation.

6. Standard definitions and symbols should be used. Reference should be made to the lists published by the American National Standards Institute and to the *Authors' Guide to the Publications of ASCE*.

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9. References cited in text should be arranged in alphabetical order in an appendix at the end of the paper, or preceding the Appendix.—Notation, as an Appendix.—References.

10. A list of key words and an information retrieval abstract of 175 words should be provided with each paper.

11. A summary of approximately 40 words must accompany the paper.

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13. Dual units, i.e., U.S. Customary followed by SI (International System) units in parentheses, should be used throughout the paper.

14. A practical applications section should be included also, if appropriate.

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Criteria for Settlement of Tanks

By W. Allen Marr,[1] M. ASCE, Jose A. Ramos,[2] and T. William Lambe,[3] F. ASCE

Abstract: Performance criteria for the settlement of large steel tanks used to store fluids at ambient temperature and pressure are presented. Where possible, the criteria include a factor of safety, defined as the ratio of tensile stress to developed stress. Permissible values of factor of safety for safe operation are indicated. The performance criteria are developed by interpreting and expanding previous studies, making approximate analyses, and analyzing the measured performance of 90 large tanks, including the tanks at Toa Nenryo Kogyo's site at Kawasaki City, Japan. For each criterion, the mechanism of failure, the structural element to which the criterion applies, and the basis for the criterion are identified.

Nature of Foundation Settlement

This paper presents performance criteria for the differential settlement of steel tanks used to store fluids. We use work of other engineers, simplified analyses, and measurements of field performance to establish these criteria.

Many engineers incorrectly believe that differential settlement poses little threat to large, flexible storage tanks. However, differential settlement has led to rupture of large tanks (Bell, 1980; Clarke, 1969; and Green and Hight, 1974). Disagreement exists among engineers, builders, and regulators on limiting values of settlement.

Fig. 1 shows criteria for settlement of various facilities. At Toa Nenryo Koygo's (TONEN) refinery in Kawasaki City, Japan, (Lambe, 1969), tanks 64 m (210 ft) in diam have undergone average settlements of up to 1.8 m (6 ft). TONEN engineers have measured average tilts of up to 1/152, and nonuniform settlement of up to 0.5 m (1.6 ft). Clearly, measured deformations of TONEN's tanks have exceeded criteria given in Fig. 1. However, surveillance has revealed no evidence of structural distress to any of these tanks.

Fig. 2 shows patterns of settlement. Most tanks settle in a combination of these patterns. The geotechnical engineer considers differential settlement as the dif-

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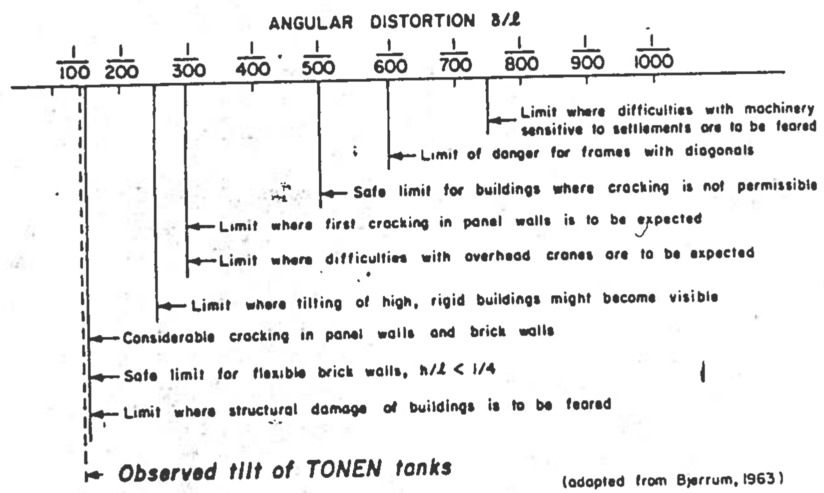


LIMITING ANGULAR DISTORTION

TYPE OF MOVEMENT	LIMITING FACTOR	MAXIMUM SETTLEMENT	MEASURED ON TONEN TANKS
TOTAL SETTLEMENT	Drainage	150-300 mm	1,800 mm
	Access	300-600 mm	
	Probability of nonuniform settlement		
	Masonry walled structure	25-50 mm	
	Framed structures	50-100 mm	
	Smokestacks, silos, mats	75-300 mm	
TILTING	Stability against over turning	Depends on height and width	0.007 ℓ
	Tilting of smokestacks, towers	0.004 ℓ	
	Rolling of trucks, etc.	0.01 ℓ	
	Stacking of goods	0.01 ℓ	
	Machine operation-cotton loom	0.003 ℓ	
	Machine operation-turbogenerator	0.0002 ℓ	
	Crane rails	0.003 ℓ	
	Drainage of floors	0.01 - 0.02 ℓ	
DIFFERENTIAL MOVEMENT	High continuous brick walls	0.0005-0.001 ℓ	0.08 ℓ
	One story brick mill building, wall cracking	0.001-0.002 ℓ	
	Plaster cracking (gypsum)	0.001 ℓ	
	Reinforced concrete building frame	0.0025-0.004 ℓ	
	Reinforced concrete building curtain walls	0.003 ℓ	
	Steel frame, continuous	0.002 ℓ	
	Simple steel frame	0.005 ℓ	

From Sowers, 1962
Note: ℓ = distance between adjacent columns that settle different amounts, or between any two points that settle differently. Higher values are for regular settlements and more tolerant structures. Lower values are for irregular settlements and critical structures. 1 inch = 25.4 mm

ALLOWABLE SETTLEMENT LIMIT

FIG. 1.—Settlement Criteria for Typical Facilities

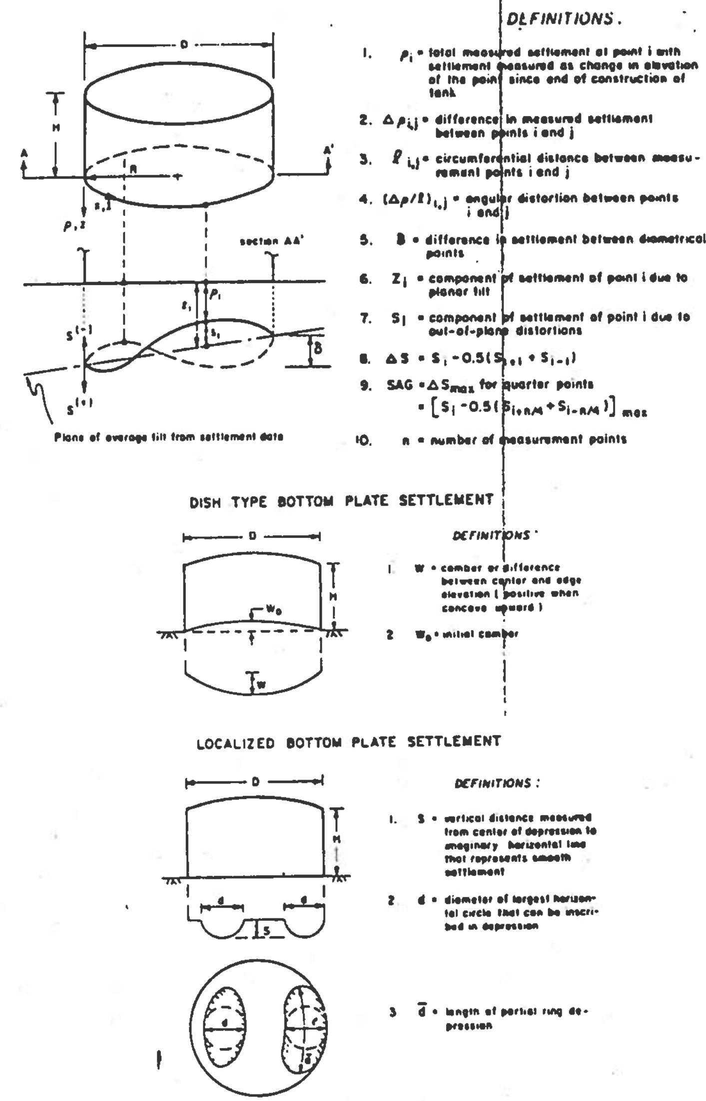


FIG. 2.—Nomenclature for Settlement of Tank

ference in vertical deformation between two points at the foundation-structure interface.

Differential settlement originates from one or more of the following causes: nonhomogeneous geometry or compressibility of the soil deposit, nonuniform distribution of the load applied to the foundation, and uniform stress acting over a limited area of the soil stratum. These causes exist to varying degrees of importance in every tank foundation. The engineer assesses their importance in developing an acceptable design. He seeks to minimize differential settlement by keeping: (1) The applied load less than the bearing capacity of the foundation; and (2) deformations arising from volume and shear strains in the foundation within permissible limits. This paper recommends permissible limits.

Differential Settlement for Steel Tanks

A storage tank consists of four main structural elements: shell, bottom plate, connection of shell to bottom plate, and roof. Each settlement pattern may influence each of these components. Fig. 3 describes detrimental settlement patterns that a tank foundation may develop, the probable foundation conditions which produce each pattern, and adverse consequences that could result from each.

Each of these settlement patterns may produce one or more modes of failure. Failures of most concern are: (1) Distortion of shell such that the floating roof malfunctions; and (2) rupture of the shell or bottom plate or shell-bottom plate connection.

Fig. 3 also relates the principal modes of failure to each settlement pattern. Criteria must protect against all failure modes.

Engineers and design groups have proposed various criteria for differential settlement of tanks. These criteria usually focus on a particular structural element of the tank and a single mode of failure for a particular set of conditions. Some practicing engineers generalize these specific criteria too far and apply them incorrectly. For the criteria described in this paper, we identify the mechanism of failure implied by each criterion, the structural element to which it applies, and the basis for each criterion. We use the symbols and notation shown in Fig. 2 and defined in Appendix III to standardize the results quoted from the literature. By settlement, we mean the change in elevation of a point from the time that point becomes structurally joined to the tank. Practically, measurements of change in elevation usually start when the bottom plate and shell are completed. Appendix I describes several methods of analyzing measured settlement data that various engineers use to calculate differential settlement parameters.

Fig. 3 summarizes the principal criteria in use today. This figure shows several apparent and significant differences in criteria for each tank component. The remaining sections of this paper use these criteria and arrive at recommended criteria for each tank component.

Criteria for Shell

Uniform Settlement.—Uniform settlement causes little concern to engineers. Uniform settlement of the shell may damage piping connections due to the differential settlement between the shell and external pipe supports, and, thus, it may hinder operations. One can avoid this minor problem by using flexible connections or by periodically repositioning the pipe supports.

Planar Tilt.—Planar tilt of the shell by itself has relatively little importance. Tilting of the shell reduces freeboard, alters the shape of the fluid surface, and places additional stresses in the shell. Using statics and geometrical considerations, one can show that the maximum tilt one can sustain without overstressing the shell equals:

	SETTLEMENT PATTERNS	PROBABLE FOUNDATION	MODE OF FAILURE and CONSEQUENCE	SETTLEMENT CRITERIA AND RECOMMENDATIONS: Lambe & Assoc. (1961)	Longeveld (1974)	Hayashi (1973) Guber (1974)	Greenwood (1974)	Others	Japanese Fire Defense Agency
SHELL	Planar Tilt (δ_{hor}, δ)	Linearly variable soil thickness or soil compressibility	Differential settlement between shell and pipe supports causes breaks in pipes	No problem	$\delta \leq 50$ cm. $\delta_{hor} \leq 30$ cm.	Little problem	$\delta \leq D/200$ for aesthetics	Little problem	$\delta \leq D/100$
SHELL	Non-Planar Settlement (ΔR(ovality); Ave Plane of tilt; ρ_i)	Variable soil thickness or compressibility	Loss of freeboard results in oil spill or distortion of cone roof; Ovalization of fluid surface causes loss of seal on floating roof; Overstress causes rupture of shell	$SAG \leq D/267$ excess stress; $SAG \leq 0.25 \frac{D}{H} \Delta R_{tol}$ ovality	$S \leq (0.2L^2/HD)\Delta R_{tol}$ ovality	$\Delta \rho / L \leq 1/180$	For $D < 50$ m $S_{max} \leq 4$ cm; For $D \geq 50$ m $S_{max} \leq 60$ cm ovality	DeBeer (1969) $\frac{\Delta S}{\ell} \leq \frac{1}{450}$; Sullivan & Nowicki (1974) $S \leq 3.0$-4.5 cm; Malik (1977) $\Delta S \leq (\ell^2/HD)\Delta R_{tol}$; Penman (1978) $SAG \leq (D/4H)\Delta R_{tol}$	$\Delta \rho \leq D/100$
BOTTOM PLATE	Dish-Type Settlement (W)	Soil of uniform thickness and compressibility	Excessive stresses rupture bottom plate spilling contents; Deformation prevents complete drainage or distorts supports for roof	No problem	$W \leq D/100$	$W <$ D/90 to D/50	Little problem	Rinne (1963) $W \leq D/90$	$W \leq D/100$
BOTTOM PLATE	Localized Settlement (S, d)	Localized pockets of highly compressible soil.		$(S/\ell) \leq 1/30$	Visual inspection of tank bottom after water test	$S <$ d/90 to d/50	Little problem	Not considered	$S \leq D/100$
SHELL-BOTTOM PLATE CONNECTION	Non-Planar Settlement (Original, Deformed, S)	Localized pockets of compressible soil. May also result from inadequate foundation preparation.	Excessive stresses rupture connection spilling contents	Use prepared foundation pad to prevent		$S \leq \frac{d}{50}$ to $\frac{d}{30}$	Not considered	Not considered	Use prepared foundation pad to prevent $S \leq D/100$

1 inch = 2.54 cm

FIG. 3.—Differential Settlement Criteria from Literature

$$\delta_{max} \le \frac{4\sigma_f (t_{max} - CA)}{FS \cdot \gamma_w \cdot G_s \cdot D} - 2(H - \Delta h_d) \qquad (1)$$

in which H = tank height; Δh_d = design freeboard; σ_f = rupture stress of the steel in the shell; t_{max} = thickness of shell; CA = corrosion allowance to shell thickness; FS = factor of safety against rupture of shell; γ_w = unit weight of water; G_s = specific gravity of stored fluid; and D = diameter of tank. Design procedures for tank shells usually require a factor of safety of 2.4. Substituting typical values into Eq. 1 shows that a 10% increase in hoop stress at the bottom of the shell results from a planar tilt of about $D/20$. Langeveld (1974) recommends limiting the maximum differential settlement between diametrical points to 50 cm (20 in.), combined with a 30 cm (12 in.) limit to the out-of-plumb projection of the tank wall. These restrictions limit the additional hoop stress to less than 2% of the maximum hoop stress from normal fluid pressure and prevent radial distortion of the top of the shell from exceeding 2.5 cm (1 in.). Failure of the shell from planar tilt alone seems unlikely. As a practical consideration, maximum tilt, δ_{max}, must not exceed $2\Delta h_d$ to avoid spilling oil from floating roof tanks or stressing the roof of cone-roof tanks. Furthermore, one must restrict the tilt for floating roofs so that the change in diameter caused by rotation of the shell relative to the roof does not exceed the tolerance of the seal. This requirement restricts tilt to

$$\delta_{max} \le 2\sqrt{R_{tol} \cdot D} \qquad (2)$$

in which ΔR_{tol} equals the tolerance of the seal.

Nonplanar Settlement.—Nonplanar settlement may radially distort the shell or overstress the shell. Radial distortion of the shell, called ovality, may cause a floating roof to malfunction. Overstress may cause rupture and spillage of contents of the tank.

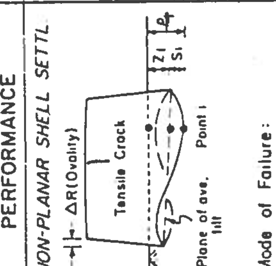
Ovality.—Fig. 4 summarizes existing criteria for nonplanar settlement of the shell, and identifies whether each criterion is based on limiting ovality or stress. Lambe (1961), Langeveld (1974), Malik, et al. (1977) and Penman (1977) give criteria for limiting ovality. These criteria differ in several aspects. The two most important are:

1. Lambe (1961) and Penman (1977) considered general dishing of the shell across the tank diameter, while Langeveld (1974) and Malik, et al. (1977) considered local distortions around the shell circumference.
2. Each used a different measure of differential settlement to establish his criterion.

In an attempt to compare these criteria for equivalent conditions, we have adapted the Langeveld and Malik, et al. criteria to fit the case of general sagging of the shell. We made the adaptation by assuming that the out-of-plane distortion caused by general sagging of the shell, S, equals approximately

$$S = -S_{max} \cdot \cos\left(\frac{4X}{D}\right) \qquad (3)$$

for all three criteria. (Fig. 5 defines these symbols.) Model tests on tanks (Malek,

ASPECT OF PERFORMANCE	REFERENCE	MECHANISM OF FAILURE	BASIS FOR CRITERION	PERFORMANCE CRITERION
NON-PLANAR SHELL SETTL.  Mode of Failure: A. Ovality: $\Delta R > \Delta R_{tol}$ B Ext. tensile stresses: $\sigma > \sigma_t$ Definitions: $\Delta S = S_i - 0.5(S_{i+1} + S_{i-1})$ $SAG = \left[S_i - 0.5(S_{i+\frac{n}{4}} + S_{i-\frac{n}{4}})\right]_{max}$ n = # of measuring points $\ell = \pi D/n$ $L = 2\pi D/n$	LAMBE (1961)	Geometrical relation based on sagging of shell $\Delta R = 3.33(H/D)SAG$	ΔR controlled by buckling of wind girder or tolerance of roof seal to radial change. Recommended F.S.=1.2	$SAG \le (D/4H)\Delta R_{tol.}$
	PENMAN (1977)	Geometrical relation based on sagging of shell $\Delta R = 4.0(H/D)SAG$	Assumes: extensional deformations of the shell can be disregarded. to find ovality; horizontal average plane of tilt	$SAG \le (D/4H)\Delta R_{tol.}$
	MALIK et al (1977)	Theory of extensionless deformations (Love, 1927) $\Delta R = -(DH/2)dS^2/dX^2$	Assumes: $d^2S/d^2X = -2\Delta S/\ell^2$ Criterion was validated with settlement data from laboratory models of tanks	$\Delta S \le (\ell^2/HD)\Delta R_{tol.}$
	LANGEVELD (1974)	Theory of extensionless deformations (Love, 1927) $\Delta R = -(DH/2)d^2S/dX^2$	Assumes: $S(X) = S_{max}.\sin(\pi X/L)$	$S_{max} < (0.20L^2/HD)\Delta R_{tol.}$
	GREENWOOD (1974)	$\Delta R = f(S_{max}, D)$	Experience with 27 fixed roof and 21 floating roof tanks	$S_{max} \le 4cm$; $D \le 50m$ $S_{max} \le 6cm$; $D \ge 50m$
	SULLIVAN & NOWICKI (1974)	$\Delta R = f(S_{max})$	Experience with 21 floating roof tanks	$S_{max} \le 3.0\text{-}4.5cm$
	DEBEER (1969)	$\sigma = f(1/R_c)$ where, $R_c = -\ell^2/2\Delta S$	Analogy with behavior of structural frames: $1/300 > \Delta S/\ell > 1/600$ Minor damage $\Delta S/\ell > 1/300$ Serious damage	$(\Delta S/\ell) \le 1/450$

1 inch = 2.54 cm
1 ft = 0.305 m

FIG. 4.—Performance Criteria for Nonplanar Settlement of Shell

et al., 1977) support the reasonableness of this assumption.

Fig. 5 graphically illustrates the basis for the modified expressions of differential settlement and compares these criteria for the equivalent condition of general sagging of the shell. The criteria differ in the expression used to define radius of curvature of the shell. Langeveld (1974) and Malik, et al. (1977) determine radius of curvature from a smooth curve fitted through three adjacent points of measurement. Langeveld (1974) fits a sine curve while Malik, et al. (1977) fit a cricle. Lambe (1961) and Penman (1977) estimate radius of curvature from a curve passing through the quarter points giving the maximum *SAG*. (Fig. 2 defines *SAG*.) Lambe (1961) and Penman (1977) use the correct radius of curvature as long as the tank settles in a sagging mode. As the number of points of measurement, n, increases, Lengeveld's estimated radius of curvature tends to underestimate the maximum radius of curvature; whereas, Malik, et al. (1977) estimate that a maximum radius of curvature that approaches the exact maximum value. The right side of Fig. 5 shows the similarity of the criteria for cases where the sagged shape approximates a cosine curve and the measuring points exceed 16.

Significant ovality may occur from local out-of-plane distortion as well as general sagging. Langeveld's assumption that a sine curve describes local distortions underestimates the local irregular distortions that actually occur. Criteria of Lambe and Penman do not consider local distortion of the shell. In Malik, et al., the criterion works with local distortion as well as general sagging, provided a sufficient number of measuring points is used.

The criterion in Malik, et al. (1977) agrees with those of Langeveld (1974), Lambe (1961) and Penman (1977) for general sagging, and considers the effect of local distortions on ovality. Therefore, we conclude that the criterion in Malik, et al.

$$S \leq \frac{\ell^2}{H \cdot D} \Delta R_{tol} \quad \text{(4)}$$

in which ℓ = the distance between points of measured settlement, gives the best current criterion for ovality resulting from out-of-plane distortion of floating-roof tanks. Little data and few analyses exist to set a criterion for the ovality of coned-roof tanks.

Data from floating roof tanks in TONEN's Kawasaki Refinery suggest large tanks can tolerate more differential settlement than indicated by Eq. 4. We think large but localized differential settlement produces localized radial distortions. The roof may shift so that binding does not occur, even for differential settlement twice the limit given by Eq. 4.

Overstress.—In the past, most foundation designers considered ovality and its effect on operation of floating roofs to dictate allowable nonplanar settlement of the shell. While this assumption probably held for old, small-diameter tanks, it may no longer apply. Many new tanks have a larger diameter-to-height ratio and, thus, can accommodate larger distortions without ovality problems. In addition, new seals for floating roofs tolerate much more radial distortion, thus, allowing larger ovalization of the shell. Consequently, overstressing of the tank shell from differential settlement becomes a distinct possibility.

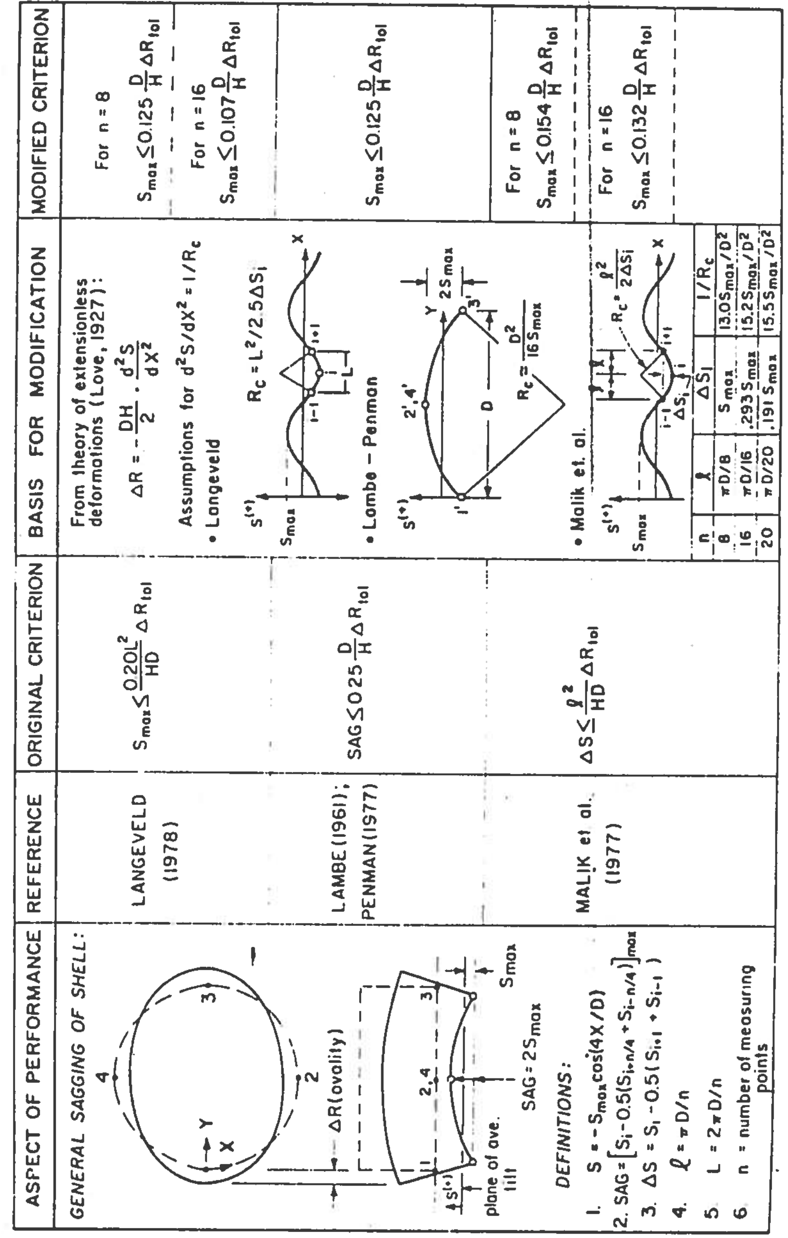


FIG. 5.—Criteria for Ovality of Shell

Fig. 4 shows that DeBeer (1969) gives the only treatment of out-of-plane settlement specifically related to overstressing of the shell. He related out-of-plane distortion at a point, ΔS_i, to radius of curvature of the shell at that point, $(R_c)_i$, as

$$(R_c)_i = -\frac{\ell^2}{2\Delta S_i} \quad \text{(5)}$$

He concluded that an R_c greater than 1,500 m (5,000 ft) causes overstress in coned-roof tanks with diameters less than 20 m (66 ft). DeBeer concluded that large, floating-roof tanks can safely sustain a $\Delta S/\ell$ of 1/450.

Our evaluation of this criterion leads us to try to develop a more direct relation between differential settlement and magnitude of overstress in the tank shell. From the theory for bending of beams, we note that the radius of curvature used by DeBeer relates to bending stress in the shell, σ, as follows:

$$(R_c)_i \propto \frac{H \cdot E}{\sigma} \quad \text{(6)}$$

in which E = the Young's modulus of the shell. Combining Eqs. 5 and 6 and replacing σ with the rupture stress of the shell, σ_f, gives

$$\Delta S_i \leq \frac{K \cdot \ell^2}{H \cdot E} \cdot \sigma_f \quad \text{(7)}$$

in which K = a constant that includes the nonelastic behavior of the shell material, secondary effects of the tank geometry, and other factors. By studying performance of tank shells, especially failures, we would hope to find empirical values of K that relate out-of-plane settlement to rupture of the shell. DeBeer's use of an R_c of 1,500 m (4,900 ft) for a tank with a diameter of 20 m (66 ft) implies a K of 2–3. His recommendation of a $\Delta S_i/\ell$ of 1/450 to prevent structural damage in large, floating-roof tanks corresponds to a K of 1.5–3.3, depending on the geometry and properties of the tanks treated by DeBeer. Greenwood's (1974) proposal of an absolute limit to differential settlement translates roughly to a K of 3.3 to prevent overstressing of the shell.

Fig. 6 summarizes data on measured shell settlements and calculated values of K for 90 tanks. The majority of these data come from published cases that emphasize large or unusual deformations. We located no documented case where the shell ruptured due to differential settlement. Thus, we could not determine maximum values of K associated with rupture of the shell.

Although several tanks were releveled for various reasons, including excessive settlement of the bottom plate, none developed rupture in the shell. The largest measured value of K is 12.1.

We can obtain one estimate of K at failure from considering beam theory. For deformation of a simple beam in the elastic and plastic range

$$\Delta S_i = \epsilon \cdot \frac{\ell^2}{H} \quad \text{(8)}$$

in which ϵ = tensile strain in the extreme fiber; ℓ = distance between points

Reference	Tank No.	Roof Type	Diameter in m	Height in m	ℓ in m	[illegible] in cm	Average tilt	$\Delta S/\ell$ max	K	Remarks
Sullivan & Nowicki (1974)	A	L	41	12	32.2	46.7	1/964	1/6830	0.027	See note 1
	B	C	41	12	32.2	32.7	1/178	1/2680	0.70	See note 1
	C	F	20.5	12	16.1	12.2	1/154	1/1010	0.37	
	D	F	20.5	12	16.1	12.0	1/195	1/1010	0.37	
Davisson & Salley (1972)	A	C	42.7	13.7	16.8	5.1	1/311	1/263	1.6	
	B	C	33.5	13.7	13.2	7.6	1/660	1/810	0.64	
	C	C	27.4	13.7	10.8	9.8	1/415	1/1370	0.45	
	D	C	38	25.6	15.0	11.7	1/882	1/2810	0.30	
Greenwood (1974)	4901	C	44.0	20.1	17.2	9.0	1/400	1/710	0.82	
	4902	C	24.6	18.3	9.6	23.0	1/140	1/1050	0.91	shell releveled to reduce tilt
	4903	C	24.6	16.5	9.6	12.0	1/726	1/1260	0.68	
	4904	C	24.6	16.5	9.6	10.0	1/445	1/630	1.5	shell releveled to reduce tilt
	4905	C	24.6	16.5	9.6	8.0	1/800	1/1050	1.1	
	4906	C	14.7	20.1	5.7	7.0	1/480	1/210	8.4	
	4908	C	19.6	20.1	7.7	8.0	1/426	1/315	4.1	
	4909	C	19.6	20.1	7.7	10.0	1/640	1/840	1.6	
	4910	C	19.6	20.1	7.7	10.0	1/815	1/1000	1.3	
	726	C	39.0	12.5	15.3	19.9	1/950	1/850	0.48	
	729	C	39.0	12.5	15.3	20.0	1/1390	1/1020	0.40	
	730	C	39.0	12.5	15.3	25.6	1/1440	1/955	0.43	
	731	C	39.0	12.5	15.3	25.6	1/355	1/364	1.18	
	158	F	70.0	14.8	27.5	28.0	1/11700	1/1960	0.18	
	159	F	51.0	14.8	13.4	36.9	1/450	1/240	2.3	
	160	F	70.0	14.8	18.4	36.9	1/1490	1/1020	0.39	
	161	F	70.0	14.8	18.4	35.5	1/666	1/334	1.2	shell shimmed to reduce ovality
	162	F	70.0	14.8	18.4	42.6	1/875	1/558	0.72	
	163	F	70.0	14.8	18.4	49.9	1/824	1/1840	0.22	
	LOF	F	44.0	16.5	17.2	12.8	1/2880	1/705	0.68	
	102	F	67.0	16.5	9.6	4.6	1/2440	1/736	1.8	
	104	F	67.0	16.5	9.6	7.6	1/4400	1/960	0.90	
	106	F	67.0	16.5	9.6	5.8	1/1570	1/960	0.90	
	109	F	55.0	16.5	9.6	14.4	1/460	1/565	1.5	
		F	70.0	14.0	27.5	66.0	1/585	1/688	0.37	
	11	C	24.4	18.5	7.7	17.1	1/666	1/250	4.0	
	12	C	19.5	18.5	7.7	13.2	1/2130	1/420	2.9	
	2	C	25.0	10.0	19.6	14.0	1/1670	1/1780	0.19	
	3	C	25.0	10.0	19.6	9.0	1/835	1/980	0.76	
	T121	C	24.4	14.7	4.8	51.4	1/320	1/216	7.1	
	N^{1n}	C	39.0	16.5	15.3	63.1	1/166	1/3140	0.17	
	S^{1n}	C	39.0	16.5	15.3	78.5	1/135	1/3350	0.16	
	1	C	18.3	15.3	7.2	19.5	1/1930	1/1130	0.94	
	2	C	18.3	15.3	7.2	13.3	1/522	1/373	2.8	
	7595	F	24.4	12.8	7.65	4.8	1/850	1/806	1.0	shell releveled
	7596	C	24.4	14.6	7.65	8.3	1/700	1/1200	0.80	
	7597	C	24.4	12.8	7.65	4.1	1/2560	1/1600	0.52	
	7598	F	29.3	12.8	7.65	2.2	1/1540	1/805	1.0	
	7599	F	24.4	16.5	7.65	4.1	1/513	1/805	1.3	
	7600	F	24.4	12.8	7.65	3.8	1/8400	1/1200	0.70	shell releveled
	650	F	42.6	14.7	33.5	14.2	NA	1/275	0.80	shell releveled (Note 1)
Aldrich & Goldberg	North	C	36.6	18.3	9.6	4.1	1/986	1/1600	0.60	
	South	C	36.6	18.3	9.6	5.2	1/1110	1/1200	0.80	
	Texaco	C	30.5	14.6	15.2	42.0	1/75	1/1790	0.27	foundation failed
Penman (1977)	Ancona	F	96.2	22.0	9.4	21.6	1/2940	1/1040	1.1	
Costa Nunes (1948)	NA[3]	NA	30.5	9.1	12.0	40.6	1/339	1/1090	0.35	
Bell (1980)	T-206	C	53.4	14.6	14.0	NA	1/588	1/5000	0.10	
	T-407	F	74.5	14.6	14.6	NA	1/3330	1/1670	0.30	
	T-8C	NA	68.6	12.2	18.0	NA	1/1000	1/2000	0.17	
	1000A	C	29.6	28.8	5.8	NA	1/116	1/385	6.4	
	1734	F	5.5	14.6	10.1	NA	1/1430	1/667	11	
	19	C	45.7	17.1	18.0	NA	1/435	1/10000	0.05	tank releveled to correct framing
	4352	C	46.6	14.6	18.3	NA	1/250	1/1430	0.28	
	4356	C	46.6	14.6	18.3	NA	1/714	1/1250	0.32	
TONEN tanks	51	F	63.9	18.3	10.0	123	1/272	1/567	1.6	releveled
	52	F	63.9	18.3	10.0	123	1/217	1/483	1.9	releveled
	53	F	63.9	18.3	10.0	180	1/402	1/320	2.9	
	54	F	63.9	18.3	10.0	136	1/338	1/290	3.1	releveled
	55	F	63.9	18.3	10.0	97	1/317	1/523	1.7	
	56	F	63.9	18.3	10.0	91	1/430	1/487	1.9	
	57	F	63.9	18.3	10.0	147	1/250	1/512	1.8	
	58	F	63.9	18.3	10.0	142	1/236	1/587	1.6	releveled
	59	F	63.9	18.3	10.0	34	1/215	1/772	1.2	releveled
	60	F	63.9	18.3	10.0	56	1/236	1/639	1.4	
	251	F	46.5	16.7	7.3	119	1/228	1/736	1.6	
	252	F	46.5	16.7	7.3	102	1/440	1/367	3.1	
	253	F	46.5	16.7	7.3	108	1/662	1/351	3.3	
	254	C	42.6	18.3	6.7	103	1/267	1/450	3.0	
	255	C	42.6	18.3	6.7	94	1/508	1/620	2.2	
	256	C	42.6	18.3	6.7	102	1/232	1/728	1.9	
	211	F	40.7	16.8	6.4	96	1/427	1/309	4.2	
	212	F	40.7	16.8	6.4	81	1/1087	1/332	4.0	
	213	F	40.7	16.8	6.4	48	1/513	1/454	2.9	
	111	C	27.5	18.3	4.3	91	1/356	1/400	5.6	
	112	C	27.5	18.3	4.3	65	1/478	1/433	3.1	
	113	C	27.5	18.3	4.3	48	1/460	1/173	12.1	
	114	C	27.5	18.3	4.3	48	1/360	1/333	8.2	
	115	C	27.5	18.3	4.3	79	1/306	1/481	4.3	
	116	C	27.5	18.3	4.3	91	1/277	1/392	5.4	
	117	C	27.5	18.3	4.3	81	1/192	1/617	3.4	
	118	C	27.5	18.3	4.3	76	1/217	1/309	6.7	
	119	C	27.5	18.3	4.3	99	1/242	1/617	2.0	
	120	C	27.5	18.3	4.3	100	1/152	1/392	5.2	
	121	C	27.5	18.3	4.3	71	1/239	1/617	3.2	
	122	C	27.5	18.3	8.6	54	1/159	1/864	1.3	

NOTES: 1) Too few measurements to deduce reliable S_i
2) Floating (F), Coned (C)
3) information not available (NA)
4) 1 m = 3.3 ft; 1 cm = 0.39 inch

FIG. 6.—Measured Performance of Shells

used to measure ΔS_i; H = the height of the beam.

For 36 ksi (1.6×10^5 N) steel, Salmon and Johnson (1971) give the following strains for various stress levels: at the yield stress, axial strain equals 0.00125; at the initiation of strain hardening, axial strain equals 0.014; and, at the tensile strength, axial strain equals 0.11. From Eq. 8, ΔS_i at tensile failure of a simple beam would be

$$\Delta S_i = 0.11 \cdot \frac{\ell^2}{H} \quad \text{(9)}$$

Multiplying Eq. 9 by 1 in the form of $\sigma_f/E \cdot E/\sigma_f$ and using $\sigma_f = 58 \times 10^3$ psi and $E = 29 \times 10^6$ psi for 36 ksi (1.6×10^5 N) steel gives

$$\Delta S_i = 0.11 \cdot \frac{\ell^2}{H} \cdot \frac{\sigma_f}{E} \cdot \frac{29 \times 10^6}{58 \times 10^3} = 55 \frac{\ell^2}{H} \cdot \frac{\sigma_f}{E} \quad \text{(10)}$$

Tensile failure of a simple beam should occur at a K of 55. The corresponding value of K at the yield strain equals 0.6 and that at the initiation of strain hardening equals 7.

Considering the results for a simple beam and the values of K calculated from measurements, we recommend a value of K of 7 for a performance criterion. Settlements reaching this criterion should produce some yielding in the upper course of the shell, but the strains should not reach the strain hardening range. Stress in the yielded zone should equal the yield stress of the steel. One of the 31 TONEN tanks exceeded this limit with no signs of distress. Two of 59 other tanks exceeded this limit with no reported ruptures of the shell.

To facilitate use of the criterion we replace σ_f in Eq. 7 with the yield stress, σ_y, to obtain

$$S_i \leq 11 \cdot \frac{\sigma_y}{E} \cdot \frac{\ell^2}{H} \quad \text{(11)}$$

for a K of 7.

Settlement equal to this criterion gives a factor of safety against rupture (defined as σ_t/σ_y) of 1.6.

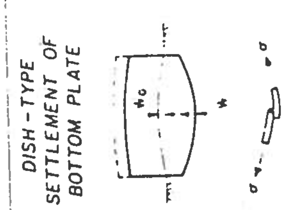
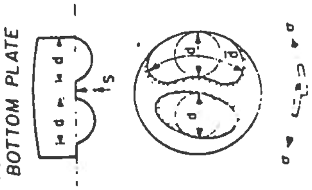
The criterion in Eq. 11 does not consider buckling of the shell. We assume that buckling resulting from differential settlement would occur in the top course, would not rupture the shell and would not result in loss of oil. However, failure by buckling requires more study.

Criteria for Bottom Plate

Uniform Settlement.—Regardless of its cause, uniform settlement of the bottom plate creates no threat to the structural integrity of a tank. Excessive uniform settlement may produce operational problems when one empties tanks, but one can best handle such deficiencies on a case-by-case basis.

Planar Tilt.—The bottom plate may tilt in a plane similar to the tilt of the shell. Such tilt appears to have no detrimental consequence to the structural integrity of the bottom plate.

Nonplanar Settlement.—Fig. 7 summarizes performance criteria for nonplanar settlement of the bottom plate. Two important deformation modes exist: dish-shaped settlement and localized depressions. All work (except Hayashi's (1973) and Guber's (1974) for local depressions adjacent to the shell) uses deflection analyses of thin circular plates with fixed-end boundary conditions to

ASPECT OF PERFORMANCE	REFERENCE	MODE OF FAILURE	BASIS FOR CRITERION	CRITERION	CRITERION REVISED *
DISH-TYPE SETTLEMENT OF BOTTOM PLATE 	Rinne (1963)	Tensile rupture of stretched bottom plate	• Membrane theory with $\sigma_{all} < \sigma_f / FS$; $\sigma_f = 280{,}000$ kN/m² for single pass lap weld; FS = 4.5	$\Delta W \leq \frac{D}{90}$	$W \leq \frac{D}{45}$
	Longeveld (1974) Herber (1955)	Tensile rupture of stretched bottom plate	• Membrane theory to give $W \leq \left[W_0^2 + \frac{0.37}{FS} \frac{\sigma_f}{E} D^2\right]^{1/2}$; $\sigma_f = 210{,}000$ kN/m² for single pass fillet weld; FS = 4.0	$\Delta W \leq \frac{D}{100}$	$W \leq \frac{D}{44}$
LOCALIZED SETTLEMENT OF BOTTOM PLATE 	Hayashi (1973)	Tensile rupture of locally stretched bottom plate	• Large defection of circular plate with rigid boundary (Timoshenko, 1955): $S \leq d\sqrt{0.28\,\sigma_f / FS \cdot E}$; $\sigma_f = 180{,}200$ kN/m² single pass fillet weld; $\sigma_f = 280{,}000$ kN/m² multiple pass fillet weld; FS = 2.0 erodable foundation; FS = 1.0 non-erodable foundation	Erodable foundation: $S \leq \frac{d}{90}$ single pass fillet weld; $S \leq \frac{d}{75}$ multiple pass fillet weld. Non-erodable foundation: $S \leq \frac{d}{63}$ single pass fillet weld; $S \leq \frac{d}{50}$ multiple pass fillet weld	$S \leq \frac{d}{51}$
	Guber (1974)		Large deflection of circular plate with flexible boundary: $S \leq d\left[\frac{2.25}{(d)^{0.75}} \cdot \frac{\sigma}{E\,FS} \cdot \frac{D}{H}\right]^{0.5}$; $\sigma_f / FS = 4219$ t/m²	$S \leq \frac{d}{19}$ to $\frac{d}{34}$	$S \leq \frac{d}{8}$ to $\frac{d}{13}$

*NOTE: σ_f = [illegible] kN/m² and $W_0 = 0$ for revised criterion to failure. 1 psi = 6.9 kN/m²

FIG. 7.—Criteria for Nonplanar Settlement of Bottom Plate

relate deformation to stress in the bottom plate. Principal differences in the results by different engineers appear to derive from differences in factor of safety and strength of the bottom plate.

Dish-Type Settlement.—Several groups have formulated criteria for dish-shaped settlement. Close examination of the work of Herber (1955) and Langeveld (1974), as adopted by Shell and the German code (Langeveld, 1974) shows them to consist of a more general extension of the work of Rinne (1963). For a factor of safety of one, a bottom plate strength of 280,000 kN/m² (40,000 psi) and zero initial camber, both expressions give a value of W of $D/45$.

We conclude that the following expression represents a reasonable criterion for the nonplanar dish-shaped settlement of the bottom plate:

$$W \leq \left(W_o^2 + \frac{0.37}{FS} \cdot \frac{\sigma_f}{E} \cdot D^2\right)^{1/2} \qquad (12)$$

with W_o equal to the initial maximum camber of the bottom plate and σ_f defined as the ultimate stress of the particular weld used to construct the bottom plate. Factor of safety, FS, equals the ratio of ultimate stress to existing stress in the bottom plate. Values of FS, as related to possible performance of the bottom plate, consist of: (1) $FS \leq 4$ in which localized yielding is possible; and (2) $FS \leq 2$ in which severe overstress and rupture is possible. Eq. 12 shows that stresses in the bottom plate relate to the square root of settlements; consequently, a factor of safety on stress of 4 corresponds to a displacement of 0.5 times that at failure. A factor of safety of 2 on stress corresponds to a displacement of 0.7 times that at failure.

Localized Depressions.—Fig. 7 gives criteria for localized settlement of the bottom plate as adopted from the work of Hayashi (1973) and Guber (1974). This work forms the basis of criteria in use by EXXON (Guber 1974). Hayashi and Guber distinguish between localized settlement remote from the shell, where settlement of the bottom plate occurs without influence of the shell, and localized settlement adjacent to the shell, where the flexibility of the shell influences stresses in the bottom plate.

For the same strength of the bottom plate, the failure criterion for localized settlement remote from the shell gives a ratio of $S/d = 1/51$, a value slightly less than the general failure criterion for dish-shaped settlement of $W/D = 1/44$ for zero initial camber. Thus, the bottom plate behaves in a similar fashion for localized and dish-shaped settlement.

Criterion of Hayashi and Guber for localized settlement of the bottom plate remote from the shell provides a basis for including the properties of the bottom plate in determining allowable differential settlement. We conclude that this criterion, expressed as

$$S \leq d\left(\frac{0.28 \cdot \sigma_f}{E \cdot FS}\right)^{1/2} \qquad (13)$$

provides a rational means of limiting localized settlement of the bottom plate at points remote from the shell. One should use the appropriate ultimate stress of the welds in the bottom plate in Eq. 13. Similar to the case of dish-shaped bottom plate settlement. we belive the factor of safety relates to the performance of the

bottom plate in the following manner: (1) $FS \leq 4$ in which localized yielding is possible; and (2) $FS \leq 2$ in which severe overstress and rupture are possible.

The flexibility of the shell relaxes part of the stresses induced in the bottom plate by local depressions adjacent to the shell. It seems reasonable to have more lenient limits in this case than for localized settlement away from the shell.

Guber (1974) established a family of control curves for depressions of the bottom plate in the proximity of the shell. These curves apply to partial ring depressions adjacent to the shell as long as $d < D/4$ and $\bar{d} > 2d$, in which d = the diameter of the largest horizontal circle that can be inscribed in the depression; $\bar{d}$ = the length of partial ring depression. These curves give limits for local depression at failure which range from $d/17$–$d/33$ for single pass fillet welds, and $d/13$–$d/26$ for multiple pass welds.

Eq. 14 describes approximately the relation expressed in Guber's family of control curves for local depression of the bottom plate adjacent to the shell:

$$S \leq d\left(\frac{2.25}{(d)^{0.75}} \cdot \frac{\sigma_f}{E \cdot FS} \cdot \frac{D}{H}\right)^{1/2} \qquad (14)$$

in which S and d are in meters. Guber recommends an allowable strength (σ_f/FS) of 42,190 kN/m (6,000 psi). This corresponds to safety factors of 4.2 and 6.6 for single pass and multiple pass fillet welds, respectively.

We conclude that Eq. 14 provides a rational approach to limit local depressions of the tank bottom in areas adjacent to the shell. Again, we belive that factor of safety relates to performance of tank bottom in the following fashion: (1) $FS \leq 4$ in which localized yielding is possible; and (2) $FS \leq 2$ in which severe overstress and rupture are possible.

The criteria for nonplanar settlement of tank floors examined to this point assume that the bottom plate shows dish-type settlement of local depression. Tank floors that deform in a combination of these two modes may experience stresses in their bottom plates larger than those predicted by either criterion alone.

Fortunately, failures of several bottom plates have been sufficiently documented to allow evaluation of these criteria. Fig. 8 summarizes data for 30 tanks with structural failures occurring in 8 cases. Fig. 8 shows that the criteria correctly predict factors of safety less than one for all cases where structural failure occurred, except T–16(0) and T–1701, which had factors of safety of 1.1. The criteria predict factors of safety greater than one for all cases of no failure except for one tank (T–39). This agreement of predicted and measured failures provides considerable support to the applicability of these criteria for settlement of the bottom plate.

Criteria for Shell-Bottom Plate Connection

Uniform Settlement.—Uniform settlement of the annular ring causes no problem to the structural integrity of the shell-bottom plate connection.

Planar Tilt.—As indicated in the section on the shell, planar tilt causes additional stresses in the shell. These effects must transfer to the shell-bottom plate connection. These additional stresses do not seem large enough to cause overstressing. Furthermore, the criterion to limit planar tilt of the shell seems suf-

Reference	Tank No.	Diameter in m.	Height in m.	DISH SHAPED w_o in cm	DISH SHAPED w in cm	DISH SHAPED FS	LOCALIZED S_l in cm.	LOCALIZED d in m.	LOCALIZED Location[2]	LOCALIZED FS	Remarks
Bell (1980)	T-270	52.3	23.7	30	25	$w<w_o$	3	4	A	6	
	T-270	52.3	23.7	30	20	$w<w_o$	6	2.5	R	0.67	bottom plate ruptured
	T-4	96.2	22.0	72	7	$w<w_o$	2	13.5	R	175	
	T-4356	46.6	14.6	0	18	31	6	4.0	R	1.7	
	T-401	74.5	14.6	0	19	78	3	14	R	84	
	T-4352	46.6	14.6	0	21	25	11	6	R	1.1	
	T-BC	68.6	12.2	41	28	$w<w_o$	NA[3]	NA			
	T-322	45.7	14.6	0	37	7.8	NA	NA			
	T-407	61.6	14.6	71	42	$w<w_o$	7	10	R	7.8	
	T-404	61.6	14.6	71	44	$w<w_o$	NA				
	T-402	61.6	14.6	0?	46	9.1	NA				
	T-212	38.4	14.6	54	68	1.6	NA				
	T-16(0)	36.6	14.6	15	9	$w<w_o$	1	6	R	1.1	bottom plate ruptured
	T-1701	45.7	20.0	21	0	$w<w_o$	36	19	R	1.1	bottom plate ruptured
Carlson et al (1961)	T-39	45.7	14.6	21	25	58	68	19	A/R	0.8/0.3	See Note 1
Clarke (1971)	R2	56.6	20 ?	0	43	8.8	8	2.2	R	0.29	structural failure
Green &	RI	14.6	12.8	15	46	0.57	NA	NA			structural failure
Hight (1974)	RI	24.4	12.8	24	82	0.49	NA	NA			structural failure
Penman (1977)		14.6	12.8	0 ?	45	0.48	NA	NA			bottom plate ruptured
		24.4	12.8	0 ?	81	0.42	NA	NA			bottom plate ruptured
	N-3000-F	39.0	16.5	0 ?	14	36+	7	15	A	70	
TONEN	51	63.9	18.3	-28	-16	$w<w_o$	2.5	6	R	16	relevelled
	52	63.9	18.3								
	53	63.9	18.3	-42	28	$w<w_o$	7.5	5.3	A	11	
	54	63.9	18.3	-37	-18	$w<w_o$	9.5	8.5	R	2	relevelled
	55	63.9	18.3	-40	40	$w<w_o$	14	27	R	10	relevelled
	56	63.9	18.3	-38	20	$w<w_o$	7.5	14.9	R	11	
	57	63.9	18.3	-42	-5	$w<w_o$	7.5	5.0	A	10	
	58	63.9	18.3	-38	37	$w<w_o$	7.0	4.2	A	4	relevelled
	59	63.9	18.3	-40	23	$w<w_o$	4.5	11.7	R	19	relevelled
	60	63.9	18.3	-44	23	$w<w_o$	14	4.8	A	2	

NOTES: 1) Soil failure, no structural failure 2) Adjacent to shell (A); Remote from shell (R)
3) Information not available (NA)
4) 1 m = 3.3 ft. 1 cm = 0.39 inch

FIG. 8.—Measured Performance of Bottom Plates

ficient to ensure that additional stresses in the connection are kept within allowable limits.

Nonplanar Settlement.—Two situations require consideration. One consists of the condition where the shell and bottom plate settle together. This condition compares to Guber's (1974) analysis for local depressions of the bottom plate adjacent to the shell. Satisfaction of the criterion for the bottom plate as shown in the bottom of Fig. 7 should also prevent overstressing of the connection due to this mode of deformation.

The second situation arises when the shell bridges over a soft spot, but the bottom plate tries to settle with the soft spot. Severe stretching of the connection can result and can lead to rupture of the connection. Engineers indicate an awareness of this potential failure mode in the literature, but we could find no criterion for design to prevent failure. Apparently, engineers consider this mode of failure so critical that one should take all practical steps during construction and operation to prevent its development. If one finds local separation of the shell from the foundation, he should immediately pack this zone with acceptable bearing material, such as sand or soil cement.

This approach seems acceptable as long as the surveillance program can detect such conditions. However, the portion of the annular ring which extends outward from the shell can possibly conceal the separation of the shell from the foundation. In this case, one can only determine bridging by measuring the settlement of the annular ring inside the tank and by comparing it with the settlement of the shell. However, removal of the tank contents before making these measurements can allow the bottom plate and connection to rebound so that one may not detect the bridging.

Performance of the connection seems important to us, yet we conclude that no criterion exists to evaluate the condition of the connection, and most surveillance programs are insufficient to indicate the severity of deformations at the connection. Performance of this component of the tank demands further study.

Other Considerations

Traditionally, engineers have not considered the effects of horizontal movements of tank foundations on the tank. Such movements occur from shear of the foundaiton, effects of *nearby* facilities, and effects of earthquake shaking. To our knowledge, no criteria exist to limit lateral movements of a tank foundation.

Several questions remain unanswered. How do horizontal displacements relate to stresses in the components of a tank? Can horizontal strains rupture the bottom plate or shell? Should engineers consider a combination of vertical and horizontal modes of displacement?

The proposed criteria consider each mode of failure as independent from other modes. Furthermore, they consider limiting stresses in the tank components determined with no consideration of stresses developed from other causes, including the stress induced by contained fluid, except the criterion in Eq. 1 which rarely controls. Approximate calculations indicate these simplifications have little influence for most cases. However, the engineer should consider the possible effects of combined overstressing if more than one stressing mode appears important. As an example, we believe the combination of large dish-shaped settlements with large localized depressions of the bottom plate is more severe then either of the settlement patterns considered separately.

Definition of settlement becomes unclear upon releveling a tank. We suggest using the original elevations to find settlement for releveling accomplished without structurally disassembling the tank components. If releveling invokes dismantling the tank, one should use new elevations for settlement determinations.

Conclusions

This paper presents performance criteria for the deformation of large tanks used to store fluids at ambient temperature and pressure. Fig. 9 summarizes these performance criteria. Where possible, the criteria include a factor of safety, defined as the ratio of tensile stress to developed stress. We indicate permissible values of factor of safety for safe operation.

We developed the performance criteria by: (1) Interpreting and extending the work of others; (2) making approximate analyses; and (3) analyzing the measured performance of large tanks, especially the tanks at TONEN's site in Kawasaki City, Japan.

Several important questions remain about the performance of tanks subjected to differential settlement. We encourage engineers to document performance of tanks, especially tanks which fail due to settlement. Such documentation should include, as a minimum, properties and dimensions of the tank; settlement of the shell for at least eight points; and settlement of the bottom plate at enough points to detect localized depressions.

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Appendix I.—Determination of Average Plane of Tilt

In order to compute the differential settlements, S_i, one must determine and remove settlements resulting from tilt. The three most widely used methods determine the average plane of tilt of the shell. Calculated settlement due to the tilting of the tank is subtracted from each measured settlement of the shell and the bottom plate.

Figs. 2 and 10 define symbols. All methods consider Eq. 15 to describe the settlement from planar tilt:

$$Z_i = A_0 + A_1 \cdot \cos(\phi_i + \beta) \qquad (15)$$

Sullivan and Nowicki (1974) assume that the orientation of the plane of tilt coincides with the diameter having the maximum difference in measured settlement. DeBeer (1969) recognized that the diameter coinciding with the plane of average tilt does not necessarily coincide with measured points. He described

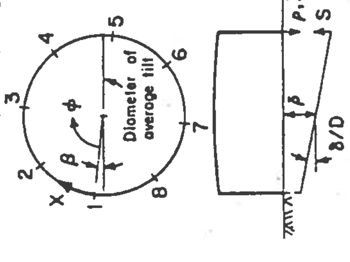
COMPONENT	ASPECT OF PERFORMANCE	MODE OF FAILURE	CRITERION *	DEFINITIONS and COMMENTS
SHELL	ΔR (ovality); Δh_d = freeboard; Tensile Crack; plane of average tilt; point i; δ; Z_i; ρ_i; S_i	PLANAR TILT		Δh_d = freeboard; H,D = tank dimensions; ΔR_{tol} controlled by: 1. tolerance of roof seal 2. buckling of wind girder 3. distortion of cone roof; σ_y = yield strength of shell; E = Young's modulus of elasticity
	Planar Tilt: $Z_i = \bar{\rho}_i + 2(\delta/D)\cos(x_i/R + \beta)$ where $\bar{\rho}$ = average shell settlement, β = orientation of plane of average tilt, δ/D = angle of average planar tilt	I Overtopping of shell	I. $\delta \le 2\Delta h_d$	
		II. Loss of roof seal	II. $\delta \le 2\sqrt{\Delta R_{tol} D}$	
	Non-Planar Settlement: $S_i = \rho_i - Z_i$; $\Delta S_i = S_i - 0.5(S_{i+1} + S_{i-1})$; $\ell = \pi D/n$	NON-PLANAR SETTLEMENT		
		III. Binding of roof seal	III. $\Delta S \le \frac{\ell^2}{HD}\Delta R_{tol}$	
		IV. Overstress of shell	IV. $\Delta S \le 11\frac{\ell^2 \sigma_y}{HE}$	
BOTTOM PLATE	Non-Planar Settlement: D; W_o; W; d; $\bar{d}$; H; S; Fillet Weld; σ	V. Rupture from dish-shaped settlement	V. $W \le \left[W_o^2 + \frac{0.37\sigma_f D^2}{FS \cdot E}\right]^{0.5}$	σ_f = ultimate strength appropriate for bottom plate; FS≤4 localized yield possible; FS≤2 severe overstress and yield possible; E = Young's modulus of elasticity; W_o = initial camber; $d, \bar{d}$ = dimensions of local depression, m; For local depressions adjacent to shell, if $\bar{d} \le D/4$ and $\bar{d} \ge 2d$, use criterion VIb. Otherwise use criterion VIa.
		VI. Rupture from localized depressions	VI.	
		a. remote from shell	a. $S \le d\left[\frac{0.28\sigma_f}{FS \cdot E}\right]^{0.5}$	
		b. adjacent to shell	b. $S \le d\left[\frac{2.25\sigma_f D}{d^{0.75} FS\,E\,H}\right]^{0.5}$; d and S in meters	
SHELL BOTTOM PLATE CONNECTION	Non-Planar Settlement: Deformed tank wall; Distorted welds; Deformed annular rings; Gap	VII. Rupture of connection as shell bridges over soft spot	VII. Surveillance and maintenance to prevent separation of shell and foundation	* Use definitions in Fig. 2 and Malik et al correction for tilt to find all settlement parameters

FIG. 9.—Recommended Performance Criteria for Differential Settlement of Tanks

a.) Characteristics of Plane of Average Tilt

REFERENCE	β	ρ (cm)	δ_{max}/D ($\times 10^{-3}$ radians)	EQUATION of PLANE of AVERAGE TILT
Sullivan & Nowicki (1974)	0°	39.0	2.61	$Z = 39.0 - 7.0\cos\phi$
DeBeer (1969)	11°	39.0	2.66	$Z = 39.0 - 7.1\cos(\phi + 11°)$
Malik et al (1977)	12.9°	39.0	2.67	$Z = 39.0 - 7.0\cos\phi + 1.6\sin\phi = 39.0 - 7.2\cos(\phi + 12.9°)$

b.) Definition of Terms



β = orientation of plane of average tilt
$\bar{\rho}$ = average shell settlement
δ_{max}/D = angle of average planar tilt
ϕ = 2X/D
ρ = measured settlement
Z = component of settlement from planar tilt
S = out-of-plane component of settlement, ρ - Z

c.) Out of Plane Distortions

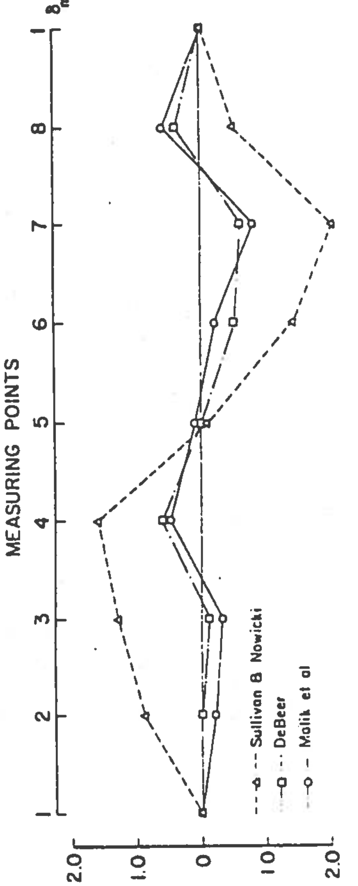


FIG. 10.—Methods to Determine Out-of-Plane Distortions

a graphical means to locate the diameter giving the plane of average tilt. Malik, Norton, and Ruiz (1977) use linear regression techniques and the measured data define, statistically, the direction of the plane of average tilt. They determine coefficients for Eq. 15 as

$$A_0 = \frac{1}{N} \cdot \sum_{i=1}^{N} \rho_i = \text{average measured settlement of shell};$$

$$A_1 = \frac{2}{N}\left[\left(\sum_{i=1}^{N} \rho_i \cdot \cos\phi_i\right)^2 + \left(\sum_{i=1}^{N} \rho_i \cdot \sin\phi_i\right)^2\right]^{0.5};$$

$$\beta = \arctan\left[\left(\sum_{i=1}^{N} \rho_i \cdot \sin\phi_i\right) \Big/ \left(\sum_{i=1}^{N} \rho_i \cdot \cos\phi_i\right)\right] \quad \ldots\ldots (16)$$

Out-of-plane settlement results from

$$S_i = \rho_i - Z_i \quad \ldots\ldots (17)$$

As shown in Fig. 10, both DeBeer (1969) and Malik, et al. (1977) give similar results for out-of-plane settlement. Sullivan and Nowicki's method tends to overestimate maximum out-of-plane settlement. We recommend Malik et al. as a consistent, reliable means to remove tilt from the measured settlements of the shell and the bottom plate.

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Appendix III.—Notation

The following symbols are used in this paper:

$A_0 . A$ = coefficients in equation for tilt;

CA = reduction to wall thickness to allow for corrosion.
D = diameter of tank;
d = diameter of circle inscribed in depression of bottomplate;
$\bar{d}$ = length of depression of bottom plate;
E = Young's Modulus of steel;
FS = factor of safety, equal to ratio of tensile strength to existing stress;
G_s = specific gravity of stored fluid;
H = height of tank;
h = height of wall;
L = twice the distance between measuring points on shell;
ℓ = distance between measuring points on shell;
n = number of points on shell at which settlement is measured;
R_c = radius of curvature;
SAG = ΔS_{max} for quarter points of shell;
S_i = out-of-plane settlement of point i;
S_{max} = maximum out-of-plane settlement of any point;
t_{max} = thickness of shell at base of tank;
W = maximum out-of-plane dish-shaped settlement of bottom plate;
W_o = initial camber of bottom plate;
X_i = coordinate along circumference of tank;
Z_i = component of settlement at point i due to planar tilt;
β = angle from reference to diameter along average plane of tilt;
γ_w = unit weight of water;
Δh_d = design freeboard of tank;
ΔR_{tol} = allowable change in radius of roof seal;
ΔS_i = $S_i - 0.5\,(S_{i-1} + S_{i+1})$;
ΔW = maximum settlement of bottom plate from initial position;
$\Delta\rho$ = difference in measured settlement between two points;
δ = difference in settlement between diametrical points;
δ_{max} = maximum difference in settlement between diametrical points;
δ_{hor} = horizontal movement of top of shell due to planar tilt;
ϵ = strain;
ρ_i = measured settlement of point i;
$\bar{\rho}$ = average measured settlement of shell;
σ = stress;
σ_{all} = allowable stress in steel;
σ_f = tensile strength of steel;
σ_y = yield strength of steel; and
ϕ_i = angle from reference to point i.