Title:	Overhaul of API 653 Differential Settlement Methods	**Agenda Item # 653-1012**
Date:	06/18/2024	
Contact:	• Name: Philip Myers Company: PEMY Consulting E-mail: phil@pemyconsulting.com • Name: Andy Wong Company: PEMY Consulting E-mail: andywong@pemyconsulting.com	
Purpose:	Address deficiencies in API 653 out-of-plane differential settlement analysis methods and accommodate "dense" laser scan data with numerous data points.	
Source:	"Commentary on API 653 Settlement Methods," PEMY Consulting white paper	
Revision:	1	
Impact:	Allows for incorporating laser scan technology surveys into differential settlement analysis, and provides more accurate engineering procedures for differential settlement.	
Rationale:	This ballot addresses several issues with the current out-of-plane differential settlement analysis methods in API 653 Annex B. 1. Remedy the inappropriate use of R^2 in determining whether a rigid tilt plane should be considered in the differential settlement analysis. "p-value" is a more appropriate statistic to determine whether rigid tilt is present. (B.2.2.4) 2. Clarify the language and improve the example calculation method to determine the rigid tilt plane cosine curve (B.2.2.4) 3. Remove the non-rigorous method to evaluate differential settlement when a rigid-tilt plane is not "well-defined". (B.2.2.5.1) 4. Modify the existing primary differential settlement evaluation method (the "Marr method") that is biased and sensitive to the spacing between measurement data points. This will be done by limiting this method's usage to appropriate spacings between data points. 5. The Marr method will be allowed as an alternative (with usage limitations as described above) to the existing secondary method (the "Andreani method") described in B.2.2.5.2. (B.2.2.4, Figure B.3). This Andreani method will be the recommended procedure for "sparse" (non-laser scan) settlement data. 6. API 653 Annex B does not have explicit accommodations for "dense" laser scan settlement data with numerous data points. An analysis method designed to accommodate this dense data should be included in the annex. 7. Explicitly allow the gold standard, finite element analysis (FEA), to be used. 8. Correct minor inaccuracies in the standard (e.g., uniform settlement in figure B.3). Detailed commentary of these issues can be found in the accompanying presentation and white paper.[1] Original proposed changes in red font. Revision 1 text in green font. Formatting notes and commentary are in blue font. An easier to read version of the proposer verbiage without marked-up deletions is included at the end of the document.	
Proposed verbiage:	**API 653 Annex B** **<u>B.2 Types of Settlement</u>**	

[1] See https://github.com/rbitip/API_Public/tree/main/ballot%20653-1012

	B.2.2 Shell Settlement Evaluation Figure B.3: remove text below plot: "Out-of-plane deflection for Point "I" is $S_i = U_i - \left(\frac{1}{2}U_{i-1} + \frac{1}{2}U_{i+1}\right)$, for example $S_{11} = U_{11} - \left(\frac{1}{2}U_{10} + \frac{1}{2}U_{12}\right)$". Remove "datum line for adjusted settlement" and accompanying lines. **B.2.2.4** While uniform settlement and rigid body tilt of a tank may cause problems as described in B.2.2.1 and B.2.2.2, the out-of-plane settlement is the important component to determine and evaluate in order to ensure the structural integrity of the shell and bottom. Based on this principle, a common approach is to determine the magnitudes of the uniform settlement and rigid body tilt (if a rigid tilt plane exists or can be identified) for each data point on the tank periphery. If a plane of rigid tilt can be distinguished, it becomes important as a datum from which to measure the magnitudes of the out-of-plane settlements. ~~When the out-of-plane settlement pattern of a tank has an easily distinguishable plane of rigid tilt, the methodology in this paragraph can be used to evaluate the acceptability of the tank's out-of-plane settlement. If a rigid tilt plane can not be readily determined, the methodology in B.2.2.5 can be used to evaluate the acceptability of the tank's out-of-plane settlement.~~ A graphical representation illustrating tank shell settlement with a rigid tilt plane well–defined by a cosine curve fit is shown in Figure B.3. The construction of this settlement plot has been developed in accordance with the following. a) The actual settlement (in most cases an irregular curve) is plotted using points around the tank circumference as the abscissa (x-coordinate). b) The vertical distance between the ~~abscissa~~ x-axis and the ~~lowest point on this curve (Point 22) is the minimum settlement, and it is called the~~ average of the settlement measurements is the uniform settlement ~~component~~. A line through this point, parallel to the ~~abscissa~~ x-axis, provides a new base or datum line for settlement measurements called adjusted settlements. c) The plane of rigid tilt settlement ~~, if well-defined,~~ is represented by the optimum (best-fit) cosine curve. Several methods exist for determining the optimum cosine curve. ~~The least accurate method is by free hand drawing techniques, a kind of trial and error procedure to fit the best cosine curve through the data. A better method is to use mathematical and graphical capabilities of a computer.~~ It is routinely performed using any linear regression software[2]. d) A commonly used and accepted method is to use a computer to solve for constants *a, b,* and *c*, to find the optimum cosine curve of the form: $$\mathrm{Elev}_{pred} = a + b \times cos(\theta + c)$$ Where Elev_{pred} is the elevation predicted by the cosine curve at angle theta. Rather than using non-linear regression, which would be required in the form of the equation above, linear regression can be applied as a least squares fit to the data of the equivalent form $d + e \times cos\,\theta + f \times sin\,\theta$, using $cos\,\theta$ and $sin\,\theta$ as basis functions and then finding the constants *a, b,* and *c* from the values of *d, e,* and *f*. $$a = d$$

[2] An example solution in Microsoft Excel for finding the cosine curve for rigid tilt can be found at https://github.com/rbitip/settlement-excel-regression-too.

$$b = sgn(e) \times \sqrt{e^2 + f^2}$$
$$c = tan^{-1}(f/e)$$

Where sgn is the signum/sign function.

~~d)~~ e) The vertical distances between the irregular curve and the cosine curve represent the magnitudes of the out-of- plane settlements (U_i at Data Point i). ~~If the previous test indicates the lack of planar tilt, the vertical distances between the irregular curve and the uniform settlement (i.e., the adjusted settlements) could be used instead.~~

The above text was removed because it is deemed unnecessary. In the proposed ballot, it is always acceptable to subtract the cosine curve even if b (cosine curve magnitude) is very small and the cosine curve has small amplitude compared to the Ui.

~~e) A commonly used and accepted method is to use a computer to solve for constants a, b, and c, to find the optimum cosine curve of the form:~~

$$\cancel{\mathrm{Elev}_{pred} = a + b \times cos(\theta + c)}$$

~~Where~~ $\cancel{\mathrm{Elev}_{pred}}$ ~~is the elevation predicted by the cosine curve at angle theta. A typical starting point for a computer best-fit cosine curve is a least-square fit where a, b, and c are chosen to minimize the sum of the square of the differences between measured and predicted elevations. The optimum cosine curve is only considered valid (i.e. accurately fits the measured data) if the value~~ $\cancel{R^2}$ ~~is greater than or equal to 0.9.~~

$$\cancel{R^2 = \frac{S - SSE}{S_{yy}}}$$

~~where~~

$\cancel{S_{yy}}$ ~~is the sum of the squares of the differences between average measured elevation and the measured elevations;~~

$\cancel{SSE}$ ~~is the sum of the square of the differences between the measured and predicted elevations.~~

~~Linear least square fitting and the R2 method of curve fitting are basic statistical tools. The use of a more rigorous statistical method to determine the optimum cosine curve, such as non-linear or iterative procedures, may be used by those experienced in their use.~~

~~Obtaining a statistically valid cosine curve may require taking more measurements than the minimums shown in Figure B.1. In many cases, the out-of-plane settlement may be concentrated in one or more areas. In such cases, the least-squares fit approach may under predict the local out-of-plane settlement and is not conservative. In these cases, R2 will typically be less than 0.9, and more rigorous curve-fitting procedures should be considered. Alternatively, the settlement may not indicate a well-defined rigid tilt plane and the procedure in B.2.2.5 should be considered.~~

~~f) The vertical distances between the irregular curve and the optimum curve represent the magnitudes of the out-of- plane settlements (Ui at Data Point i). Si is the out-of-plane deflection at Point i (see Figure B.3).~~

~~NOTE When determining the optimum cosine curve described in B.2.2.4 e), taking additional measurements around the shell will result in a more accurate cosine curve fit. However, using all of the measurement points in the equation shown in B.3.2.1 will result in very small allowable out-of-plane settlements, Smax since the arc length L between measurement points is small. It is acceptable to all measurement points to develop the optimum cosine curve, but only use a subset of these points spaced no~~

~~further than 32 ft (8 minimum) when calculating Si and Smax.The points used must include the points furthest from the optimum cosine curve. For example, if 8 points are required, but 16 measurements are taken, and the arc length between measurement points is only 15 ft, calculate the optimum cosine curve using all 16 points, but use only 8 points to calculate Si . The equations in Figure B.3 would be revised to read:~~

~~Si = Ui – (1/2 Ui – 2 + 1/2 Ui + 2)~~
~~S11 = U11 – (1/2 U9 + 1/2 U13)~~

B.2.2.5
~~If a well-defined rigid tilt plane can not be determined or the maximum out-of-plane deflection determined in accordance with B.3.2.1 is exceeded, the procedures given in this section may be used in lieu of more rigorous analysis or repair.~~ The method used to evaluate the out-of-plane settlement is dependent on the number of settlement data points collected. This distinction is especially important for settlement data collected via laser scanning. ~~For "sparse" data, where~~ If the number of settlement data points is less than or equal to 64, either of the methods described in B.2.2.6 or B.2.2.7 shall be used, if applicable. ~~For "dense" data, where~~ If the number of settlement data points is greater than 64, the method in B.2.2.8 shall be used~~ instead~~. In lieu of these methods, or if the maximum out-of-plane settlement determined in accordance with these methods is exceeded, it is always acceptable to use finite-element methods to evaluate the settlement instead.

~~B.2.2.5.1~~
~~For settlement profiles without a well-defined rigid tilt plane, the settlement arc length, Sarc, and out-of- plane deflection at the point under consideration, Si, must be determined from a plot of the measurement data. Figure B.4 is a graphical illustration of the various measurement terms and procedures for determining estimates of the settlement arc length and the corresponding out-of-plane deflection, including the refinement of measurements, when needed.~~

~~a) The actual settlement is plotted using points around the tank circumference as the abscissa.~~

~~b) An initial settlement arc length and maximum settlement is determined from the points on the plotted data that indicate a change in direction of settlement slope (see Figure B.4).~~

~~c) Additional settlement measurement points may be needed halfway between the points indicating a change in direction of the settlement slope to further refine the settlement arc length and location and magnitude of maximum settlement.~~

~~d) Step c) may need to be repeated. The best estimate of the settlement arc length and maximum out-of-plane deflection shall be considered in the procedure given in B.3.2.2.~~

~~B.2.2.5.2~~ **B.2.2.6**
~~If a valid cosine fit of the rigid tilt plane can be determined, but the maximum out-of-plane deflection determined in accordance with B.3.2.1 is exceeded~~ If ~~the settlement data is "sparse" (~~the number of settlement data points is less than or equal to 64~~)~~, the procedure in B.3.2.~~2~~1 ~~may~~shall be used to evaluate the settlement. In this case, see Figure B.4 for a graphical illustration of the determination of the settlement arc length and the corresponding out-of-plane deflection. As seen in the figure, each settlement arc

length is determined from where the linearly interpolated curve of the out-of-plane settlement crosses the cosine curve.

Remove figure B.4. Renumber figure B.5 to the new B.4 and fix references to Figure B.5 as appropriate.

~~B.2.2.5.3~~

~~If an examination of the measured settlement plot indicates a fold pattern about a diameter of the tank, the maximum out-of-plane settlement should be determined using a settlement arc length of 50 % of the tank's circumference.~~

B.2.2.7

The procedure in B.3.2.2 may be used to evaluate ~~the~~ settlement ~~for "sparse" settlement data~~ as an alternative to the method in B.2.2.6. In this case, see Figure B.5 for a graphical illustration of the various measurement terms and procedures for determining estimates of the settlement arc length and the corresponding out-of-plane deflection, including the refinement of measurements, when needed.

Include the version of figure B.3 in the current API 653 standard, including the "out of plane deflection" equation text removed in the version of the figure earlier in the ballot; remove the "datum line for adjusted settlement" as before. Renumber to figure B.5.

B.2.2.7.1

The procedure in B.2.2.7 ~~may only~~ shall not be used as an alternative to the procedure in B.2.2.6 if the spacing between station measurements is greater than or equal to 15 ft and less than or equal to 22 ft. Examples of applicable tank diameters and number of measurement stations are shown in the table below.

See Table 1 at end of this ballot document.

Examples of applicable tank diameters and number of measurement stations when removing every other data point (as discussed in Figure B.5) are shown in the table below.

See Table 2 at end of this ballot document.

B.2.2.8

If the ~~settlement data is "dense" (~~the number of settlement data points is greater than 64~~)~~, the following method ~~should~~ shall be used to evaluate the settlement in conjunction with B.3.2.3:

The method estimates the tank bottom perimeter curvature (second derivative of differential settlement) by using a computer program to fit ~~fitting~~ a finite Fourier series ~~like the following equation~~ to the observed deflections from the rigid tilt plane using the following equation.

$$\cancel{u(\ell) = \sum_{2 \le k \le k_{\max}} A_k \cdot \cos\left(k \cdot \frac{\ell}{R} - \varphi_k\right)}$$

~~This equation can be written in a better form for multiple linear regression,~~

$$u(\ell) = \sum_{2 \le k \le k_{\max}} a_k \cdot \cos\left(k \cdot \frac{\ell}{R}\right) + b_k \cdot \sin\left(k \cdot \frac{\ell}{R}\right)$$

where ℓ is the arc length (ft) from the origin, *R* is the diameter, ℓ / R is the angle in radians, and ~~p~~ $p = 2\pi R / k$ is the period, ~~A_p is the amplitude of the Fourier term for that period, and φ_p is the phase angle.~~

There is no constant or first-order term in this equation because these were removed by subtracting the tilt plane.

~~Note: The maximum permitted period, p_{max} is selected so that the shortest half-wave is 20 feet,~~

$$\cancel{p_{min} = 2 \cdot \pi \cdot R_{ft} \big/ \left(2 \cdot 20_{ft}\right) = \pi \cdot R_{ft} / 20_{ft}}$$

When this equation contains at least the first six ~~most statistically significant~~ terms it is defined as a *trigonometric fit* (trig fit, for short). Sinusoidal terms after the first six are only included if they increase the model's overall adjusted R^2.

The curvature is determined by analytically calculating the second derivative of the fitted Fourier series. The second derivative of the trigonometric regression fit *u(l)* is

$$\frac{d^2}{dl^2} u(l) = -\left(\frac{k}{R}\right)^2 u(l)$$

The permissible magnitude (absolute value) of the second derivative of the trigonometric regression fit, D2max, is given in B.3.2.3.

The "API Revision Draft Insert" document contains an example calculation for this method in the R programming language.

API Revision Draft Insert.docx

B.2.2.8.1

The procedure in B.2.2.8 shall not be used if the tank diameter is less than 61 feet. In that case, the ~~dense~~ settlement dataset ~~should~~ shall be reduced to the appropriate size necessary to perform the procedures detailed in B.2.2.6 or B.2.2.7.

B.3 Determination of Acceptable Settlement

B.3.2 Permissible Out-of-plane Settlement

Switch places of B.3.2.1 and B.3.2.2 (as done below)

~~B.3.2.2~~ **B.3.2.1**

When using the procedure in B.2.2.~~5~~6 to determine out-of-plane deflection, the permissible out-of-plane deflection is given by the following equation (see Note):

$$S_{max,in} = min\left(K \times S_{arc} \times \left(\frac{D}{H}\right) \times \left(\frac{Y}{E}\right), 4.0\right)$$

Tank Diameter ft	Open Top Tanks, K	Fixed Roof Tanks, K
$D \le 50$	28.7	10.5
$50 < D \le 80$	7.8	5.8
$80 < D \le 120$	6.5	3.9
$120 < D \le 180$	4.0	2.3
$180 < D \le 240$	3.6	Not applicable
$240 < D \le 300$	2.4	Not applicable
$300 < D$	Not applicable	Not applicable

where

$S_{max, in}$ is permissible out-of-plane deflection, in inches;

S_{arc} is effective settlement arc, see B.2.2.~~5.1~~6, in feet;

D is tank diameter, in feet (ft);

Y is yield strength of the shell material, in pound force per square inch (lbf/in.2);

E is Young's Modulus, in pound force per square inch (lbf/in.2);

H is tank height, in feet.

NOTE This equation is based on "Final Report on the Study of Out-of-Plane Tank Settlement," J. Andreani, N. Carr, Report to API SCAST, May, 2007.

~~B.3.2.1~~ B.3.2.2

When using the procedure ~~with an optimal cosine curve approach defined~~ in B.2.2.~~4~~7 to determine out-of-plane deflection, the permissible out-of-plane deflection is given by the following equation (see Note):

$$S_{max,ft} = \frac{L^2 \times Y \times L}{2[(E \times H)]}$$

where

Smax, ft is permissible out-of-plane deflection, in feet;

L is arc length between measurement points, in feet;

Y is yield strength of the shell material, in pound force per square inch (lbf/in.2);

E is Young's Modulus, in pound force per square inch (lbf/in.2);

H is tank height, in feet.

NOTE This equation is based on "Criteria for Settlement of Tanks," W. Allen Marr, M. ASCE, Jose A. Ramos, and T. William Lambe, F. ASCE, Journal of Geotechnical Engineering Division, Proceedings of the American Society of Civil Engineers, Vol. 108, August, 1982.

B.3.2.3

When using the procedure in B.2.2.~~7~~8 to determine out-of-plane settlement, the permissible magnitude (absolute value) of the second derivative of the trigonometric regression fit $u(l)$, $\frac{d^2}{dl^2}u(l)$, is given by the following equation (see Note):

$$\mathrm{D2}_{max} = 22 \times \frac{Y}{E \times H}$$

where

$\mathrm{D2}_{max}$ is the permissible second derivative of the trigonometric regression fit $u(l)$, in feet/feet^2;

Y is yield strength of the shell material, in pound force per square inch (lbf/in.^{2});

E is Young's Modulus, in pound force per square inch (lbf/in.^{2});

H is tank height, in feet.

	NOTE The details of the trigonometric regression method and an example computation can be found at https://github.com/rbitip/API_Public/tree/main/ballot%20653-1012.

		Number of manual measurement stations																									
Diam	Circum	6	8	10	12	14	16	18	20	22	24	26	28	32	36	40	42	44	46	48	50	52	54	56	58	60	62
32	100	17															XX	Distance between points, ft									
48	150		19																								
64	200			20	17																						
80	250				21	18	16																				
95	300					21	19	17																			
111	350						22	19	18	16																	
127	400								20	18	17																
143	450									20	19	17	16														
159	500										21	19	18	16													
175	550											21	20	17	15												
191	600												21	19	17												
207	650													20	18	16											
223	700													22	19	18	17	16									
239	750														21	19	18	17	16								
255	800															20	19	18	17	17	16						
271	850																20	19	18	18	17	16					
286	900																	20	20	19	18	17	17	16			
302	950																		21	20	19	18	18	17	16	16	
318	1000																				20	19	19	18	17	17	16

Table 1 Example of applicable tank diameters and number of measurement stations for the procedure in B.2.2.7.

		Number of manual measurement stations																									
Diam	Circum	6	8	10	12	14	16	18	20	22	24	26	28	32	36	40	42	44	46	48	50	52	54	56	58	60	62
32	100				17												XX	Distance between every point, after removing									
48	150						19	17										every other data point, ft									
64	200								20	18	17																
80	250										21	19	18	16													
95	300												21	19	17												
111	350													22	19	18	17	16									
127	400															20	19	18	17	17							
143	450																	20	20	19	18	17	17	16			
159	500																				20	19	19	18	17	17	16
175	550																						20	20	19	18	18
191	600																									20	19

Table 2

“Clean Copy” of Proposed Verbiage

API 653 Annex B

B.2 Types of Settlement

B.2.2 Shell Settlement Evaluation

Figure B.3: remove text below plot: “Out-of-plane deflection for Point “I” is $S_i = U_i - \left(\frac{1}{2}U_{i-1} + \frac{1}{2}U_{i+1}\right)$, for example $S_{11} = U_{11} - \left(\frac{1}{2}U_{10} + \frac{1}{2}U_{12}\right)$”. Remove “datum line for adjusted settlement” and accompanying lines.

B.2.2.4
While uniform settlement and rigid body tilt of a tank may cause problems as described in B.2.2.1 and B.2.2.2, the out-of-plane settlement is the important component to determine and evaluate in order to ensure the structural integrity of the shell and bottom. Based on this principle, a common approach is to determine the magnitudes of the uniform settlement and rigid body tilt (if a rigid tilt plane exists or can be identified) for each data point on the tank periphery. If a plane of rigid tilt can be distinguished, it becomes important as a datum from which to measure the magnitudes of the out-of-plane settlements.

A graphical representation illustrating tank shell settlement with a rigid tilt plane well–defined by a cosine curve fit is shown in Figure B.3. The construction of this settlement plot has been developed in accordance with the following.

a) The actual settlement (in most cases an irregular curve) is plotted using points around the tank circumference as the abscissa (x-coordinate).

b) The vertical distance between the x-axis and the average of the settlement measurements is the uniform settlement. A line through this point, parallel to the x-axis, provides a new base or datum line for settlement measurements called adjusted settlements.

c) The plane of rigid tilt settlement is represented by the optimum (best-fit) cosine curve. Several methods exist for determining the optimum cosine curve. It is routinely performed using any linear regression software[3].

d) A commonly used and accepted method is to use a computer to solve for constants *a, b,* and *c*, to find the optimum cosine curve of the form:

$$\mathrm{Elev}_{pred} = a + b \times cos(\theta + c)$$

Where Elev_{pred} is the elevation predicted by the cosine curve at angle theta. Rather than using non-linear regression, which would be required in the form of the equation above, linear regression can be applied as a least squares fit to the data of the equivalent form $d + e \times cos\,\theta + f \times sin\,\theta$, using $cos\,\theta$ and $sin\,\theta$ as basis functions and then finding the constants *a, b,* and *c* from the values of *d, e,* and *f*.

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[3] An example solution in Microsoft Excel for finding the cosine curve for rigid tilt can be found at https://github.com/rbitip/settlement-excel-regression-too.

$$a = d$$
$$b = sgn(e) \times \sqrt{e^2 + f^2}$$
$$c = tan^{-1}(f/e)$$

Where sgn is the signum/sign function.

e) The vertical distances between the irregular curve and the cosine curve represent the magnitudes of the out-of- plane settlements (U_i at Data Point i).

B.2.2.5

The method used to evaluate the out-of-plane settlement is dependent on the number of settlement data points collected. This distinction is especially important for settlement data collected via laser scanning. If the number of settlement data points is less than or equal to 64, either of the methods described in B.2.2.6 or B.2.2.7 shall be used, if applicable. If the number of settlement data points is greater than 64, the method in B.2.2.8 shall be used. In lieu of these methods, or if the maximum out-of-plane settlement determined in accordance with these methods is exceeded, it is always acceptable to use finite-element methods to evaluate the settlement instead.

B.2.2.6

If the number of settlement data points is less than or equal to 64, the procedure in B.3.2.1 shall be used to evaluate the settlement. In this case, see Figure B.4 for a graphical illustration of the determination of the settlement arc length and the corresponding out-of-plane deflection. As seen in the figure, each settlement arc length is determined from where the linearly interpolated curve of the out-of-plane settlement crosses the cosine curve.

Remove figure B.4. Renumber figure B.5 to the new B.4 and fix references to Figure B.5 as appropriate.

B.2.2.7

The procedure in B.3.2.2 may be used to evaluate settlement as an alternative to the method in B.2.2.6. In this case, see Figure B.5 for a graphical illustration of the various measurement terms and procedures for determining estimates of the settlement arc length and the corresponding out-of-plane deflection, including the refinement of measurements, when needed.

Include the version of figure B.3 in the current API 653 standard, including the “out of plane deflection” equation text removed in the version of the figure earlier in the ballot; remove the “datum line for adjusted settlement” as before. Renumber to figure B.5.

B.2.2.7.1

The procedure in B.2.2.7 shall not be used as an alternative to the procedure in B.2.2.6 if the spacing between station measurements is greater than or equal to 15 ft and less than or equal to 22 ft. Examples of applicable tank diameters and number of measurement stations are shown in the table below.

See Table 1 at end of this ballot document.

Examples of applicable tank diameters and number of measurement stations when removing every other data point (as discussed in Figure B.5) are shown in the table below.

See Table 2 at end of this ballot document.

B.2.2.8
If the number of settlement data points is greater than 64, the following method shall be used to evaluate the settlement in conjunction with B.3.2.3:

The method estimates the tank bottom perimeter curvature (second derivative of differential settlement) by using a computer program to fit a finite Fourier series to the observed deflections from the rigid tilt plane using the following equation.

$$u(\ell) = \sum_{2 \le k \le k_{\max}} a_k \cdot \cos\left(k \cdot \frac{\ell}{R}\right) + b_k \cdot \sin\left(k \cdot \frac{\ell}{R}\right)$$

where ℓ is the arc length (ft) from the origin, *R* is the diameter, ℓ / R is the angle in radians, and $p = 2\,\pi R\ /\ k$ is the period.

There is no constant or first-order term in this equation because these were removed by subtracting the tilt plane.

When this equation contains at least the first six terms it is defined as a *trigonometric fit* (trig fit, for short). Sinusoidal terms after the first six are only included if they increase the model's overall adjusted R^2.

The curvature is determined by analytically calculating the second derivative of the fitted Fourier series. The second derivative of the trigonometric regression fit *u(l)* is

$$\frac{d^2}{dl^2} u(l) = -\left(\frac{k}{R}\right)^2 u(l)$$

The permissible magnitude (absolute value) of the second derivative of the trigonometric regression fit, D2max, is given in B.3.2.3.

B.2.2.8.1
The procedure in B.2.2.8 shall not be used if the tank diameter is less than 61 feet. In that case, the settlement dataset shall be reduced to the appropriate size necessary to perform the procedures detailed in B.2.2.6 or B.2.2.7.

B.3 Determination of Acceptable Settlement

B.3.2 Permissible Out-of-plane Settlement

B.3.2.1
When using the procedure in B.2.2.6 to determine out-of-plane deflection, the permissible out-of-plane deflection is given by the following equation (see Note):

$$S_{max,in} = min\left(K \times S_{arc} \times \left(\frac{D}{H}\right) \times \left(\frac{Y}{E}\right), 4.0\right)$$

Tank Diameter ft	Open Top Tanks, K	Fixed Roof Tanks, K
$D \leq 50$	28.7	10.5
$50 < D \leq 80$	7.8	5.8
$80 < D \leq 120$	6.5	3.9
$120 < D \leq 180$	4.0	2.3
$180 < D \leq 240$	3.6	Not applicable
$240 < D \leq 300$	2.4	Not applicable
$300 < D$	Not applicable	Not applicable

where

$S_{max,\ in}$ is permissible out-of-plane deflection, in inches;
S_{arc} is effective settlement arc, see B.2.2.6, in feet;
D is tank diameter, in feet (ft);
Y is yield strength of the shell material, in pound force per square inch (lbf/in.2);
E is Young's Modulus, in pound force per square inch (lbf/in.2);
H is tank height, in feet.

NOTE This equation is based on "Final Report on the Study of Out-of-Plane Tank Settlement," J. Andreani, N. Carr, Report to API SCAST, May, 2007.

B.3.2.2

When using the procedure in B.2.2.7 to determine out-of-plane deflection, the permissible out-of-plane deflection is given by the following equation (see Note):

$$S_{max,ft} = \frac{L^2 \times Y \times L}{2[(E \times H)]}$$

where

Smax, ft is permissible out-of-plane deflection, in feet;
L is arc length between measurement points, in feet;
Y is yield strength of the shell material, in pound force per square inch (lbf/in.2);
E is Young's Modulus, in pound force per square inch (lbf/in.2);
H is tank height, in feet.

NOTE This equation is based on "Criteria for Settlement of Tanks," W. Allen Marr, M. ASCE, Jose A. Ramos, and T. William Lambe, F. ASCE, Journal of Geotechnical Engineering Division, Proceedings of the American Society of Civil Engineers, Vol. 108, August, 1982.

B.3.2.3

When using the procedure in B.2.2.8 to determine out-of-plane settlement, the permissible magnitude (absolute value) of the second derivative of the trigonometric regression fit $u(l)$, $\frac{d^2}{dl^2}u(l)$, is given by the following equation (see Note):

$$\mathrm{D2}_{max} = 22 \times \frac{Y}{E \times H}$$

where

$\mathrm{D2}_{max}$ is the permissible second derivative of the trigonometric regression fit $u(l)$, in feet/feet^2;
Y is yield strength of the shell material, in pound force per square inch (lbf/in.^{2});
E is Young's Modulus, in pound force per square inch (lbf/in.^{2});

H is tank height, in feet.

NOTE The details of the trigonometric regression method and an example computation can be found at https://github.com/rbitip/API_Public/tree/main/ballot%20653-1012.