Step-by-Step Markup of Trig-Reg Example

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Define tank properties.

```
# trig-reg example

R <- 136 # tank radius, ft
H <- 66.4 # shell height, ft
Y <- 36e3 # yield strength, psi
E <- 30e6 # young's modulus, psi
```

Load in example laser scan data .csv file.

```
# load in data and process ----
df <- read.csv("XYZ Example 1.csv")
```

Re-center X and Y coordinate data. Convert X, Y, and Z coordinates from meters to feet. Convert X and Y coordinates into angle about center.

```
df$x <- (df$X - mean(df$X)) * 3.28084 # re-center + meter-ft conversion
df$y <- (df$Y - mean(df$Y)) * 3.28084
df$z <- df$Z * 3.28084
theta <- atan2(df$y, df$x) # determine angle about center, radians
df$azimuth <- (theta + (theta < 0) * 2 * pi) # only positive angles
df <- df[order(df$azimuth),] # reorder by angle
```

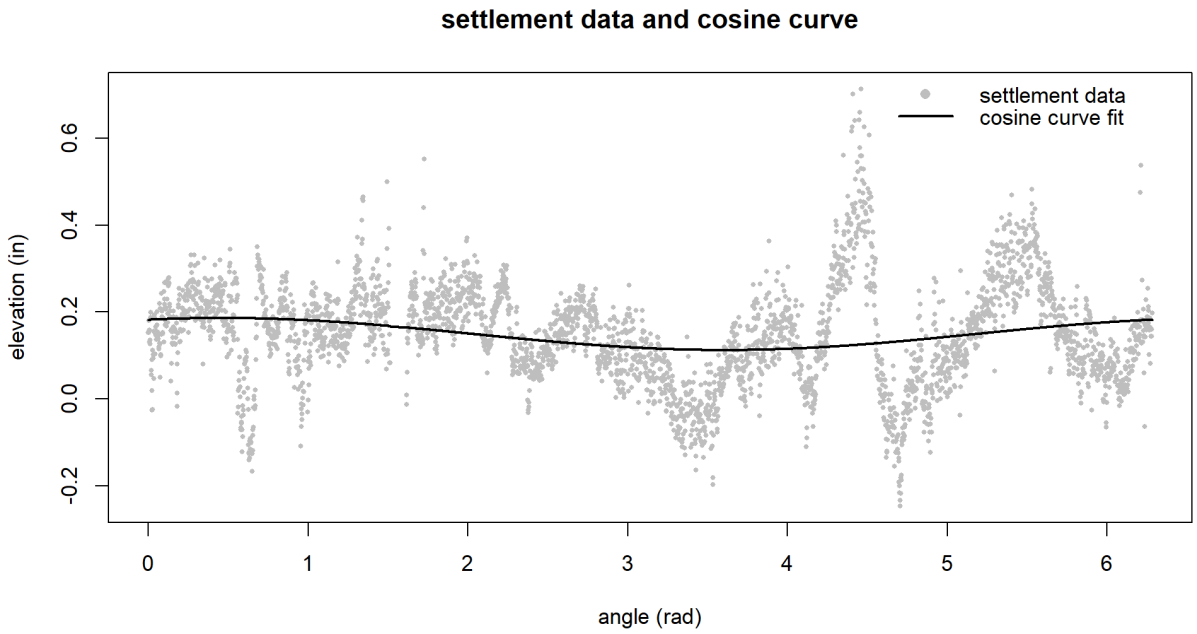
Perform a cosine curve fit of the data to determine the out-of-plane settlement.

```
# cosine curve fit ----

lm <- lm(z ~ cos(azimuth) + sin(azimuth), data = df)
df$zhat <- fitted(lm) # cosine curve fit
df$z1 <- residuals(lm) # out of plane settlement
```

Generate plot of settlement elevations vs cosine curve.

```
# settlement data vs cosine curve plot
plot(df$azimuth, df$z*12, pch=16, cex = 0.5, col = "gray", xlab="angle (rad)",
ylab="elevation (in)")
lines(df$azimuth, df$zhat*12, lwd = 2, col = "black")
legend("topright", legend = c("settlement data", "cosine curve fit"), pch = c(16,
NA), lwd = c(NA, 2), col = c("gray", "black"), bty = "n")
title(main = "settlement data and cosine curve")
```



Generate cosine/sine basis functions for use in linear regression.

```
# out-of-plane settlement ----

# forward variable selection

# create data frame of basis cos and sin functions, B
# (easier to define them as columns in a matrix than defining them individually)
maxfreq = floor(2 * pi * R / (2 * 20)) # max frequency = circum / (2 * 20 ft)
B <- as.data.frame(matrix(ncol = 1 + 2 * maxfreq, nrow = nrow(df)))
B[,1] <- df$z1
colnames(B)[1] <- "z1"
for(freq in 1:maxfreq){
  B[,2 * (freq - 1) + 2] <- cos(freq * df$azimuth)
  B[,2 * (freq - 1) + 3] <- sin(freq * df$azimuth)
  colnames(B)[2 * (freq - 1) + 2] <- paste0("c", freq)
  colnames(B)[2 * (freq - 1) + 3] <- paste0("s", freq)
  # B$c1 is cos(azimuth), B$s2 is sin(2*azimuth), and so on...
}
```

Begin forward variable selection: perform linear regression of out-of-plane settlement data with cosine and sine basis functions of successively higher frequencies.

```
# try adding each frequency and determine when to stop using more cos/sin terms
adjR2 <- vector(mode="numeric", length = maxfreq)
for(freq in seq(from = 1, to = maxfreq)){
  vars <- colnames(B)[seq(from = 2, to = 2 * (freq - 1) + 3)]
  frm <- as.formula(paste("z1 ~ -1 + ", paste(vars, collapse = " + ")))

  lmx <- lm(frm, data = B)
  adjR2[freq] <- summary(lmx)$adj.r.squared
}
```

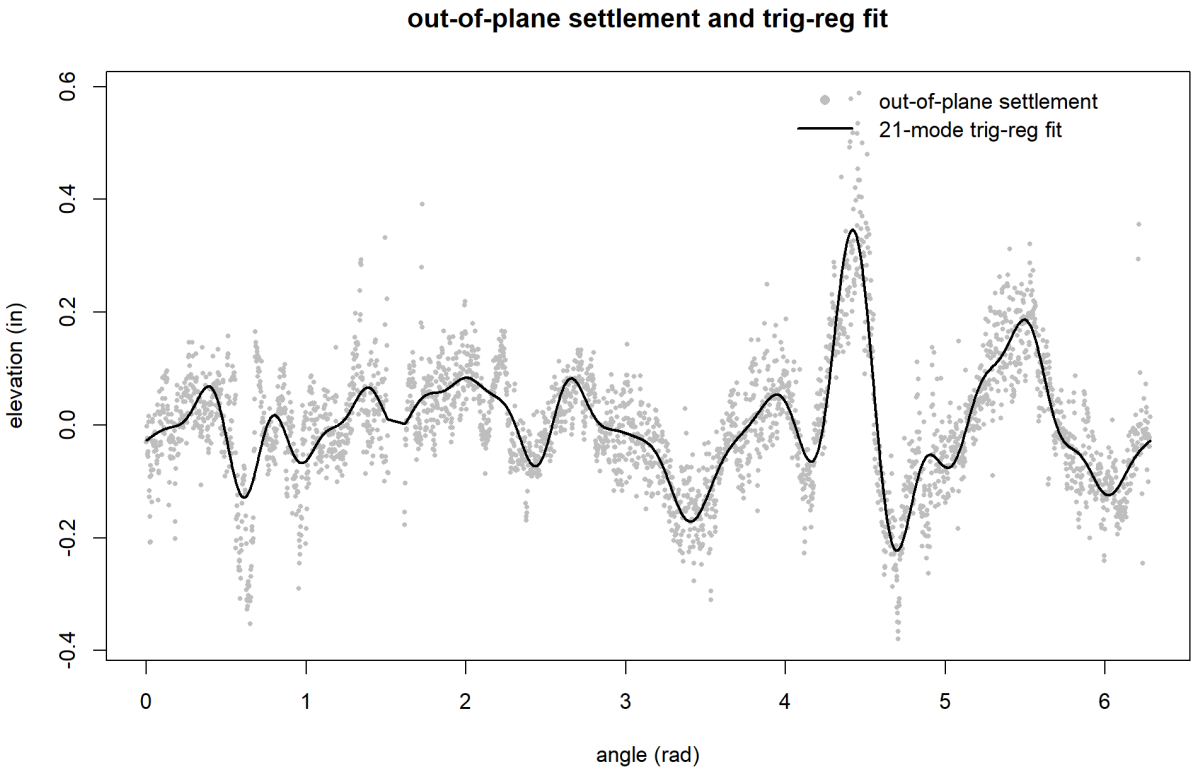
Determine the trigonometric fit frequency with maximum adjusted R^2. This is the trigonometric fit to the out-of-plane settlement.

```
# find the maximum frequency that increases adj. R^2
endfreq <- max(which(diff(adjR2) > 0)) + 1
vars <- colnames(B)[seq(from = 2, to = 2 * (endfreq - 1) + 3)]
frm <- as.formula(paste("z1 ~ -1 + ", paste(vars, collapse = " + ")))
lmx <- lm(frm, data = B)

df$z1hat <- fitted(lmx)
```

Generate plot of out-of-plane settlement compared to selected trigonometric fit curve.

```
# out-of-plane-settlement plot
plot(df$azimuth, df$z1*12, pch=16, cex = 0.5, col = "gray", xlab="angle (rad)",
ylab="elevation (in)")
lines(df$azimuth, df$z1hat*12, lwd = 2, col = "black")
legend("topright", legend = c("out-of-plane settlement", paste0(endfreq, "-mode
trig-reg fit")), pch = c(16, NA), lwd = c(NA, 2), col = c("gray", "black"), bty =
"n")
title(main = "out-of-plane settlement and trig-reg fit")
```



Calculate second derivative of the trigonometric fit. This is a proxy for the curvature of the tank bottom circumference. Calculate the maximum permissible second derivative of the trigonometric fit. This is based on the allowable settlement criteria of the Marr method.

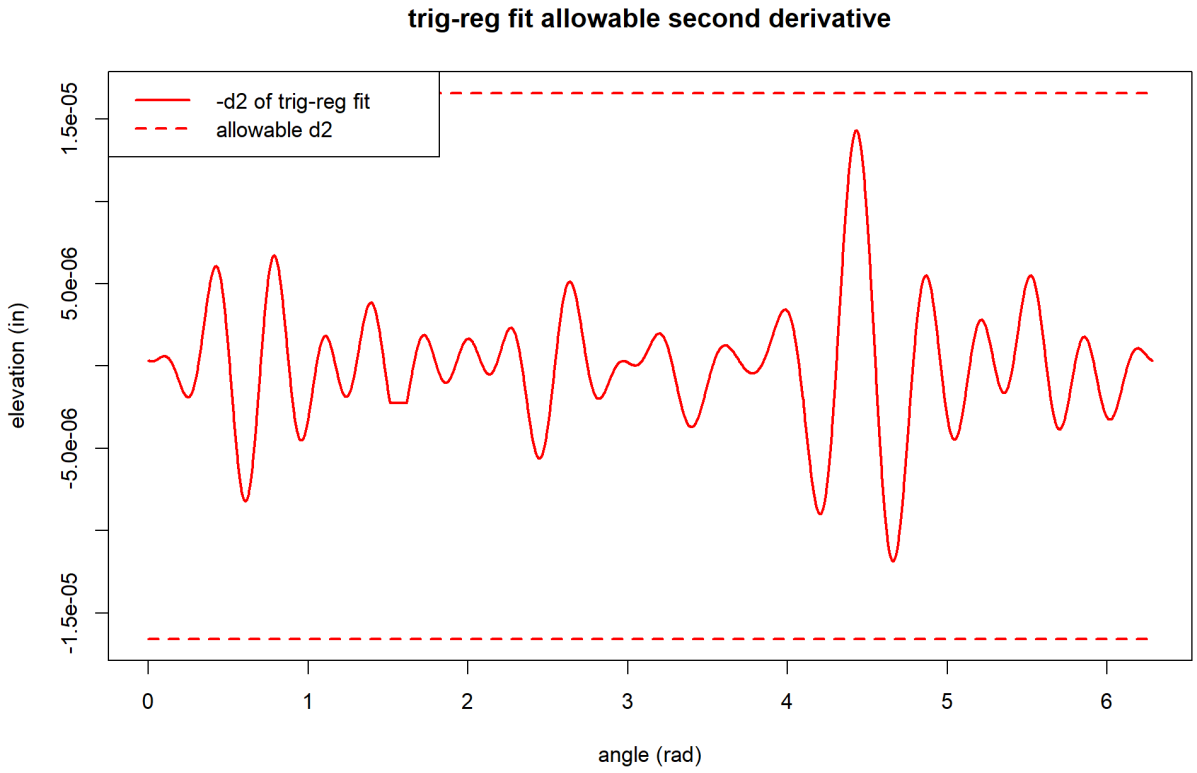
```
# allowable out-of-plane settlement ----

# calculate second derivative of trig-reg fit
coeffs <- lmx$coefficients
d2 <- vector(mode = "numeric", length = nrow(df))
for(freq in 1:endfreq){
  d2 <- d2 + freq^2 * coeffs[2 * (freq - 1) + 1] * B[,2 * (freq - 1) + 2] * -1 /
R^2
  d2 <- d2 + freq^2 * coeffs[2 * (freq - 1) + 2] * B[,2 * (freq - 1) + 3] * -1 /
R^2
}

# allowable second derivative based on modified Marr
d2max <- 11 * Y / (12 * H * E) # in/in^2
```

Generate plot of the second derivative of the trigonometric fit compared to the permissible second derivative.

```
# second derivative and allowable plot
plot(df$azimuth, -d2 * 12 / 144, type = "l", lwd = 2, col = "red", xlab="angle
(rad)", ylab="elevation (in)",
     ylim = c(min(-d2 * 12 / 144, -d2max), max(-d2 * 12 / 144, d2max)))
lines(c(0,2*pi), c(d2max, d2max), lty = 2, lwd = 2, col = "red")
lines(c(0,2*pi), c(-d2max, -d2max), lty = 2, lwd = 2, col = "red")
legend("topleft", legend = c("-d2 of trig-reg fit", "allowable d2"), lwd = c(2, 2),
lty = c(1, 2), col = c("red", "red"))
title(main = "trig-reg fit allowable second derivative")
```



Generate plot of trigonometric fit second derivative with permissible overlayed with out-of-plane settlement data and the trigonometric fit.

```
# overlay out-of-plane settlement with trig-reg second derivative
scalefactor <- signif(diff(range(df$z1 * 12)) / diff(range(d2 * 12 / 144)), 1)/2

plot(df$azimuth, df$z1*12, pch=16, cex = 0.5, col = "gray", xlab="angle (rad)",
ylab="elevation (in)")
lines(df$azimuth, df$z1hat*12, lwd = 2, col = "black")
lines(df$azimuth, -d2 * 12 / 144 * scalefactor, lwd = 2, col = "red")
lines(c(0,2*pi), c(d2max, d2max) * scalefactor, lty = 2, lwd = 2, col = "red")
lines(c(0,2*pi), c(-d2max, -d2max) * scalefactor, lty = 2, lwd = 2, col = "red")
legend("topright", legend = c("out-of-plane settlement", paste0(endfreq, "-mode
trig-reg fit"), "-d2 of trig-reg fit", "allowable d2"),
       pch = c(16, NA, NA, NA), lwd = c(NA, 2, 2, 2), lty = c(NA, 1, 1, 2), col =
c("gray", "black", "red", "red"), bty = "n")
title(main = "out-of-plane settlement: trig-reg fit and second derivative")
```