

7th Asian-Pacific Conference on Aerospace Technology and Science, 7th APCATS 2013

A guidance to grid size design for CFD numerical simulation of hypersonic flows

H. R. Zhang^{a,*}, Y. Yu^b

^{a,b}*School of Aerospace Engineering, Beijing Institute of Technology, 5 South Zhongguancun Street, Haidian District, Beijing, 100081, China*

Abstract

Studies showed that grids are significant to the accuracy of numerical simulation in hypersonic flows, but there is no definite method to determine the magnitudes of grid sizes yet. This paper proposes theoretical guidance for grid size design from the physical phenomena, small-scale structures generated in hypersonic flows with interaction between viscous/non-viscous flows, which can be proved by interaction shear flows (ISF) theory. The small-scale structure's physical scale can be a reference to determine the grid sizes in flow and normal direction for CFD calculation. The results show that the accuracy of numerical simulation is partly related to the grid size, and the better simulation results can be obtained when using grid designed by the small-scale structure of ISF theory. By the guidance, we can determine the magnitudes of grid size directly without refining the grids times to try the best grid size.

© 2013 The Authors. Published by Elsevier Ltd. Open access under [CC BY-NC-ND license](#).
Selection and peer-review under responsibility of the National Chiao Tung University

Keywords: CFD; Numerical simulation; ISF; Grid size; Aerodynamic characteristic

Nomenclature

M	mach number
Re	reynolds number
T	temperature
<i>Greek symbols</i>	
ρ	density
<i>Subscripts</i>	
∞	incoming flow

* Corresponding author.

E-mail address: 851475786@qq.com

c critical

1. Introduction

In the modern hypersonic aero-craft's aerodynamic characteristics studies, Computational Fluid Dynamics (CFD) has been played a major role. A large amount of work [1-3] has been done in the CFD computation of aerodynamic, but there are still some problems need to study, such as the grid effect.

Following the past studies, the grid size has an important effect on the accuracy of aerodynamic calculation, especially, the wall-surface normal grid size and flow grid spacing. Pan [4] showed that grid effect is a key factor on the heat flux computations, especially the grid spacing near wall-surface. Hoffmann [5] illustrated the dependency of the heat flux qualities on the grid system, in particular, grid line distribution near the wall-surface. In addition, he also shows that changes in flow Mach number and/or Reynolds number may require further refinement of the grid system. Stephen [6] estimated the influence of structured-grid quality on the results of calculation, and demonstrated that the orthogonality of grid is helpful to calculate the heat flux and friction coefficient in hypersonic flows. Yan [7] demonstrated that one can obtain a better simulation result of flow field from the refinement of grid, but when the simulation results are accepted there will be no obvious improvement with increasing the grid quantities.

Grid size has a great effect on the results of CFD calculation, which has become a common view among most researchers. But how long the first wall-surface normal grid size is? How long the flow direction grid size should be? There is no definite answer to these questions. Most of the researchers design grids by experience or calculating several times then choose the appropriate grid by the results of grid convergence analysis. So it is necessary to have a theoretical guidance for the grid design. While this paper is just about the question of guidance for the grid design.

2. ISF theory

Gao Zhi put forward the theory of interaction shear flows (ISF) basing the following hypothesis: convection is dominant in the flow direction while convection competed with diffusion in the normal direction, which can be mathematically described as equation (1)、(2) for two-dimension compressible flows.

$$\rho \frac{\partial f}{\partial t} \cong \rho v \frac{\partial f}{\partial y} \cong \frac{\partial}{\partial y} \left(\mu \frac{\partial f}{\partial y} \right) \quad (1)$$

$$\rho u \frac{\partial f}{\partial x} \gg \frac{\partial}{\partial x} \left(\mu \frac{\partial f}{\partial x} \right) \quad (2)$$

Where $f = (u, v)$, u, v are the flow and normal velocity components of shear layer respectively; x, y are the flow and normal coordinate variations of shear layer respectively; ρ is density; μ is viscosity coefficient; t is time.

From ISF theory, a physical phenomena, a small-scale structure will generate when the viscous/non-viscous flows interaction is strong enough, can be predicted. The theory presents the magnitude estimation of small-scale structures in flow and normal directions [8].

The physical sizes of small-scale structure in compressible ISF flows can be summarized as formula (3)

$$(t_c; x_c, y_c; u_c, v_c) = \left(\text{Re}^{-m}; \text{Re}^{-3m/2}, \text{Re}^{-(1+m)/2}; \text{Re}^{-m/2}, \text{Re}^{-(1-n_p+\omega n_T-m)/2} \right) \quad (3)$$

Where $Re = \frac{\rho UL}{\mu}$, $t_c; x_c, y_c; u_c, v_c$ are the time/length/velocity scales of strong viscous shear flows;

$n_t; n_x, n_y, n_u, n_v$ are five quantities needed to be confirmed; ω is a parameter in $\mu = \mu_r \left(\frac{T}{T_r} \right)^\omega$ and

$0.7 \leq \omega \leq 1.0$; L is reference length; U is reference velocity; m is interaction parameter; the parameter of n_ρ

and n_T are respectively determined by the formulas: $Re^{-n_\rho} = \rho_2 / \rho_1$ and $Re^{-n_T} = T_2 / T_1$.

To illustrate the question concisely, we consider it as incompressible flow, where $n_\rho = \omega = 0$, then the physical sizes of small-scale structure can be summarized as equation (4)

$$(t_c; x_c, y_c; u_c, v_c) = (Re^{-m}; Re^{-3m/2}, Re^{-(1+m)/2}; Re^{-m/2}, Re^{-(1-m)/2}) \quad (4)$$

Where new scales generated between strong viscous/inviscid interaction flows differ from the reference scales. This theory can be partly used to design grid sizes as guidance from the equation (4). The minimum grid size should be smaller than the physical scales of small-scale structure. While this paper tries to use the ISF theory to design grids and prove the theory's superiority.

3. Models and test conditions

The models, hollow cylinder extended flare (HCEF) and sharp double cone (SDC), are numerically simulated by commercial CFD software FASTRAN. Both are involved by hypersonic flows. For HCEF and SDC cases, lots of experiments and numerical calculations [9-10] have been done. Sketches of HCEF and SDC's geometry [9] and representative pressure fields are respectively shown in Figures1-2. Flows are from left to right.

The test conditions are presented in Table 1. Fundamental quantities in Table1 (velocity, density, and temperature) are taken from the Calspan's report. Mach number and Reynolds number are acquired using appropriate thermodynamics and transport physical property of molecular nitrogen. All experiments were conducted in the Large Energy National Shock tunnel at Buffalo Research Center in the Calspan-University [11]. Steady, laminar, axisymmetric flow was used for all tests considered here. Experimental data of HCEF and SDC include wall-surface pressure and heat flux [1].

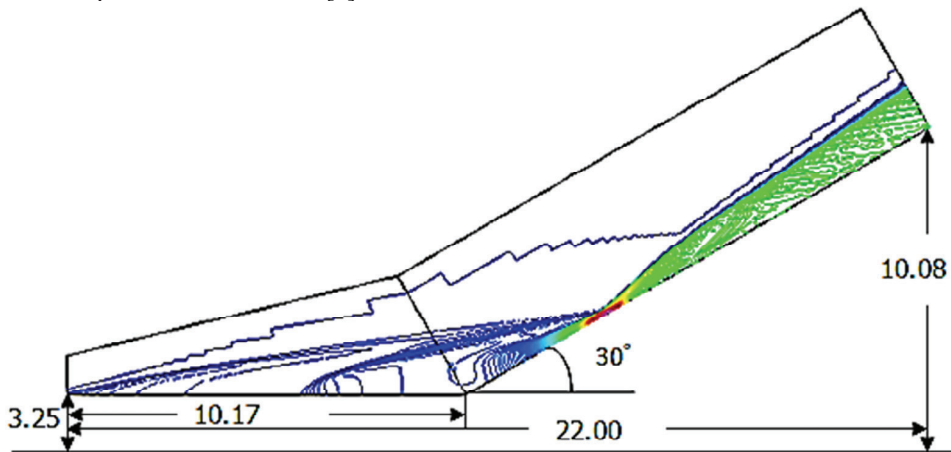


Fig. 1. HCEF's computational domain and pressure contour sketch

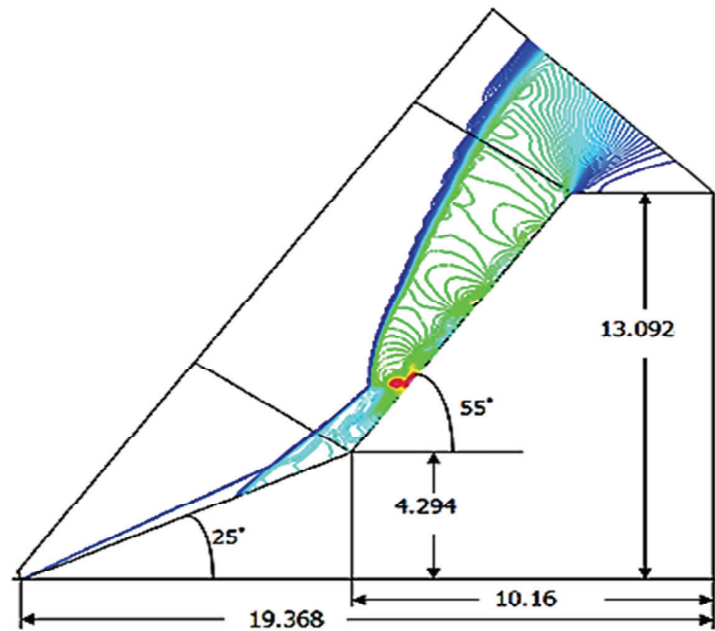


Fig. 2. SDC's computational domain and pressure contour sketch

Table 1. Test Conditions

Model	Run	$v_{\infty}, m/s$	$\rho_{\infty}, kg/m^3$	T_{∞}, K	T_w, K	M_{∞}	Re_{∞}, m^{-1}
HCEF	8	2667	0.001206	132.8	296.7	11.35	359600
SDC	28	2664	0.000655	185.6	293.3	9.59	144010

4. Grid

As Figures 1-2 show, the computational domains for HCEF and SDC is designed to fully contain the shock wave. The design of grid adopts the theory of near wall interaction shear flows (ISF) [8]. The gridlines in normal direction should be refined locally from the wall-surface as well as, especially, the gridlines in flow direction should be also refined at the strong interaction area.

For the strong viscous shear flow, a small-scale structure will be caused by strong interaction between viscous and non-viscous flows. According to the equation (4), the sizes of small-scale structures are $L Re^{-3m/2}$ in the flow direction and $L Re^{-(1+m)/2}$ in the normal direction, where

m: Interaction parameter, $m \leq 1/2$;

L: Reference length.

For the hollow cylinder extended flare (HCEF) and sharp double cone (SDC), according to the Reynolds numbers, we can estimate the small-scale structure sizes by ISF theory with supposing interaction parameter $m=1/4$. Then the flow and normal sizes are defined respectively as l_x, l_y , which are shown in table 2.

Table. 2

$m = 1/4$	Re_∞, m^{-1}	l_x	l_y
HCEF	359600	8.25×10^{-3}	3.37×10^{-4}
SDC	144010	1.16×10^{-2}	5.97×10^{-4}

In this paper, only the grid effect in the flow direction is considered, so the grid size in the normal direction remains a fixed value. For the HCEF and SDC, respectively, four sets of grids are determined in different grid density.

According to the small-scale structure of ISF theory, we can obtain in Table 2 that the normal sizes' magnitudes are both 10^{-4} , so the first wall-surface normal grid interval employs $\Delta y = 5 \times 10^{-5} \text{m}$ and remains unchanged, and then a hyperbolic tangent function is used to distribute gridlines in the normal direction. The flow direction grid interval is changed at different grid density and equally distributed. Then among the four sets of grids, respectively, one set of grid is fitted to the ISF theory. (The physical sizes of small-scale structure are defined approximately 10 times bigger of the minimum grid sizes in this paper.) The layouts of grid for HCEF and SDC are respectively defined in Tables 3-4.

Table. 3. Grids for HCEF(run 8)

Case	$(I \times J)$	$\Delta y_{\min}/\text{m}$	$\Delta x/\text{m}$
1	136x96	5E-5	2E-3
2*	272x96	5E-5	1E-3
3	544x96	5E-5	5E-4
4	1088x96	5E-5	2.5E-4

Table .4. Grids for SDC(run 28)

Case	$(I \times J)$	$\Delta y_{\min}/\text{m}$	$\Delta x/\text{m}$
1	136x96	5E-5	4.18E-3
2	272x96	5E-5	2.09E-3
3*	544x96	5E-5	1.045E-3
4	1088x96	5E-5	5.2E-4

*: Grid fitted to ISF theory.

Footnotes should be avoided if possible. Necessary footnotes should be denoted in the text by consecutive superscript letters. The footnotes should be typed single spaced, and in smaller type size (8 pt), at the foot of the page in which they are mentioned, and separated from the main text by a short line extending at the foot of the column.

5. Result and discussion

Figure3 and Figure4 respectively show the normalized wall-surface pressure coefficient C_p and the Stanton number St for HCEF. The separation region is characterized by the first pressure plateau and the attachment shock causes the second pressure rise, which is obviously showed in the pictures. Computations in the four sets of grid all agree with the experimental data well. While the results obtained on the four sets of grids give nearly identical solutions for both C_p and St . So for the HCEF case, the grid has little effect on the results of numerical simulation.

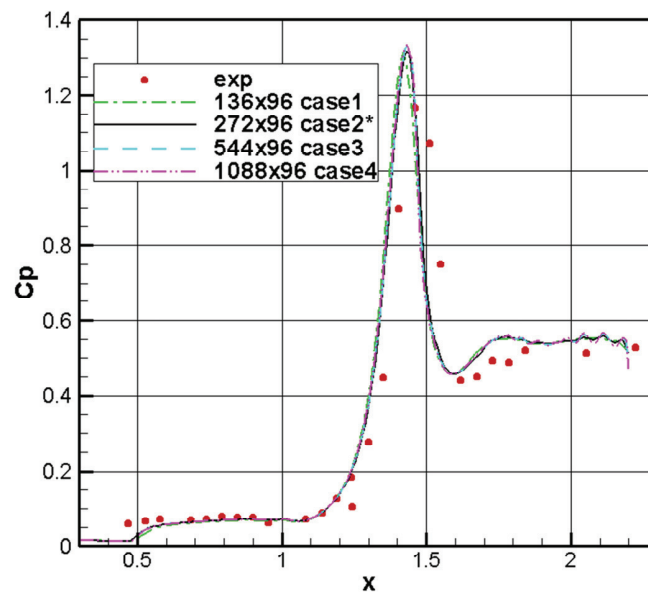


Fig. 3. Surface pressure coefficient for Run 8 over HCEF

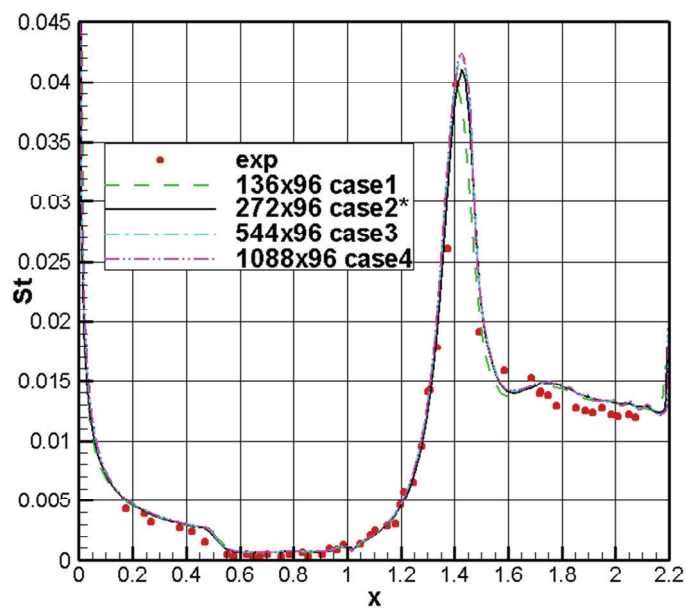


Fig. 4. Surface Stanton number for Run 8 over HCEF

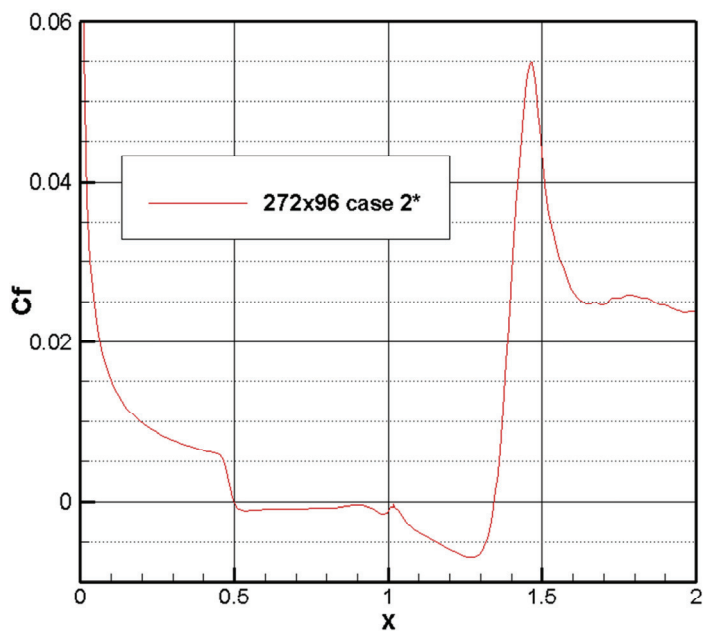


Fig. 5. Wall-surface friction coefficient for HCEF

Table. 5.

Location	CFD/Case2*
Separation	0.5
Reattachment	1.46

From wall–surface friction coefficient we can distinguish the separation and reattachment point. The Figure 5 shows the wall-surface friction coefficient of case2 for HCEF. From the picture, we can see the separation and reattachment point which have an estimated location at 0.5 and 1.46 respectively presented in Table 5.

Figure6 and Figure7 show the details of Figure3 around the separation and Reattachment points. There are very slight differences between the four cases though not very obviously in the global view. The solutions changes significantly from the coarsest to the finest grid. The solutions obtained on the case2* and case3 and case4 are similarly better than that of case1 obviously. The solution obtained on the case2* is close to the grid converged answer. So we can directly design the grid sizes by the small-scale structure of ISF theory for HCEF. In addition, relative differences of Stanton number follow the same trend as presented in corresponding figures for C_p and are not presented.

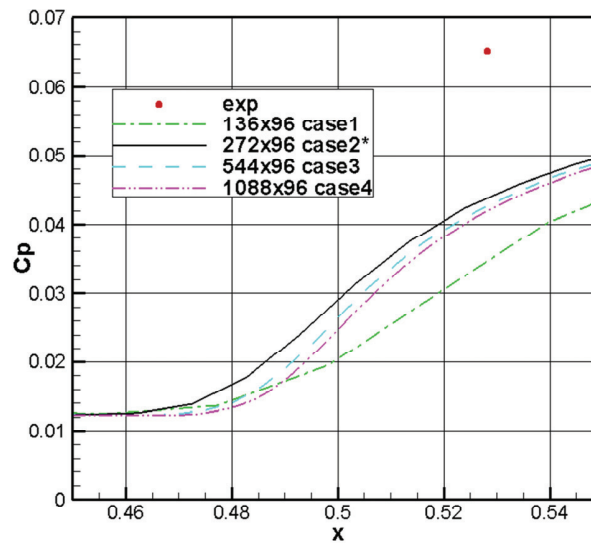


Fig. 6. Detail of Figure 3 around separation point

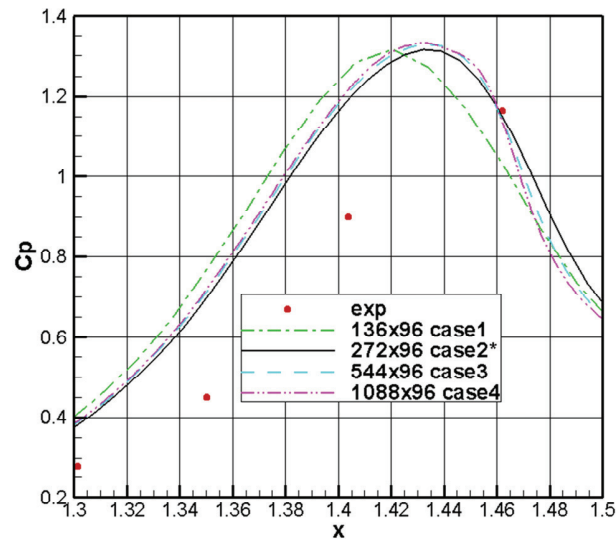


Fig. 7. Detail of Figure 3 near attachment point

Figure 8 and Figure 9 respectively show the normalized wall-surface pressure coefficient C_p and the Stanton number St for SDC. The separation and attachment points move forward obviously as the grid is refined, which illustrates that grid has a great influence on the results of CFD numerical simulation for SDC. From the pictures, we can see that the results of case3* and case4 are similarly close to the experimental data though they do not agree with the experimental data completely. Refining grid further is needed in this calculation for better solutions.

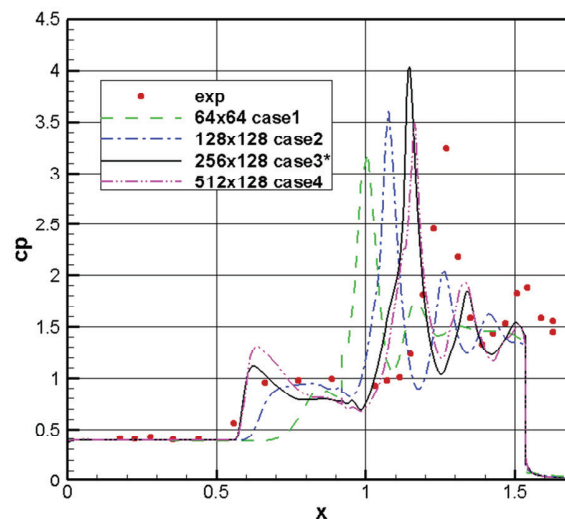


Fig. 8. Surface pressure coefficient for Run 28 over SDC

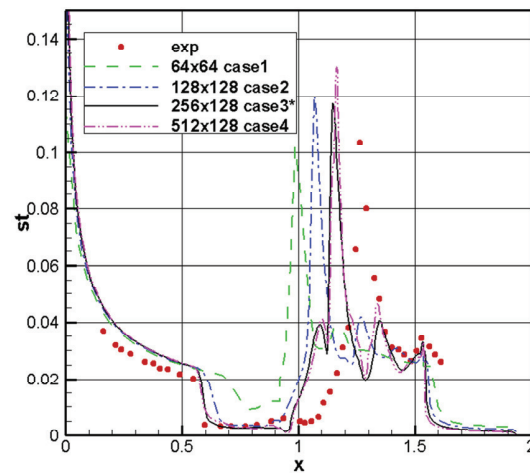


Fig. 9. Surface Stanton number for Run 28 over SDC

6. Conclusion

The calculations for HCEF and SDC show that the accuracy of numerical simulation results is partly related to the grid density and much better numerical simulation results can be obtained when the grid is designed under the guidance of small-scale structure of ISF theory. But the accuracy and applicability of the small-scale structures of ISF theory for grid design need to test and verify further.

Acknowledgements

This research work was supported by the Science Foundation of Beijing Institute of Technology 20120142017.

References

- [1] J. K. Harvey, Code validation study of laminar shock/boundary layer and shock/shock interactions in hypersonic flow, Part B: Comparison with Navier-Stokes and DSMC Solutions, AIAA, 2001-1031.
- [2] M. Holden, Experimental studies of laminar separated flows induced by shock wave/boundary layer and shock/shock interaction in hypersonic flows for CFD validation, AIAA, 2000-0930.
- [3] M. J. Wright, K. Sinha, J. Olejniczak, and G. V. Candler, Numerical and experimental investigation of double-cone shock interactions, AIAA, Vol. 38, No. 12, December 2000.
- [4] Pan Sha, Feng Dinghua, Ding Guohao, Tian Zhengyu, Yang Yueming, Li Hua, Grid dependency and convergence of hypersonic aerothermal simulation, Chinese journal of aeronautics, 2010, 31(3):493-499.
- [5] K. A. Hoffmann, Difficulties associated with the heat flux computations of high speed flows by the navier-stocks equations, AIAA, 91-0467.
- [6] Stephen J. Alter, A structured-grid quality measure for simulated hypersonic flows, AIAA, 2004-612.
- [7] Yan Chao, Yu Jianjun, Li Junzhe, Scheme effect and grid dependency in CFD computations of heat transfer, ACTA Aerodynamica Sinica, 2006, 24(1): 125-130.
- [8] Gao Zhi, Strong viscous flow theory with application to computation of high Reynolds number flows, ACTA Aerodynamica Sinica, 2001, 19(4): 420-426.
- [9] Peter A. Gnoffo, CFD validation studies for hypersonic flow prediction, AIAA, 2001-1025.
- [10] Holden M, Harvey J, Comparisons between experimental measurements over cone/cone and cylinder/flare configurations and predictions employing DSMC and Navier-Stokes solvers, AIAA, 2001-1031.
- [11] "Experimental Database from CUBRC Studies in Hypersonic Laminar and Turbulent Interacting Flows including Flowfield Chemistry," Prepared for RTO Code Validation of DSMC and Navier-Stokes Code Validation Studies, Calspan-University at Buffalo Research Center, Buffalo, NY, June 2000.