see any startling changes, such as the airfoil that was patented this year.

Figure 4

Contests will still be won by N.A.C.A. 6409's and its variations, my own favorite is a K&E/Davis.

Figure 5

The ubiguitous fiberglass tail boom will be replaced with a complete glass fiber moulded fuselage, ready for mounting the equipment.

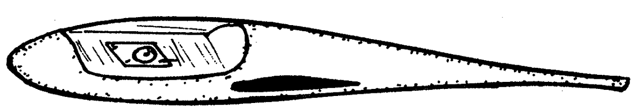


Figure 6

This brings us to a decision we as free flighters will have to face. This is the builder of the model rule. A moment's reflection on the Mattel "Superstar" which flies as well as many scratch built rubber jobs gives the key to the future. One of the R/C glider makers will produce a plane which although intended for R/C will also fit Nordic specs and fly better than most of the planes at the field today. Many of us will be able to pay the hundred dollars or so such a ship will cost. And lots of today's "Flyers" as opposed to the "Builder/Flyers" will be willing to pay such a sum for the ready to fly Nordic.

I will cry when this happens, but I can see the "Hobie" Nordic on the way.

Now for equipment. Today we have the timer, and the circular tow hook, how about a homing device "Estes" to find the errant model in a cornfield $25.

As long as we have batteries on board, why not an automatic thermal flying device. A thermal sniffing device (March '69) Flying Model) could be made to fly the ship straight until lift is found, and then work its way into the thermal core. When that lift is played out the plane would proceed hunting again until lift is found or a max occurs. ($150)

The familiar Seelig timer, will be replaced with a transistor timing circuit that will d/t to within a thousandth of a second of its assigned time. Such devices are already being worked on by my friends at N.A.S.A. ($50)

If you have been following Manyard/Hill.s electrostatic autopilot work (Feb '73 Flying Models), one can see that it is only a matter of time and money to adapt the system to Nordic size. This will enable the designer to reduce the stab area to as little as 5%. This will lower the effective wing loading and perhaps even make the Flying Wing Nordic flyable.

If we have the servos already in the ship, we can make a device that will tape-program the launch into a zero/zero zoom like a hand-launch glider, with a turn on top. Say an extra fifty foot altitude. ($50)

If all this sounds expensive and complex, remember that the flyers that are really out to win will spare no expense to get the finest equipment. Bob Sifleet once told me that an F.A.I. power ship costs $200 per. Not to mention the starter, the thermal detector, etc.

So the competition flyer will pay the tab.

My projection of all this is shown in the illustration.

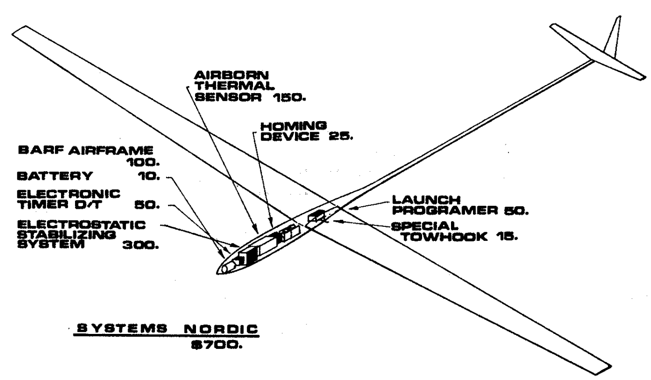



Figure 7

If all this sounds reprehensible. Remember that I might be wrong and all this is a horrible dream.

A happy thought is that the mass of expensive equipment might still be beaten by a seasonal flyer flying a standard ship. And this is what makes Nordic and free-flight the joy that it is...

# MODEL AIRCRAFT MOMENT OF INERTIA

**Doug Wilson**


### SUMMARY

A method for measuring moment of inertia of compound pendulums is applied to model aircraft.


### CONCLUSIONS

The method is successful. More theoretical work would help to explain the relation of moment of inertia and airplane geometry to dynamic behaviour. A typical figure for the non-dimensional moment of inertia of an FAI power model is .138.

### LONGITUDINAL MOMENT OF INERTIA

A model fuselage or a baseball bat, held betwixt thumb and forefinger, can be made to swing to and fro. As the pivot point is moved from the end of the pendulum towards the cg, frequency of oscillation increases to a maximum at a distance, k, radius of gyration of mass, from the cg. Closer than k to the cg frequency falls off again rapidly. This maximum position can be estimated, without ever touching a stopwatch, to within 20% or so. For a more accurate technique, see Appendix.

The radius of gyration is a measure of the moment of inertia, Ig, of the bat or fuselage or complete model. Ig is a parameter of the dynamic behaviour of airplanes. It is similar to inertia except that it is the natural tendency of an object to continue rotating once started, unless retarded by some force. See for example last year's article on propeller inertia. Weighing the stab is a common method of estimating Ig. If there were no weight in the body or wing Ig would be the tail weight times tail moment arm,lt, squared.

On a motionless beam balance the moment exerted by the pans is just weight times distance to the fulcrum. Movement alters this static situation and a moment due to rotation acceleration arises. The inertia force on the weight pans is a function of their straight line or tangential acceleration. This acceleration depends on the distance to the fulcrum, much as the rim of a wheel moves more quickly than the surface of the axle. Not only the acceleration, but the moment of the force arising from the acceleration is larger at greater radius. This double dependence on radius accounts for the appearance of tail moment arm squared in the argument above.

Doubling the weight of the stab doubles the moment of inertia; doubling the tail arm quadruples it.

The advantage of the pendulum approach over weighing the stab is that it includes the total integrated effect of each little bit of mass at all their various distances from the cg. Just as balancing the model on a finger is a faster way of locating the cg than weighing each part and calculating its contribution, swinging the model is an easy way of finding the moment of inertia.

It might be nice to compare different sizes of models with each other, like a series of Satellites 600, 800, etc. If the model is scaled up by a factor of 2 the area increases by 4 and the weight by 8. The wing loading doubles, the moment of inertia goes up by 32! To be fair to the larger model then, Ig can be divided by $\mathrm{mlt}^2$. The quotient is,

$$ib = Ig/\mathrm{mlt}^2$$

Numbers like ib are called dimensionless parameters. For any scale factor applied to the original design ib will be the same if each piece of structure is increased by the same ratio, length, width, and thickness.

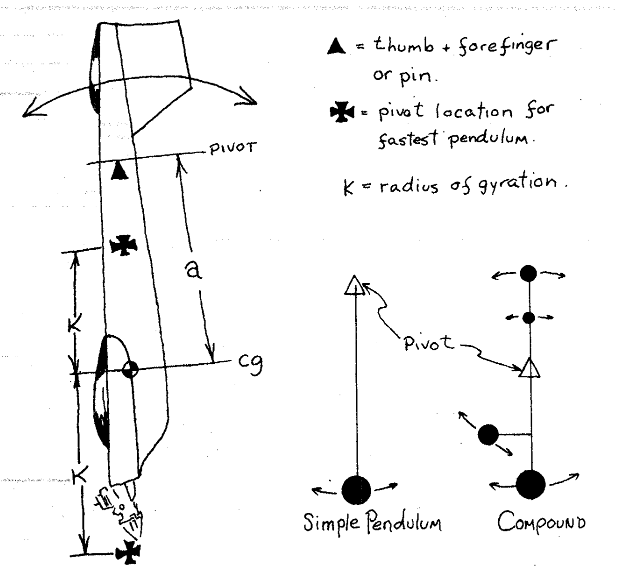
To save any confusion over the notation so far, it should be restated that Ig, lt and ib are intended to represent single quantities not multiplications of I times g and so forth.

Using the swinging pendulum, a shortcut to finding ib is to take the ratio

$$ib = (k/lt)^2$$

where k is the distance from the cg to the fastest pivot as before and lt is tail arm. The number ib has to be seen in relation to the model geometry. Dynamic stability and response analysis used in full scale work can actually predict experimental results. This theory, sadly, would have to be extensively modified or replaced to be useful to modellers.

Full scale theory might be stretched to cover large disturbances but it could not be used past the stall, which is what model flyers are most concerned about anyway. Information is needed on accelerated flow and separated flow. For the time being it is difficult to say what effect a change such as lengthening the tail boom might have on flight characteristics. One needs a sophisticated analog computer, a model, to find out.



## APPENDIX

### ACCURATE DETERMINATION OF RADIUS OF GYRATION

Analogous to a Schuler pendulum used for inertial navigation, the most accurate compound pendulum is one whose pivot is found one radius of gyration, k, from the cg. A model is a compound pendulum, differing from a simple pendulum in that the mass is not concentrated in dense blob. The practical result is that a pivot too far from or too close to the cg is inaccurate. The moment of inertia of a compound pendulum is

$$I_g = W_a \left( \frac{1}{4\pi^2 f^2} - \frac{a}{g} \right) \qquad (1)$$

$$4\pi^2 \doteq 39.49 \quad \text{Ref 1 p235}$$

The radius of gyration is

$$k = gIg/W$$

A physical interpretation of k is that it is the length of an equivalent simple pendulum having the same mass concentrated at a point. When the model is suspended far from the cg the bracketed term in equation 1 is a small difference of tow large numbers, incurring great percentage error in spite of careful measurements of f and a. Hung at radius near k however, the bracketed expression is more manageable. To start with, though, k is not known. The procedure is then to pivot the model anywhere convenient and calculate a first approximation of k. This figure locates the next trial pivot from which a more precise k is found, and so on until satisfaction is obtained. Two or three iterations develop 10% accuracy or at least mutual agreement.

Table 1 shows the tabulated measurements and sums done for an FAI gas job and a HLG. The k found in one line is the a used in the next. Fistances are in feet to accord with the usual units of g. Converting to inches the gassie has k = 12.5" and the HLG k @ 5.2". Taking the ratio $k^2/lt^2$ gives

ib = $(12.5/33.63)^2$ = .138

and ib = $(5.2/13)^2$ @ .160

It might be informative to have a statistical survey or current models, comparing ib to the tail volume..

The tail arm usually is measured from cg to stabilizer 25% chord. Irving (Ref. 2) suggests using the tail arm taken from wing/body aerodynamic center to stabilizer 25% chord. For conventional models to stab ¼ chord. This is a longer distance than the usual one and is appropriate where the lift of the stab is included in the total lift (as Bill Bogart suggested be done in lasy year's Report). Irving is a very good book on the subject of static stability.

If this new way of measuring tail arm is used, tail volume coefficient is raised somewhat. Tail volume is a geometrical concept.

$$V' = \frac{\text{stab area}}{\text{wing area}} \times \frac{\text{tail arm}}{\text{wing mean chord}}$$

For tapered wings an average wing chord is used. V' stands for a tail volume based on Irving's way of measuring lt. This is quite a good indicator of static stability. If anyone were interested enough to do these calculations and send the results for V' and ib to

Doug Wilson
11 North Square
Newport Pagnell
Bucks, England.

a graph could be produced for next year's Symposium. Please include any other information on the model that you care to reveal.

It involves more work, but a second modification to the tail volume coefficient can make it more representative of the static stability.

$$\bar{V} = \frac{\bar{V}}{1+\tau} \qquad \text{Ref 2 p 42}$$

$$\tau = \frac{S_t}{S} \frac{a_t}{a_w} \left( 1 - \frac{\partial \varepsilon}{\partial \alpha} \right) \qquad (2)$$

Tail volume coefficient of the F1C, forinstance, was

V' = 1.54
V'' = 1.28

### NOTES ON FREQUENCY MEASUREMENTS

PIVOT — Model is pivoted about pitch axis, which is an axis in the general direction of the wing span. A thumb and forefinger pivot worked on heavy models but for light ones the friction may be too great. On the power model it was possible to put pins through the fuselage and support them on fingernails. To avoid weakening the glider fuselage the pin was pushed in just a short way on one side. For the power model it happened that the engine was about k from the cg, so that the hole in the spinner nut was a lucky feature. (k can be taken forward or aft of cg as required.)

SWINGS — The mathematics behind the method breaks down if the swings are too large. If too small, pivot friction seems to become objectionable. About 5 degrees either side of plumb was used. The pivot support was moved slightly to start the oscillation and repeated small movements were made to keep feeding in energy required to maintain it. A swing, back and forth, counts as one cycle, and the frequency is the number of cycles divided by the total time to complete the number, say 30 cycles.

RADIUS – The pivot to cg separation, a, is measured, making allowance for the vertical location when the pivot is near the cg.

WEIGHT – in finding k or ib it is not necessary to know weight explicitly.

The compound pendulum is suitable for yaw and roll moments of inertia as well.

## NOMENCLATURE

| | | |
|---|---|---|
| a | = | pivot to cg radius (feet) |
| at | = | 3-d tail lift curve slope (per rad) |
| aw | = | 3-d wing lift curve slope (per rad) |
| $\alpha$ | = | incidence (degrees or radians) |
| $1-\frac{\partial\epsilon}{\partial\alpha}$ | = | downwash factor, Ref. 3 p.45 (no units) |
| $\epsilon$ | = | downwash angle (degrees or radians) |
| f | = | frequency (cycles per second, Hz) |
| g | = | gravitational constant (ft/sec$^2$) |
| ib | = | non-dimensionalized moment of inertia |
| Ig | = | moment of inertia (slug ft$^2$ or lb. ft. sec$^2$) |
| k | = | radius of gyration of mass (feet) |
| lt | = | tail moment arm (feet) |
| m | = | W/g, mass (slugs) |
| S | = | Wing area (ft$^2$ of in$^2$) |
| St | = | Stab area (ft$^2$ or in$^2$) |
| $\tau$ | = | A factor |
| V | = | Tail volume coefficient (no units) |
| V' | = | Modified tail volume (no units) |
| V" | = | V'/l+$\tau$ tail volume (no units) |
| W | = | Weight (lb) |

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## ACKNOWLEDGEMENTS


There is nothing original in this essay but some material may not be as well known as it could be. The compound pendulum was first explained to me by G.T. Downer, lecturer at Ryerson Polytechnical Institute. Dr. P.A.T. Christopher at Cranfield has been trying, against long odds, to help me understand airplane dynamics.


### TABLE 1

SAMPLE ITERATIONS FOR K.

FAI GAS

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | (1 ÷ 2) | $(3)^2$ | $\frac{1}{4\pi^2 \times (4)}$ | | (6) ÷ 12 | (7) ÷ 32.2 | (5 − 8) | a × (9) | g × (10) | $\sqrt{(11)}$ |
| # of swings | TIME | f | $f^2$ | $\frac{1}{4\pi^2 f^2}$ | a INCHES | a FEET | a/g | | | $K^2$ | K FEET |
| 30 | 49 | .613 | .375 | .0677 | 15.25 | 1.27 | .0395 | .0282 | .0359 | 1.16 | 1.08 |
| 30 | 48 | .625 | .390 | .0650 | 13.00 | 1.08 | .0336 | .0314 | .0339 | 1.09 | 1.04 |
| 30 | 47 | .638 | .406 | .0623 | 12.50 | 1.04 | .0324 | .0299 | .0312 | 1.01 | 1.00 |
| 30 | 48 | .625 | .390 | .0650 | 12.00 | 1.00 | .0312 | .0338 | .0338 | 1.09 | 1.04 |
| HLG | | | | | | | | | | | |
| 30 | 31.0 | .954 | .915 | .0277 | 3.75 | .313 | .0097 | .0180 | .0056 | .181 | .425 |
| 30 | 31.0 | .954 | .915 | .0277 | 5.1 | .425 | .0137 | .0140 | .0060 | .192 | .437 |
| 30 | 31.5 | .967 | .940 | .0269 | 5.25 | .437 | .0136 | .0133 | .0058 | .187 | .432 |