# Math Magik Project

Estimating Indoor Model Flight Times

Roie R. Black

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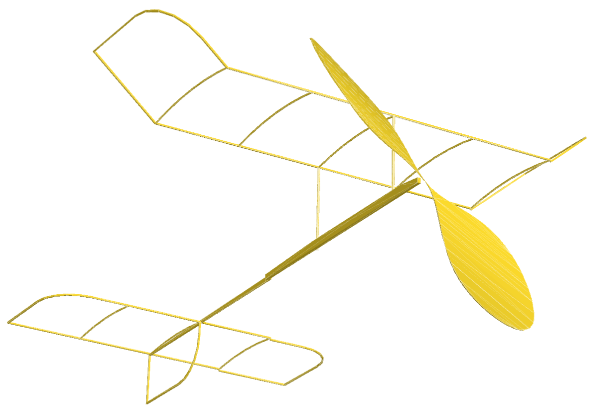

## 1 Introduction

In the 1976 edition of the *National Free Flight Society Symposium*, Doug McLean authored an article titled *A Method for Predicting Indoor Model Duration* [2]. Doug's method involves data from actual model flights and simplified formulas for things like the aerodynamics and efficiency of the model, propeller, and the rubber powering the airplane. In this article, we will revisit this approach and create some Python code to help predict the flight times for models we design using *OpenSCAD* [1].

## 1.1 About This Document

This document is produced using LaTeX, a typesetting program very popular in academia and science. All math in this document has been checked using *Python SymPy*.

# 2 Python Packages

We will be using two nice Python packages to support this article: —SYMPY and *Pint*. These packages are easily installed using the *Python* **pip** tool. Here is the line I used to install them on my system:

```
$ pip install sympy, pint
```

## 2.1 SymPy

We will be working through a bit of math in this article. I will show the equations in their “normal” form but I will be using a nice *Python* package called *SymPy* to generate the actual results. *SymPy* is a symbolic math engine, it manipulates *symbols* the same way you did back in your algebra class in high school. The really nice thing about *SymPy* is that it knows a lot about math, and it does not make mistakes (well, unless you do when you set things up!)

As a simple example of what *SymPy* can do, let’s show a bit of that algebra. Suppose you were asked to expand $(x + y)^3$. Here is the *Python* code that uses *SymPy* to figure this out for you:

```
import sympy
sympy.var("x, y")

sol = sympy.expand((x + y)**3)

print(r"\begin{align*}")
print(sympy.latex(sol))
print(r"\end{align*}")
```

Those **print** statements are strictly for use in this document, so things display nicely in the final *PDF file*. The result is this:

$$x^3 + 3x^2y + 3xy^2 + y^3$$

I bet you wish you had access to this tool back in your school days!

## 2.2 *Pint*

In engineering, we absolutely must pay attention to the units we use for physical properties. In the U.S. we commonly use Imperial units (inches or feet, and ounces or pounds). Elsewhere it is more common to use Metric units (centimeters, or meters, and grams or kilograms). Indoor modelers seem to mix the two systems, using inches for lengths, and grams for weight. Failure to pay attention to all of this can lead to embarrassing failures, as NASA found out in one of their Mars expeditions when some subcontractors were using Metric units and everyone else was using Imperial units!

Fortunately, another *Python*— package can make tracking units and converting them as needed easy. The —PINT— package has a lot of power we can use here:

```
from pint import UnitRegistry
ureg = UnitRegistry()

rho = 0.002308 * (ureg.slug / ureg.ft**3)
print('{:~P}'.format(rho))
```

Once again, we see some trickery to get the output to look nice in a document generated using LaTeX.

Here is the result:

0.002308 slug/ft³

Who came up with "slug" as a unit anyway? Let's convert that to the Metric system:

```
rho_m = rho.to_base_units()
print('{:~P}'.format(rho_m))
```

*Pint* uses the Metric system internally, so we just asked it to show those “base” units:

1.1894943128514974 kg/m³

This is really nice! No more digging out a calculator and googling for unit conversion factors!

Armed with these tools, let’s take a look at the atmosphere we will be flying through next.

# 3 Air Properties

We will need a few basic properties of air for our calculations:

The density of air, which we looked ta briefly in the last section, is assumed to be constant in our flying sites (well, unless you fly in very tall buildings!)

$$\rho = 33.6 \frac{g}{ft^3} \tag{1}$$

The kinematic viscosity, a measure of the ”air’s resistance to motion is given next:

$$\nu` = 15.88x : math : `10^{-5} \frac{ft^2}{sec}` \tag{2}$$

# 4 Symbols

| | |
|---|---|
| $AF$ | Aspect ratio of forward wing |
| $AR$ | aspect ratio of rear lifting surface |
| $b$ | wingspan |
| $c_w$ | mean wing chord |
| $c_t$ | mean stab chord |
| $b_r$ | stab span |
| $C_d$ | overall drag coefficient |
| $C_{di}$ | overall induced drag coefficient |
| $C_{dif}$ | induced drag coefficients of forward wing(s) |
| $C_{dir}$ | induced drag coefficient of tail |
| $C_{dp}$ | total profile drag coefficient |
| $C_{dpf}$ | profile drag of forward wing(s) |
| $C_{dpr}$ | profile drag coefficient of tail |
| $C_{dw}$ | profile drag coefficient of bracing wires |
| $\Delta C_d$ | total drag of wing posts and bracing wire |
| $C_f$ | mean aerodynamic chord of forward wing |
| $C_l$ | overall lift coefficient |
| $C_{lf}$ | lift coefficient of forward wing(s) |
| $C_{lr}$ | lift coefficient of horizontal stab |
| $dw$ | diameter of bracing wire |
| $E$ | initial energy stored in rubber motor |
| $F$ | emperical efficiency factor |
| $g$ | grams |
| $G$ | vertical gap between wings of a biplane or tandem |
| $h$ | ceiling height in feet |
| $H$ | dimensionless ceiling height |
| $J$ | propeller advance ratio |
| $K_m$ | rubber motor constant |
| $l$ | tail moment arm |
| $l_w$ | total length of bracing wire |
| $n$ | propeller rotational speed (revolutions/second) |
| $N$ | maximum turns for rubber motor |
| $P$ | power required for level flight (thrust * speed) |
| $Re_f$ | Reynolds number for forward wing(s) |
| $Re_r$ | Reynolds number for tail |
| $Re_w$ | Reynolds number for bracing wire |
| $S$ | total horizontal projected area of model |
| $S_f$ | total horizontal projected area of forward wing(s) |
| $S_p$ | propeller disc area |
| $S_r$ | horizontal projected area of tail |
| $S_{wp}$ | area of wing posts projected in flight direction |
| $t$ | flight time (duration) |
| $T$ | propeller thrust force |
| $V$ | flight speed |

# References

[1] M. Kintel. OpenSCAD - The Programmers Solid 3D CAD Modeller, 2021. URL `https://www.openscad.org/`.

[2] D. McLean. A method for predicting indoor model duration. *National Free Flight Association Symposium*, 1976.