PROPELLER PERFORMANCE

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ABSTRACT

The basic mechanics of propeller operation are described from a physical point of view, so that such important parameters as the power input, the thrust, the slip velocity, the pitch angle, and airfoil frictional losses may be related to each other by simple mathematical expressions. A calculation procedure for obtaining performance is outlined, and the procedure is illustrated by showing the results for a FIC model propeller as a function of forward speed.

PROPELLER MECHANICS

In general, for understanding the propeller performance of any model, we can proceed as follows. As described by Prandtl[1], when a propeller generates thrust, it loses or wastes energy through momentum losses and through friction effects. To understand the momentum losses, assume that the propeller is moving forward at a speed V with respect to the undisturbed air. Now we simplify the problem by assuming that the forward thrust is distributed evenly over the entire circle described by the propeller tips. The thrust causes the air going past the propeller to speed up so that instead of passing through the propeller plane at speed V, the air is passing through at a speed V + v, and, since the thrust, is uniformly distributed, v is approximately uniform over the propeller plane.

Think about this for a minute. Since the propellor is sucking air at a speed V + v, the mass of air going by per second can be computed from the equation

$$\dot{m} = \rho A (V + v) \tag{1}$$

where ρ is the air density and A is the area of the circle. Now, by Newton's law we know that the force of thrust is equal to m times some velocity.

As shown by Prandtl, the air just ahead of the propeller plane is at a pressure less than atmospheric so that, by Bernoulli's Theorem, the airspeed relative to the propeller plane is V + v; **but as soon as the air gets past the propeller plane, still going at speed V + v, its pressure is greater than** atmospheric. This pressure jump across the propeller plane, illustrated in Figure 1, acts on the propeller blades, providing the thrust. Meanwhile, the air downstream of the propeller plane gains speed as it moves further downstream until the speed gain is sufficient, as given by Bernoulli's Theorem again, to cause the pressure to match the pressure of the surrounding air. The speed gain is illustrated on Figure 1 as q. This added speed, q, represents a power loss, which is m $q^2/2$. The propeller thrust, by Newton's law, is

$$T = \dot{m}\, q \tag{2}$$

and the power required to accelerate the air is

$$P_{th} = T (V + v) \tag{3}$$

so that v is really the "slip velocity" at the propeller plane. The useful work we get out of the propeller per unit time is, of course,

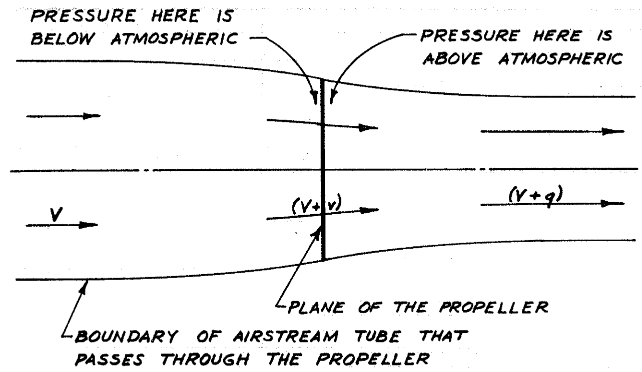


FIGURE 1
SKETCH SHOWING STREAMLINES PAST A PROPELLER MOVING AT SPEED V RELATIVE TO THE ATMOSPHERE. ARROWS INDICATE AIRSPEEDS V, (V+v), AND (V+q) RELATIVE TO THE PROPELLER.

$$P_{useful} = T\, V \tag{4}$$

By dividing (4) into (3) we get the ideal thrust efficiency

$$\eta_{th} = \frac{V}{V + v} \tag{5}$$

So, we want to make the slip velocity v as small as possible for good thrust efficiency.

By equating the power loss in $q^2/2$ to the lost power at the propeller plane, T v, and using equation (2) we find that

$$q = 2\, v \tag{6}$$

From now on we can combine (2) and (6) to eliminate q, which we no longer need. Then

$$T = 2\, \dot{m}\, v = 2 \rho A (V + v)\, v \tag{7}$$

This we can use directly to calculate T in terms of v and vice versa.

Now we can consider the details of what happens right at the propeller blades themselves, taking into account frictional losses of the airflow over the blades. Consider a single blade element at some radius, r, turning at n revolutions per second. The blade moves at a speed $2\pi n r$ within the propeller plane while the surrounding air is moving at the speed (V + v) perpendicular to the propeller plane. From this we can define a blade angle ϕ as

$$\tan\phi = \frac{V + v}{2\pi n r} \tag{8}$$

The angle ϕ, the velocity components, and the blade element forces are illustrated on Figure 2. VA is the apparent wind velocity; L and D are the lift and drag per unit length along the blade. L and D depend on the blade angle of attack α as well as VA. The blade element efficiency, as derived by Prandtl, is

$$\eta_b = \frac{1 - \left(\frac{D}{L}\right)\tan\phi}{1 + \left(\frac{D}{L}\right)\cot\phi} \tag{9}$$

Now, in principle, we could add up the forces of the blade elements all along the length of each blade and solve for the thrust and torque on the propeller shaft, and from this obtain an overall propeller blade efficiency, which accounts for not only frictional losses, as represented by D, but also losses due to the nonuniformity of the blade loading across the propeller plane. This overall blade efficiency is called the "hydraulic efficiency." To simplify matters we can get a reasonable approximation to η_h by using the expression.

$$\eta_h = \eta_r \left\{ \frac{1 - \left(\frac{D}{L}\right) \tan \phi}{1 + \left(\frac{D}{L}\right) \cot \phi} \right\}_{r = 0.7R} \tag{10}$$

where ϕ and D/L are evaluated at a blade radius equal to 0.7 R, where R is the radius to the blade tip or half the propeller diameter. The factor η_r accounts for the nonuniformity of the blade loading across the propeller plane.

The power P required to operate the propeller is P_{th}/η_h which is

$$P = \frac{T\,V}{\eta} \tag{11}$$

where

$$\eta = \eta_{th}\,\eta_h \tag{12}$$

is the propeller overall efficiency factor.

CALCULATIONAL PROCEDURE FOR OBTAINING PROPELLER PERFORMANCE

We will begin by assuming that we have a propeller with a given diameter, 2R, operating at a given rotational speed, n, at a speed, V, relative to the undisturbed air; furthermore, the shaft torque, Q, is also given so that the power input, P, is known. That is,

$$P = 2\pi n Q \tag{13}$$

Next we assume a value for the slip velocity, v, which normally turns out to be a small fraction, say on the order of one-quarter or less,-of the model forward speed, V. Then we compute the thrust from equation (7), tan ϕ from equation (8), the hydraulic efficiency from equation (10), the thrust efficiency from equation (5), and finally the power required using equations (11) and (12). Notice that the value of P obtained from equation (11) may be different from the power input that we specified as known at the outset. This is because we did not assume the correct value of v. Increasing v will increase the value of P obtained from equation (11) and vice versa; we must pick new values of v and repeat the calculations until the correct value of P is obtained. Generally only three or four iterations are required.

This procedure implies that the propeller used had just the proper amount of blade area, pitch angles ($\phi + \alpha$), and airfoil shapes to absorb the power P which was given. This problem of shaping the blades to absorb the available power can best be understood by referring to a specific calculational example such as given below.

A CALCULATION OF F1C PROPELLER PERFORMANCE

Here we illustrate the above procedure by predicting propeller performance of a 7.0 inch diameter propeller on the

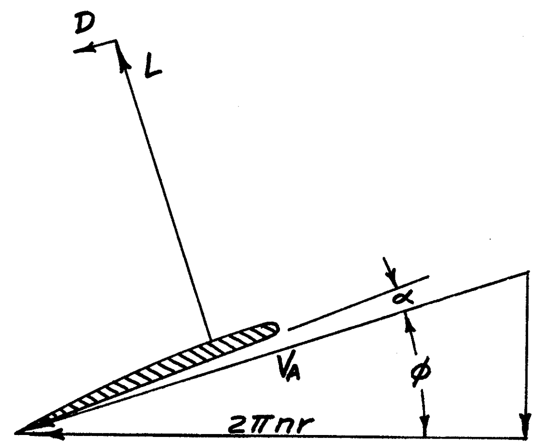


FIGURE 2
SKETCH SHOWING AIR VELOCITY COMPONENTS AND AERODYNAMIC FORCES ON A PROPELLER BLADE ELEMENT

Rossi F-15 engine turning at 25,000 rpm where the power output P is a maximum of 0.65 horsepower[2] or 357.5 foot-pounds per second.

If all this power were used to mechanically lift a 1.653 pound load, equal to the weight of a typical F1C model, the upward terminal speed would be 216.3 feet per second; the energy available in a 7.0 second run would be sufficient to raise the weight to a height of 1514 feet! Obviously, drag and propeller inefficiency factors will not permit this performance on an F1C model, but we can come closer to this ideal by first understanding where the energy losses occur and then setting about to minimize these losses.

Before carrying through the calculational procedure, we need to specify values for the parameters ρ, η_r, and D/L. We will use the sea level standard value for air density, ρ = 0.002378 slugs per cubic foot. We know that η_r should be near 1.0, but the propeller disc area covered by the spinner does not supply any thrust, and the propeller elements nearest the spinner do not carry much load, since both $2\pi nr$ and VA are smallest there. Hence, we have a non-uniform load distribution over the propeller disc area. To account for this in a crude sort of way and to add, hopefully, a bit of conservatism, I have used η_r = 0.90. For D/L, Prandtl suggests that values of 0.02 to 0.05 are typical; since our propeller blade is on the small side, corresponding to a blade Reynolds number of about 150,000, I have chosen the more conservative or greater skin friction end of the spectrum, D/L = 0.05, for the calculations. Here it should be pointed out that the ratio D/L is that for a two-dimensional airfoil, not including induced drag effects, since the propeller "induced drag" has already been accounted for by the slip velocity v.

Following the above procedure, we obtain values of T, v, ϕ, η_h, η_{th}, and η as a function of model speed as shown in Figures 3 and 4. Notice that T is 3.0 pounds at V = 50 feet per second, so a vertical climb is quite possible. Also, notice

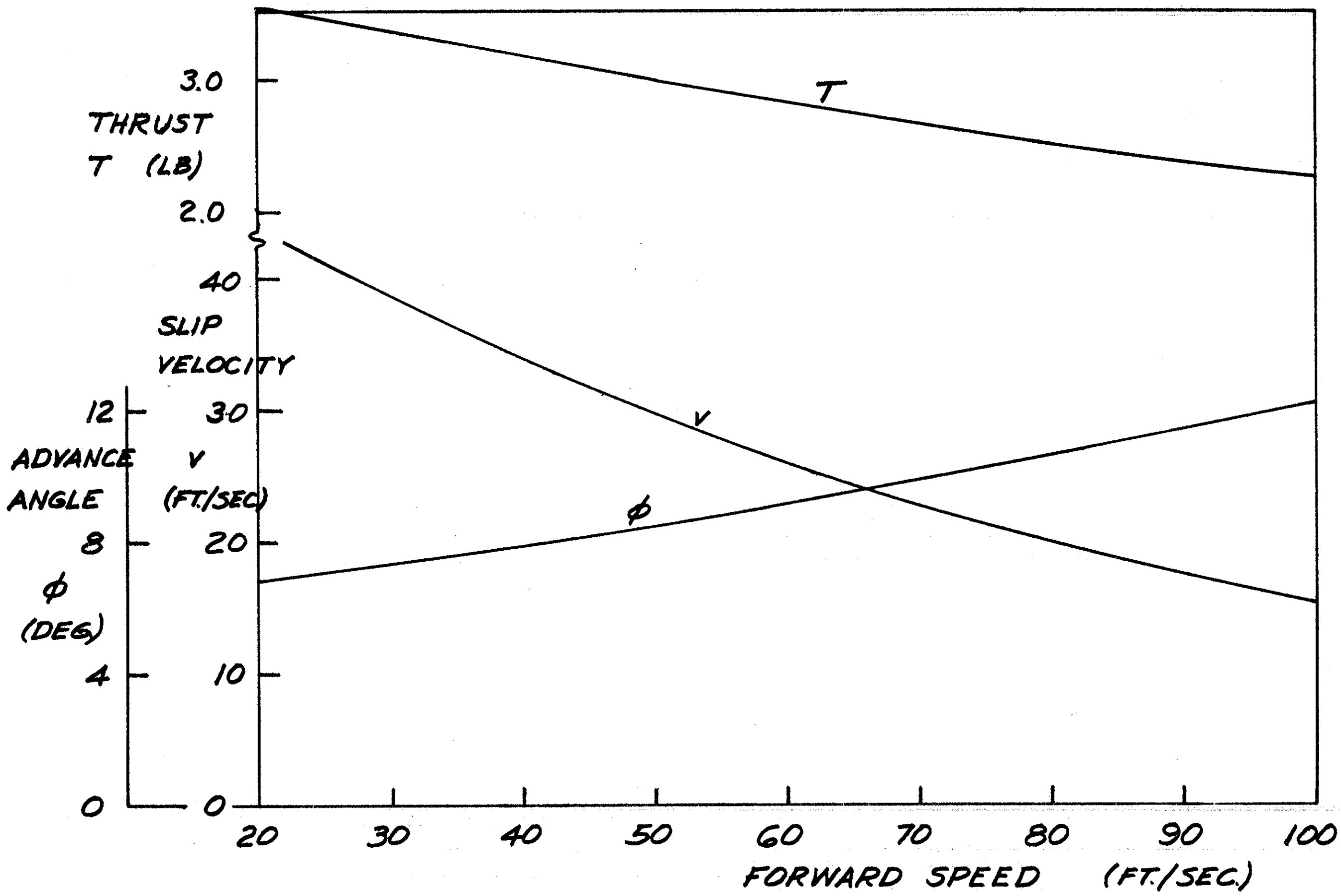


FIGURE 3

CALCULATED VALUES OF T, v, AND ϕ FOR A ROSSI F-15 ENGINE WITH A 7.0 INCH DIAMETER PROPELLER AT 25,000 RPM

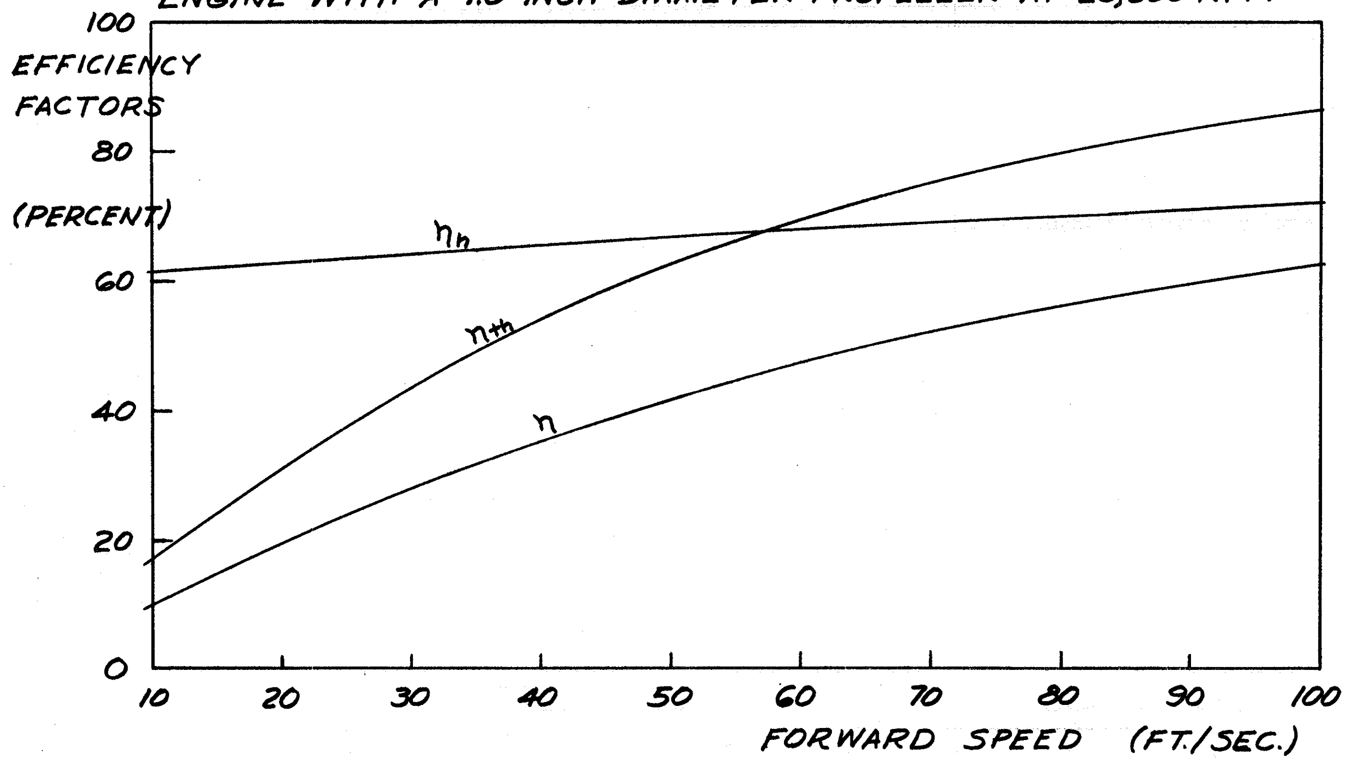


FIGURE 4

CALCULATED VALUES OF η_h, η_{th}, AND η FOR A ROSSI F-15 ENGINE WITH A 7.0 INCH DIAMETER PROPELLER AT 25,000 RPM that ϕ varies with V. This points out the fact that what we have calculated is the performance of a *series* of optimumually-shaped propellers, each propeller tailored to a particular value of V. For example, if we suppose that we have the propeller tailored for V = 70 feet per second and we pick an optimum α of 3.0 degrees for this speed, the propeller pitch angle measured at r = 0.70 R will be $(\phi + \alpha) = 12.8$ degrees and the corresponding geometric ptich is

$$P = 2\pi r \tan(\phi + \alpha) = 3.66 \text{ inches.}$$

Notice that this propeller corresponds closely to the one used by Lars Olofsson, which had a diameter d = 6.97 inches (177 mm) and p = 3.74 inches (95 mm). However, Figure 4 shows that ϕ should be smaller than 9.8 degrees for V less than 70 feet per second and vice versa. Hence, actual performance of one single propeller will not be quite the same as on Figures 4 and 5 except at one particular value of V. Nevertheless, since ϕ varies rather slowly with V, only 3.0 degrees over the range of speeds at which most F1C models fly, the performance should be almost the same as predicted by Figures 3 and 4.

Another limitation of Figures 3 and 4 is the assumption that engine speed is constant for all values of V; in reality, the engine speed increases somewhat with increases in V, partly because an increase in V with a fixed pitch angle $(\phi + \alpha)$ causes a decrease in α and a decrease in the blade L and D values.

Actual performance of a single propeller as a function of V could be calculated using the above procedure if we adjusted both D/L and engine rpm as a function of V. Obtaining the engine rpm change would require integrating the forces over the entire propeller. Since integrating the forces greatly complicates the calculations, such refinements are not attempted here.

COMMENTS ON F1C PROPELLER EFFICIENCY

As shown by Figure 4, F1C propeller efficiency factors are quite low; at V = 60 feet per second η is only 47.5 percent. Such low values are the direct result of using a large amount of power on such a small propeller disc area and at such low forward speeds. That is, ηth is low simply because the slip velocity is so large compared to V, as shown directly by equation (5). Nevertheless, if we want a large amount of thrust, the slip velocity must be correspondingly large, as shown by equation(7). Also, η h is small because ϕ is small, and ϕ is small because the engine rpm is so large. What is implied is that large increases in efficiency can be obtained by gearing down the propeller rpm and at the same time providing more torque to drive a larger propeller.

As an example, suppose that the propeller shaft were geared down to 2500 rpm, increasing torque by a factor of 10, and suppose that the propeller diameter were increased to 14 inches. Then, for V = 60 feet per second, v = 11.2 feet per second, $\phi = 33.7$ degrees, ηh = .0.809, ηth = 0.842, η = 0.682, and T = 4.06 pounds. Hence, the changes would be dramatic; efficiency and thrust would be increased by more than 43 perecent.

Although this sounds attractive, the negative factors also should be considered. Some thrust would be lost because of gearing power losses; the gearing and propeller would add weight; the large torque would bring new trim problems, and the large propeller would, unless folded, be a quite significant penalty during the glide. Hence, it might be better to retain our small, high speed, and inefficient F1C propellers.

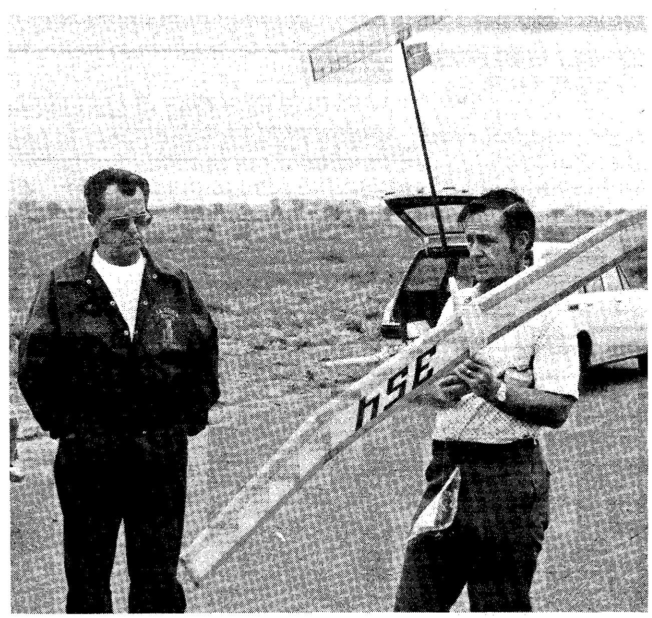
The obvious area for F1C propeller improvement is to improve the blade airfoil shapes, since ϕ is so small that η b is also small unless blade drag D is made small. The next step is to evaluate typical values of blade lift coefficients by relating the thrust to the lift per unit length along the blade. Then airfoil section data may be used to evaluate D and to study the best means of reducing D, either through changes in blade chord or airfoil shape.

CONCLUDING REMARKS

Although the details of the airflow along a propeller blade are not considered here, it has been possible to calculate propeller performance by using a blade station at 70 percent of the blade radius as typical for calculating the blade frictional losses. Fortunately, the resulting thrust is not very sensitive to the particular blade station chosen, which was taken closer to the tip than the hub because of the greater blade speed at the tip. The calculations for an F1C model demonstrate substantial power losses due to both frictional and momentum losses. Further work can be done on evaluating a specific propeller design by relating the thrust and torque as calculated above to an integration of the forces along the blade length. By this means, the blade lift coefficient can be estimated for a given chord distribution along the blade; conversely, lift and drag coefficients obtained from airfoil characteristics may be used to calculate the blade chord distribution required to obtain the given values of torque and thrust.

REFERENCES

1. PRADTL, L., "ESSENTIALS OF FLUID DYNAMICS," HAFNER PUBLISHING CO., PP. 221 - 238.
2. PEARCE, F.; ROSSI, F.; AND HARTILL, B.; "ROSSI F-15," NFFS SYMPOSIUM, 1974, P.83.



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