3.5.6. Ideal Gases

One particularly important class of substances that is the ideal gas. An ideal gas is a model describing the behavior of real gases in the limit of zero pressure and infinite temperature (i.e., zero density). It does not account for the interaction between molecules of the gas (e.g., inter-molecular forces). Nevertheless, the ideal gas model is a reasonably accurate model for gases where the system pressure is less than 0.05 times the critical pressure or the system temperature is more than twice the critical temperature (to be discussed).

3.5.6.1. Equation of State

An equation of state is a relationship between properties of a particular substance or class of substances. Equations of state cannot be obtained from thermodynamics but are obtained either from experimental measurements or from some molecular model. Note that there can be various types of equations of state, e.g., two equations of state for an ideal gas include a thermal equation of state which is the ideal gas law, $p = \rho R T$, and a caloric equation of state which describes the relationship between the internal energy and temperature, $du = c_v(T)dT$.

Thermal Equation of State
The thermal equation of state for an ideal gas is given by what is commonly referred to as the ideal gas law,

$$p = \rho R T \quad \text{or} \quad pv = RT \quad \text{or} \quad p\bar{v} = \bar{R}_U T \quad \text{or} \quad pV = mRT \quad \text{or} \quad pV = n\bar{R}_U T, \qquad (3.79)$$

where p is the *absolute* pressure of the gas, ρ is the gas density ($= 1/v$, i.e., the inverse of the specific volume), R is the <u>gas constant</u> for the gas of interest, and T is the *absolute* temperature. The quantity $\bar{v}$ is the specific volume on a per mole basis, e.g., $[\bar{v}] = \text{m}^3/\text{mol}$, V is the volume, e.g., $[V] = \text{m}^3$, m is the mass, e.g., $[m] = \text{kg}$, and n is the amount of the substance, e.g., $[n] = \text{kmol}$. The quantity $\bar{R}_U$ is the universal gas constant (discussed in the following notes).

Notes:

(1) Absolute pressures and temperatures must be used when using the ideal gas law or anything derived from the ideal gas law.
(2) The gas constant, R, will be different for different gases. The gas constant can be determined in terms of the <u>universal gas constant</u>,

$$R = \frac{\bar{R}_U}{MW}, \qquad (3.80)$$

where $\bar{R}_U = 8314\,\text{J}/(\text{kmol}\cdot\text{K}) = 1545.4\,(\text{ft}\cdot\text{lb}_\text{f})/(\text{lb}_\text{mol}\cdot\text{°R}) = 1.986\,\text{Btu}/(\text{lb}_\text{mol}\cdot\text{°R})$ and MW is the molecular weight of the gas. The gas constant for air is $R_\text{air} = 287\,\text{J}/(\text{kg}\cdot\text{K})$ or, in English units, $R_\text{air} = 53.3\,(\text{ft}\cdot\text{lb}_\text{f})/(\text{lb}_\text{m}\cdot\text{°R})$. Note that $MW_\text{air} = 28.98\,\text{kg}\,\text{kmol}^{-1}$.
(3) The <u>compressibility factor</u>, Z, is defined as,

$$Z := \frac{pv}{RT} = \frac{p}{\rho RT}. \qquad (3.81)$$

If $Z \approx 1$ for a gas, then it can be modeled well with the ideal gas model. The compressibility factor, Z, is plotted in Figure 3.27 for a variety of substances as a function of the reduced pressure, p/p_c, and reduced temperature, T/T_c, where p_c and T_c are the <u>critical pressure and temperature</u> for the substance. Note that the critical temperature is the temperature above which a gas cannot be liquefied no matter how large a pressure is applied. The critical pressure is the minimum pressure for liquefying a gas at the critical temperature.
  (a) For values of $p/p_c < 0.05$ or $T/T_c > 2$, $Z \approx 1$ so in this range the ideal gas model works well.
  (b) Table 3.1 lists the critical temperature and pressure values for various substances.
(4) One should be careful in treating superheated water vapor, i.e., steam, as an ideal gas. Figure 3.28 shows a T-v plot for water. The points in the plot are the relative error between a specific volume found using the thermodynamic property table, which is very accurate, and the specific volume using the Ideal Gas Law, which is approximate ($[|v_\text{table} - v_\text{ideal}|/v_\text{table}] * 100\%$). The shaded area in
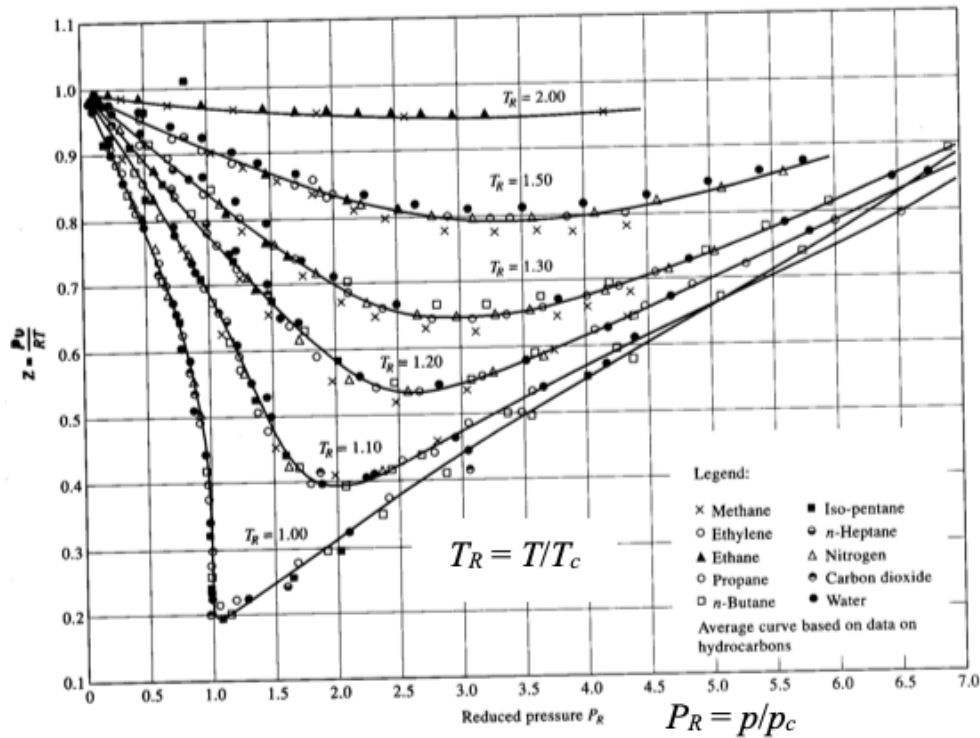
FIGURE 3.27. A plot of the compressibility factor Z as a function of the reduced pressure $P_R = p/p_c$ for different reduced temperatures $T_R = T/T_c$.

TABLE 3.1. Critical temperatures and critical pressures for various gases.

Gas	T_c [K]	p_c [atm]
air	132.41	37.25
He	5.19	2.26
H_2	33.24	12.80
N_2	126.2	33.54
O_2	154.78	50.14
CO_2	304.20	72.90
CO	132.91	34.26

the plot is the region where the relative error is less than 1%, i.e., the region where steam can be reasonably modeled using the Ideal Gas Law. As a rule of thumb, it's better to use the property tables for water in the superheated vapor region rather than assuming ideal gas behavior.
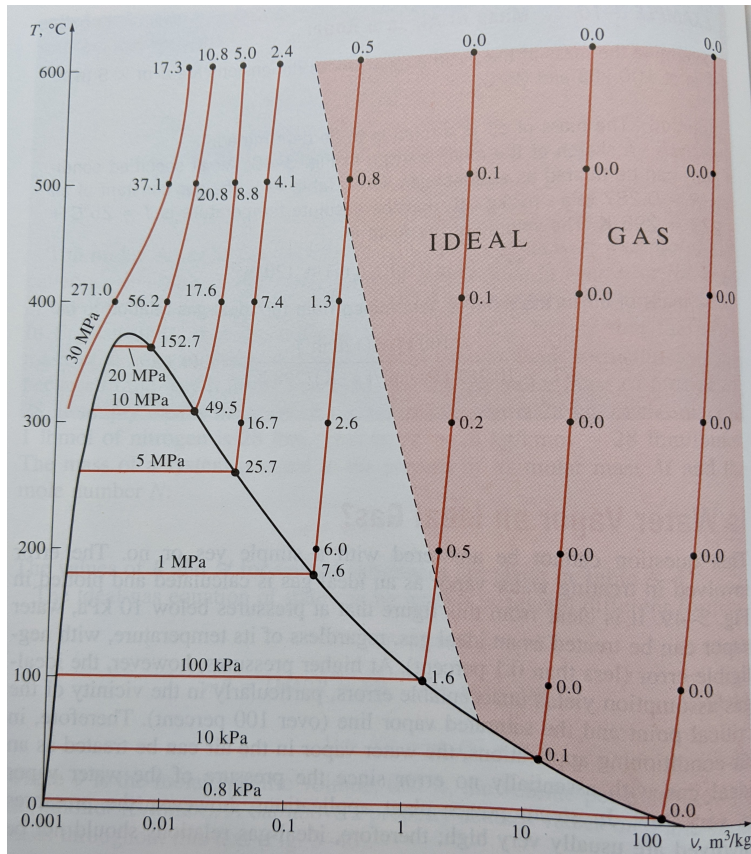
FIGURE 3.28. A T-v plot for water. The numbers on the isobars indicate the relative error in the specific volume found from the table vs. the Ideal Gas Law, i.e., $[|v_{\text{table}} - v_{\text{ideal}}|/v_{\text{table}}]*$ 100%. The shaded error is the region where the relative error is less than 1%, i.e., using the Ideal Gas Law in this region will give an accurate prediction of the specific volume. The figure is Figure 3.49 from Çengel, Y.A. and Boles, M.A., *Thermodynamics - An Engineering Approach*, 5th ed., McGraw-Hill.

A tank contains 0.5 m³ of nitrogen (N_2) at -71 °C and 1356 kPa (abs). Determine the mass of nitrogen, in kg, using,
a.   the ideal gas model, and
b.   data from the compressibility chart.

SOLUTION:
Using the ideal gas model,

$$p\bar{v} = \bar{R}T \quad \Rightarrow \quad \bar{v} = \frac{\bar{R}T}{p}, \tag{1}$$

Using the given data,
p = 1356*10³ Pa
T = -71 °C = 202 K
$\bar{R}$ = 8314 J/(kmol.K)
$\Rightarrow \bar{v}$ = 1.23 m³/kmol

The molecular weight of N_2 is M_{N2} = 28.01 kg/kmol. Hence,

$$v = \frac{\bar{v}}{M} \quad \Rightarrow \quad v = 0.0442 \text{ m}^3/\text{kg} \tag{2}$$

The mass may be found from the specific volume and the volume of the tank,

$$m = \frac{V}{v} \quad \Rightarrow \quad \boxed{m = 11.31 \text{ kg}} \tag{3}$$

using V = 0.5 m³.

In order to use the compressibility chart, we must first calculate the reduced temperature and reduced pressure. Note that the critical temperature and critical pressure for N_2 are T_c = 126 K and p_c = 33.9 bar (= 33.9*10⁵ Pa). Hence,

$$T_R = \frac{T}{T_c} \quad \Rightarrow \quad T_R = 1.603, \tag{4}$$

$$p_R = \frac{p}{p_c} \quad \Rightarrow \quad p_R = 0.400. \tag{5}$$

Using the compressibility chart (e.g., Figure A-1 from Moran et al., 7th ed.) with these reduced values,
    Z = 0.98. (6)
Hence, the specific volume using the compressibility chart will be 0.98 times the specific volume using the ideal gas law (where Z = 1, Eq. (2)),
    v = 0.0433 m³/kg. (7)
Thus, from Eq. (3), the mass is,
    $\boxed{m \quad = 11.54 \text{ kg}}$ (8)

The relative error in using the ideal gas model is,

$$\varepsilon = \frac{m_{\text{ideal gas}} - m_{\text{compressibility}}}{m_{\text{compressibility}}}, \tag{9}$$

    ε = 0.020. (10)
Assuming ideal gas behavior only results in a 2% error in the calculation of the mass, which is sufficiently small for most engineering calculations.

Caloric Equation of State

Now let's return to a discussion of the caloric equation of state for an ideal gas. Since an ideal gas is considered a simple, compressible system, the internal energy, u, is uniquely determined by two properties. Here we'll use the properties of temperature, T, and specific volume, $v(=1/\rho)$,

$$u = u(T, v), \tag{3.82}$$

so that any change in the internal energy is given by,

$$du = \left.\frac{\partial u}{\partial T}\right|_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv. \tag{3.83}$$

Utilizing our definition for specific heat given in Eq. (3.67) we have,

$$du = c_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv. \tag{3.84}$$

The second term on the right-hand side of Eq. (3.84) is zero for an ideal gas. (This can be shown using Maxwell's relations, a topic not addressed in these notes. See for example, Moran and Shapiro, Fundamentals of Engineering Thermodynamics, 3rd ed., Wiley, Section 11.4.2.) Thus, the internal energy of an ideal gas, u, is a function only of the temperature,

$$u = u(T) \implies du = \frac{du}{dT} dT. \tag{3.85}$$

Using the definition given in Eq. (3.67) we have,

$$du = c_v(T) dT. \tag{3.86}$$

Integrating both sides and noting that the specific heat can be a function of temperature in general,

$$u - u_{\text{ref}} = \int_{T_{\text{ref}}}^{T} c_v(T) dT, \tag{3.87}$$

where the subscript "ref" indicates some reference state. Evaluating at two different states,

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T) dT. \tag{3.88}$$

The specific enthalpy is also only a function of temperature for an ideal gas as shown below,

$$h = u + \frac{p}{\rho} = u + RT \implies h = h(T). \tag{3.89}$$

Taking the derivative of the specific enthalpy with respect to the temperature and using the definition given in Eq. (3.68) we see that,

$$dh = \frac{dh}{dt} dT = c_p(T) dT, \tag{3.90}$$

$$h - h_{\text{ref}} = \int_{T_{\text{ref}}}^{T} c_p(T) dT, \tag{3.91}$$

or,

$$h_2 - h_1 = \int_{T_1}^{T_2} c_p(T) dT, \tag{3.92}$$

where c_p is the specific heat at constant pressure which can, in general, be a function of temperature and the subscript "ref" indicates some reference state. In addition,

$$dh = du + RdT = c_v dT + RdT, \tag{3.93}$$

$$= (c_v + R) dT. \tag{3.94}$$

Comparing Eqs. (3.90) and (3.94) we see that the specific heats for an ideal gas are related in the following manner,

$$\boxed{c_p = c_v + R}. \tag{3.95}$$

The specific heat ratio, k, defined as,

$$\boxed{k := \frac{c_p}{c_v}},\tag{3.96}$$

appears frequently in thermo-fluid systems. Other helpful relations that can be derived by combining Eqs. (3.95) and (3.96) include,

$$c_p = \frac{kR}{k-1},\tag{3.97}$$

$$c_v = \frac{R}{k-1}.\tag{3.98}$$

Notes:

(1) When the temperature change for a process is sufficiently small (small depends on the substance, but a few hundred Kelvin is a good rule of thumb), the dependence of c_v and c_p on temperature can be reasonably neglected, i.e., $c_v = \text{constant}_1$ and $c_p = \text{constant}_2$. As a result, Eqs. (3.88) and (3.92) may be written as,

$$u_2 - u_1 = c_v(T_2 - T_1),\tag{3.99}$$

$$h_2 - h_1 = c_p(T_2 - T_1).\tag{3.100}$$

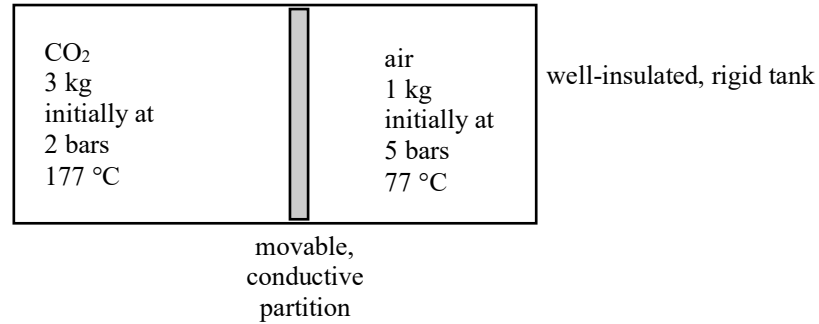An ideal gas with constant specific heats is referred to as a perfect gas.

(2) The values for c_v and c_p as a function of temperature have been tabulated for various gases in the back of some textbooks, e.g., Table A-20 in Moran et al., 8th ed.

(3) Since air is a frequently used substance in engineering, the values for several properties, such as specific internal energy and specific enthalpy, taking into account the temperature dependence of c_v and c_p have been tabulated in some textbooks, e.g., Table A-22 in Moran et al., 8th ed.

(4) Plots of the variation in the specific heat at constant pressure made dimensionless by the universal gas constant ($\bar{c}_p/\bar{R}_U$) for various gases are shown in Figure 3.29. Note that for air the specific heat does not vary more than approximately 12% over a few hundred Kelvin in the range $T > 270\,\text{K}$. Hence, a perfect gas assumption for air flows in this range and a temperature change of less than a few hundred Kelvin is reasonable. Obviously it would be more accurate to account for the variation due to temperature. The specific heat temperature dependence can be predicted from statistical mechanics models. The variations in specific heat are due to the activation of different energy storage modes (e.g., vibration and rotation) within the gas molecules at different temperatures. More detail on this topic can be found in Callen, H.B., *Thermodynamics and an Introduction to Thermostatistics*, Wiley.

One kilogram of air, initially at 5 bars (abs) and 77 °C, and 3 kg of carbon dioxide (CO_2), initially at 2 bars (abs) and 177 °C, are confined to opposite sides of a rigid, well-insulated container. The partition is free to move and allows conduction from one gas to the other without energy storage in the partition itself. Determine:
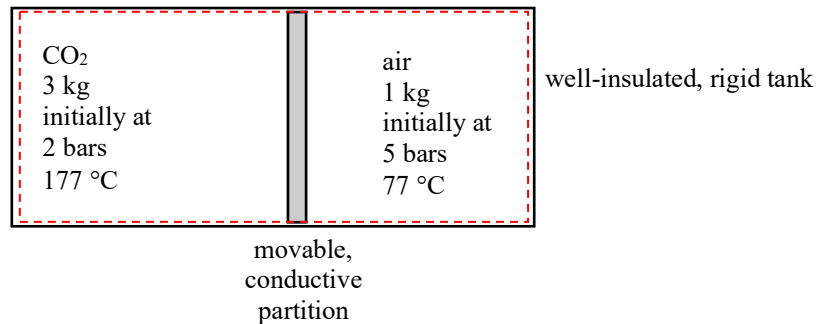a) the final equilibrium temperature
b) and the final pressure
You may assume that the specific heats for both the air and CO_2 remain constant over the range of temperatures: $c_{v,air}$=0.726 kJ/(kg·K); $c_{v,CO2}$=0.750 kJ/(kg·K); $c_{p,air}$=1.013 kJ/(kg·K); $c_{p,CO2}$=0.939 kJ/(kg·K)

CO_2
3 kg
initially at
2 bars
177 °C

air
1 kg
initially at
5 bars
77 °C

well-insulated, rigid tank

movable,
conductive
partition

SOLUTION:

Apply the 1st Law to the following CV.

CO_2
3 kg
initially at
2 bars
177 °C

air
1 kg
initially at
5 bars
77 °C

well-insulated, rigid tank

movable,
conductive
partition

$$\Delta E_{CV} = Q_{\underset{CV}{into}} + W_{\underset{CV}{on}} \tag{1}$$

where

$$\Delta E_{CV} = \Delta U_{CV} = U_f - U_i = m_{CO_2}\left(u_{CO_2,f} - u_{CO_2,i}\right) + m_{air}\left(u_{air,f} - u_{air,i}\right)$$

$$Q_{\underset{CV}{into}} = 0 \quad \text{(well-insulated tank)}$$

$$W_{\underset{CV}{on}} = 0 \quad \text{(rigid tank)}$$

Assuming constant specific heats (perfect gases) and simplifying COE gives:

$$m_{CO_2}c_{v,CO_2}\left(T_{CO_2,f} - T_{CO_2,i}\right) + m_{air}c_{v,air}\left(T_{air,f} - T_{air,i}\right) = 0$$

Since the partition is conductive, $T_{CO_2,f} = T_{air,f} = T_f$ resulting in:

$$m_{CO_2}c_{v,CO_2}\left(T_f - T_{CO_2,i}\right) + m_{air}c_{v,air}\left(T_f - T_{air,i}\right) = 0$$

$$\boxed{\therefore T_f = \frac{m_{CO_2}c_{v,CO_2}T_{CO_2,i} + m_{air}c_{v,air}T_{air,i}}{m_{CO_2}c_{v,CO_2} + m_{air}c_{v,air}}} \tag{2}$$

Using the given values:

m_{CO2} = 3 kg
$c_{v,CO2}$ = 0.750 kJ/(kg·K)
$T_{CO2,i}$ = 177 °C = 450 K
m_{air} = 1 kg
$c_{v,air}$ = 0.726 kJ/(kg·K)
$T_{air,i}$ = 77 °C = 350 K
$\Rightarrow$ $\boxed{T_f = 426\ \text{K} = 153\ °C}$

The final pressure in each compartment will be the same otherwise the partition would continue to move. Use the ideal gas law to determine the final densities of the gases in terms of the final temperature and pressure.

$$p = \rho R T \Rightarrow \rho = \frac{p}{RT} \tag{3}$$

In addition, the total volume of the tank is the sum of the final volumes occupied by each gas.

$$V_{tank} = \underbrace{\frac{m_{CO_2}}{\rho_{CO_2,f}}}_{=V_{CO_2,f}} + \underbrace{\frac{m_{air}}{\rho_{air,f}}}_{=V_{air,f}} \tag{4}$$

Combine Eqns. (3) and (4) and simplify.

$$V_{tank} = m_{CO_2}\frac{R_{CO_2}T_f}{p_f} + m_{air}\frac{R_{air}T_f}{p_f}$$

$$p_f = m_{CO_2}\frac{R_{CO_2}T_f}{V_{tank}} + m_{air}\frac{R_{air}T_f}{V_{tank}}$$

$$\boxed{p_f = \frac{T_f}{V_{tank}}\left(m_{CO_2}R_{CO_2} + m_{air}R_{air}\right)} \tag{5}$$

The tank volume is known from the initial masses, pressures, and temperatures.

$$\boxed{V_{tank} = \underbrace{\frac{m_{CO_2}}{\rho_{CO_2,i}}}_{=V_{CO_2,i}} + \underbrace{\frac{m_{air}}{\rho_{air,i}}}_{=V_{air,i}} = m_{CO_2}\frac{R_{CO_2}T_{CO_2,i}}{p_{CO_2,i}} + m_{air}\frac{R_{air}T_{air,i}}{p_{air,i}}} \tag{6}$$

Using the given data:

m_{CO2} = 3 kg
R_{CO2} = 0.1889 kJ/(kg·K)
$T_{CO2,i}$ = 177 °C = 450 K
$p_{CO2,i}$ = 2 bar = 202.7 kPa
m_{air} = 1 kg
R_{air} = 0.287 kJ/(kg·K)
$T_{air,i}$ = 77 °C = 350 K
$p_{air,i}$ = 5 bar = 506.63 kPa
T_f = 426 K (from previous part of the problem)
$\Rightarrow$ $\boxed{V_{tank} = 1.46\ \text{m}^3}$ and $\boxed{p_f = 250\ \text{kPa} = 2.46\ \text{bar}}$

A piston-cylinder assembly contains carbon monoxide modeled as an ideal gas with a constant specific heat ratio of $k = 1.4$. The carbon monoxide undergoes a polytropic expansion with $n = k$ from an initial state, where the temperature is 200 °F and pressure of 40 psia, to a final state where the volume is twice the initial volume. Determine:
a. the final temperature, in °F, and final pressure, in psia, and
b. the work done by the gas and heat transfer into the gas, each in Btu/lb$_m$.

SOLUTION:
Treat the CO as an ideal gas. Hence,
$$pV = mRT \quad \text{or} \quad pv = RT . \tag{1}$$
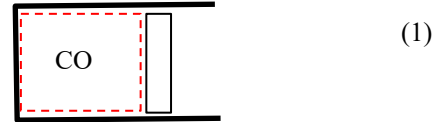
Since the process is polytropic,
$$pV^n = c \quad \text{or} \quad p = \frac{c}{V^n} \quad \Rightarrow \quad \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n , \tag{2}$$

where $n = k = 1.4$, for this case, and c is a constant, which can be determined from the initial state. Note that the mass of CO remains constant, so combining Eqs. (1) and (2) gives,
$$\frac{p_2}{p_1}\frac{V_2}{V_1} = \frac{T_2}{T_1} \Rightarrow \left(\frac{V_1}{V_2}\right)^n \frac{V_2}{V_1} = \frac{T_2}{T_1} \Rightarrow \left(\frac{V_1}{V_2}\right)^{n-1} = \frac{T_2}{T_1}. \tag{3}$$

Using the given parameters and Eqs. (2) and (3),
$p_1 = 40$ psia
$T_1 = 200$ °F $= 660$ °R
$n = k = 1.4$
$V_2 = 2V_1$
$\Rightarrow$ $\boxed{p_2 = 15.16 \text{ psia}}$, $\boxed{T_2 = 500.2 \text{ °R} = 40.19 \text{ °F}}$

The work done by the gas may be found using,
$$W_{\text{by gas}} = \int_{V_1}^{V_2} p\, dV = c\int_{V_1}^{V_2} V^{-n}\, dV = \frac{c}{1-n}\left(V_2^{1-n} - V_1^{1-n}\right) = \frac{p_1 V_1^n}{1-n}\left(V_2^{1-n} - V_1^{1-n}\right) \quad (n \neq 1), \tag{4}$$

$$W_{\text{by gas}} = \frac{p_1 m^n v_1^n}{1-n} m^{1-n}\left(v_2^{1-n} - v_1^{1-n}\right), \tag{5}$$

$$\frac{W_{\text{by gas}}}{m} = \frac{p_1 v_1^n}{1-n}\left(v_2^{1-n} - v_1^{1-n}\right). \tag{6}$$

where the initial conditions have been used to determine the constant c. The specific volume v_2 may be found using Eq. (1),
$$v = \frac{RT}{p} . \tag{7}$$

Using the given data,
$M_{\text{CO}} = 28.01$ lb$_m$/lb$_m$mol
$\bar{R}_{\text{CO}} = 1.986$ Btu/(lb$_m$mol.°R)
$p_1 = 40$ psia $= 5760$ lb$_m$/(ft.s^2)
$T_1 = 660$ °R
$v_2 = 2v_1$
$n = k = 1.4$
$\Rightarrow R_{\text{CO}} = 0.07090$ Btu/(lb$_m$.°R), $v_1 = 8.124*10^{-3}$ ft^3/lb$_m$, $v_2 = 1.625*10^{-2}$ ft^3/lb$_m$
$\Rightarrow$ $\boxed{W_{\text{by gas}}/m = 28.33 \text{ Btu/lb}_m}$.

The heat added to the gas may be found using the 1$^{\text{st}}$ Law applied to the gas,
$$\underbrace{\Delta E_{\text{sys}}}_{=\Delta U_{\text{sys}}} = Q_{\text{into sys}} - W_{\text{by sys}} \quad \Rightarrow \quad m\Delta u_{\text{sys}} = Q_{\text{into sys}} - W_{\text{by sys}} \quad \Rightarrow \quad \frac{Q_{\text{into sys}}}{m} = \Delta u_{\text{sys}} + \frac{W_{\text{by sys}}}{m}, \tag{8}$$

where kinetic and potential energies have been neglected. The change in specific internal energy of the gas, assuming ideal gas behavior, may be found either through ideal gas tables at the appropriate temperatures (e.g., Table A-23E of Moran et al., 7th ed.),

$\bar{u}_1$ = 3275.8 Btu/lb$_m$mol (at T_1 = 660 °R) $\Rightarrow$ u_1 = 117.0 Btu/lb$_m$

$\bar{u}_2$ = 2479.2 Btu/lb$_m$mol (at T_2 = 500 °R) $\Rightarrow$ u_2 = 88.51 Btu/lb$_m$

Note that $u = \bar{u}/M$ where M_{CO} = 28.01 lb$_m$/lb$_m$mol. Hence, from Eq. (8) and the previously calculated value for specific work,

$\boxed{Q_{\text{into gas}}/m = \text{-0.16 Btu/lb}_m}$.

If we instead assume that the CO behaves as a perfect gas (an ideal gas with constant specific heats),

$$u_2 - u_1 = c_v\left(T_2 - T_1\right),\tag{9}$$

where c_v = 0.178 Btu/(lb$_m$.°R) (from Table A-20E in Moran et al., 7th ed., at a temperature of 100 °F). Thus,

$Q_{\text{into gas}}/m$ = -0.11 Btu/lb$_m$.

We get approximately the same result using either method.

It can be shown that the heat transfer is, in fact, identically zero for a polytropic expansion of an ideal gas when $n = k$, as is the case here. Combining Eqs. (6) and (7) gives,

$$\frac{W_{\text{by gas}}}{m} = \frac{p_1 v_1^n}{1-n}\left(v_2^{1-n} - v_1^{1-n}\right) = \frac{\left(RT_1/v_1\right)v_1^n}{1-n}\left(v_2^{1-n} - v_1^{1-n}\right) = \frac{RT_1 v_1^{n-1}}{1-n}\left(v_2^{1-n} - v_1^{1-n}\right) = \frac{RT_1}{1-n}\left[\left(\frac{v_2}{v_1}\right)^{1-n} - 1\right],\tag{10}$$

$$\frac{W_{\text{by gas}}}{m} = \frac{RT_1}{1-n}\left[\left(\frac{v_2}{v_1}\right)^{1-n} - 1\right] = \frac{RT_1}{1-n}\left[\left(\frac{v_2}{v_1}\right)\left(\frac{v_2}{v_1}\right)^{-n} - 1\right] = \frac{RT_1}{1-n}\left[\left(\frac{v_2}{v_1}\right)\left(\frac{p_2}{p_1}\right) - 1\right],\tag{11}$$

where Eq. (2) has been used in the last step. Continuing to simplify,

$$\frac{W_{\text{by gas}}}{m} = \frac{RT_1}{1-n}\left[\left(\frac{RT_2}{RT_1}\right) - 1\right] = \frac{R}{1-n}\left(T_2 - T_1\right).\tag{12}$$

Since we're told that $n = k$ for this polytropic process,

$$\frac{W_{\text{by gas}}}{m} = \frac{R}{1-k}\left(T_2 - T_1\right) = c_v\left(T_2 - T_1\right),\tag{13}$$

where the relationship between the specific heat at constant volume, the gas constant, and the specific heat ratio has been used for an ideal gas. Note that the right hand side of this expression is the change in the specific internal energy, assuming a perfect gas (ideal gas with constant specific heats),

$$\frac{W_{\text{by gas}}}{m} = u_2 - u_1.\tag{14}$$

Thus, from the 1st Law, we must have $Q = 0$. Our previous answers were close to zero, but there is some numerical error. Plus, the $Q = 0$ result assumes constant specific heats, which isn't exactly true.
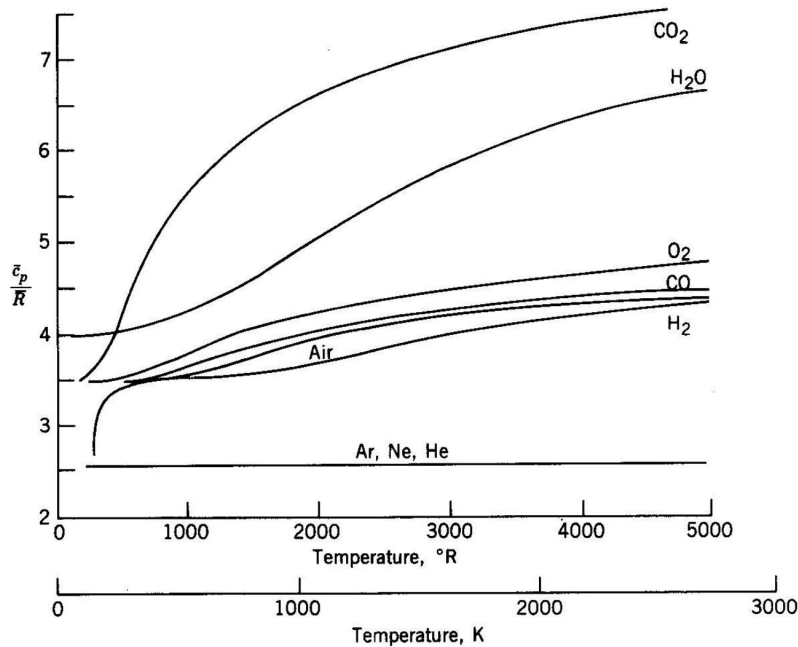
(A)
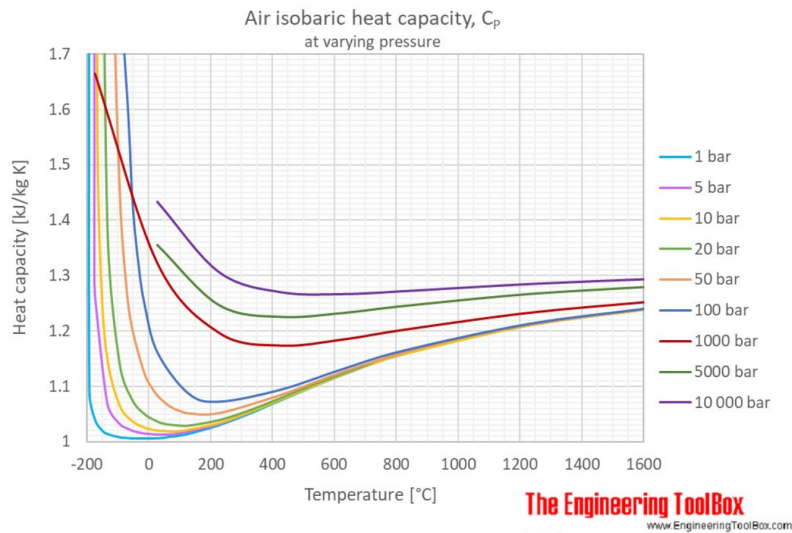
(B)

FIGURE 3.29. Plots of (A) $\bar{c}_p/\bar{R}_U$ as a function of temperature for various gases. (Figure from ???.) (B) c_p as a function of temperature for air at various pressures. (Figure from *The Engineering ToolBox*.)

3.6. The Second Law of Thermodynamics

The First Law must be satisfied for a process to occur, but it doesn't indicate if the process will occur. For example, consider a system consisting of a block sliding down an inclined surface under the action of gravity. The block is initially at rest and is at rest at the final state so the change in kinetic energy is zero. Assume