Q1_var_credito

August 21, 2022

ENUNCIADO

Considere um título com valor de Face igual a 1000 (reais) e 40 (reais) de coupon

O título vence em cinco anos, paga coupons anuais e possui rating BB.

Estime o VaR de Crédito pela distribuição empírica e também usando a distribuição normal.

(utilize um ano como horizonte temporal)

Dados do problema:

[ ]:
```
import pandas
matriz_transicao = pandas.read_csv("Q1_matriz_transicao.tsv", sep="\t").
 ↪set_index("Rating")
matriz_transicao
```

[ ]:

Rating	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0.00	0.00	0.00
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0.00
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0.00	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0.00	0.22	1.30	2.38	11.24	64.86	19.79

[ ]:
```
forward_zero_curves = pandas.read_csv("Q1_forward_zero_curves.tsv", sep="\t").
 ↪set_index("Rating")
forward_zero_curves
```

[ ]:

Rating	Y1	Y2	Y3	Y4
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

```
Default    NaN    NaN    NaN    NaN
```

A precificação do título após um ano é dada pela seguinte equação:

$$P = 40 + \frac{40}{(1+r_1)^1} + \frac{40}{(1+r_2)^2} + \frac{40}{(1+r_3)^3} + \frac{1040}{(1+r_4)^4}$$

Agora vamos computar os fluxos de caixa para cada taxa e precificar o título para cada cenário:

[ ]:
```
cashflow = forward_zero_curves.copy()
cashflow["CF0"] = 40.00
cashflow["CF1"] = round(40 / (1 + forward_zero_curves["Y1"] / 100)**1, 2)
cashflow["CF2"] = round(40 / (1 + forward_zero_curves["Y2"] / 100)**2, 2)
cashflow["CF3"] = round(40 / (1 + forward_zero_curves["Y3"] / 100)**3, 2)
cashflow["CF4"] = round(1040 / (1 + forward_zero_curves["Y4"] / 100)**4, 2)
cashflow["P"] = cashflow[["CF0", "CF1", "CF2", "CF3", "CF4"]].sum(axis=1)
cashflow.iloc[-1] = 0
cashflow
```

[ ]:
```
            Y1     Y2     Y3     Y4   CF0    CF1    CF2    CF3     CF4  \
Rating
AAA       3.60   4.17   4.73   5.12  40.0  38.61  36.86  34.82  851.71
AA        3.65   4.22   4.78   5.17  40.0  38.59  36.83  34.77  850.09
A         3.72   4.32   4.93   5.32  40.0  38.57  36.76  34.62  845.26
BBB       4.10   4.67   5.25   5.63  40.0  38.42  36.51  34.31  835.38
BB        5.55   6.02   6.78   7.27  40.0  37.90  35.59  32.85  785.45
B         6.05   7.02   8.03   8.52  40.0  37.72  34.92  31.73  749.88
CCC      15.05  15.02  14.03  13.52  40.0  34.77  30.24  26.98  626.24
Default   0.00   0.00   0.00   0.00   0.0   0.00   0.00   0.00    0.00

                P
Rating
AAA       1002.00
AA        1000.28
A          995.21
BBB        984.62
BB         931.79
B          894.25
CCC        758.23
Default      0.00
```

Como é um título BB, transpomos a linha correspondente da matriz de transição numa nova coluna, levando as probabilidades de mudança do rating do título:

[ ]:
```
cashflow["p"] = matriz_transicao.iloc[4]
cashflow
```

[ ]:

Rating	Y1	Y2	Y3	Y4	CF0	CF1	CF2	CF3	CF4	\
AAA	3.60	4.17	4.73	5.12	40.0	38.61	36.86	34.82	851.71	
AA	3.65	4.22	4.78	5.17	40.0	38.59	36.83	34.77	850.09	
A	3.72	4.32	4.93	5.32	40.0	38.57	36.76	34.62	845.26	
BBB	4.10	4.67	5.25	5.63	40.0	38.42	36.51	34.31	835.38	
BB	5.55	6.02	6.78	7.27	40.0	37.90	35.59	32.85	785.45	
B	6.05	7.02	8.03	8.52	40.0	37.72	34.92	31.73	749.88	
CCC	15.05	15.02	14.03	13.52	40.0	34.77	30.24	26.98	626.24	
Default	0.00	0.00	0.00	0.00	0.0	0.00	0.00	0.00	0.00	

Rating	P	p
AAA	1002.00	0.03
AA	1000.28	0.14
A	995.21	0.67
BBB	984.62	7.73
BB	931.79	80.53
B	894.25	8.84
CCC	758.23	1.00
Default	0.00	1.06

Para precificar o título daqui a um ano, multiplica-se os preços com as probabilidades e achar o preço esperado em cada cenário:

[ ]:
```
cashflow["P*p"] = round(cashflow["P"] * cashflow["p"] / 100, 2)
cashflow
```

[ ]:

Rating	Y1	Y2	Y3	Y4	CF0	CF1	CF2	CF3	CF4	\
AAA	3.60	4.17	4.73	5.12	40.0	38.61	36.86	34.82	851.71	
AA	3.65	4.22	4.78	5.17	40.0	38.59	36.83	34.77	850.09	
A	3.72	4.32	4.93	5.32	40.0	38.57	36.76	34.62	845.26	
BBB	4.10	4.67	5.25	5.63	40.0	38.42	36.51	34.31	835.38	
BB	5.55	6.02	6.78	7.27	40.0	37.90	35.59	32.85	785.45	
B	6.05	7.02	8.03	8.52	40.0	37.72	34.92	31.73	749.88	
CCC	15.05	15.02	14.03	13.52	40.0	34.77	30.24	26.98	626.24	
Default	0.00	0.00	0.00	0.00	0.0	0.00	0.00	0.00	0.00	

Rating	P	p	P*p
AAA	1002.00	0.03	0.30
AA	1000.28	0.14	1.40
A	995.21	0.67	6.67
BBB	984.62	7.73	76.11
BB	931.79	80.53	750.37
B	894.25	8.84	79.05
CCC	758.23	1.00	7.58

```
Default     0.00   1.06    0.00
```

Somando esta coluna encontra-se o preço esperado:

[ ]:
```
expected_price = cashflow["P*p"].sum()
expected_price
```

[ ]:
```
921.48
```

Com isso, e supondo distribuição normal de preços, vamos calcular primeiro a variância:

[ ]:
```
cashflow["Variance"] = round(cashflow["p"] / 100 * (cashflow["P"] -␣
 ↪expected_price)**2, 2)
cashflow
```

[ ]:

Rating	Y1	Y2	Y3	Y4	CF0	CF1	CF2	CF3	CF4	P	p	P*p	Variance
AAA	3.60	4.17	4.73	5.12	40.0	38.61	36.86	34.82	851.71	1002.00	0.03	0.30	1.95
AA	3.65	4.22	4.78	5.17	40.0	38.59	36.83	34.77	850.09	1000.28	0.14	1.40	8.69
A	3.72	4.32	4.93	5.32	40.0	38.57	36.76	34.62	845.26	995.21	0.67	6.67	36.42
BBB	4.10	4.67	5.25	5.63	40.0	38.42	36.51	34.31	835.38	984.62	7.73	76.11	308.17
BB	5.55	6.02	6.78	7.27	40.0	37.90	35.59	32.85	785.45	931.79	80.53	750.37	85.60
B	6.05	7.02	8.03	8.52	40.0	37.72	34.92	31.73	749.88	894.25	8.84	79.05	65.55
CCC	15.05	15.02	14.03	13.52	40.0	34.77	30.24	26.98	626.24	758.23	1.00	7.58	266.51
Default	0.00	0.00	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	1.06	0.00	9000.73

E com isso podemos calcular o desvio padrão:

[ ]:
```
import math
stdev = round(math.sqrt(cashflow["Variance"].sum()), 2)
stdev
```

[ ]:
```
98.86
```

E o VaR de crédito no IC 99%:

[ ]:
```
from scipy.stats import norm
var = round(norm.ppf(0.01) * stdev, 2)
```

```
var
```

[ ]:
```
-229.98
```

Pela distribuição empírica, calculamos a probabilidade acumulada:

[ ]:
```
cashflow["Cumulative"] = cashflow["p"].loc[::-1].cumsum().loc[::-1]
cashflow
```

[ ]:

Rating	Y1	Y2	Y3	Y4	CF0	CF1	CF2	CF3	CF4	\
AAA	3.60	4.17	4.73	5.12	40.0	38.61	36.86	34.82	851.71	
AA	3.65	4.22	4.78	5.17	40.0	38.59	36.83	34.77	850.09	
A	3.72	4.32	4.93	5.32	40.0	38.57	36.76	34.62	845.26	
BBB	4.10	4.67	5.25	5.63	40.0	38.42	36.51	34.31	835.38	
BB	5.55	6.02	6.78	7.27	40.0	37.90	35.59	32.85	785.45	
B	6.05	7.02	8.03	8.52	40.0	37.72	34.92	31.73	749.88	
CCC	15.05	15.02	14.03	13.52	40.0	34.77	30.24	26.98	626.24	
Default	0.00	0.00	0.00	0.00	0.0	0.00	0.00	0.00	0.00	

Rating	P	p	P*p	Variance	Cumulative
AAA	1002.00	0.03	0.30	1.95	100.00
AA	1000.28	0.14	1.40	8.69	99.97
A	995.21	0.67	6.67	36.42	99.83
BBB	984.62	7.73	76.11	308.17	99.16
BB	931.79	80.53	750.37	85.60	91.43
B	894.25	8.84	79.05	65.55	10.90
CCC	758.23	1.00	7.58	266.51	2.06
Default	0.00	1.06	0.00	9000.73	1.06

Daí pegamos o percentil mais próximo (Default) e o VaR será:

[ ]:
```
var = cashflow["P"].loc["Default"] - expected_price
var
```

[ ]:
```
-921.48
```