GRAVITATIONAL LENSING

6 - THE LENS MAP, TIME-DELAYS

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CRITICAL LINES AND CAUSTICS

Both convergence and shear are functions of position on the lens plane:

$\kappa = \kappa(\vec{\theta})$

$\gamma = \gamma(\vec{\theta})$

The determinant of the lensing Jacobian is

$$\det A(\vec{\theta}) = [1 - \kappa(\vec{\theta}) - \gamma(\vec{\theta})][1 - \kappa(\vec{\theta}) + \gamma(\vec{\theta})]$$

*The **critical lines** are the lines made of the points where the eigenvalues of the Jacobian are zero:*

$1 - \kappa(\vec{\theta}_t) - \gamma(\vec{\theta}_t) = 0$ *tangential critical line*

$1 - \kappa(\vec{\theta}_r) + \gamma(\vec{\theta}_r) = 0$ *radial critical line*

Along these lines the magnification diverges!

*Via the lens equations, they are mapped into the **caustics**…* $\vec{\beta}_{t,r} = \vec{\theta}_{t,r} - \vec{\alpha}(\vec{\theta}_{t,r})$

TYPES OF IMAGES

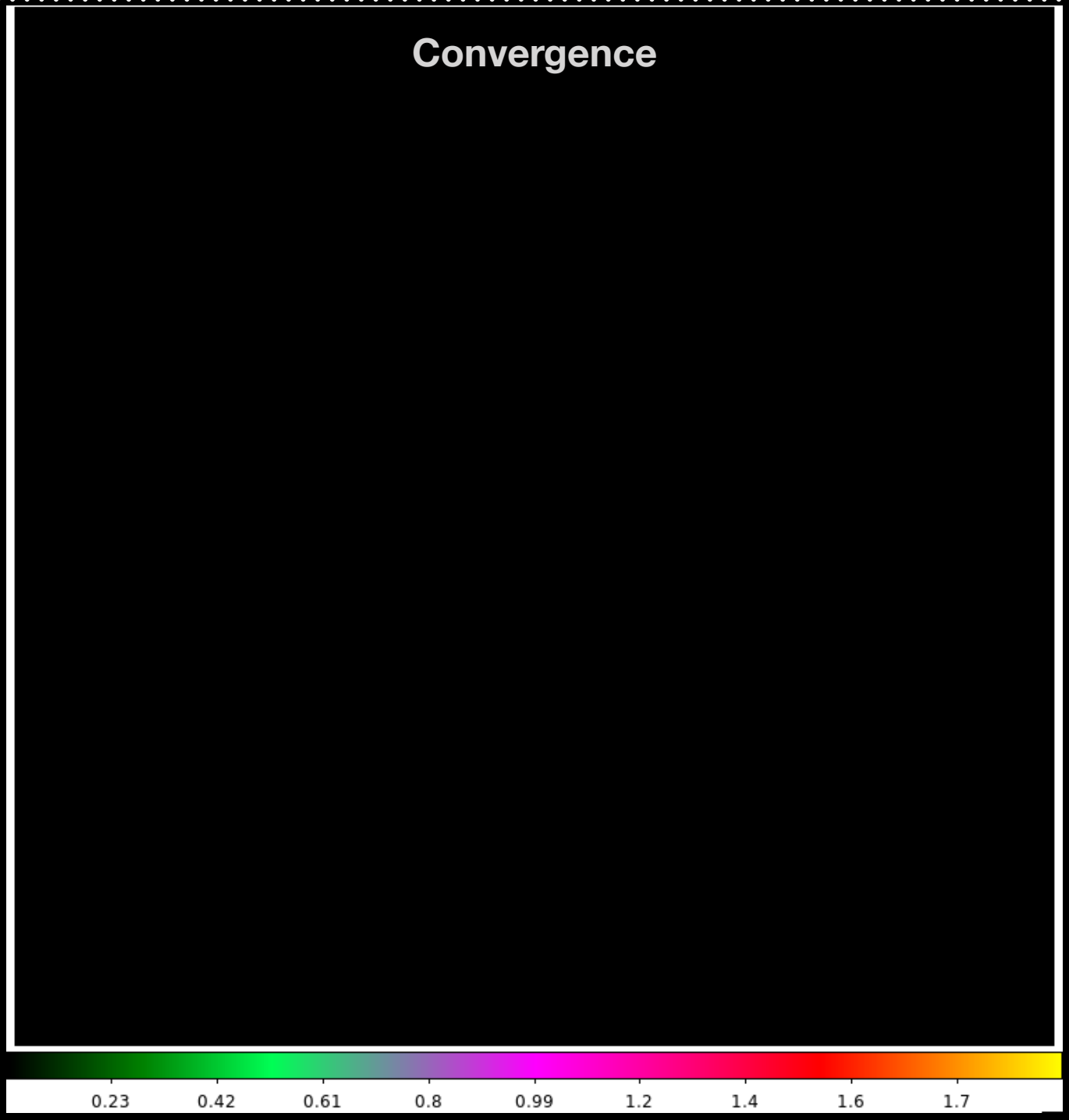
There are three types of images: eigenvalues $1-\kappa-\gamma$ $1-\kappa+\gamma$

	magnification	
Both eigenvalues are positive.	$\lvert A\rvert > 0 \quad \mu > 0$	Normal image or "minimum image"
One of the eigenvalues is negative.	$\lvert A\rvert < 0 \quad \mu < 0$	Inverted in one dimension. "saddle point image"
Both of the eigenvalues is negative.	$\lvert A\rvert > 0 \quad \mu > 0$	Inverted in both dimension. "maximum image"

LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES

cluster z = 0.507

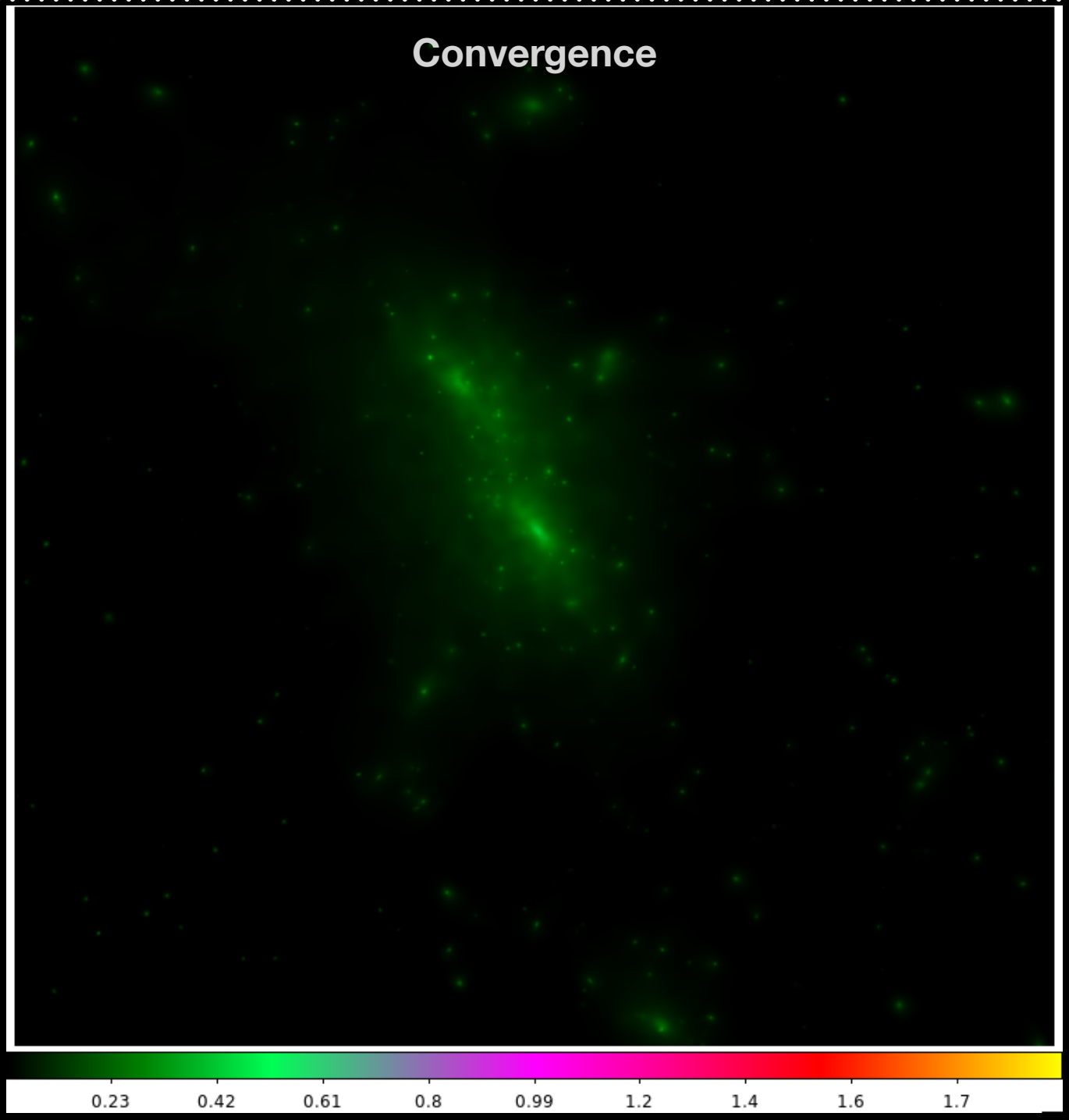
source z = 0.5



LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES

cluster z = 0.507

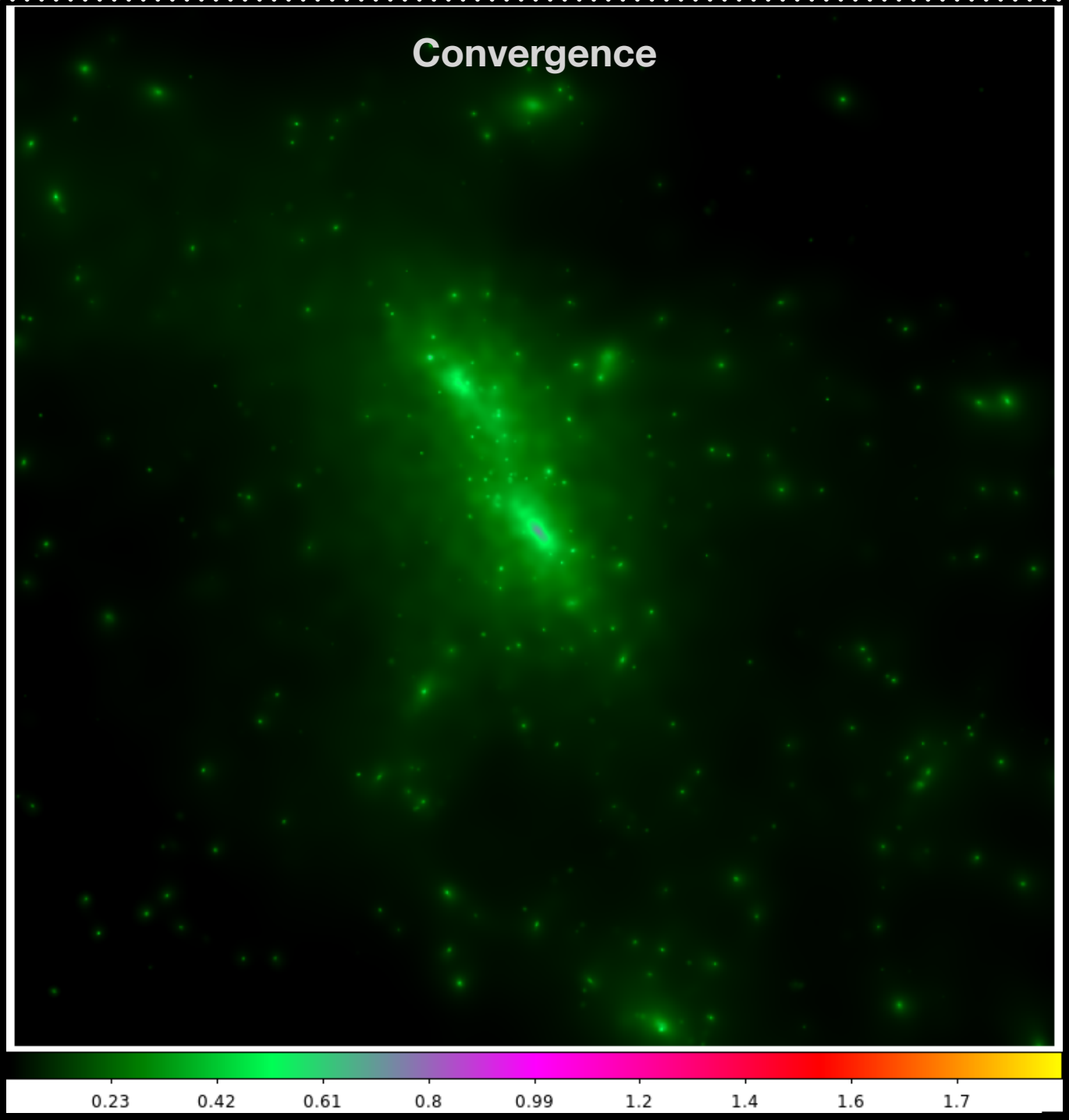
source z = 0.6



LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES

cluster z = 0.507

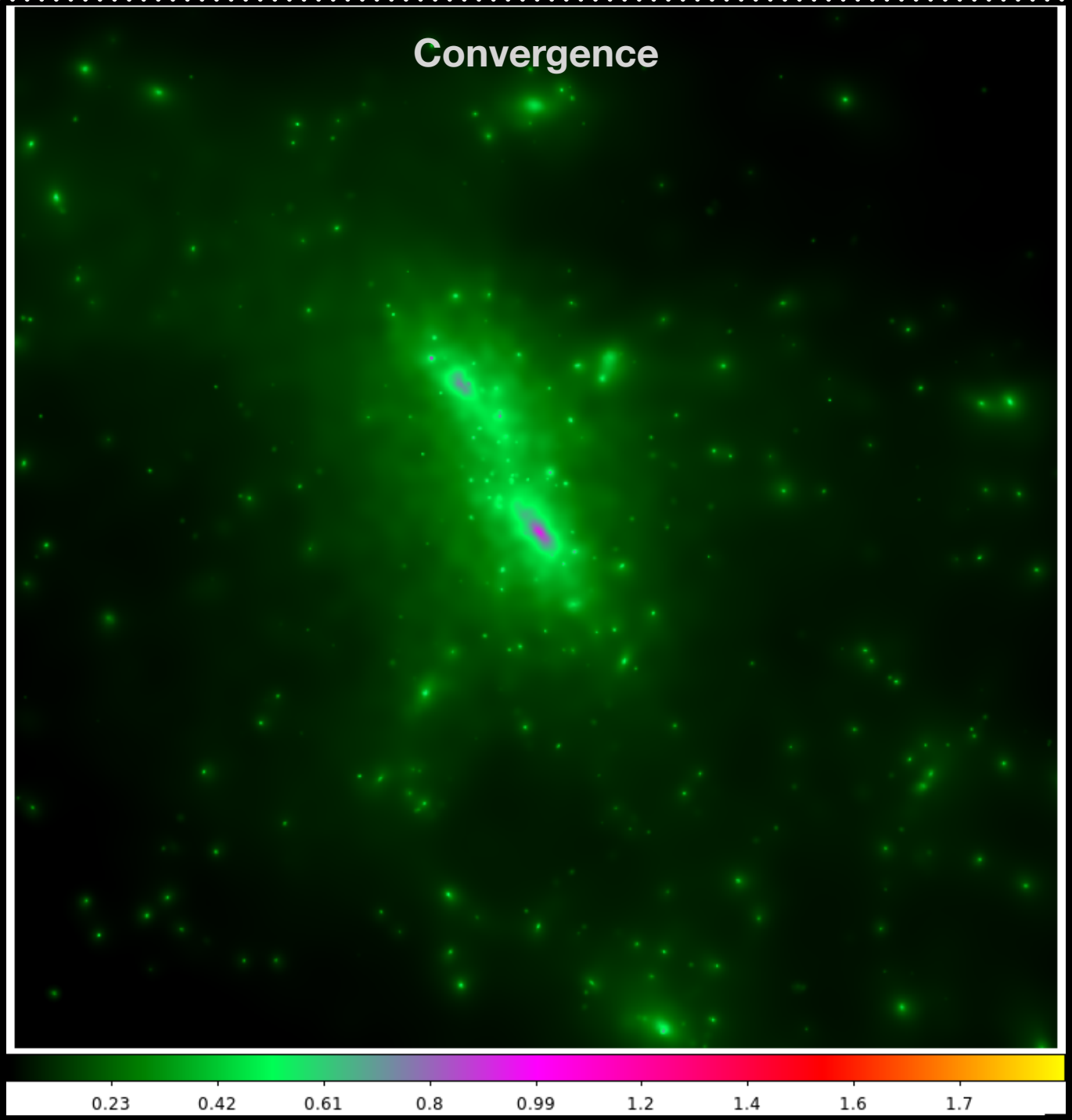
source z = 0.7



LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES

cluster z = 0.507

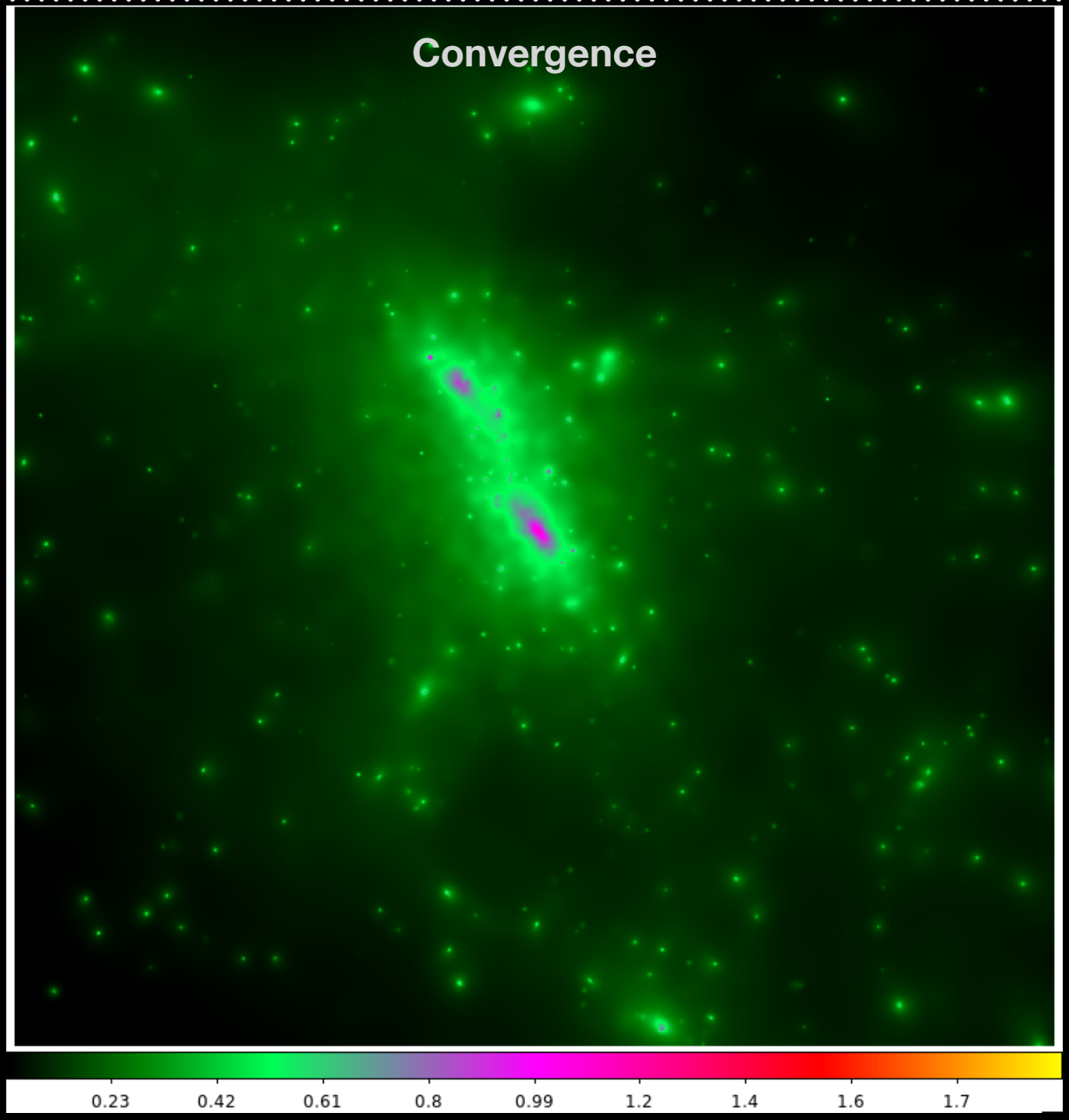
source z = 0.8



LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES

cluster z = 0.507

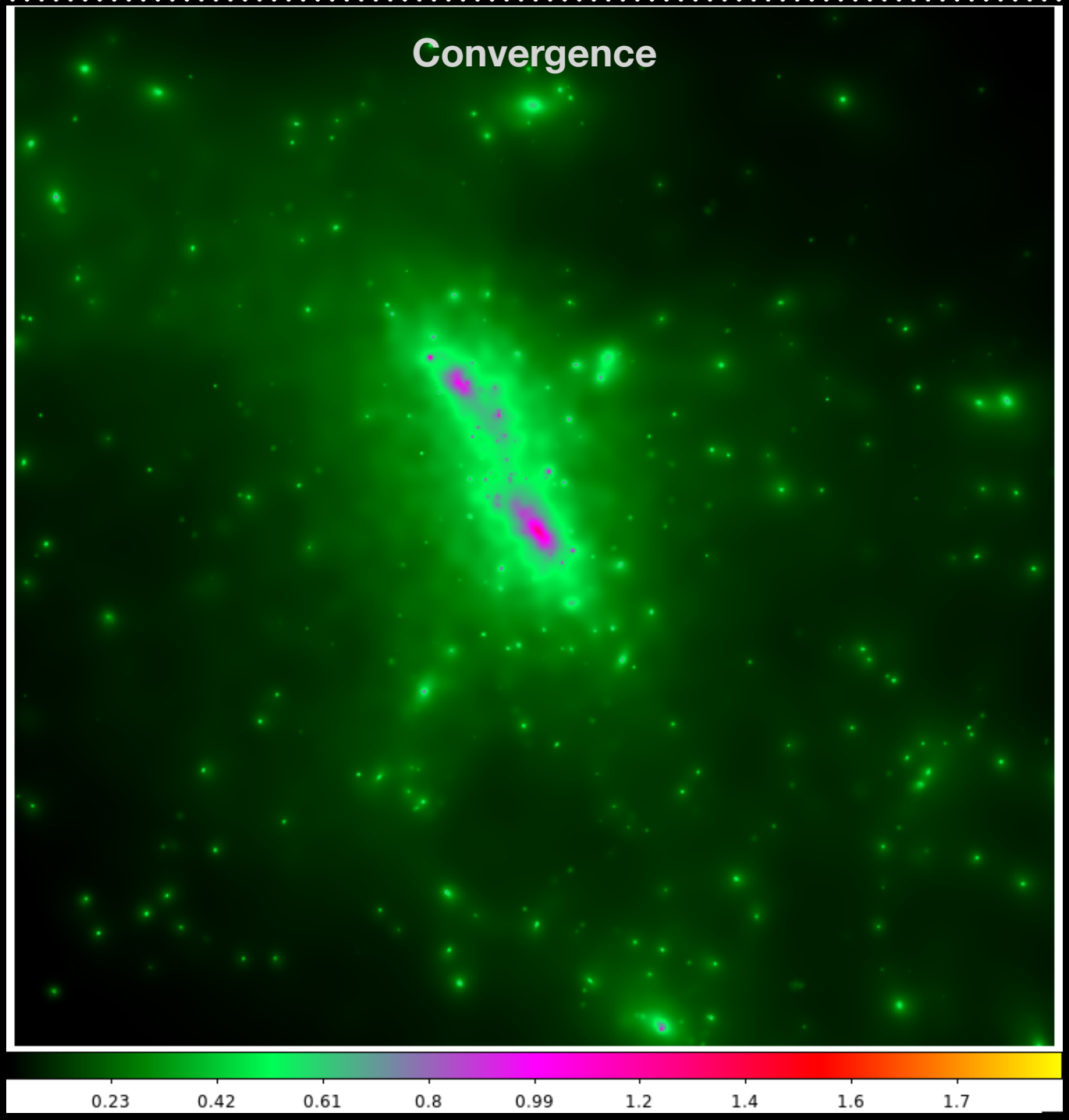
source z = 0.9



LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES

cluster z = 0.507

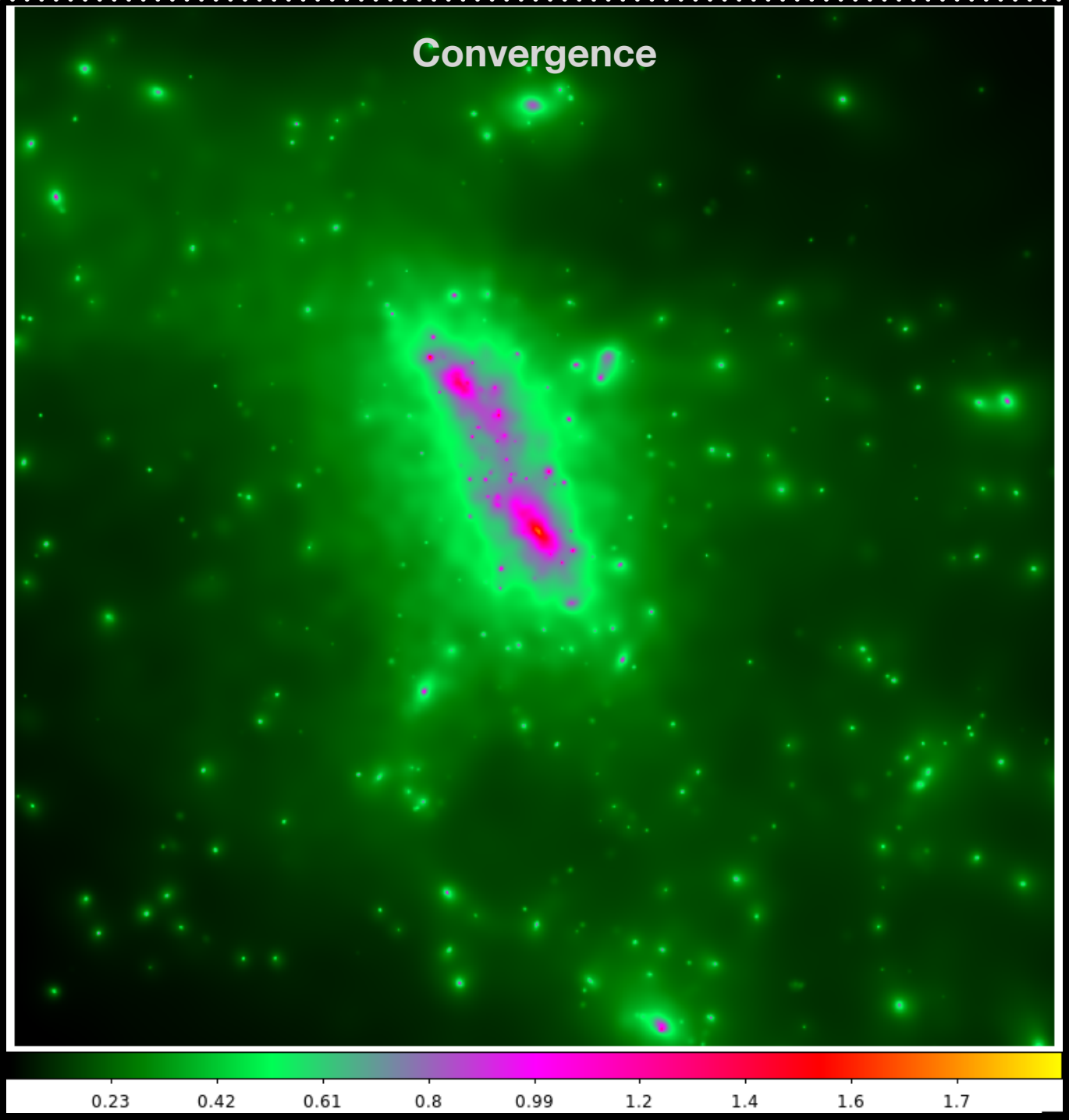
source z = 1.0



LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES

cluster z = 0.507

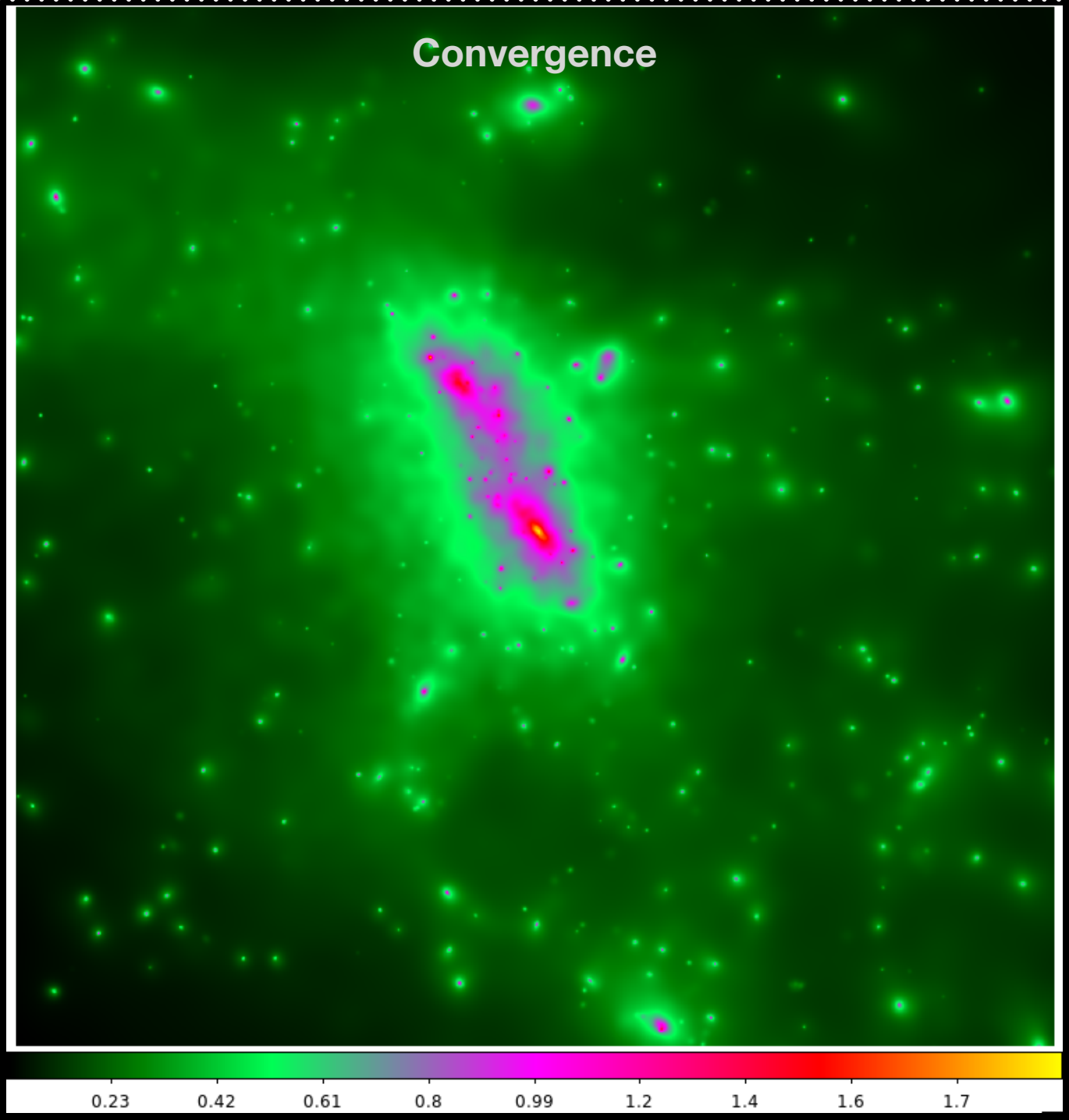
source z = 1.5



LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES

cluster z = 0.507

source z = 2.0



LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES

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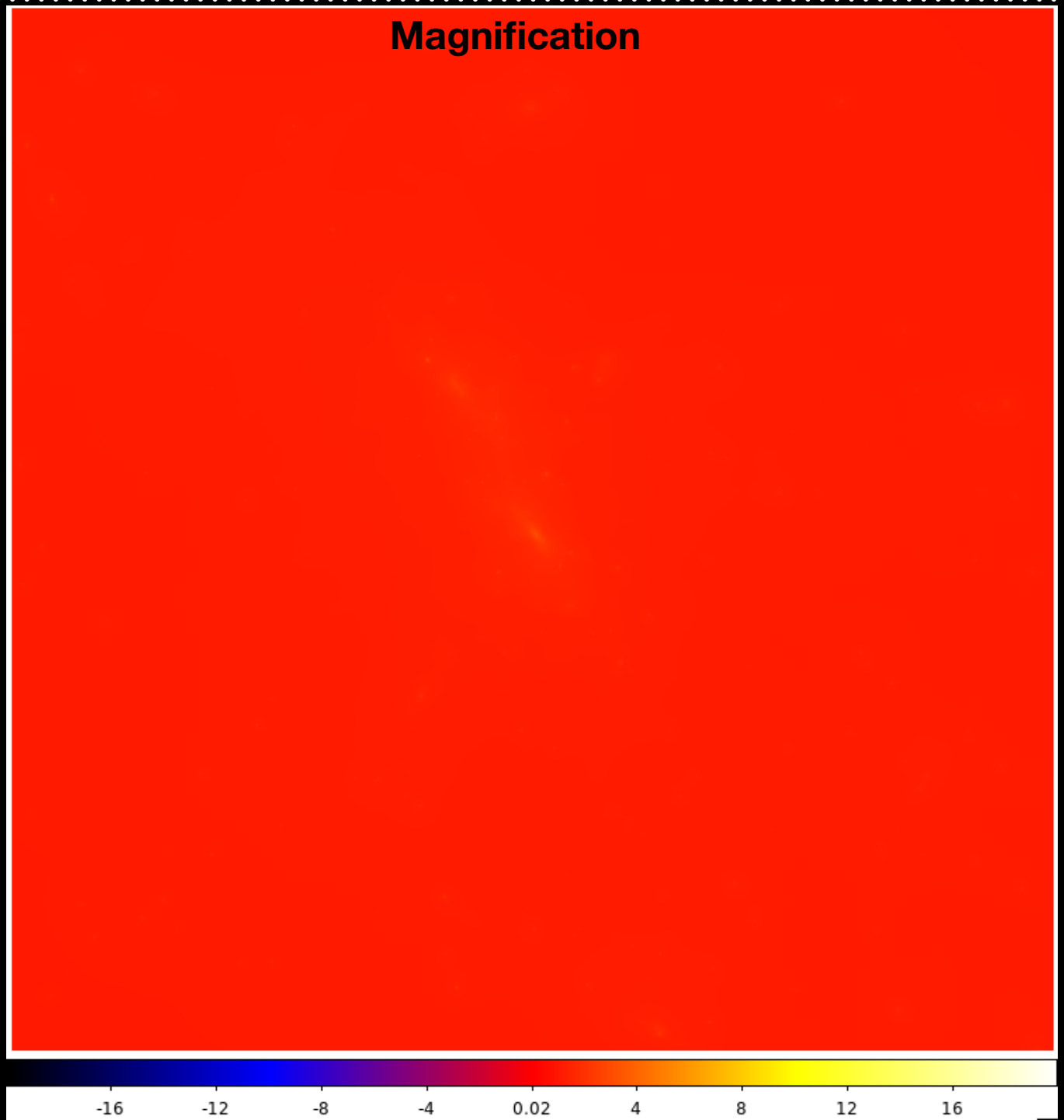
source z = 0.6



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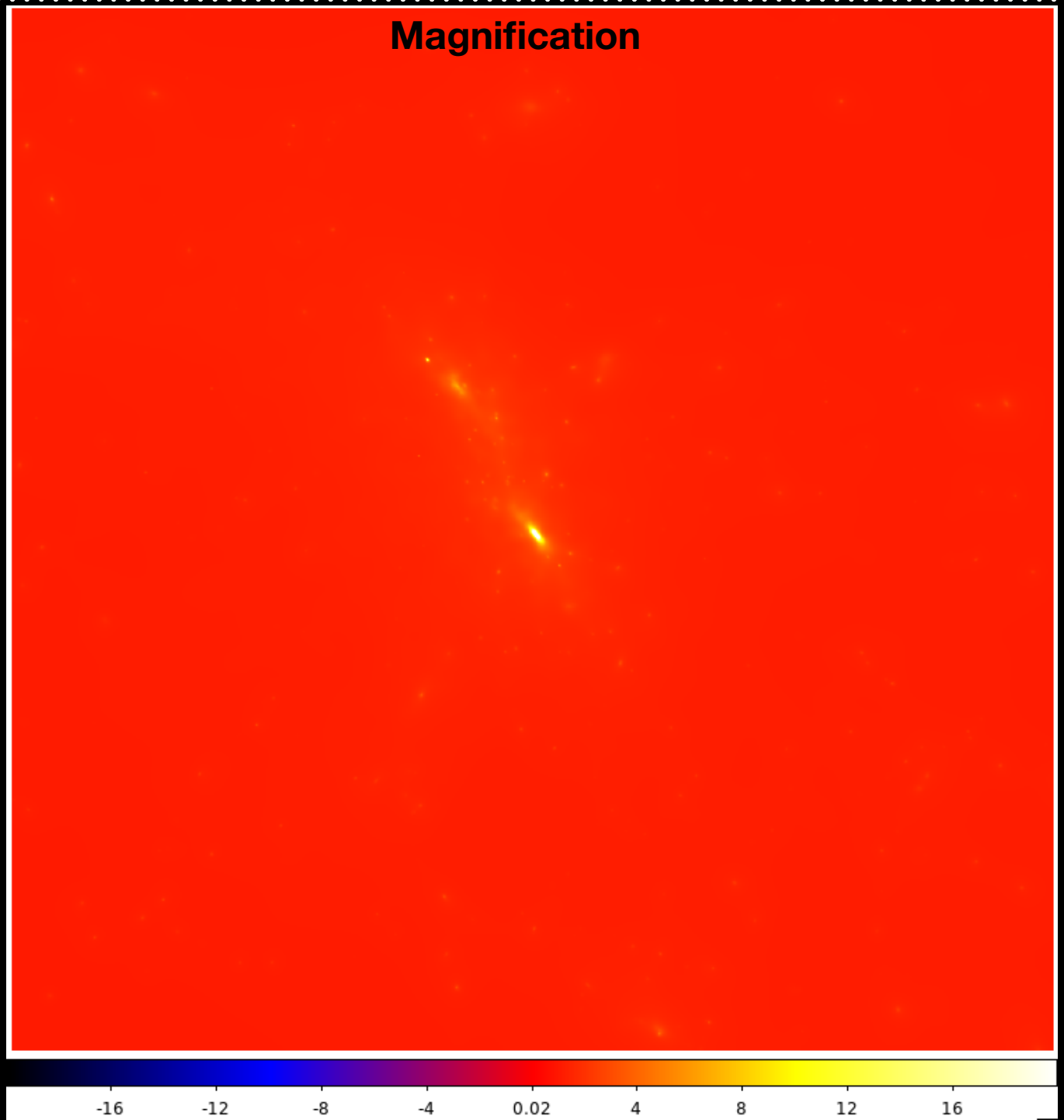
source z = 0.7



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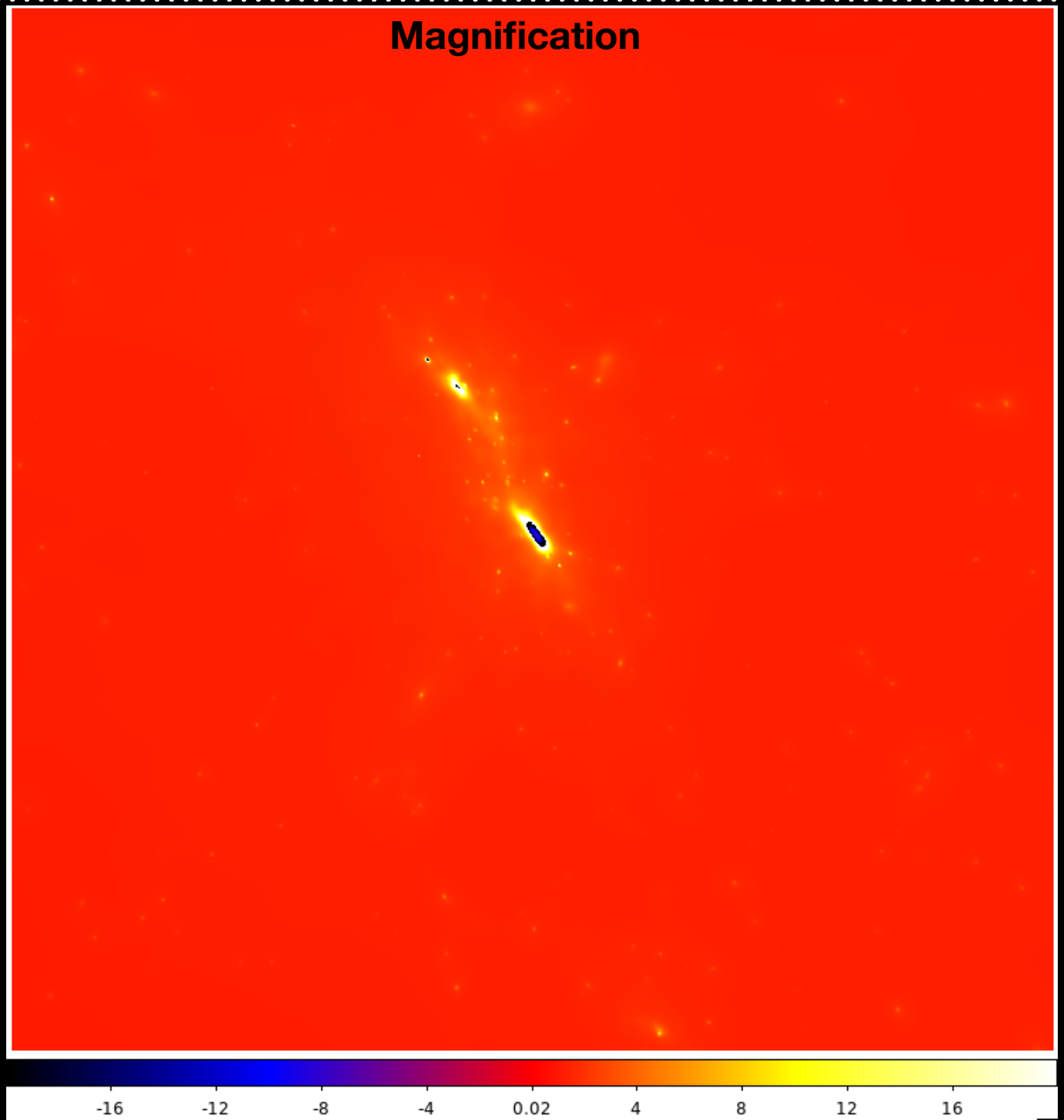
source z = 0.8



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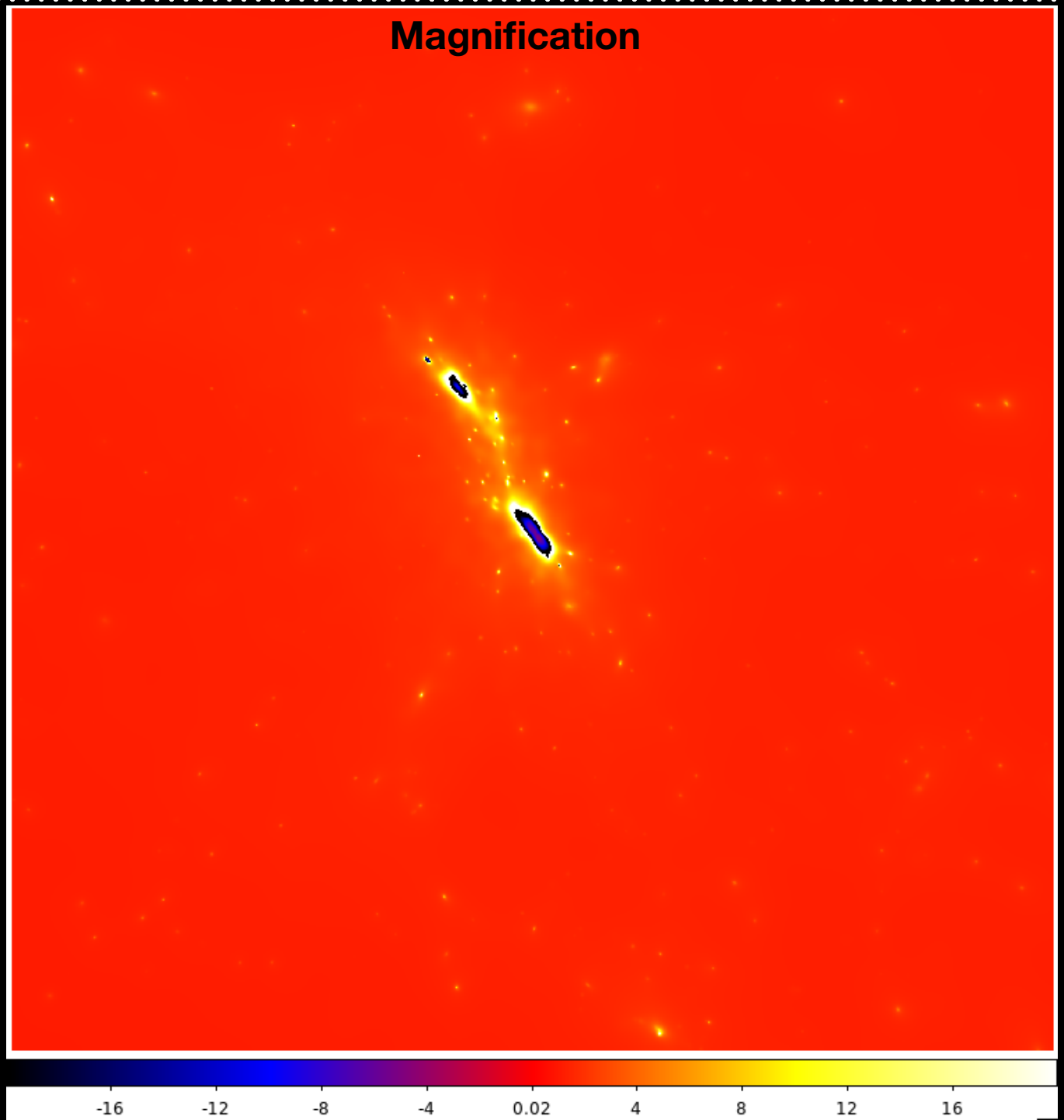
source z = 0.9



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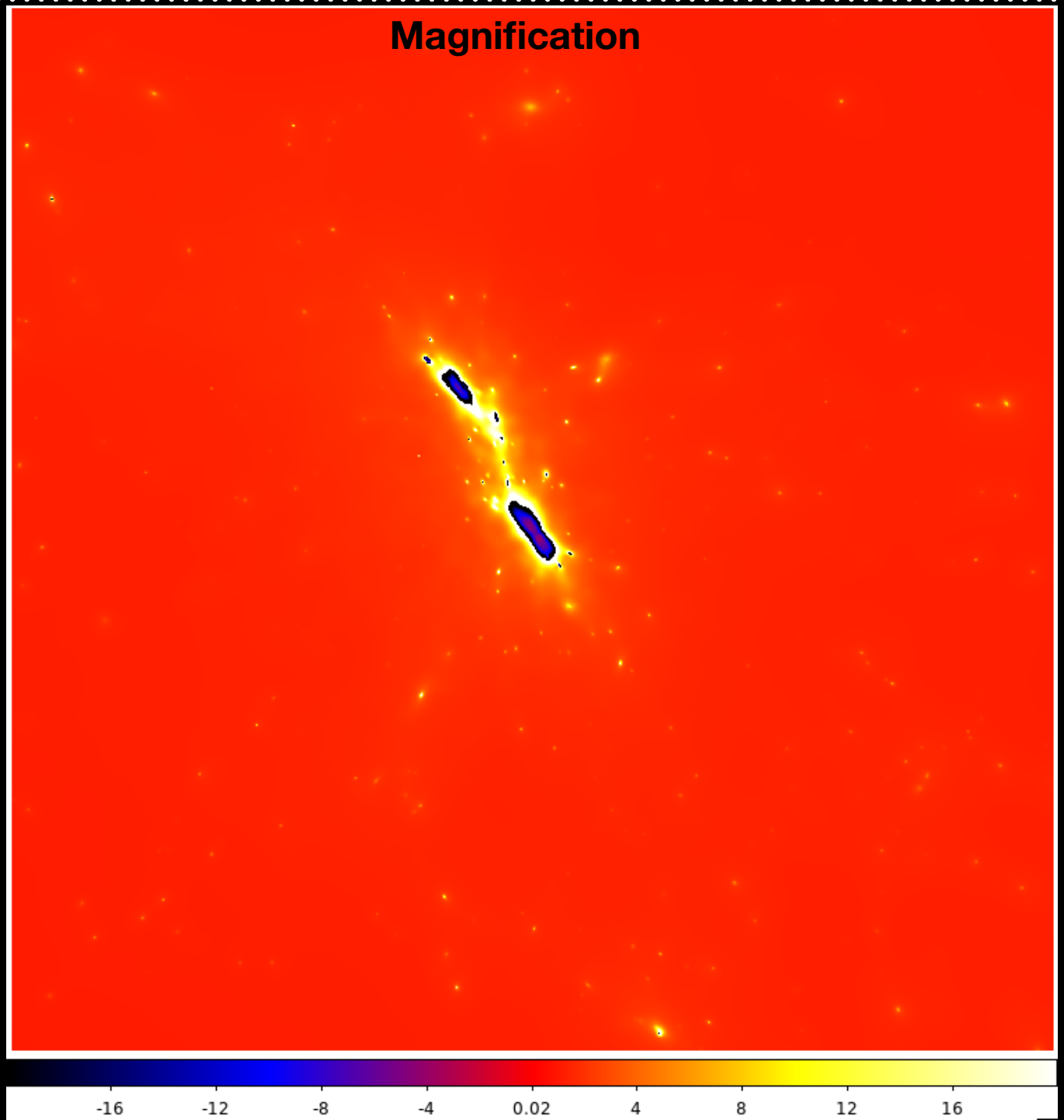

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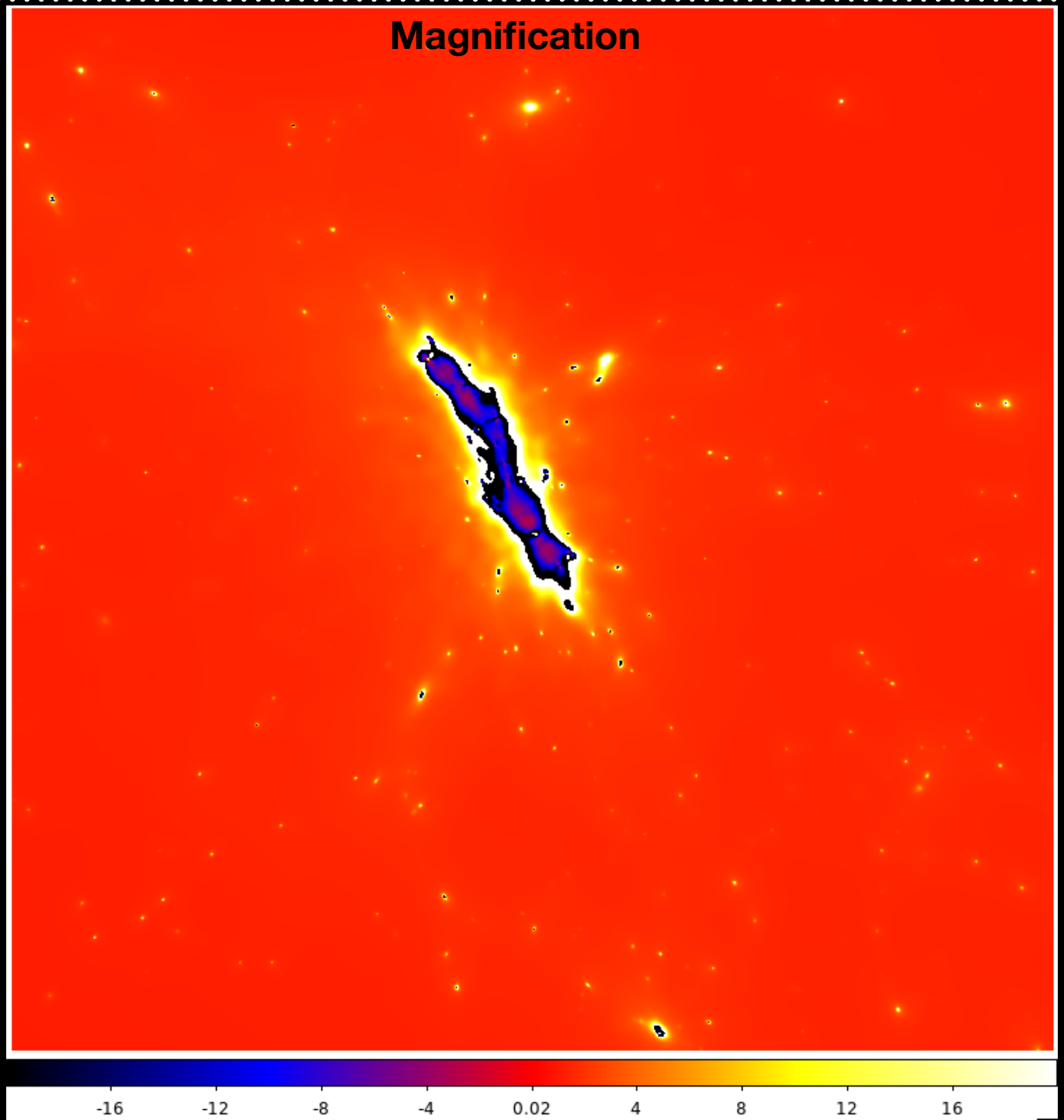
source z = 1.5



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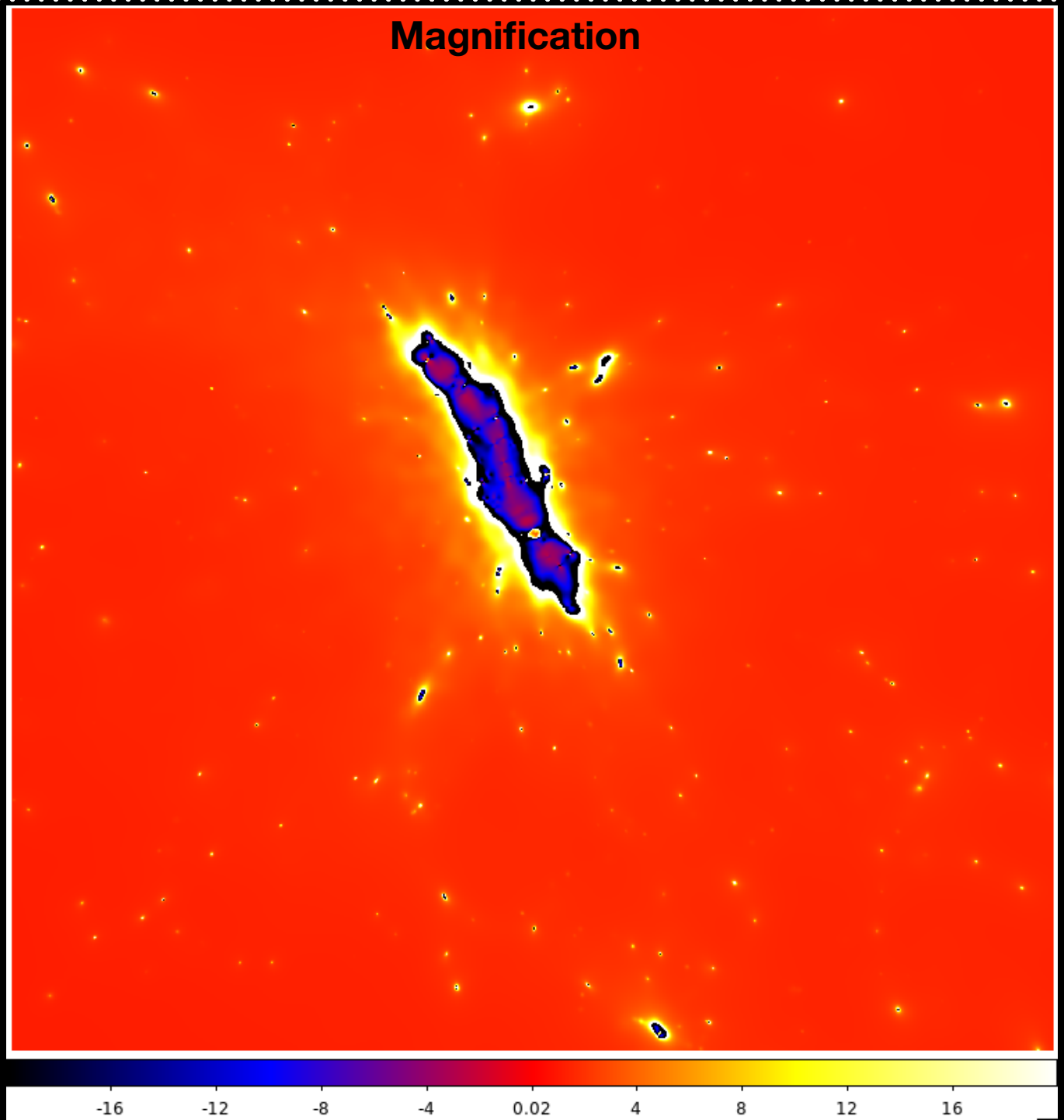
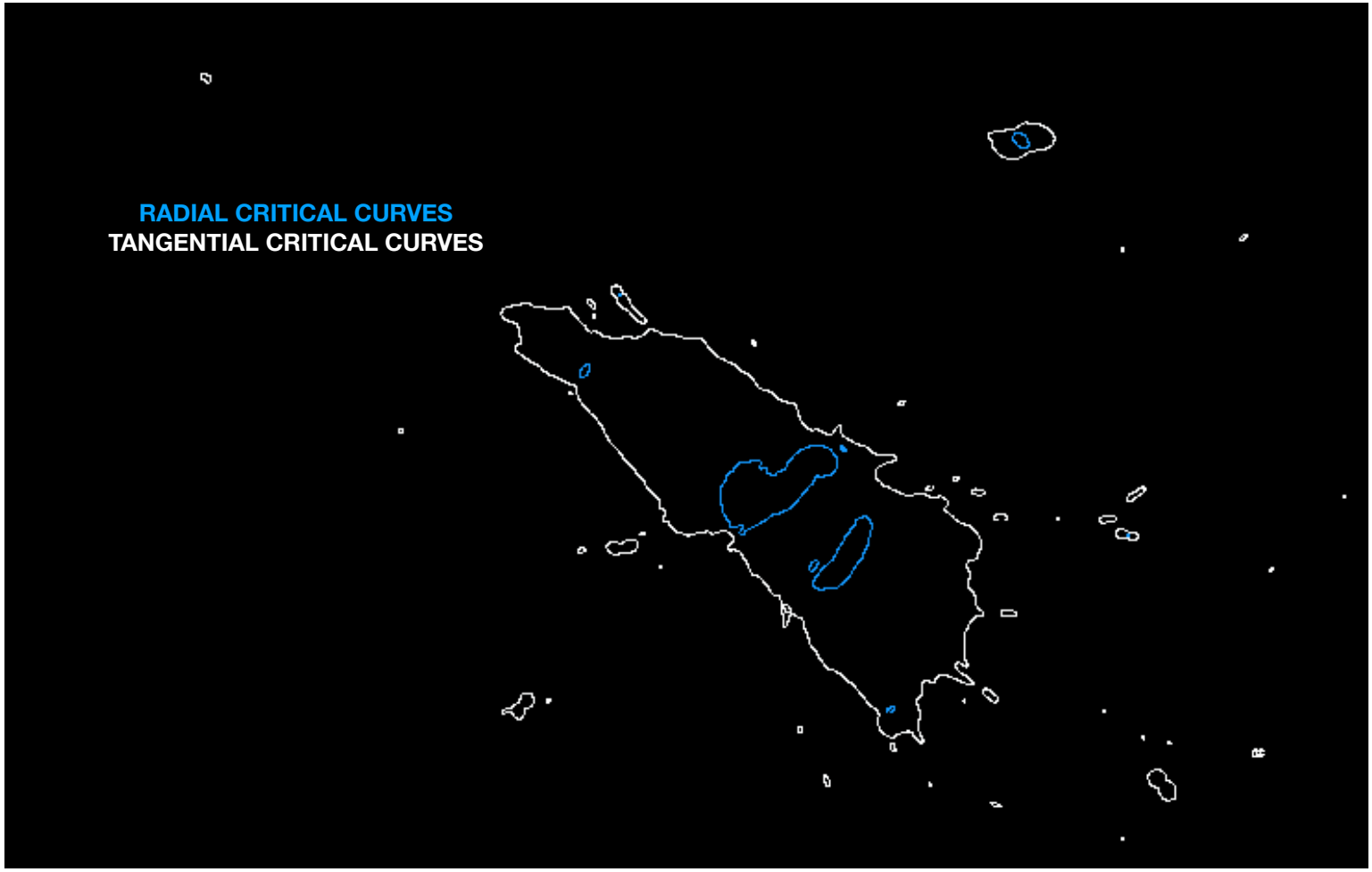
source z = 2.0

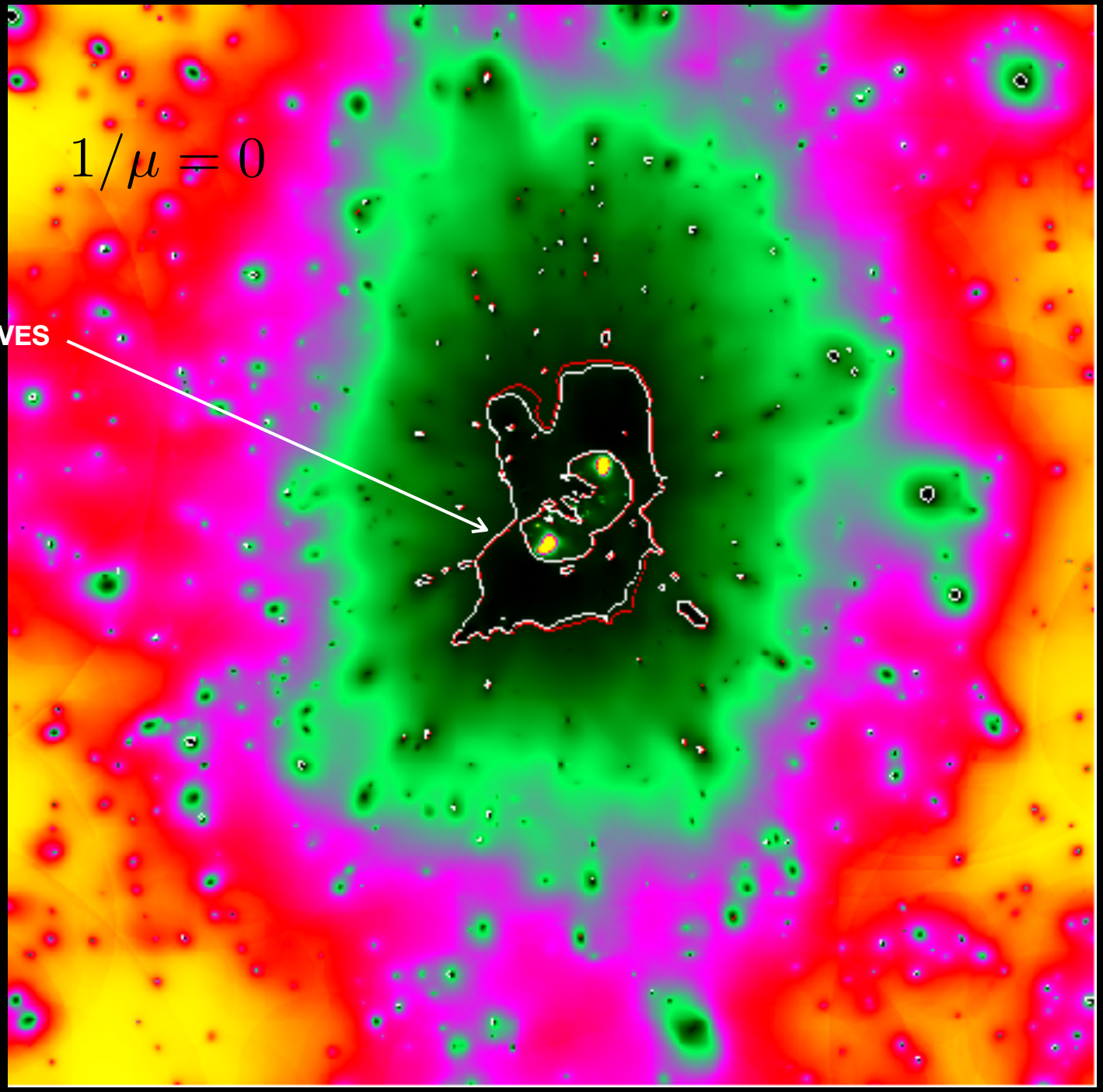


LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES

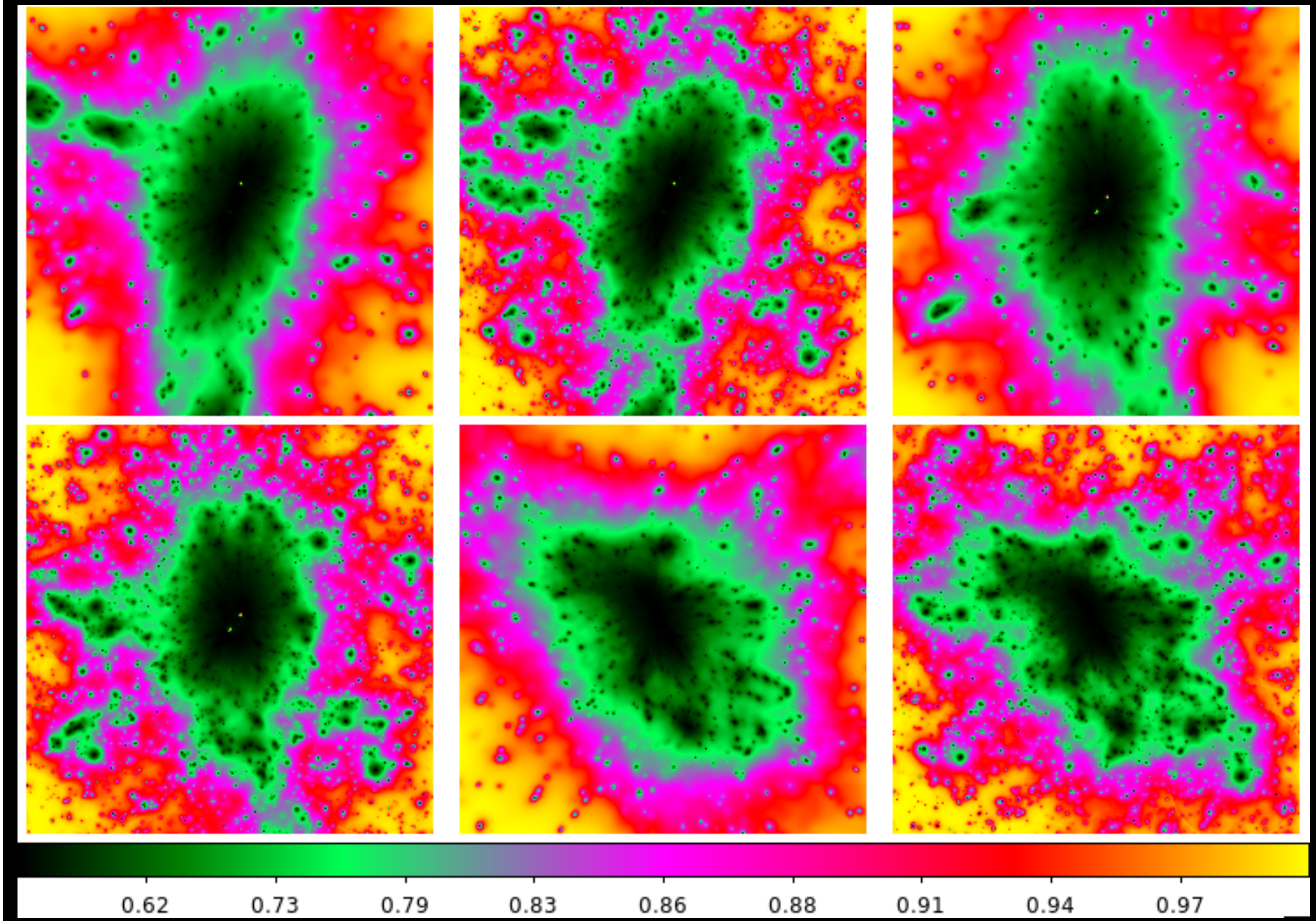
CRITICAL CURVES OF A GALAXY CLUSTER



LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES

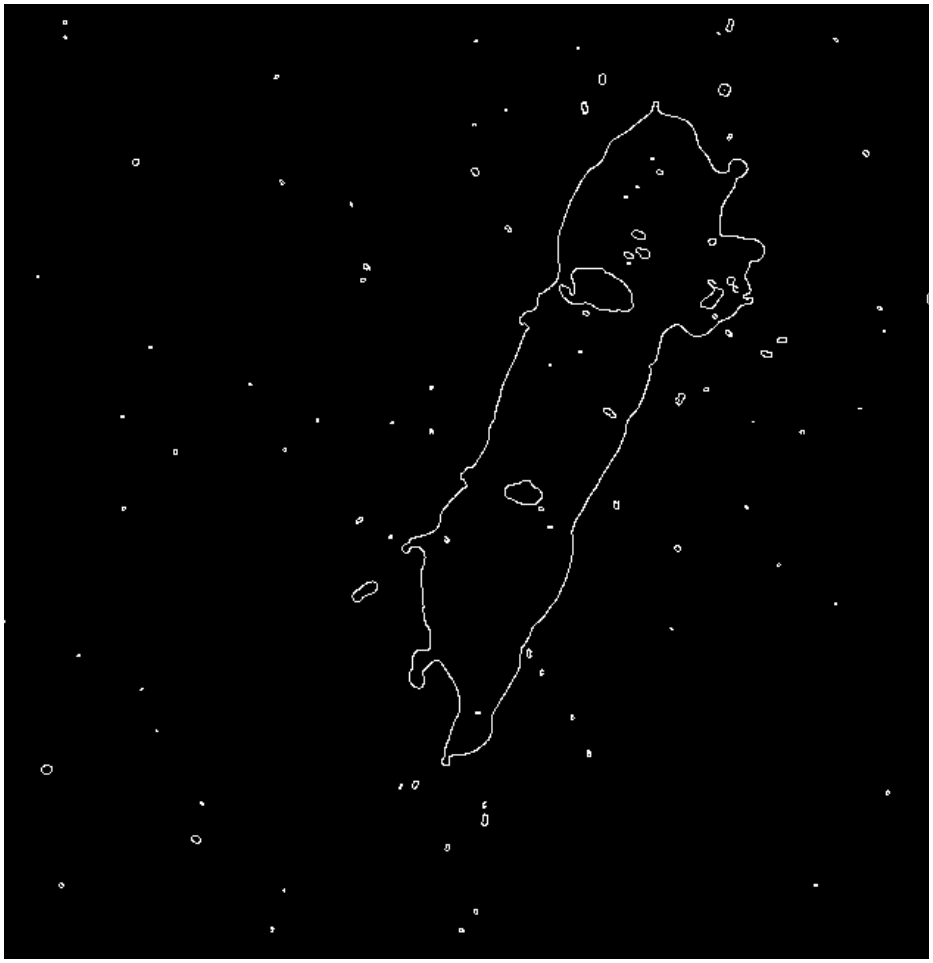


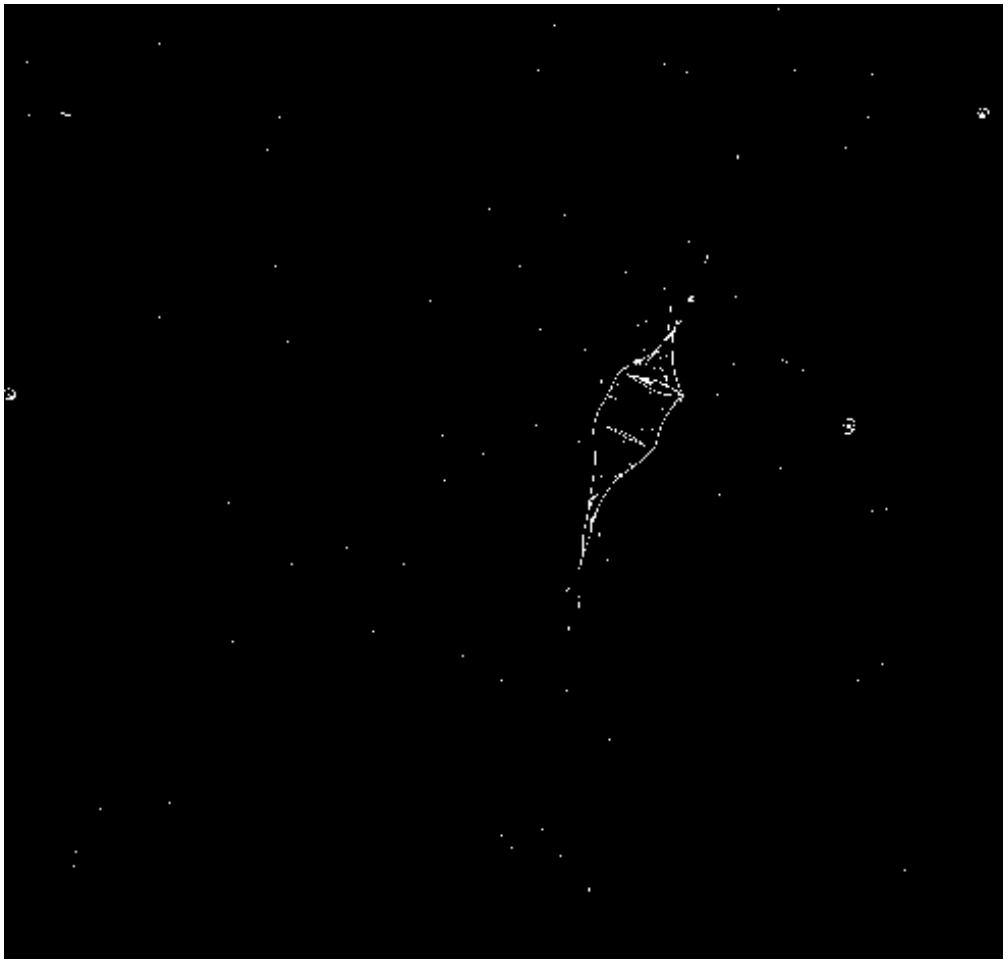
LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES



INVERSE MAGNIFICATION

LENS MAPPING : CRITICAL CURVES & CAUSTIC CURVES





POSITIVE AND NEGATIVE MAGNIFICATION

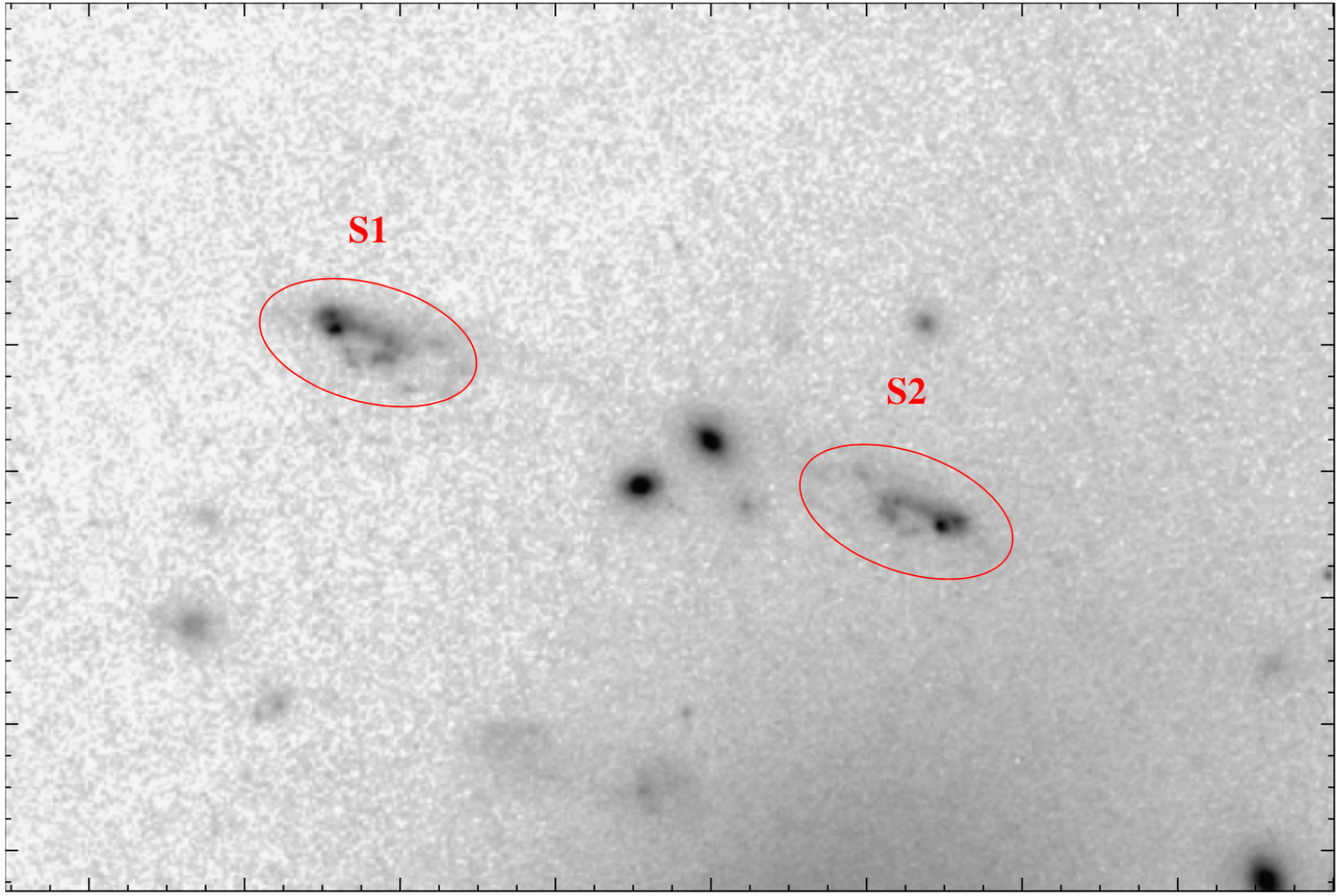
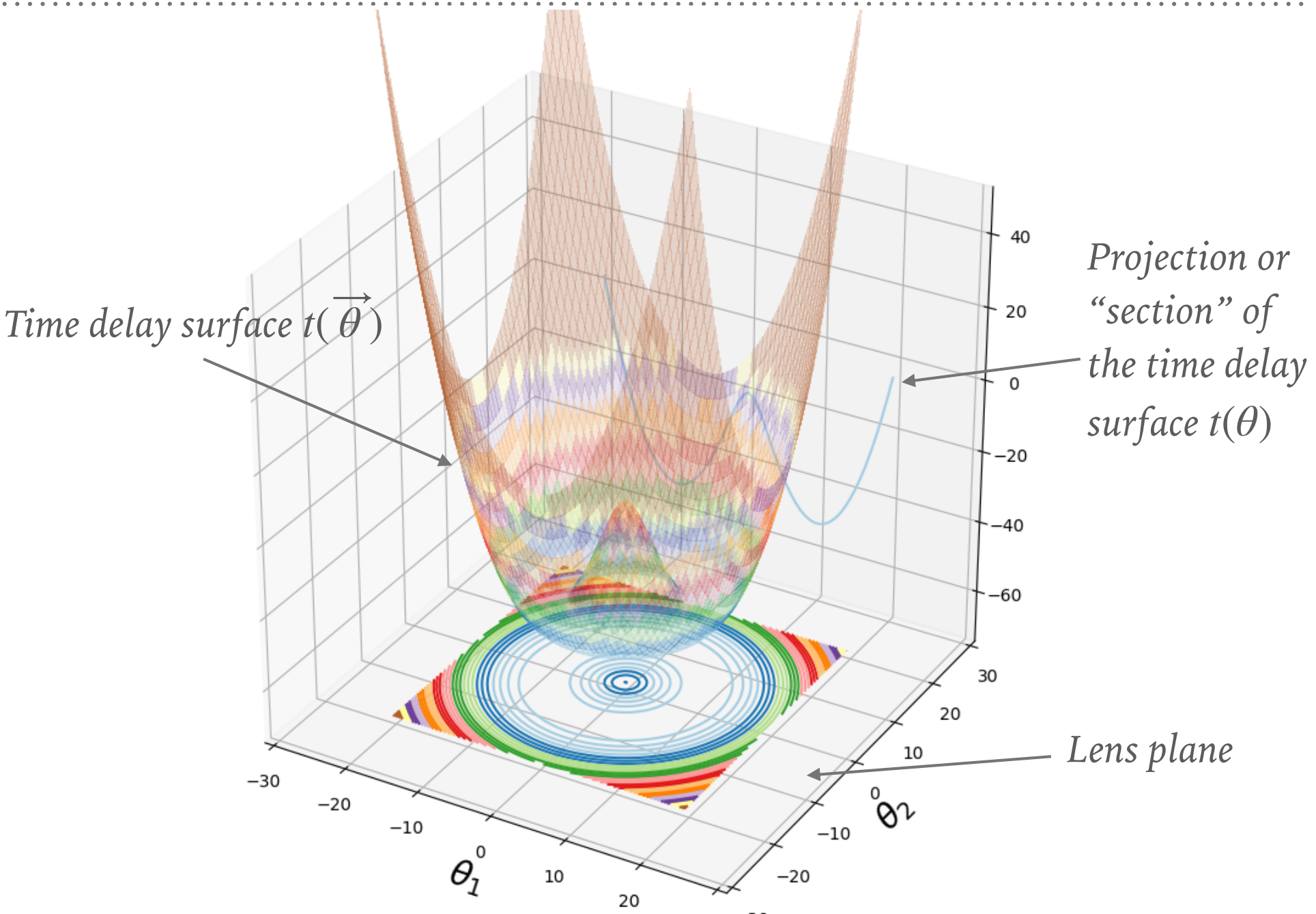


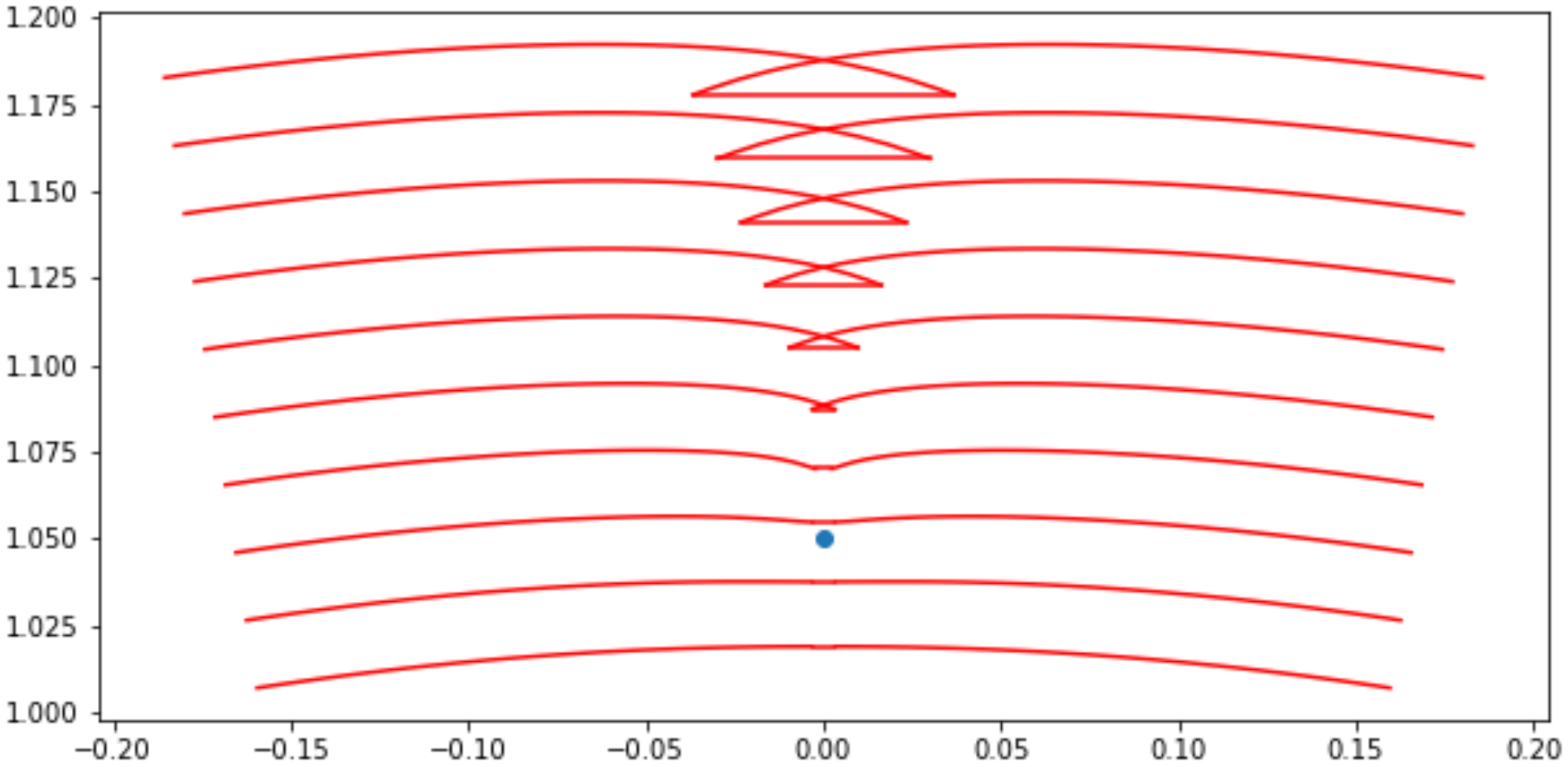
Fig. 10 The lensed pair S1–S2 in AC114. This galaxy at $z = 1.867$ displays the surprising morphology of a hook, with an obvious change in parity (Smail et al. 1995; Campusano et al. 2001)

Kneib & Natarajan 2012

EXAMPLE OF TIME DELAY SURFACE FOR A CIRCULAR LENS



WAVEFRONTS


1.200
1.175
1.150
1.125
1.100
1.075
1.050
1.025
1.000
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20

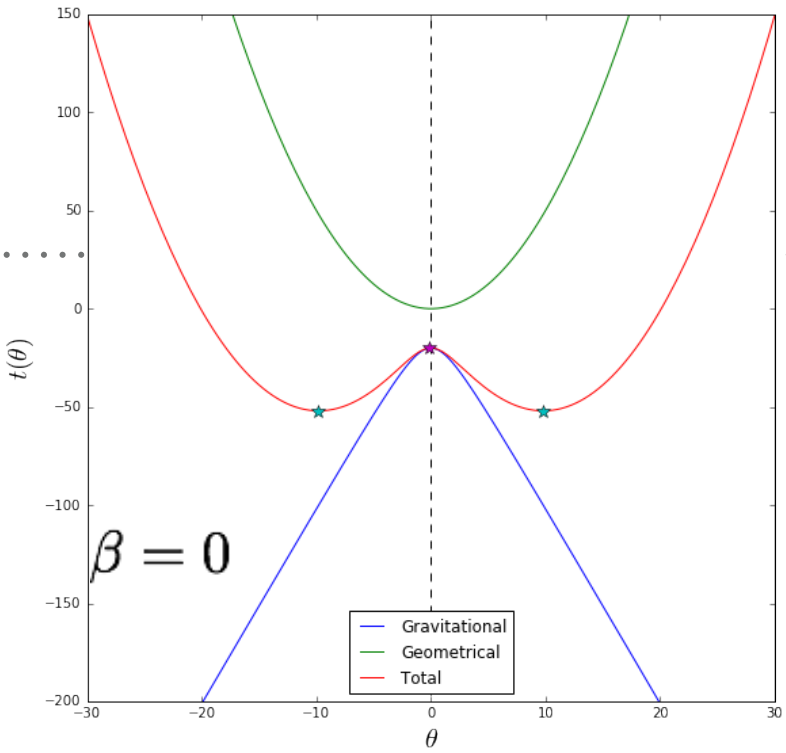
EXAMPLE OF TIME DELAY SURFACE

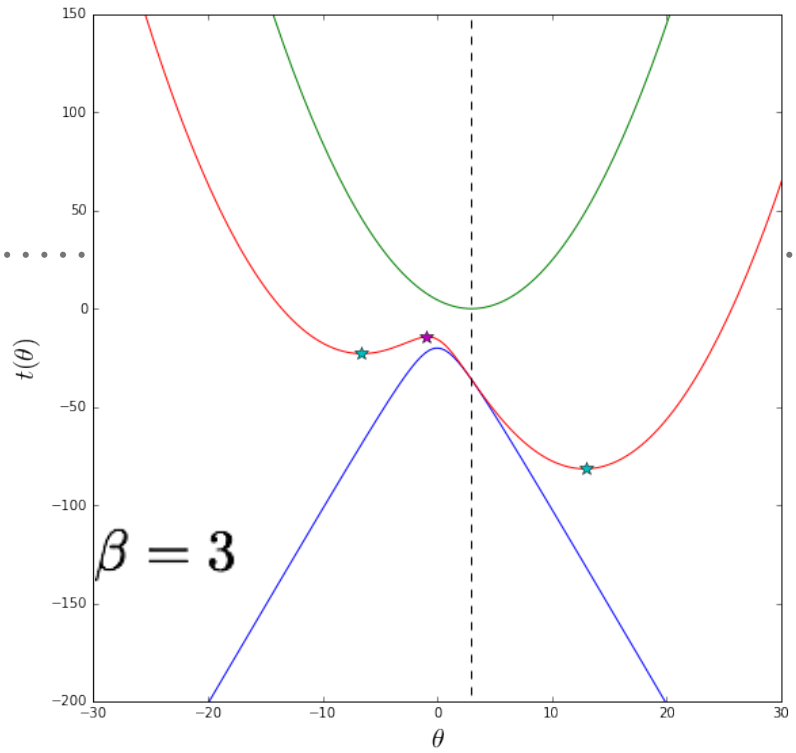
Toy potential:

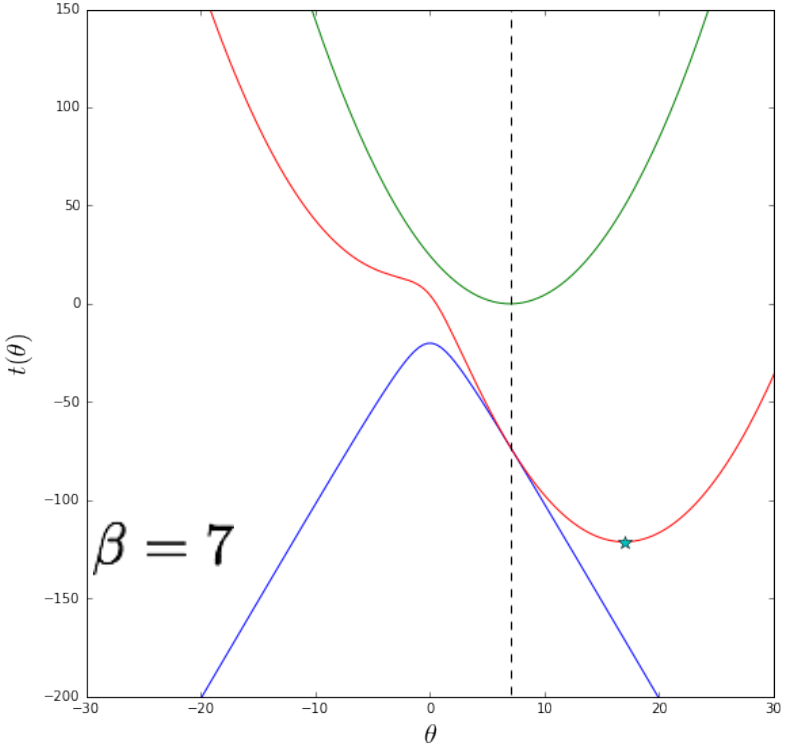
$$\hat{\Psi}(\theta) = K\sqrt{\theta^2 + \theta_c^2}$$

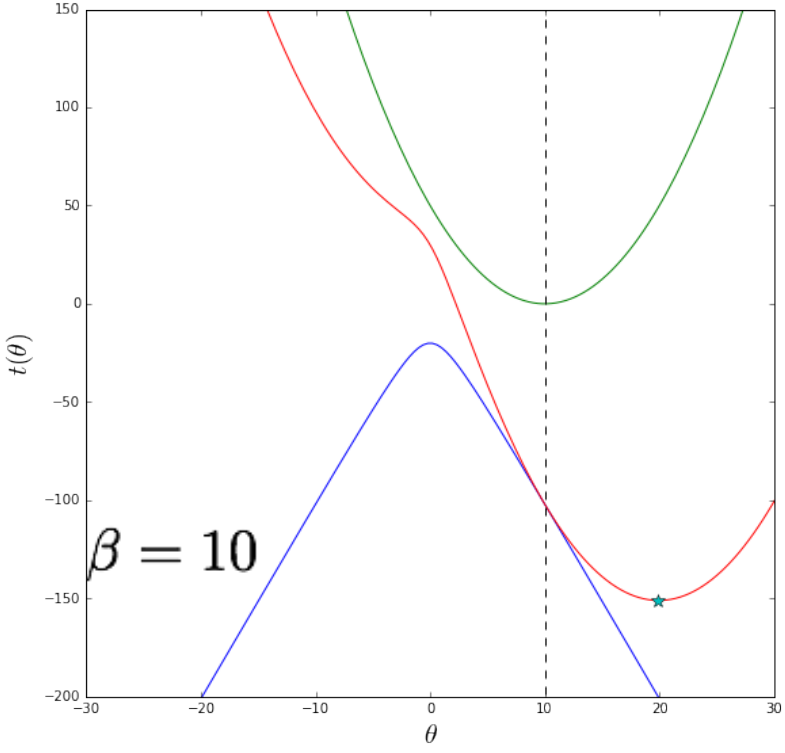
Assuming axial-symmetry, we can discuss the time-delay function instead of the time delay surface.

$$t(\theta) \propto \frac{1}{2}(\theta - \beta)^2 - K\sqrt{\theta^2 + \theta_c^2}$$


$\beta = 0$
Gravitational
Geometrical
Total
$t(\theta)$
θ


$\beta = 3$
$t(\theta)$
θ


$\beta = 7$
$t(\theta)$
θ

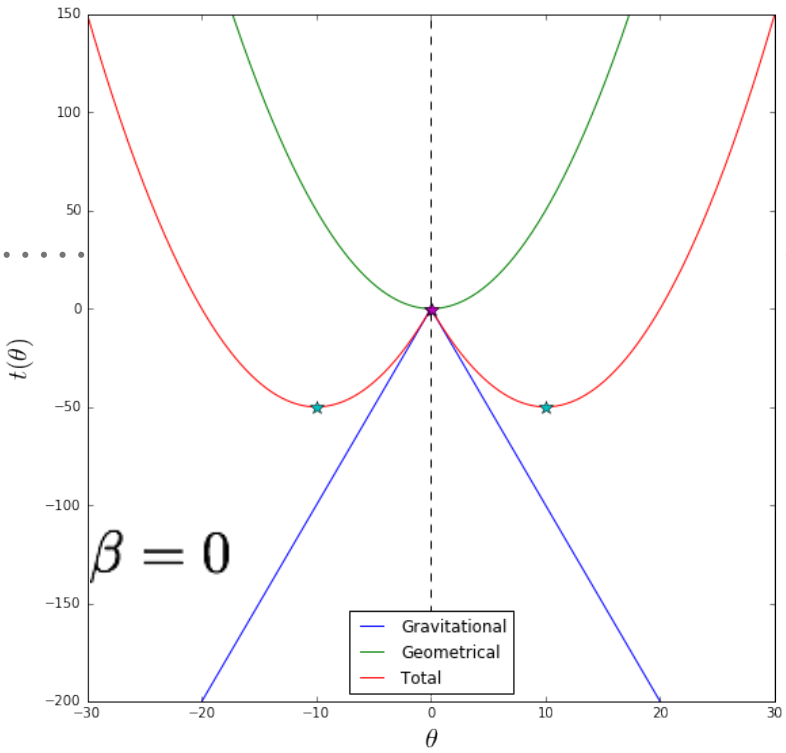

$\beta = 10$
$t(\theta)$
θ

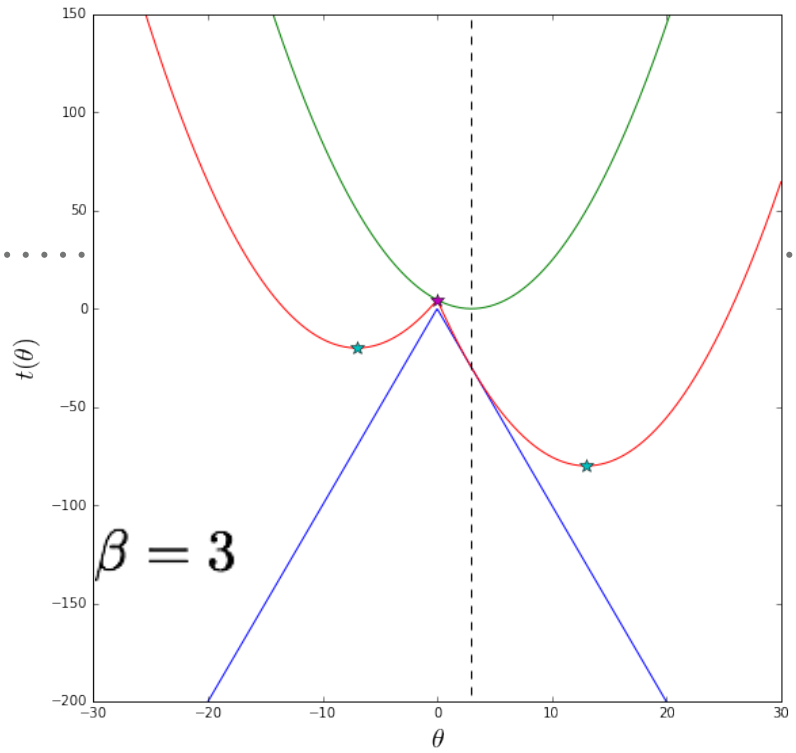
EXAMPLE OF TIME DELAY SURFACE

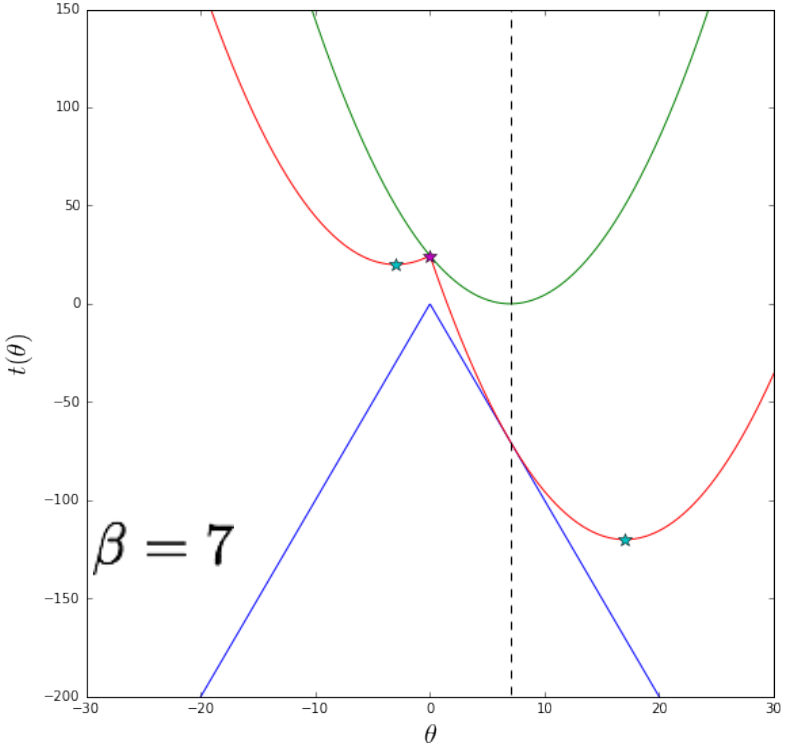
Let's change potential:

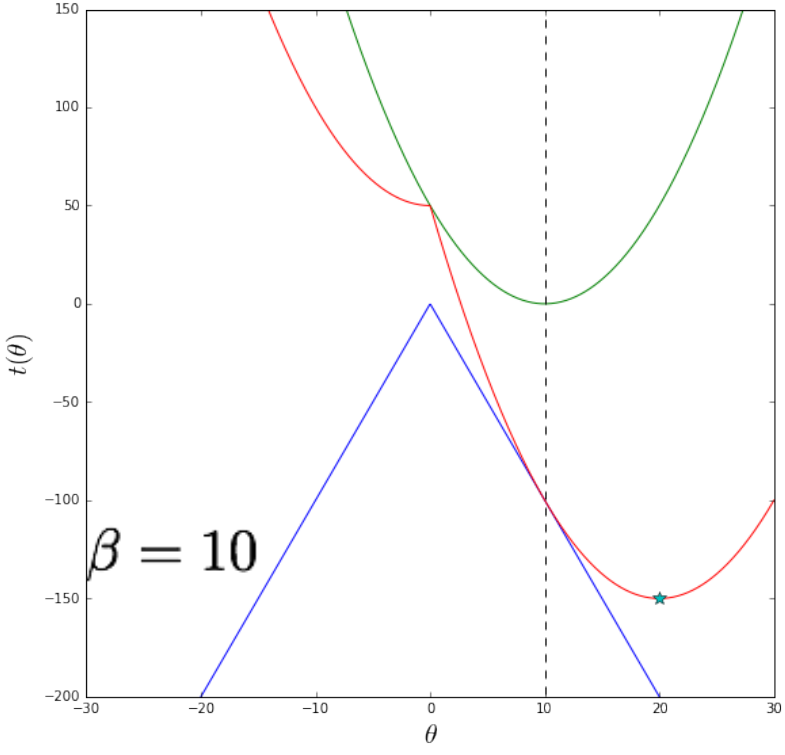
$$\hat{\Psi}(\theta) = K|\theta|$$

The lens model is the same as before, but the core has been removed


$\beta = 0$
Gravitational
Geometrical
Total
$t(\theta)$
θ


$\beta = 3$
$t(\theta)$
θ


$\beta = 7$
$t(\theta)$
θ

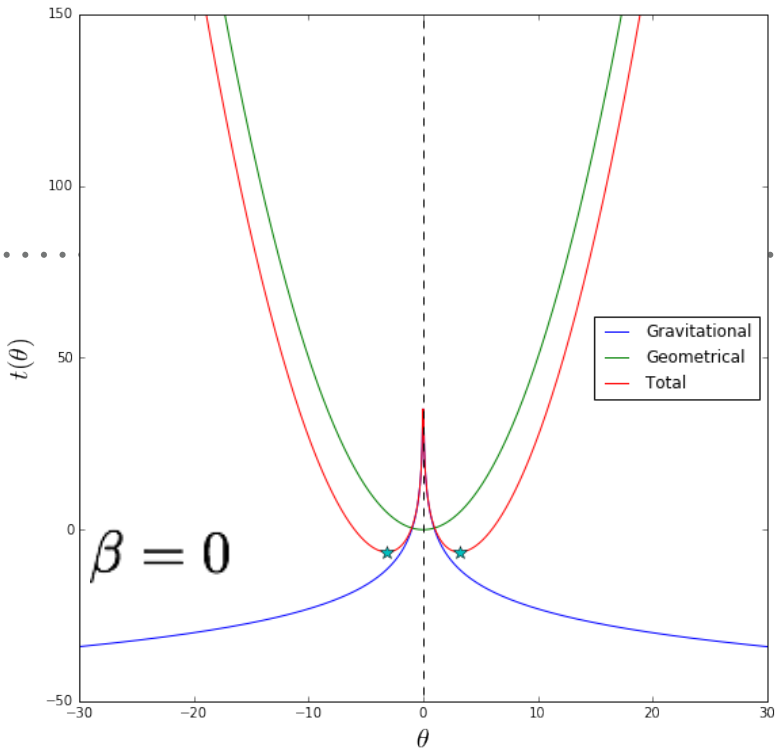

$\beta = 10$
$t(\theta)$
θ

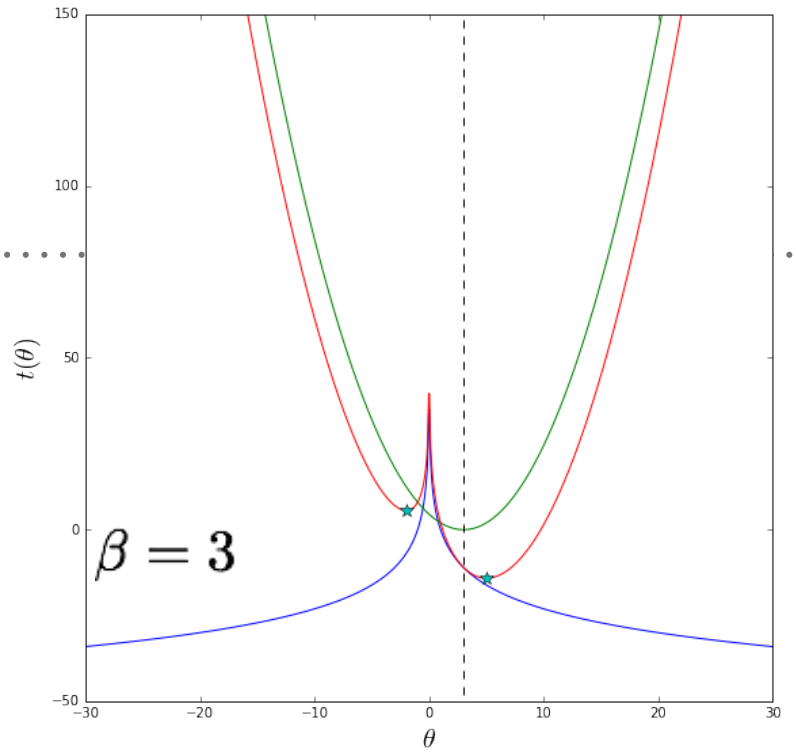
EXAMPLE OF TIME DELAY SURFACE

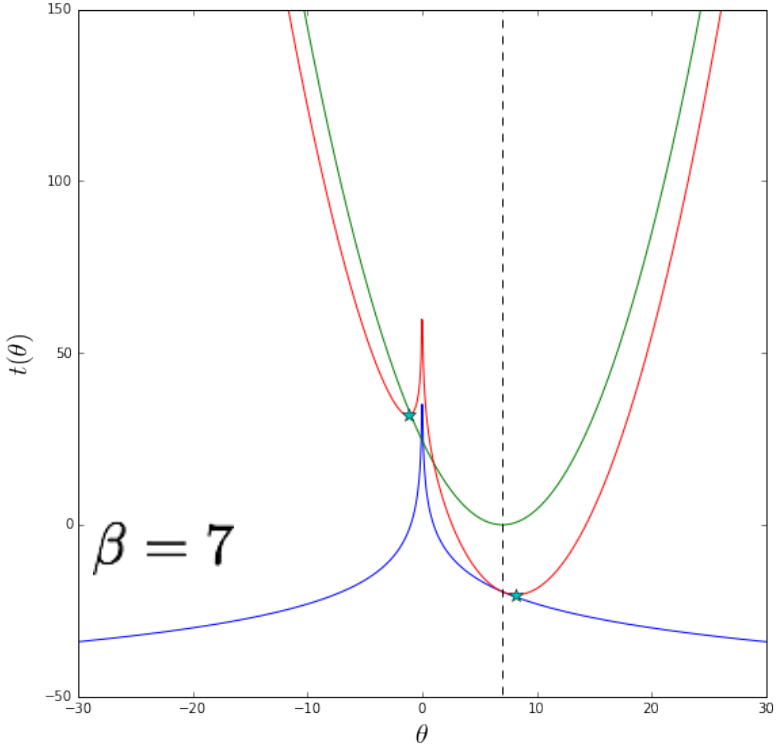
Yet another potential

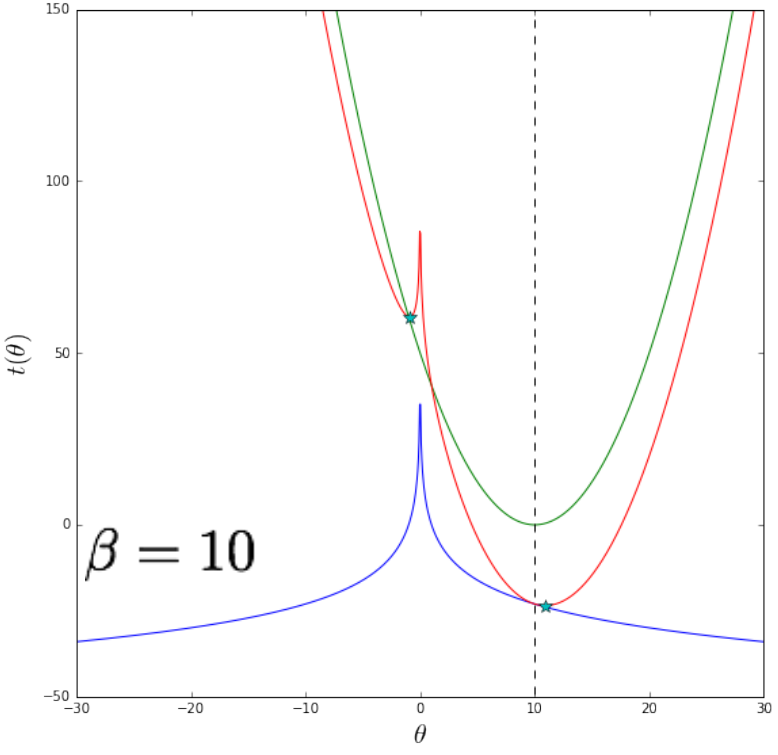
$$\hat{\Psi}(\theta) = K \ln|\theta|$$

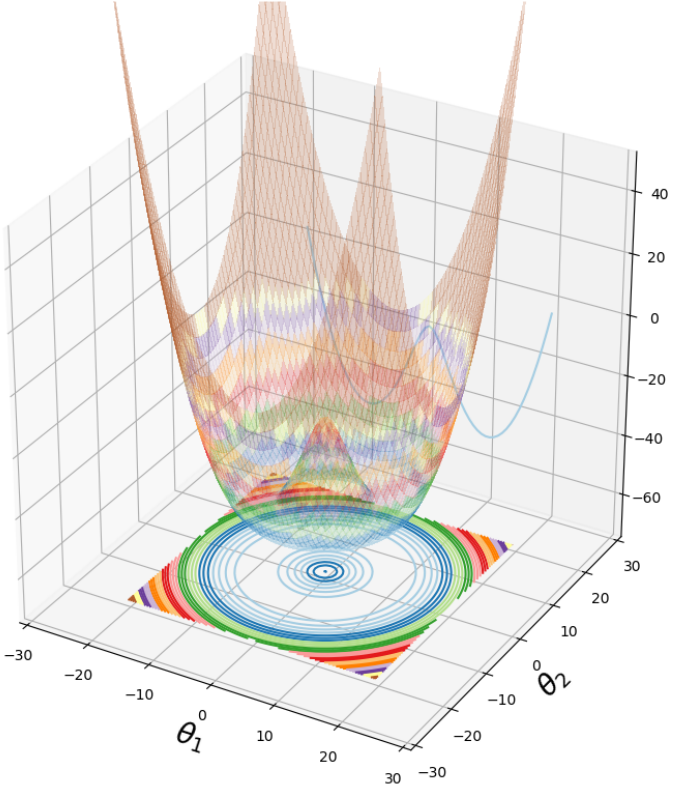
This is the lensing potential of the point mass…

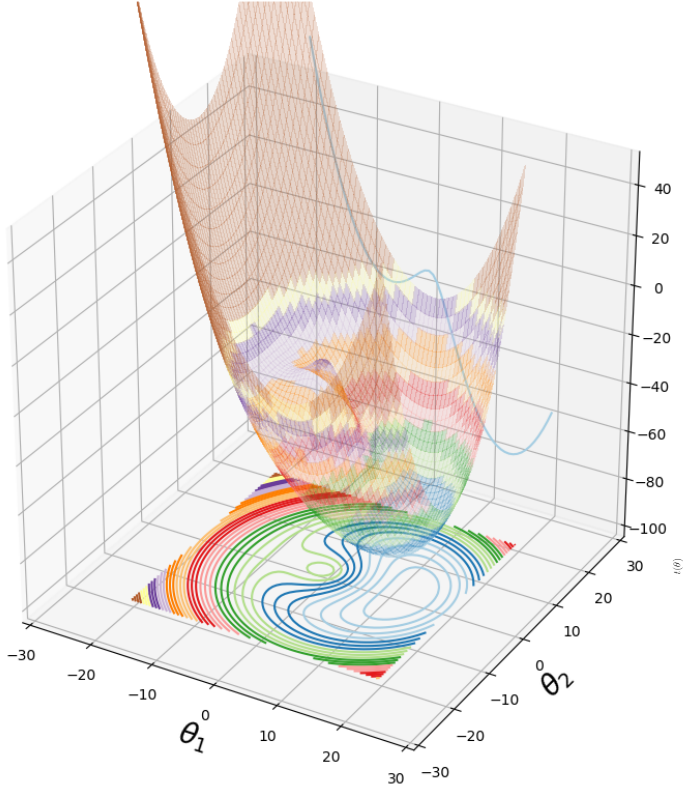

Gravitational
Geometrical
Total
$\beta = 0$
$t(\theta)$
θ


$\beta = 3$
$t(\theta)$
θ

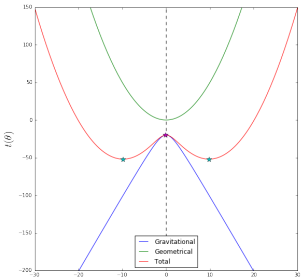

$\beta = 7$
$t(\theta)$
θ

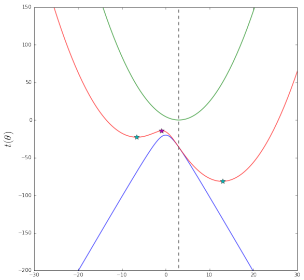

$\beta = 10$
$t(\theta)$
θ

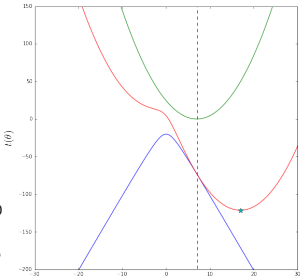


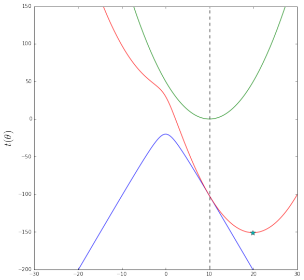


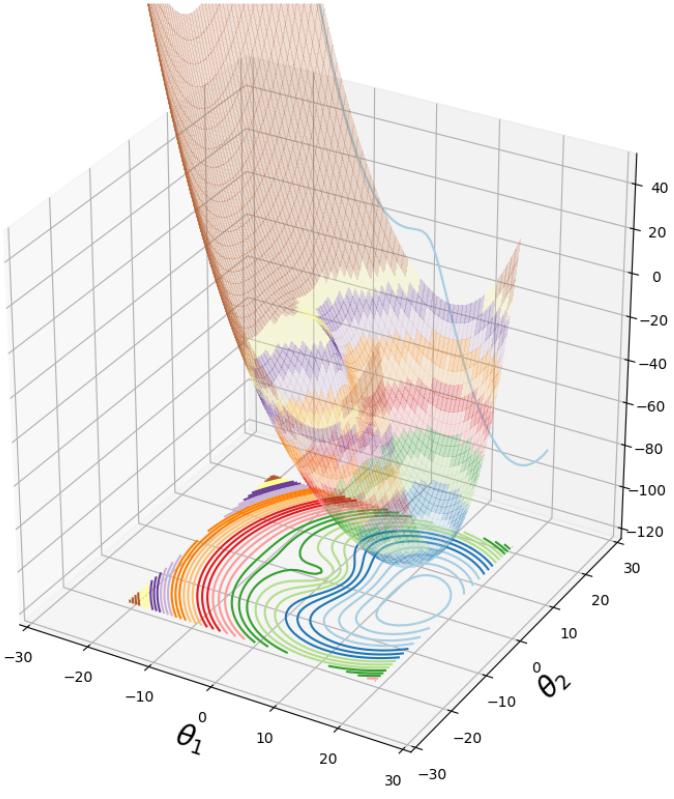
…but $t(\theta)$ is a surface!

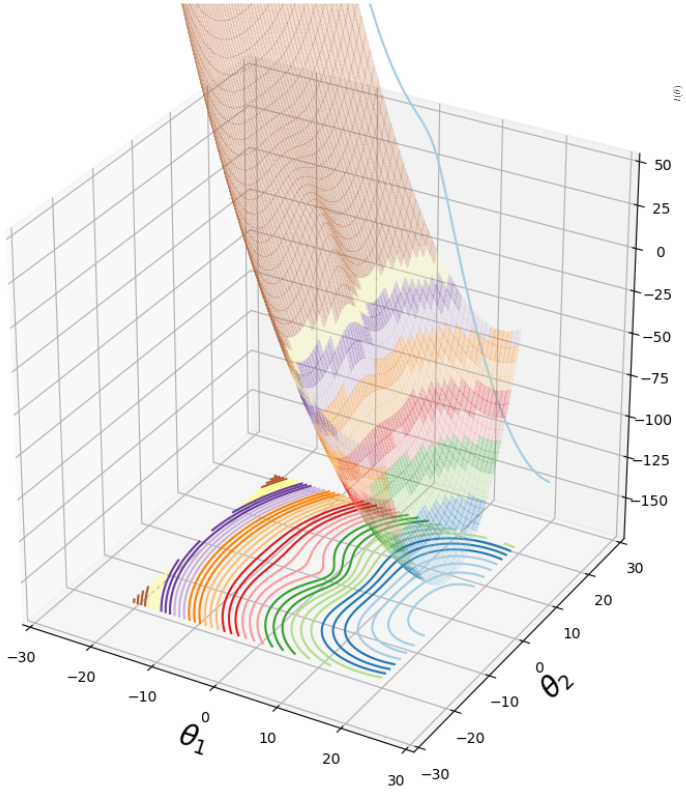


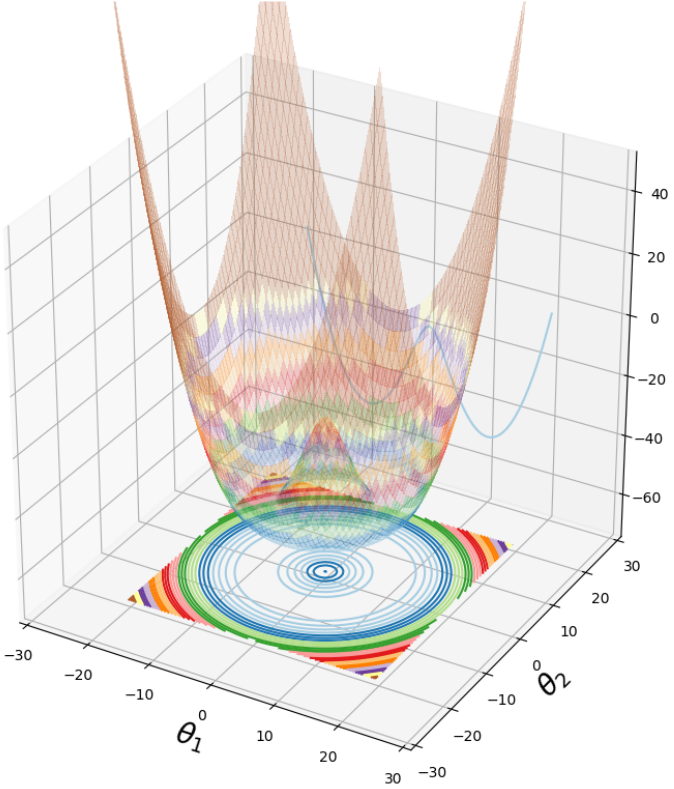


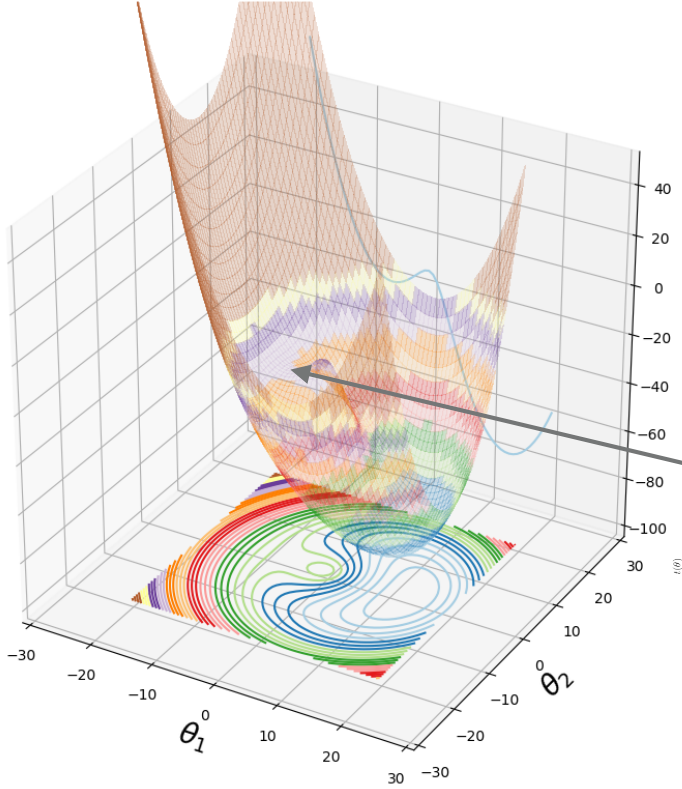




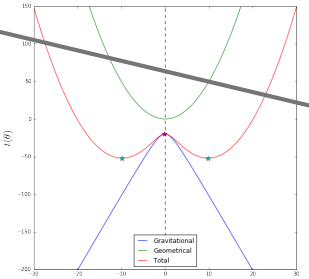


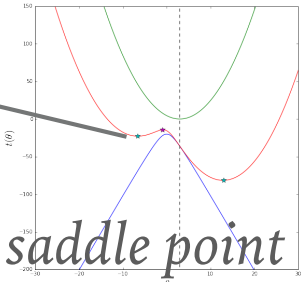




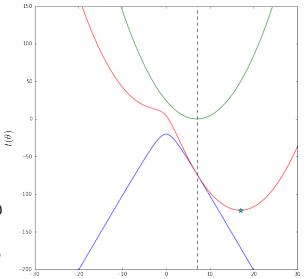


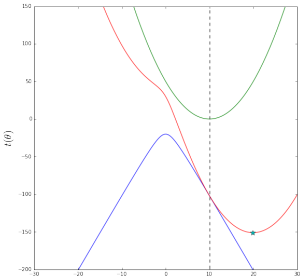
…but $t(\theta)$ is a surface!

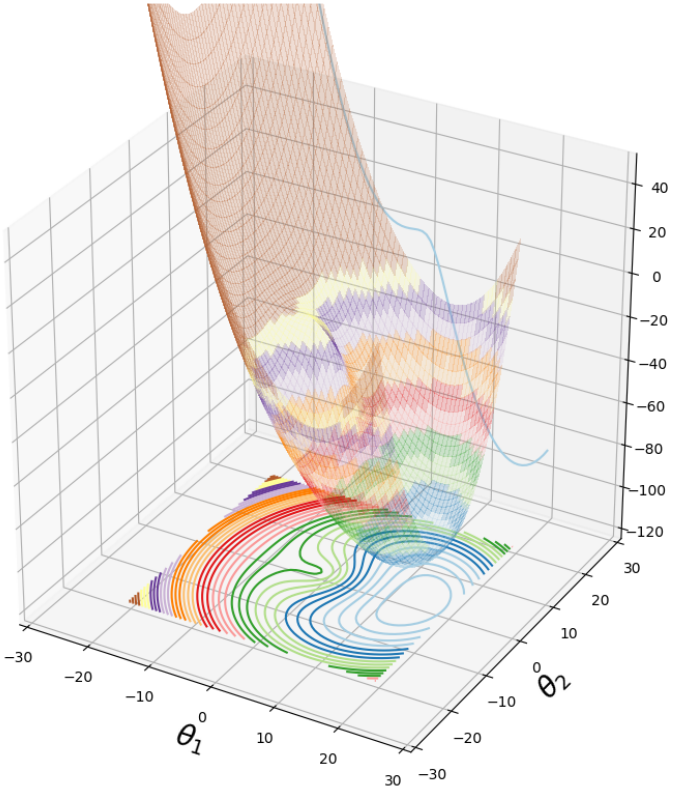


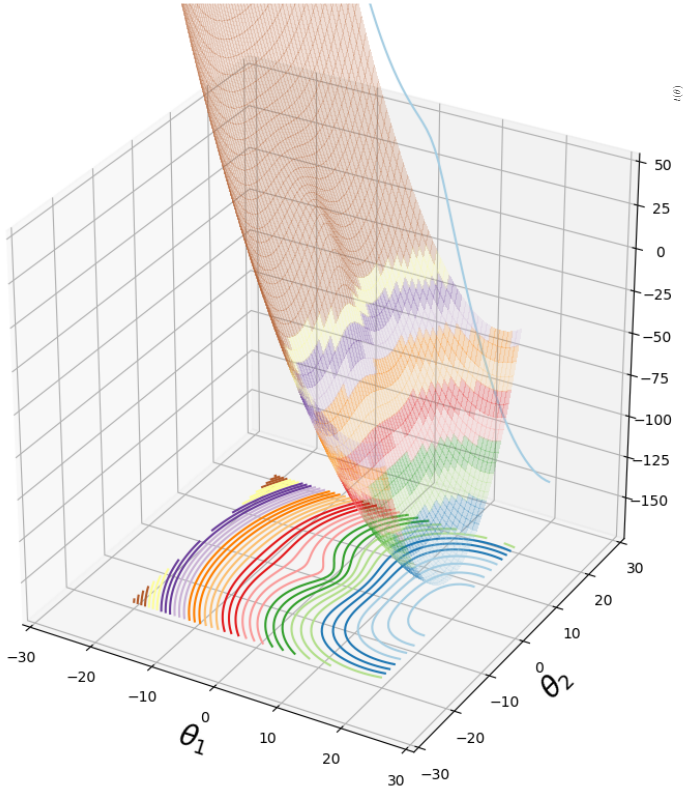


saddle point







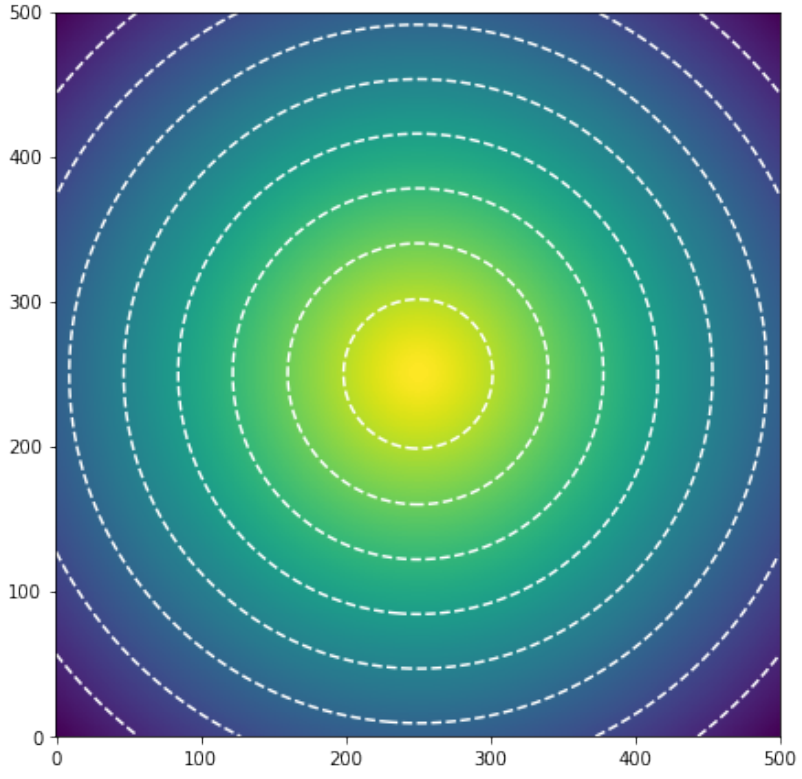


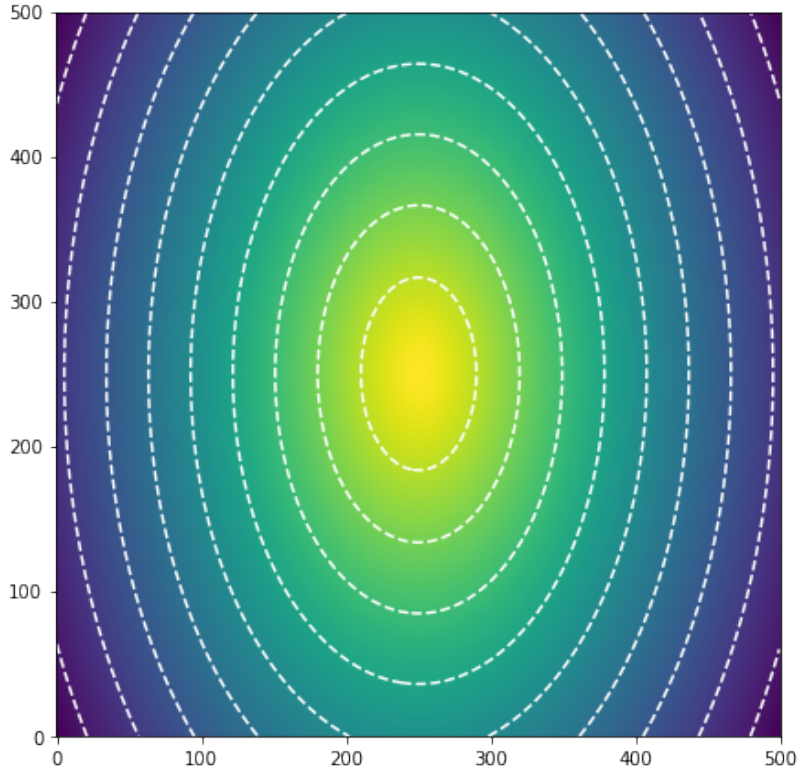
INTRODUCING ELLIPTICITY

One easy way to make a lens elliptical is by modifying the potential as follows:

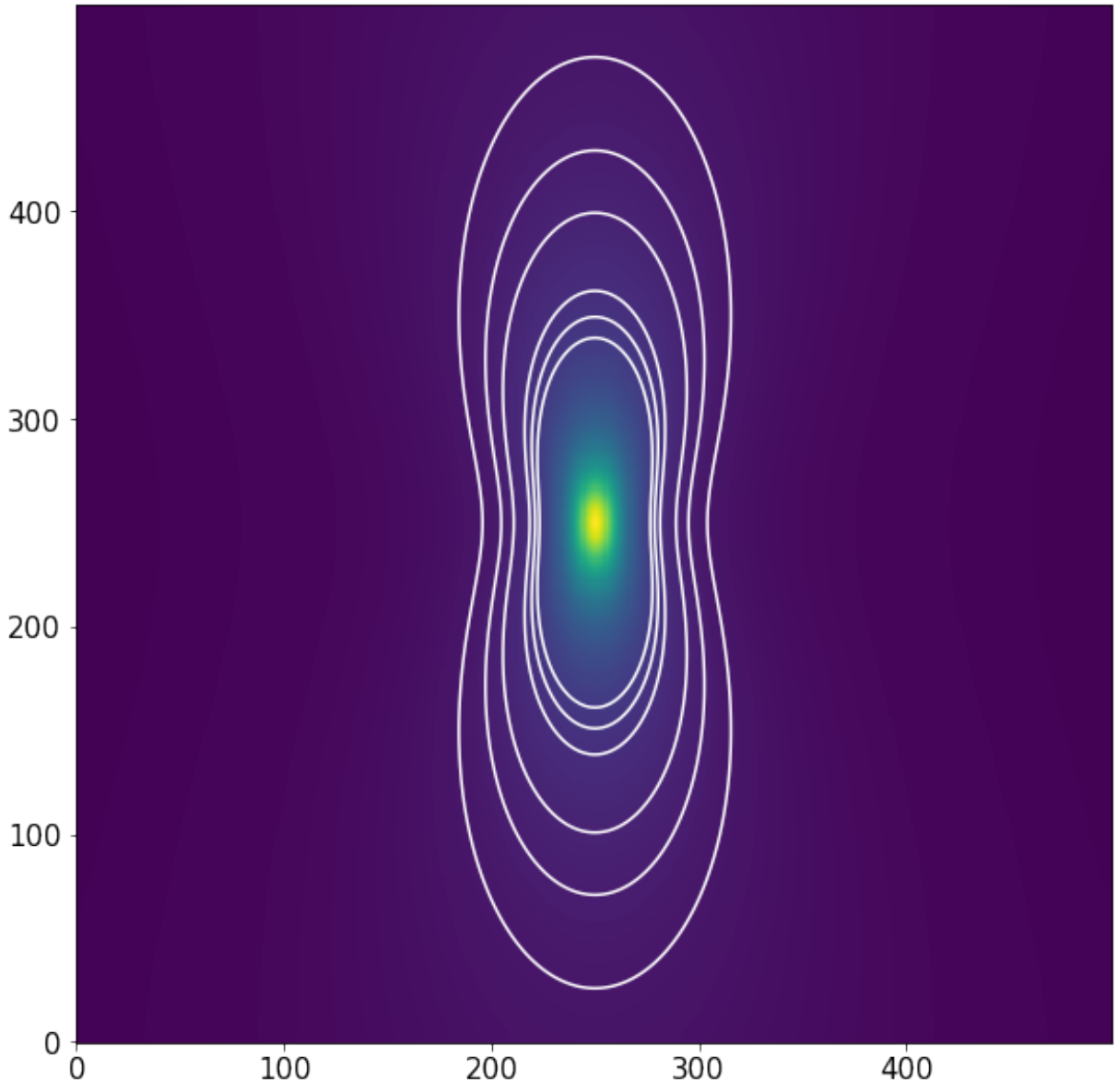
$$|\theta| \rightarrow \sqrt{\frac{\theta_1^2}{1-\epsilon} + \theta_2^2(1-\epsilon)}$$

This makes the potential constant over ellipses rather than on circles.





CAUTION: PSEUDO ELLIPTICAL LENSES



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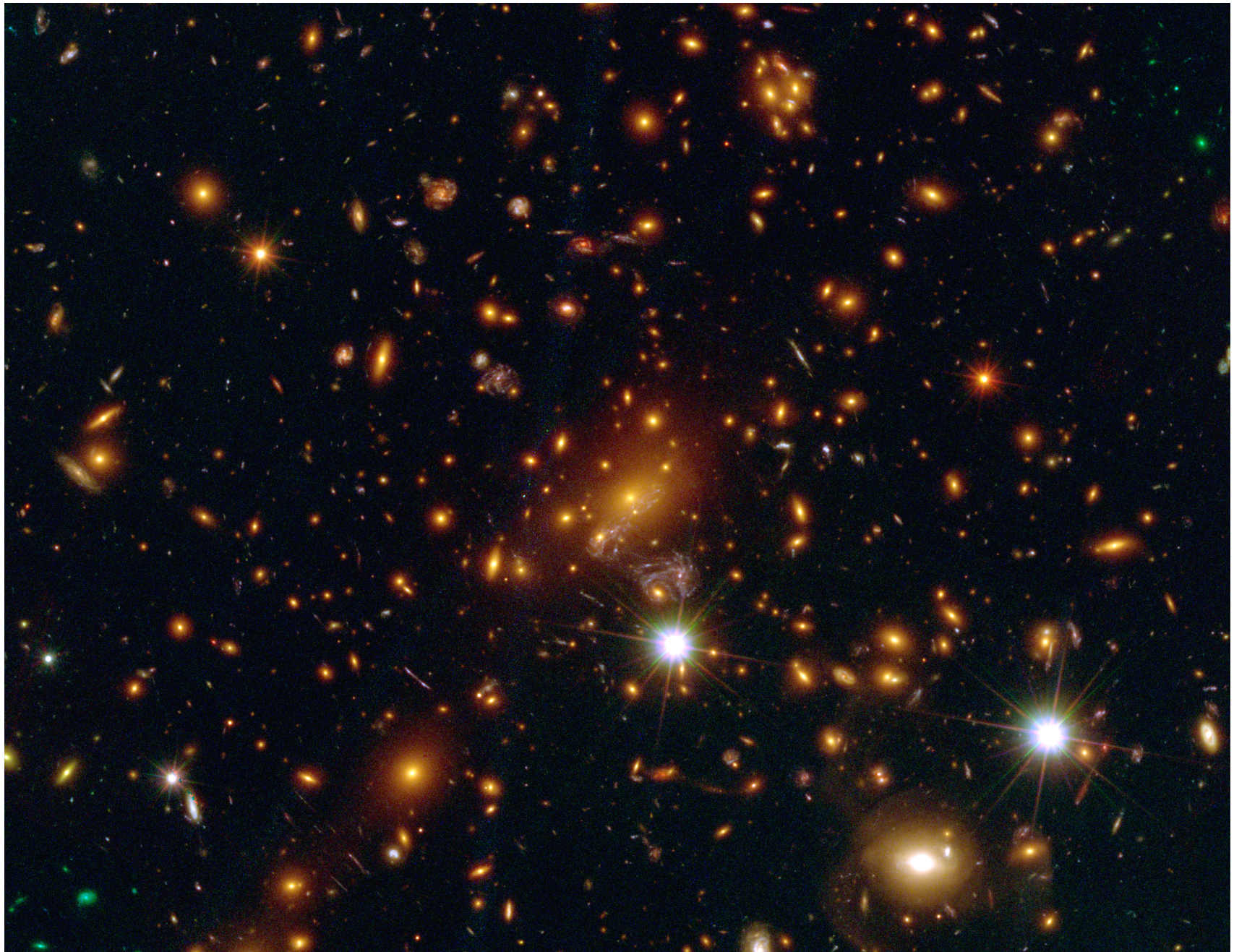

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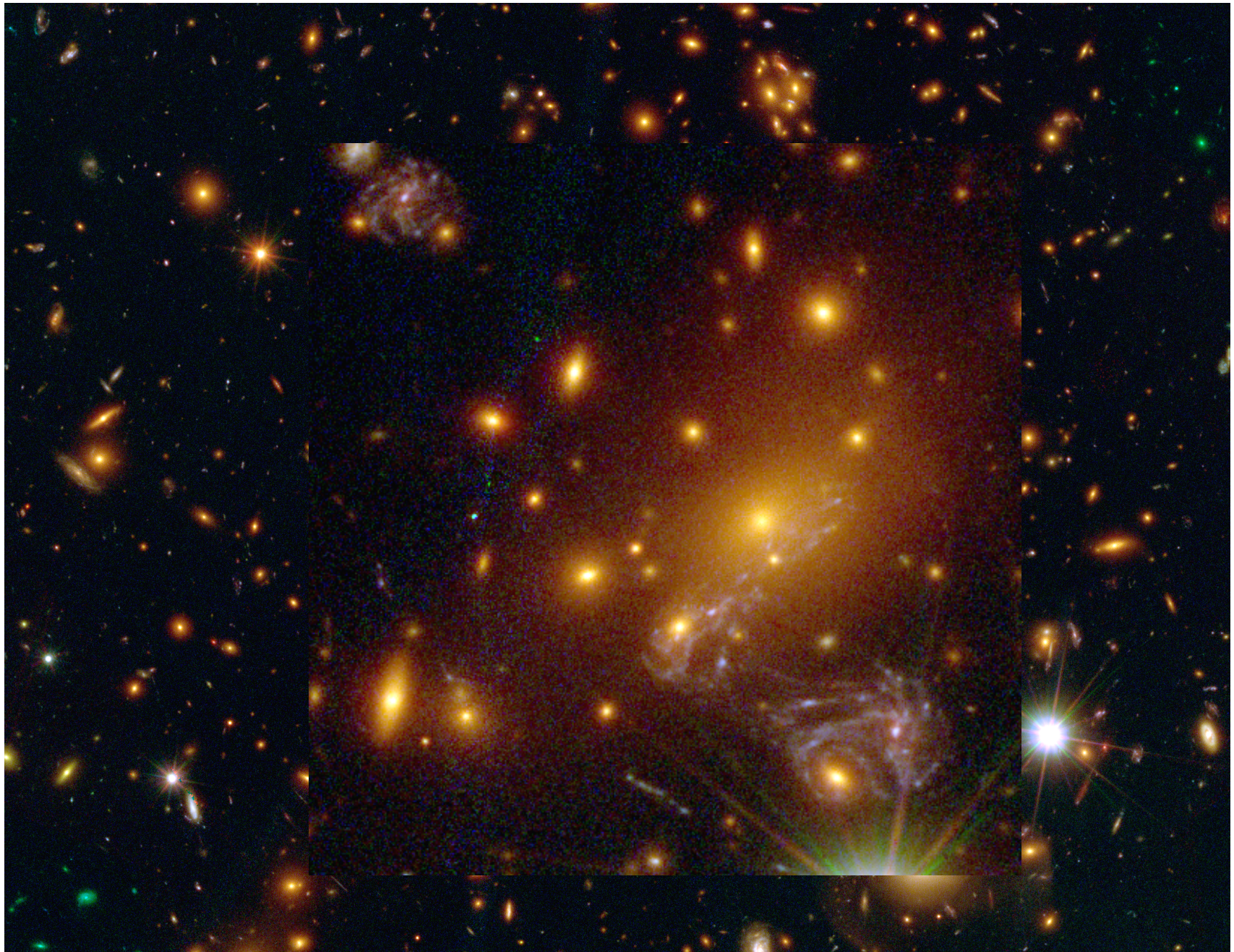

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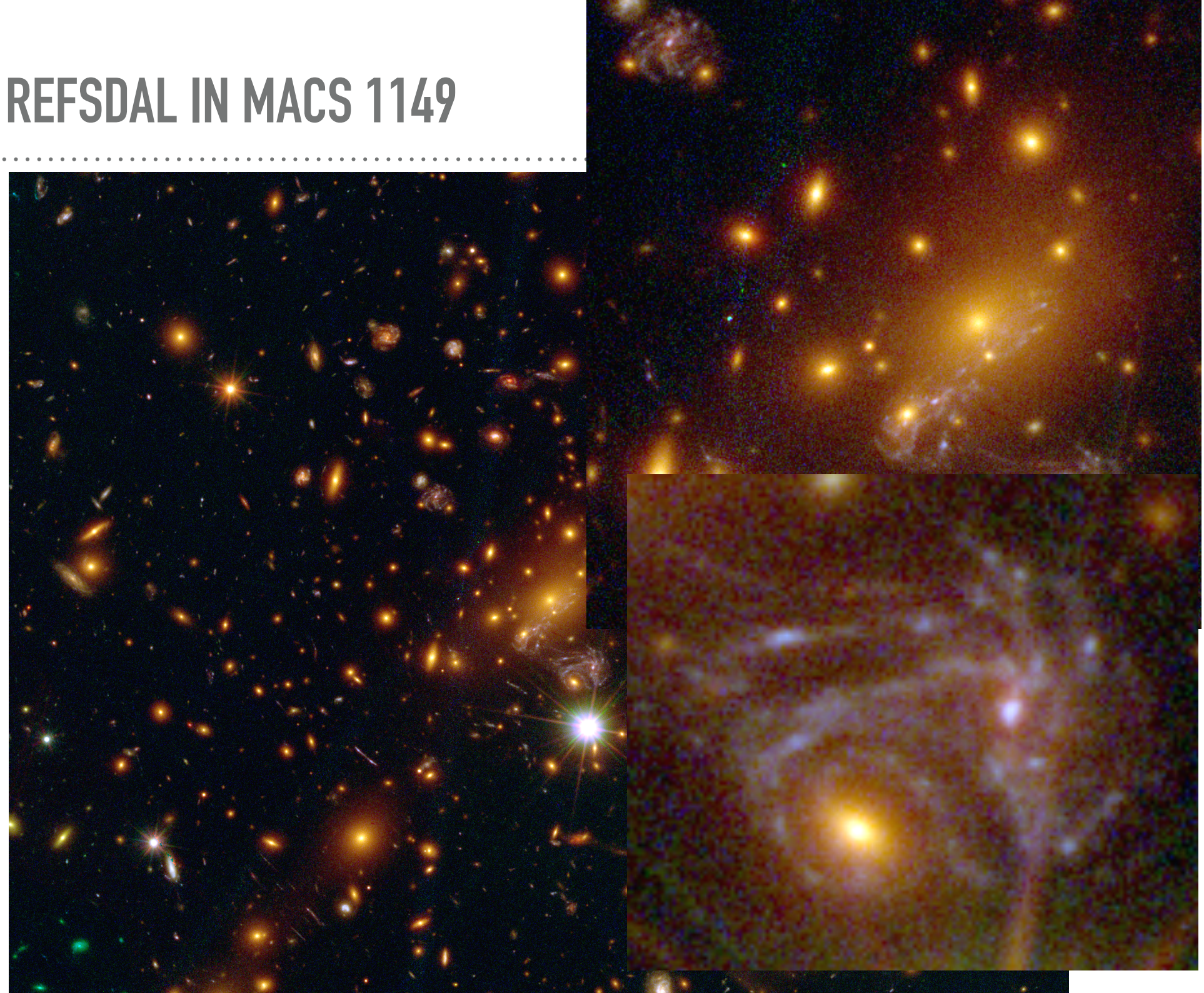

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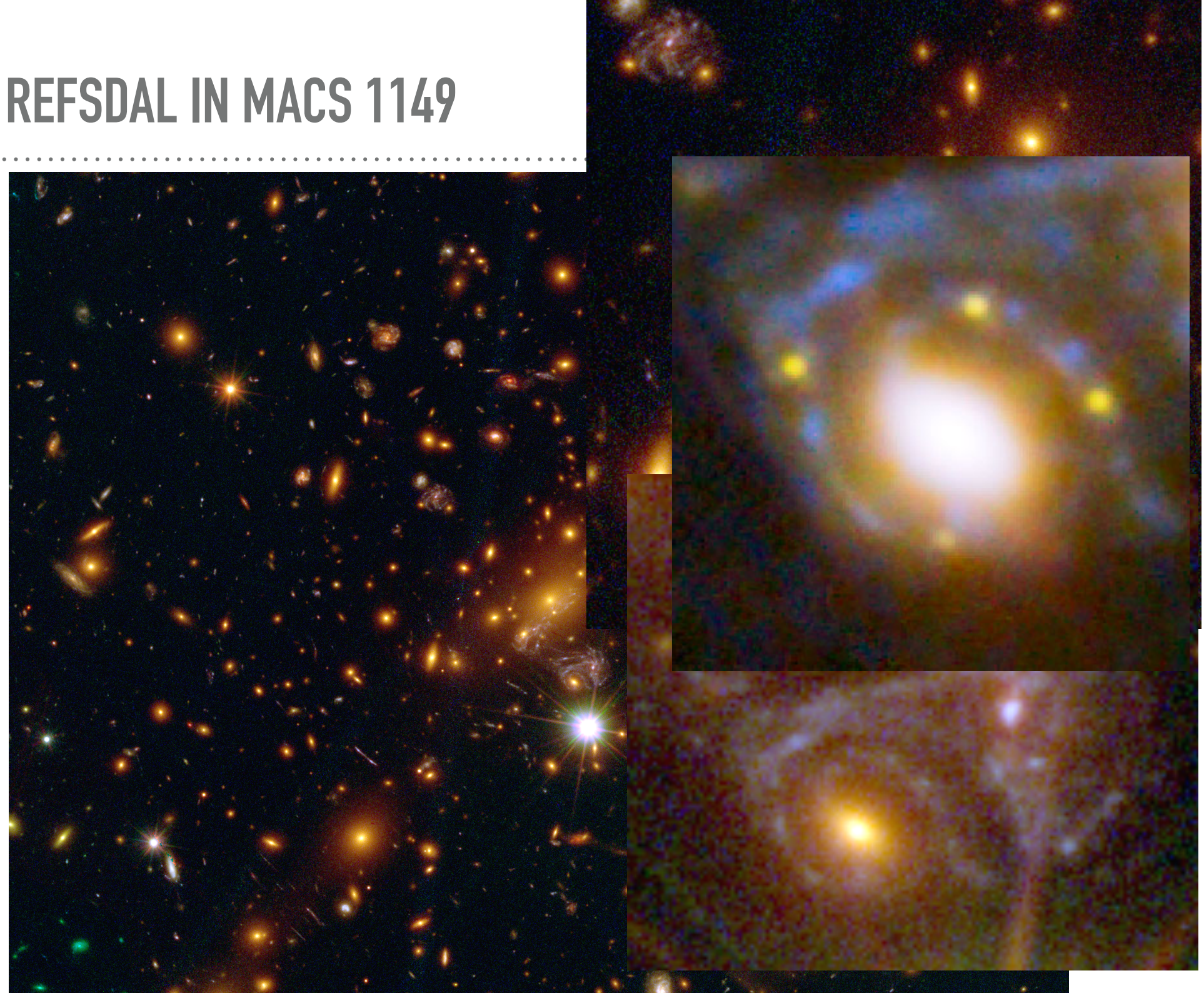

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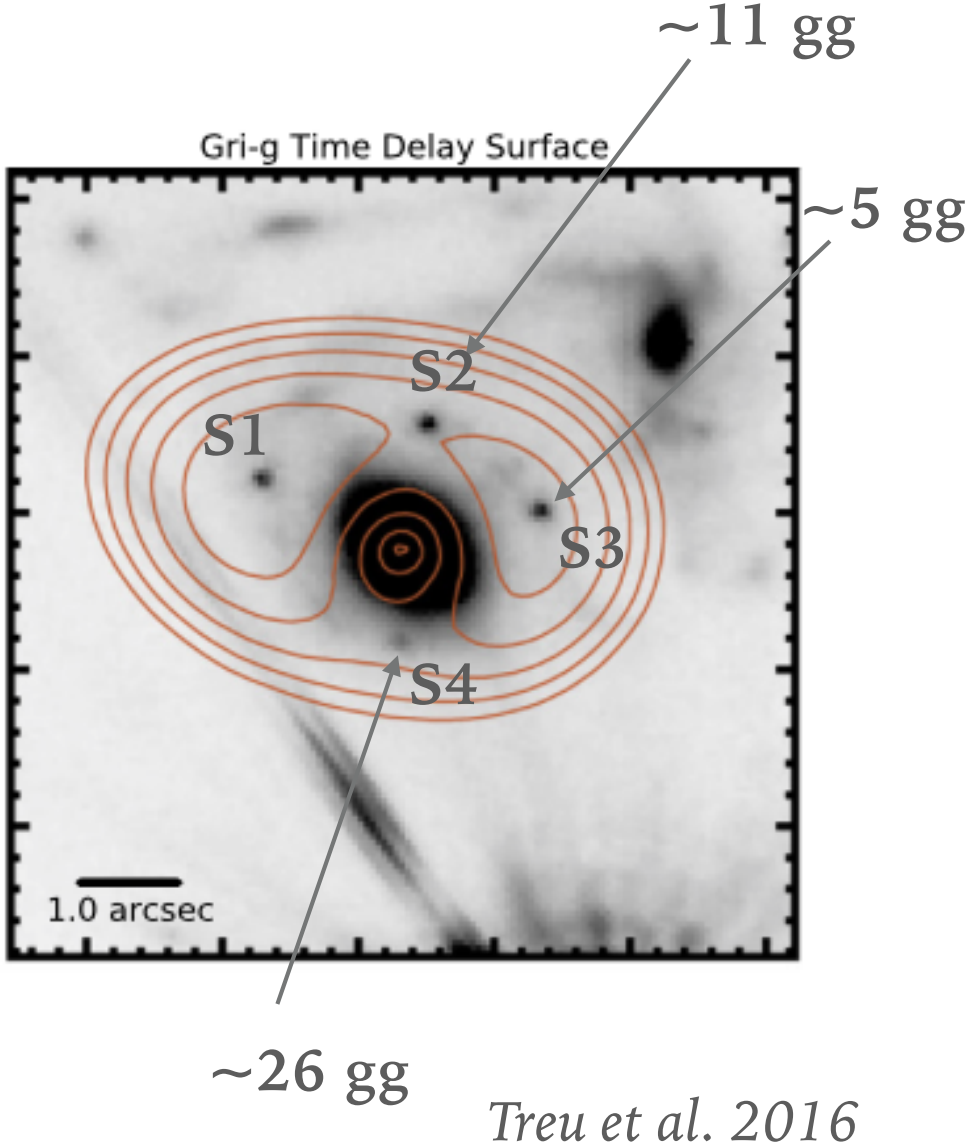
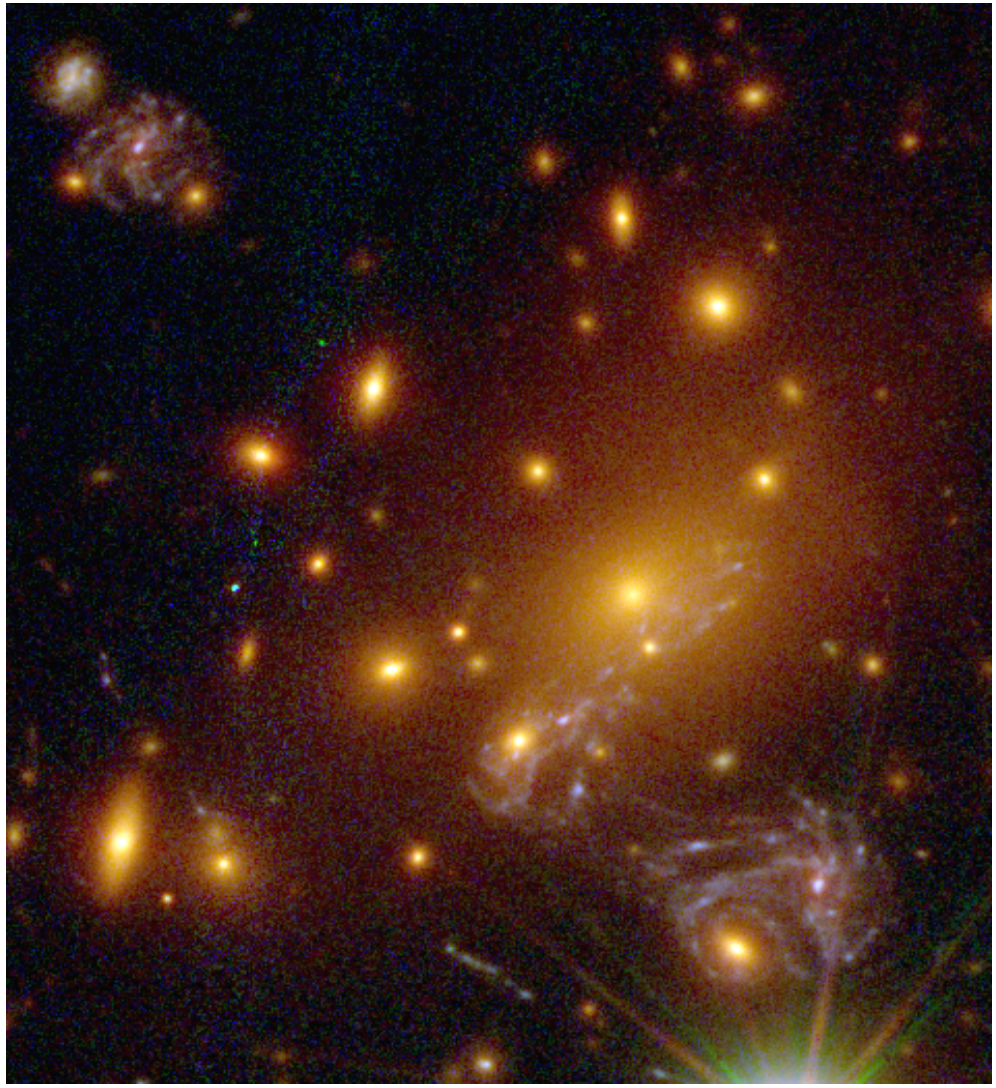
Nov. 2014 (Kelly et al.)



Treu et al. 2016

SN REFSDAL IN MACS 1149



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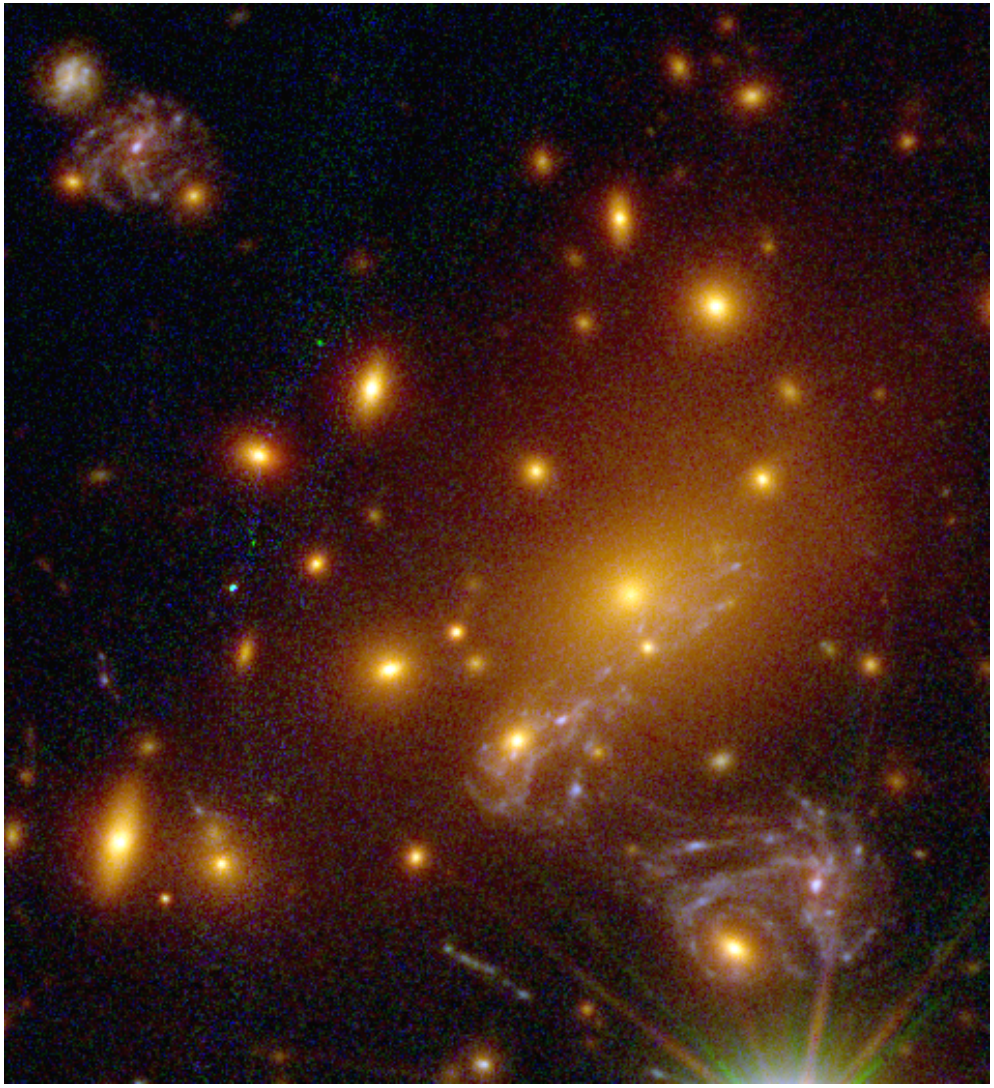

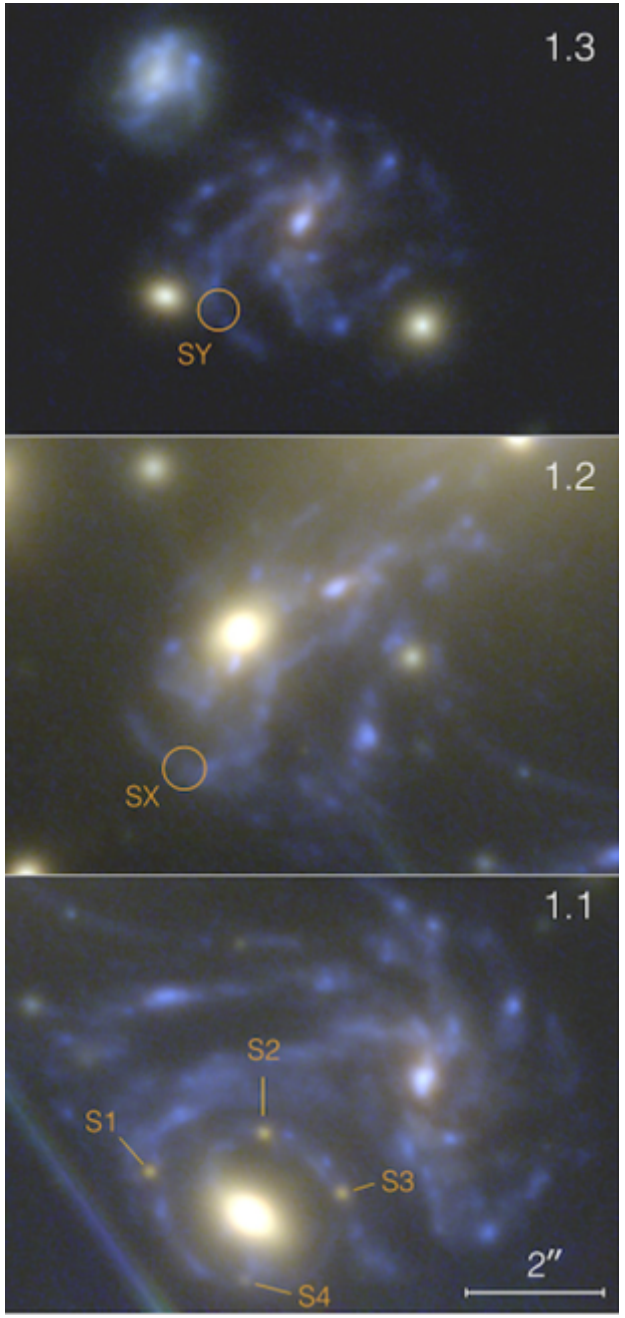

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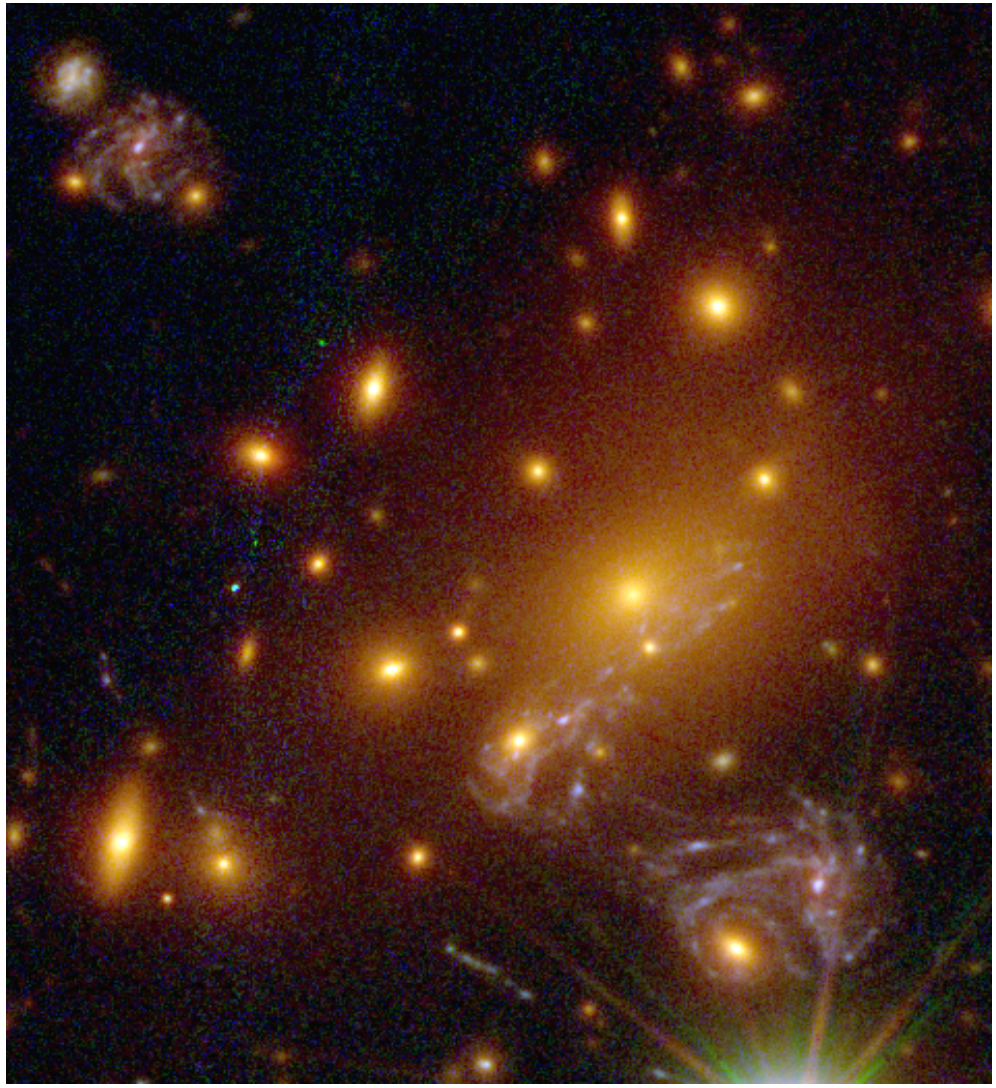

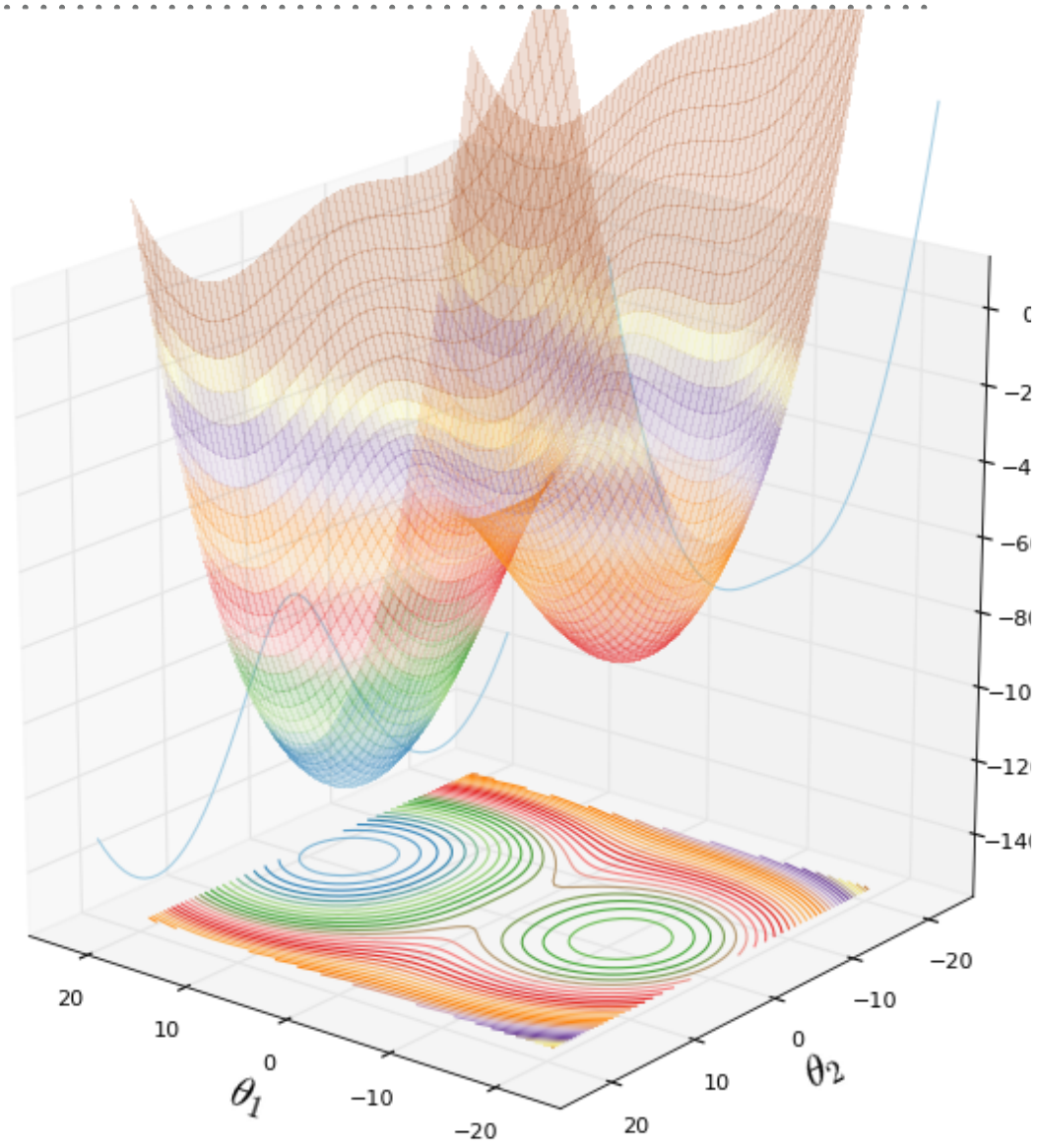

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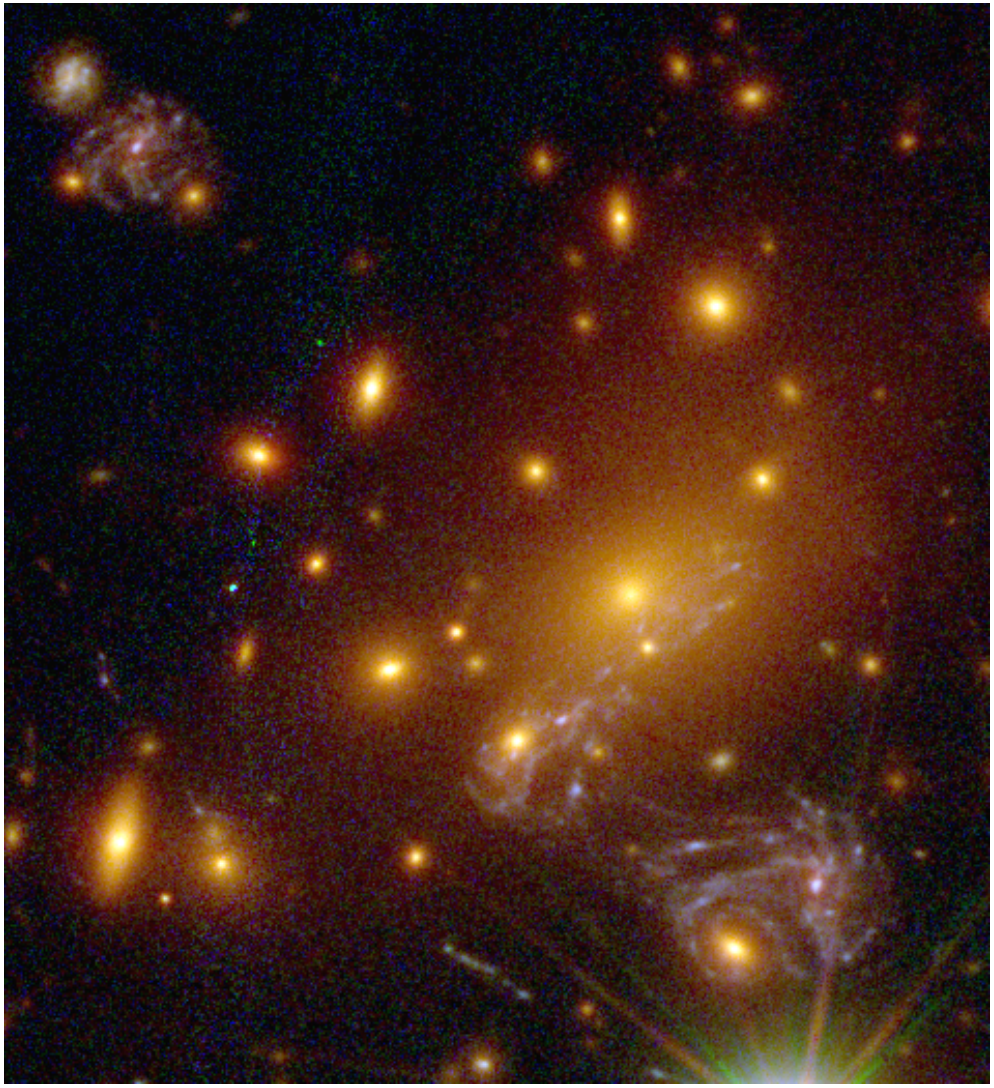

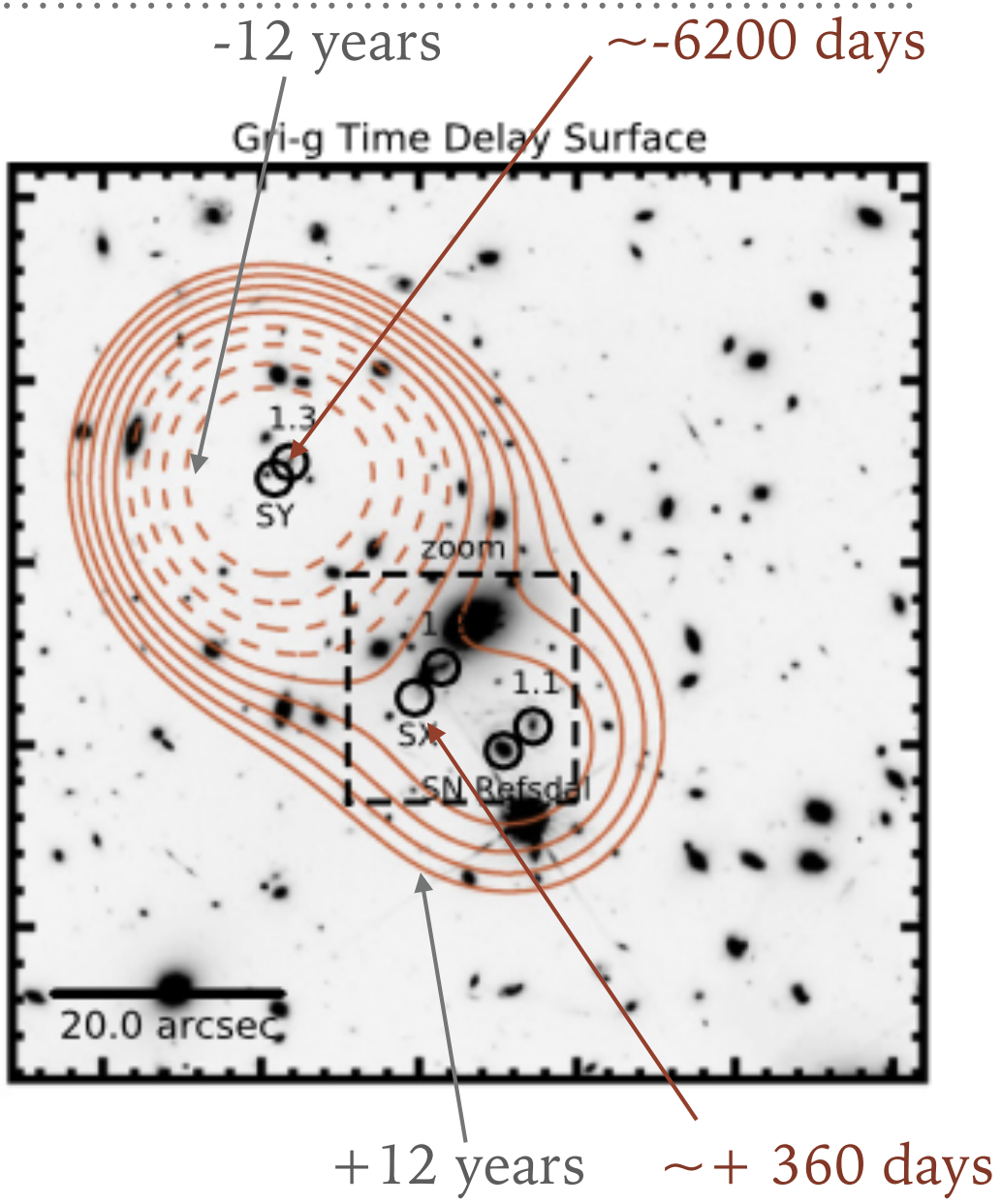

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16/12/2016…

Time delay

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