GRAVITATIONAL LENSING

3 – THE LENSING EQUATION I

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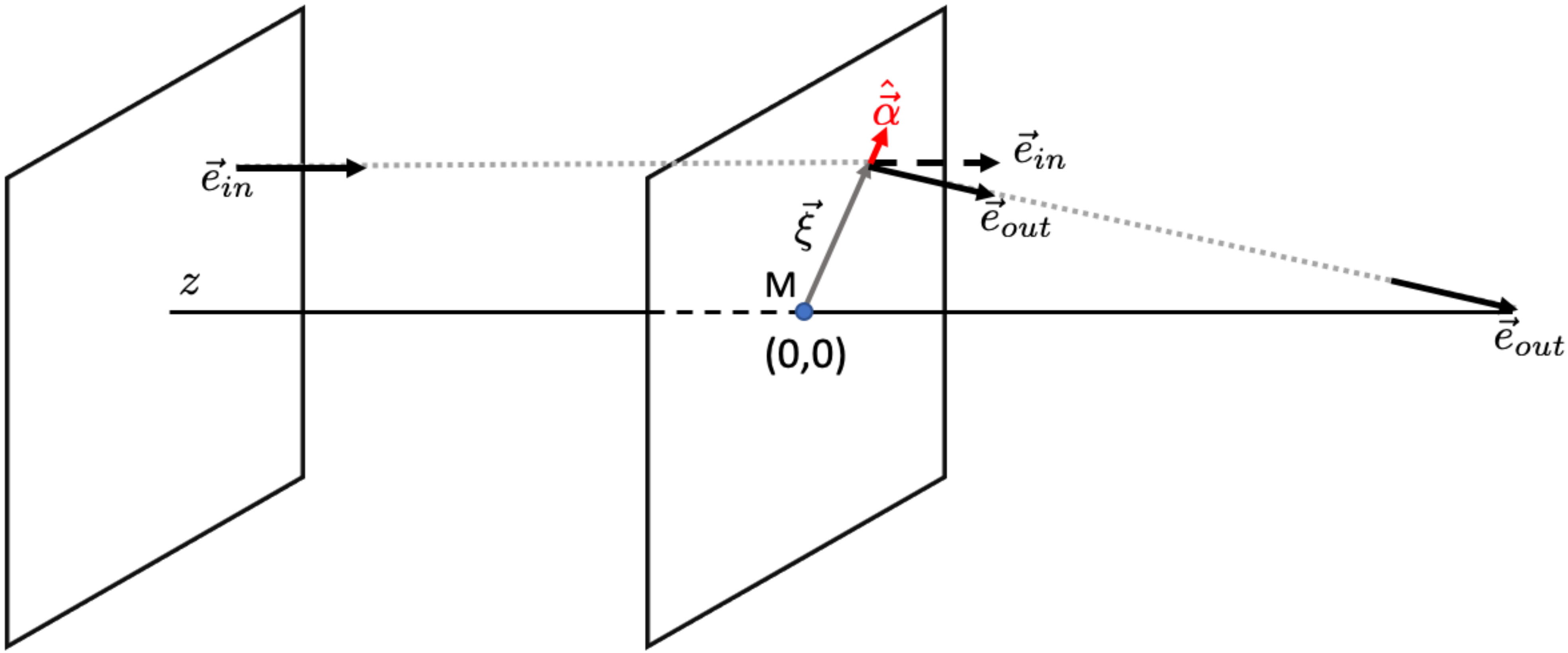
GENERALISATION OF THE DEFLECTION ANGLE FORMULA

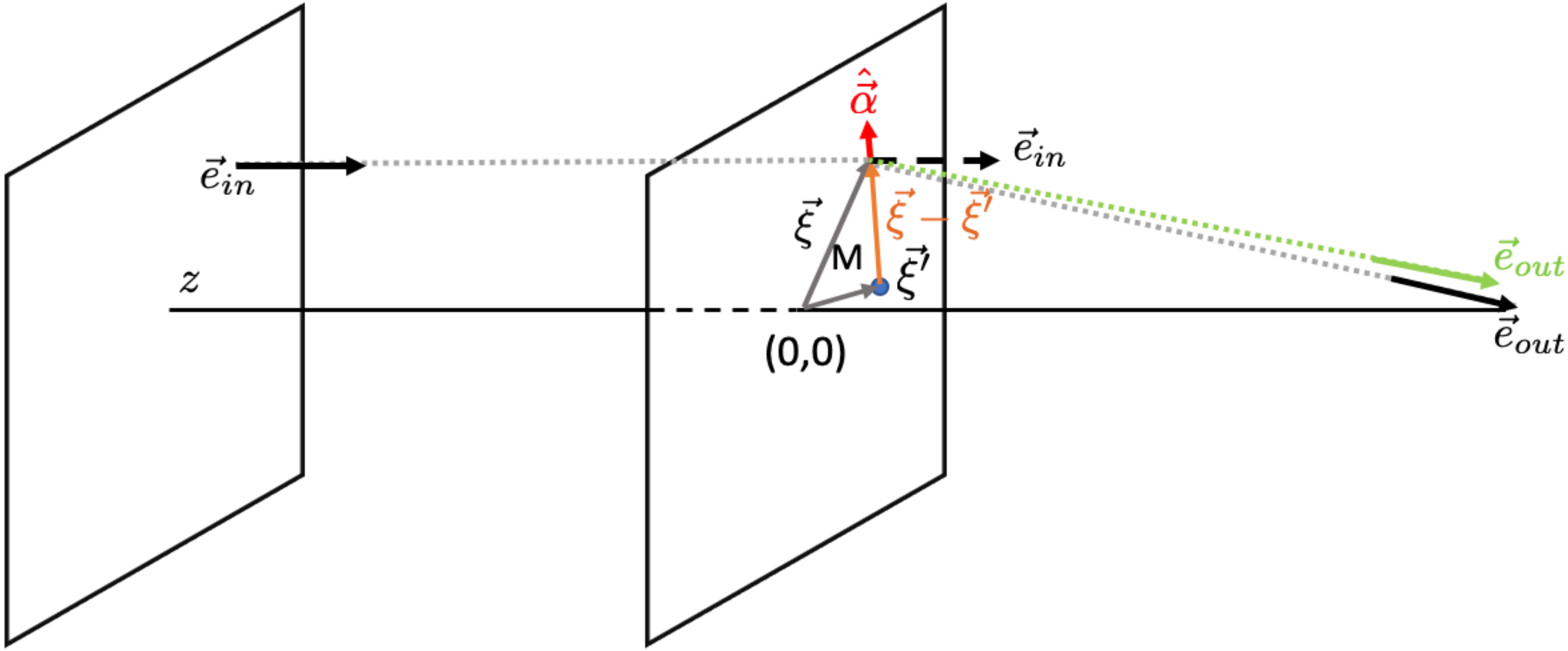
$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4GM}{c^2\xi}\vec{e}_{\xi} = \frac{4GM}{c^2}\frac{\vec{\xi}}{|\xi|^2}$$



Using "Thin screen approximation"

GENERALISATION OF THE DEFLECTION ANGLE FORMULA

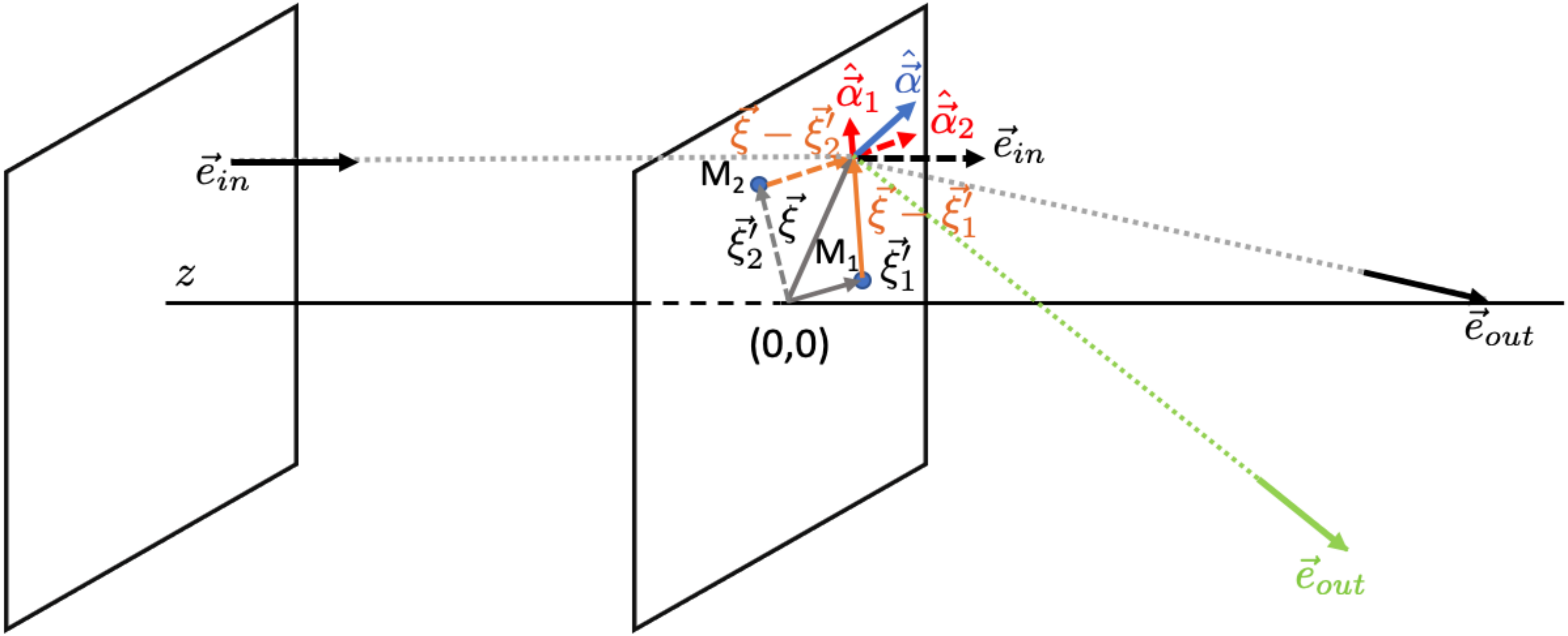
$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4GM}{c^2} \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$



Using "Thin screen approximation"

GENERALISATION OF THE DEFLECTION ANGLE FORMULA

$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \sum_i M_i \frac{\vec{\xi} - \vec{\xi^i}}{|\vec{\xi} - \vec{\xi^i}|^2}$$



Using "Thin screen approximation"

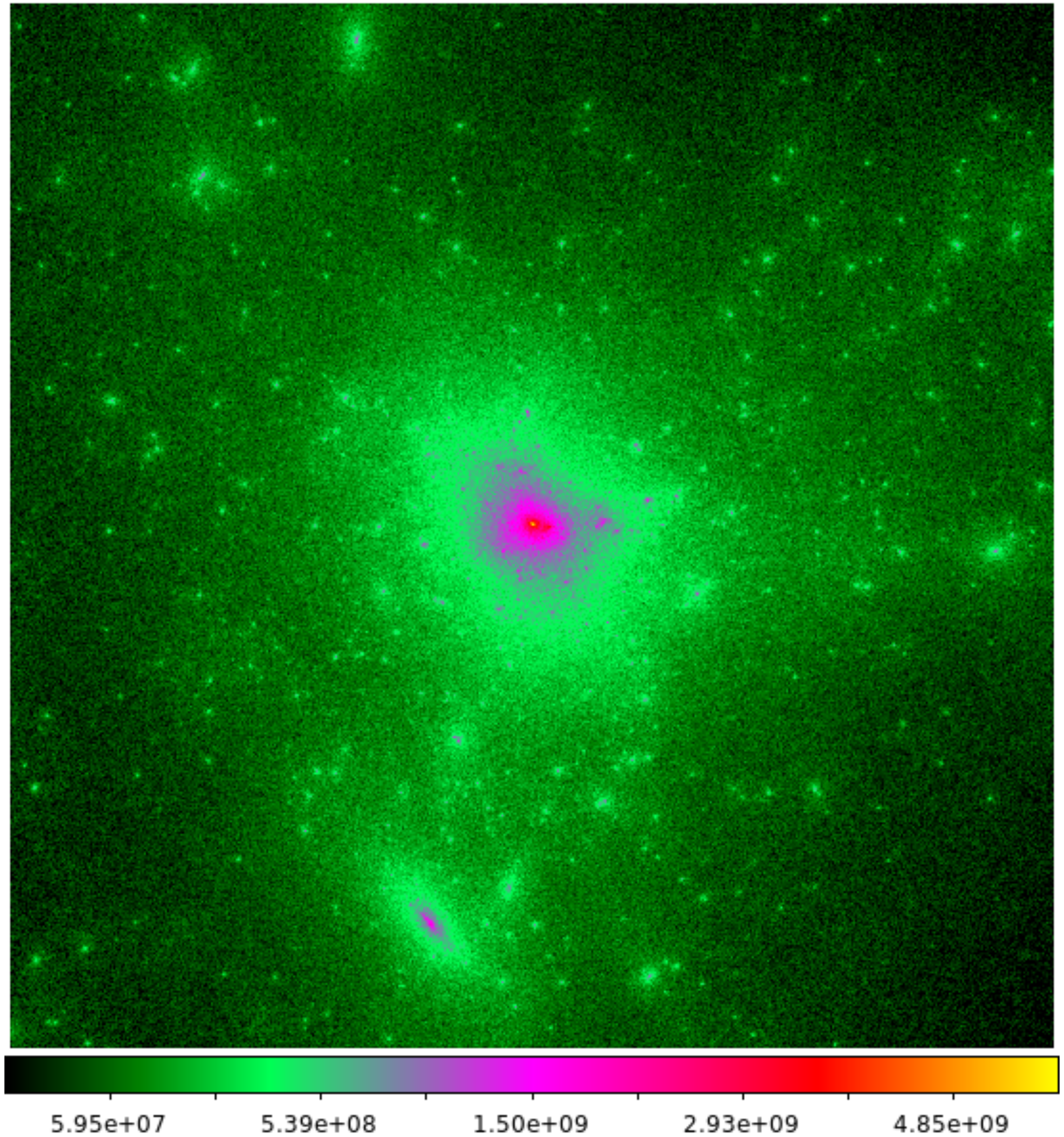
DEFLECTION BY AN EXTENDED MASS DISTRIBUTION

- This can be easily generalized to the case of a continuum distribution of mass

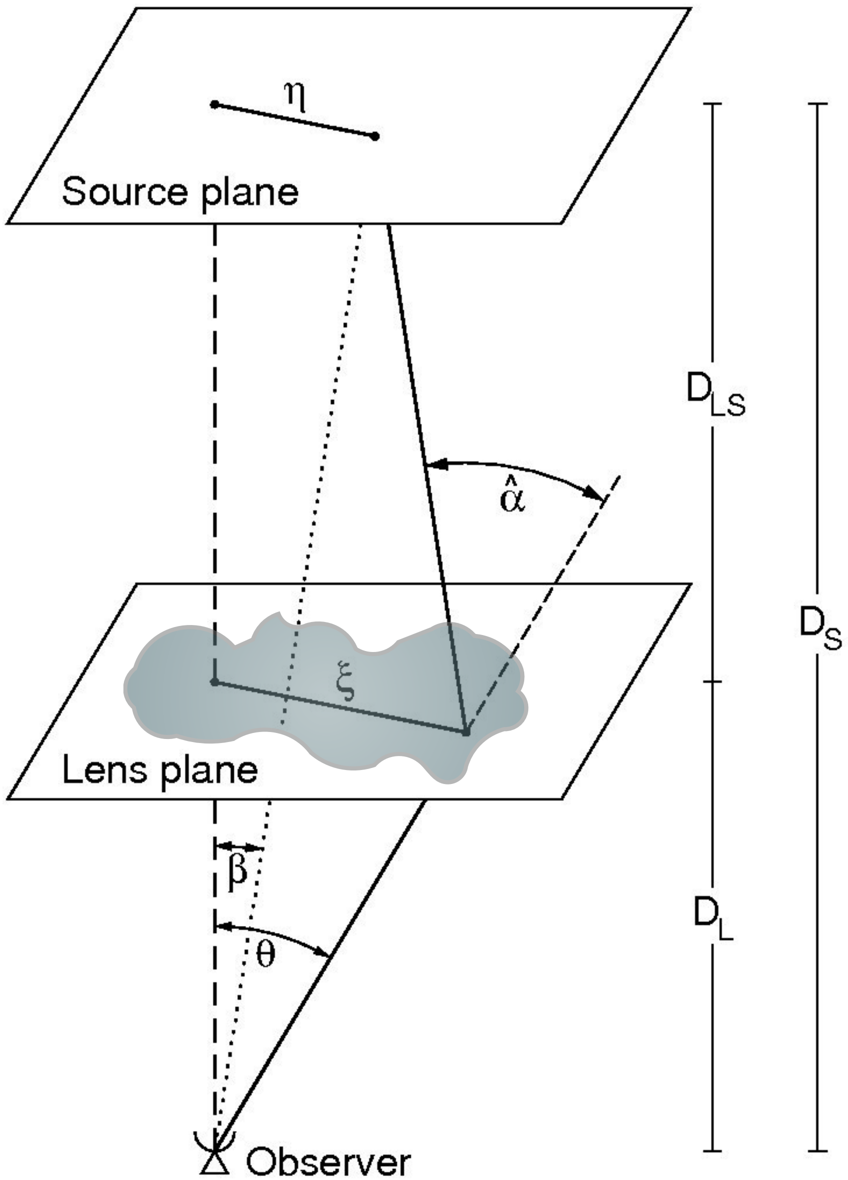
$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z) dz$$

$$d\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2} \Sigma(\vec{\xi}') d\xi'^2$$

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2} \Sigma(\vec{\xi}') d\xi'^2$$



LENS EQUATION



Remember that:

1) *positions on the lens and source planes are defined by vectors*
2) *the deflection angle itself is a vector*

$$\vec{\theta} = \frac{\vec{\xi}}{D_L} \qquad \vec{\beta} = \frac{\vec{\eta}}{D_S}$$

$$\vec{\beta} = \vec{\theta} - \frac{D_{LS}}{D_S}\hat{\vec{\alpha}}(\vec{\theta}) = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

POINT MASS LENS I

$$\hat{\alpha} = \frac{4GM}{c^2} \frac{\vec{x} - \vec{x}_o}{|\vec{x} - \vec{x}_o|^2}$$

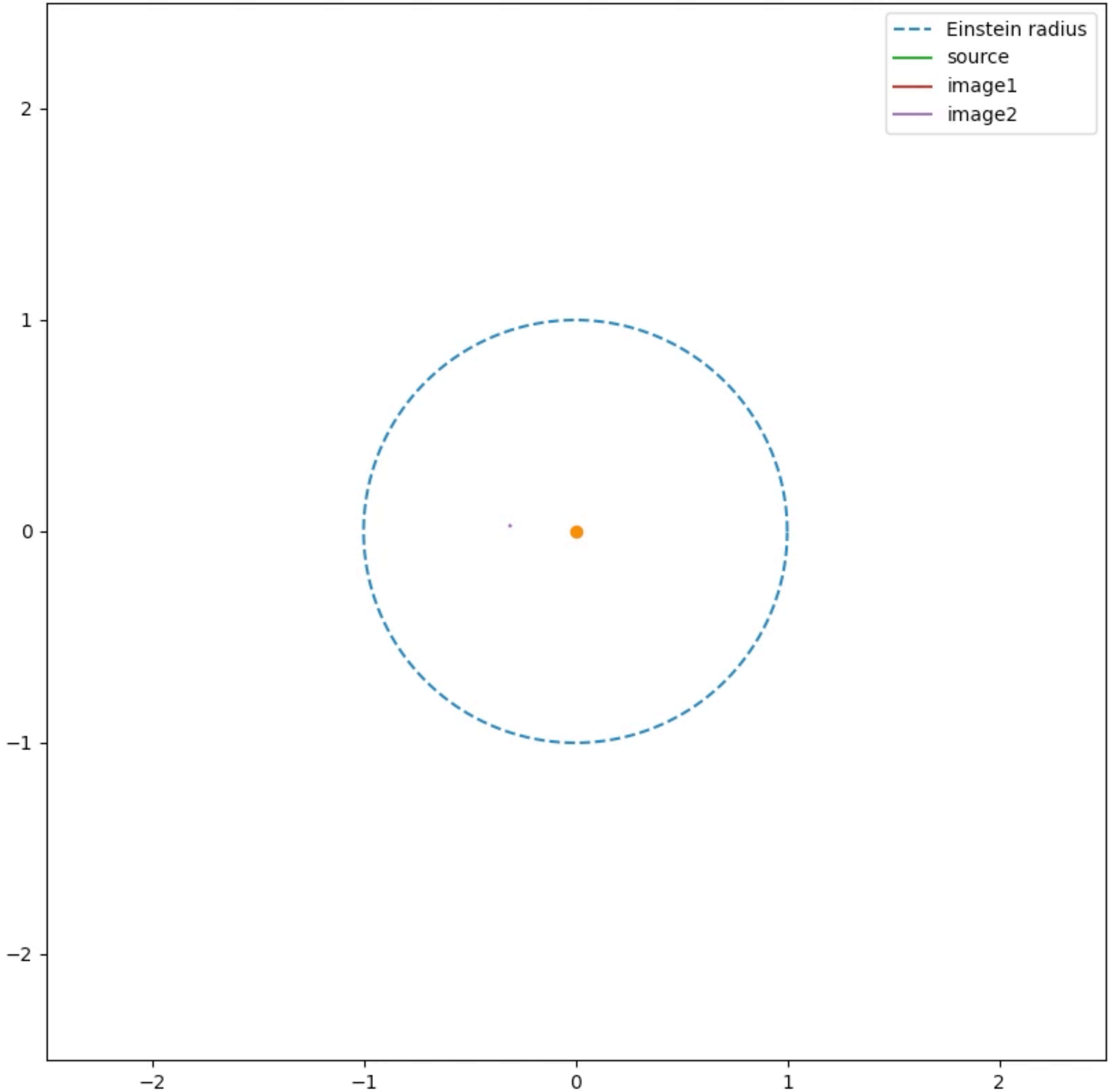
Deflection

$$\vec{\alpha}(\vec{\theta}) = \frac{D_{ls}}{D_s}\hat{\alpha} = \frac{4GM}{c^2}\left(\frac{D_{ls}}{D_s D_l}\right)\frac{\vec{\theta} - \vec{\theta}_o}{|\vec{\theta} - \vec{\theta}_o|^2}$$

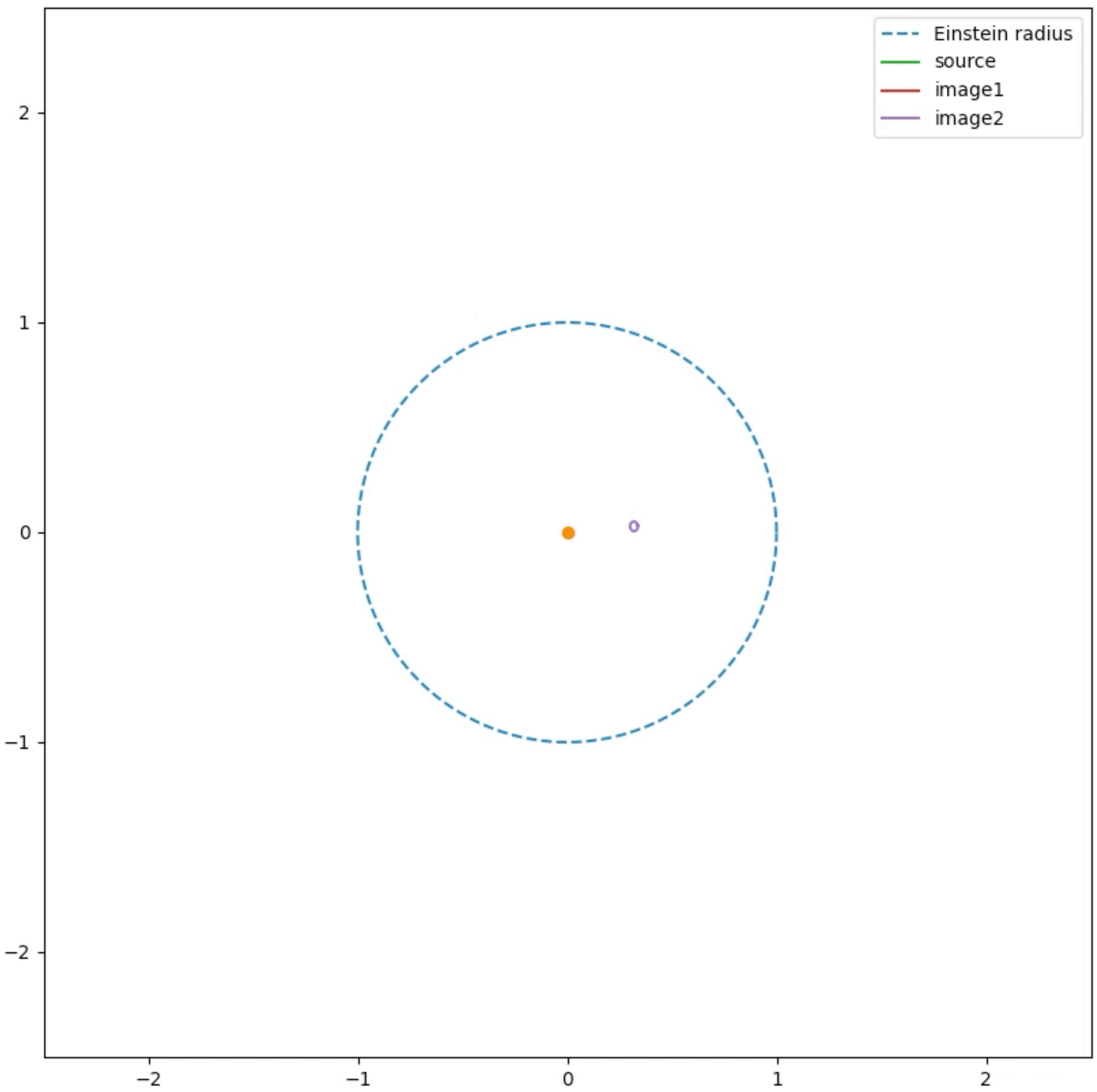
$$= \theta_E^2 \frac{\vec{\theta} - \vec{\theta}_o}{|\vec{\theta} - \vec{\theta}_o|^2}$$

Einstein ring radius $$\theta_E^2 = \frac{4GM}{c^2}\left(\frac{D_{ls}}{D_s D_l}\right)$$

Image positions $$\theta = \frac{\beta \pm \sqrt{\beta^2 + 4\theta_E}}{2}$$ β **source postion**



Einstein radius
source
image1
image2
2
1
0
−1
−2
−2
−1
0
1
2


Einstein radius
source
image1
image2
2
1
0
−1
−2
−2
−1
0
1
2