GRAVITATIONAL LENSING

7 - THE LENS MAP II

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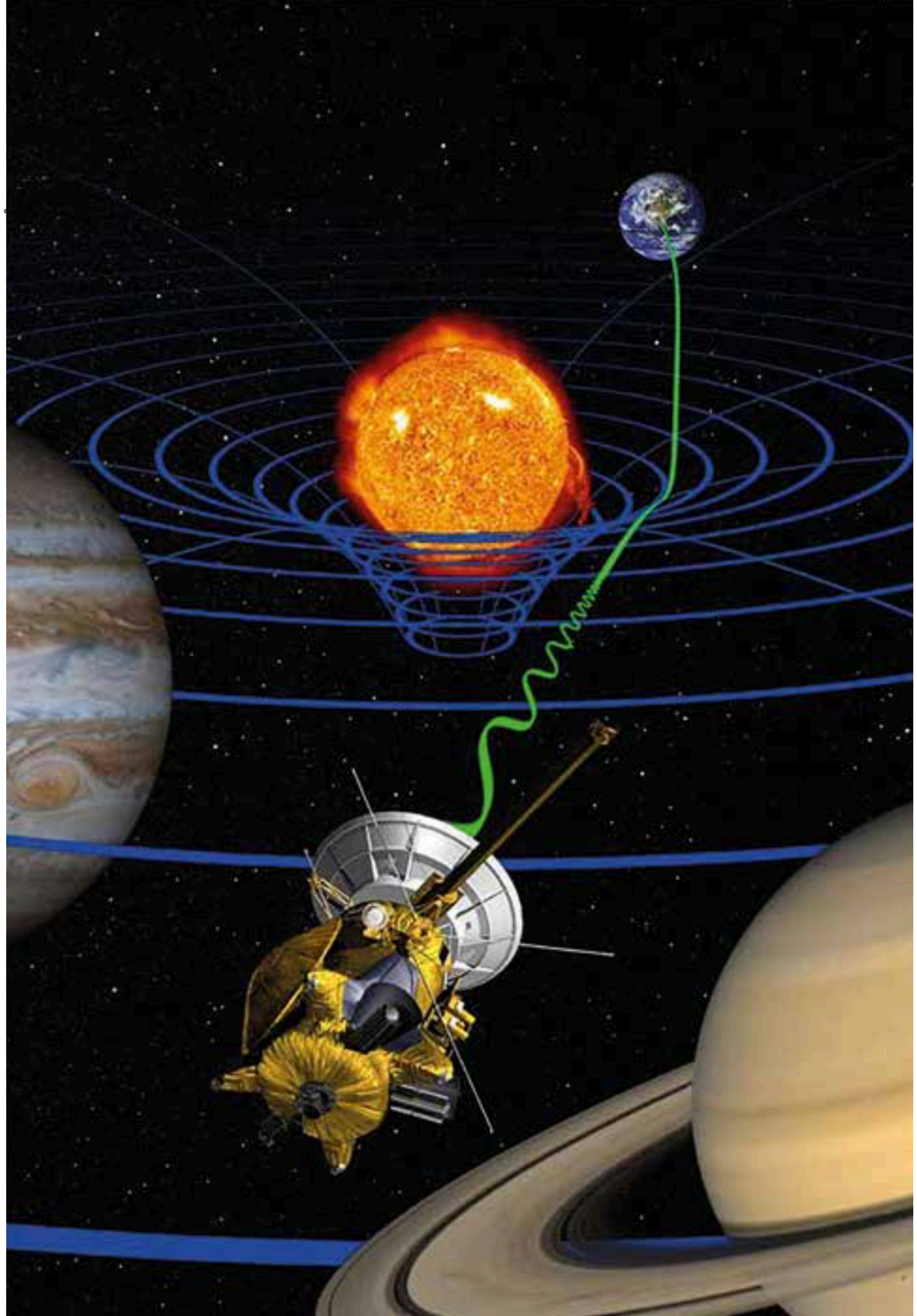
SHAPIRO DELAY AS A TEST OF GR

- Send a radio signal towards another planet (Mercury, Venus, Mars) behind the sun
- Measure time needed to the signal to come back after being reflected
- Measurement done in 2003 with the Cassini spacecraft
- Delay is few ~100 microseconds
- GR confirmed at the level of 0.002%



TOY LENS POTENTIALS

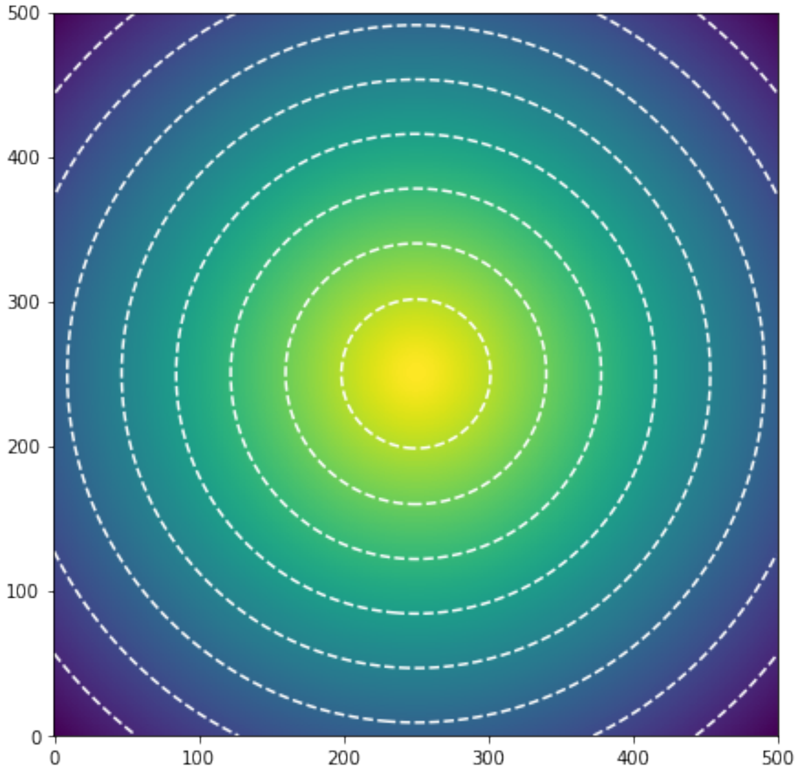
		LENSING POTENTIAL
POINT MASS		$\psi(\vec{\theta}) = C \ln \lvert\vec{\theta}\rvert$
SINGULAR ISOTHERMAL SPHERE	SIS	$\psi(\vec{\theta}) = C\lvert\vec{\theta}\rvert$
SINGULAR ISOTHERMAL ELLIPSOID	SIE	$\psi(\vec{\theta}) = C\sqrt{\frac{\theta_1^2}{(1-\epsilon)} + \theta_2^2(1-\epsilon)}$
PSEUDO ISOTHERMAL ELLIPSOID	PSIE	$\psi(\vec{\theta}) = C\sqrt{\frac{\theta_1^2}{(1-\epsilon)} + \theta_2^2(1-\epsilon) + \theta_c^2}$

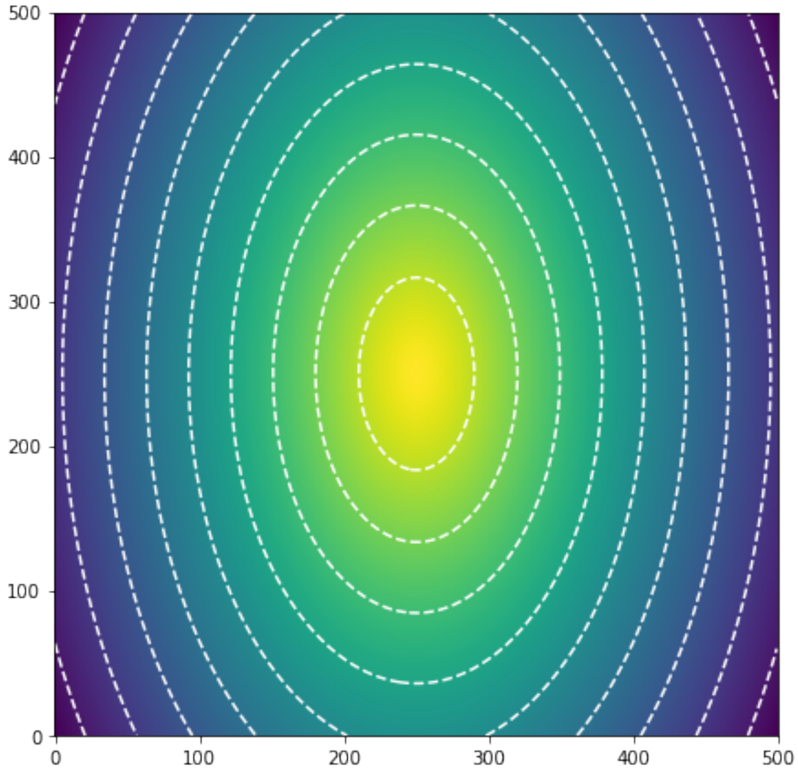
INTRODUCING ELLIPTICITY

SINGULAR ISOTHERMAL ELLIPSOID

$$|\theta| \rightarrow \sqrt{\frac{\theta_1^2}{1-\epsilon} + \theta_2^2(1-\epsilon)}$$

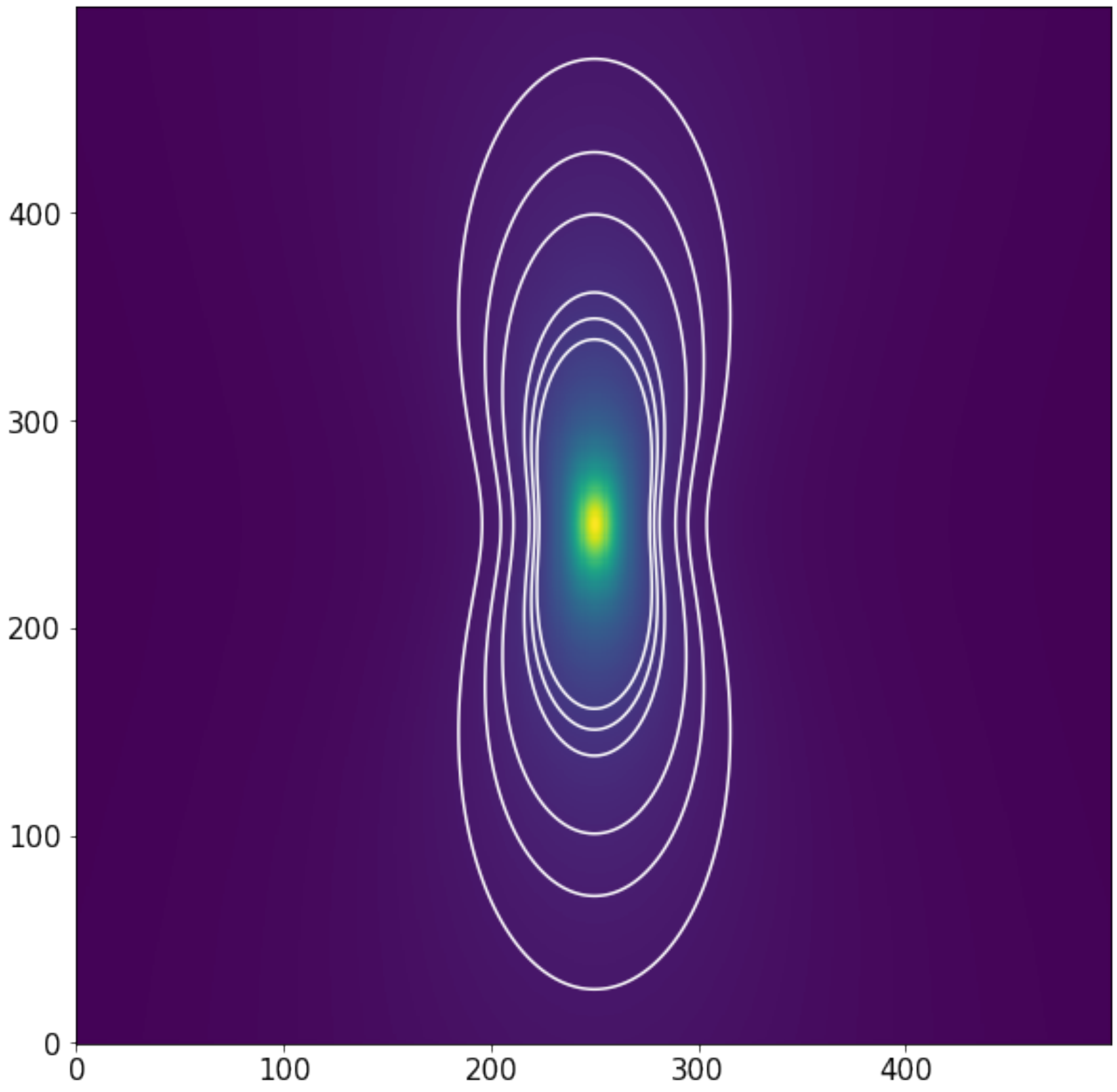
This makes the potential constant over ellipses rather than on circles.





CAUTION: PSEUDO ELLIPTICAL LENSES

The surface density is not always so realistic for theses simple potentials.



TYPES OF IMAGES

There are three types of images:

eigenvalues

$$a_1 = 1 - \kappa - \gamma$$

$$a_2 = 1 - \kappa + \gamma$$

magnification

Type I

$$a_1 > 0$$
$$a_2 > 0$$

$$\mu = \frac{1}{|\boldsymbol{A}|} > 0$$

Minimum of the time-delay,

Type II

One of the eigenvalues is negative and one positive.

$$\mu = \frac{1}{|\boldsymbol{A}|} < 0$$

Saddle point of the time delay.

Type III

$$a_1 < 0$$
$$a_2 < 0$$

$$\mu = \frac{1}{|\boldsymbol{A}|} > 0$$

Maximum of time-delay.

TYPES OF IMAGES

There are three types of images: eigenvalues

$$a_1 = 1 - \kappa - \gamma$$

$$a_2 = 1 - \kappa + \gamma$$

Theorem : Type I images have a magnification equal to or larger than 1.

Theorem :

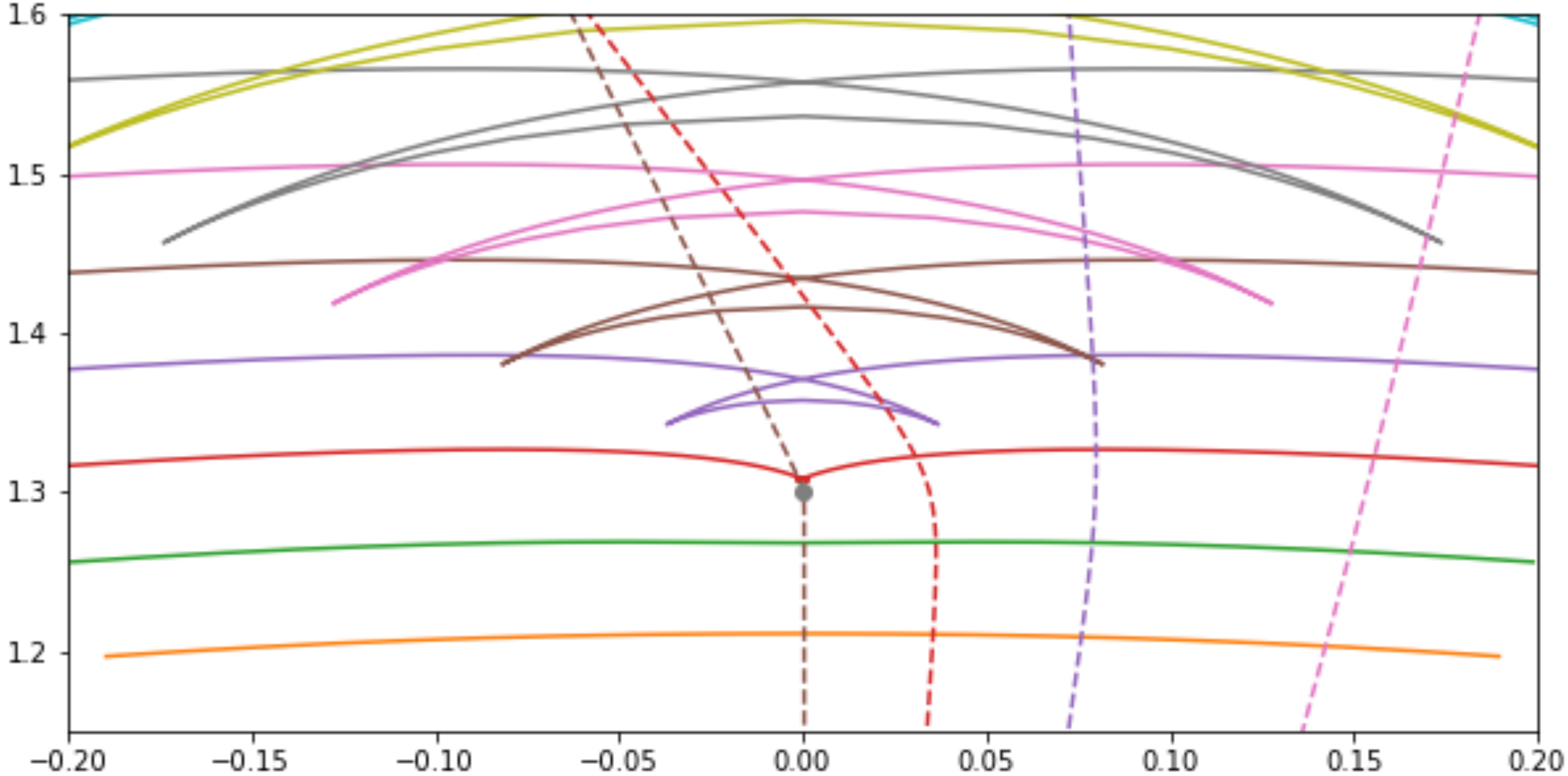
a) $n_{\mathrm{I}} \geq 1$ - There is at least one ordinary image.

b) $n_{\mathrm{I}} + n_{\mathrm{III}} = n_{\mathrm{II}} + 1$

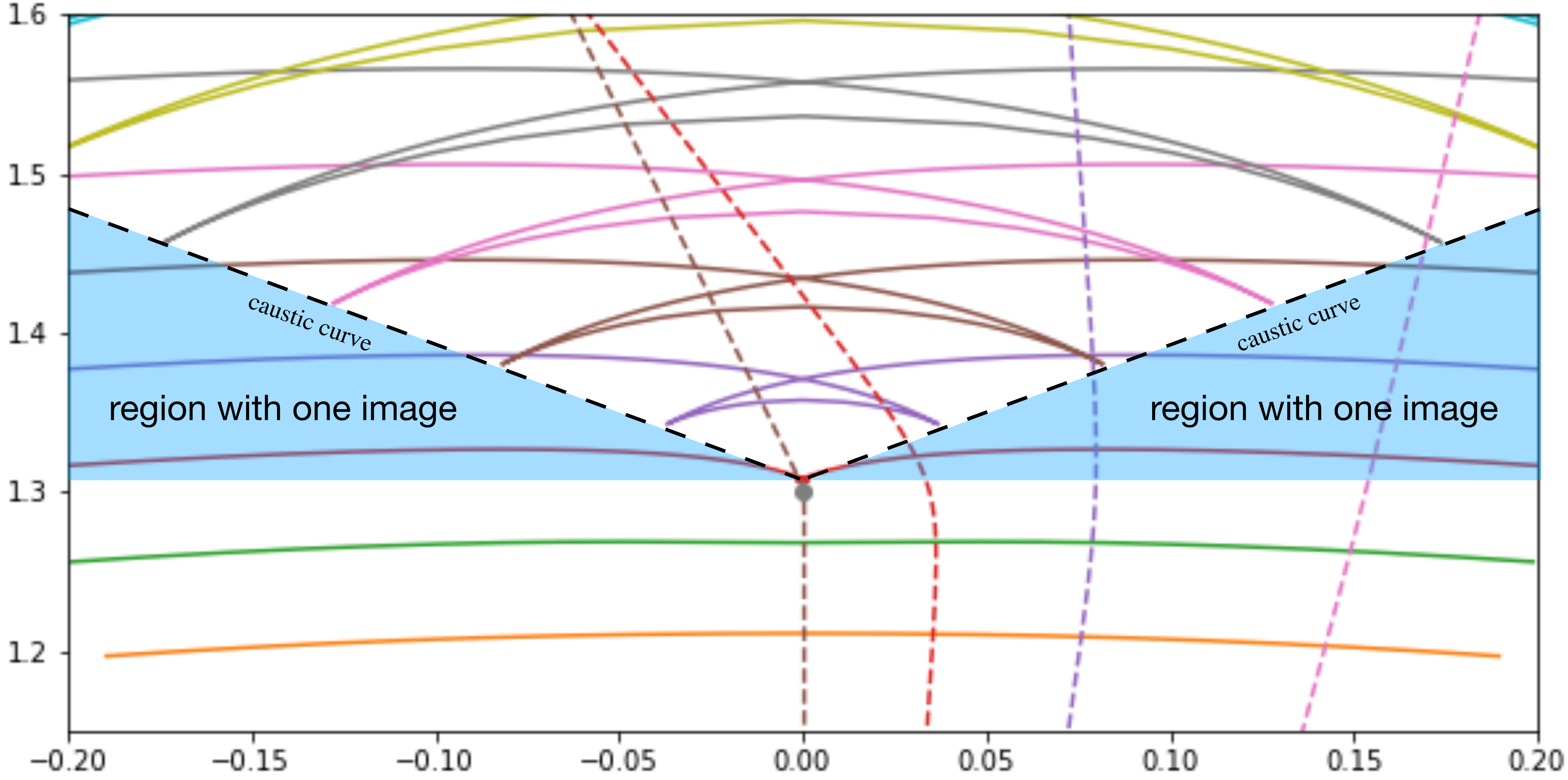
c) $n = n_{\mathrm{I}} + n_{\mathrm{II}} + n_{\mathrm{III}}$ is odd - The total number of images is odd.

Theorem : Image with the shortest time-delay will be type I.

WAVEFRONTS



WAVEFRONTS


1.6
1.5
1.4
1.3
1.2
caustic curve
caustic curve
region with one image
region with one image
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20