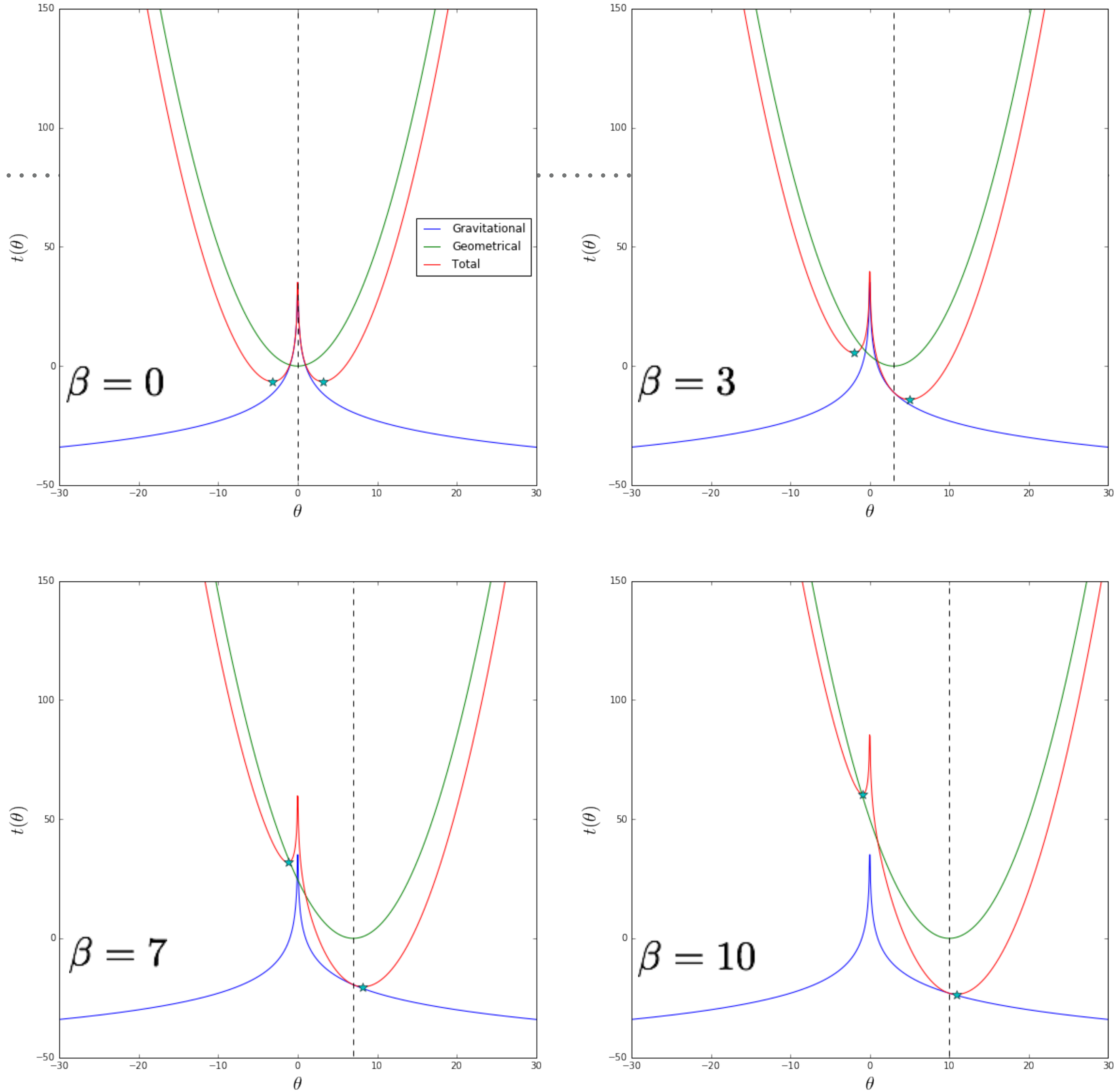
GRAVITATIONAL LENSING

8 – GRAVITATIONAL MICROLENSING I

R. Benton Metcalf

2022-2023

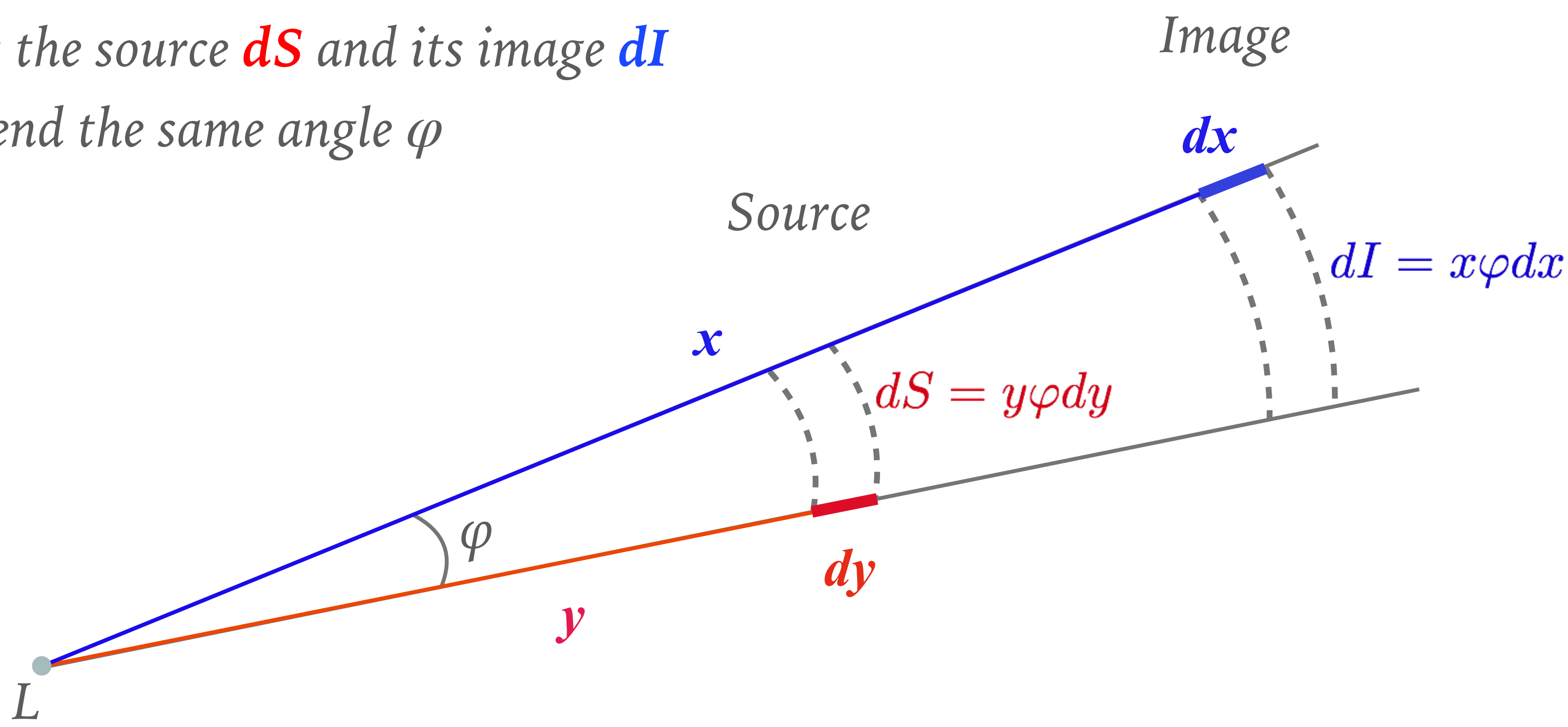
time-delay surfaces for a point mass



MAGNIFICATION

Remeber: $\overrightarrow{x}$, $\overrightarrow{y}$, $\overrightarrow{\alpha}(\overrightarrow{x})$ *are parallel!*

Thus the source ***dS*** *and its image* ***dI***
subtend the same angle φ



The figure shows that $\mu(x) = \frac{x}{y}\frac{dx}{dy}$ *or* $\det A(x) = \frac{y}{x}\frac{dy}{dx}$

CRITICAL LINES AND CAUSTICS

$$y = x - \frac{1}{x}$$

From the lens equation, it follows that:

$$\lambda_t(x) = \frac{y}{x} = 1 - \frac{1}{x^2}$$

$$\lambda_r(x) = \frac{dy}{dx} = 1 + \frac{1}{x^2}$$

The second eigenvalue is always positive (no critical line). The first is zero on the circle

$$x^2 = 1$$

Thus, the Einstein ring is the tangential critical line! The corresponding caustic is a point at $y=0$

IMAGE MAGNIFICATION

Clearly,

$$\det A(x) = \frac{y}{x}\frac{dy}{dx}$$

$$\mu(x) = \det A^{-1}$$

$$\lambda_t(x) = \frac{y}{x} = 1 - \frac{1}{x^2}$$

$$\lambda_r(x) = \frac{dy}{dx} = 1 + \frac{1}{x^2}$$

$$\mu(x) = \left(1 - \frac{1}{x^4}\right)^{-1}$$

$$\det A(x) = \frac{y}{x}\frac{dy}{dx} = \left(1 - \frac{1}{x^4}\right)$$

IMAGE PARITY

Note that:

$$y > 0 \quad \Rightarrow \quad \begin{array}{l} x_+ > 0 \\ x_- < 0 \end{array}$$

$$\mu_t = \frac{x}{y} \quad \Rightarrow \quad \begin{array}{l} \mu_t(x_+) > 0 \\ \mu_t(x_-) < 0 \end{array}$$

$$\mu_r = \frac{dx}{dy} > 0$$

Thus the parity of the images is different!

Not surprising given that the two images are separated by the critical line

SOURCE MAGNIFICATION

Let's compute now the image magnification as a function of the source position:

$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right] \quad \Rightarrow \quad \frac{x}{y} = \frac{1}{2}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)$$

$$\frac{dx}{dy} = \frac{1}{2}\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right)$$

Thus, the magnifications at the two image positions are

$$\mu = \frac{x}{y}\frac{dx}{dy}$$

$$\begin{aligned}
\mu_\pm(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\
&= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\
&= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right)
\end{aligned}$$

SOURCE MAGNIFICATION

The total magnification is obtained by summing the magnifications of the images:

$$
\begin{aligned}
\mu_\pm(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\
&= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\
&= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right)
\end{aligned}
$$

$$
\boxed{\mu(y) = \mu_+(y) + |\mu_-(y)| = \frac{y^2+2}{y\sqrt{y^2+4}}}
$$

The sum of the signed magnification is one!

We can take a series expansion of the magnification to see that $\mu \propto 1 + 2/y^4$ for $y \to \infty$

Thus, the magnification drops quickly as the source moves away from the lens!

SOURCE MAGNIFICATION

In addition:

$$\left|\frac{\mu_+}{\mu_-}\right| = \frac{1+\frac{y^2+2}{y\sqrt{y^2+4}}}{\frac{y^2+2}{y\sqrt{y^2+4}}-1} = \frac{y^2+2+y\sqrt{y^2+4}}{y^2+2-y\sqrt{y^2+4}}$$

$$\frac{1}{2}\left(y+\sqrt{y^2+4}\right)^2 = y^2+2+y\sqrt{y^2+4}$$

$$\frac{1}{2}\left(y-\sqrt{y^2+4}\right)^2 = y^2+2-y\sqrt{y^2+4}$$

$$\left|\frac{\mu_+}{\mu_-}\right| = \left(\frac{y+\sqrt{y^2+4}}{y-\sqrt{y^2+4}}\right)^2 = \left(\frac{x_+}{x_-}\right)^2 .$$

Laurent series expansion at infinity:

$$\left|\frac{\mu_+}{\mu_-}\right| \propto y^4$$

As we move the source away from the lens, the image in x_+ dominates the flux budget very soon.

$$\lim_{y\to\infty}\mu_- = 0$$

$$\lim_{y\to\infty}\mu_+ = 1$$

A SOURCE ON THE EINSTEIN RING

For a source on the Einstein ring:

$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right] \rightarrow y = 1 \rightarrow x_\pm = \frac{1}{2}(1 \pm \sqrt{5}) \Rightarrow \mu_\pm = \left[1 - \left(\frac{2}{1 \pm \sqrt{5}}\right)^4\right]^{-1}$$

Therefore $\mu = \mu_+ + |\mu_-| = 1.17 + 0.17 = 1.34$*:*

$$\Delta m = -2.5\log_{10}\mu \simeq 0.3$$

Given how quickly the magnification drops by moving the source away from the lens, we can assume that only sources within the Einstein radius are magnified in a significant way.

For this reason, the circle within the Einstein radius is assumed to be the cross section for microlensing.

MICROLENSING OBSERVABLES?

- typical Einstein radii for lenses in the MW are $\sim$1 mas
- thus, the image separation is too small to resolve the images
- magnification is small also for relatively close pairs of lenses and sources
- how to detect a microlensing event?

MICROLENSING LIGHT CURVE

Assume a linear trajectory of the source relative to the lens, with impact parameter y_0

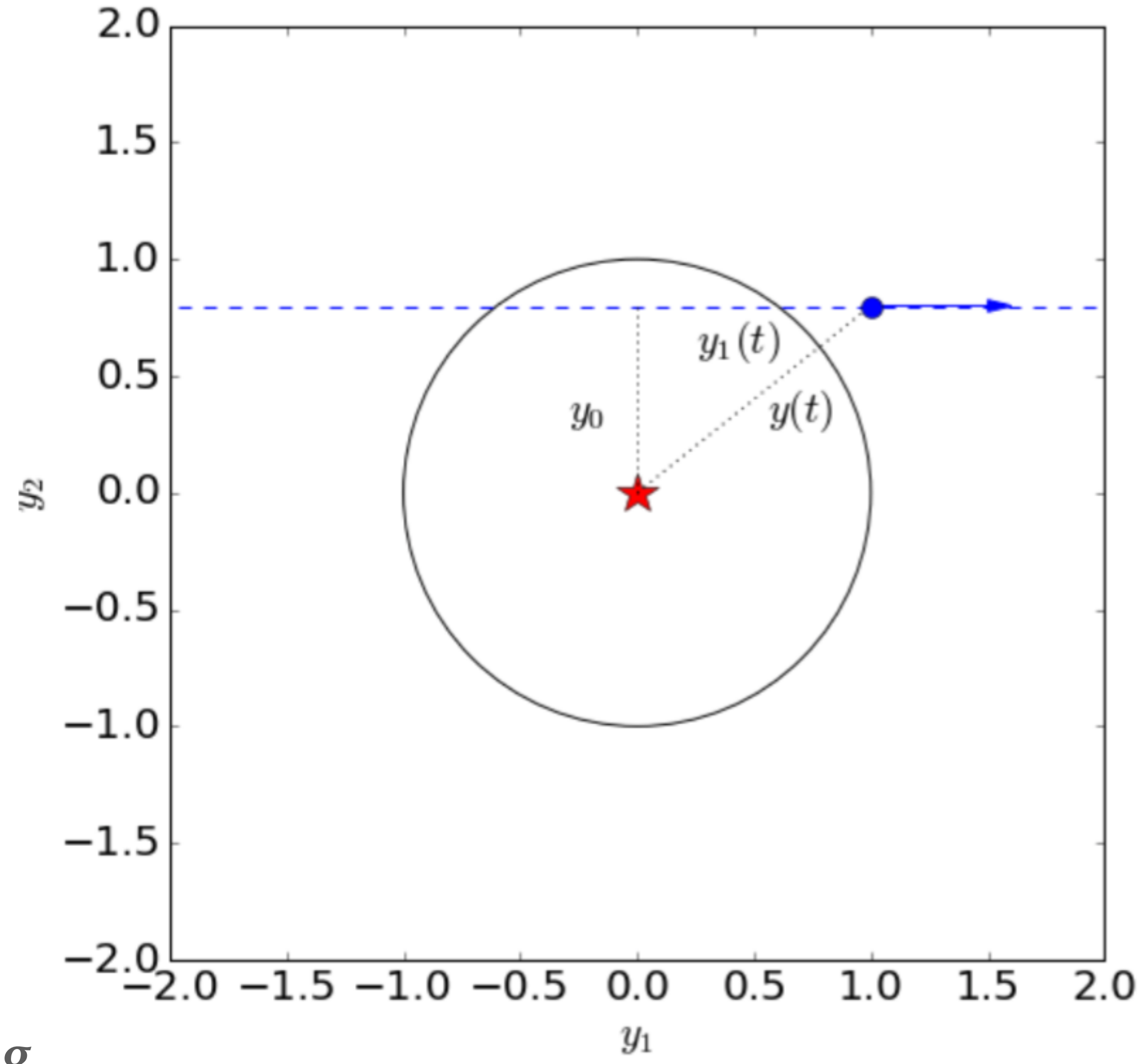
Assume also constant transverse velocity v:

$$y_1(t) = \frac{v(t - t_0)}{D_L \theta_E}$$

We can define a characteristic time of the event:

$$t_E = \frac{D_L \theta_E}{v} = \frac{\theta_E}{\mu_{rel}}$$

This is the ***Einstein radius crossing time***

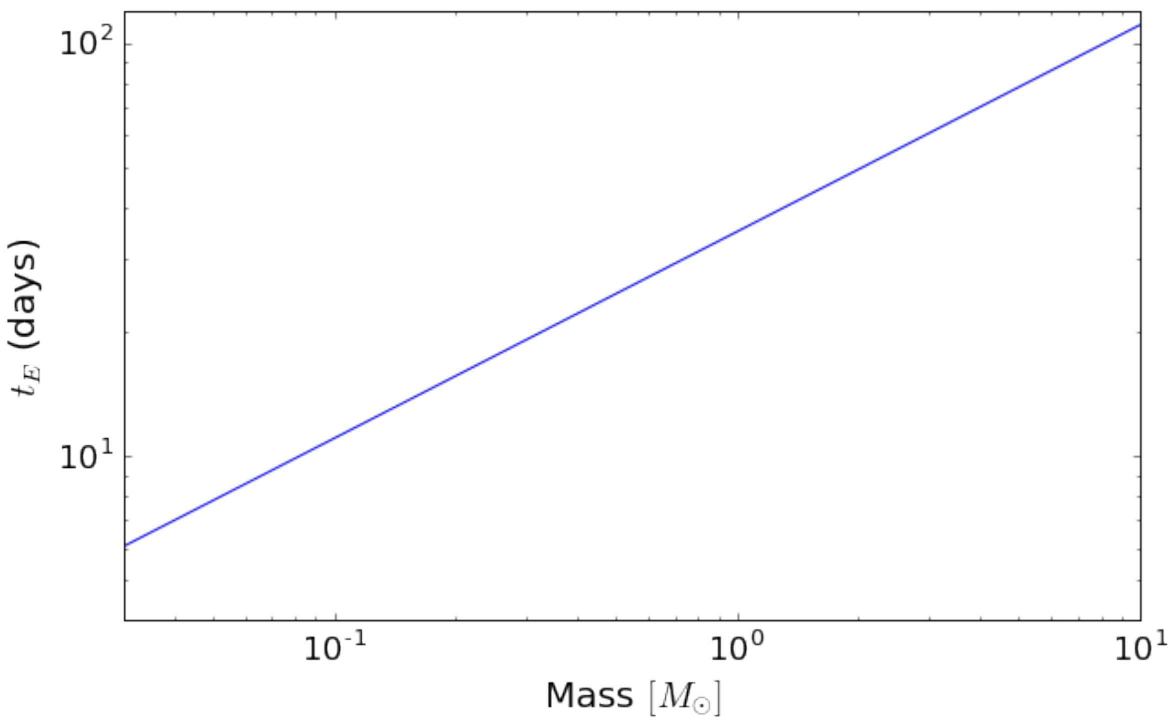


MICROLENSING LIGHT CURVE

Given the definition of Einstein radius $$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}}$$

The order of magnitude of the t_E is

$$t_E \approx 19 \text{ days} \sqrt{4\frac{D_L}{D_S}\left(1-\frac{D_L}{D_S}\right)} \left(\frac{D_S}{8\text{kpc}}\right)^{1/2} \left(\frac{M}{0.3M_\odot}\right)^{1/2} \left(\frac{v}{200\text{km}/s}\right)^{-1}$$



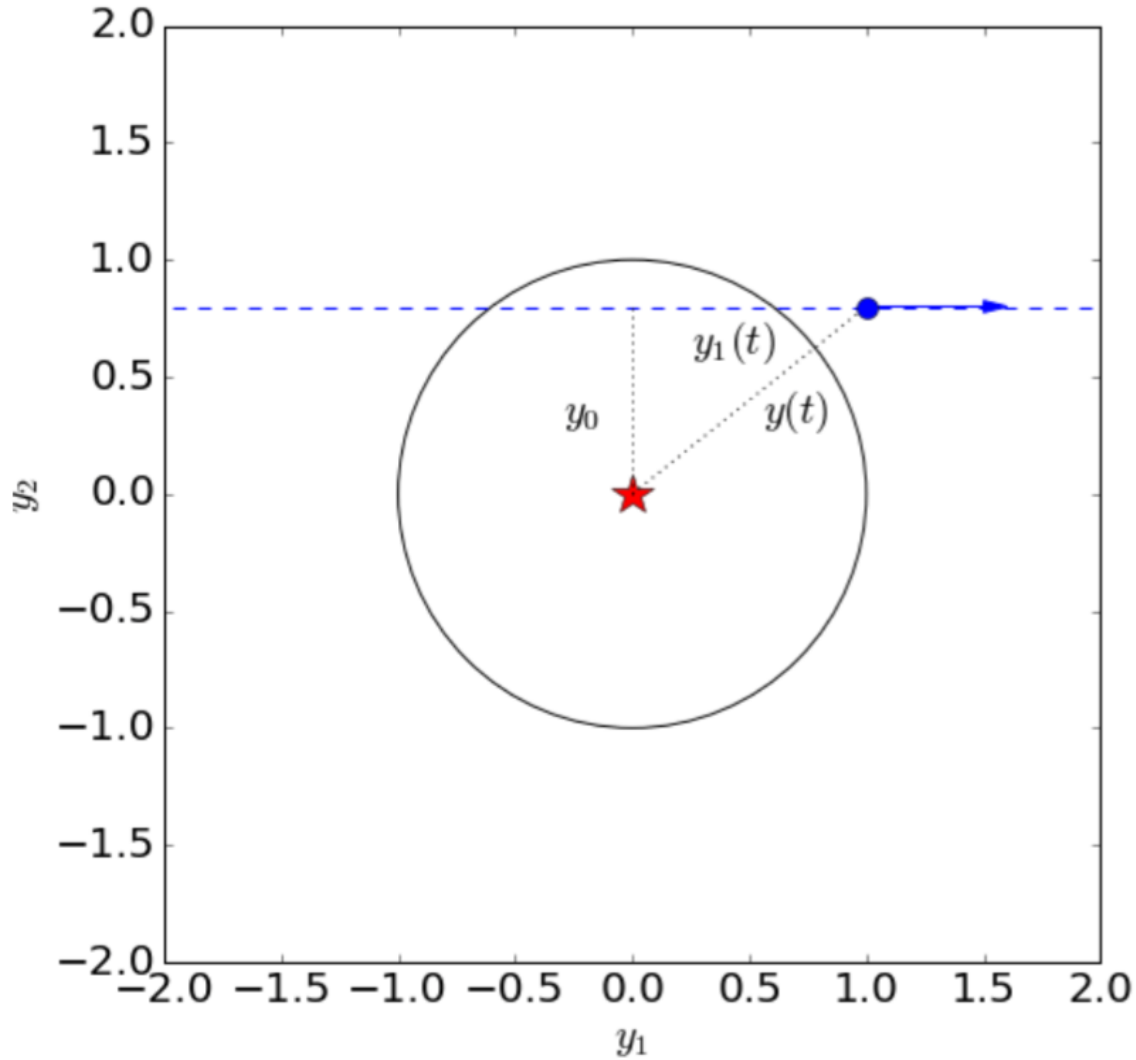
MICROLENSING LIGHT CURVE

We obtain

$$y_1(t) = \frac{v(t - t_0)}{D_L \theta_E} = \frac{t - t_0}{t_E}$$

Thus:

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t - t_0)^2}{t_E^2}}$$



MICROLENSING LIGHT CURVE

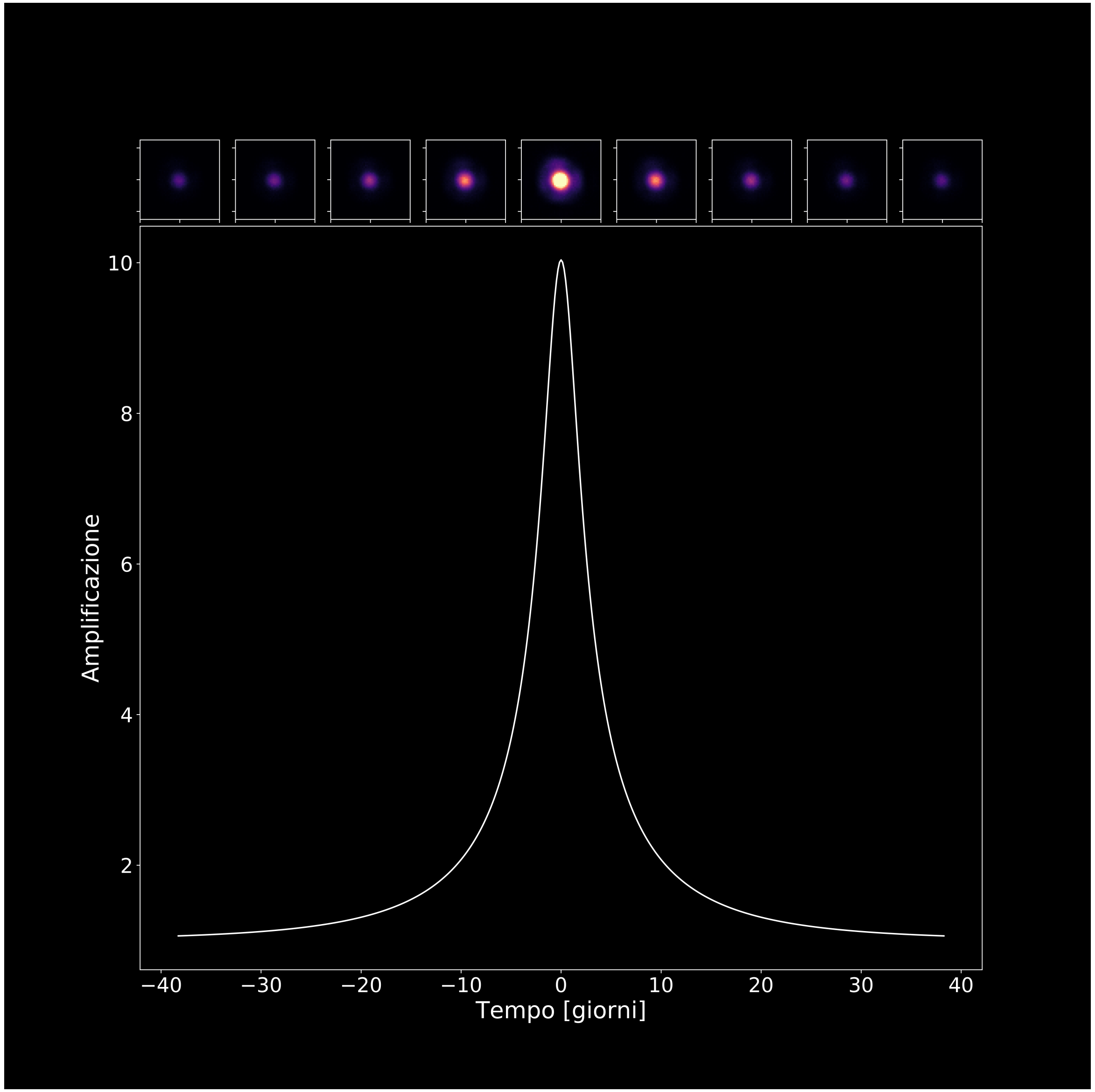
Combine

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t - t_0)^2}{t_E^2}}$$

with

$$\mu(y) = \frac{y^2 + 2}{y\sqrt{y^2 + 4}}$$

And obtain $\mu(t)$



Possible gravitational microlensing of a star in the Large Magellanic Cloud

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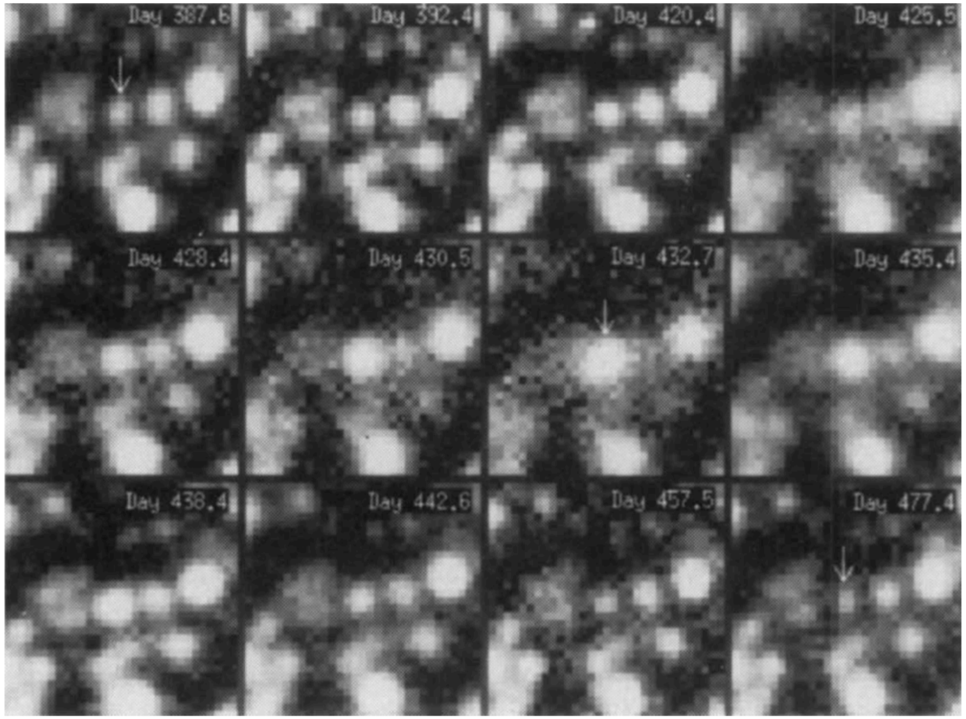
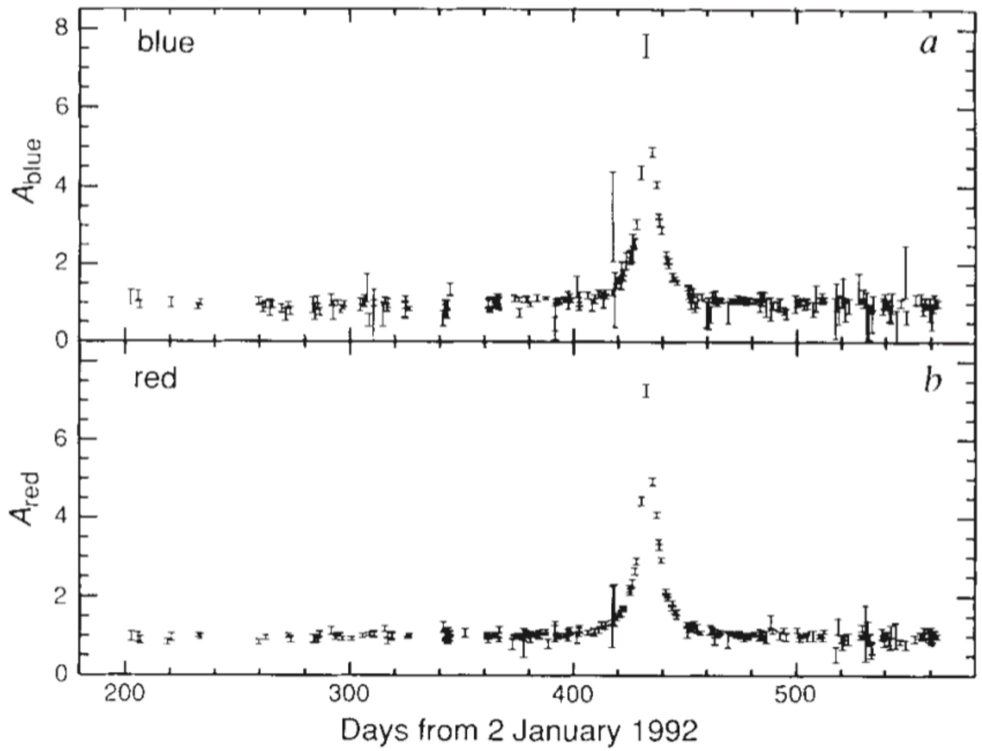
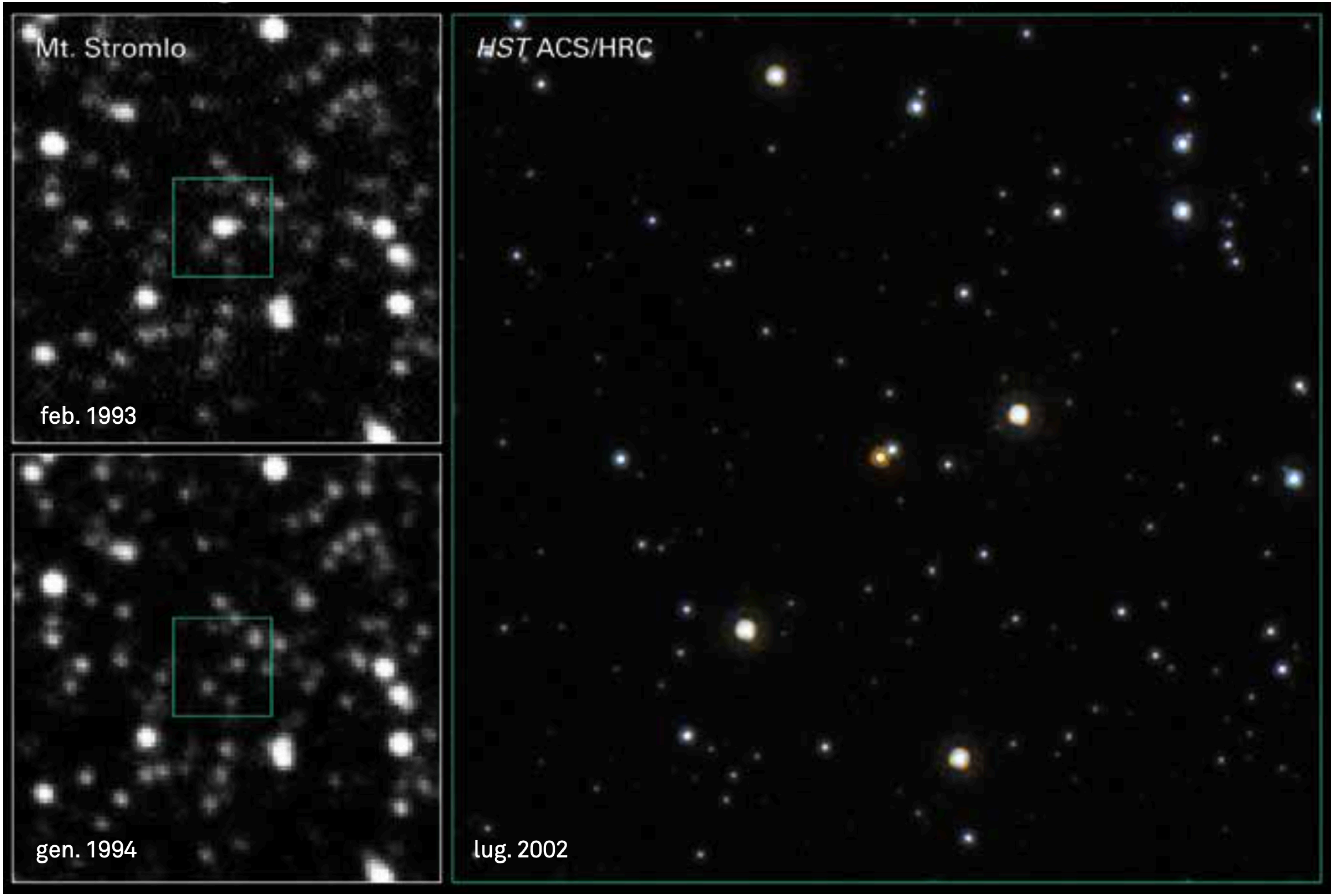
THERE is now abundant evidence for the presence of large quantities of unseen matter surrounding normal galaxies, including our own[1,2]. The nature of this 'dark matter' is unknown, except that it cannot be made of normal stars, dust or gas, as they would be easily detected. Exotic particles such as axions, massive neutrinos or other weakly interacting massive particles (collectively known as WIMPs) have been proposed[3,4], but have yet to be detected. A less exotic alternative is normal matter in the form of bodies with masses ranging from that of a large planet to a few solar masses. Such objects, known collectively as massive compact halo objects[5] (MACHOs), might be brown dwarfs or 'jupiters' (bodies too small to produce their own energy by fusion), neutron stars, old white dwarfs or black holes. Paczynski[6] suggested that MACHOs might act as gravitational microlenses, temporarily amplifying the apparent brightness of background stars in nearby galaxies. We are conducting a microlensing experiment to determine whether the dark matter halo of our Galaxy is made up of MACHOs. Here we report a candidate for such a microlensing event, detected by monitoring the light curves of 1.8 million stars in the Large Magellanic Cloud for one year. The light curve shows no variation for most of the year of data taking, and an upward excursion lasting over 1 month, with a maximum increase of ~2 mag. The most probable lens mass, inferred from the duration of the candidate lensing event, is ~0.1 solar mass.



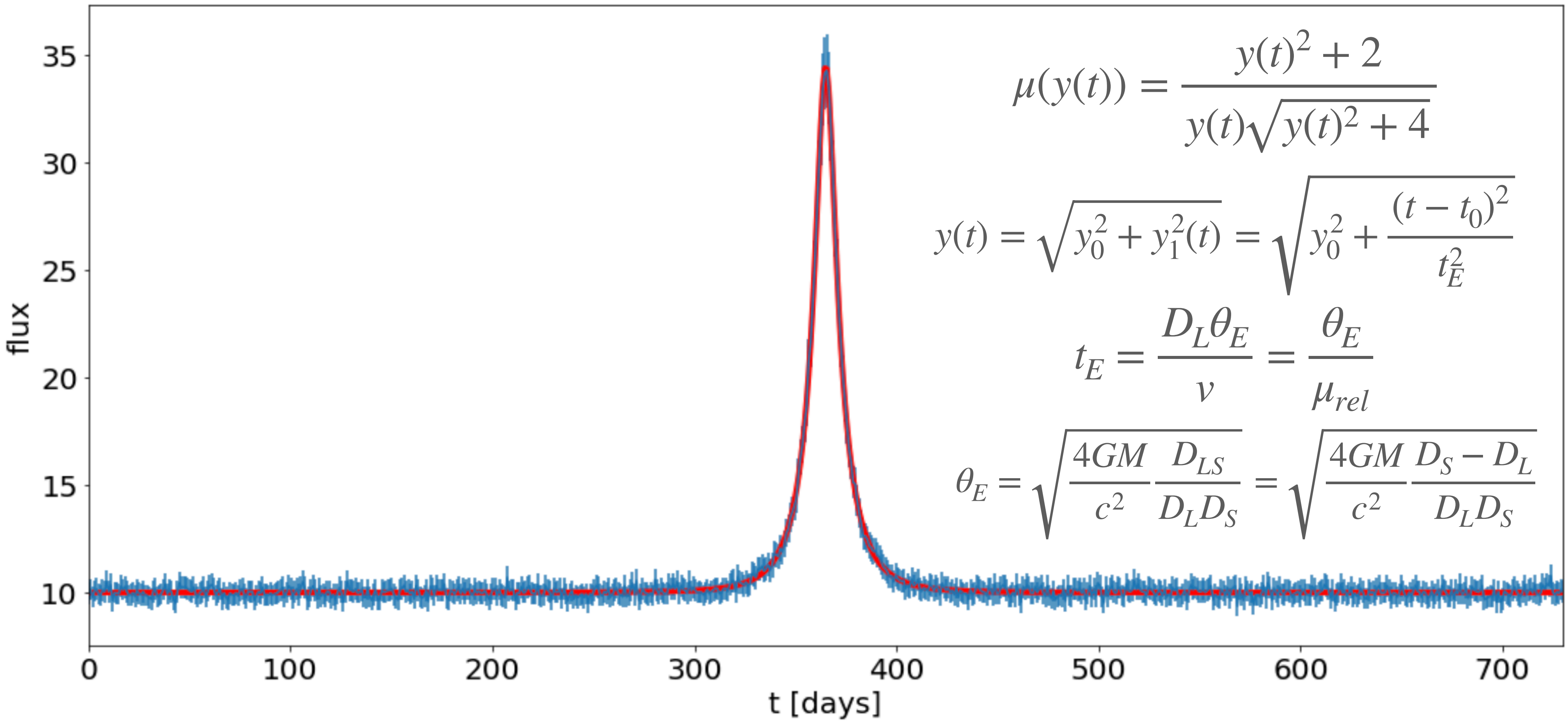


Alcock et al. 1993, Nature, 365, 621

DETECTION OF THE LENS STAR IN A MICROLENSING EVENT



EXAMPLE DISCUSSED IN THE NOTEBOOK



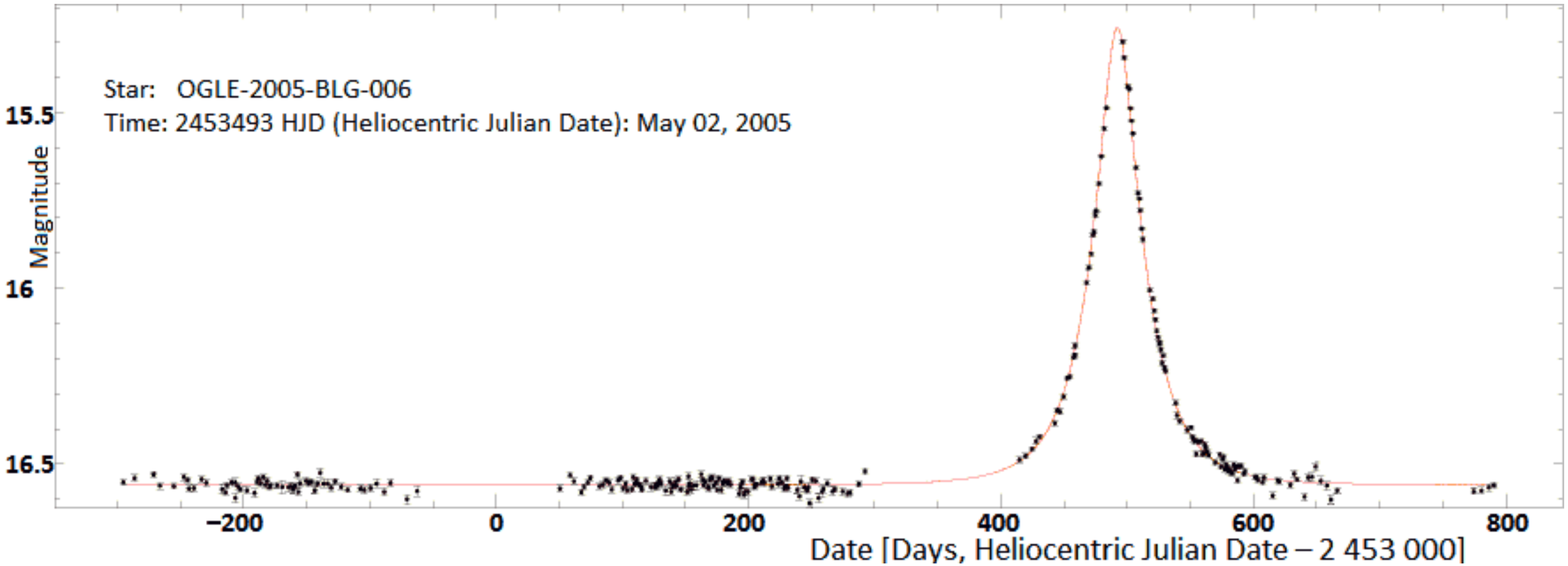
$$\mu(y(t)) = \frac{y(t)^2 + 2}{y(t)\sqrt{y(t)^2 + 4}}$$

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t - t_0)^2}{t_E^2}}$$

$$t_E = \frac{D_L \theta_E}{v} = \frac{\theta_E}{\mu_{rel}}$$

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}} = \sqrt{\frac{4GM}{c^2}\frac{D_S - D_L}{D_L D_S}}$$

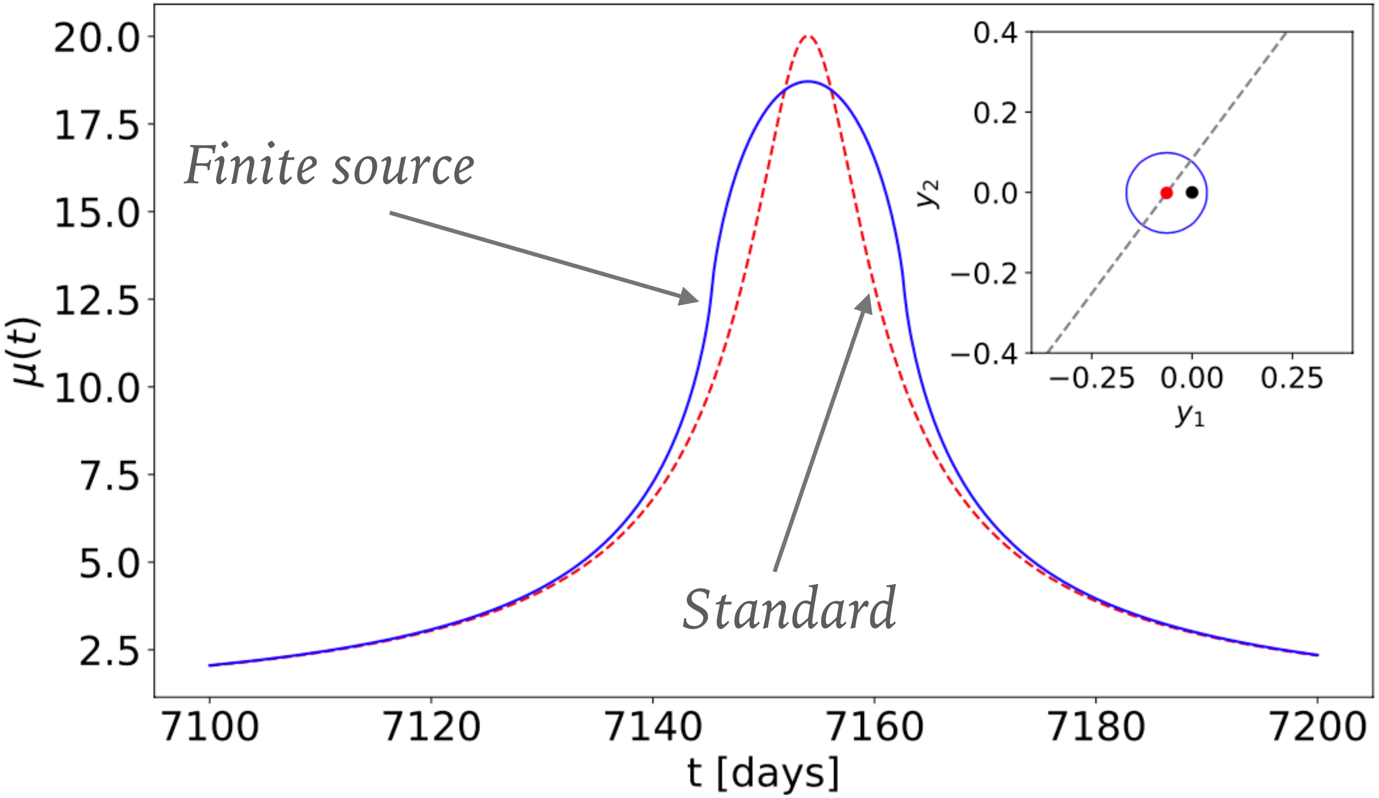
EXAMPLE OF REAL STANDARD LIGHT CURVE



MICROLENSING DEGENERACY AND ITS BREAKERS

- As seen, from the standard light curve we can measure the Einstein crossing time, which is a degenerate combination of the lens mass, distance and velocity
- Thus it is impossible to characterise the lens and the source of a single event through light curve measurement alone…
- … unless special circumstances are present!

FINITE SOURCE EFFECTS

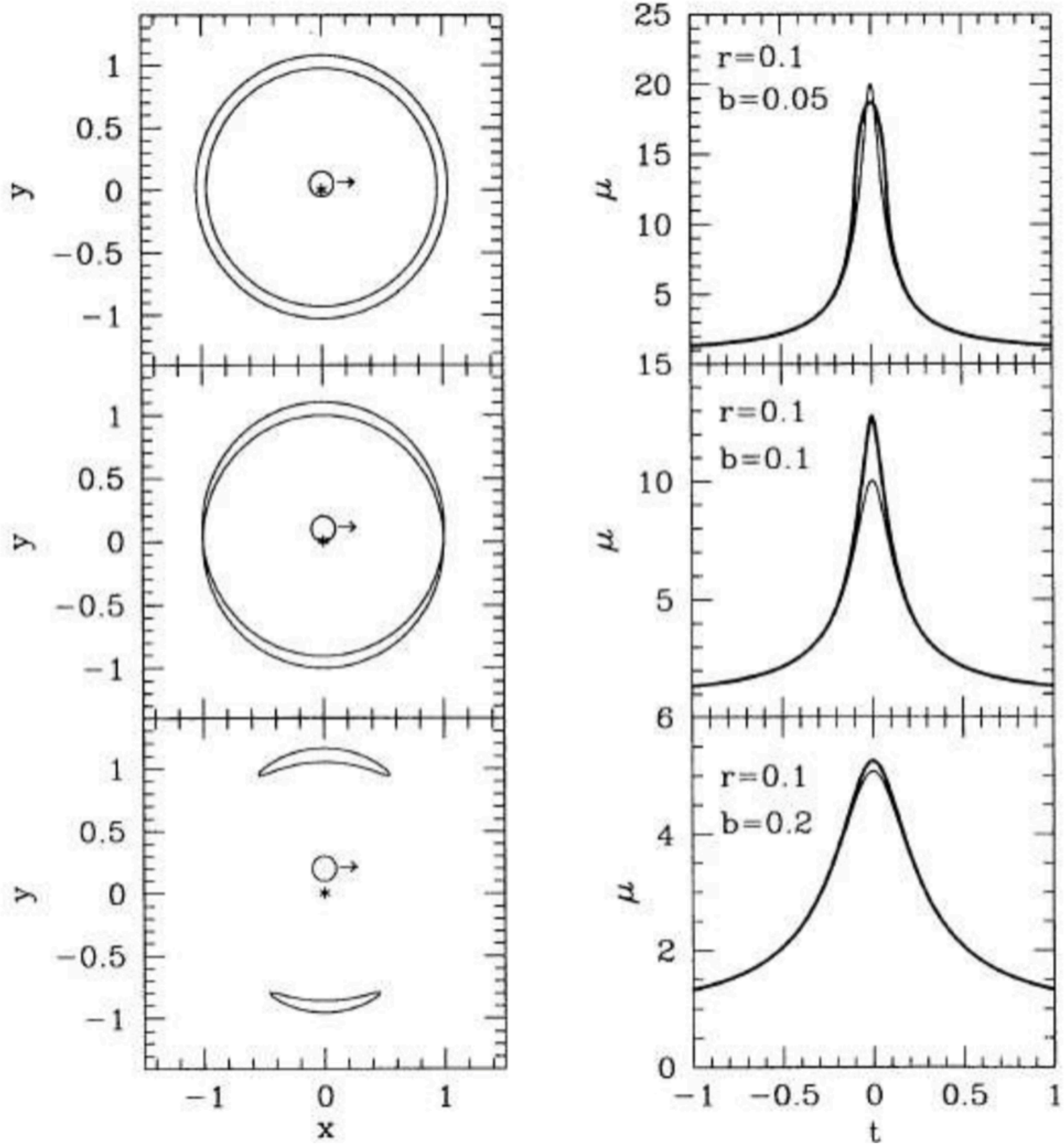


We can fit the light curve with an additional parameter (ρ_S), and use some empirical relation to measure the source size from the source color and magnitude. For example, Kervella et al. (2004) find:

$$\log(2\beta_*) = 0.0755(V - K) + 0.517 - 0.2K$$

Then, we can infer the Einstein radius.

FINITE SOURCE EFFECTS



MICROLENSING PARALLAX

As seen:

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}} = \sqrt{\frac{4GM}{c^2}\frac{D_S - D_L}{D_L D_S}} = \sqrt{\frac{4GM}{c^2}\left(\frac{1}{D_L} - \frac{1}{D_S}\right)} = \sqrt{\frac{4GM}{c^2}\pi_{rel}}$$

This illustrates that in a microlensing event we are sensitive to the relative parallax of lens and source.

In addition, note that:

$$\theta_E = \frac{4GM}{c^2}\frac{\pi_{rel}}{\theta_E} = \frac{4GM}{c^2}\pi_E \quad , \quad \pi_E = \frac{\pi_{rel}}{\theta_E}$$

This gives us another relation between the Einstein radius and the mass of the lens:

$$\theta_E = kM\pi_E \Rightarrow M = \frac{\theta_E}{k\pi_E} \quad k = 8.14 \text{ mas } \mathrm{M}_\odot^{-1}$$

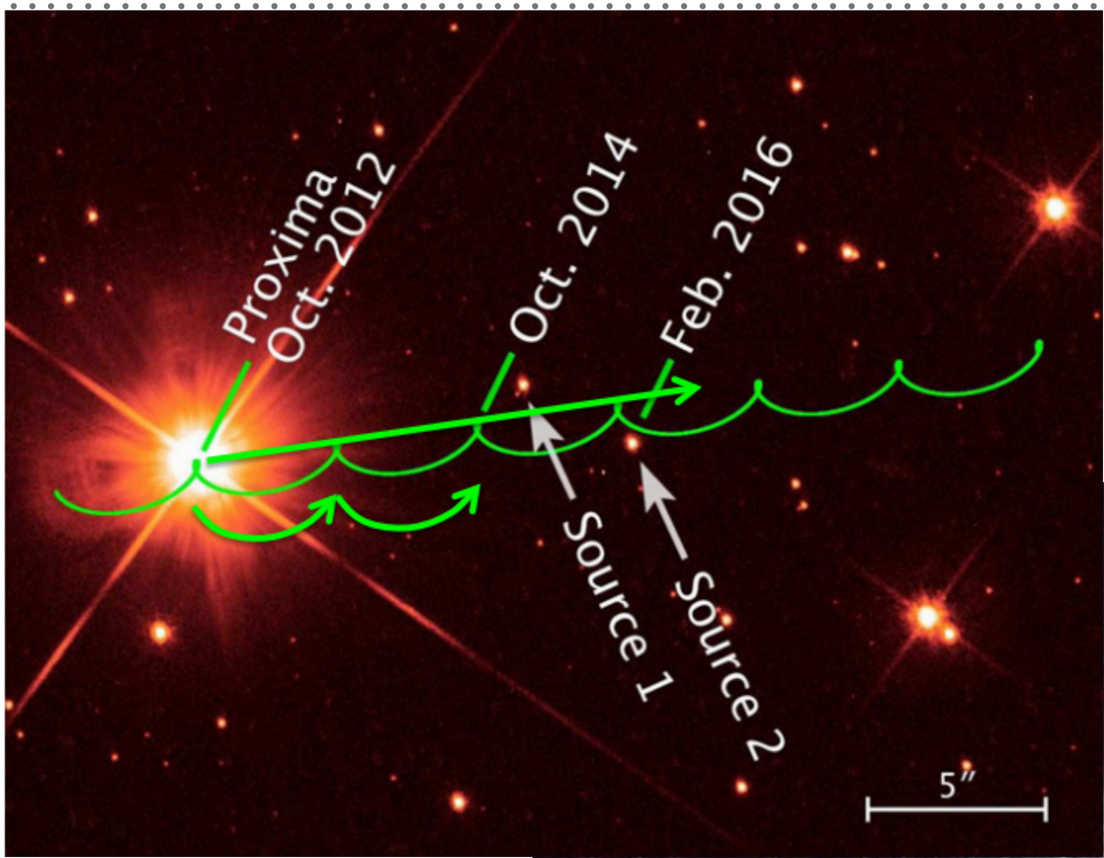
WHAT KIND OF MEASURABLE EFFECTS DOES PARALLAX CAUSE?

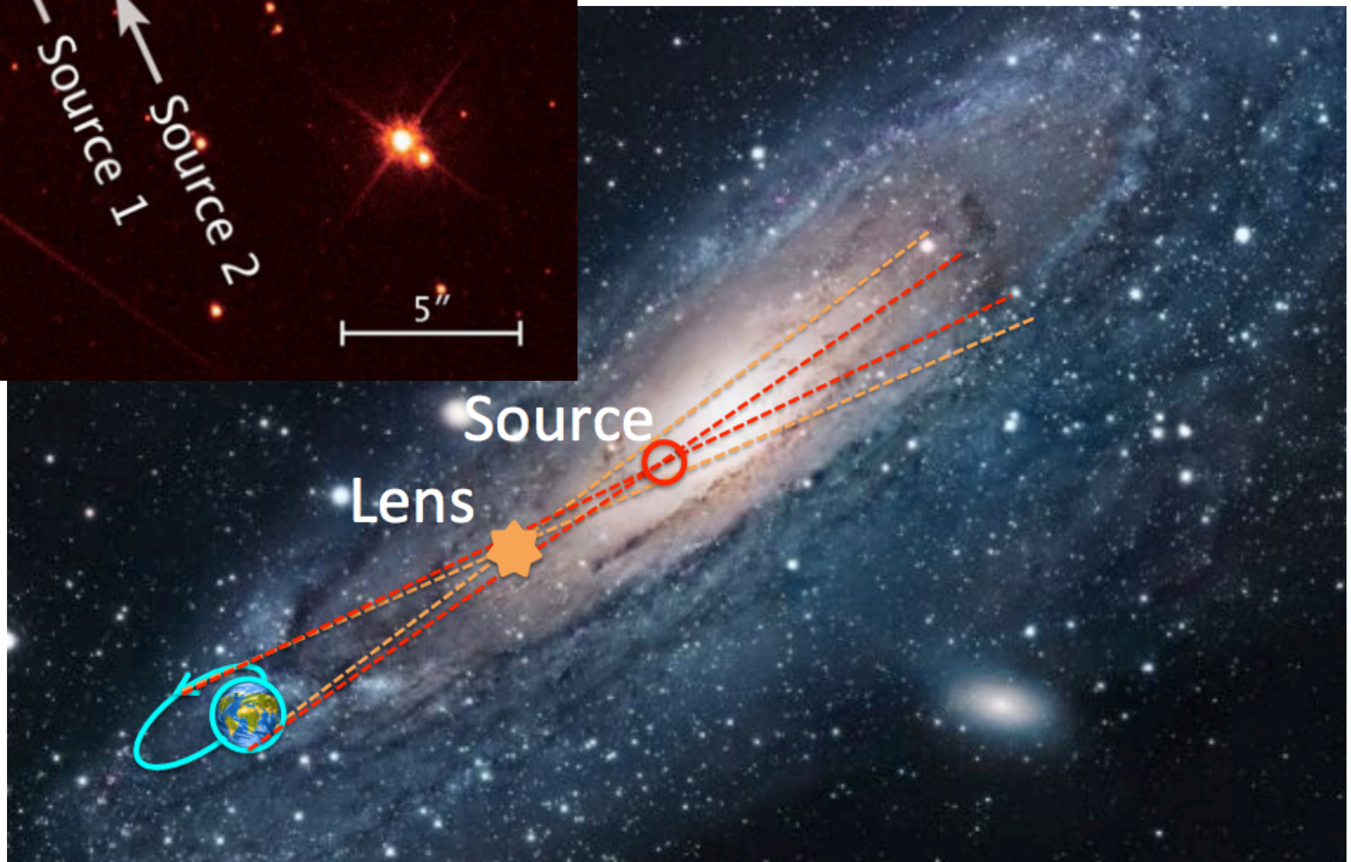
Parallax causes a difference in the relative position of lens and source depending on the observer point of view. Since the observable in a microlensing event depends on the relative separation between source and lens, parallax effects are very relevant.

In microlensing, parallax effects can be observed in two ways:

- *Because the observer move during the microlensing event*
- *Because two observers look at the same pair of lens and source from different positions*

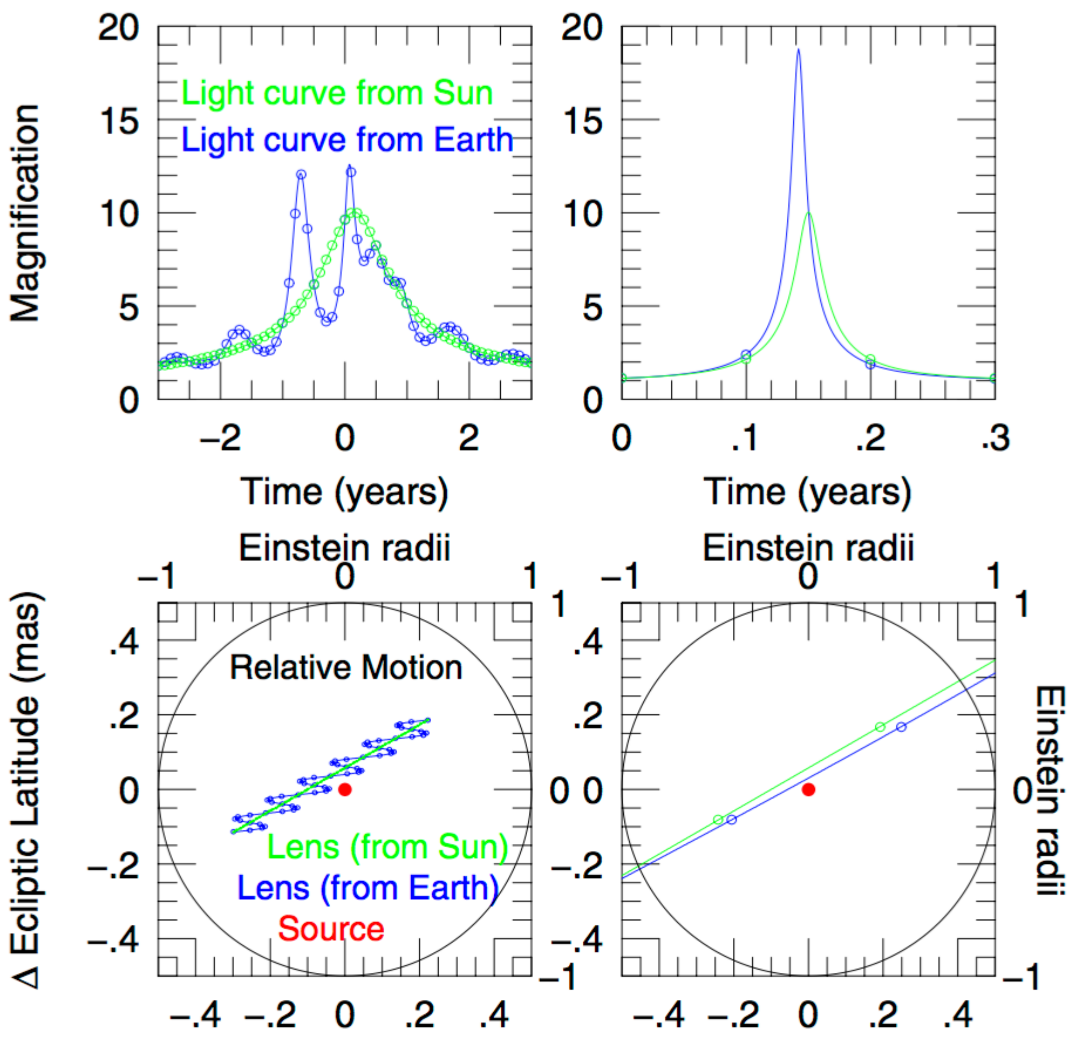
ORBITAL PARALLAX





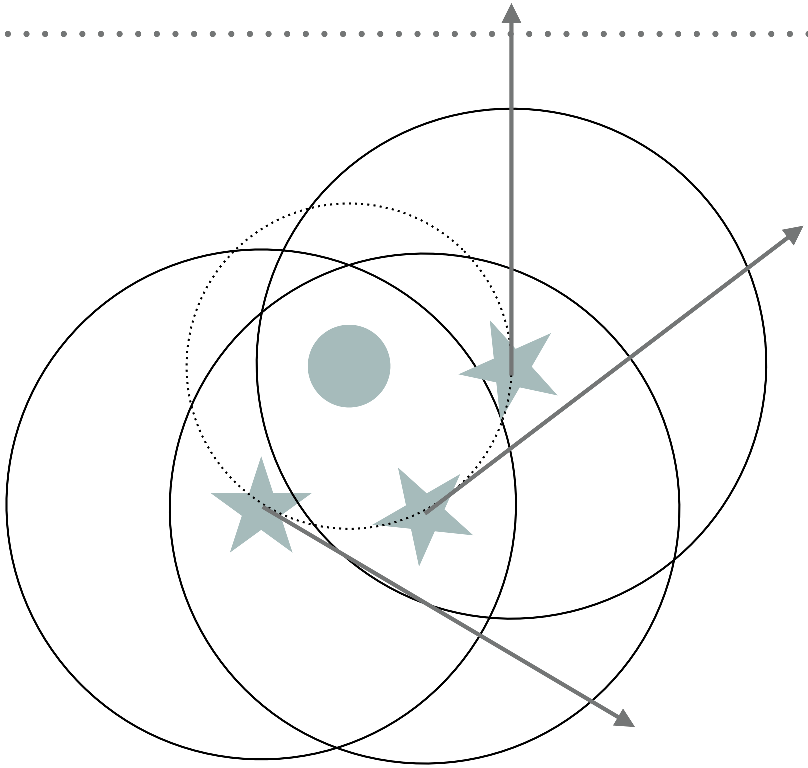
ORBITAL PARALLAX

- on the left: what we would see if the μ_{hel}=0.1 mas/year
- on the right: the typical μ_{hel}=5 mas/year



Gould & Horne, 2013

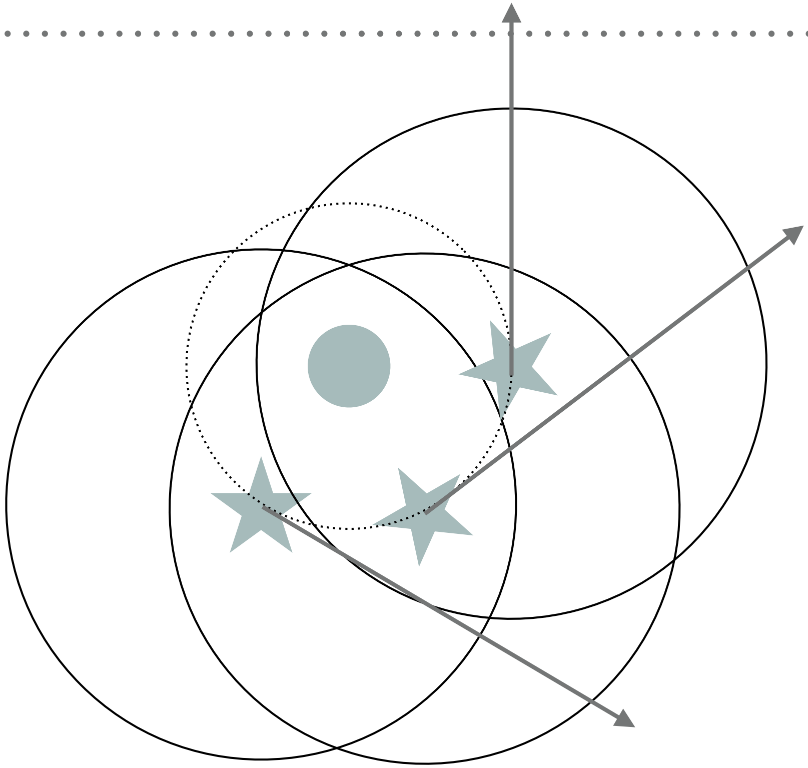
IMPORTANT: MICROLENSING PARALLAX IS A VECTOR!



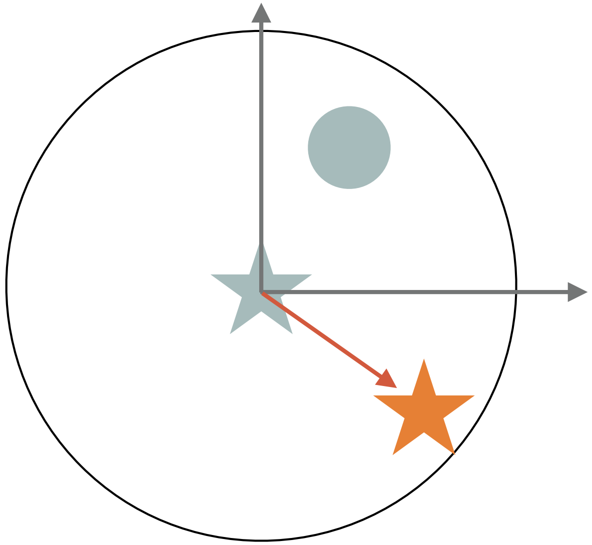
Magnification does not depend on the direction of proper motion

$$\mu(t) = \frac{y^2(t) + 2}{y(t)\sqrt{y^2(t) + 4}}$$

IMPORTANT: MICROLENSING PARALLAX IS A VECTOR!



$$\vec{\pi}_E = \pi_E \frac{\vec{\mu}_{rel}}{\mu_{rel}}$$



Magnification does not depend on the direction of proper motion

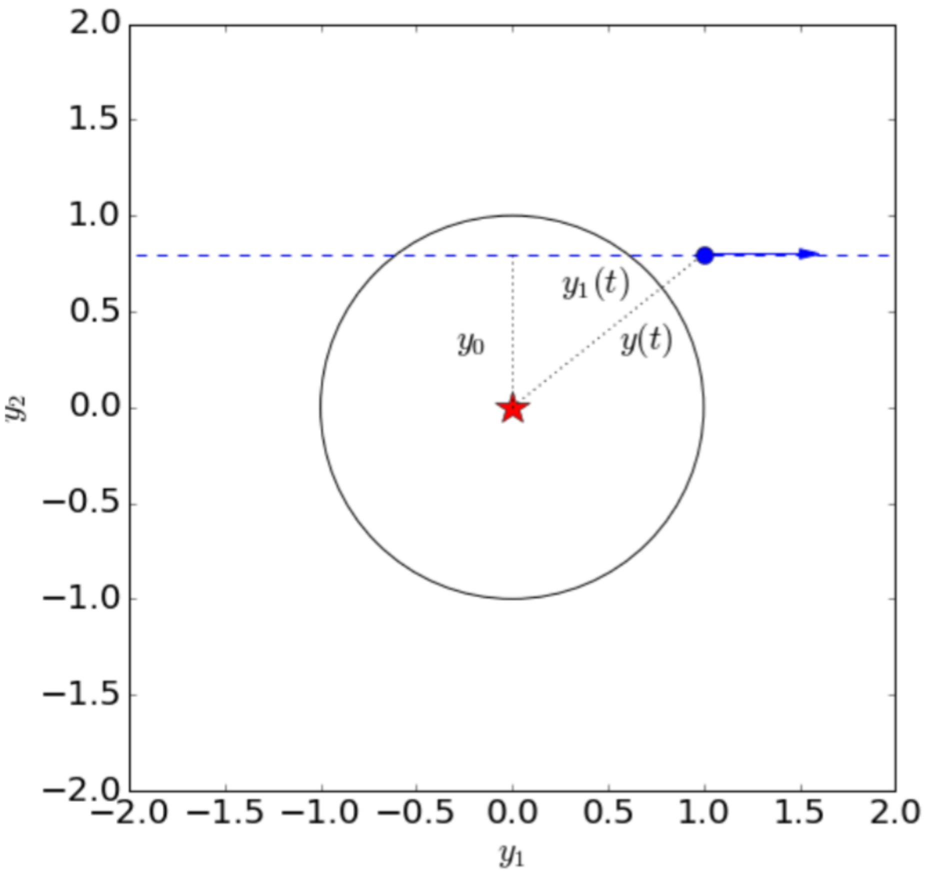
$$\mu(t) = \frac{y^2(t) + 2}{y(t)\sqrt{y^2(t) + 4}}$$

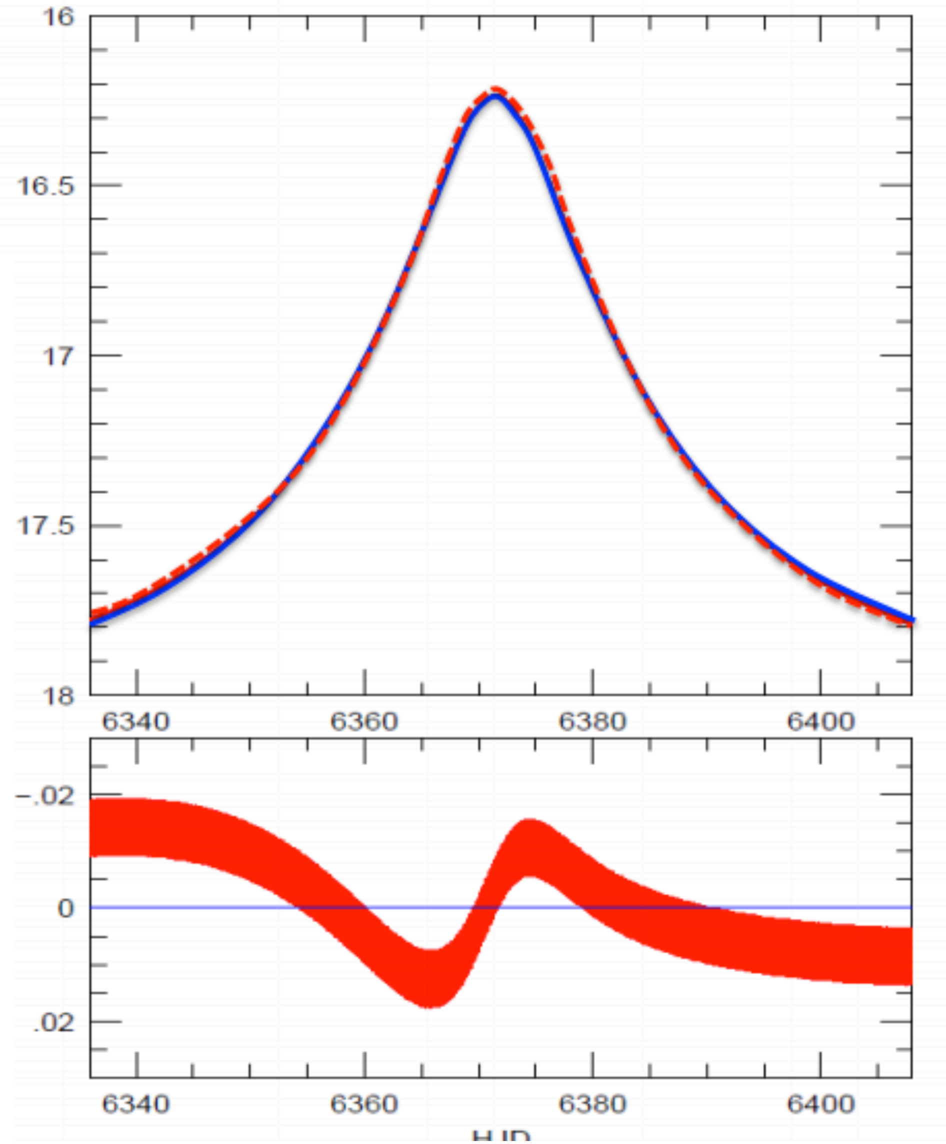
…but microlensing parallax does!

Depending on the lens displacement relative to the source (parallel or perpendicular to the proper motion), we will see different effects

COMPONENT PARALLEL TO THE LENS TRAJECTORY

Asymmetric distortion of the light curve due to acceleration of the lens





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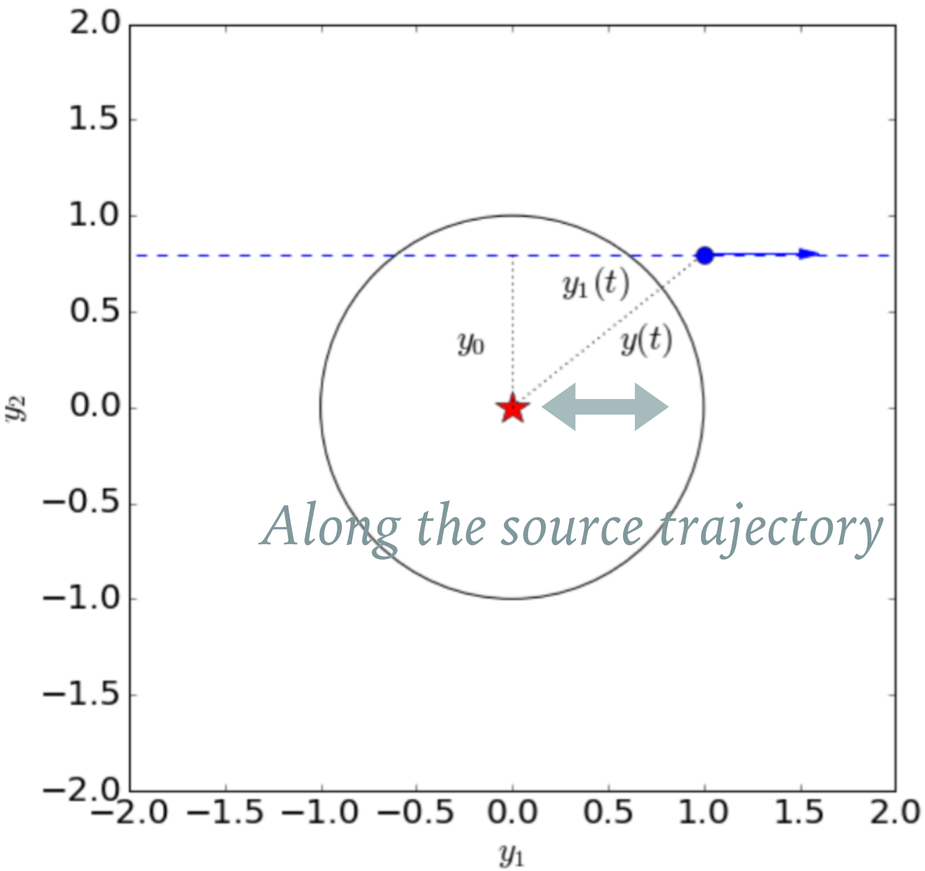
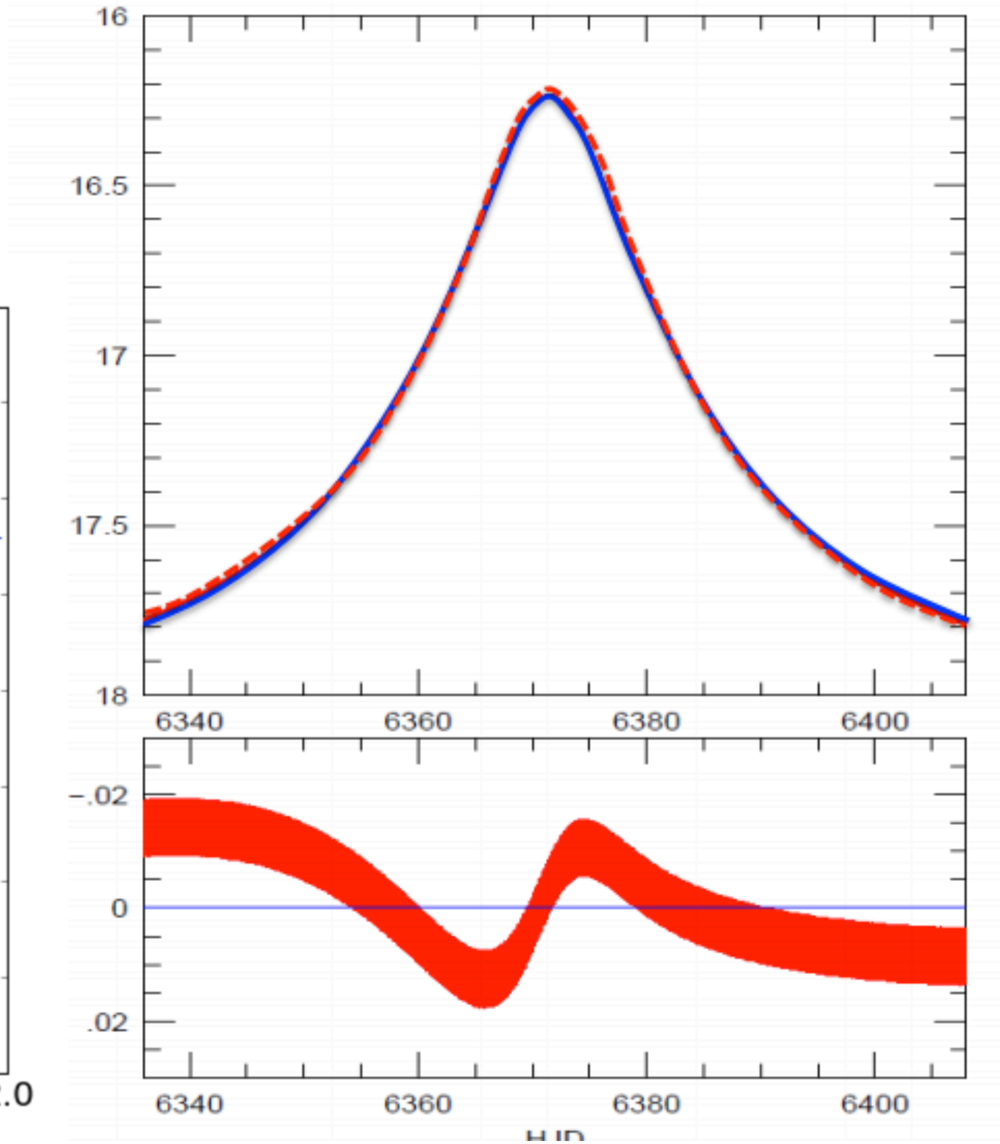
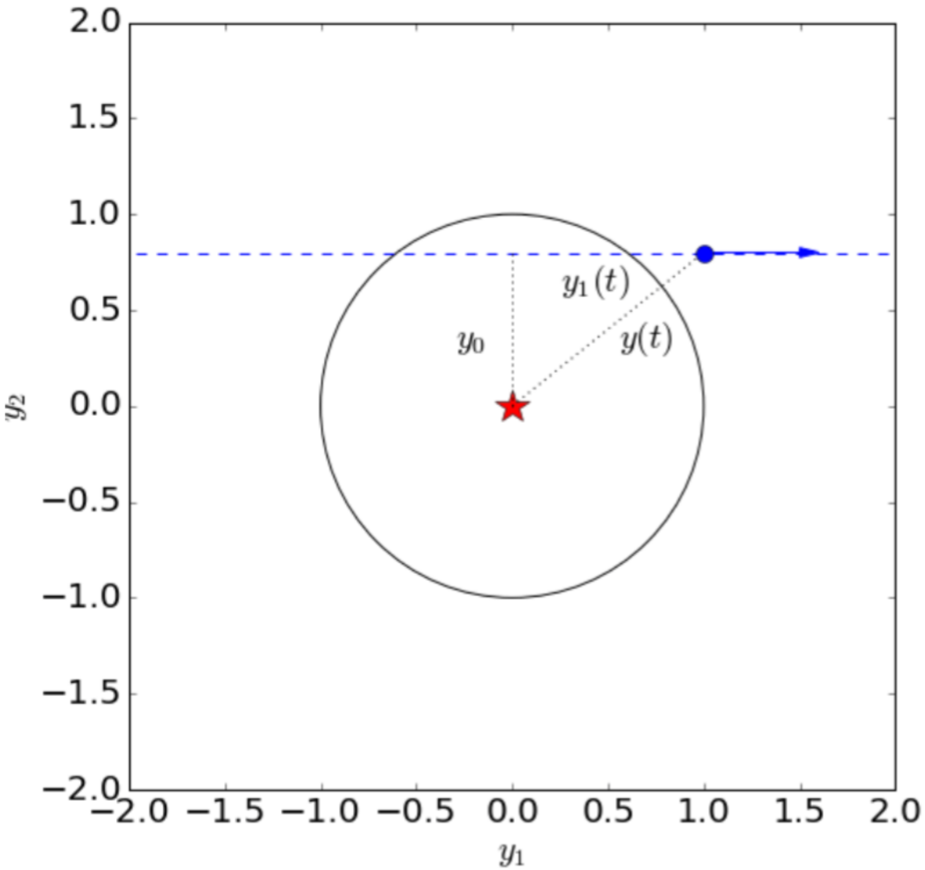
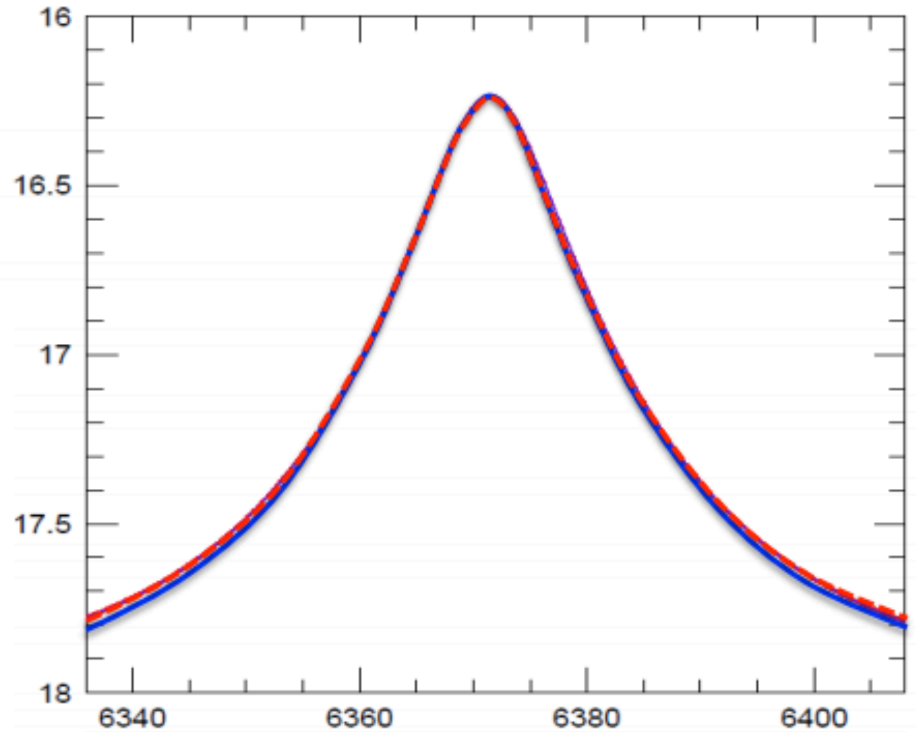




COMPONENT PERPENDICULAR TO THE LENS TRAJECTORY

Symmetric distortion of the light curve due to motion perpendicular to lens trajectory





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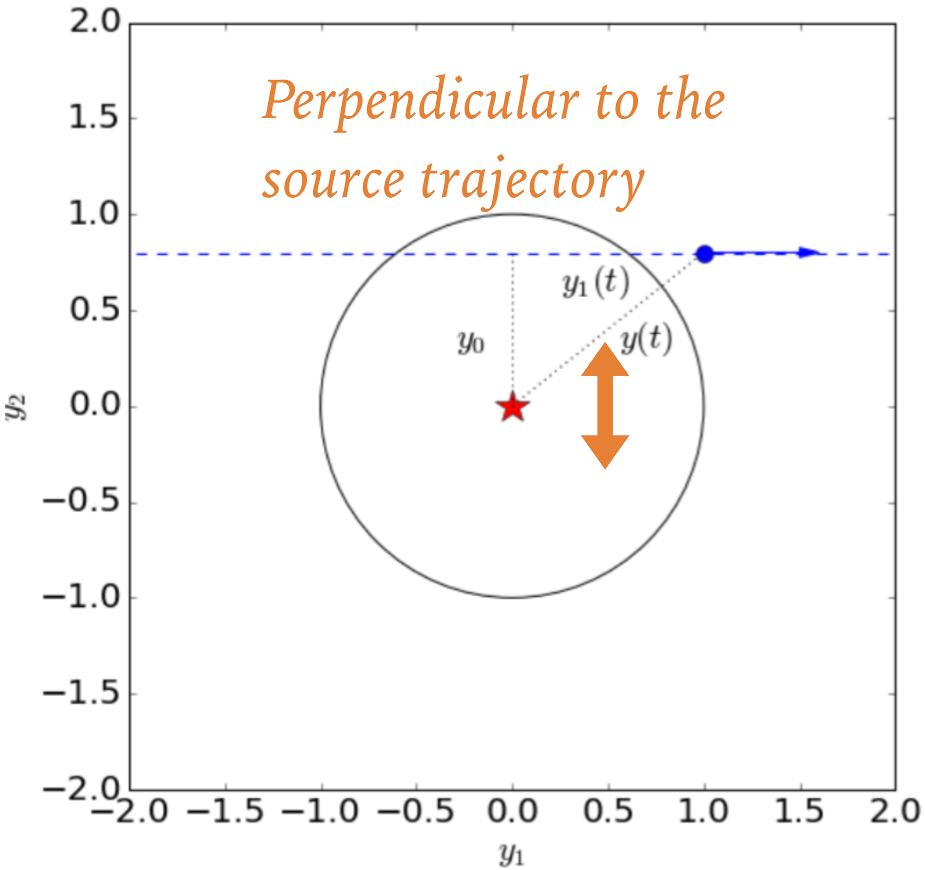
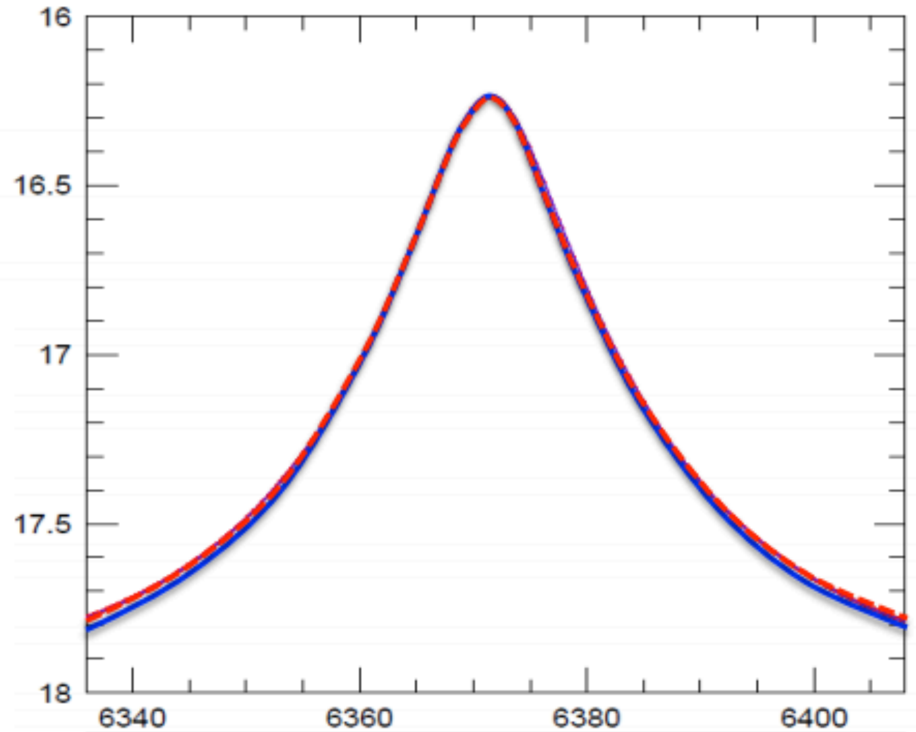
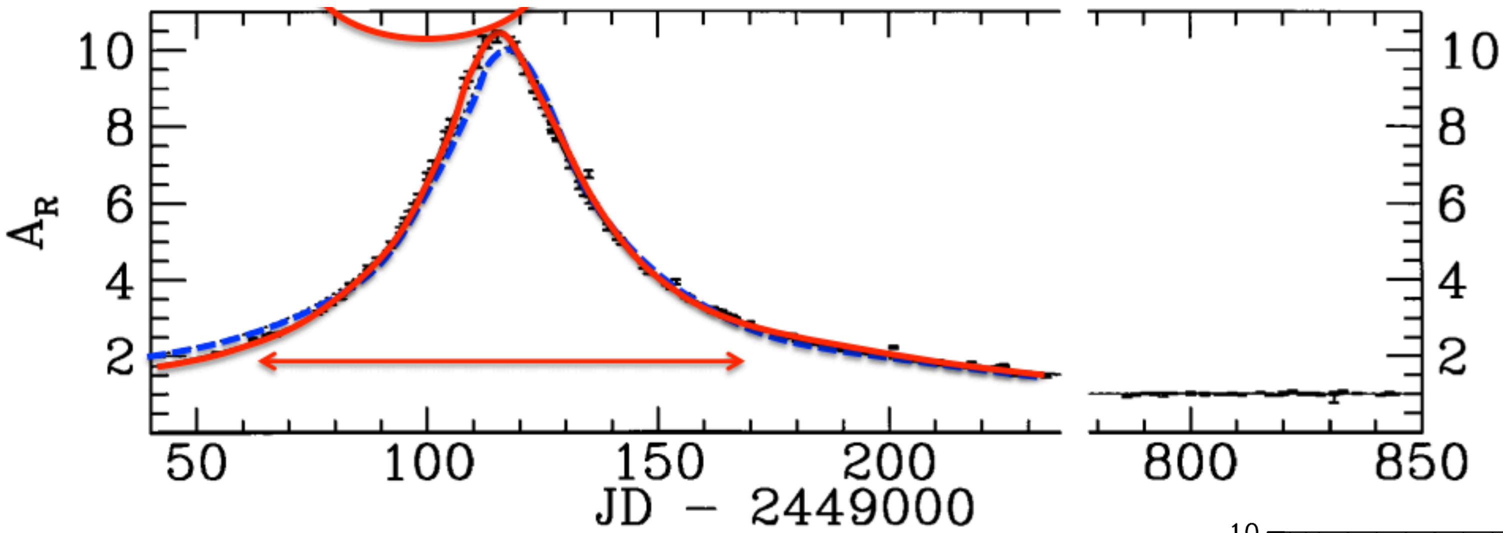


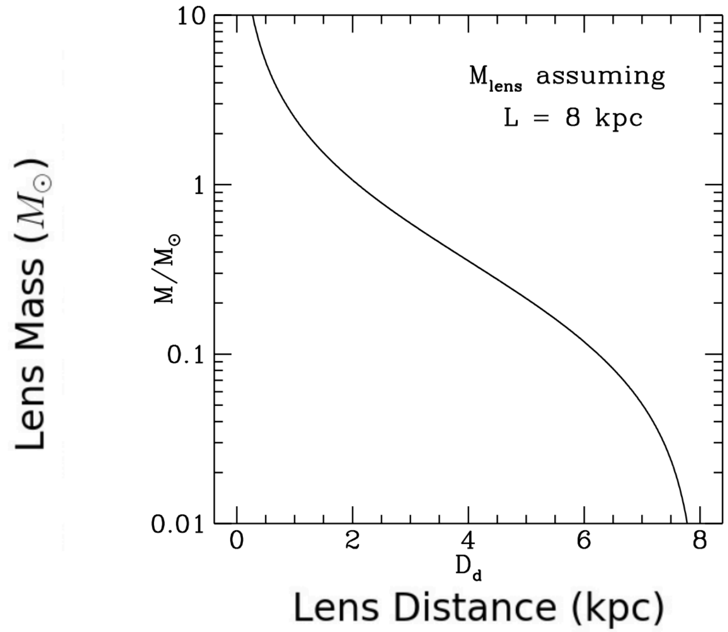


FIRST DETECTION OF ORBITAL PARALLAX

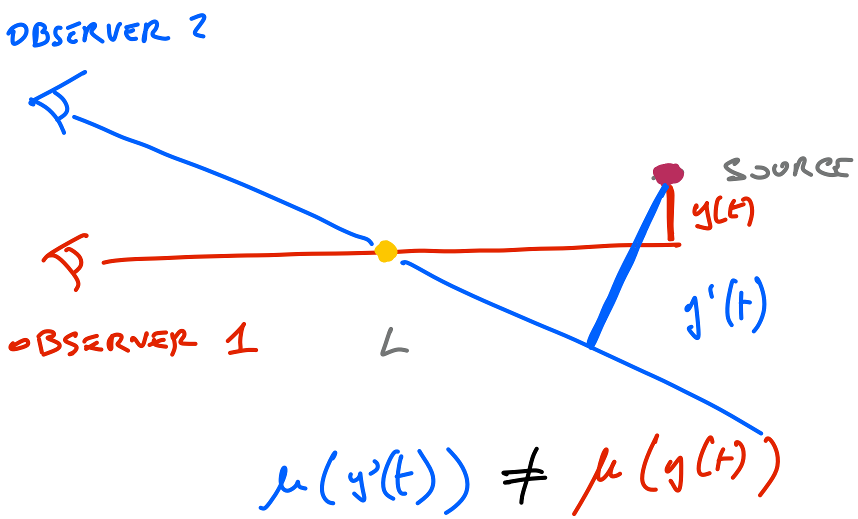
Alcock et al. 1995

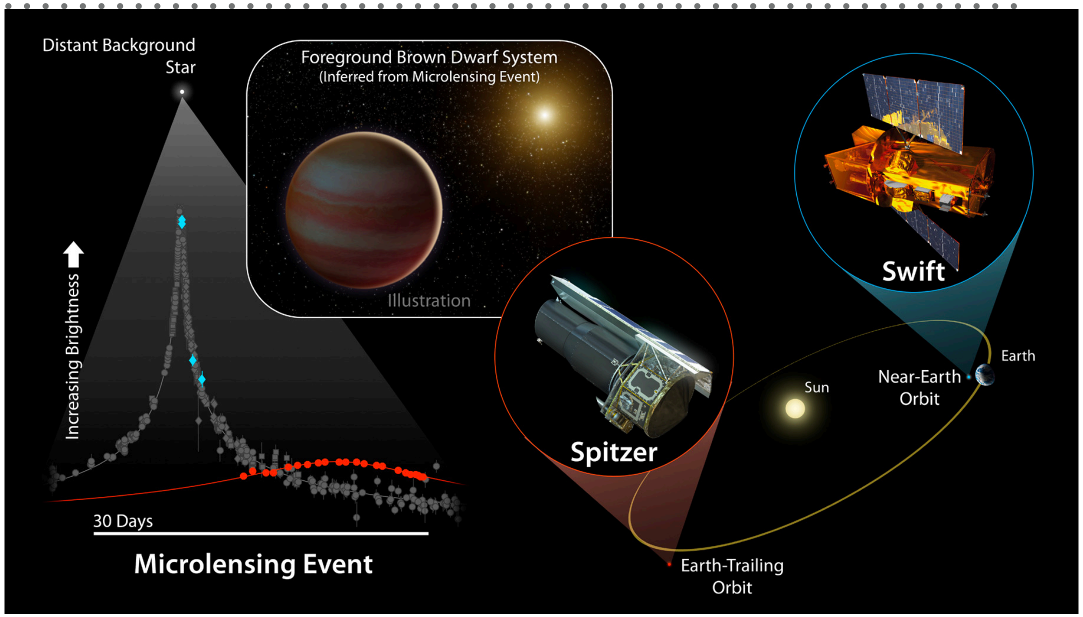


Even without an estimate of θ_E, measuring the parallax still allows to measure the mass vs distance

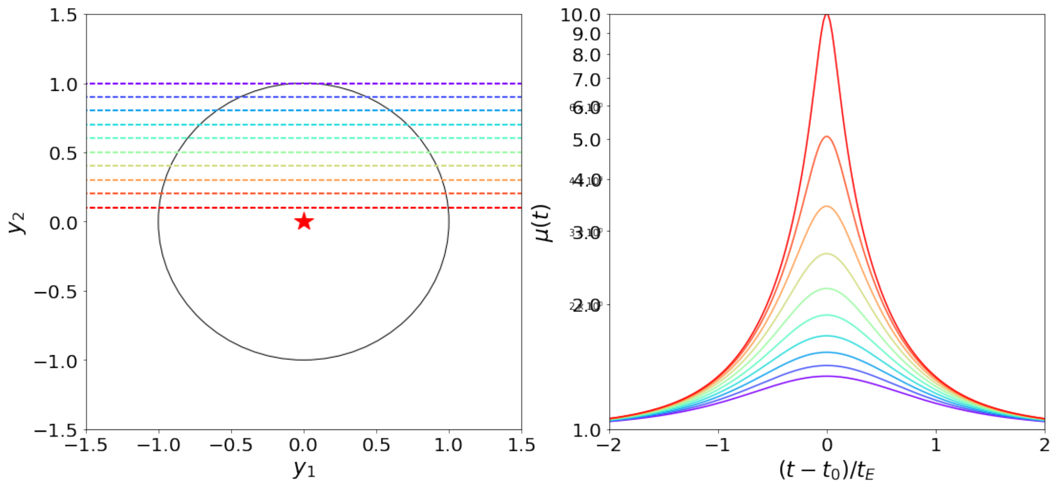


SATELLITE PARALLAX

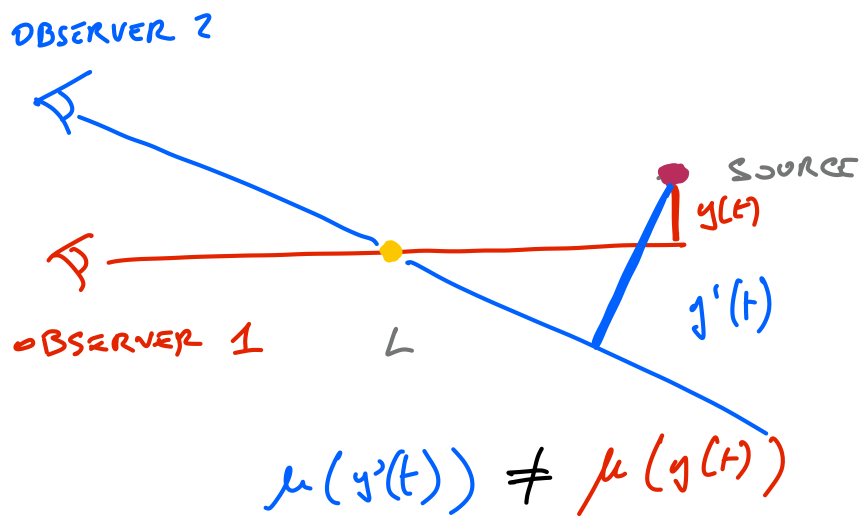


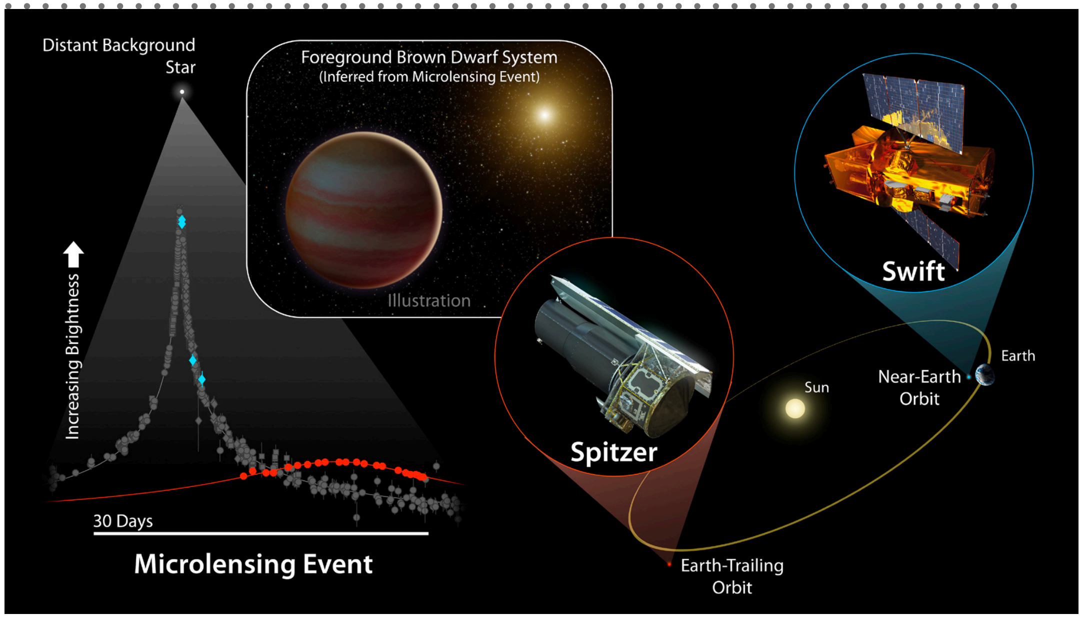


$$\mu(y(t)) = \frac{y(t)^2 + 2}{y(t)\sqrt{y(t)^2 + 4}}$$



SATELLITE PARALLAX





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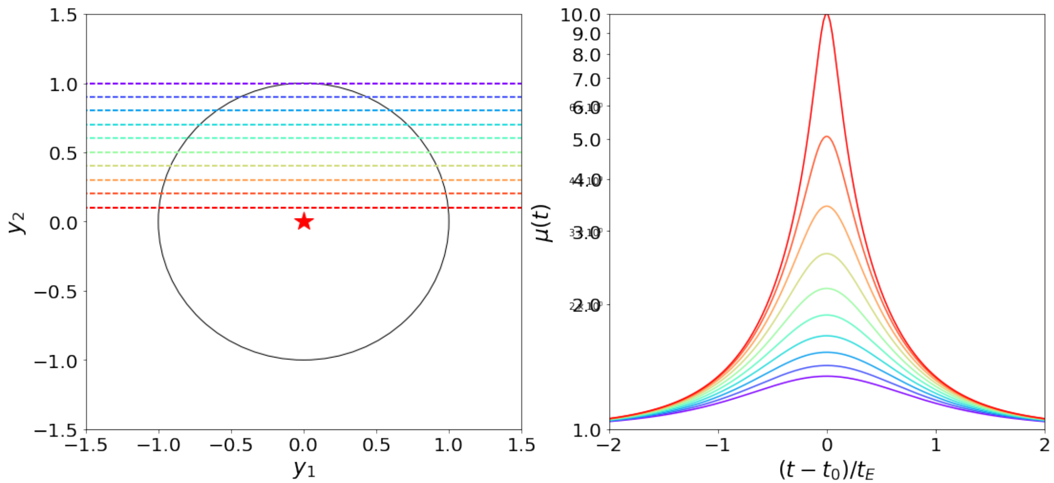
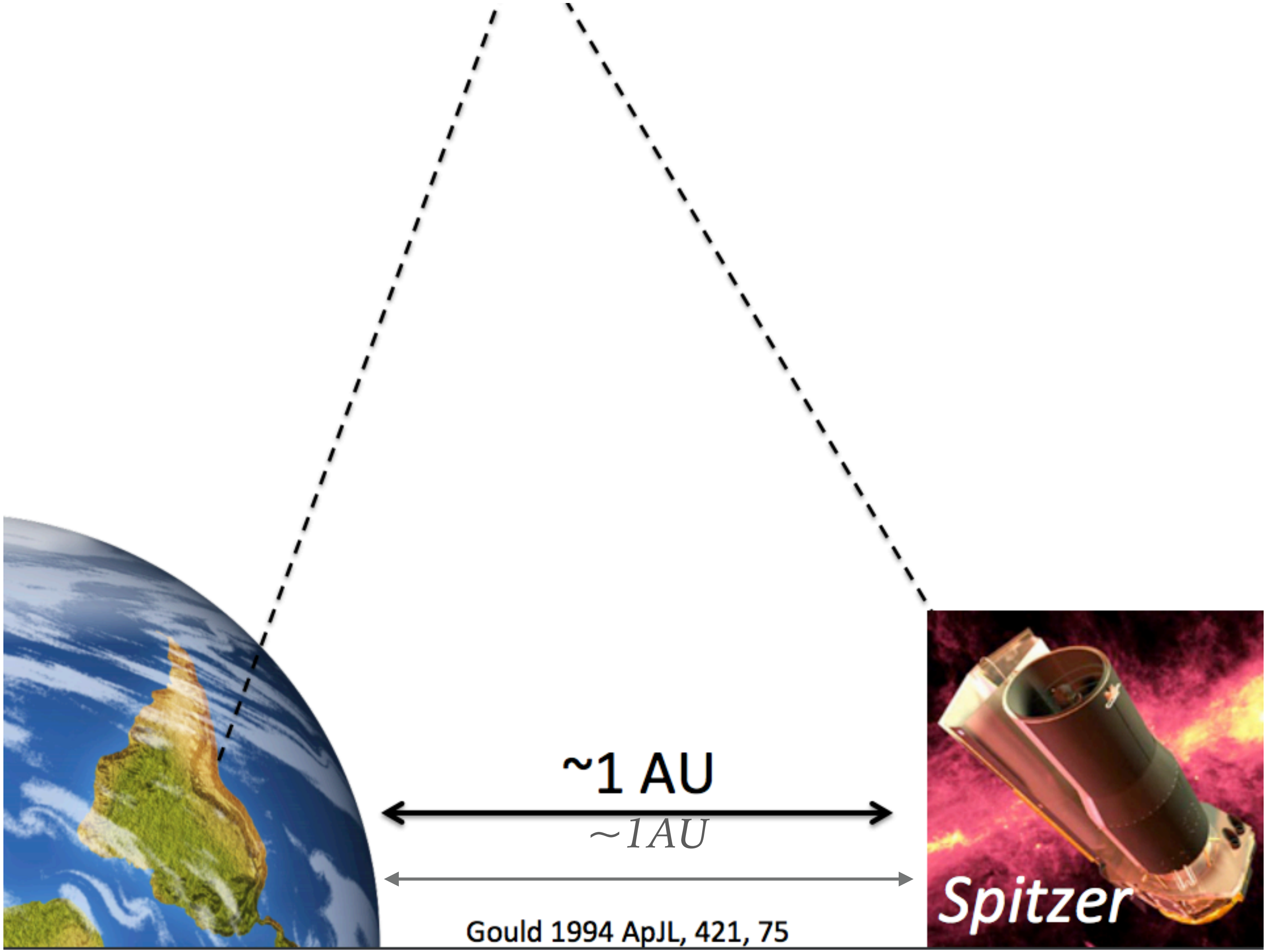
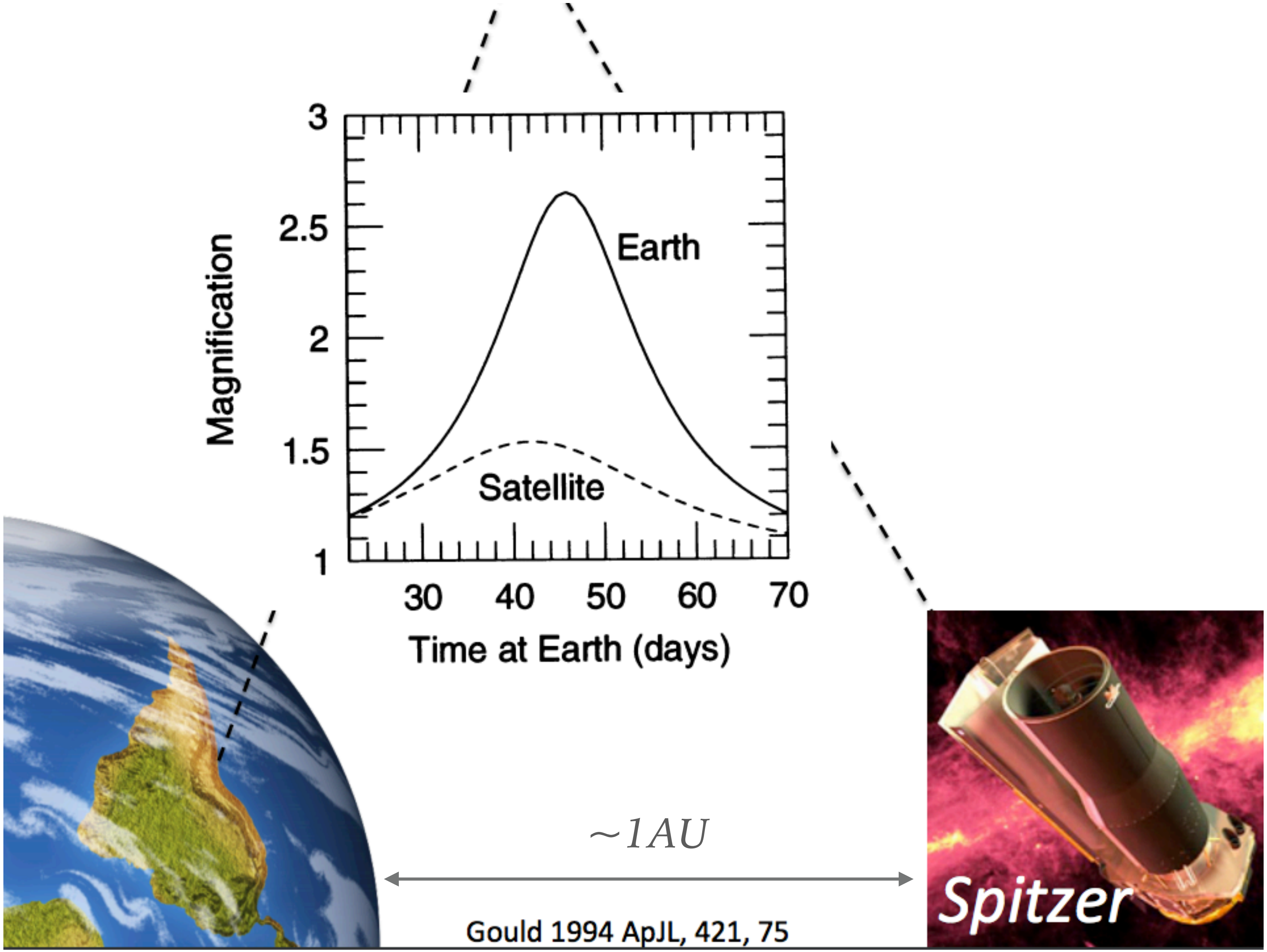


Two observers looking at the same Microlensing event will see two different light curves (under some circumstances).

SATELLITE PARALLAX



SATELLITE PARALLAX



Gould 1994 ApJL, 421, 75

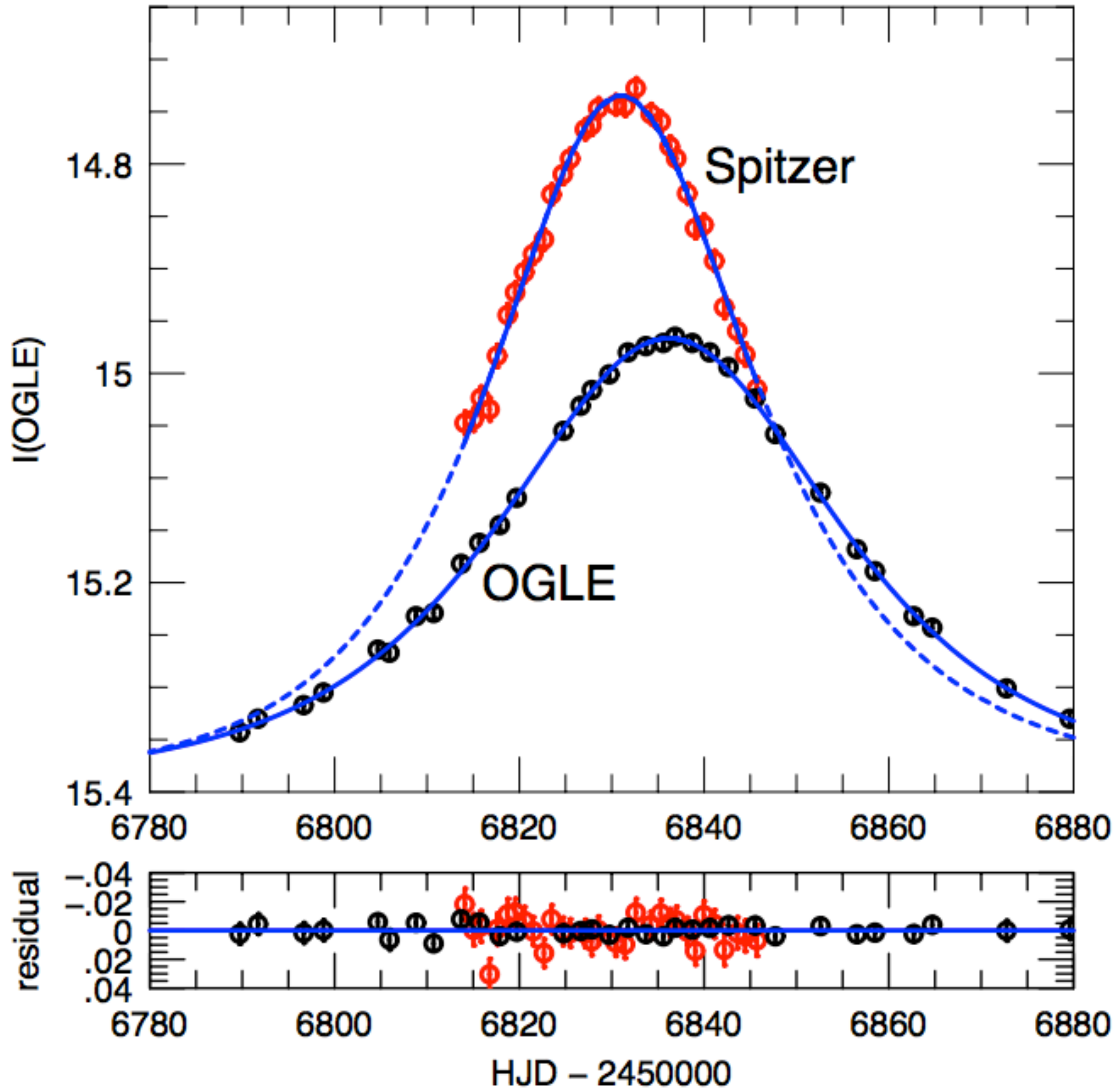
ORBITAL PARALLAX

Similarly, in the case of orbital parallax, what matters is the Einstein crossing time:

Are we likely to see annual parallax effects for an event with $t_E \sim 10$ *days?*

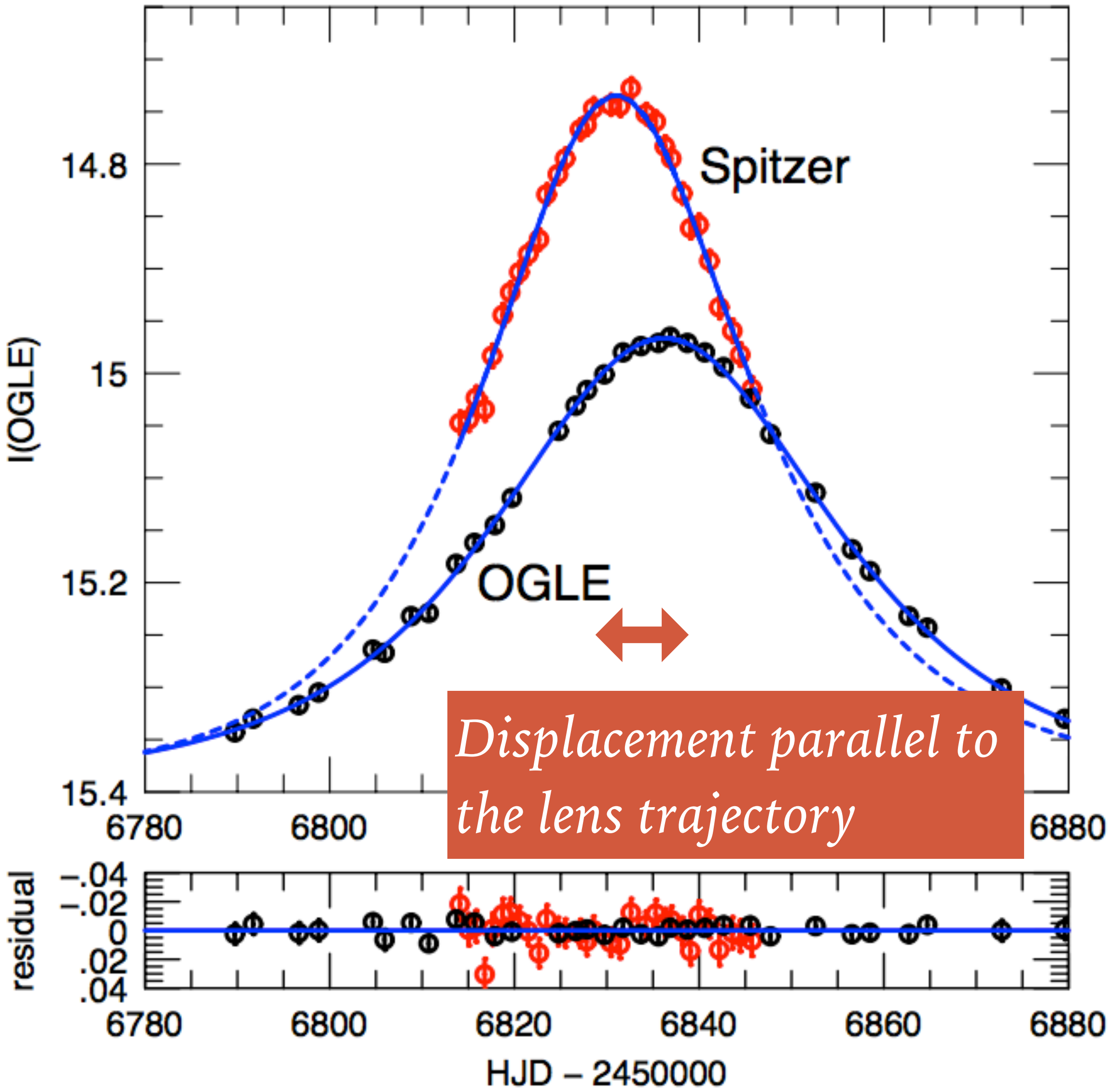
And for an event with $t_E \sim 100$ *days?*

SATELLITE PARALLAX



Yee et al. 2015, ApJ, 802, 76

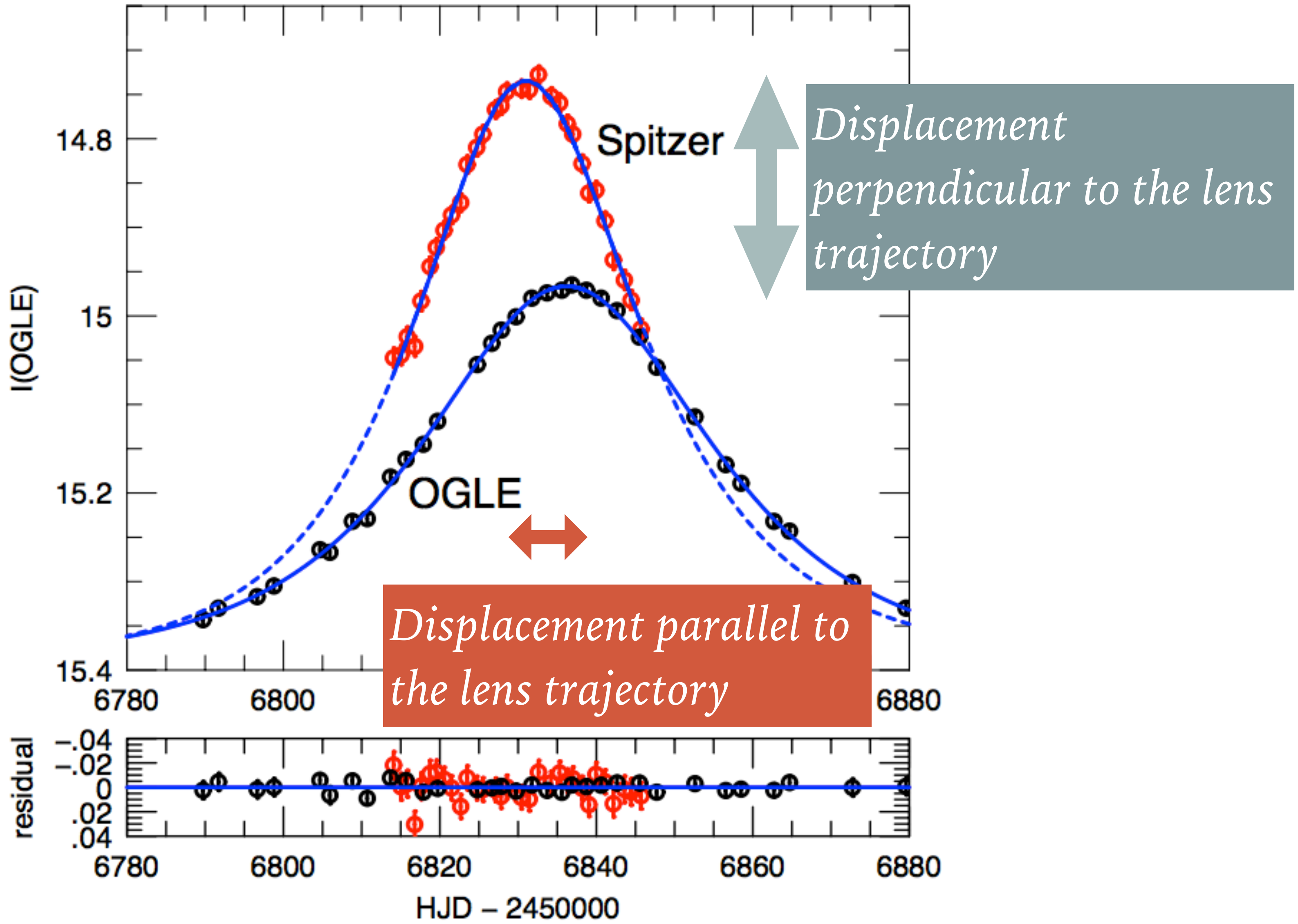
SATELLITE PARALLAX



Displacement parallel to the lens trajectory

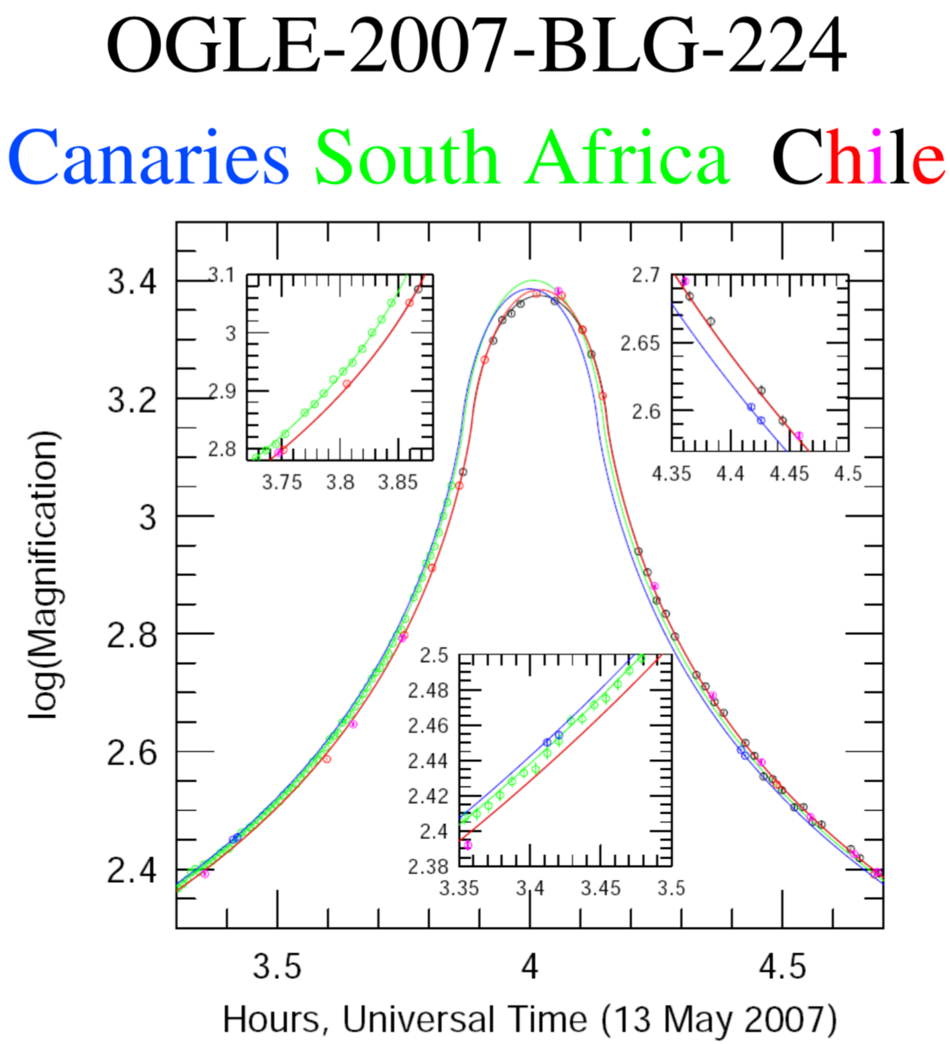
Yee et al. 2015, ApJ, 802, 76

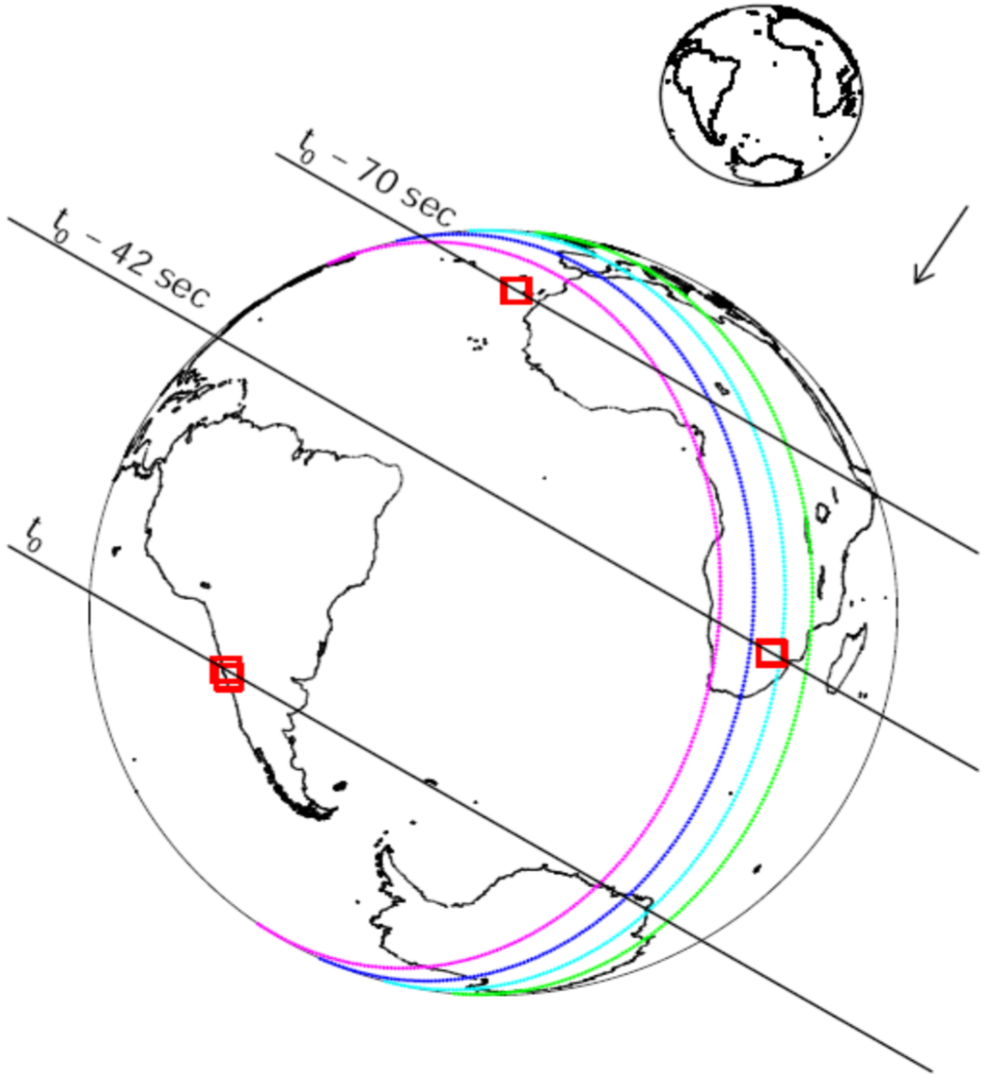
SATELLITE PARALLAX



Yee et al. 2015, ApJ, 802, 76

MICROLENS PARALLAX (TERRESTRIAL)

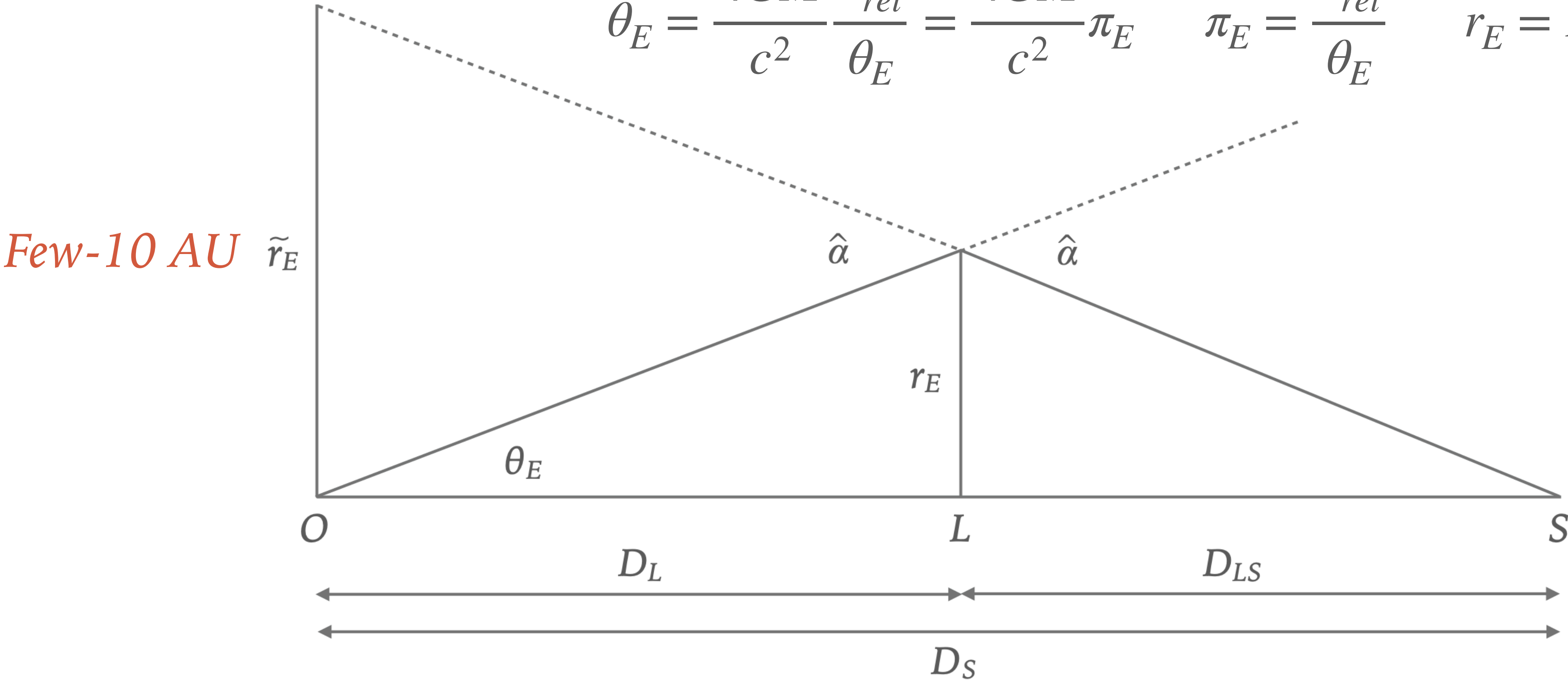




Gould & Horne, 2013

RELEVANT BASELINE

$$\theta_E = \frac{4GM}{c^2}\frac{\pi_{rel}}{\theta_E} = \frac{4GM}{c^2}\pi_E \qquad \pi_E = \frac{\pi_{rel}}{\theta_E} \qquad r_E = D_L\theta_E$$



The relevant scale for parallax is the size of the Einstein radius on the observer plane

$$\tilde{r}_E = D_L\hat{\alpha}(\theta_E) = D_L\frac{4GM}{c^2 D_L \theta_E} = \frac{1}{\pi_E}$$

ASTROMETRIC MEASUREMENT OF THE RELATIVE PROPER MOTION

- Another method to measure the Einstein radius is by measuring the relative proper motion of source and lens.
- This requires to see the lens!
- If we have high resolution imaging with HST or ground based AO, we can measure the position of lens and source at some time after the maximum of the light curve Δt
- If we measure a shift $\Delta\theta$ then $\mu_{rel} = \Delta\theta / \Delta t$ and

$$\theta_E = t_E \times \mu_{rel}$$

ASTROMETRIC MICROLENSING

Position of the image centroid relative to the unlensed position.

$$\delta\vec{x} = \frac{|\mu_1|\vec{x}_1 + |\mu_2|\vec{x}_2}{|\mu_1| + |\mu_2|} - \vec{y}$$

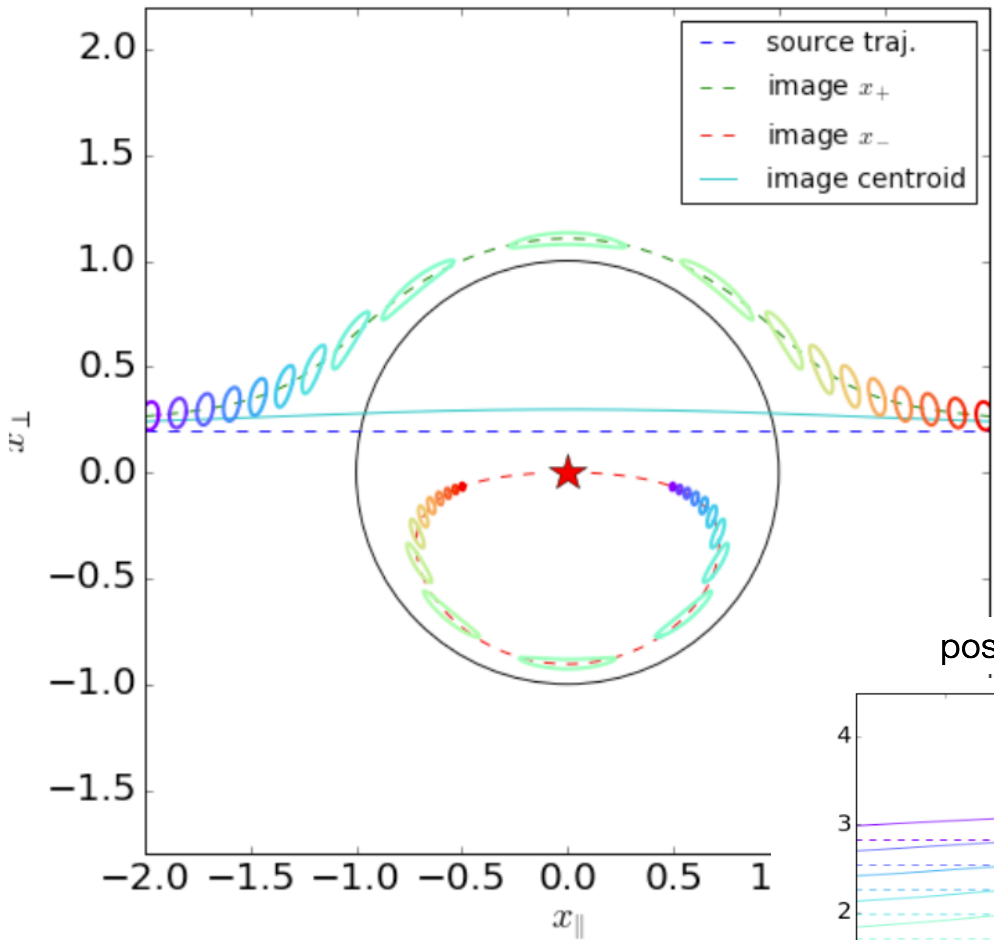
$$|\delta\vec{x}| = \frac{y}{y^2 + 2}$$

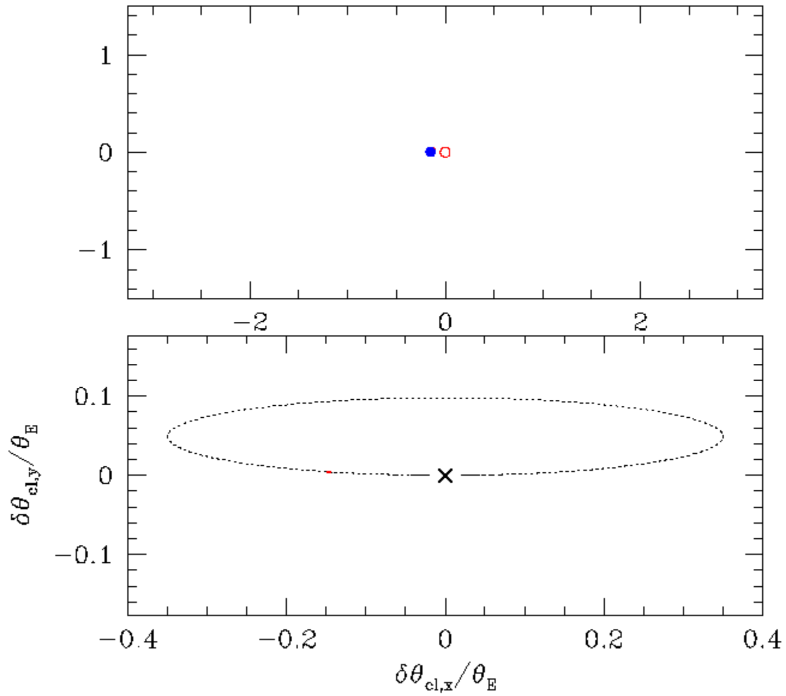
Components that are parallel and perpendicular to the motion of the star relative to the lens.

$$\vec{x}_{\|} = \delta\vec{x} \cdot \hat{y} = \frac{y_{\|}}{y^2 + 2} = \frac{(t - t_o)/t_E}{(t - t_o)^2/t_E^2 + y_o^2 + 2}$$

$$\vec{x}_{\perp} = \delta\vec{x} - (\delta\vec{x} \cdot \hat{y})\hat{y} = \frac{y_{\perp}}{y^2 + 2} = \frac{y_o}{(t - t_o)^2/t_E^2 + y_o^2 + 2}$$

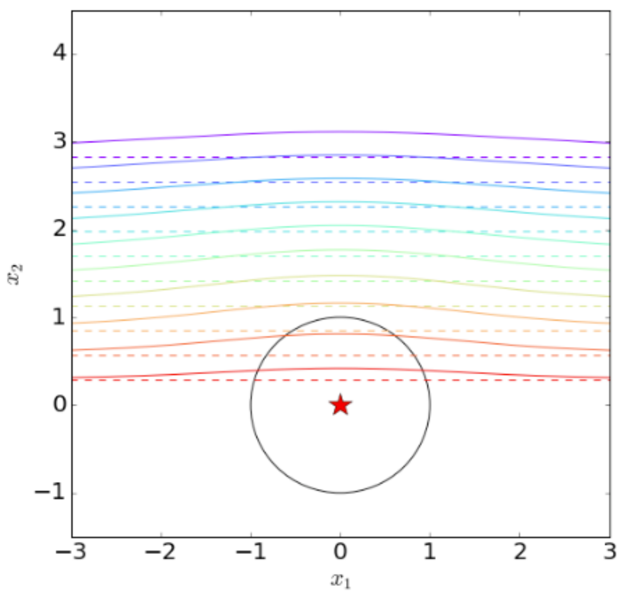
ASTROMETRIC MICROLENSING





$$\vec{x}_c = \frac{\vec{x}_+ \mu_+ + \vec{x}_- |\mu_-|}{\mu_+ + |\mu_-|}$$

position relative to lens



position relative to unlensed source position