GRAVITATIONAL LENSING

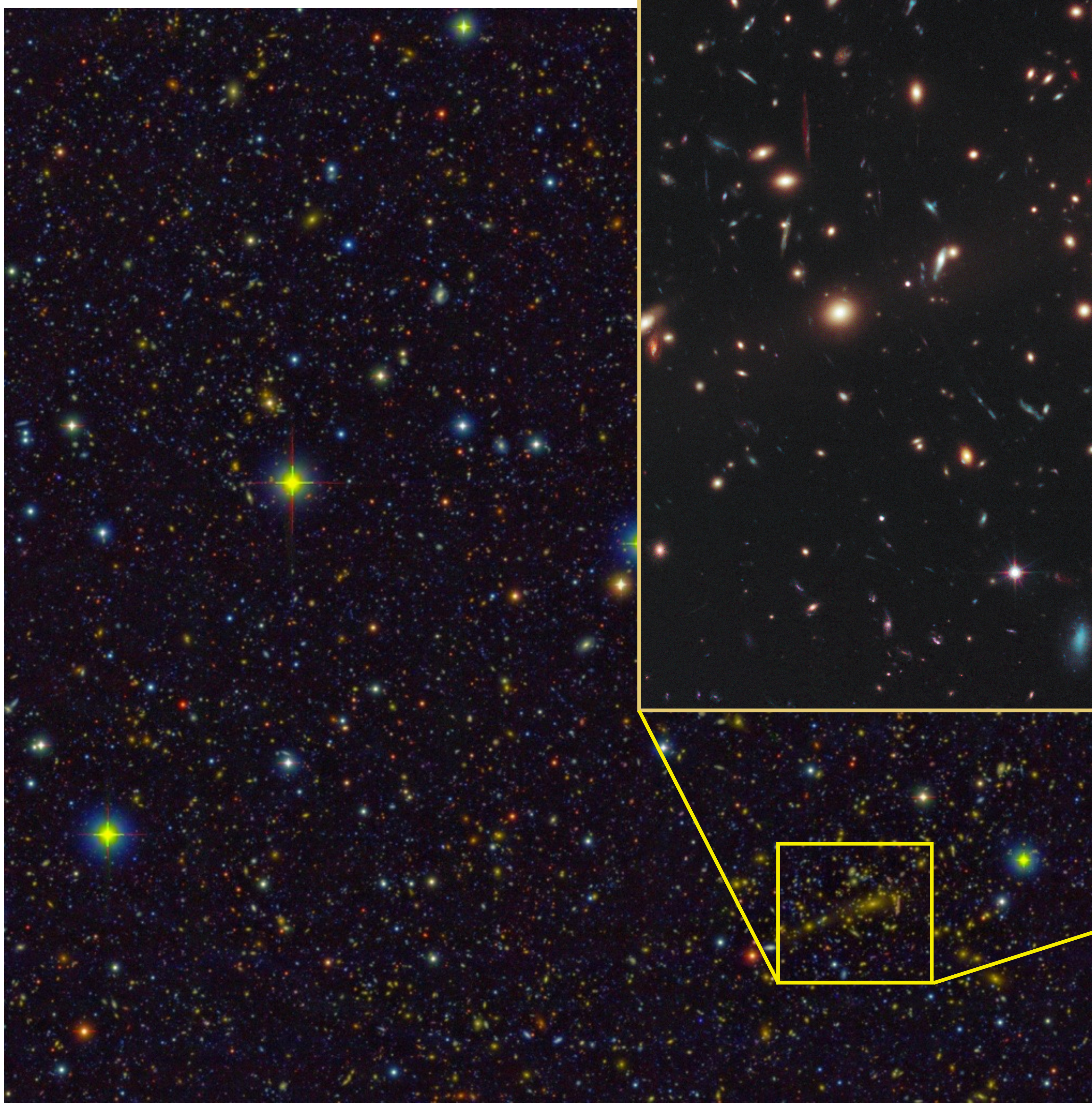
WEAK LENSING BY GALAXY CLUSTERS

R. Benton Metcalf

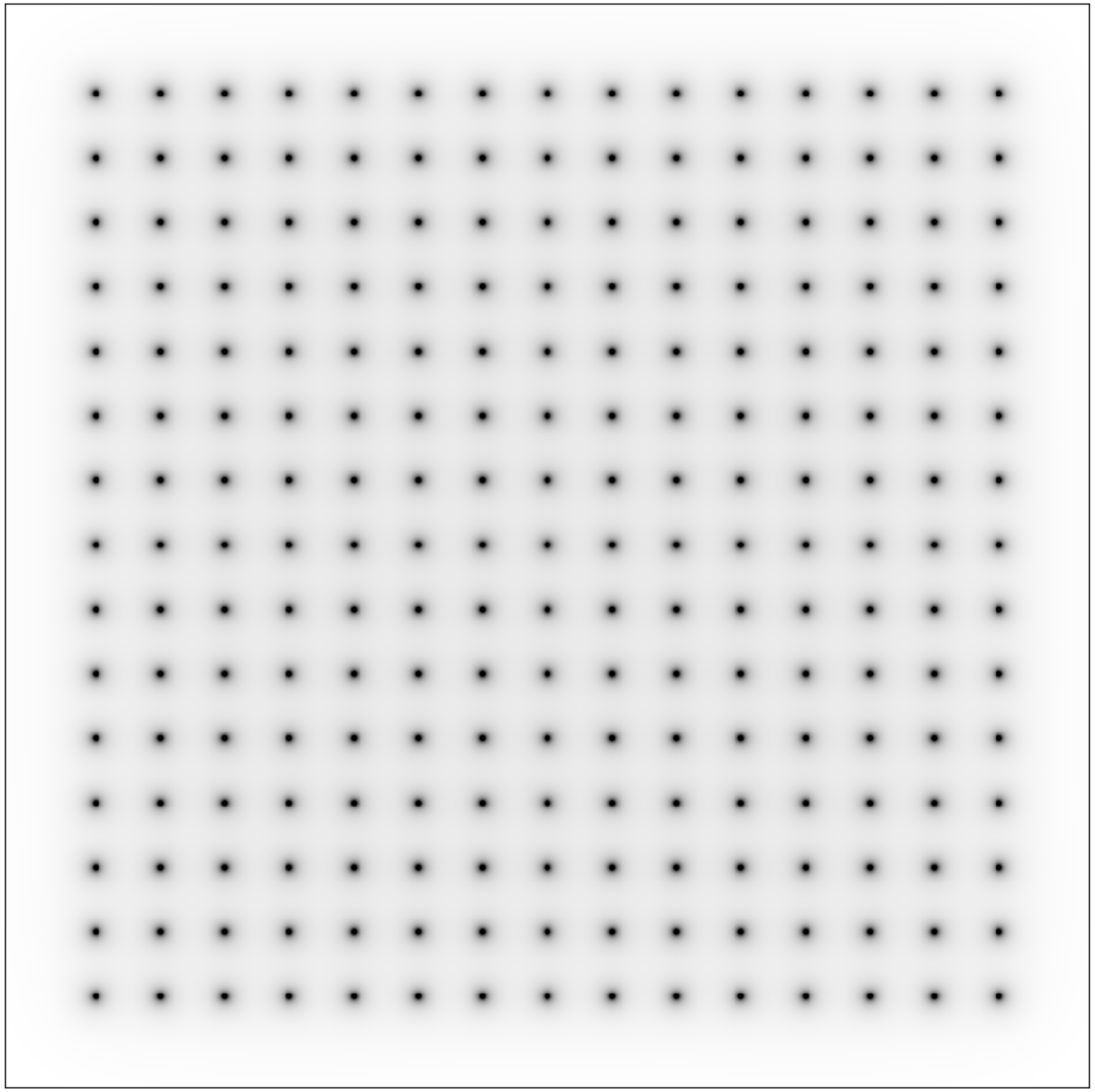
2022-2023

A WIDER FIELD OF VIEW AROUND A CLUSTER

A WIDER FIELD OF VIEW



EXAMPLE WITH AN ELLIPTICAL LENS



1) *Arcs and large distortions near the critical lines*
2) *As we move away from this region, images become similar to ellipses*

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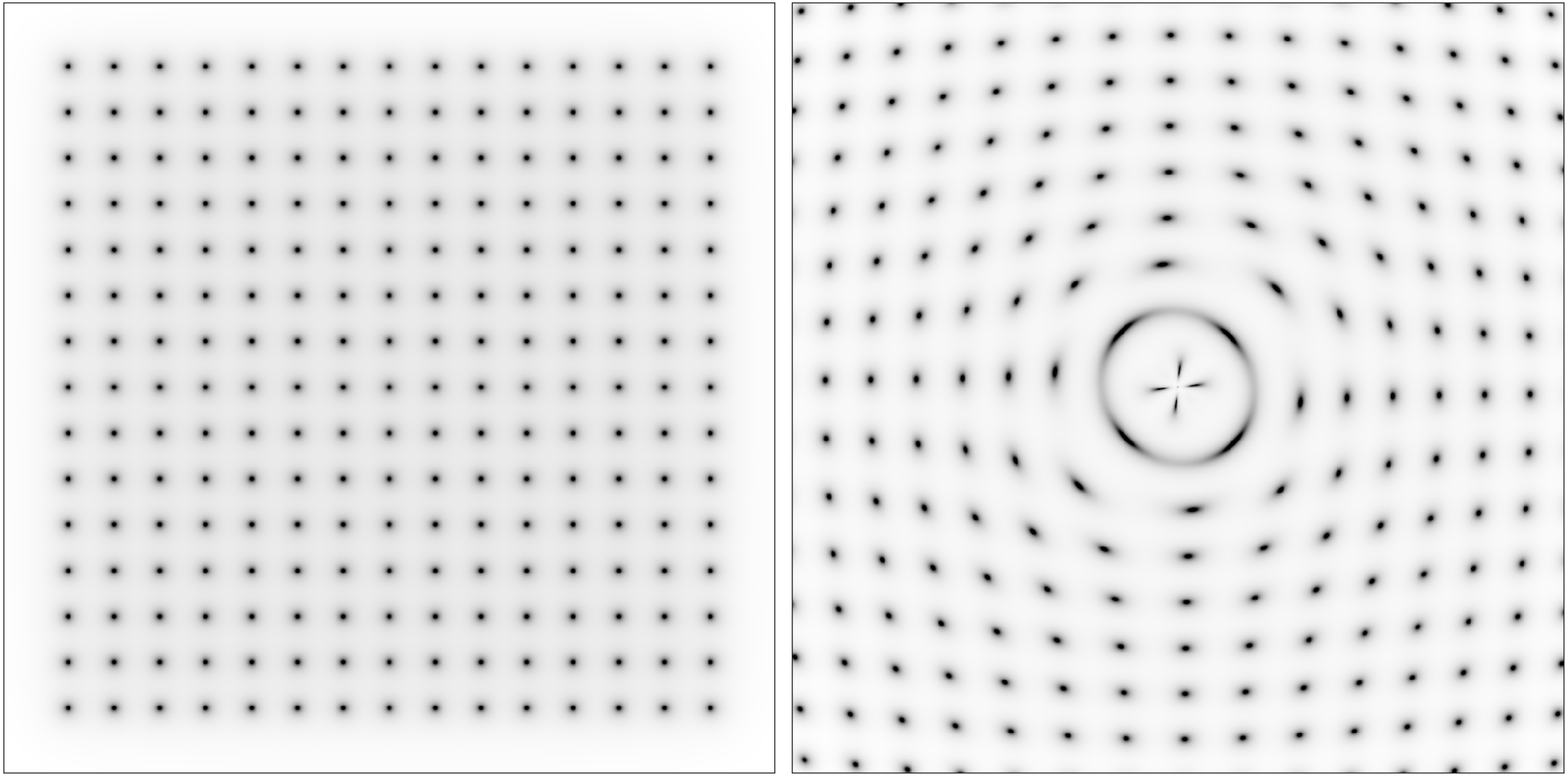

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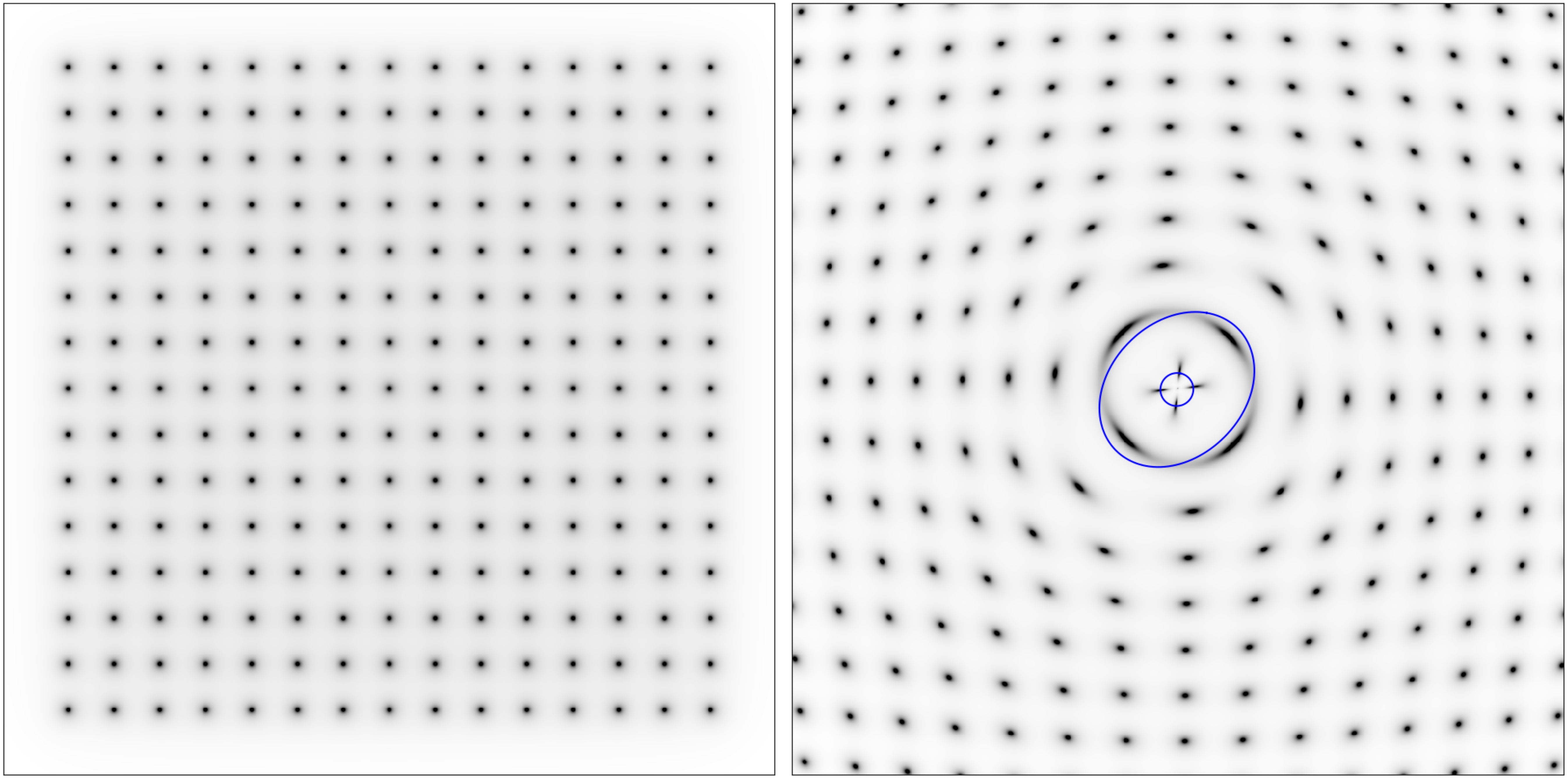


1) *Arcs and large distortions near the critical lines*
2) *As we move away from this region, images become similar to ellipses*

WEAK LENSING BY GALAXY CLUSTERS

- Far from the critical lines, the deflection angle varies very slowly
- Over regions comparable to the size of distant galaxies the convergence and the shear can be considered to be constant
- This means that the first derivatives of the deflection field are constant and the second derivatives are zero
- i.e. the lens mapping is linear and fully described by the lensing Jacobian

EXAMPLE: FIRST ORDER DISTORTION OF A CIRCULAR SOURCE

$$\beta_1^2 + \beta_2^2 = \beta^2$$

In the reference frame where A is diagonal:

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 1-\kappa-\gamma & 0 \\ 0 & 1-\kappa+\gamma \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

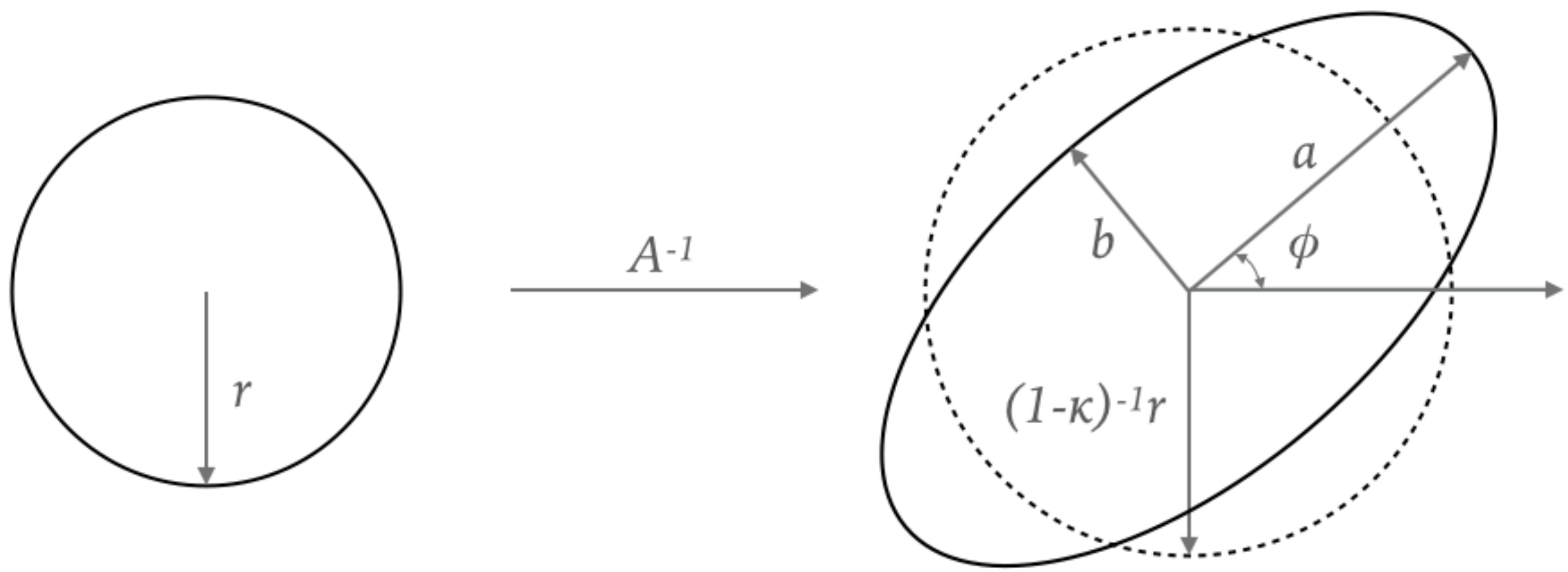
$$\begin{aligned} \beta_1 &= (1-\kappa-\gamma)\theta_1 \\ \beta_2 &= (1-\kappa+\gamma)\theta_2 \end{aligned}$$

$$\beta^2 = \beta_1^2 + \beta_2^2 = (1-\kappa-\gamma)^2\theta_1^2 + (1-\kappa+\gamma)^2\theta_2^2$$

This is the equation of an ellipse with semi-axes:

$$a = \frac{\beta}{1-\kappa-\gamma} \qquad b = \frac{\beta}{1-\kappa+\gamma}$$

EXAMPLE: FIRST ORDER DISTORTION OF A CIRCULAR SOURCE

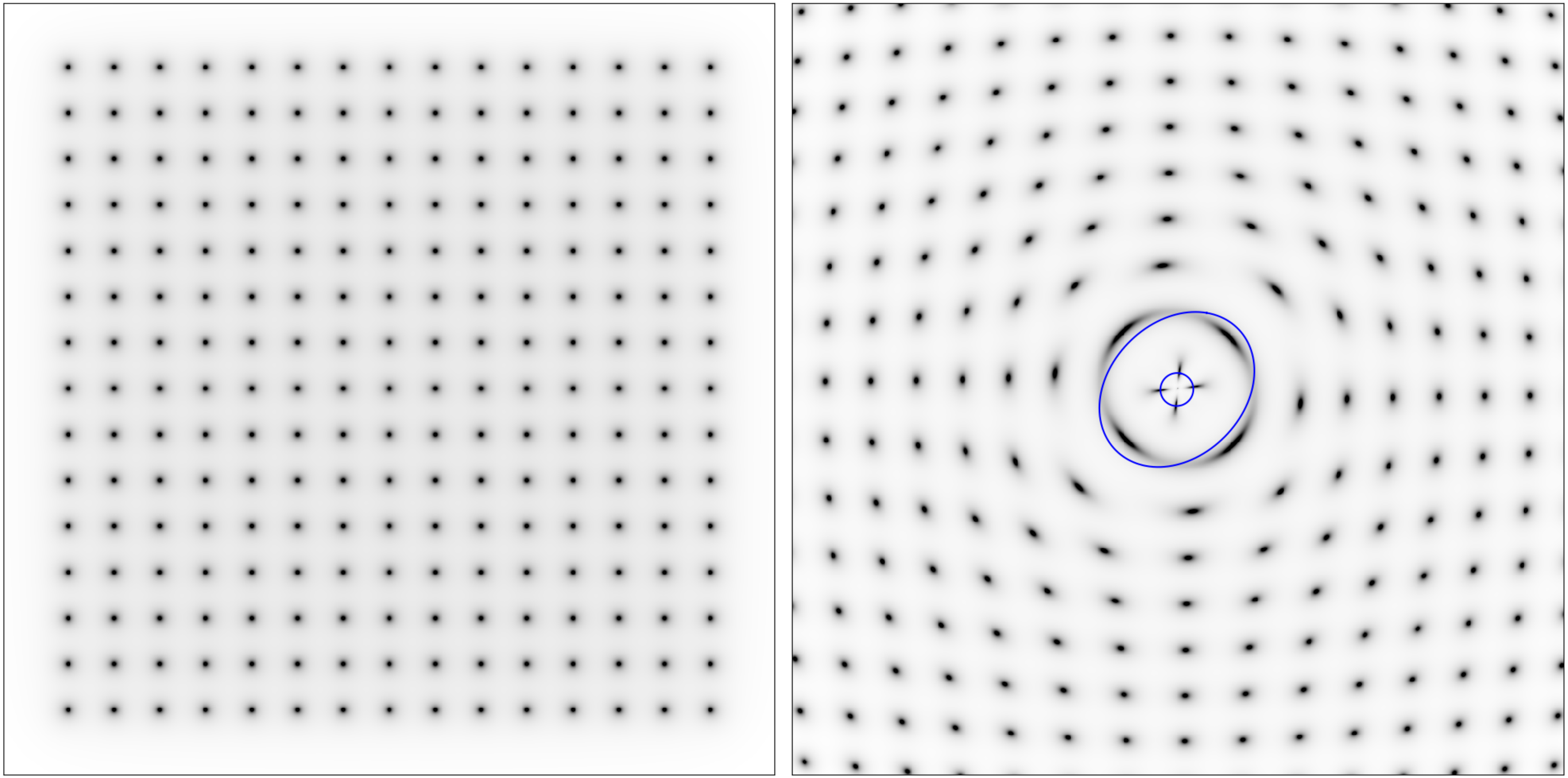


convergence: *responsible for isotropic expansion or contraction*

shear: *responsible for anisotropic distortion*

Ellipticity: $$\epsilon = \frac{a-b}{a+b} = \frac{\gamma}{1-\kappa} = g$$

EXAMPLE WITH AN ELLIPTICAL LENS



1) *the modulus of the ellipticity decreases as we move away from the lens*
2) *There is a coherent alignment of the lensed images: ellipticity is not a scalar, but a tensor!*

SHEAR AND ELLIPTICITY AS TENSORS

By analogy with the shear, we can define the reduced shear and the ellipticity tensors:

$$\Gamma = \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix} \Rightarrow \tilde{g} = \begin{pmatrix} g_1 & g_2 \\ g_2 & -g_1 \end{pmatrix} \quad \tilde{\epsilon} = \begin{pmatrix} \epsilon_1 & \epsilon_2 \\ \epsilon_2 & -\epsilon_1 \end{pmatrix}$$

To define the ellipticity, we need two components (or the modulus and the angle φ between the θ_1 axis and the eigenvectors of the tensor with eigenvalue ϵ).

$$\begin{aligned} g_1 &= g \cos 2\varphi \\ g_2 &= g \sin 2\varphi \end{aligned} \qquad \begin{aligned} \epsilon_1 &= \epsilon \cos 2\varphi \\ \epsilon_2 &= \epsilon \sin 2\varphi \end{aligned}$$

We can also define the complex reduced shear and the complex ellipticity:

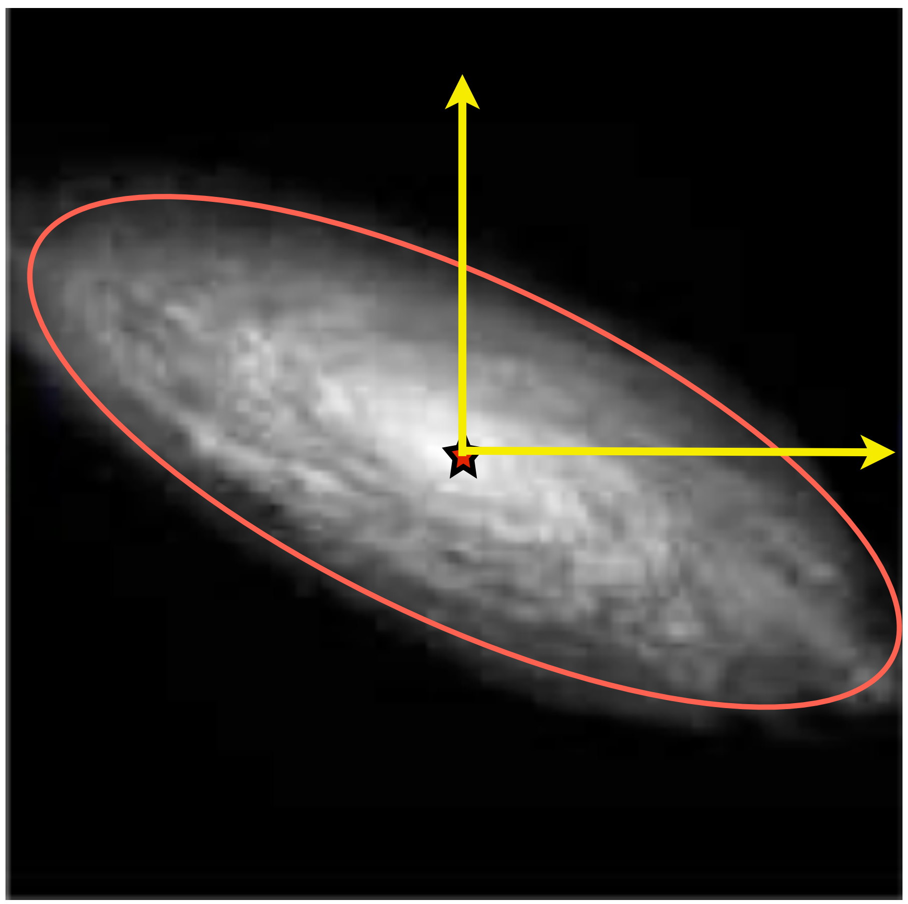
$$g = g_1 + ig_2$$

$$\epsilon = \epsilon_1 + i\epsilon_2$$

THE IDEA

- Measuring the ellipticity tensor (complex ellipticity), we measure a combination of γ and κ
- These are both related to the lensing potential:
$$\kappa(\overrightarrow{\theta}) = \frac{\Sigma(\overrightarrow{\theta})}{\Sigma_{cr}} = \frac{1}{2}\Delta_\theta \hat{\Psi}(\overrightarrow{\theta}),\ \gamma_1 = \frac{1}{2}(\hat{\Psi}_{11} - \hat{\Psi}_{22}),\ \gamma_2 = \hat{\Psi}_{12} = \hat{\Psi}_{21}$$
- Thus, by measuring the ellipticities of lensed sources we can infer the lens mass distribution.

ELLIPTICITY FROM BRIGHTNESS MOMENTS



Observable: brightness distribution

Image centroid $$\bar{\boldsymbol{\theta}} \equiv \frac{\int \mathrm{d}^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})] \, \boldsymbol{\theta}}{\int \mathrm{d}^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})]}$$

Weight function $$q_I(I) = \mathrm{H}(I - I_{\mathrm{th}}).$$

Define a tensor of second order brightness moments:

$$Q_{ij} = \frac{\int \mathrm{d}^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})] \, (\theta_i - \bar{\theta}_i) \, (\theta_j - \bar{\theta}_j)}{\int \mathrm{d}^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})]} \, , \quad i, j \in \{1, 2\}$$

For an image with circular isophotes, $Q_{11}=Q_{22}$ and $Q_{12}=Q_{21}=0$

COMPLEX ELLIPTICITY AND REDUCED SHEAR

From Q_{ij}, one can define the complex ellipticity:

eigenvalues

$$\lambda_+ = \frac{1}{2}\left(Q_{11} + Q_{22} + \sqrt{(Q_{11} - Q_{22})^2 + 4Q_{12}^2}\right)$$

$$\lambda_- = \frac{1}{2}\left(Q_{11} + Q_{22} - \sqrt{(Q_{11} - Q_{22})^2 + 4Q_{12}^2}\right)$$

modulus of ellipticity

$$|\epsilon| = \frac{a^2 - b^2}{a^2 + b^2} = \frac{\sqrt{(Q_{11} - Q_{22})^2 + 4Q_{12}^2}}{Q_{11} + Q_{22}}$$

ellipticity

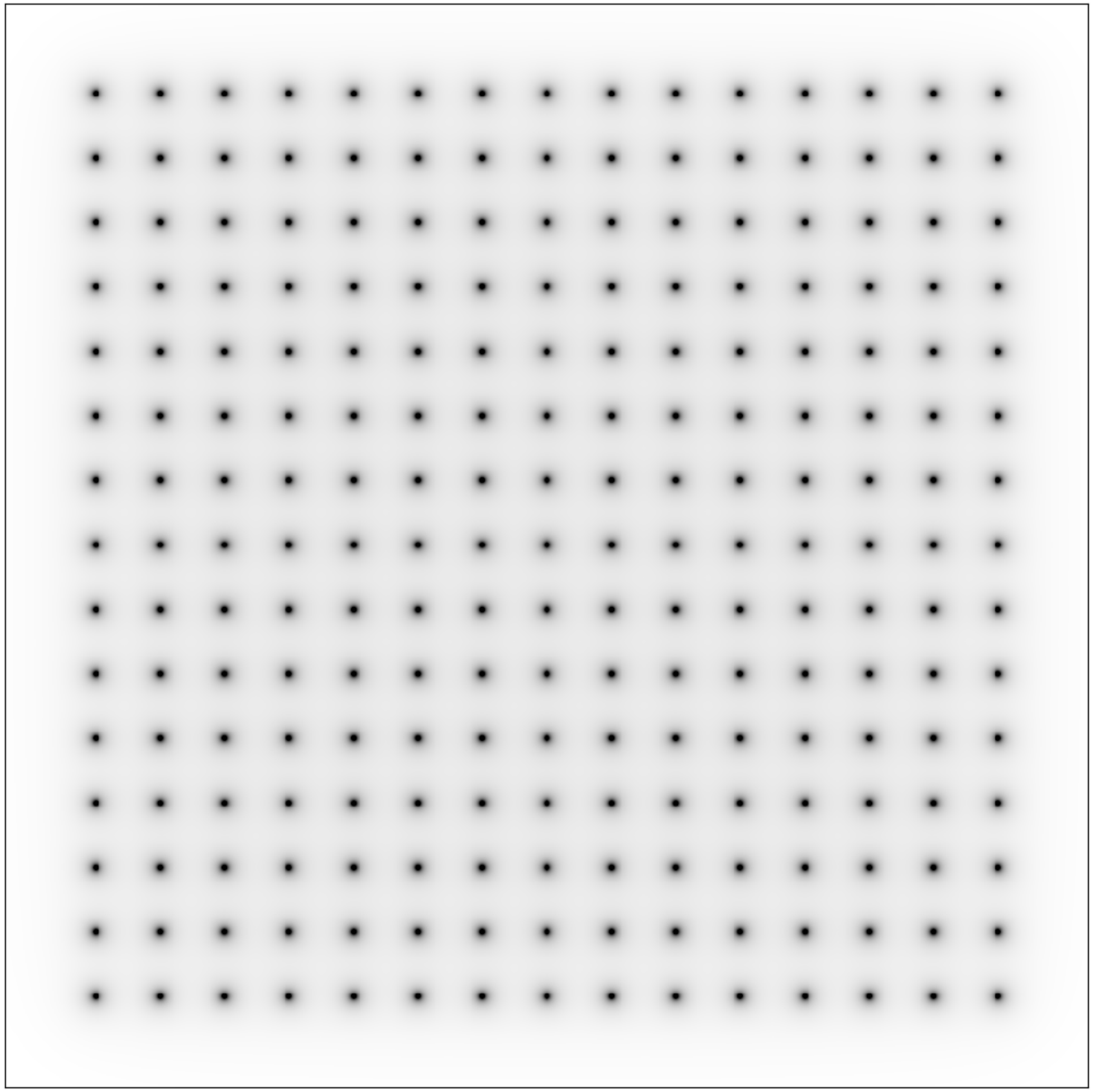
$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = \frac{1}{Q_{11} + Q_{22}} \begin{pmatrix} Q_{11} - Q_{22} \\ 2Q_{12} \end{pmatrix}$$

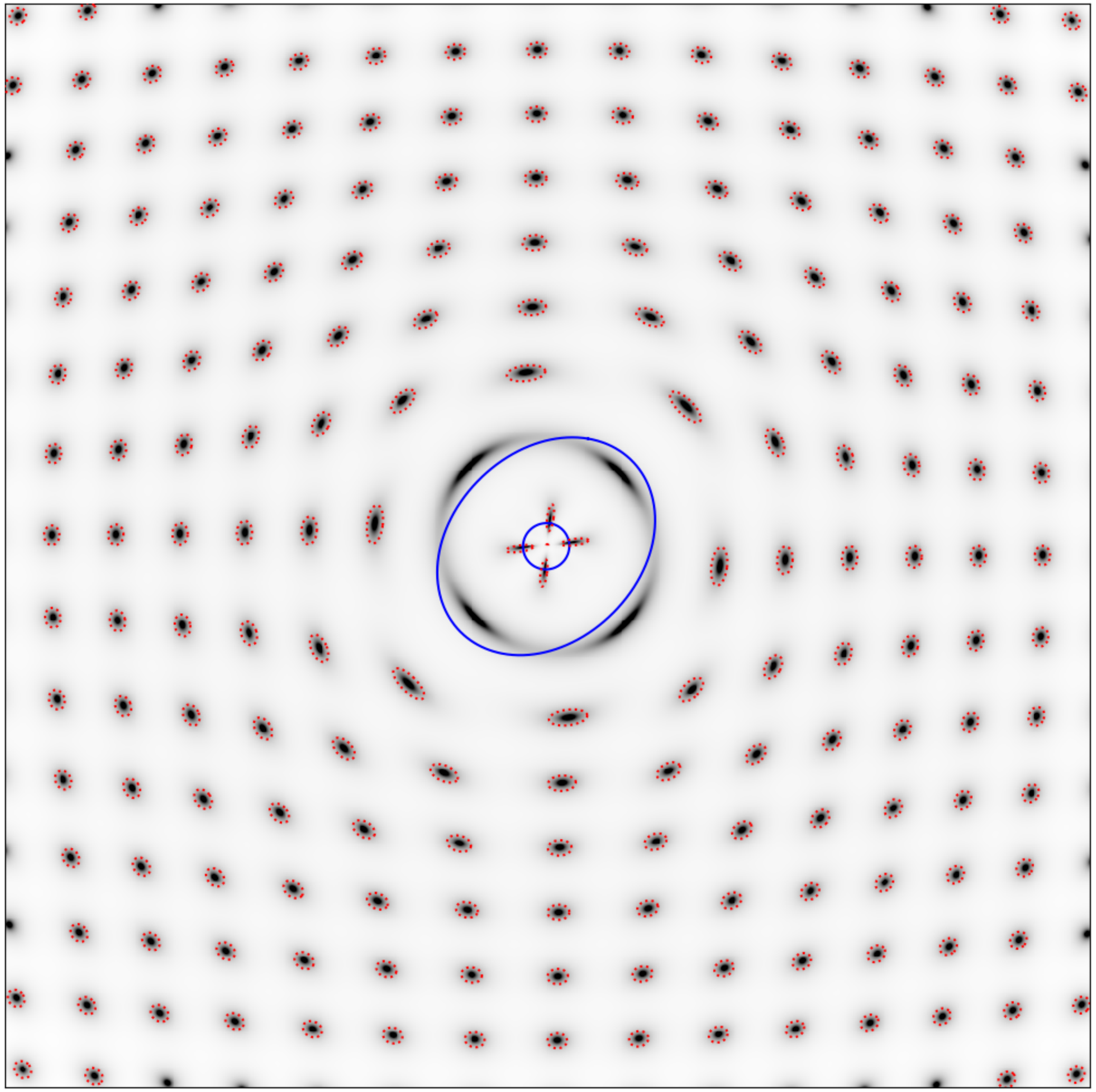
complex ellipticity

$$\epsilon = \epsilon_1 + i\epsilon_2 = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22}}$$

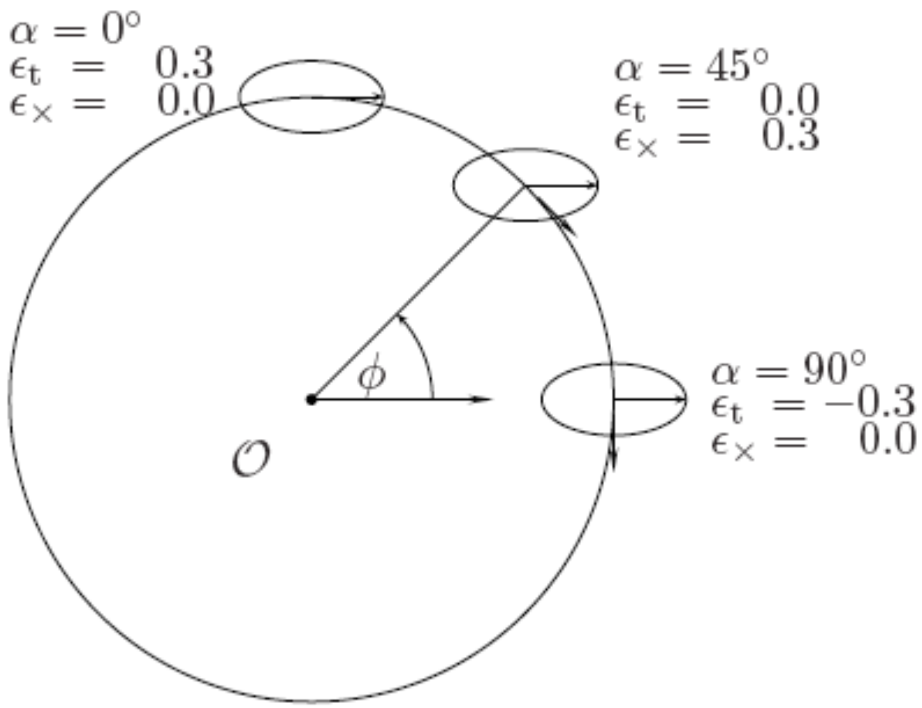
APPLYING TO THE PREVIOUS EXAMPLE

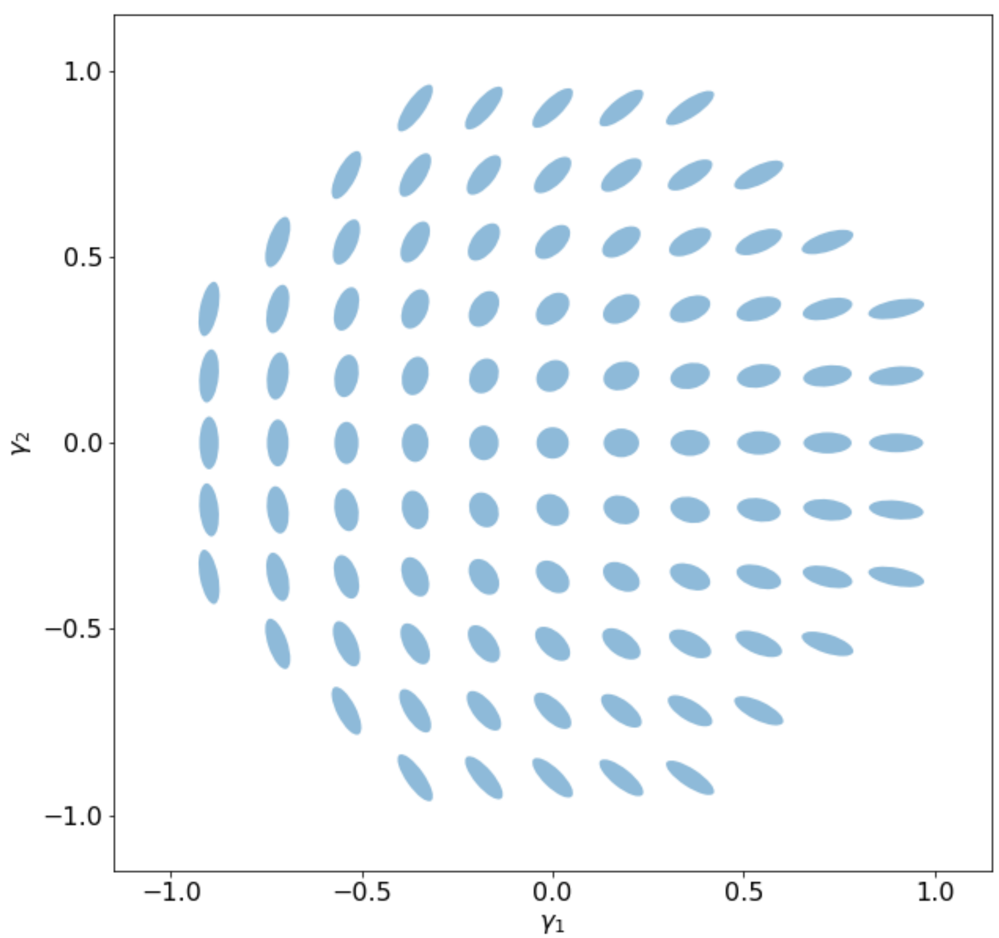
Chapter 6, Sect. 5.5.2





TANGENTIAL AND CROSS COMPONENTS OF THE SHEAR





Lensing only causes radial or tangential distortions!

It is convenient to work in a rotated framework:

$$R(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

Using the transforming rule of tensors:

$$\Gamma' = R^T(\phi)\Gamma R(\phi)$$

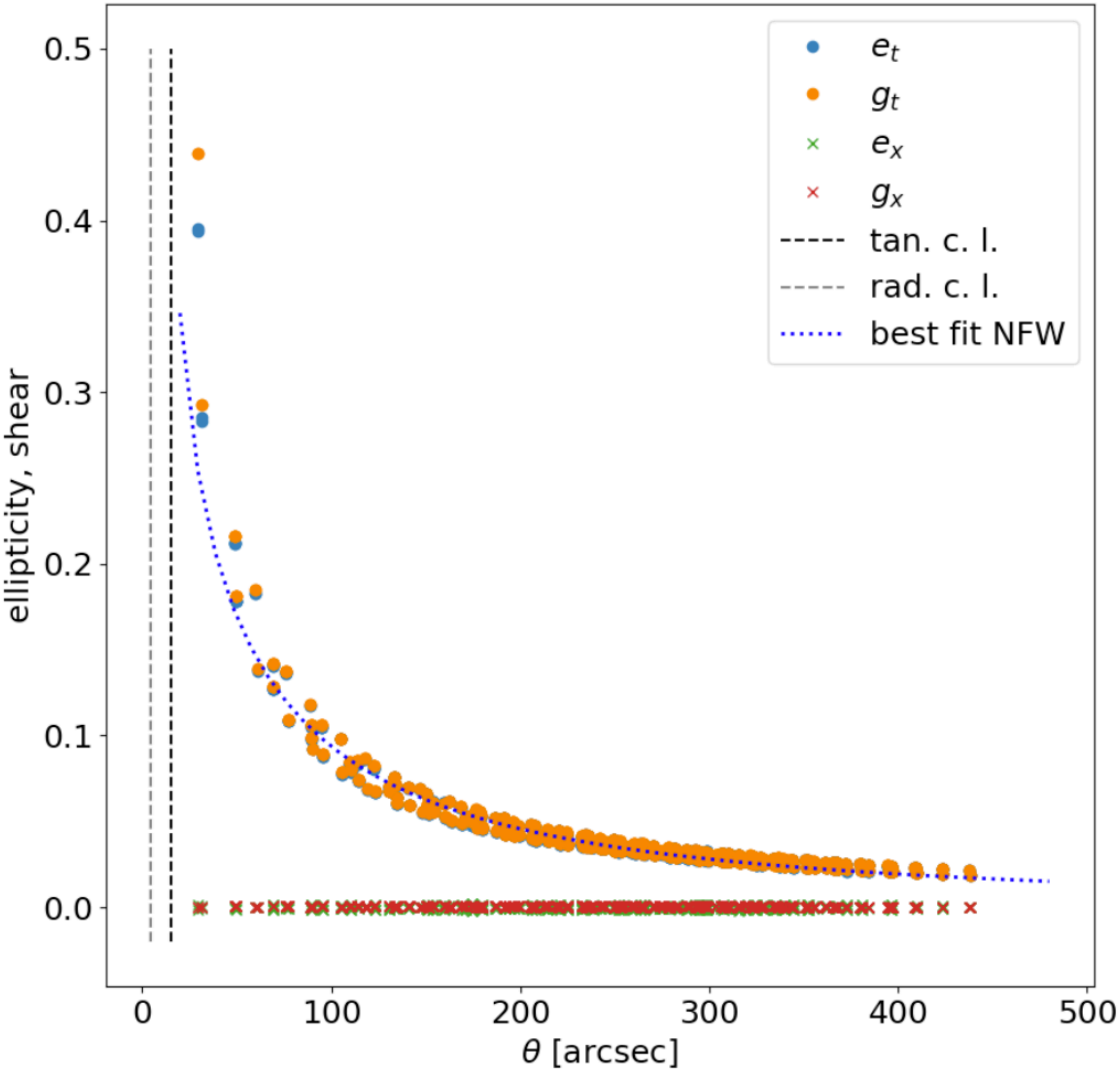
$$\gamma_1' = \gamma_1 \cos 2\phi + \gamma_2 \sin 2\phi$$

$$\gamma_2' = -\gamma_1 \sin 2\phi - \gamma_2 \cos 2\phi$$

Radial or tangential distortions correspond to $\gamma_2' = 0.$ *We define the tangential and the cross components of the shear as:*

$$\gamma_t = -\gamma_1'$$
$$\gamma_\times = -\gamma_2'$$

APPLICATION TO THE PREVIOUS EXAMPLE



Tangential ellipticity decreases as function of distance from the center (one point per galaxy).

Cross component is consistent with zero (good!)

Deviations near the critical lines (no weak-lensing)

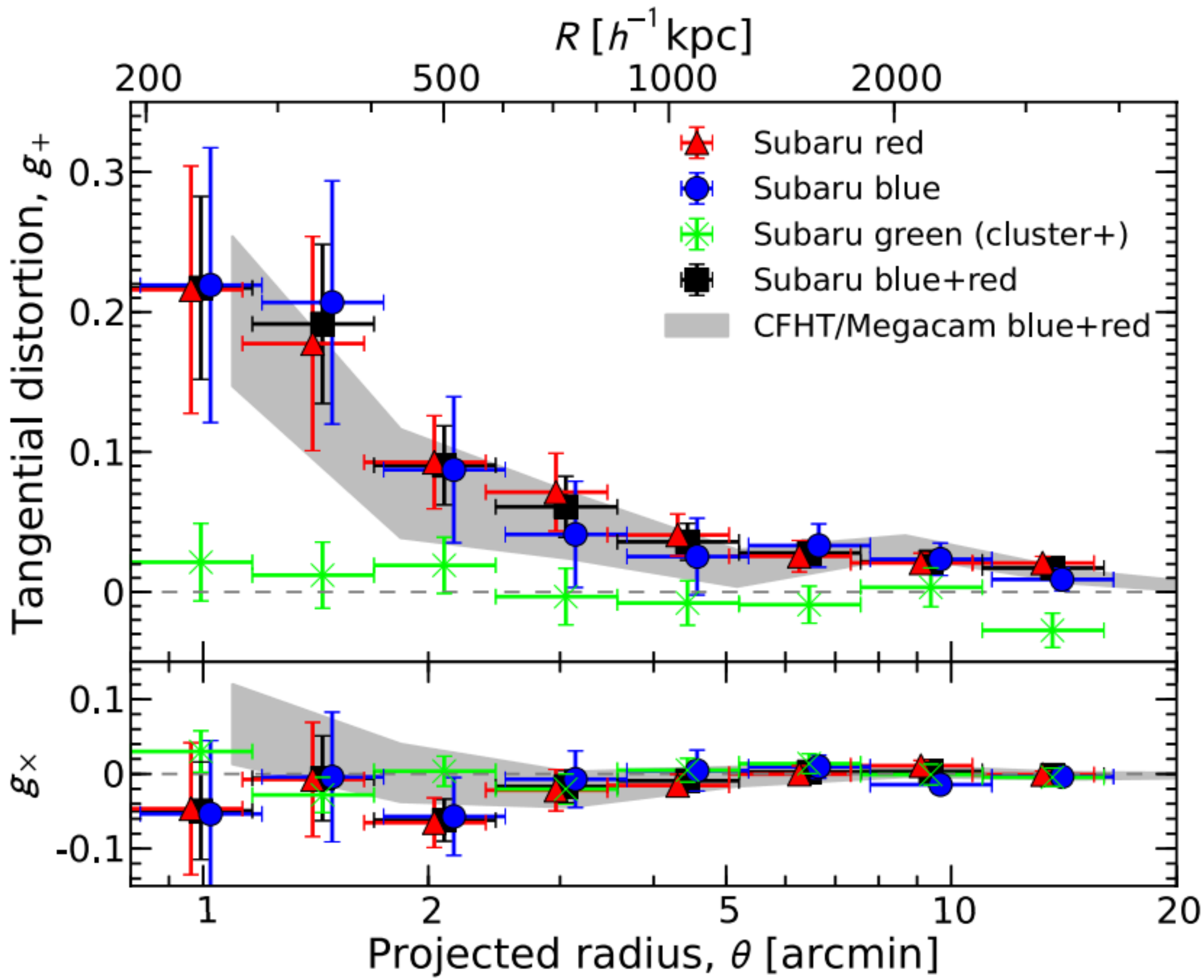
We can use a lens model (e.g. dPIE) and predict the reduced shear for a set of parameters $\overrightarrow{p}$:

$$g_{t,i}(\overrightarrow{\theta}_i \,|\, \overrightarrow{p})$$

We can fit the model to the data:

$$\chi^2_{WL}(\overrightarrow{p}) = \sum_{i=1}^{N_{ima}} \frac{[\epsilon_{t,i} - g_{t,i}(\overrightarrow{p})]^2}{\sigma_i^2}$$

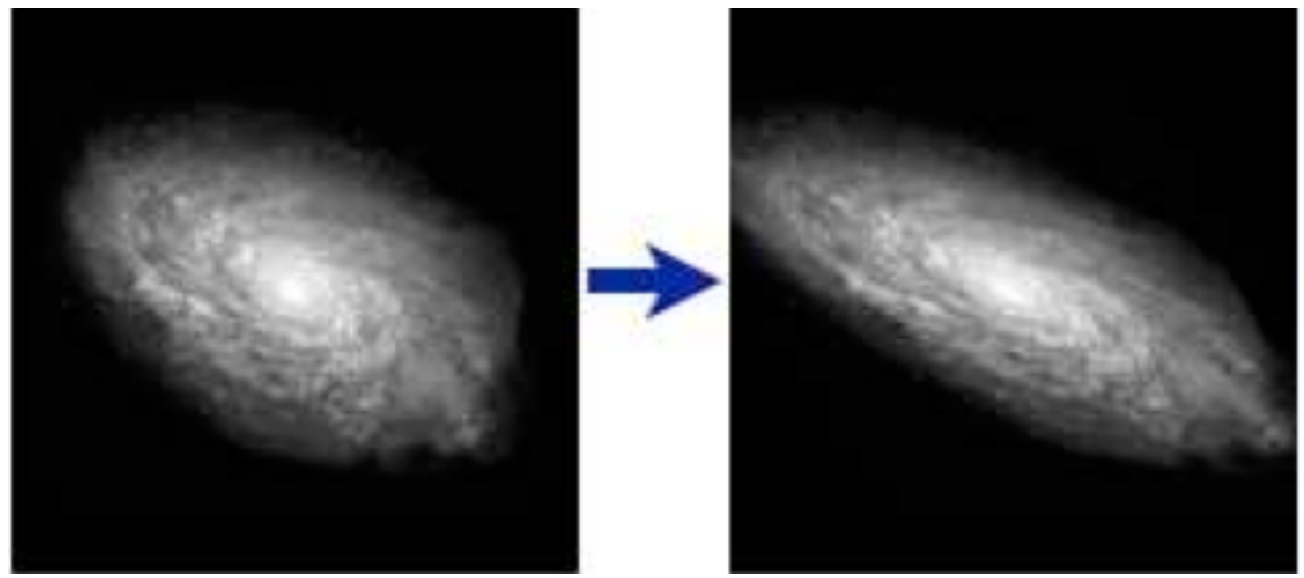
TANGENTIAL SHEAR PROFILE OF MACSJ1206



Umetsu et al. 2012

UNFORTUNATELY GALAXIES ARE NOT CIRCULAR…

$I_s(\vec{\beta})$



$I(\vec{\theta})$

We can use the lens mapping (at first order) to find the transformation between observed and intrinsic ellipticity

$$Q_{ij}^{(s)} = \frac{\int d\beta^2 q_I[I^{(s)}(\beta)] I^{(s)}(\beta)(\beta_i - \overline{\beta}_i)(\beta_j - \overline{\beta}_j)}{\int d\beta^2 q_I[I^{(s)}(\beta)] I^{(s)}(\beta)} \quad i,j \in \{1,2\}$$

With $\boldsymbol{\beta} - \bar{\boldsymbol{\beta}} = \mathcal{A}\,(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}})$ $\quad \mathrm{d}^2\beta = \det\mathcal{A}\,\mathrm{d}^2\theta,$ $\quad \mathcal{A} \equiv \mathcal{A}(\bar{\boldsymbol{\theta}})$

We can show that $Q^{(\mathrm{s})} = \mathcal{A}\,Q\,\mathcal{A}^T = \mathcal{A}\,Q\,\mathcal{A}$

UNFORTUNATELY GALAXIES ARE NOT CIRCULAR…

Using the fact that $\mathcal{A}(\boldsymbol{\theta}) = (1-\kappa)\begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix}$

and that $$\epsilon = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22}}$$

we can show that $Q^{(\mathrm{s})} = \mathcal{A}\, Q\, \mathcal{A}^T = \mathcal{A}\, Q\, \mathcal{A}$ *implies*

$$\epsilon^{(\mathrm{s})} = \begin{cases} \dfrac{\epsilon - g}{1 - g^* \epsilon} & \text{if } |g| \le 1 \; ; \\[2ex] \dfrac{1 - g\epsilon^*}{\epsilon^* - g^*} & \text{if } |g| > 1 \; . \end{cases}$$

The inverse relation can be obtained by changing g with -g.

ESTIMATING THE REDUCED SHEAR

We assume that the intrinsic orientations of galaxies (phases of the complex ellipticity) are random. In this case,

$$E(\epsilon^{(s)}) = 0$$

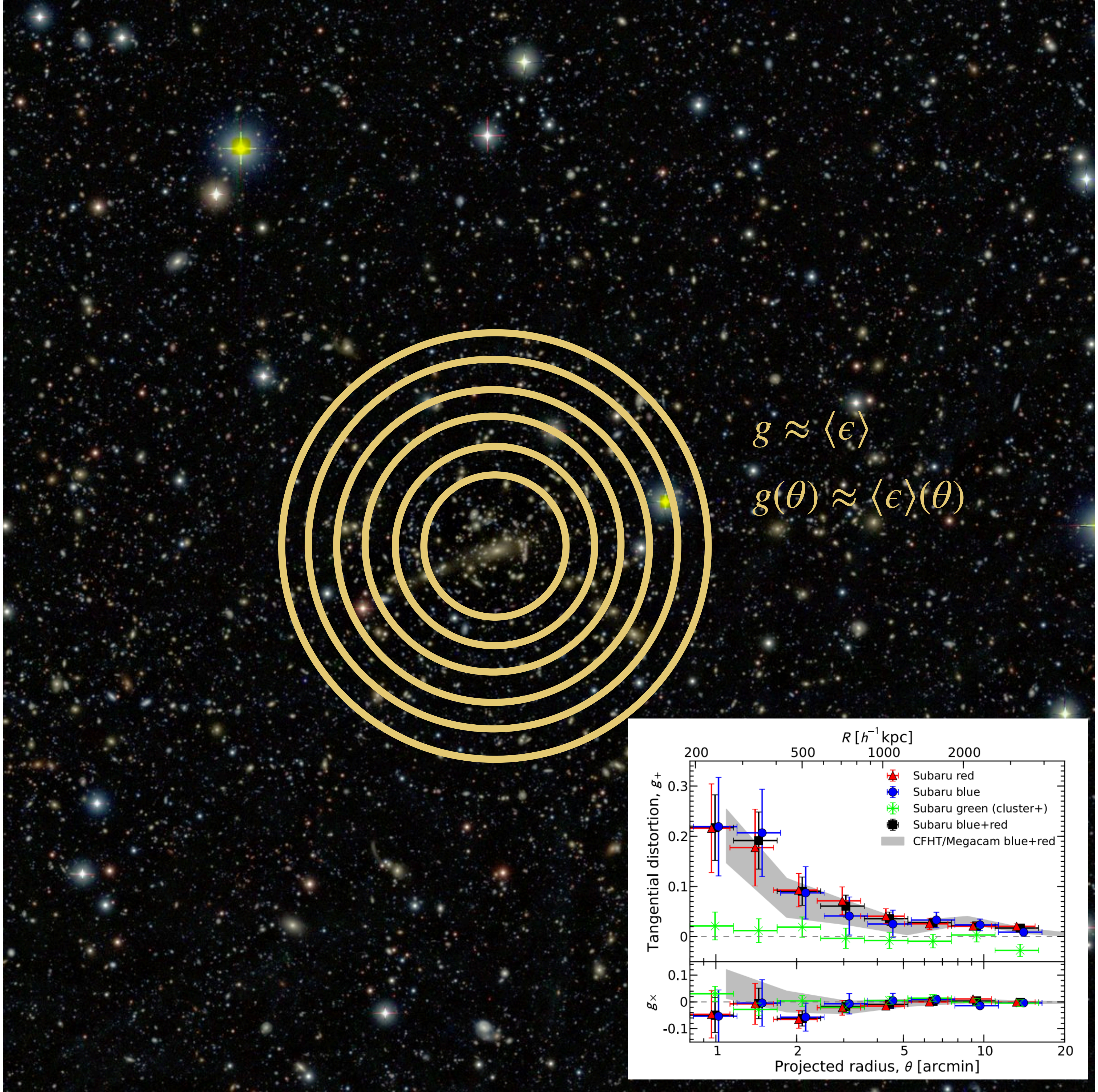
$$\epsilon^{(\mathrm{s})} = \begin{cases} \dfrac{\epsilon - g}{1 - g^* \epsilon} & \text{if } |g| \leq 1\ ; \\ \\ \dfrac{1 - g\epsilon^*}{\epsilon^* - g^*} & \text{if } |g| > 1\ . \end{cases} \qquad \mathrm{E}(\epsilon) = \begin{cases} g & \text{if } |g| \leq 1 \\ 1/g^* & \text{if } |g| > 1 \end{cases}$$

Each image ellipticity provides an un-biased estimate of the local shear. However this is very noisy. The noise is determined by the intrinsic ellipticity dispersion

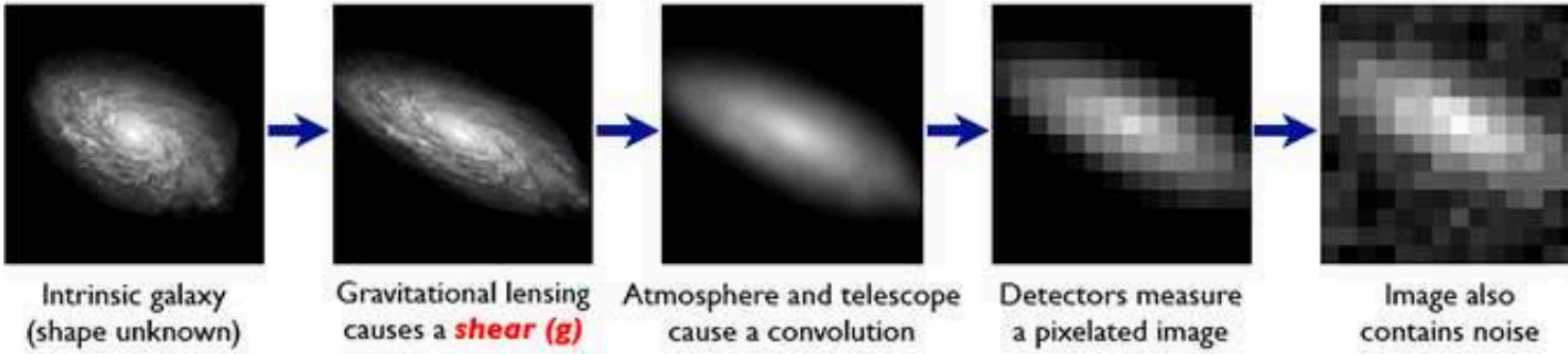
$$\sigma_\epsilon = \sqrt{\left\langle \epsilon^{(\mathrm{s})} \epsilon^{(\mathrm{s})*} \right\rangle}$$

Noise can be beaten down by averaging over many galaxy images. The accuracy of the shear estimate depends on the local density of galaxies for which shape can be measured. Thus, deep imaging observations are required.

$$\gamma \approx g \approx \langle \epsilon \rangle$$

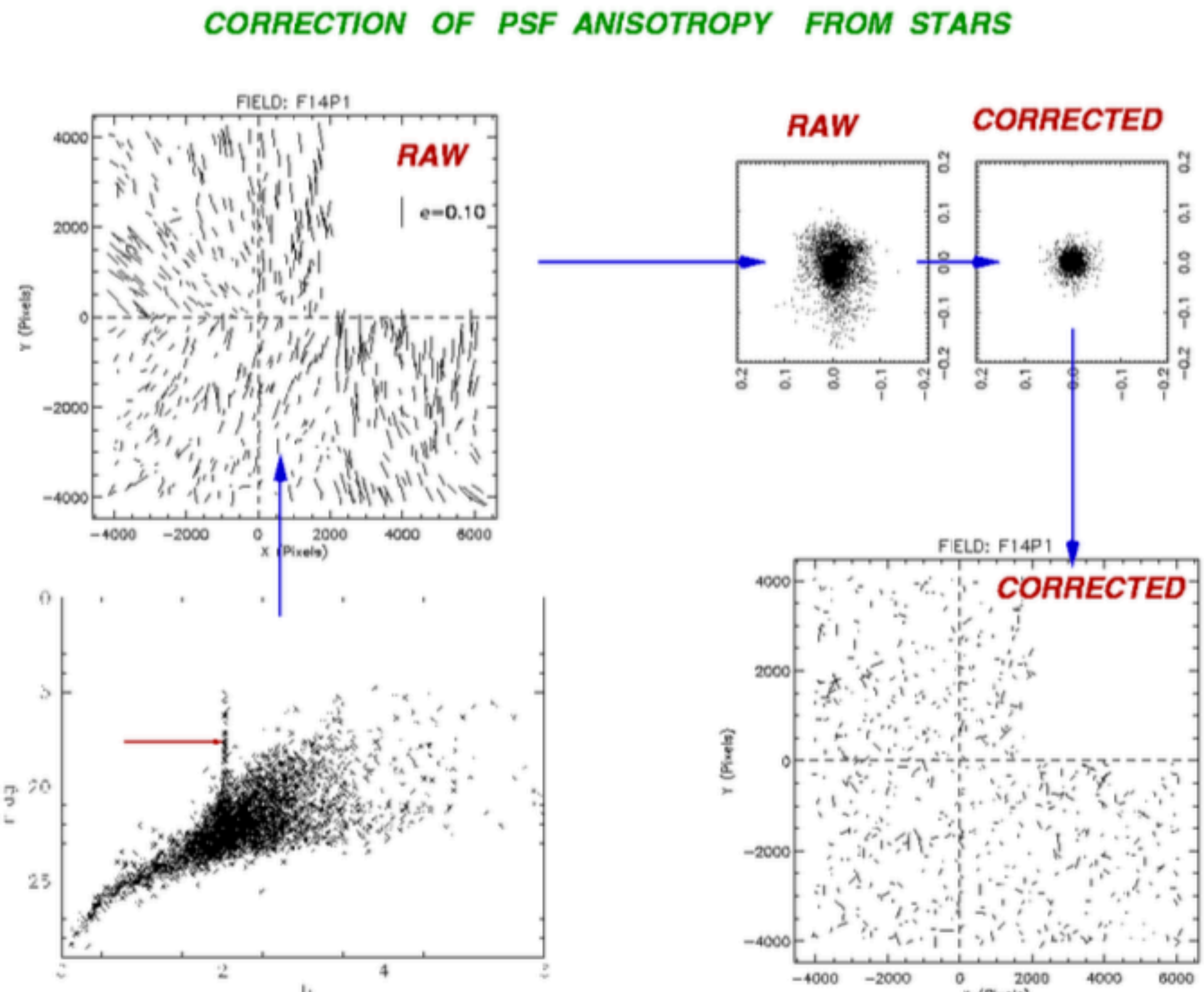

$g \approx \langle \epsilon \rangle$
$g(\theta) \approx \langle \epsilon \rangle(\theta)$
$R\ [h^{-1}\,\mathrm{kpc}]$
200
500
1000
2000
Subaru red
Subaru blue
Subaru green (cluster+)
Subaru blue+red
CFHT/Megacam blue+red
Tangential distortion, g_+
0.3
0.2
0.1
0
$g_\times$
0.1
0
-0.1
1
2
5
10
20
Projected radius, θ [arcmin]

NOT SO EASY… :-(



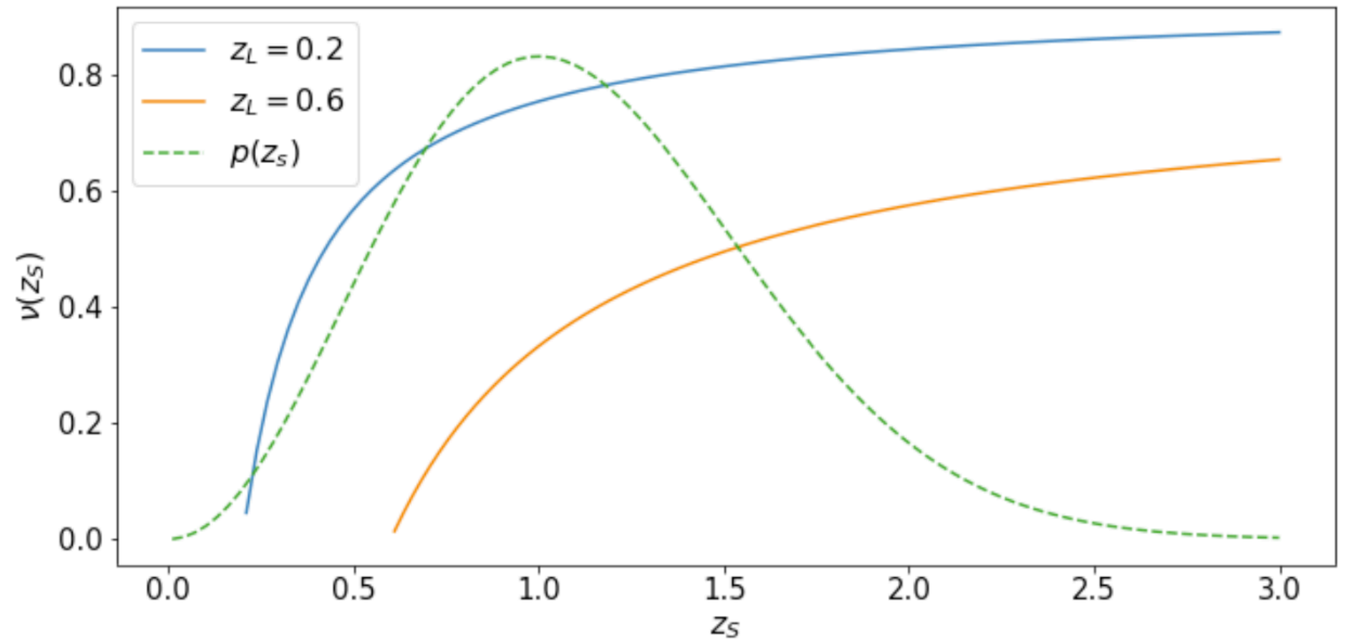
- images are blurred by instrument **PSF** and by the atmosphere. PSF tends to circularize shapes but can also introduce artificial elongations, i.e. spurious signal
- images are **pixelated**
- sometimes galaxies are **blended**
- several **instrumental effects** can mimic as shear signal (e.g. CTE, bad tracking, star saturations, ghosts, cosmic rays)

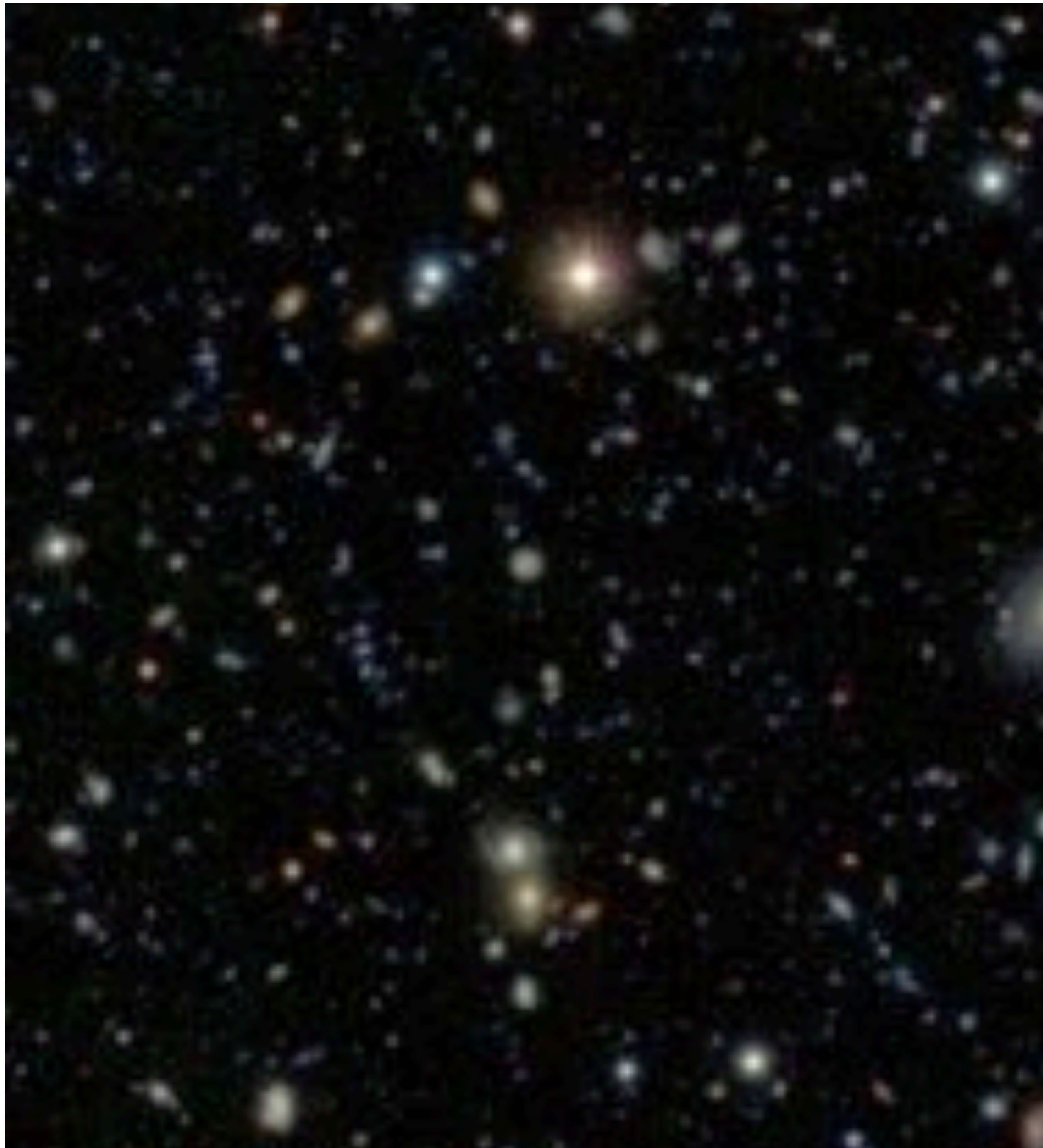
CORRECTION OF PSF ANISOTROPY



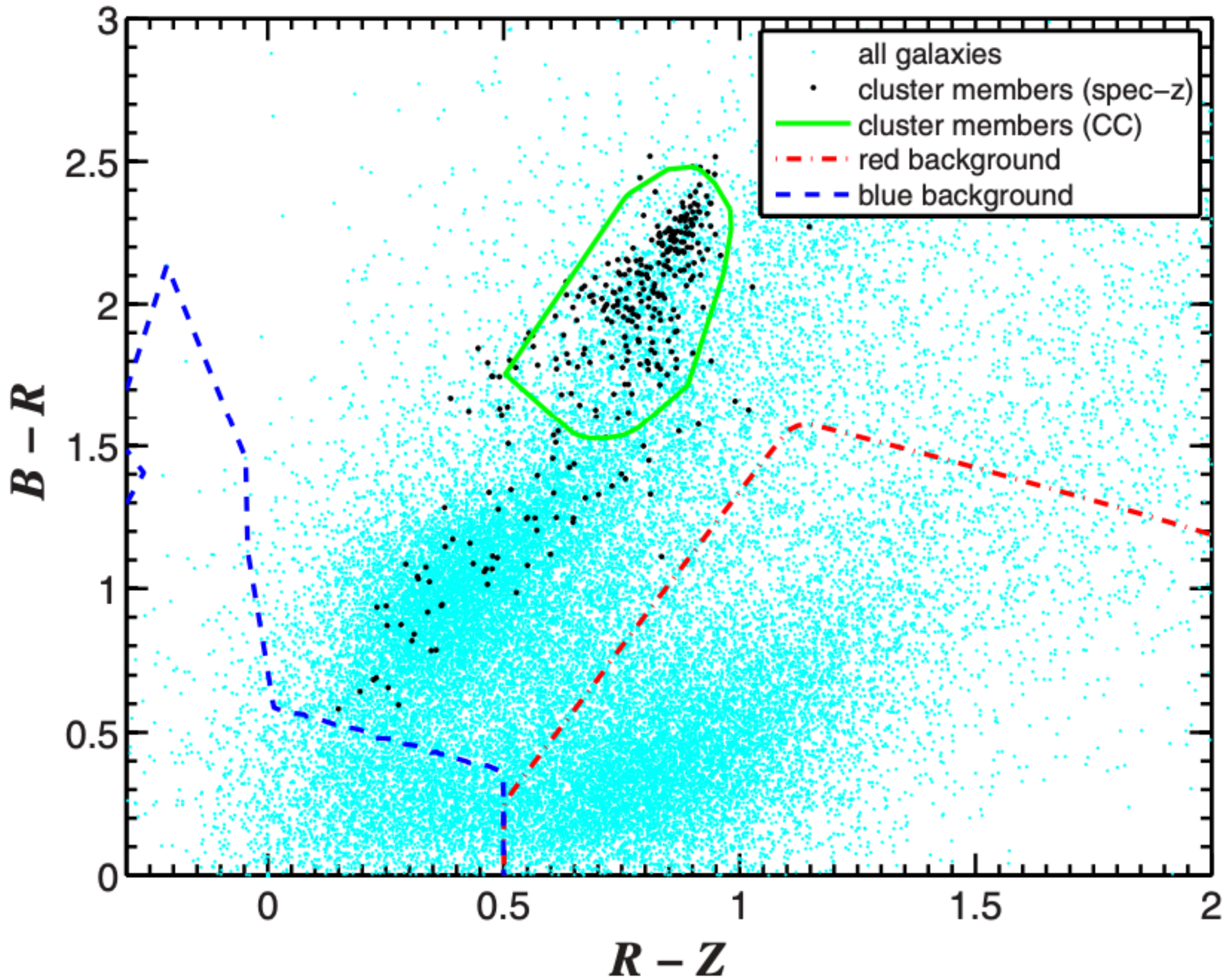
MORE CHALLENGES

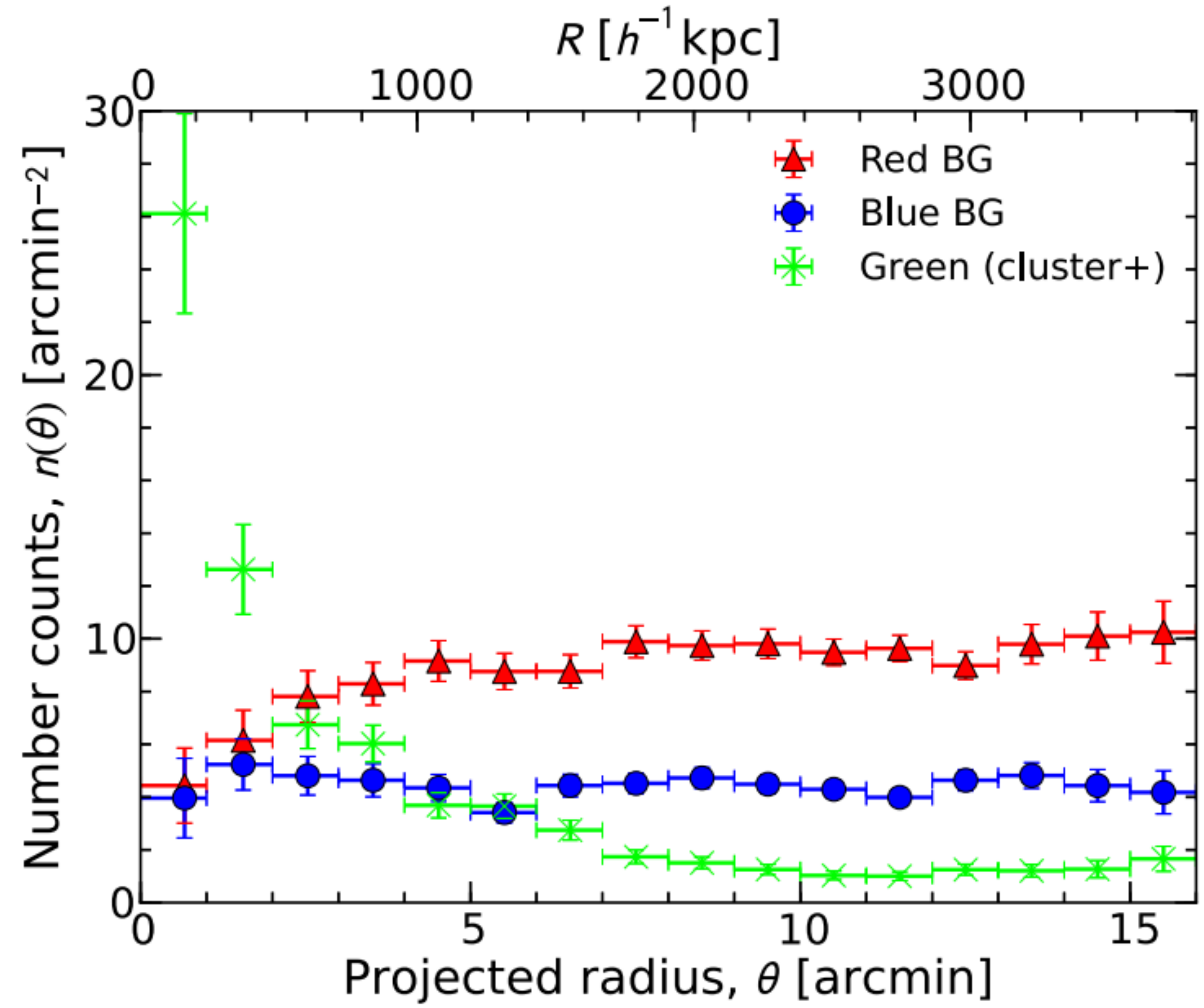
- if there are **intrinsic alignments**, then the shear measurement will be biased
- lensing signal (shear) is **redshift** dependent. This needs to be taken into account, especially when dealing with deep observations
- only galaxies behind the lens (cluster) are lensed. Averaging over galaxies that are erroneously classified as background but in fact are in the cluster foreground or within the cluster causes **signal dilution**
- and more…





COLOR SELECTION OF LENSED SOURCES (MACSJ1206)





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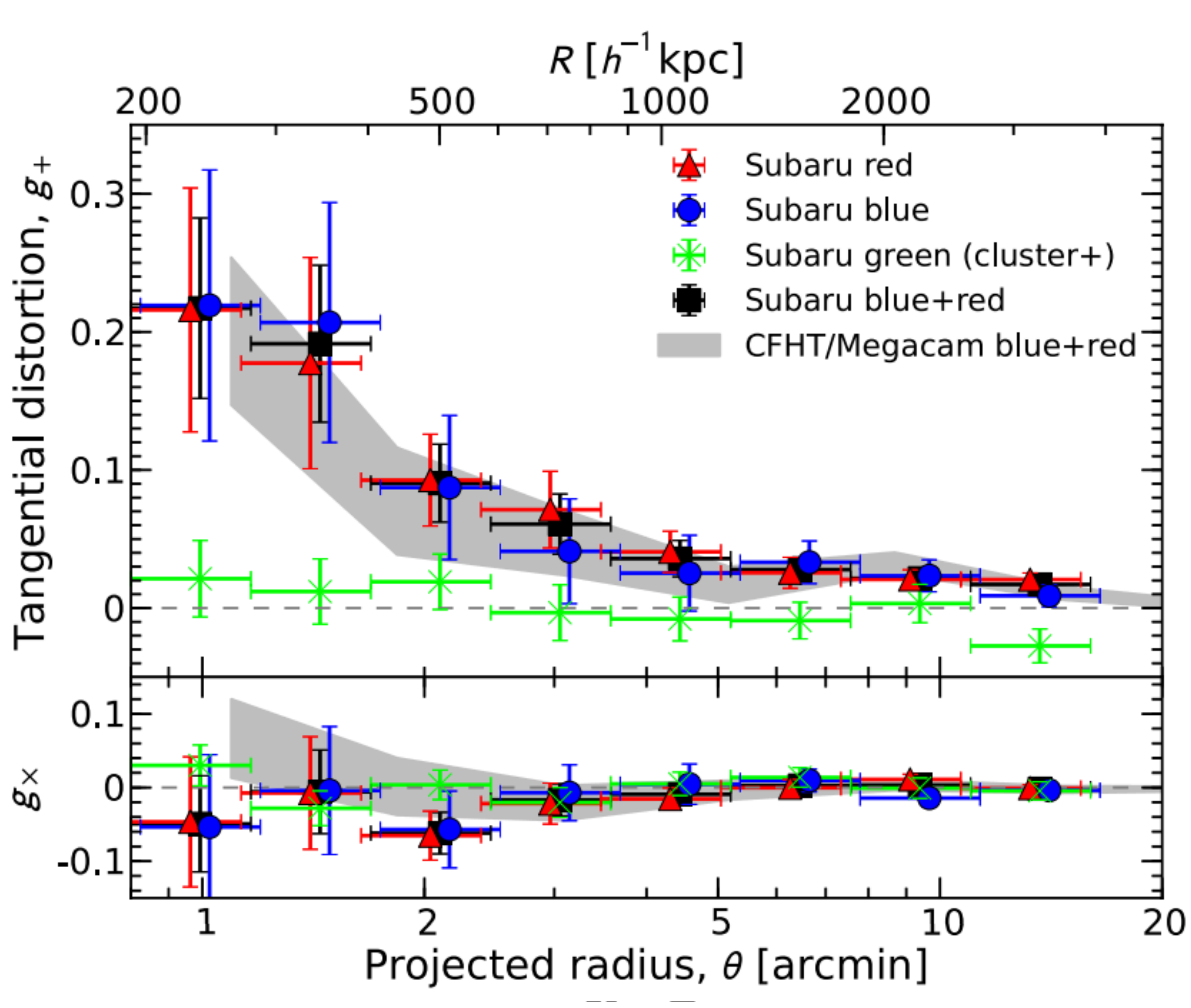
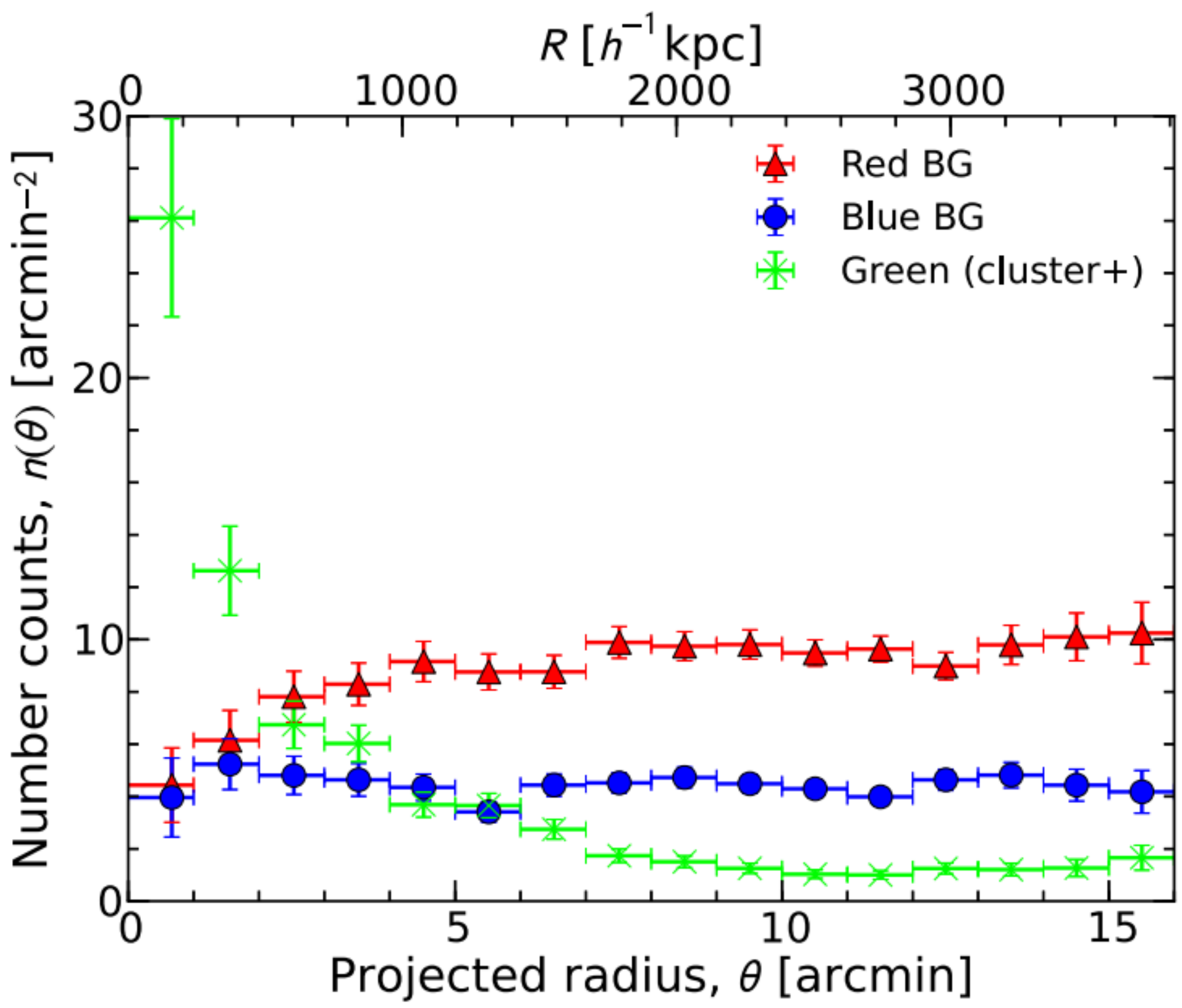
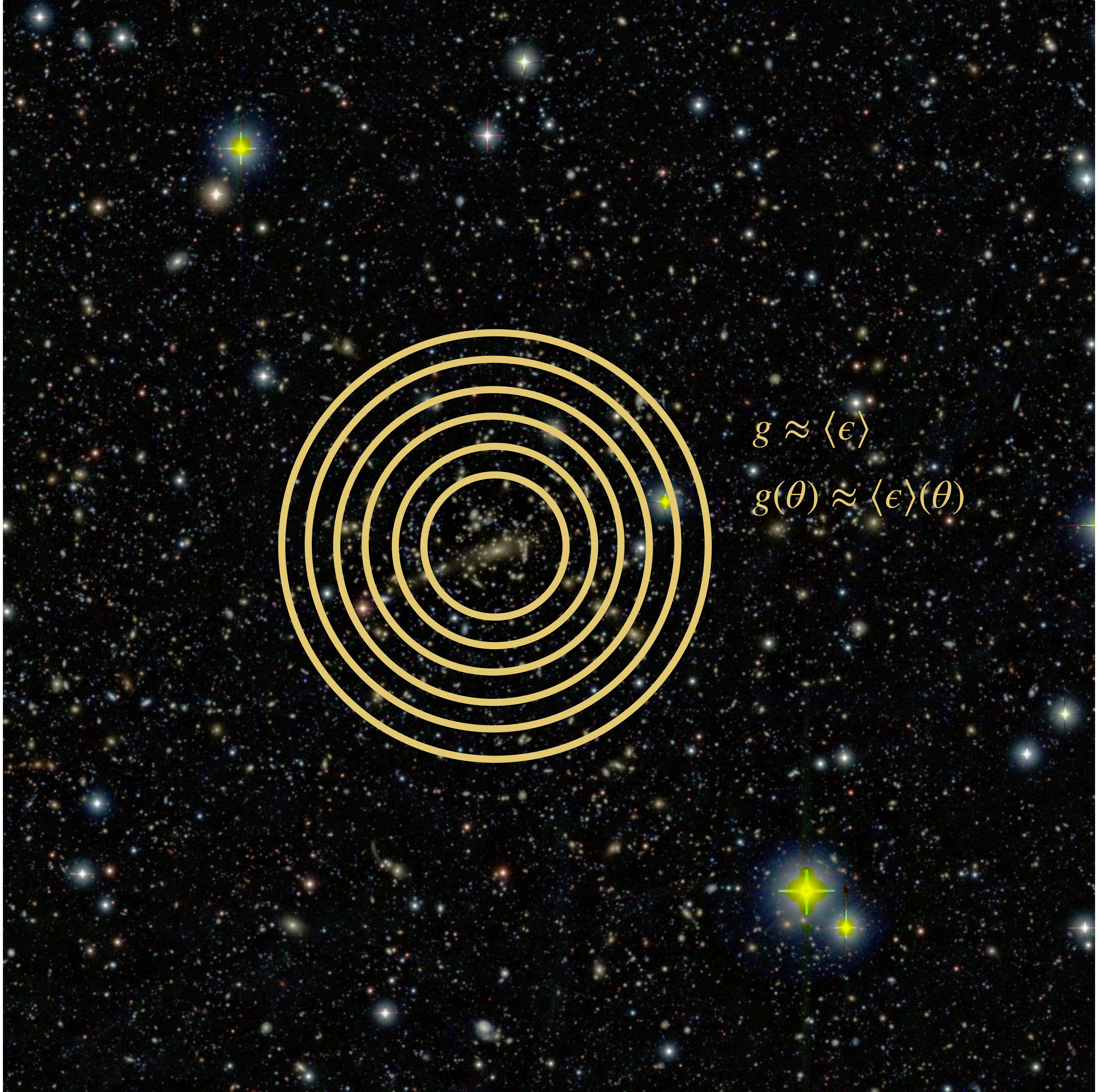
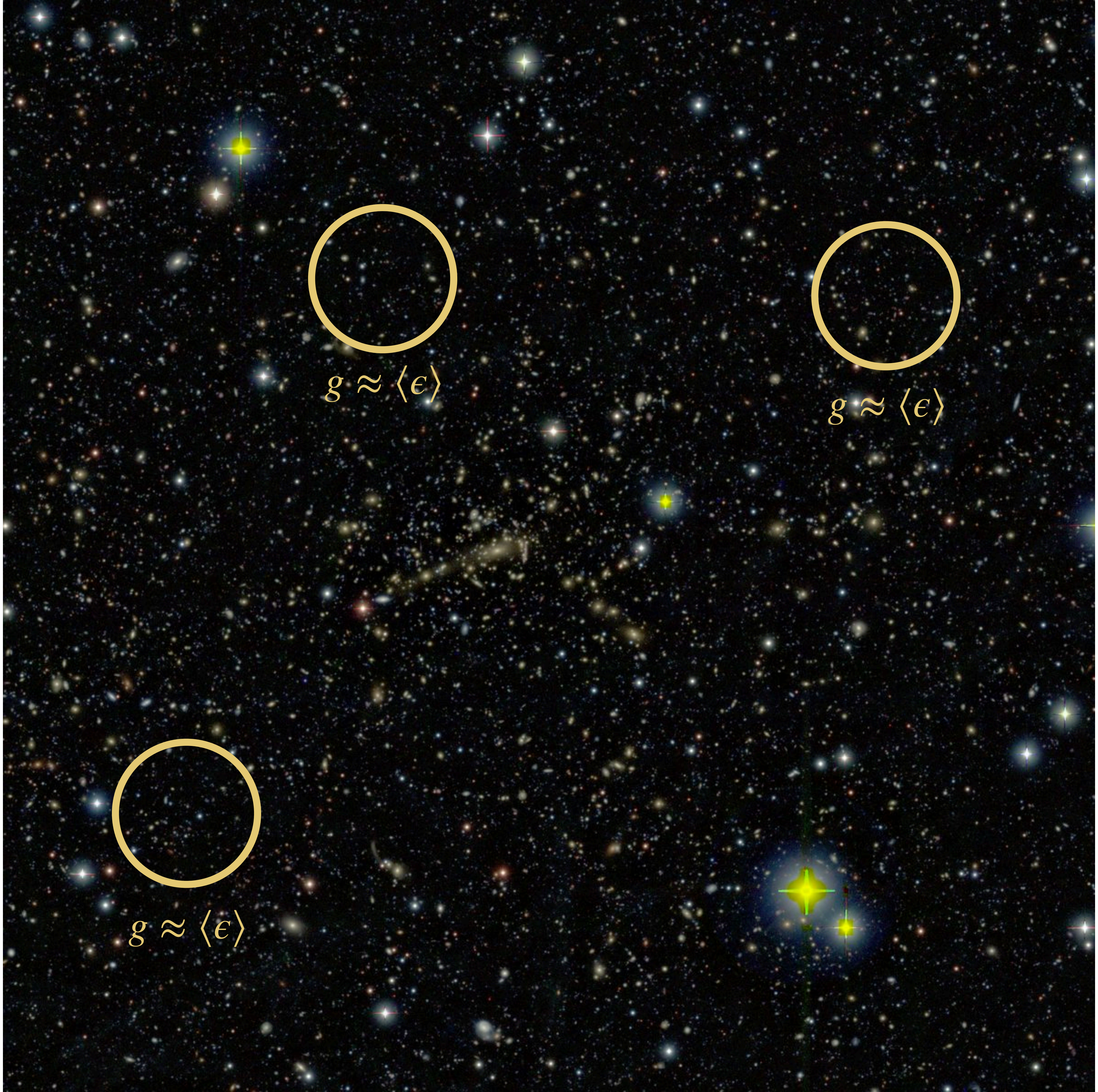





$g \approx \langle \epsilon \rangle$
$g(\theta) \approx \langle \epsilon \rangle (\theta)$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THE KAISER & SQUIRES INVERSION ALGORITHM

Shear and convergence in Fourier space

$$\kappa = \frac{1}{2}(\psi_{11} + \psi_{22}) \quad \Rightarrow \quad \hat{\kappa} = -\frac{1}{2}(k_1^2 + k_2^2)\hat{\psi}$$

$$\gamma_1 = \frac{1}{2}(\psi_{11} - \psi_{22}) \quad \Rightarrow \quad \hat{\gamma}_1 = -\frac{1}{2}(k_1^2 - k_2^2)\hat{\psi}$$

$$\gamma_2 = \psi_{12} \quad \Rightarrow \quad \hat{\gamma}_2 = -k_1 k_2 \hat{\psi} \ ,$$

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using:

$$\left[k^{-2} \begin{pmatrix} k_1^2 - k_2^2 \\ 2k_1k_2 \end{pmatrix}\right] [k^{-2} (\; k_1^2 - k_2^2 \quad 2k_1k_2 \;)] = 1$$

$$\hat{\kappa} = k^{-2} (\; k_1^2 - k_2^2 \quad 2k_1k_2 \;) \begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2}[(k_1^2 - k_2^2)\hat{\gamma}_1 + 2k_1k_2\hat{\gamma}_2]$$

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$$(f \hat{*} g) = \hat{f}\hat{g}$$

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$$\gamma_2 = \psi_{12} \quad \Rightarrow \quad \hat{\gamma}_2 = -k_1 k_2 \hat{\psi} \; ,$$

$$\boxed{\kappa(\vec{\theta}) = \frac{1}{\pi}\int \mathrm{d}^2\theta' [D_1(\vec{\theta} - \vec{\theta}')\gamma_1 + D_2(\vec{\theta} - \vec{\theta}')\gamma_2]}$$

$$\begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2} \begin{pmatrix} k_1^2 - k_2^2 \\ 2k_1k_2 \end{pmatrix} \hat{\kappa}$$

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Noise in the shear

Ideally $$\epsilon^{\mathrm{ob}} = \epsilon^{\mathrm{int}} + \gamma + n$$ *where* $$\langle \epsilon^{\mathrm{int}} \rangle = 0$$ $$\langle n \rangle = 0$$

The shear is estimated with $$\hat{\gamma} = \frac{1}{n_g} \sum_i \epsilon_i^{\mathrm{ob}}$$

The standard deviation of this estimator is $$\sigma_{\hat{\gamma}} = \frac{\sqrt{\sigma_\epsilon^2 + \sigma_n^2}}{n_g}$$

In actuality there can be bias, both additive and multiplicative

$$\epsilon^{\mathrm{ob}} = (1 + m_i)\epsilon_i^{\mathrm{true}} + c_i$$ *where m_i and c_i are not zero on average.*

THE KAISER & SQUIRES INVERSION ALGORITHM

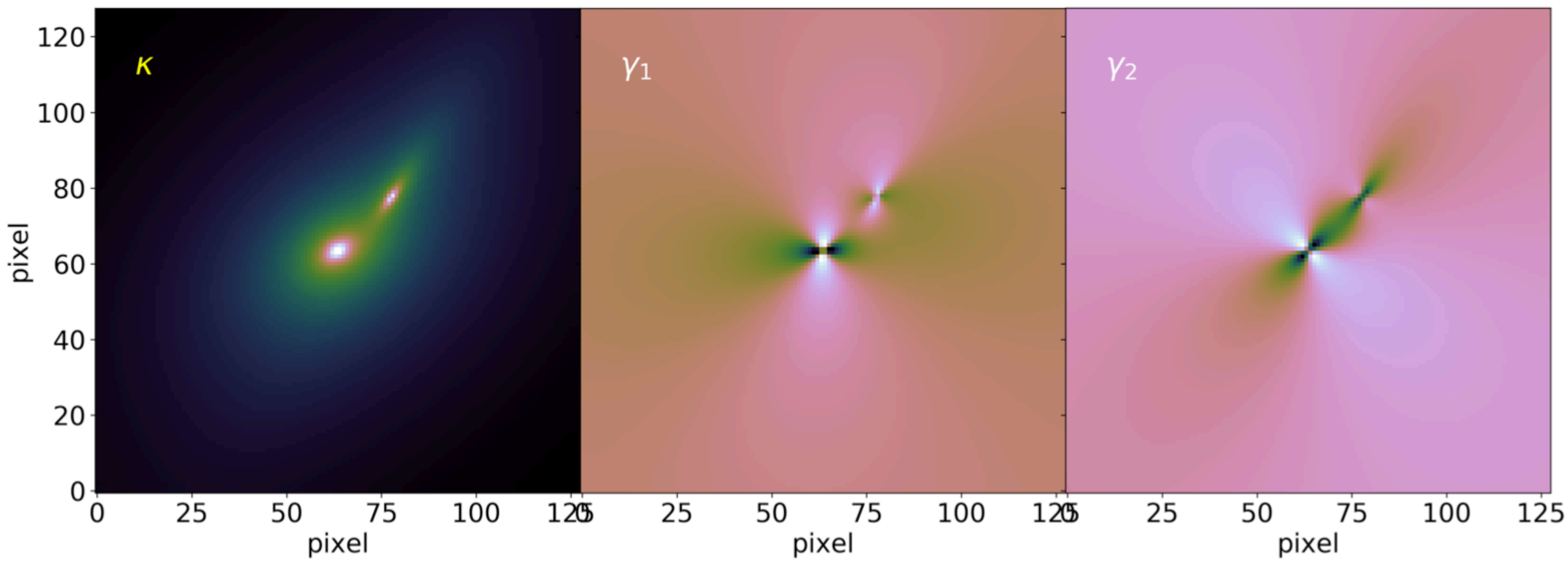
- infinite fields would be required: wide field + boundary conditions.
- ellipticity measures the reduced shear, not the shear:

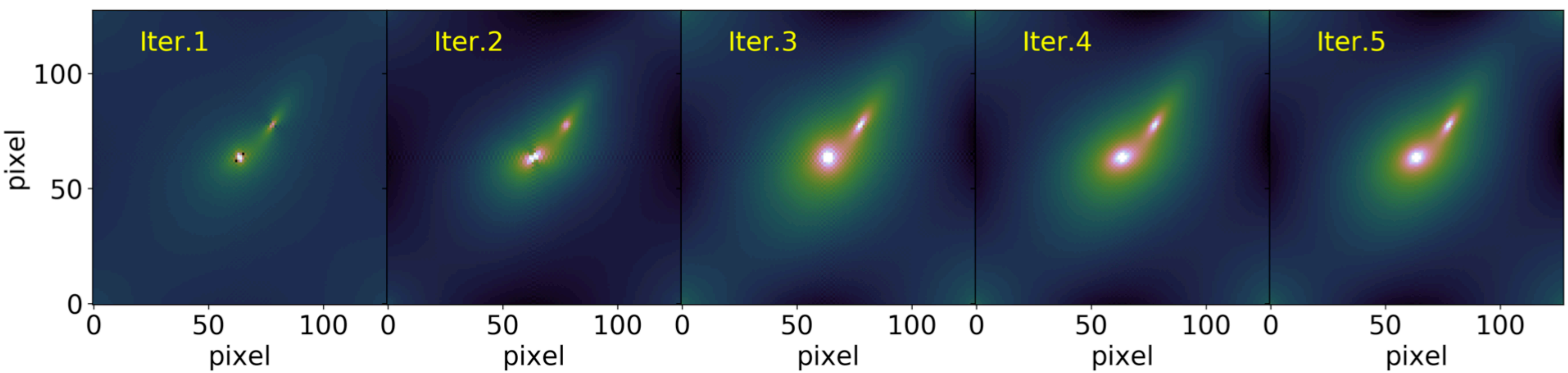
$$\kappa(\vec{\theta}) = \frac{1}{\pi} \int \mathrm{d}^2\theta' [D_1(\vec{\theta} - \vec{\theta}')g_1(1-\kappa) + D_2(\vec{\theta} - \vec{\theta}')g_2(1-\kappa)]$$

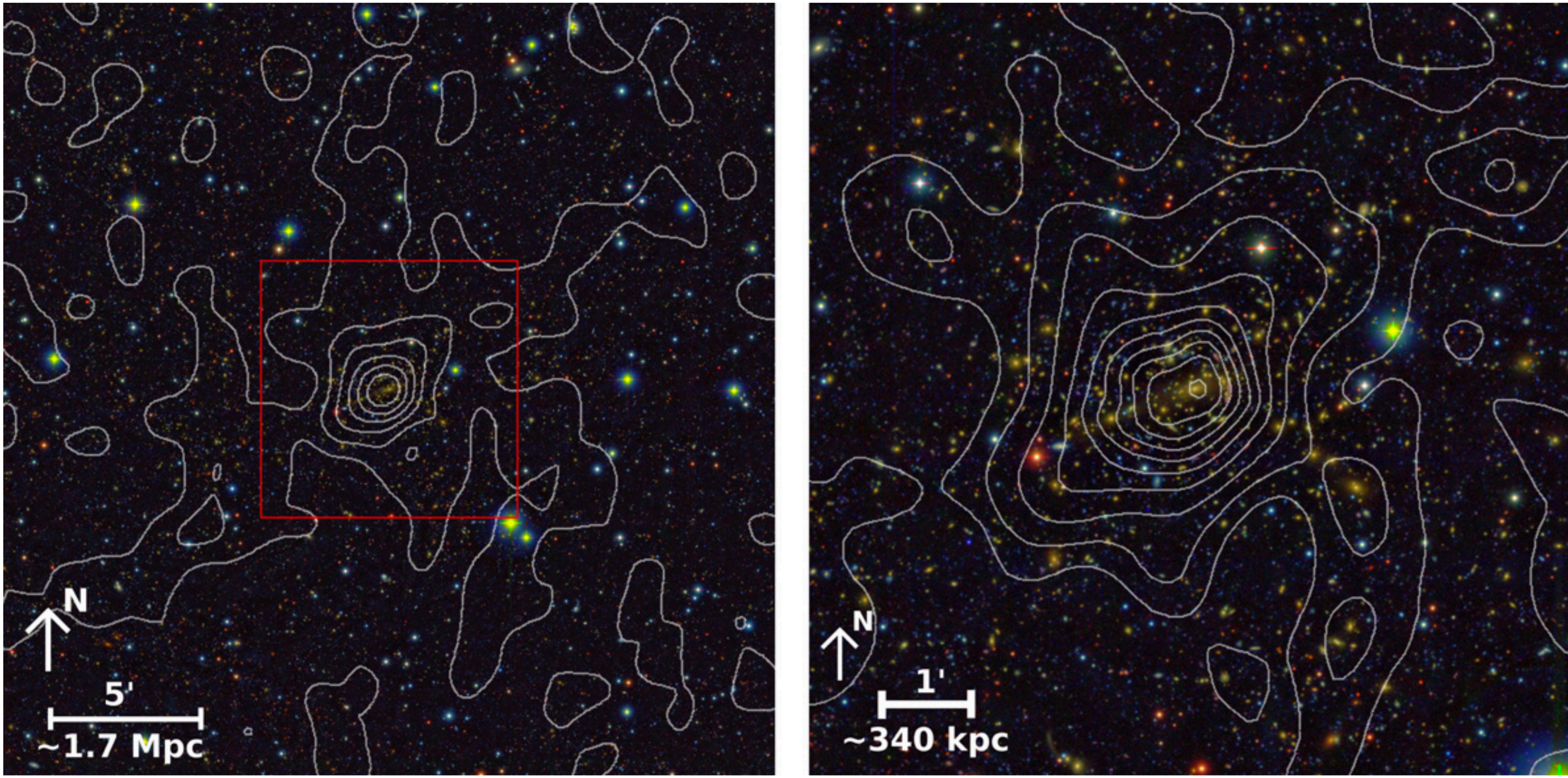
This equation can be solved iteratively starting from $\kappa=0$

EXAMPLE

Chapter 6, Sect. 6.4.3






N
5′
~1.7 Mpc
N
1′
~340 kpc