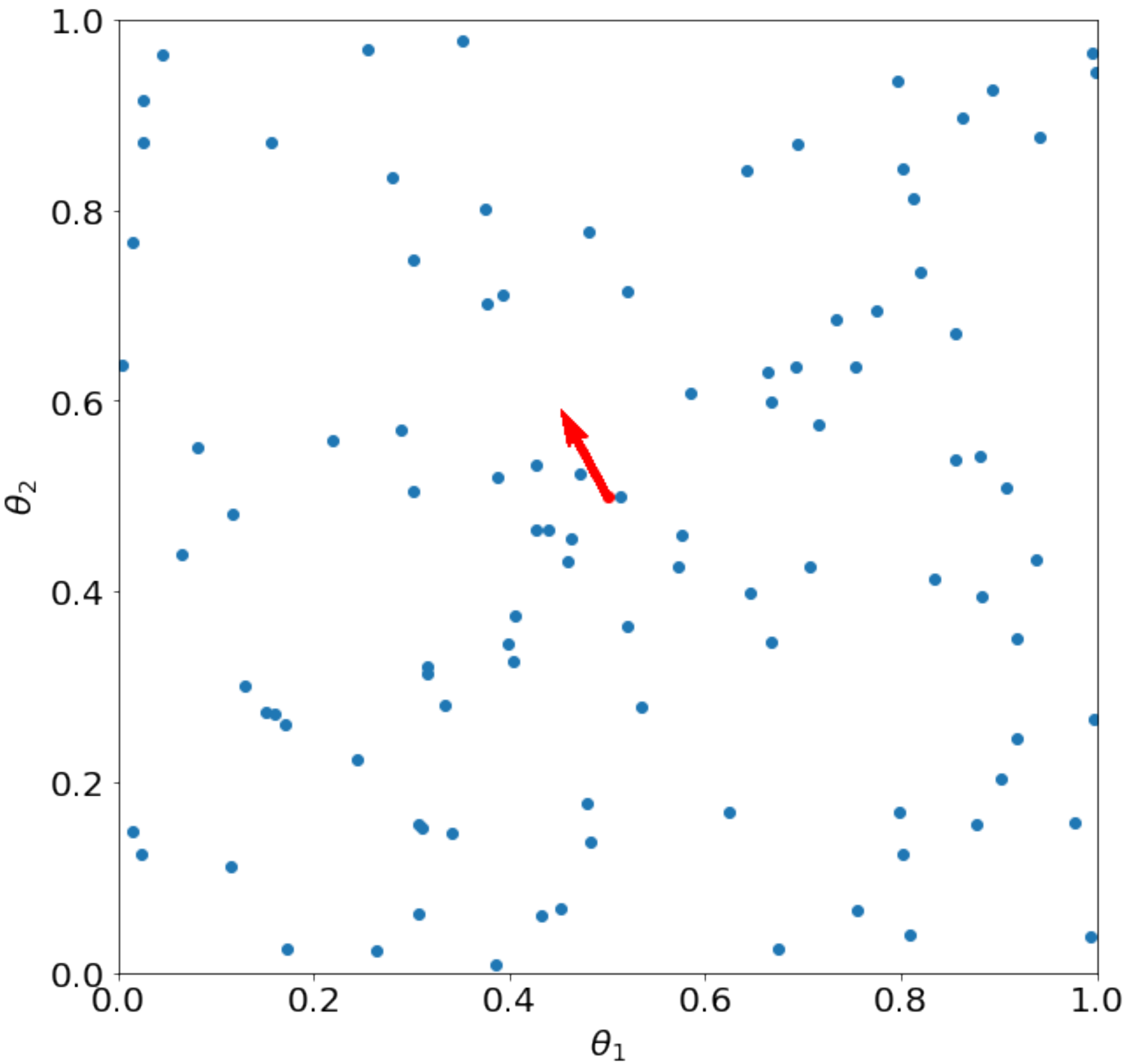
GRAVITATIONAL LENSING

MICROLENSING WITH COMPLEX LENSES

R. Benton Metcalf

2022-2023

MULTIPLE POINT MASSES



We consider a system of N point masses at the same distance D_L.

As seen, a light ray crossing the lens plane at the portion θ will experience the deflection

$$\hat{\alpha}(\vec{\theta}) = \frac{4G}{c^2 D_L} \sum_{i=1}^{N} \frac{M_i}{|\vec{\theta} - \vec{\theta}_i|^2} (\vec{\theta} - \vec{\theta}_i)$$

MULTIPLE POINT MASSES

- compared to an individual point mass, the spatial symmetry is broken
- The mass scale of the system is the total mass=sum of the individual masses
- We may use this mass to define an equivalent Einstein radius and use it to scale all angles

MULTIPLE POINT MASSES

$$M_{tot} = \sum_{i=1}^{N} M_i. \qquad\qquad m_i = M_i / M_{tot}$$

$$\vec{\alpha}(\vec{\theta}) = \sum_{i=1}^{N} \frac{D_{\mathrm{LS}}}{D_{\mathrm{L}} D_{\mathrm{S}}} \frac{4GM_i}{c^2} \frac{(\vec{\theta} - \vec{\theta}_i)}{|\vec{\theta} - \vec{\theta}_i|^2} \frac{M_{tot}}{M_{tot}} = \sum_{i=1}^{N} m_i \frac{\theta_E^2}{|\vec{\theta} - \vec{\theta}_i|^2} (\vec{\theta} - \vec{\theta}_i)$$

dividing by θ_E:

$$\vec{\alpha}(\vec{x}) = \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{x}_i|^2} (\vec{x} - \vec{x}_i)$$

$$\vec{y} = \vec{x} - \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{x}_i|^2} (\vec{x} - \vec{x}_i)$$

COMPLEX LENS EQUATION (WITT, 1990)

$$\vec{y} = \vec{x} - \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{x}_i|^2} (\vec{x} - \vec{x}_i)$$

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$$\vec{y} = \vec{x} - \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{x}_i|^2} (\vec{x} - \vec{x}_i)$$

$$z = x_1 + ix_2 \qquad z_s = y_1 + iy_2$$

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$$\vec{y} = \vec{x} - \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{x}_i|^2} (\vec{x} - \vec{x}_i)$$

$$z = x_1 + ix_2 \qquad\qquad z_s = y_1 + iy_2$$

$$\alpha(z) = \sum_{i=1}^{N} m_i \frac{(z - z_i)}{(z - z_i)(z^* - z_i^*)} = \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

COMPLEX LENS EQUATION (WITT, 1990)

$$\vec{y} = \vec{x} - \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{x}_i|^2} (\vec{x} - \vec{x}_i)$$

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$$z_s = z - \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

Complex lens equation

COMPLEX LENS EQUATION (WITT, 1990)

- Thus:

$$z_s = z - \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

- Taking the conjugate:

$$z^* = z_s^* + \sum_{i=1}^{N} \frac{m_i}{z - z_i}$$

- We obtain z^* and substitute it back into the original equation, which results in a (N^2+1)th order complex polynomial in the unknown z, $\boldsymbol{p}^{N^2+1}(z) = 0$

- This equation can be solved only numerically, even in the case of a binary lens

COMPLEX LENS EQUATION (WITT, 1990)

- Note that the solutions are not necessarily solutions of the lens equations (spurious solutions)
- One has to check if the solutions are solutions of the lens equation
- Rhie (2001,2003): maximum number of images is 5(N-1) for N>2

JACOBIAN DETERMINANT

The Jacobian determinant is (on the real plane):

$$\det A = \frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2} - \left(\frac{\partial y_1}{\partial x_2}\right)^2$$

How do we write it in complex notation?

JACOBIAN DETERMINANT

The complex derivatives (Wirtinger derivatives) of z_s are:

$$\begin{aligned}\frac{\partial z_s}{\partial z} &= \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} + \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} - \frac{\partial y_1}{\partial x_2}\right)\\ \frac{\partial z_s}{\partial z^*} &= \frac{1}{2}\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} - \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} + \frac{\partial y_1}{\partial x_2}\right)\end{aligned}$$

JACOBIAN DETERMINANT

Note that in lensing these two derivatives are equal!

The complex derivatives (Wirtinger derivatives) of z_s are:

$$\frac{\partial z_s}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} + \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} - \frac{\partial y_1}{\partial x_2}\right)$$

$$\frac{\partial z_s}{\partial z^*} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} - \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} + \frac{\partial y_1}{\partial x_2}\right)$$

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$$
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\end{aligned}
$$

Thus:

$$
\left(\frac{\partial z_s}{\partial z}\right)^2 = \frac{1}{4}\left[\left(\frac{\partial y_1}{\partial x_1}\right)^2 + \left(\frac{\partial y_1}{\partial x_2}\right)^2 + 2\frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2}\right]
$$

$$
\left(\frac{\partial z_s}{\partial z^*}\right)\left(\frac{\partial z_s}{\partial z^*}\right)^* = \frac{1}{4}\left[\left(\frac{\partial y_1}{\partial x_1}\right)^2 + \left(\frac{\partial y_1}{\partial x_2}\right)^2 - 2\frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2}\right] + \left(\frac{\partial y_1}{\partial x_2}\right)^2
$$

JACOBIAN DETERMINANT

Note that in lensing these two derivatives are equal!

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\frac{\partial z_s}{\partial z} &= \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} + \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} - \frac{\partial y_1}{\partial x_2}\right) \\
\frac{\partial z_s}{\partial z^*} &= \frac{1}{2}\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} - \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} + \frac{\partial y_1}{\partial x_2}\right)
\end{aligned}
$$

Thus:

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\left(\frac{\partial z_s}{\partial z}\right)^2 = \frac{1}{4}\left[\left(\frac{\partial y_1}{\partial x_1}\right)^2 + \left(\frac{\partial y_1}{\partial x_2}\right)^2 + 2\frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2}\right]
$$

$$
\left(\frac{\partial z_s}{\partial z^*}\right)\left(\frac{\partial z_s}{\partial z^*}\right)^* = \frac{1}{4}\left[\left(\frac{\partial y_1}{\partial x_1}\right)^2 + \left(\frac{\partial y_1}{\partial x_2}\right)^2 - 2\frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2}\right] + \left(\frac{\partial y_1}{\partial x_2}\right)^2
$$

By taking the difference of these two equations:

$$
\left(\frac{\partial z_s}{\partial z}\right)^2 - \left(\frac{\partial z_s}{\partial z^*}\right)\left(\frac{\partial z_s}{\partial z^*}\right)^* = \frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2} - \left(\frac{\partial y_1}{\partial x_2}\right)^2 = \det A
$$

JACOBIAN DETERMINANT (OR INVERSE MAGNIFICATION)

Now, we can use the lens equation:

$$z_s = z - \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

To obtain:

$$\frac{\partial z_s}{\partial z} = 1 \qquad \frac{\partial z_s}{\partial z^*} = \sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2}$$

so that

$$\left(\frac{\partial z_s}{\partial z}\right)^2 - \left(\frac{\partial z_s}{\partial z^*}\right)\left(\frac{\partial z_s}{\partial z^*}\right)^* = \frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2} - \left(\frac{\partial y_1}{\partial x_2}\right)^2 = \det A$$

$$\Rightarrow \quad \det A = 1 - \left|\sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2}\right|^2$$

CRITICAL LINES

From this equation:

$$\det A = 1 - \left| \sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2} \right|^2$$

We see that on the critical lines (detA=0)

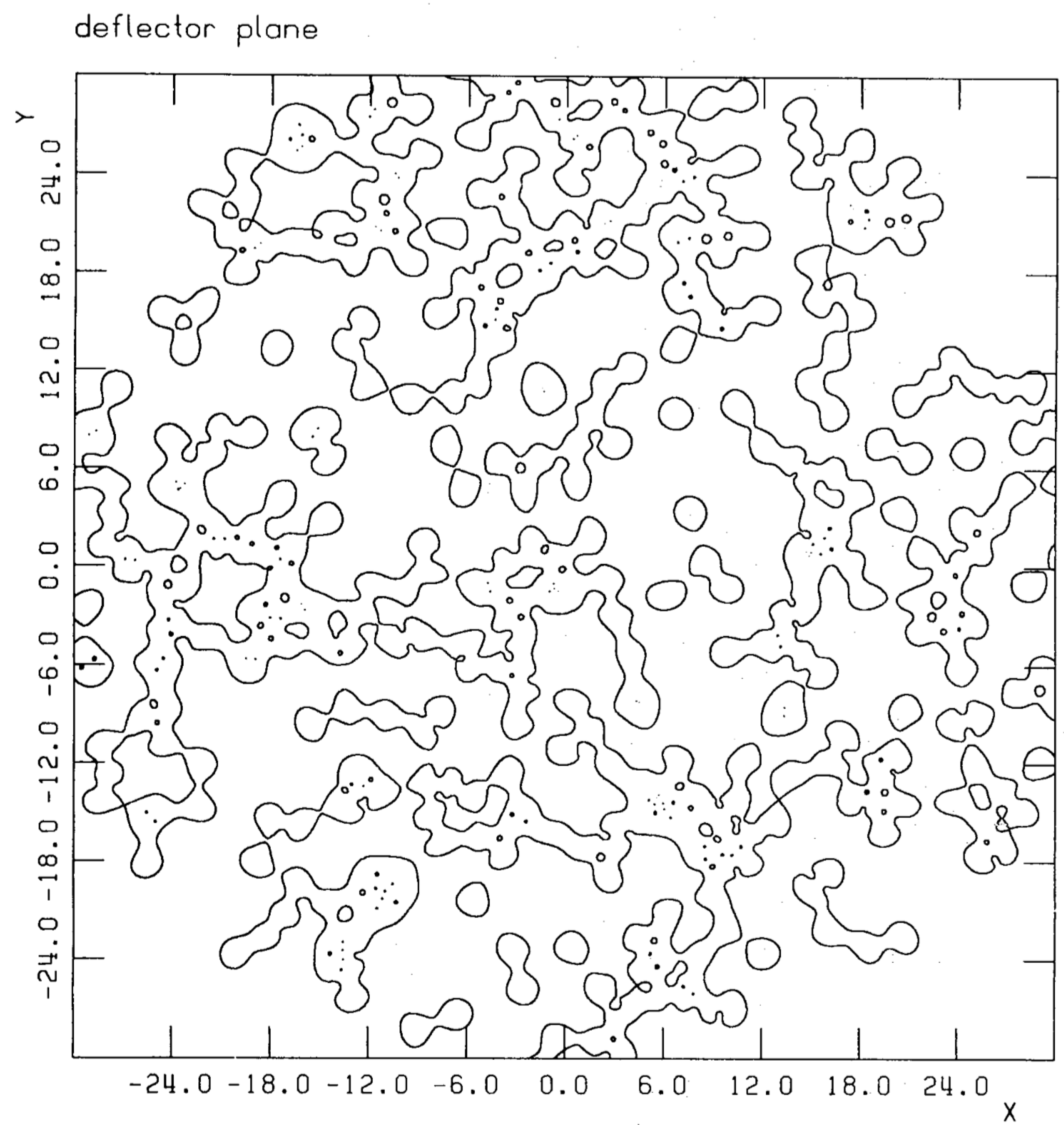
$$\left| \sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2} \right|^2 = 1$$

This sum has to be satisfied on the unit circle:

$$\sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2} = e^{i\phi} \qquad \phi \in [0, 2\pi)$$

Getting rid of the fraction, this equation can be turned into a polynomial of degree 2N: for each phase, there are <=2N critical points. Solving for all phases, we find up to 2N critical lines.

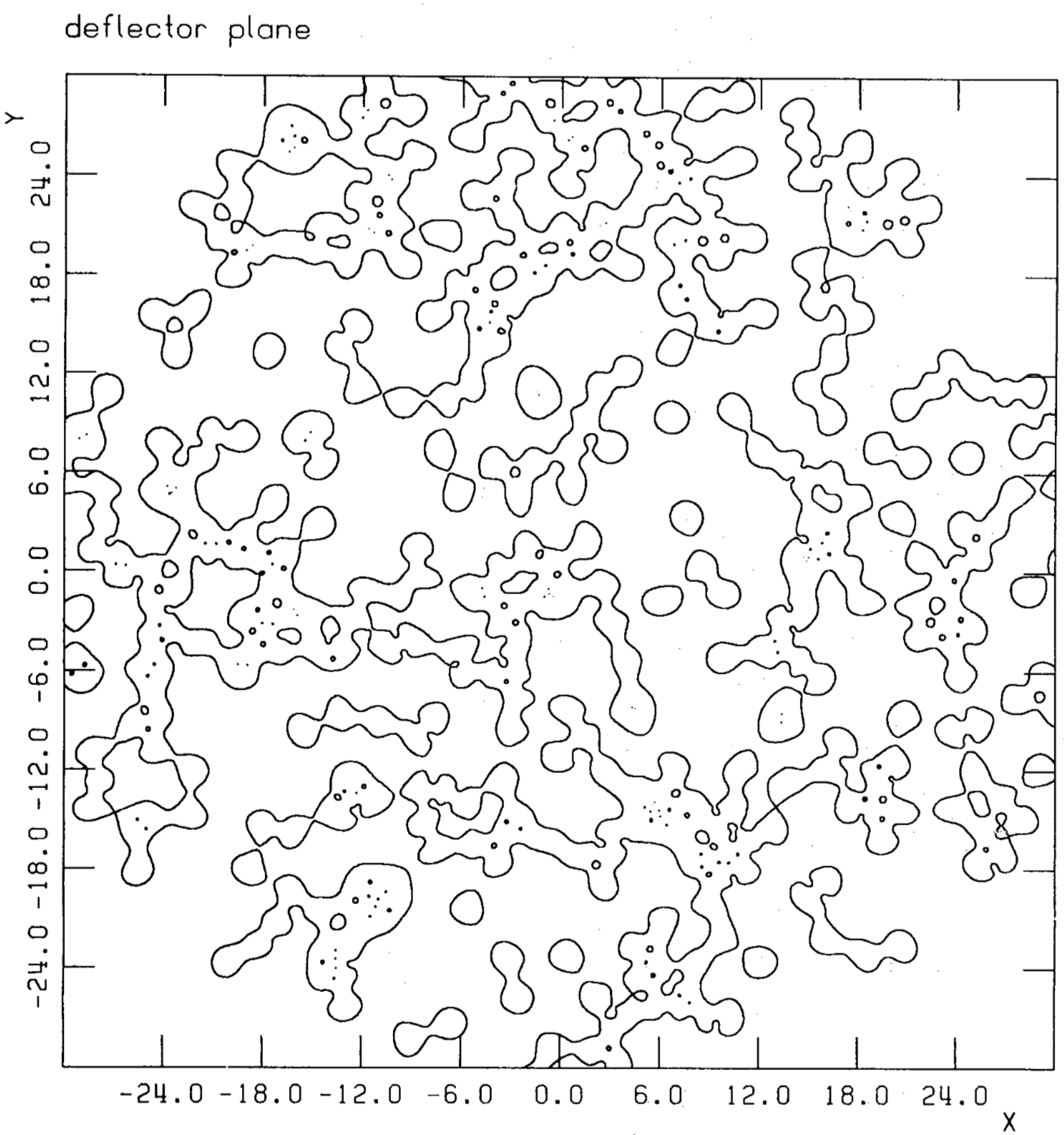
CRITICAL LINES

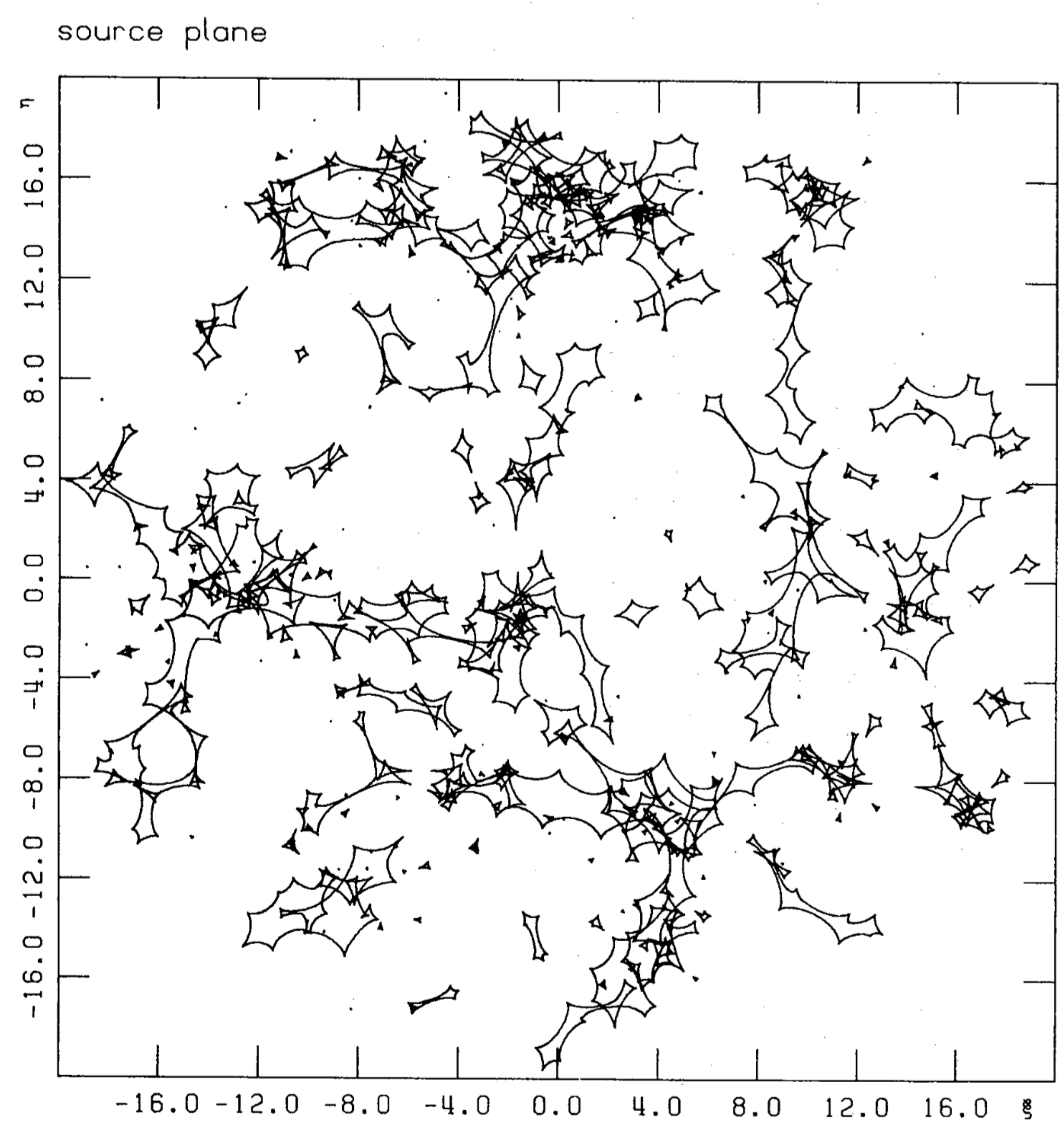


critical lines originated by 400 stars

Witt, 1990, A&A, 236, 311

CRITICAL LINES AND CAUSTICS





critical lines and caustics originated by 400 stars

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GRAVITATIONAL LENSING

10- GRAVITATIONAL MICROLENSING III : BINARY LENSES

R. Benton Metcalf

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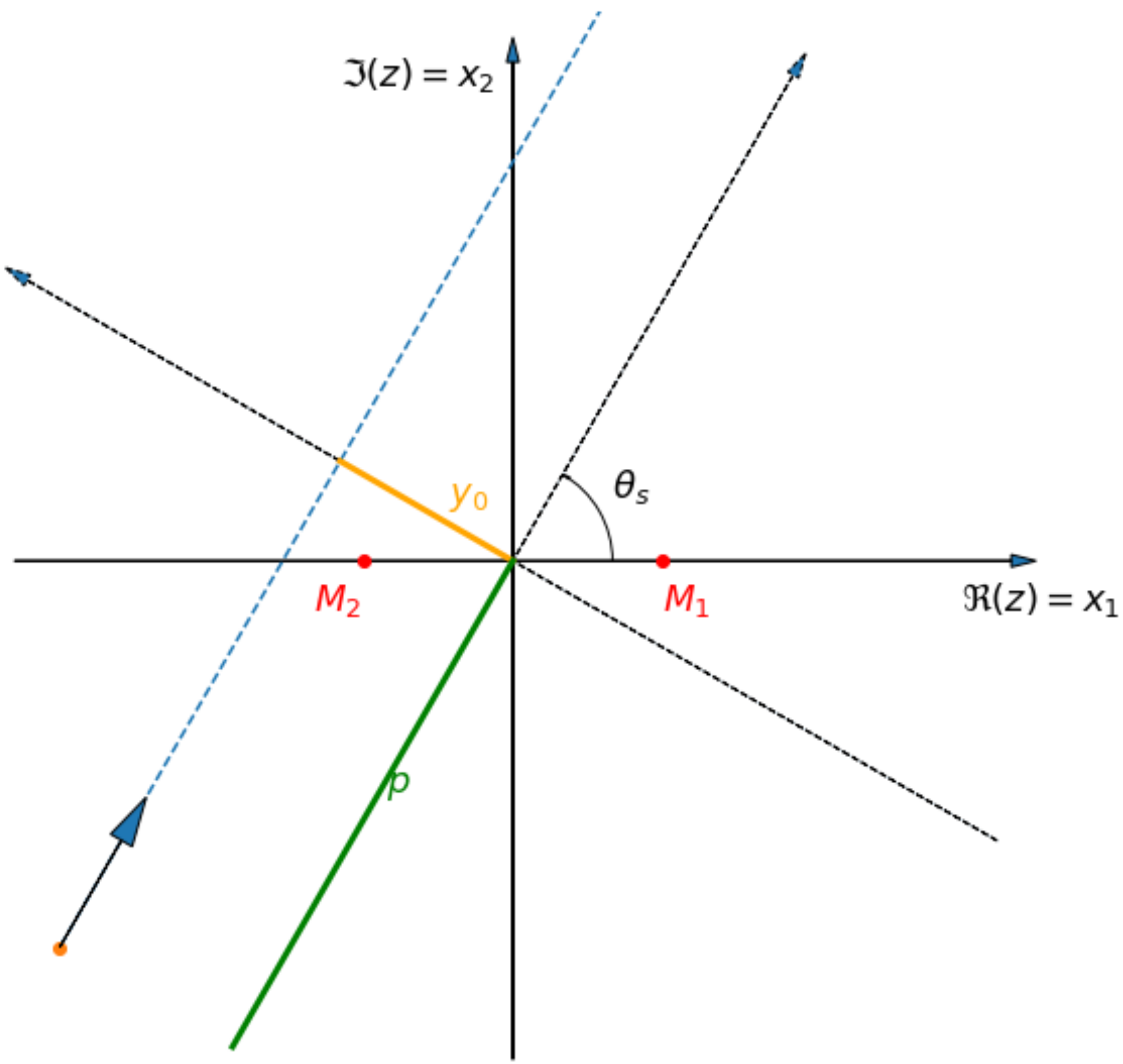
GEOMETRY OF A BINARY LENS



- Two point masses $M_1$and M_2
- Origin is chosen to coincide with the midpoint between the two masses Real axis passes through the two lenses.
- θ_S inclination of the source trajectory relative to the real axis
- y_0: impact parameter with respect to the origin
- t_0: time of minimum distance from the origin
- $p = \dfrac{t - t_0}{t_E}$: time from t_0 in units of t_E
- Position of the source in the complex plane:

$$\Re(z) = p \cos \theta_S - y_0 \sin \theta_S$$

$$\Im(z) = p \sin \theta_S + y_0 \cos \theta_S$$

LENS EQUATION FOR THE BINARY LENS

$$z_s = z - \sum_{i=1}^{2} \frac{m_i}{z^* - z_i^*} = z - \frac{m_1}{z^* - z_1^*} - \frac{m_2}{z^* - z_2^*}$$

Take the complex conjugate to compute z^* and insert it back… we obtain a polynomial equation of degree $(N^2+1) = 5$:

$$p_5(z) = \sum_{i=0}^{5} c_i z^i = 0 \qquad \Delta m = \frac{m_1 - m_2}{2} \quad m = \frac{m_1 + m_2}{2} \quad z_2 = -z_1 \quad z_1 = z_1^*$$

$$c_0 = z_1^2[4(\Delta m)^2 z_s + 4m\Delta m z_1 + 4\Delta m z_s z_s^* z_1 + 2m z_s^* z_1^2 + z_s z_s^{*2} z_1^2 - 2\Delta m z_1^3 - z_s z_1^4]$$

$$c_1 = -8m\Delta m z_s z_1 - 4(\Delta m)^2 z_1^2 - 4m^2 z_1^2 - 4m z_s z_s^* z_1^2 - 4\Delta m z_s^* z_1^3 - z_s^{*2} z_1^4 + z_1^6$$

$$c_2 = 4m^2 z_s + 4m\Delta m z_1 - 4\Delta m z_s z_s^* z_1 - 2z_s z_s^{*2} z_1^2 + 4\Delta m z_1^3 + 2z_s z_1^4$$

$$c_3 = 4m z_s z_s^* + 4\Delta m z_s^* z_1 + 2z_s^{*2} z_1^2 - 2z_1^4$$

$$c_4 = -2m z_s^* + z_s z_s^{*2} - 2\Delta m z_1 - z_s z_1^2$$

$$c_5 = z_1^2 - z_s^{*2}$$

Up to 5 images

CRITICAL LINES AND CAUSTICS

$$\det A = 1 - \left| \sum_{i=1}^{2} \frac{m_i}{(z^* - z_i^*)^2} \right|^2 = 1 - \left| \frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2} \right|^2 = 0$$

Or

$$\left| \frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2} \right|^2 = 1 \Rightarrow \frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2} = e^{i\phi} \quad \forall \ \phi \in [0,2\pi)$$

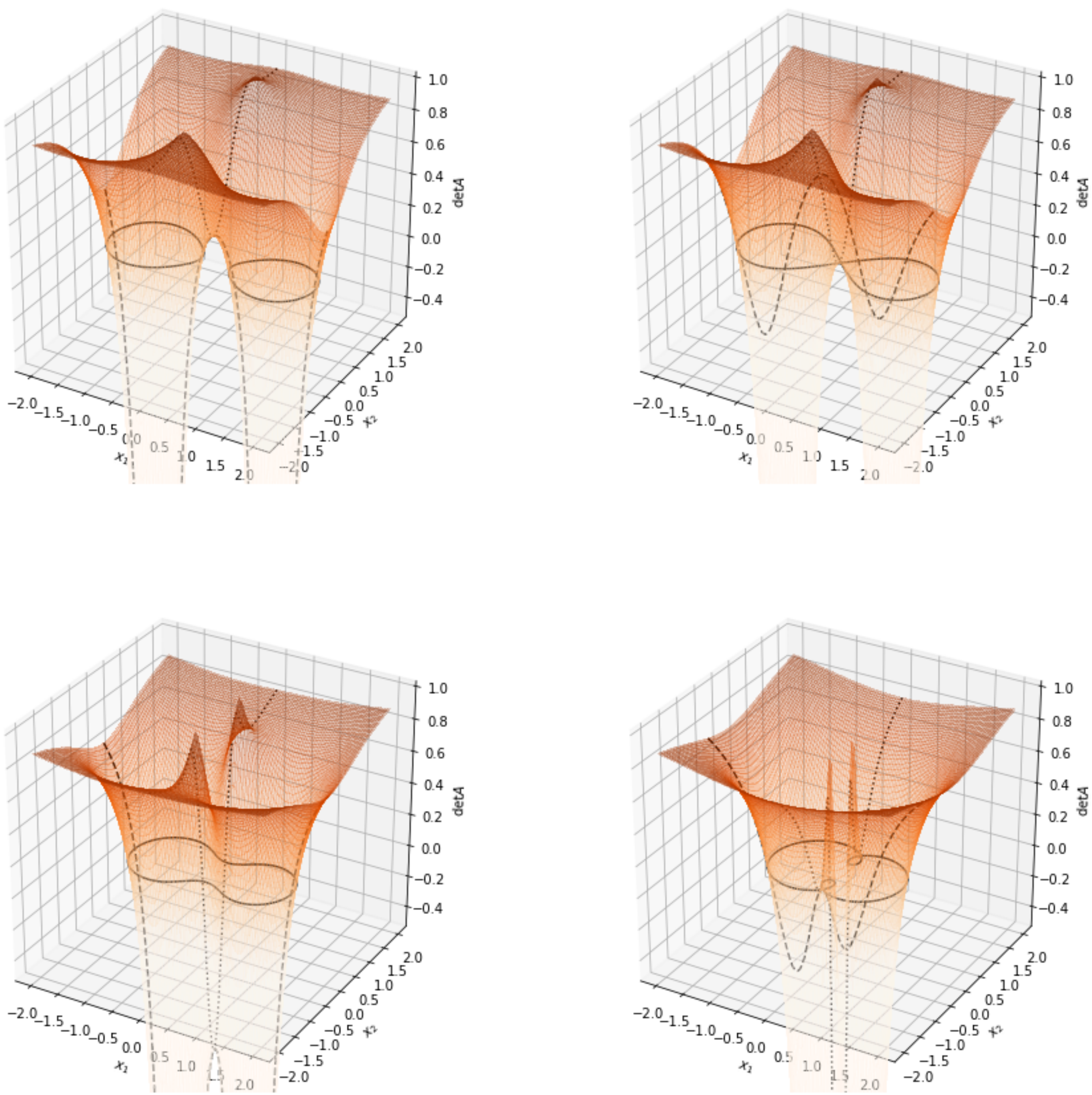
Which can be reduced to

$$p_4(z) = z^4 - z^2(2z_1^{*2} + e^{i\phi}) - zz_1^* 2(m_1 - m_2)e^{i\phi} + z_1^{*2}(z_1^{*2} - e^{i\phi}) = 0$$

Once the roots are found (critical points), the caustics can be obtained using the lens equation:

$$z_{cau} = z_{crit} - \frac{m_1}{z_{crit}^* - z_1^*} - \frac{m_2}{z_{crit}^* - z_2^*}$$

$\det A$ SURFACES



Depending on the distance between the two lenses

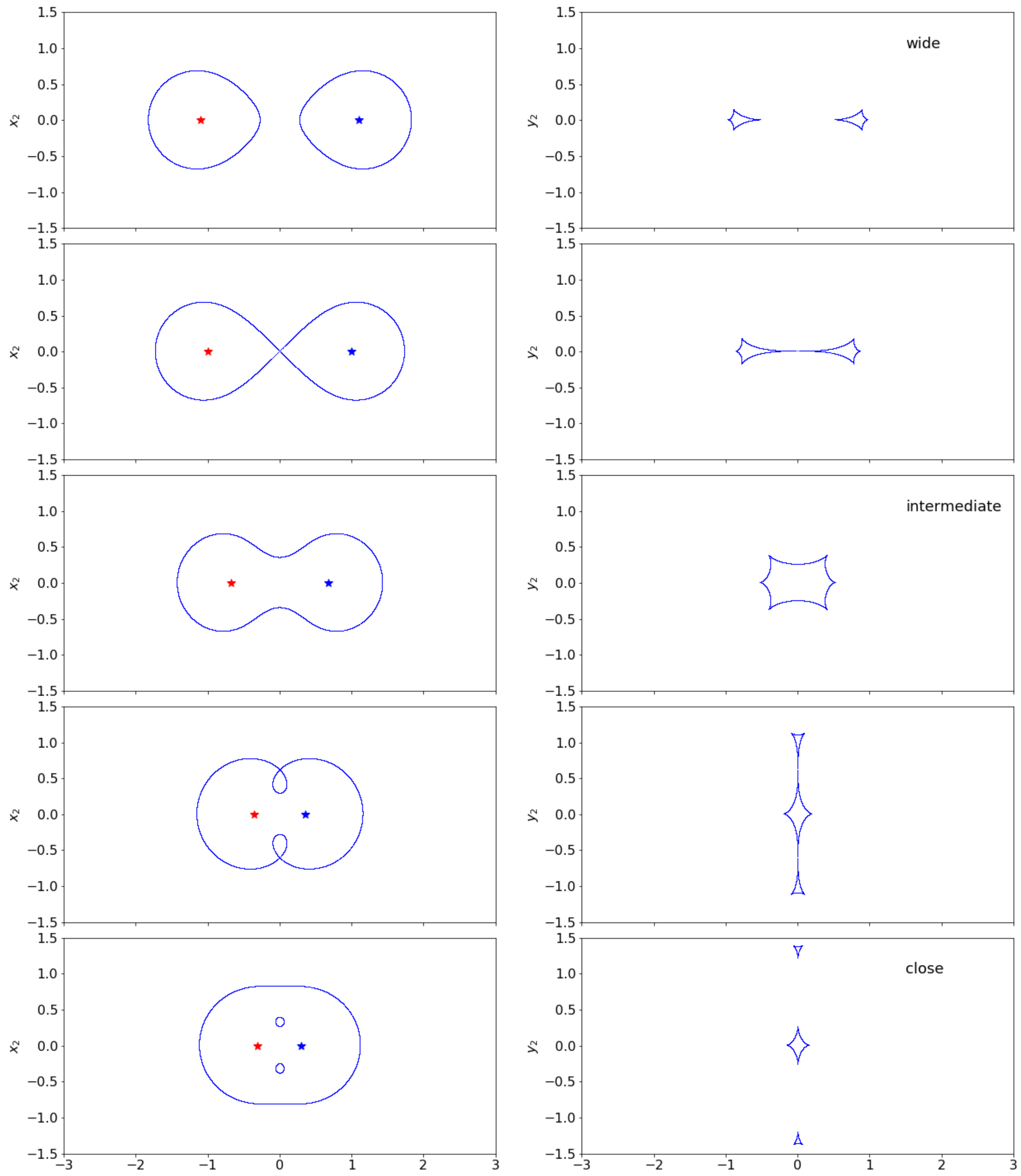
$$d = z_1 - z_2 = 2z_1$$

we can have one, two, or three critical lines. Note that the transitions between these three regimes happen when critical lines touch.

This happens at saddle points of the $\det A$ *surface!*

$$\frac{\partial \det A}{\partial z^*} = 0$$

CAUSTIC (AND CRITICAL LINE) TOPOLOGIES



$$d > d_{WI}$$

$$d = d_{WI} = (m_1^{1/3} + m_2^{1/3})^{3/2}$$

$$d_{IC} < d < d_{WI}$$

$$d = d_{IC} = (m_1^{1/3} + m_2^{1/3})^{-3/4}$$

$$d < d_{IC}$$

MAGNIFICATION

On the lens plane, the magnification is

$$\det A(z) = 1 - \left| \sum_{i=1}^{2} \frac{m_i}{(z^* - z_i^*)^2} \right| \qquad \mu(z) = \det A(z)^{-1}$$

Remember that even microlensing by binary lenses will be revealed through magnification effects.

No single images will be observed! Thus, what matters is the total magnification of all images of a given source:

$$\mu(z_s) = \sum_{images} |\mu_{ima}(z_{ima})|$$

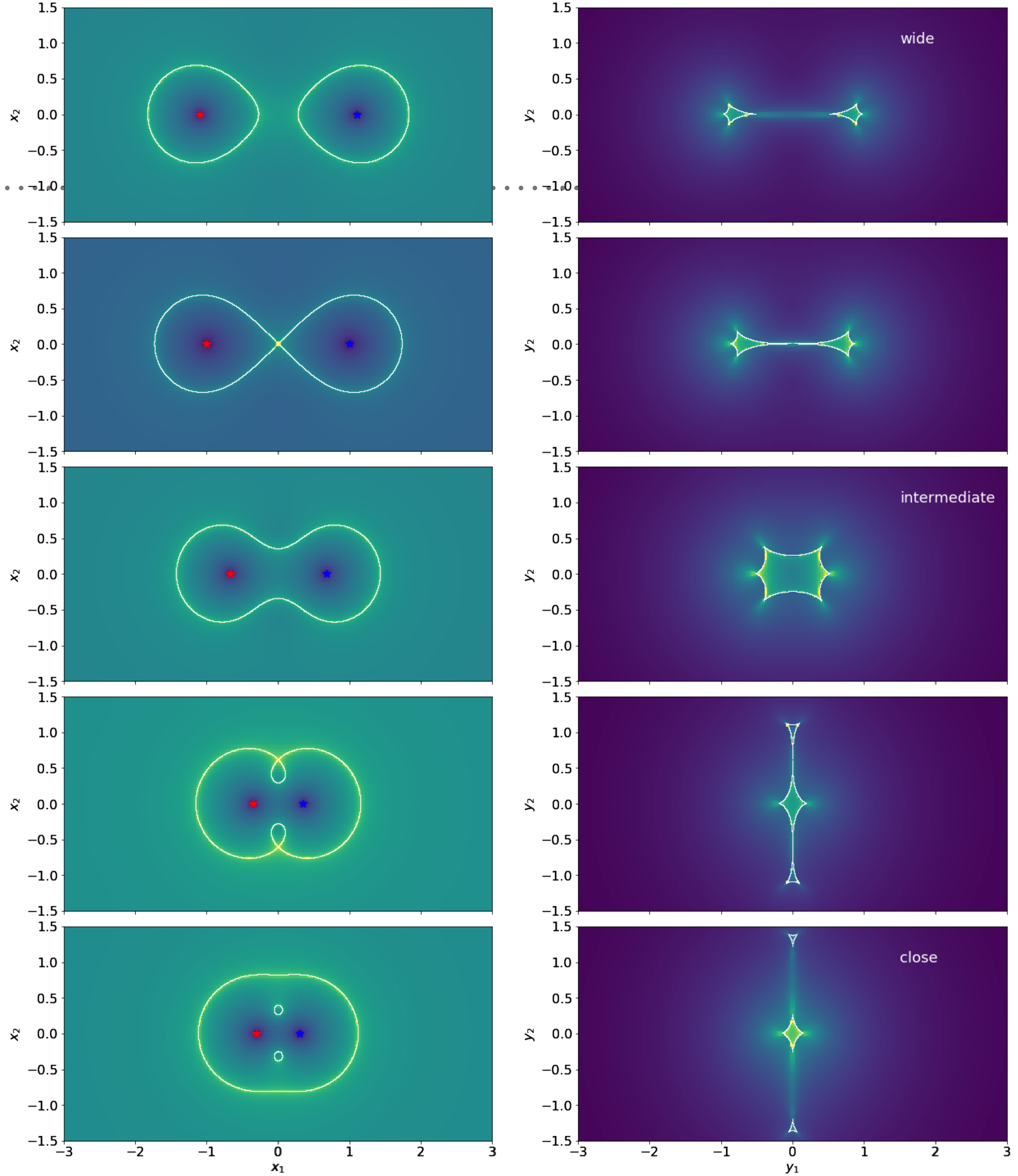
where z_{ima} are now the positions of the images of the source at z_s

EXAMPLES

Shown are maps of the magnification on the lens (left) and on the source plane (right)

We can recognise the critical lines and the caustics (where magnification diverges)

These maps are very important: depending on the trajectory on the source relative to the lens, several of these features will be impressed in the light-curves!



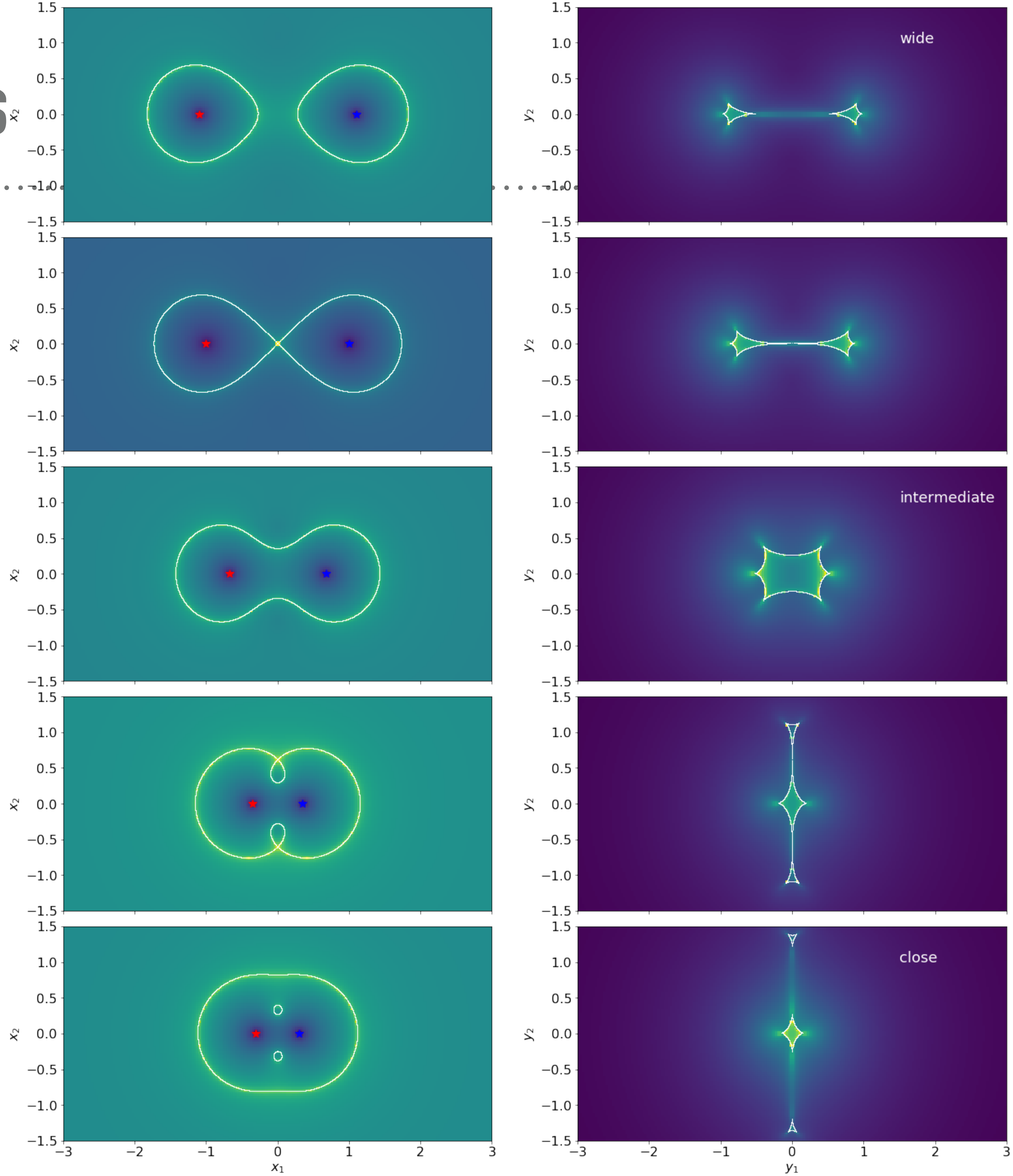
INTERESTING PROPERTIES

Lobes of high magnification near the ***cusps***

Sharp magnification changes on the ***folds***

Inside the caustics: *moderately high magnification* $\mu > 3$ *(Witt & Mao 1995)*

Extended regions of high magnification in ***between the caustics*** *in wide and close systems*



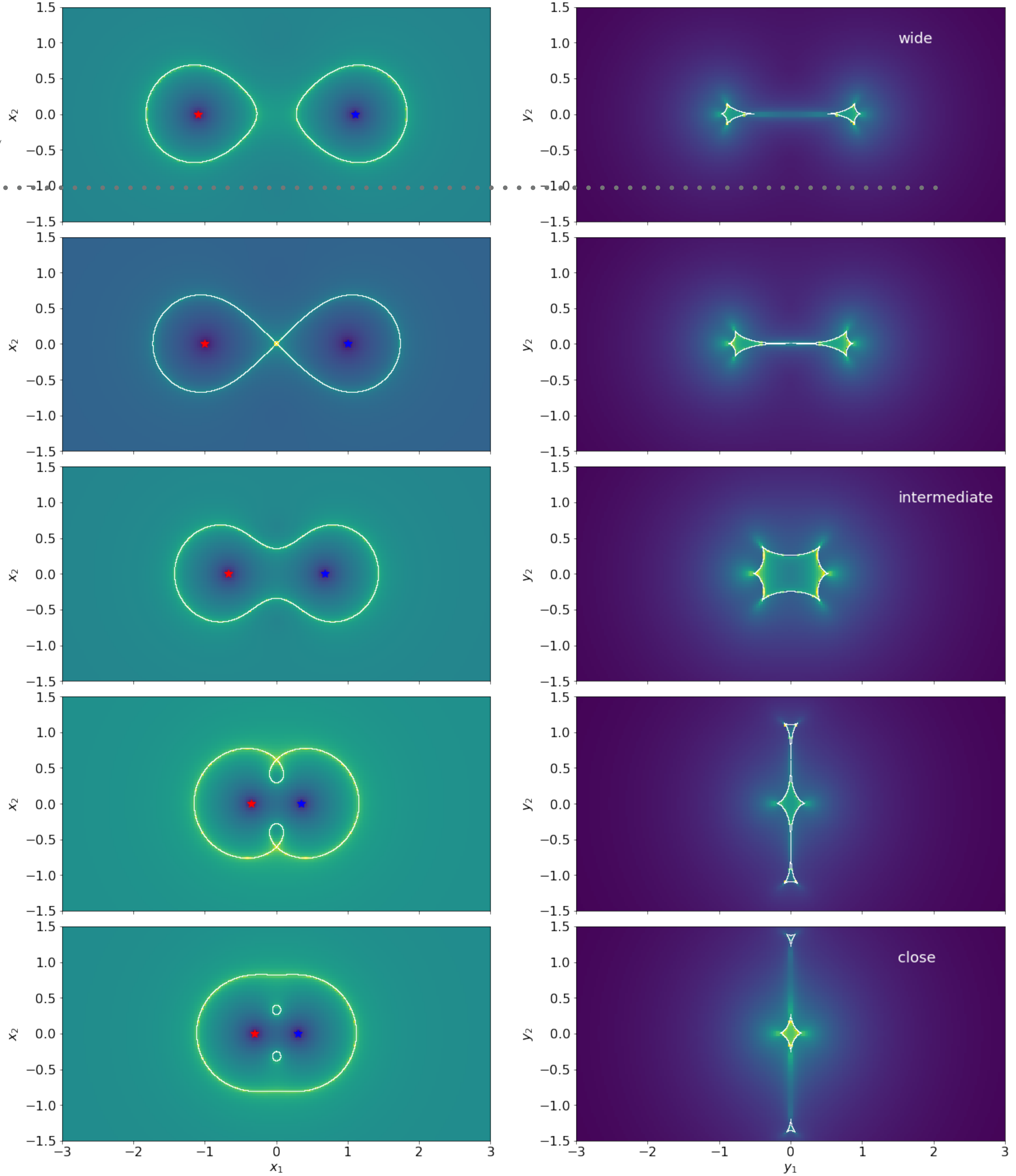
DEPENDENCE ON $q = M_1/M_2$

As q changes, the morphologies of the magnification maps, critical lines and caustics change

Critical lines: larger around the primary lens, smaller around the secondary

Wide systems: smaller caustic for the primary, larger for the secondary

Close systems: secondary caustics move to the back of the primary



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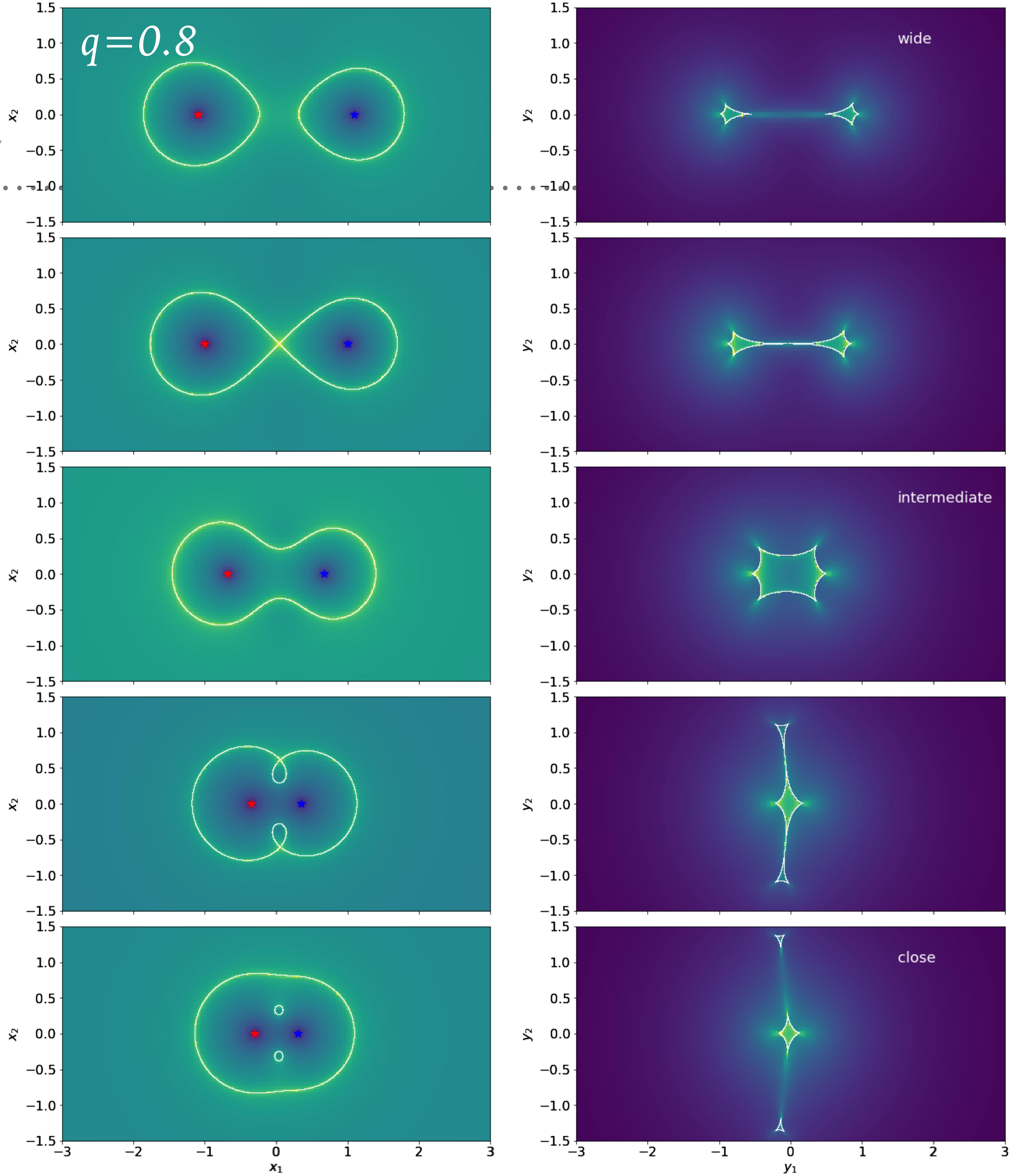

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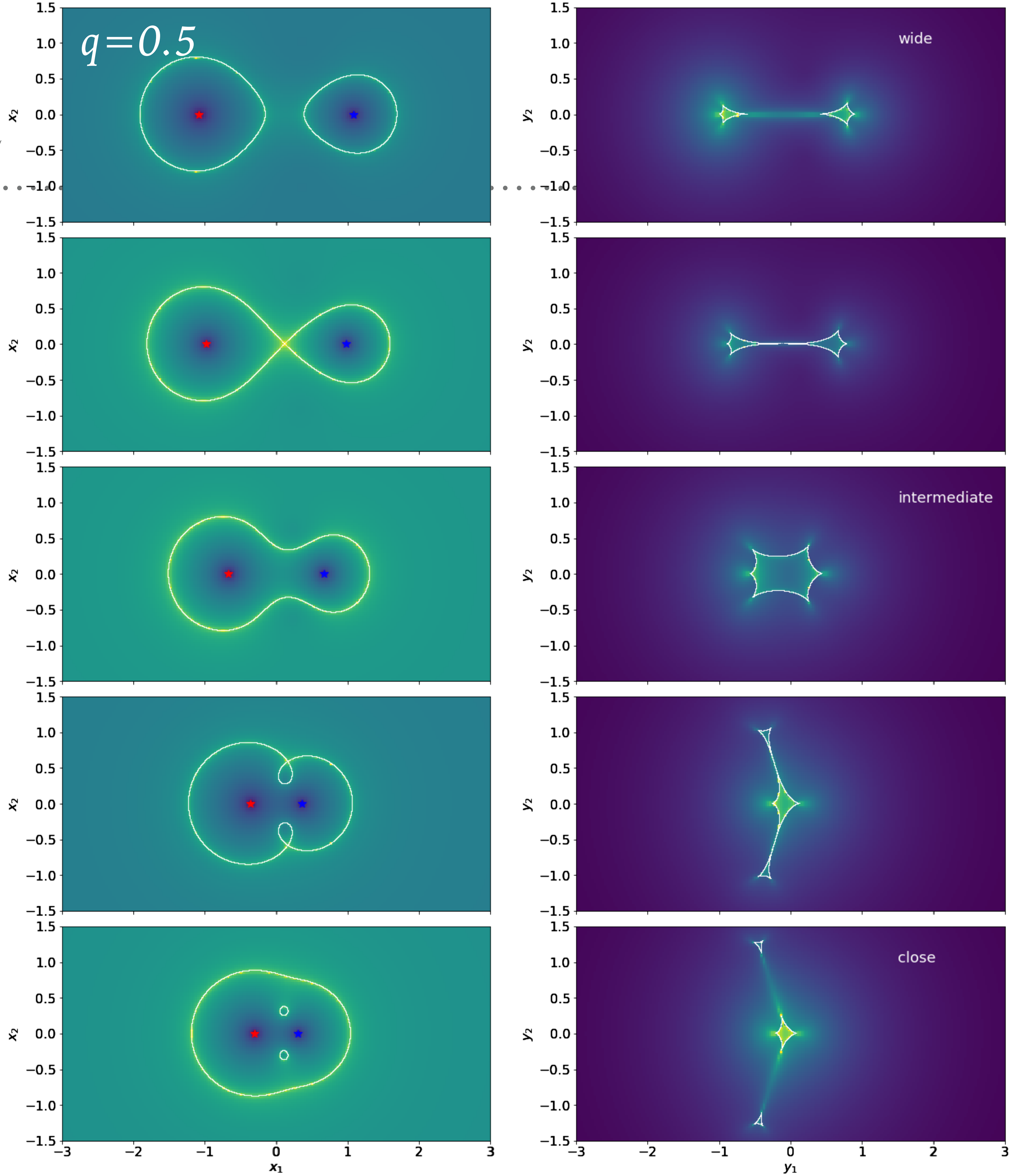

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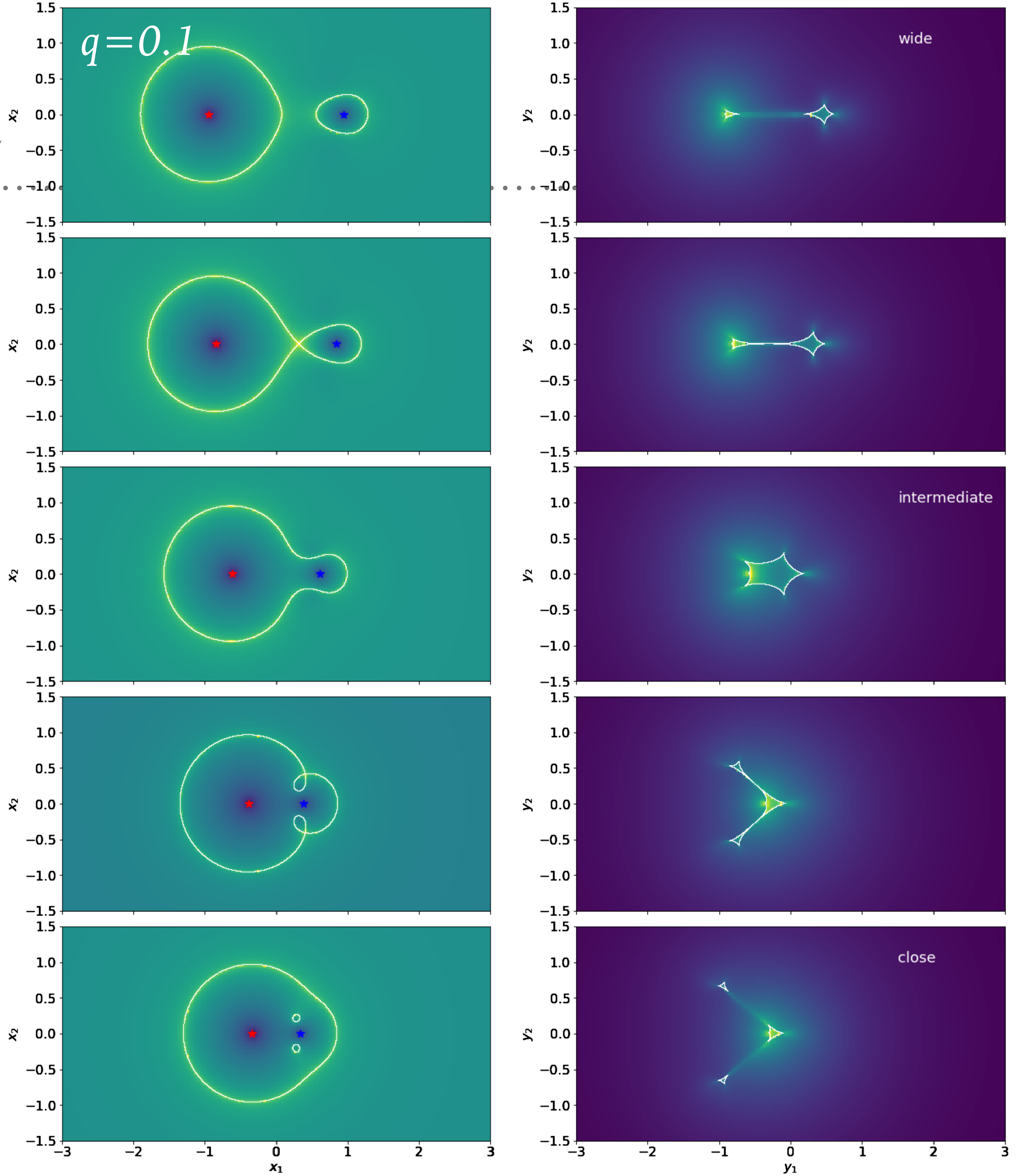
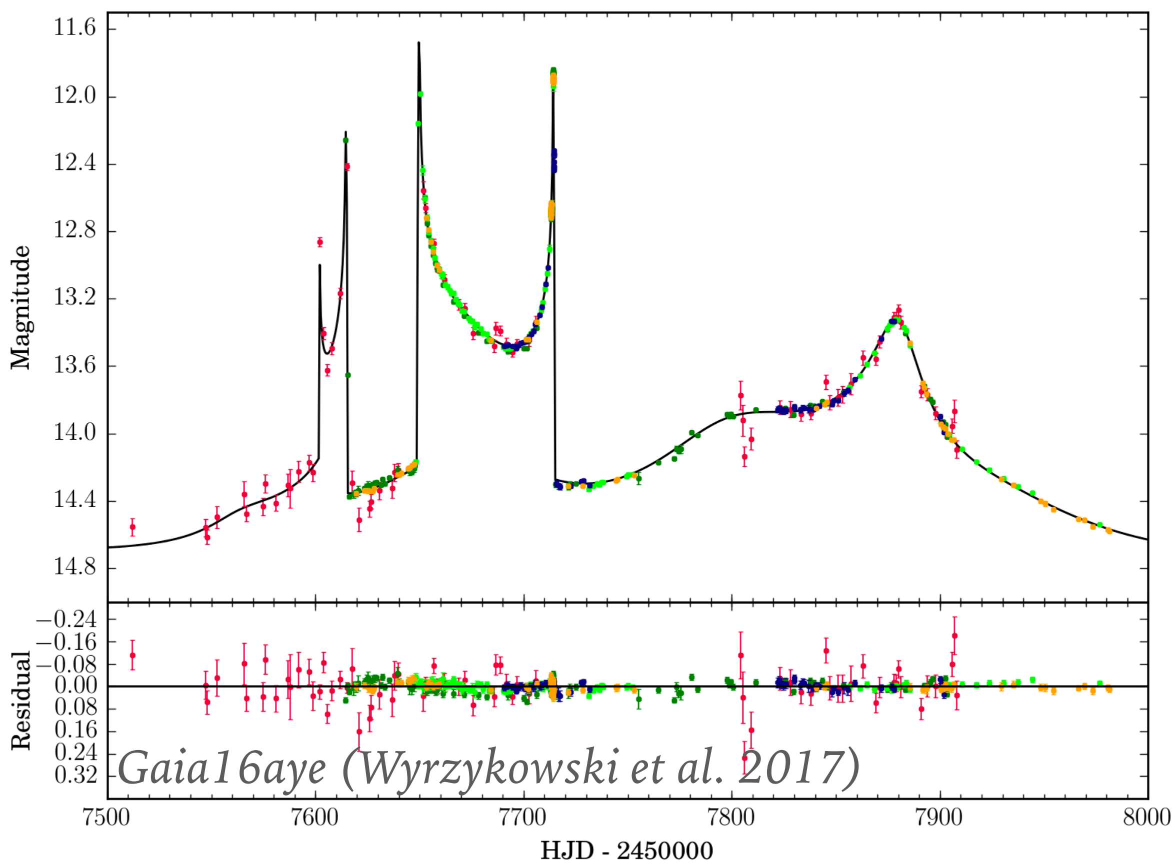
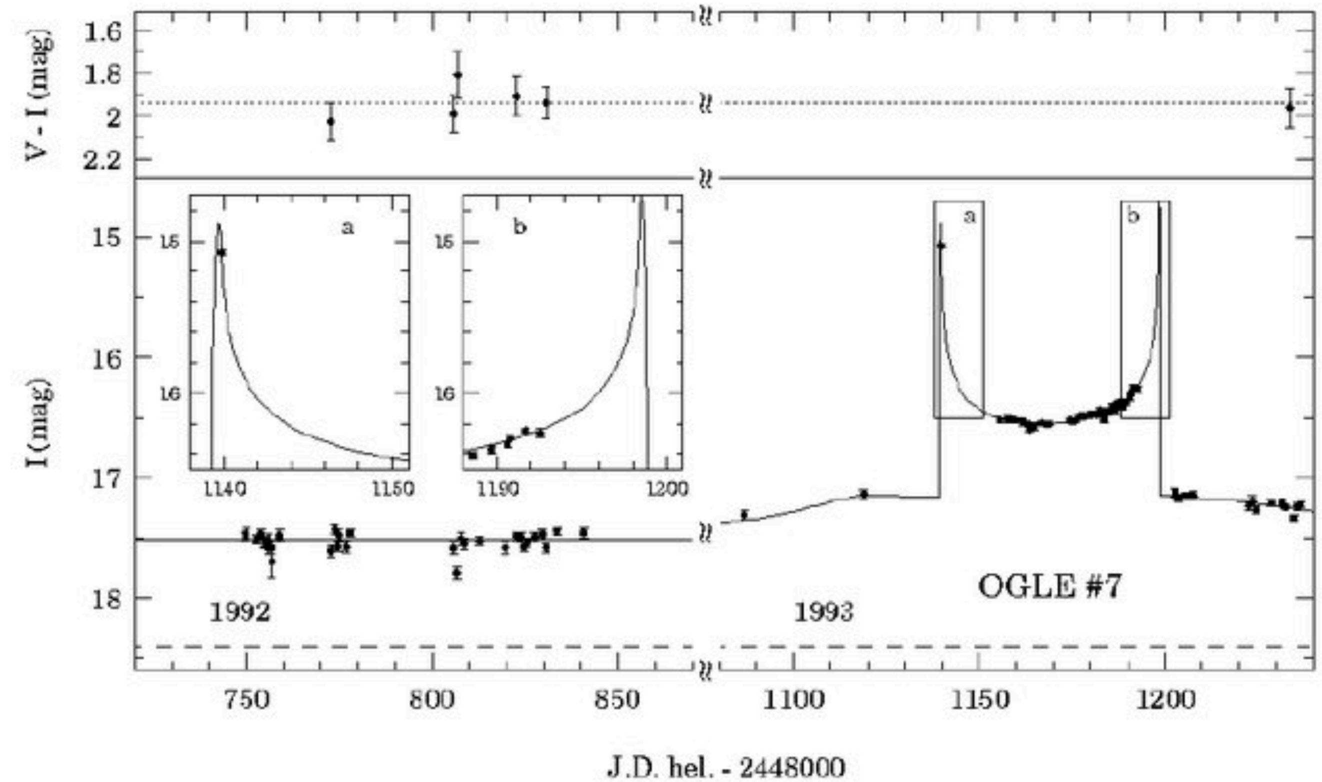
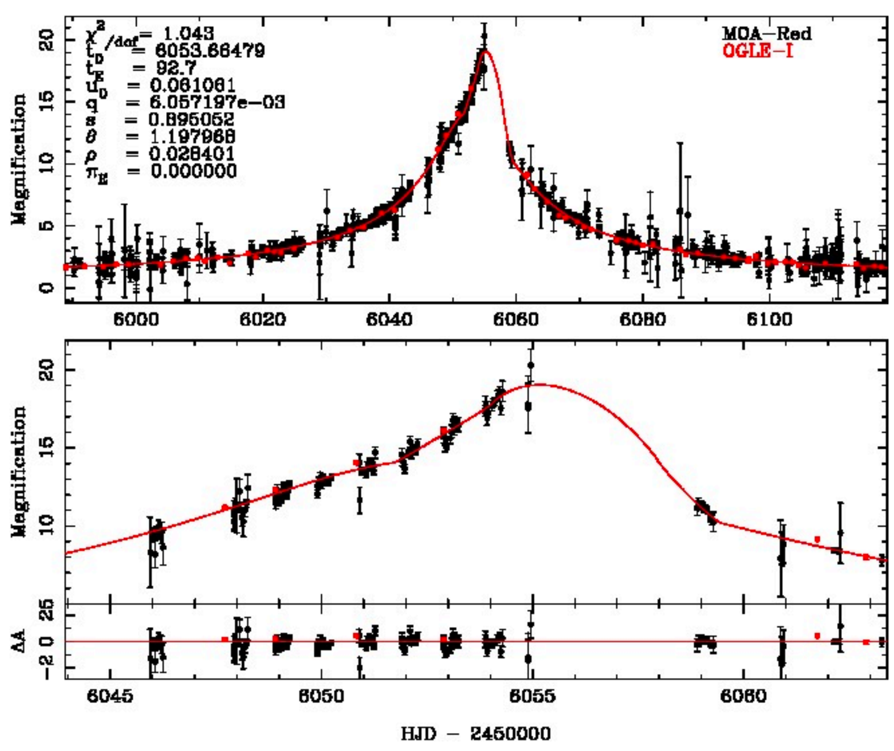
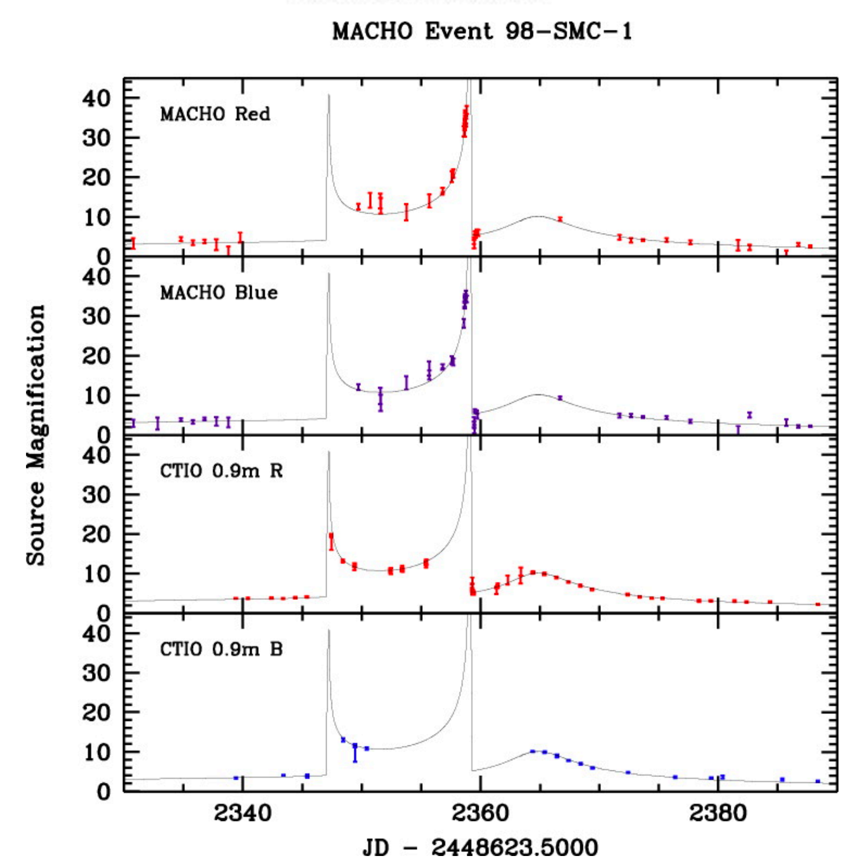


SOME OBSERVED LIGHT CURVES









PLANETARY MICROLENSING

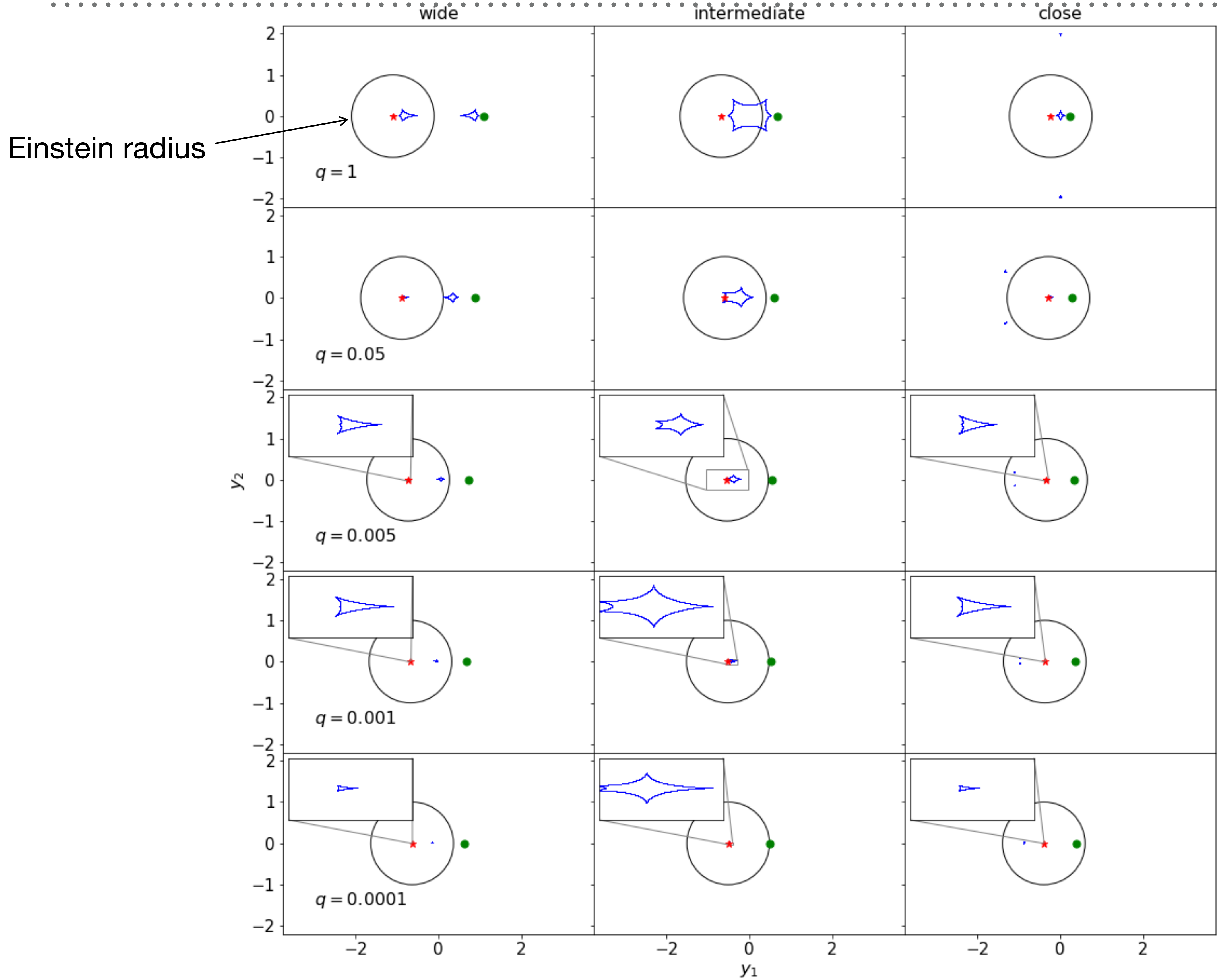
Consider a system of a host star and a planet orbiting it.

This is a binary system where the mass ratio q is very small.

example : For the Jupiter- Sun system $q = 0.001$.

example : For the Earth- Sun system $q = 3 \times 10^{-6}$.

CAUSTICS AS A FUNCTION OF $q = m_2/m_1$



PLANETARY MICROLENSING

Light curve is mostly that of an isolated star, but …

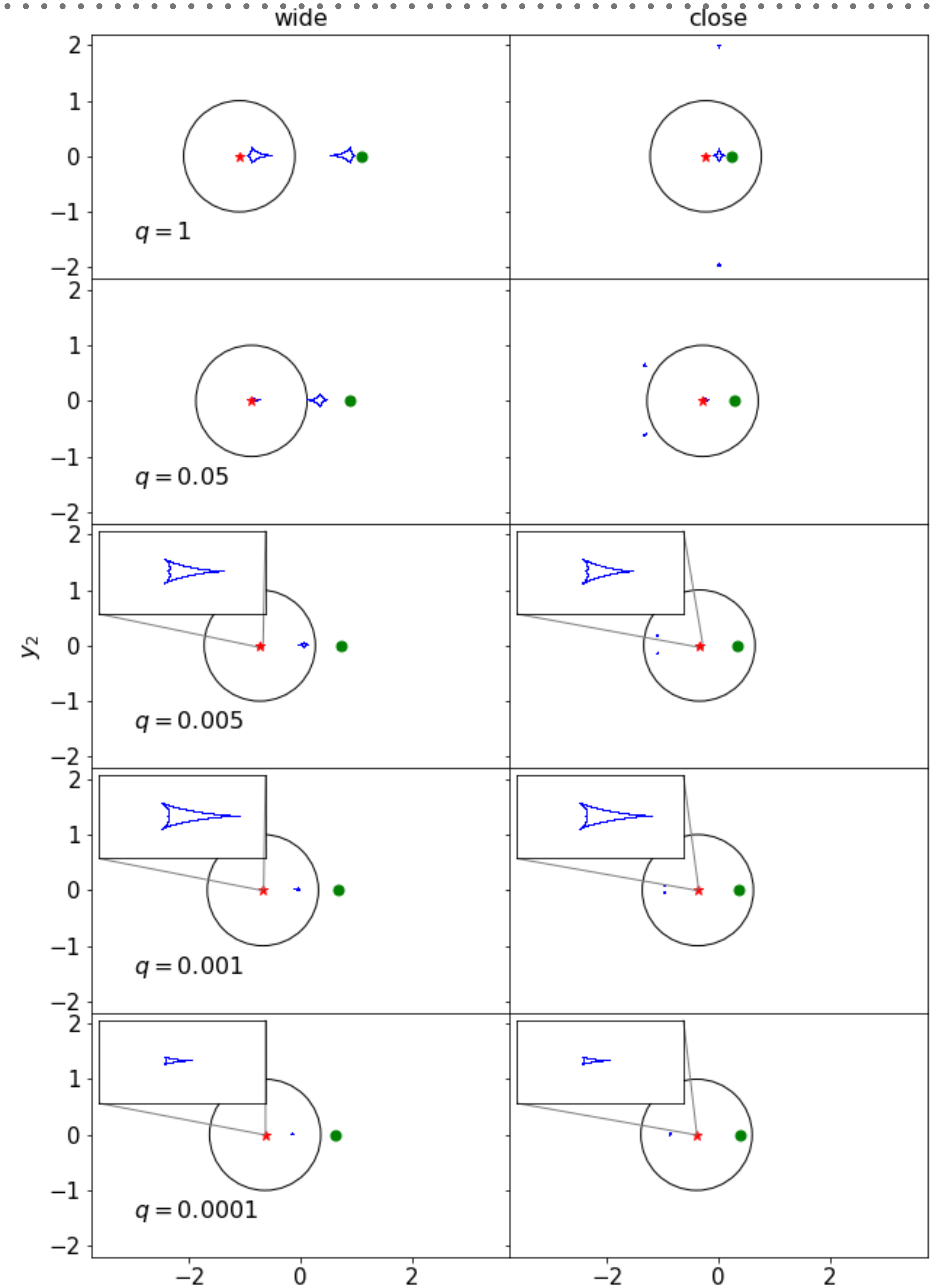
The planets produce small perturbations to the magnification patter, localized to a small region of near the caustics.

The source must cross one of these perturbed region to produce a detectable effect.

The shape of the perturbation to the light curve is determined by the caustic configuration.

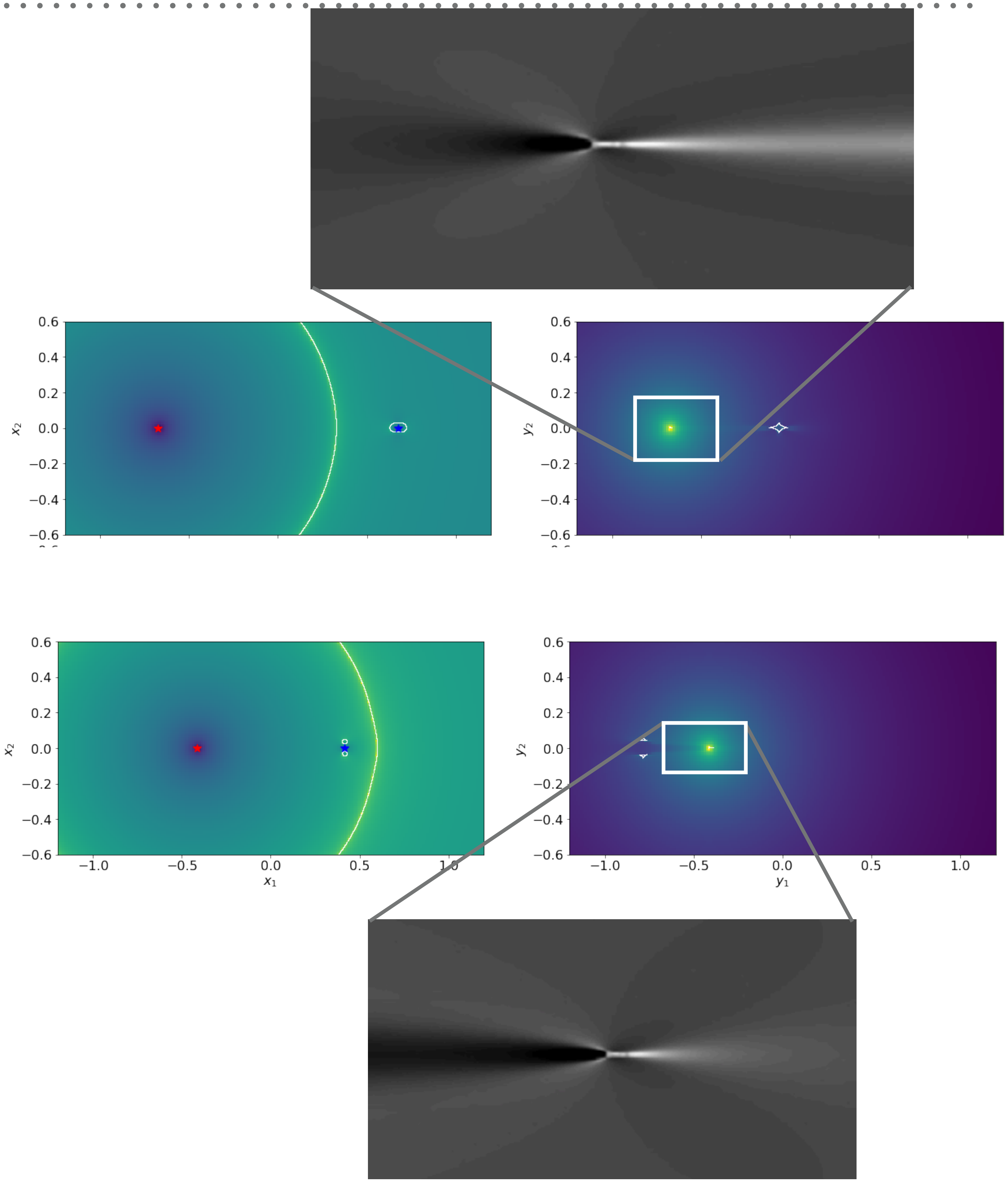
PERTURBATIONS OF THE CENTRAL CAUSTIC (WIDE/CLOSE SYSTEMS)

- As q decreases, we see that one caustic shrinks and approaches the primary lens (i.e. the star)
- This is what we call the ***"central caustic"*** in wide and close systems
- Four cusps and four folds
- One cusp is elongated towards the planet
- Three cusps on the back
- **Different from point-like caustic of a point lens!**

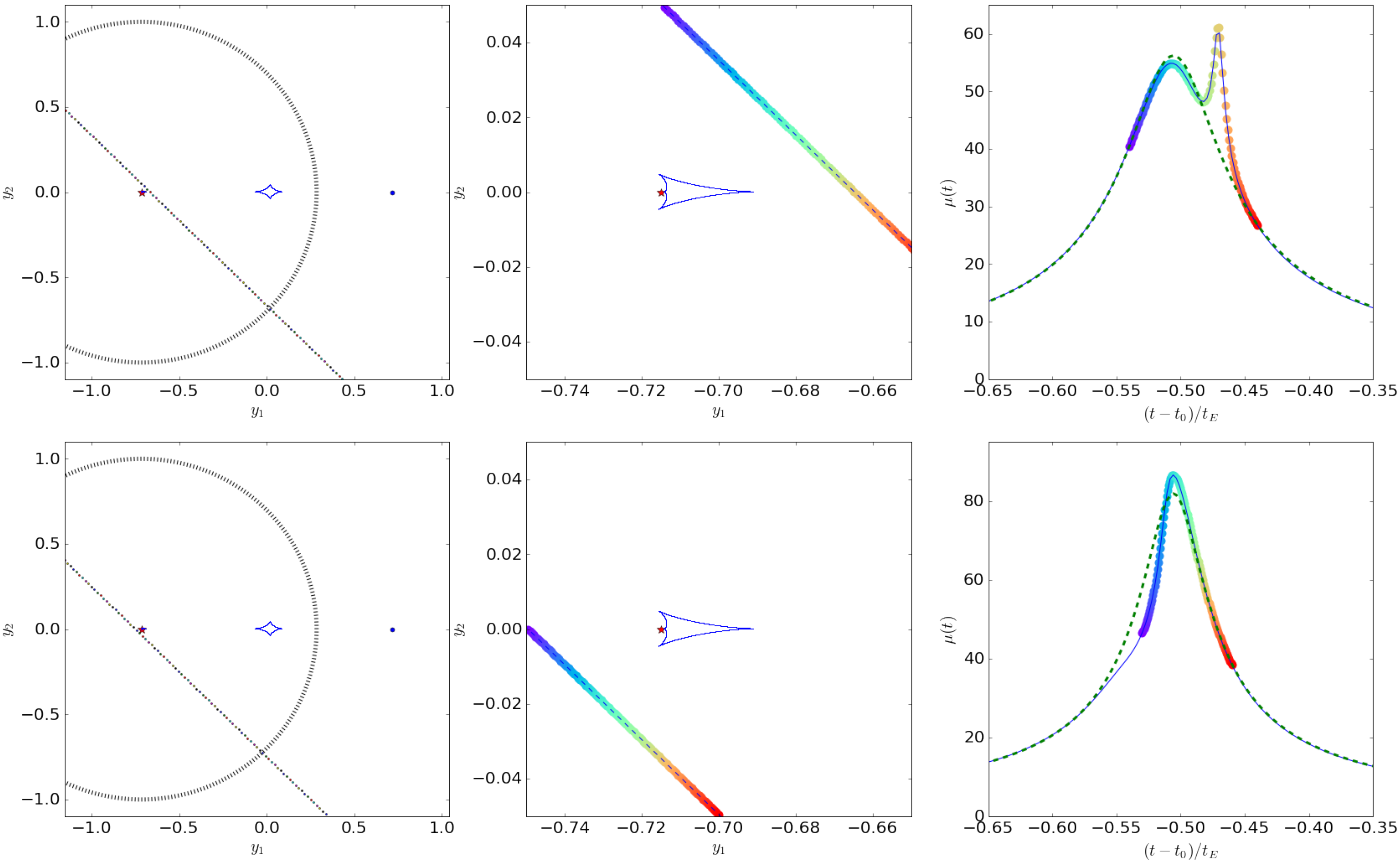


WHAT KIND OF SIGNATURES?

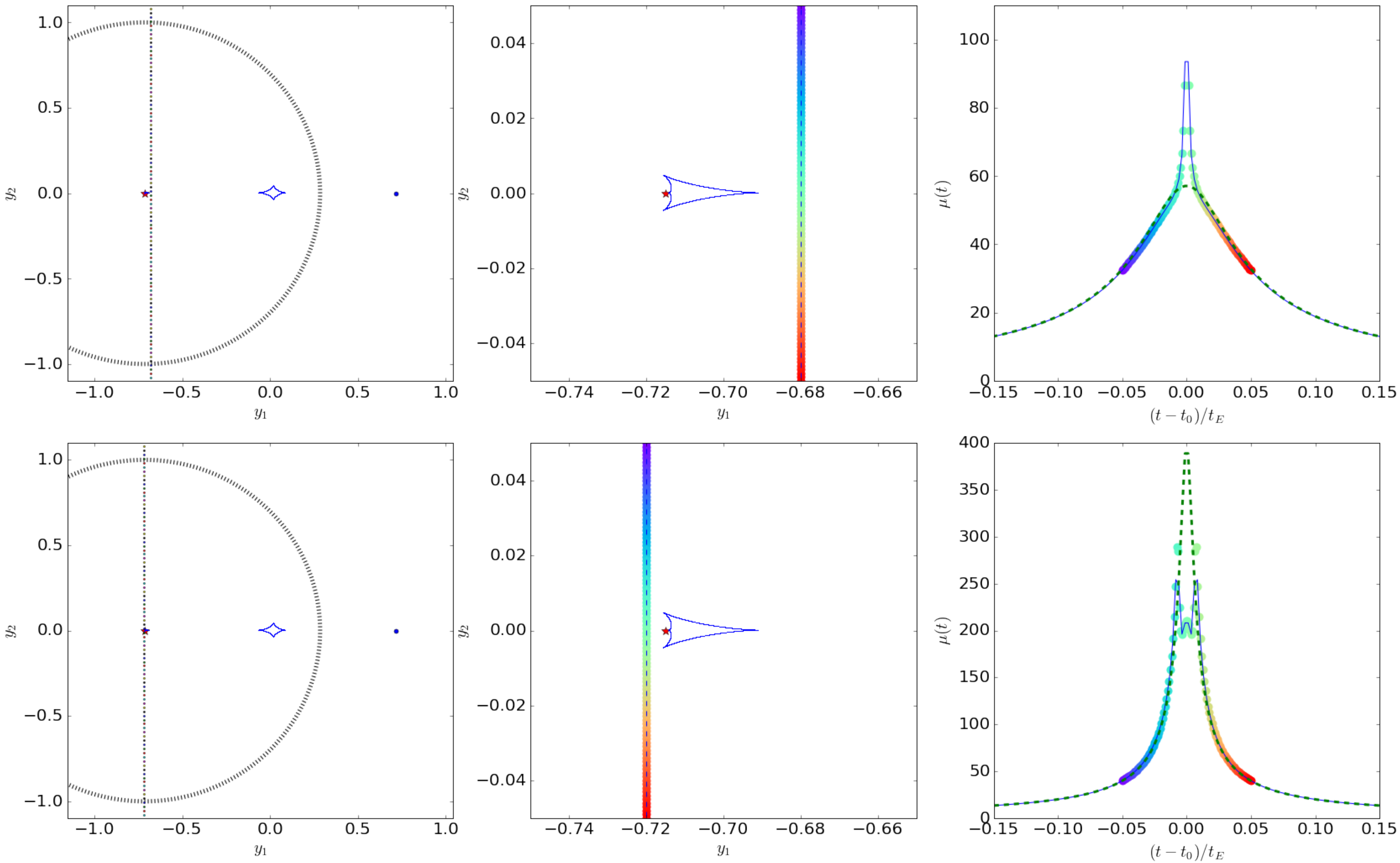
- Magnification if the sources passes in between the planet and the star
- De-magnification if the source passes on the back of the caustic!
- Some examples…



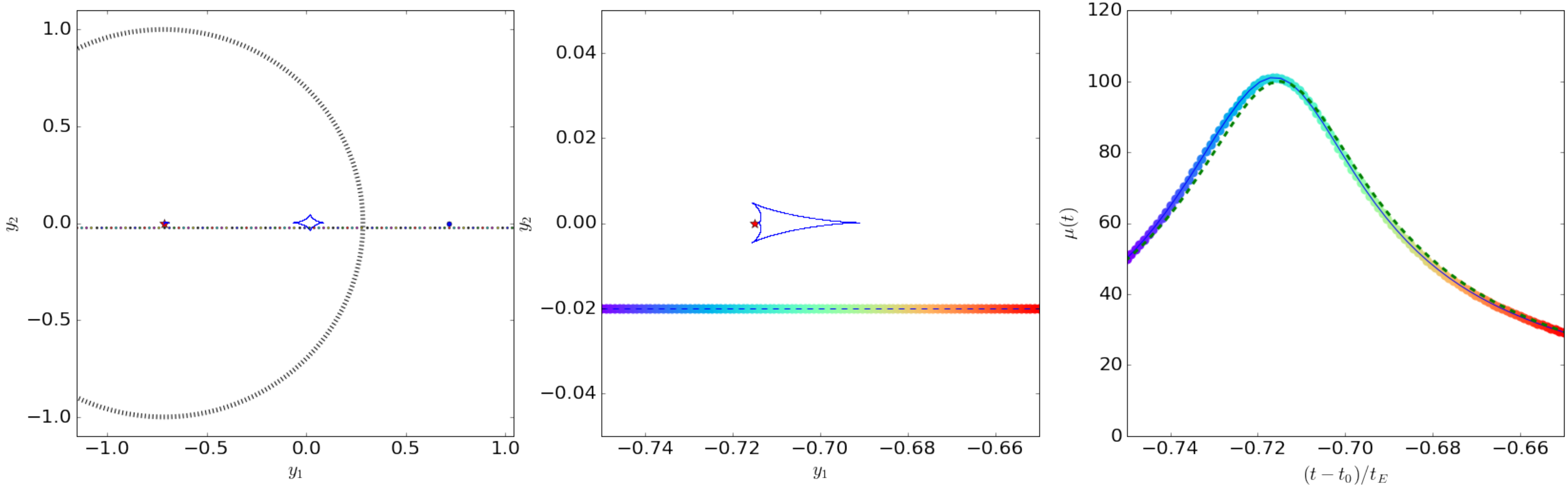
CENTRAL CAUSTIC PERTURBATIONS



CENTRAL CAUSTIC PERTURBATIONS

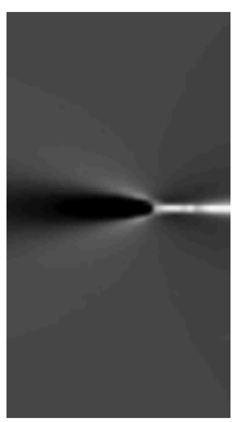


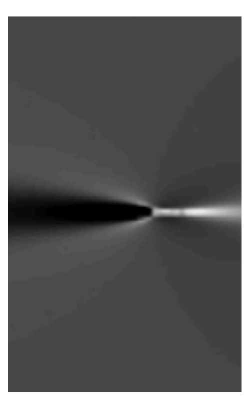
CENTRAL CAUSTIC PERTURBATIONS

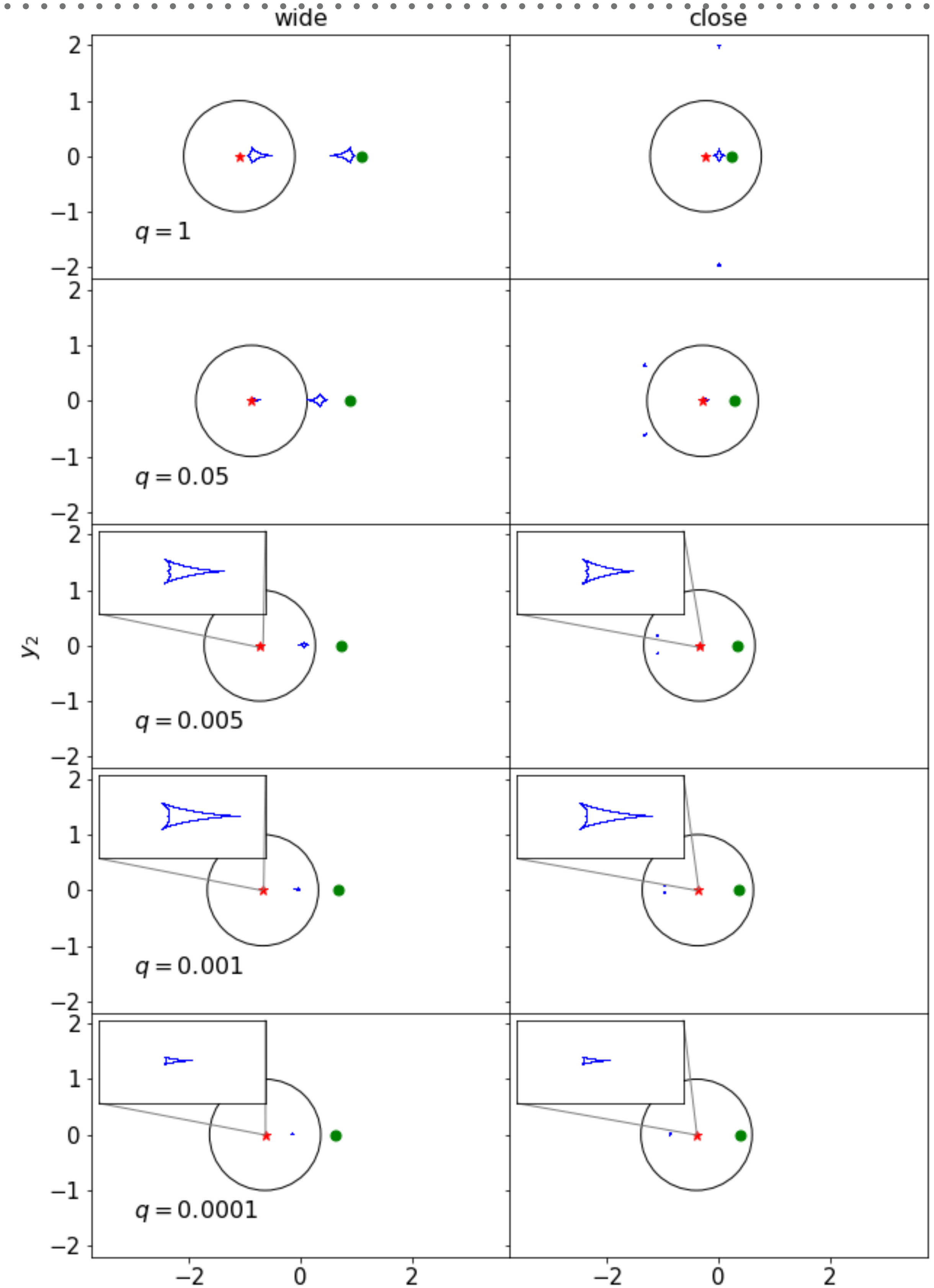


WIDE-CLOSE DEGENERACY!

- Caustics and magnification patterns are identical in wide and close systems with $d_w = d_c^{-1}$





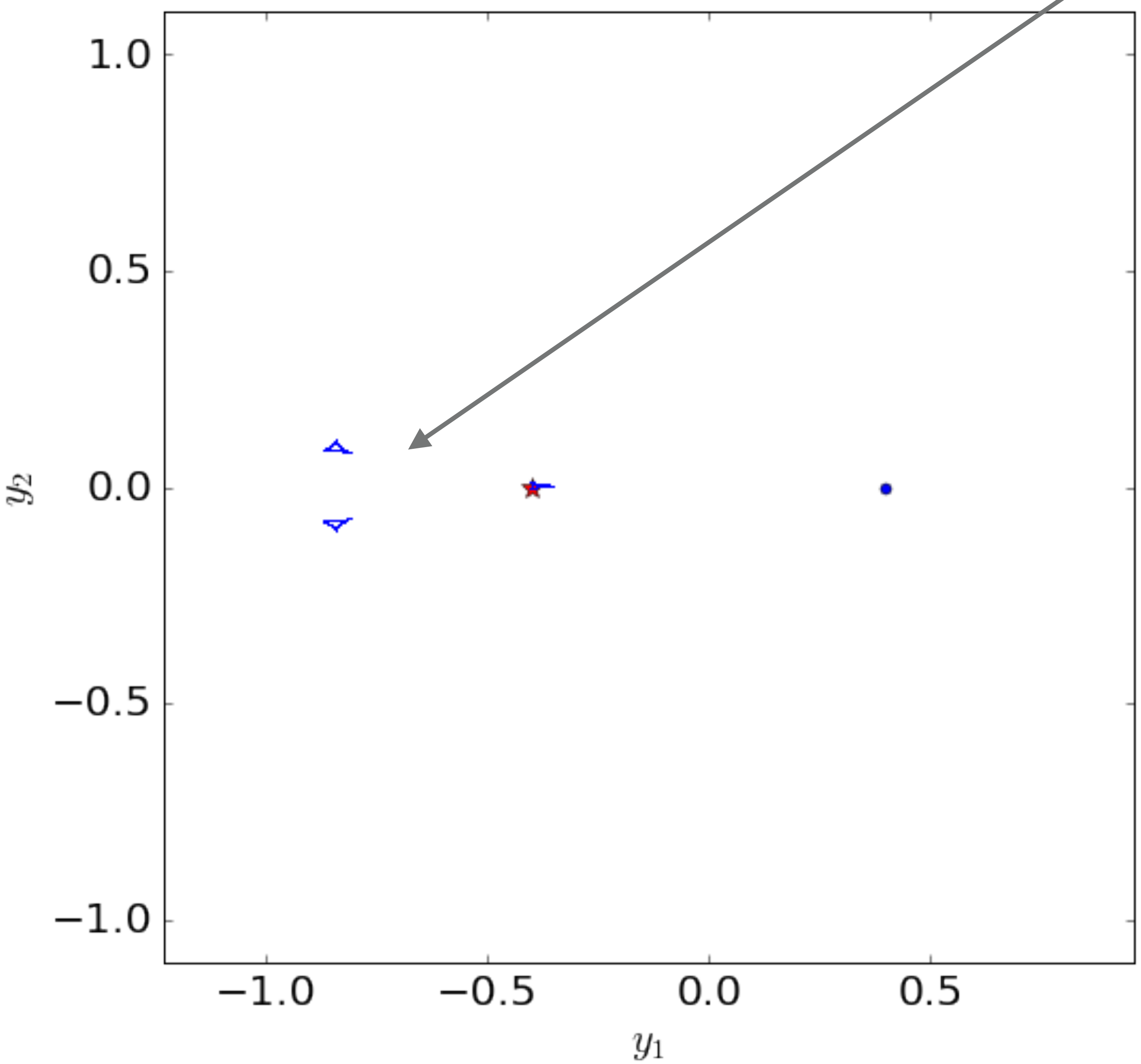


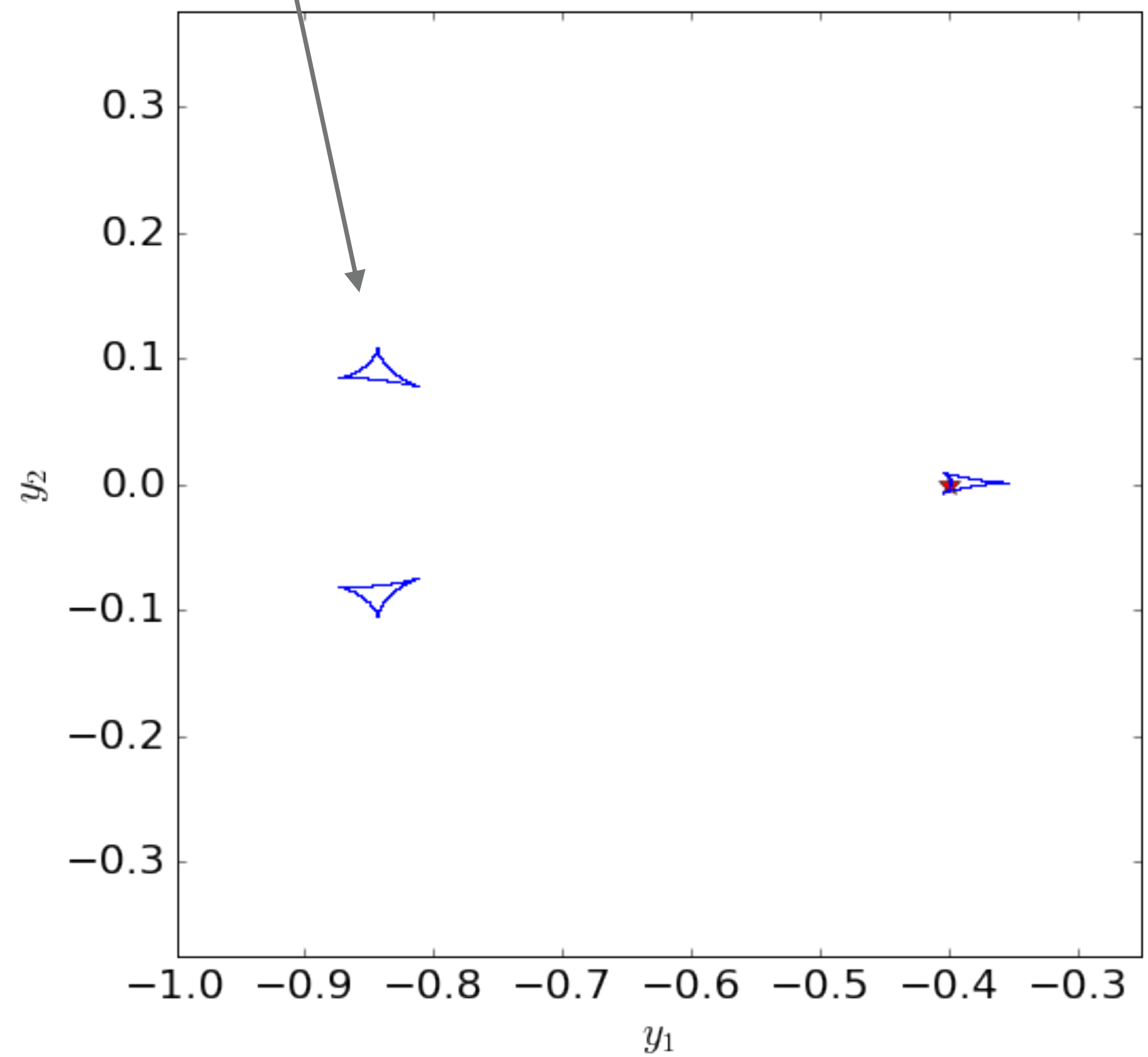
PLANET DETECTION THROUGH CENTRAL CAUSTICS PERTURBATIONS

- Only possible in the case of high magnification events (sources passing very close to the host stars)
- For this reason, they are rare events
- Advantages:
  - near the peak of the event
  - can sometimes be predicted in advance
  - high magnification makes possible to follow-up the events using small telescopes
  - more accurate photometry
- Disadvantages:
  - degeneracy wide-close topologies

PLANETARY CAUSTICS IN CLOSE TOPOLOGIES

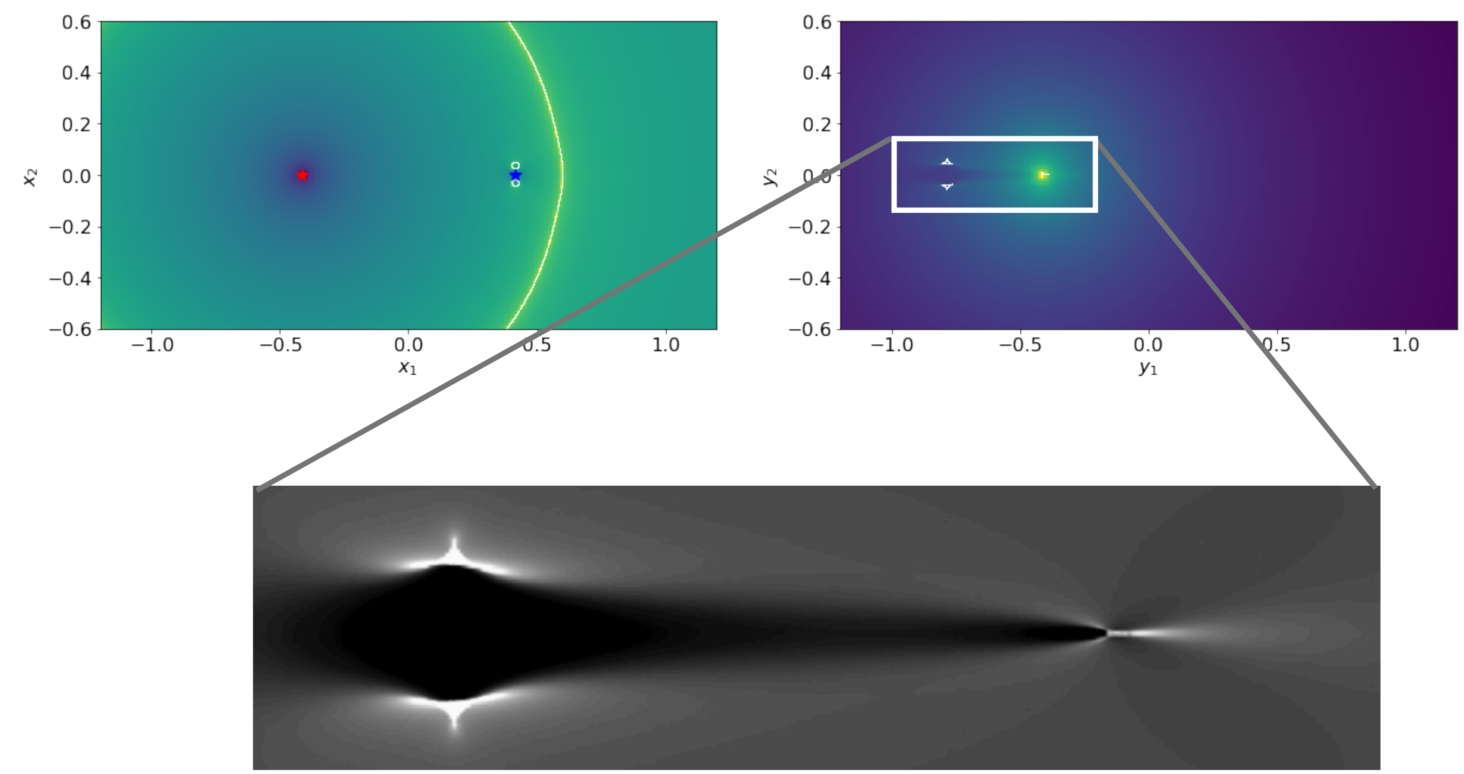
planetary caustics



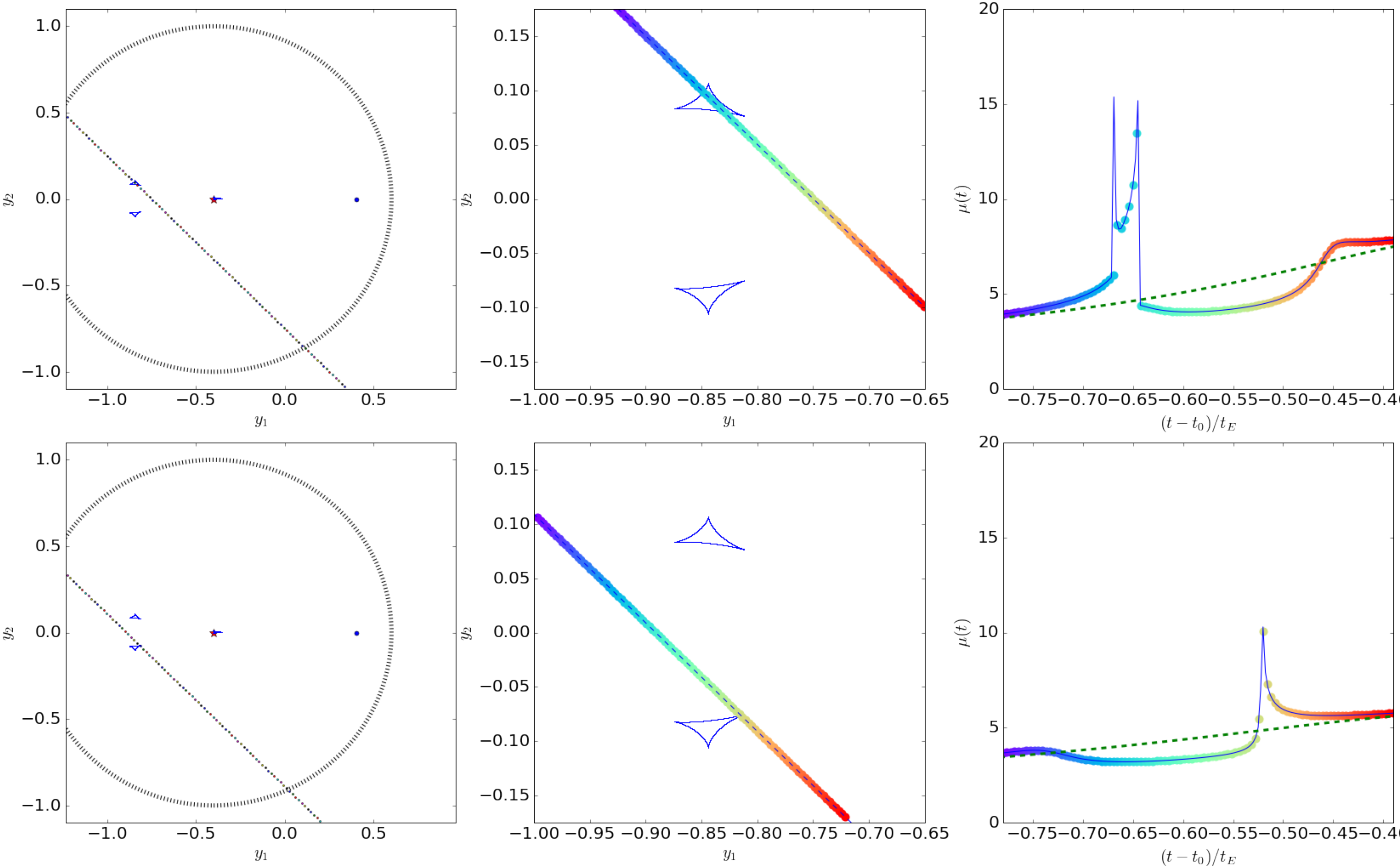


WHAT KIND OF SIGNATURES?

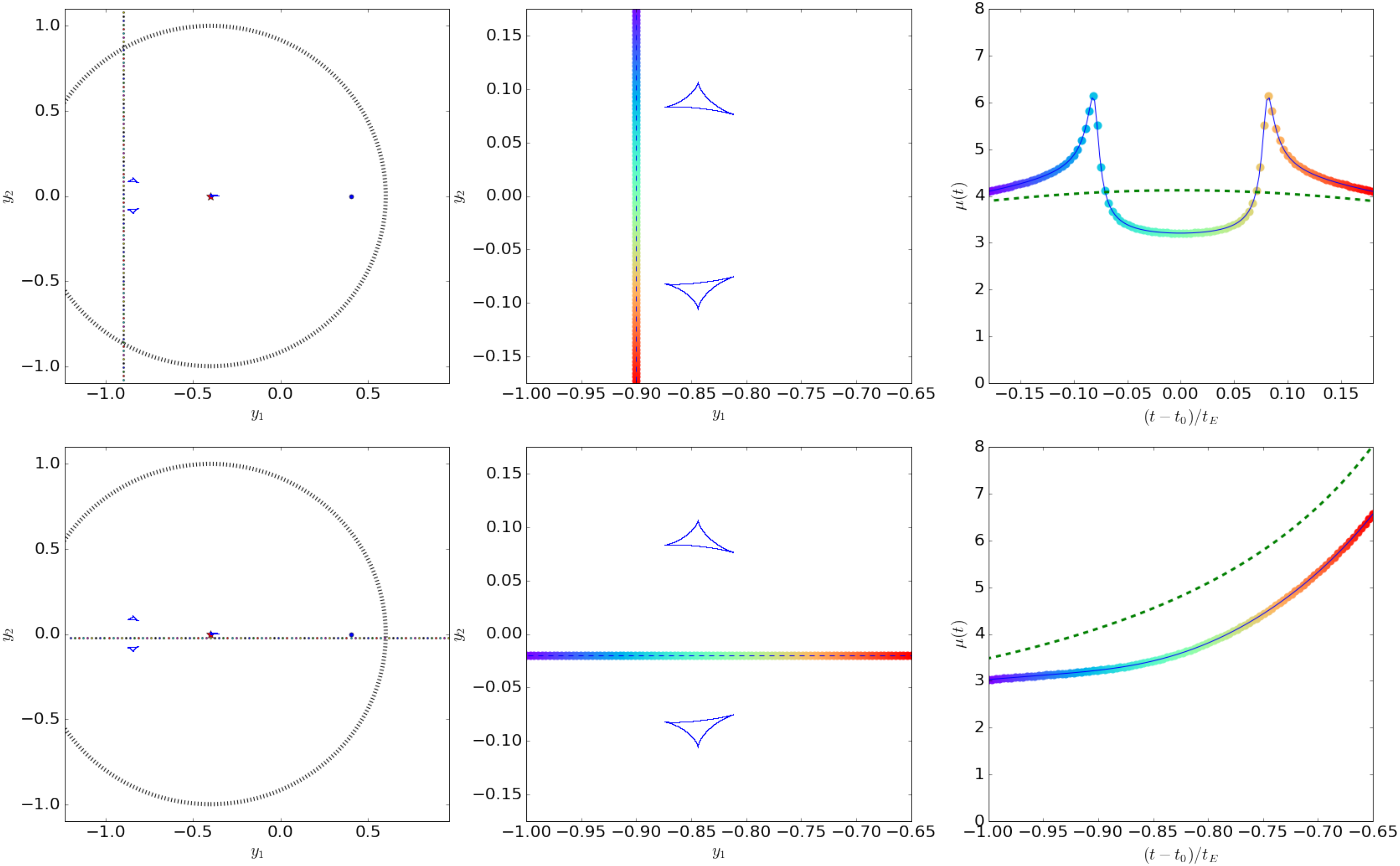
- Caustics have three cusps: magnification near the cusps
- They also have three folds: sudden magnification jumps
- Strong demagnification in between the planetary caustics



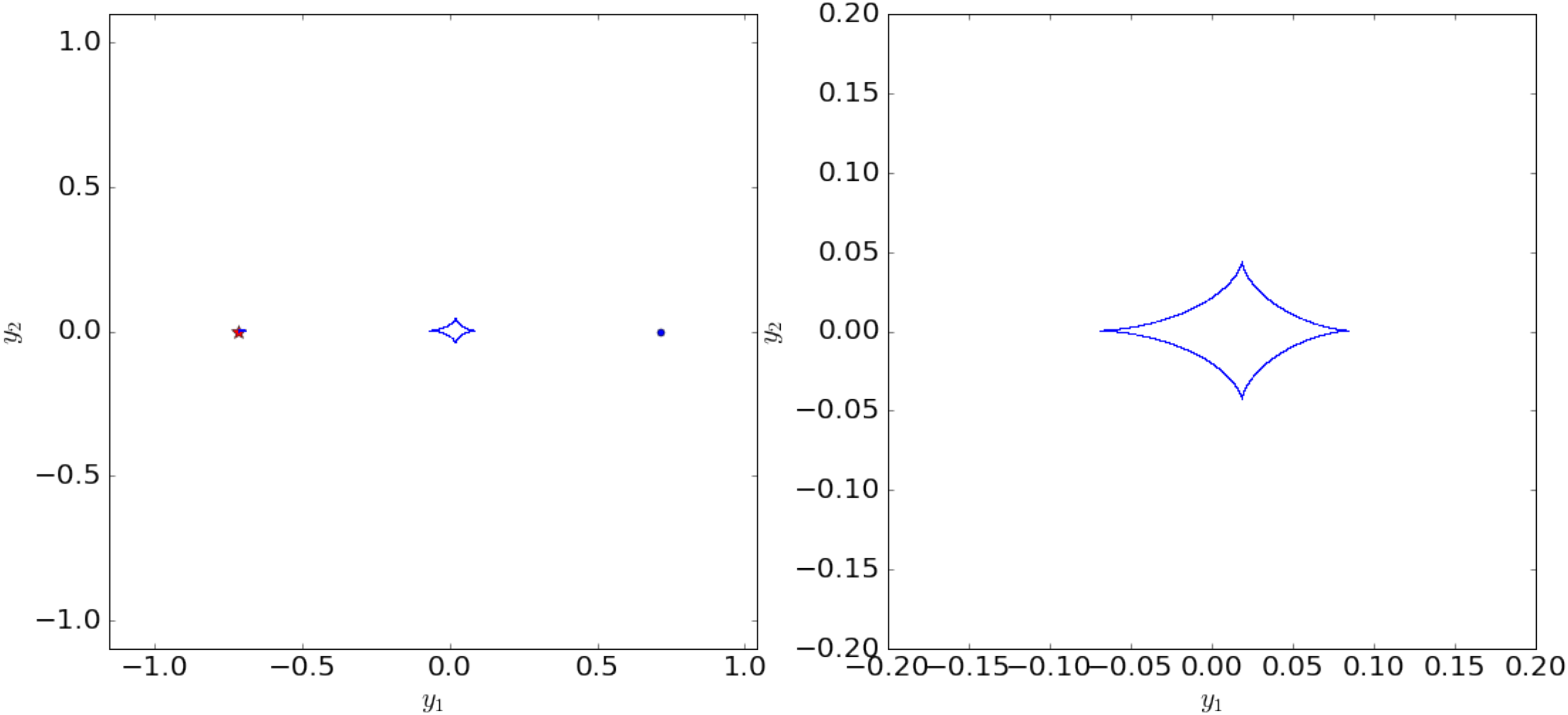
PLANETARY CAUSTICS PERTURBATIONS IN CLOSE TOPOLOGIES



PLANETARY CAUSTICS PERTURBATIONS IN CLOSE TOPOLOGIES

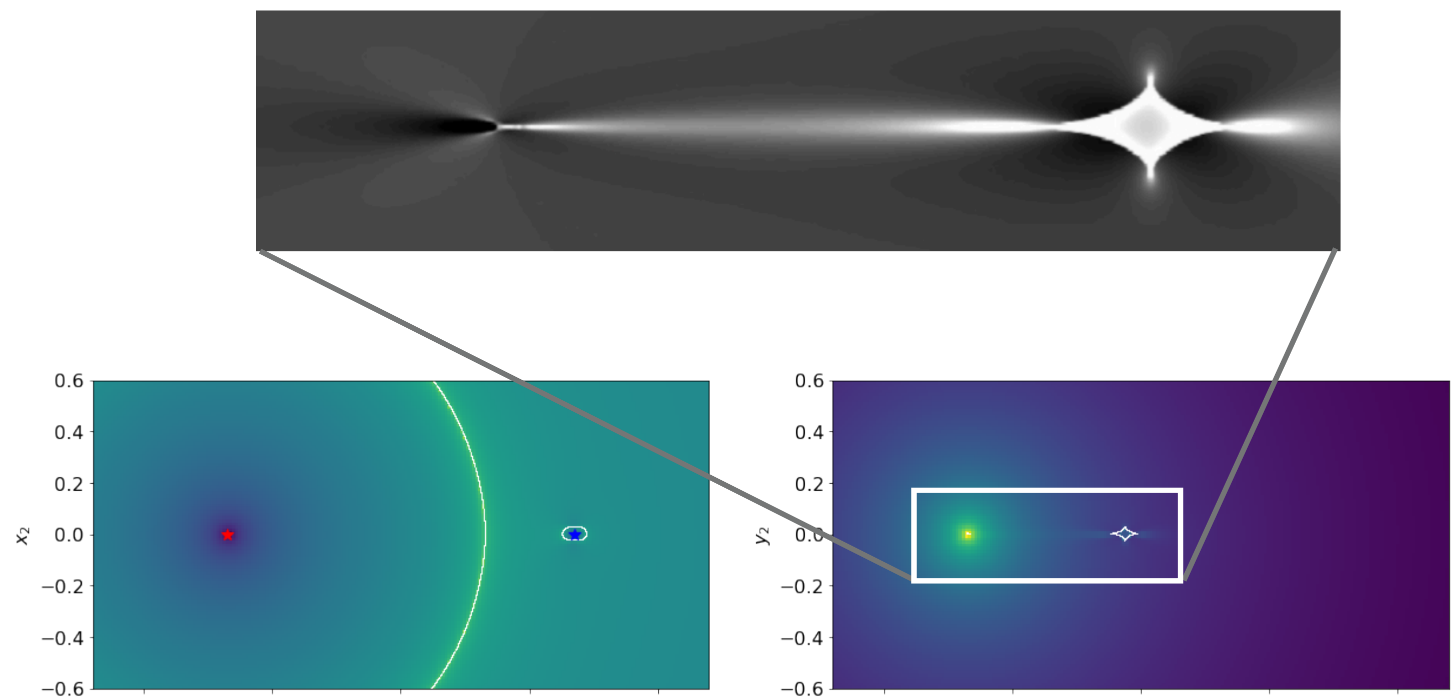


PLANETARY CAUSTICS IN WIDE TOPOLOGIES

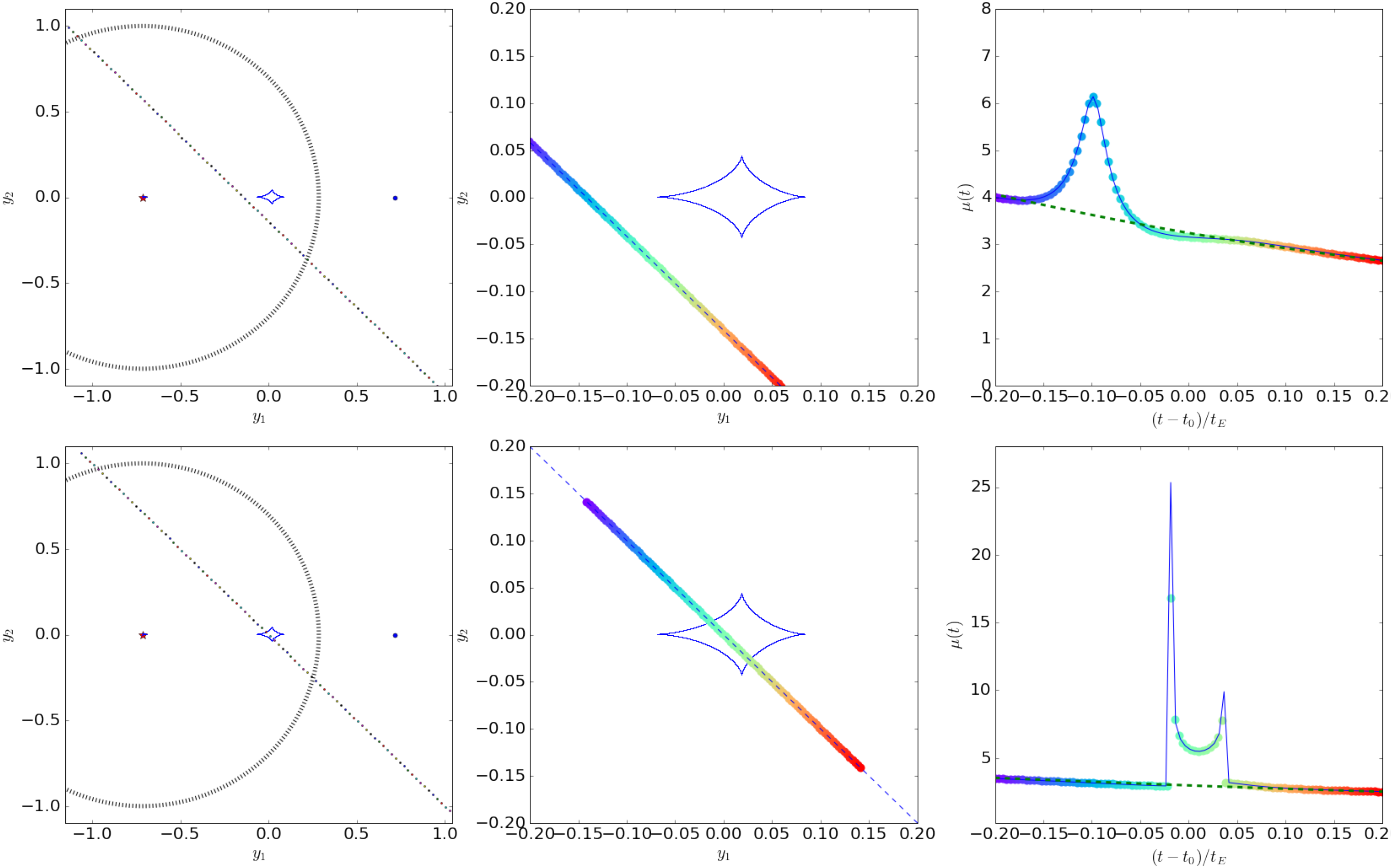


WHAT KIND OF SIGNATURES?

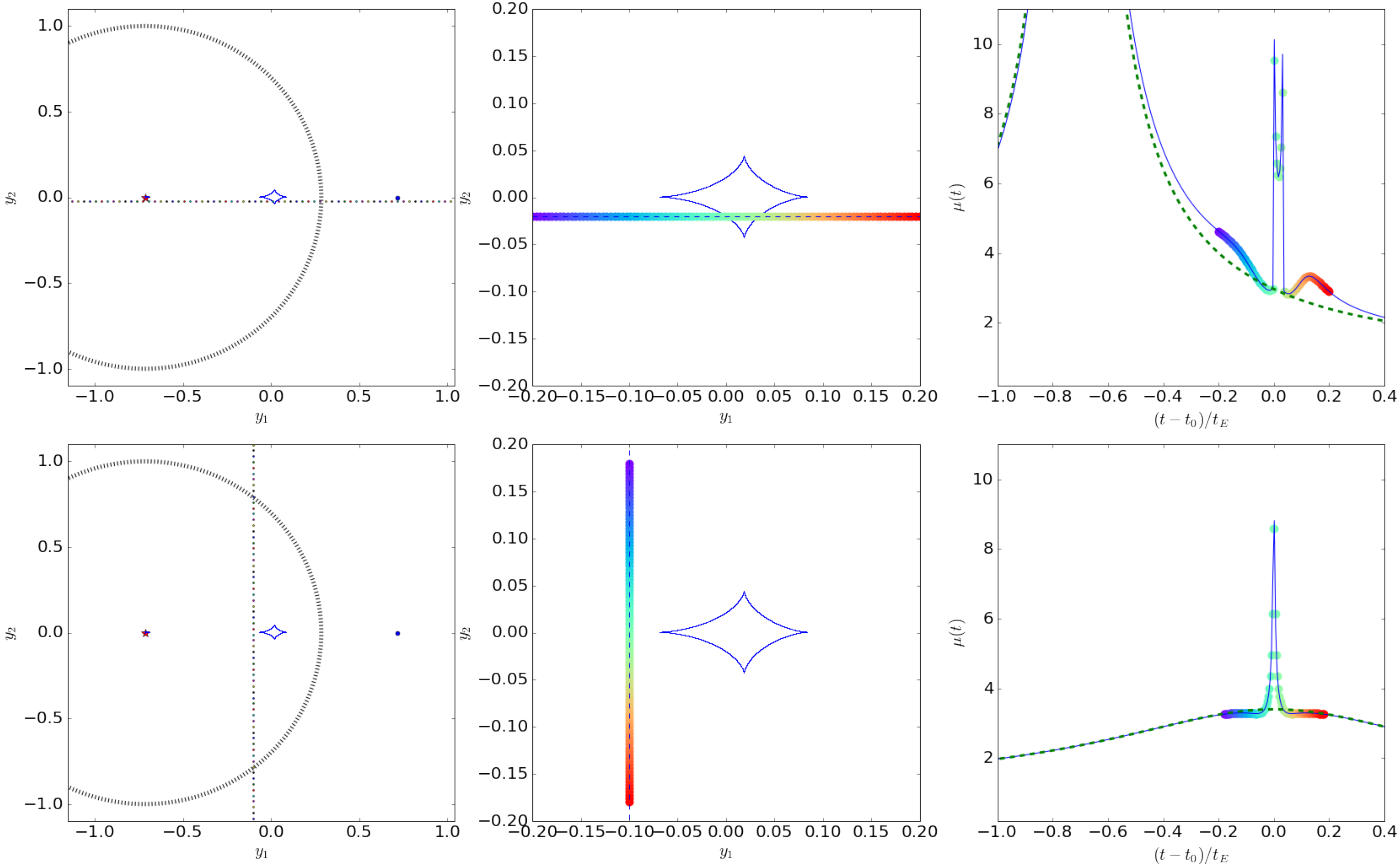
- Four cusps, four folds
- Strong magnification near the cusps
- Sudden magnification jumps when crossing the folds
- De-magnification on the outer parts of the folds
- Mild magnification inside the caustic



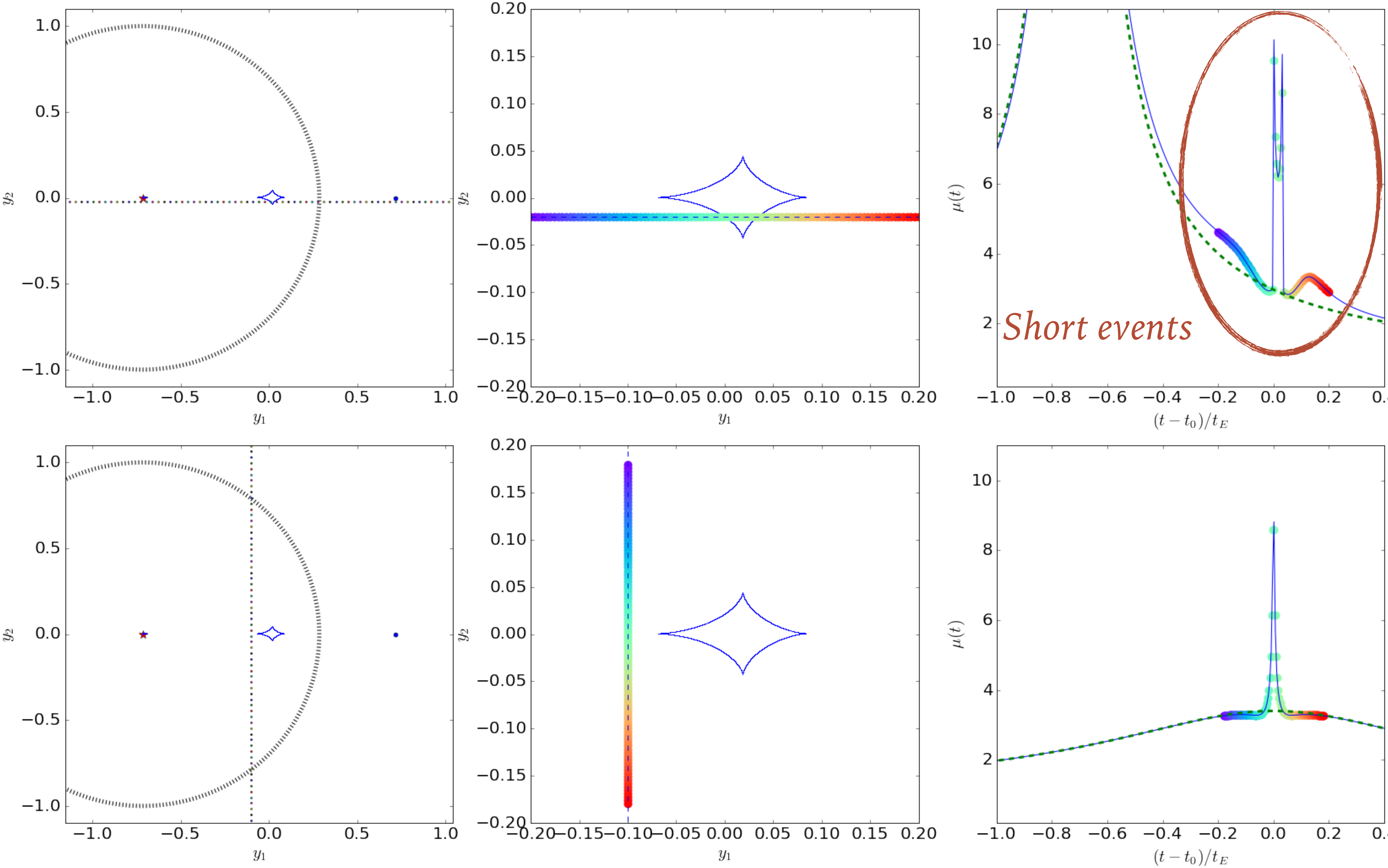
PLANETARY CAUSTICS PERTURBATIONS IN WIDE TOPOLOGIES



PLANETARY CAUSTICS PERTURBATIONS IN WIDE TOPOLOGIES



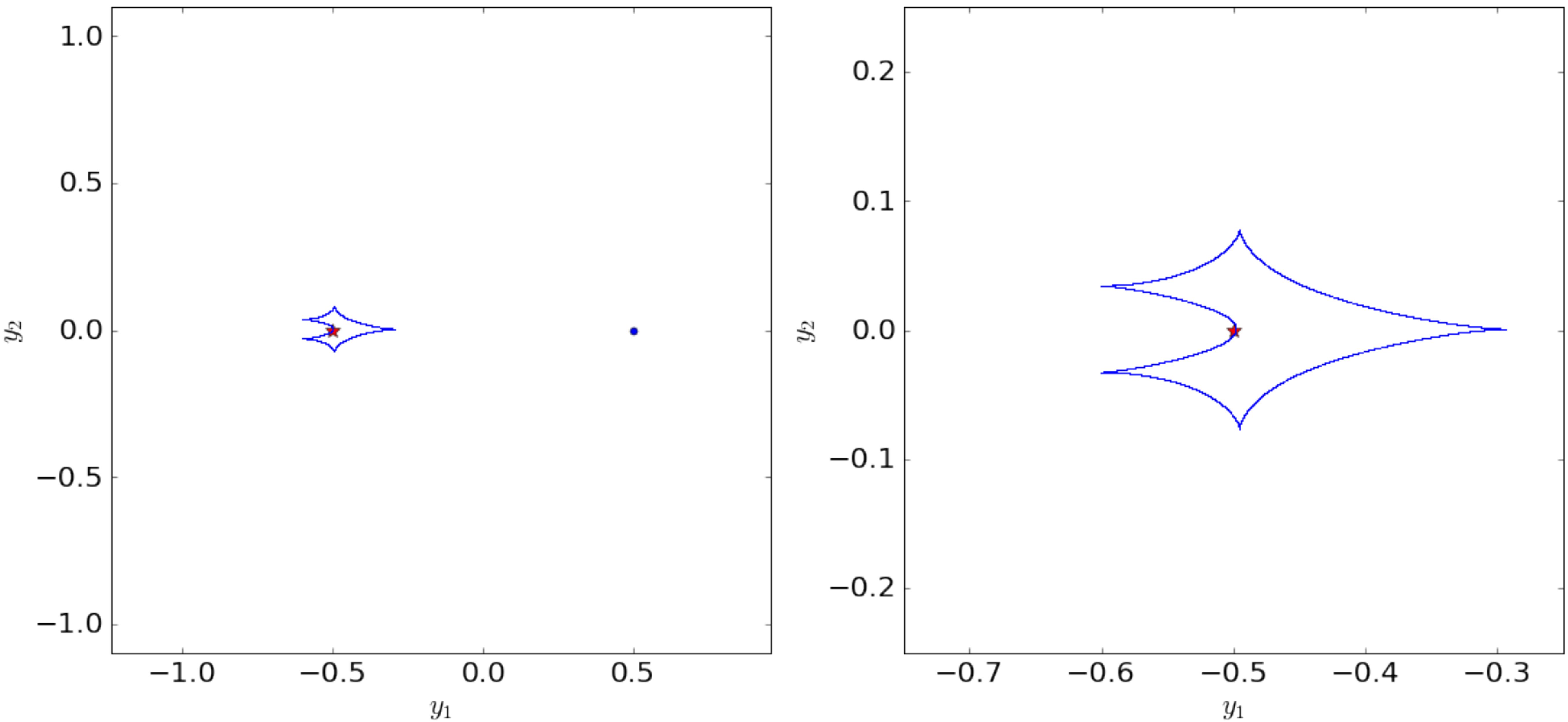
PLANETARY CAUSTICS PERTURBATIONS IN WIDE TOPOLOGIES



PLANET DETECTION THROUGH PLANETARY CAUSTICS PERTURBATIONS

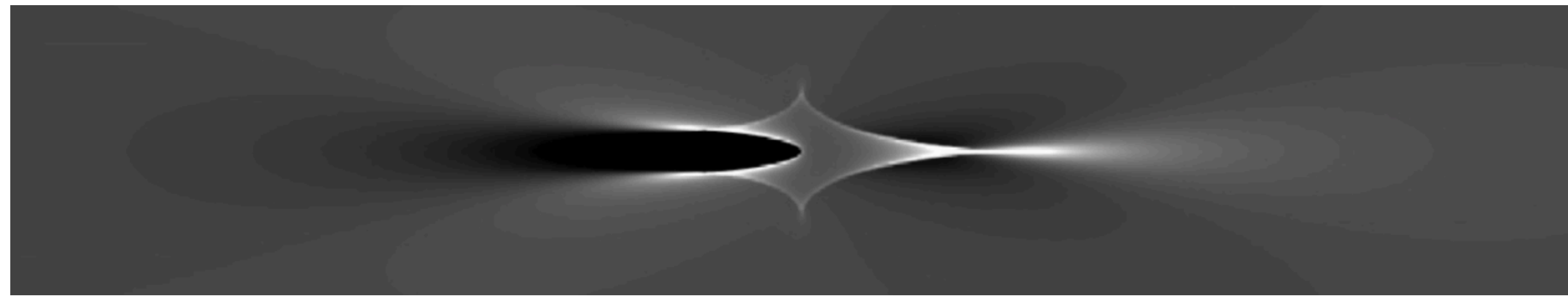
- Typically in low-to-mid magnification regimes, as the source passes at relatively large distance from the primary lens
- Short!
- More frequent
- Less predictable

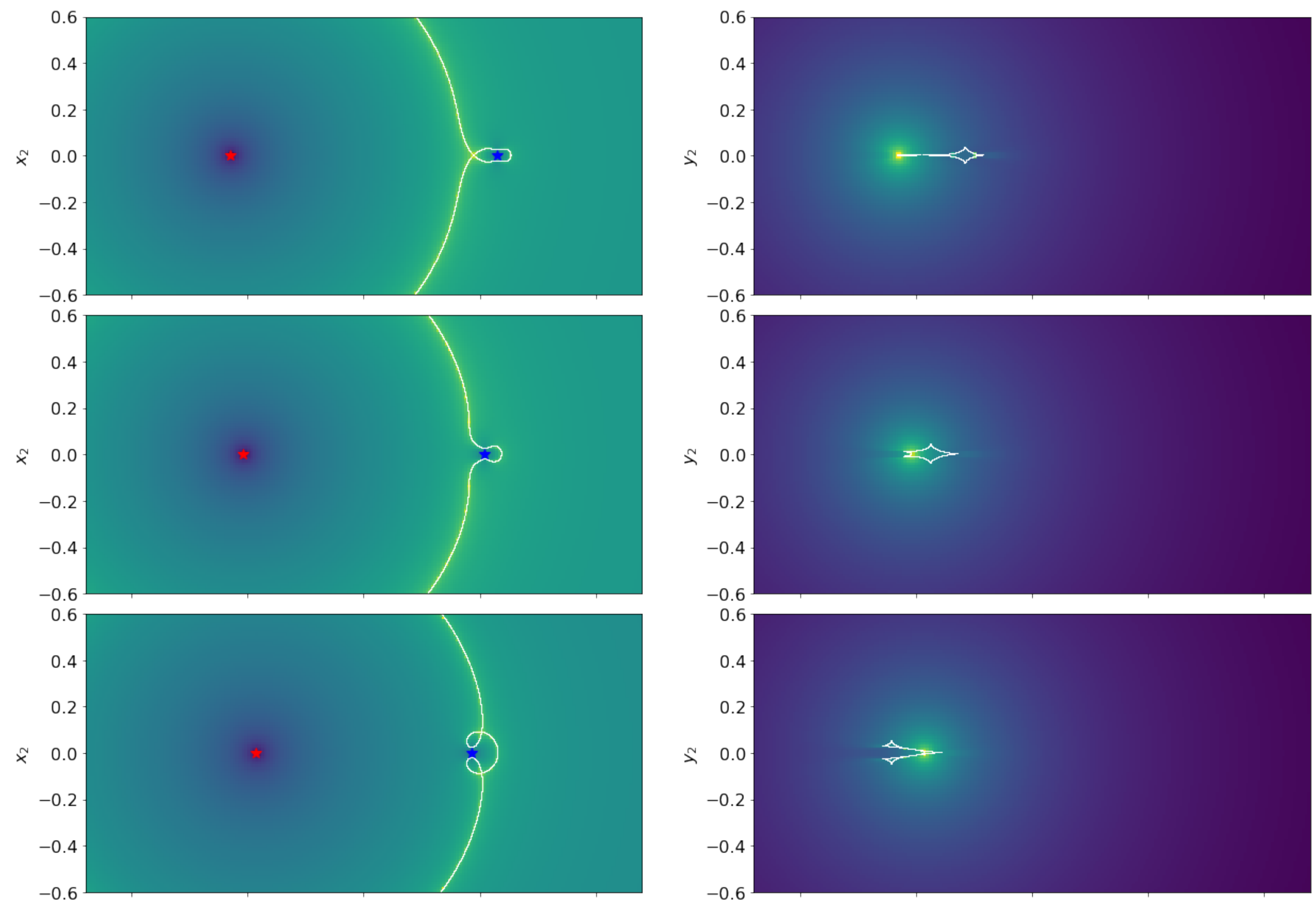
PLANETARY CAUSTICS IN INTERMEDIATE TOPOLOGIES



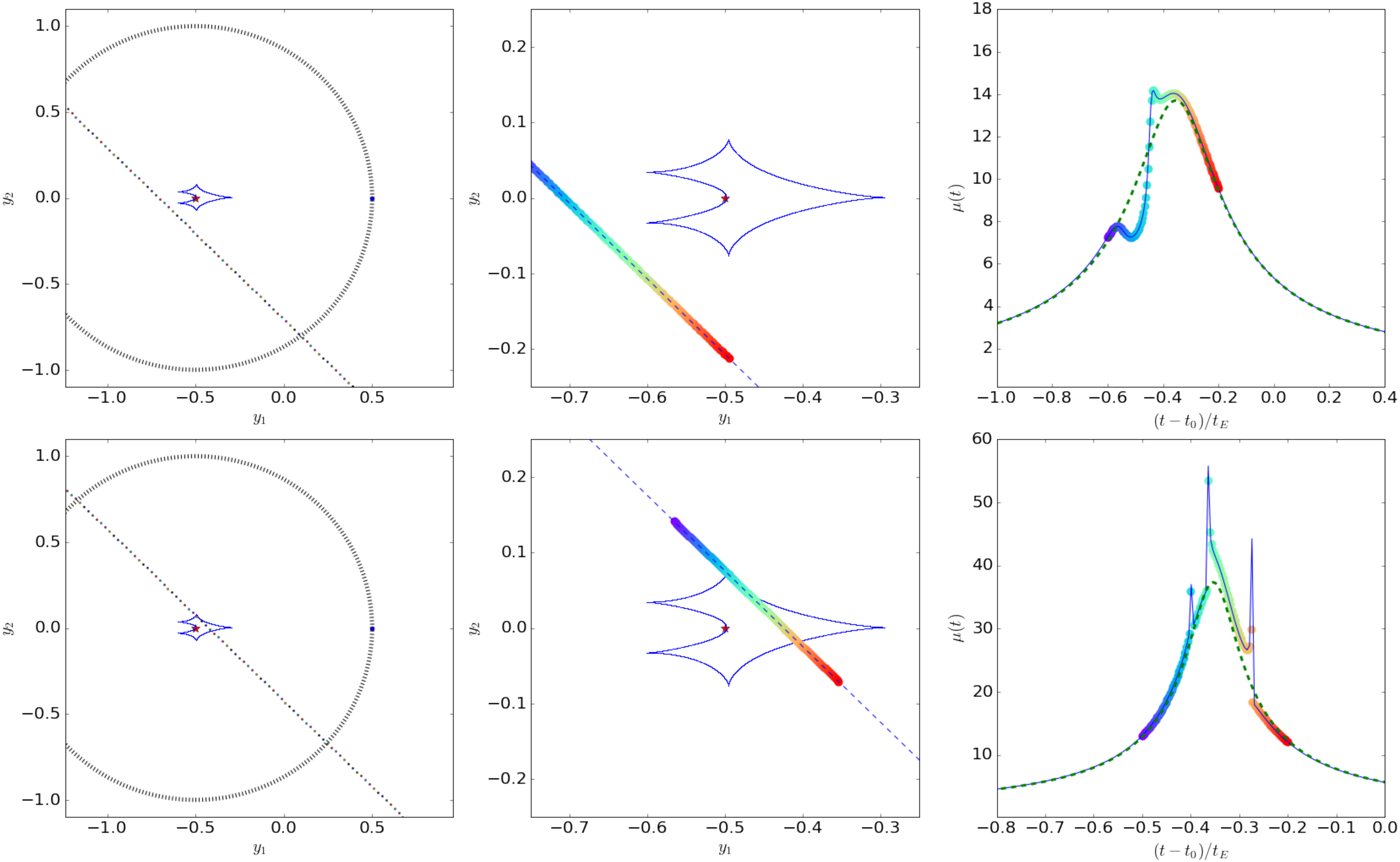
WHAT KIND OF SIGNATURES?

- 6 cusps, 6 folds
- Weak and strong cusps
- Extended
- De-magnification on the back of the caustic
- Examples:

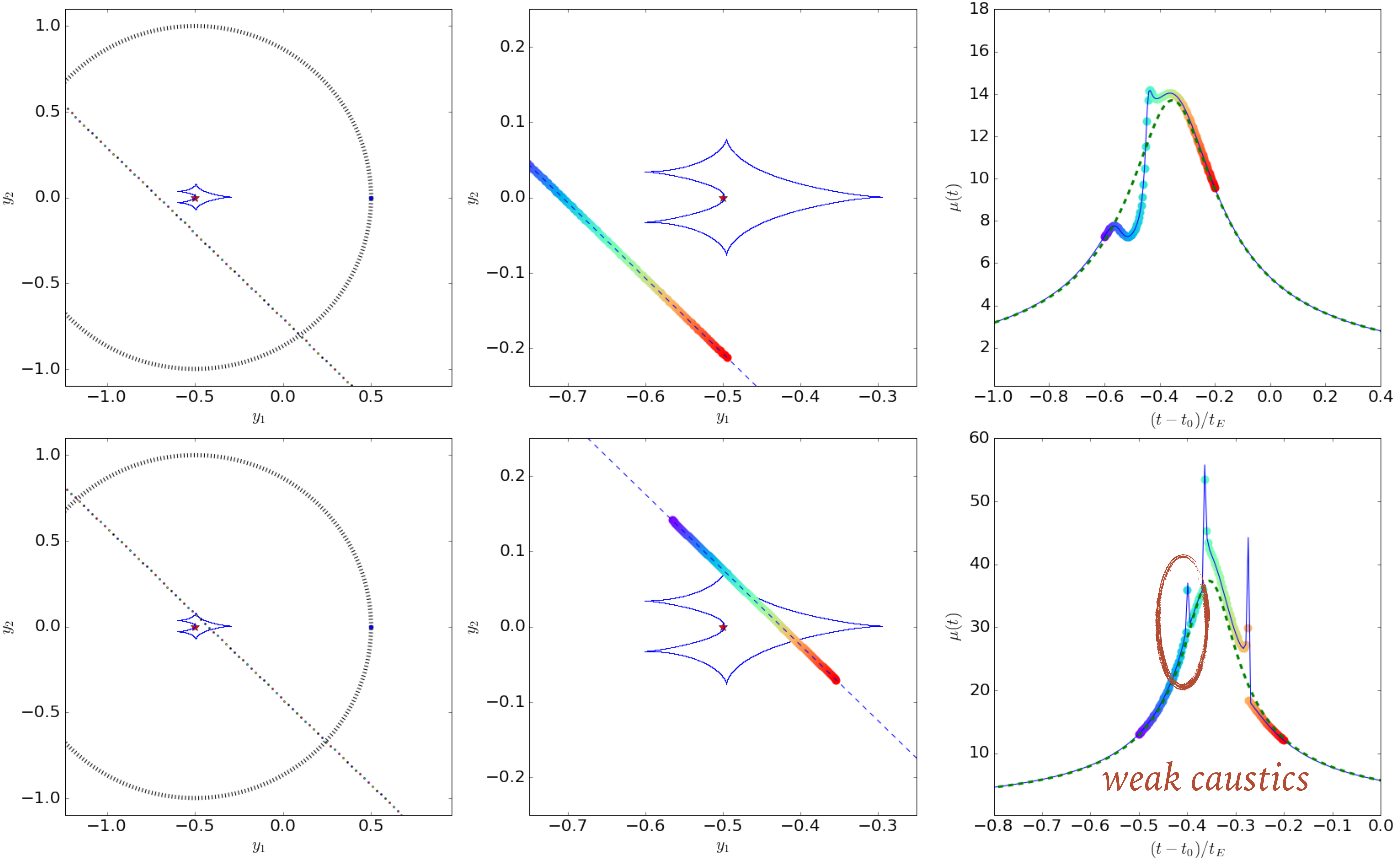




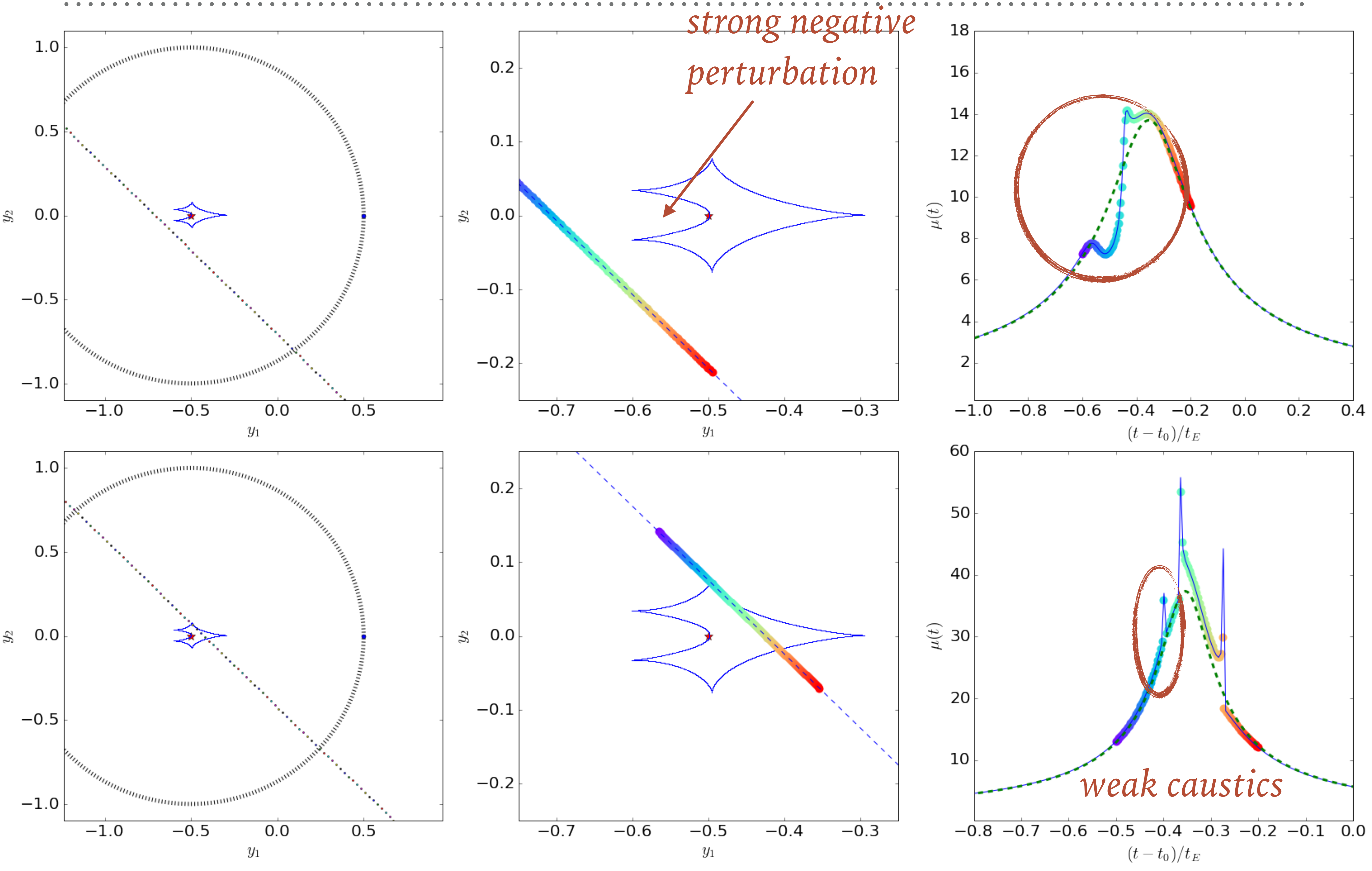
PLANETARY CAUSTICS PERTURBATIONS IN INTERMEDIATE TOPOLOGIES



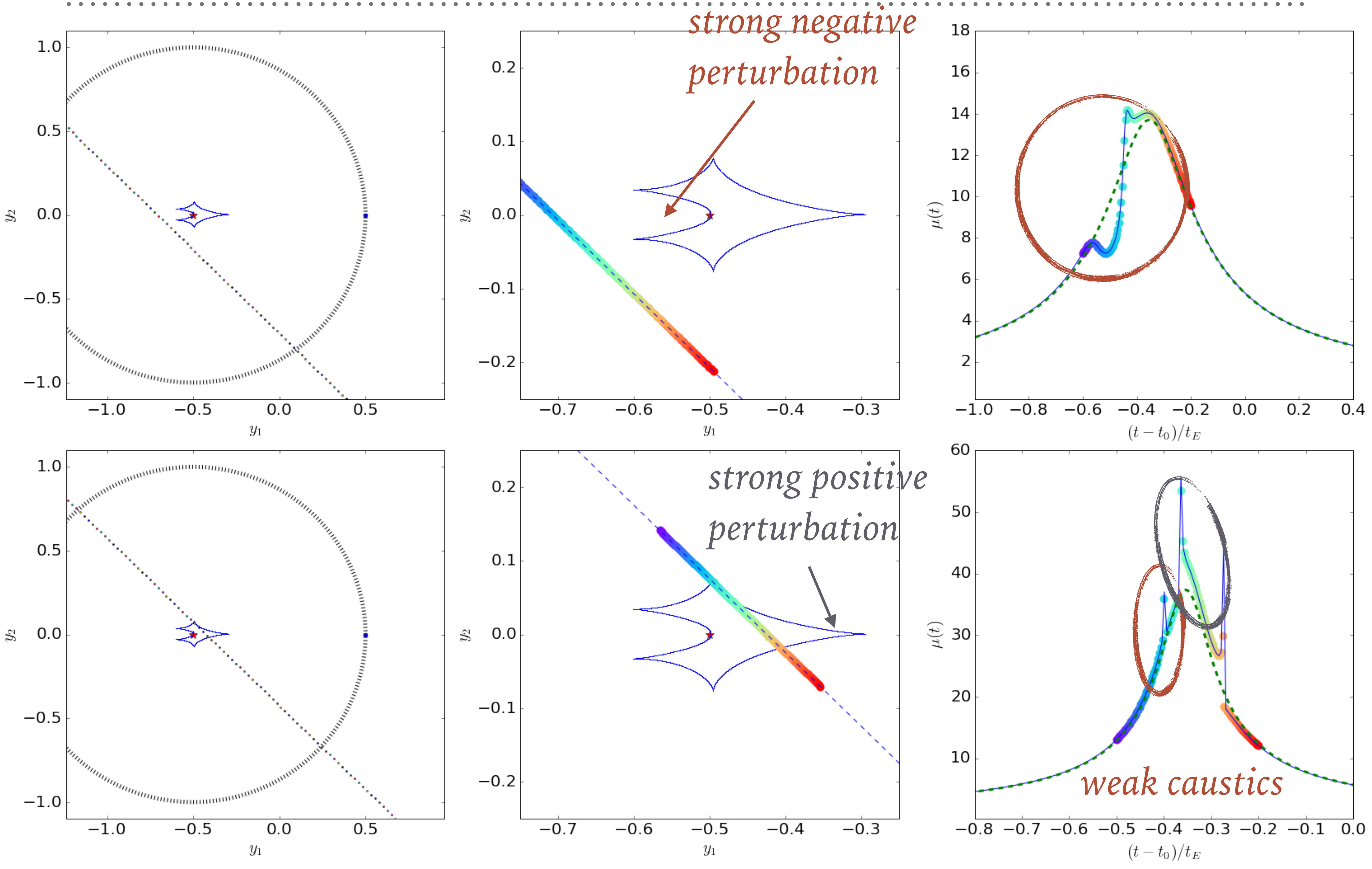
PLANETARY CAUSTICS PERTURBATIONS IN INTERMEDIATE TOPOLOGIES



PLANETARY CAUSTICS PERTURBATIONS IN INTERMEDIATE TOPOLOGIES

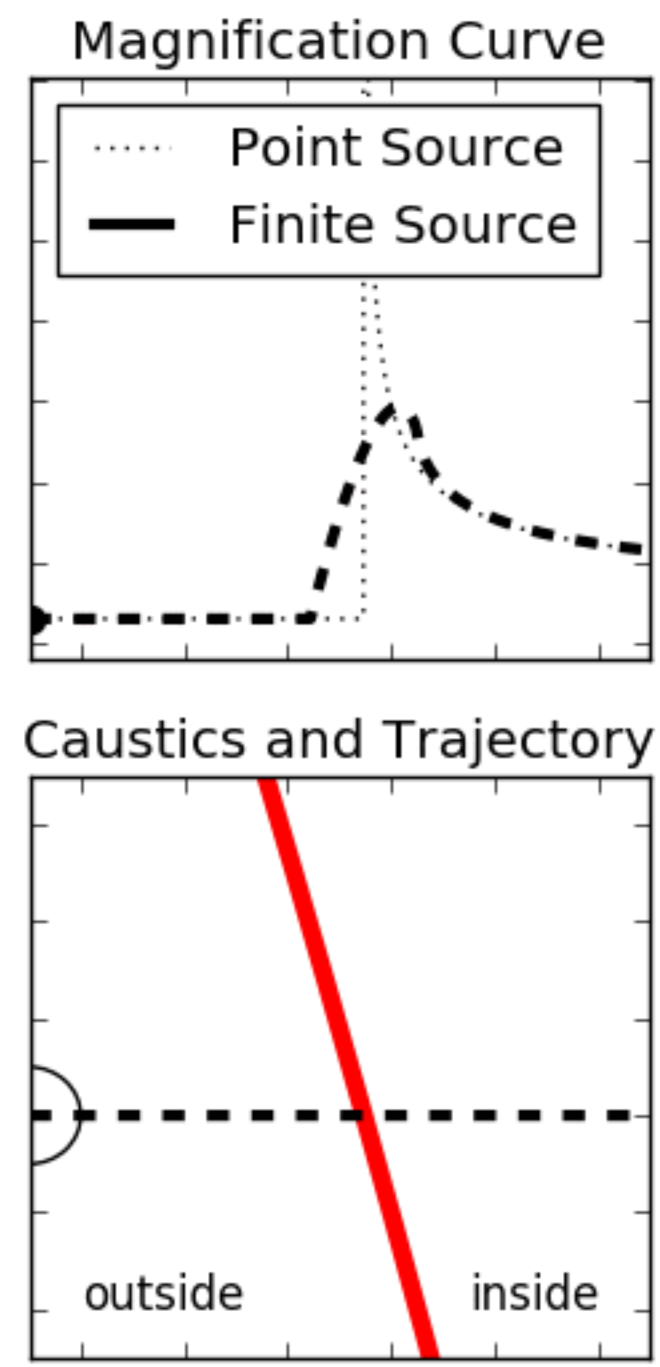


PLANETARY CAUSTICS PERTURBATIONS IN INTERMEDIATE TOPOLOGIES



PLANET DETECTION THROUGH RESONANT CAUSTIC PERTURBATIONS

- Typically in intermediate-to-high magnification regimes, as the source passes close the primary lens
- Weak caustics: sharp peaks that can be washed out by finite source effects
- Long events
- Sensitive to orbital motions



Credit: J. Yee

INTERMEDIATE OR RESONANT CAUSTICS

In order to have a resonant caustic the distance between the binary components must be

$$d_{IC} < d < d_{WI}$$

where $\quad d_{WI} = (m_1^{1/3} + m_2^{1/3})^{3/2} \qquad d_{IC} = (m_1^{1/3} + m_2^{1/3})^{-3/4}$

These two relations can be written in a different form:

$$d_{WI} = m_2^{1/2}(1 + q^{1/3})^{3/2} \qquad d_{IC} = m_2^{-1/4}(1 + q^{1/3})^{-3/4}$$

Considering that $q \ll 1$, *we have that* $m_2 \sim 1$ *and that both* d_{WI} *and* d_{IC} *differ very little from unity:*

$$d_{WI} \sim \left(1 + \frac{3}{2}q^{1/3}\right) \qquad d_{IC} \sim \left(1 - \frac{3}{4}q^{1/3}\right)$$

INTERMEDIATE OR RESONANT CAUSTICS

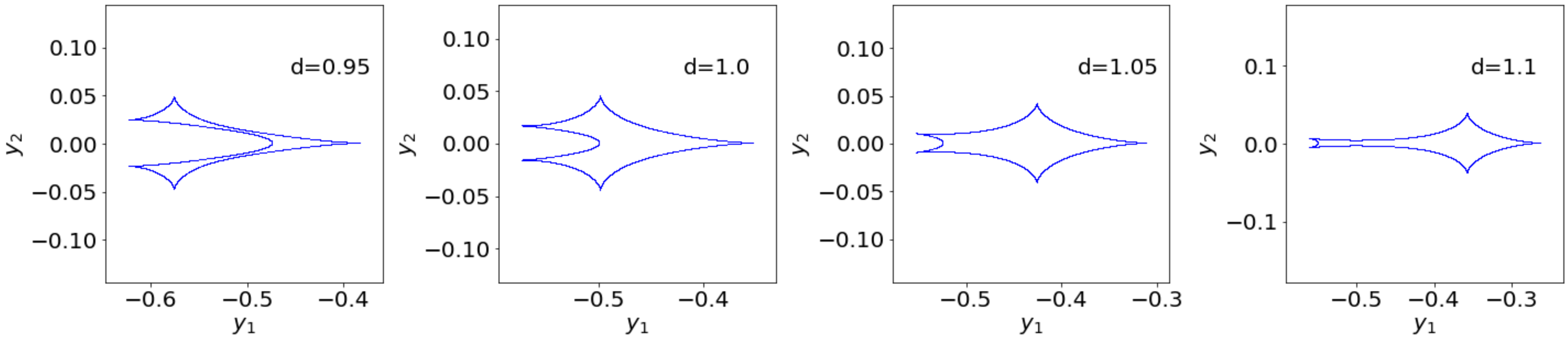
Therefore, in the case of star-planet pairs, resonant caustics exist within a very narrow range of distances around the Einstein radius of the star:

$$d_{WI} \sim \left(1 + \frac{3}{2} q^{1/3}\right) \qquad\qquad d_{IC} \sim \left(1 - \frac{3}{4} q^{1/3}\right)$$

$$\frac{d_{WI} + d_{IC}}{2} \sim 1$$

$$d_{WI} - d_{IC} \sim \frac{9}{4} q^{1/3}$$

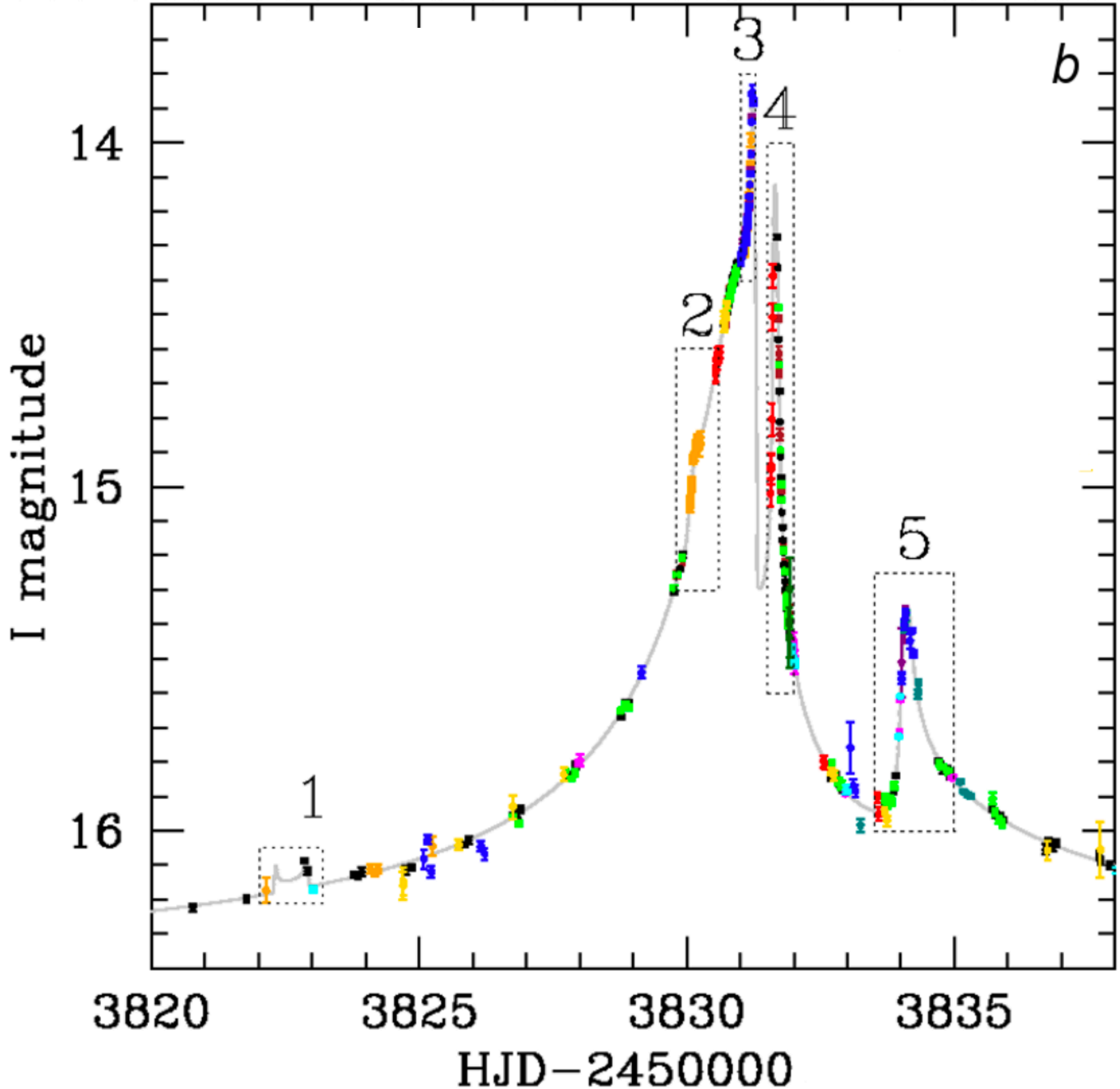
RESONANT CAUSTIC DEPENDENCE ON d



The shape of the resonant caustic changes dramatically if d varies.

Since events are long, it happens that, on the timescale of the event the distance between the star and the planet changes due to the planet orbital motion.

Consequently, the structure of the caustic changes. This effect can be modelled.

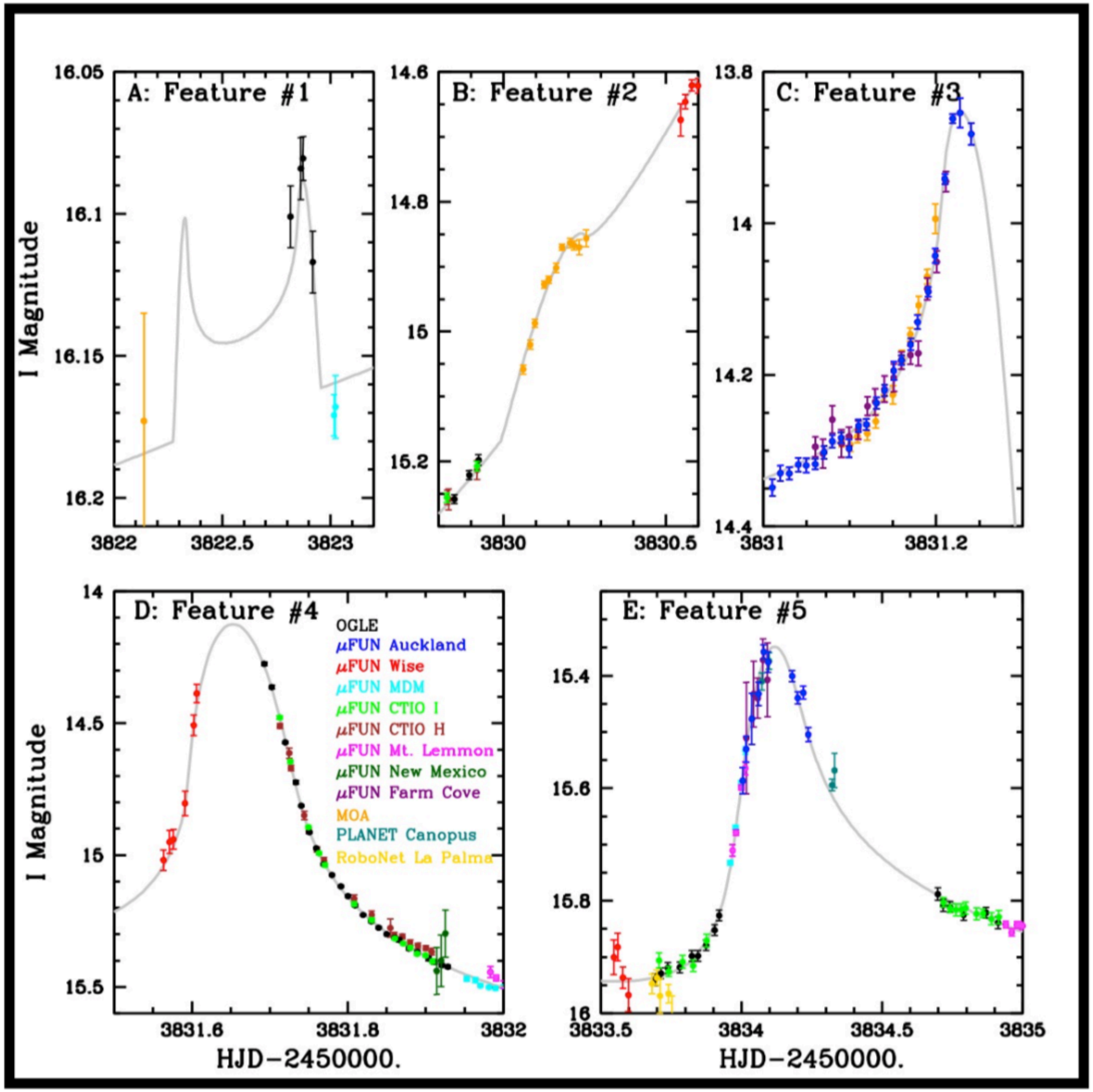


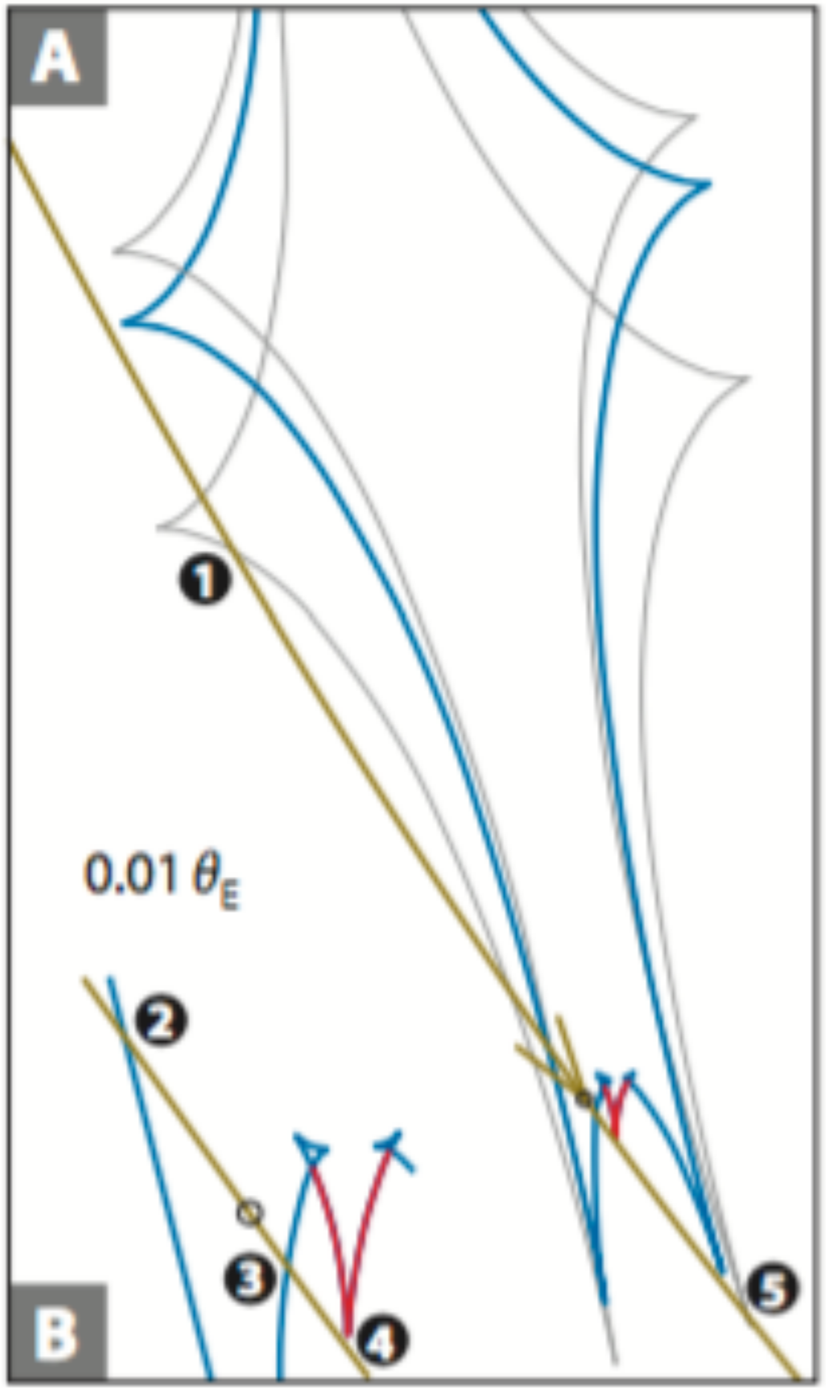
OGLE-2006-BLG-109

Several deviations from standard microlensing LC.

Features are typical of a resonant caustic perturbation.

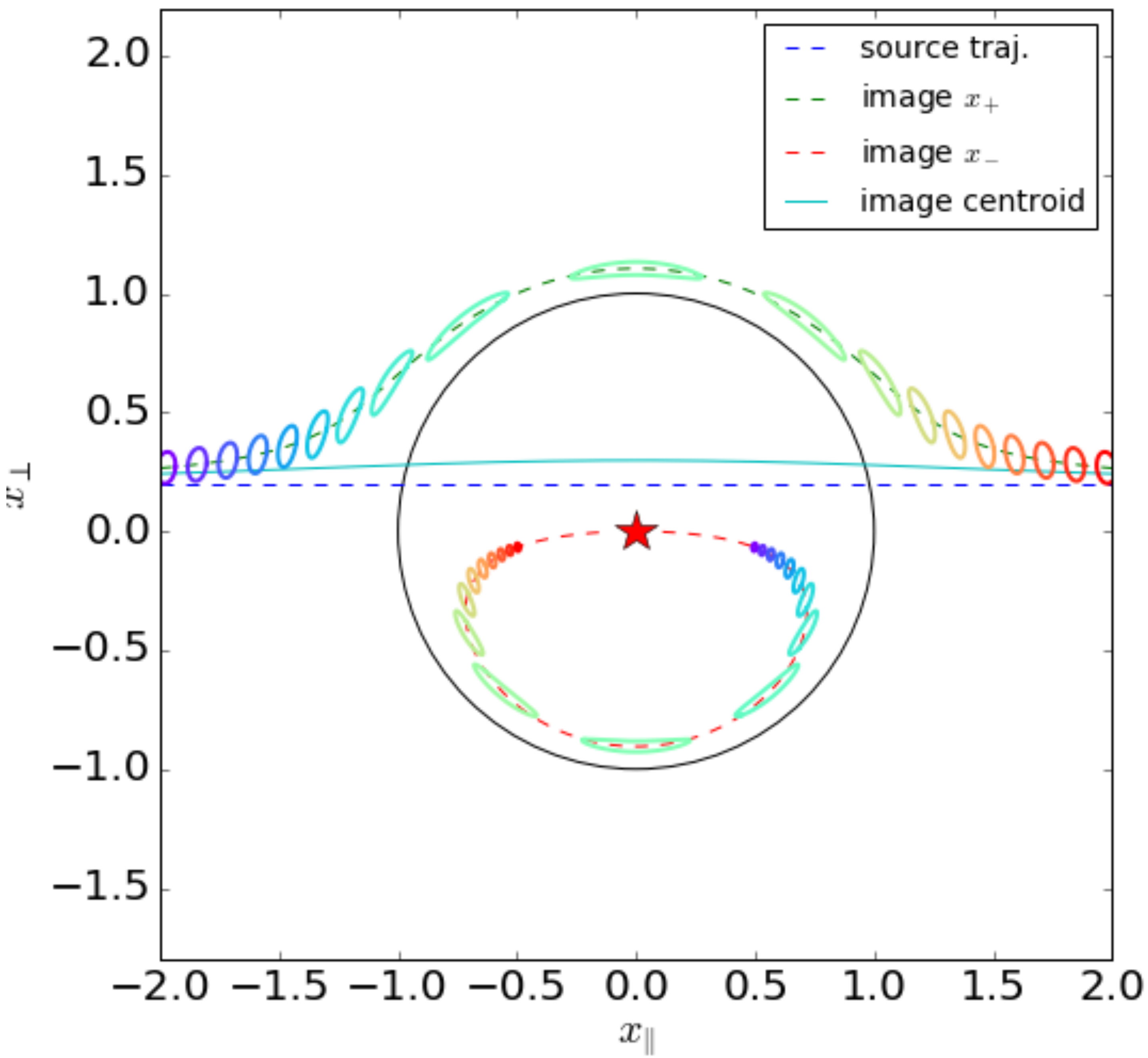
MULTIPLE PLANETS AND EVOLVING CAUSTIC





ANOTHER WAY TO LOOK AT THE LIGHT CURVES: POINT MASS LENS

Lensing of an extended circular source



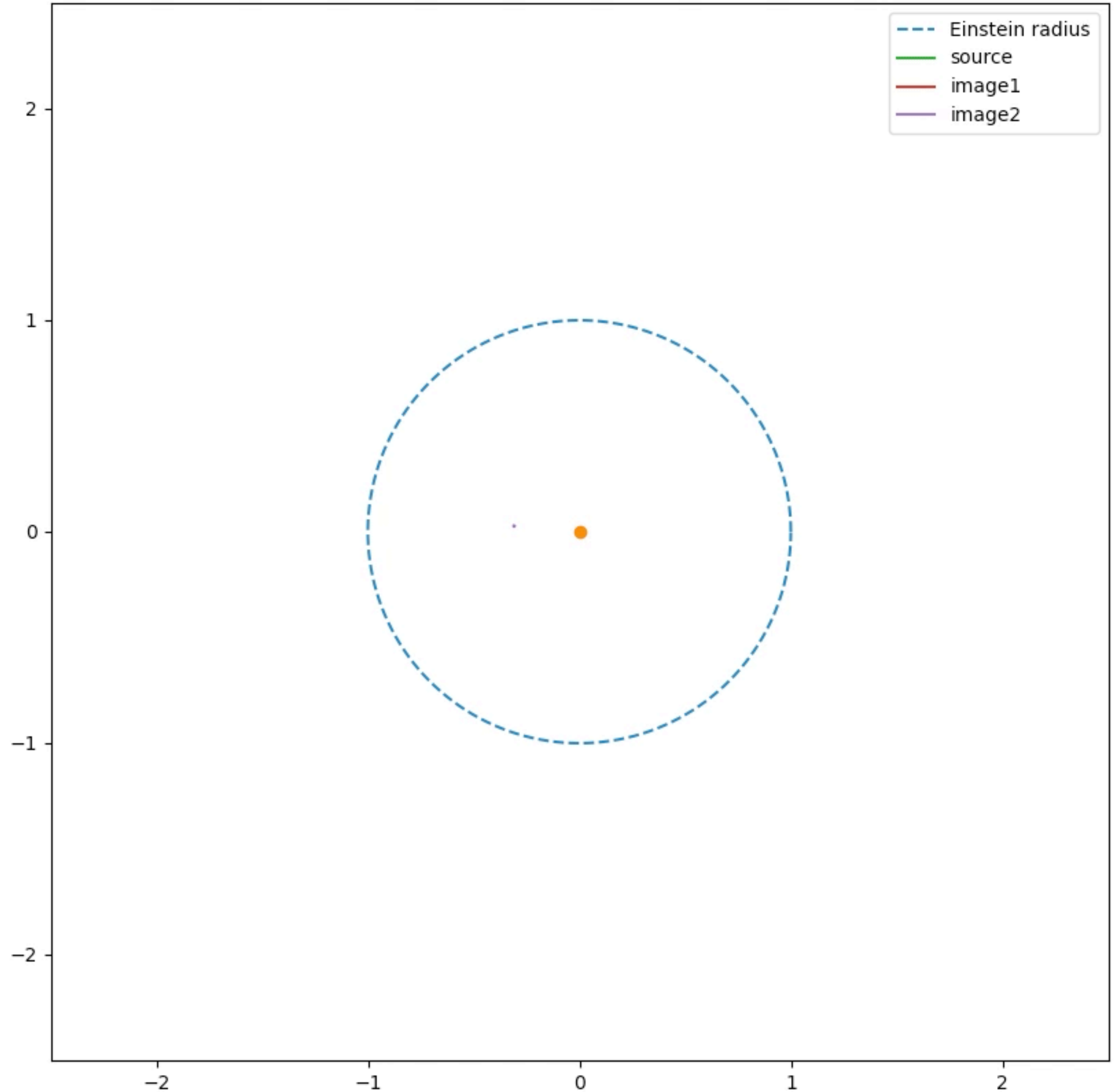
A point lens always produces two images, namely

$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right]$$

Imagine a source moving along the dashed line in the figure…

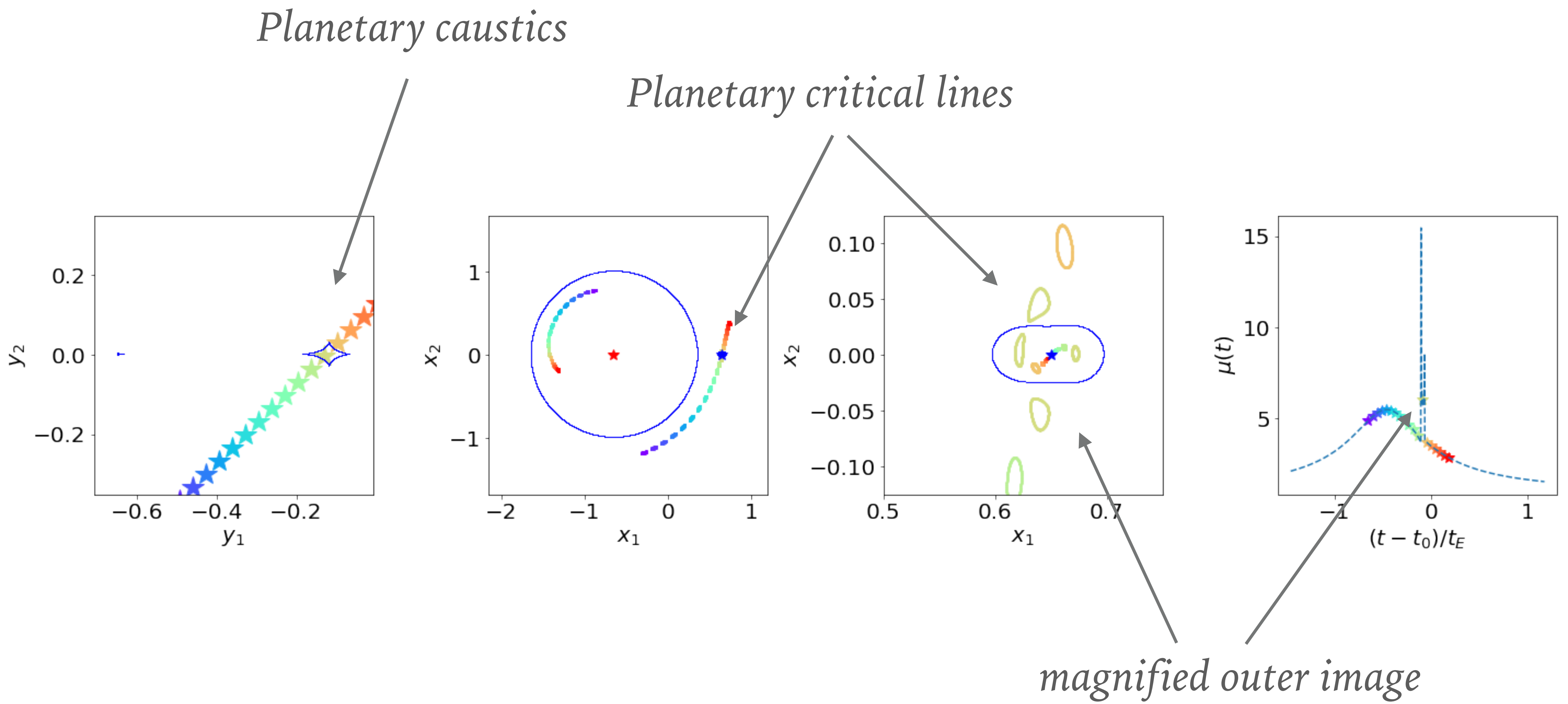
Remember: the two images are always aligned with the lens and the source!

As the source move, also the images move…



Einstein radius
source
image1
image2
2
1
0
−1
−2
−2
−1
0
1
2

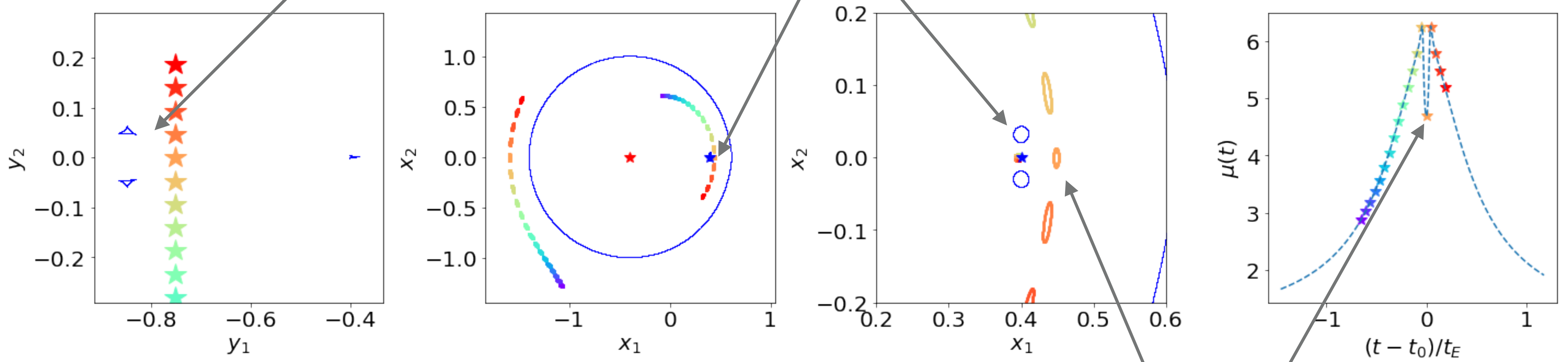
PLANETARY PERTURBATIONS AS PERTURBATIONS OF SINGLE IMAGES



PLANETARY PERTURBATIONS AS PERTURBATIONS OF SINGLE IMAGES

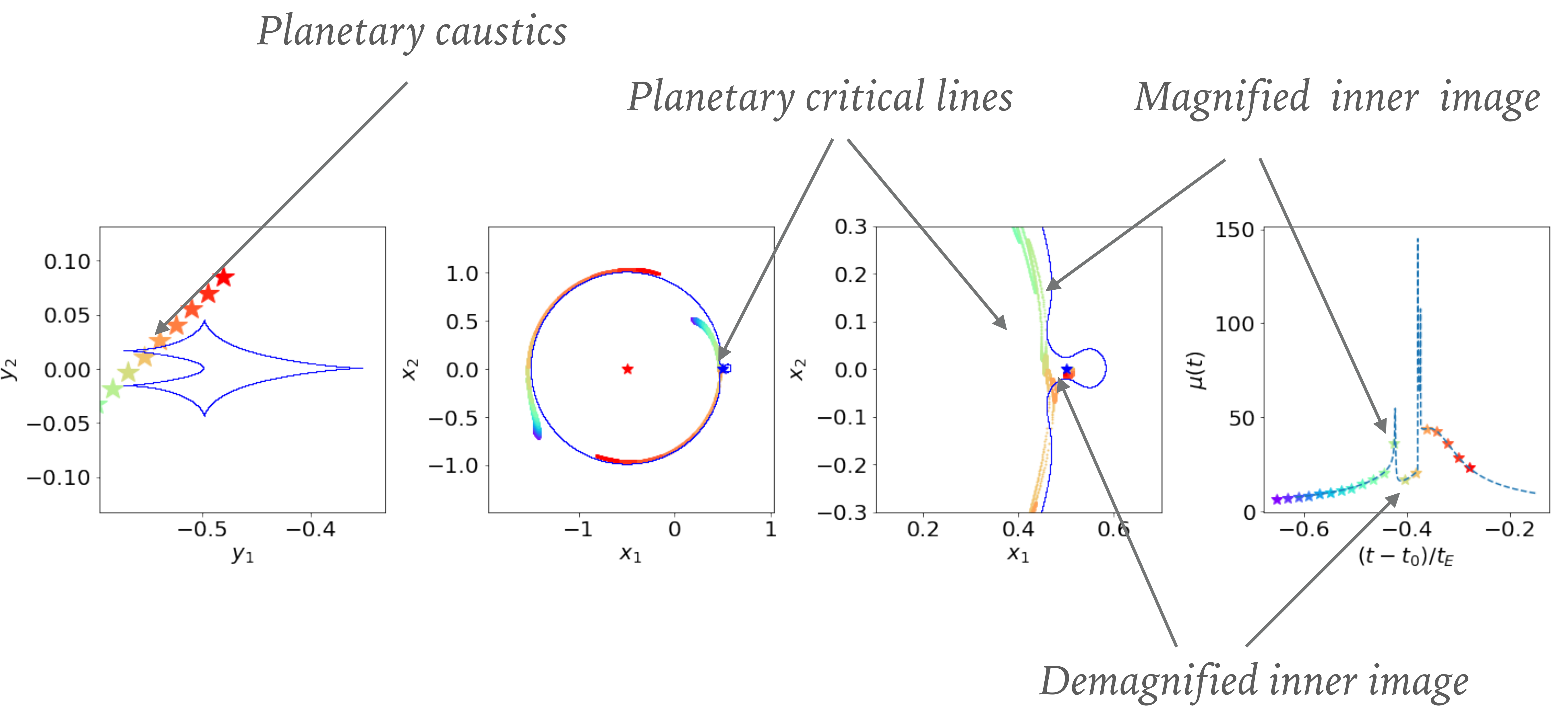
Planetary caustics

Planetary critical lines

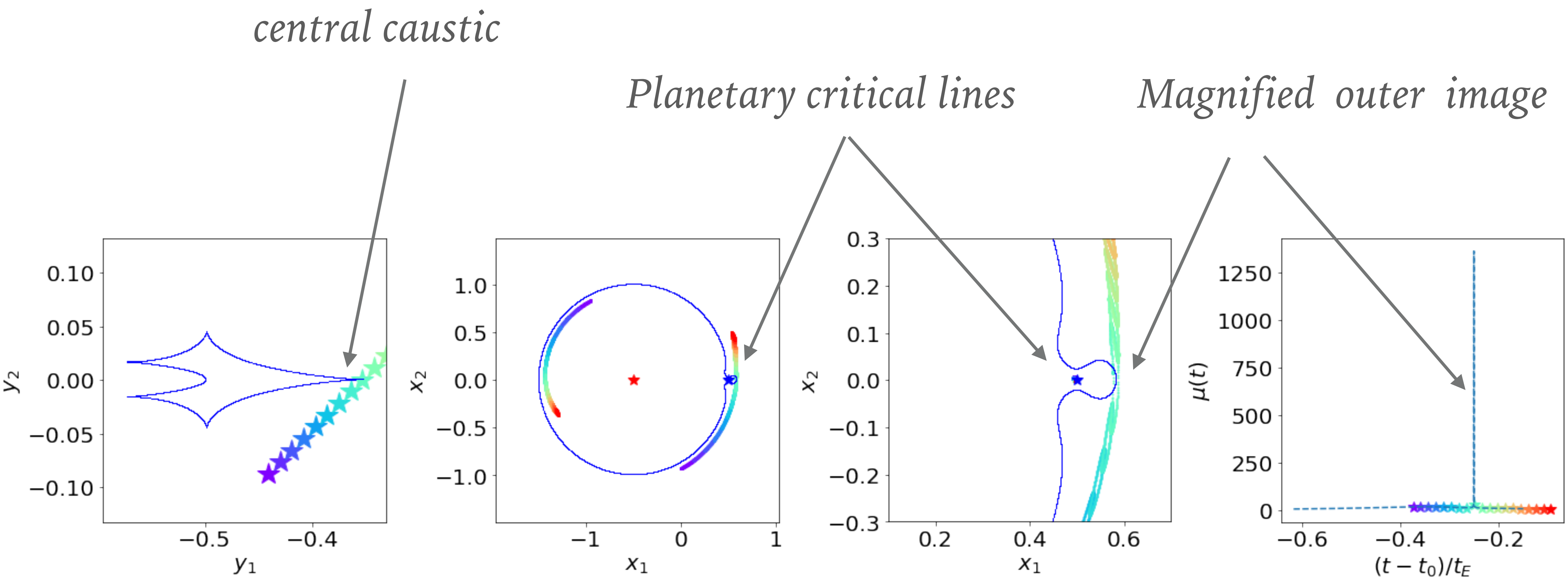


Demagnified inner image

PLANETARY PERTURBATIONS AS PERTURBATIONS OF SINGLE IMAGES



PLANETARY PERTURBATIONS AS PERTURBATIONS OF SINGLE IMAGES



Some interesting movies of this are available at:

https://www.astronomy.ohio-state.edu/gaudi.1/movies.html

INTERPRETING THE LIGHT CURVES

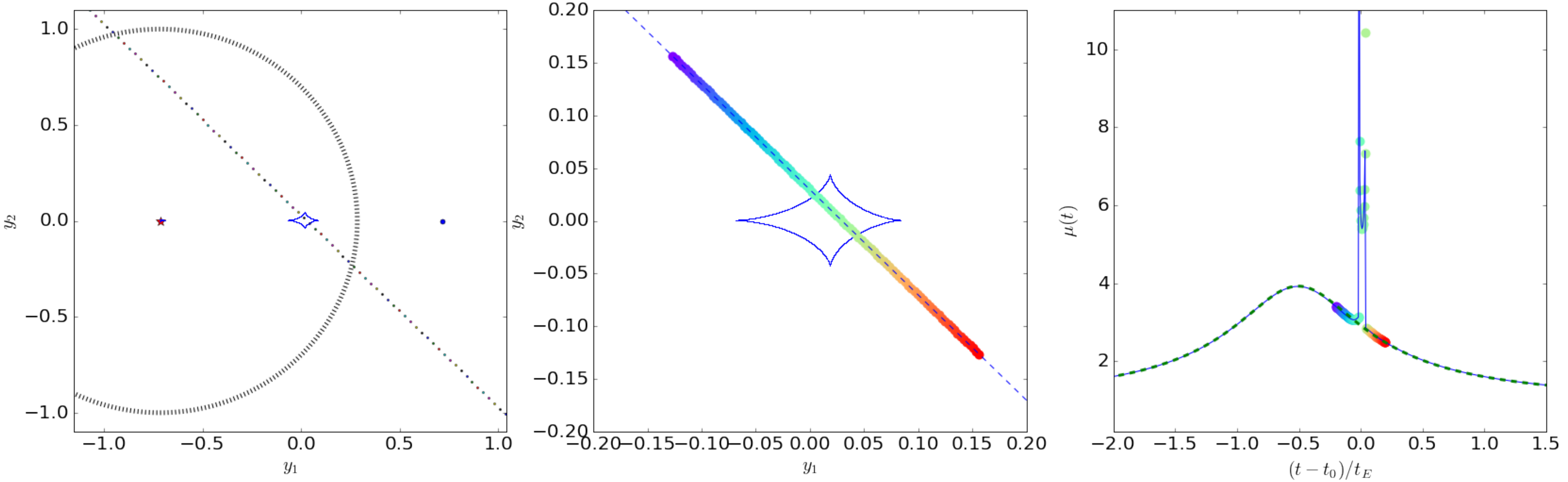
If we notice a planetary caustic perturbation, it means that the planet is located at the position of one of the images:

$$|x_{\pm}| = d$$

Consequently the position of the caustic which is being crossed can be derived from the lens equation (which is satisfied by the images)

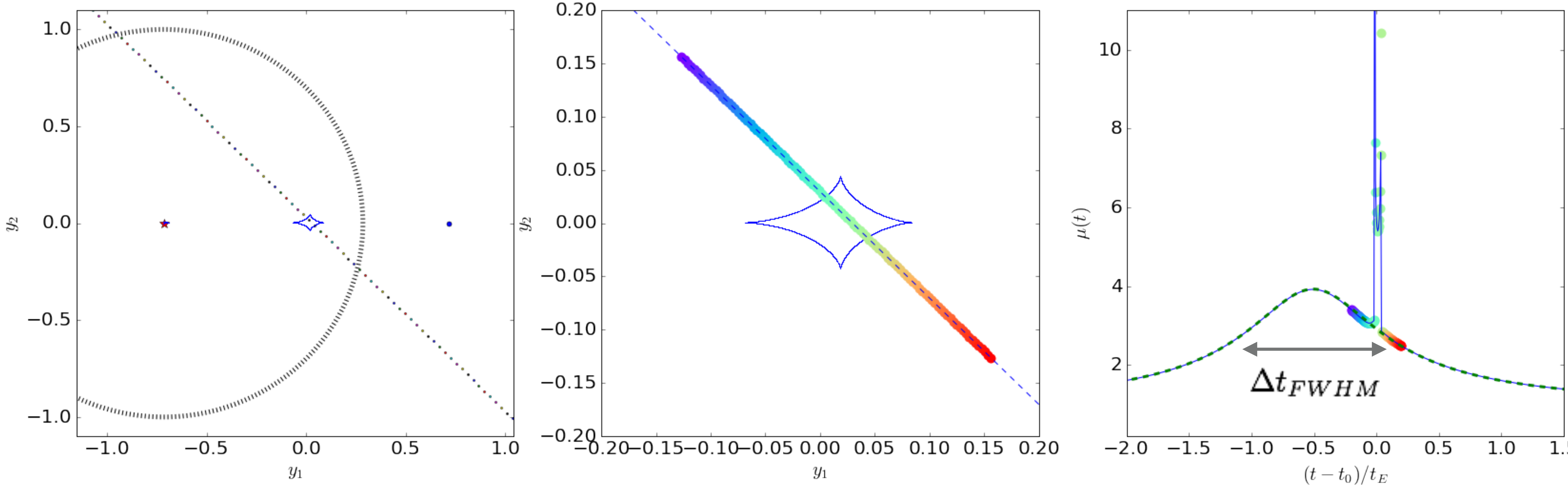
$$x_{\pm} = \frac{1}{2}\left[y \pm \sqrt{y^2 + 4}\right]$$

PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



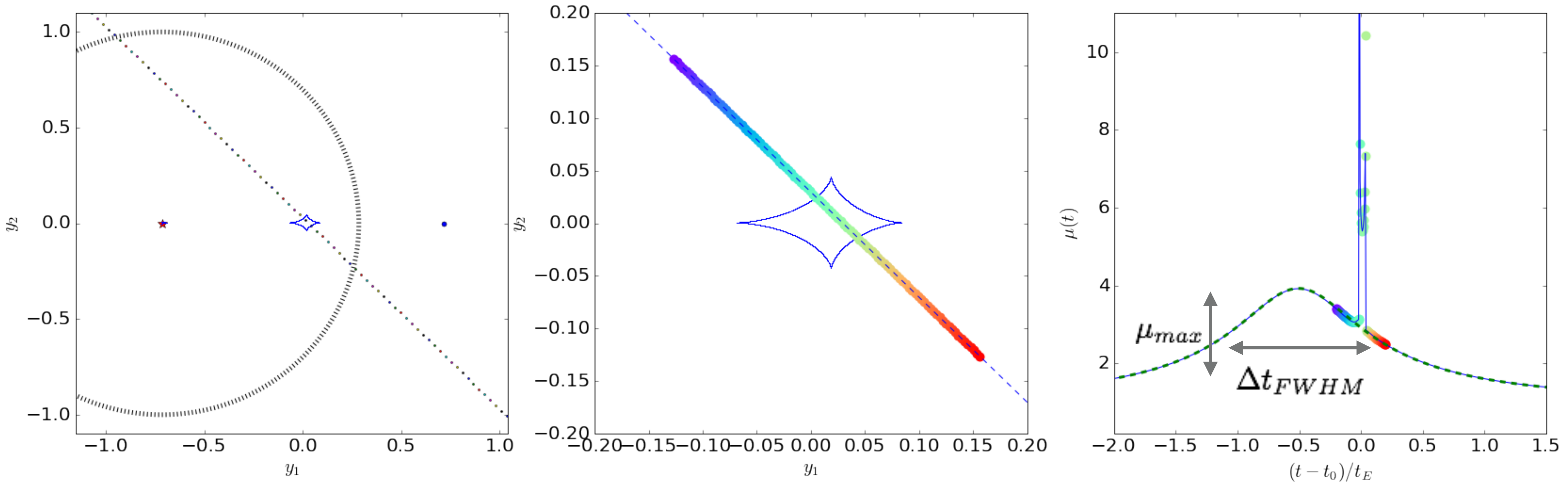
- primary event:
- planetary perturbation:

PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



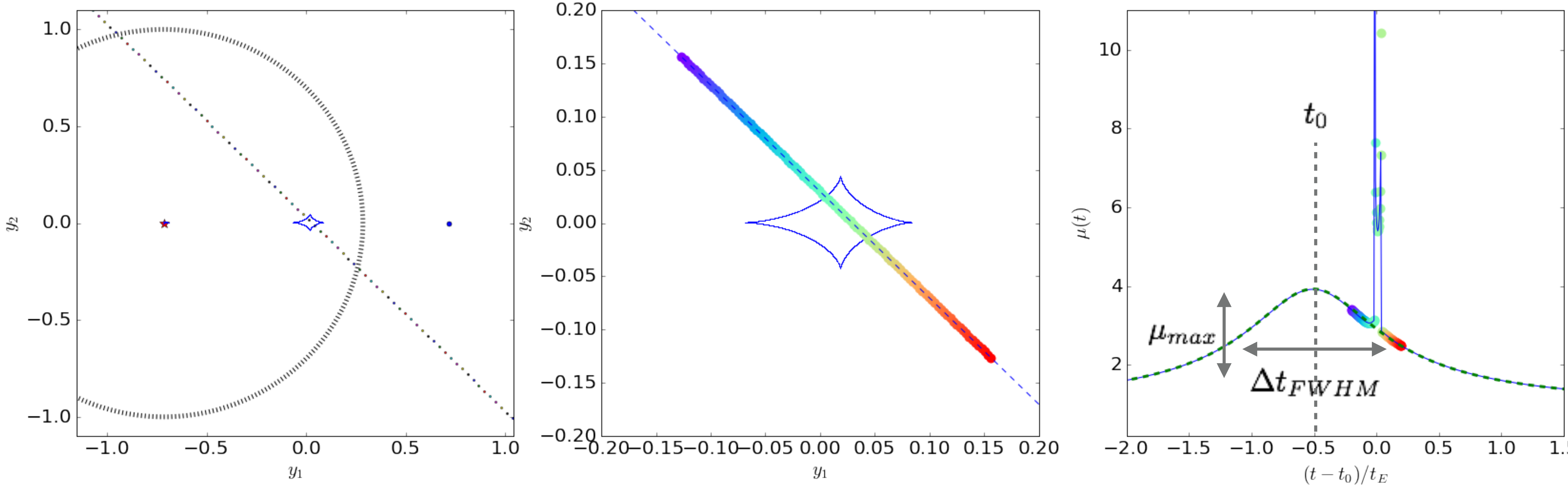
- primary event: Δt_{FWHM}
- planetary perturbation:

PLANET PROPERTIES "READ OFF" OF THE LIGHT CURVES



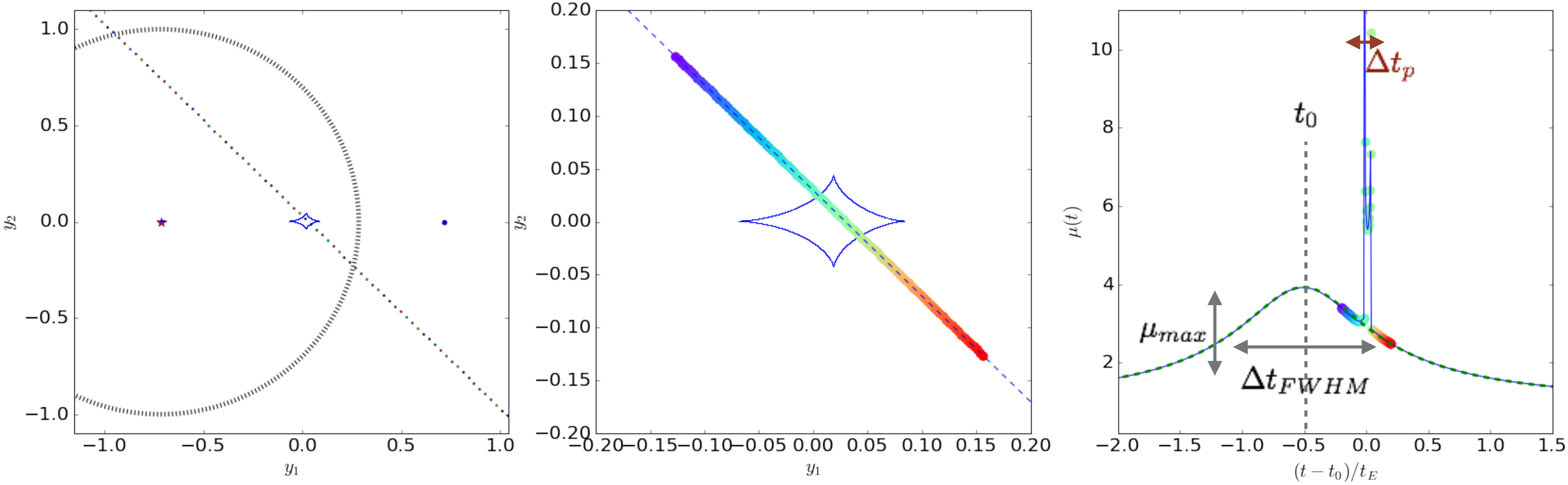
- primary event: Δt_{FWHM} μ_{max}
- planetary perturbation:

PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



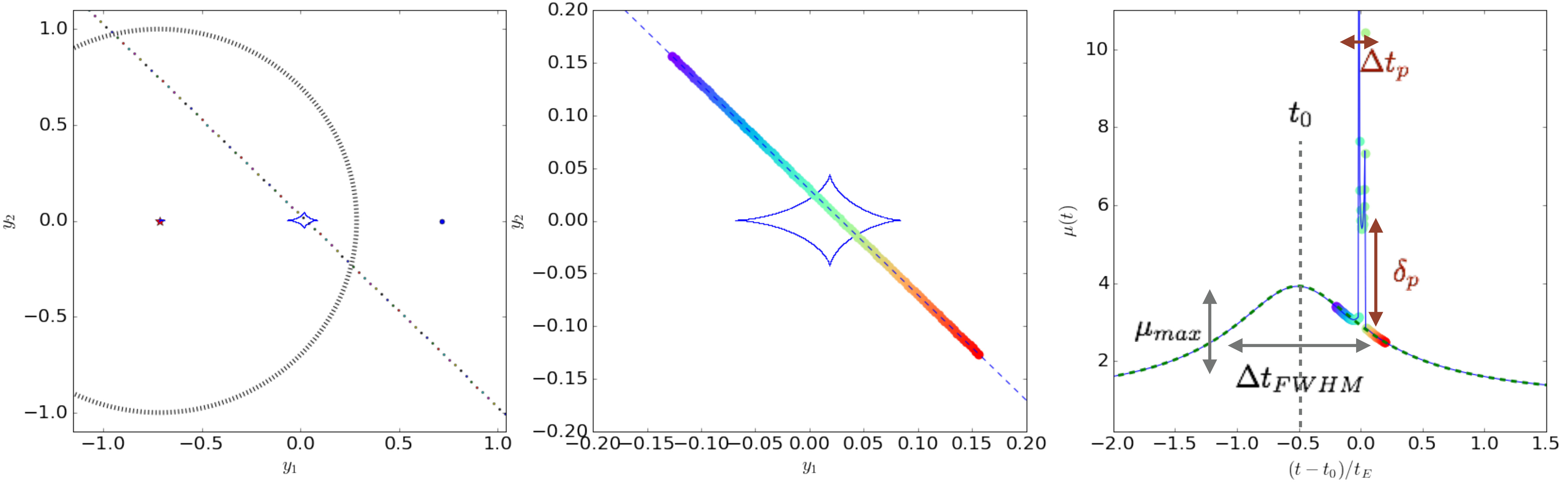
- primary event: Δt_{FWHM} μ_{max} t_0
- planetary perturbation:

PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



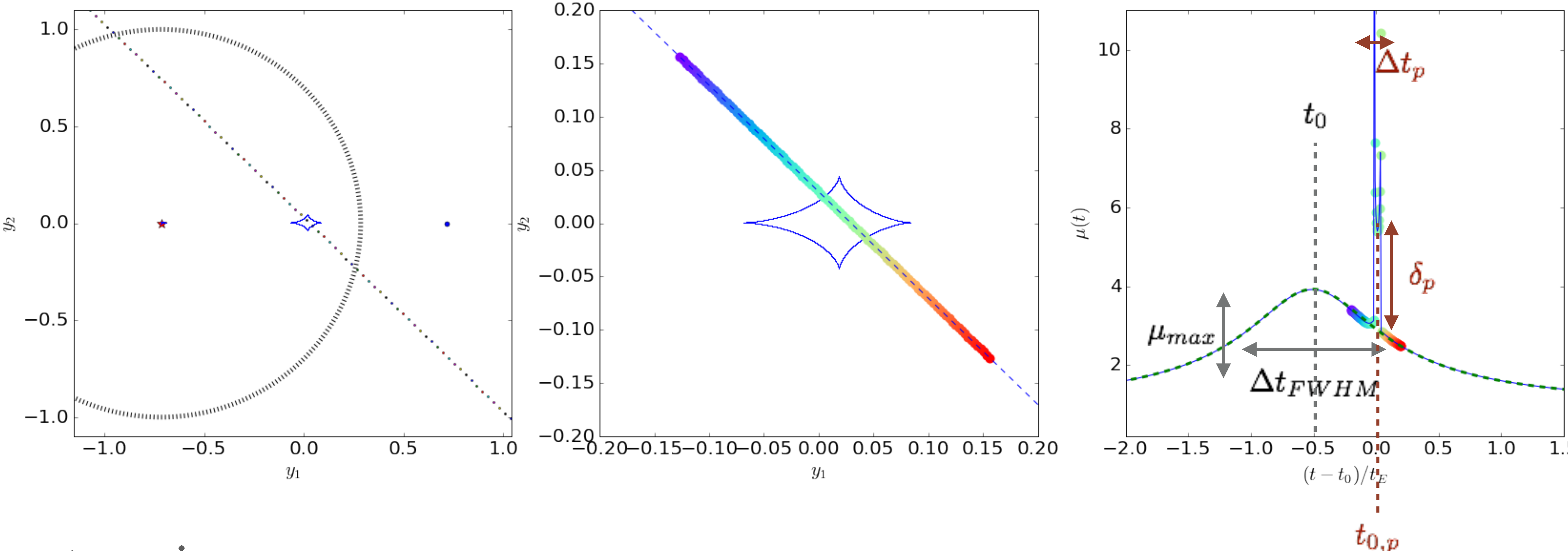
- primary event: Δt_{FWHM} μ_{max} t_0
- planetary perturbation: Δt_p

PLANET PROPERTIES "READ OFF" OF THE LIGHT CURVES



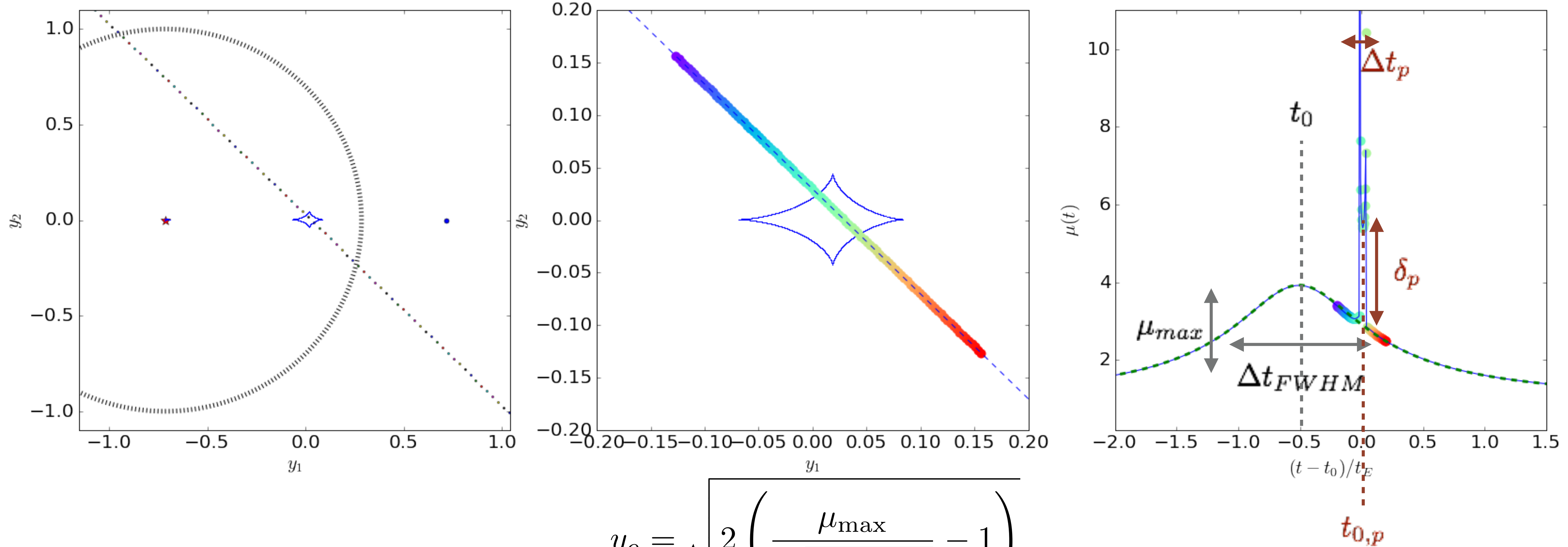
- primary event: Δt_{FWHM} μ_{max} t_0
- planetary perturbation: Δt_p δ_p

PLANET PROPERTIES "READ OFF" OF THE LIGHT CURVES



- primary event: Δt_{FWHM} μ_{max} t_0
- planetary perturbation: Δt_p δ_p $t_{0,p}$

PLANET PROPERTIES "READ OFF" OF THE LIGHT CURVES

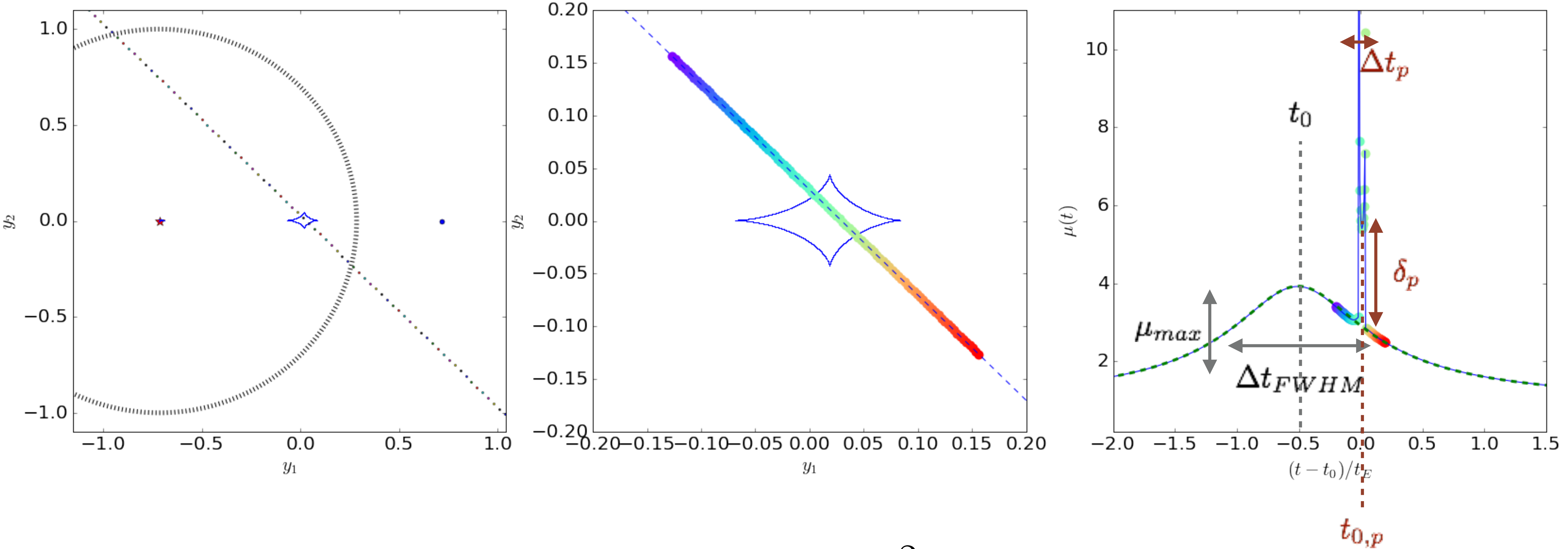


$$y_o = \sqrt{2\left(\frac{\mu_{\max}}{\sqrt{\mu_{\max}^2 - 1}} - 1\right)}$$

$$\Delta t_{FWHM},\ \mu_{max},\ t_0 \ \Rightarrow\ \mu(y) = \frac{y^2+2}{y\sqrt{y^2+4}} \qquad y(t) = \sqrt{y_0^2 + \left(\frac{t-t_0}{t_E}\right)^2}$$

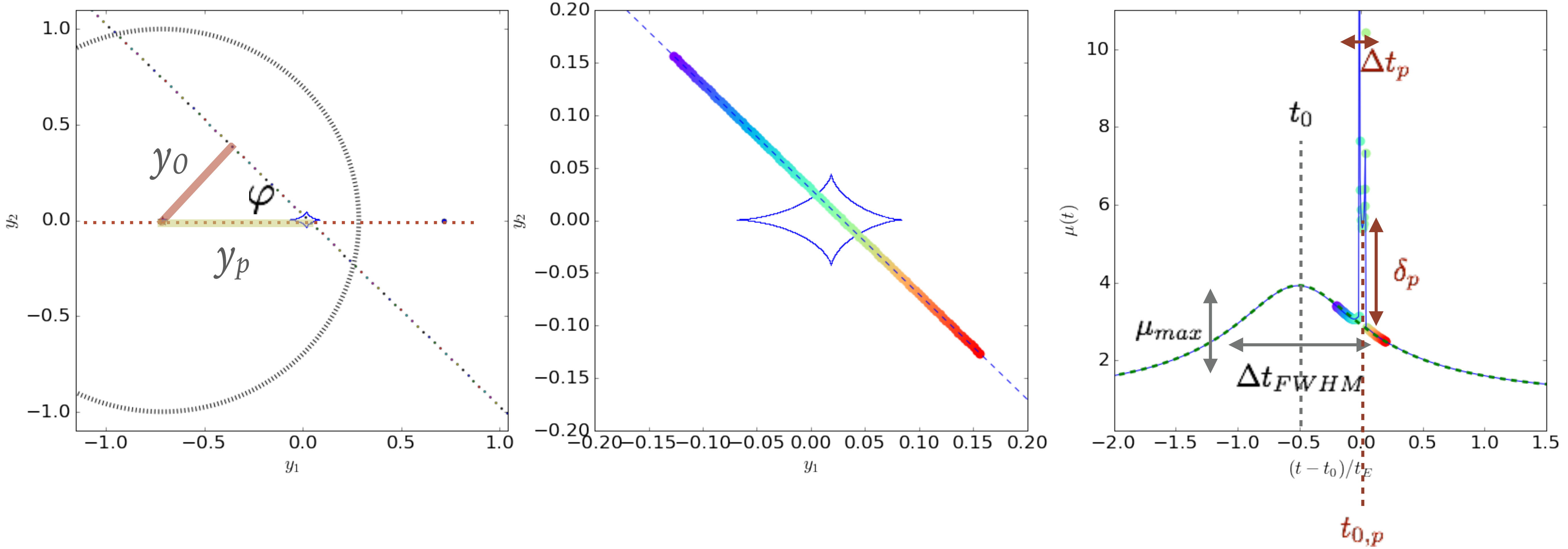
$$\Rightarrow \quad y_0 \quad t_E$$

PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



$$\Delta t_p \sim t_{E,p} \;\Rightarrow t_E \Rightarrow \quad q \simeq \left(\frac{t_{E,p}}{t_E}\right)^2$$

PLANET PROPERTIES "READ OFF" OF THE LIGHT CURVES



$$y_0, y_p \Rightarrow \varphi = \sin^{-1}\frac{y_0}{y_p}$$

$$y(\mu) = \sqrt{2\left(\frac{\mu}{\sqrt{\mu^2-1}} - 1\right)}$$

TO SUMMARIZE

- different caustic topologies give rise to different kind of perturbations on the light curves
- planets can be detected in only a few qualitatively different ways:
  - at relatively low magnification of the primary, if the source crosses the planetary caustics from close or wide planets
  - near the peak of the light curve, if the source has a small impact parameter, in both cases of wide and close planets
  - at modest to high-magnification, through the perturbations from the resonant caustic.
  - in the case of free-floating planets, as single, short time-scale events.
- Light curves can be used to extract parameters of the primary and of the planet
- As seen for single point masses, degeneracies can be broken (finite source effects, microlensing parallax)
- Some cases are more difficult than others…

WHY SEARCHING PLANETS?

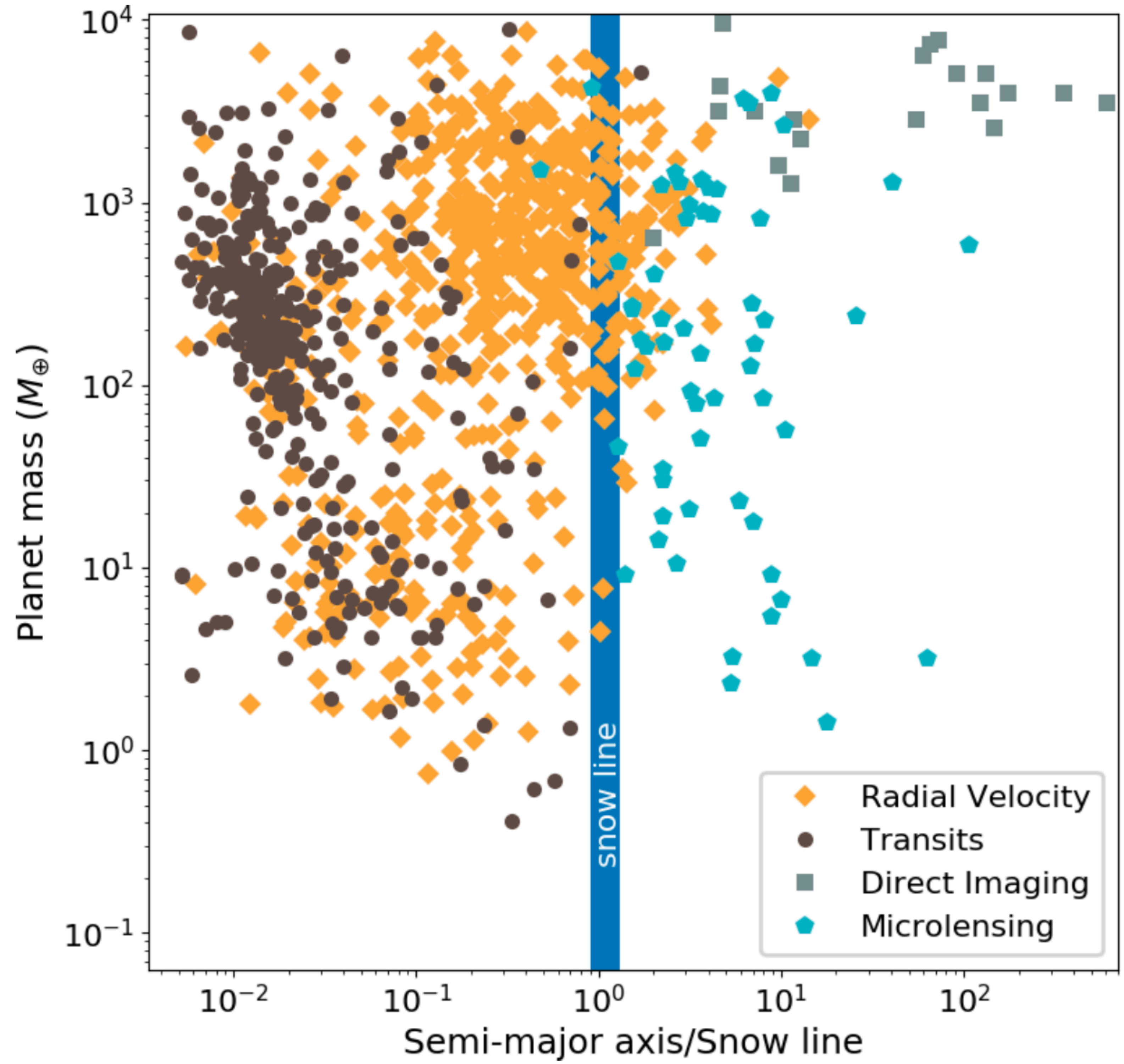
- Planet formation and evolution is still not fully understood
- Different models predict different distributions of planet masses and distances from the host stars
- Sizes of planets may depend on the mass of the host star, on the position in the galaxy (e.g. on the local metallicity), etc
- Models are thus tested by counting planets of different masses and orbital parameters

METHODS TO SEARCH PLANETS OTHER THAN MICROLENSING

- **Radial velocity shift**: relies on the fact that a star does not remain completely stationary when it is orbited by a planet. The star moves, ever so slightly, in a small circle or ellipse, responding to the gravitational tug of its smaller companion. This produces periodic shifts of lines in the stellar spectra. This method was very successful in ***discovering giant planets near their hosts***.
- **Transit surveys:** identify exoplanets with near edge-on orbits that pass in front of their host stars. **Hundreds of planets within ~1 AU from their hosts** were found by e.g. the NASA's Kepler space mission.
- **Direct imaging:** Massive planets at large distances from their hosts can be visually identified in planetary systems that are not too far away.
- **Astrometry**: involves very precise measurements of a star's position in the sky. If the star is orbited by a planet, periodic shifts of its position occur (for the same reasons that cause the radial velocity shifts).
- **Pulsar timing:** pulsars are rapidly rotating neutron stars that emit periodic pulses of electromagnetic radiation. These pulses become irregular in case the pulsar is orbited by a planet.

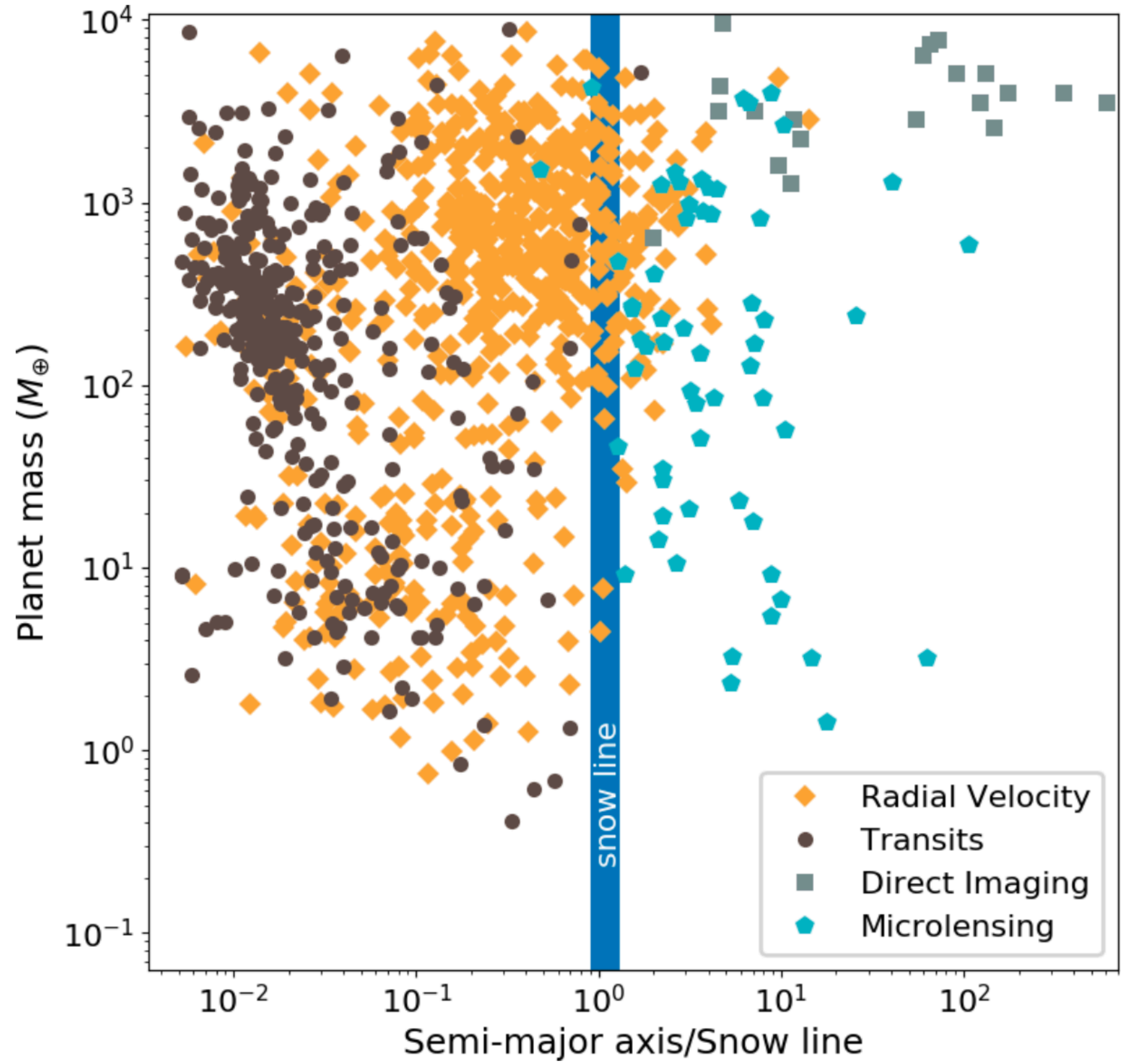
THE CURRENT PICTURE

- ~4400 confirmed exoplanets
- Transits and radial velocity shifts are by far the most successful techniques
- Astrometry: very precise measurements are necessary. Only one detection (not confirmed). Possibly GAIA will revolutionise the picture
- Vast majority of detections are massive planets close to their hosts (<1-2 AU)
- To explore the outer parts of planetary systems, only direct imaging works but it is limited to very massive planets



USING MICROLENSING FOR PLANET SEARCHES

- ~90 planets discovered via microlensing so far
- ~1% of all microlensing events contain planetary anomalies
- d_{min}=0.66 AU
- bulk of planets at d~3 AU
- wide range of masses
- complementary technique to others that are most sensitive to planets near their host stars



USING MICROLENSING FOR PLANET SEARCHES

- planets are most easily identified when they are at a distance ~ER
- example: 1 mas at ~5kpc=5AU
- peak sensitivity beyond the snow line
- the snow line marks a very important region for planet formation!
- Boundary distance from the star beyond which the ambient temperature drops below ~160K and water turns to ice

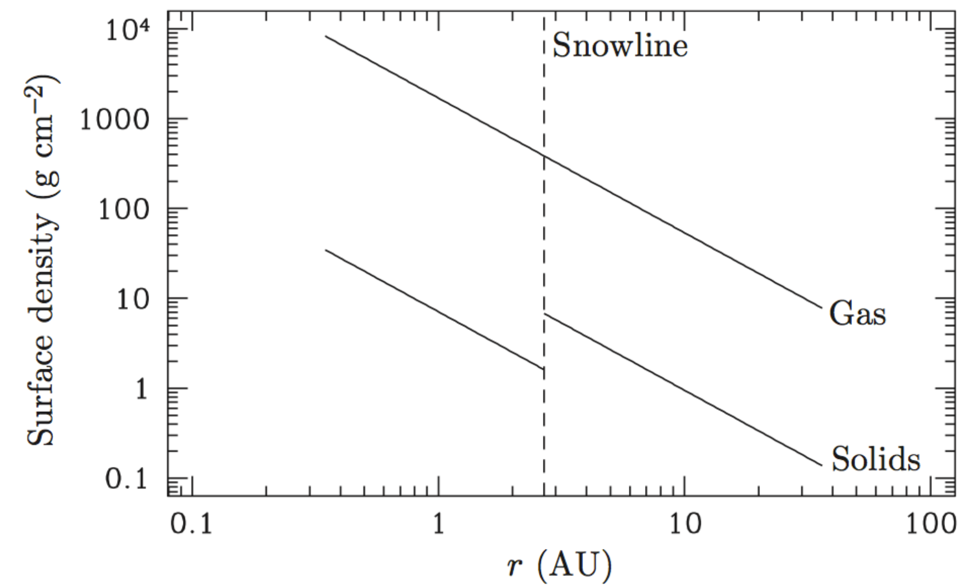
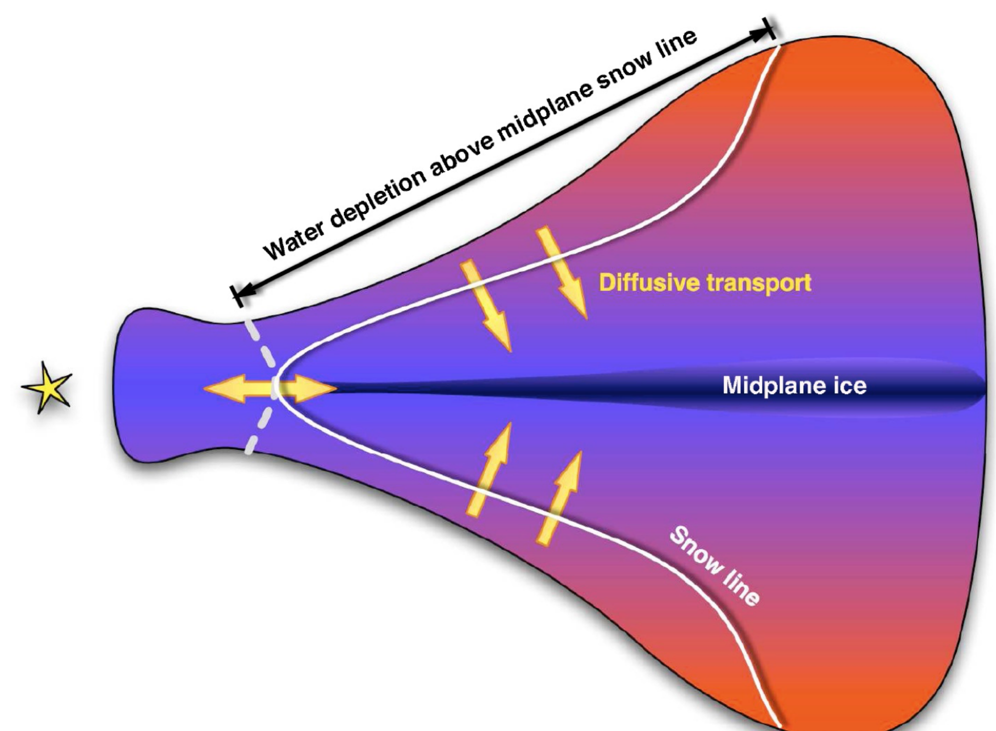
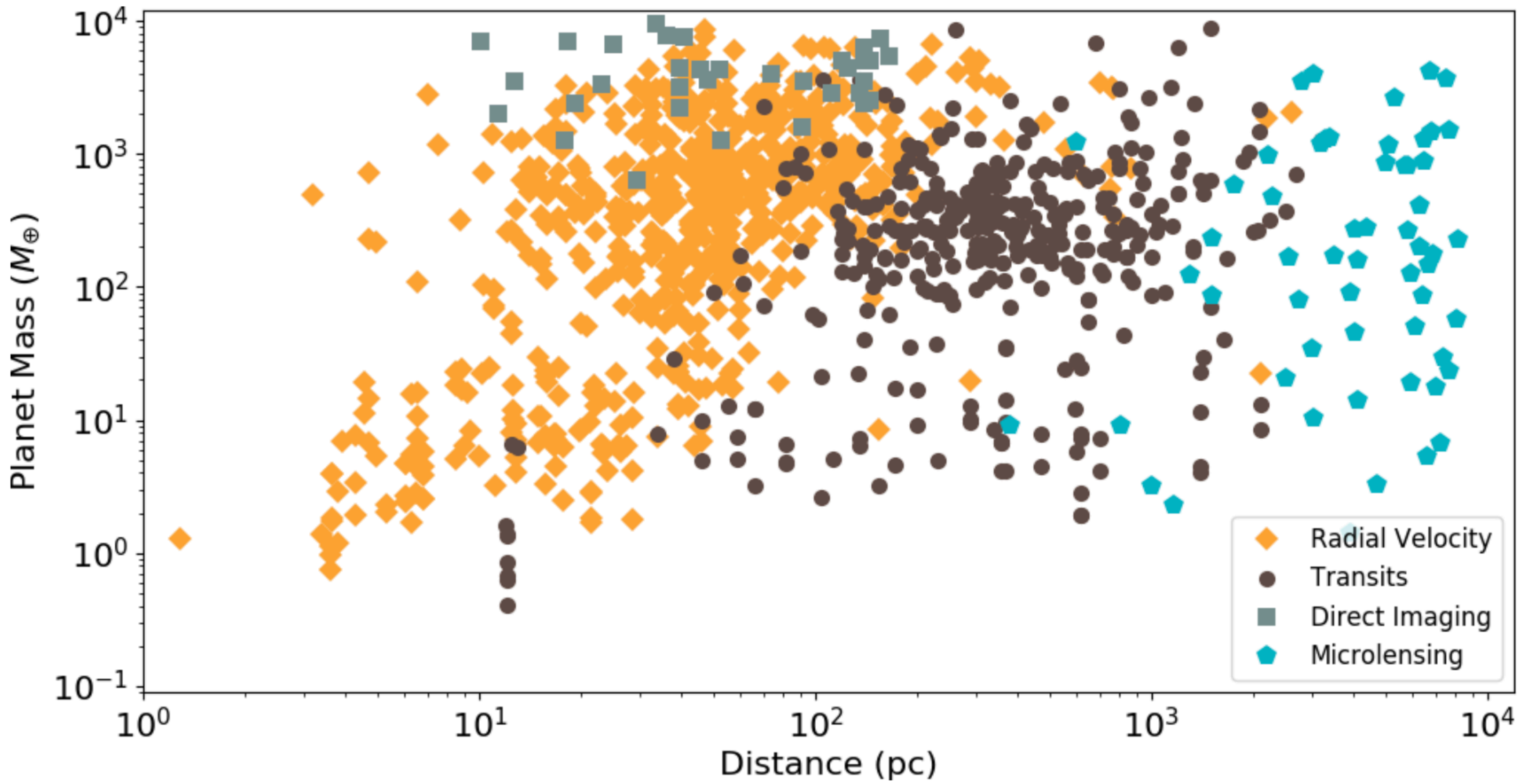


Fig. 1.1. The surface density in gas (upper line) and solids (lower broken line) as a function of radius in Hayashi's minimum mass Solar Nebula. The dashed vertical line denotes the location of the snowline.



IN ADDITION…



- sensitivity to low-mass planets
- sensitivity to long period and free-floating planets
- sensitivity to a wide range of host stars over a wide range of galacto-centric distances
- sensitivity to multiple planets and orbital parameters…

STRATEGIES TO FIND PLANETS WITH MICROLENSING

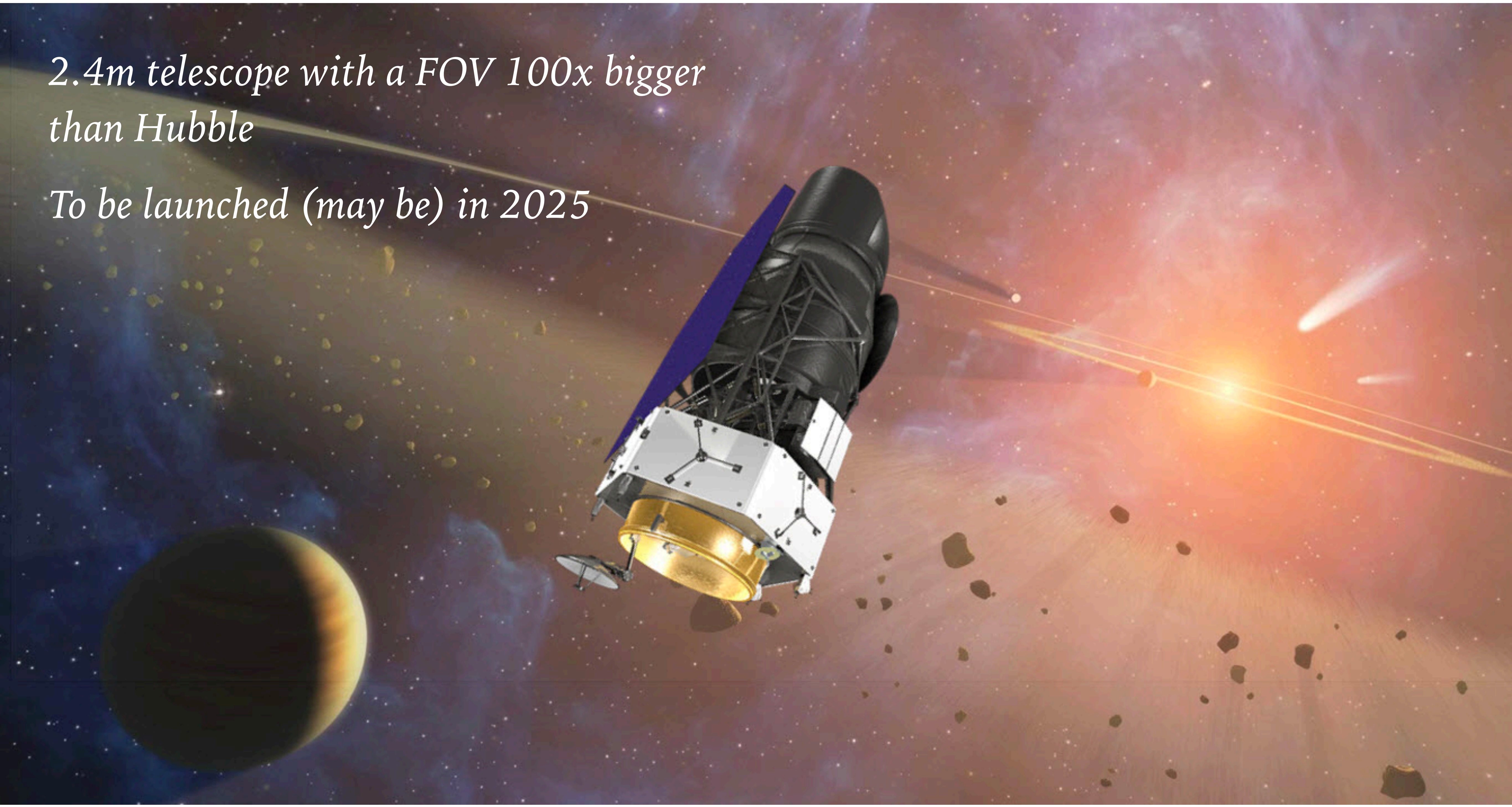
By-product of microlensing searches!

1. Find microlensing event: OGLE, MOA, now 10-30 min cadence; Korean Microlensing Telescope Network (KMTNet), 3 telescopes in Australia, South-Africa, and Chile, 1.6 m each monitoring 16 sq. degrees on the sky with a cadence of ~10 mins

2. Planets found as anomalies in the standard light curves: automatic modelling

3. trigger follow-up from other telescopes (e.g. Spitzer)

THE FUTURE: WFIRST



2.4m telescope with a FOV 100x bigger than Hubble

To be launched (may be) in 2025