GRAVITATIONAL LENSING

2 - Deflection of Light II

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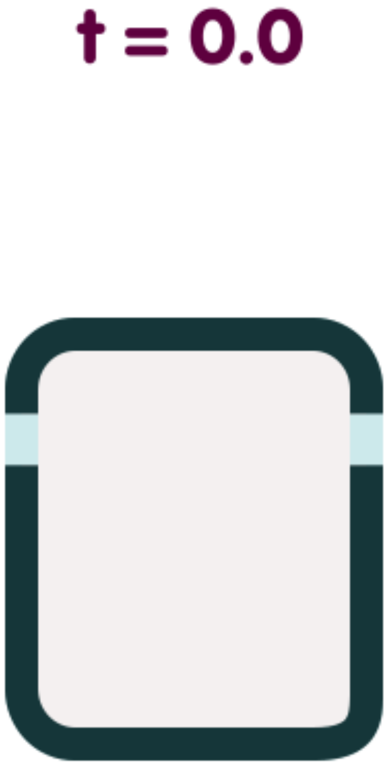
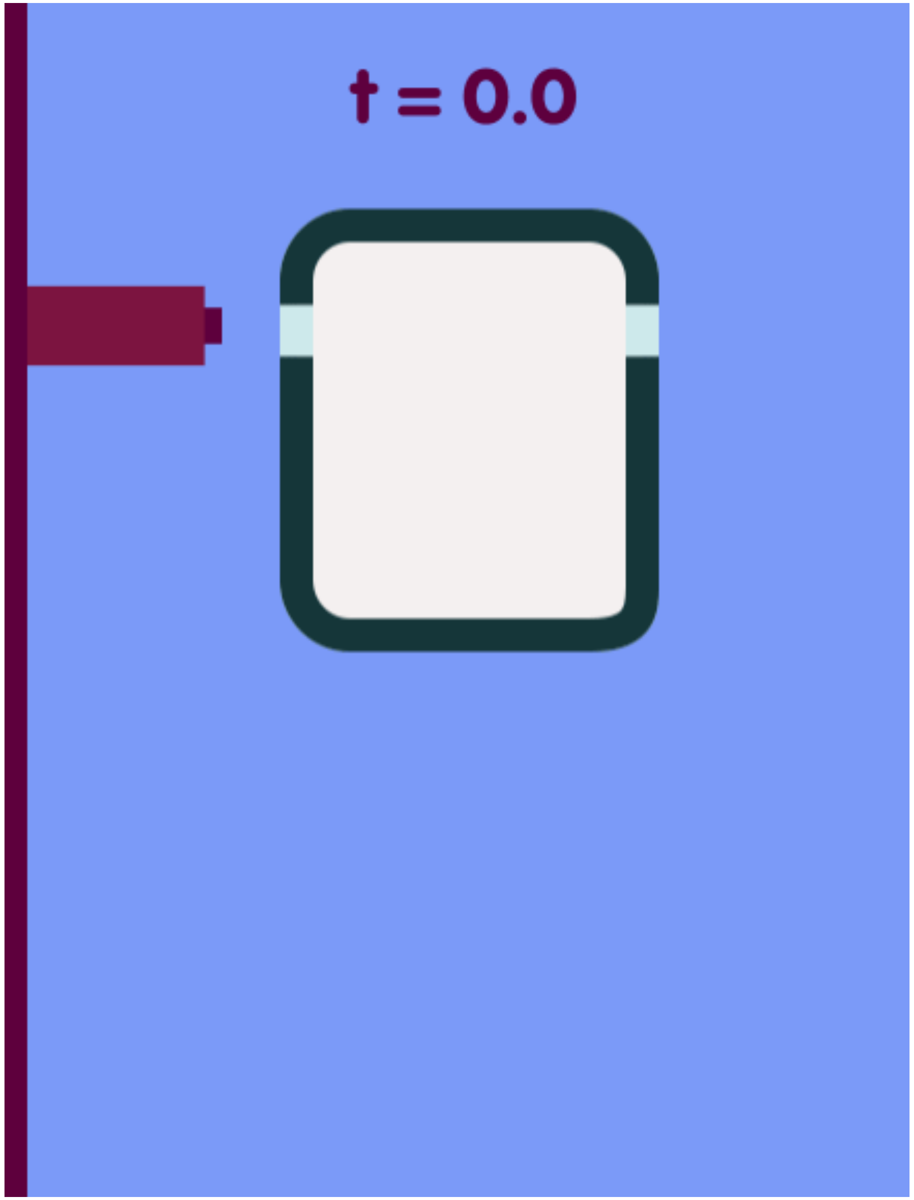
2022-2023

TIMELINE

- 1907-1911: Einstein resumes the idea of light deflection using special relativity and equivalence principle: "*In an arbitrary gravitational field, at any given spacetime point, we can choose a locally inertial reference frame such that, in a sufficiently small region surrounding that point, all physical laws take the same form they would take in absence of gravity, namely the form prescribed by Special Relativity*"

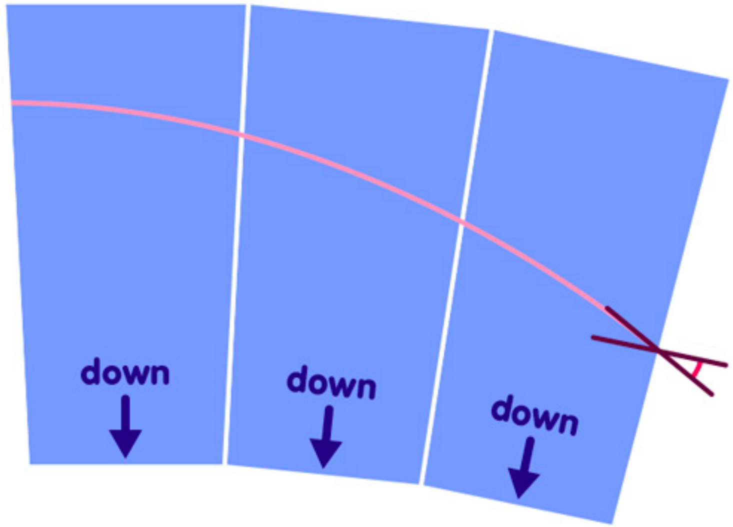
EINSTEIN'S EQUIVALENCE PRINCIPLE

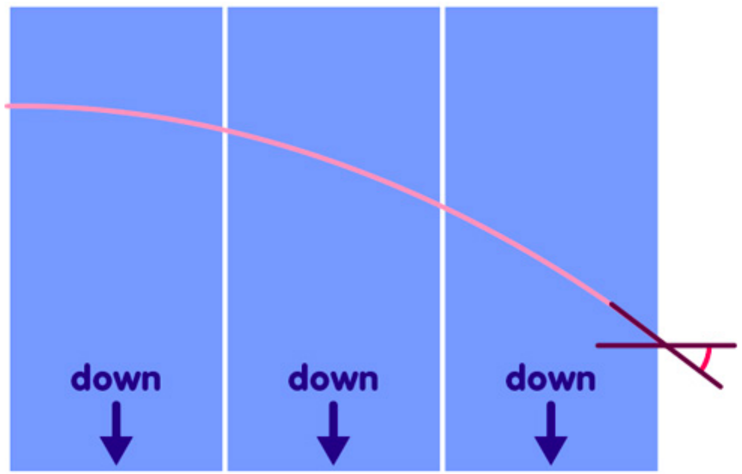




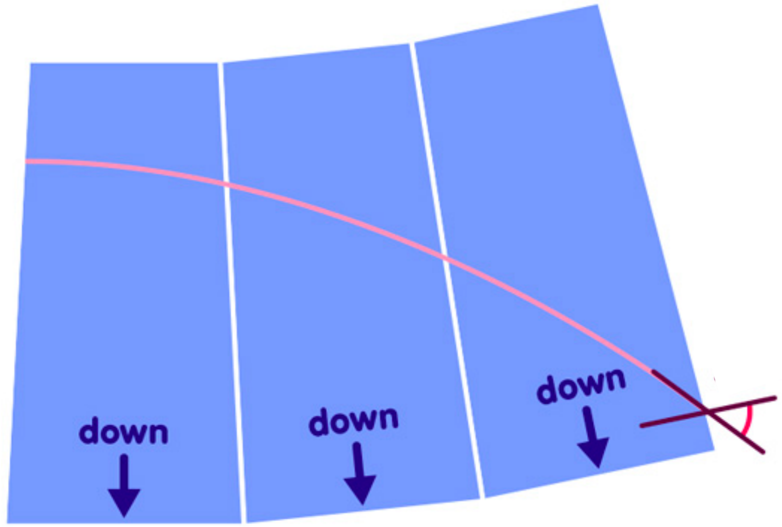
CURVING SPACE-TIME AND LIGHT DEFLECTION

Bend away from light





Euclidean geometry



https://www.einstein-online.info/en/spotlight/equivalence_light

Bend toward light

CURVED SPACE

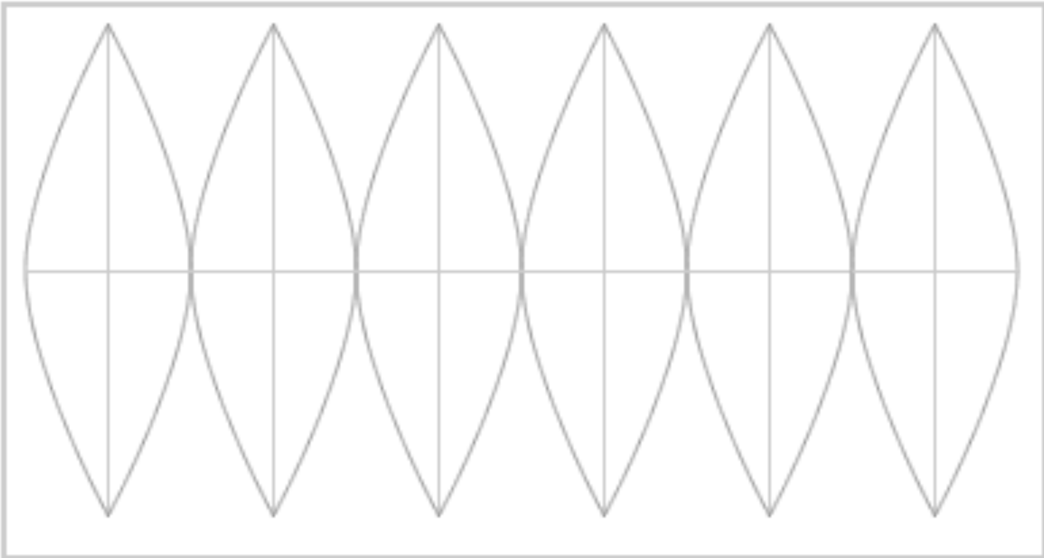
- If space is curved, there is no coordinate system that is Cartesian everywhere.

- However, at each point there is a coordinate system that is locally Cartesian.

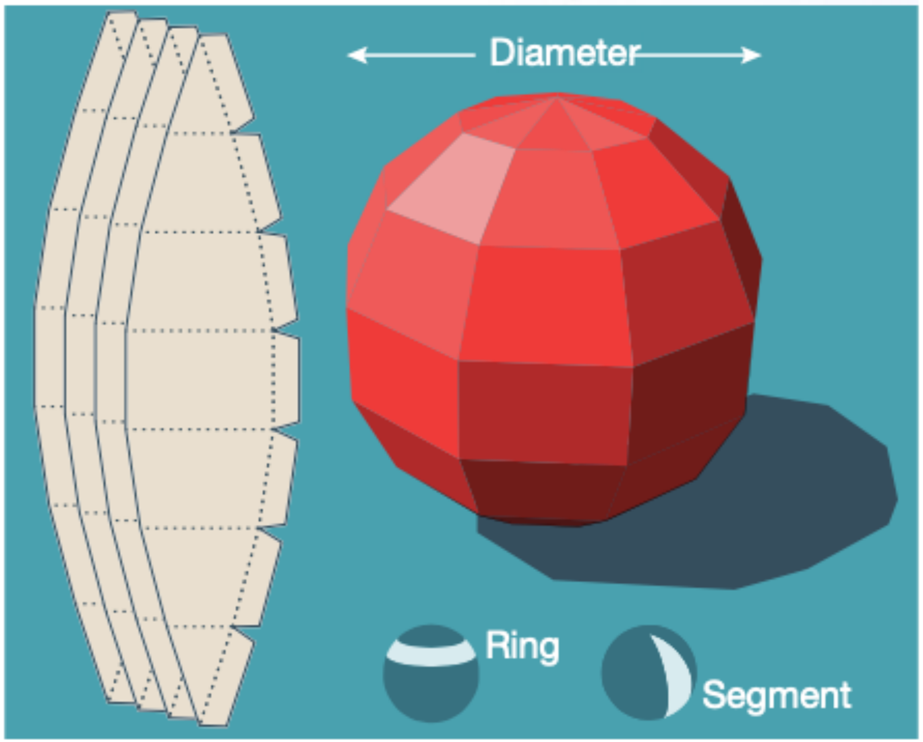
- Straight lines can meet.

- Parallel transported vectors will not agree if they are transported around different paths.

CURVED SPACE



Covering of a sphere with paper.



NULL GEODESICS

ξ^μ - Minkowski (flat space-time) coordinates

tangent vector $v^\nu \equiv \dfrac{dx^\nu}{d\lambda}$

metric $$g_{\mu\nu} = \frac{\partial \xi^\alpha}{\partial x^\mu} \frac{\partial \xi^\beta}{\partial x^\nu} \eta_{\alpha\beta}$$

geodesic equation

$$\frac{d^2 x^\gamma}{d\lambda^2} = -\Gamma^\gamma_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}$$

$$\frac{dv^\gamma}{d\lambda} = -\Gamma^\gamma_{\alpha\beta} v^\alpha v^\beta$$

Affine connection
Christoffel symboles
Levi-Civita connection

$$\Gamma^\gamma_{\alpha\beta} = \frac{\partial^2 \xi^\mu}{\partial x^\alpha \partial x^\beta} \frac{\partial x^\gamma}{\partial \xi^\mu}$$

NULL GEODESICS

covariant derivative

$$\overline{\boldsymbol{\nabla}}_\nu v^\mu = \frac{\partial v^\mu}{\partial x^\nu} + \Gamma^\mu_{\nu\gamma} v^\gamma$$

tangent vector

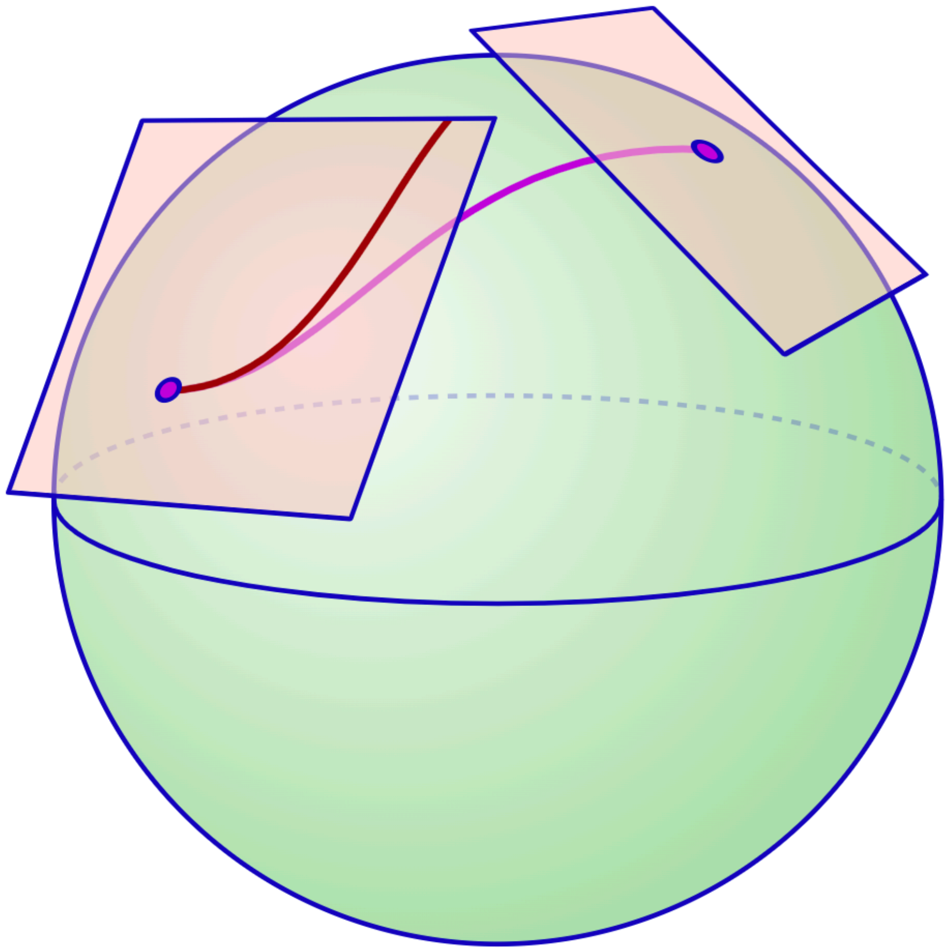
$$v^\nu \equiv \frac{dx^\nu}{d\lambda}$$

Parallel transport of tangent vector

$$\begin{aligned}
\frac{Dv^\mu}{D\lambda} = v^\nu \overline{\boldsymbol{\nabla}}_\nu v^\mu &= v^\nu \frac{\partial v^\mu}{\partial x^\nu} + \Gamma^\mu_{\nu\gamma} v^\gamma v^\nu \\
&= \frac{dx^\nu}{d\lambda} \frac{\partial v^\mu}{\partial x^\nu} + \Gamma^\mu_{\nu\gamma} v^\gamma v^\nu \\
&= \frac{dv^\mu}{d\lambda} + \Gamma^\mu_{\nu\gamma} v^\gamma v^\nu \\
&= 0
\end{aligned}$$

geodesic equation

CURVED SPACE



NULL GEODESICS

Newtonian gauge

$$d\tau^2 = \left(1 + \frac{2\phi_N}{c^2}\right) dt^2 - c^{-2}\left(1 - \frac{2\phi_N}{c^2}\right) dl^2$$

$$= 0 \qquad \text{for light}$$

$$\frac{d\vec{x}}{d\lambda} = \left(\frac{dt}{d\lambda}, v^1, v^2, v^3\right) \qquad\qquad 0 = g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} \simeq \left(\frac{dt}{d\lambda}\right)^2 - \frac{1}{c^2}|\vec{v}|^2$$

$$\Gamma^i_{\mu\nu} = \frac{-1}{c^2\left(1 - \frac{2\phi}{c^2}\right)}\left[\delta_{i\nu}\frac{\partial\phi}{\partial x^\mu} + \delta_{i\mu}\frac{\partial\phi}{\partial x^\nu} - \delta_{\mu\nu}\frac{\partial\phi}{\partial x^i}\left(c^2\delta_{\mu 0} + \delta_{\mu[1,2,3]}\right)\right]$$

NULL GEODESICS

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$$\frac{d\vec{v}}{d\lambda} = -\frac{1}{c^2(1-2\phi_N)}\left[\vec{\nabla}\phi_N\left(\left(\frac{dt}{d\lambda}\right)^2 c^2 + |\vec{v}|^2\right) - 2\left(\vec{v}\cdot\vec{\nabla}\phi_N\right)\vec{v}\right]$$

NULL GEODESICS

Newtonian gauge

$$d\tau^2 = \left(1 + \frac{2\phi_N}{c^2}\right) dt^2 - c^{-2}\left(1 - \frac{2\phi_N}{c^2}\right) dl^2$$

$$= 0 \quad \text{for light}$$

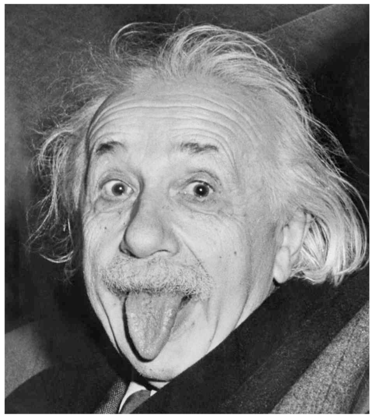
$$\frac{1}{|\vec{v}|}\frac{d\vec{v}}{d\lambda} = -\frac{2}{c^2(1-2\phi_N)}\left[\vec{\nabla}\phi_N - \left(\hat{v}\cdot\vec{\nabla}\phi_N\right)\hat{v}\right]$$

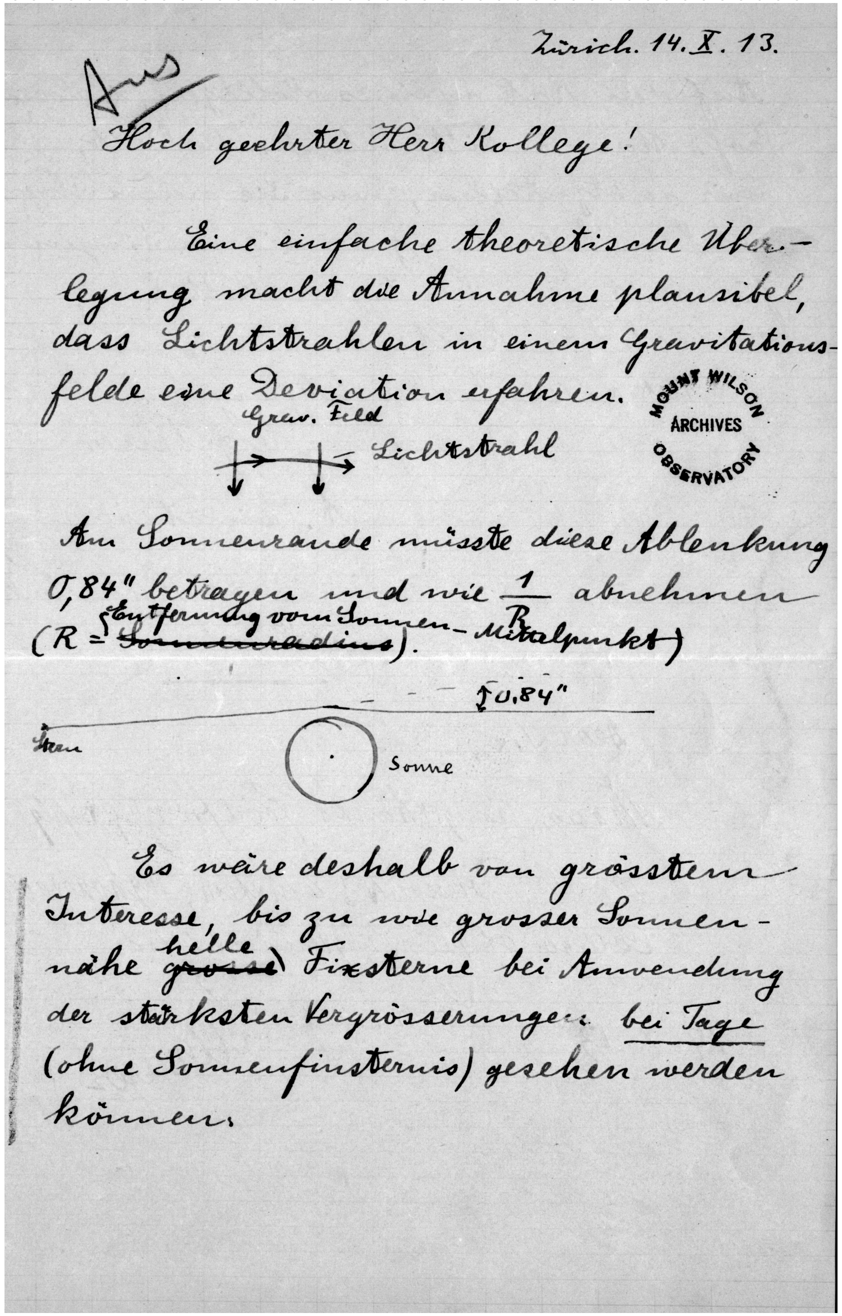
$$\simeq -\frac{2}{c^2}\left[\vec{\nabla}\phi_N - \left(\hat{v}\cdot\vec{\nabla}\phi_N\right)\hat{v}\right]$$

$$= -\frac{2}{c^2}\vec{\nabla}_\perp\phi_N$$

TIMELINE

- result of calculation of deflection is identical to that from Newtonian gravity
- 1913: A. Einstein writes to George Ellery Hale (Director of Mount Wilson Observatory), asking for advice on how to observe the position of stars in sun-light…





Zürich. 14. X. 13.

Aus

Hoch geehrter Herr Kollege!

Eine einfache theoretische Überlegung macht die Annahme plausibel, dass Lichtstrahlen in einem Gravitationsfelde eine Deviation erfahren.

Grav. Feld

Lichtstrahl

MOUNT WILSON ARCHIVES OBSERVATORY

Am Sonnenrande müsste diese Ablenkung 0,84" betragen und wie $\frac{1}{R}$ abnehmen (R = ~~Sonnenradius~~ Entfernung vom Sonnen-Mittelpunkt).

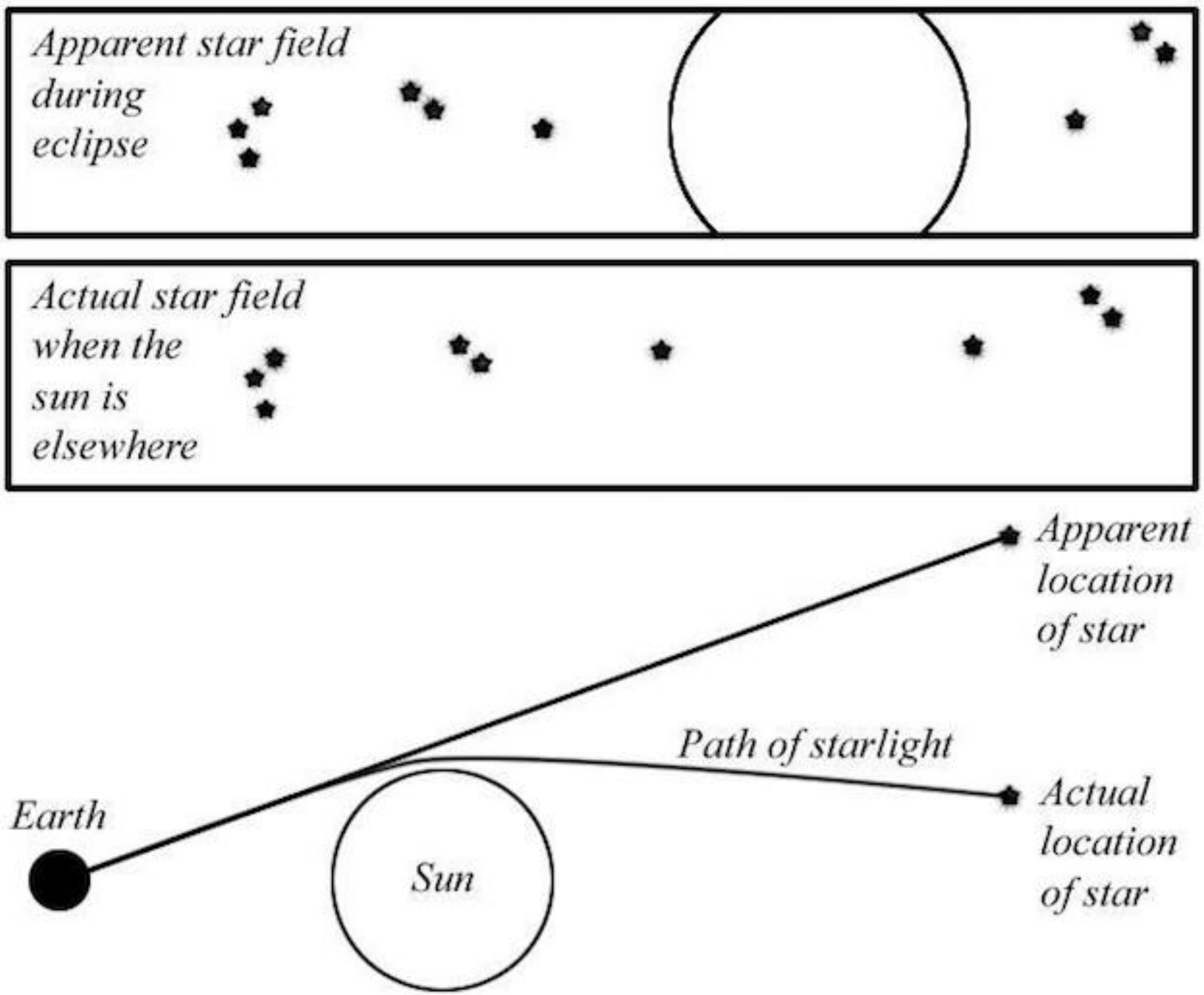
0,84"

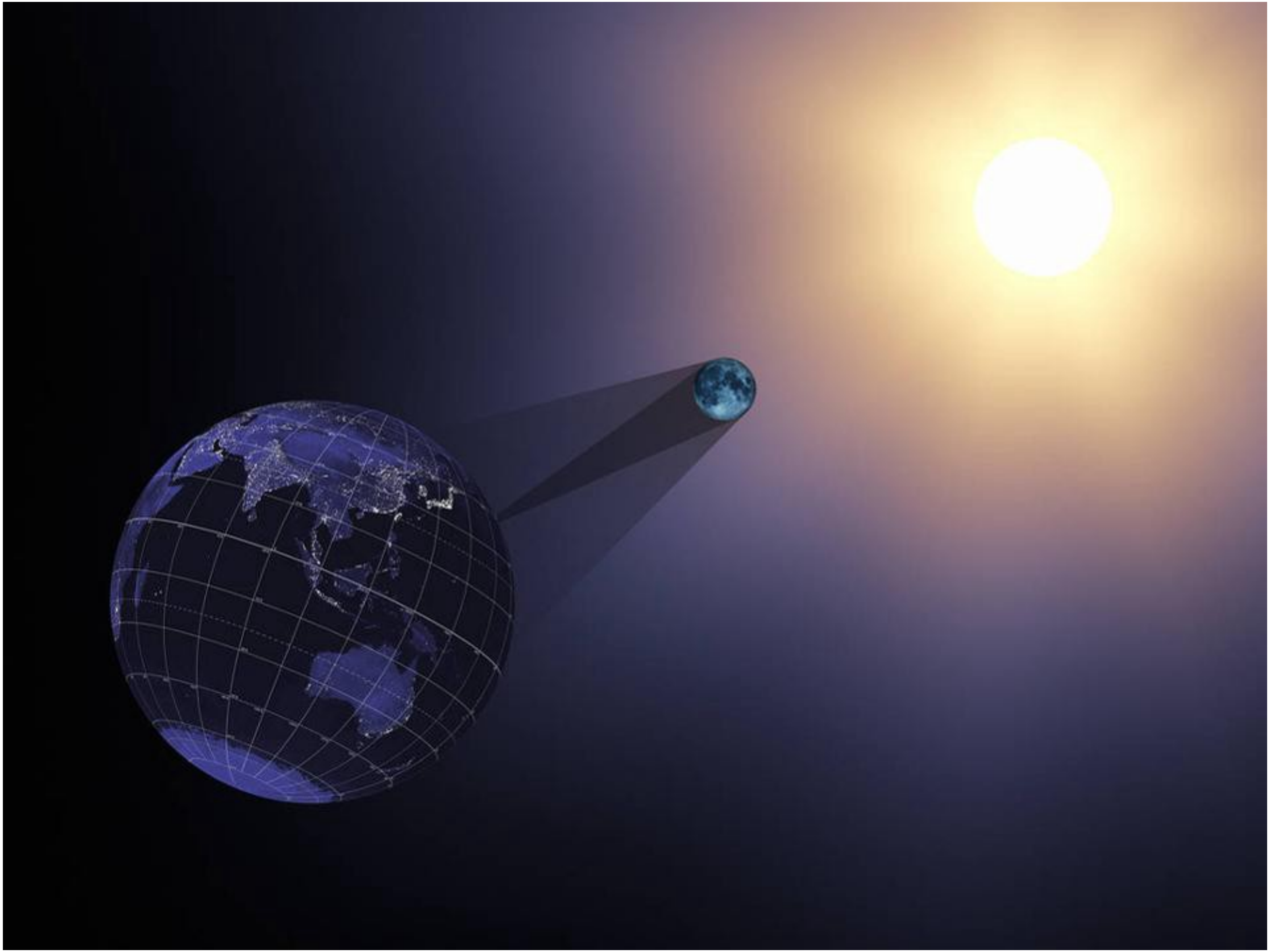
Stern

Sonne

Es wäre deshalb von grösstem Interesse, bis zu wie grosser Sonnennähe ~~grosse~~ helle Fixsterne bei Anwendung der stärksten Vergrösserungen bei Tage (ohne Sonnenfinsternis) gesehen werden können.

TESTING THE DEFLECTION OF LIGHT DURING A SOLAR ECLIPSE





TIMELINE

- 1914 (August): Total solar eclipse in Crimea:
  - Attempts by Erwin Finlay-Freundlich and William Wallace Campbell
  - Unfortunately, WWI began and Russia entered into the war on Aug. 1st
  - Erwin Finlay-Freundlich (German citizen) arrested
  - William Wallace Campbell had his instrumentation confiscated
- 1915: Einstein publishes the Theory of General Relativity…

A LIGHT RAY GRAZING THE SURFACE OF THE SUN

General relativity:

$$\hat{\alpha} = \frac{4GM_\odot}{c^2 R_\odot} = 1.75''$$

Newtonian gravity:

$$\hat{\alpha} = \frac{2GM_\odot}{c^2 R_\odot} = 0.875''$$

GRAVITATIONAL LENSING AS TEST OF GR

A. Einstein presented his Theory at the Prussian Academy of Science in Berlin in November 1915.

Using GR, Einstein had already explained successfully the perihelion precession of Mercury's orbit, first recognised by Le Verrier in 1859.

Measuring the correct value of the deflection of light by the gravitational field of the sun was not only important to proof that light could be bent by gravity, as expected from the equivalence principle, but also that the correct theory of gravity is General Relativity.

Despite the difficulties due to the ongoing WWI, Sir Arthur Eddington came in possession of Einstein's publications.

FIRST OBSERVATION OF GRAVITATIONAL LENSIN & CONFIRMATION OF GR

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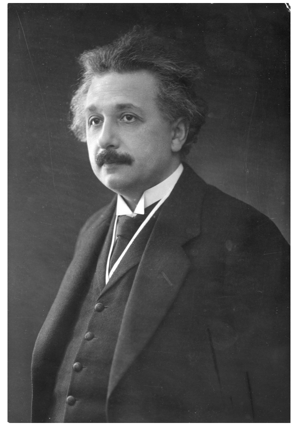


World War I - July, 1914 to Nov. 1918

Einstein's theory already successfully explained the precession of the perihelion of Mercury, first observed by Le Verrier in 1859.

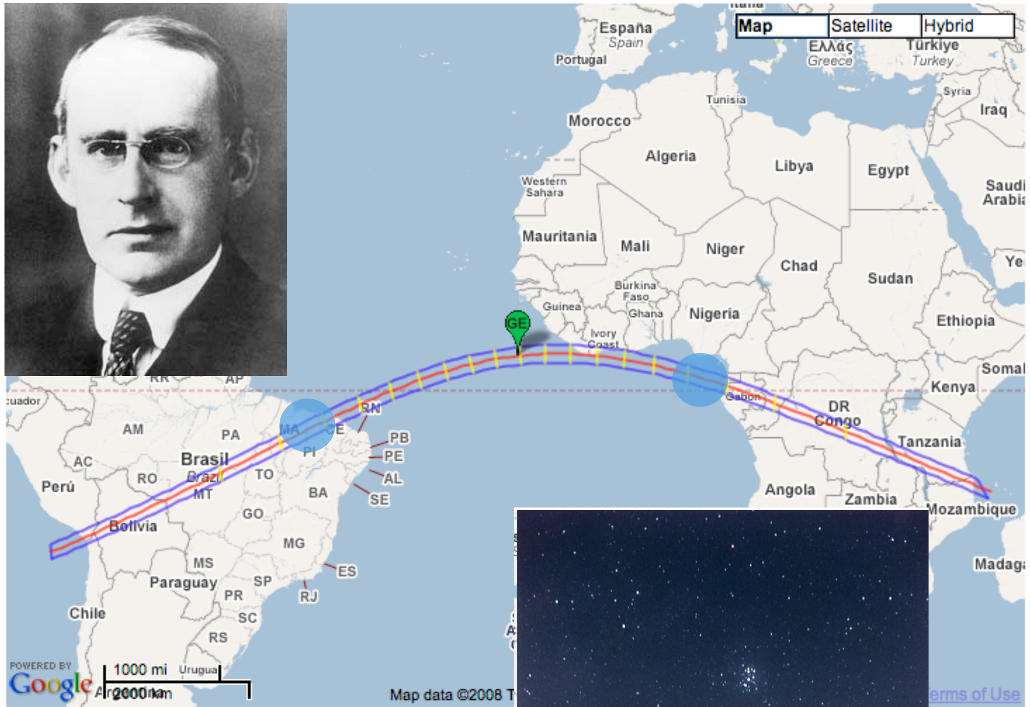
Arthor Eddington of Cambridge received Einstein's paper through neutral Netherlands because of wartime embargo.

EDDINGTION EXPEDITION

Next solar eclipse - May 29th 1919

Totality was observable from Africa to South America

An unusually long, 6 min duration and the location of the Hyades star cluster nearby making it specially advantageous.



Expeditions were sent to two locations:

Hyades

Principe Island off Equatorial Guinea

Eddington & Cottingham

scattered clouds

with 2 stars measure ~ 1.61 +/- 0.31”

Sobral in northern Brazil

Crommelin & Davidson

sun’s heat deformed mirror of main 16-inch telescope

backup 4-inch telescope was able to measure the positions of 7-stars 1.98 +/- 0.12”

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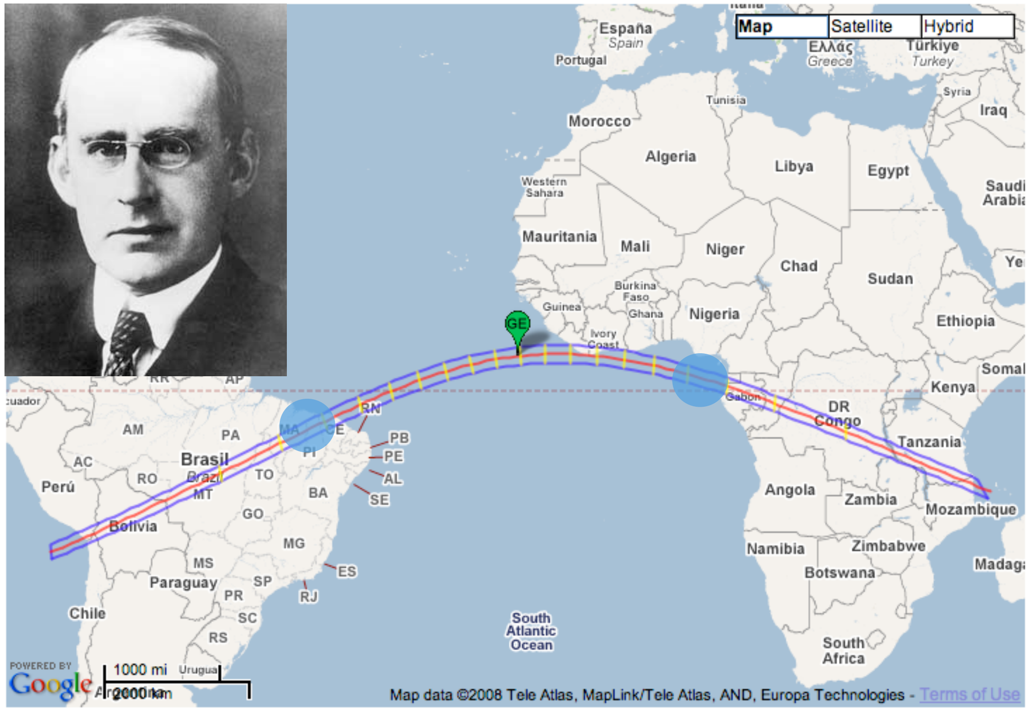

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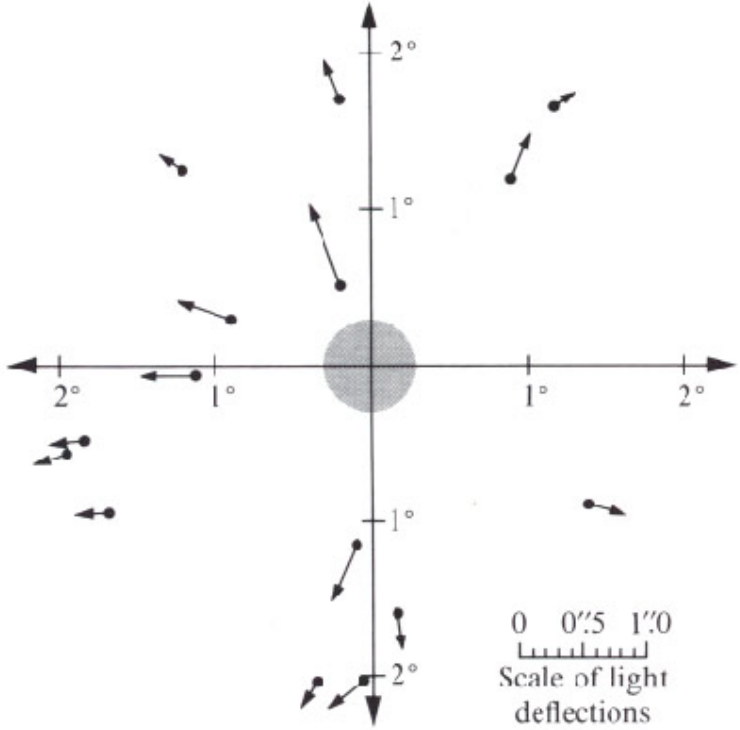

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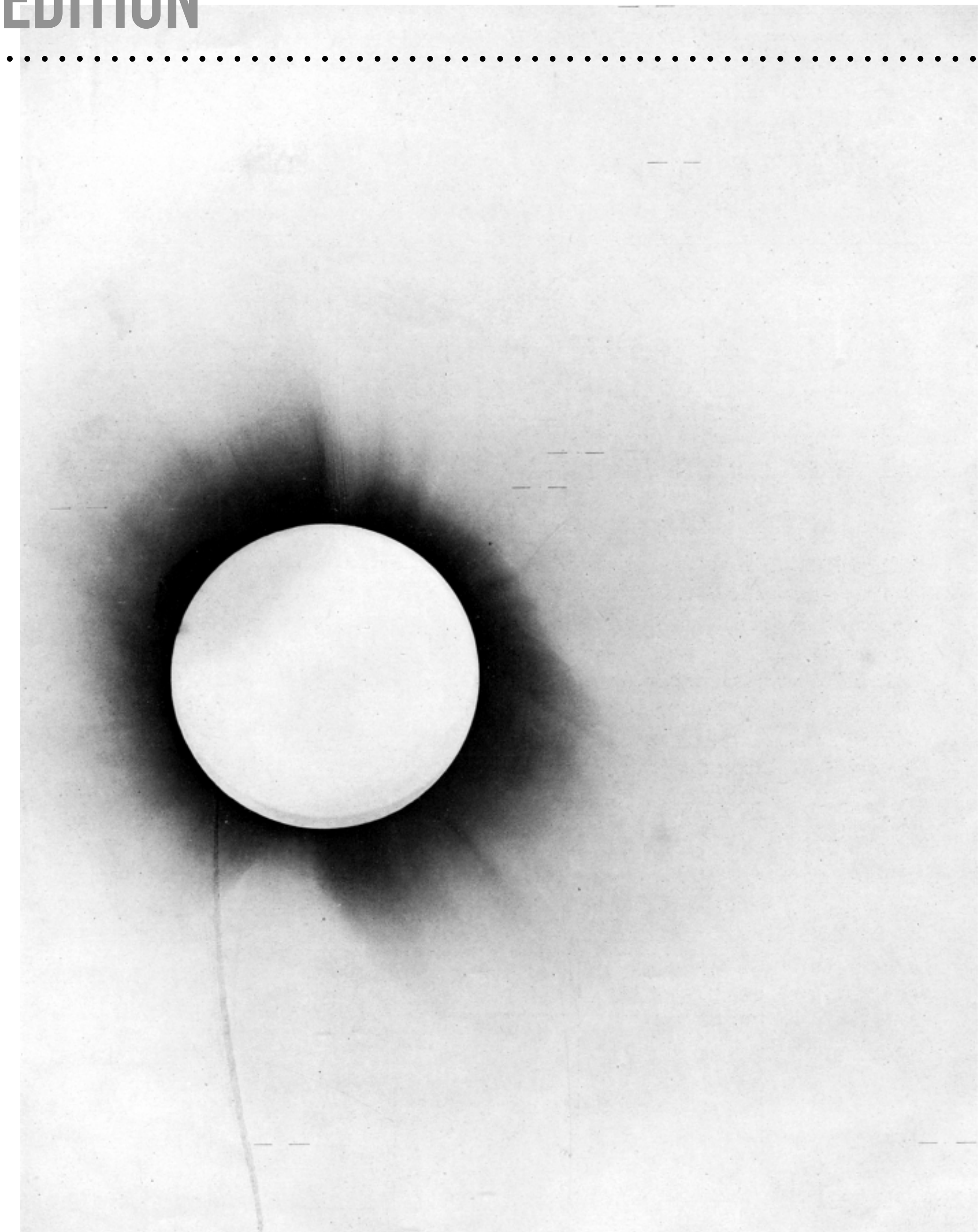
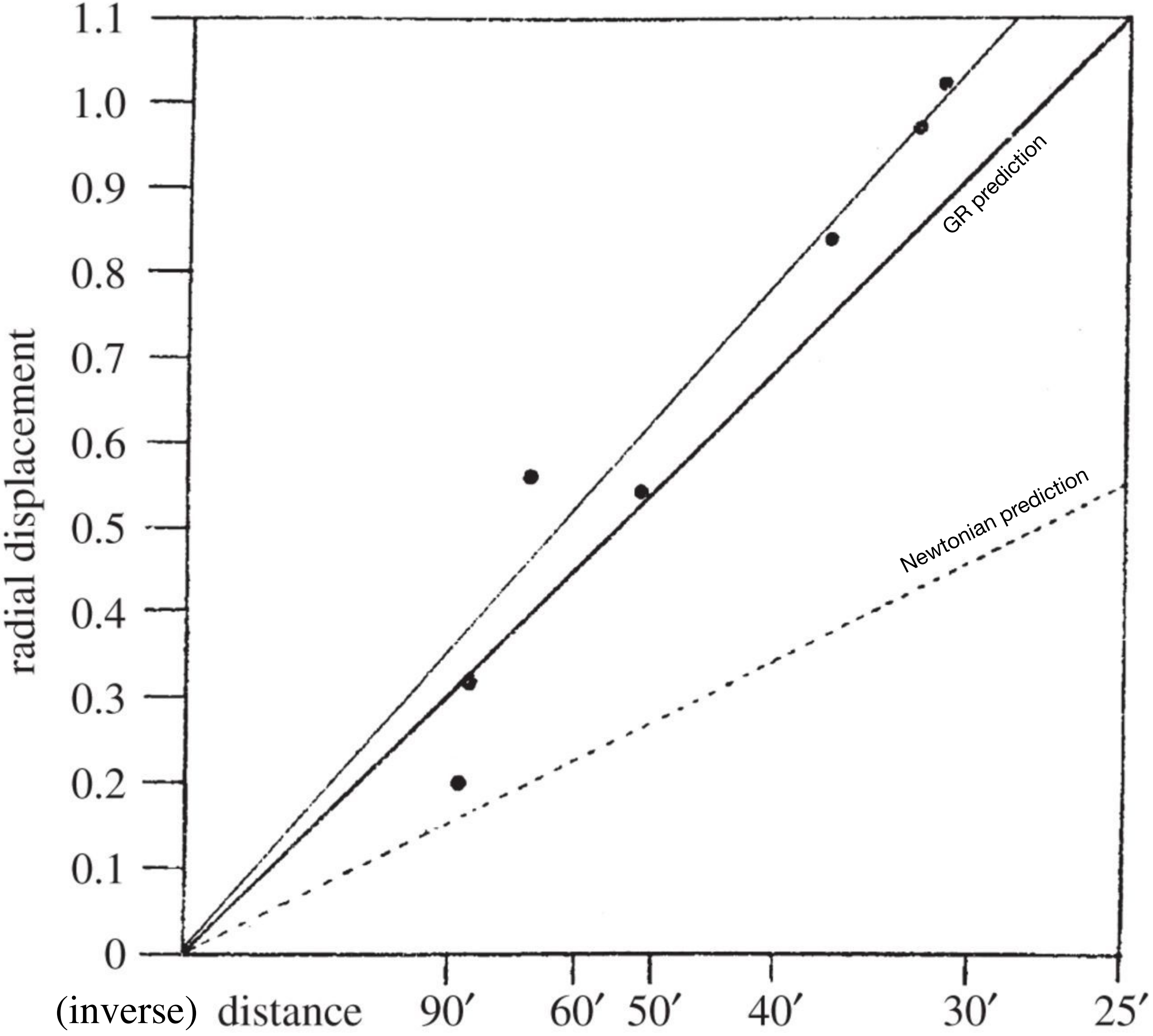


EDDINGTION EXPEDITION

Digitized and enhanced image
from the original expedition.



Data from Sobral, meeting of the Royal Society and the Royal Astronomical Society in London on 6 November 1919

GRAVITY BENDS LIGHT! (7/11/1919)

LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

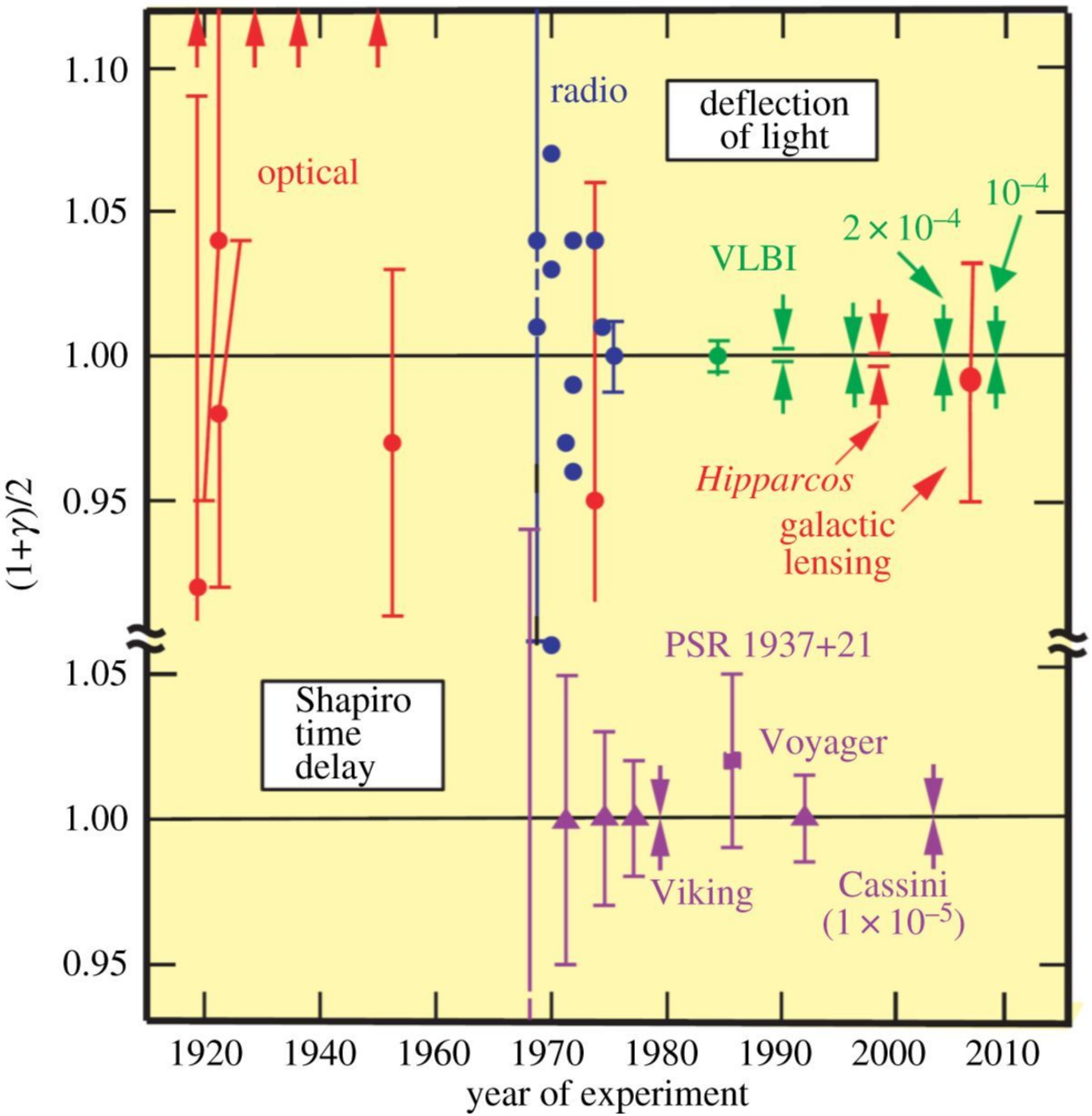
NEW THEORY OF THE UNIVERSE.

NEWTONIAN IDEAS OVERTHROWN.

Yesterday afternoon in the rooms of the Royal Society, at a joint session of the Royal and Astronomical Societies, the results ob-

"STARLIGHT BENT BY THE SUN'S ATTRACTION": THE EINSTEIN THEORY.

the parameter $(1+\gamma)/2$



Will, et al, 2014

STRONG FIELD GRAVITATIONAL LENSING

General static isotropic metric :

$$d\tau^2 = B(r)dt^2 - A(r)dr^2 - r^2\left(d\theta^2 + \sin^2(\theta)d\phi^2\right)$$

Schwarzschild's black hole solution

$$\begin{cases} B(r) = \left(1 - \dfrac{R_{Sch}}{r}\right) \\ A(r) = \left(1 - \dfrac{R_{Sch}}{r}\right)^{-1} \end{cases}$$

Constants of motion

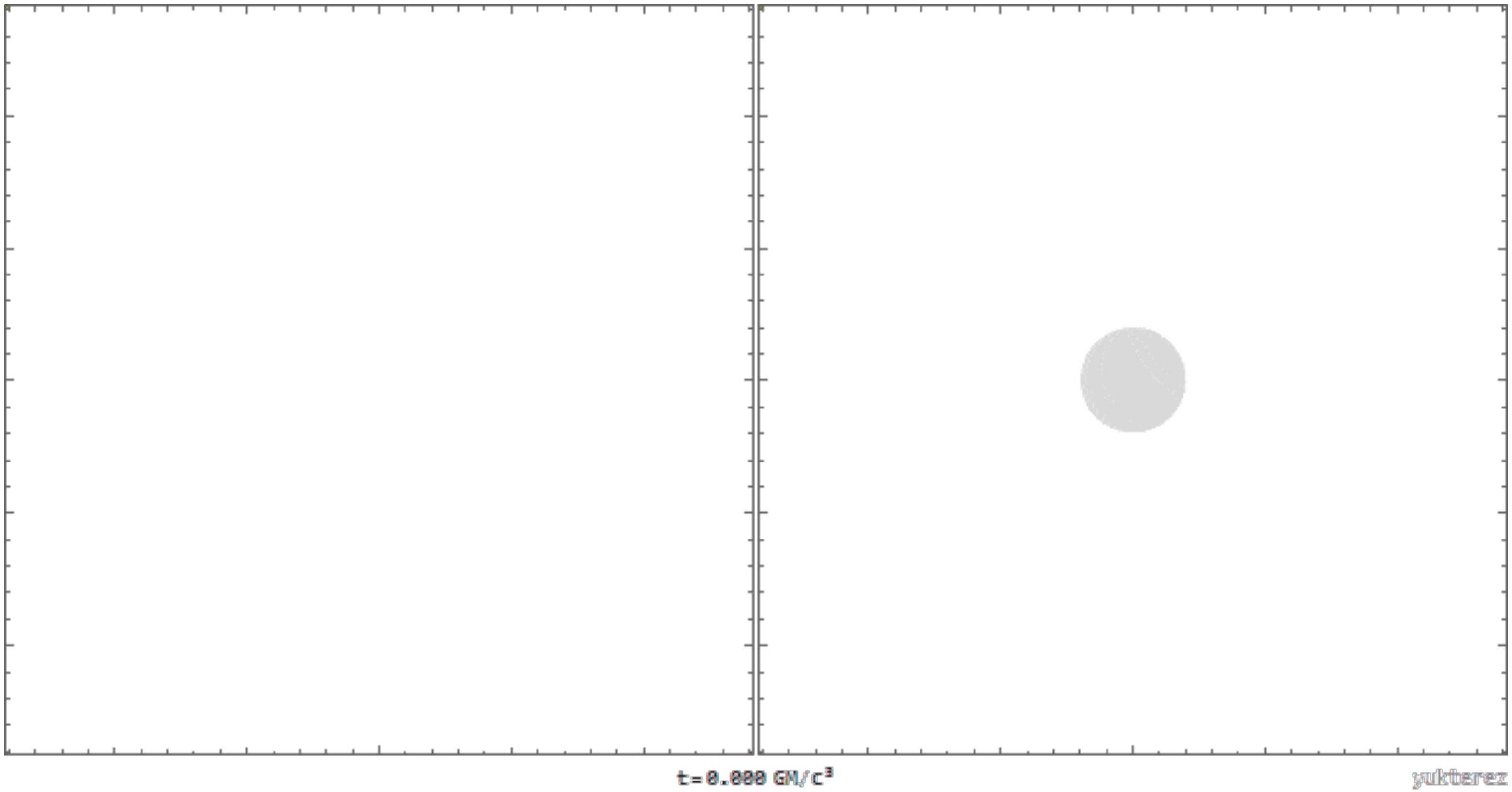
$$\begin{cases} J = r^2 \dfrac{d\phi}{d\lambda} \\ -E = A(r)\left(\dfrac{dr}{d\lambda}\right)^2 + \dfrac{J^2}{r^2} - \dfrac{1}{B(r)} \end{cases}$$

$E > 0$ massive particles

$E = 0$ photons

$$\frac{1-E}{2} \simeq \text{ Newtonian energy}$$

STRONG FIELD GRAVITATIONAL LENSING



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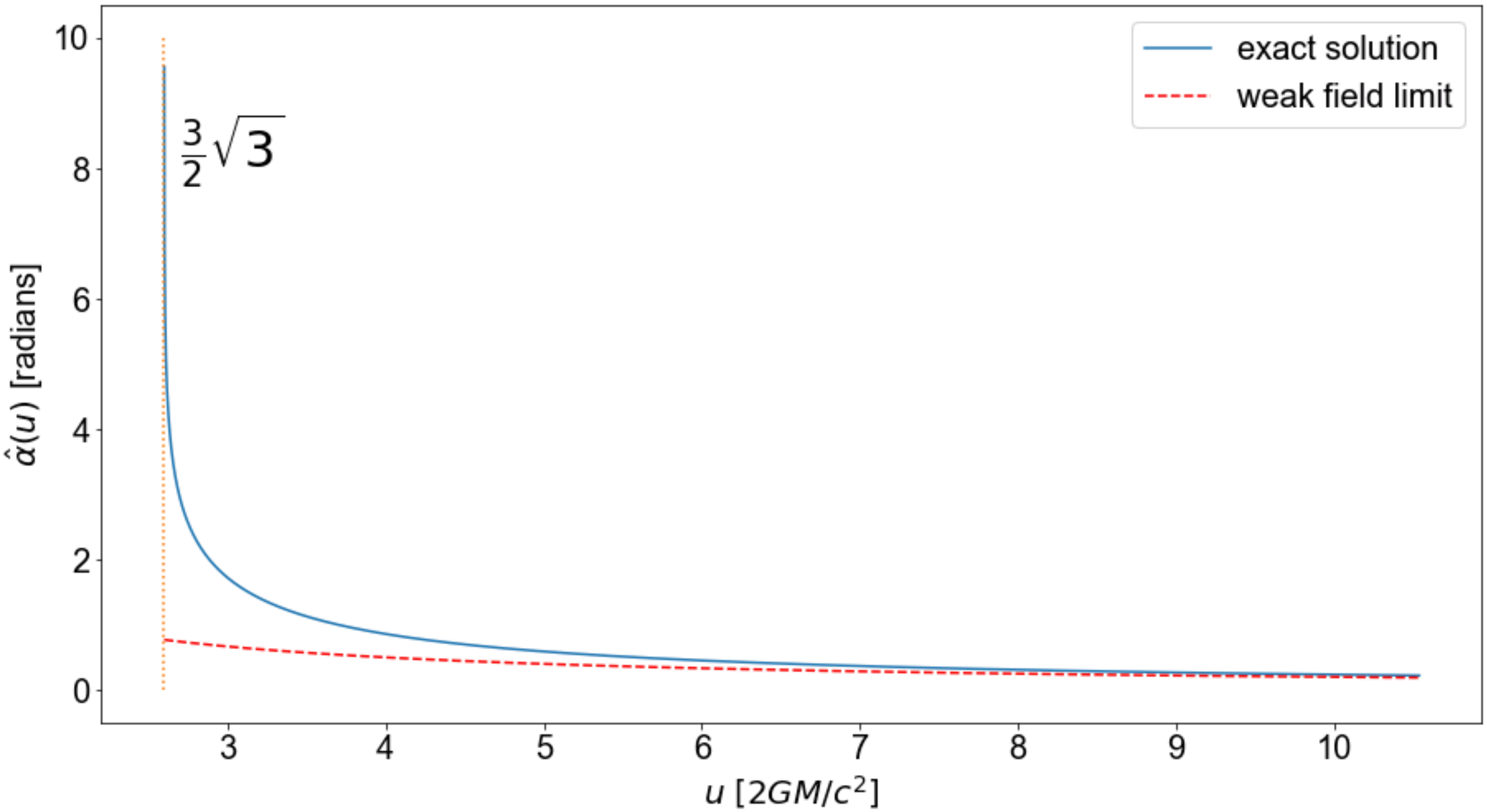
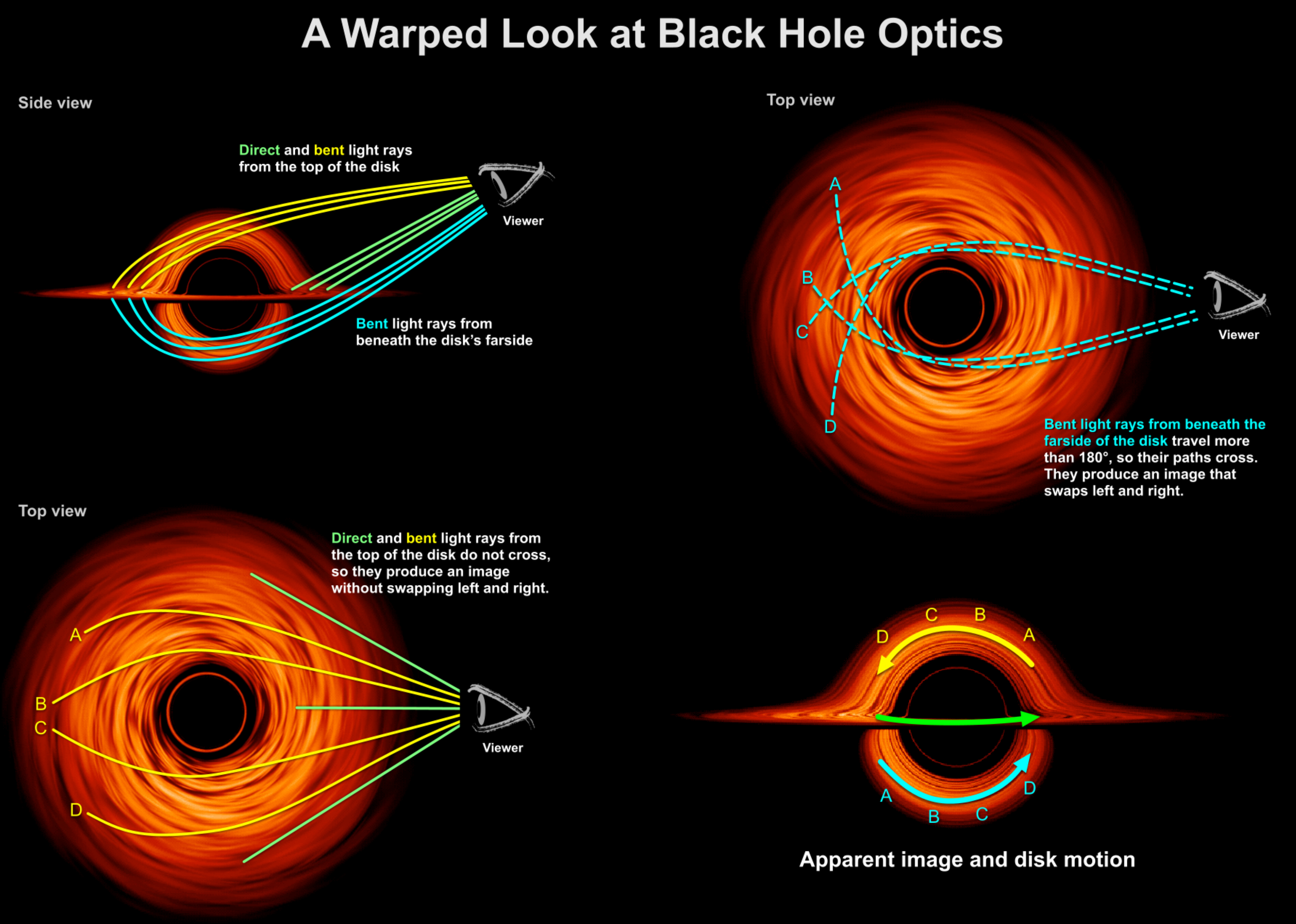


$$u = \frac{b}{R_{\mathrm{Sch}}}$$ impact parameter

STRONG FIELD GRAVITATIONAL LENSING



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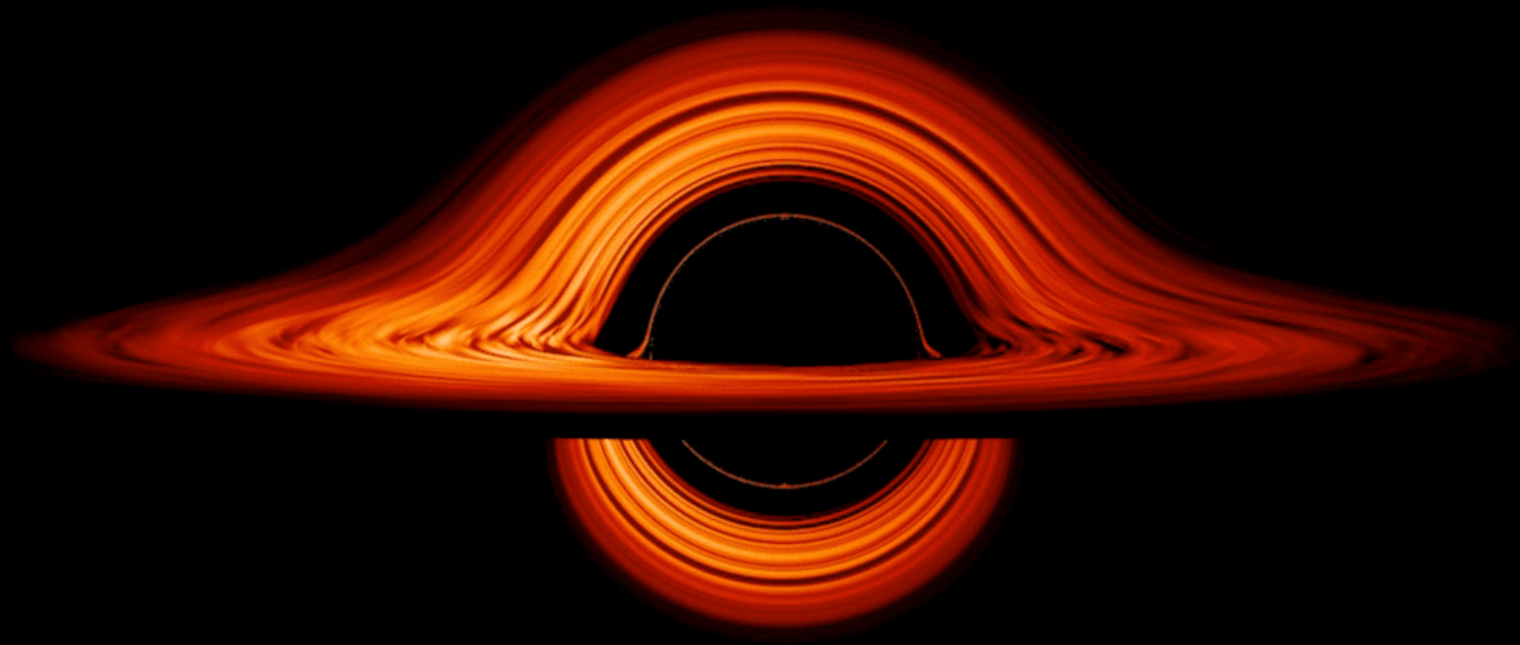

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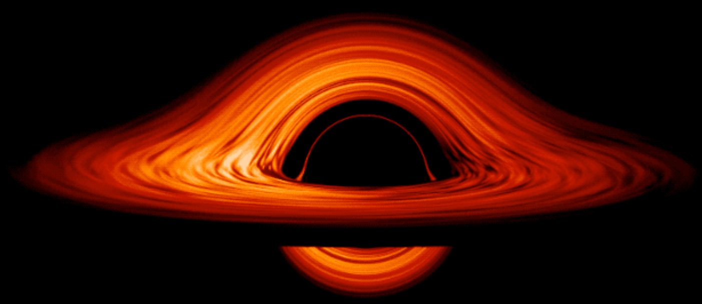
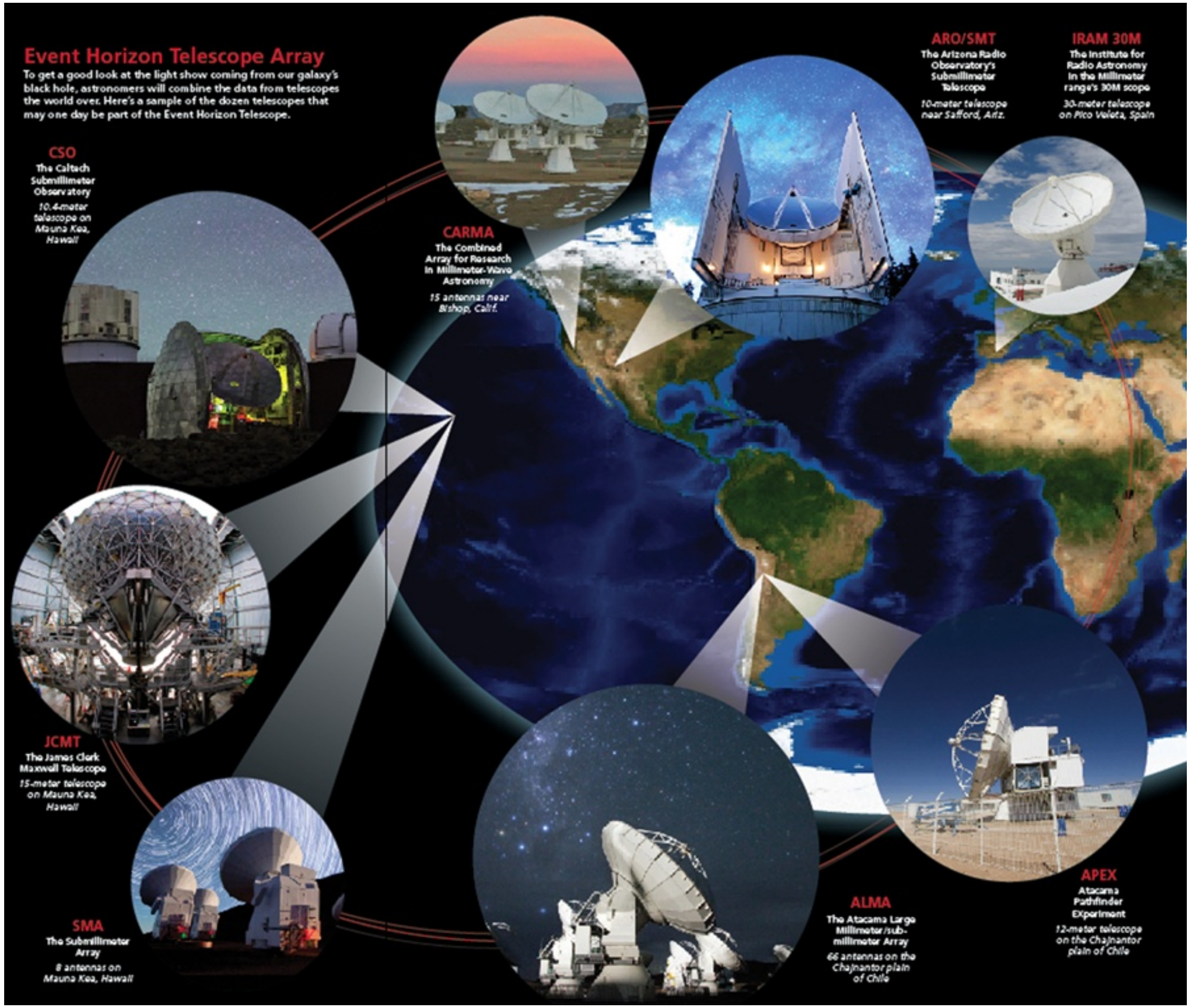


Event Horizon Telescope (EHT)

Collaboration between many observatories to make a **global radio interferometer array.**



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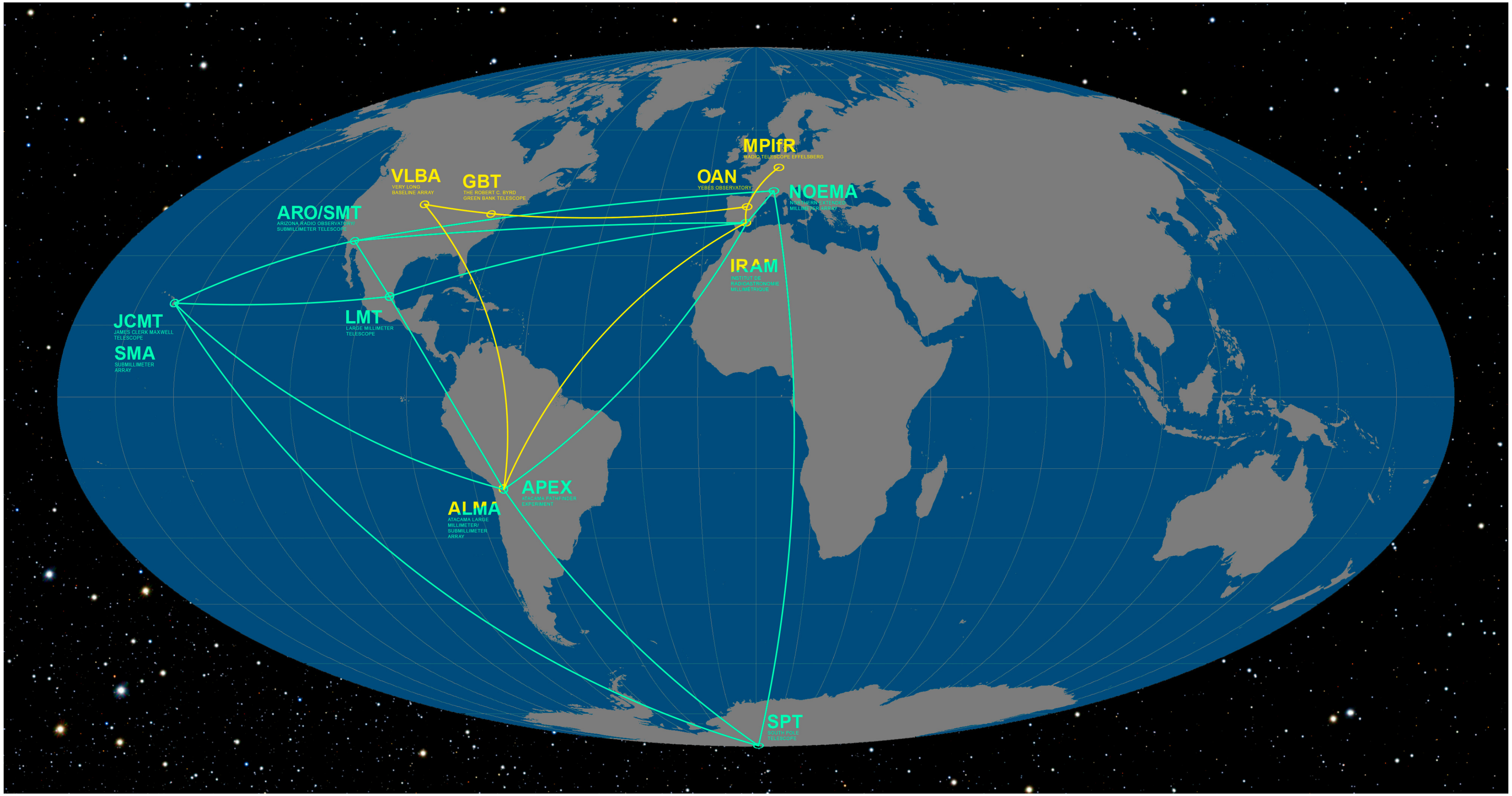

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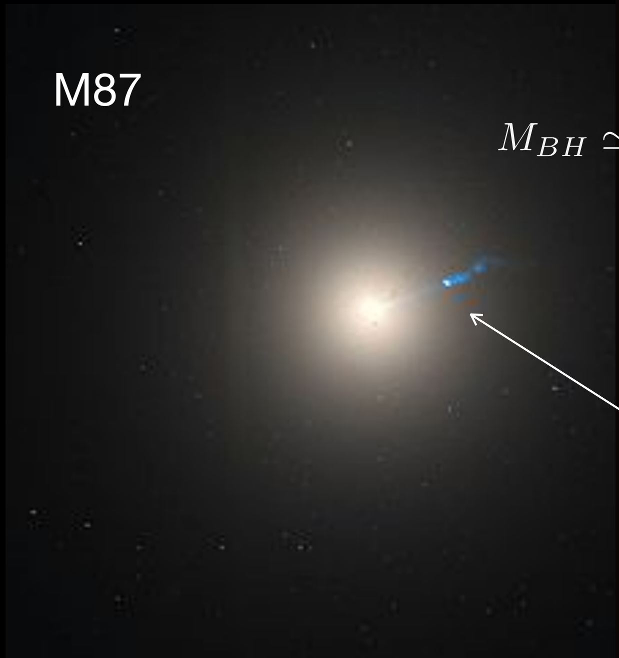
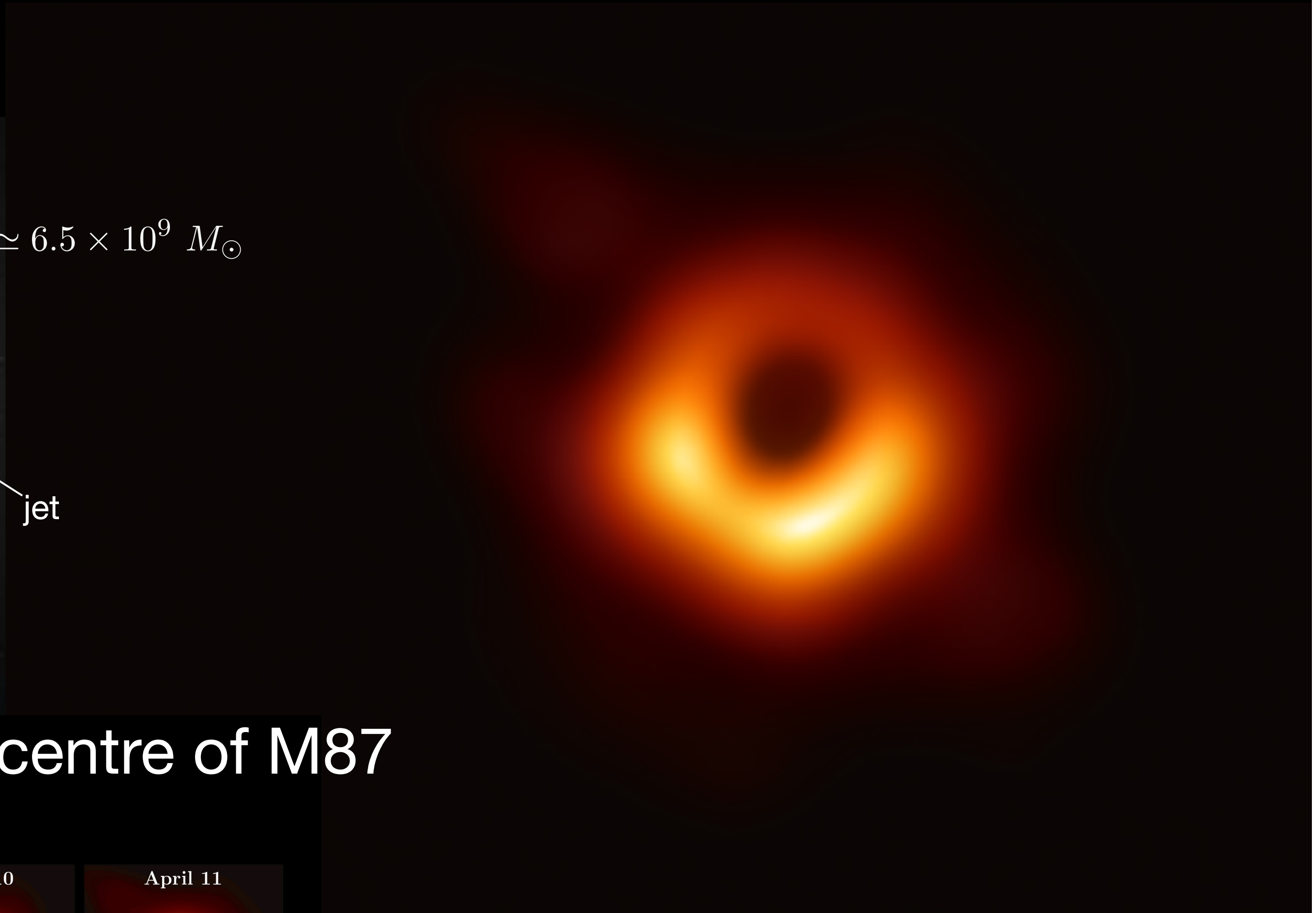
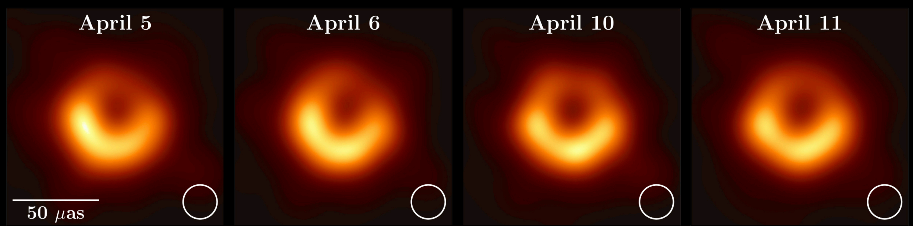


$M_{BH} \simeq 6.5 \times 10^9 \ M_\odot$



Black hole in the centre of M87



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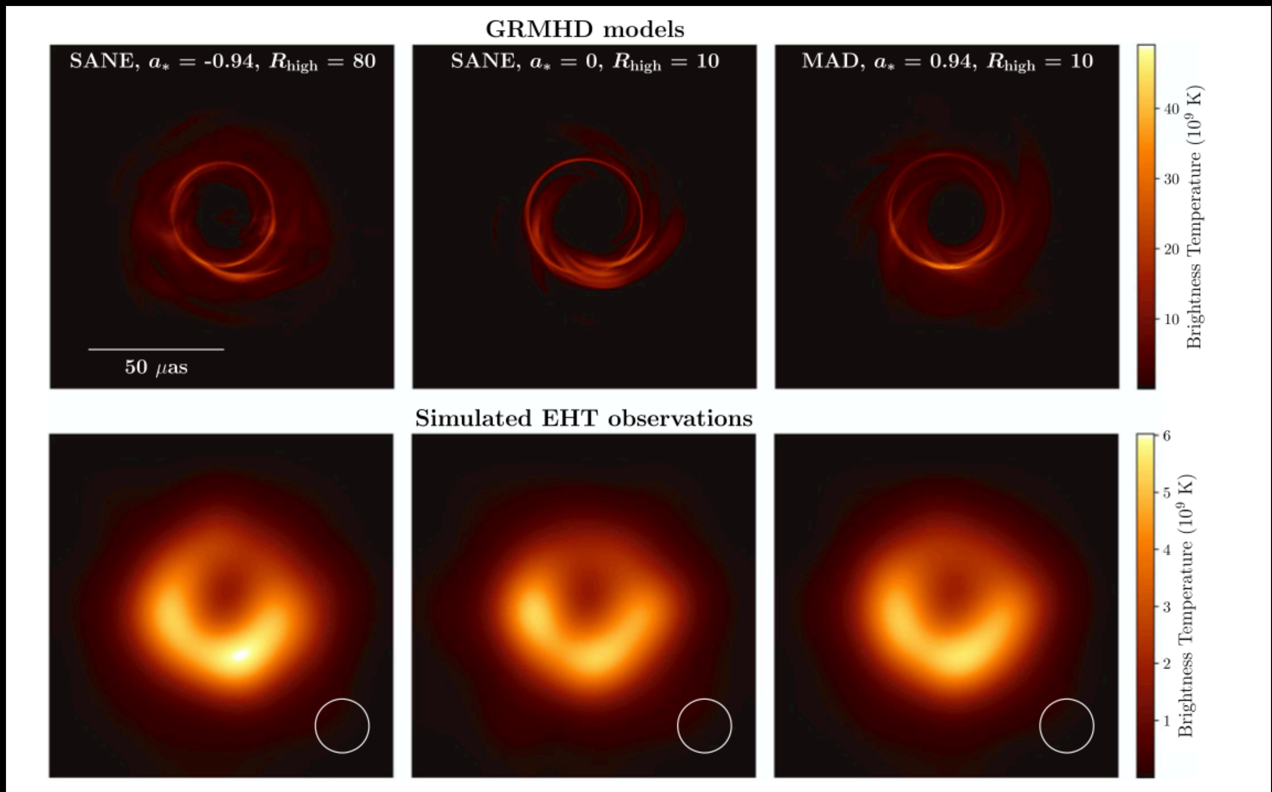

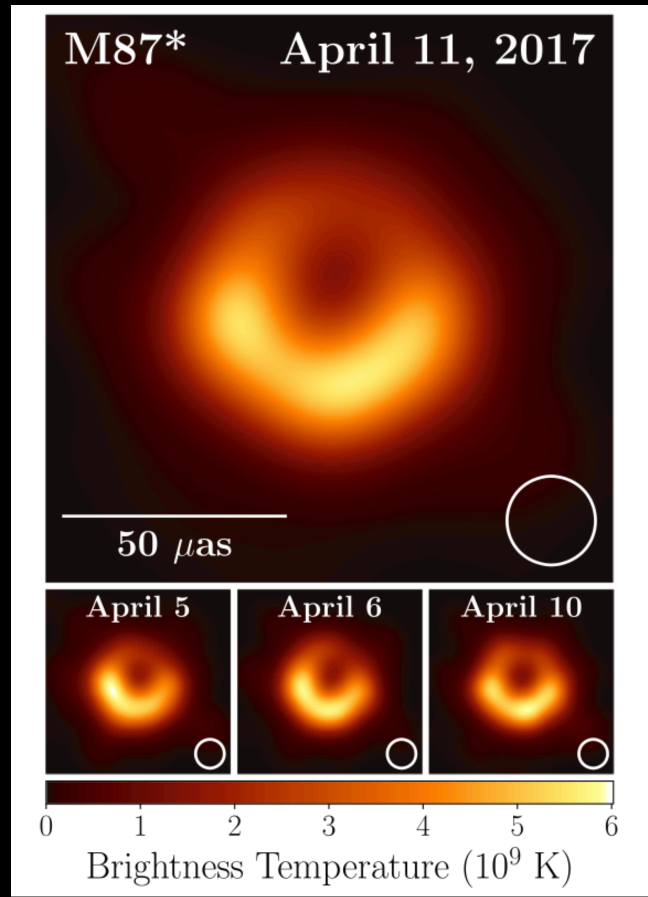

FERMAT'S PRINCIPLE IN GENERAL RELATIVITY

- Assumptions:
  - the deflection occurs in small region of the universe and over time-scales where the expansion of the universe is not relevant
  - the weak-field limit can be safely applied: $|\Phi|/c^2 \ll 1$
  - perturbed region can be described in terms of an effective refractive index
  - Fermat principle