GRAVITATIONAL LENSING

LENS MODELLING I

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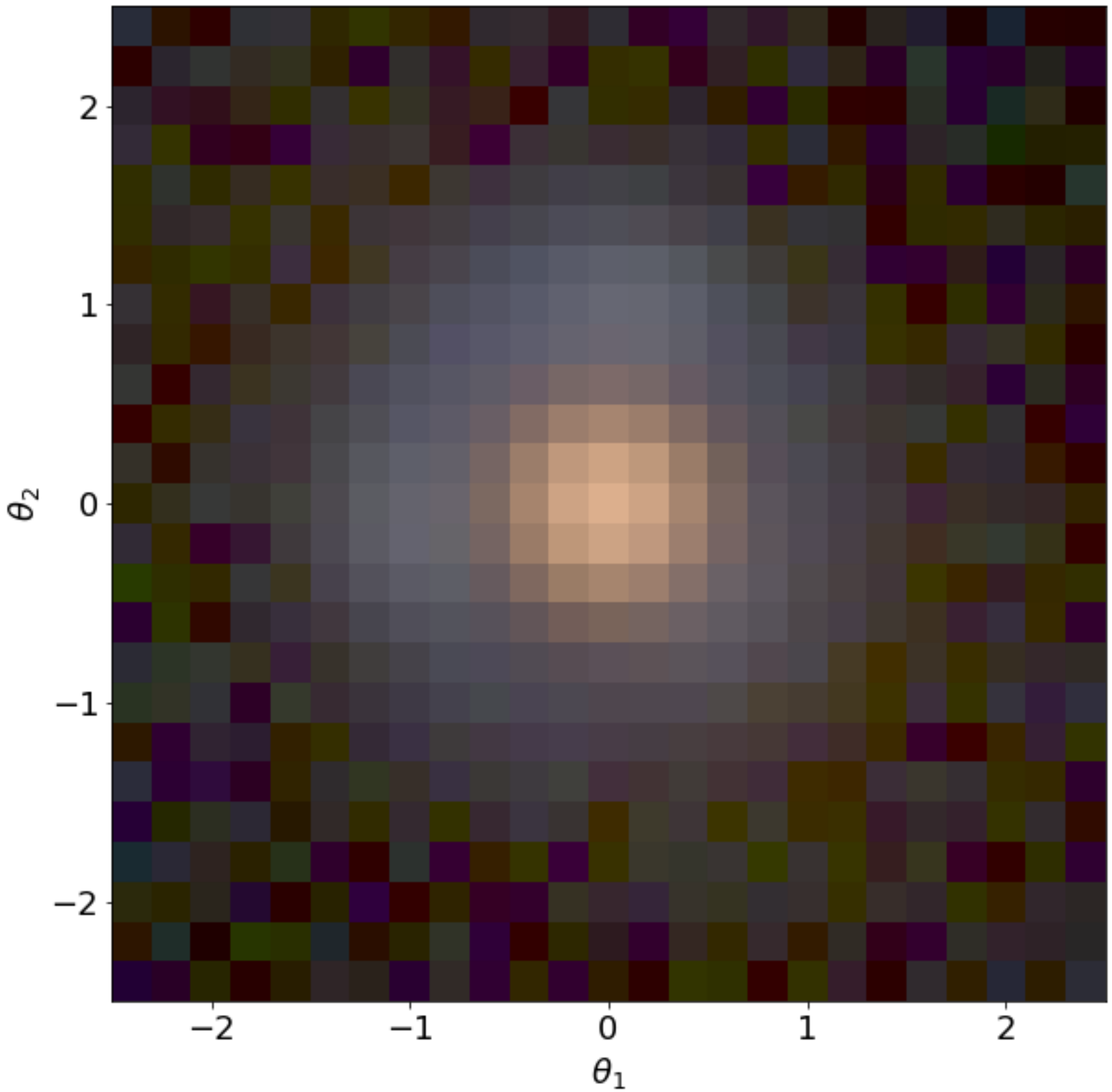
MODELING A GALAXY USING STRONG LENSING

- Once a lens has been found, we may want to use the strong lensing observables to investigate what is its content
- Possible observables are:
  - Image positions
  - Image fluxes/distortions
  - Image time-delays
- Note that these observables provide different constraints on the potential and on its derivatives
- Strategies: **parametric** or free-form

PARAMETRIC RECONSTRUCTION OF A STRONG LENSING GALAXY



Example: suppose we have discovered a lens candidate like the one in the Figure on the left.

We perform HST follow-up observations and we confirm the lensing nature of the object

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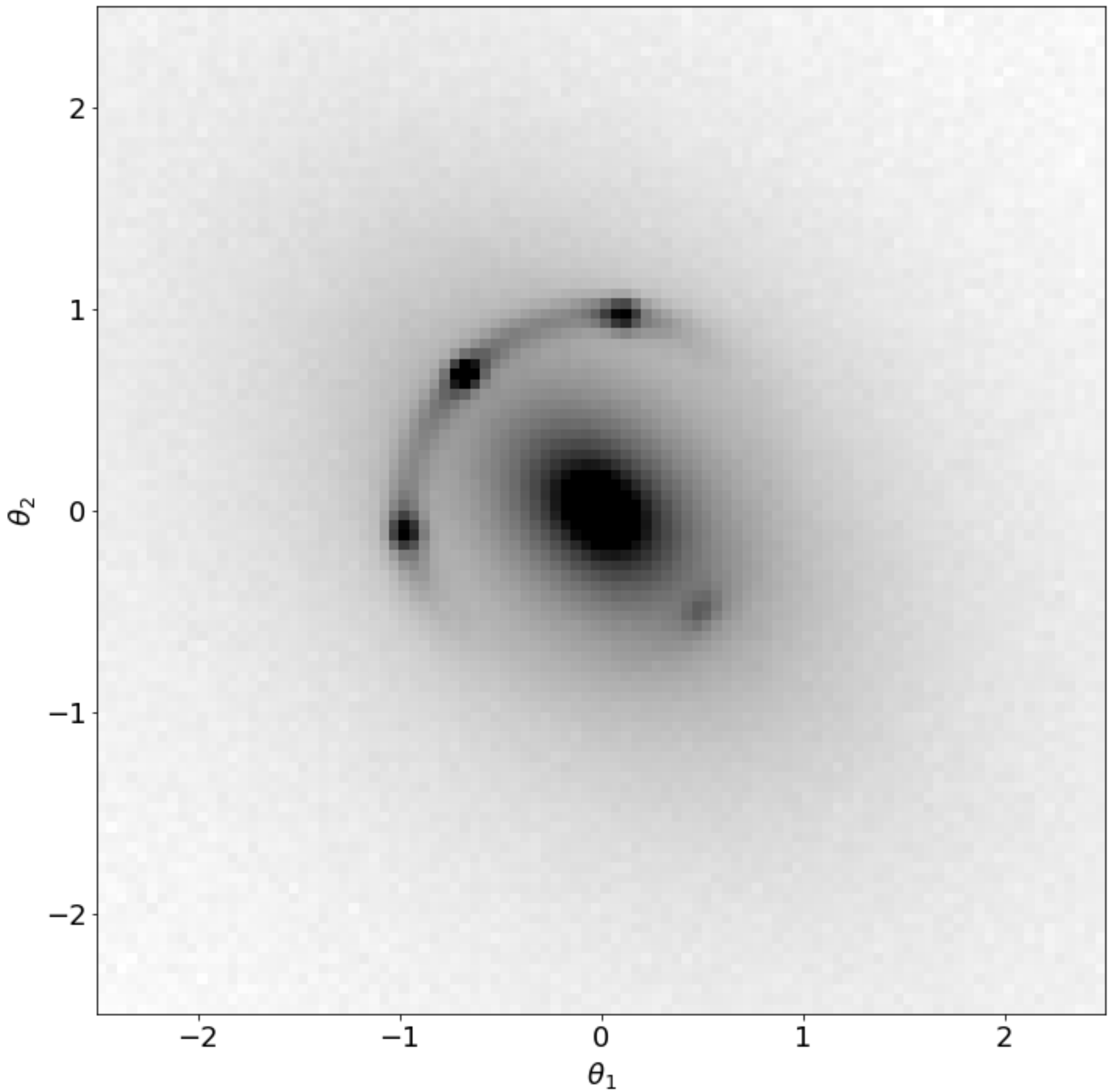

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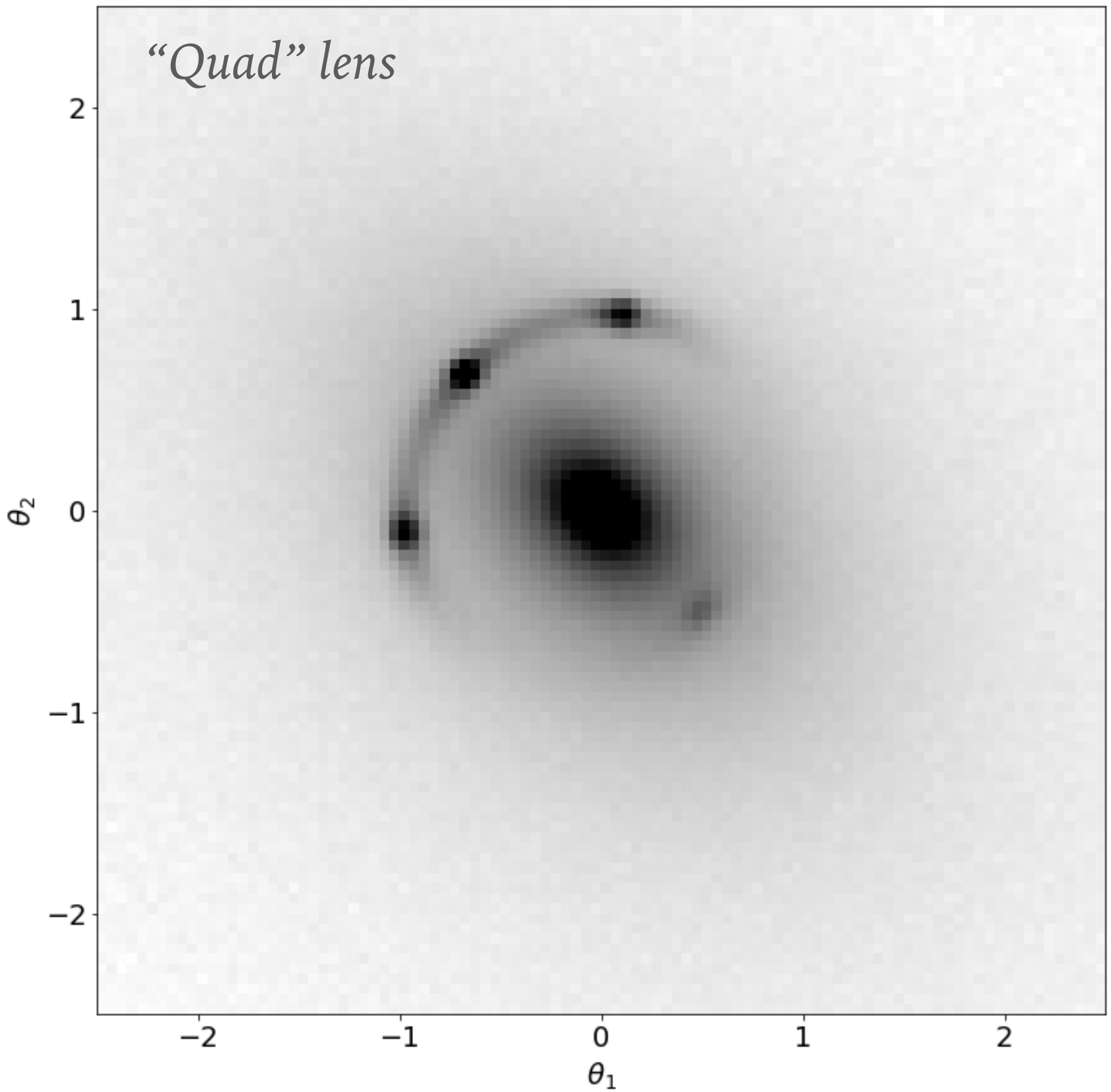

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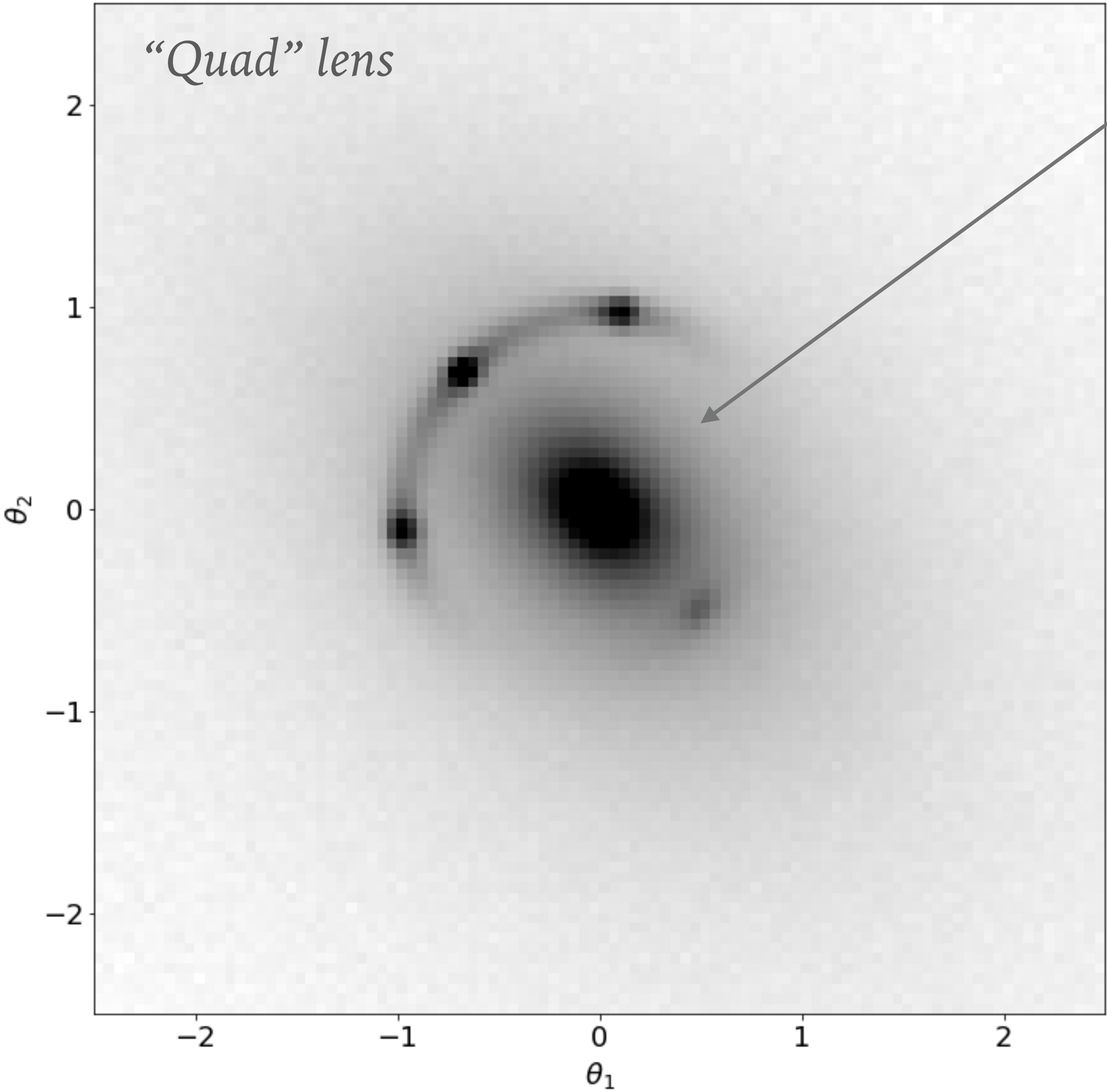

The lens is an early type galaxy

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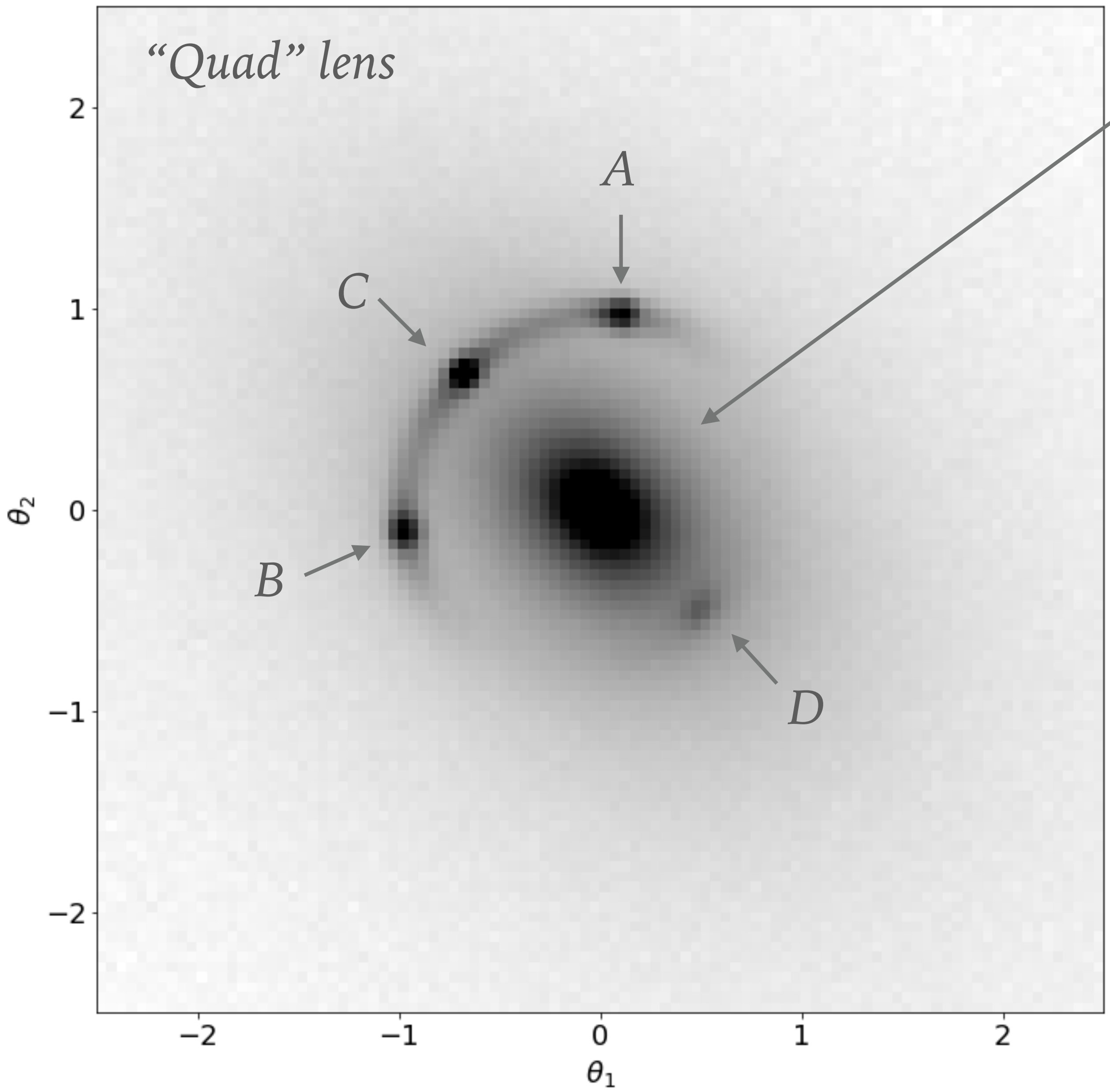

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We see four images of a QSO and a lensed galaxy host

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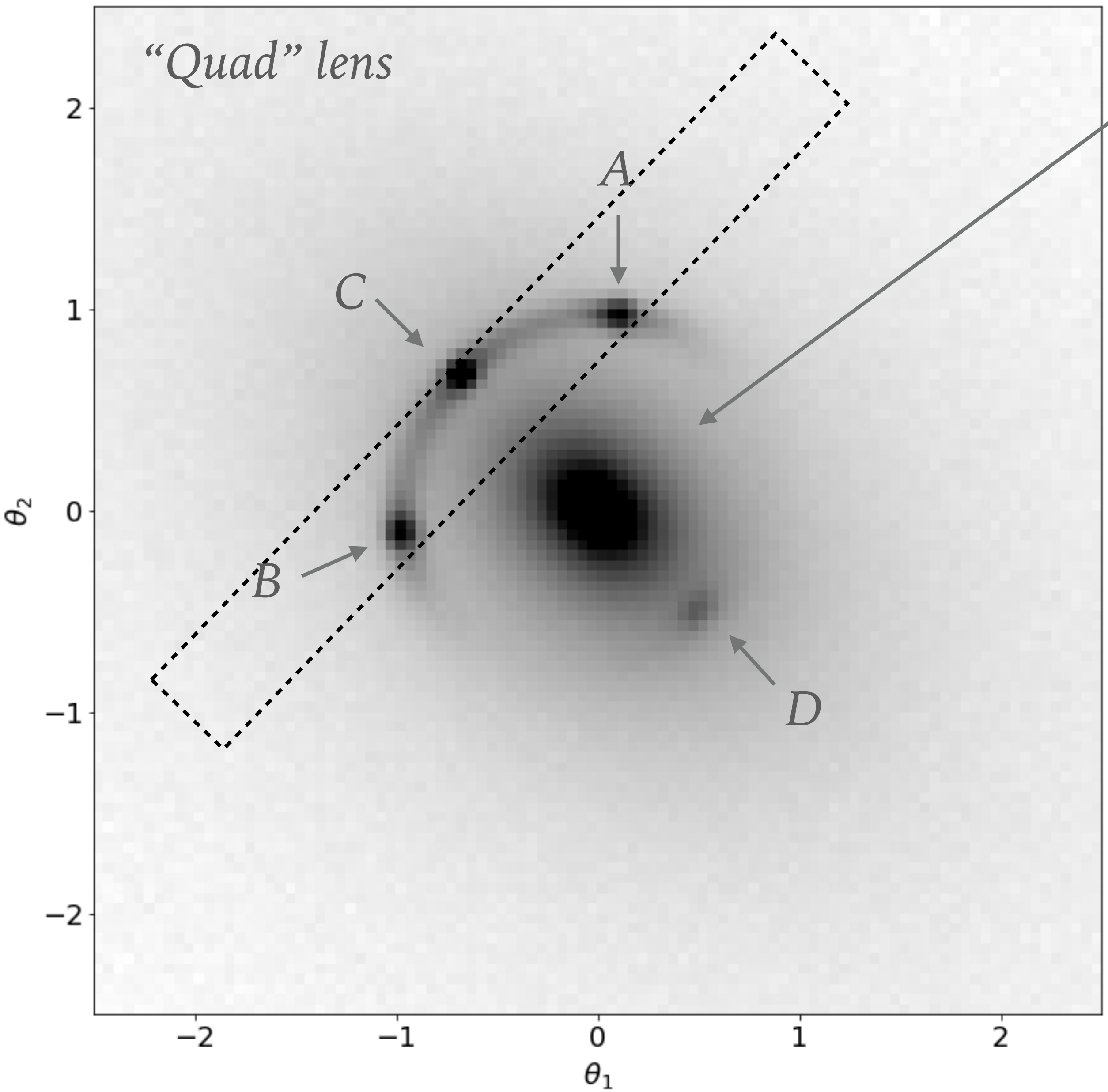

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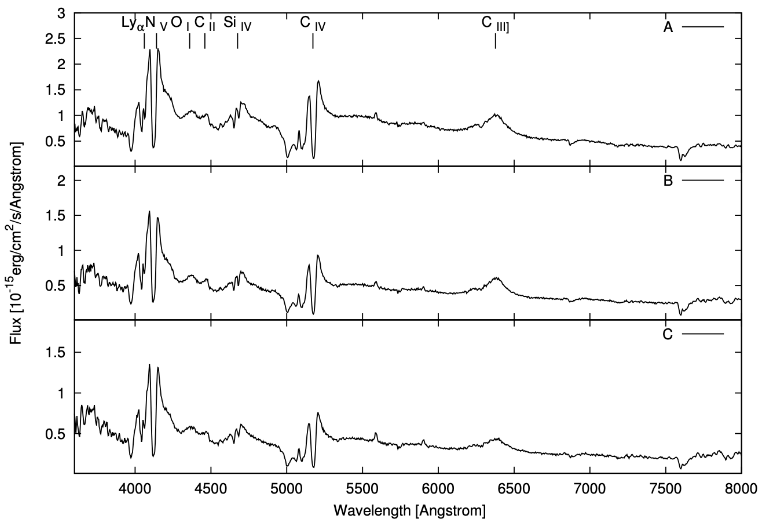
Spectroscopic confirmation and redshifts: $z_L = 0.3$, $z_S = 2$



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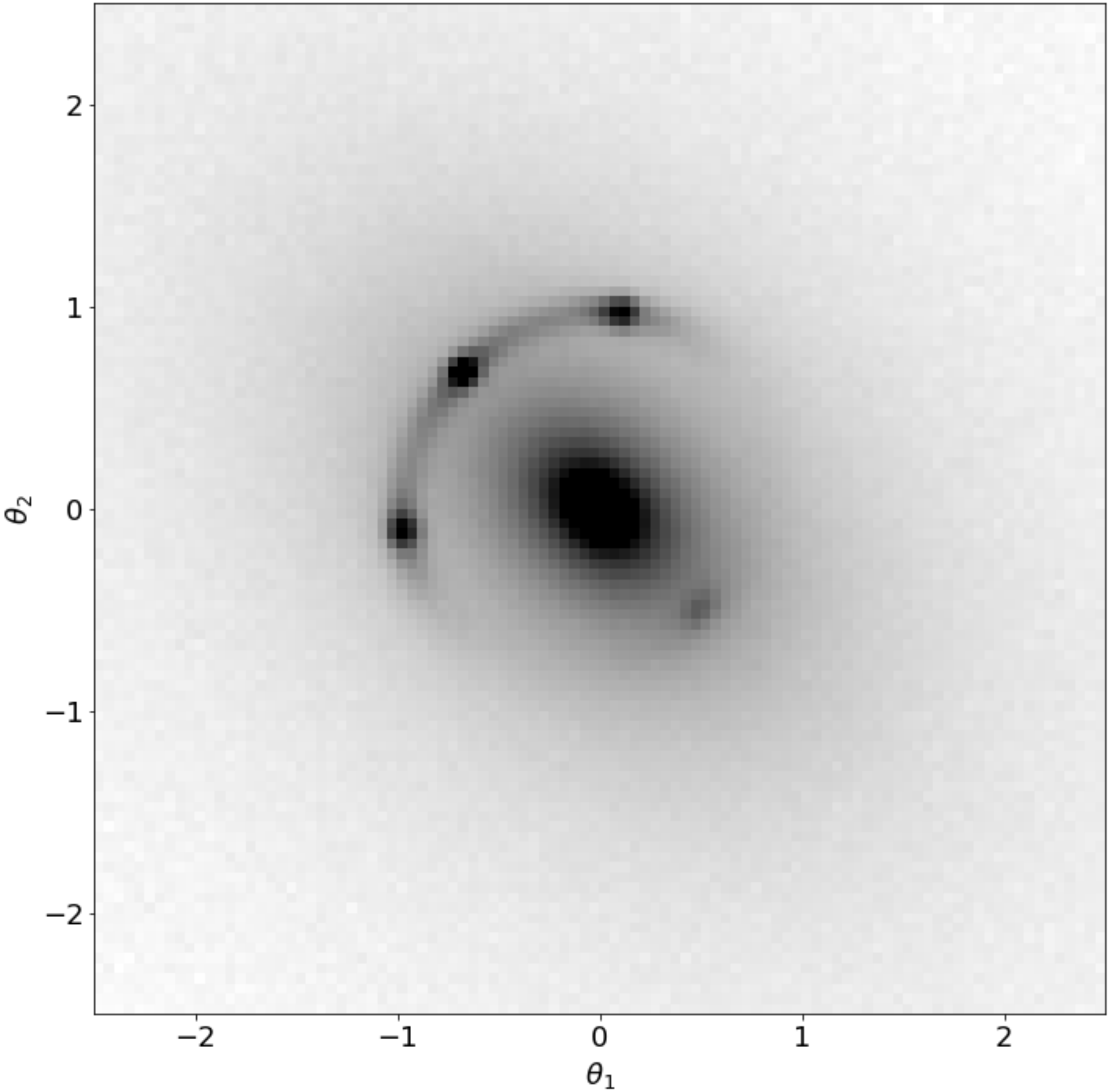
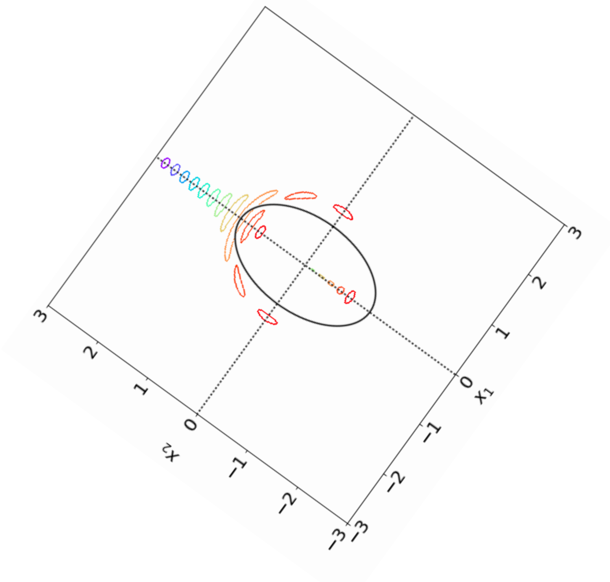


Looking at the lens, we realise that the image configuration is very similar to what we could reproduce with an elliptical lens model.



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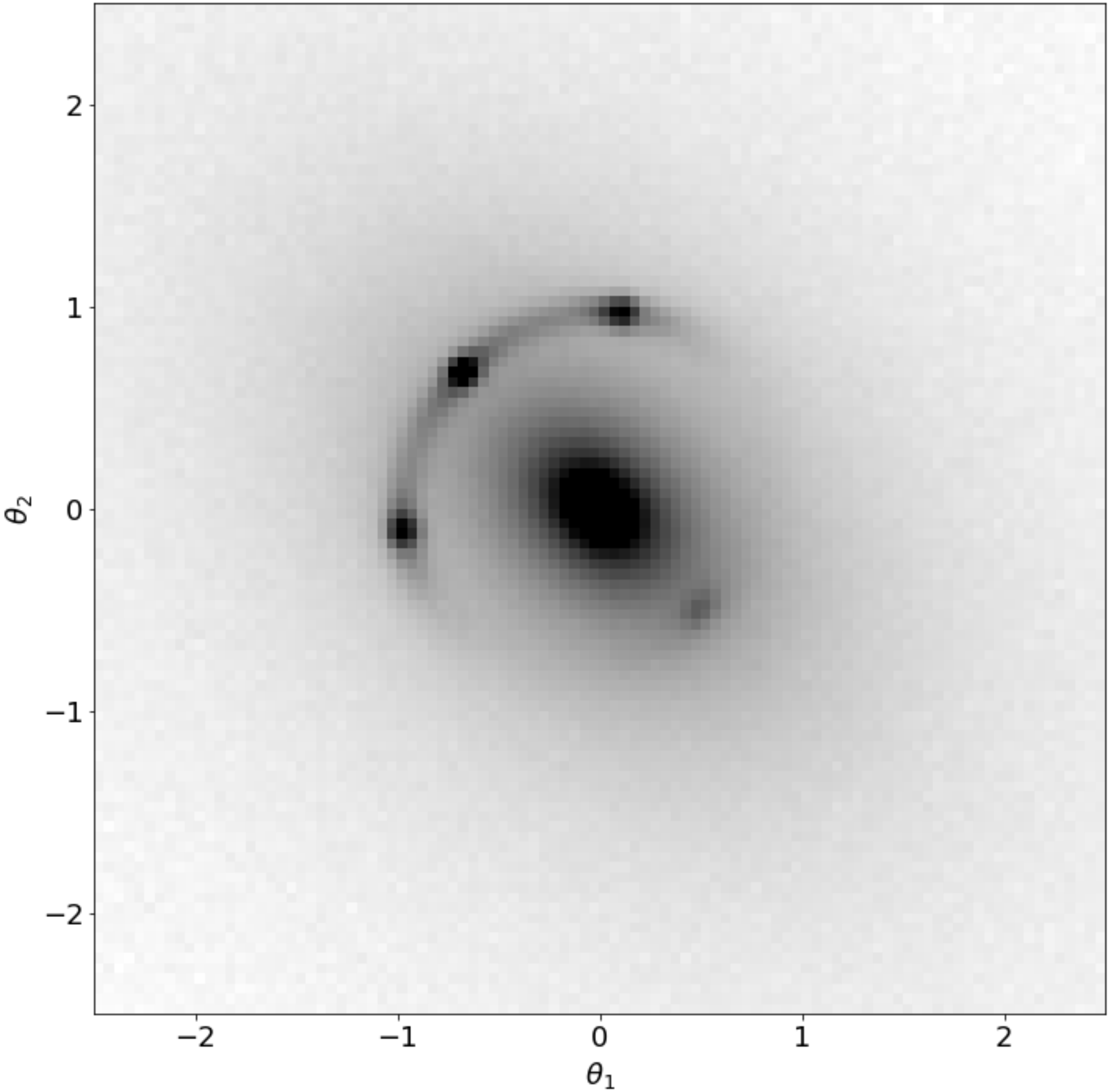

We decide to model the lens with a SIE

$$\Sigma(\vec{\xi}) = \frac{\sigma_v^2}{2G} \frac{\sqrt{f}}{\sqrt{\xi_1^2 + f^2 \xi_2^2}}$$

Two model parameters: σ_v, f + orientation φ

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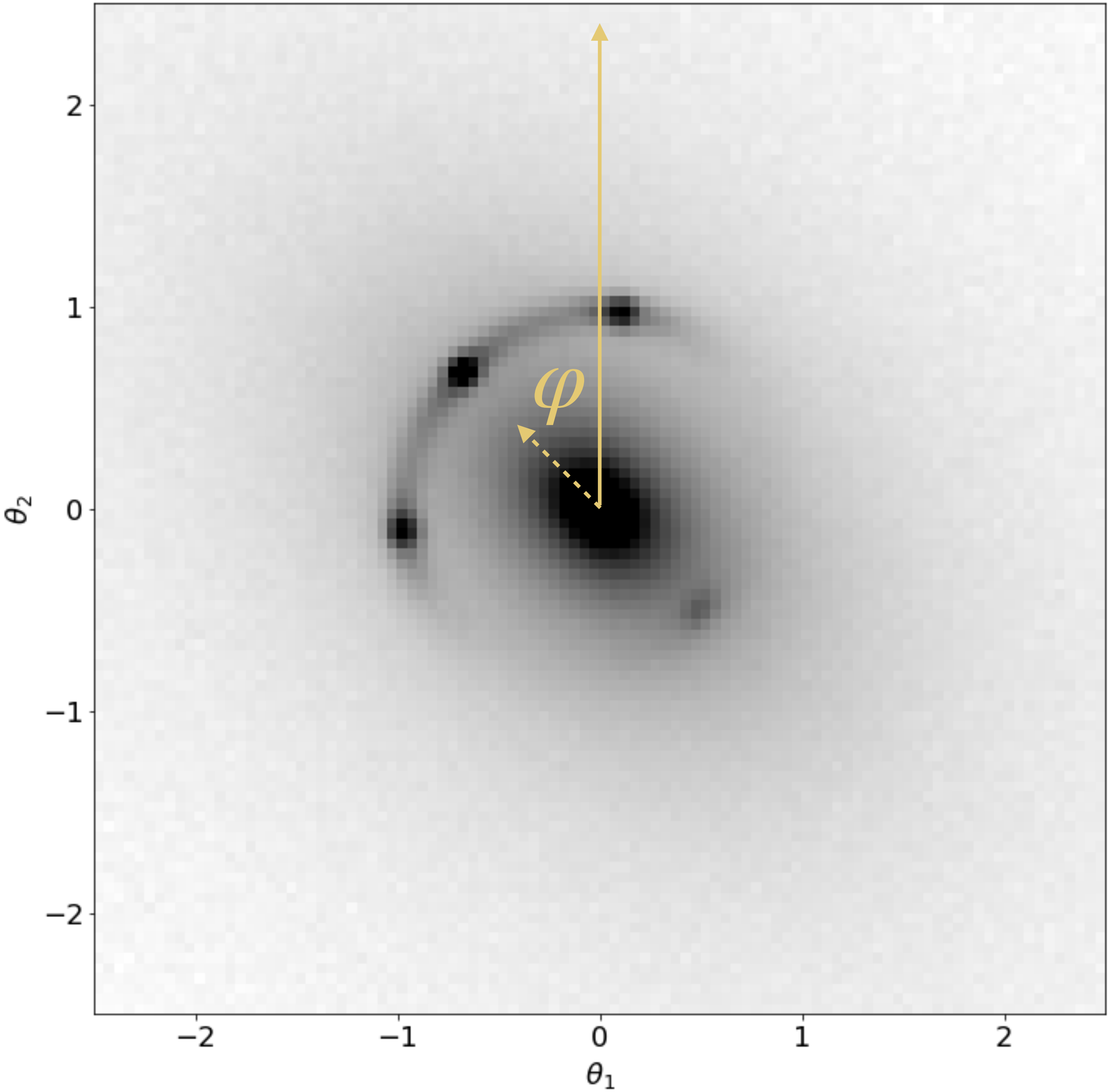

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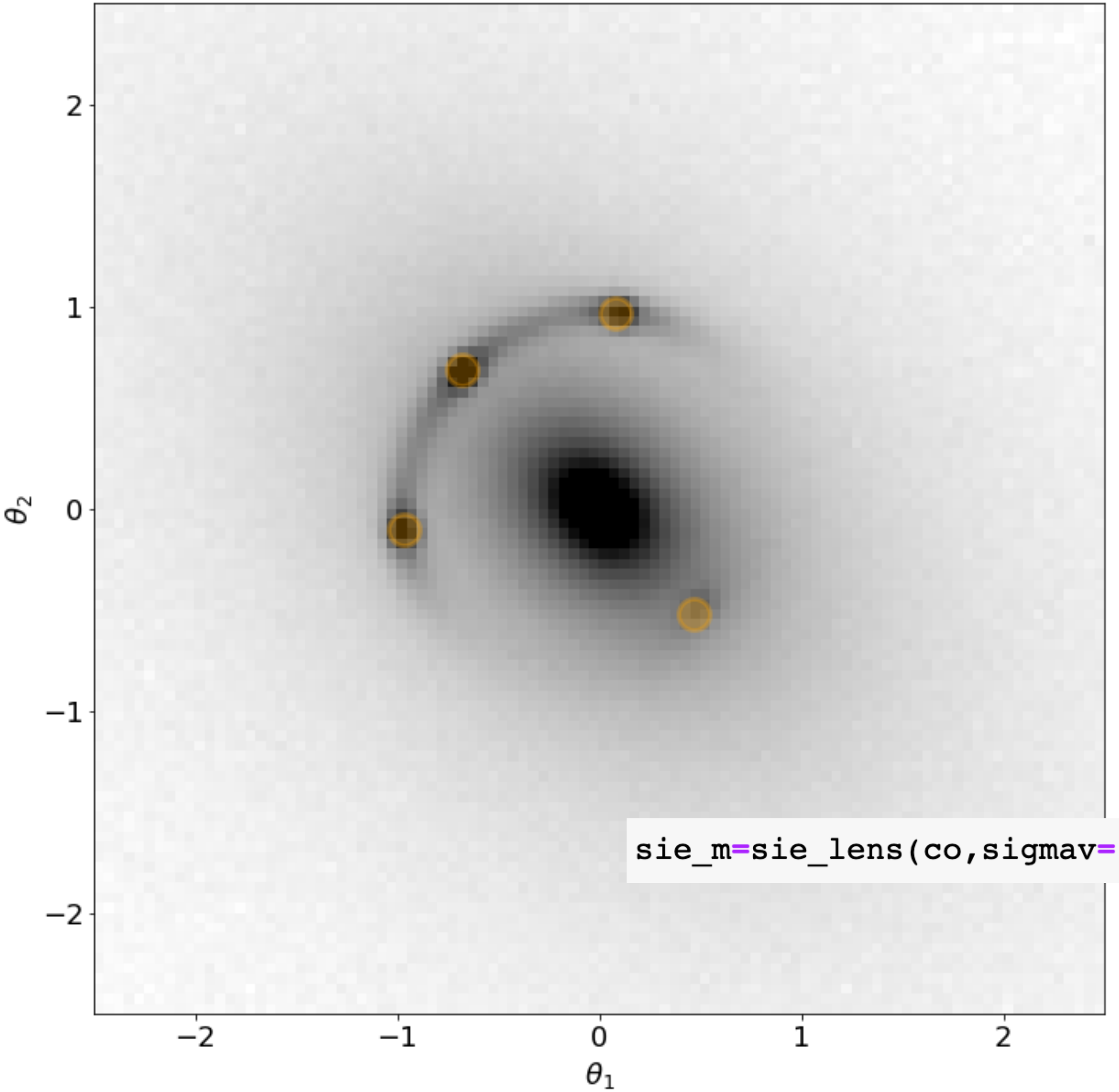


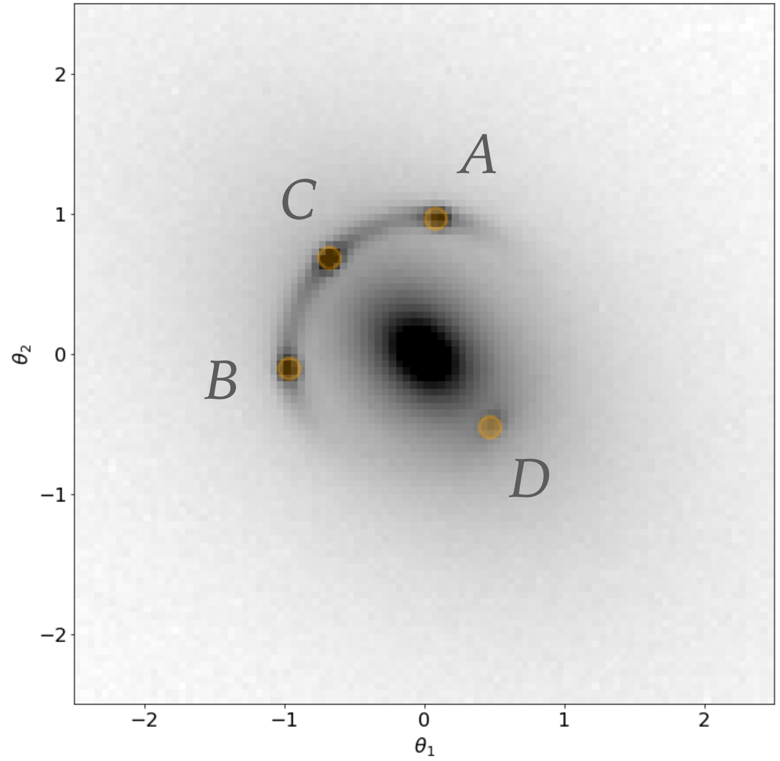
We start by using the image positions:

```
[ 0.07890278  0.96488912]
[-0.67984394  0.68741001]
[-0.96560759 -0.10242888]
[ 0.46841478 -0.51791398]
```

And we make a guess about the lens parameters:

```
sie_m=sie_lens(co,sigmav=180.0,zl=0.3,zs=2.0,f=0.3,pa=40.0)
```

PARAMETRIC RECONSTRUCTION OF A STRONG LENSING GALAXY



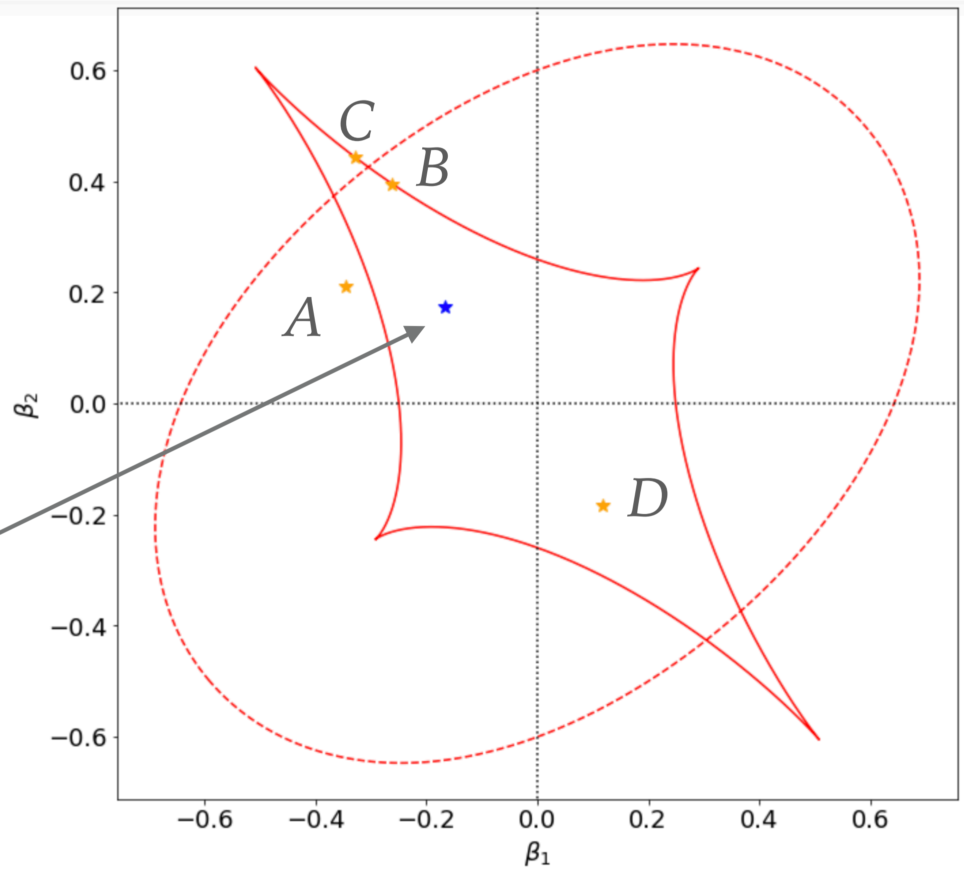
Use the model to de-lens the images:

```
sie_m=sie_lens(co,sigmav=180.0,zl=0.3,zs=2.0,f=0.3,pa=40.0)
```

$$\alpha_1(\vec{x}) = \frac{\sqrt{f}}{f'}\operatorname{arcsinh}\left(\frac{f'}{f}\cos\varphi\right)$$

$$\alpha_2(\vec{x}) = \frac{\sqrt{f}}{f'}\arcsin(f'\sin\varphi)$$

$$\overrightarrow{y} = \overrightarrow{x} - \overrightarrow{\alpha}(\overrightarrow{x})$$



Images end up in different "source" positions…

Let's take the average source position

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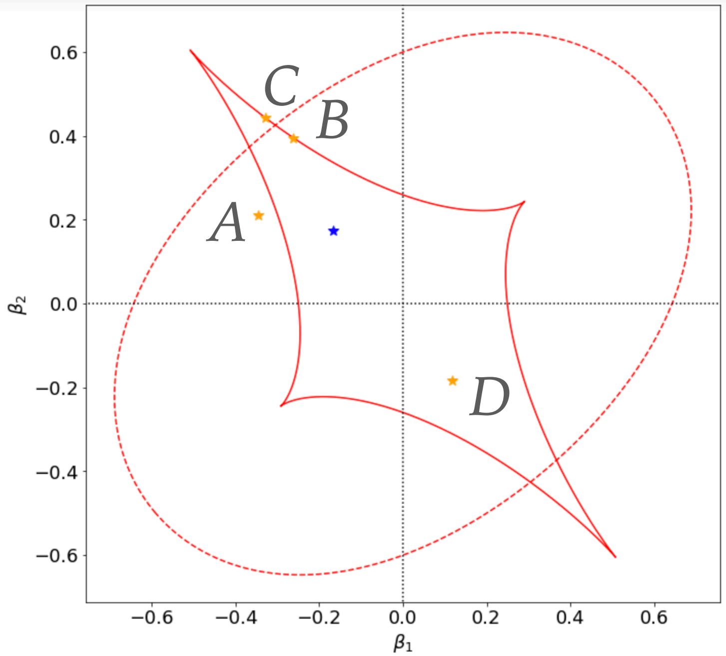
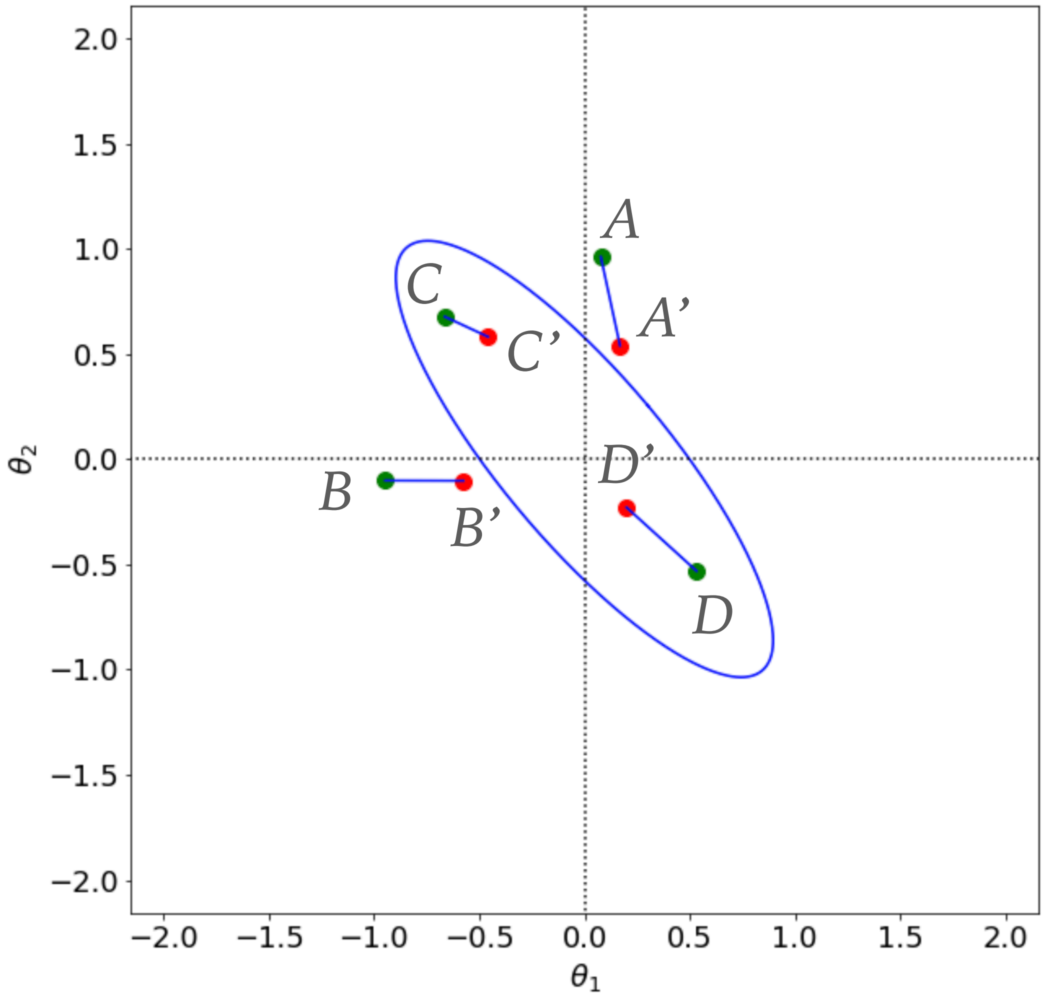


What kind of images would the average source produce?

Let's solve the lens equation… (see notebook 19)



Predicted images (red) are different from the observed images (green)!

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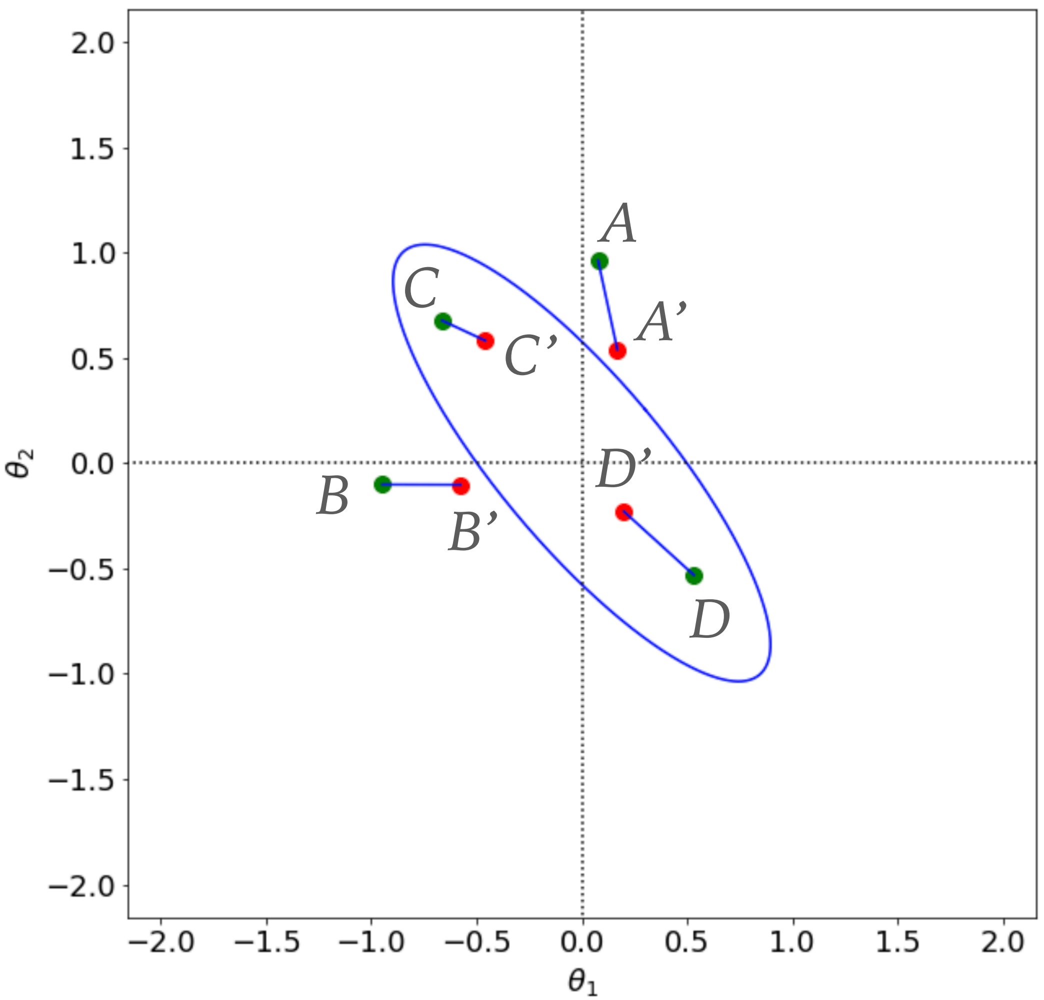

Likelihood of the model:

$$\Pr(D \mid \vec{p}) = \frac{1}{\Pi_{j=1}^{n} \sigma_j \sqrt{2\pi}} \exp - \frac{\chi^2}{2}$$

$$\chi^2 = \sum_{j=1}^{n} \frac{[\vec{\theta}_j - \vec{\theta}_j'(\vec{p})]^2}{\sigma_j^2}$$

$$\vec{p} = \{\sigma_v, f, \varphi\}$$

We need to find the best combination of parameters $\vec{p}$ such to minimise the lengths of the blue sticks, or, in other words, the χ^2. This also maximises the likelihood of the model!

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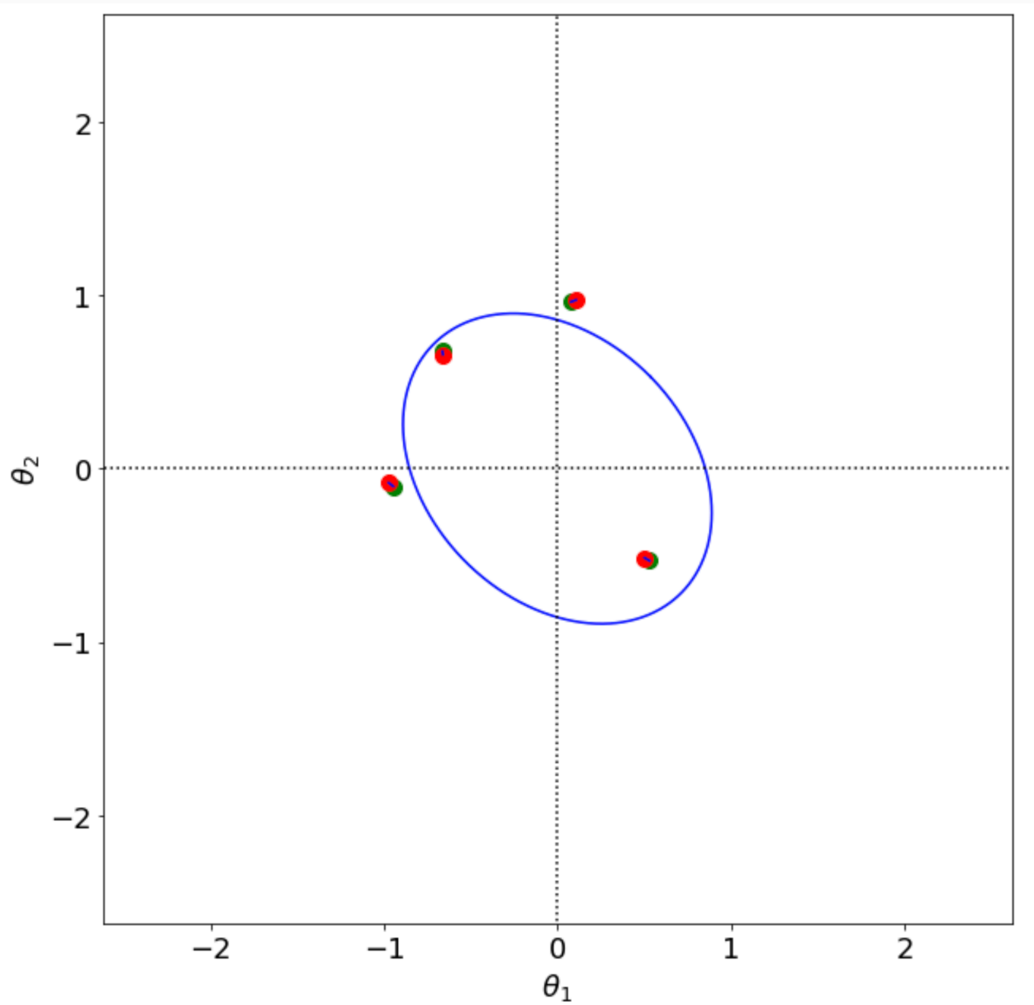

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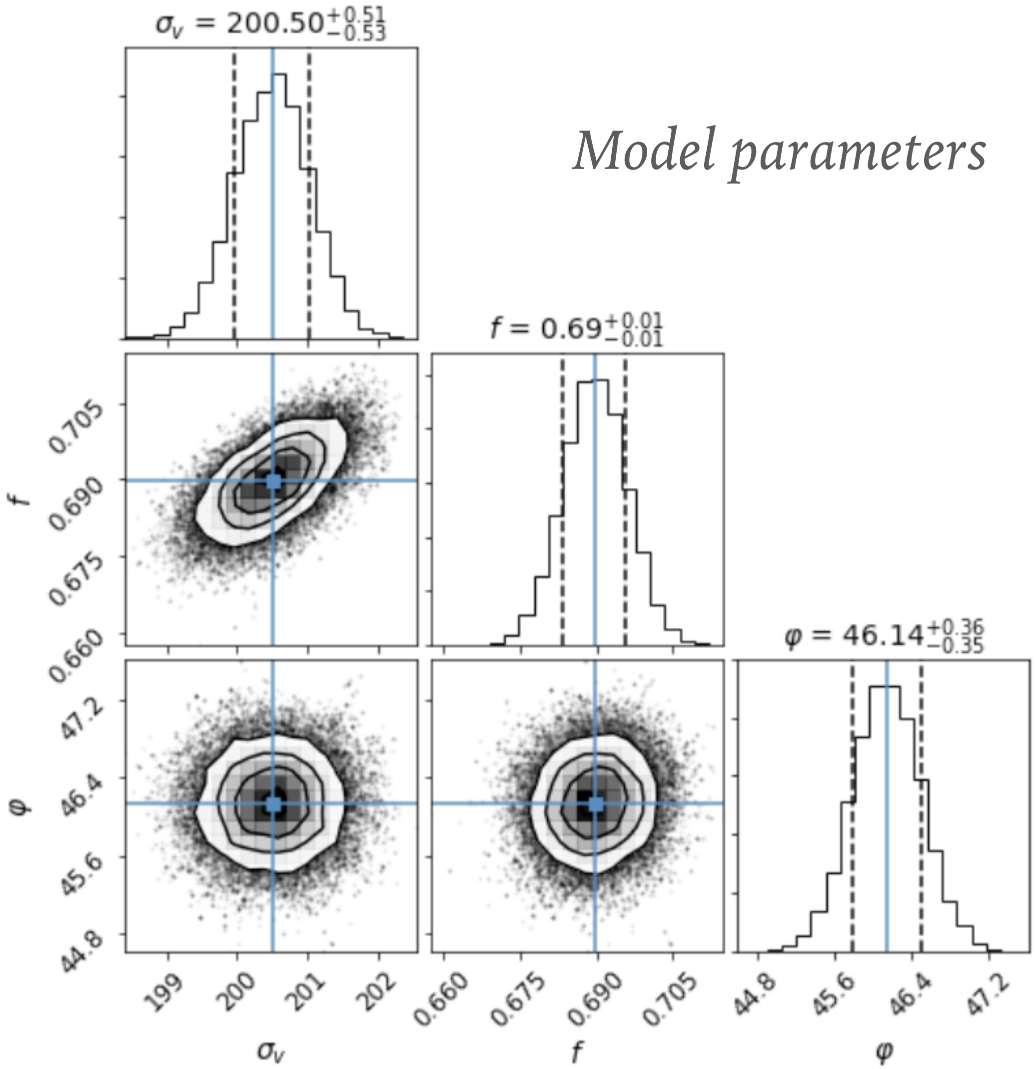
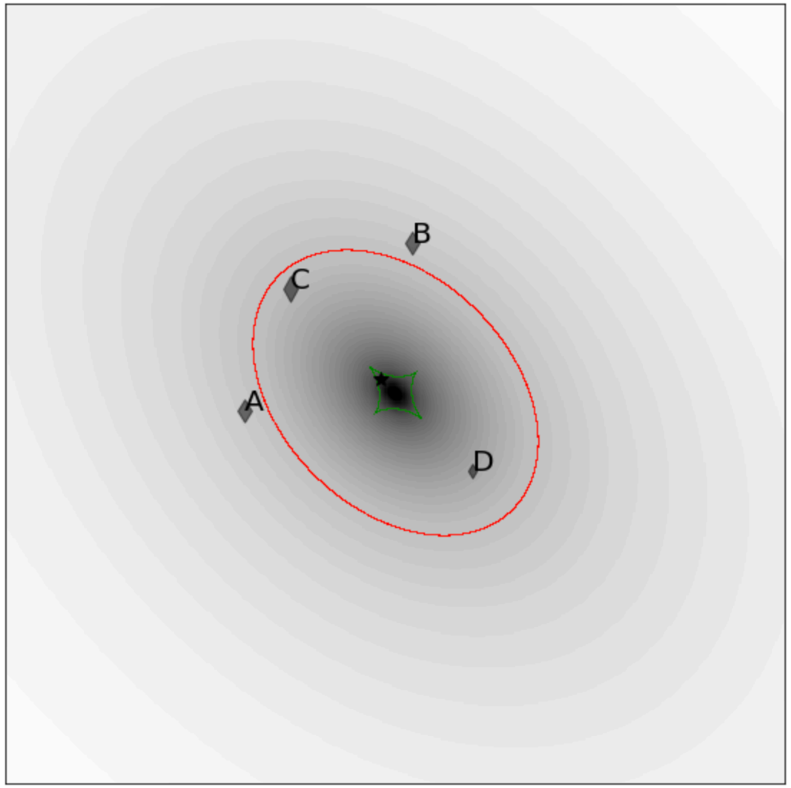
FIT RESULTS



Model parameters



Mass map, mass profile, etc.

LENS PLANE OPTIMIZATION

1. Using the observed images , thing their source positions.

$$\boldsymbol{y}_i = \boldsymbol{x}_i - \alpha(x_i; \boldsymbol{p})$$

2. Using the average source position find the image positions.

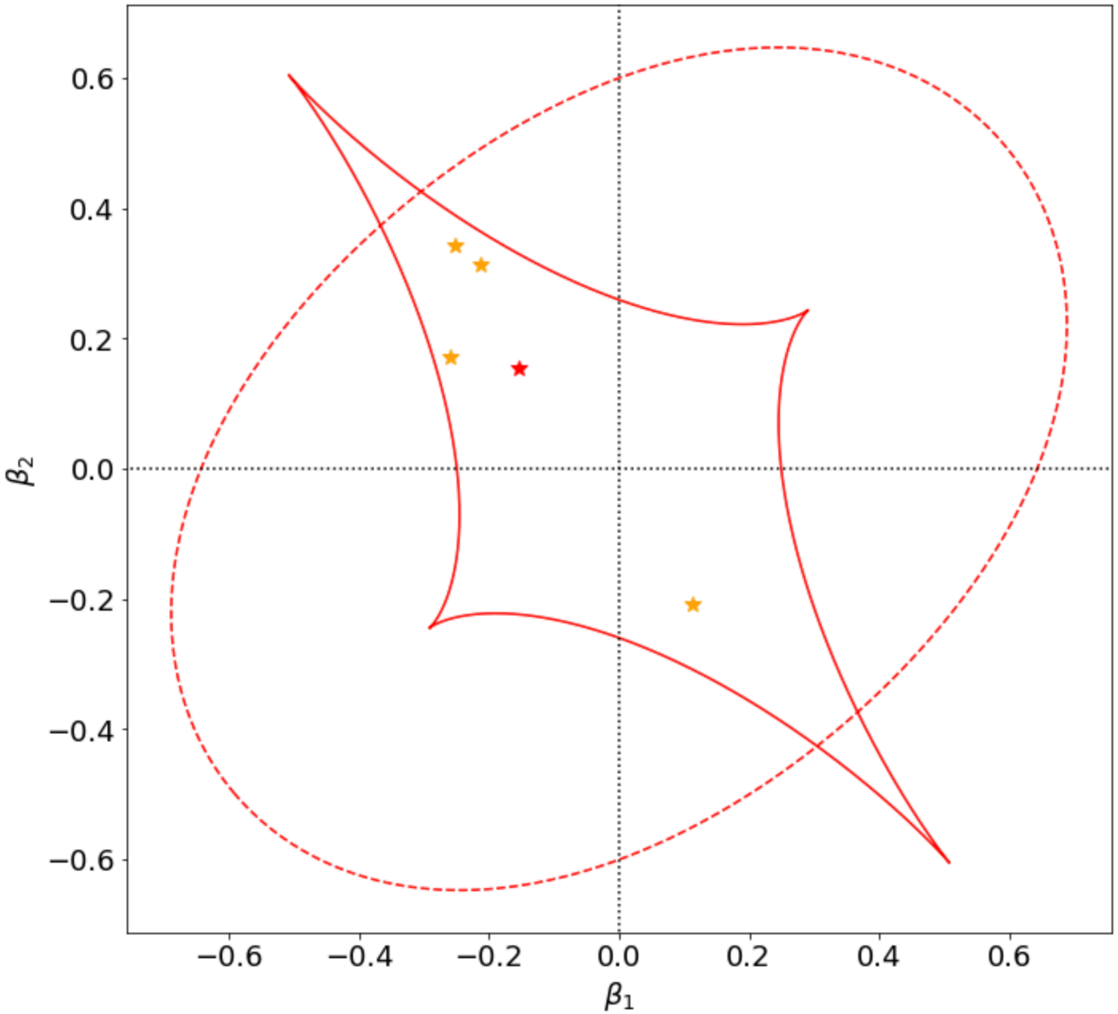
$$\overline{\boldsymbol{y}} = \frac{1}{n}\sum_i^n \boldsymbol{y}_i \qquad\qquad \overline{\boldsymbol{y}} \to \{\boldsymbol{x}_1, \boldsymbol{x}_2, ...\}$$

3. Calculate $\chi^2(\boldsymbol{p})$ *for the images.*

$$\chi^2(\boldsymbol{p}) = \sum_i^n \frac{|\boldsymbol{x}_i^{ob} - \boldsymbol{x}_i(\overline{\boldsymbol{y}}, \boldsymbol{p})|^2}{\sigma_i^2}$$

4. numerically minimize $\chi^2(\boldsymbol{p})$ *with respect to the lens parameters*

ALTERNATIVE STRATEGY: OPTIMISATION IN THE SOURCE PLANE



Instead of minimising the distance between observed and predicted image position, one could try to find the parameters that bring all the images onto the same source position.

$$\chi^2(\boldsymbol{p}) = \sum_i^n \frac{\mu_i^2 |\boldsymbol{y}_i - \overline{\boldsymbol{y}}|^2}{\sigma_i^2}$$

- *No need to solve the lens equation: much faster!*
- *But prone to introduce biases: higher ellipticity, shallower slope*

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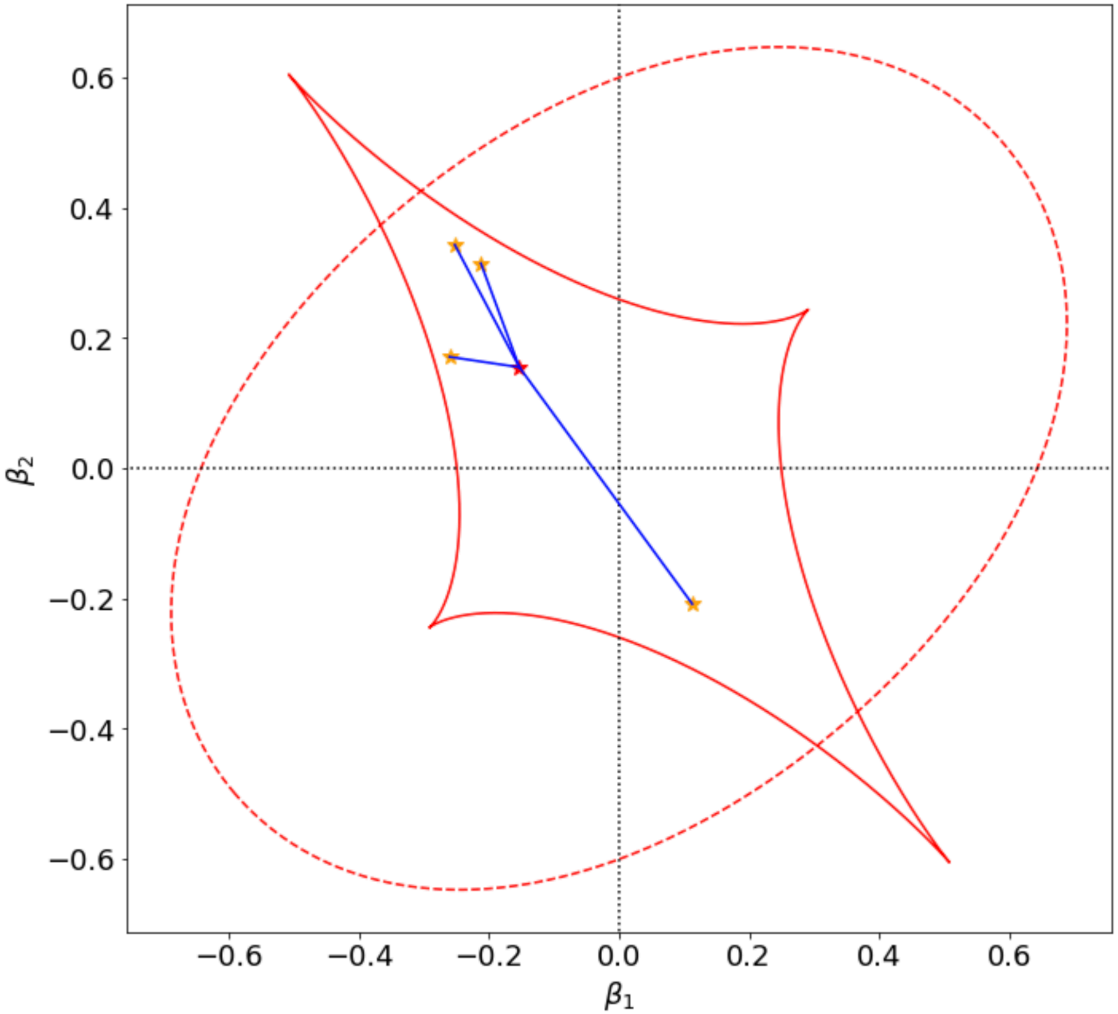

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SOURCE PLANE OPTIMIZATION

1. *Using the observed images, thing their source positions.*

$$\boldsymbol{y}_i = \boldsymbol{x}_i - \alpha(x_i; \boldsymbol{p})$$

2. *Calculate a* $\chi^2(\boldsymbol{p})$ *that forces the source positions to be the same*

$$\overline{\boldsymbol{y}} = \frac{1}{n}\sum_i^n \boldsymbol{y}_i \qquad \chi^2(\boldsymbol{p}) = \sum_i^n \frac{\mu_i^2 |\boldsymbol{y}_i - \overline{\boldsymbol{y}}|^2}{\sigma_i^2}$$

3. *numerically minimize* $\chi^2(\boldsymbol{p})$ *with respect to the lens parameters*

SOURCE PLANE OPTIMIZATION

Advantage:

No need to solve the lens equation! faster

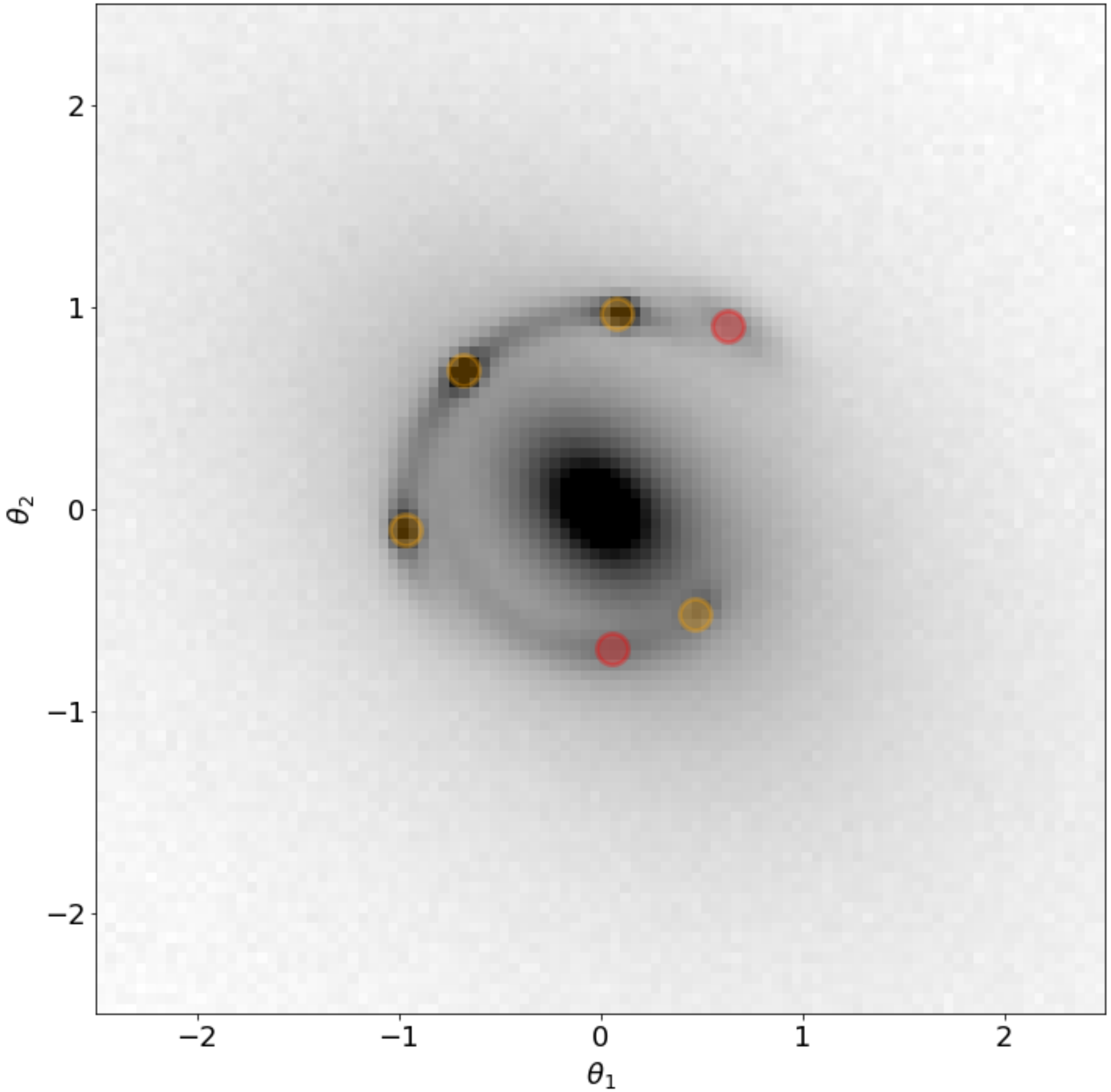
Disadvantages:

Prone to introduce biases.

Chi-squared is not well motivated from a statistical point of view.

Can find a model that has images that are not observed.

ADDING OTHER CONSTRAINTS: MULTIPLE SOURCES



*If there are more families of multiple images (**compound lenses**), they can be fitted jointly:*

$$\chi^2_{tot} = \sum_{i=1}^{N_{fam}} \sum_{j=1}^{n} \frac{[\vec{\theta}_{ij} - \vec{\theta}'_{ij}(\vec{p})]^2}{\sigma^2_{ij}}$$

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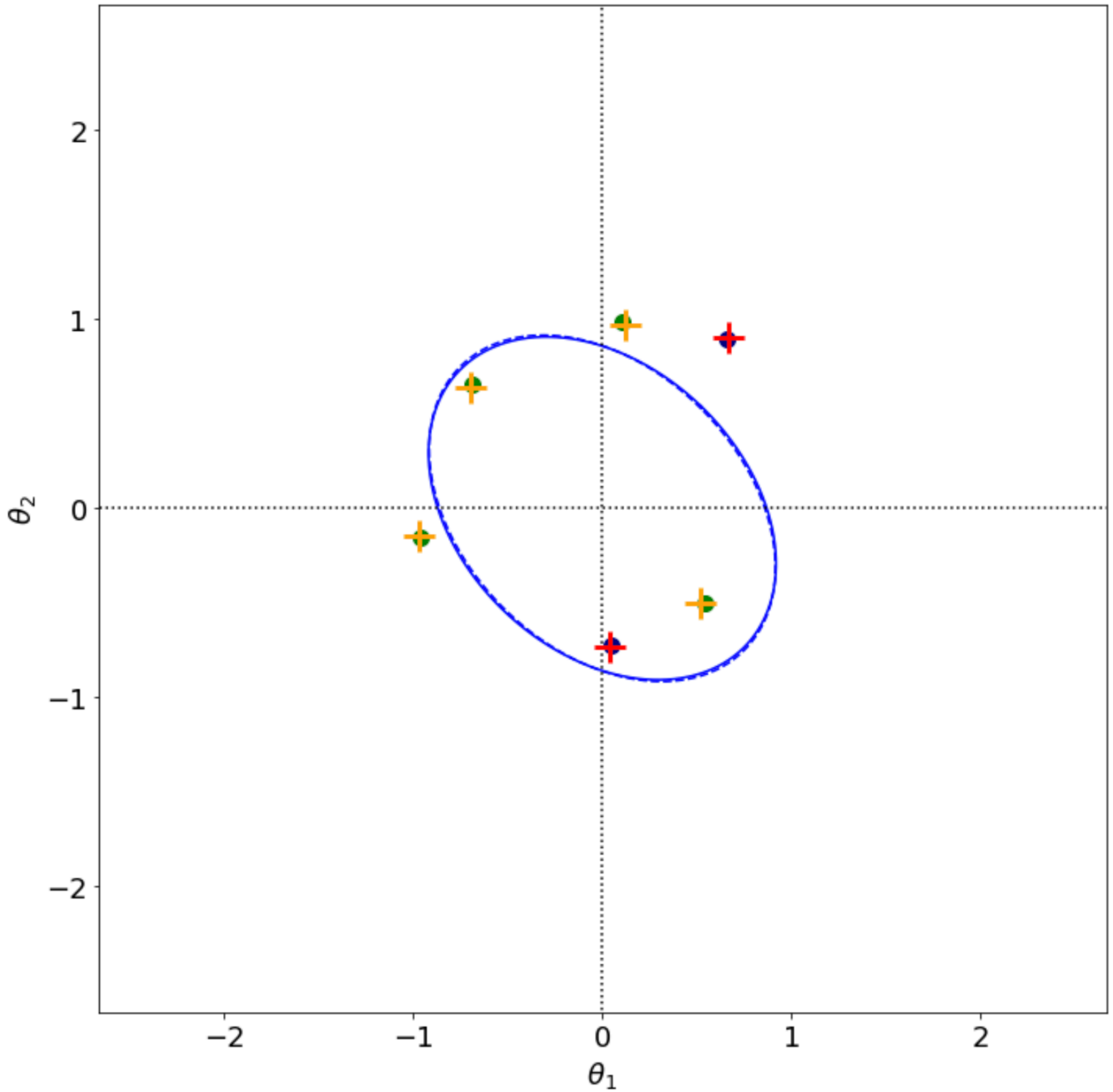

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ADDING OTHER CONSTRAINTS: FLUX RATIOS AND/OR TIME DELAYS

To add other constraints into the model, we just have to rewrite the likelihood function:

$$\chi^2_{tot} = \chi^2_{pos} + \chi^2_{flux} + \chi^2_{\Delta t}$$

Where the χ^2 *terms are:*

$$\chi^2_{flux} = \sum_{j=1}^{n} \frac{[F_j - F'_j(\overrightarrow{p})]^2}{\sigma^2_{F,j}}$$

Flux-ratios

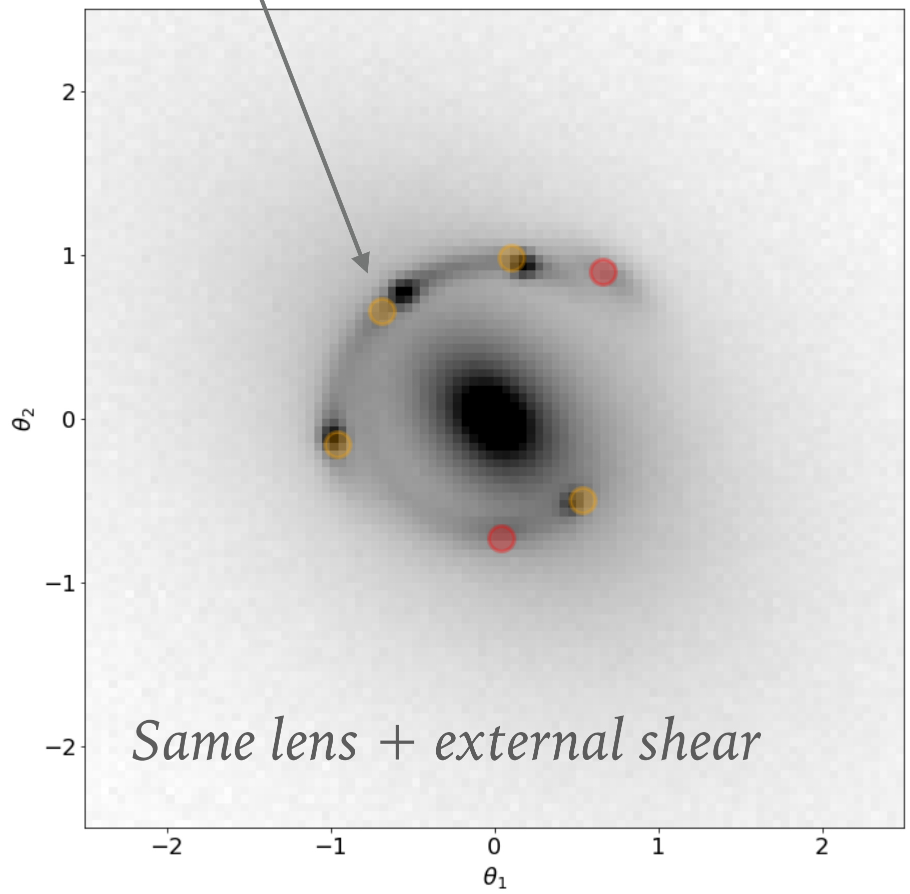
$$\chi^2_{\Delta t} = \sum_{j=1}^{n} \frac{[\Delta t_j - \Delta t'_j(\overrightarrow{p})]^2}{\sigma^2_{\Delta t,j}}$$

Time delays

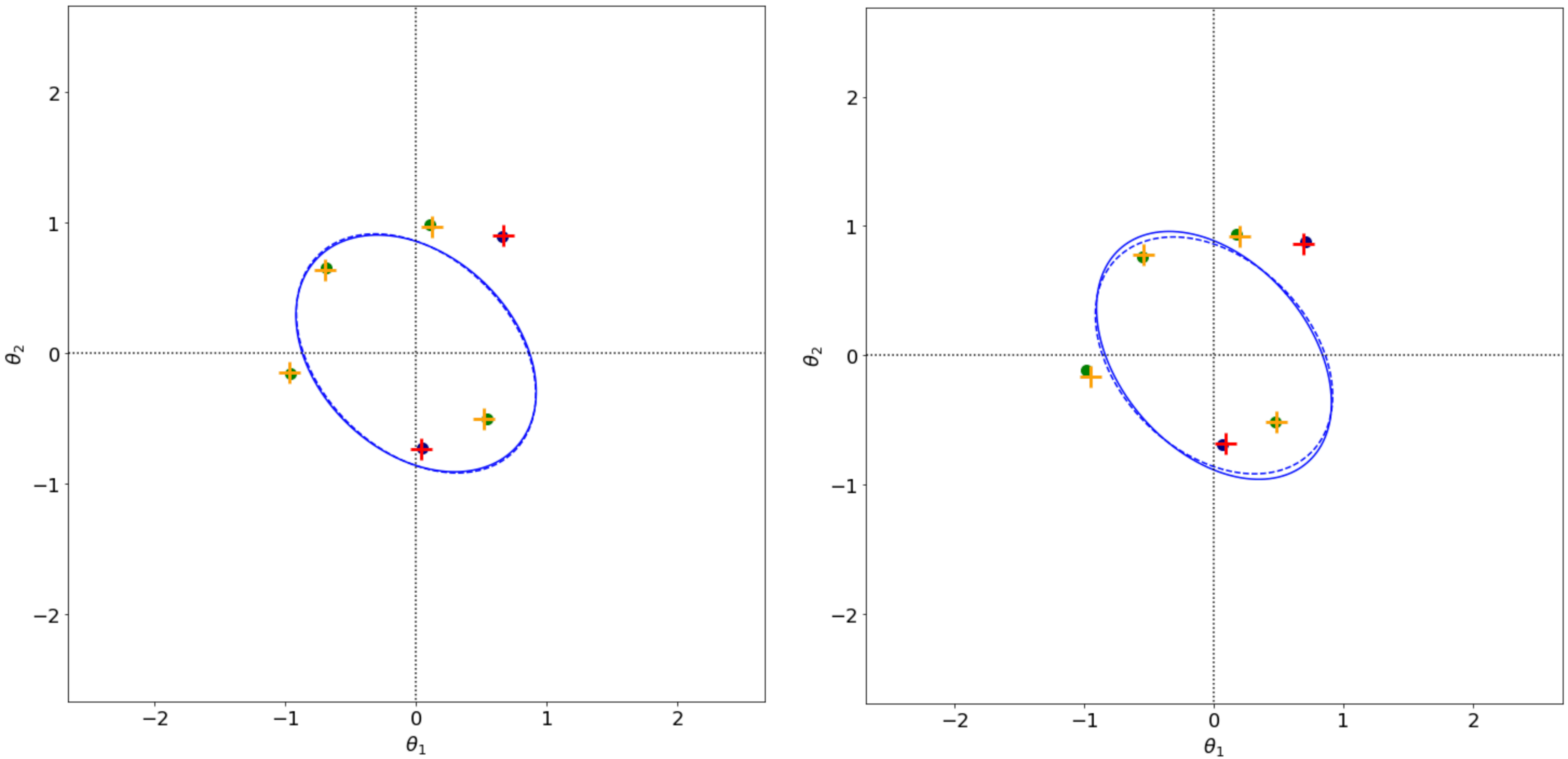
ADDITIONAL REMARKS

- The choice of the lens model is arbitrary: we chose to use a SIE because of the simplicity of the lens and of the number of multiple images (no 5th image!)
- However, additional mass components may be necessary! Example: external shear
- Or other profiles might work better! Example: power-law, softened profiles
- Caveat: the complexity of the model is limited by the number of constraints available

Note the shifts in the image positions!



EXAMPLE: NEGLECTING INGREDIENTS



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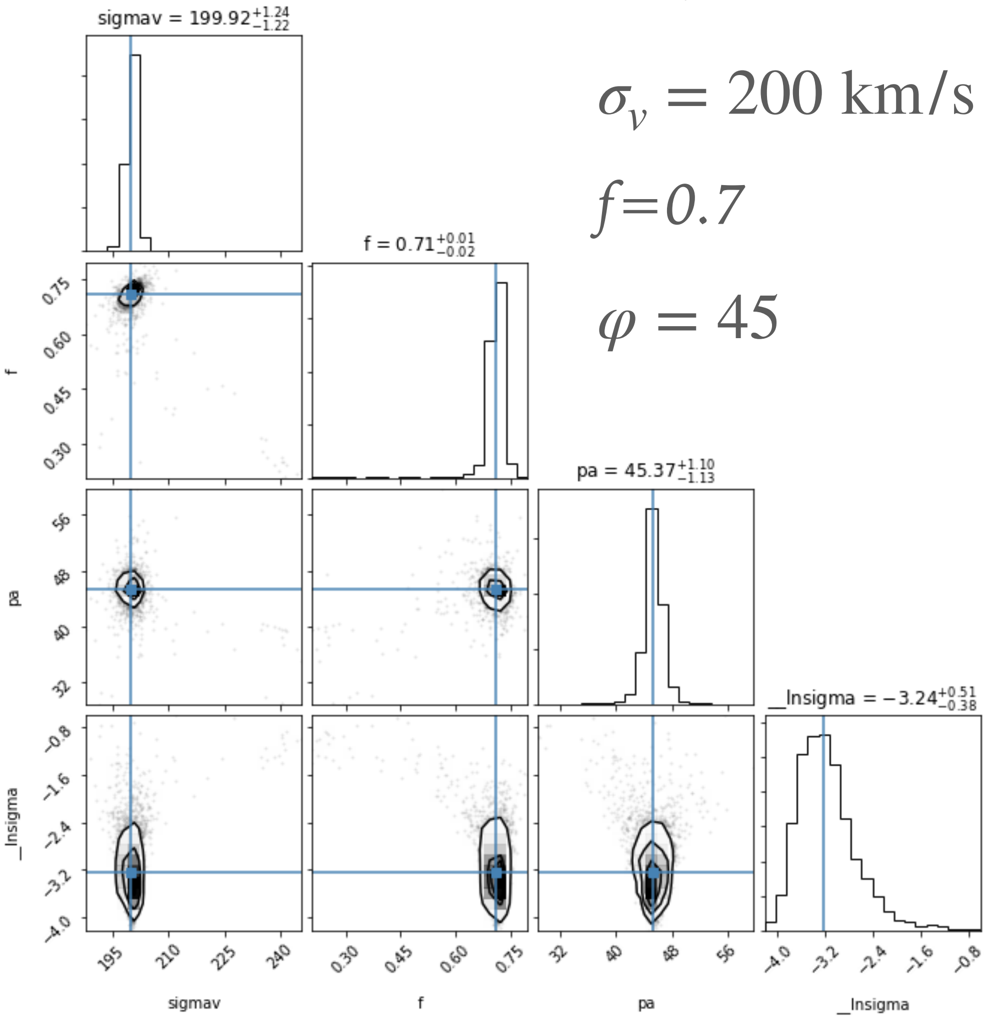
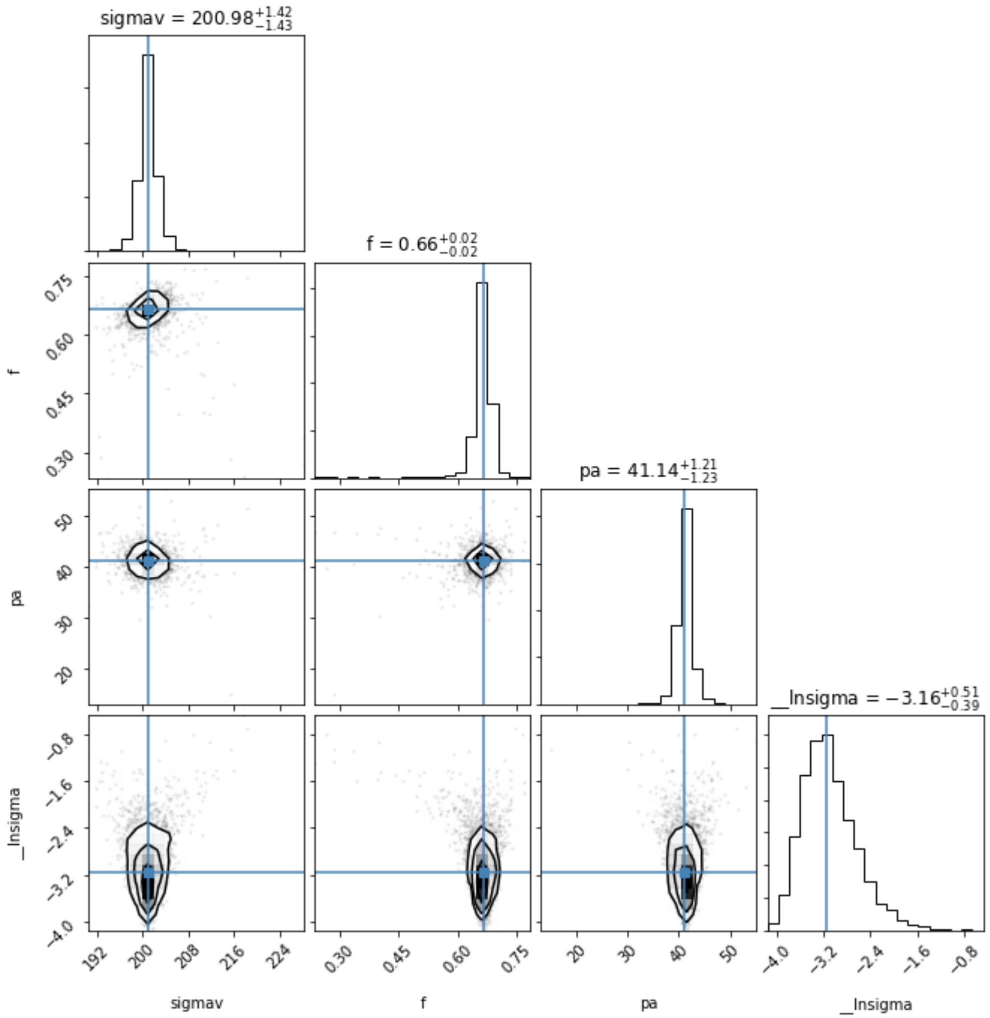


Truth:

$\sigma_v = 200$ km/s

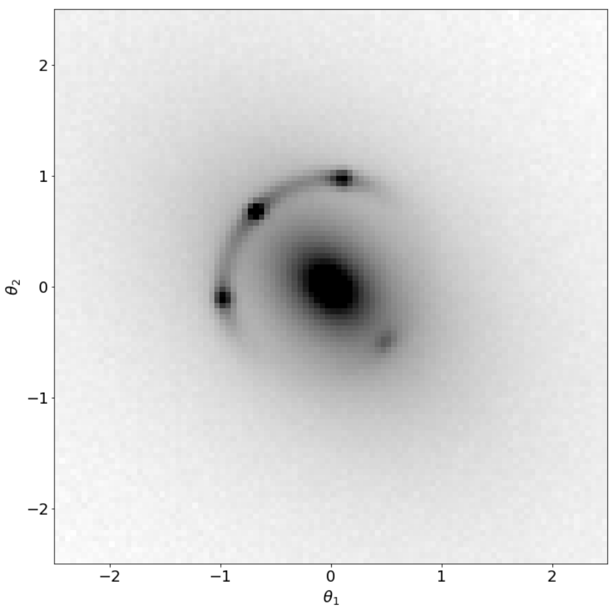
f=0.7

$\varphi = 45$



VERY IMPORTANT

- Image positions, magnifications, time delays: all depend on lens and source distances
- Without knowing the distances, we cannot measure physical quantities!
- To measure the distances, we must measure redshifts and assume a cosmological model



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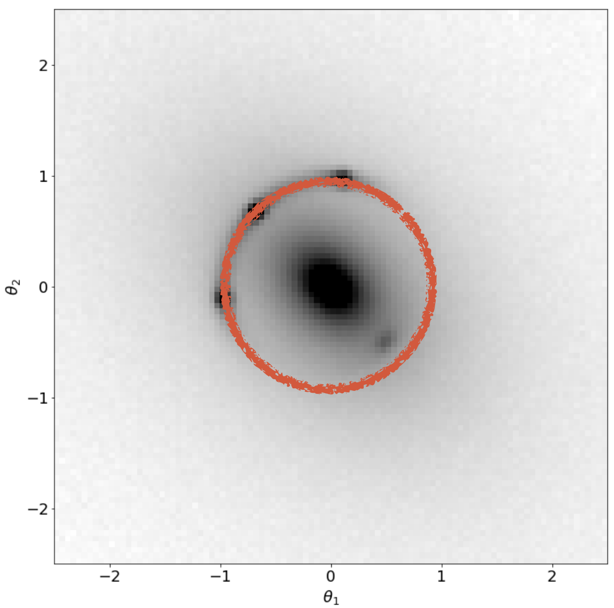

Einstein radius: $\theta_{E,SIS} = \frac{4\pi\sigma_v^2}{c^2}\frac{D_{LS}}{D_S}$

convergence: $\bar{\kappa}(<\theta_E) = \frac{\bar{\Sigma}(\theta)}{\Sigma_{cr}} = 1$

mass: $M(<\theta_E) = \pi\theta_E^2 D_L^2 \Sigma_{cr} = \pi\theta_E^2 D_L^2 \frac{c^2}{4\pi G}\frac{D_S}{D_L D_{LS}}$