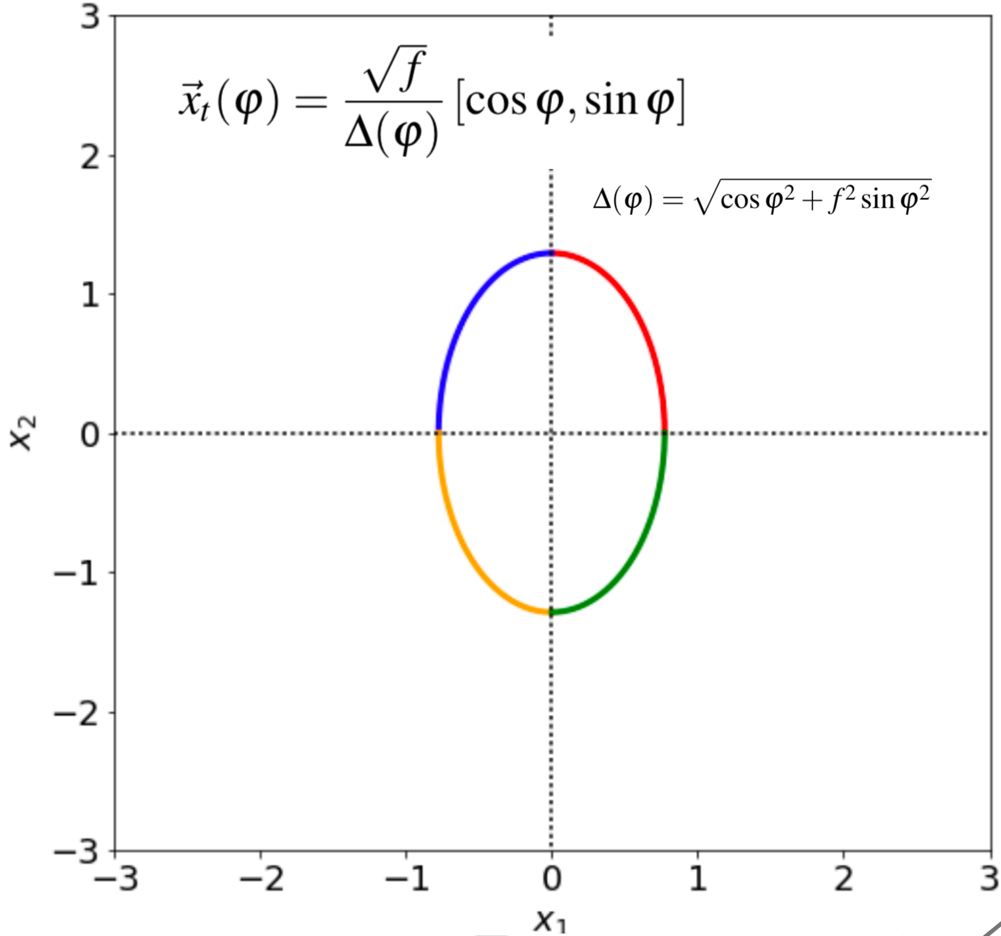
GRAVITATIONAL LENSING

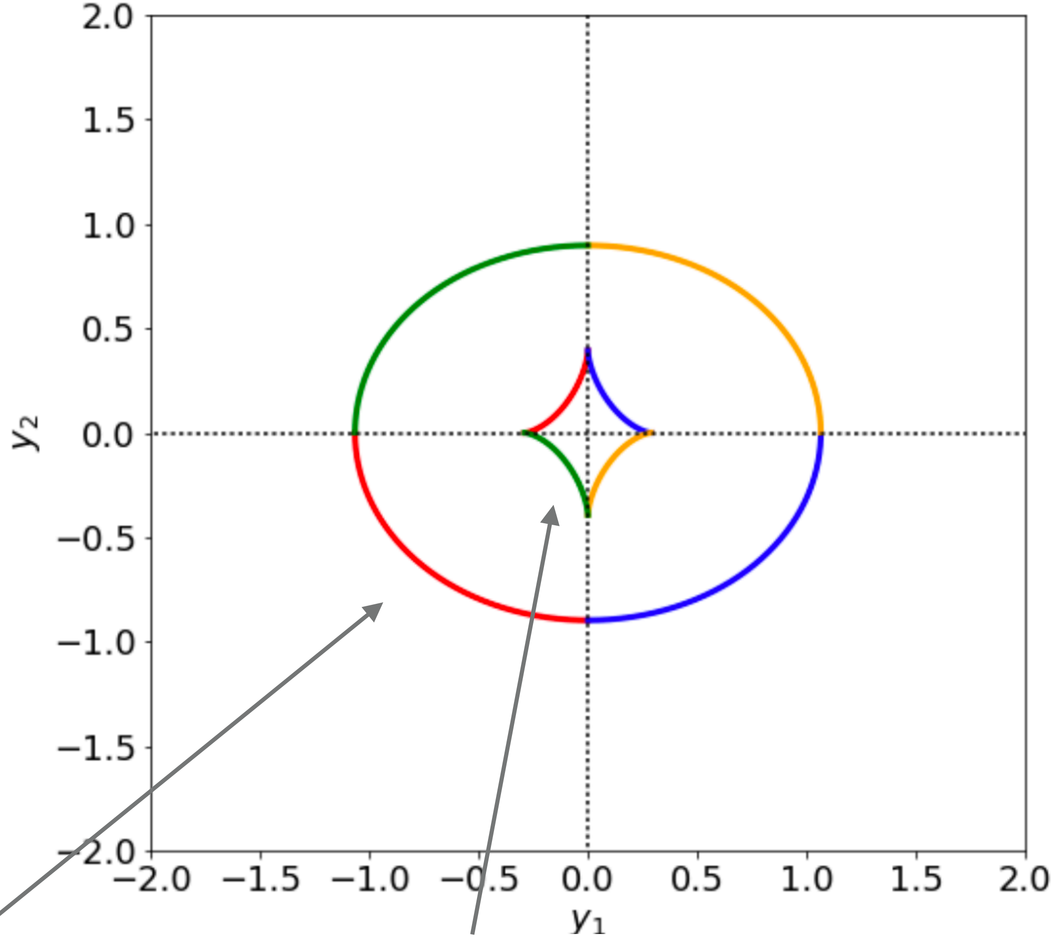
ELLIPTICAL MODELS II

R. Benton Metcalf

2022-2023

MAPPING RULES FOR CAUSTICS AND CUT

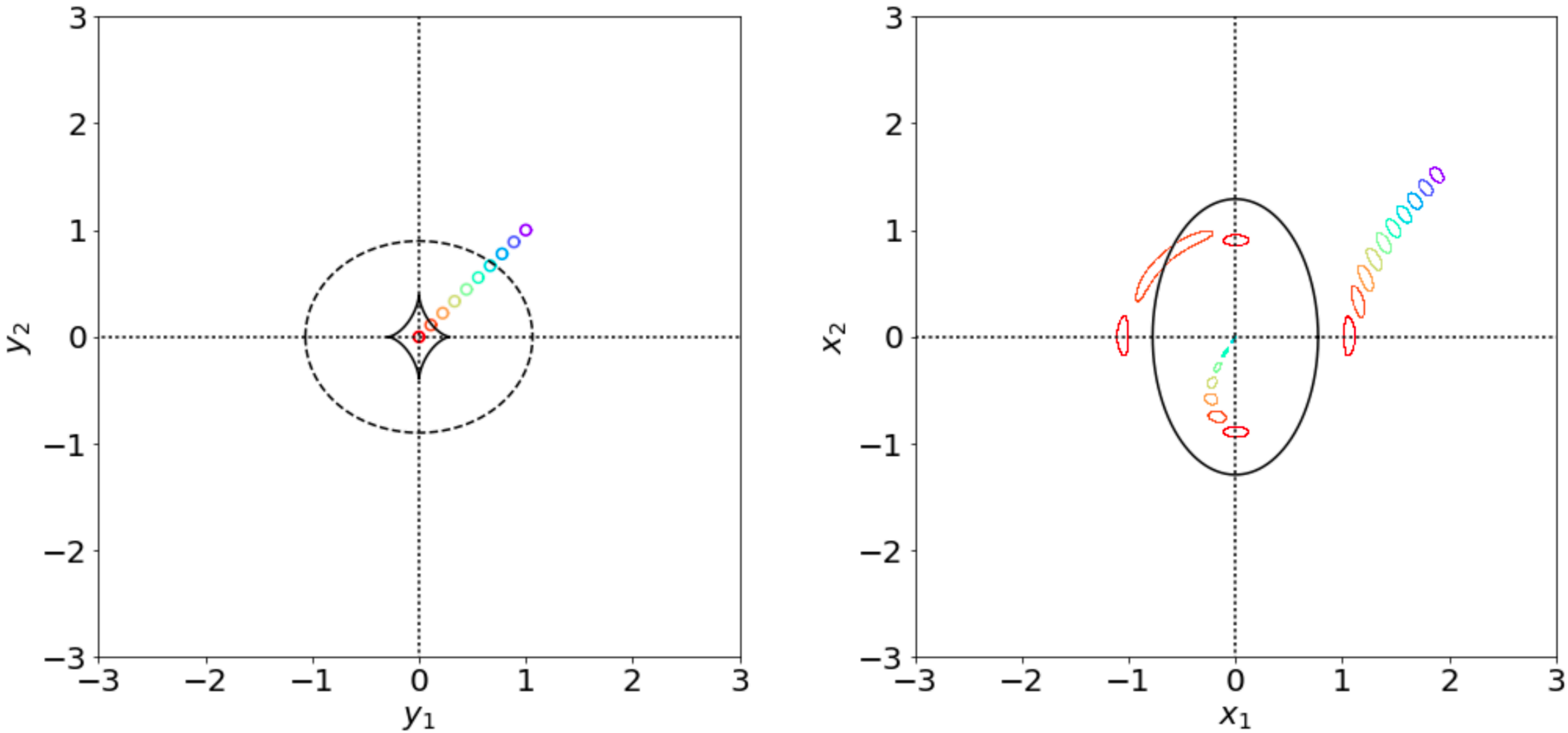




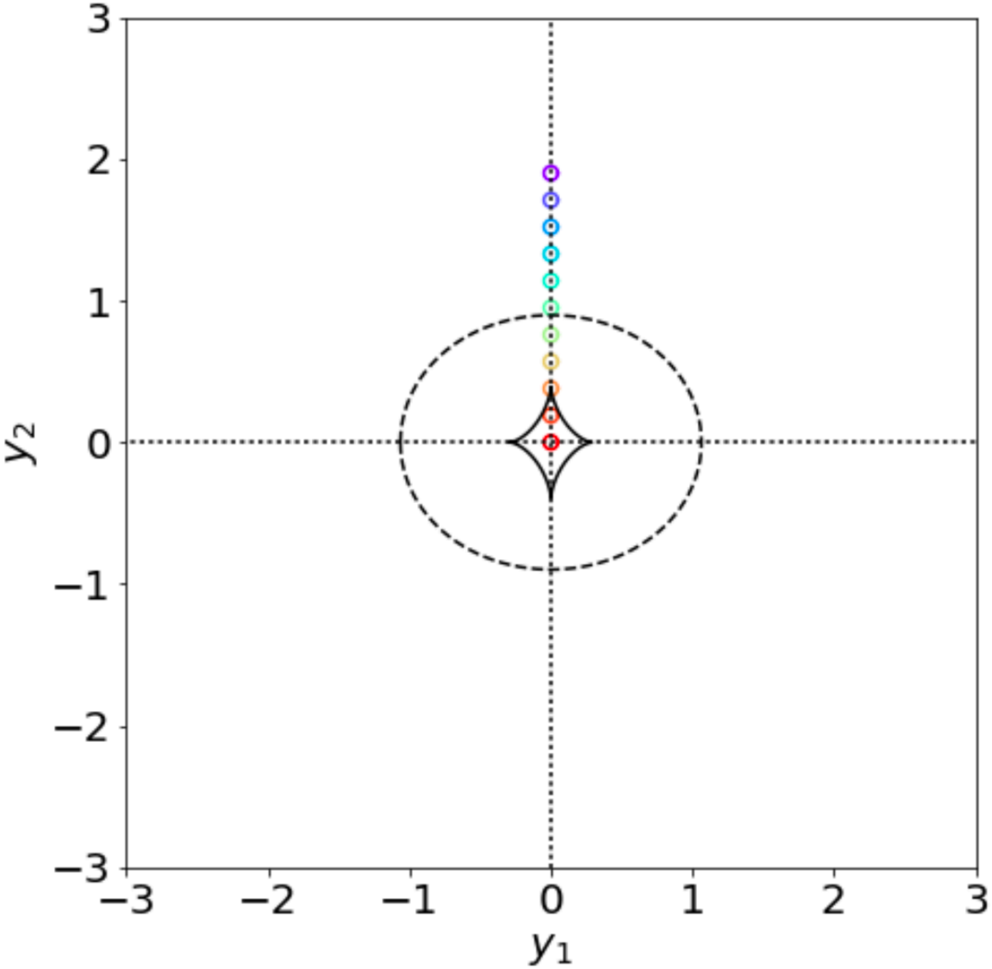
$$
\begin{aligned}
y_{c,1} &= -\frac{\sqrt{f}}{f'}\operatorname{arcsinh}\left(\frac{f'}{f}\cos\varphi\right) \\
y_{c,2} &= -\frac{\sqrt{f}}{f'}\arcsin(f'\sin\varphi)\ .
\end{aligned}
$$

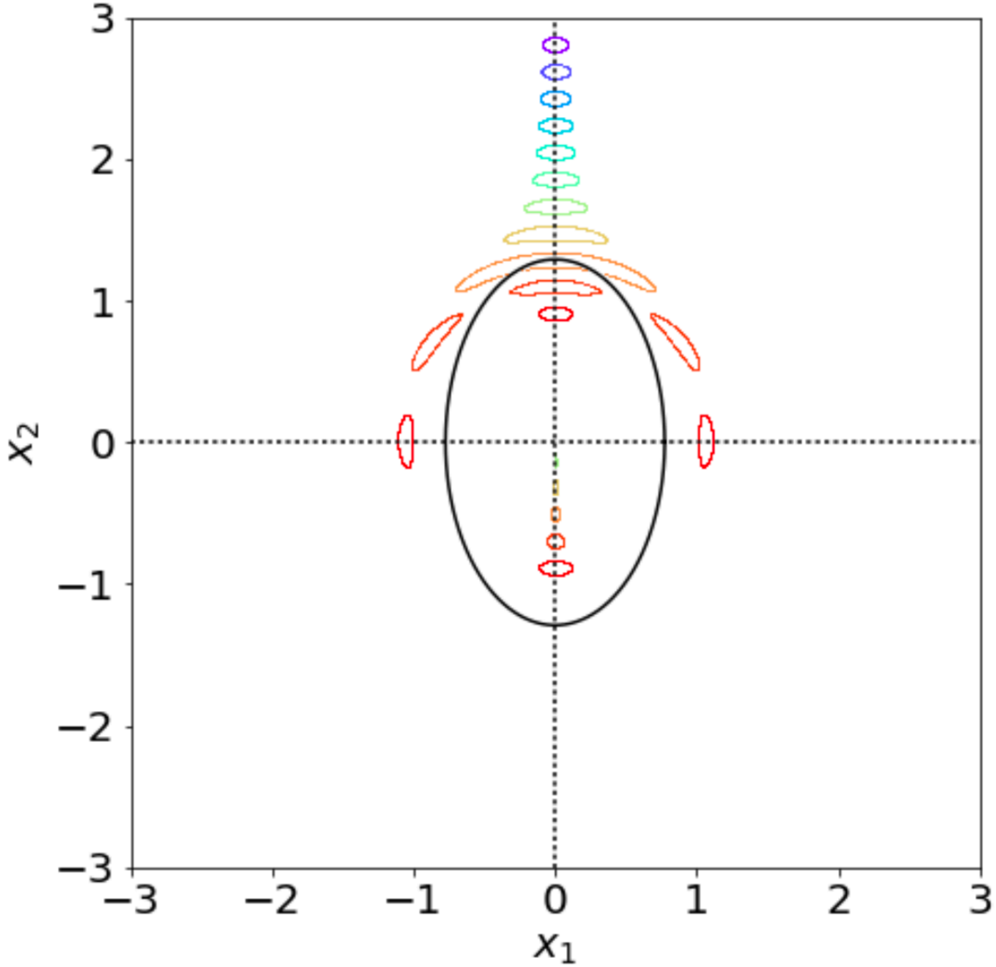
$$
\begin{aligned}
y_{t,1} &= \frac{\sqrt{f}}{\Delta(\varphi)}\cos\varphi - \frac{\sqrt{f}}{f'}\operatorname{arcsinh}\left(\frac{f'}{f}\cos\varphi\right) \\
y_{t,2} &= \frac{\sqrt{f}}{\Delta(\varphi)}\sin\varphi - \frac{\sqrt{f}}{f'}\arcsin(f'\sin\varphi)\ .
\end{aligned}
$$

MULTIPLE IMAGES BY SINGULAR ISOTHERMAL ELLIPSOIDS

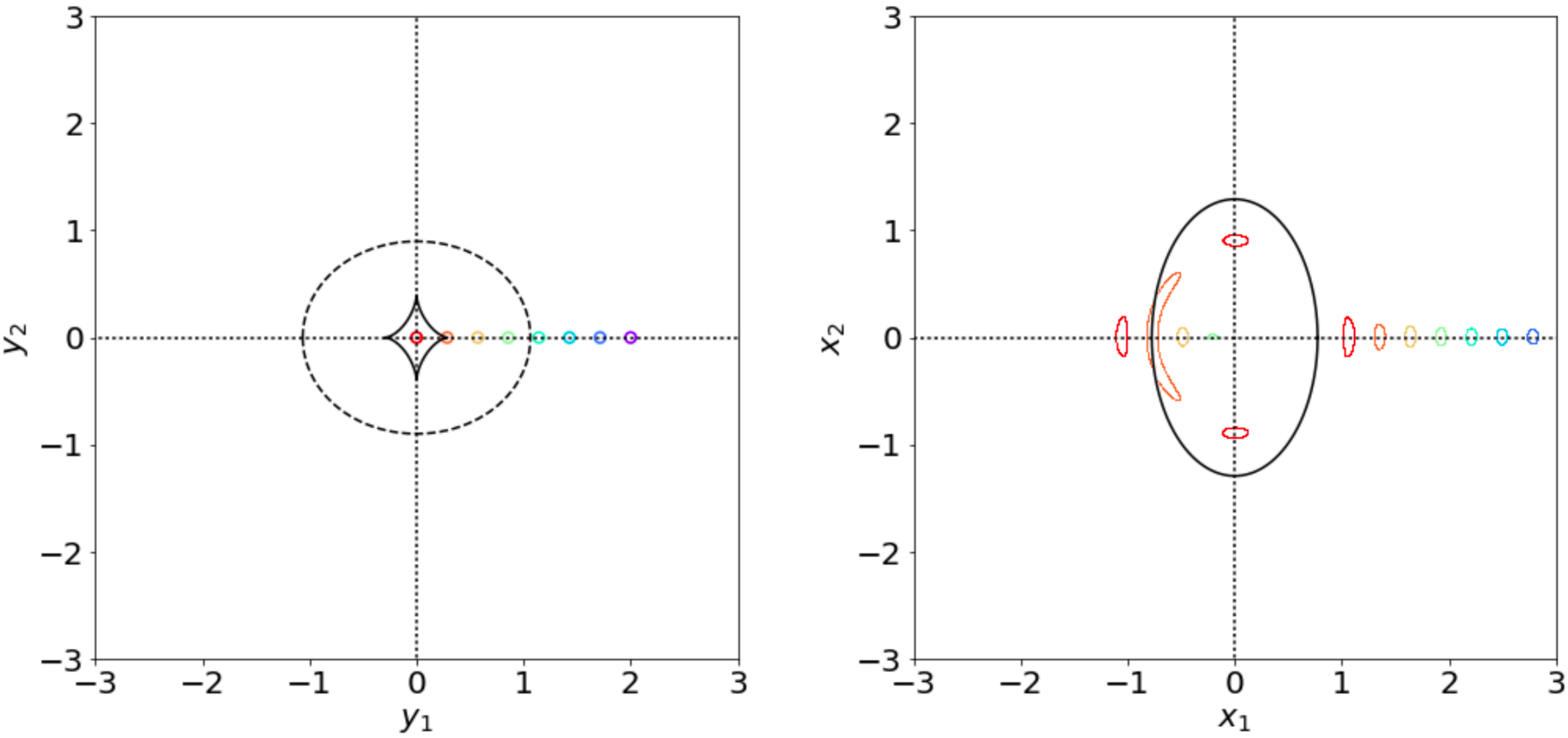


MULTIPLE IMAGES BY SINGULAR ISOTHERMAL ELLIPSOIDS

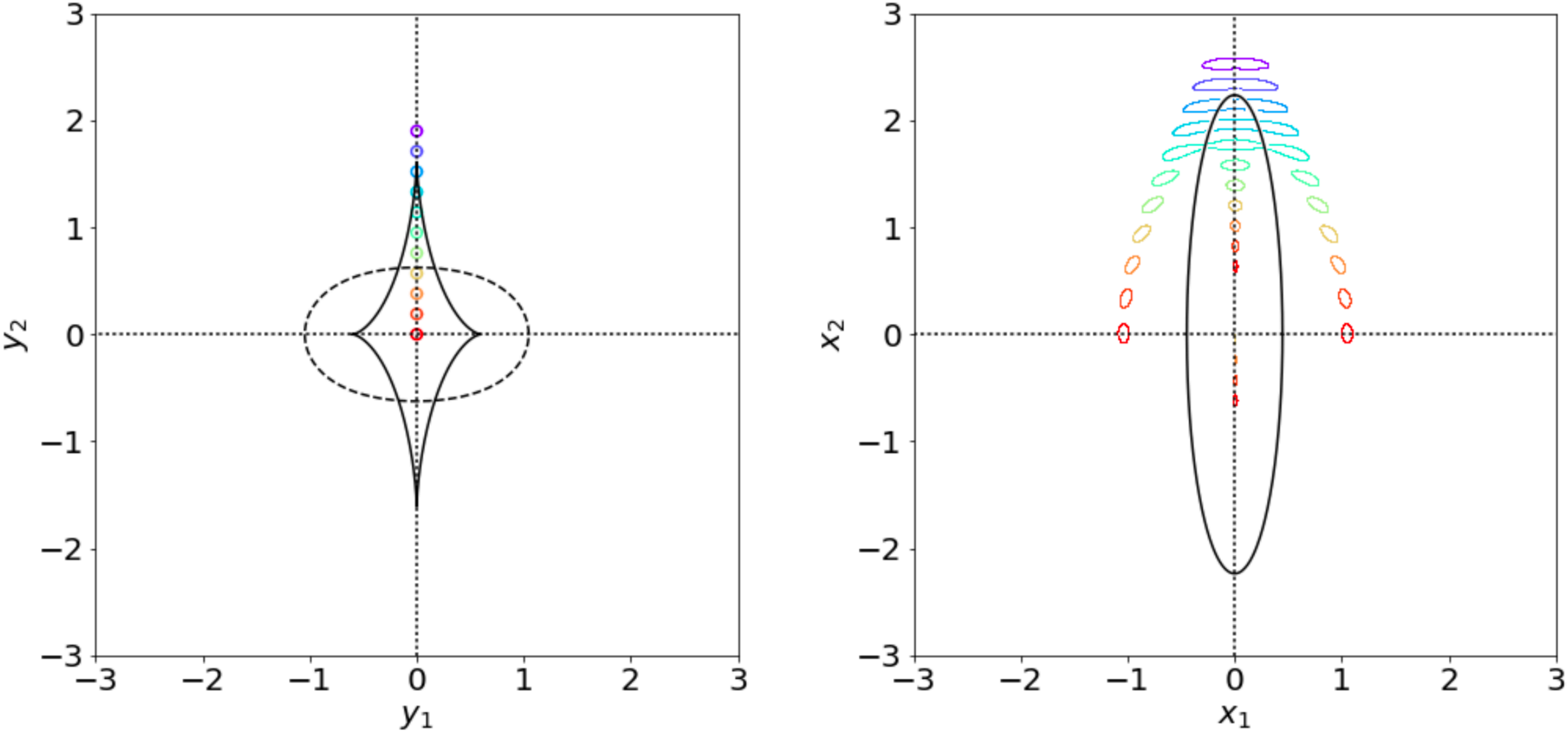




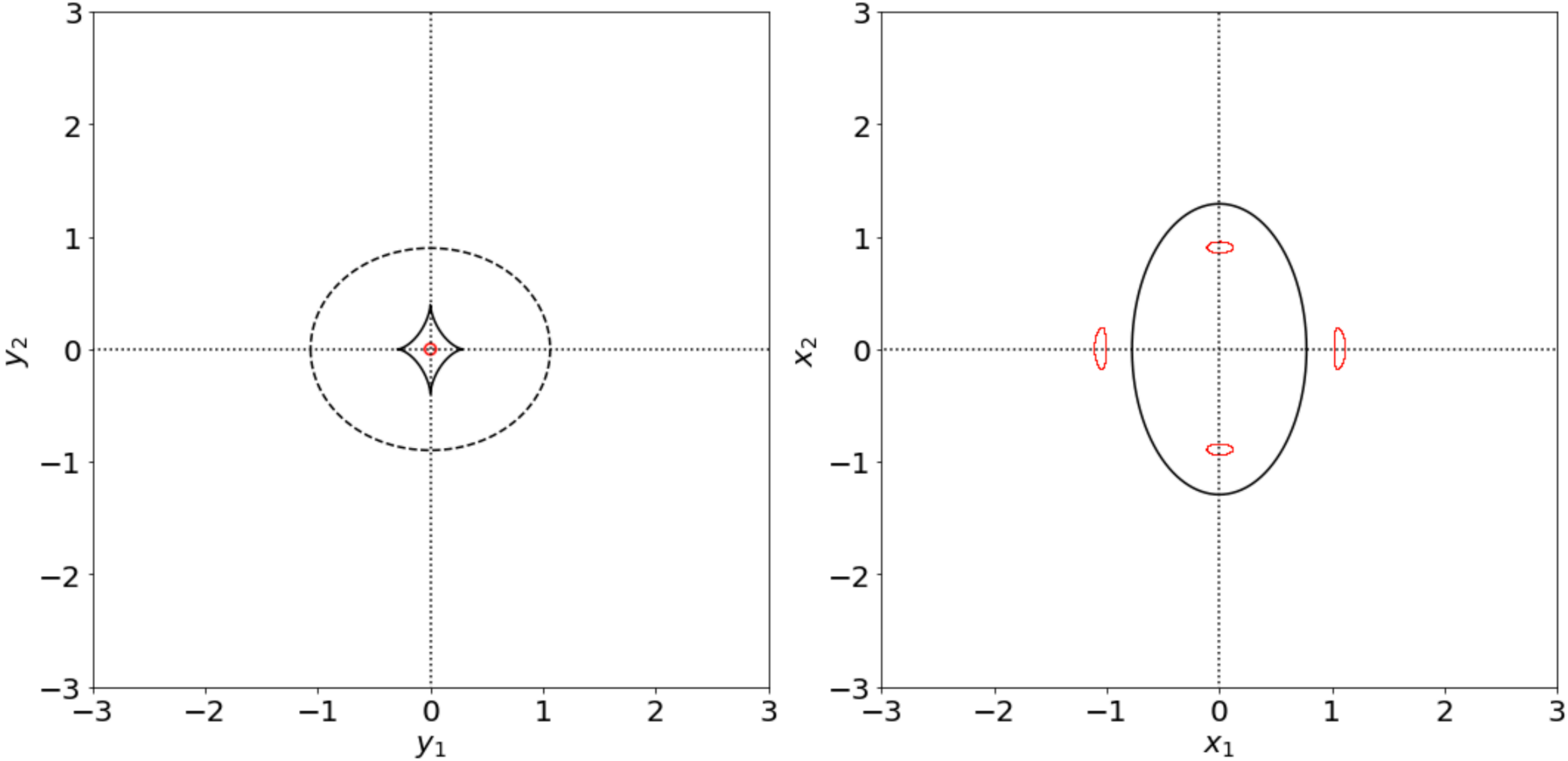
MULTIPLE IMAGES BY SINGULAR ISOTHERMAL ELLIPSOIDS



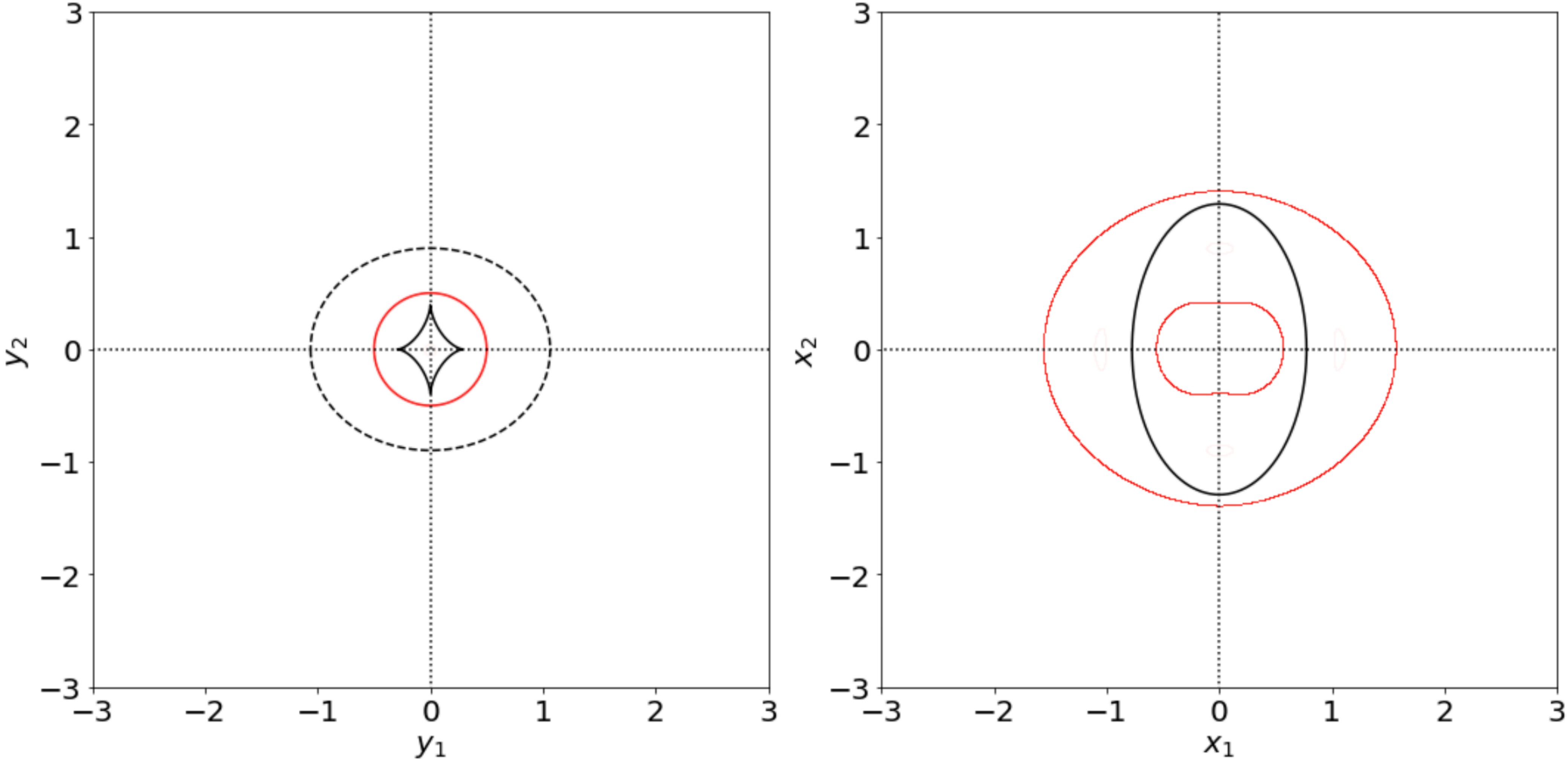
NAKED CUSPS



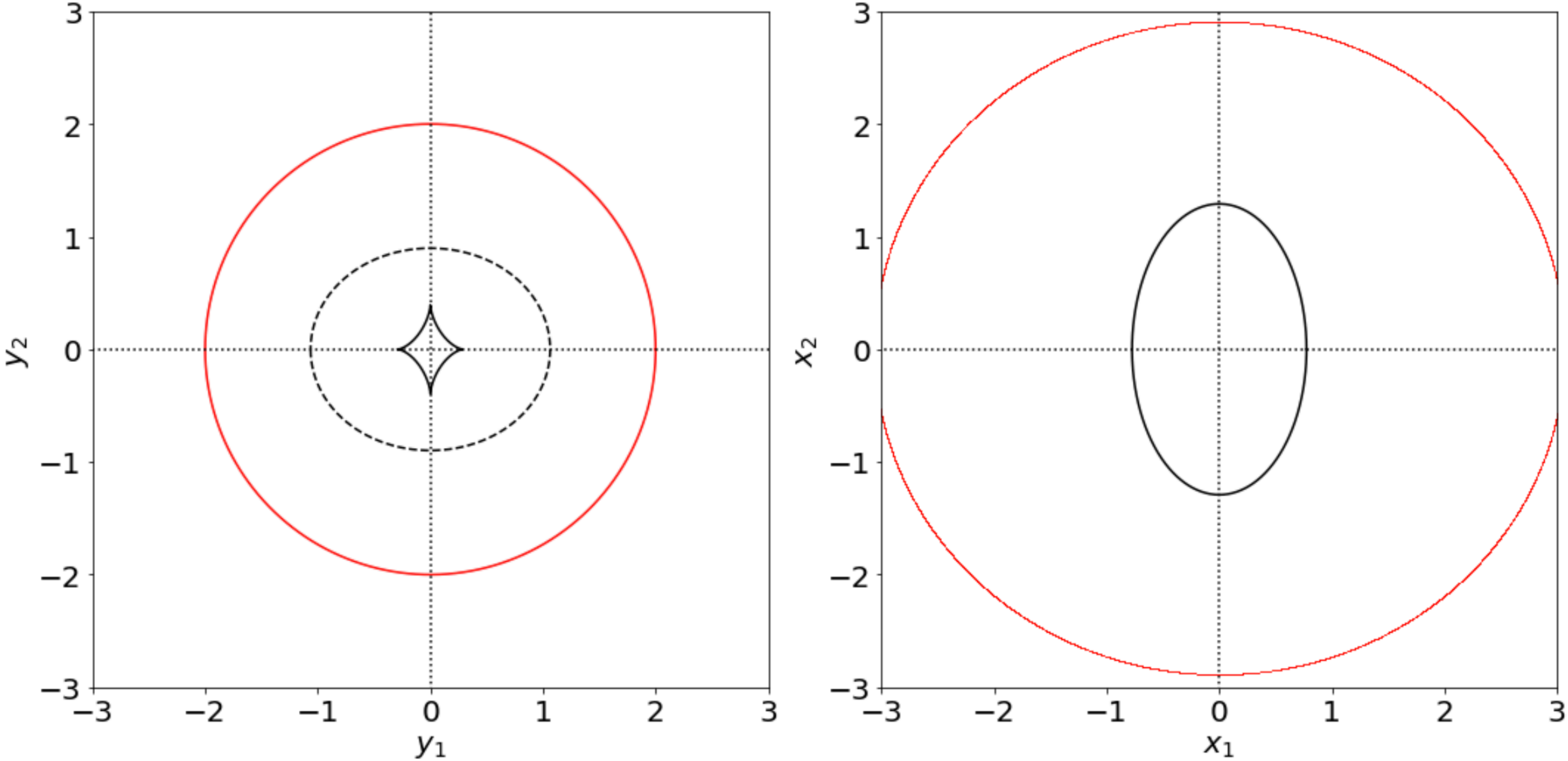
SOURCE SIZE EFFECTS



SOURCE SIZE EFFECTS



SOURCE SIZE EFFECTS



EXAMPLE 1

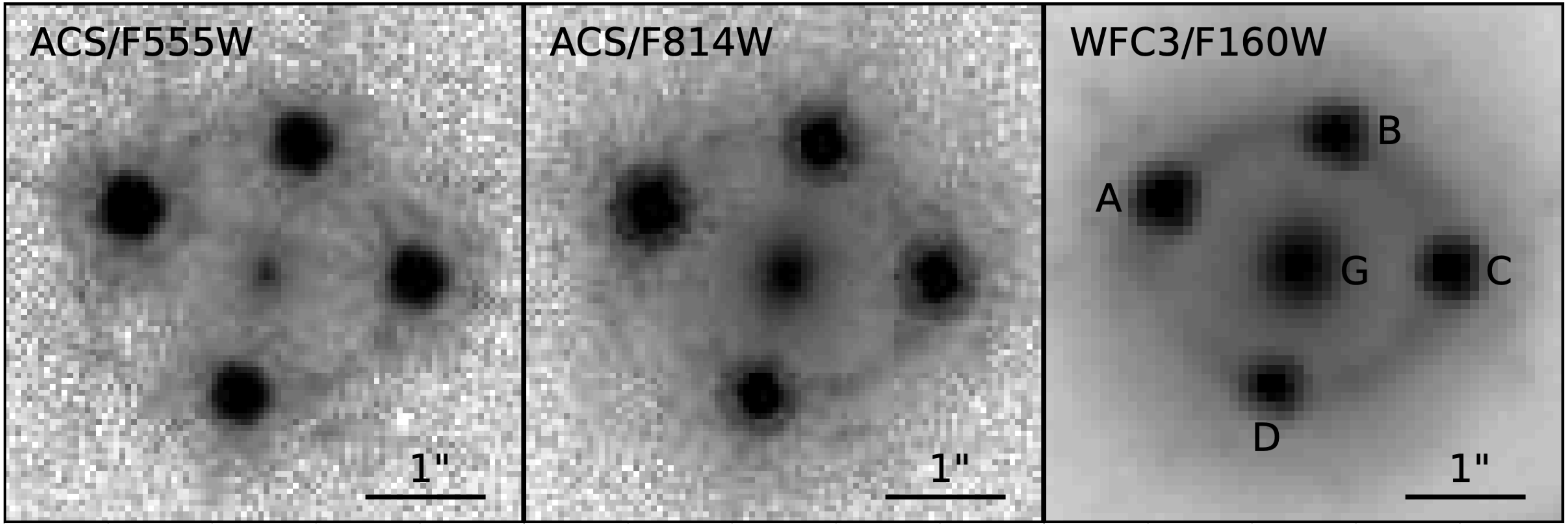
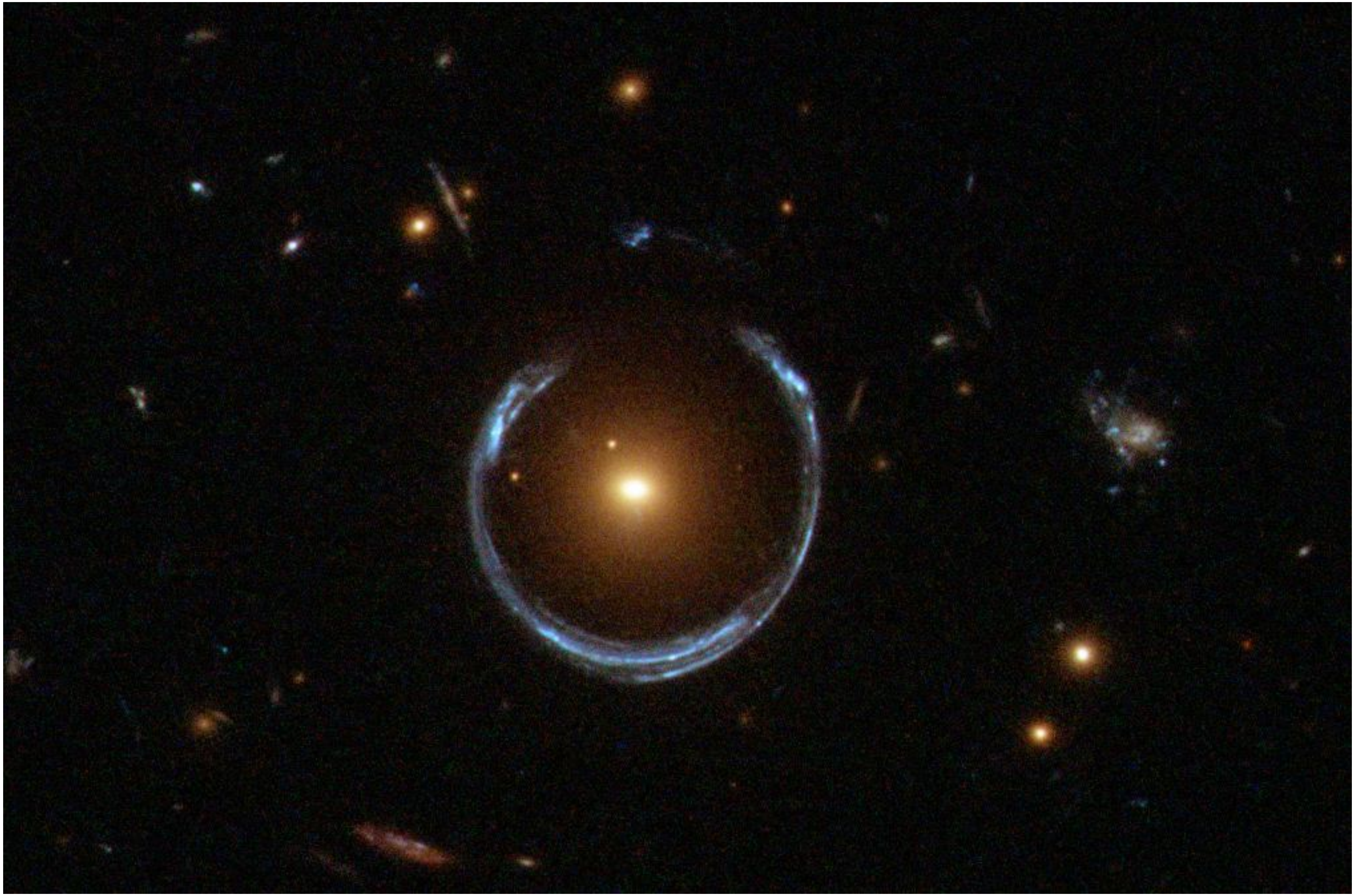
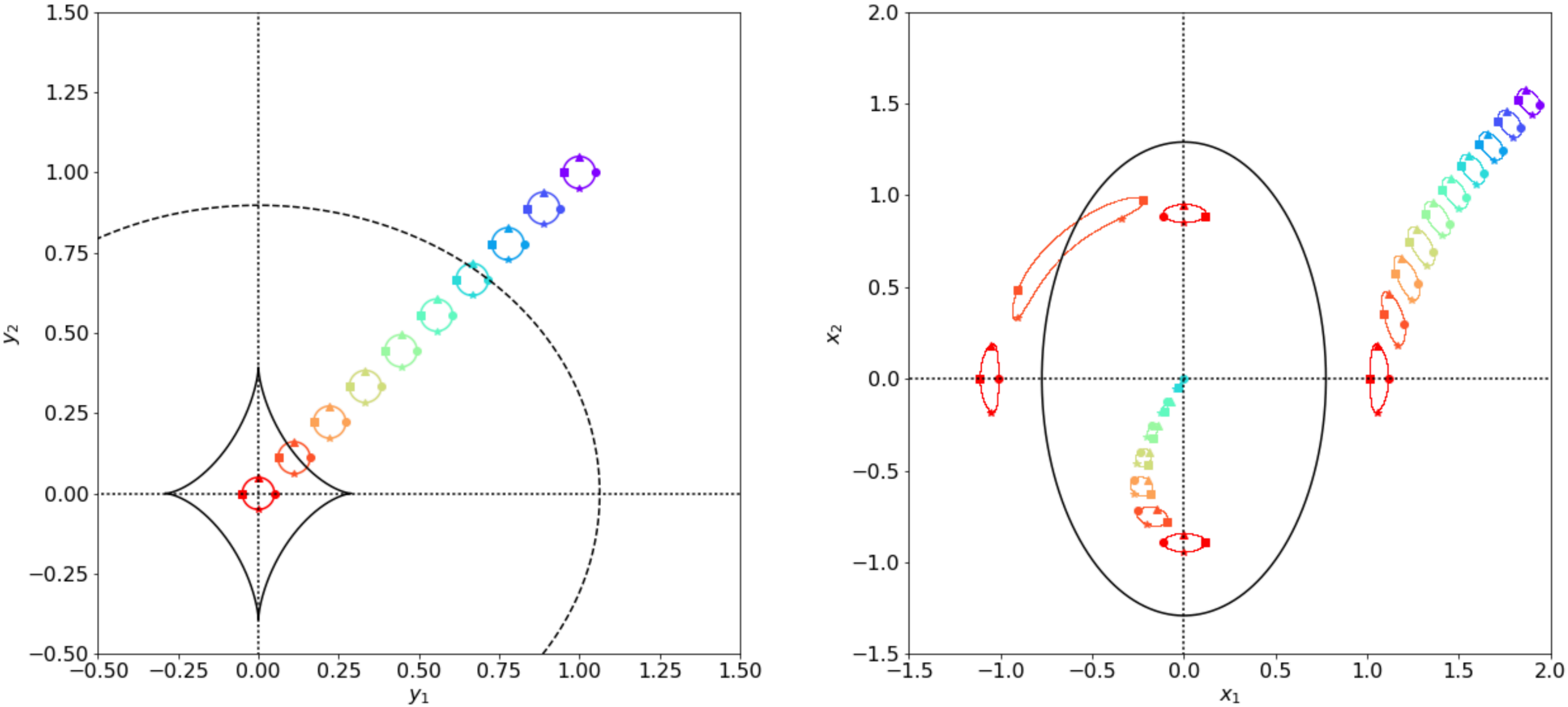


Figure 1. *HST* images of HE 0435−1223. Shown are cutouts of the lens system used for lens modeling in the ACS/F555W (left), ACS/F814W (middle), and WFC3/F160W (right) bands. The images are 4.″5 on a side. The scale is indicated in the bottom right of each panel. The main lens galaxy (G) and lensed quasar images (A, B, C, D) are marked.

EXAMPLE 2



PARITY INVERSION ACROSS THE CRITICAL LINES

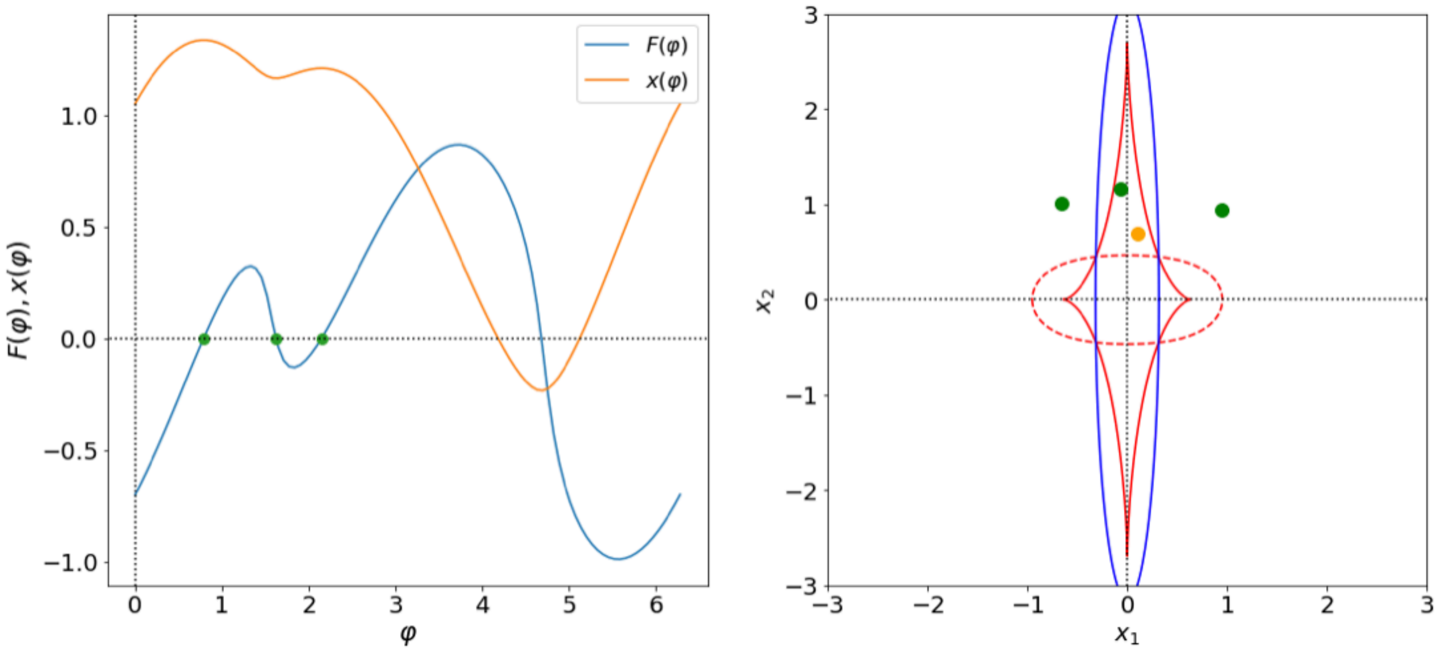


EXAMPLE 3

SIE LENSES : IMAGE POSITIONS

$$x = (y_1 + \alpha_1(\phi)) \cos\phi + (y_2 + \alpha_2(\phi)) \sin\phi$$

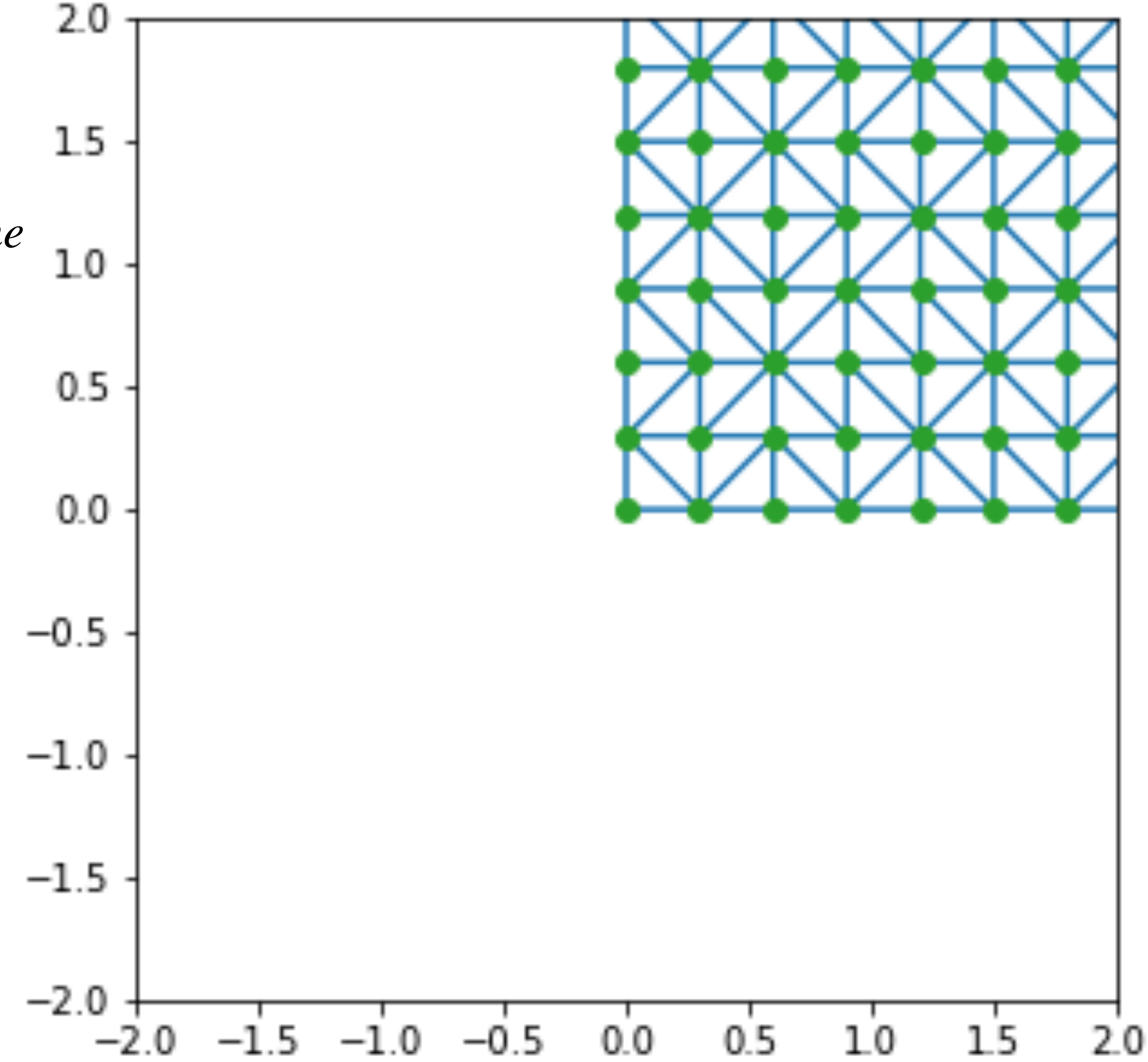
$$F(\phi) = (y_1 + \alpha_1(\phi)) \sin\phi - (y_2 + \alpha_2(\phi)) \cos\phi$$



NUMERICAL IMAGE POSITIONS

Tessellation of image plane.

Divide image plane into triangles.

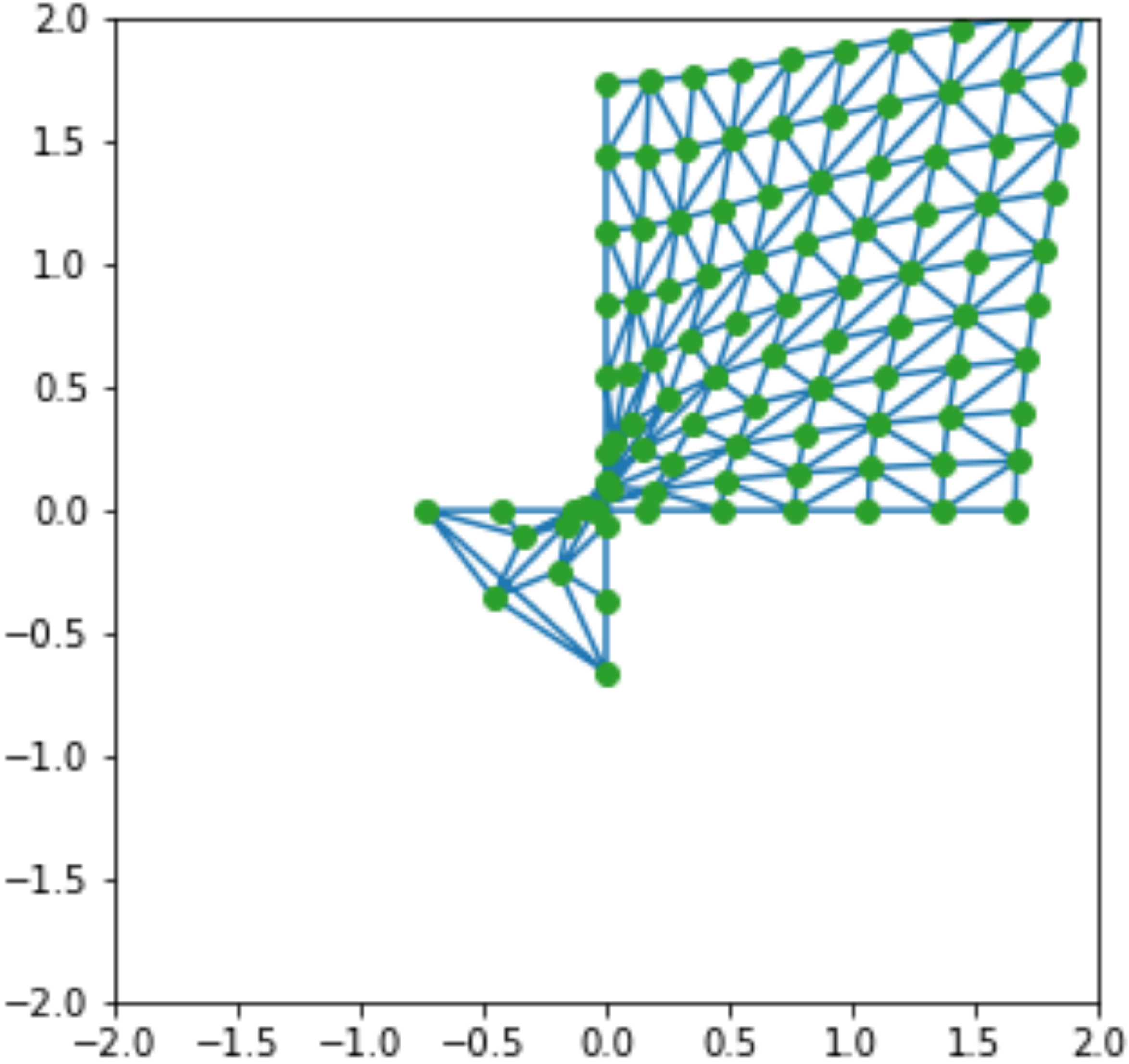


NUMERICAL IMAGE POSITIONS

Tessellation of image plane.

Divide image plane into triangles.

Find the source position of each vertex point by adding the deflection.



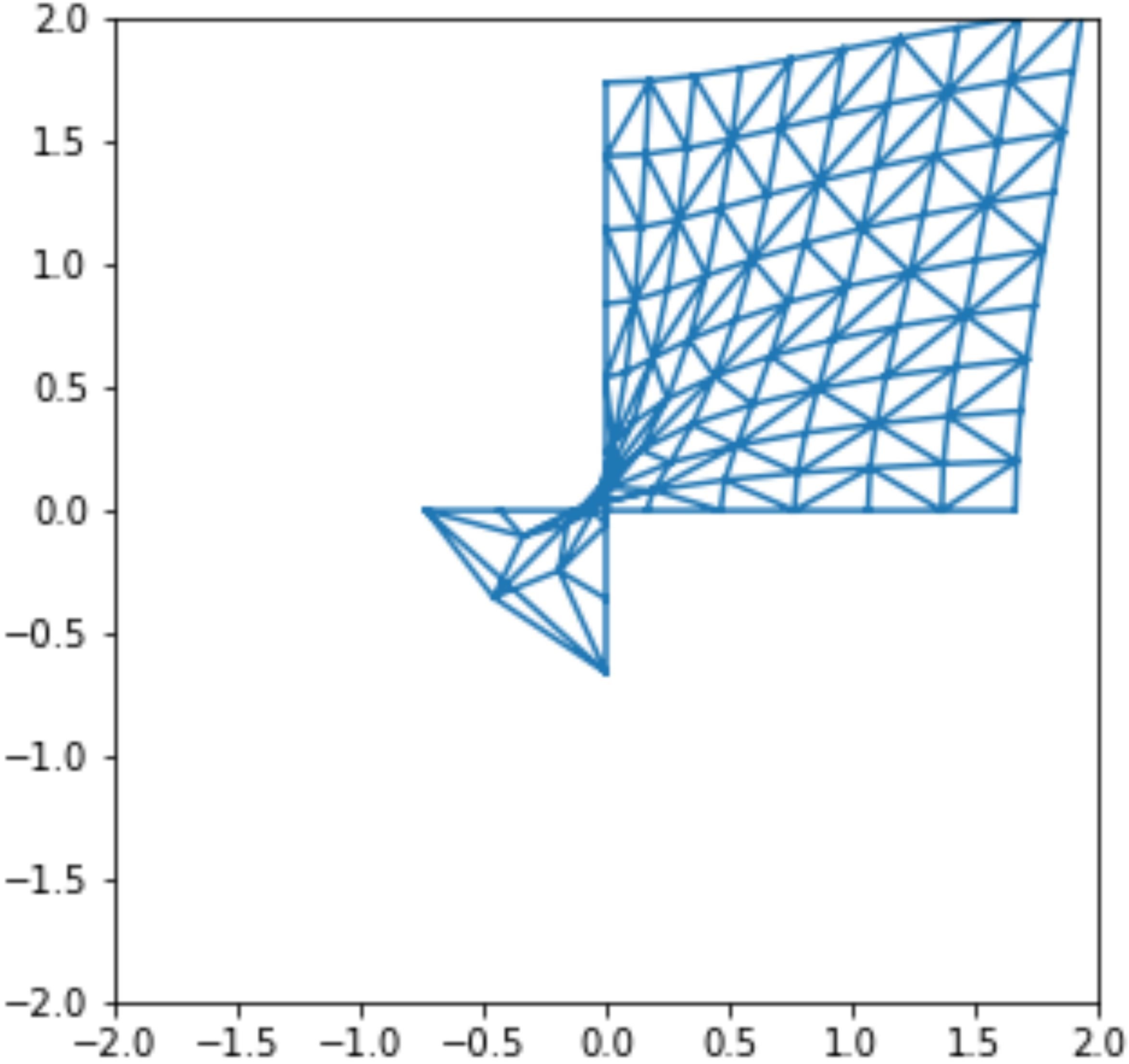
NUMERICAL IMAGE POSITIONS

Tessellation of image plane.

Divide image plane into triangles.

Find the source position of each vertex point by adding the deflection.

Find which triangle the source is in.



NUMERICAL IMAGE POSITIONS

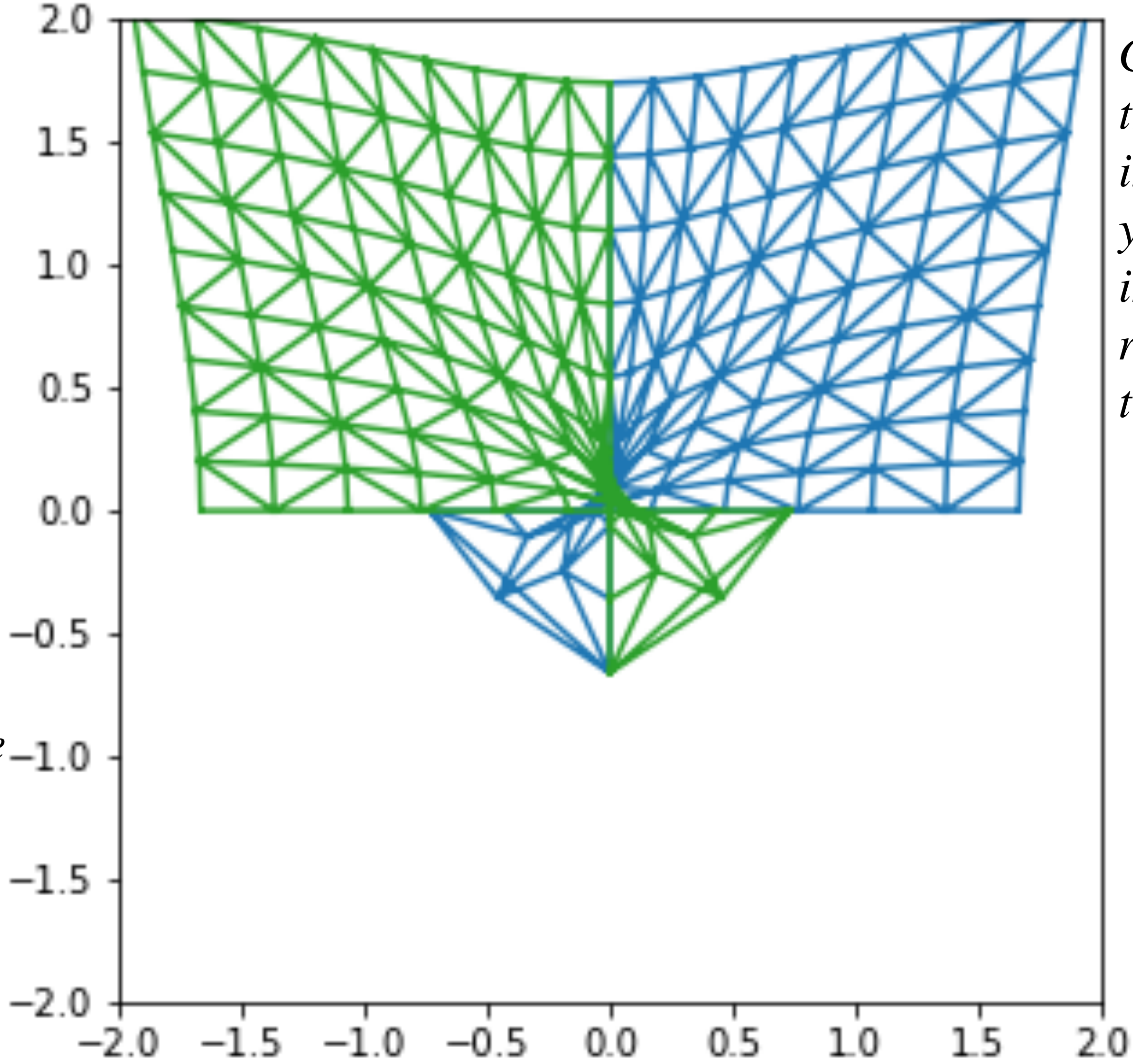
Tessellation of image plane.

Divide image plane into triangles.

Find the source position of each vertex point by adding the deflection.

Find which triangle the source is in.

Since the triangle will overlap, the source will be in more than one triangle.



Going back to the triangle on the image plane will tell you where the images are, to the resolution of the tessellation.

NUMERICAL IMAGE POSITIONS

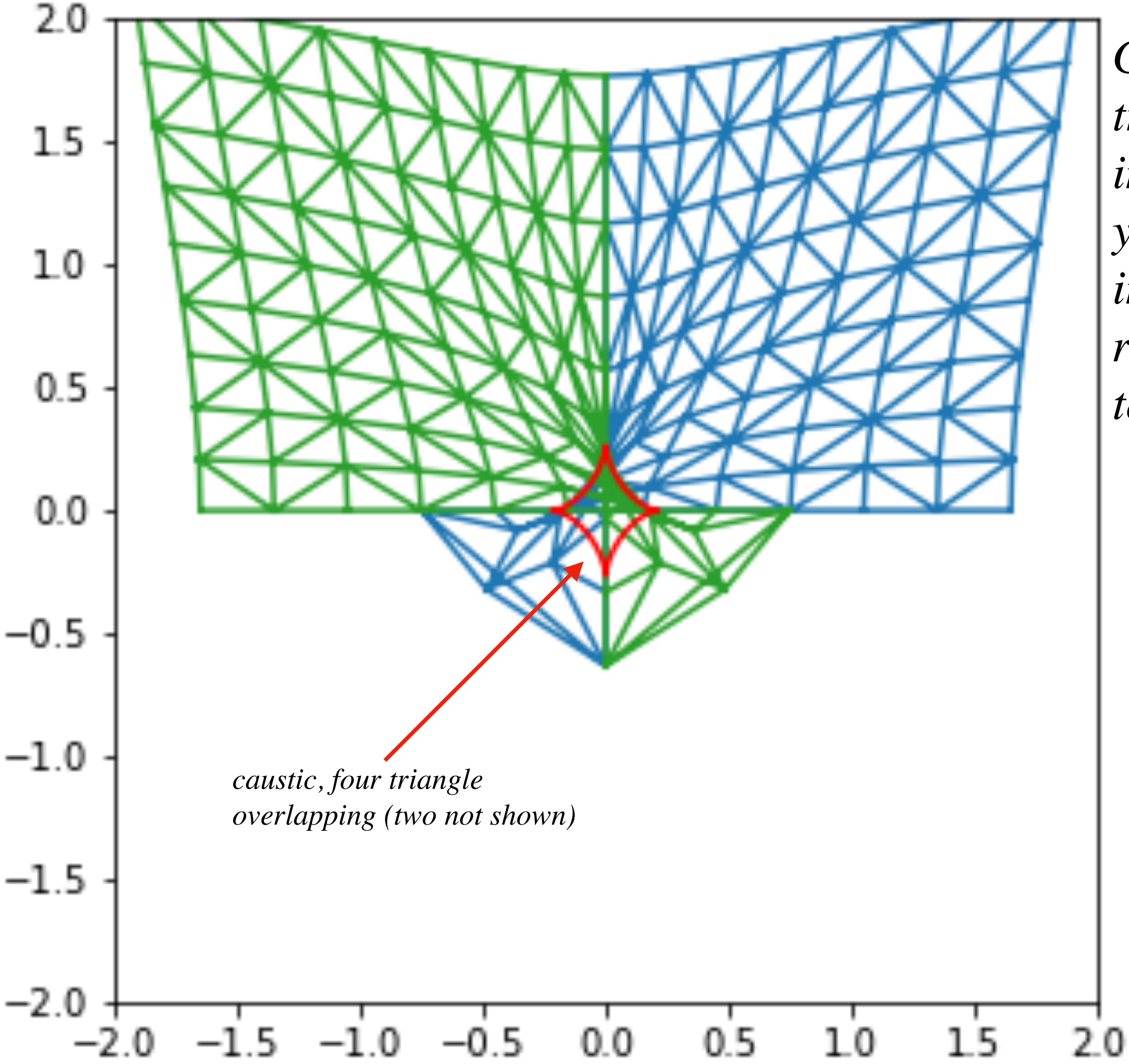
Tessellation of image plane.

Divide image plane into triangles.

Find the source position of each vertex point by adding the deflection.

Find which triangle the source is in.

Since the triangle will overlap, the source will be in more than one triangle.



Going back to the triangle on the image plane will tell you where the images are, to the resolution of the tessellation.

NUMERICAL IMAGE POSITIONS

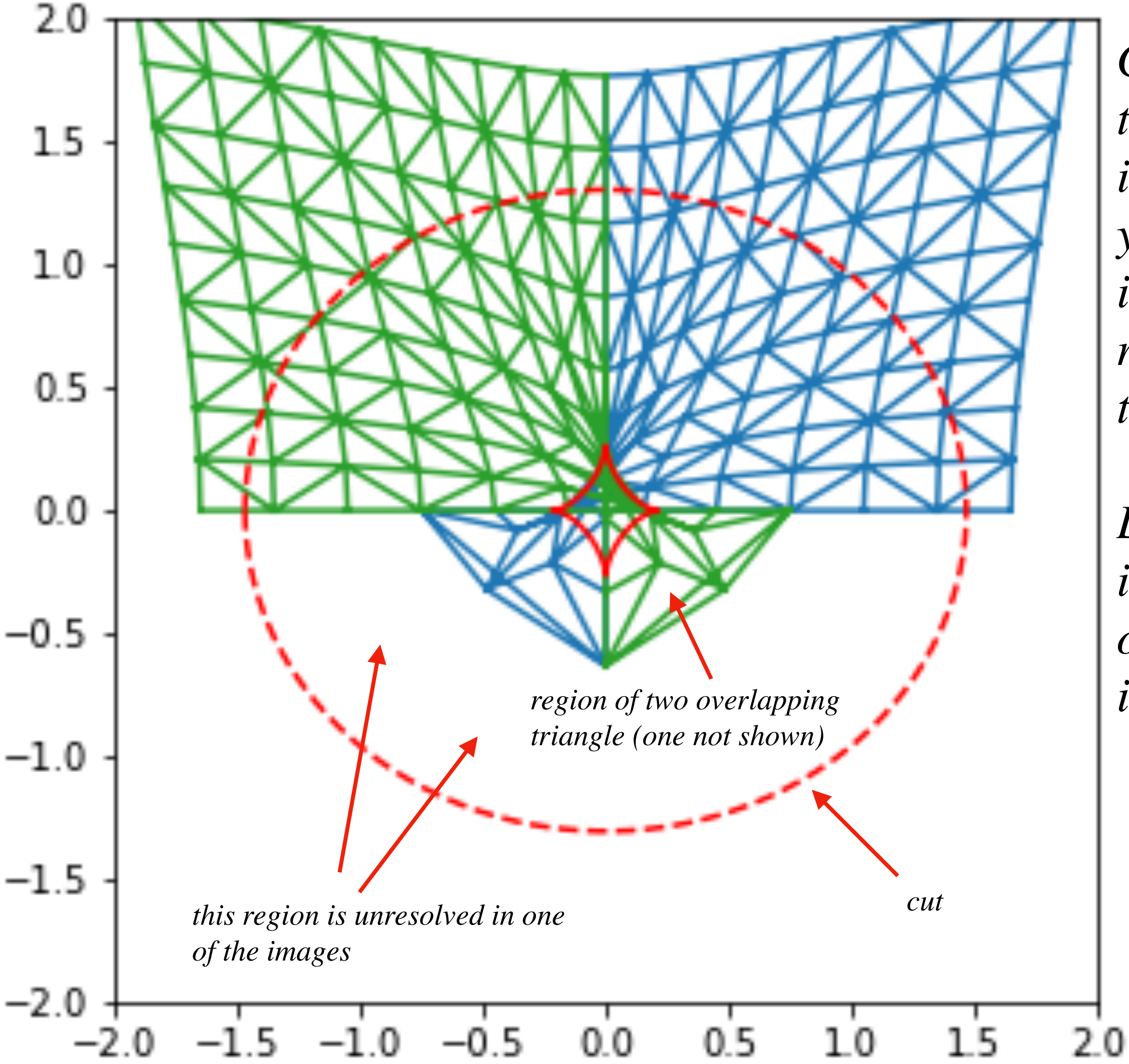
Tessellation of image plane.

Divide image plane into triangles.

Find the source position of each vertex point by adding the deflection.

Find which triangle the source is in.

Since the triangle will overlap, the source will be in more than one triangle.



Going back to the triangle on the image plane will tell you where the images are, to the resolution of the tessellation.

Low magnification images can be lost or spurious included.

ELLIPTICAL POWER-LAW LENS

In general, if $\kappa(x) = \kappa(R)$ $R = \sqrt{q^2x^2 + y^2}$

$$z = x + iy$$

$$\kappa(z) = \frac{\partial \alpha *}{\partial z} \Rightarrow \alpha(z)^* = \frac{2}{qz}\int_0^{R(z)} dr \frac{r\ \kappa(r)}{\sqrt{1 - \frac{1-q^2}{q^2}\frac{r^2}{z^2}}}$$

For an elliptical power-law $\kappa(x) = \frac{3-n}{2}\left(\frac{b}{R}\right)^{n-1}$

$$\alpha(z)^* = 2\frac{\partial \Psi}{\partial z} \Rightarrow \Psi(z) = \frac{1}{3-n}\frac{z\alpha^*(z) + z^*\alpha(z)}{2}$$

$$\Psi(x, y) = \frac{1}{3-n}(x\alpha_x + y\alpha_y)$$

ELLIPTICAL POWER-LAW LENS

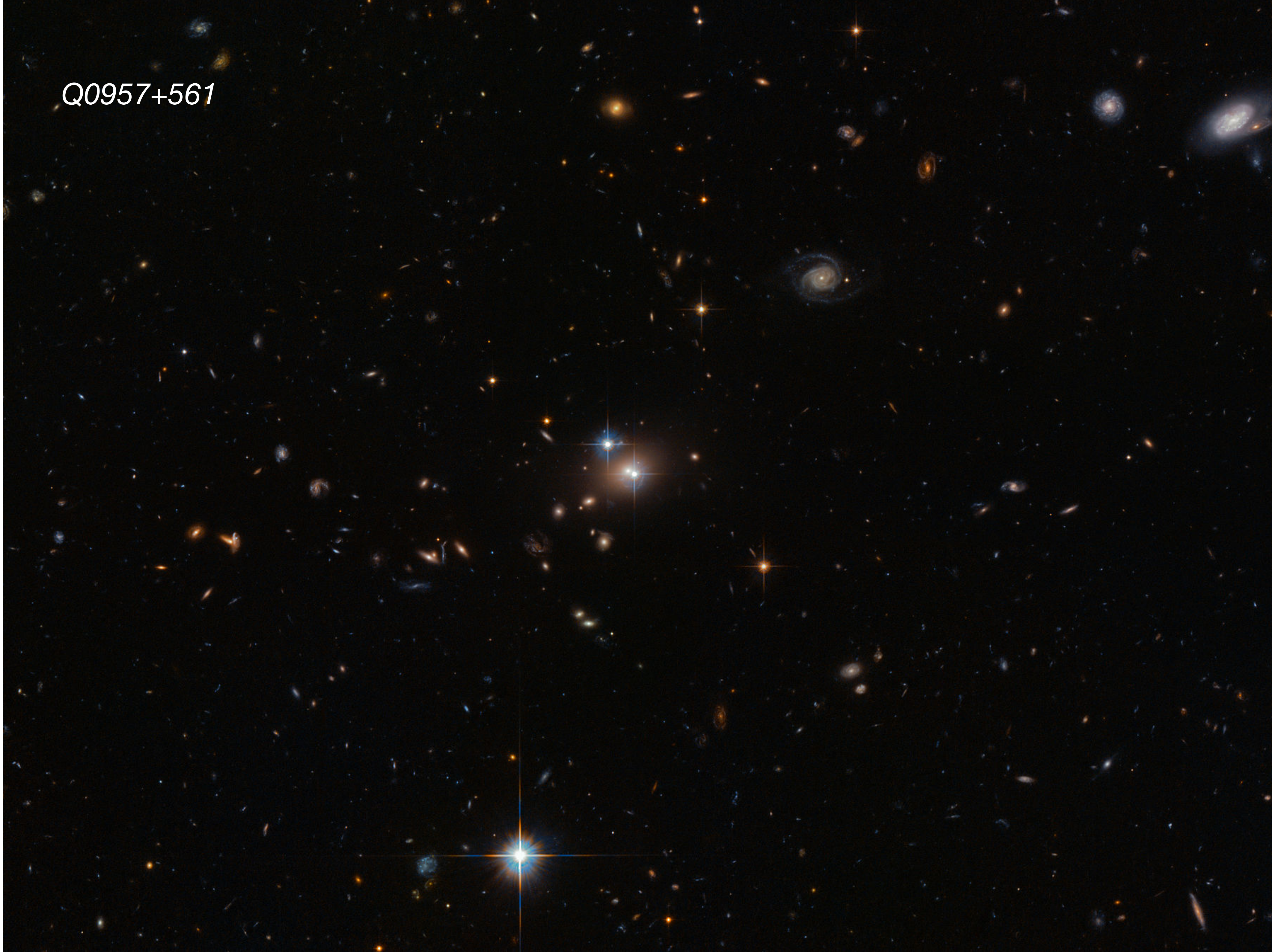
In general, if $\kappa(x) = \kappa(R)$ $R = \sqrt{q^2 x^2 + y^2}$

$$z = x + iy$$

$$\kappa(z) = \frac{\partial \alpha *}{\partial z} \Rightarrow \alpha(z)^* = \frac{2}{qz} \int_0^{R(z)} dr \frac{r \ \kappa(r)}{\sqrt{1 - \frac{1-q^2}{q^2} \frac{r^2}{z^2}}}$$

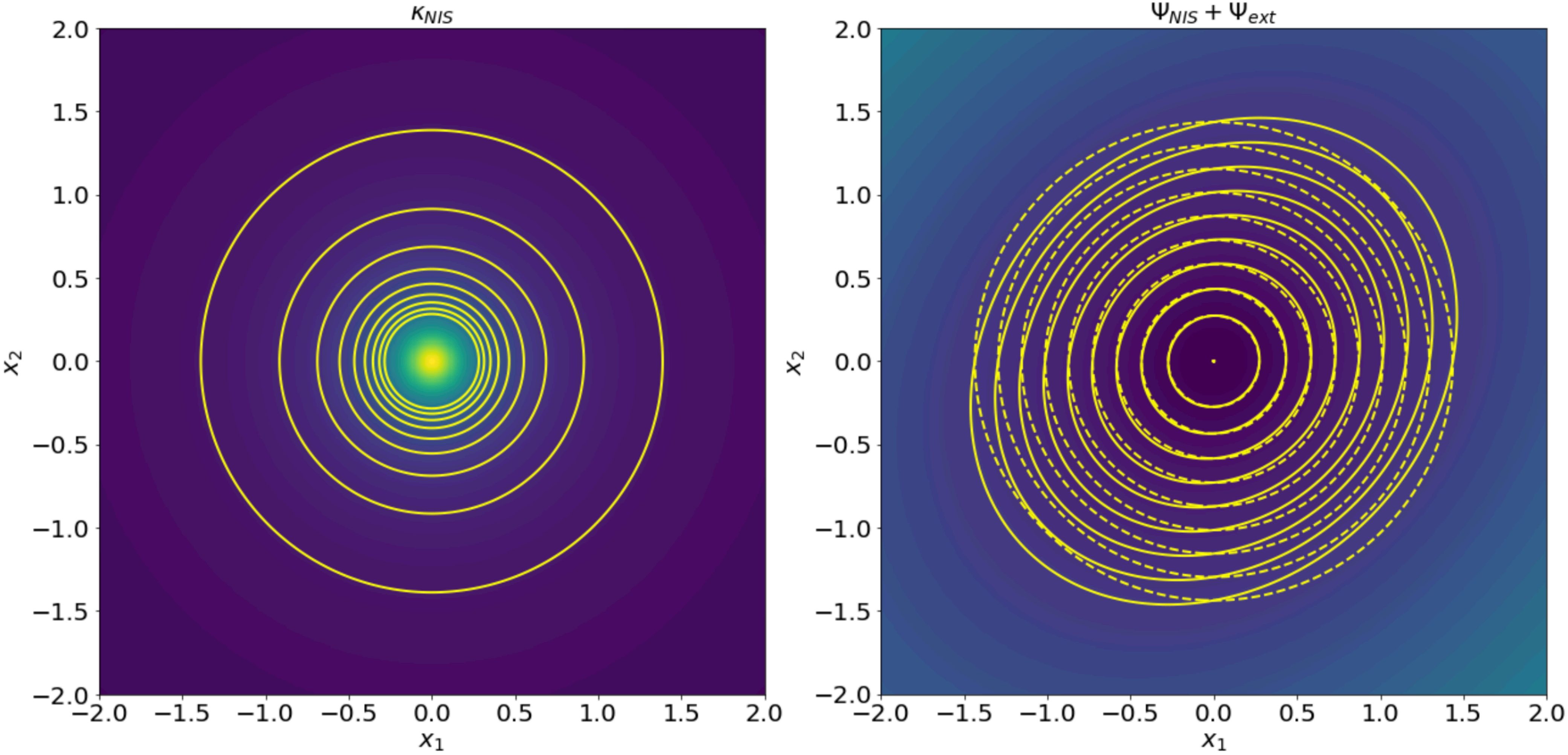
For an elliptical power-law $\kappa(x) = \frac{3-n}{2} \left(\frac{b}{R} \right)^{n-1}$

$$\gamma^*(z) = \frac{\partial \alpha^*}{\partial z} = -\kappa(z) \frac{z^*}{z} + (2-n) \frac{\alpha^*}{z}$$


Q0957+561

EFFECTS OF EXTERNAL SHEAR

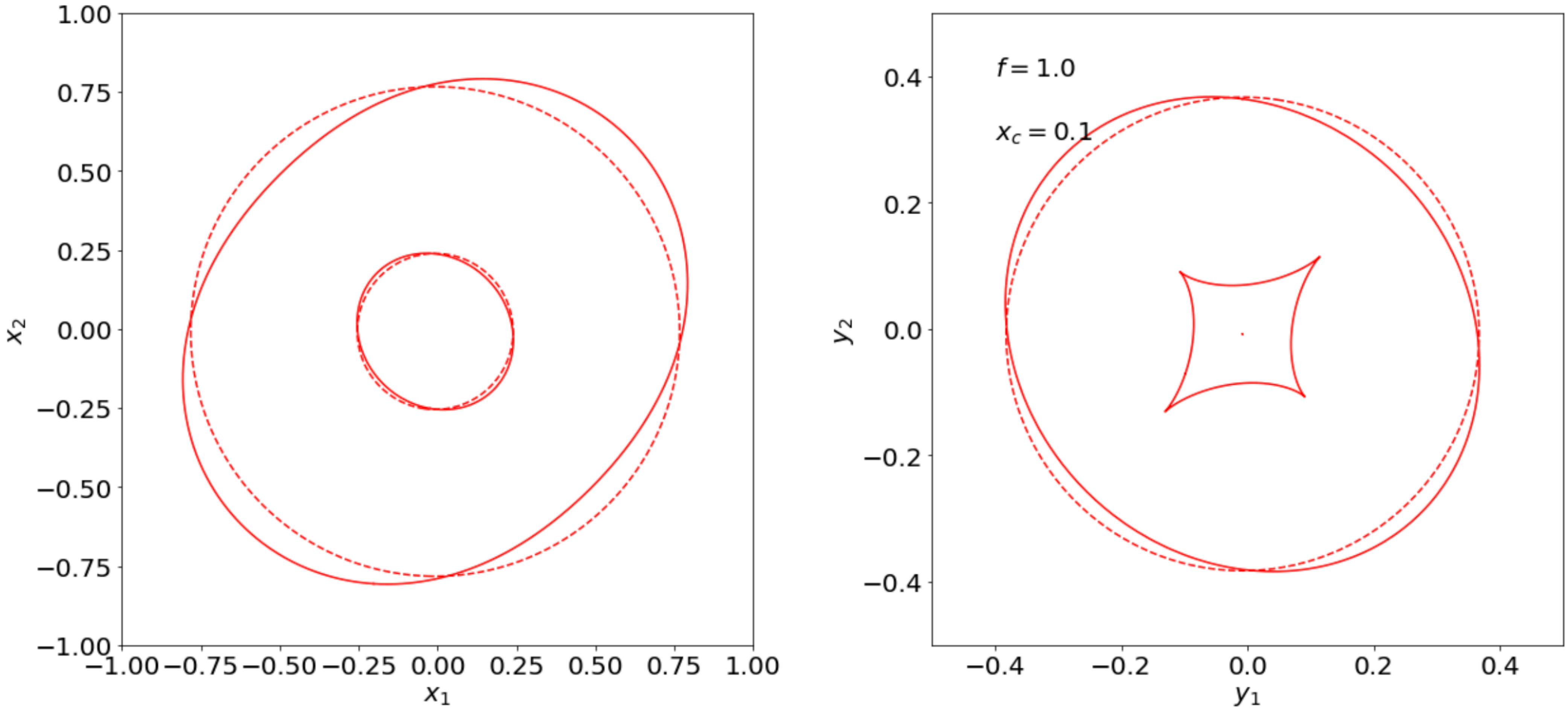
$$\Psi_\gamma(\overrightarrow{x}) = \Psi_{ext}(\overrightarrow{x}) = \frac{\gamma}{2}x^2\cos 2(\phi - \phi_\gamma)$$



Introducing an external shear, the lensing potential becomes ~elliptical

EFFECTS OF EXTERNAL SHEAR

$$\Psi_\gamma(\overrightarrow{x}) = \Psi_{ext}(\overrightarrow{x}) = \frac{\gamma}{2} x^2 \cos 2(\phi - \phi_\gamma)$$



We obtain a Pseudo-Elliptical lens!

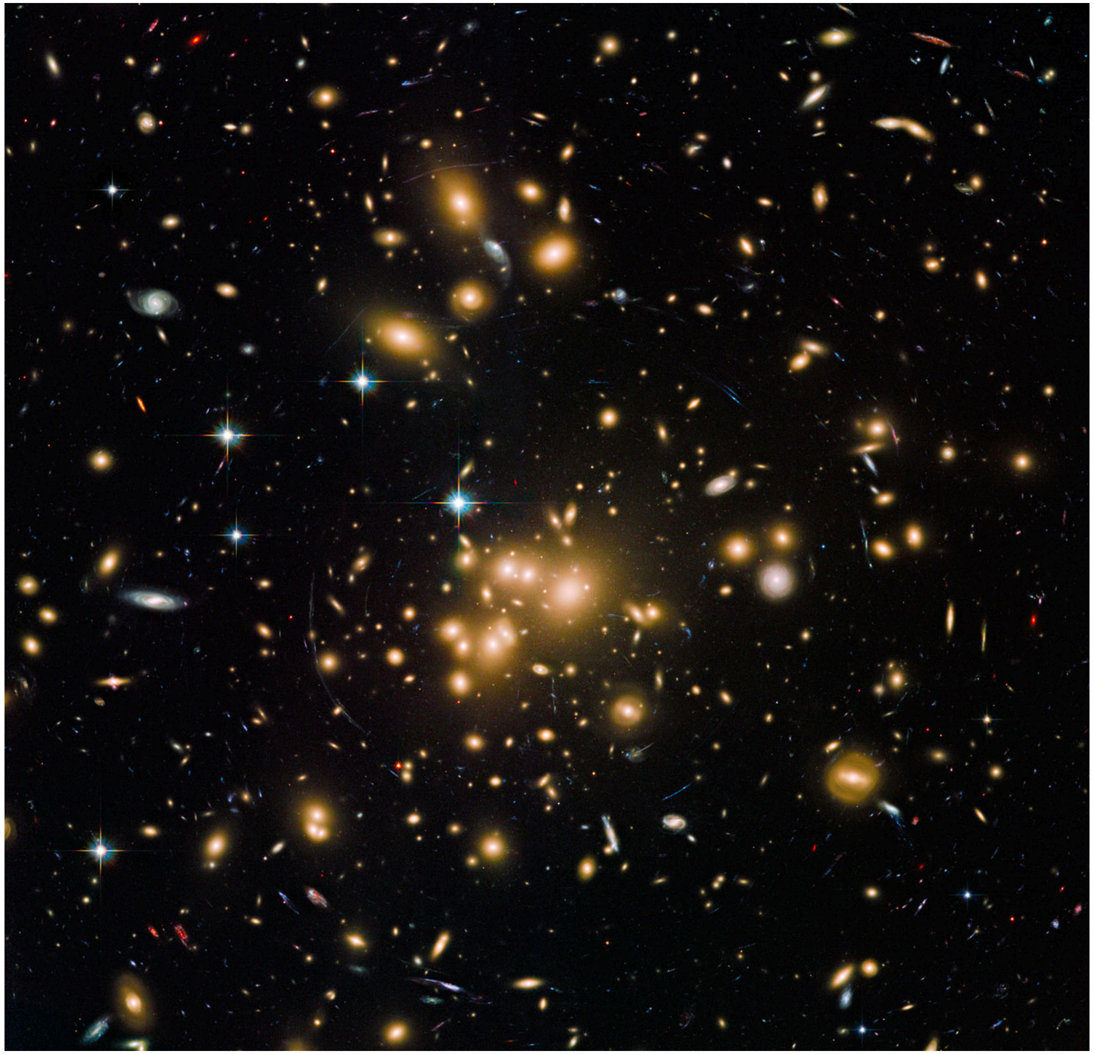
IN PRACTICE…

Realistic lens systems generally must include both intrinsic ellipticity and external shear.

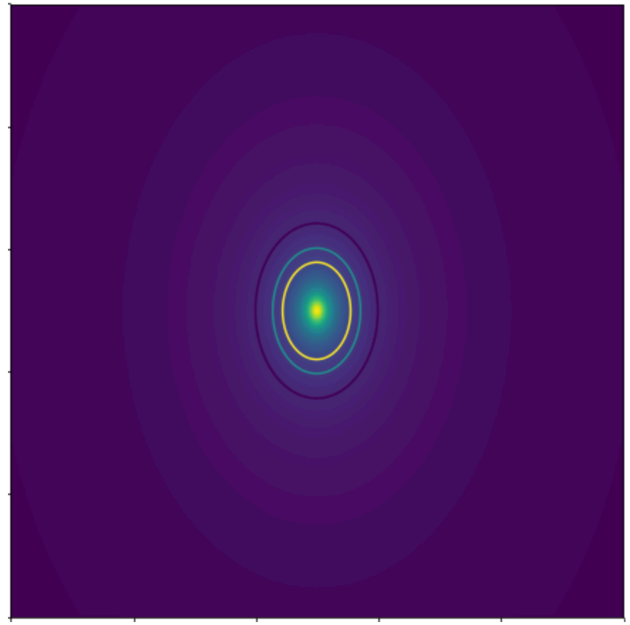
Uniform mass surface density from the lens' environment might exist, but is not detectable without further information. For example the velocity dispersion of the lens.

The direction of the external shear may not be related to the intrinsic ellipticity.

MULTIPLE MASS COMPONENTS



Often a simple lens model composed by a single, smooth mass distribution is not sufficient to describe realistic lenses

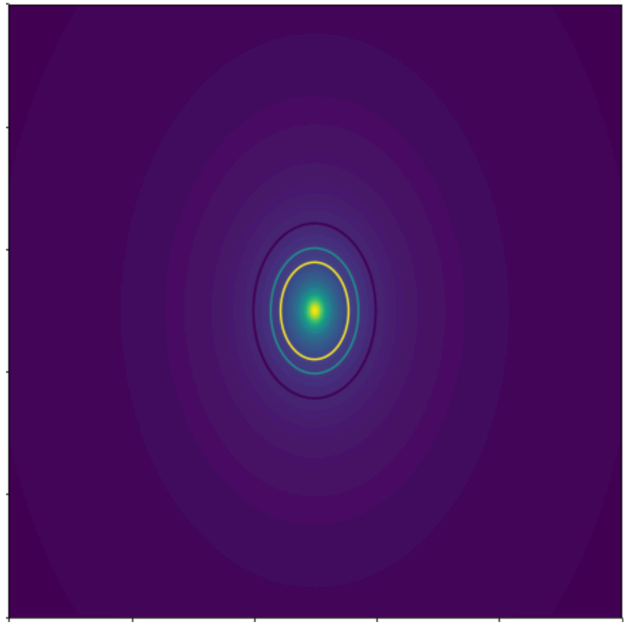


Hierarchy of mass components and combined potential:

$$\Psi(\vec{x}) = \sum_{i=1}^{n_{smooth}} \Psi_{smooth,i}(\vec{x} - \vec{x}_{smooth,i}) + \sum_{i=1}^{n_{sub}} \Psi_{sub,i}(\vec{x} - \vec{x}_{sub,i})$$

MULTIPLE MASS COMPONENTS

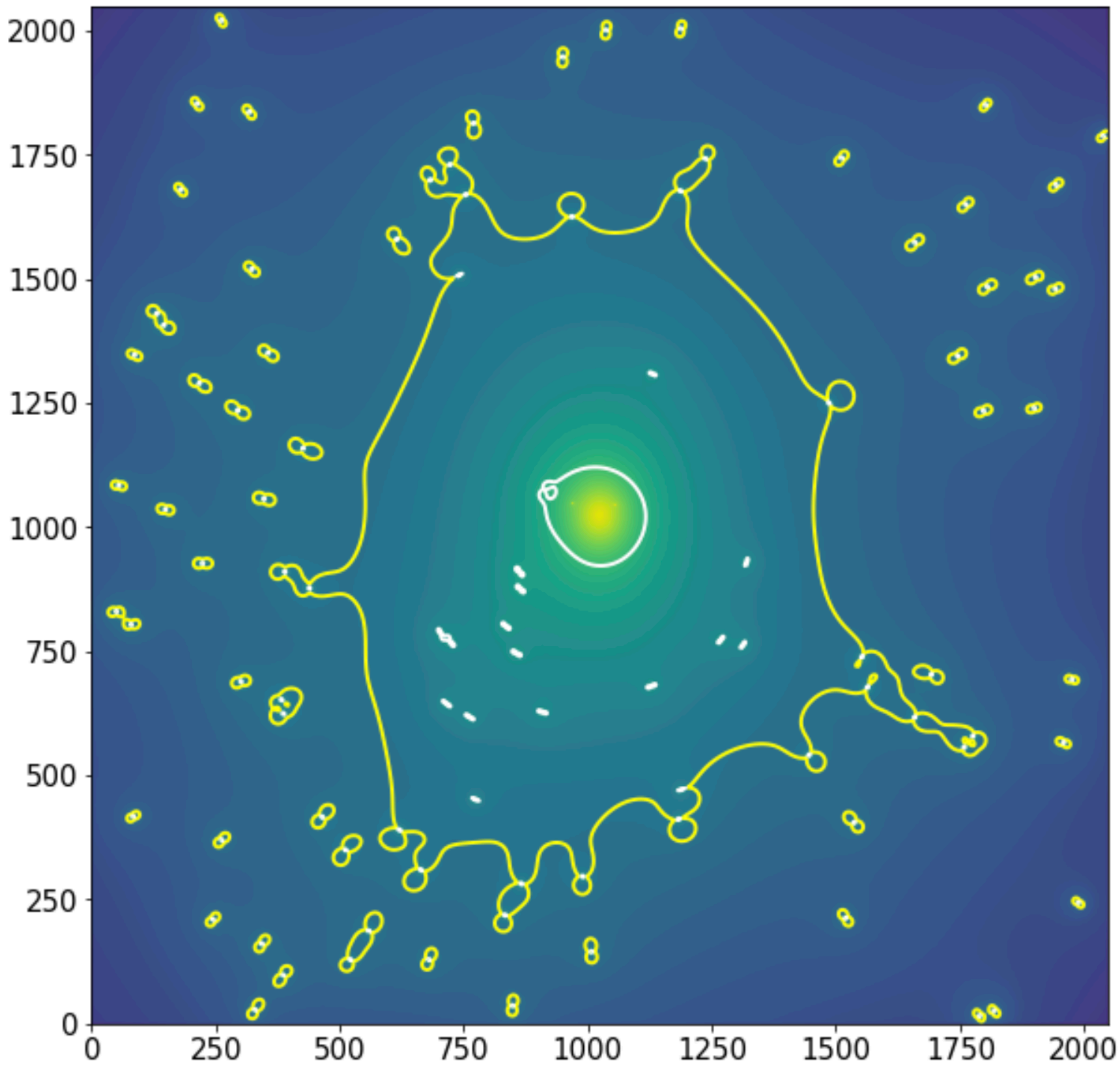
Often a simple lens model composed by a single, smooth mass distribution is not sufficient to describe realistic lenses



Hierarchy of mass components and combined potential:

$$\Psi(\vec{x}) = \sum_{i=1}^{n_{smooth}} \Psi_{smooth,i}(\vec{x} - \vec{x}_{smooth,i}) + \sum_{i=1}^{n_{sub}} \Psi_{sub,i}(\vec{x} - \vec{x}_{sub,i})$$

EXAMPLE



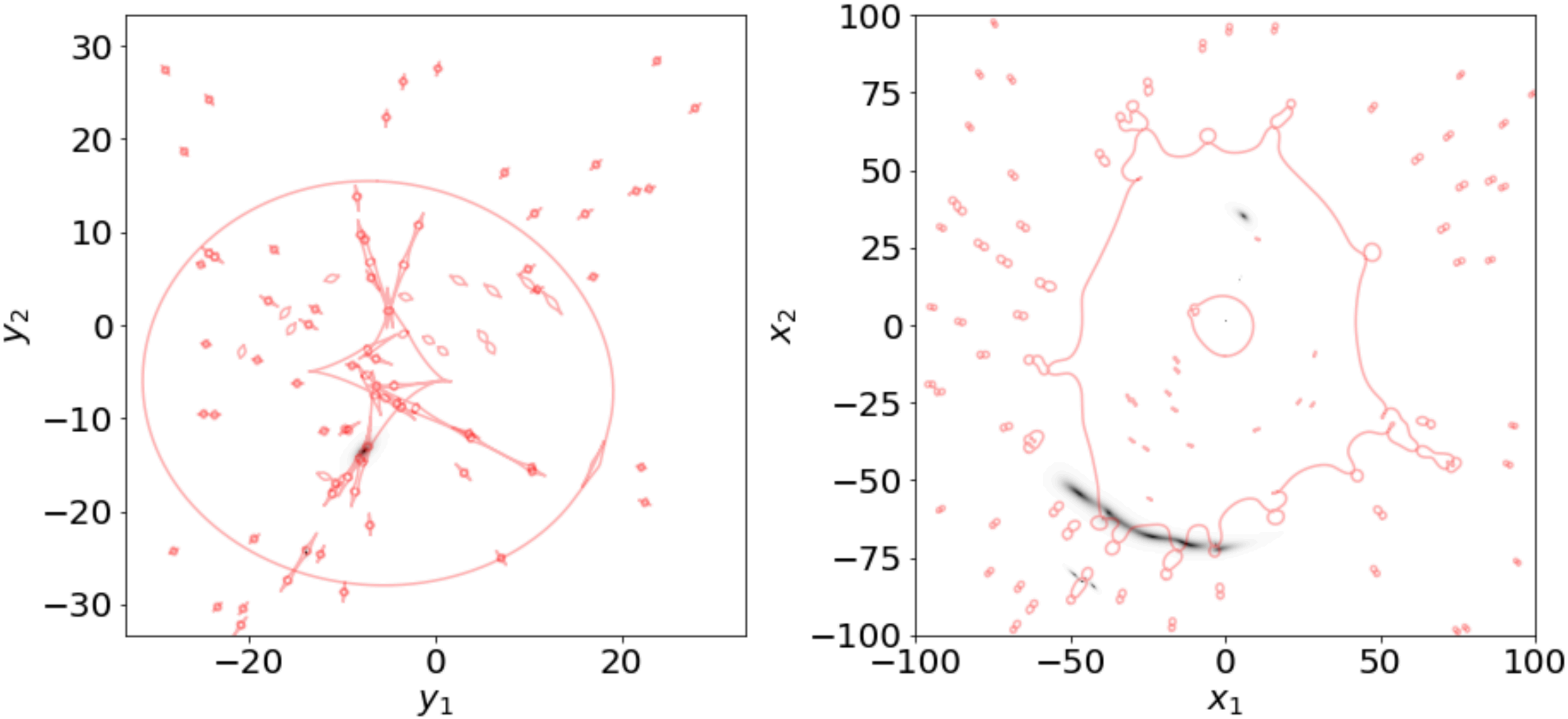
NIE lens + 100 substructures

Primary critical lines show a lot of wiggles

Many secondary critical lines

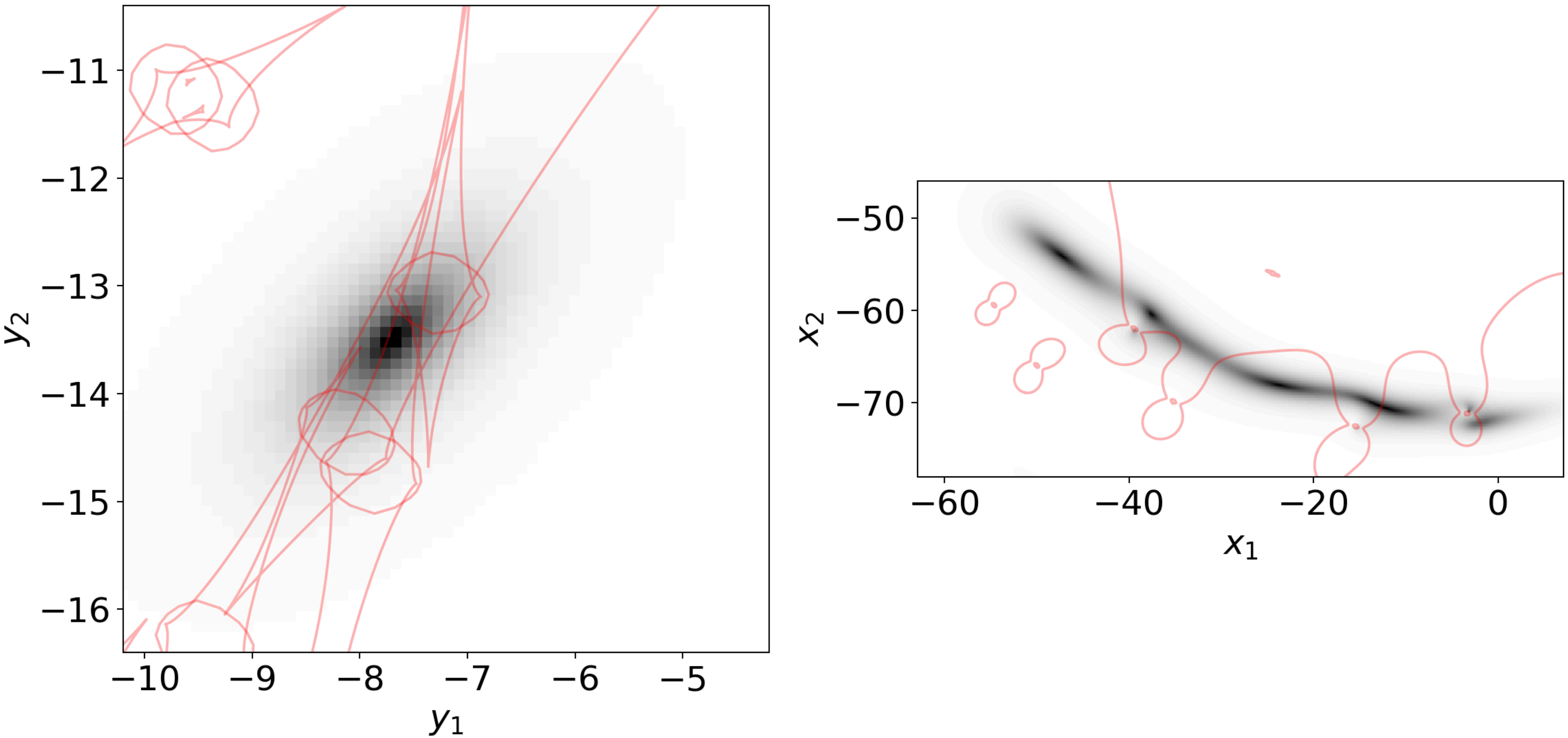
Effects of substructures can be of different kind…

EXAMPLE

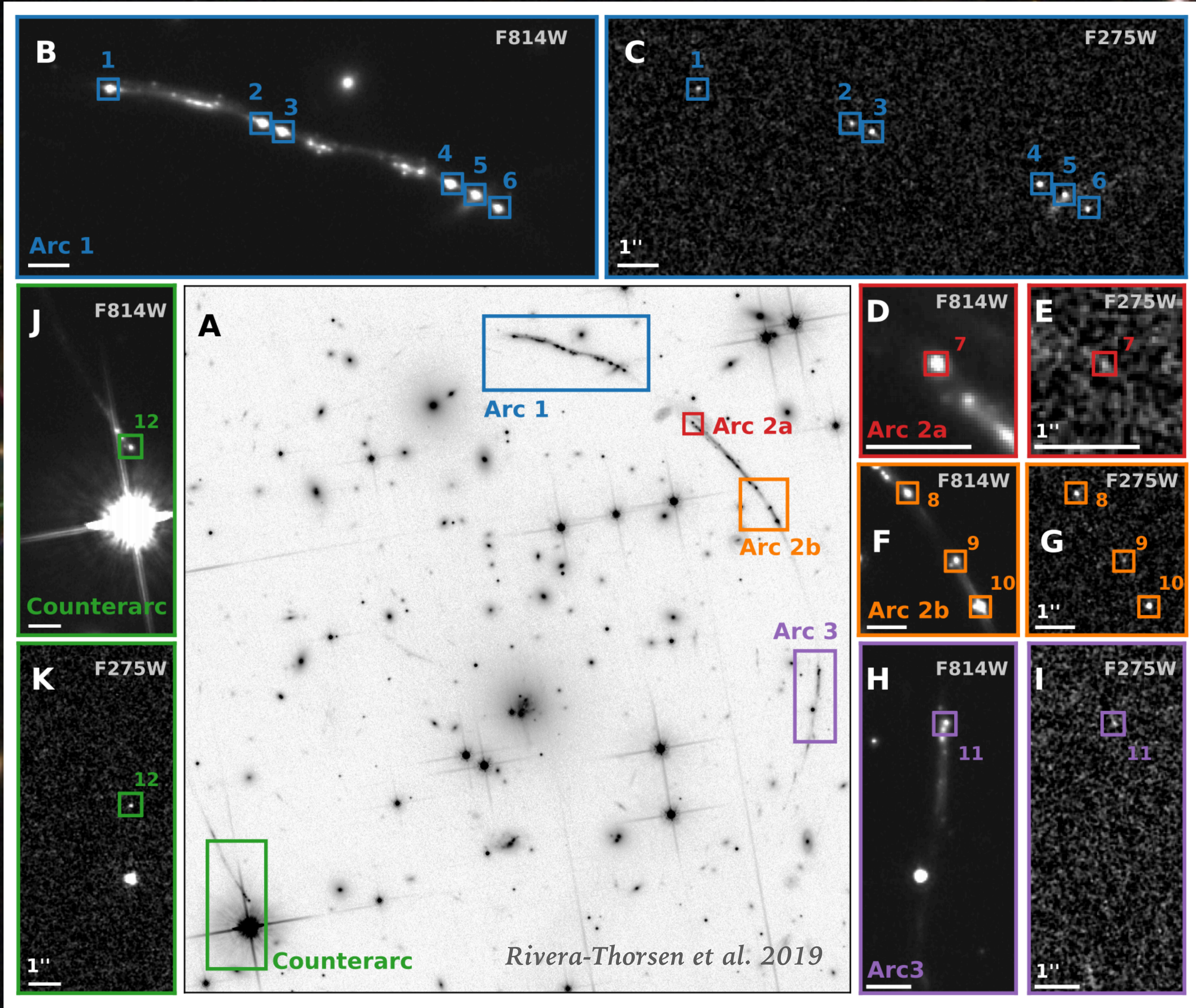


In this case, substructures cause a long cusp-arc (which would be made of 3 images) to split into several more multiple images

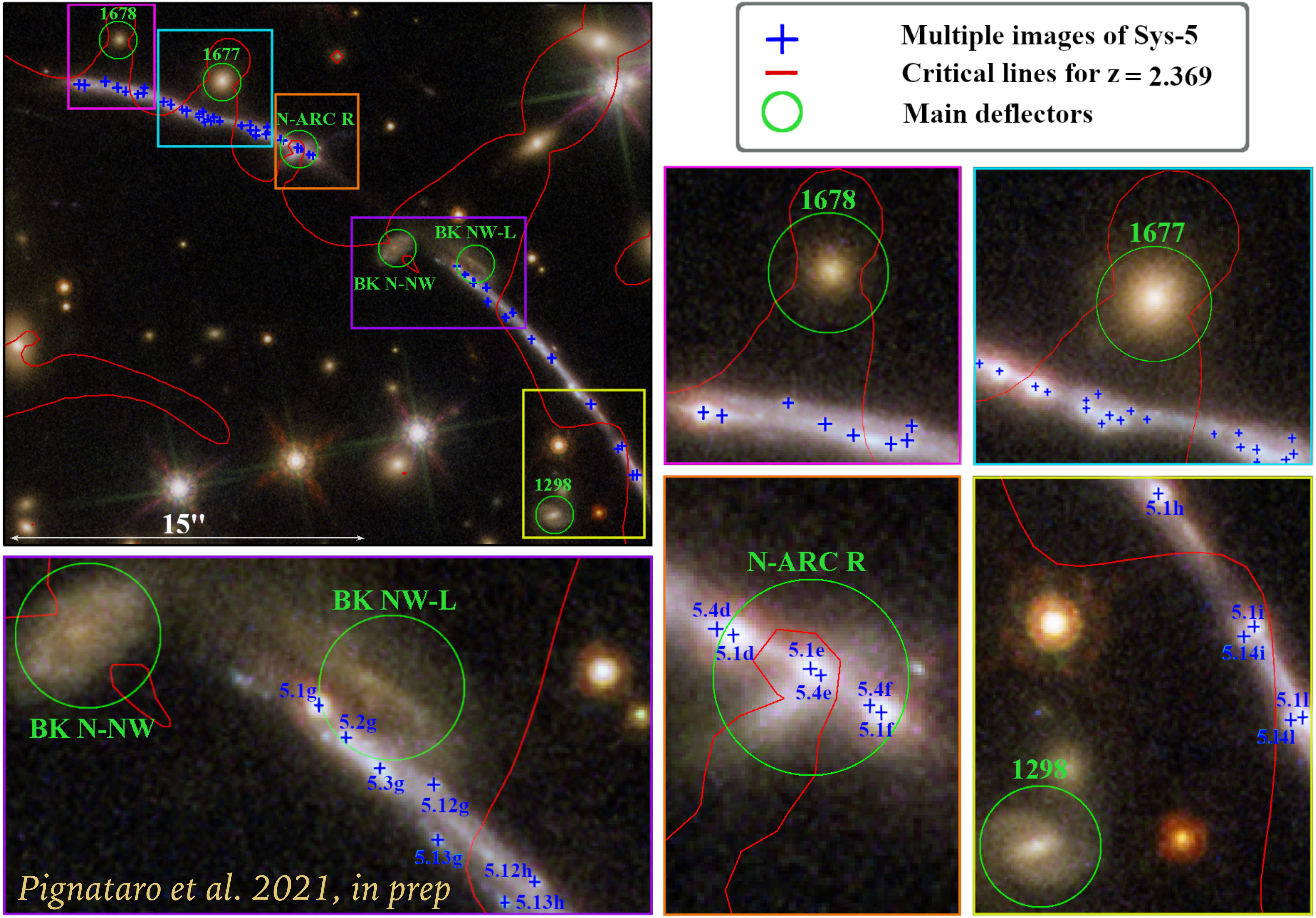
EXAMPLE



In this case, substructures cause a long cusp-arc (which would be made of 3 images) to split into several more multiple images



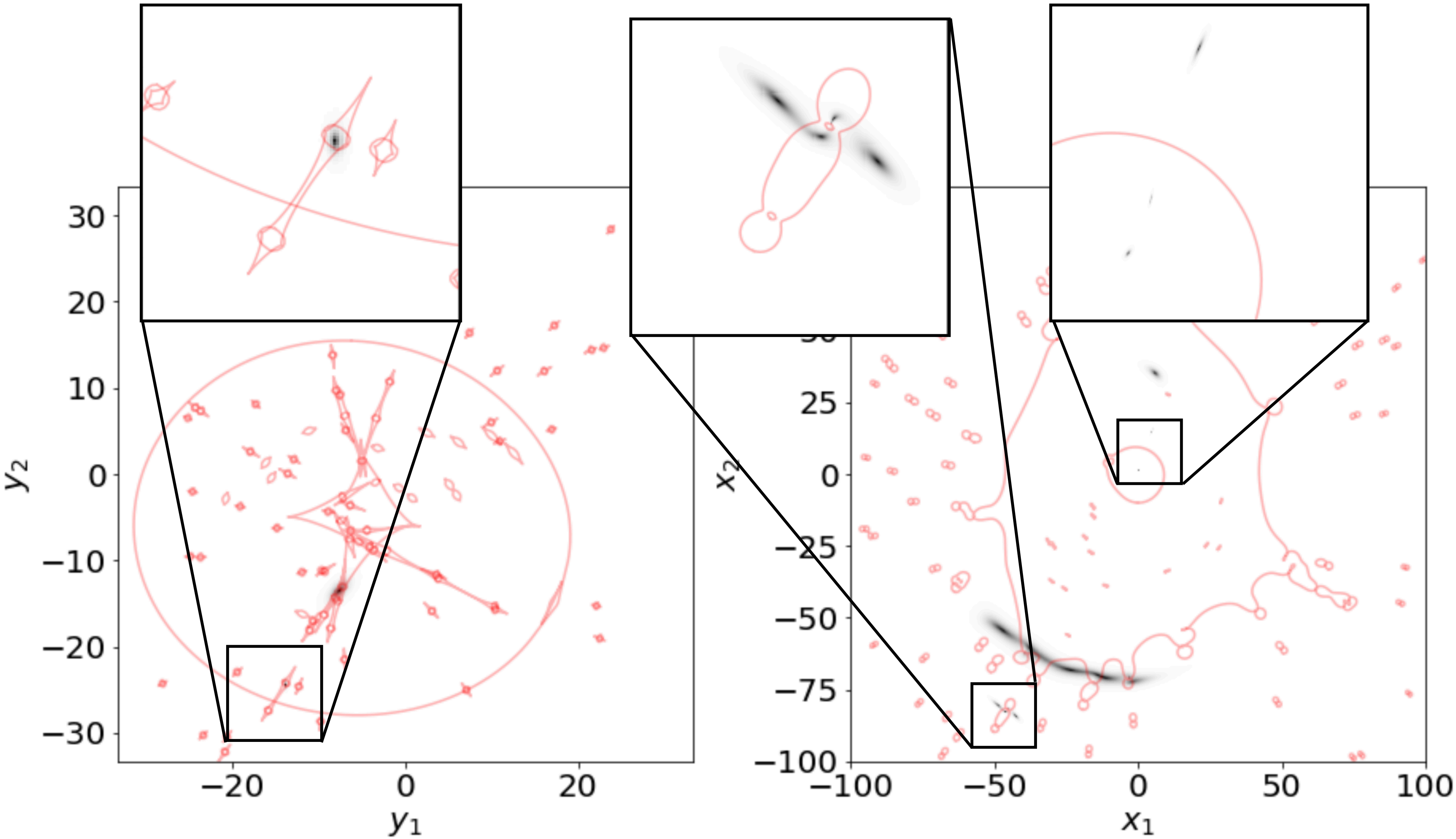
B
F814W
1
2
3
4
5
6
Arc 1
C
F275W
1''
J
12
Counterarc
K
A
Arc 1
Arc 2a
Arc 2b
Arc 3
Counterarc
Rivera-Thorsen et al. 2019
D
7
E
F
8
9
10
G
H
11
I
Arc3

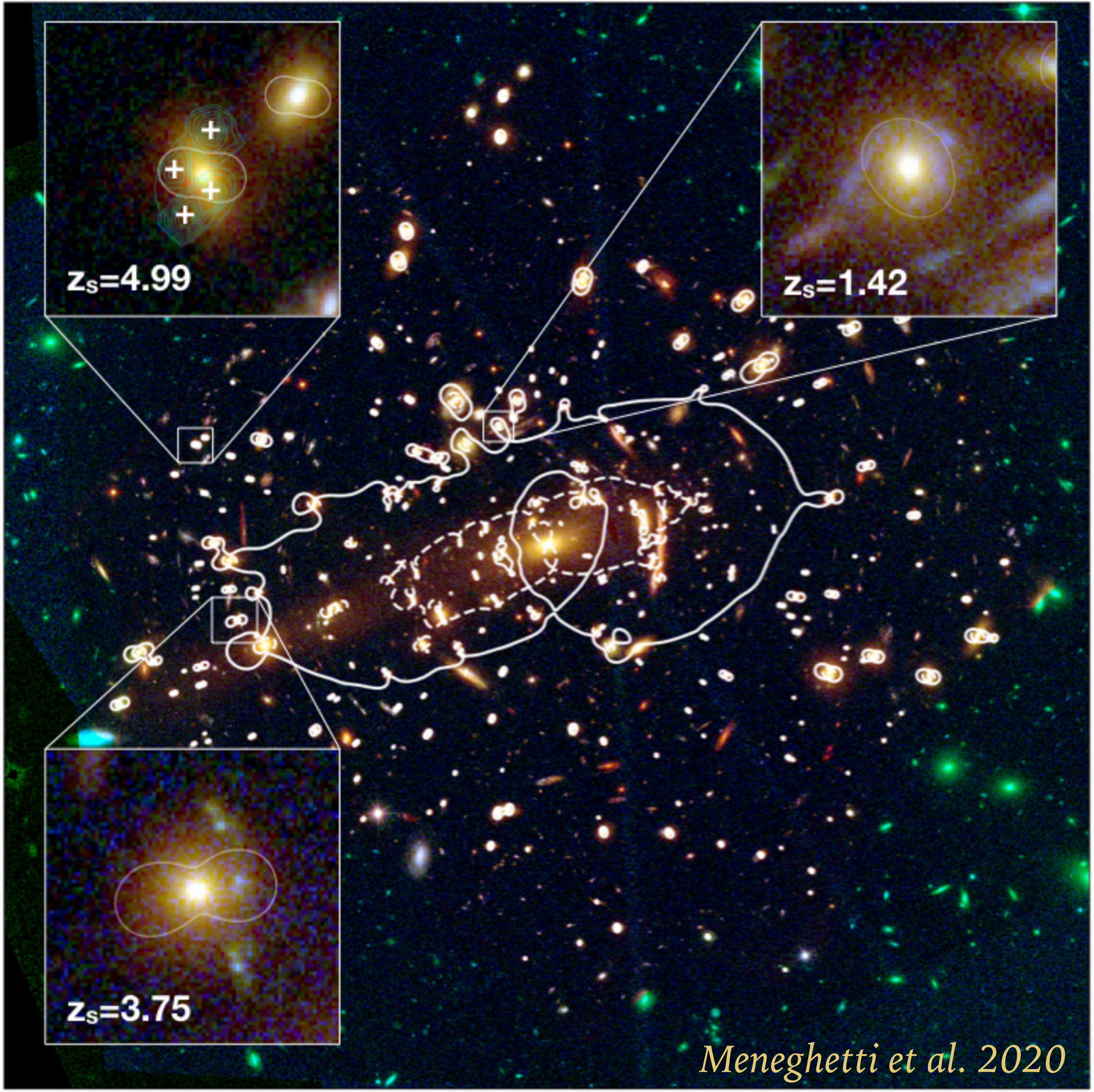


Multiple images of Sys-5
Critical lines for z = 2.369
Main deflectors
1678
1677
N-ARC R
BK NW-L
BK N-NW
1298
15"
5.1g
5.2g
5.3g
5.12g
5.13g
5.12h
5.13h
5.4d
5.1d
5.1e
5.4e
5.4f
5.1f
5.1h
5.1i
5.14i
5.1l
5.14l
Pignataro et al. 2021, in prep

EXAMPLE

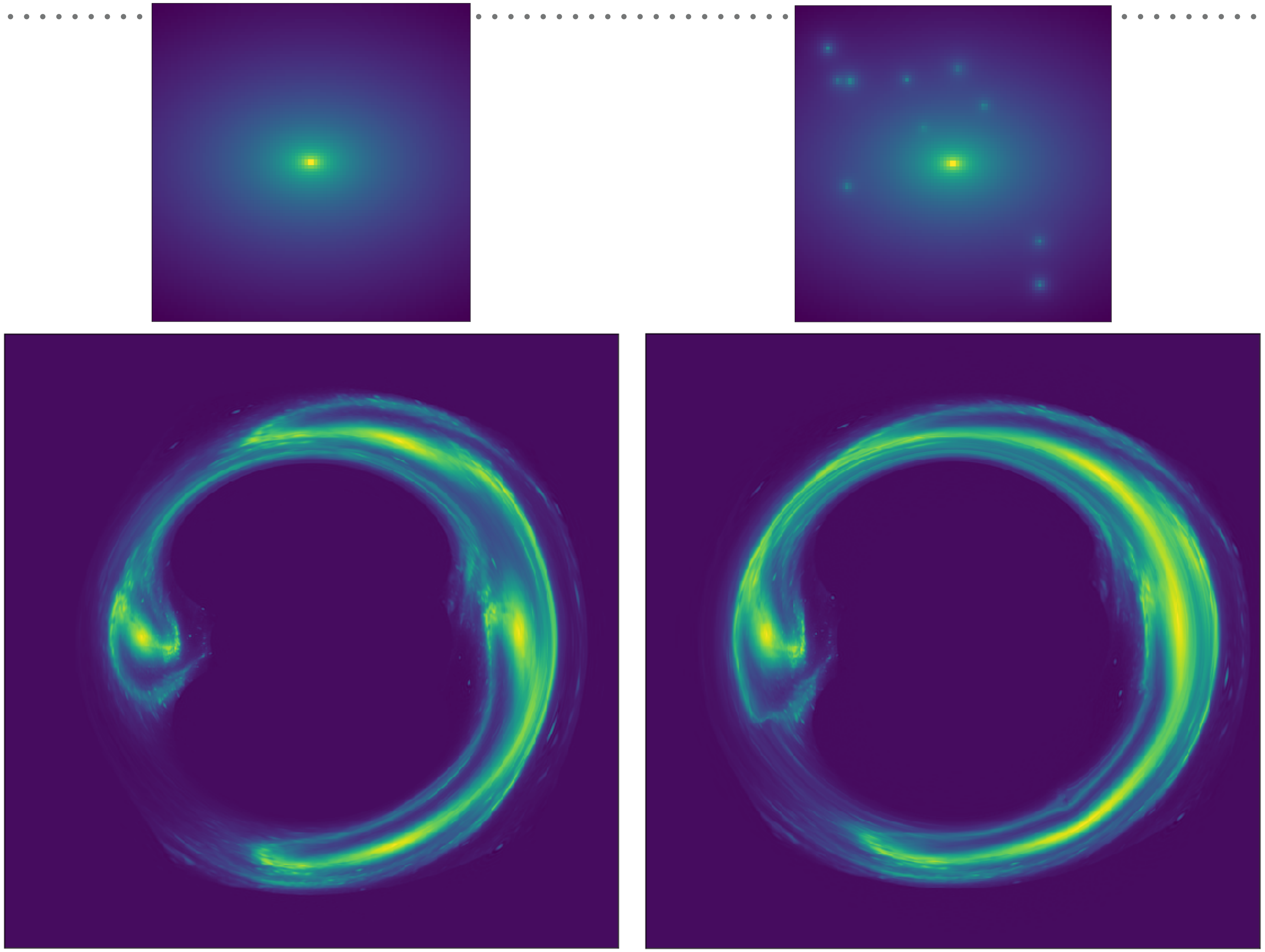
Substructures can individually act as strong lenses





Meneghetti et al. 2020

EXAMPLE



If they are much smaller than the size of the source the effects can be shifts or variation of surface brightness.