

# GRAVITATIONAL LENSING

## 3 - HISTORICAL INTRODUCTION

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*R. Benton Metcalf*  
2022-2023

# AFTER EDDINGTON'S EXPEDITIONS

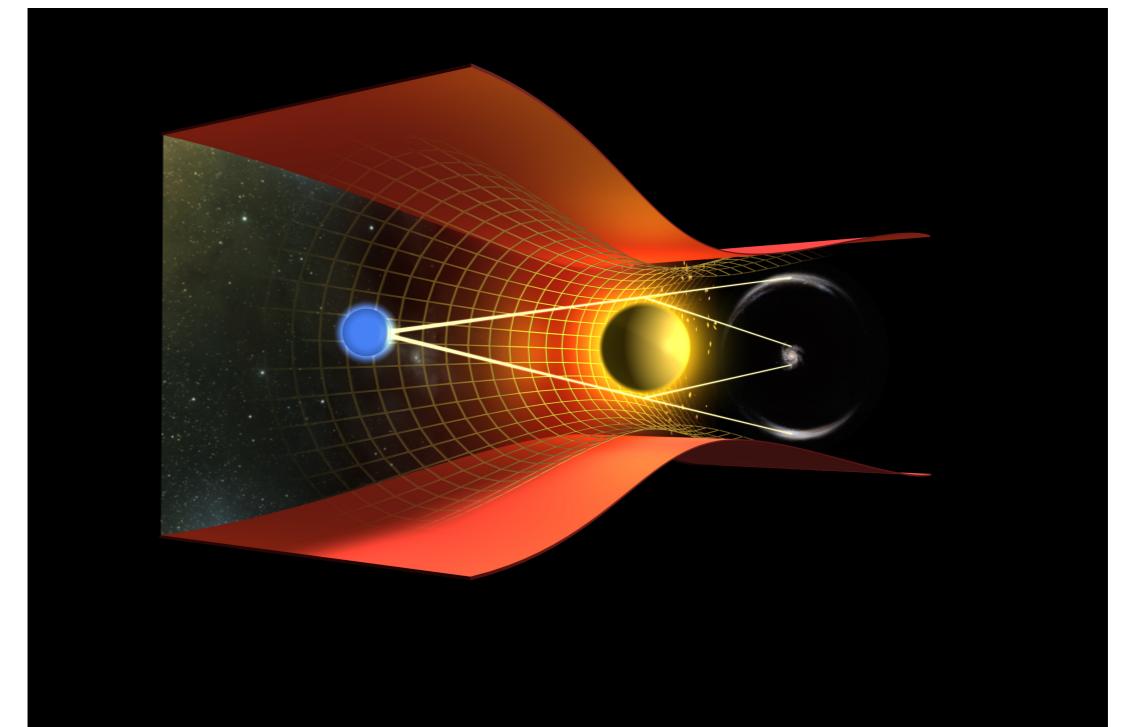
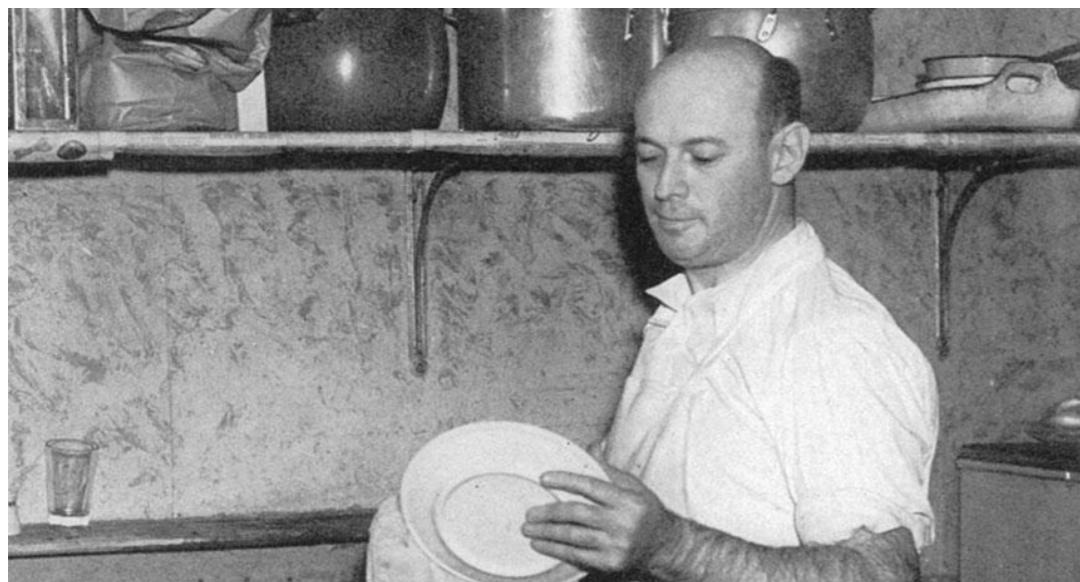
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- After the Eddington expeditions, it was a common belief that light detection was a phenomenon of scarce utility
- Einstein himself shared this belief...
- Very little progress in gravitational lensing in the ~17 years following Eddington expeditions.

# THE ENCOUNTER BETWEEN EINSTEIN AND RUDY MANTLE

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- 1936: Rudy Mandl visits the Science Service office in Washington.
- In particular, he suggests that distant stars might be amplified by closer stars.
- Personnel from the Science Service put him in contact with A. Einstein.



# THE ENCOUNTER BETWEEN EINSTEIN AND RUDY MANDL

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## SCIENCE SERVICE

THE INSTITUTION FOR THE POPULARIZATION OF SCIENCE ORGANIZED 1921 AS A NON-PROFIT CORPORATION, WITH TRUSTEES NOMINATED BY THE NATIONAL ACADEMY OF SCIENCES.



THE NATIONAL RESEARCH COUNCIL, THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, THE E. W. MARSH ESTATE AND THE JOURNALISTIC PROFESSION. WATSON DAVIS, DIRECTOR.

2101 CONSTITUTION AVENUE

WASHINGTON, D.C.

CABLE ADDRESS: SCISERVICE

Sept. 16, 1936

Prof. Albert Einstein  
Institute for Advanced Study  
Princeton, N.J.

Dear Prof. Einstein:

Last spring an apparently sincere layman in science, Rudy Mandl, came into our offices here in the building of the National Academy of Sciences and discussed a proposed test for the relativity theory based on observations during eclipses of the stars.

We supplied Mr. Mandl with a small sum of money to enable him to visit you at Princeton and discuss it with you. On his return he showed us what were apparently authentic letters from you to him regarding his suggestion.

Mr. Mandl has since moved to New York City (108-11 Roosevelt Ave., Corona, L.I.) but before he left he told us that you had agreed to publish his ideas, or at least incorporate some of them in a technical paper to be prepared by you for some scientific journal.

A letter has today come from Mr. Mandl asking us if this paper has yet been published.

Could you tell us what is the status of the Mandl proposal from your point of view, with the promise that anything you would write would be completely confidential?

beautwritter.

Sincerely yours,

Robert D. Potter  
Science Service

# 1936: EINSTEIN PAPER PUBLISHED ON SCIENCE

## LENS-LIKE ACTION OF A STAR BY THE DEVIATION OF LIGHT IN THE GRAVITATIONAL FIELD

SOME time ago, R. W. Mandl paid me a visit and asked me to publish the results of a little calculation, which I had made at his request. This note complies with his wish.

The light coming from a star *A* traverses the gravitational field of another star *B*, whose radius is  $R_o$ . Let there be an observer at a distance  $D$  from *B* and at a distance  $x$ , small compared with  $D$ , from the extended central line  $\overline{AB}$ . According to the general theory of relativity, let  $\alpha_o$  be the deviation of the light ray passing the star *B* at a distance  $R_o$  from its center.

For the sake of simplicity, let us assume that  $\overline{AB}$  is large, compared with the distance  $D$  of the observer from the deviating star *B*. We also neglect the eclipse (geometrical obscuration) by the star *B*, which indeed is negligible in all practically important cases. To permit this,  $D$  has to be very large compared to the radius  $R_o$  of the deviating star.

It follows from the law of deviation that an observer situated exactly on the extension of the central line  $\overline{AB}$  will perceive, instead of a point-like star *A*, a luminous circle of the angular radius  $\beta$  around the center of *B*, where

$$\beta = \sqrt{\alpha_o \frac{R_o}{D}}.$$

It should be noted that this angular diameter  $\beta$  does

not decrease like  $1/D$ , but like  $1/\sqrt{D}$ , as the distance  $D$  increases.

Of course, there is no hope of observing this phenomenon directly. First, we shall scarcely ever approach closely enough to such a central line. Second, the angle  $\beta$  will defy the resolving power of our instruments. For,  $\alpha_o$  being of the order of magnitude of one second of arc, the angle  $R_o/D$ , under which the deviating star *B* is seen, is much smaller. Therefore, the light coming from the luminous circle can not be distinguished by an observer as geometrically different from that coming from the star *B*, but simply will manifest itself as increased apparent brightness of *B*.

The same will happen, if the observer is situated at a small distance  $x$  from the extended central line  $\overline{AB}$ . But then the observer will see *A* as two point-like light-sources, which are deviated from the true geometrical position of *A* by the angle  $\beta$ , approximately.

The apparent brightness of *A* will be increased by the lens-like action of the gravitational field of *B* in the ratio  $q$ . This  $q$  will be considerably larger than unity only if  $x$  is so small that the observed positions of *A* and *B* coincide, within the resolving power of our instruments. Simple geometric considerations lead to the expression

$$q = \frac{l}{x} \cdot \frac{1 + \frac{x^2}{2R^2}}{\sqrt{1 + \frac{x^2}{4R^2}}},$$

where

$$l = \sqrt{\alpha_o D R_o}.$$

# ...BUT OREST CHWOLSON HAD ALREADY CONSIDERED THE EFFECT BACK IN 1924

## Über eine mögliche Form fiktiver Doppelsterne. Von *O. Chwolson*.

Es ist gegenwärtig wohl als höchst wahrscheinlich anzunehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche eines Sternes vorbeigeht, eine Ablenkung erfährt. Ist  $\gamma$  diese Ablenkung und  $\gamma_0$  der Maximumwert an der Oberfläche, so ist  $\gamma_0 \geq \gamma \geq 0$ . Die Größe des Winkels ist bei der Sonne  $\gamma_0 = 1.^{\circ}7$ ; es dürften aber wohl Sterne existieren, bei denen  $\gamma_0$  gleich mehreren Bogensekunden ist; vielleicht auch noch mehr. Es sei  $A$  ein großer Stern (Gigant),  $T$  die Erde,  $B$  ein entfernter Stern; die Winkeldistanz zwischen  $A$  und  $B$ , von  $T$  aus gesehen, sei  $\alpha$ , und der Winkel zwischen  $A$  und  $T$ , von  $B$  aus gesehen, sei  $\beta$ . Es ist dann

$$\gamma = \alpha + \beta.$$

Ist  $B$  sehr weit entfernt, so ist annähernd  $\gamma = \alpha$ . Es kann also  $\alpha$  gleich mehreren Bogensekunden sein, und der Maximumwert von  $\alpha$  wäre etwa gleich  $\gamma_0$ . Man sieht den Stern  $B$  von der Erde aus an zwei Stellen: direkt in der Richtung  $TB$  und außerdem nahe der Oberfläche von  $A$ , analog einem Spiegelbild. Haben wir mehrere Sterne  $B, C, D$ , so würden die Spiegelbilder umgekehrt gelegen sein wie in

Petrograd, 1924 Jan. 28.

## Antwort auf eine Bemerkung von *W. Anderson*.

Daß ein Elektronengas einer Substanz mit negativem Brechungsvermögen optisch äquivalent sein müßte, kann bei dem heutigen Stand unserer Kenntnisse nicht zweifelhaft sein, da dasselbe einer Substanz von verschwindend kleiner Eigenfrequenz äquivalent ist.

Aus der Bewegungsgleichung

$$\epsilon X = \mu d^2x/dt^2$$

eines Elektrons von der elektrischen Masse  $\epsilon$  und der ponderablen Masse  $\mu$  folgt nämlich für einen sinusartig pendelnden Prozeß von der Frequenz  $\nu$  die Gleichung

$$\epsilon X = -(2\pi\nu)^2 \mu x.$$

Berücksichtigt man, daß  $\epsilon x$  das »Moment« eines schwingenden Elektrons ist, so erhält man für die Polarisation  $p = n\epsilon x$  eines Elektronengases mit  $n$  Elektronen pro Volumeinheit

einem gewöhnlichen Spiegel, nämlich in der Reihenfolge  $D, C, B$ , wenn von  $A$  aus gerechnet wird ( $D$  wäre am nächsten zu  $A$ ).



Der Stern  $A$  würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestände aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um  $A$  in der Entfernung  $\gamma_0 - \beta$  liegen, würden von dem Stern  $A$  gleichsam eingefangen werden. Sollte zufällig  $TAB$  eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern  $A$  von einem Ring umgeben erscheinen.

Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

*O. Chwolson.*

$$p = -\epsilon^2 n / [\mu (2\pi\nu)^2] \cdot X.$$

Hieraus folgt, daß die scheinbare Dielektrizitätskonstante

$$D = 1 + 4\pi p/X = 1 - \epsilon^2 n / (\pi\mu\nu^2)$$

ist.  $\sqrt{D}$  ist in diesem Falle der Brechungsexponent, also jedenfalls kleiner als 1. Es erübrigert sich bei dieser Sachlage, auf das Quantitative einzugehen.

Es sei noch bemerkt, daß ein Vergleich des Elektronengases mit einem Metall unstatthaft ist, weil die bei der elementaren Theorie der Metalle zugrundegelegte »Reibungskraft« bei freien Elektronen fehlt; das Verhalten der letzteren ist allein durch die Einwirkung des elektrischen Feldes und durch die Trägheit bedingt.

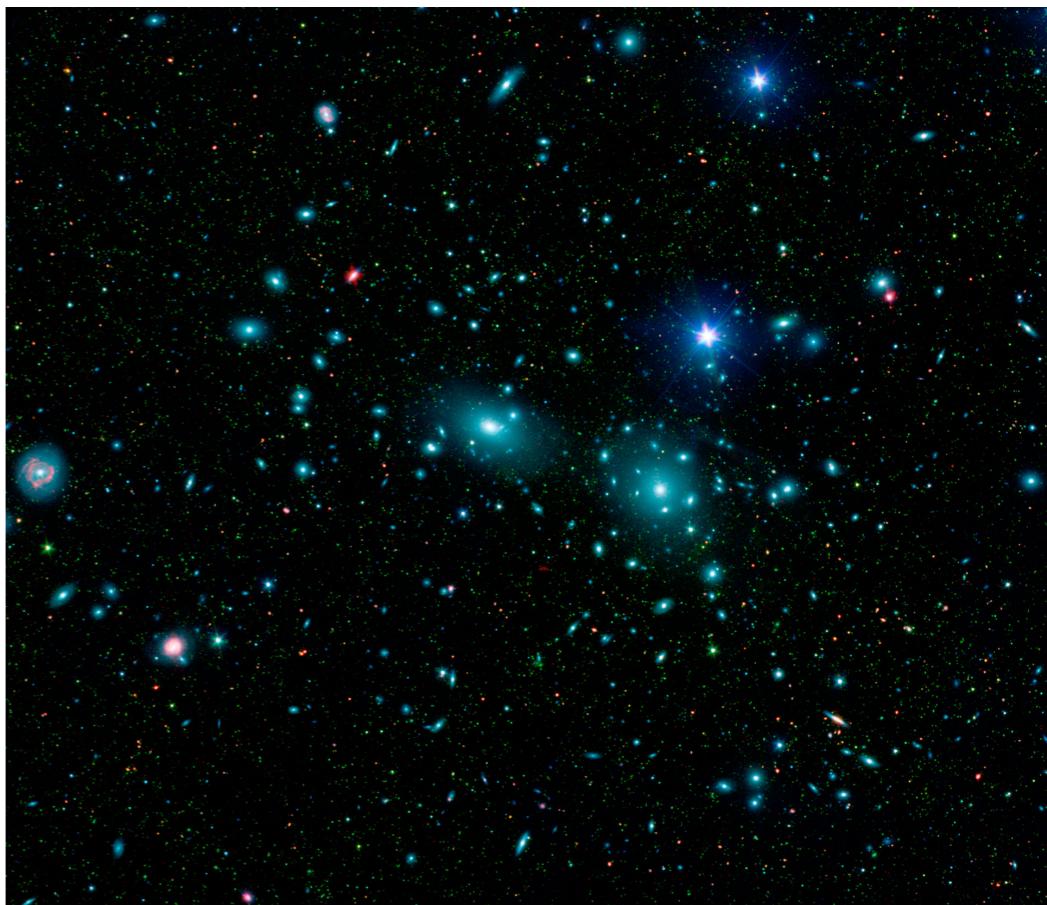
Berlin, 1924 April 15.

*A. Einstein.*

# OBSERVABLE OR NOT? USEFUL OR NOT?

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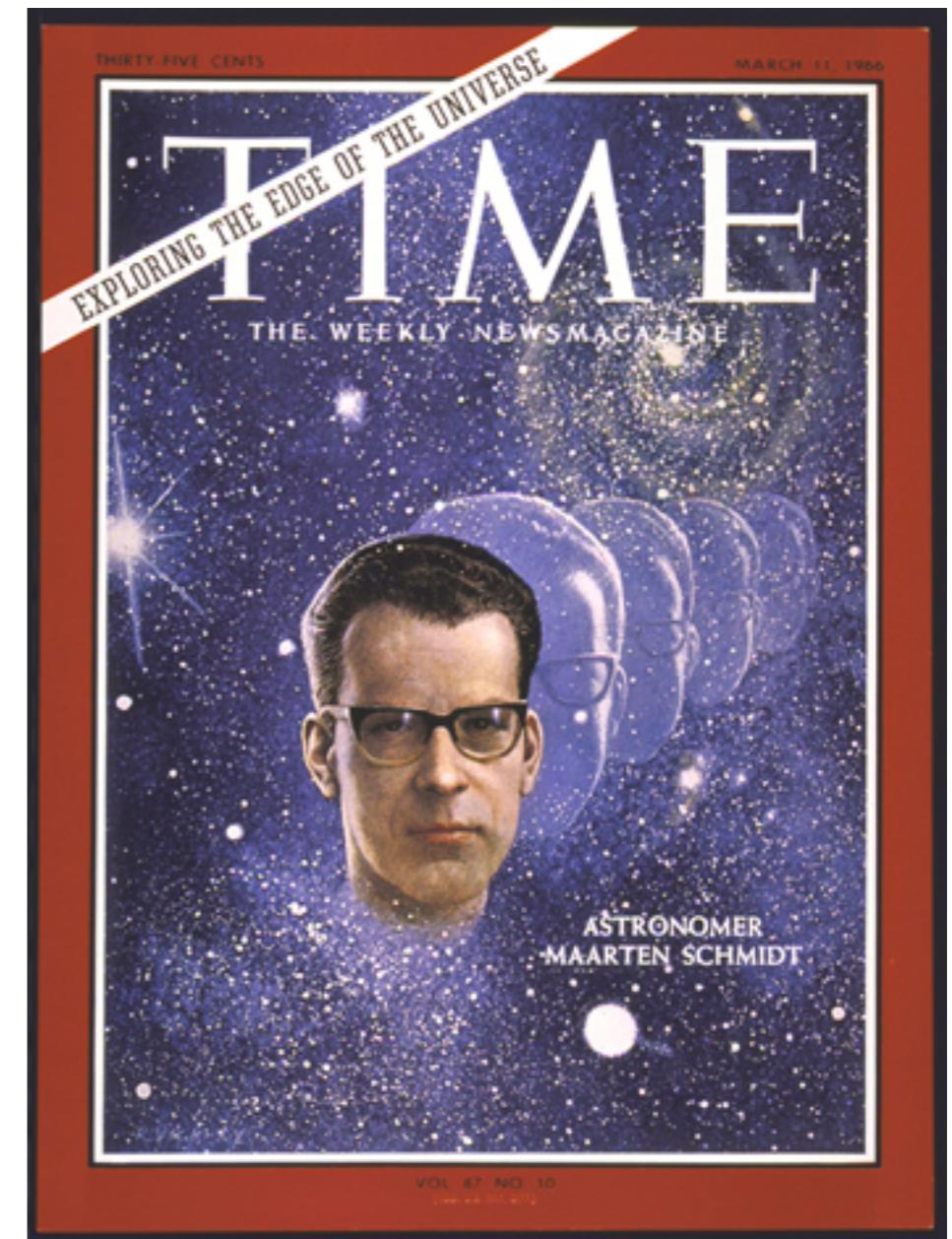
- In 1924, Hubble had shown the existence of extragalactic nebulae...
- 1937: Fritz Zwicky pointed out that there are lenses that are far more massive than stars: galaxies and clusters of galaxies
- In 1937, he also anticipated that gravitational lensing could be used measure the mass of nebulae (that he realised to be much larger than inferred from their light, when he studied the motion of galaxies in the Coma cluster, in 1933)



# TOWARD THE FIRST OBSERVATIONS OF GRAVITATIONAL LENSING

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- 1959: Maarten Schmidt discovers QSOs
- 1963: Yuri Klimov considers galaxy-galaxy lensing: multiple images, rings
- 1964: Sidney Liebes discusses lensing of stars on stars, of stars on globular clusters, of stars in GC on other stars in GC, of stars in our galaxy on stars in the Andromeda galaxy, of stars on gravitational waves
- 1964: Sjur Refsdal, lensing of Supernovae and their usage to measure the Hubble constant



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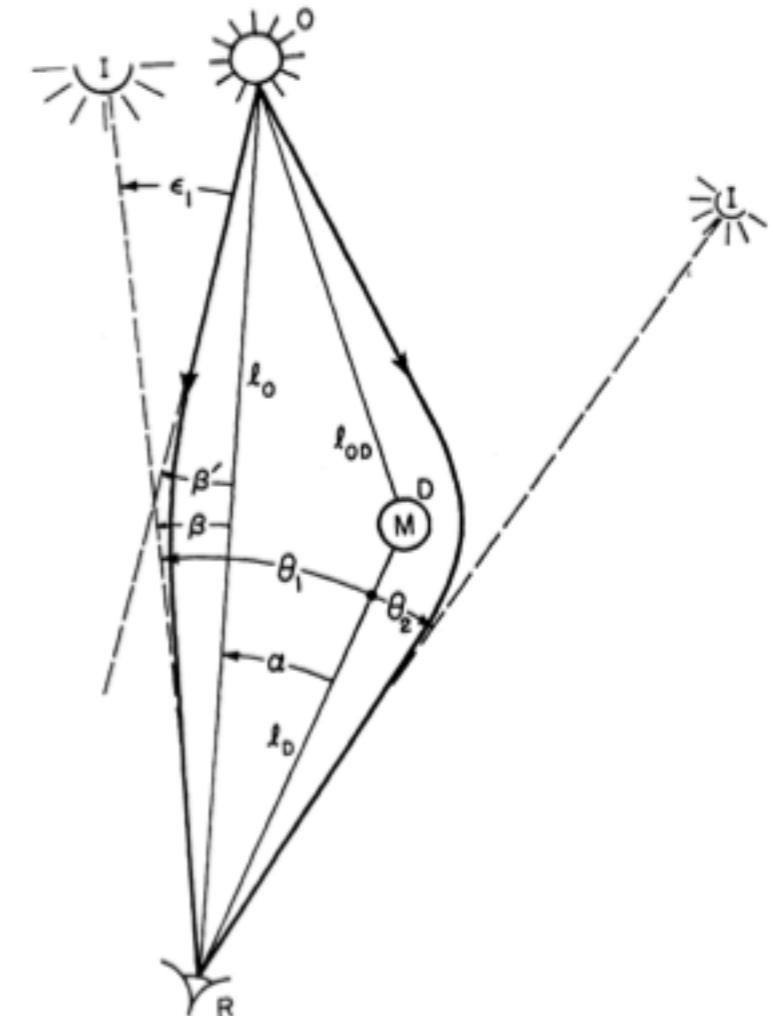


FIG. 1. The gravitational deflection of light by the deflector  $D$  of mass  $M$  causes the receiver  $R$  to detect two images  $I$  of the object  $O$ .

Liebes, 1964

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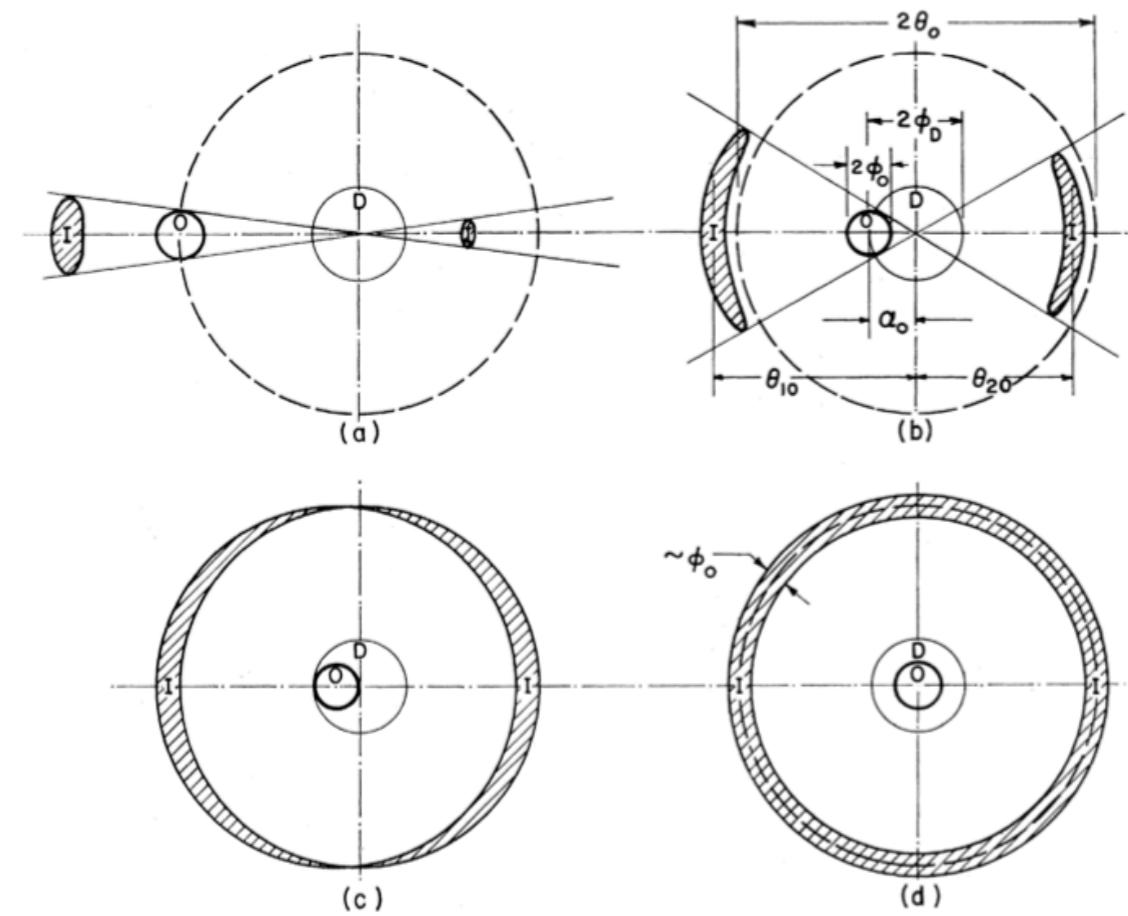


FIG. 2. Transformation of the image configuration  $I$  of the object  $O$  as the object moves from left to right behind the deflector  $D$ .

Liebes, 1964

ON THE POSSIBILITY OF DETERMINING HUBBLE'S PARAMETER  
AND THE MASSES OF GALAXIES FROM THE GRAVITATIONAL  
LENS EFFECT\*

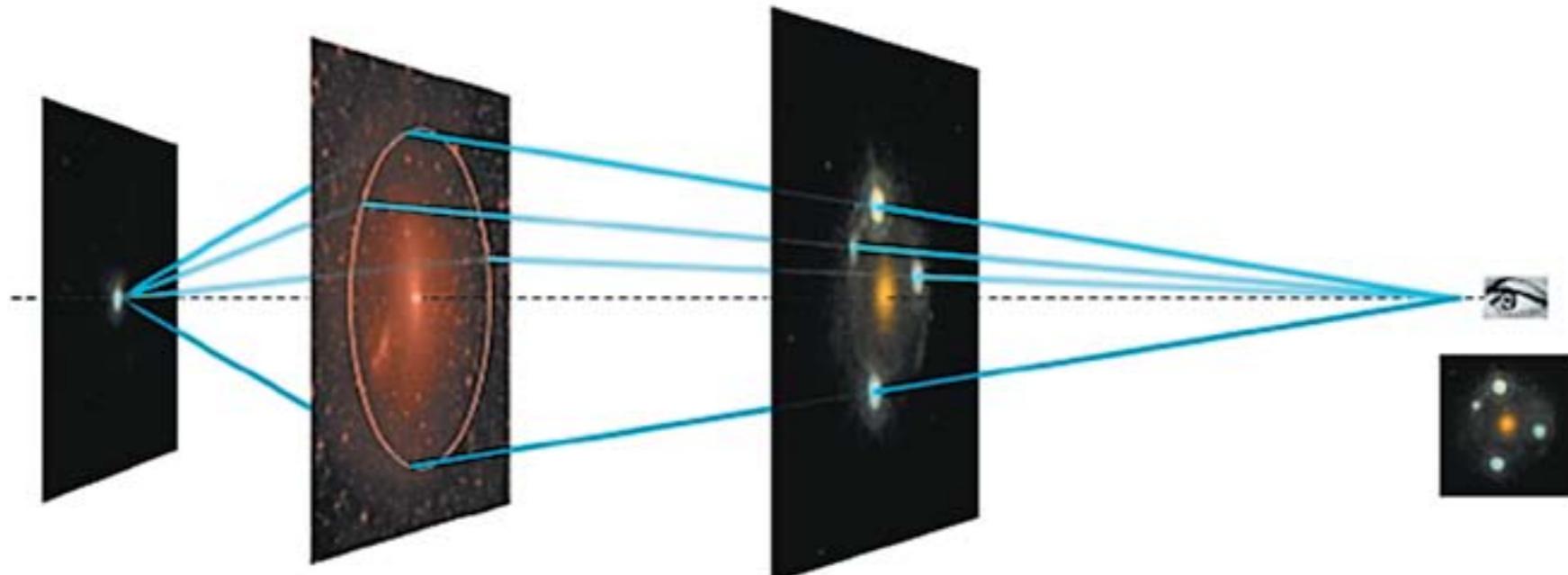
*Sjur Refsdal*

(Communicated by H. Bondi)

(Received 1964 January 27)

*Summary*

The gravitational lens effect is applied to a supernova lying far behind and close to the line of sight through a distant galaxy. The light from the supernova may follow two different paths to the observer, and the difference  $\Delta t$  in the time of light travel for these two paths can amount to a couple of months or more, and may be measurable. It is shown that Hubble's parameter and the mass of the galaxy can be expressed by  $\Delta t$ , the red-shifts of the supernova and the galaxy, the luminosities of the supernova "images" and the angle between them. The possibility of observing the phenomenon is discussed.



# THE FIRST OBSERVATION

*Discovery of the first galaxy-QSO lens system (1979)*

## 0957+561 A, B: twin quasistellar objects or gravitational lens?

D. Walsh

University of Manchester, Nuffield Radio Astronomy Laboratories, Jodrell Bank, Macclesfield, Cheshire, UK

R. F. Carswell

Institute of Astronomy, Cambridge, UK

R. J. Weymann

Steward Observatory, University of Arizona, Tucson, Arizona 85721

0957+561 A, B are two QSOs of mag 17 with 5.7 arc s separation at redshift 1.405. Their spectra leave little doubt that they are associated. Difficulties arise in describing them as two distinct objects and the possibility that they are two images of the same object formed by a gravitational lens is discussed.

SPECTROSCOPIC observations have been in progress for several years on QSO candidates using a survey of radio sources made at 966 MHz with the MkIA telescope at Jodrell Bank. Many of the identifications have been published by Cohen *et al.*<sup>1</sup> with interferometric positions accurate to ~2 arc s and a further list has been prepared by Porcas *et al.*<sup>2</sup>. The latter list consists of sources that were either too extended or too confused for accurate interferometric positions to be measured, and these were observed with the pencil-beam of the 300 ft telescope at NRAO, Green Bank at  $\lambda$  6 cm and  $\lambda$  11 cm. This gave positions with typical accuracy 5–10 arc s and the identifications are estimated as ~80% reliable.

The list of Porcas *et al.* includes the source 0957+561 which has within its field a close pair of blue stellar objects, separated by ~6 arc s, which are suggested as candidate identifications. Their positions and red and blue magnitudes,  $m_R$  and  $m_B$ , estimated from the Palomar Observatory Sky Survey (POSS) are given in Table 1 and a finding chart is given in Fig. 1. Since the images on the POSS overlap, the magnitude estimates may

be of lower accuracy than normal, but they are very nearly equal and object A is definitely bluer than object B. The mean position of the two objects is 17 arc s from the radio position, so the identification is necessarily tentative.

### Observations

The two objects 0957+561 A, B were observed on 29 March 1979 at the 2.1 m telescope of the Kitt Peak National Observatory (KPNO) using the intensified image dissector scanner (IIDS). Sky subtraction was used with circular apertures separated by 99.4 arc s. Some observational parameters are given in Table 2. The spectral range was divided into 1,024 data bins, each bin 3.5 Å wide, and the spectral resolution was 16 Å. After 20-min integration on each object it was clear that both were QSOs with almost identical spectra and redshifts of ~1.40 on the basis of strong emission lines identified as C IV  $\lambda$ 1549 and C III]  $\lambda$ 1909. Further observations were made on 29 March and on subsequent nights as detailed in Table 2. By offsetting to observe empty sky a few arc seconds from one object on both 29 and 30 March it was confirmed that any contamination of the spectrum of one object by light from the other was negligible.

Table 1 Positions and magnitudes of 0957+561 A, B

Object	RA	Dec (1950.0)	$M_R$	$M_B$
0957+561A	09 57 57.3	+56 08 22.9	17.0	16.7
0957+561B	09 57 57.4	+56 08 16.9	17.0	17.0

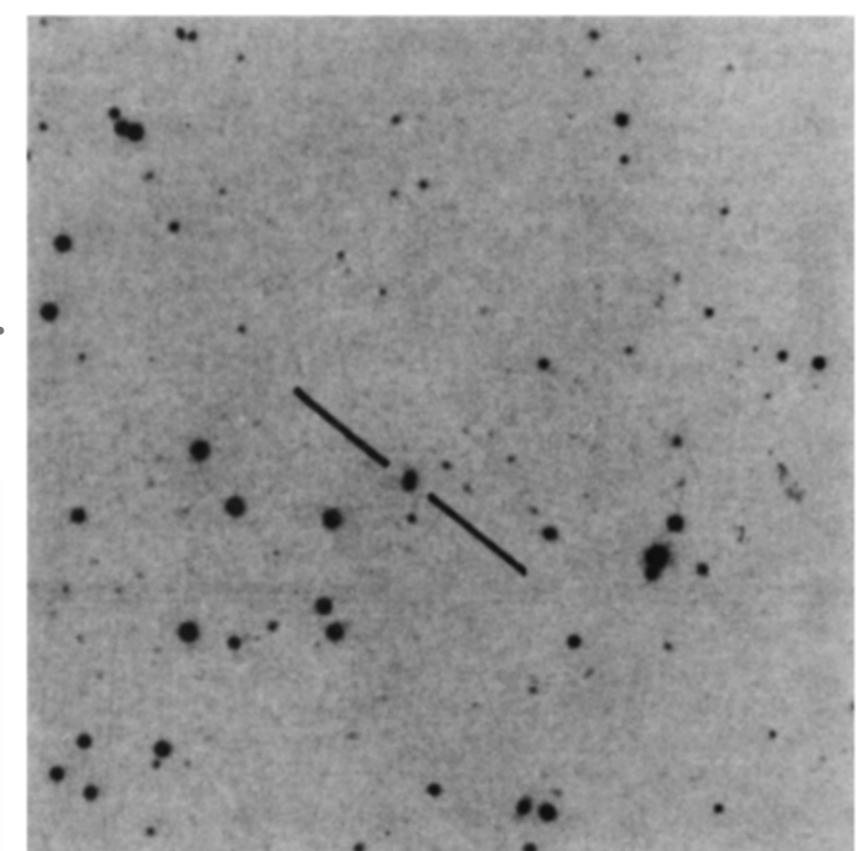


Fig. 1 Finding chart for the QSOs 0957+561 A and B. The chart is 8.5 arc min square with the top right hand corner north preceding and is from the E print of the POSS.

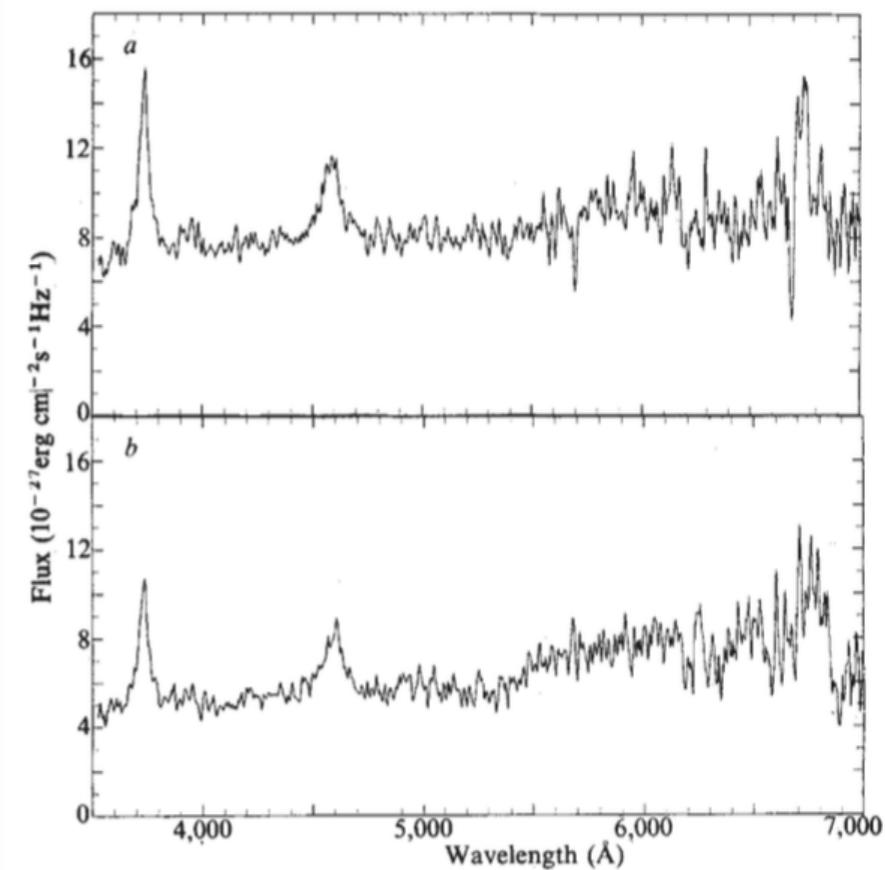


Fig. 2 IIDS scans of 0957+561 A(a) and B(b). The data are smoothed over 10 Å and the spectral resolution is 16 Å.

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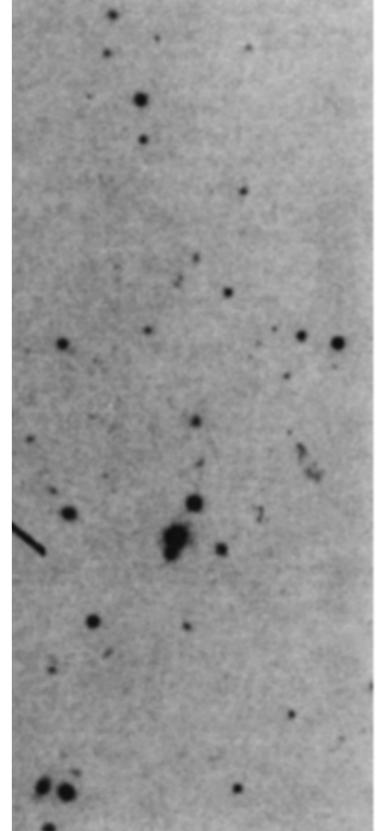
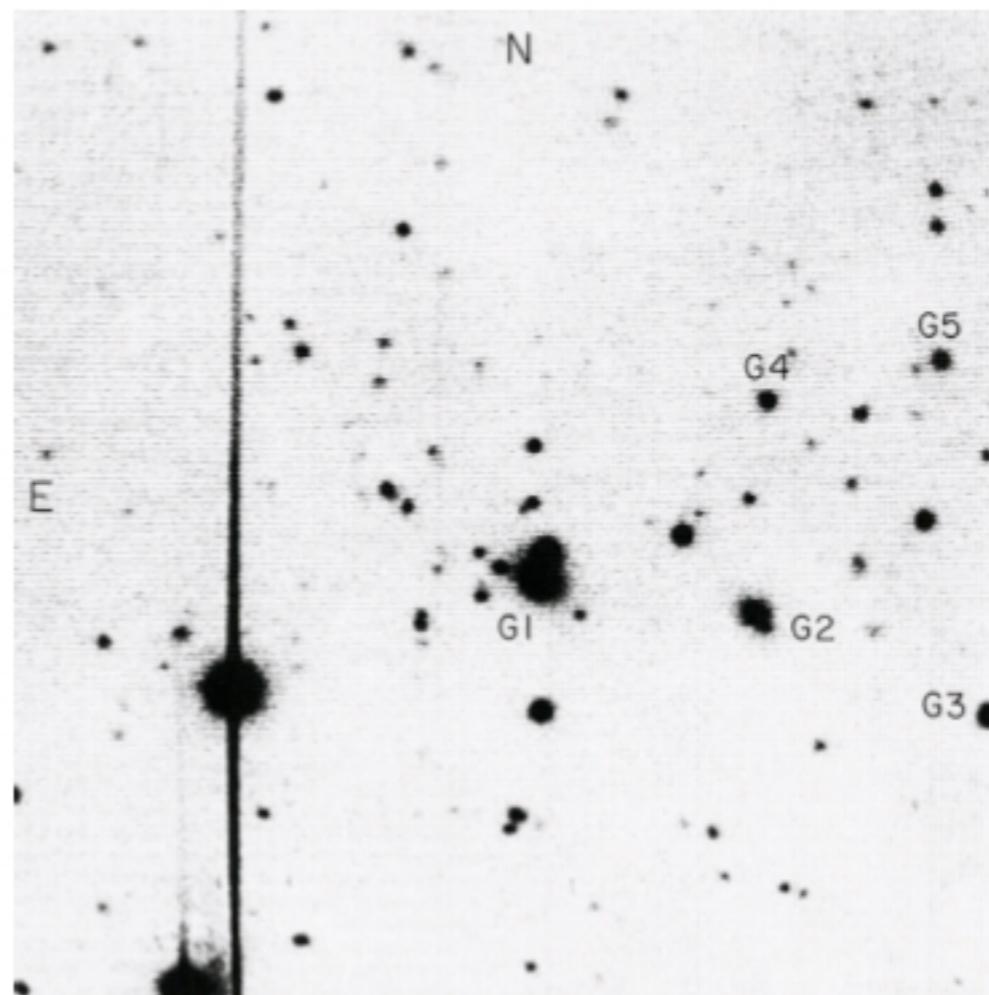
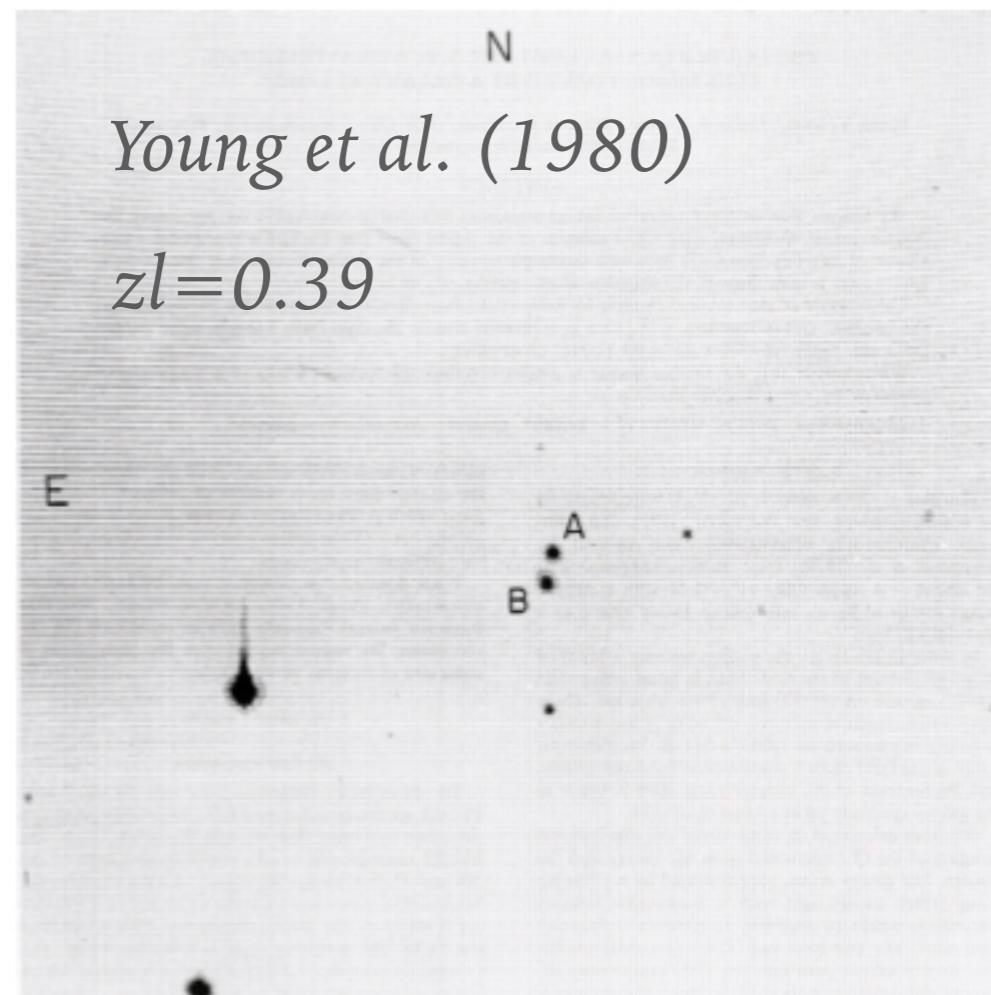
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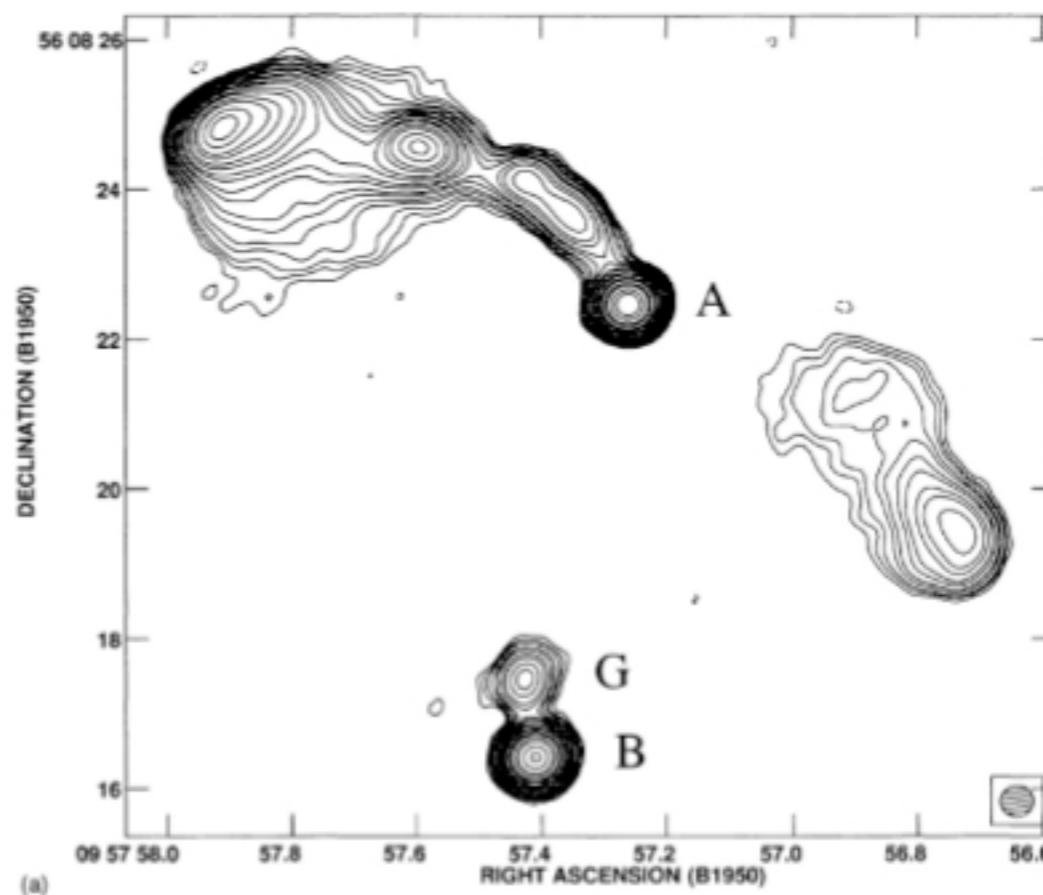
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957+561 A and B. The chart at hand corner north preceding it of the POSS.



Gorenstein et al. (1984)

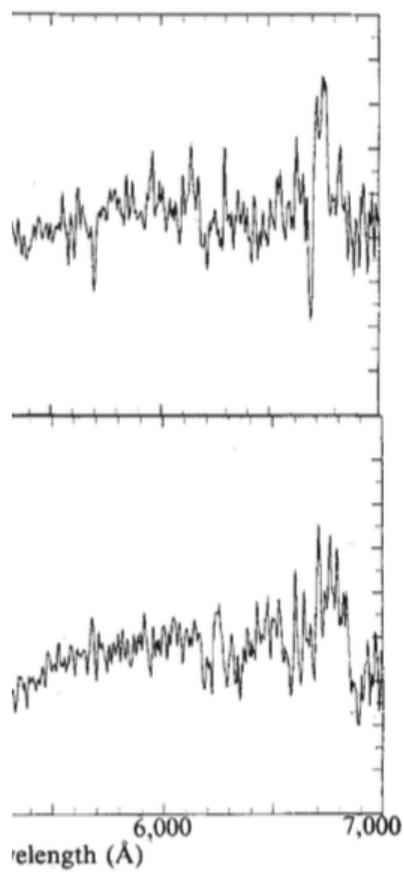
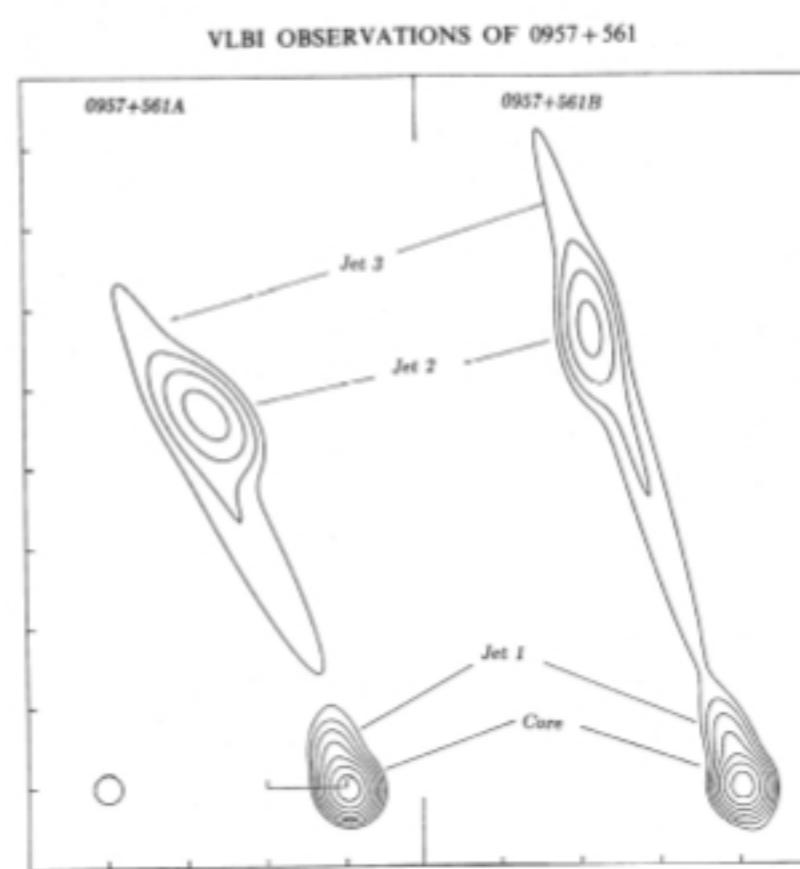
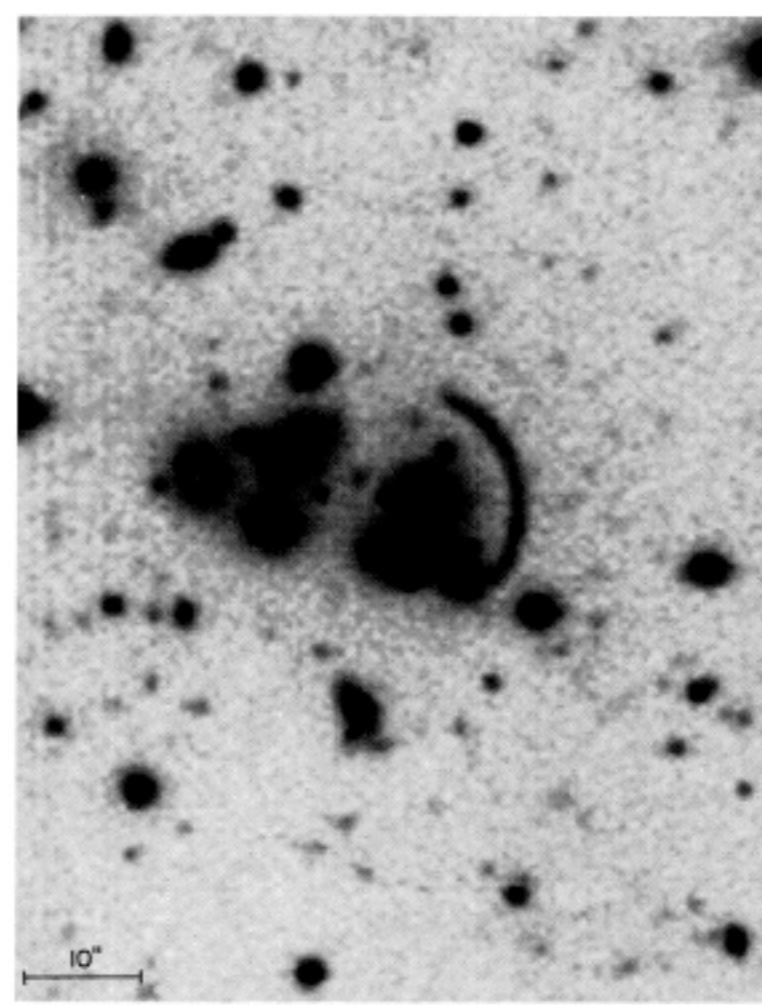
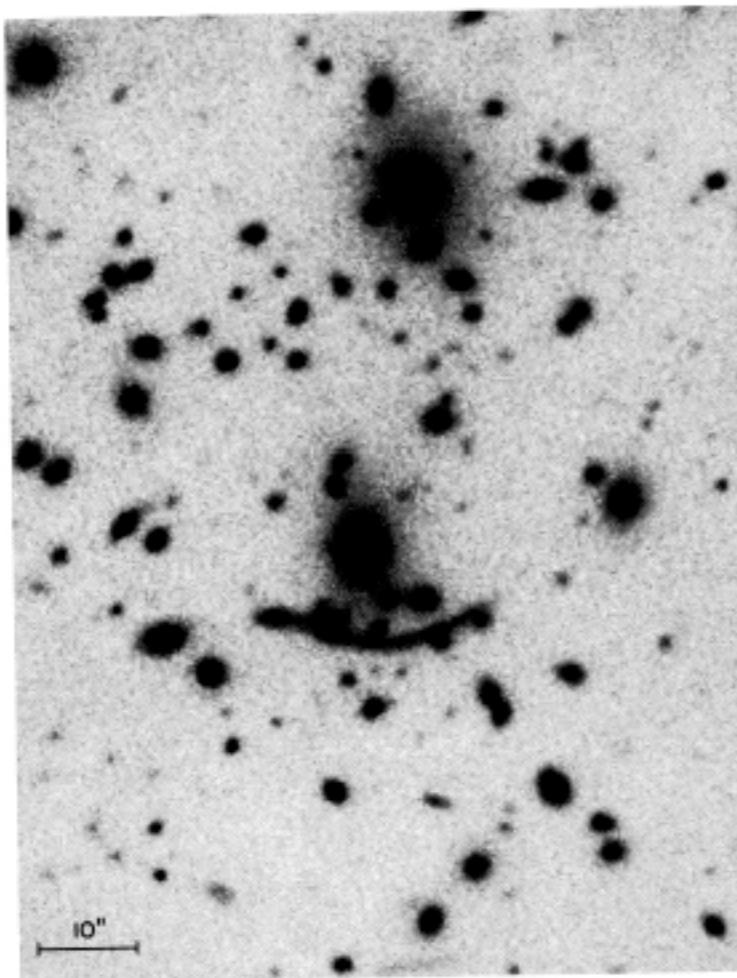


Fig. 2 IIDS scans of 0957+561 A(a) and B(b). The data are smoothed over 10 Å and the spectral resolution is 16 Å.

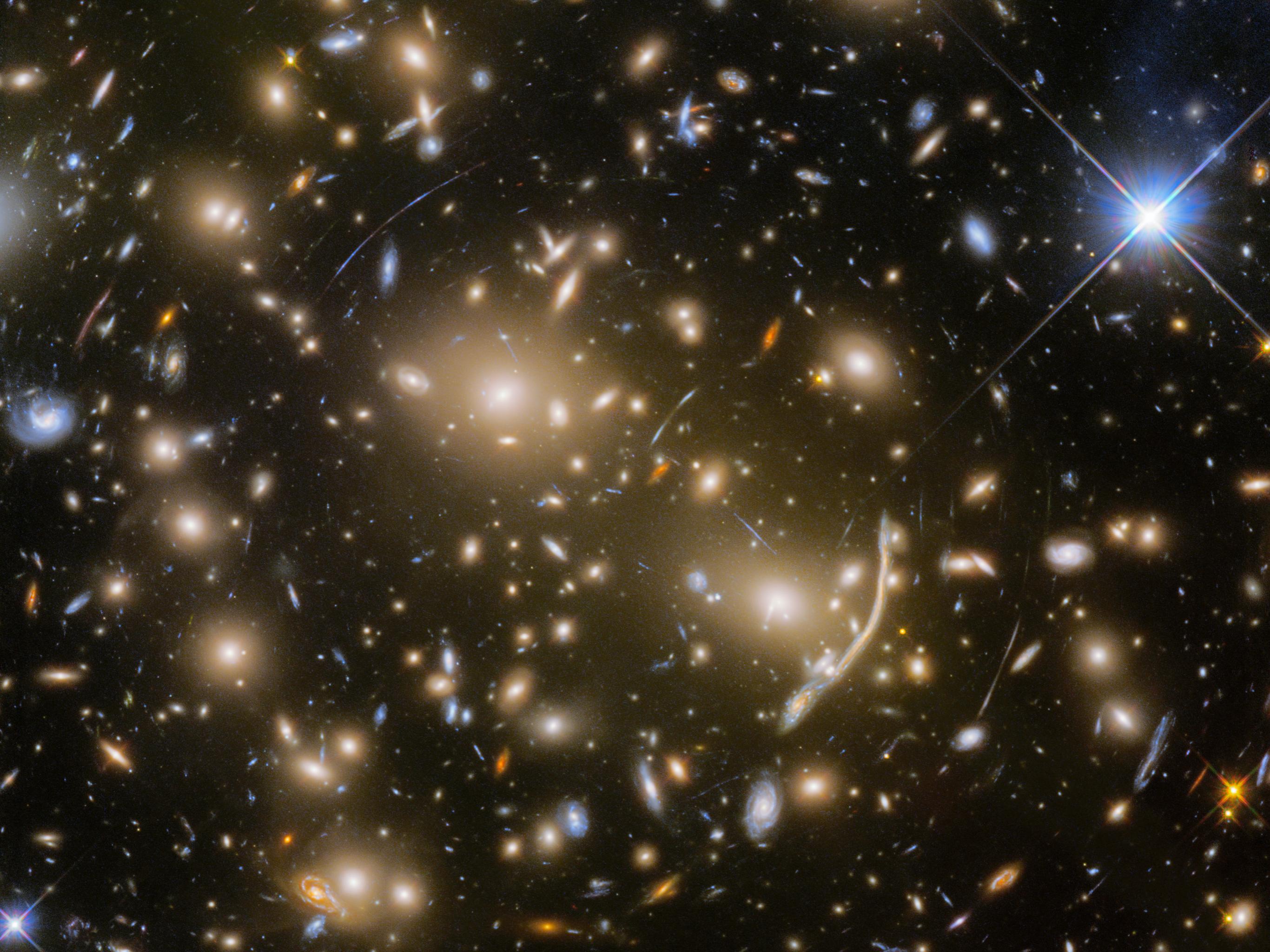


### Historical remark: The first gravitational arcs

The first detection of gravitational arcs in galaxy clusters is dated 1986. In this year, two groups independently discovered strongly elongated, curved features around two clusters of galaxies (Soucail et al., 1987; Lynds & Petrosian, 1989): A370 (left panel below) and CL2244-02 (right panel).



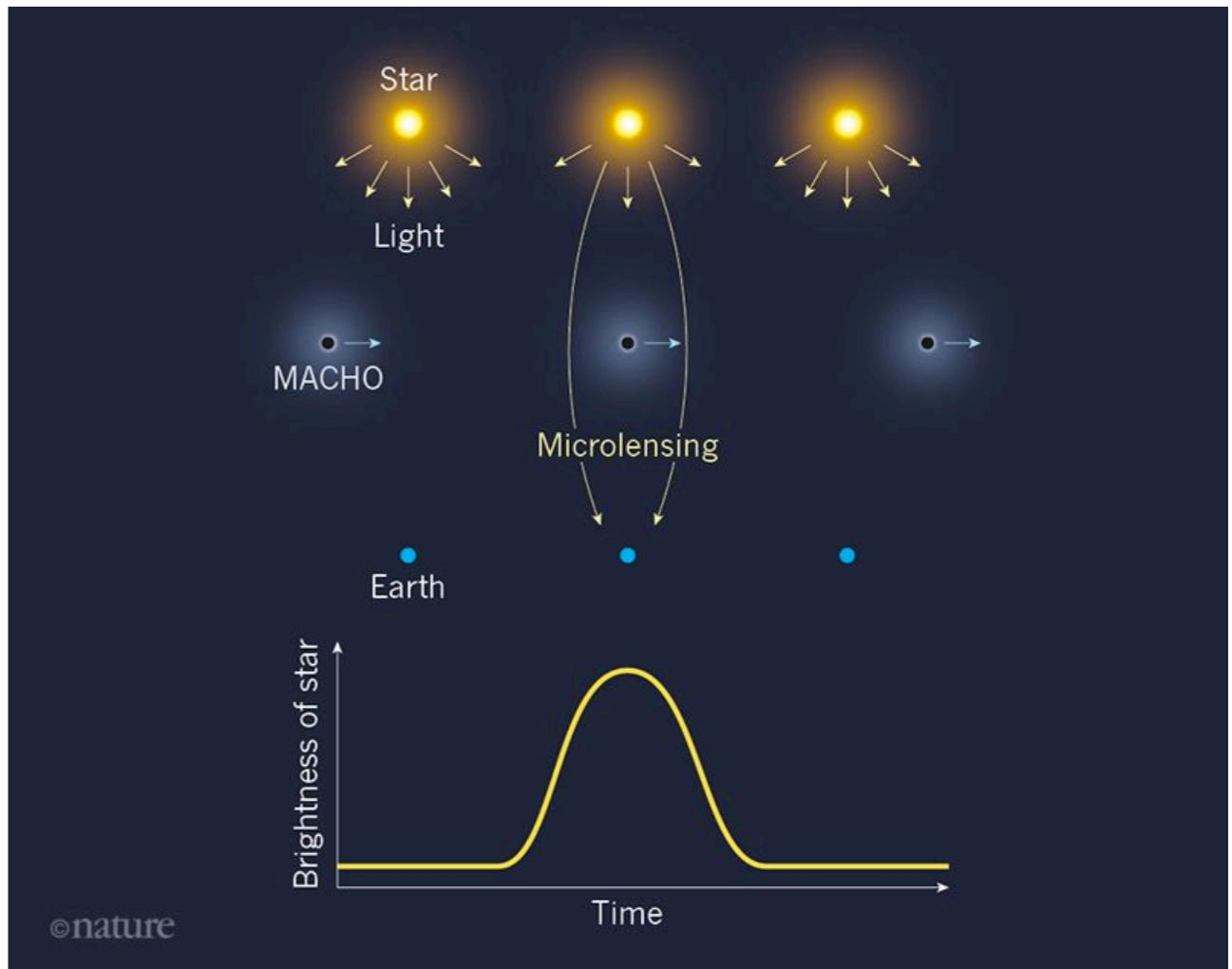
They were seen displaced from the cluster center and curving around it. Several hypothesis were put forward about the nature of these features, all proven wrong. The correct interpretation of these observations as gravitational lensing effects was made by Paczynski (1987), when the redshift of the arc in A370 was measured and discovered to be much larger than the redshift on the cluster. In particular, A370 is at redshift  $z_d = 0.374$ , while the arc is at redshift  $z_s = 0.724$ . The arc in CL2244-02 ( $z_d = 0.3$ ) is at redshift  $z_s = 2.24$ . The figures below show color higher quality images of the same clusters observed with HST and with ISAAC@VLT.



# MICROLENSING

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- 1986: Bohdan Paczynski estimates that 1 star every one million is micro-lensed in the galaxy
- These events occur on very small scales (milliarcsec), but produce significant increment of the source flux
- Paczynki proposes to monitor the stars in the Magellanic Clouds searching for such events, in particular for those produced by dark lenses (candidate dark matter)



# Possible gravitational microlensing of a star in the Large Magellanic Cloud

C. Alcock<sup>\*†</sup>, C. W. Akerlof<sup>†¶</sup>, R. A. Allsman<sup>\*</sup>,  
T. S. Axelrod<sup>\*</sup>, D. P. Bennett<sup>\*†</sup>, S. Chan<sup>‡</sup>,  
K. H. Cook<sup>\*†</sup>, K. C. Freeman<sup>‡</sup>, K. Griest<sup>†||</sup>,  
S. L. Marshall<sup>†§</sup>, H-S. Park<sup>\*</sup>, S. Perlmutter<sup>†</sup>,  
B. A. Peterson<sup>†</sup>, M. R. Pratt<sup>†§</sup>, P. J. Quinn<sup>†</sup>,  
A. W. Rodgers<sup>†</sup>, C. W. Stubbs<sup>†§</sup>  
& W. Sutherland<sup>†</sup>

\* Lawrence Livermore National Laboratory, Livermore, California 94550, USA

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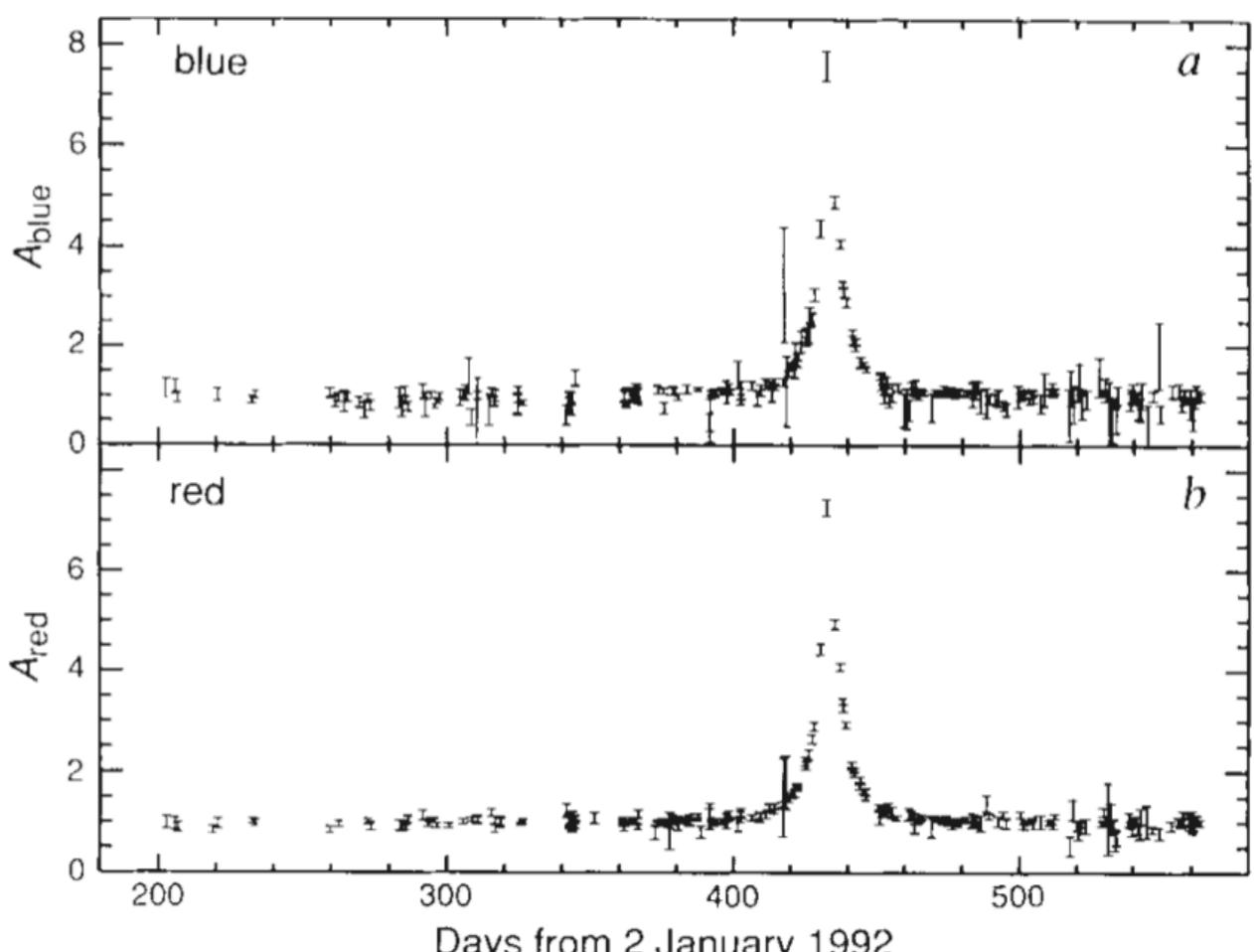
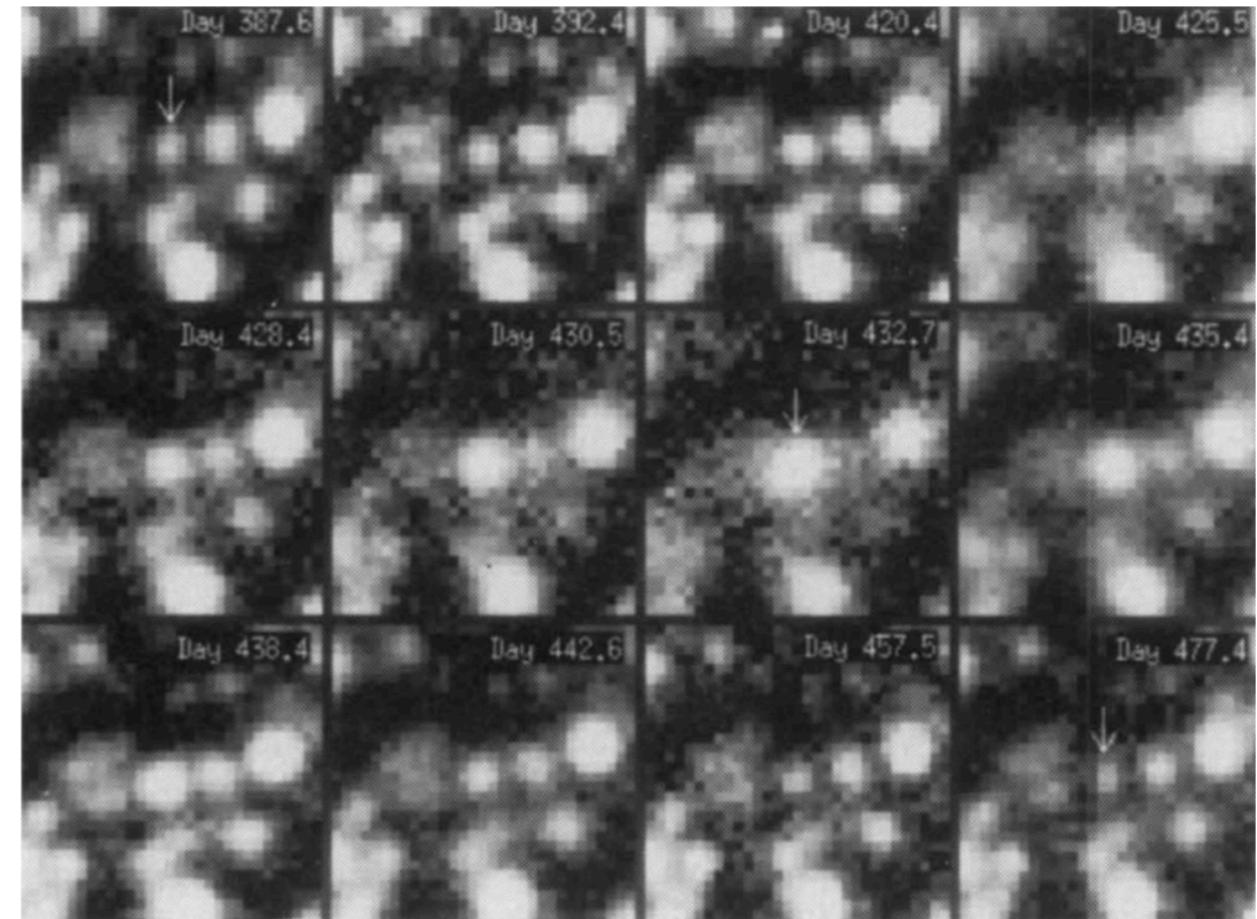
‡ Mt Stromlo and Siding Spring Observatories, Australian National University, Weston, ACT 2611, Australia

§ Department of Physics, University of California, Santa Barbara, California 93106, USA

¶ Department of Physics, University of California, San Diego, California 92039, USA

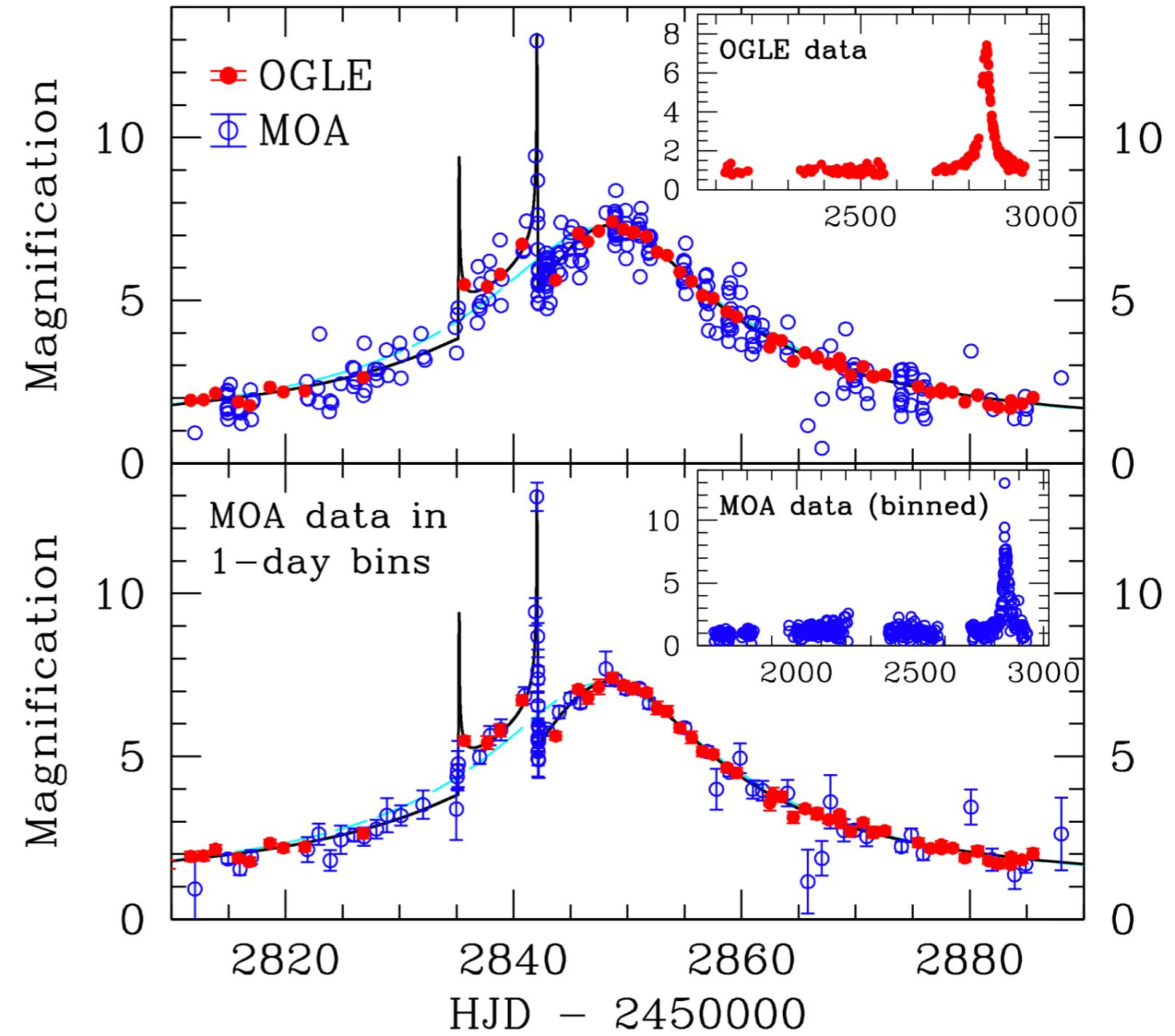
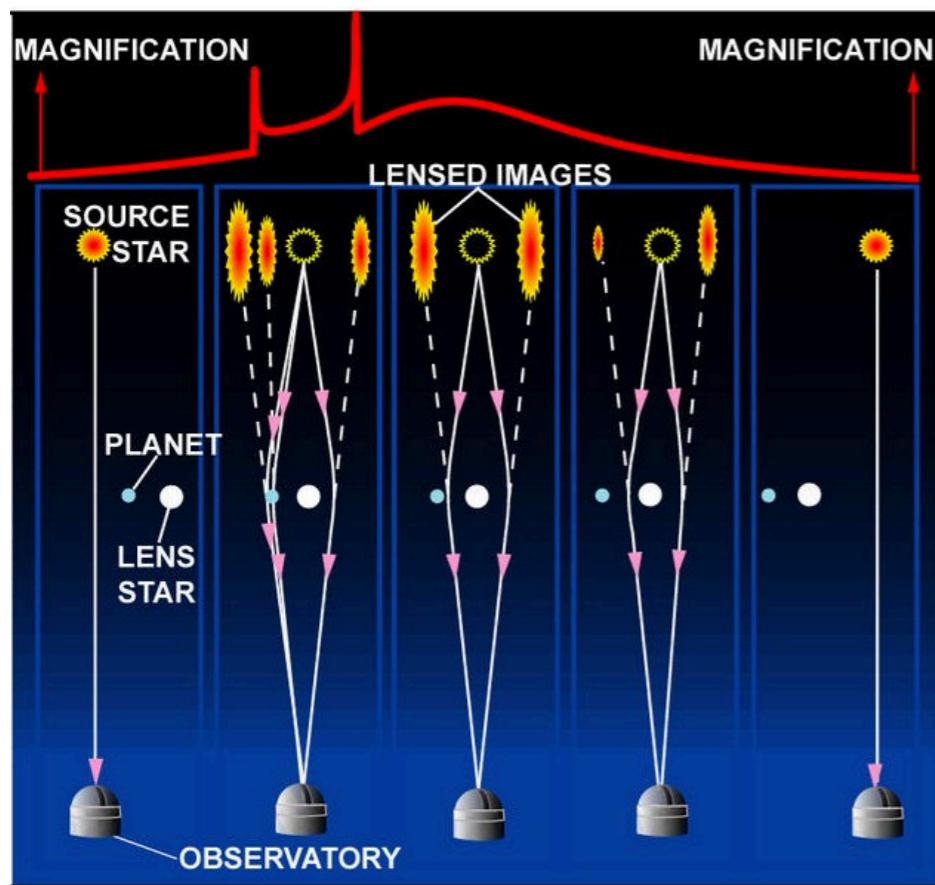
|| Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA

THERE IS NOW ABUNDANT EVIDENCE FOR THE PRESENCE OF LARGE QUANTITIES OF UNSEEN MATTER SURROUNDING NORMAL GALAXIES, INCLUDING OUR OWN<sup>1,2</sup>. THE NATURE OF THIS 'DARK MATTER' IS UNKNOWN, EXCEPT THAT IT CANNOT BE MADE OF NORMAL STARS, DUST OR GAS, AS THEY WOULD BE EASILY DETECTED. EXOTIC PARTICLES SUCH AS AXIONS, MASSIVE NEUTRINOS OR OTHER WEAKLY INTERACTING MASSIVE PARTICLES (COLLECTIVELY KNOWN AS WIMPS) HAVE BEEN PROPOSED<sup>3,4</sup>, BUT HAVE YET TO BE DETECTED. A LESS EXOTIC ALTERNATIVE IS NORMAL MATTER IN THE FORM OF BODIES WITH MASSES RANGING FROM THAT OF A LARGE PLANET TO A FEW SOLAR MASSES. SUCH OBJECTS, KNOWN COLLECTIVELY AS MASSIVE COMPACT HALO OBJECTS<sup>5</sup> (MACHOS), MIGHT BE BROWN DWARFS OR 'JUPITERS' (BODIES TOO SMALL TO PRODUCE THEIR OWN ENERGY BY FUSION), NEUTRON STARS, OLD WHITE DWARFS OR BLACK HOLES. PACZYNSKI<sup>6</sup> SUGGESTED THAT MACHOS MIGHT ACT AS GRAVITATIONAL MICROLENSES, TEMPORARILY AMPLIFYING THE APPARENT BRIGHTNESS OF BACKGROUND STARS IN NEARBY GALAXIES. WE ARE CONDUCTING A MICROLENSING EXPERIMENT TO DETERMINE WHETHER THE DARK MATTER HALO OF OUR GALAXY IS MADE UP OF MACHOS. HERE WE REPORT A CANDIDATE FOR SUCH A MICROLENSING EVENT, DETECTED BY MONITORING THE LIGHT CURVES OF 1.8 MILLION STARS IN THE LARGE MAGELLANIC CLOUD FOR ONE YEAR. THE LIGHT CURVE SHOWS NO VARIATION FOR MOST OF THE YEAR OF DATA TAKING, AND AN UPWARD EXCURSION LASTING OVER 1 MONTH, WITH A MAXIMUM INCREASE OF ~2 MAG. THE MOST PROBABLE LENS MASS, INFERRED FROM THE DURATION OF THE CANDIDATE MICROLENSING EVENT, IS ~0.1 SOLAR MASS.



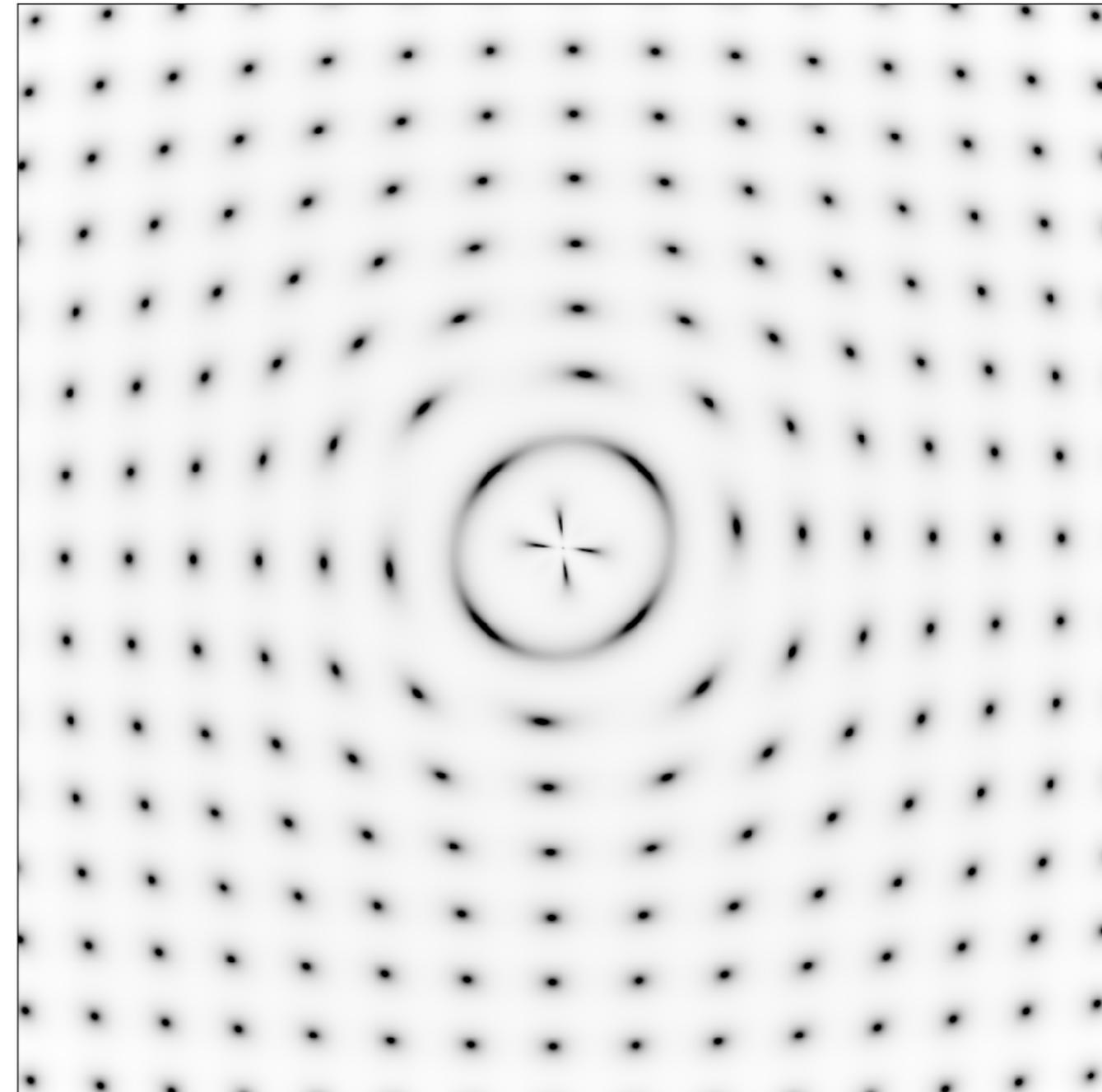
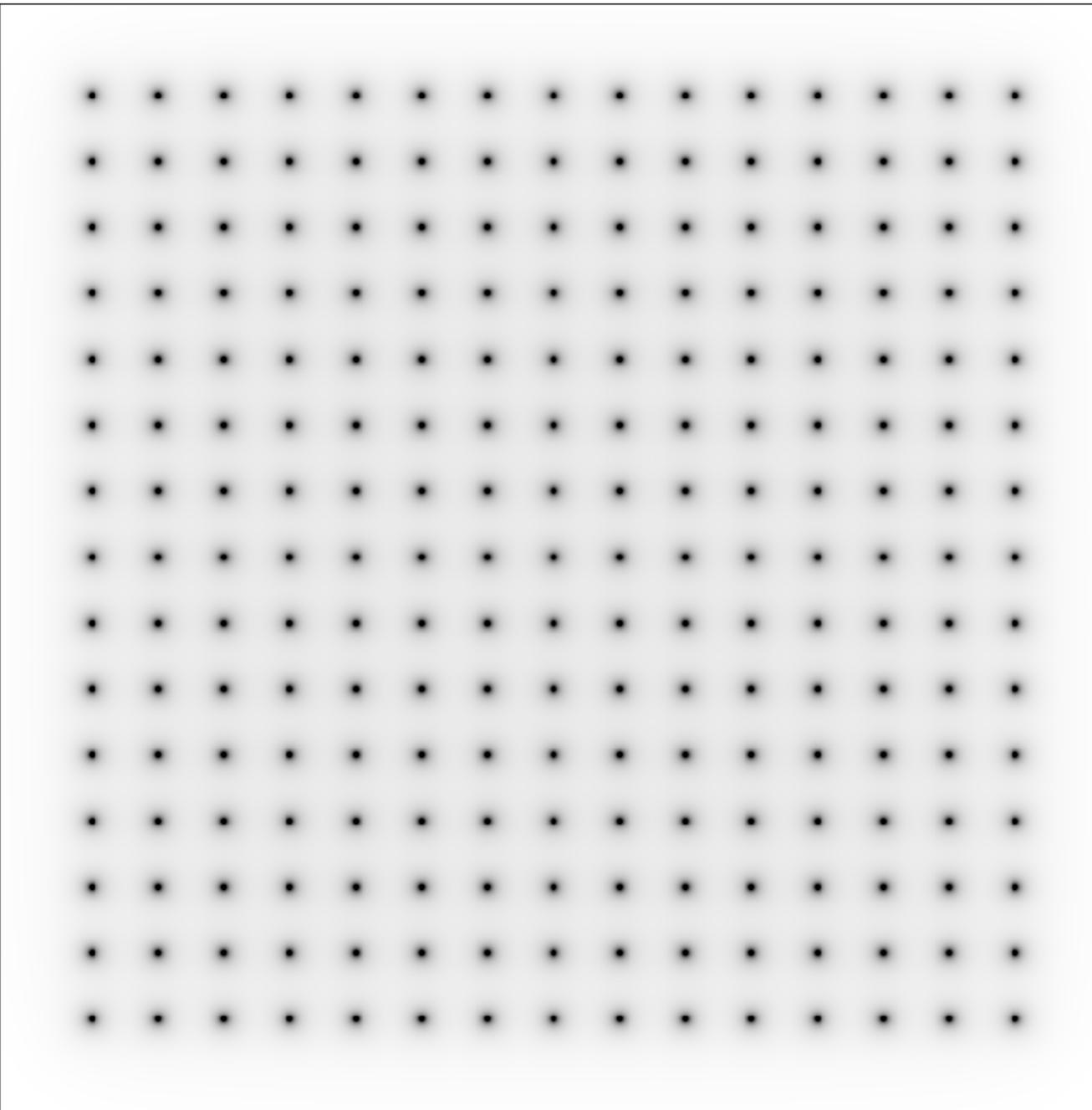
# FIRST DETECTION OF PLANETARY MICROLENSING

2003: first planet discovered  
via Microlensing (Bond et al.  
2003)



# WEAK LENSING: NO MULTIPLE IMAGES OR LARGE DISTORTIONS

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# FIRST DETECTION OF WEAK LENSING IN A1689 (TYSON ET AL. 1990)

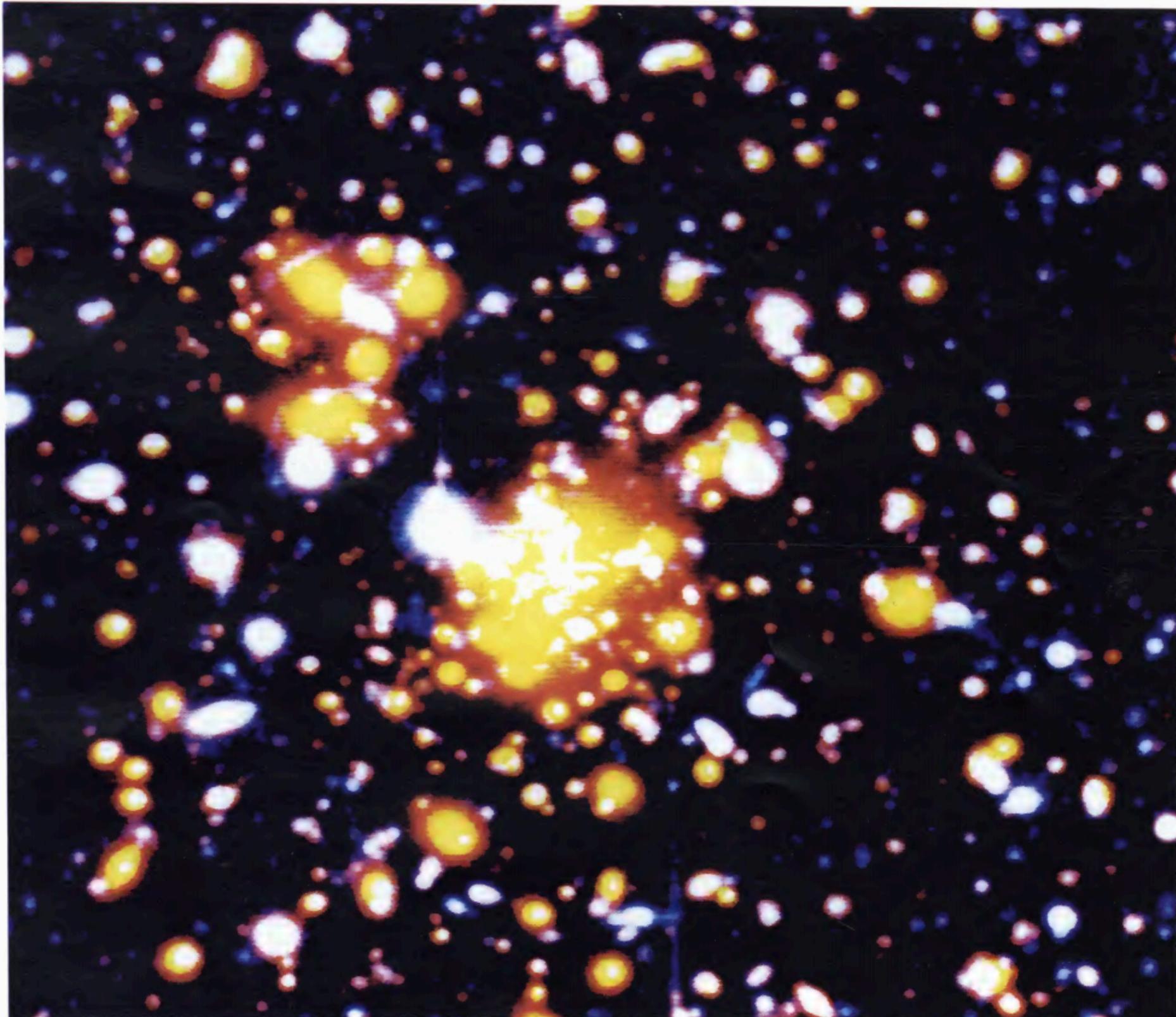


FIG. 1.—A composite color image of the A1689 field [red, green, blue] =  $[R, B_J, B_J - 0.35R]$ . Lens-distorted background blue galaxies become arcs and mini-arcs along circles centered on the cluster lens.

# FIRST DETECTION OF WEAK LENSING IN A1689 (TYSON ET AL. 1990)

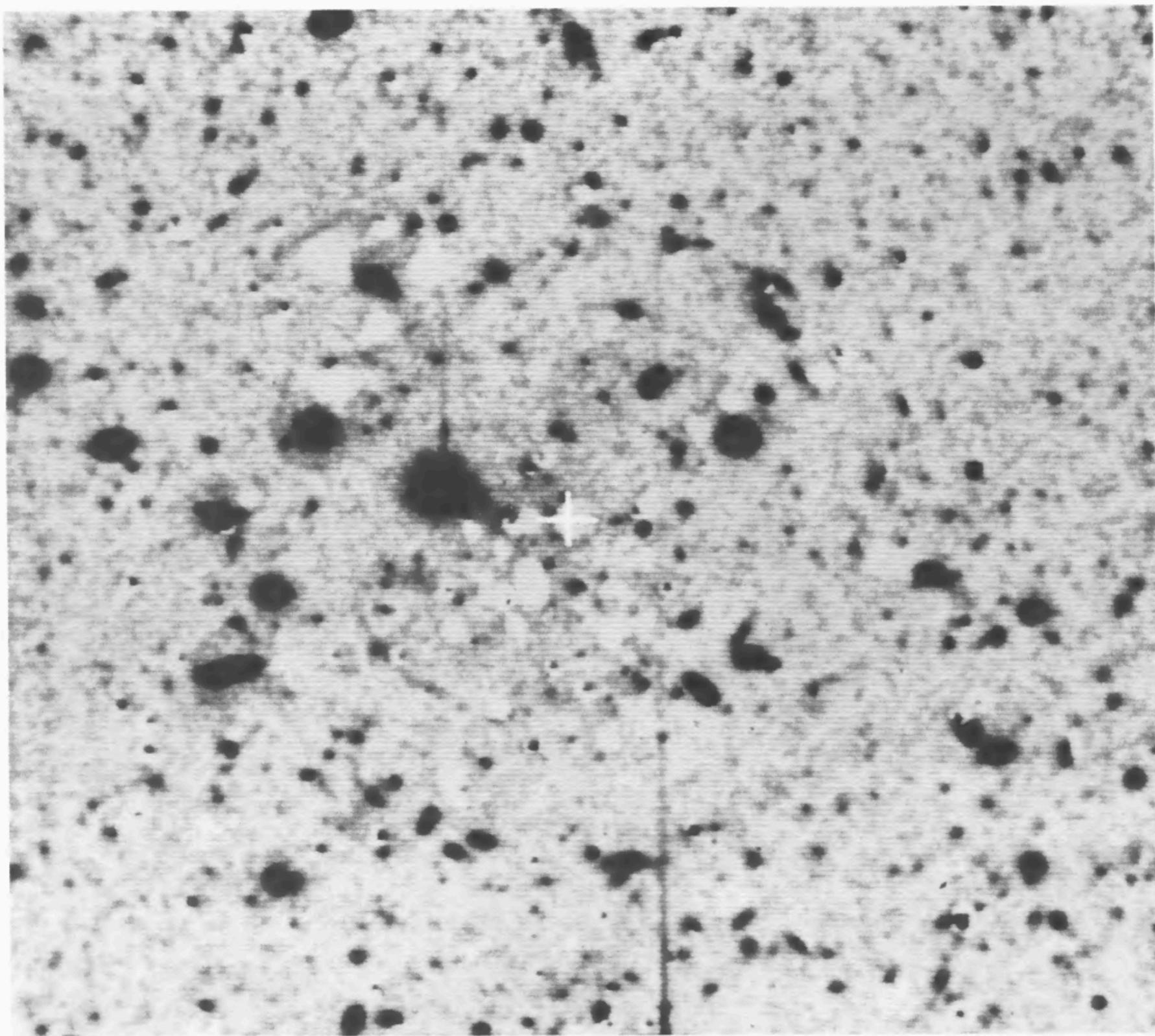
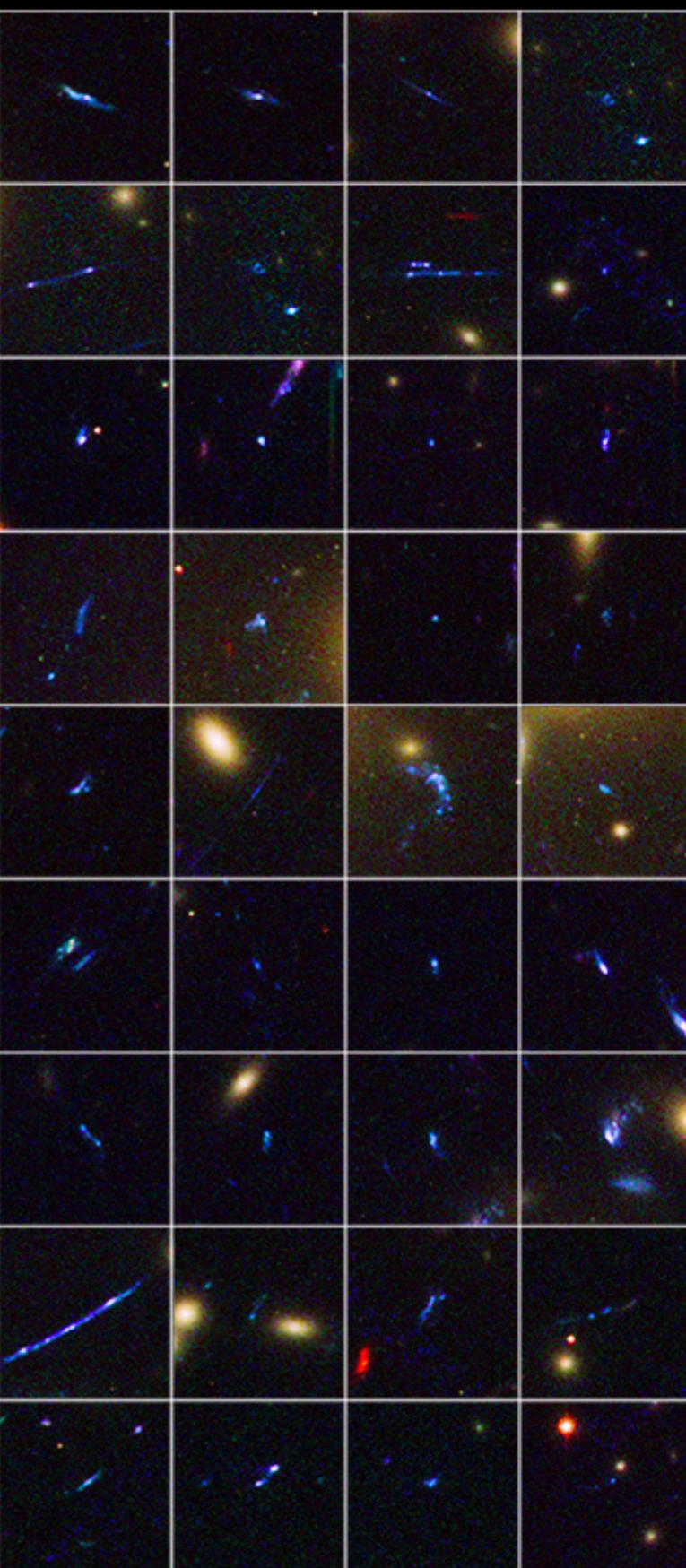
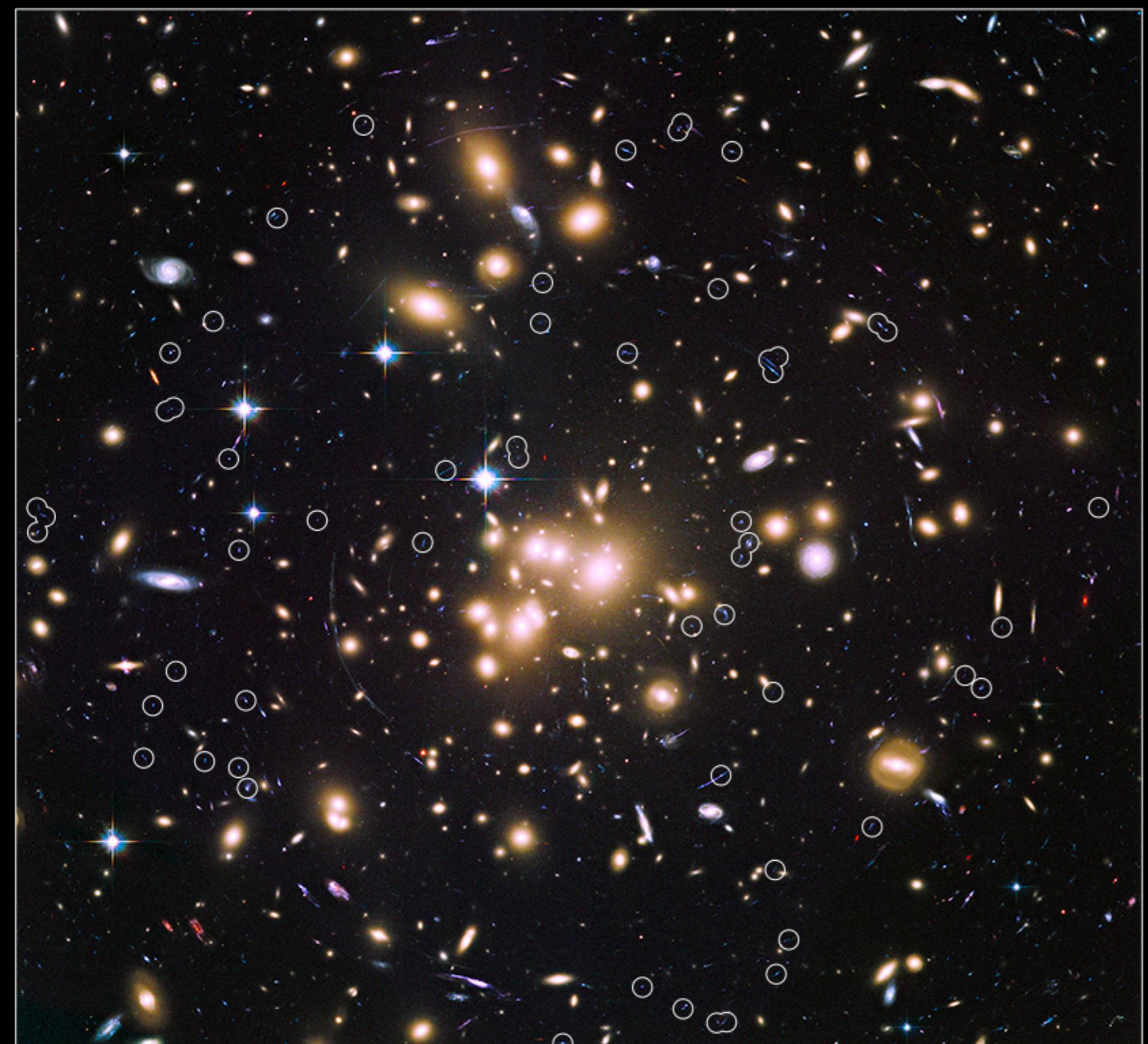


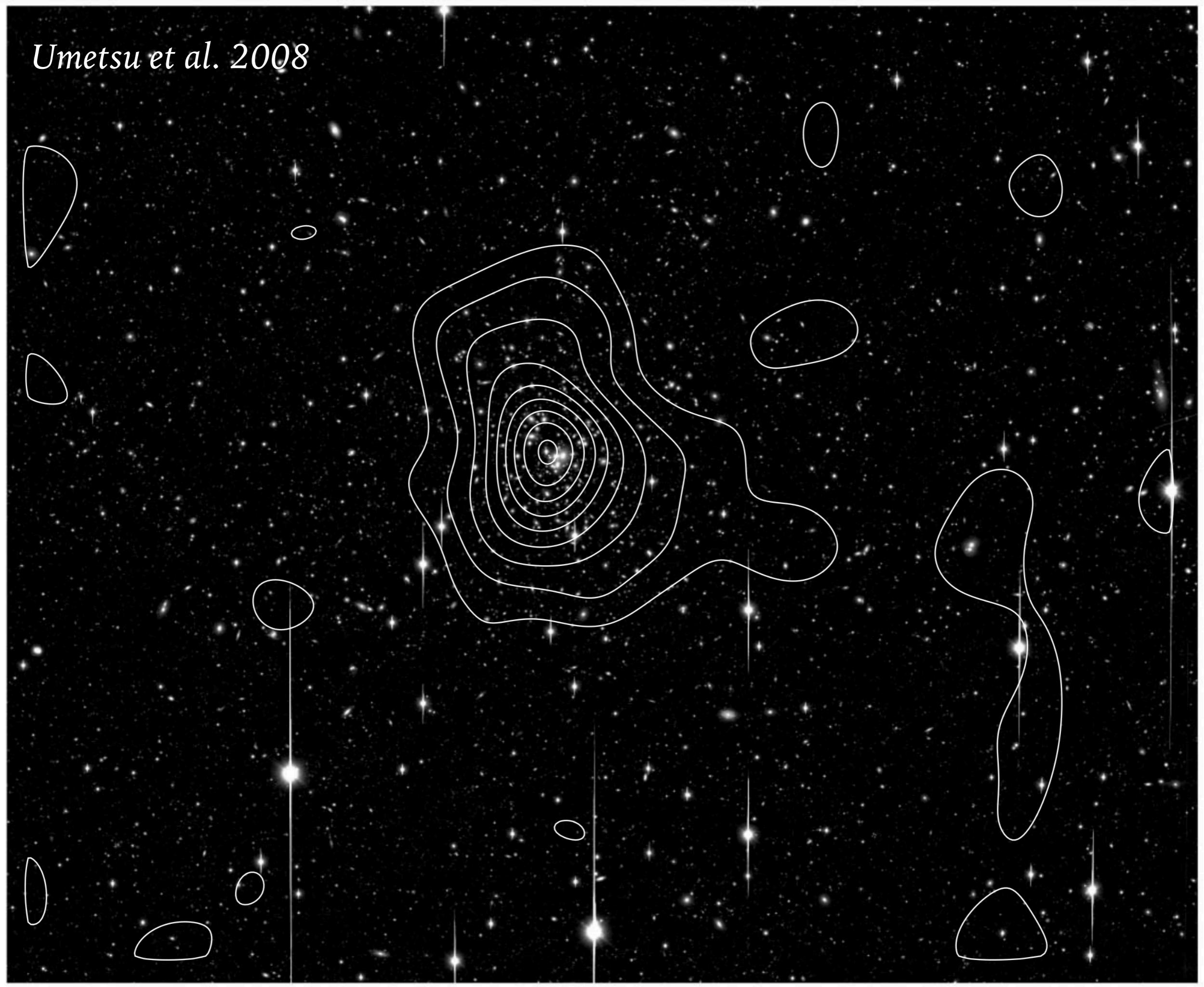
FIG. 2.—The monochrome blue image of A1689 with a scaled red image subtracted to eliminate much of the cluster light [ $\log(B_j - 0.35R)$ ]. Several large blue arcs are seen. However, the coherent alignment of the more numerous faint blue (background) galaxy major axes gives a stronger signal. In this  $3.8' \times 4'$  field over 50 lens-distorted galaxy images are also stretched along circles about the lens center. The cross is centered on the brightest cluster galaxy.

Galaxy Cluster Abell 1689

Hubble Space Telescope ■ ACS/WFC ■ WFC3/UVIS



Umetsu et al. 2008



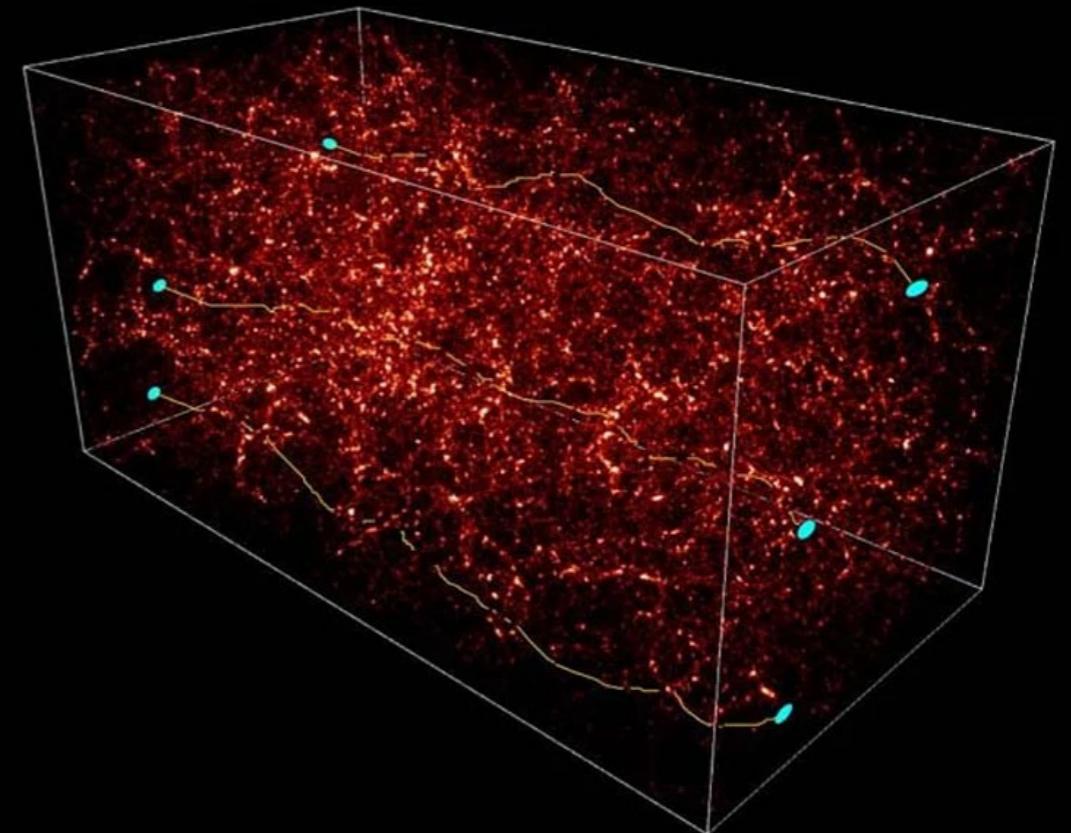
# COSMIC SHEAR

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Distortion of the images of distant galaxies by the large-scale distribution of mass in the Universe.

Detected statistically by correlations in the shapes of galaxies.

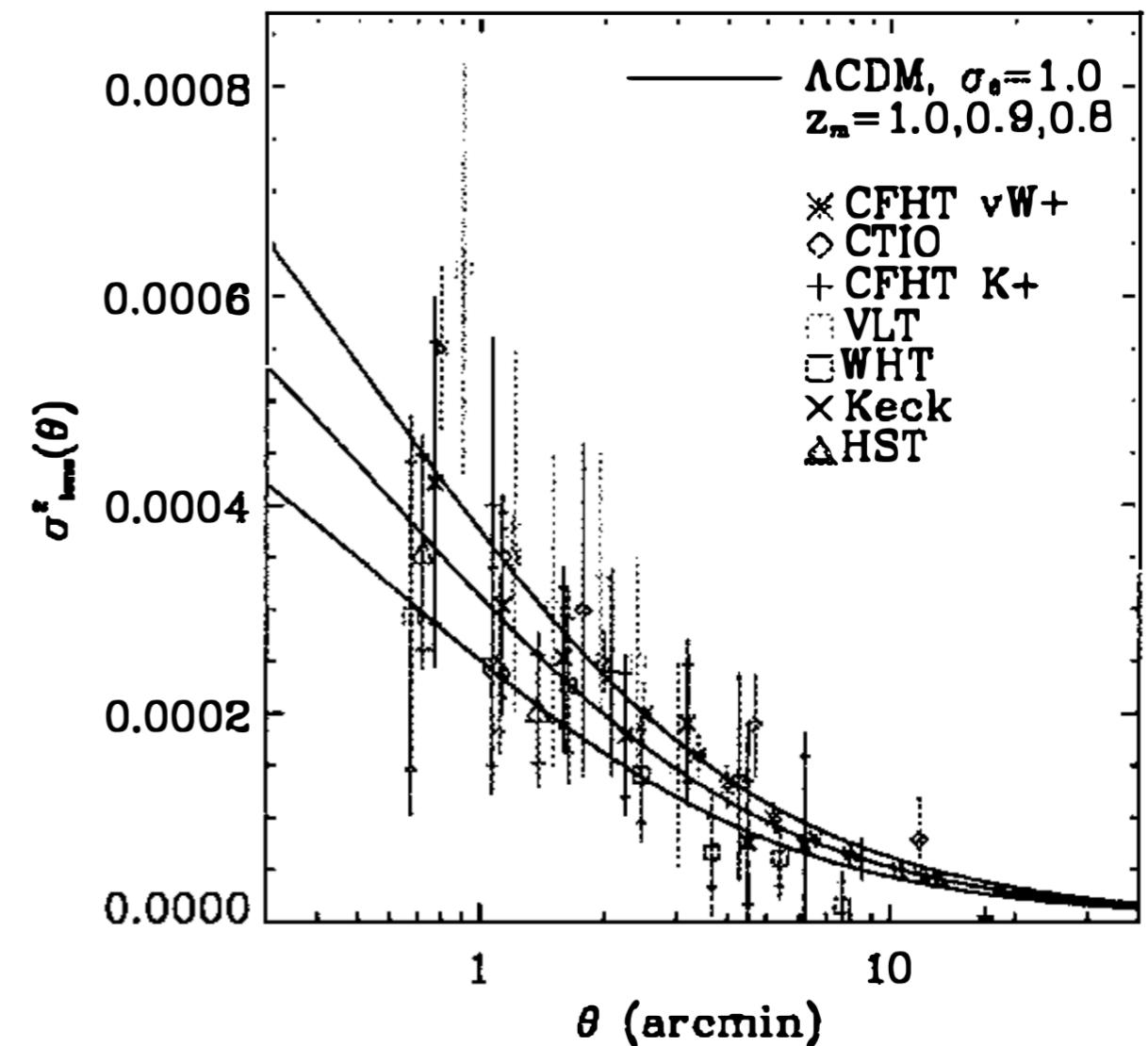
Sensitive to dark matter and the evolution of the expansion of the Universe.



# COSMIC SHEAR

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- Lensing effects are imprinted in the shape of sources even when they pass relatively far from cosmic structures
- This signal is tiny but still detectable in a statistical sense: for example measuring the alignments of galaxies within apertures of a given size, or by comparing the orientation of many pairs of galaxies at a given distance
- The first detections of the cosmic shear signal are dated at the beginning of 2000s and were produced by three groups independently: Bacon, Refregier & Ellis (2000), Van Waerbeke et al. (2000), Wittman et al. (2000)
- Cosmic shear is nowadays one of the most important cosmological tools and it is central in several experiments aiming at constraining the nature of dark matter and dark energy



Van Waerbeke et al. 2002

# GRAVITATIONAL LENSING

## 4- THE LENSING EQUATION I

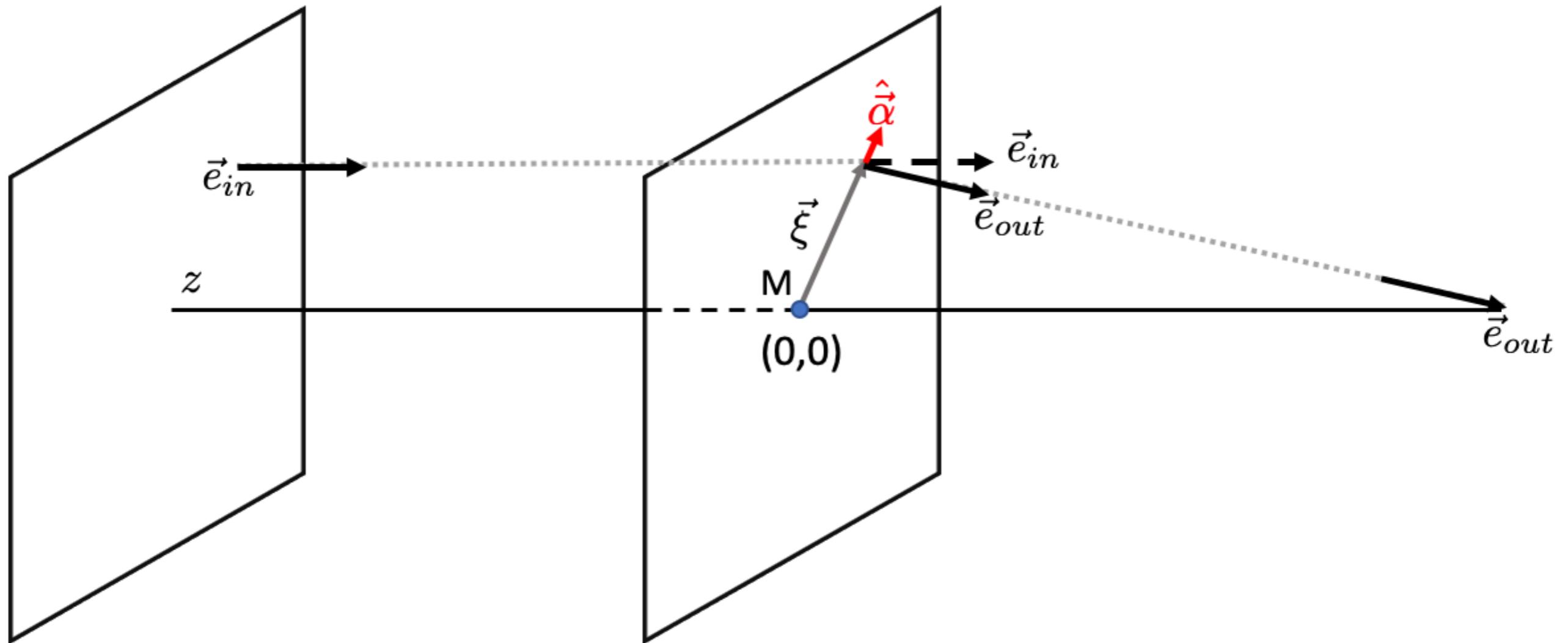
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*R. Benton Metcalf*  
2022-2023

# GENERALISATION OF THE DEFLECTION ANGLE FORMULA

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$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4GM}{c^2\xi} \vec{e}_\xi = \frac{4GM}{c^2} \frac{\vec{\xi}}{|\xi|^2}$$

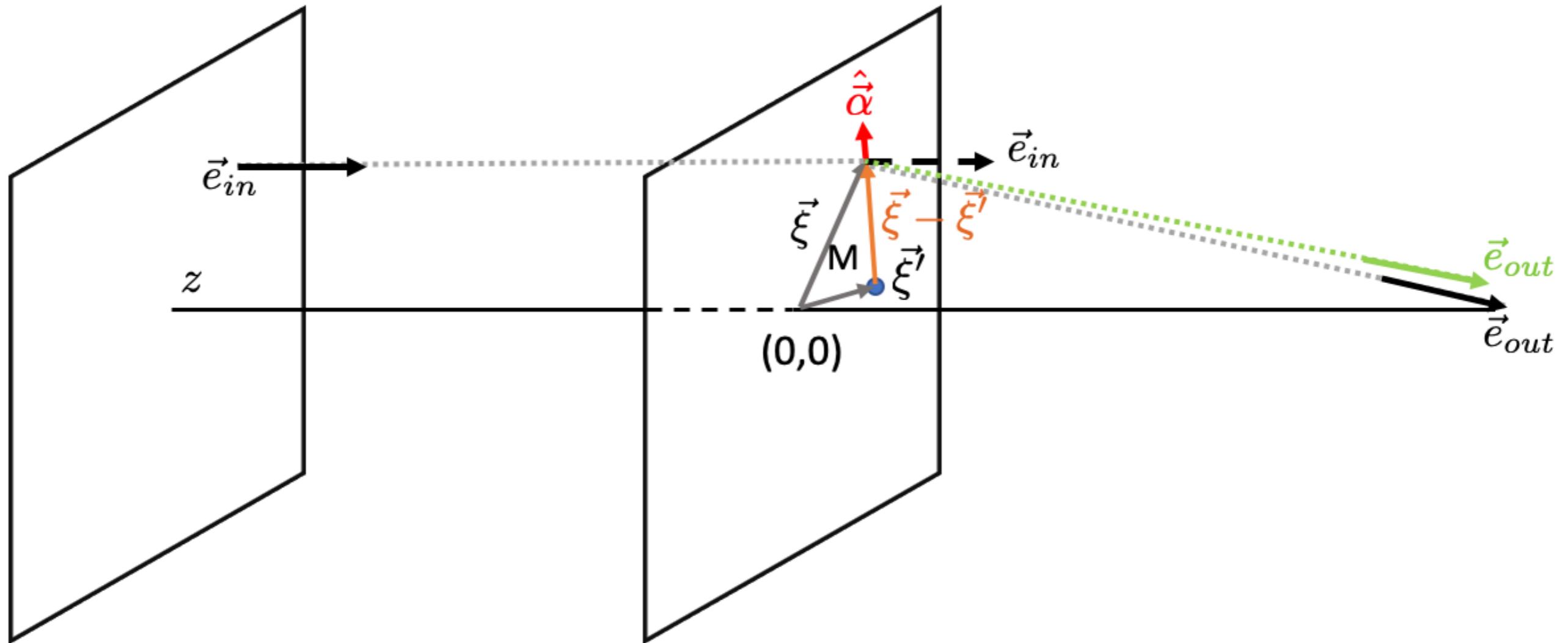


Using “Thin screen approximation”

# GENERALISATION OF THE DEFLECTION ANGLE FORMULA

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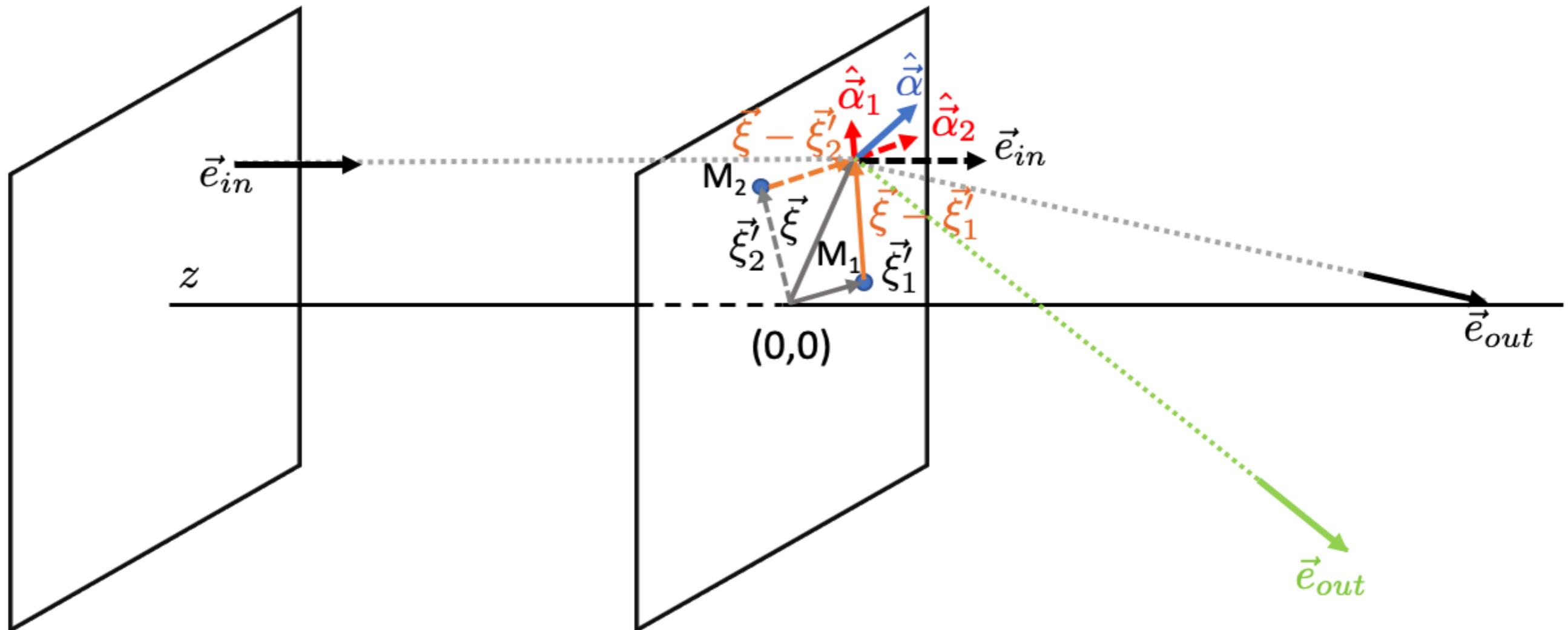
$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4GM}{c^2} \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$



Using “Thin screen approximation”

# GENERALISATION OF THE DEFLECTION ANGLE FORMULA

$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \sum_i M_i \frac{\vec{\xi} - \vec{\xi}^i}{|\vec{\xi} - \vec{\xi}^i|^2}$$



Using “Thin screen approximation”

# DEFLECTION BY AN EXTENDED MASS DISTRIBUTION

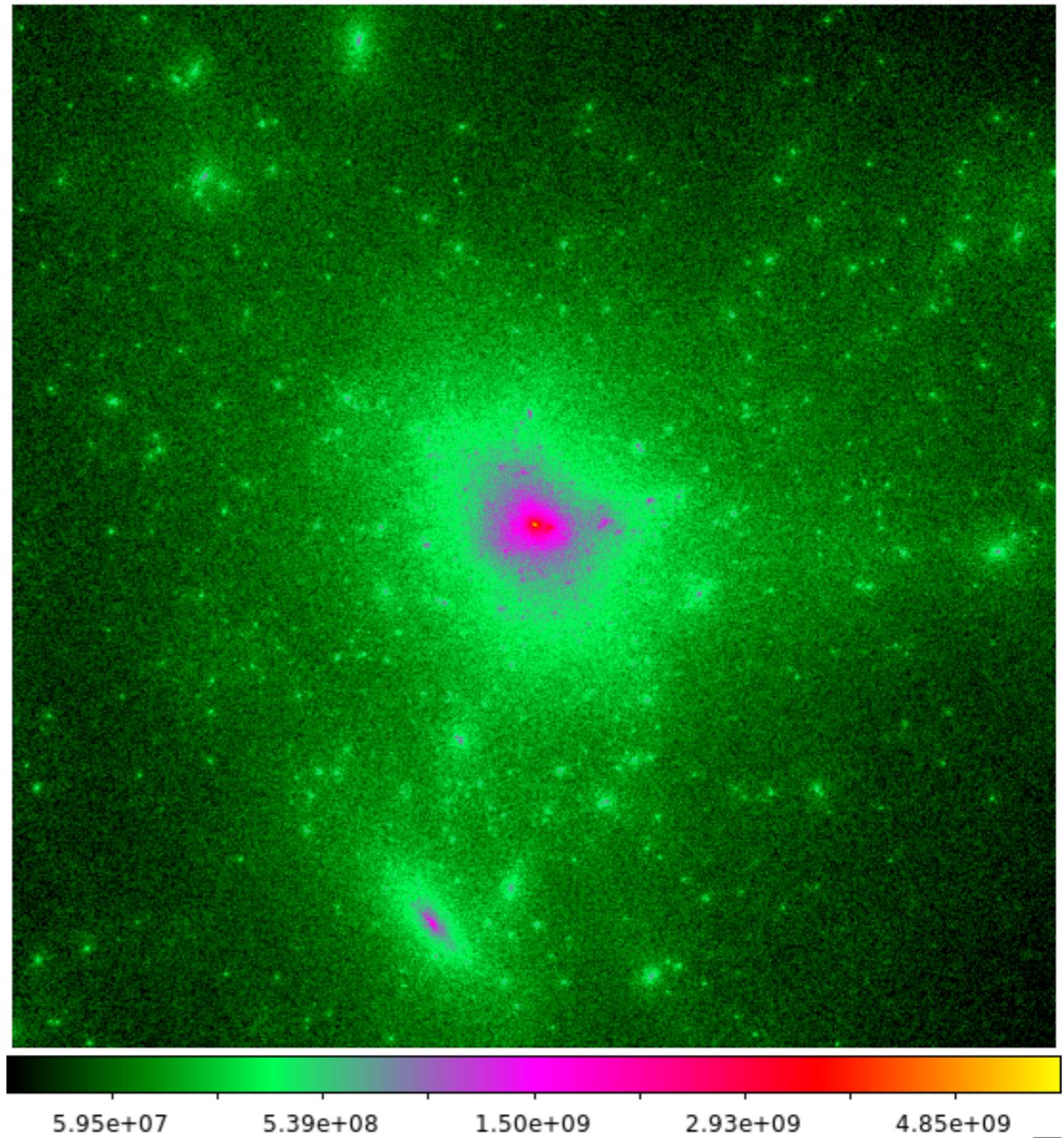
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- This can be easily generalized to the case of a continuum distribution of mass

$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z) dz$$

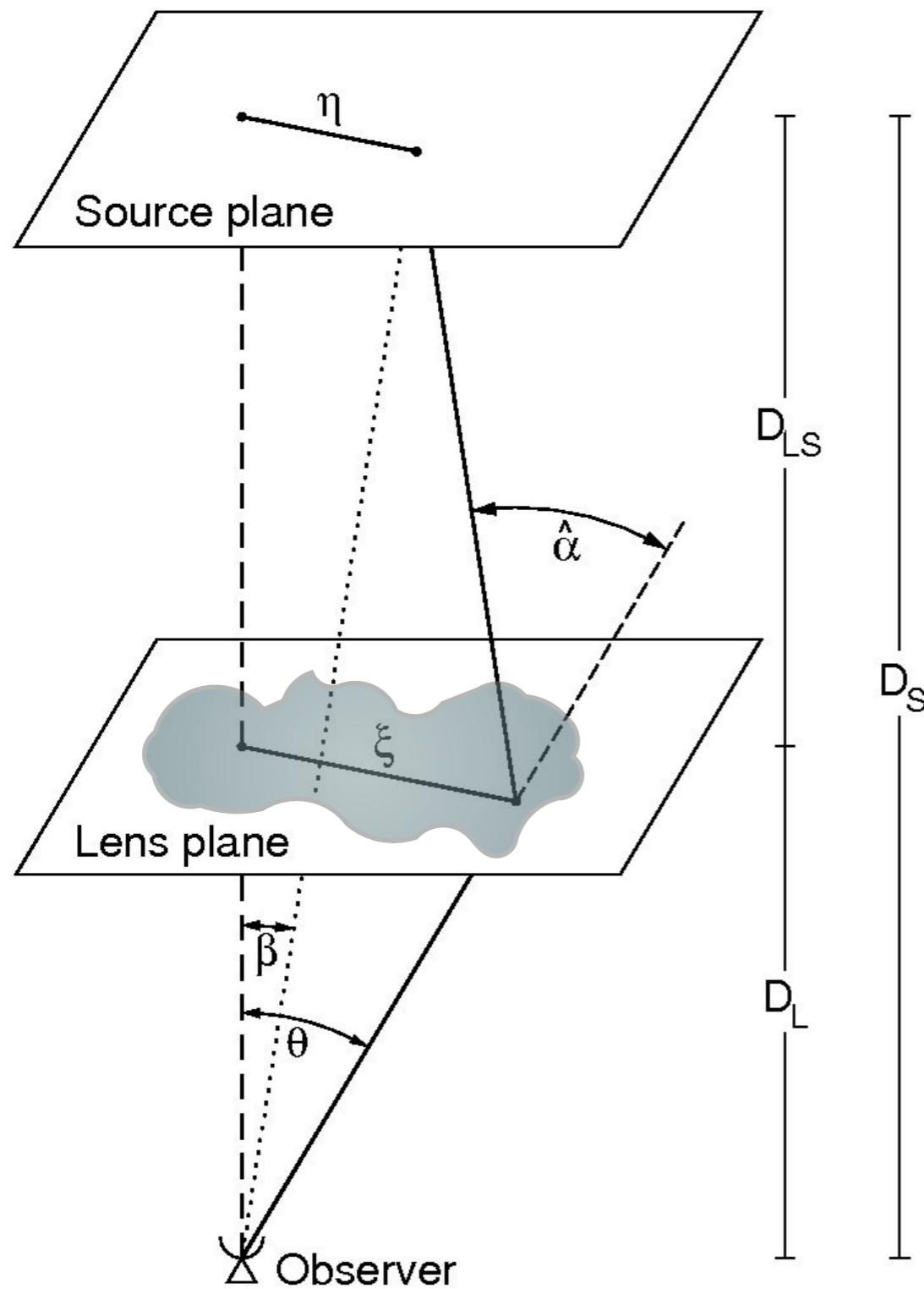
$$d\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2} \Sigma(\vec{\xi}') d\xi'^2$$

$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2} \Sigma(\vec{\xi}') d\xi'^2$$



# LENS EQUATION

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Remember that:

- 1) positions on the lens and source planes are defined by vectors
- 2) the deflection angle itself is a vector

$$\vec{\theta} = \frac{\vec{\xi}}{D_L} \quad \vec{\beta} = \frac{\vec{\eta}}{D_S}$$

$$\vec{\beta} = \vec{\theta} - \frac{D_{LS}}{D_S} \hat{\alpha}(\vec{\theta}) = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

# POINT MASS LENS I

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$$\hat{\alpha} = \frac{4GM}{c^2} \frac{\vec{x} - \vec{x}_o}{|\vec{x} - \vec{x}_o|^2}$$

**Deflection**

$$\begin{aligned}\vec{\alpha}(\vec{\theta}) &= \frac{D_{ls}}{D_s} \hat{\alpha} = \frac{4GM}{c^2} \left( \frac{D_{ls}}{D_s D_l} \right) \frac{\vec{\theta} - \vec{\theta}_o}{|\vec{\theta} - \vec{\theta}_o|^2} \\ &= \theta_E^2 \frac{\vec{\theta} - \vec{\theta}_o}{|\vec{\theta} - \vec{\theta}_o|^2}\end{aligned}$$

**Einstein ring radius**

$$\theta_E^2 = \frac{4GM}{c^2} \left( \frac{D_{ls}}{D_s D_l} \right)$$

**Image positions**

$$\theta = \frac{\beta \pm \sqrt{\beta^2 + 4\theta_E}}{2} \quad \beta \quad \text{source position}$$

