CSE section 4

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2. (a) Let the domain of discourse be integers. $\mathtt{Greater}(x, y) = x > y$, and $\mathtt{Even}(x) = \exists k x = 2k$. The statement is now $\exists x(\mathtt{Greater}(x^2, 10) \wedge \mathtt{Even}(3x))$.

$$\begin{aligned} \mathtt{Greater}(4^2, 10) && \text{(Definition of Greater)} \\ \mathtt{Even}(3 \cdot 4) && \text{(Definition of Even)} \\ \exists x(\mathtt{Greater}(x^2, 10) \wedge \mathtt{Even}(3x)) && \text{(Intro } \wedge\text{)} \end{aligned}$$

   (b) Let the domain of discourse be integers. Let $\mathtt{Prime}(x) = x$ is prime, our statement is now $\forall n \exists p(\mathtt{Prime}(p) \wedge \mathtt{Greater}(p, n))$

$$\begin{aligned} &\text{Let } a \text{ be arbitrary} \\ \mathtt{Prime}(p) && \text{(Definition of prime)} \\ \mathtt{Greater}(p) && \text{(Definition of greater)} \\ \forall n \exists p(\mathtt{Prime}(p) \wedge \mathtt{Greater}(p, n)) && \text{(Intro } \forall\text{)} \end{aligned}$$

3. (a) 3 elements.

   (b) Infinitely many elements.

4. Given any $(x, y) \in (A \cap B) \times C$, we know that $x \in (A \cap B)$, and that $y \in C$. Clearly then $y \in C \cup D$, and also that $x \in A$, because $x \in A$ and $x \in B$. So we have figured out that $(x, y) \in A \times (C \cup D)$.

5. $A \cap (A \cup B) = \{x \mid x \in A \wedge (x \in A \vee x \in B)\} = \{x \mid x \in A\}$, as $p \wedge (p \vee q) \equiv p$. (Also, the first part is just writing out what the set operations mean).