CSE Template

Rohan Mukherjee

July 30, 2024

(a) We have the following.

```
let R: string[][] = [];
let i: number = 0;
{{P_1: R = [], i = 0}}
{{Inv: R = replace(A[0 .. i-1], M)}}
while (i !== A.length) {
    if (contains_key(A[i], M)) {
        const val = get_value(A[i], M);
        {{Inv, contains-key(A[i], M), val = get-value(A[i], M)}}
        R.push(val);
        {{P_2: R[0 .. i-1] = replace(A[0 .. i-1], M), R[i] = val,
        contains-key(A[i], M), val = get-value(A[i], M)}}
    } else {
        {{Inv, !contains-key(A[i], M)}}
        R.push([A[i]]);
        {{P_3: R[0 .. i-1] = replace(A[0 .. i-1], M),
        not contains-key(A[i], M), R[i] = [A[i]]}}
    }
    {{P_2 or P_3}}
    {{Q: R = replace(A[0 .. i], M)}}
    i = i+1;
    {{Inv}}
}
{{P_4: R = replace(A[0 .. i-1], M) and i = n}}
{{R = replace(A, M)}}
```

(b) We prove that $P_1 \implies$ the loop invariant. Since $i = 0$, we know that $A[0..-1] = []$, the empty list. By definition of replace, we have that $\mathit{replace}([], M) = []$. Thus, $R = [] = \text{replace}(\text{A}[0..\text{i}-1], \text{M})$ and the loop invariant holds.

Next we prove that $P_4 \implies$ Post. Since $i = n$, we know that $A[0..i-1] = A[0..n-1] = A$, since n is the length of A. Thus $R = \text{replace}(\text{A}[0..\text{i}-1], \text{M}) = \text{replace}(\text{A}, \text{M})$ and the postcondition holds.

Finally we prove that P_2 or P_3 implies Q. We do this by cases. In the first case, we assume P_2, which is that $R[0..i-1] = \text{replace}(\text{A}[0..\text{i}-1, \text{M}])$, $R[i] = \text{val}$, $\text{contains}-\text{key}(\text{A}[\text{i}], \text{M})$ is true, and that $\text{val} = \text{get}-\text{value}(\text{A}[\text{i}], \text{M})$. We have that:

$$
\begin{aligned}
\text{replace}(\text{A}[0..\text{i}], \text{M}) &= \text{replace}(\text{A}[0..\text{i}-1] + \text{A}[\text{i}], \text{M}) && A[0..i] = A[0..i-1] + A[i] \\
&= \text{replace}(\text{A}[0..\text{i}-1], \text{M}) + [\text{get}-\text{value}(\text{A}[\text{i}], \text{M})] && \text{def replace, contains-key(A[i], M)} \\
&= R[0..i-1] + [\text{val}] && \text{val} = \text{get}-\text{value}(\text{A}[\text{i}], \text{M}) \\
&= R[0..i-1] + [R[i]] && R[i] = val \\
&= R && R = R[0..i-1] + R[i]
\end{aligned}
$$

Thus $P_2 \implies Q$.

In the second case, we assume P_3, which is that $R[0..i-1] = \text{replace}(\text{A}[0..\text{i}-1, \text{M}])$, $!\text{contains}-\text{key}(\text{A}[\text{i}], \text{M})$ is true, and that $R[i] = [A[i]]$. We have that:

$$
\begin{aligned}
\text{replace}(\text{A}[0..\text{i}], \text{M}) &= \text{replace}(\text{A}[0..\text{i}-1] + \text{A}[\text{i}], \text{M}) && A[0..i] = A[0..i-1] + A[i] \\
&= \text{replace}(\text{A}[0..\text{i}-1], \text{M}) + [\text{A}[\text{i}]] && \text{def replace, } !\text{contains}-\text{key}(\text{A}[\text{i}], \text{M}) \\
&= R[0..i-1] + [\text{A}[\text{i}]] && \text{R[0..i-1] = replace(A[0.. i-1])} \\
&= R[0..i-1] + [R[i]] && R[i] = [A[i]] \\
&= R && R = R[0..i-1] + [R[i]]
\end{aligned}
$$

Thus $P_3 \implies Q$. Combining these together shows that P_2 or $P_3 \implies Q$.