Math Template

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June 17, 2024

procedure FINDMATRIX(m, n)
  Initialize an $m \times n$ matrix A with all rows as vectors in $\{1\} \times \{\pm 1\}^{n-1}$
  while number of rows in $A > m$ **do**
    Find a row a of A with the fewest number of zeros
    Find another row b of A that differs in only one entry of a
    Replace both a and b with $(a + b)/2$
  end while
  return A
end procedure

We have to answer a few questions. The only thing to prove is that such a b always exists, and more specifically, if i is the entry that a and b differ in, then $a_i = 1$ and $b_i = -1$ or vice versa. We prove this by induction. Initially this is true, since for each row $a \in \{1\} \times \{\pm 1\}^{n-1}$, we could flip the second entry to get a b that differs in only one entry, where the entry of neither a nor b is 0. Now suppose the claim is true inductively, that at the previous step of the algorithm, all rows a with the minimal number of zeros have at least one corresponding b that differs in only one entry at position i where $a_i = 1$ and $b_i = -1$ or vice versa (neither a_i nor b_i are 0).