# CSE 311 Quiz 5

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5. a) If $a = 0$, we see that $0 \mid b$, which forces b to be 0, as the only multiple of 0 is 0 itself. Clearly then $a = b$. The exact same scenario plays out if $b = 0$, so we may exclude the case where $a = 0$ or $b = 0$. Therefore, given $a, b \in \mathbb{Z} \setminus 0$, if $a \mid b$, then there is some $k \in \mathbb{Z}$ so that $b = ak$. Similarly, as $b \mid a$, there is some $l \in \mathbb{Z}$ so that $a = bl$. Plugging the second equation into the first one, we see that $b = blk$, and as $b \neq 0$, we see can divide both sides by b to get that $1 = lk$. The only way this is possible is if $l, k \in \{\pm 1\}$ (either -1 · -1, or $1 \cdot 1$). Plugging in the value for $l$, we see that $a = b$ or $a = -b$, so we are done.

6. a) Let $P(n) = \sum_{k=0}^{n} k = \frac{n(n+1)}{2}$. Clearly $\sum_{k=0}^{0} k = 0 = 0(0+1)/2$, so the base case is finished. Now suppose for some arbitrary $l \in \mathbb{N}$ that $P(l)$ holds. Then

$$\sum_{k=0}^{l+1} k = \left(\sum_{k=0}^{l} k\right) + (l+1) = \frac{l(l+1)}{2} + (l+1)$$

Where for the last equality we have used the inductive hypothesis. Simplifying the RHS, we get that

$$\begin{aligned} \frac{l(l+1)}{2} + (l+1) = \frac{l(l+1)}{2} + \frac{2l+2}{2} &= \frac{l^2 + l + 2l + 2}{2} \\ &= \frac{l^2 + 3l + 2}{2} \\ &= \frac{(l+1)(l+2)}{2} \end{aligned}$$

Which shows that $P(l+1)$ holds. By the principal of mathematical induction, $P(n)$ must be true for all $n \in \mathbb{N}$, and we are done.