Math Template

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As per the usual, let $H_1 = \begin{pmatrix} 1 \end{pmatrix}$ and define the nth hadamard matrix of size $2^n \times 2^n$ recursively by:

$$H_n = \begin{pmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -H_{n-1} \end{pmatrix}$$

Decompose the vector $x \in \{-1, 1\}^{2^n}$ as $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ where $x_1, x_2 \in \{-1, 1\}^{2^{n-1}}$. Then, we can write:

$$H_n x = \begin{pmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -H_{n-1} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} H_{n-1}x_1 + H_{n-1}x_2 \\ H_{n-1}x_1 - H_{n-1}x_2 \end{pmatrix}$$

If the max coordinate in $H_{n-1}x_1$ has the same sign as the max coordinate in $H_{n-1}x_2$, then the max coordinate in $H_n x$ is just $|H_{n-1}x_1 + H_{n-1}x_2|$. On the other hand, if they have differing signs, the max coordinate is just $|H_{n-1}x_1 - H_{n-2}x_2|$. Since $|H_{n-1}x_1| \approx \beta(H_{n-1})$, we have that that $|H_n x| \approx 2\beta(H_{n-1})$. Normalizing H_n wil give that $|\frac{1}{\sqrt{2^n}} H_n x| \approx \frac{2}{\sqrt{2}} \beta\Big(\frac{1}{\sqrt{2^{n-1}}} H_{n-1}\Big)$. This would say that $\beta(H_n) \approx \sqrt{2^n}$ up to constants.

We write

$$2^8 = 2^4 \cdot 7 + 2^7 + 2^4$$

$$2^{16} = 2^7 \cdot 7 + 2^{11} \cdot 7 + 2^5 \cdot 5^3 \cdot 7 + 2^9 \cdot 5 \cdot 7 + 2^8 \cdot 3 \cdot 5 + 2^9 + 2^5$$