CSE Section 8

Rohan Mukherjee

February 23, 2023

1. (a) $0 \cup ([1,9][0,9]^*)$.
   (b) $0 \cup (1 \cup 2)(0 \cup 1 \cup 2)^* 0$.
   (c) $[(01 \cup 10) \cup (001 \cup 100 \cup 1)^*]111[(01 \cup 10) \cup (001 \cup 100 \cup 1)^*]$.
   (d) $((0 \cup \varepsilon)(10)^*) \cup ((01)^*(0 \cup \varepsilon))$
   (e) $(1[10^* \cup 0^*1])^*(0 \cup 1)^*$

   (a) $S \to T00$
   $T \to 1T \mid 0T \mid T1 \mid T0 \mid 0 \mid 1 \mid \varepsilon$
   (b) $S \to T1T1T1T$
   $T \to 1T \mid 0T \mid T1 \mid T0 \mid 0 \mid 1 \mid \varepsilon$
   (c) $S \to 1S0 \mid 0S1 \mid \varepsilon$
   (d) $S \to 0S0 \mid 1T0 \mid 0T1 \mid 1S1$
   $T \to 0T0 \mid 1T0 \mid 0T1 \mid 1T1 \mid 00 \mid 01 \mid 10 \mid 11$ This last one does this: keep adding stuff to both sides until you pick one where they are different, then just go to the next level, where you can just do whatever you want and eventually end (guaranteed different though, because to get there you must've picked at least 1 digit different).

5. (a) The bug in this proof is that they only proved it for one of the recursive steps given, but there are 4. The author only proved this for trees that have both left and right child nodes always, which is not necessarily the case.
   (b) Let T be an arbitrary tree of height k. Pick an arbitrary leaf node at the bottom level of this tree. There are three ways to make this tree height $k+1$:
   (1) We add a node to the right of this leaf,
   (2) We add a node to the left of this leaf,
   (3) We could add both a left node and a right node.
   ...So we see that in any case, the tree has an odd number of nodes. As we chose a leaf node arbitrarily, we could do case 1 2 or 3 to any leaf node, our proposition would still hold true, as our proof only depended on the tree currently having an odd number of nodes.

6. For a **Tree** T, let $P(T) \coloneqq \text{sum}(T) = \text{sum}(\text{reverse}(T))$.

   Base case: We notice that $\text{sum}(\text{Nil}) = \text{sum}(\text{reverse}(\text{Nil})) = 0$, because $\text{reverse}(\text{Nil}) = \text{Nil}$.

   Inductive Hypothesis: Suppose for some arbitrary **Trees** L and R, $P(L)$ and $P(R)$ hold, and that $x \in \mathbb{Z}$ is an arbitrary integer.

   Inductive Step: $\boxed{\text{Goal: } P(\text{Tree}(x, L, R))}$

   We notice that $\text{reverse}(\text{Tree}(x, L, R)) = \text{Tree}(x, \text{reverse}(L), \text{reverse}(R))$. Then, $\text{sum}(\text{Tree}(x, L, R)) = x + \text{sum}(L) + \text{sum}(R)$ by definition, and we also see that $\text{sum}(\text{reverse}(\text{Tree}(x, L, R))) = \text{sum}(\text{Tree}(x, \text{reverse}(L), \text{reverse}(R))) = x + \text{sum}(\text{reverse}(L)) + \text{sum}(\text{reverse}(R))$. By the inductive hypothesis, $\text{sum}(\text{reverse}(L)) = \text{sum}(L)$, and $\text{sum}(\text{reverse}(R)) = \text{sum}(R)$, so we see that these quantities are indeed equal.

   By the principal of induction, we see that $P(T)$ holds for all trees T.