Math 338 - Homework 2

Due Friday 1/21

Answer the following questions. You are encouraged to work with other students and to seek help from the instructor while working on these problems, but please write up your answers on your own.

1. (Boyce 2.1) Look at these axioms for length:

> **Axioms of Length:** Each segment $\overline{AB}$ can be assigned a positive number, called the **length** of $\overline{AB}$, denoted AB, such that the following properties hold:
>
> - The length of the unit segment is 1.
> - $AB = BA$.
> - If A, B, and C are three points with B between A and C, then AC is equal to $AB + BC$.
> - There exists a unique point M between A and B, called the *midpoint* of $\overline{AB}$, such that $AM = MB$.

Suppose $A = B$. What number must be assigned as the measure of "segment" $\overline{AB}$ in order to be consistent with the axioms of length? Explain why assigning any other number would result in a contradiction to one or more of the axioms of length.

Solution: The length of $\overline{AB}$ must be 0. Suppose by way of contradiction that $\overline{AB}$ has length $d > 0$. There must be a unique midpoint M between A and B such that $\overline{AM} = \overline{MB}$. We also know that $\overline{AM} + \overline{MB} = d$. But because M is between A and B, $M = A = B$. So then we have $\overline{AM} = \overline{MB} = \overline{AB}$. This meanse $2d = d$, which is a contradiction when $d \neq 0$.

2. (Boyce 2.2) Look at these axioms for angle measure:

> **Axioms of Angle Measure:** Every minor angle $\angle ABC$ can be assigned a positive number between 0 and 180, called the degree measure of the angle, so that the following properties hold:
>
> - The degree measure of a right angle is 90°.
> - Measure of $\angle ABC$ = Measure of $\angle CBA$.
> - If point D is in the interior of $\angle ABC$, then the measure of $\angle ABC$ is equal to the sum of the measures of $\angle ABD$ and $\angle DBC$.
> - There exists a unique ray that is the angle bisector of $\angle ABC$.

Suppose $A = C$. What number must be assigned as the measure of angle $\angle ABC$ in order to be consistent with the axioms of angle measure? Why?

Solution:

The angle $\angle ABC$ must have measure 0. Since A is in the interior of $\angle ABC$, the measure of $\angle ABA+$ the measure of $\angle ABC =$ the measure of $\angle ABC$. This shows that $\angle ABA$ has measure 0. But $A = C$ so $\angle ABC$ must have measure 0.

3. (Boyce 2.3) Prove that two segments are congruent if and only if they have the same length.

   Solution: First suppose two segments are congruent. That means there is an isometry that takes one segment to the other. Since isometries don't change distance, the two segments must have the same length.

   Now suppose two segments, say $\overline{AB}$ and $\overline{CD}$, have the same length. We can translate so that point A lies on point C. Then rotate the segments so they overlap. Since they are the same length, this will send point B to point D. So after a series of isometries, $\overline{AB}$ can be moved to $\overline{CD}$. This means they are congruent.

4. (Boyce 2.6) Suppose $\angle ABC$ and $\angle CBD$ are supplementary, and $\angle ABC \cong \angle EBD$. Prove $\angle ABE \cong \angle CBD$. (Feel free to use the Congruence and Angle Measure Theorem)

   Solution: Let x be the measure of angle $\angle ABC$, and let y be the measure of $\angle CBD$. Since they are supplementary, $x + y = 180°$. Since $\angle EBD$ is congruent to $\angle ABC$, its measure is also x.

   Since $\angle ABC$ and $\angle CBD$ are supplementary, A, B, and D must be collinear, with B between A and D. This means $\angle ABD$ and $\angle EBD$ are supplementary. That means $x+$ the measure of $\angle ABE$ is $180°$. Therefore, $\angle ABE$ has measure y.

   Using the Congruence and Angle Measure Theorem, we can say that since $\angle ABE$ and $\angle CBD$ have the same measure, they are congruent.