Math 338 - Homework 6

Due Friday 2/18

Answer the following questions. You are encouraged to work with other students and to seek help from the instructor while working on these problems, but please write up your answers on your own.

1. (Boyce 6.3) Prove the Inscribed Quadrilateral Theorem: Let $ABCD$ be a quadrilateral inscribed in a circle. Then $\angle ABC$ and $\angle CDA$ are supplementary, and $\angle BCD$ and $\angle DAB$ are supplementary.

   Solution: Let O be the center of the circle. Then the Incribed Angle Theorem shows that $m\angle ABC$ is $1/2$ of the measure of the corresponding central angle $\angle AOC$. Let $\theta = \angle AOC$. Then there is another central angle with endpoints A and C and measure $360° - \theta$. The Inscribed Angle Theorem says $m\angle ADC$ is $(1/2)(360° - \theta) = 180° - (1/2)\theta) = 180° - m\angle ABC$.

   This shows that $\angle ABC$ and $\angle ADC$ are supplementary. The same argument shows that $\angle BCD$ and $\angle DAB$ are supplementary.

2. (Boyce 6.4) Prove that if A and B are points on a circle such that $\overline{AB}$ is a diameter of the circle, and C is any other point on the circle, then ΔABC is a right triangle.

   Solution: Suppose $\overline{AB}$ is a diameter of a circle. Then if O is the center of the circle, $\angle AOB$ has measure $180°$. If C is a point on the circle that is not A or B, then $\angle ACB$ is an inscribed angle with central angle $\angle AOB$. So by the Incribed Angle Theorem, the measure of $\angle ACB$ is half of the measure of $\angle AOB$, so it is $90°$.

3. (Boyce 6.8) Prove the Tangent-Radius Theorem: Let P be a point on a circle centered at point O, and let $\overleftrightarrow{PQ}$ be a line through P that is tangent to the circle. Then $\overline{OP}$ is perpendicular to $\overleftrightarrow{PQ}$.

   Solution: Suppose P is a point on the circle. Let C be the point on the circle so that $\overline{PC}$ is a diameter. Choose any third point B. If Q is a point on the tangent line to the circle at P, by the Lemma (Boyce 6.5), $\angle QPB$ is congruent to $\angle PBC$. By construction $\angle PBC$ is a right angle (since $\overline{PC}$ is a diameter). That means $\angle QPB \cong \angle QPO$ is a right angle as well.

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