Math 338 - Homework 2

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Answer the following questions. You are encouraged to work with other students and to seek help from the instructor while working on these problems, but please write up your answers on your own.

1. (Boyce 2.1) Look at these axioms for length:

Axioms of Length: Each segment $\overline{AB}$ can be assigned a positive number, called the **length** of $\overline{AB}$, denoted AB, such that the following properties hold:

- The length of the unit segment is 1.
- AB = BA
- If A, B, C are three points with B between A and C, then AC is <u>equal</u> to AB+BC.
- There exists a uniques point M between A and B, called *midpoint* of $\overline{AB}$, such that $AM = MB$

Suppose $A = B$. What number must be assigned as the measure of "segment" $\overline{AB}$ in order to be consistent with the axioms of length? Explain why assigning any other number would result in a contradiction to one or more of the axioms of length.

0 is the only measure of "segment" of $\overline{AB}$ that would be consistent with each axiom.

Conj: If $A = B$ then $AB = 0$.

<u>Proof</u>:

Suppose by way of contradiction (BWOC) that $AB \neq 0$.

Then $AB > 0$.

Suppose we choose C so B is between A and C.

Then $AB + BC = AC$ (length axiom 3)

$\Rightarrow AB + BC = BC$ (A=B)

$\Rightarrow AB = 0$ (subtract BC from both sides)

This contradicts our assumption that $AB > 0$. So $AB = 0$. $\quad\square$

2. (Boyce 2.2) Look at these axioms for angle measure:

Axioms of Angle Measure: Every minor angle $\angle ABC$ can be assigned a positive numeber between 0 and 180, called the drgree measure of the angle, so that the following properties hold:

- The degree measure of a right angle 90°.
- Meausure of $\angle ABC$ = Measure of $\angle CBA$
- If point D is in the interior of $\angle ABC$, then the measure of $\angle ABC$ is equal to the sum of the measures of $\angle ABD$ and $\angle DBC$
- There exists a unique ray that is the angle bisector of $\angle ABC$.

Suppose $A = C$. What number must be assigned as the measure of $\angle ABC$ in order to be consistent with the axioms of angle measure? Why?

0 is the only measure of $\angle ABC$ that would be consistent with all of the axioms of angle measure.

Conj: If $A = C$ then $m\angle ABC = 0$.

<u>Proof</u>:

Suppose BWOC that $m\angle ABC \neq 0$.

Then $m\angle ABC > 0$.

Suppose we choose a unique ray $\overline{BD}$ that creats a right angle with $\overline{AB}$ such that $m\angle ABD = 90°$.

Then $90° = m\angle ABD = m\angle ABC + m\angle CBD$.

$$\begin{aligned} \Rightarrow m\angle ABC &= m\angle ABD + m\angle CBD \\ &= m\angle ABD + m\angle ABD \\ &= 0 \end{aligned} \tag{1}$$

This contradicts our assumption that $m\angle ABC > 0$. So $m\angle ABC = 0$. $\qquad \square$

3. (Boyce 2.3) Prove that two segments are congruent if and only if they have the same length.

<u>Proof:</u>

1. $\overline{AB} \cong \overline{CD} \Rightarrow AB = CD$.

Suppose $\overline{AB} \cong \overline{CD}$.

Then there is an isometry that superimposes $\overline{AB}$ on $\overline{CD}$.

So let f be that isometry : $f(\overline{AB}) = \overline{CD}$.

Isometries preserve distance.

The length of $f(\overline{AB})$ equals the length of $\overline{AB}$.

So by substitution $CD = AB$.

2. $AB = CD \Rightarrow \overline{AB} \cong \overline{CD}$

Suppose $AB = CD$.

There is a translation along the $\vec{v}$ from A to C, $f_1(A) = C$.

Then f_2 rotates $f_1(\overline{AB})$ by $\theta = \angle DC f_1(B)$, center $f_1(A)$.

Then by the definition of rotation then $f_2(f_1(B)) = D$ takes $f_2(f_1(\overline{AB})) = \overline{CD}$.

Therefore they are congruent.   □

4. (Boyce 2.6) Suppose $\angle ABC$ and $\angle CBD$ are supplementary, and $\angle ABC \cong \angle EBD$. Prove $\angle ABE \cong \angle CBD$. (Feel free to use the Congruence and Angle Measure Theorem)

Proof:

1. $\angle ABE \cong \angle CBD \Rightarrow m\angle ABE = m\angle CBD$.

Suppose $\angle ABE \cong \angle CBD$.

There is an isometry that superimposes $\angle ABE$ onto $\angle CBD$.

Let f be that isometry : $f(\angle ABE) = \angle CBD$.

Isometries preserve angle.

The angle measure of $f(\angle ABE)$ equals the angle measure of $\angle CBD$.

So by substitution $m\angle ABE = m\angle CBD$.

2. $m\angle ABE = m\angle CBD \Rightarrow \angle ABE \cong \angle CBD$.

Suppose $m\angle ABE = m\angle CBD$.

There is a rotation $f_1(m\angle ABE)$ about B by $\theta = 180°$.

It is given that $\angle ABC$ and $\angle CBD$ are supplementary, so we know that point $f_1(A) = D$, $f_1(C) = E$, and $f_1(B) = B$.

It is also given that $\angle ABC \cong \angle EBD$, which preserves angles, so we also know that $f_1(E) = C$.

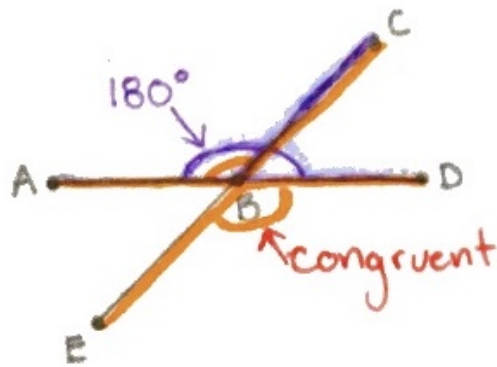Then by the definition of rotation $f_1(m\angle ABE) = m\angle CBD$.

Therefore $\angle ABE \cong \angle CBD$ □.

Figure 1: Image of Problem 4