Math 338 - Homework 1

Due Friday 1/14

Answer the following questions. You are encouraged to work with other students and to seek help from the instructor while working on these problems, but please write up your answers on your own.

1. (Boyce 1.1) Create truth tables for 'X and Y' and 'X or Y'

   Solution:

X	Y	X and Y	X or Y
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

2. (Boyce 1.2) Consider the following statement, for which n represents an arbitrary positive integer.

   $$\text{If } n < 2 \text{ then } n < 5.$$

   (a) Is the conditional statement true (for any value of n)?

   (b) State the negation, converse, contrapositive, and inverse of the statement. Which are true? Which are false? Why?

   Solution:

   (a) It is true. For any $n < 2$, $n < 2 < 5$ so $n < 5$.

   (b)
   - *Negation: There exists an n which is smaller than 2 but not smaller than 5.* This is false, since the original statement was true.
   - *Converse: If $n < 5$ then $n < 2$.* This is false, since the number $n = 4$ is less than 5 but is not less than 2.
   - *Contrapositive: If $n \geq 5$ then $n \geq 2$.* This is true, since if $n \geq 5 > 2$ then $n \geq 2$.
   - *Inverse: If $n \geq 2$ then $n \geq 5$.* This is false, since the number $n = 4$ is greater than 2 but not greater than or equal to 5.

3. (Boyce 1.4) We defined "collinear" as follows: three points in a plane are **collinear** if one point lies on the line that connects the other two points.

   (a) Create a definition for **non-collinear** points in a place.

   (b) According to your definition, can two points ever be non-collinear? Explain.

Solution:

   (a) One option: three points are not collinear if for each line connecting two of the points, the remaining point does not lie on that line.

   (b) Based on this definition, they cannot, since there is no third point to lie off of the line connecting them.

4. (Barsamian 1) Look at the following axiom system. Which of the following interpretations is a model of Axiom System 1? Explain. (Hint: Two of the following are models and one is not.)

Axiom System:	Axiom System #1
Primitive Relations:	relation on the set of integers spoken "x is related to y"
Axioms:	<1> 5 is related to 7 <2> 5 is related to 8 <3> For all integers x and y, if x is related to y, then y is related to x. <4> For all integers x, y, and z, if x is related to y and y is related to z, then x is related to z.

   (a) Interpret the words 'x is related to y' to mean '$xy > 0$'.

   (b) Interpret the words 'x is related to y' to mean $xy \neq 0$'.

   (c) Interpret the words 'x is related to y' to mean 'x and y are both even or both odd'.

Solution:

   (a) 'x is related to y' to mean '$xy > 0$': This is a model. The rule that $xy > 0$ holds for any pair of integers of the same sign. $5, 7, 8$ all have the same sign so $\langle 1 \rangle$ and $\langle 2 \rangle$ hold. $\langle 3 \rangle$ holds since order is not important in $xy > 0$. And $\langle 4 \rangle$ holds since if x and y share a sign, then if z shares a sign with y it also shares a sign with x.

(b) 'x is related to y' to mean $xy \neq 0$': This is a model. The rule that $xy \neq 0$ requires only that both x and y are not zero. This holds in $\langle 1 \rangle$ and $\langle 2 \rangle$. In $\langle 3 \rangle$ it holds since order is not important. And $\langle 4 \rangle$ holds since x is related to y and y is related to z tells us all of x, y, z are nonzero, so x is related to z.

(c) 'x is related to y' to mean 'x and y are both even or both odd': This is not a model. $\langle 2 \rangle$ fails since 5 is odd but 8 is even.