Math 338 - Homework 8

Due Friday 3/4

Answer the following questions. You are encouraged to work with other students and to seek help from the instructor while working on these problems, but please write up your answers on your own.

1. (Boyce 8.4) A definition of a *radian* angle measure is the following: the radian unit is the angle at the center of a circle whose arc is equal in length to the circle's radius. Compare an angle of radian measure $\pi/2$ in Euclidean geometry and an angle of radian measure $\pi/2$ in taxicab geometry.

   Solution: In Euclidean geometry, and angle of measure $\pi/2$ is a right angle ($90°$). In Taxicab geometry, we saw the circumference of a circle is $8r$. So a radian would be the angle corresponding to 1/8 of the circle ($45°$). An angle of $\pi/2$ radians would be $4/2 = 2$ radians, which is $90°$.

2. Draw a Taxicab triangle that violates the following rule for Euclidean triangles: The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

   Solution: The triangle with vertices $(0,0)$, $(1,0)$ and $(0,1)$ has side lengths 1, 1, and 2. This violates the rule stated above.

3. (Boyce 8.5) Come up with examples of each, or explain why such an example is not possible:

   (a) Two triangles that are congruent in both Euclidean geometry and Taxicab geometry.

   (b) Two triangles that are congruent in Euclidean geometry but not in Taxicab geometry.

   (c) Two triangles that are congruent in Taxicab geometry but not in Euclidean geometry.

   Solution:

(a) One example is the triangle formed by the vertices $A = (0, 0)$, $B = (1, 0)$ and $C = (0, 1)$, and the triangle formed by the vertices $D = (4, 0)$, $E = (5, 0)$ and $F = (4, 1)$. The translation of ΔABC on the vector $(4, 0)$ superimposes it on ΔDEF. Since translations are isometries in both Euclidean and Taxicab geometry, $\Delta ABC \cong \Delta DEF$ in both geometries.

(b) One example is ΔABC with vertices $A = (0, 0)$, $B = (1, 0)$, and $C = (0, 1)$, and ΔDEF with vertices $D = (0, 0)$, $E = (\sqrt{2}/2, -\sqrt{2}/2)$, and $F = (\sqrt{2}/2, \sqrt{2}/2)$. For $\Delta = ABC$, the side lengths are 1, 1, and $\sqrt{2}$. For ΔDEF the side lengths are 1, 1, and $\sqrt{2}$. So by the SSS Congruence Theorem, $\Delta ABC \cong \Delta DEF$.

In Taxicab geometry, ΔABC has side lengths 1, 1, and 2. But ΔDEF has side lengths $\sqrt{2}$, $\sqrt{2}$, and $\sqrt{2}$. The triangles cannot be congruent, since length is preserved by isometries.

(c) This is not possible. If two triangles are congruent in Taxicab geometry, the isometry that superimposes one on the other is necessarily also a Euclidean isometry.