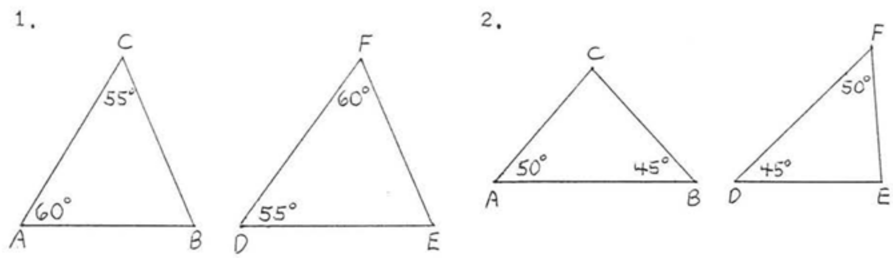
Math 338 - Homework 5

Due Friday 2/11

Answer the following questions. You are encouraged to work with other students and to seek help from the instructor while working on these problems, but please write up your answers on your own.

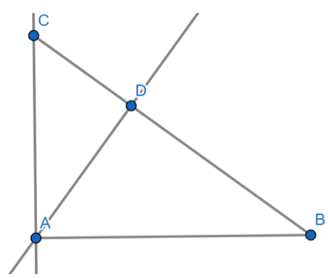
1. (Africk 4.2 1,2) Write down the similarity relations for each pair of triangles:



Solution: By the AA Similarity Theorem, we can say that in 1, $\Delta ABC \sim \Delta FED$ and in 2, $\Delta ABC \sim \Delta FDE$.

2. (Barsamian 10.2.10) Prove that the altitude to the hypotenuse of a right triangle creates two smaller triangles that are each similar to the larger triangle.

Solution: Let D be the point on the hypotenuse of right triangle ΔABC where the segment $\overline{AD}$ is perpendicular to the hypotenuse.



Then triangle ΔDAC is a right triangle by construction, with right angle $\angle CDA$. Also, ΔABC and ΔDAC share $\angle ACD$. So by the AA Similarity Theorem, $\Delta ABC \sim \Delta DAC$.

Similarly, the triangle ΔDAB is a right triangle, and it shares $\angle ABD$ with ΔABC So again by the AA Similarity Theorem, $\Delta DAB \sim ACB$.

The similarity relation is transitive, since compositions of dilations and isometries can be constructed as one dilation composed with isometries, so $\Delta DAC \sim DBA$.

3. (Boyce 5.6) Prove the SSS Similarity Theorem: Let ΔABC and ΔDEF be triangles such that
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}.$$
Then $\Delta ABC \sim \Delta DEF$.

Solution: Consider ΔABC. We can apply a dilation with center A and scaling factor $k = DE/AB$, so that B' is now in a position where $AB' = kAB = DE$, and C' is in a position where $AC' = kAC = (DE/AB)AC$. By assumption, $DE/AB = FD/CA$, so $AC' = FD$.

The triangle $\Delta AB'C'$ now shares an angle with ΔABC ($\angle ABC$), and satisfies $AB/AB' = 1/k = AC/AC'$. So the SAS Simlarity Theorem tells us $\Delta ABC \sim AB'C'$.

Since ΔABC is similar to $\Delta AB'C'$, we know that $B'C'/BC = AB'/AB = k = DE/AB$. But by assumption, $DE/AB = EF/BC$, so $B'C'/BC = EF/BC$. This tells us that $B'C' = EF$.

Now, by the SSS Congruence Theorem, $\Delta AB'C' \cong \Delta DEF$. Therefore, $\Delta ABC \sim \Delta DEF$.