

Name: \_\_\_\_\_

## Math 338 Practice Final Exam

*Answer each question in the space provided. To receive full credit, you must clearly present your solution and any necessary computations.*

1. (15 points) Decide if each of the following statements is True or False. Explain your answer.

- (a) If two triangles are congruent in Euclidean geometry, then they're congruent in Taxicab geometry.

- (b) If  $ABCD$  is a parallelogram, then  $\triangle ABC \cong \triangle CDA$ .

- (c) If  $ABCD$  is a quadrilateral inscribed in a circle, then  $\angle ABC$  and  $\angle CDA$  are supplementary.

2. (10 points) Define the following terms.

(a) Euclidean metric (give the formula too!)

(b) Circle inversion

3. (15 points) Prove that if  $ABCD$  is a parallelogram inscribed in a circle,  $ABCD$  is a rectangle.

4. (10 points) Explain why it isn't possible to construct a square in the Poincare disk.

5. (25 points) Fill in the missing steps in the following proof.

Claim: Suppose  $\triangle ABC$  and  $\triangle DEF$  are triangles such that  $\angle ABC \cong \angle DEF$ , and  $\angle BCA \cong \angle EFD$ , then  $\triangle ABC \sim \triangle DEF$ .

Proof:

- Apply a dilation to  $\triangle ABC$  with center  $B$  and scaling factor  $k = \underline{\hspace{2cm}}$ .
- After the dilation, call the triangle  $\triangle A'B'C'$  where  $B = B'$  and  $A$  moved to  $A'$ ,  $C$  moved to  $C'$ .
- Then  $A'B' = DE$  by our choice of  $k$ .
- We also know that  $\angle A'B'C' \cong \angle DEF$  because  $\underline{\hspace{2cm}}$ .
- Further,  $\angle B'C'A' \cong \angle EFD$  because  $\underline{\hspace{2cm}}$ .
- So by the  $\underline{\hspace{2cm}}$  Triangle Congruence Theorem,  $\triangle A'B'C' \cong \triangle DEF$ .
- Therefore  $\underline{\hspace{2cm}}$ .

6. (25 points) Suppose  $ABCD$  is a parallelogram. Prove that if  $X$  is a point on  $\overline{CD}$  such that  $\overline{BX}$  is perpendicular to  $\overline{CD}$ , then the area of  $ABCD$  is  $BX \cdot CD$ . (You can use the axioms for area, including the area of a rectangle formula, but **not** the area of a triangle formula.)