

# MTH 344-001 Winter 2022 HW 1

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1. Do the following define operations on the given set? Why or why not? (Your answers only need to be a sentence or two; if these do define operations, you do not need to check if they're commutative, associative, ect. in this problem.)

(a)  $a * b = 4a - b$  on  $\mathbb{Z}$

(b)  $a * b = 3^b$  on  $\mathbb{Q}$

(c)  $a * b = \frac{a}{ab+2}$  on  $\mathbb{R}$

(d)  $a * b = \frac{1}{ab+2}$  on  $\mathbb{R}^+$

2. Define an operation  $*$  on  $\mathbb{R}$  by  $a * b = 3a - 4b + 2$

(a) Is  $*$  commutative?

$$\begin{aligned} a * b &= a + b - ab \\ &= b + a - ab \quad \text{since } + \text{ is commutative} \\ &= b + a - ba \quad \text{since } \cdot \text{ is commutative} \\ &= b * a \end{aligned} \tag{1}$$

(b) Is  $*$  associative?

(c) Is there an identity element  $e \in \mathbb{R}$  w.r.t.  $*$ ?

(d) Does every element  $a \in \mathbb{R}$  have an inverse w.r.t.  $*$ ?

3. Define an operation  $*$  on  $\mathbb{R}$  by  $a * b = a + b - ab$ .

(a) Is  $*$  commutative?

(b) Is  $*$  associative?

(c) Is there an identity element  $e \in \mathbb{R}$  w.r.t.  $*$ ?

(d) Does every element  $a \in \mathbb{R}$  have an inverse w.r.t.  $*$ ?

4. Define an operation  $*$  on the set  $G = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x \neq 0\}$  by

$$(a, b) * (c, d) = (ac, ad + bc).$$

Prove that  $\langle G, * \rangle$  is an abelian group. (You do not need to check associativity. We will show this in class.)

5. Let  $a, b, c, x$  be elements of a group  $G$ . Solve the following system of equations for  $x$  :

(a)  $bx^2 = ax^{-1}$     and  $x^4 = c$

(b)  $x^2c = bxa^{-1}$     and  $xca = cax$

6. This problem asks you to consider the importance of using proper notation for inverses in groups. Your answer only needs to be a sentence or two.

Suppose  $a, b$ , and  $x$  are elements of a *nonabelian* group  $G$ , and that we want to solve the equation  $ax = b$  for  $x$ . Why would it be incorrect and unclear to say that the solution is  $x = \frac{b}{a}$