MTH 344-001 Winter 2022 HW 2
Due: Friday January 28th

1. In each of the following parts, briefly explain why H is *not* a subgroup of G. (The operations in the groups $\mathbb{Z}$ and $\mathbb{R}$ are addition. The operation in $\mathbb{R} \times \mathbb{R}$ is coordinate-wise addition.)
   (a) $H = \{0, 1, 2, 3, 4, 5\}$, $G = \mathbb{Z}$
   (b) $H = \mathbb{R}^+$, $G = \mathbb{R}$
   (c) $H = \{(x, y) \mid y = \sqrt{x}\}$, $G = \mathbb{R} \times \mathbb{R}$

2. In each of the following parts, prove that H is a subgroup of G. (The operation in $\mathbb{R}^*$ is multiplication and the operation in $\mathbb{R} \times \mathbb{R}$ is coordinate-wise addition.)
   (a) $H = \{3^m 4^n \mid m, n \in \mathbb{Z}\}$, $G = \mathbb{R}^*$
   (b) $H = \{(x, y) \mid y = -3x\}$, $G = \mathbb{R} \times \mathbb{R}$

3. Let G be a group. We say that elements $a, x \in G$ *commute* if $ax = xa$. Prove that if a and x commute, then a^{-1} and x also commute, i.e. $a^{-1}x = xa^{-1}$.

4. Let G be a group. The *center* of G is the set C of all elements of G that commute with every other element of G:
$$C = \{g \in G \mid xg = gx \text{ for all } x \in G\}.$$
   Prove that C is a subgroup of G. (Hint: Use problem 3 when showing C is closed under inverses.)

5. Let H be a subgroup of a group G and let $a \in G$ be a constant. Show that
$$K = \{aha^{-1} \mid h \in H\}$$
   is also a subgroup of G.

6. Let G be a group. For each $a \in G$, let $f_a \colon G \to G$ be the function defined by
$$f_a(x) = ax,$$
   i.e. the function f_a is "left multiplication by a."
   (a) Prove that f_a is a bijection.
   (b) Show that $f_a \circ f_b = f_{ab}$.
   (c) Find a formula for f_a^{-1}.