HW 3

MTH 344-001 Winter 2022

Randi Bolt

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1. Consider the following permutations in S_8:

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 1 & 3 & 8 & 2 & 6 & 4 & 7 \end{pmatrix} \qquad \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 2 & 1 & 6 & 5 & 3 & 8 & 4 \end{pmatrix}.$$

(a) Compute $\alpha \circ \beta$ and β^{-1}

$\alpha \circ \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 7 & 1 & 4 & 2 & 3 & 6 & 8 \end{pmatrix}$

$\beta^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 2 & 6 & 8 & 5 & 4 & 1 & 7 \end{pmatrix}$

(b) Write α as a composition of disjoint cycles. Then show that $\alpha^3 = \epsilon$.

Permutation:

$\alpha = (487)(152)$

Show $\alpha^3 = \epsilon$..

$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 1 & 3 & 8 & 2 & 6 & 4 & 7 \end{pmatrix}$

$\alpha^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 5 & 3 & 7 & 1 & 6 & 8 & 4 \end{pmatrix}$

$\alpha^3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{pmatrix}$

$\Rightarrow \alpha^3 = \epsilon$.

(c) Write β as a composition of transpositions. Is β an even or odd permutation?

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Since β can be written $\beta = (17)(78)(84)(46)(63)(31)$ implies that β is an even permutation.

2. Let G be a group. Prove that the function: $f : G \to G$ given by $f(x) = x^{-1}$ is a permutation of G.

3. Let G be a group. Prove G is abelian if and only if the function $f(x) = x^{-1}$ is an isomorphism from G to G. (Note: By #2 we already know that f is a bijection.)

4. Let G_1 and G_2 be groups and let $f : G_1 \to G_2$ be an isomorphism. Prove that if H is a subgroup of G_1, then

$$f(H) = \{f(h) | h \in H\}$$

is a subgroup of G_2.

5. Determine whether each of the following groups of size 4 is isomorphic to $\mathbb{Z}_4$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$. (Recall that $D_4 = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$ is the group of symmetriese of a square, where r denotes a 90° rotation clockwise and s denotes a reflection about a vertical axis.)

(a) $G_1 = \{\epsilon, (12), (34), (12)(34)\} \leq S_4$

(b) $G_2 = \{\epsilon, (1234), (13)(24), (1432)\} \leq S_4$

(c) $G_3 = \{1, r, r^2, r^3\} \leq D_4$

(d) $G_4 = \{1, s, r^2, sr^2\} \leq D_4$