MTH 344-001 Winter 2022 HW 1

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1. Do the following define operations on the given set? Why or why not? (Your answers only need to be a sentance or two; if these do define operations, you do not need to check if they're commutative, associatve, ect. in this problem.)

(a) $a * b = 4a - b$ on $\mathbb{Z}$

(b) $a * b = 3^b$ on $\mathbb{Q}$

(c) $a * b = \frac{a}{ab+2}$ on $\mathbb{R}$

(d) $a * b = \frac{1}{ab+2}$ on $\mathbb{R}^+$

2. Define an operation * on $\mathbb{R}$ by $a * b = 3a - 4b + 2$

(a) Is * commutative?

(b) Is * associative?

(c) Is there an identity element $e \in \mathbb{R}$ w.r.t. *?

(d) Does every element $a \in \mathbb{R}$ have an inverse w.r.t. *?

3. Define an operation * on $\mathbb{R}$ by $a * b = a + b - ab$.

(a) Is * commutative?

(b) Is * associative?

(c) Is there an identity element $e \in \mathbb{R}$ w.r.t. *?

(d) Does every element $a \in \mathbb{R}$ have an inverse w.r.t. *?

4. Define an operation * on the set $G = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x \neq 0\}$ by

$$(a, b) * (c, d) = (ac, ad + bc).$$

Prove that $\langle G, * \rangle$ is an abelian group. (You do not need to check associativity. We will show this in class.)