MTH 344-001 Winter 2022 HW 1

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1. Do the following define operations on the given set? Why or why not? (Your answers only need to be a sentance or two; if these do define operations, you do not need to check if they're commutative, associatve, ect. in this problem.)

(a) $a * b = 4a - b$ **on** $\mathbb{Z}$

(b) $a * b = 3^b$ **on** $\mathbb{Q}$

(c) $a * b = \frac{a}{ab+2}$ **on** $\mathbb{R}$

(d) $a * b = \frac{1}{ab+2}$ **on** $\mathbb{R}^+$

2. Define an operation * on $\mathbb{R}$ **by** $a * b = 3a - 4b + 2$

(a) Is * commutative?

(b) Is * associative?

(c) Is there an identity element $e \in \mathbb{R}$ **w.r.t. *?**

(d) Does every element $a \in \mathbb{R}$ **have an inverse w.r.t. *?**

3. Define an operation * on $\mathbb{R}$ **by** $a * b = a + b - ab$**.**

(a) Is * commutative?

(b) Is * associative?

(c) Is there an identity element $e \in \mathbb{R}$ **w.r.t. *?**

(d) Does every element $a \in \mathbb{R}$ **have an inverse w.r.t. *?**

4. Define an operation * on the set $G = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x \neq 0\}$ **by**

$$(a, b) * (c, d) = (ac, ad + bc).$$

Prove that $\langle G, * \rangle$ is an abelian group. (You do not need to check associativity. We will show this in class.)

5. Let a,b,c,x be elements of a group G. Solve the following system of equations for x :

(a) $bx^2 = ax^{-1}$ **and** $x^4 = c$

(b) $x^2c = bxa^{-1}$ **and** $xca = cax$

6. This problem asks you to consider the importance of using proper notation for inverses in groups. Your answer only needs to be a sentence or two.

Suppose a,b, and x are elements of a *nonabelian* group G, and that we want to solve thte equation $ax = b$ for x. Why would it be incorrect and unclear to say that the solution is $x = \frac{b}{a}$