Note on Jan 11

Relationship between Bivariate CDFs and PDFs

- Bivariate CDFs and PDFs have the following relationship:

$$f_{Y_1,Y_2}(y_1, y_2) = \frac{\partial^2 \left[F_{Y_1,Y_2}(y_1, y_2)\right]}{\partial y_1 \partial y_2};$$

$$-\infty < y_1 < \infty, \qquad -\infty < y_2 < \infty$$

where f_{Y_1,Y_2} is a **joint (bivariate) probability density function (joint pdf).**

- This means that …

$$f_1(y_1) = \int_{-\infty}^{\infty} f_{Y_1,Y_2}(y_1, y_2)\, dy_2$$

- That is, if we **integrate** Y_2 continuous variable **out** of the bivariate/joint pdf, we are left with the pdf of the Y_1 variable. This is the **marginal** probability density function of Y_1 variable**.**

- The **marginal** probability density function of Y_2 variable is obtained by **integrating** Y_1 continuous variable **out** of the bivariate/joint pdf as shown below.

$$f_2(y_2) = \int_{-\infty}^{\infty} f_{Y_1,Y_2}(y_1, y_2)\, dy_1$$

Example 1:

$$f(y_1, y_2) = \begin{cases} \frac{6}{5}\left(y_1 + y_2^2\right); & 0 \le y_1 \le 1, \qquad 0 \le y_2 \le 1 \\ 0\ ; & Otherwise \end{cases}$$

Compute and report the marginal pdf of Y_1.

Independence of Random Variables

Two random variables, Y_1 and Y_2, are **independent** if and only if

- **Using CDF**:

$$\boxed{F_{Y_1,Y_2}(y_1, y_2) = F_1(y_1) \times F_2(y_2)}$$

where $F_1(y_1)$ and $F_2(y_2)$ **marginal CDFs**.

- **Using PDF**:

$$\boxed{f_{Y_1,Y_2}(y_1, y_2) = f_1(y_1) \times f_2(y_2)}$$

where $f_1(y_1)$ and $f_2(y_2)$ **marginal PDFs**.

- If the support is **dependent**, then variables are **NOT independent.**
- Further, we **don't need the marginal CDFs and pmfs/pdfs to show independence**! We can show independence using non-negative functions of the variables themselves.

Example 1 Continued:

$$f(y_1, y_2) = \begin{cases} \frac{6}{5}\left(y_1 + y_2^2\right) & ; \quad 0 \le y_1 \le 1, \quad 0 \le y_2 \le 1 \\ 0 \; ; \; Otherwise \end{cases}$$

Are the random variables Y_1 and Y_2 independent according to probability? Explain.

Example 2 : Consider the following function.

$$f_{Y_1,Y_2}(y_1, y_2) = 0.3\left(1 - y_1^2 + y_2\right) \; ; \; 0 \le y_1 \le y_2 \le 2$$

(a) Draw and shade the support of this function on Cartesian plane.

(b) Find and report the marginal pdf of Y_1.

(c) Find the probability that Y_2 is higher than 1.5. That is, find $P(Y_2 > 1.5)$.

(d) Are the random variables Y_1 and Y_2 independent according to probability? Explain.