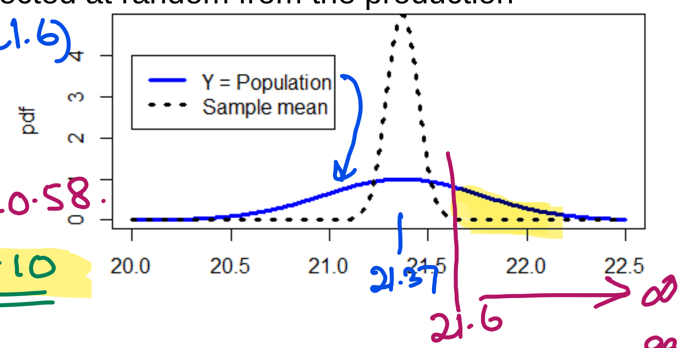
Example 3: A candy maker produces mints that have a label weight of 20.4 grams. Assume that the distribution of the weights of these mints has a normal distribution with mean = 21.37 and variance = 0.16.

⇒ Population has a normal distribution with $\mu = 21.37$, $\sigma^2 = 0.16$.

(a) Let Y be the weight (in grams) of a **single** mint selected at random from the production line. Find the probability that $Y > 21.6$. $P(Y>21.6)$



Compute Z for 21.6.

$Z = \frac{21.6-21.37}{\sqrt{0.16}} = 0.575$

after rounding to 2 decimals, Z=0.58.

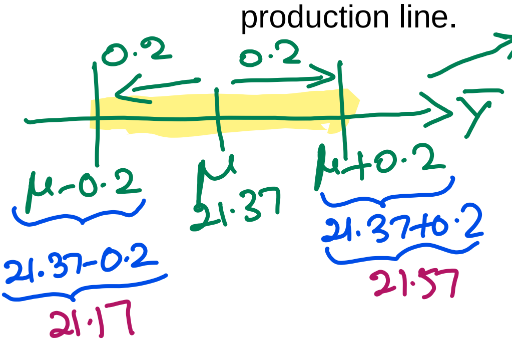
$P(Y>21.6) = P(Z>0.58) = 0.2810$
from Table 4.

On Ti calculator: normalcdf(21.6, 10^{99}, 21.37, $\sqrt{0.16}$)

In R: pnorm(21.6, 21.37, sqrt(0.16), lower.tail=FALSE)
0.2826456

(b) Find the probability that the **average** weight (in grams) of a random sample of 25 mints from the production line will be within 0.~~5~~2 grams of the population mean.
Let $\bar{Y}$ be the **average** weight (in grams) of a random sample of 25 mints from the production line.

Find $P(21.17 < \bar{Y} < 21.57)$.

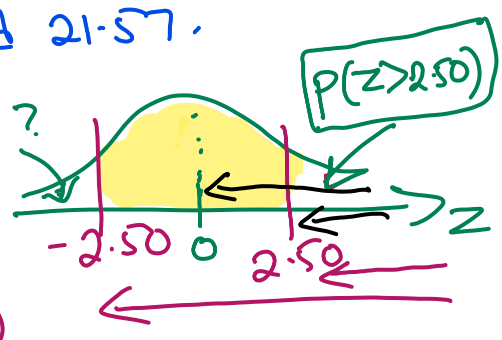


Since the sample comes from population with a normal distribution, From Theorem 7.1, $\bar{Y}$ has a normal distribution with mean $= \mu = 21.37$,
variance $= \frac{\sigma^2}{n} = \frac{0.16}{25} = 0.0064$

If using Table 4, compute Z for 21.17 and 21.57.

Z for 21.17 is $Z = \frac{21.17-21.37}{\sqrt{0.0064}} = -2.50$

Z for 21.57 is $Z = \frac{21.57-21.37}{\sqrt{0.0064}} = 2.50$



$\Rightarrow P(21.17<\bar{Y}<21.57) = P(-2.50<Z<2.50)$
$= 1-2P(Z>2.50)$
$= 1-2(0.0062) = 0.9876$ highly likely

On Ti calculator: normalcdf(21.17, 21.57, 21.37, $\sqrt{0.0064}$)

In R: pnorm(21.17, 21.37, sqrt(0.0064)) − pnorm(21.57, 21.37, sqrt(0.0064))

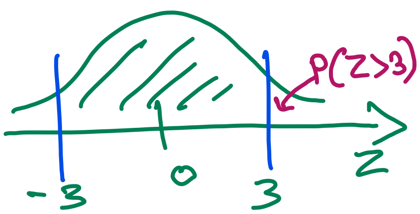
Example 4: A company manufactures electoral resistors and assume that the distribution of resistance is a normal distribution with variance of 0.01. $\sigma^2 = 0.01$

If a random sample of 25 resistors are selected for inspection, then find the probability that sample mean resistance will **differ from the population mean** by **no more than** ~~2.5.~~ 0.06

Let $\bar{Y}$ be the sample mean resistance of 25 resistors.

Find $P(|\bar{Y}-\mu| \le 0.06) = P(-0.06 \le \bar{Y}-\mu \le 0.06)$

Since population has a normal distribution, from Theorem 7.1, $\bar{Y}$ has a normal distribution with mean $= \mu$ and variance $= \frac{\sigma^2}{n} = \frac{0.01}{25} = 0.0004$. $\Rightarrow \frac{\sigma}{\sqrt{n}} = \sqrt{0.0004}$



First compute z as

$P(-0.06 \le (\bar{Y}-\mu) \le 0.06) = P\left(\frac{-0.06}{\sqrt{0.0004}} \le \frac{\bar{Y}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} \le \frac{0.06}{\sqrt{0.0004}}\right)$

$= P(-3 < Z < 3)$

$= 1 - 2P(Z>3)$ Table 4

$= 1 - 2(0.00135)$

$= 0.9973$ highly likely.

$Z = \frac{\bar{Y}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$

On Ti calculator: normalcdf(-3, 3, 0, 1) (−)

In R: 1-2*pnorm(3, 0, 1, lower.tail=FALSE).

What about the sampling distribution of other statistics?

Recall: Sample Variance $= S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$

Theorem 7.3:

Let $Y_1, Y_2, \ldots, Y_n$ be a random sample of size n from a normal distribution with mean μ and variance σ^2. Then

New variable "kite"

$$\boxed{\frac{(n-1)S^2}{\sigma^2}} = \frac{1}{\sigma^2}\sum_{i=1}^{n}[Y_i - \bar{Y}]^2 \sim \chi^2\,(v = \text{n} - 1)$$

degrees of freedom

where χ^2 is a **Chi-square** distribution.

Further, $\bar{\boldsymbol{Y}}$ and $\boldsymbol{S^2}$ are **independent** random variables.

Right-skewed

S^2 is a random variable

σ^2 is a constant

$\Gamma(\alpha) = (\alpha-1)!$ only if α is an integer.

Facts about the χ^2 Distribution:

- **Definition 4.10**: The $\chi^2(v)$ pdf is a special case of the gamma with $\alpha = v/2$ and $\beta = 2$.
  If $Y \sim \chi^2\,(v)$, then the pdf of Y is

$$f(y) = \frac{1}{\Gamma(v/2)\, 2^{v/2}} y^{(v/2)-1}\, e^{-y/2} \qquad ; \qquad y > 0$$

  where v is **a positive integer** that represents the **degrees of freedom (df).** It follows that $E(Y) = v$ and $Var(Y) = 2v$.

- If $Z \sim N(0,1)$, then $Z^2 \sim \chi^2\,(v = 1)$. (You did prove this in Homework 3)
- **Theorem 7.2**: If $X_1, X_2, \dots, X_n$ are **independent** and $X_i \sim \chi^2\,(v_i)$, then

Sum of independent chi. square variables

$$\sum_{i=1}^{n} X_i \sim \chi^2\left(\sum_{i=1}^{n} v_i\right)$$

Use Table 6 from the textbook.

Ti 83 o4 84 Calculator instructions

If $Y \sim \chi^2(v)$, to get probability that Y is between a and b (that is, find $P(a < Y < b)$):

1. Press [2nd] button and then [VARS] button.
2. Press arrow down to χ^2**cdf(** and press [ENTER].
3. Enter the values for a, b, and v with a comma between each. Close parenthesis and press [ENTER]. That is, a is **lower**, b is **upper,** and v is **df.**
4. If lower bound is $-\infty$, then enter -10^{99} and if upper bound is ∞, then enter 10^{99}

In R : use pchisq(a, v, lower.tail = FALSE) to get $P(\chi^2 > a)$.

Example 5: A candy maker produces mints that have a label weight of 20.4 grams. Assume that the distribution of the weights of these mints has a normal distribution with mean = 21.37 and variance = 0.16. $\sigma^2 = 0.16$

Let S^2 be the **variance** of weight (in grams) of a **random sample of 25** mints from the production line. Find the probability that sample variance is within 0.05 of 0.16. $\leftarrow \sigma^2$ $\leftarrow n$

That is, find $P(|S^2 - \sigma^2| < 0.05) = P(-0.05 < S^2 - 0.16 < 0.05)$

$= P(0.11 < S^2 < 0.21)$

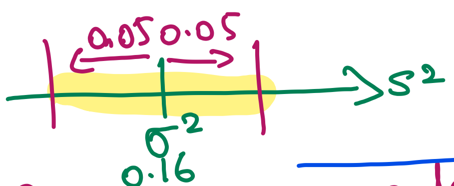
$= P\left(\frac{(25-1)\,0.11}{0.16} < \frac{(n-1)\,S^2}{\sigma^2} < \frac{(25-1)\,0.21}{0.16}\right)$

$= P(16.5 < \chi^2 < 31.5)$.

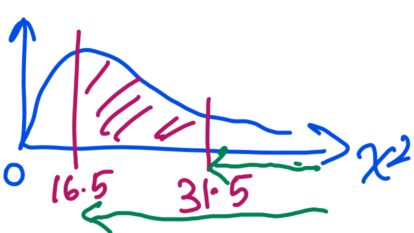
$= P(\chi^2 > 16.5) - P(\chi^2 > 31.5)$

$= 0.900 - 0.100$ From Table 6

$= 0.8$ or 80%.



Note : since population has a normal distribution, from Theorem 7.3, $\frac{(n-1)S^2}{\sigma^2}$ has a $\chi^2\,(v = n-1 = 25-1 = 24$



On Ti calculator: χ^2cdf(16.5, 31.5, 24)

In R: pchisq(31.5, 24) − pchisq(16.5, 24).

$\boxed{0.7293}$