STAT462 Winter 2022 Midterm Exam-Practice Problems (Chapter 5)

- **Please do not expect exact same questions on the actual exam.**
- **This is longer than the actual exam to give you more practice.**
- **The actual exam is much shorter than this.**
- **The answers are at the end of this document.**
- **Please see similar problems on the lecture note if you cannot think of how to solve these problems.**
- **If your final answer is a probability function, then provide a complete function.**
- **Be able to do similar questions.**

1. Consider the following function of random variables Y_1 and Y_2 where k is a constant:

$$p_{Y_1,Y_2}(y_1,y_2) = \begin{cases} k(\,2y_2 - y_1) & ;\ y_1 = 0,1, \quad ;\ y_2 = 1,2 \\ 0\,;\ Otherwise \end{cases}$$

   a. Find the value of k so that the above function is a valid joint pmf.
   b. Are the random variables Y_1 and Y_2 independent according to probability? Explain.
   c. Find and report the conditional pmf of $Y_1 \mid Y_2 = y_2$.
   d. Find $E(Y_1^2 \mid Y_2 = y_2)$

2. Consider the following table:

	y_1	
y_2	0	1
0	.38	.17
1	.14	.02
2	.24	.05

   a. Are the random variables Y_1 and Y_2 independent according to probability? Explain.
   b. Compute the probability that Y_1 is more than zero.
   c. Find and report the marginal pmf of Y_2.
   d. Find and report the conditional pmf of $Y_1 \mid Y_2 = 0$.
   e. Compute $E[\,Y_2 - Y_1\,]$
   f. Compute $E[\,Y_1Y_2\,]$
   g. Find $E(Y_2 \mid Y_1 = 1)$

3.

Let Y_1 and Y_2 have the joint probability density function given by

$$f(y_1, y_2) = \begin{cases} ky_1y_2, & 0 \le y_1 \le 1, 0 \le y_2 \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a. Find the value of k that makes this a probability density function.
b. Compute the probability that Y_1 is smaller than 0.8
c. Compute the probability that Y_2 is higher than 0.8
d. Find and report the marginal pdf of Y_1.
e. Find and report the marginal pdf of Y_2.
f. Are the random variables Y_1 and Y_2 independent according to probability? Explain.
g. Compute $E[\,(Y_1 - 1)Y_2^2\,]$

4. Consider the following function of random variables Y_1 and Y_2 where k is a constant:

$$f(y_1, y_2) = \begin{cases} k(2 - y_2) \ ; \ 0 \le y_1 \le y_2 \le 2 \\ \quad 0 \ ; \ Otherwise \end{cases}$$

a. Find the value of k that makes this a probability density function.
b. Compute the probability that Y_1 is smaller than 1
c. Compute the probability that Y_2 is higher than 1.5
d. Are the random variables Y_1 and Y_2 independent according to probability? Explain.
e. Find and report the conditional pdf of $Y_2 \mid Y_1 = y_1$.
f. Compute $P(Y_1 < 0.7 \mid Y_2 = 1.2)$
g. Compute $P(0.8 < Y_2 < 1 \mid Y_1 = 0.5)$
h. Compute $E\left[\frac{Y_1+Y_2}{Y_2}\right]$

5. Consider the following function:

$$f(y_1, y_2) = \begin{cases} 3y_1, & 0 \le y_2 \le y_1 \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a. Compute the probability that Y_1 is smaller than 0.4
b. Compute the probability that Y_2 is higher than 0.4
c. Compute $P(0.6 < Y_1 < 0.8 \mid Y_2 = 0.4)$
d. Compute $P(Y_2 > 0.2 \mid Y_1 = 0.5)$
e. Find $E(Y_2^2 \mid Y_1 = y_1)$

6. $Let \quad E(Y_1) = 6, \qquad Var(Y_1) = 3, \qquad E(Y_1 \mid Y_2) = \frac{Y_2^2}{5}, \qquad E(Y_2 \mid Y_1) = \frac{Y_1}{2}$
   a. Compute $E(Y_2)$
   b. Then compute $Var(Y_2)$

7.

Let the discrete random variables Y_1 and Y_2 have the joint probability function

$$p(y_1, y_2) = 1/3, \qquad \text{for } (y_1, y_2) = (-1, 0), (0, 1), (1, 0).$$

Find $\mathrm{Cov}(Y_1, Y_2)$. Notice that Y_1 and Y_2 are dependent. (Why?) This is another example of uncorrelated random variables that are not independent.

8. Let $E(Y_1) = 2, \quad E(Y_2) = -1, \quad Var(Y_1) = 4, \quad Var(Y_2) = 6, \quad E(Y_1Y_2) = -1.$
   a. Compute $E(3Y_1 - 4Y_2)$
   b. Compute $Var(3Y_1 - 4Y_2)$
   c. Compute $E(2Y_1 - Y_2^2)$

9. Let $E(Y_1) = -1, \quad E(Y_2) = 4 \quad Var(Y_1) = 6, \quad Var(Y_2) = 8, \quad E(Y_1Y_2) = -4.$
   a. Compute $E(9Y_1 + 2Y_2)$
   b. Compute $Var(9Y_1 + 2Y_2)$

Answers:

1.
   a. 0.1
   b. No
   c. $\begin{cases} \frac{2y_2-y_1}{4y_2-1} & ; \ y_1 = 0,1 \ , y_2 = 1,2 \\ 0 \ ; \ Otherwise \end{cases}$
   d. $\frac{2y_2-1}{4y_2-1}$

2.
   a. No
   b. 0.24
   c. $\begin{cases} 0.55 \ ; \ y_2 = 0 \\ 0.16 \ ; \ y_2 = 1 \\ 0.29 \ ; \ y_2 = 2 \\ \text{Zero} \ ; \ \text{otherwise} \end{cases}$
   d. $\begin{cases} 38/55 \ ; \ y_1 = 0 \\ 17/55 \ ; \ y_1 = 1 \\ 0 \ ; \ Otherwise \end{cases}$
   e. 0.5
   f. 0.12
   g. 0.5

3.
   a. 4
   b. 0.64
   c. 0.36
   d. $\begin{cases} 2y_1 \ ; \ 0 \le y_1 \le 1 \\ 0 \ ; \ Otherwise \end{cases}$
   e. $\begin{cases} 2y_2 \ ; \ 0 \le y_2 \le 1 \\ 0 \ ; \ Otherwise \end{cases}$
   f. Yes
   g. −1/6

4.

a. 3/4
b. 7/8 0.875
c. 0.15625
d. No
e. $\begin{cases} \dfrac{2-y_2}{2-2y_1+\frac{y_1^2}{2}} & ; \ 0 \le y_1 \le y_2 \le 1 \\ 0 & ; \ Otherwise \end{cases}$
f. 7/12 = 0.583333
g. 0.1955556
h. 1.5

5.

a. 0.064
b. 0.432
c. 1/3 = 0.3333
d. 0.6
e. $\dfrac{y_1^2}{3}$

6.

a. 3
b. 21

7. Zero

8.

a. 10
b. 108
c. −3

9.

a. −1
b. 518