Note on Jan 4

Example : From 461 last Lecture Note

Example

Two-way table, Pivot table, Contingency table

A car dealership sells 0, 1, or 2 luxury cars on any day. When selling a car, the dealer also tries to persuade the customer to buy an extended warranty for the car.

Let Y_1 denote the *number of luxury cars sold* on a given day and let Y_2 denote the *number of extended warranties sold*.

The joint probability function of Y_1 and Y_2 can be given by the following table or equation.

		Number of warranties (Y_2)		
		0	1	2
Number of cars (Y_1)	0	1/6	0	0
	1	1/12	1/6	0
	2	1/12	1/3	1/6

$$p(y_1, y_2) = \begin{cases} 1/6, & (y_1, y_2) = (0,0) \\ 1/12, & (y_1, y_2) = (1,0) \\ 1/6, & (y_1, y_2) = (1,1) \\ 1/12, & (y_1, y_2) = (2,0) \\ 1/3, & (y_1, y_2) = (2,1) \\ 1/6, & (y_1, y_2) = (2,2) \\ 0, & \text{otherwise} \end{cases}$$

(a) Compute the probability that only 1 car will be sold on a given day (regardless of number of warranties sold).

$$P(Y_1 = 1) = P(Y_1 = 1 \text{ and } Y_2 = 0 \text{ OR } 1)$$
$$= P(Y_1 = 1, Y_2 = 0) + P(Y_1 = 1, Y_2 = 1)$$
$$= \frac{1}{12} + \frac{1}{6}$$
$$= \underline{\underline{\frac{3}{12} = \frac{1}{4} = 0.25 \text{ or } 25\%}}$$

(b) Compute the probability that 1 warranty will be sold **given that (or knowing that or if)** 2 cars were sold on a given day.

$$P(Y_2 = 1 \mid Y_1 = 2) = \frac{P(Y_1 = 2, Y_2 = 1)}{P(Y_1 = 2)}$$
$$= \frac{1/3}{7/12} = \underline{\underline{\frac{4}{7}}}$$

$1/12$

Note: $P(Y_1=2) = P(Y_1=2, Y_2=0) + P(Y_1=2, Y_2=1) + P(Y_1=2, Y_2=2)$

$$= \frac{1}{12} + \frac{1}{3} + \frac{1}{6}$$

$$= \frac{7}{12}$$

(c) Find and report the marginal pmf of Y_1.

		Number of warranties (Y_2)			
		0	1	2	Total
Number of cars (Y_1)	0	1/6	0	0	1/6
	1	1/12	1/6	0	3/12 = 1/4
	2	1/12	1/3	1/6	7/12
Total		1/2	3/6	1/6	1

Marginal pmf of Y_1 is

$$p_1(y_1) = \begin{cases} 1/6 & ; y_1 = 0 \\ 1/4 & ; y_1 = 1 \\ 7/12 & ; y_1 = 2 \\ 0 & ; \text{otherwise.} \end{cases}$$

This is importat.

If you were to find marginal pmf of Y_2,

$$p_2(y_2) = \begin{cases} 4/12 & ; y_2 = 0 \\ 3/6 & ; y_2 = 1 \\ 1/6 & ; y_2 = 2 \\ 0 & ; \text{otherwise} \end{cases}$$

You may simplify these probabilities.

(d) Are the random variables Y_1 and Y_2 independent according to probability? Explain.

		Number of warranties (Y_2)			
		0	1	2	Total
Number of cars (Y_1)	0	1/6	0	0	1/6
	1	1/12	1/6	0	3/12
	2	1/12	1/3	1/6	7/12
Total		4/12	3/6	1/6	1

Y_1 and Y_2 are independent $\Leftrightarrow$ The joint pmf is a product of marginals for each pair of (y_1, y_2).

That is,

$$p_{1,2}(y_1, y_2) = p_1(y_1) \times p_2(y_2)$$

for each (y_1, y_2).

Consider $y_1 = 0$, $y_2 = 0$;

$$p_1(y_1) \times p_2(y_2) = p_1(0) \times p_2(0) = \frac{1}{6} \times \frac{4}{12} = \frac{1}{18}$$

However, $p_{1,2}(y_1, y_2) = p_{1,2}(0,0) = \frac{1}{6}$

And $p_{1,2}(y_1, y_2) \neq p_1(y_1) \times p_2(y_2)$ for at least one pair of (y_1, y_2).

Therefore, Y_1 and Y_2 are NOT independent.

Does it make sense in terms of meaning of Y_1, Y_2?

Yes, because the number warranties sold depends on number of cars sold.

If you found it is true $p(y_1, y_2) = p_1(y_1) \times p_2(y_2)$ for one pair of (y_1, y_2), then check for each pair of (y_1, y_2) until you find it is true for all or it is false for at least one pair.