Another way to do Example 2 part c on Jan 13 Note:

We are given

$f(y_1, y_2) = y_1 y_2 \; ; \; 0 \le y_1 \le 2 \, , \, 0 \le y_1 \le 1$

and need to find $E(Y_1^2 Y_2)$.

Since we did show that Y_1, Y_2 are independent, we can write

$$f(y_1, y_2) = y_1 y_2$$
$$= f_1(y_1) \times f_2(y_2)$$

where $f_1(y_1) = a y_1$ and $f_2(y_2) = b y_2$ such that $ab = 1$.

Then

$$E(Y_1^2) = \int_{-\infty}^{\infty} y_1^2 f_1(y_1)\, dy_1$$
$$= \int_0^2 y_1^2 (a y_1)\, dy_1$$
$$= a\left(\frac{y_1^4}{4}\right)\Big|_0^2$$
$$= 4a$$

$$E(Y_2) = \int_{-\infty}^{\infty} y_2 f_2(y_2)\, dy_2$$
$$= \int_0^1 y_2 (b y_2)\, dy_2$$

$$= b \left. \frac{y_2^3}{3} \right|_0^1$$

$$= \frac{b}{3}$$

Then since Y_1, Y_2 are indcpendent

$$E(Y_1^2 Y_2) = E(Y_1^2)E(Y_2)$$

$$= (4a)\left(\frac{b}{3}\right)$$

$$= \frac{4ab}{3}$$

Since we said $ab = 1$ earlier,

$$E(Y_1^2 Y_2) = \frac{4}{3}$$

The only difference in this method is that we did not find the exact marginal pdf of Y_1 and the exact marginal pdf of Y_2 because Y_1, Y_2 are independent.

Again, this method can be used only if Y_1, Y_2 are independent.