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Midterm Exam #2: Astronomy 160b, spring 2007, March 27, 2007

The test relates to some data about the supermassive black hole in the center of the galaxy given below. Note that you do **NOT** need to know anything particular about supermassive black holes — all the questions can be answered using our standard equations of orbital motion, and information about relativity and so forth that we have have considered in other contexts. The test consists of four problems (worth a total of 30 points) and will last 50 minutes — plan your time accordingly! Please put your name on *both* pages of the test, and note that we're using both sides of the test paper. Please do all problems on the test paper — if you need more space, continue on a separate piece of paper labelled with your name and the problem number. Use a different piece of paper for each problem you need to continue. **The test is open book, but electronic devices such as calculators are not allowed.**

$1 \text{ year} = 3 \times 10^7$ seconds

$1 \text{ A.U.} = 1.5 \times 10^{11}\text{m}$

$1M_\odot = 2 \times 10^{30}$ kg

$1M_J = 10^{-3}M_\odot$

$1M_E = 3 \times 10^{-6}M_\odot$

$P_J \approx 11$ years

$a_J \approx 5$ A.U.

$c = 3 \times 10^8$ m/s

$G = 7 \times 10^{-11}$ in mks units

$1 \text{ parsec} = 3 \times 10^{16}\text{m}$

$1 \text{ radian} = 2 \times 10^5$ arcseconds

$a^3 = P^2GM/(4\pi^2)$

$\alpha = D_2/D_1$

$V = 2\pi a/P$

$V_* M_* = V_p M_p$

$z = \Delta P_p/P_p = \Delta\lambda/\lambda_0 = \left\{\frac{1+V_R/c}{1-V_R/c}\right\}^{1/2} - 1 \approx V_r/c$

$(1+\epsilon)^n \approx 1 + n\epsilon$

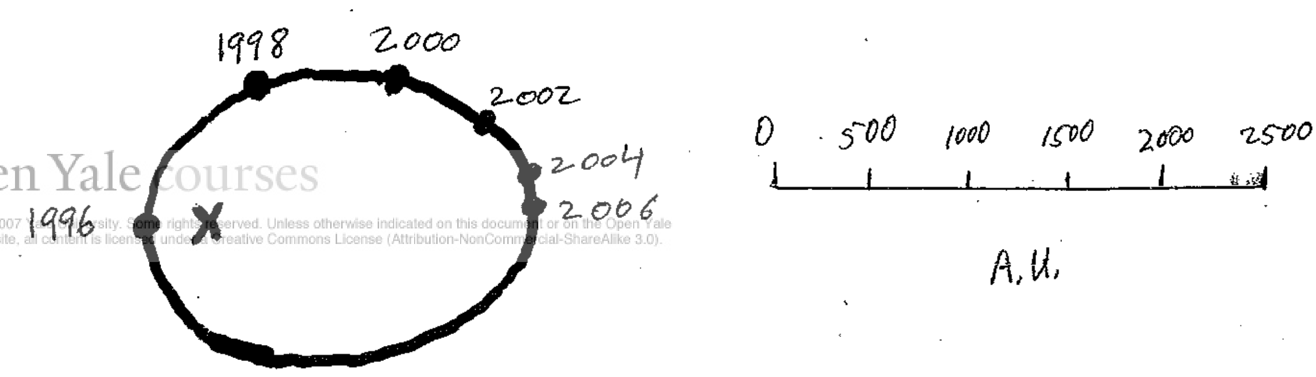
$V_{esc} = (2GM/R)^{1/2}$

$R_s = 2GM/c^2$

$v_{tot} = (v_1 + v_2)/(1 + v_1 v_2/c^2)$

$z_{\text{grav}} = \frac{1}{\sqrt{1-R_s/R}} - 1$

Prof. Andrea Ghez and her collaborators at UCLA have been observing stars near the center of our Galaxy for a number of years. During that time they have seen the stars closest to the galactic center orbiting around an unseen object located at the exact center of the galaxy. From the orbits of these stars, Ghez and her colleagues and competitors have inferred the presence of a massive black hole at the center of the galaxy. Below I have sketched an example of the kind of data they have obtained (I modified the details a bit for the purposes of this test — for the real thing see Ghez et al. 2000, *Nature* vol. 407, p. 349). The points are observations of the position of a particular star over the years (the year of the observation is noted), the cross marks the center of the galaxy, and the line is the inferred orbit of the star. Note that the orbital period of the star around the galactic center is 20 years. Note also that there is a size scale indicated in astronomical units.



Problem 1 (10 points). From the information provided on the other side of the paper, calculate the Schwarzschild radius of the black hole.

$P = 20$ yr

$a = \frac{1}{2} \times$ distance from 1996 to 2006 (2) (1 pt. off for $\frac{1}{2}$ missing)

$= 1000$ AU

$a^3 = P^2 M$ (2)

$M = \frac{(10^3)^3}{(2\times10^1)^2} = \frac{10^9}{4\times10^2} = 2.5\times10^6 M_\odot$

$= 2.5\times10^6 \times 2\times10^{30}$

$= 5\times10^{36}$ kg (2)

(2) $R_s = \frac{2GM}{c^2}$

$= \frac{2\times\cancel{7}\times10^{-11}\times\cancel{5}\times10^{36}}{(3\times10^8)^2} = \frac{70\times10^{25}}{9\times10^{16}}$

$= 8\times10^9$ m (2)

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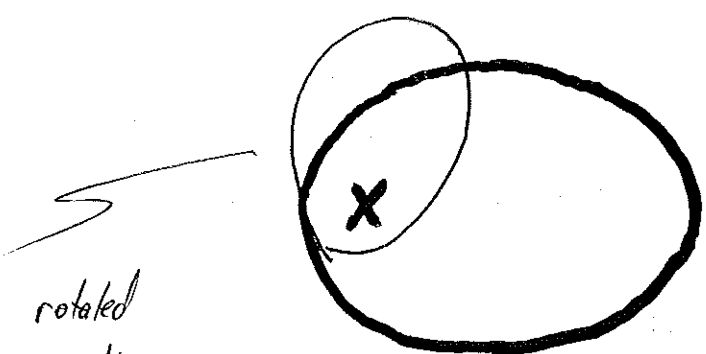
Problem 2 (8 points). If the star's motion were perfectly Newtonian, it would follow exactly the same path each orbit, and the shape of the orbit wouldn't change. However, there are two post-Newtonian effects that will cause a change in the shape of the orbit. List the two post-Newtonian effects, and explain the ways in which they change the shape of the orbit. Then on the diagram below (which shows the current orbit) sketch the orbit as it might appear when re-observed many years from now once these post-Newtonian phenomena have had time to make a noticable difference.

1

1. precession of periastron (in this case "perigalacticon"). The closest point to the galactic center rotates with time, so the whole orbit rotates with it. 2

1

2. gravitational wave radiation, causes the period, and hence the size of the orbit, to shrink. 2



1 pt. rotated

1 pt. smaller a

Problem 3 (10 points). Suppose the star is actually a pulsar, and the pulse period was observed to be precisely 3 seconds in 1996. Assume the orbit is face-on, so that there is no Doppler shift (the motion is neither toward nor away from us at any point in the orbit). Nevertheless, the pulse period will change during the orbit due to changes in the gravitational redshift.

a) Starting from the full relativistic formula for the gravitational redshift given on the first page of the test, derive a post-Newtonian approximation. Note that Newtonian physics predicts no redshift due to gravity at all.

b) Using the post-Newtonian formula derived above and other information provided throughout the test, calculate how much the observed pulse period has increased or decreased between 1996 and 2006.

zero pts. for wrong a equation

c) Is the use of the post-Newtonian formula here justified? Why or why not?

8

a) 2 $z_{grav} = \frac{1}{\sqrt{1 - R_s/r}} - 1 = (1 - \frac{R_s}{r})^{-1/2} - 1 \simeq 1 + \frac{R_s}{2r} - 1 = \frac{R_s}{2r}$

b) Since the pulsar is further from the gal. center in 2006, there will be less grav. redshift, so the observed period is shorter 1 (full credit for right sign even without explanation)

1 specifically the change in pulse period will be $\Delta P_{96} - \Delta P_{06}$.

in 1996 distance is $300\,AU = 3\times10^2 \times 1.5\times10^{11} = 5\times10^{13}\,m$

in 2006 distance is $1700\,AU = 1.7\times10^3 \times 1.5\times10^{11} = 3\times10^{14}\,m$

$R_s = 8\times10^9\,m$ (from part 1).

for subtracting only 1 pt off if they don't do this

2 (calculation)

$\left(\frac{R_s}{2r}\right)_{96} = \frac{8\times10^9}{\cancel{4}\,2\times10^{14}} = \cancel{8}\times10^{\cancel{-4}-5} = 8\times10^{-5}$ $\left(\frac{R_s}{2r}\right)_{06} = \frac{8\times10^9}{6\times10^{14}} = 10^{-5}$

~~so~~ so $\frac{\Delta P}{P}$ = that value, so change in period is

for 1 using P $(8\times10^{-5} - 1\times10^{-5})\times3 = 3\times7\times10^{-5} = 2\times10^{-4}\,s$

c) Use is justified because ~~Rs~~ $\frac{R_s}{R_{close}} = \frac{8\times10^9}{5\times10^{13}} \ll 1$ 2

1 pt for statement without numbers

Problem 4 (2 points). Which of the following observations helped prompt Einstein to develop the *general* theory of relativity? (Circle *all* that apply)

a) The constancy of the speed of light for all observers.
b) The equality of the inertial and gravitational mass.
c) The deflection of light observed from background stars during a solar eclipse.
d) The discovery of the binary pulsar.

2 pts for b only

1 pt for b and some other answer

0 pt for b and 2 other answers, or no (b).