① a $M = -19$

$d = 10^9$ lightyears $= 3 \times 10^8$ pc

(where 1 ly $= 3.3$ pc)

$$m - M = 5 \log \left(\frac{3 \times 10^8}{10} \right) = 5 \log (3 \times 10^7) = 5(\log 3 + \log 7)$$

$$= 5(0.5 + 7) = 37.5$$

$m - M = 37.5$

$m - (-19) = 37.5$

$\boxed{m = 18.5 \text{ magnitudes}}$

b $\lambda_0 = 0.65$ microns

$$z = \frac{1}{a} - 1 = \frac{1}{0.9} - 1 = 1.11 - 1 = 0.11$$

$$z = \frac{\lambda - \lambda_0}{\lambda_0}$$

$0.11 \lambda_0 = \lambda - \lambda_0$

$0.11 \lambda_0 + \lambda_0 = \lambda$

$\lambda_0 (1.11) = \lambda$

$\lambda = 0.65 (1.11) = \boxed{0.72 \text{ microns}}$

OR

$$\frac{a_{now}}{a_{then}} = \frac{\lambda_{obs}}{\lambda_{emit}}$$

$$\frac{1}{0.9} = \frac{\lambda_{obs}}{0.65 \text{ microns}}$$

$\boxed{\lambda_{obs} = 0.72 \text{ microns}}$

(II) $z=1.5$

$m-M=42.5$

$a = \frac{1}{1+z} = \frac{1}{1+1.5} = 0.4$

$m-M = 5\log\left(\frac{d}{10}\right)$

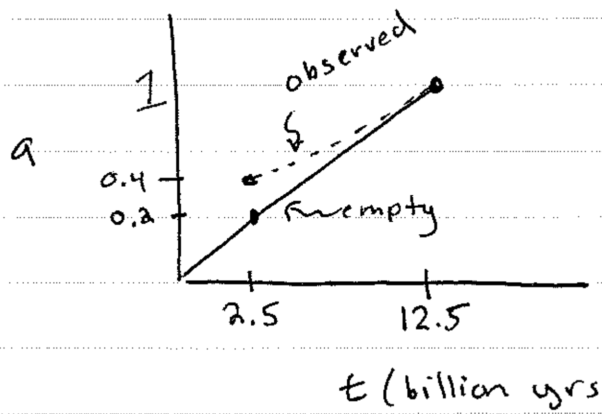
$42.5 = 5\log\left(d/10\right)$

$d = 3\times10^{9}\,pc = 10^{10}$ lightyears

point "now" $a=1$ $t=12.5$ billion yrs

point "then" $a=0.4$ $t=2.5$ billion yrs

↑ or 10^{10} billion yrs IN THE PAST

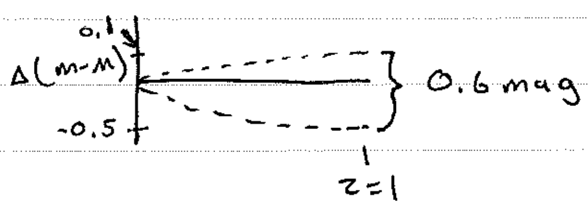


universe is <u>accelerating</u>

we know this because at $t=2.5$ billion years the expansion of the empty universe had a rate of $z=4$ ($a=0.2$ at 2.5 billion on the empty line), while the observed point has a slower expansion at the same time, $z=1.5$. Since all points must meet at "now" the observed universe expansion needs to speed up to end up matching the empty universe "now".

III a We need dark energy (DE) to explain <u>why</u> the observations of SNe Ia appear fainter at high z than would be expected in a pure $\Omega_M = 1$ universe. This assumes that Ia's are STANDARD CANDLES! If distance SNe Ia's are fainter than nearby ones, than the fact that high z Ia's <u>appear</u> faint may be due merely to the fact that they <u>ARE</u> faint. In that case, DE need not be invoked to explain the observed points.

b from $\Delta(m-M)$ plot at z = 1 the magnitude difference between the $\Omega_M = 0.25$ $\Omega_\Lambda = 0.75$ line and the $\Omega_M = 1$ line is 0.6 magnitudes



$$0.6 = 2.5 \log\left(\frac{b_{z=1}}{b_{z=0}}\right)$$

$$\boxed{b_{z=1} = 0.58\, b_{z=0}}$$

$\Rightarrow$ nearby SNe are about twice as bright as the distance SNe