# Astronomy 160: Frontiers and Controversies in Astrophysics
Homework Set # 4 Solutions

1) **The formula for the relativistic Doppler shift is**

$$\Delta\lambda/\lambda_0 = \left(\frac{1+v_R/c}{1-v_R/c}\right)^{1/2} - 1.$$

**Show that the post-Newtonian approximation reproduces the result we used before, namely that $\Delta\lambda/\lambda_0 = v_R/c$. What happens when $v_R$ approaches $+c$ or $-c$? Does this result make sense?**

$$\frac{\Delta\lambda}{\lambda_0} = \left(\frac{1+\frac{V_R}{c}}{1-\frac{V_R}{c}}\right)^{1/2} - 1$$

if $\frac{V_R}{c} << 1$, then we can use the approximation $(1+\epsilon)^n \approx 1 + n\epsilon$ as follows:

$$\frac{\Delta\lambda}{\lambda_0} = \left(1+\frac{V_R}{c}\right)^{1/2}\left(1-\frac{V_R}{c}\right)^{-1/2} - 1 = \left(1+\frac{V_R}{2c}\right)\left(1+\frac{V_R}{2c}\right) - 1$$

$$\frac{\Delta\lambda}{\lambda_0} = 1 + \frac{V_R}{c} + \left(\frac{V_R}{2c}\right)^2 - 1$$

since $\left(\frac{V_R}{2c}\right)^2$ is negligible:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{V_R}{c}$$

As $V_R \to c$

$$\frac{\Delta\lambda}{\lambda_0} = \left(\frac{1+\frac{c}{c}}{1-\frac{c}{c}}\right)^{1/2} - 1 = \left(\frac{2}{0}\right)^{1/2} - 1 = \infty$$

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda-\lambda_0}{\lambda_0} = \infty$$

$$\lambda = \infty$$

As $V_R \to -c$

$$\frac{\Delta\lambda}{\lambda_0} = \left(\frac{1+\frac{-c}{c}}{1-\frac{-c}{c}}\right)^{1/2} - 1 = \left(\frac{0}{2}\right)^{1/2} - 1 = -1$$

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda-\lambda_0}{\lambda_0} = -1$$

$$\lambda - \lambda_0 = -\lambda_0$$

$$\lambda = 0$$

As $V_R \to c$, $\lambda_{observed} \to \infty$ and as $V_R \to -c$, $\lambda_{observed} \to 0$. This makes sense because the observed wavelength will range from 0 to $\infty$ as $V_R$ goes from $-c$ all the way to $c$.

If a light emitting object is receding from you at the speed of light ($V_R = c$), then you observe the light to be infinitely redshifted (spacing between wavelengths is infinite). If a light emitting object is approaching you at the speed of light ($V_R = -c$), then you observe the light to be infinitely blueshifted (spacing between the wavelengths is zero).

2) **Consider a black hole and a neutron star in a circular orbit around each other, where the black hole is twice as massive as the neutron star. Use the equations we derived in the first part of the course to answer the following questions.**

a. **If the black hole is moving at a speed of $0.2c$, how fast is the neutron star moving?**

If the black hole is moving at $0.2c$ and the black hole is twice as massive as the neutron star, we can figure out the velocity of the neutron star:

$$\begin{aligned} v_{NS} M_{NS} &= v_{BH} M_{BH} \\ v_{NS} M_{NS} &= 0.2c \times M_{BH} \\ v_{NS} M_{NS} &= 0.2c \times 2M_{NS} \\ v_{NS} &= 0.2c \times 2 \\ v_{NS} &= 0.4c \end{aligned}$$

b. **Given the information in (a), how far apart are these two objects? What is the orbital period of the system?**

To find the distance between these two objects, we can use the equation that relates the distance between two objects to their mass and velocity. However, we do need to assume something about the mass of the neutron star. Since we know from stellar evolution that $1.4M_\odot < M_{NS} < 3M_\odot$, we can say that the neutron star has a mass of $2M_\odot$ and, since the problem tells us that $M_{BH} = 2M_{NS}$, the mass of the black hole must be $4M_\odot$. This means that the total mass of the system is $6M_\odot$. The total velocity of the system is just $v_{BH} + v_{NS} = 0.6c$ (since we're using the Newtonian approximation).

$$\begin{aligned} a &= \frac{GM}{v^2} \\ a &= \frac{(7 \times 10^{-11})(6 \times 2 \times 10^{30})}{(0.6 \times 3 \times 10^8)^2} \text{ [m]} \\ a &= \frac{84 \times 10^{19}}{(1.8 \times 10^8)^2} \text{ [m]} \\ a &= \frac{8.4 \times 10^{20}}{3 \times 10^{16}} \text{ [m]} \\ a &= 3 \times 10^4 \text{ m} = 30 \text{ km} = 2 \times 10^{-7} \text{ AU} \end{aligned}$$

Since the black hole and neutron star are in a circular orbit between the center of mass, the velocity in the orbit can be described as:

$$v = \frac{2\pi a}{P}$$

or,

$$\begin{aligned}
P &= \frac{2\pi a}{v} \\
P &= \frac{6 \times 3 \times 10^4}{1.8 \times 10^8} \text{ [s]} \\
P &= \frac{18 \times 10^4}{1.8 \times 10^8} \text{ [s]} \\
P &= \frac{1.8 \times 10^5}{1.8 \times 10^8} \text{ [s]} \\
P &= 10^{-3} \text{ s} = 3 \times 10^{-11} \text{ yr}
\end{aligned}$$

3) **The assumption that you can use the equations derived in the first part of the course in the situation described in problem II is clearly wrong. Explain.**

Because the velocities in problem 1 are significant fractions of the speed of light (0.2c and 0.4c), you cannot use Newtonian equations to solve the problem. The quantity v/c is thus too large and you will need to take into account relativistic effects.

4) **Read the articles on the classes server about the discovery of Neptune and the search for "Vulcan". At the time, the discovery of Neptune was hailed as a "proof" that Newton's theories of motion and gravity were correct. To what extent did this discovery in fact constitute such a proof? To what extent did the failure to discover Vulcan prove Newton's theories?**

The discovery of Neptune using Newton's theory provided a support to its validity. It did not prove it just as any single supporting evidence cannot by itself prove any theory. The failure to discover "Vulcan" did not disprove Newtonian mechanics, but helped in setting a limit to its accuracy.