Astronomy 160: Frontiers and Controversies in Astrophysics
Homework Set # 6 Solutions

1) a) **Faint "brown-dwarf" stars have absolute magnitudes of around 17.5. How many times fainter than the Sun are these stars?**

$M_{BD} = 17.5$
$M_{sun} = 5$
$17.5 - 5 = \frac{-5}{2} \log \frac{b_1}{b_2}$
$\frac{12.5 \times 2}{-5} = \log \frac{b_1}{b_2}$
$-5 = \log \frac{b_1}{b_2}$
$10^{-5} = \frac{b_1}{b_2}$
So brown dwarfs are 10^5 *fainter* than the sun.

b) **If one observes a nearby galaxy at a distance of 1 Mpc ($= 10^6$ parsecs) what is the apparent magnitude of Sun-like stars in that galaxy?**

distance $= 10^6$ parsecs
$m - M = 5 \log \frac{D}{10pc}$
$m - 5 = 5 \log 10^5$
$m - 5 = 25$
So the apparent magnitude of Sun-like stars at a distance of 10^6 parsecs is 30.

c) **The magnitude of the full Moon is around -14.5. How much brighter does the Sun appear than the Moon?**

The apparent magnitude of the full moon is -14.5. So we can use the brightness equation along with the apparent magnitude of the sun (-27) to find the ratio in brightnesses.
$-27 + 14.5 = \frac{-5}{2} \log \frac{b_1}{b_2}$
$-12.5 \times \frac{-2}{5} = 5 = \log \frac{b_1}{b_2}$
$\frac{b_1}{b_2} = 10^5$
The sun is 10^5 times brighter than the full moon.

d) **The brightest stars are around 10^5 times brighter than the Sun. If the apparent magnitude of these bright stars in some galaxy is 22.5, how far away is the galaxy?**

First we need to find the absolute magnitude of the bright stars.
$M_1 - 5 = \frac{-5}{2} \log 10^5$
$M_1 = \frac{-5}{2} \times 5 + 5 = -12.5 + 5 = -7.5$
Now use the distance equation and the given apparent magnitude to find the distance.
$22.5 - (-7.5) = 5 \log \frac{D}{10pc}$

$$22.5 + 7.5 = 30 = 5 \log \tfrac{D}{10pc}$$
$$\tfrac{D}{10pc} = 10^6 \text{ so } D = 10^7 parsecs$$

e) **White dwarfs are 10^4 times fainter than "A-type stars". the nearest example of which is Sirius. Sirius has a distance of 3 parsecs, and an absolute magnitude of $M = 1$. Sirius is orbited by a white dwarf known as Sirius B. What is the apparent magnitude of Sirius B (note that you can assume that the two stars in the Sirius system are at the same distance from Earth).**

First we ned to calculate the apparent magnitude of Sirius. Then we can use the brightness equation to find the apparent magnitude of the companion white dwarf (Sirius B).

$$m_1 - 1 = 5 \log \tfrac{3}{10}$$
$$m_1 - 1 = 5 \times \tfrac{-1}{2}$$
$$m_1 = \tfrac{-5}{2} + 1 = -1.5$$

$$-1.5 - m_2 = \tfrac{-5}{2} \log 10^4$$
$$-1.5 - m_2 = \tfrac{-5}{2} \times 4$$
$$-10 = -1.5 - m_2$$
$$m_2 = 8.5$$

f) **The faintest galaxies observed by the Hubble Space Telescope have apparent magnitudes around 30. Suppose these galaxies are ≈ 3 giga-parsecs away (3×10^9 parsecs). Assuming every star in these galaxies emits about the same amount of light as the Sun (a false assumption, but let's make it just the same), how many stars would these galaxies contain? (Hint, the number of stars in each galaxy will be equal to amount by which the galaxy is intrinsically brighter than the Sun — e.g. a galaxy with 10 Sun-like stars has a brightness 10 times that of the Sun).**

First we find the absolute magnitude of the faint galaxy, and then we can calculate how much brighter it is than the sun.

$$D = 3 \times 10^9 \text{ parsecs}$$
$$30 - M_{gal} = 5 \log \tfrac{D}{10pc}$$
$$30 - M_{gal} = 5 \log (10^{\frac{1}{2}} \times 10^8)$$
$$30 - M_{gal} = 5 \times 8.5 = 42.5$$
$$M_{gal} = -12.5$$

$$-12.5 - 5 = \tfrac{-5}{2} \log \tfrac{b_1}{b_2}$$
$$-17.5 \times \tfrac{-2}{5} = \log \tfrac{b_1}{b_2}$$
$$7 = \log \tfrac{b_1}{b_2}$$

The galaxy is 10^7 times brighter than the Sun, so it contains 10^7 Sun-like stars.

2) a) **Suppose the entire difference between Hubble's measurement of the Hubble constant and the currently known value of 70 km/s/Mpc was due to observing the wrong kind of Cepheid. What is the difference in absolute magnitude between the two types of Cepheids? That is, compute $\Delta\mathrm{Cep} = M_{C1} - M_{C2}$, where M_{C1} and M_{C2} are the absolute magnitudes of the two types of Cepheids.**

If we assume that the entire difference between Hubble's measurement of the Hubble constant and the currently known value is due to the difference in absolute magnitude, we need to determine how the absolute magnitude can affect the Hubble constant. We can write down the distance modulus equation for both types of Cepheids:

$$m_{C1} - M_{C1} = 5\log(d_1/10\mathrm{pc})$$

$$m_{C2} - M_{C2} = 5\log(d_2/10\mathrm{pc})$$

If we then subtract the two equations from one another, we end up with:

$$
\begin{aligned}
(m_{C1} - M_{C1}) - (m_{C2} - M_{C2}) &= 5\log(d_{C1}/10\mathrm{pc}) - 5\log(d_{C2}/10\mathrm{pc}) \\
m_{C1} - m_{C2} - M_{C1} + M_{C2} &= 5\log(d_{C1}/d_{C2})
\end{aligned}
$$

However, we know that the *apparent* magnitude of the two types of Cepheids are the same, since this is just a measured quantity. This means that $m_{C1} - m_{C2} = 0$. So, we then have:

$$
\begin{aligned}
-M_{C1} + M_{C2} &= 5\log(d_{C1}/d_{C2}) \\
M_{C1} - M_{C2} &= -5\log(d_{C1}/d_{C2}) \\
\Delta\mathrm{Cep} = M_{C1} - M_{C2} &= -5\log(d_{C1}/d_{C2})
\end{aligned}
$$

In order to find the difference in the absolute magnitude of the two types of Cepheids, we need to relate the two distances to the two values of the Hubble constant:

$$v = Hd$$

$$d = \frac{v}{H}$$

There are two versions of this equation in this problem (one for d_{C1} and one for d_{C2}). Hubble assumed that he was looking at Type II Cepheids and got a value of $H = 500\mathrm{km/s/Mpc}$. Therefore, his version of this equation is:

$$d_{C2} = \frac{v_{C2}}{500\mathrm{km/s/Mpc}}$$

Of course, we now know that he was looking at Type I Cepheids, and we get a value of $H = 70\text{km/s/Mpc}$. So, our version of the equation is:

$$d_{C1} = \frac{v_{C1}}{70\text{km/s/Mpc}}$$

Plugging these two values into the equation with ΔCep, we get:

$$\Delta\text{Cep} = M_{C1} - M_{C2} \; = \; -5\log\left(\frac{\frac{v_{C1}}{70\text{km/s/Mpc}}}{\frac{v_{C2}}{500\text{km/s/Mpc}}}\right)$$

$$= \; -5\log\left(\frac{500}{70}\frac{v_{C1}}{v_{C2}}\right)$$

However, once again, v is a quantity measurable from Earth and the intrinsic properties of the Cepheid are not going to change our *measurements*. This means that $v_{C1} = v_{C2}$ and:

$$\Delta\text{Cep} = M_{C1} - M_{C2} \; = \; -5\log\left(\frac{500}{70}\right)$$

$$= \; -5\log(7)$$

$$= \; -5(0.8)$$

$$= \; -4$$

So, the Type I Cepheids would have to be ~ 4 magnitudes brighter than Type II Cepheids to fully account for Hubble's error.

b) **Suppose there had been no problem with the standard candles Hubble used, including the Cepheids, and that the error in the determination of Hubble constant was due to some bizarre mistake in determine the value of an Astronomical Unit (that is, the distance from the Earth to the Sun). Explain why such an error could result in a mistaken value for the Hubble constant. What would the mistaken value of an AU have to be (in meters) to explain Hubble's error?**

This question is somewhat ambiguous and there are (at least) two possible answers that I saw that got full credit.

The first answer (which accounted for the majority of the correct answers) had to do with the use of parallax as the base of the distance ladder. The idea is that the length of the AU is critical in determining the distance to nearby variable stars, which can then be used to determine the distance to progressively further and further away "standard candles." Hubble's value was too large by a factor of 7, which means the distances he calculated were too *small* by a factor of 7. These distance errors could be explained by an error in parallax (the base of the distance ladder). We can consider two values for the AU: AU_{right} ($= 1.5 \times 10^{11}\text{m}$) and AU_{wrong}. Since, in each question, the parallax angle (as something that can be measured), will be the same, we can write down two equations:

$$\alpha = \frac{\mathrm{AU_{right}}}{d_{\mathrm{right}}}$$

$$\alpha = \frac{\mathrm{AU_{wrong}}}{d_{\mathrm{wrong}}}$$

Since they both have the same value for α, we can set the right sides equal to one another:

$$\frac{\mathrm{AU_{right}}}{d_{\mathrm{right}}} = \frac{\mathrm{AU_{wrong}}}{d_{\mathrm{wrong}}}$$

Since Hubble's distances were too small by a factor of 7, this means that $d_{\mathrm{right}} = 7d_{\mathrm{wrong}}$. Therefore,

$$\frac{\mathrm{AU_{right}}}{7d_{\mathrm{wrong}}} = \frac{\mathrm{AU_{wrong}}}{d_{\mathrm{wrong}}}$$

$$\frac{\mathrm{AU_{right}}}{7} = \mathrm{AU_{wrong}}$$

$$\frac{1.5 \times 10^{11}\mathrm{m}}{7} = \mathrm{AU_{wrong}}$$

$$\mathrm{AU_{wrong}} = 2 \times 10^{10}\mathrm{m}$$

The second answer to this problem (which was submitted by several people) pointed out that, by redefining the AU, we redefine what the length of the parsec is. The parallax equation is basically as:

$$\alpha = \frac{\text{Earth-Sun Distance}}{\text{Distance in parsecs}}$$

If the definition of the numerator on the right hand side of that equation changes, the denominator must also change by the same amount (to keep the parallax angle on the left hand side of the equation the same). A larger unit means that the numerical value measured is smaller (for example, 24 inches is 2 feet, the smaller unit going with the bigger number). Thus, if the measured AU is too big, the number in the denominator (for a fixed length) is too small, making Hubble's constant too big. Since Hubble measured the constant to be too large by a factor of 7, his AU (by this reasoning) must have been 7 times too big, or $10^{12}\mathrm{m}$.

Finally, I'll point out that these two effects exactly cancel one another (i.e. $\frac{1}{7} * 7$), so the net effect is that changing the value of the AU does *not* change the value of the Hubble constant, in the units commonly used (km/s/Mpc).