Astronomy 160: Frontiers and Controversies in Astrophysics
Homework Set # 2 Solutions

1) **How big a shift in the wavelength of green light will this instrument be able to detect?**

We know (from class) that the wavelength of green light is 5000 Å ($= 5 \times 10^{-7}$ m). So, if the Doppler precision of the Rocky Planet Finder is 1 m/s, that means we need to find what wavelength shift a 1 m/s motion will induce in 5000 Å light.

$$\begin{aligned}
\frac{\Delta\lambda}{\lambda_0} &= \frac{v}{c} \\
\frac{\Delta\lambda}{5 \times 10^{-7}\ [\mathrm{m}]} &= \frac{1\ [\mathrm{m/s}]}{3 \times 108\ [\mathrm{m/s}]} \\
\Delta\lambda &= \frac{5 \times 10^{-7}}{3 \times 108}\ [\mathrm{m}] \\
\Delta\lambda &= 2 \times 10^{-15}\ \mathrm{m}
\end{aligned}$$

So, a 1 m/s velocity produces a 2×10^{-15} m change in the wavelength of green light.

2) **Show that, for a nearly circular orbit, $a = GM/V^2$, where V is the total velocity of the system ($V = V_{planet} + V_{star}$).**

We know that:

$$a^3 = \frac{GMP^2}{4\pi^2}$$

For a circular orbit, we know that the circumference of the circular orbit is just $2\pi a$ and that the planet covers that distance in one period (P). So, the velocity of the planet in its orbit is:

$$v_P = \frac{2\pi a}{P}$$

If we re-arrange the equation, we get:

$$P = \frac{2\pi a}{v_P}$$

Now, if we plug in this way of expressing the period into the first equation, we get:

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$$\begin{aligned} a^3 &= \frac{GM(\frac{2\pi a}{v_P})^2}{4\pi^2} \\ a^3 &= \frac{GM(4\pi^2 a^2)}{4\pi^2 v_P^2} \\ a^3 &= \frac{GMa^2}{v_P^2} \\ a &= \frac{GM}{v_P^2} \end{aligned}$$

3) **In question 2, why is it important to specify that the orbit is nearly circular?**

It is important to specify that because the equation we use to relate velocities, orbits and periods ($V = 2\pi a/P$) only holds when the orbit is circular. Other, non-circular orbits, will require a modified version of the above equation.

4) **Suppose you observe a Sun-like star with a planet in an Earth-like orbit. How massive does the planet have to be for it to be detected by the Rocky Planet Finder?**

Earth-like orbit:

$$a = 1AU$$

The period is then:

$$a^3 = P^2 M$$

$$P = (A^3/M)^{1/2} \; [yr]$$

$$P = (1AU)^3/1M_\odot$$

$$P = 1year$$

Minimum stellar velocity (V_S) for detection is $1m/s$ and the planet velocity is:

$$V_P = \frac{2\pi a}{P} = \frac{2 \times 3 \times 1.5 \times 10^{11}}{3 \times 10^7}$$

$$V_P = 3 \times 10^4 m/s$$

The mass of the planet is then:

$$V_S M_S = V_P M_P$$

$$M_P = \frac{V_S}{V_P} M_S$$

$$M_P = \frac{1}{3 \times 10^4} 1M_\odot = \frac{1}{3} \times 10^{-4} M_\odot$$

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$$M_P = 3 \times 10^{-5} M_\odot \ (\textit{in Solar masses})$$

Or alternatively:

$$M_P = 3 \times 10^{-5} \times 2 \times 10^{30} kg = 6 \times 10^{25}$$

Which is Earth-like mass.

5) **What is the maximum orbital period and semi-major axis of an Earth-like planet that could be detected by the Rocky Planet Finder?**

An Earth-like planet (6×10^{24} kg) around a Sun-like star ($1M_\odot = 2 \times 10^{30}$ kg) can be detected if the planet imparts a 1 m/s velocity on the star.

$$V_p = \frac{M_*}{M_p} V_* = \frac{2 \times 10^{30}\,\mathrm{kg}}{6 \times 10^{24}\,\mathrm{kg}} 1\,\mathrm{m/s} = 3 \times 10^5\,\mathrm{m/s}$$

$$a = \frac{GM}{V^2} = \frac{7 \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}} \times 2 \times 10^{30}\,\mathrm{kg}}{(3 \times 10^5\,\mathrm{m/s})^2} = \frac{14 \times 10^{19}\,\mathrm{m^3s^{-2}}}{9 \times 10^{10}\,\mathrm{m^2/s^2}} = 2{\times}10^9\,\mathrm{m} = 10^{-2}\,\mathrm{AU}$$

$$P = \left(\frac{a^3}{M}\right)^{1/2} = \left(\frac{(10^{-2}\,\mathrm{AU})^3}{1M_\odot}\right)^{1/2} = (10^{-6})^{1/2}\,\mathrm{yrs} = 10^{-3}\,\mathrm{yrs} \sim 9\,\mathrm{hrs}$$

6) **Given the results you have obtained, evaluate the claim that "the Rocky Planet Finder permits the detection of planets of 1-20 Earth masses." (Fun fact: the radius of the Sun is 7×10^8 meters).**

The Rockey Planet Finder's claim that it can detect planets 1-20 Earth masses is true (2 pts). This statement is misleading, however, because a 1 Earth mass planet can only be detected when it is VERY close to its star (1 pt). So even though it can detect Earth mass planets, these planets are not very Earth-like.

7) **Arguments that there are likely to be many instances of life ("as we know it") throughout the Universe tend to depend on the premise that there are lots of Earth-like planets in Earth-like orbits for such life to evolve on. To what extent are arguments of this kind strengthened or weakened by the discovery of Hot Jupiters?**

Here are some of the main points I was looking for in the essay:

- The argument for "life as we know it" on lots of Earth-like planets in Earth-like orbits is, in general, *weakened* by the discovery of hot Jupiters. If you disagreed with that and backed up your statement, that's OK.
- I wanted to see that you understood how hot Jupiters were different from Earth-like planets, and how the existence of these hot Jupiters may make Earth-like planets less likely (at least in the same system).
- I also wanted to see some discussion of selections effects – that we are more likely to detect hot Jupiters because they are larger with short periods and thus induce higher stellar velocities. Therefore, there may still be lots of Earth-like planets that exist that we have been yet unable to detect.

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- If you said these things, then you probably got 4 out of the 5 points. If you had read the websites and talked about migration or other theories, then you may have received another point.

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