Astronomy 160: Frontiers and Controversies in Astrophysics
Homework Set # 1 Solutions

1) **Read the problem set policy (linked from the syllabus). Then answer the following true/false questions:**

a) You can drop your lowest problem set score.

False

b) You can work on problem sets with other students, provided you split up before you write down your answers.

True

c) You can hand in a problem set 24 hours late and lose only one point.

True

d) You can consult with a tutor or TF about the precise wording of your answers on the problem sets.

False

e) You can hand in a problem set a week late and only lose two points.

False

2) **Express the following in simple scientific notation to one-digit accuracy (that is, in the form $N \times 10^m$, where N and m are integers). Please do not use calculators, and show your work.**

a) 8×365.24

$$8 \times 365.24 = 8 \times (4 \times 10^2) = 3 \times 10^3$$

b) $5 \times 6 \times 10^{-3}$

$$5 \times 6 \times 10^{-3} = 30 \times 10^{-3} = 3 \times 10^{-2}$$

c) $\sqrt{4 \times 10^6}$

$$\sqrt{4 \times 10^6} = \sqrt{4} \times (10^6)^{1/2} = 2 \times 10^{6/2} = 2 \times 10^3$$

d) $1/(3 \times 10^{-2})$

$$\frac{1}{(3 \times 10^{-2})} = (3 \times 10^{-2})^{-1} = 3^{-1} \times 10^2 = 0.3 \times 10^2 = 3 \times 10^1$$

e) $(1.2 \times 10^8)^{1/3}$

$$(1.2 \times 10^8)^{1/3} = (120 \times 10^6)^{1/3} = 120^{1/3} \times (10^6)^{1/3} = 5 \times 10^2$$

3) **Neptune's moon Nereid has an orbital period of almost exactly one Earth year. If the mass of Neptune is around 10^{26} kilograms, what is the semi-major axis of Nereid's orbit?**

$M_{Nep} = 10^{26}$ kg $= 10^{-4}$ $M_\odot$
$a^3 = P^2 \times M_{Nep}$
$a^3 = 10^{-4}$ since P = 1 year
$a = 10^{-4/3}$ AU
$a = 10^{-1/3} \times 10^{-3/3}$
$a = 10^{-1/3} \times 10^{-1}$
$a = \frac{1}{10^{1/3} \times 10}$
$a = \frac{1}{2 \times 10^1}$ AU

OR

$M_{Nep} = 10^{26}$ kg $= \frac{1}{2 \times 10^{-4} M_\odot} = 5 \times 10^{-5} M_\odot$
so $a^3 = P^2 \times M_{Nep}$
$a^3 = 5 \times 10^{-5}$
$a^3 = 50 \times 10^{-6}$
$a = 50^{\frac{1}{3}} \times 10^{-2}$
$a = 4 \times 10^{-2}$

OR

$M_{Nep} = 10^{26}$ kg
do problem in mks units
$G = 7 \times 10^{-11}$
$a^3 = \frac{P^2 \times G \times M}{4\pi^2}$
$a^3 = \frac{(3 \times 10^7)^2 \times 7 \times 10^{-11} \times 10^{26}}{4 \times 3 \times 3}$
$a^3 = \frac{10 \times 10^{14} \times 7 \times 10^{-11} \times 10^{26}}{4 \times 10^1}$
$a^3 = \frac{7 \times 10^{29}}{4}$

$a^3 = 2 \times 10^{29}$
$a = (2 \times 10^{29})^{\frac{1}{3}}$
$a = (200 \times 10^{27})^{\frac{1}{3}}$
$a = 6 \times 10^9$ meters

4) **Consider a Sun-like star, which is orbited by a planet with a period of 80 years. If the separation of the star and the planet appears to be 20 arc seconds, how far away is the star?**

We are give that $M = 1M_\odot$, $P = 80\,\text{yrs}$ and $\alpha = 20$". The distance to the star is given by the equation α(in arcseconds)= d(in AU)/D(in parsecs), where $d = a$ in the equation $a(\text{in AU})^3 = P(\text{in years})^2 \times M(\text{in solar masses})$.

First solve for the star-planet separation:

$$d = a = (P^2 M)^{1/3} = ((80\,\text{yrs})^2 (1M_\odot))^{1/3} = ((8\times10^1)^2\times1)^{1/3} = (64\times10^2)^{1/3} = (6.4\times10^3)^{1/3} = 2\times10^1\ \text{AU}$$

Then solve for the distance to the star:

$$D = \frac{d}{\alpha} = \frac{2 \times 10^1\ \text{AU}}{20"} = \frac{2 \times 10^1}{2 \times 10^1} = 1\,\text{pc}$$

5) **In fact, there is such a star (α Centauri) except that its companion isn't a planet, but another Sun-like star. Does this fact make any difference in the forgoing calculation? Explain.**

If you "replace" the planet with the star in the above scenario, you will need to change your calculation. Specifically, a second star significantly increases the mass of the system from $1\text{M}_\odot$ to $2\text{M}_\odot$. Remeber that the equation

$$a3 = p2M$$

takes as its input the *total* mass of the system. If you increase the total mass of the system from $1\text{M}_\odot$ to $2\text{M}_\odot$, the seperation between the two objects is larger by a factor of $(2)^{1/3}$. This, in turn, means that the system is more distant by a factor of $(2)^{1/3}$.