Astronomy 160: Frontiers and Controversies in Astrophysics
Homework Set # 5 Solutions

1) **Consider a binary star system consisting of two $1.5M_\odot$ neutron stars. Suppose one of the neutron stars is a pulsar with an average observed pulse period of exactly 2 seconds. If the orbit is circular and edge-on to our line of sight, what is the maximum and minimum pulse period observed? (Note: the pulse period P_p obeys the same Doppler shift formulas as wavelength does).**

Total Mass: $M = M_{NS} + M_P = 1.5M_\odot + 1.5M_\odot = 3M_\odot = 6 \times 10^{30}\,\mathrm{kg}$, ORBITAL period: $P = 8\,\mathrm{hr} = 8\,\mathrm{hr} \times \frac{3600\,\mathrm{s}}{\mathrm{hr}} = 3 \times 10^4\,\mathrm{s}$. Now use this information to solve for the orbital velocity of the pulsar. In order to do that, we must first find the total orbital velocity of the system, where $V = V_{NS} + V_P$. Because $V_{NS}M_{NS} = V_P M_P$ and $M_{NS} = M_P$, then $V_P = V/2$.

Now combine the following equations

$$a^3 = \frac{P^2GM}{4\pi^2} \text{ and } a = \frac{VP}{2\pi}$$

$$\left(\frac{VP}{2\pi}\right)^3 = \frac{P^2GM}{4\pi^2}$$

$$\frac{V^3P^3}{8\pi^3} = \frac{P^2GM}{4\pi^2}$$

$$V^3 = \frac{2\pi GM}{P}$$

$$V = \left(\frac{2\pi GM}{P}\right)^{1/3}$$

$$V = \left(\frac{2\pi \times 7 \times 10^{-11} \times 6 \times 10^{30}}{3 \times 10^4}\right)^{1/3}$$

$$V = \left(\frac{80 \times 10^{19}}{10^4}\right)^{1/3}$$

$$V = \left(80 \times 10^{15}\right)^{1/3}$$

$$V = 4 \times 10^5\,\mathrm{m/s}$$

therefore

$$V_P = \frac{V}{2} = \frac{4 \times 10^5\,\mathrm{m/s}}{2} = 2 \times 10^5\,\mathrm{m/s}$$

This isn't very fast so we can use the Newtonian Doppler formula. We are total that the average pulse period is $P_p = 2\,\mathrm{s}$:

$$\frac{\Delta P_p}{P_p} = \frac{v}{c}$$

$$\Delta P_p = \frac{v}{c} \times P_p = \frac{2 \times 10^5\,\mathrm{m/s}}{3 \times 10^8\,\mathrm{m/s}} \times 2\,\mathrm{s} = \frac{4}{3} \times 10^{-3}\,\mathrm{s}$$

$$\Delta P_p = \pm 1 \times 10^{-3}\,\mathrm{s} = \pm 0.001\,\mathrm{sec}$$

So the longest pulse period is 2.001 s and the shortest is 1.999 s.

2) **Suppose a malevolent intelligence is trying to deceive us into thinking that a binary system of the kind described in problem 1 exists, when in fact that isn't the case. Suppose this evil being lives on a neutron star that is *not* part of a binary system, and its scheme for deceiving us is to put a pulse-making machine on a movable platform that can go up and down. When the platform is high off the surface of the neutron star, the pulses suffer less gravitational redshift, and thus the observed pulse period is shorter — when the platform is on the surface of the neutron star, there will be more gravitational redshift, and P_p will be longer. If the radius of the neutron star is $(4/3)R_s$, where R_s is the Schwarzshild radius appropriate for the mass of the neutron star, it turns out that the difference in the observed pulse period due to the gravitational redshift between a high platform position and the surface is $\Delta P_p/P_p = 3h/R_n$, where h is the height of the platform and R_n is the radius of the neutron star. If the neutron star has mass $M_n = 1.5M_\odot$, how high does the platform have to be to reproduce the results of problem 1?**

The equation we're given for the height of the platform is

$\frac{\Delta P_p}{P_p} = \frac{3h}{R_n}$.

First find R_n:

$R_n = (4/3)R_s = \frac{4}{3}\frac{2GM}{c^2} = \frac{4}{3}\frac{2\times7\times10^{-11}\times3\times10^{30}}{9\times10^{16}} = \frac{4}{3} \times 5 \times 10^3 = 7 \times 10^3$ meters

Now since moving the platform up and down accounts for the entire dopper shift we found in the first problem, we need to *double* our ΔP_p. So

$\frac{\Delta P_p}{P_p} = \frac{2\times3\times10^{-3}}{2} = \frac{3h}{7\times10^3}$.

The 2's and the 3's cancel in the above equation, and you're left with
$h = 7$ meters.

3) **The two situations in problems 1 and 2 could both provide a perfect fit to an observed sinusoidal velocity curve. But if such a velocity curve were observed from a pulsar, most people (and essentially all astronomers) would believe the existence of a binary system, rather than a malevolent intelligence. Why? Is this belief justified?**

While both stories provide *possible* explanations for the observed change in the pulse period, the explanation in the first problem (i.e. binary orbit of a pulsar) provides a much simpler, more straightforward explanation. We have observed many binary star systems throughout our galaxy and we have strong observational evidence both for pulsars and neutron stars. The second story (the aliens) requires not ONLY that alien life exists, but also that these aliens have the ability to live on a neutron star, have the ability to construct moveable platforms in *very* high gravity, have the ability to control those platforms with high precision, and have nothing better to do than to trick unsuspecting Earthlings.

While the alien story is not something we can scientifically rule out, at some point it becomes reasonable to invoke Occam's Razor and state that the simplest explanation is likely the truth.

4) **The 1993 Nobel Prize for physics was awarded to Joe Taylor and Russell Hulse for the discovery of the first binary pulsar. Read Hulse's entertaining account of their discovery (on the classes server resources page). What particular abilities and personal characteristics does Hulse have that enabled him to make this discovery? How do Hulse's talents and discoveries compare with those of someone like Einstein? Do you think Hulse deserved the Nobel Prize, or did he just get lucky?**

This is a 6 point problem. 5 of the 6 points were awarded as described below - the sixth was given for especially interesting thoughtful, or well-expressed ideas at any point along the way.

Hulse's characteristics (2 points): 1 point for a list of adjectives; 1 point for pertinent examples and analysis.

Hulse vs. Einstein (2 points): This could be approached from a number of points of view, including theory vs. observation; generality vs specifics; both dedicated to explaining anomalies, etc. One point for any one of these things; the full two points for clearly expressed discussion of several possibilities. Saying that Einstein was "obviously" superior without saying why, or of pointing out only similarities without noting the (quite significant) differences tended to get less than full credit.

Did H deserve the prize: 1 point for a clearly stated and supported view, in either direction.