PROBLEM SET 1
GMM

Exercise 1 Consider the model with $y_i = x_i'\beta + e_i$ and $\mathbb{E}(z_i e_i) = 0$, with $\{e_i\}$ being i.i.d.

a) Derive the efficient GMM estimator

b) Compare with 2SLS. Are they always equal?

c) Under what conditions the GMM estimator is consistent and normally distributed?

d) Find the asymptotic distribution of the GMM estimator.

Exercise 2 Let y_i be given by $y_i = \beta + u_i$ with $\{u_i\}$ i.i.d. where the first two moments of u_i are:

$$\mathbb{E}(u_i) = 0\,,\ \mathbb{E}(u_i^2) = m_2$$

a) Suppose you want to estimate β using GMM. What moment conditions would you use? Under the moments conditions' you choose, derive the efficient GMM estimator and find the asymptotic distribution

b) Now, suppose that $\mathbb{E}(u_i^3) = 0$, $\mathbb{E}(u_i^4) = m_4$ and $\mathbb{E}(u_i^6) = m_6$ and the moments conditions you use are:

$$g(y_i, \beta) = \begin{bmatrix} y_i - \beta \\ (y_i - \beta)^3 \end{bmatrix}$$

Derive the efficient GMM estimator and find the asymptotic distribution.

Exercise 3 You observe (y_i, x_i, z_i) which are i.i.d. Take the model

$$y_i = x_i'\beta + e_i \ \ ; \ \ \mathbb{E}(x_i e_i) = 0 \ \ ; \ \ e_i^2 = z_i'\gamma + \eta_i \ \ ; \ \ \mathbb{E}(z_i \eta_i) = 0$$

Find the method of moments estimators $(\hat{\beta}, \hat{\gamma})$ for (β, γ)

Exercise 4 For the case described in exercise 1, suppose $R'\beta = c$. Consider $R_{k\times q}$ and $c \in \mathbb{R}^q$. Derive the constrained GMM estimator and it's asymptotic distribution.

Exercise 5 Let $r(\beta) : \mathbb{R}^k \to \Theta \subset \mathbb{R}^q$ and $\theta = r(\beta)$. The GMM estimator of θ is $\hat{\theta}_{gmm} = r(\hat{\beta}_{gmm})$. Show how to construct a Wald statistic W to test

$$\mathbb{H}_0 : \theta = \theta_0 \ \ against \ \ \mathbb{H}_0 : \theta \neq \theta_0$$

and derive the asymptotic distribution

Exercise 6 In the linear model estimated by GMM with general weight matrix W, the asymptotic variance of $\hat{\beta}_{GMM}$ is:

$$V = (Q'WQ)^{-1}Q'W\Omega WQ(Q'WQ)^{-1}$$

a) Let V_0 be this matrix when $W = \Omega^{-1}$. Show that $V_0 = (Q'\Omega^{-1}Q)^{-1}$

b) We want to show that for any W, $V - V_0$ is positive semi-definite (for then V_0 is the smaller possible covariance matrix, and $W = \Omega^{-1}$ is the efficient weight matrix). To do this, start by finding matrices A and B such that $V = A'\Omega A$ and $V_0 = B'\Omega B$.

c) Show that $B'\Omega A = B'\Omega B$ and therefore that $B'\Omega(A - B) = 0$.

d) Use the expressions $V = A'\Omega A$, $A = B + (A - B)$, and $B'\Omega(A - B) = 0$ to show that $V \geq V_0$.