Notes

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November 3, 2024

1 3D Dynamics

3D Dynamics with fixed mass, using a rotation matrix to represent orientation:

$$
\begin{aligned}
\mathbf{x} &= \begin{bmatrix} \mathbf{p} \\ \mathbf{R} \\ \mathbf{v} \\ \omega \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \\
\dot{\mathbf{p}} &= \mathbf{v} \\
\dot{\mathbf{R}} &= \mathbf{R}\hat{\omega} \\
\dot{\mathbf{v}} &= \frac{1}{m}\mathbf{R}\begin{bmatrix} f_T \\ 0 \\ 0 \end{bmatrix} - \frac{f_D}{m\|\mathbf{v}\|}\mathbf{v} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \\
\dot{\omega} &= \mathcal{I}^{-1}\left(\mathbf{u} - \omega \times \mathcal{I}\omega\right)
\end{aligned}
$$

Where $\hat{\omega}$ is the skew-symmetric matrix of ω, f_T is the thrust force, f_D is the drag force, m is the mass, g is the acceleration due to gravity, and $\mathcal{I}$ is the inertia matrix. We can combine these equations to get the full dynamics:

$$
\begin{aligned}
\dot{\mathbf{x}} &= f(\mathbf{x}, \mathbf{u}, t) \\
\begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{R}} \\ \dot{\mathbf{v}} \\ \dot{\omega} \end{bmatrix} &= \begin{bmatrix} \mathbf{v} \\ \mathbf{R}\hat{\omega} \\ \frac{f_T(t)}{m}\mathbf{R}\mathbf{e}_1 - \frac{f_D(\mathbf{x})}{m\|\mathbf{v}\|}\mathbf{v} - g\mathbf{e}_3 \\ \mathcal{I}^{-1}\left(\mathbf{u} - \omega \times \mathcal{I}\omega\right) \end{bmatrix} \\
f_D(\mathbf{x}) &= \frac{1}{2}\rho\|\mathbf{v}\|^2 C_D A
\end{aligned}
$$