MATH540-HW5

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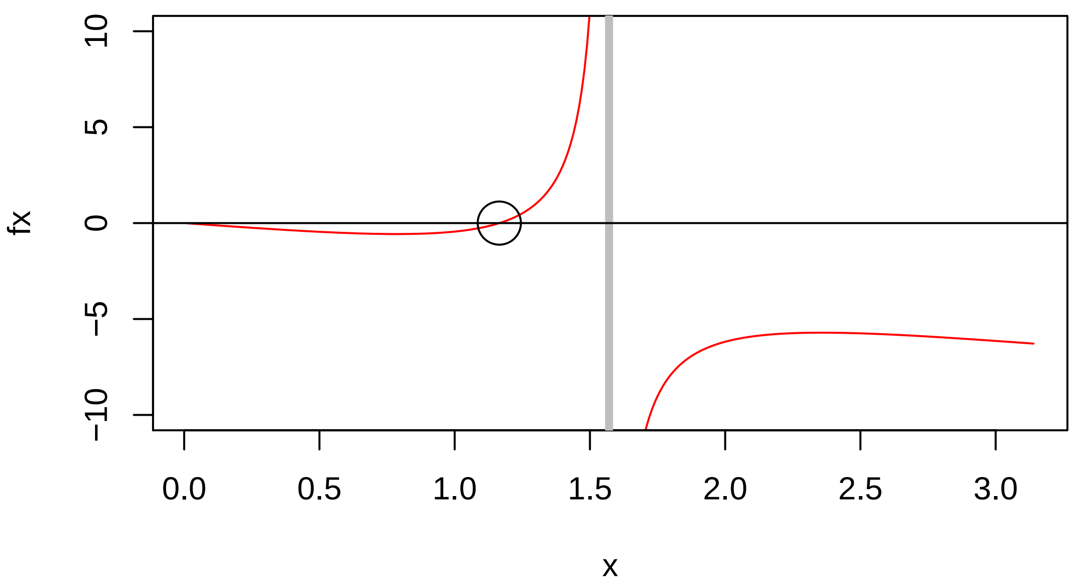
Problem 1.a

Show that the function f(x)=tan(x)-2x has a root r with $0 < r < pi/2$ (Hint: sketch a graph)

```
x=seq(0.01,3.14,by=0.01)
fx=tan(x) - 2*x
plot(x,fx,type='l',col='red',ylim=c(-10,10))
abline(v=pi/2,lwd=4,col='gray')
abline(h=0)
points(1.165,0,pch=1,cex=3)
```



Problem 1.b

Use a calculator or computer and the bisection method to find the root to some reasonable accuracy.

Note: From the previous graph, it is observed that the sign of f(0+e) is the same as the sign of f(pi-epsilon). In order to start the bisection method, we need endpoints of differing sign, so we start the search over the range of (0,pi/2)

```r
n=0
epsilon=.000001
delta=.00001
a=0.0+epsilon
b=pi/2-epsilon
x=seq(a,b,by=epsilon)
fx=tan(x) - 2*x
while (n<100 & (b-a)>delta) {
  n = n+1
  if ( fx[1]/abs(fx[1]) == fx[round(length(fx)/2)]/abs(fx[round(length(fx)/2)]) ) {
    a=x[round(length(x)/2)]
    b=b
  } else {
    a=a
    b=x[round(length(x)/2)]
  }
  x=seq(a,b,by=epsilon)
  fx=tan(x) - 2*x
}
sprintf("The number of iteration is: %d",n)
```

```
## [1] "The number of iteration is: 18"
```

```r
sprintf("The interval length is:  %f",(b-a))
```

```
## [1] "The interval length is:  0.000007"
```

```r
sprintf("The approximate root is at:  %f",(b+a)/2)
```

```
## [1] "The approximate root is at:  1.165564"
```

```r
sprintf("Which evaluates as:  %f",tan((b+a)/2) - 2*(b+a)/2)
```

```
## [1] "Which evaluates as:  0.000010"
```

Problem 1.c

Use a calculator or computer and Netwon's method to find the root to some reasonable accuracy.

Note: In order to start the Newton's method, we need an initial point somewhere "near" the root. For this exercise, 1.0 is used as the intital point.

```r
f <- function(x) {
  return(tan(x) - 2*x)
}

epsilon=.0001
delta=.001
```

```
x0 = 1.0
f.x = f(x0)
n=0
while (n<20 & abs(f.x)>epsilon) {
  n=n+1
  df.dx = (f(x0+delta)-f(x0))/delta
  x1 = (x0 - (f(x0)/df.dx))
  x0 = x1
  f.x = f(x0)
}
sprintf("The number of iteration sis: %d",n)
sprintf("The approximate root is at: %f",x0)
sprintf("Which evaluates as: %f",f(x0))
```

```
## [1] "The number of iterations is: 5"

## [1] "The approximate root is at: 1.165563"

## [1] "Which evaluates as: 0.000008"
```

However, even a small step away from the root can cause the algorithm to diverge. For x=pi/4, I obtain the following result.

```
## [1] "The number of iterations is: 20"

## [1] "The approximate root is at: 858459693823.194946"

## [1] "Which evaluates as: -1716919387645.491455"
```