CS240, Spring 2022
Assignment 3: Question 2

Q2a) The best case will occur when $random.size(A.size) = 0$. This will result in only a constant number of operations will run for each call of ArrayAlg. The array reduces by 1, which means that we will have a runtime of:

$$T(n) = \sum_{i=0}^{n} 1$$
$$T(n) = n$$
$$T(n) \in O(n)$$

Q2b) The worst case will occur when $random.size(A.size) = n - 1$. This will result in only a loop which will have $n - 1$ of operations will run for each call of ArrayAlg. The array reduces by 1, which means that we will have a runtime of:

$$T(n) = \sum_{i=0}^{n} (n-1)$$
$$T(n) = \frac{(n+1)(n)}{2} - n$$
$$T(n) = \frac{n^2}{2} - \frac{n}{2}$$
$$T(n) = O(n^2)$$

Q2c) For each iteration of ArrayAlg, we have a 2 possibilities: Either $random.size(A.size) = 0$ and so we will only do 1 operation, otherwise if $random.size(A.size) = x$ we will do x operations as we will print x elements. Therefore our good and bad case looks like:

$$T(A[0, ..., A.Size]) = \begin{cases} 1 + T(A[0, ..., A.Size - 2] \text{ where } random.size(A.size) = 0 \\ x + T(A[0, ..., A.Size - 2] \text{ where } random.size(A.size) = x) \end{cases}$$

Note that for each iteration of ArrayAlg, the "good case" will only happen once, and the base case will happen the remaining times. Thus:

$$\text{number of good outcomes} = 1 \quad \text{number of bad outcomes} = n - 1$$

It thus follows that:

$$T^{exp}(n) = \frac{1}{\prod_n}\Big(\sum_{\pi \in \prod_n : \pi good} T(\pi) + \sum_{\pi \in \prod_n : \pi bad} T(\pi)\Big)$$
$$T^{exp}(n) = \frac{1}{\prod_n}\Big(\sum_{\pi \in \prod_n : \pi good} 1 + T(n-2) + \sum_{\pi \in \prod_n : \pi bad} x + T(n-2)\Big)$$
$$T^{exp}(n) = \frac{1}{n}(1 \times (1 + T(n-2)) + n \times (x + T(n-2)))$$

Howver since $1 <= x <= n-1$ it follows that $x < n$ thus we get:

$$T^{exp}(n) = \frac{1}{n}(1 \times (1 + T(n-1)) + (n-1) \times (x + T(n-1)))$$
$$T^{exp}(n) < \frac{1}{n}(1 \times (1 + T(n-1)) + (n-1) \times (n + T(n-1)))$$
$$T^{exp}(n) < \frac{1}{n}(1 + T(n-1) + n^2 + nT(n-1) - n - T(n-1))$$
$$T^{exp}(n) < \frac{1}{n}(1 + n^2 + nT(n-1) - n)$$
$$T^{exp}(n) < \frac{1}{n} + n + T(n-1) - 1$$

So our function will run n times and each time it will do $n + \frac{1}{n} - 1$ operations and since:

$$n(n + \frac{1}{n} - 1) \in O(n^2)$$

Therefore $T^{exp}(n) \in O(n^2)$