CS240, Spring 2022
Assignment 2: Question 1

Q1) From the lectures we are told the are equation for the average-case run-time is:

$$T^{avg}(n) = \frac{\sum_{I:size(I)=n} T(I)}{\text{(number of instances with size n)}}$$

However, we know that each arrangement of the array will be equally likely. Since the array has a total of $n!$ permutations our equation for the average run-time becomes:

$$T^{avg}(n) = \frac{1}{n!} \sum_{n \in \prod} T(\pi)$$

Each time we run through the array we can go through i elements before we terminate, therefore out of n elements we need to choose i and the rest can be in any order. Thus our equation becomes:

$$T^{avg}(n) = \frac{1}{n!} \sum_{i=0}^{n} \binom{n}{i} (n-i)!$$

$$T^{avg}(n) = \frac{1}{n!} \sum_{i=0}^{n} \frac{n!}{i!}$$

$$T^{avg}(n) = \sum_{i=0}^{n} \frac{1}{i!}$$

Note that the Taylor series expansion of e^x is given by:

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

$$e^1 = \sum_{i=0}^{\infty} \frac{1^i}{i!}$$

$$e = \sum_{i=0}^{\infty} \frac{1}{i!}$$

Thus it follows that:

$$\sum_{i=0}^{n} \frac{1}{i!} \leq \sum_{i=0}^{\infty} \frac{1}{i!}$$

$$T^{avg}(n) \leq e$$

And since e is a constant it follows that:

$$T^{avg}(n) \in O(1)$$