CS240, Spring 2022
Assignment 5: Question 1

Q1) Give an algorithm to find the point with the maximum x-value in a 2D kd-tree, and analyze its complexity. For maximum credit, your algorithm must run in o(n) time, where n is the number of points in the kd-tree. For simplicity, you may assume that n is a power of 4. If the run-time T(n) of your algorithm satisfies a recurrence relation that has already been seen in class, you can take for granted the corresponding growth rate for T(n), without giving the proof.

Assume that we already have a KD tree called K which for each split (both in terms of x and y) somewhat equally separates values into two parts.

When our algorithm encounters a split in terms of x, we will always recurse on the right child of K. This is because we are looking for the largest x-value, which should always be larger then our current split. When we reach a split in terms of y we should recurse on both sides as the largest x value can have any y value.

We will assume that we build the KD tree by doing an x split first, and we will use a boolean to represent the type of split. Our algorithm is thus:

Algorithm 1 KD Tree Max X Search

```
findMax(kDTree K, Bool xSplit) {
if K is leaf then
    return k.x
end if
if xSplit == true then
    return findMax(k.rightChild, false)
end if
// if we reach this stage, it means we should do a Y split
return max(findMax(k.rightChild, true), findMax(k.leftChild, true))
```

Note that half the values will go under the x split, and the other half will go under the y split and will additionally be split twice, thus our relation is (simplifying using known relations):

$$T(n) = \frac{1}{2}T(n/2) + \frac{1}{2} \times 2T(n/4) + O(1) \tag{1}$$

$$= \frac{1}{2}\sqrt{n} + \frac{1}{2} \times 2T(n/4) + O(1) \tag{2}$$

$$= \frac{1}{2}\sqrt{n} + \frac{1}{2}\log(n) \tag{3}$$

Since we know $\sqrt{n}$ dominates $\log(n)$ we will do our limit test using $\frac{\sqrt{n}}{n}$:

$$\begin{align}
\text{limit} &= \lim_{n\to\infty} \frac{\sqrt{n}}{n} \tag{4}\\
&= \lim_{n\to\infty} \frac{1}{\sqrt{n}} \tag{5}\\
&= 0 \tag{6}
\end{align}$$

And so by the limit test we have:

$$\frac{1}{2}\sqrt{n} + \frac{1}{2}\log(n) = o(n)$$