CS370, Winter 2023
Assignment 5: Question 6

Q5a) Provide a solution to the system of equations

A accurate solution to the system of equations is:

$$\begin{bmatrix} 4 & -4.2 & 0 \\ -2 & 2.099 & 7 \\ 4 & -1.7 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4.2 \\ 4.901 \\ 5.7 \end{bmatrix}$$

and so our solution is:

$$x_0 = 0, x_1 = -1, x_2 = 1$$

Q5b) Provide a solution to the system of equations using Gaussian elimination with no row pivoting

To begin we will start with the matrix:

$$\begin{bmatrix} 4 & -4.2 & 0 \\ -2 & 2.099 & 7 \\ 4 & -1.7 & 4 \end{bmatrix}, \begin{bmatrix} 4.2 \\ 4.901 \\ 5.7 \end{bmatrix}$$

We will then add 1/2(1) into (2):

$$\begin{bmatrix} 4 & -4.2 & 0 \\ 0 & -0.001 & 7 \\ 4 & -1.7 & 4 \end{bmatrix}, \begin{bmatrix} 4.2 \\ 7.001 \\ 5.7 \end{bmatrix}$$

We will then subtract (1) into (3):

$$\begin{bmatrix} 4 & -4.2 & 0 \\ 0 & -0.001 & 7 \\ 0 & 2.5 & 4 \end{bmatrix} =, \begin{bmatrix} 4.2 \\ 7.001 \\ 1.5 \end{bmatrix}$$

We will then add 2500(2) to (3):

$$\begin{bmatrix} 4 & -4.2 & 0 \\ 0 & -0.001 & 7 \\ 0 & 0 & 17504 \end{bmatrix}, \begin{bmatrix} 4.2 \\ 7.001 \\ 17503(roundingdown) \end{bmatrix}$$

We can now use back substitution starting with solving x_3 which gives us:

$$170504x_2 = 170503$$

$$x_2 = 0.9999$$

Using this we thus get:

$$-0.001x_1 + 7(0.9999) = 7.001$$

$$x_1 = -1.7$$

We can finally get:

$$4x_0 - 4.2(-1.7) = 4.2$$

$$x_0 = -0.735$$

Which is very far off from our original estimate as we have:

$$x_0 = -0.735, -1.7, 0.9999$$

Q5b) Provide a solution to the system of equations using Gaussian elimination with row pivoting

To begin we will start with the matrix:

$$\begin{bmatrix} 4 & -4.2 & 0 \\ -2 & 2.099 & 7 \\ 4 & -1.7 & 4 \end{bmatrix}, \begin{bmatrix} 4.2 \\ 4.901 \\ 5.7 \end{bmatrix}$$

We will then switch (1) and (3):

$$\begin{bmatrix} 4 & -1.7 & 4 \\ -2 & 2.099 & 7 \\ 4 & -4.2 & 0 \end{bmatrix}, \begin{bmatrix} 5.7 \\ 4.901 \\ 4.2 \end{bmatrix}$$

We will add 1/2(1) to (2):

$$\begin{bmatrix} 4 & -1.7 & 4 \\ 0 & 1.249 & 9 \\ 4 & -4.2 & 0 \end{bmatrix}, \begin{bmatrix} 5.7 \\ 7.751 \\ 4.2 \end{bmatrix}$$

We will then subtract (1) to (3):

$$\begin{bmatrix} 4 & -1.7 & 4 \\ 0 & 1.249 & 9 \\ 0 & -2.5 & -4 \end{bmatrix}, \begin{bmatrix} 5.7 \\ 7.751 \\ -1.5 \end{bmatrix}$$

We will then multiply by 2.0016 (2) to (3):

$$\begin{bmatrix} 4 & -1.7 & 4 \\ 0 & 1.249 & 9 \\ 0 & 0 & 14.014 \end{bmatrix}, \begin{bmatrix} 5.7 \\ 7.751 \\ 14.014 \end{bmatrix}$$

Solving now we thus get:

$$14.014x_2 = 14.014$$

$$x_2 = 1$$

Plugging this in to the next equation we get:

$$1.249x_1 + 9 = 7.751$$

$$x_1 = -1$$

Lastly we get:

$$4x_0 + 4.2 = 4.2$$

$$x_0 = 0$$

Which is very similar to the accurate answer we had at the start. **Q5a)** Which system was more accurate

In this case it would appear that system c is more accurate then system b, and by switching the rows we were able to avoid the rounding error!