

CS370, Winter 2023

Assignment 2: Question 2

(1) Do two Euler steps for the following IVP:

To be being we will define the following variables, $z_1 = x$, $z_2 = y$. From these we can derive that:

$$\frac{dz_1(t)}{dt} = 4z_1 - 2z_2 + t$$

$$\frac{dz_2(t)}{dt} = 3z_1 + 5t$$

We can then derive f by doing the following:

$$f(t, z) = \frac{d}{dt} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 4z_1 - 2z_2 + t \\ 3z_1 + 5t \end{bmatrix}$$

Solving for $t = 1.2$ (first step) gives us:

$$\begin{aligned} Z^{(1)} &= Z^{(0)} + hf(t_0, Z^{(0)}) \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0.2 \begin{bmatrix} 4(1) - 2(1) + 1 \\ 3(1) + 5(1) \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0.2 \begin{bmatrix} 3 \\ 8 \end{bmatrix} \\ &= \begin{bmatrix} 1.6 \\ 2.6 \end{bmatrix} \end{aligned}$$

Solving now for $t = 1.4$ (second step) gives us:

$$\begin{aligned} Z^{(2)} &= Z^{(1)} + hf(t_0, Z^{(1)}) \\ &= \begin{bmatrix} 1.6 \\ 2.6 \end{bmatrix} + 0.2 \begin{bmatrix} 4(1.6) - 2(2.6) + 1.2 \\ 3(1.6) + 5(1.2) \end{bmatrix} \\ &= \begin{bmatrix} 1.6 \\ 2.6 \end{bmatrix} + 0.2 \begin{bmatrix} 2.4 \\ 10.8 \end{bmatrix} \\ &= \begin{bmatrix} 2.08 \\ 4.76 \end{bmatrix} \end{aligned}$$