r2knowle_a2q3

February 4, 2023

1 A2-Q3: Parametric Spline

[1]:
```
import numpy as np
from scipy.interpolate import make_interp_spline
import matplotlib
import matplotlib.pyplot as plt
```

1.1 (a) Write your nickname and display it

[2]:
```
# [1] Display nickname image

f = plt.imread('nickname.png')
plt.figure(figsize=(9,5));
plt.imshow(f);
```

1.2 (b) Hardcode interpolation points

[3]:
```
# === YOUR CODE HERE ===
R = [(90.15973715651137, 488.2908004778973), (83.12795698924731, 415.
↪9639187574672), (73.08255675029869, 330.57801672640386), (72.07801672640383,␣
↪218.0695340501792), (86.14157706093187, 126.65639187574669), (193.
↪6273596176822, 114.6019115890083), (222.75902031063322, 197.97873357228195),␣
↪(137.3731182795699, 276.3328554360813), (85.13703703703703, 288.
↪3873357228196), (209.7, 442.0819593787336)]
O = [(406.58984468339304, 434.0456391875747), (330.2448028673835, 359.
↪7096774193549), (297.09498207885304, 226.10585424133814), (328.
↪2357228195938, 116.61099163679808), (394.5353643966547, 46.293189964157705),␣
↪(476.9076463560334, 104.55651135005974), (492.9802867383512, 217.
↪06499402628435), (488.96212664277175, 319.52807646356035), (467.
↪8667861409797, 382.8140979689367), (403.5762246117084, 438.06379928315414)]
B = [(641.6522102747908, 428.0183990442055), (630.6022700119473, 355.
↪69151732377543), (626.5841099163679, 273.31923536439666), (621.
↪5614097968936, 198.98327359617684), (616.5387096774193, 104.55651135005974),␣
↪(694.8928315412186, 118.62007168458786), (706.9473118279569, 198.
↪98327359617684), (651.6976105137395, 252.22389486260454), (717.
↪9972520908004, 318.52353643966546), (692.8837514934288, 387.836798088411)]
dash = [(707.9518518518518, 69.39761051373955), (735.074432497013, 93.
↪50657108721623), (755.1652329749103, 111.5882915173238)]
D = [(871.6918757467145, 98.52927120669051), (877.7191158900835, 163.
↪8243727598566), (891.7826762246116, 246.19665471923537), (895.8008363201911,␣
↪311.49175627240146), (906.8507765830345, 419.98207885304663), (980.
↪1821983273596, 369.7550776583035), (1026.3910394265233, 252.22389486260454),␣
↪(1013.33201911589, 148.7562724014337), (951.0505376344086, 108.
↪57467144563918), (901.8280764635604, 98.52927120669051)]

R_y = []
O_y = []
B_y = []
dash_y = []
D_y = []

R_x = []
O_x = []
B_x = []
dash_x = []
D_x = []


for val in R:
    R_y.append(val[1])
    R_x.append(val[0])
```

```python
for val in O:
    O_y.append(val[1])
    O_x.append(val[0])

for val in B:
    B_y.append(val[1])
    B_x.append(val[0])

for val in dash:
    dash_y.append(val[1])
    dash_x.append(val[0])

for val in D:
    D_y.append(val[1])
    D_x.append(val[0])
```

1.3 (c) ParametricSpline

[4]:
```python
def MySpline(x, y):
    '''
     S = MySpline(x, y)

     Input:
       x and y are arrays (or lists) of corresponding x- and y-values,
       specifying the points in the x-y plane.  The x-values
       must be in increasing order.

     Output:
       S is a function that takes x or an array (or list) of x-values
         It evaluates the cubic spline and returns the interpolated value.

     Implementation:

       Hence...
         a[0] = a_0          b[0] = b_1        c[0] = c_1
         a[1] = a_1          b[1] = b_2        c[1] = c_2
              :                   :                 :
         a[n-2] = a_(n-2)    b[n-2] = b_(n-1)  c[n-2] = c_(n-1)
         a[n-1] = a_(n-1)

       The polynomial piece is evaluated at xx using

         p_i(xx) = a[i]*(x[i+1]-xx)**3/(6*hi) + a[i+1]*(xx-x[i])**3/(6*hi) +
                   b[i]*(x[i+1]-xx) + c[i]*(xx-x[i])

       where hk = x[k+1] - x[k] for k = 0, ... , n-2
    '''
```

```python
n = len(x)
h = np.zeros(n-1)
b = np.zeros(n-1)
c = np.zeros(n-1)
a = np.zeros(n)

M = np.zeros((n,n))
r = np.zeros(n)



# === YOUR CODE HERE ===

# Determine h first:

for i in range(0, n-1):
    h[i] = x[i+1] - x[i]


# Now we need to determine a, starting with the first matrix

M[0][0] = h[0]/3
M[0][1] = h[0]/6

for i in range(1, n-1):
    M[i][i-1] = h[i-1]/6
    M[i][i] = (h[i-1] + h[i])/3
    M[i][i+1] = h[i]/6

M[n-1][n-2] = h[0]/6
M[n-1][n-1] = h[0]/3

# Now we need to determine the second matrix:
for i in range (1, n-1):
    r[i] = (y[i+1] - y[i])/h[i] - (y[i] - y[i-1])/h[i-1]

# Lets now solve the array for the values of a
a = np.linalg.solve(M, r)
a[0] = 0           # make sure ending points are zero for natural BCs
a[n-1] = 0         # make sure ending points are zero for natural BCs



# Now we can determine both b and c
for i in range(0, n-1):
    b[i] = y[i]/h[i] - a[i]*h[i]/6
```

```
    for i in range(0, n-1):
        c[i] = y[i+1]/h[i] - a[i+1]*h[i]/6



    #=======================================
    #
    # This is the function that gets returned.
    # It evaluates the cubic spline at xvals.
    #
    def spline(xvals, x=x, a=a, b=b, c=c):
        '''
         S = spline(xvals)

         Evaluates the cubic spline at xvals.

         Inputs:
          xvals can be list-like, or a scalar (**must be in ascending order**)

         Output:
          S is a list of values with the same number of elements as x
        '''
        # Turn non-list-like input into list-like
        if type(xvals) not in (list, np.ndarray,):
            xvals = [xvals]

        S = []  # The return list of values

        #
        k = 0   # this is the current polynomial piece
        hk = x[k+1] - x[k]



        for xx in xvals:

            # If the next x-value is not on the current piece...
            if xx>x[k+1]:
                # ... Go to next piece
                k += 1
                hk = x[k+1] - x[k]

            S_of_x = a[k]*(x[k+1]-xx)**3/(6*hk) + a[k+1]*(xx-x[k])**3/(6*hk) +
  ↪b[k]*(x[k+1]-xx) + c[k]*(xx-x[k])

            S.append(S_of_x)
```

```python
        return S
    #======================================

    return spline


def ParametricSpline(Sx,Sy):
    '''
     x_cs, y_cs, t = ParametricSpline(Sx,Sy)

      Takes an array of x- and y-values, and returns a parametric
      cubic spline in the form of two piecewise-cubic data structures
      (one for the x-component and one for the y-component), as well as
      the corresponding parameter values.

      The splines use natural boundary conditions.

      Input:
       Sx   array of x-values
       Sy   array of y-values

      Output:
       x_cs function that evaluates the cubic spline for x-component
       y_cs function that evaluates the cubic spline for y-component
       t is the array of parameter values use for the splines

      Note that x_cs(t) and y_cs(t) give Sx and Sy, respectively.
    '''

    # These lines are just placeholders... replace them
    t = []
    for i in range(0,len(Sx)):
        t.append(i);
    x_cs = MySpline(t, Sx)
    y_cs = MySpline(t, Sy)

    # === YOUR CODE HERE ===

    return x_cs, y_cs, t
```

1.4 (d) Find parametric splines for each segment

[5]:
```python
# === YOUR CODE HERE ===
Rcalc = ParametricSpline(R_x, R_y)
Ocalc = ParametricSpline(O_x, O_y)
Bcalc = ParametricSpline(B_x, B_y)
dashcalc = ParametricSpline(dash_x, dash_y)
```

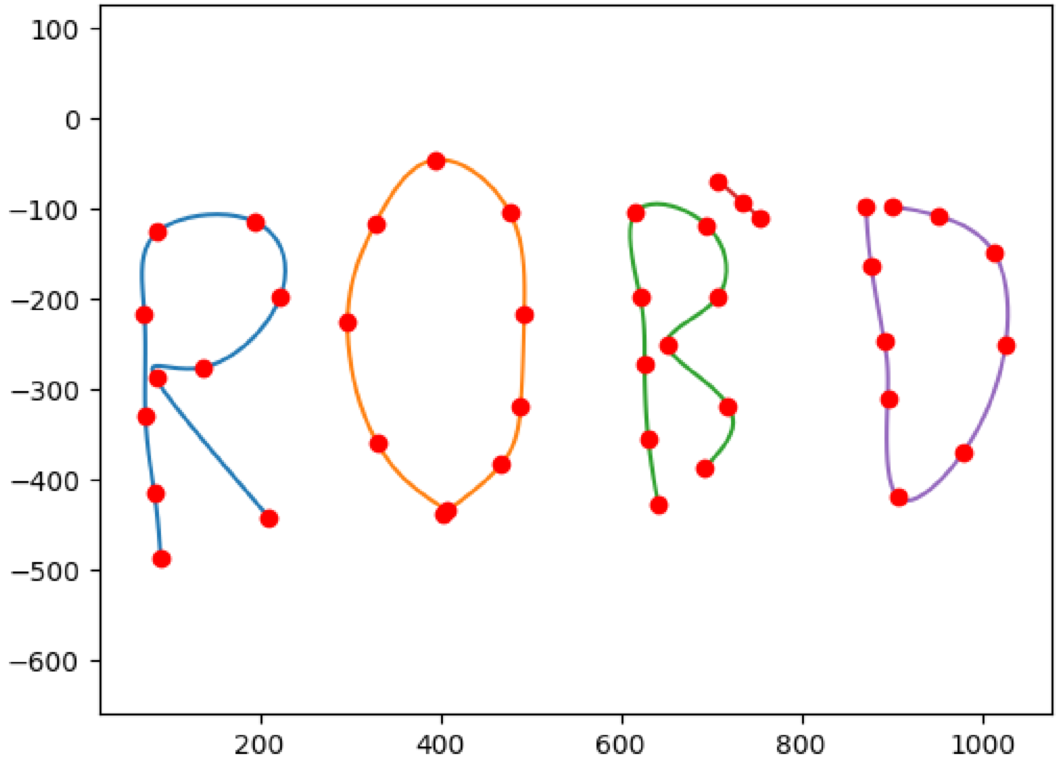
```
Dcalc = ParametricSpline(D_x, D_y)
```

1.5 (e) Plot the segments

[16]:
```
# === YOUR CODE HERE ===
pointsForLetters = np.linspace(Rcalc[2][0], Rcalc[2][-1], 100)
pointsForDash = np.linspace(dashcalc[2][0], dashcalc[2][-1], 100)
plt.axis('equal')
plt.plot(Rcalc[0](pointsForLetters), list(map(lambda val: -val,␣
 ↪Rcalc[1](pointsForLetters)))) # This is for flipping it so it displays␣
 ↪correctly!
plt.plot(Ocalc[0](pointsForLetters), list(map(lambda val: -val,␣
 ↪Ocalc[1](pointsForLetters))))
plt.plot(Bcalc[0](pointsForLetters), list(map(lambda val: -val,␣
 ↪Bcalc[1](pointsForLetters))))
plt.plot(dashcalc[0](pointsForDash), list(map(lambda val: -val,␣
 ↪dashcalc[1](pointsForDash))))
plt.plot(Dcalc[0](pointsForLetters), list(map(lambda val: -val,␣
 ↪Dcalc[1](pointsForLetters))))

plt.plot(R_x, list(map(lambda val: -val, R_y)), 'ro')
plt.plot(O_x, list(map(lambda val: -val, O_y)), 'ro')
plt.plot(B_x, list(map(lambda val: -val, B_y)), 'ro')
plt.plot(dash_x, list(map(lambda val: -val, dash_y)), 'ro')
plt.plot(D_x, list(map(lambda val: -val, D_y)), 'ro')
#plt.plot(t,y,'ro');
```

[16]:
```
[<matplotlib.lines.Line2D at 0x19ea6b919c0>]
```



100
0
−100
−200
−300
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600
800
1000

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