r2knowle_a2q1

February 3, 2023

0.1 A2-Q1: MySpline

[5]:
```python
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

0.2 MySpline

[65]:
```python
def MySpline(x, y):
    '''
     S = MySpline(x, y)

     Input:
       x and y are arrays (or lists) of corresponding x- and y-values,
       specifying the points in the x-y plane.  The x-values
       must be in increasing order.

     Output:
       S is a function that takes x or an array (or list) of x-values
         It evaluates the cubic spline and returns the interpolated value.

     Implementation:

       Hence...
         a[0] = a_0          b[0] = b_1        c[0] = c_1
         a[1] = a_1          b[1] = b_2        c[1] = c_2
              :                   :                 :
         a[n-2] = a_(n-2)    b[n-2] = b_(n-1)  c[n-2] = c_(n-1)
         a[n-1] = a_(n-1)

       The polynomial piece is evaluated at xx using

         p_i(xx) = a[i]*(x[i+1]-xx)**3/(6*hi) + a[i+1]*(xx-x[i])**3/(6*hi) +
                   b[i]*(x[i+1]-xx) + c[i]*(xx-x[i])

       where hk = x[k+1] - x[k] for k = 0, ... , n-2
    '''
```

```python
n = len(x)
h = np.zeros(n-1)
b = np.zeros(n-1)
c = np.zeros(n-1)
a = np.zeros(n)

M = np.zeros((n,n))
r = np.zeros(n)



# === YOUR CODE HERE ===

# Determine h first:

for i in range(0, n-1):
    h[i] = x[i+1] - x[i]


# Now we need to determine a, starting with the first matrix

M[0][0] = h[0]/3
M[0][1] = h[0]/6

for i in range(1, n-1):
    M[i][i-1] = h[i-1]/6
    M[i][i] = (h[i-1] + h[i])/3
    M[i][i+1] = h[i]/6

M[n-1][n-2] = h[0]/6
M[n-1][n-1] = h[0]/3

# Now we need to determine the second matrix:
for i in range (1, n-1):
    r[i] = (y[i+1] - y[i])/h[i] - (y[i] - y[i-1])/h[i-1]

# Lets now solve the array for the values of a
a = np.linalg.solve(M, r)
a[0] = 0            # make sure ending points are zero for natural BCs
a[n-1] = 0          # make sure ending points are zero for natural BCs



# Now we can determine both b and c
for i in range(0, n-1):
    b[i] = y[i]/h[i] - a[i]*h[i]/6
```

```
    for i in range(0, n-1):
        c[i] = y[i+1]/h[i] - a[i+1]*h[i]/6



    #======================================
    #
    # This is the function that gets returned.
    # It evaluates the cubic spline at xvals.
    #
    def spline(xvals, x=x, a=a, b=b, c=c):
        '''
         S = spline(xvals)

         Evaluates the cubic spline at xvals.

         Inputs:
          xvals can be list-like, or a scalar (**must be in ascending order**)

         Output:
          S is a list of values with the same number of elements as x
        '''
        # Turn non-list-like input into list-like
        if type(xvals) not in (list, np.ndarray,):
            xvals = [xvals]

        S = []  # The return list of values

        #
        k = 0   # this is the current polynomial piece
        hk = x[k+1] - x[k]


        for xx in xvals:

            # If the next x-value is not on the current piece...
            if xx>x[k+1]:
                # ... Go to next piece
                k += 1
                hk = x[k+1] - x[k]

            S_of_x = a[k]*(x[k+1]-xx)**3/(6*hk) + a[k+1]*(xx-x[k])**3/(6*hk) + b[k]*(x[k+1]-xx) + c[k]*(xx-x[k])

            S.append(S_of_x)

        return S
```

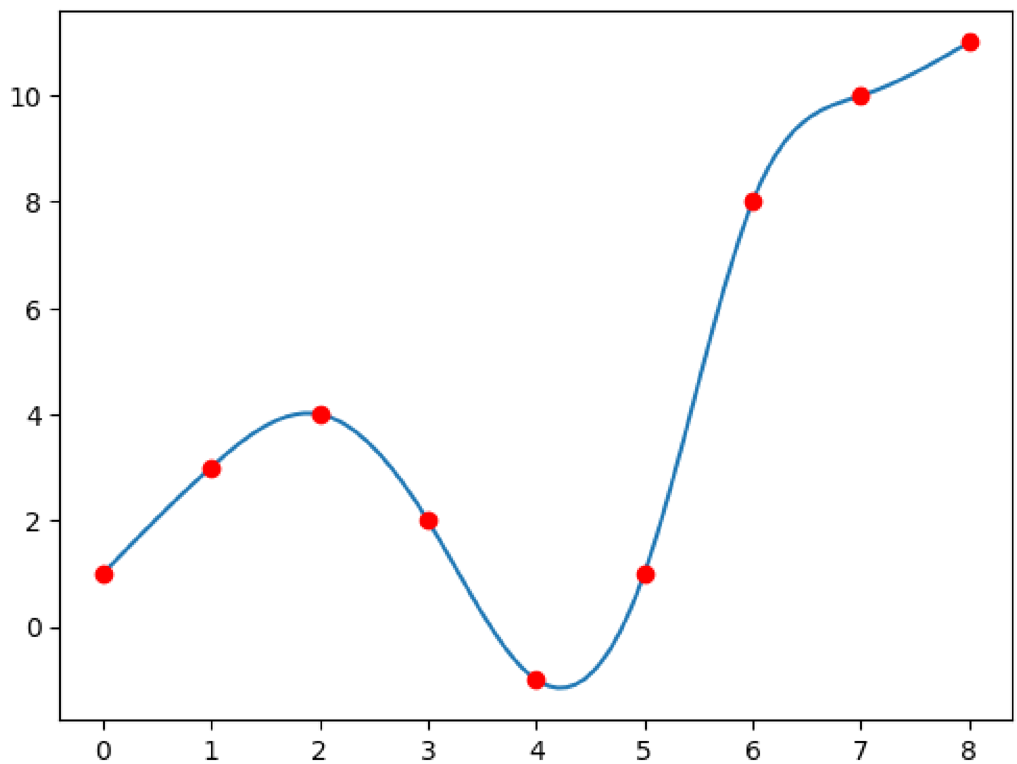
```
    #======================================

    return spline
```

0.3 Test MySpline

[69]:
```
# Simple data points to interpolate
y = [1, 3, 4, 2, -1, 1, 8, 10, 11]
t = [0, 1, 2, 3,  4, 5, 6, 7, 8]
```

[70]:
```
# Call the function
sp = MySpline(t,y)
```

[71]:
```
# Plot the spline and the interpolation points
xx = np.linspace(t[0], t[-1], 100)
plt.plot(xx, sp(xx))
plt.plot(t,y,'ro');
```



[ ]:
```
```

[ ]:
```
```