CS370, Winter 2023
Assignment 2: Question 2

(a) Finding the conditionals for a,b,c,e,f,g

We are told that our piecewise function must pass through (1,2), (2,1), (3,1). To begin we will solve for a,e b by plugging points into certain functions. Lets use (1,2) and plug it into S(x):

$$\begin{aligned}
\mathrm{S}(1) &= a + b(x-1) + c(x-1)^2 + \frac{1}{4}(x-1)^2(x-2) \\
2 &= a + b((1)-1) + c((1)-1)^2 + \frac{1}{4}((1)-1)^2((1)-2)) \\
2 &= a \\
a &= 2
\end{aligned}$$

Plugging (2,1) into a equation for S(x) we then get:

$$\begin{aligned}
\mathrm{S}(2) &= e + f(x-2) + g(x-2)^2 - \frac{1}{4}(x-2)^2(x-3) \\
1 &= e + f((2)-2) + g((2)-2)^2 - \frac{1}{4}((2)-2)^2((2)-3) \\
1 &= e \\
e &= 1
\end{aligned}$$

If we plug (2,1) into the other equation for S(x) it tells us the conditions for the other variables:

$$\begin{aligned}
\mathrm{S}(2) &= a + b(x-1) + c(x-1)^2 + \frac{1}{4}(x-1)^2(x-2) \\
1 &= a + b((2)-1) + c((2)-1)^2 + \frac{1}{4}((2)-1)^2((2)-2)) \\
1 &= a + b + c \\
b + c &= -1
\end{aligned}$$

If we plug (3,1) into the equation for S(x) we get:

$$\begin{aligned}
\mathrm{S}(3) &= e + f(x-2) + g(x-2)^2 - \frac{1}{4}(x-2)^2(x-3) \\
1 &= e + f((3)-2) + g((3)-2)^2 - \frac{1}{4}((3)-2)^2((3)-3) \\
1 &= e + f + g \\
f + g &= 0
\end{aligned}$$

Which gives us the conditions for b,c,f,g and the exact values for a and e.

(b) Find condition of coefficients such that S'(2) is 0

In order for equation to be continuous both equations for S'(2) must be equal. This implies that we have the following:

$$\begin{aligned}
\text{S'}(2) &= \text{S'}(2) \\
\frac{dS}{dx}(a + b(x-1) + c(x-1)^2 + \frac{1}{4}(x-1)^2(x-2)) &= \frac{dS}{dx}(e + f(x-2) + g(x-2)^2 - \frac{1}{4}(x-2)^2(x-3)) \\
b + 2c(x-1) + \frac{(x-1)(3x-5)}{4} &= f + 2g(x-2) - \frac{(x-2)(3x-8)}{4}) \\
b + 2c((2)-1) + \frac{((2)-1)(3(2)-5)}{4} &= f + 2g((2)-2) - \frac{((2)-2)(3(2)-8)}{4}) \\
b + 2c + \frac{1}{4} &= f \\
b + 2c - f &= -\frac{1}{4}
\end{aligned}$$

(c) Enforce the boundary conditions and solve for c,g

To begin with we are told that the second derivative of S(1) and S(3) is 0. To get an equation for c we will then rewrite this as:

$$\begin{aligned}
\text{S}(1) &= \frac{d^2S}{d^2x}(a + b(x-1) + c(x-1)^2 + \frac{1}{4}(x-1)^2(x-2)) \\
0 &= 2c + \frac{3x-4}{2} \\
0 &= 2c + \frac{3(1)-4}{2} \\
0 &= 2c - \frac{1}{2} \\
c &= \frac{1}{4}
\end{aligned}$$

$$\begin{aligned}
\text{S}(3) &= \frac{d^2S}{d^2x}(e + f(x-2) + g(x-2)^2 - \frac{1}{4}(x-2)^2(x-3)) \\
0 &= g - \frac{3x-7}{2} \\
0 &= g - \frac{3(3)-7}{2} \\
0 &= g - 1 \\
g &= \frac{1}{2}
\end{aligned}$$

Which proves the coefficients values as necessary.

(d) Compute the values of b and f

Now that we have the values for g and c from (c), we can plug it into the following equations:

$$\begin{aligned} b + c &= -1 \\ b + \frac{1}{4} &= -1 \\ b &= -\frac{5}{4} \end{aligned}$$

$$\begin{aligned} f + g &= 0 \\ f + \frac{1}{2} &= 0 \\ f &= -\frac{1}{2} \end{aligned}$$

Thus we have have solved for both f and b.

(e) What conditions are needed for a cubic spline

We need to check that S”(x) is continuous when x = 2.