CS370, Winter 2023
Assignment 5: Question 5

Q5) How to solve the system xA= b in $O(N^2)$ flops

In class we are told that to generate the LU factorization takes $O(N^3)$ flops and that back substitution and forward substitution takes $O(N^2)$ flops. To begin with we are given the LU factorization for A, so our equation can immediately become:

$$xLU = b$$

We can then define a new vector y and define it such that:

$$y = xL$$

For now we will leave y populated with variables $y_0, y_1 ... y_n$. We can substitute this back into our equation to get:

$$yU = b$$

Since y and b are both vector and U is a upper triangular matrix, we can use back substitution to solve for Y in O(N^2) flops are proved in class. Now that we have solved Y we can return to our previous equation:

$$y = xL$$

Since we know y now, and x is a vector and L is a lower triangular matrix. We can solve for x using forward substitution which will give us our solution in O(N^2). Thus our this algorithm will solve for x in O(N^2) flops