University of Waterloo
CS 480
r2knowle: 2023-11-28

Exercise # 2

Q2) To begin we are given the following inequality to try and minimize:

$$\min_{w \in \mathbb{R}} \max_{\forall j, ||z_j|| \leq \lambda} ||(X+Z)w - y||_2$$

We can expand this to get:

$$\min_{w \in \mathbb{R}} \max_{\forall j, ||z_j|| \leq \lambda} ||Xw - y + Zw||_2$$

Now by triangle inequality it follows that:

$$||Xw - y + Zw||_2 <= ||Xw - y||_2 + ||Zw||_2$$

Since we are first looking at the inner maximization problem, this gives us an upper bound on Z. Thus our equation becomes:

$$\min_{w \in \mathbb{R}} \left(||Xw - y||_2 + \max_{\forall j, ||z_j|| \leq \lambda} ||Zw||_2 \right)$$

By the definition of the L2 norm we get that this is equivalent to:

$$\min_{w \in \mathbb{R}} \left(||Xw - y||_2 + \max_{\forall j, ||z_j||_2 \leq \lambda} \sqrt{\sum_j^d z_j w_j} \right)$$

$$\min_{w \in \mathbb{R}} ||Xw - y||_2 + \max_{\forall j, ||z_j||_2 \leq \lambda} \sum_j^d ||z_j||_2 |w_j|$$

From our definition of math this becomes:

$$\min_{w \in \mathbb{R}} ||Xw - y||_2 + \sum_j^d \lambda |w_j|$$

We can expand the value out to get:

$$\min_{w \in \mathbb{R}} ||Xw - y||_2 + \lambda \sum_j^d |w_j|$$

2 From definition of L1 norm we can simplify to get:

$$\min_{w \in \mathbb{R}} ||Xw - y||_2 + \lambda ||w||_1$$

Thus showing how we can get (3) from (2) as required.