University of Waterloo
CS 486
r2knowle: 2023-11-11

Assignment # 3

Q1.1) To begin we are going to assume that eating soups on days 1, 2 and 3 are independent of each other. Let θ_x represent the true probability that we have soup on day x where $x \in (1, 2, 3)$. We will also let S_x and N_x represent the number of observations where we had and didnt have soup (respectively on day x). Thus in class we get the following derivation for each parameter:

$$\begin{aligned}
\theta_1 &= \frac{S_1 + 1}{N_1 + S_1 + 2} \\
&= \frac{3+1}{2+3+2} \\
&= \frac{4}{7} \\
\theta_2 &= \frac{S_2 + 1}{N_2 + S_2 + 2} \\
&= \frac{2+1}{3+2+2} \\
&= \frac{3}{7} \\
\theta_3 &= \frac{S_3 + 1}{N_3 + S_3 + 2} \\
&= \frac{3+1}{2+3+2} \\
&= \frac{4}{7}
\end{aligned}$$

The probability we enjoy of meal d is given by:

$$P(d|h_{\theta_1,\theta_2,\theta_3}) = \theta_1^{S_1}(1-\theta_1)^{N_1}\theta_2^{S_2}(1-\theta_2)^{N_2}\theta_3^{S_3}(1-\theta_3)^{N_3}$$

Q1.2) To begin we are going to assume that eating soups on days 1, 2 and 3 are independent of each other. Let θ_x represent the true probability that we have soup on day x where $x \in (1, 2, 3)$. We will also let S_x and N_x represent the number of observations where we had and didnt have soup (respectively on day x). Therefore the probability we enjoy the meal d is given by:

$$P(d|h_{\theta_1,\theta_2,\theta_3}) = \theta_1^{S_1}(1-\theta_1)^{N_1}\theta_2^{S_2}(1-\theta_2)^{N_2}\theta_3^{S_3}(1-\theta_3)^{N_3}$$

Taking the log likelihood of this we get:

$$\begin{aligned}
&= \log\left(\theta_1^{S_1}(1-\theta_1)^{N_1}\theta_2^{S_2}(1-\theta_2)^{N_2}\theta_3^{S_3}(1-\theta_3)^{N_3}\right) \\
&= S_1\log(\theta_1) + N_1\log(1-\theta_1) + S_2\log(\theta_2) + N_2\log(1-\theta_2) + S_3\log(\theta_3) + N_3\log(1-\theta_3)
\end{aligned}$$

We will take the derivative of each of the paramters $(\theta_1, \theta_2, \theta_3)$ to find the values for which they are maximized. At the end we will apply Lapache smoothing as taught in class. Starting with θ_1 we get:

$$0 = \frac{S_1}{\theta_1} + \frac{N_1}{1-\theta_1}$$
$$0 = \frac{S_1(1-\theta)}{\theta_1(1-\theta_1)} + \frac{N_1\theta_1}{\theta_1(1-\theta_1)}$$
$$0 = \frac{S_1 - S_1\theta_1 + N_1\theta}{\theta_1(1-\theta_1)}$$
$$0 = S_1 - S_1\theta_1 + N_1\theta_1$$
$$-S_1 = (-S_1 + N)\theta_1$$
$$\theta_1 = \frac{-S_1}{(-S_1 + N)}$$

Q1.2) If we only look at day 1 we get the following observations:

	Had soup on day 1	Didn't have soup on day 1
Liked the meal	2	0
Didn't like the meal	1	2

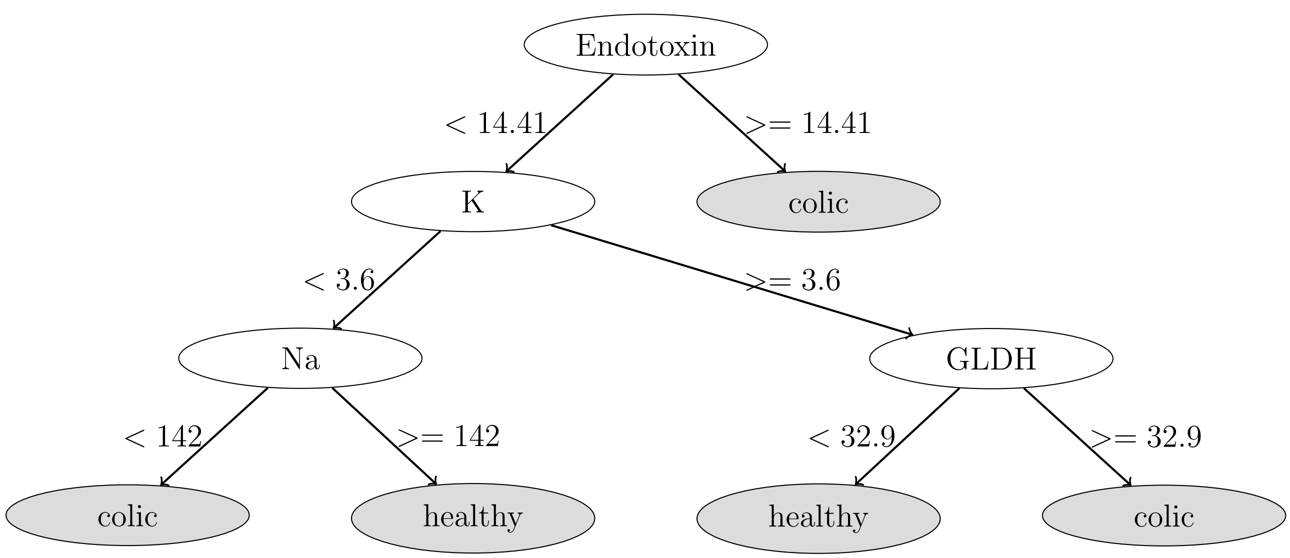
Q4.1) Below is the code used to produce the decision tree, as well as get the number of values predicted correctly vs incorrectly:

```
import numpy as np

# ============= Loss Implementation =============
def entropyLoss(vals_y):

    count = 0.0
    for val in vals_y:
        if val == "healthy.\n":
            count += 1.0
    p = 0
    if count != 0:
        p = count / float(len(vals_y))

    if p == 1 or p == 0:
        return 0

    return -p * np.log2(p) - (1 - p) * np.log2(1 - p)

# ============= Decision Tree Implementation =============

class DecisionTree:
    threshold = 0
    thresholdFeature = 0
    values = []
    leftTree = -1
    rightTree = -1
    classification = 0
    numLeft = 0
    numRight = 0
    counter = 0

    # This method determines if all values are homogenous
    def allTheSame(self, vals):
        if len(vals) == 0:
            return True
        originalValue = vals[0][-1]
        for val in vals:
            if (val[-1] != originalValue):
                return False
        return True


    # Given a dataset, this function will determine where the split is to minimize entropy loss
        (maximize info gain)
    def findBestThreshold(self, vals, loss):
        bestLoss = 9999999
        bestFeature = 0
        bestThreshold = 0

        width = len(vals[0]) - 1
        for test_entry in vals:
            for idx in range(0, width):
                left = []
                right = []

                for entry in vals:
```

```python
                    # Note that picking a threshold that is <= two consecutive elements is the same as >= the top element.
                    #   as no elements by definition can be between the threshold and the proposed threshold
                    if (float(entry[idx]) >= float(test_entry[idx])):
                        right.append(entry[-1])
                    else:
                        left.append(entry[-1])

                leftWeight = len(left) / (len(left) + len(right))
                rightWeight = len(right) / (len(left) + len(right))

                totalLoss = loss(left) * leftWeight + loss(right) * rightWeight
                if totalLoss < bestLoss:
                    bestLoss = totalLoss
                    bestFeature = idx
                    bestThreshold = test_entry[idx]

        return [bestFeature, bestThreshold]

    def __init__(self, vals, loss):
        if self.allTheSame(vals):
            self.values = vals
        else:
            x = self.findBestThreshold(vals, loss)
            self.thresholdFeature = x[0]
            self.threshold = x[1]

            leftTreeVals = []
            rightTreeVals = []
            for val in vals:
                if float(val[self.thresholdFeature]) >= float(self.threshold):
                    rightTreeVals.append(val)
                else:
                    leftTreeVals.append(val)

            self.leftTree = DecisionTree(leftTreeVals, loss)
            self.rightTree = DecisionTree(rightTreeVals, loss)

    def classify(self, item):
        if self.leftTree == -1:
            if item[-1] == self.values[0][-1]:
                return 1
            print(item[-1])
            return 0
        else:
            if float(item[self.thresholdFeature]) >= float(self.threshold):
                return self.rightTree.classify(item)
            return self.leftTree.classify(item)

    def printTree(self, indx, cols):
        if len(self.values) == 0:
            print("LV ", indx, ": ", cols[self.thresholdFeature], " ", self.threshold)
        else:
            print("LV", indx, ": ", self.values[0][-1].split("\n")[0])
        if self.leftTree != -1:
            self.leftTree.printTree(indx+1, cols)
        if self.rightTree != -1:
            self.rightTree.printTree(indx+1, cols)
```

```
    def predict(self, items):
        count = 0
        for val in items:
            count += self.classify(val)
        return count


# ============= Testing of Values =============

trainingFile = open("horseTrain.txt", "r")
lines = trainingFile.readlines()
training = []
for line in lines:
    training.append(line.split(","))

columns = ["K", "Na", "CL", "HCO3", "Endotoxin", "Aniongap", "PLA2", "SDH", "GLDH", "TPP", "Breath
    rate",
          "PCV", "Pulse rate", "Fibrinogen", "Dimer", "FibPerDim"]

dt = DecisionTree(training, entropyLoss)
dt.printTree(1, columns)
print("Correct Predictions:", dt.predict(training), "out of", len(training))

testFile = open("horseTest.txt", "r")
lines = testFile.readlines()
testing = []
for line in lines:
    testing.append(line.split(","))
print("Correct Predictions:", dt.predict(testing), "out of", len(testing))
```

Q4.2) This gives us the following decision tree:



Q4.3) For the training set we get that we correctly classifiy:

132 out of 132 training data points

Q4.4) For the test set we get that we correctly classify:

13 our of the 13 testing data points