

**Question 3** [8 marks]

**a)** let  $x_1, x_2, x_3 \in \mathbb{C}^n$ , thus by Lemma 1(i) we know that:

$$\langle x_1, x_2 + x_3 \rangle = \overline{\langle x_2 + x_3, x_1 \rangle}$$

By Lemme 1(ii) this will thus become:

$$\overline{\langle x_2 + x_3, x_1 \rangle} = \overline{\langle x_2, x_1 \rangle} + \overline{\langle x_3, x_1 \rangle}$$

By using Lemma 1(i) again (reversing the original conjugate) this equation will thus become:

$$\begin{aligned} \langle \overline{x_2}, \overline{x_1} \rangle + \langle \overline{x_3}, \overline{x_1} \rangle &= \langle x_1, x_2 \rangle + \langle x_3, x_1 \rangle \\ &= \langle x_1, x_2 \rangle + \langle x_1, x_3 \rangle \end{aligned}$$

Thus proving the original equation, using only the first Lemma(i-ii).

**b)** let  $x_1, x_2, c \in \mathbb{C}^n$ , thus by Lemma 1(i) we know that:

$$\langle x_1, cx_2 \rangle = \overline{\langle cx_2, x_1 \rangle}$$

By Lemme 1(iii) this will thus become:

$$\overline{\langle cx_2, x_1 \rangle} = \bar{c} \overline{\langle x_2, x_1 \rangle}$$

By using Lemma 1(i) again (reversing the original conjugate) this equation will thus become:

$$\bar{c} \overline{\langle x_2, x_1 \rangle} = \bar{c} \langle x_1, x_2 \rangle$$

Thus proving the original equation, using only the first Lemma(i and iii).

**c)** let  $x_1, x_2, x_3, c \in \mathbb{C}^n$

$$\langle x_1, c(x_2 + x_3) \rangle = \bar{c} \langle x_1, x_2 \rangle + \langle x_1, x_3 \rangle$$