Question 3 [8 marks]

a) let $x_1, x_2, x_3 \in \mathbb{C}^n$, thus by Lemma 1(i) we know that:

$$\langle x_1, x_2 + x_3 \rangle = \langle \overline{x_2 + x_3, +x_1} \rangle$$

By Lemme 1(ii) this will thus become:

$$\langle \overline{x_2 + x_3, +x_1} \rangle = \langle \overline{x_2, x_1} \rangle + \langle \overline{x_3, x_1} \rangle$$

By using Lemma 1(i) again (reversing the original conjugate) this equation will thus become:

$$\begin{aligned} \langle \overline{x_2, x_1} \rangle + \langle \overline{x_3, x_1} \rangle &= \langle x_1, x_2 \rangle + \langle \overline{x_3, x_1} \rangle \\ &= \langle x_1, x_2 \rangle + \langle x_1, x_3 \rangle \end{aligned}$$

Thus proving the original equation, using only the first Lemma(i-ii).

b) let $x_1, x_2, c \in \mathbb{C}^n$, thus by Lemma 1(i) we know that:

$$\langle x_1, cx_2 \rangle = \langle \overline{cx_2, x_1} \rangle$$

By Lemme 1(iii) this will thus become:

$$\langle \overline{cx_2, x_1} \rangle = \overline{c} \langle \overline{x_2, x_1} \rangle$$

By using Lemma 1(i) again (reversing the original conjugate) this equation will thus become:

$$\overline{c} \langle \overline{x_2, x_1} \rangle = \overline{c} x_1, x_2 \rangle$$

Thus proving the original equation, using only the first Lemma(i and iii).

c) let $x_1, x_2, x_3 c \in \mathbb{C}^{n}$”

$$\langle x_1, c(x_2 + x_3) \rangle = \overline{c} \langle x_1, x_2 \rangle + \overline{c} \langle x_1, x_3 \rangle$$