Question 1 [15 marks]

a) We will start by assuming that $u \bullet v = 0$, it thus follows that:

$$\begin{aligned}(u \bullet v) &= 0\\(u \bullet v) + (u \bullet v) &= 0 + 0\\(u \bullet v) + (v \bullet u) &= 0\end{aligned}$$

We will now add $(u \bullet u)$ and $(v \bullet v)$ to both sides:

$$\begin{aligned}(u \bullet u) + (u \bullet v) + (v \bullet u) + (v \bullet v) &= (u \bullet u) + 0 + (v \bullet v)\\(u \bullet u) + (u \bullet v) + (v \bullet u) + (v \bullet v) &= (u \bullet u) + (v \bullet v)\end{aligned}$$

We know from the propities of the dot product that $u \bullet u = ||u||^2$, so our equation becomes:

$$\begin{aligned}(u \bullet u) + (u \bullet v) + (v \bullet u) + (v \bullet v) &= ||u||^2 + (v \bullet v)\\(u \bullet u) + (u \bullet v) + (v \bullet u) + (v \bullet v) &= ||u||^2 + ||v||^2\end{aligned}$$

We know from the propities of the dot product that $(u \bullet v) + (u \bullet u) = u \bullet (u \bullet v)$, so we will distrabute out u and then v to get:

$$\begin{aligned}u \bullet (u \bullet v) + (v \bullet u) + (v \bullet v) &= ||u||^2 + ||v||^2\\u \bullet (u \bullet v) + v \bullet (u \bullet v) &= ||u||^2 + ||v||^2\end{aligned}$$

As well since $(u + v) \bullet (u + v) = u \bullet (u \bullet v) + v \bullet (u \bullet v)$, it implies that our equation will become:

$$(u + v) \bullet (u + v) = ||u||^2 + ||v||^2 ||u + v||^2 \qquad = ||u||^2 + ||v||^2$$