Q5. Let m be an integer such that $m > 1$. Since $m > 1$, Unique Factorization Theorem (UFT) tells us that we can express m as a product of prime factors uniquely such that (for some $z \in \mathbb{Z}$, $z \geq 1$):

$$m = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_z^{\alpha_z}$$

where p_i (for $1 \leq i \leq z$) represent the prime divisors of m, and where the positive integer α_i could be zero. This can be re-written in product notation such that:

$$m = \prod_{i=1}^{z} (p_i^{\alpha_i})$$

We know that α_i is a positive integer, thus the Division Algorithm (DA) tells us that the integers g_i and r_i exist such that:

$$\alpha_i = 5g_i + r_i \ (0 \leq r < 5)$$

Thus we can rewrite n as:

$$m = \prod_{i=1}^{z} (p_i^{5g_i + r_i})$$

$$m = \prod_{i=1}^{z} (p_i^{5g_i} p_i^{r})$$

$$m = \prod_{i=1}^{z} p_i^{5g_i} \cdot \prod_{i=1}^{z} p_i^{r}$$

$$m = (\prod_{i=1}^{z} p_i^{g_i})^5 \cdot \prod_{i=1}^{z} p_i^{r}$$

Using the reverse of Unique Factorization Theorem (UFT) we know we can define a positive integer s to be:

$$s = \prod_{i=1}^{z} p_i^{r} \ (0 \leq r < 5)$$

Therefore we know that r is always less then 5, so this means that s contains no fifth powers so s is five-free as it cant be divisible by any fifth power.

Plugging a positive five-free integer s back into our equation we get that:

$$m = (\prod_{i=1}^{z} p_i^{g_i})^5 \cdot s$$

Using the reverse of Unique Factorization Theorem (UFT) we know we can define a positive integer t to be:

$$t = \prod_{i=1}^{z} p_i^{q_i}$$

Thus plugging it back into our original equation we get:

$$m = (t)^5 \cdot \prod_{i=1}^{z} p_i^r$$

where s and t are positive integers and s is five-free.