Q1 a) Let n be an arbitrary integer and assume n is odd, therefore let [1] $n = 2k+1$ for some integer k.

$n^3 - n + 7$

$\Rightarrow (2k+1)^3 - (2k+1) + 7$ ← from our assumption

$\Rightarrow (2k+1)(4k^2+4k+1) - (2k+1) + 7$

$\Rightarrow (8k^3 + 12k^2 + 6k + 1) - 2k - 1 + 7$

$\Rightarrow 8k^3 + 12k^2 + 4k + 7$ ← let a be $4k^3$ (an integer)

$\Rightarrow 2a + 12k^2 + 4k + 7$ ← Let b be $6k^2$ (an integer)

$\Rightarrow 2a + 2b + 4k + 7$

$\Rightarrow 2\underbrace{(a+b+2k+3)}_{\text{integer}} + 1 = 2y + 1$ (for some integer y)

[1] an odd number is equal to 2 times some integer plus one, since our simplification is $2y+1$ for some integer y, we can say that $n^3 - n + 7$ will be odd for any n. ∴ proved

b) Let n be an arbitrary integer and assume n is even, therefore $n = 2k$ for some integer k

$n^3 - n + 7$

$\Rightarrow (2k)^3 - 2k + 7$ ← from our definition of an even number

$\Rightarrow 8k^3 - 2k + 7$ ← Let a be $4k^3$ (an integer)

$\Rightarrow 2a - 2k + 7$

$\Rightarrow 2(a - k + 3) + 1$ ← let b be $a-k+3$ (an integer)

$\Rightarrow 2b + 1$

∴ Since b is an integer our result will always be odd we have disproved the statement