

Q4) Assume that x is $m^2 + 2$ where m and x are integers, we will split m into even and odd integers.

Case 1: m is odd $(2k+1) = m$ for some integer k

$$x = m^2 + 2$$

$$\Rightarrow x = (2k+1)^2 + 2 \quad \leftarrow \text{from the definition of odd}$$

$$\Rightarrow x = 4k^2 + 2k + 1 + 2$$

$$\Rightarrow x = 2(2k^2 + k) + 3 \quad \leftarrow \text{let } a = 2k^2 + k \text{ (an integer)}$$

$$\Rightarrow x = 2(2k^2 + k) + 3 \quad \leftarrow \text{let } x = k^2 \text{ (an integer) and } 4 = k \text{ an integer}$$

$$\Rightarrow x = 2(2n) + 3 \quad \leftarrow \text{let } 2n = 2k^2 + k \text{ (an integer)}$$

$$\Rightarrow x = 4n + 3$$

\therefore if m is odd then $x = 4n + 3$ for some integer n

Case 2: m is even $(2k) = m$ for some integer k

$$x = m^2 + 2$$

$$\Rightarrow x = (2k)^2 + 2$$

$$\Rightarrow x = 4k^2 + 2$$

$$\Rightarrow x = 4a + 2 \quad \leftarrow \text{Since } k \text{ is an integer if } a = k^2 \text{ then } a \text{ will also be an integer}$$

\therefore if m is even then $x = 4a + 2$ for some integer a

\therefore for any x that can be expressed as $m^2 + 2$ for some integer m , it can also be expressed as $4k + 3$ or $4k + 2$ for some integer k