

We will prove the the binomial equation is divisible by $n + 1$, by showing that for any $n \geq 1$ that $n + 1$ is a factor of the binomial equation

$$\begin{aligned}\binom{2n}{n} &= \frac{2n!}{n!(2n-n)!} \\ &= \frac{2n!}{n! * n!}\end{aligned}$$

We will divide the numerator by $n!$, notice that this will produce a series of multiplies from $(n + 1)$ to $(2n)$ on the numerator and we are left with a series of multiples from 1 to n on the denominator:

$$\begin{aligned}\frac{2n!}{n! * n!} &= \frac{(2n) * 2n - 1 * (2n - 2) * \dots * (n + 2) * (n + 1)}{n!} \\ &= \prod_{k=1}^n \left(\frac{n + k}{k}\right) \\ &= \prod_{k=1}^n \left(\frac{n}{k} + \frac{k}{k}\right) \\ &= \prod_{k=1}^n \left(\frac{n}{k} + 1\right)\end{aligned}$$

We will now factor out the first term (when $k = 1$) to show that if $n \geq 2$, the binomial expression will equal some real number times $(n + 1)$

$$\begin{aligned}\prod_{k=1}^n \left(\frac{n}{k} + 1\right) &= \left(\frac{n}{1} + 1\right) \prod_{k=2}^n \left(\frac{n}{k} + 1\right) \\ &= (n + 1) \prod_{k=2}^n \left(\frac{n}{k} + 1\right)\end{aligned}$$

However if we have $n = 1$, we will get $(n + 1)$ multiplied by 1

$$(n + 1) \prod_{k=2}^1 \left(\frac{n}{k} + 1 \right) = (n + 1) * 1$$

Therefore as long as $n \geq 1$, the binomial expression will be some real number times $n + 1$. Since $n + 1$ is thus a factor of the binomial equation, it will always be divisible by $n + 1$.