Q04. We will prove the the binomial equation is divisible by $n+1$, by showing that for any $n \geq 1$ that $n+1$ is a factor of the binomial equation

$$\binom{2n}{n} = \frac{(2n)!}{n!(2n-n)!}$$
$$= \frac{2n!}{n! * n!}$$

We will divide the numerator by $n!$, notice that this will produce a series of multiplies from $(n+1)$ to $(2n)$ on the numerator and we are left with a series of multiples from 1 to n on the denominator:

$$\frac{2n!}{n! * n!} = \frac{(2n) * 2n - 1 * (2n-2) * ... * (n+2) * (n+1)}{n!}$$
$$= \prod_{k=1}^{n} (\frac{n+k}{k})$$
$$= \prod_{k=1}^{n} (\frac{n}{k} + \frac{k}{k})$$
$$= \prod_{k=1}^{n} (\frac{n}{k} + 1)$$

We will now factor out the first term (when $k = 1$) to show that if $n \geq 2$, the binomial expression will equal some integer times $(n+1)$ (we know this because all binomial coefficients are integers):

$$\prod_{k=1}^{n} (\frac{n}{k} + 1) = (\frac{n}{1} + 1) \prod_{k=2}^{n} (\frac{n}{k} + 1)$$
$$= (n+1) \prod_{k=2}^{n} (\frac{n}{k} + 1)$$

However if we have $n = 1$, we will get $(n+1)$ multiplied by 1

$$(n+1) \prod_{k=2}^{1} (\frac{n}{k} + 1) = (n+1) * 1$$

Therefore as long as $n \geq 1$, the binomial expression will be some integer times $n+1$. Since $n+1$ is thus a factor of the binomial equation, it will always be divisible by $n+1$.