MATH 135 Fall 2020: Written Assignment 10 (WA10)
Due at 11:55 PM EST on Friday December 4th, 2020
Covers the contents of Lessons 9.1, 9.2, 9.3, 10.1, 10.2 and 10.3

Q01. (5 marks) Alice's RSA public key is given by $(e, n) = (47, 1147)$.

(a) (2 marks) Determine Alice's private key (d, n). Show your work.

(b) (1 mark) Bob sends his first message $M_1 = 2$ to Alice, encrypting it with RSA using Alice's public key. He obtains a ciphertext C_1 that gets forwarded to Alice. What is C_1?

(c) (2 marks) Bob sends his second message M_2 to Alice, encrypting it with RSA using Alice's public key. Alice received the ciphertext $C_2 = 3$ that originated from M_2. What is M_2?

Q02. (5 marks)

(a) (2 marks) Find, with proof, all complex numbers w and z such that $w = 1 + z$ and $|w| = |z| = 1$.

(b) (3 marks) Prove that, for all complex numbers z and w, $|z+iw| = |z-iw|$ if and only if $z\overline{w} \in \mathbb{R}$.

Q03. (5 marks) Shade the following region of the complex plane: $\{i + \overline{z} \colon 1 < |z| < 4, z \in \mathbb{C}\}$. Make sure to justify your answer.

Q04. (5 marks) For a real number x, let $\lfloor x \rfloor$ denote the largest integer that does not exceed x.

(a) (3 marks) Prove that, for every non-negative integer n,

$$(1+i)^n + (1-i)^n = 2\sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n}{2k}(-1)^k.$$

(b) (2 marks) Prove that, for every non-negative integer n, $(1+i)^n + (1-i)^n = 0$ if and only if $n \equiv 2 \pmod 4$.

Hint: Note that $(1+i)^4 = (1-i)^4 = -4$.

Note: Although these questions look similar, they are unrelated.

Q05. (5 marks) Let $z_0 = i$, $z_1 = 1 + i$, and for $n \geq 2$ define $z_n = z_{n-1}z_{n-2}$. Prove that, for every non-negative integer n, $\mathrm{Re}(z_n)\,\mathrm{Im}(z_n) = 0$ if and only if $3 \mid n$.