a) We will begin by proving that the recursive sequence is monotonic, or more specifically that is entirely non decreasing:

$(a_1 = 1) \qquad a_{n+1} \geq a_n \qquad (n \in \mathbb{N})$

We will prove this via induction, starting with the base case:

$\boxed{1}$ Base case: $a_{(1)+1} \geq a_1 \qquad (a_1 = 1$ from definition$)$

$a_2 \geq a_1 \qquad (a_2 = 4 - \frac{2}{a_n} = 4 - \frac{2}{1} = 2)$

$2 \geq 1$

$\boxed{2}$ Inductive hypothesis: Assume $a_{k+1} \geq a_k$ will be true for any $k = n$

$\boxed{3}$ Inductive Step: we will now show that for any $k+1$ this will be true:

$a_{(k+1)+1} \geq a_{(k+1)}$ ← plug $k+1$ into the recursive sequence

$4 - \frac{2}{a_{k+1}} \geq 4 - \frac{2}{a_k}$ ← from $a_n = 4 - \frac{2}{a_{n-1}}$

$-\frac{2}{a_{k+1}} \geq -\frac{2}{a_k}$ ← Subtract 4 from both sides

$\frac{1}{a_{k+1}} \leq \frac{1}{a_k}$ ← divide by $-2$

$a_k \leq a_{k+1}$ ← multiply by $a_k$ and $a_{k+1}$

∴ Since $a_{k+2} \geq a_{k+1}$ results in our inductive hypothesis $\boxed{2}$, we have proved that the $k+1$ case is true. Since the base case is also true, we have proved that for every $n$ $(n \in \mathbb{N})$ $a_{n+1} \geq a_n$ or that the series $\{a_n\}$ is monotonic non decreasing.

* Conditions for MCT met as seen above

Using the monotone convergence theory we will now show that the series converges, we will pick an arbitrary upper bound of 4, such that for any $n \in \mathbb{N}$:

$(a_1 = 1) \qquad a_n < 4$

We will prove this via induction starting with the base case

$\boxed{1}$ Base case: $a_1 < 4$

$1 < 4$ ← from definition of the base case

$\boxed{2}$ Inductive hypothesis: for any $k = n$, we will assume $a_k < 4$ will be true

Hilroy