2) We will prove via counter example:

take the function $f(x) = \frac{x}{2} + \frac{1}{2}$ (shown on the first graph).

It satisfys the following conditions:

1) $f(3) = 2$ and $f(7) = 4$

Proof: $f(3) = \frac{3}{2} + \frac{1}{2} = 2$

$f(7) = \frac{7}{2} + \frac{1}{2} = 4$

2) $f(x)$ achieves each value once

Proof: Since $f(x)$ is a positive linear function, if $a > b$ & $a, b \in \mathbb{R}$ then $f(a) > f(b)$, if $a < b$ then $f(a) < f(b)$. So for any different a, b $f(a) \neq f(b)$!

$\therefore$ we have shown that a continuous $f(x)$ exsists on $[3, 7]$ that satisfys the condition.