3 Inductive Step: we will now prove any $K+1$ Case

$a_{k+1} < 4$

$4 - \frac{2}{a_k} < 4$ ← from definition of the subsequent term $\left(a_{n+1} = 4 - \frac{2}{a_n}\right)$

$-\frac{2}{a_k} < 0$ * a_K must be ≥ 1 as the base case is 1 and any

$-\frac{2}{a_k} \leq -\frac{2}{1} < 0$ term after must be larger or equal as its monotonic non decreasing

$-\frac{2}{a_k} \leq -2 < 0$

$-2 < 0$

∴ Since this equality holds for any natural value of K and since the base case is true, we have proven that for all natural values of n, that a_n is bounded by 4

From the monotone convergence theory the series $\{a_n\}$ must have a limit as its monotone non-decreasing and has an upper bound of 4, this can also be expressed as:

$$\lim_{n\to\infty} a_n = L \quad \text{or} \quad \lim_{n\to\infty} a_{n+1} = L$$

We can then say that:

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty}\left(4 - \frac{2}{a_n}\right)$$

$$L = 4 - \lim_{n\to\infty} \frac{2}{a_n}$$ ← distributive property of limits

$$L = 4 - \frac{2}{L}$$ ← $\lim \frac{g(x)}{f(x)} = \frac{\lim g(x)}{\lim f(x)}$ property

$$L^2 = 4L - 2$$ ← multiply by L

$$L^2 - 4L + 2 = 0$$

apply the quadratic formula

$$L = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

- however $2 - \sqrt{2}$ is less then 1, and since $\{a_n\}$'s base case is 1 and every term is greater then the last, this is impossible!

∴ The limit of the sequence is $2 + \sqrt{2}$.