**Q03** Let $x$ be an integer, we know that 9797 can be expressed in prime factors:

$$9797 = 97^1 \cdot 101^1$$

These prime factors are co-prime to each other ($\gcd(97, 101) = 1$), thus we can apply SMT to the original equation. In other words an $x$ value will solve the original equation if that same $x$ solves both:

$$\begin{cases} x^2 + 5145x + 2332 & \equiv 0 \pmod{97} \\ x^2 + 5145x + 2332 & \equiv 0 \pmod{101} \end{cases}$$

Simplifying into congruence classes our two equations become:

$$\begin{cases} [x^2] + [4][x] + [4] & \equiv 0 \pmod{97} \\ [x^2] + [95][x] + [9] & \equiv 0 \pmod{101} \end{cases}$$

If we take the equation for the (mod 97) equation we know that it can be simplifed to:

$$[x^2] + [4][x] + [4] \equiv 0 \pmod{97} \implies ([x] + [2])([x] + 2) \equiv 0 \pmod{97}$$

Thus we know that the only possible x solution would be of the form:

$$[x] \equiv -[2] \equiv [-2] \equiv [95] \pmod{97}$$

We can thus rewrite $x$ as:

$$x = 95 + 97n \text{ (for some n} \in \mathbb{Z})$$

If we substitute x for this in the (mod 101) and simplify we get:

$$\begin{aligned} x^2 + [95]x + [9] &\equiv 0 \pmod{101} \\ (95 + 97n)(95 + 97n) + 95(95 + 97n) + [9] &\equiv 0 \pmod{101} \\ 9025 + 18430n + 9409n^2 + 9025 + 9215n + [9] &\equiv 0 \pmod{101} \\ [16]n^2 + [72]n + [81] &\equiv 0 \pmod{101} \end{aligned}$$

We notice that the final equation can be rewritten as:

$$([4][n] + [9])([4][n] + [9]) = 0$$

This would imply that:

$$[4][n] \equiv -[9] \equiv [92] \pmod{101}$$

Before we contuine we will solve for the multiplictive inverse of [4], we know that:

$$[4][4]^{-1} = [1] \pmod{101}$$

We also know that:

$$\begin{aligned} [4][76] &\equiv 304 \pmod{101} \\ [4][76] &\equiv [1] \pmod{101} \end{aligned}$$

Thus we know the multiplictive inverse of [4] is [76]. if we multiple both sides by the multiplictive inverse we get:

$$\begin{aligned} [4][n] &\equiv [92] \pmod{101} \\ [4]^{-1}[4][n] &\equiv [92][4]^{-1} \pmod{101} \\ [n] &\equiv [92][76] \pmod{101} \\ [n] &\equiv [23] \pmod{101} \end{aligned}$$

We can thus rewrite $n$ as:

$$n \equiv 23 + 97z \text{ (for some z} \in \mathbb{Z})$$

We now have two equatoins for x and n:

$$\begin{cases} n \equiv 23 + 101z \\ x \equiv 95 + 97n \end{cases}$$

Thus by CTR the solution to solution of the origonal equatoin will be of the form:

$$\begin{aligned} x &\equiv [95] + [97][n] \pmod{9797} \\ x &\equiv [95] + [97]([23] + [101][z] \pmod{9797} \\ x &\equiv [95] + [2231] + [9797][z] \pmod{9797} \\ x &\equiv [95] + [2231] + [0][z] \pmod{9797} \\ x &\equiv [2326] \pmod{9797} \end{aligned}$$

Since we know that $x_0$ is a particlar solution to our congruence, we know that by LCT the set of all solutions will be given by:

$$\{x \in \mathbb{Z} : x \equiv 2326 \pmod{9797}\}$$