

Q4: if  $(a+b)^n - b^n$  we can be written as:

$$\Rightarrow a \mid \left( \sum_{m=0}^n \binom{n}{m} a^{n-m} b^m - b^n \right) \quad (a, n, m \in \mathbb{Z})$$

$$\Rightarrow a \mid \left( \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n-1} a b^{n-1} + b^n - b^n \right)$$

$$\Rightarrow a \mid \left( \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n-1} a b^{n-1} \right)$$

$$\Rightarrow a \mid a \cdot K$$

- We can take  $a$  out of every term in the sum

$$\Rightarrow a \mid a \left( \binom{n-1}{0} a^{n-1} b + \binom{n-1}{1} a^{n-2} b^2 + \dots + \binom{n-1}{n-1} b^{n-1} \right)$$

$$\Rightarrow a \mid K = a \left( \sum_{m=0}^{n-1} \binom{n-1}{m} a^{n-1-m} b^m \right) \quad \leftarrow \text{Some integer } K$$

$$K = \sum_{m=0}^{n-1} \binom{n-1}{m} a^{n-1-m} b^m$$

- Since  $a$  is the summation,  $a \mid K$  will be equal to it. This means that  $a$  will be able to divide it!