Q02a Let n be a positive integer (greater then 1), then by the Unique Factorization Theorem, n can be expressed as:

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

where p_1,p_2,...,p_k, $k \geq 1$ are a list of prime divisors of n and a_1,a_2,...,a_k are all non-negative integers. From the DFPF, we know that all of n's divisors (known as the integer c) are of the form:

$$d = p_1^{\beta_1} p_2^{\beta_2} \cdots p_k^{\beta_k}, \text{ where } 0 \leq \beta_i \leq \alpha_i \text{ for } i = 1, 2, ..., k$$

We know from DFPF that for each prime factor of d (known as p_i), that its exponent (β_i) must be bounded by: $0 \leq \beta_i \leq \alpha_i$. In other words this means that the for p_i the choice is between 0, 1, 2,...,α_i which means that for each p_i the total combinations it can have is $\alpha_i + 1$.

$$\text{Divisor (d) combinations for each } p_i = \alpha_i + 1$$

Let c_i corrisponds to the combinations of p_i for $1, 2, ..., k$, the total combinations of d will be given by:

$$\text{Total Combinations of d } = (c_1)(c_2)...(c_k)$$

Since we know $c_i = \alpha_i + 1$, this can be replaced by:

$$\text{Total Combinations of d } = (\alpha_1 + 1)(\alpha_2 + 1)...(\alpha_k + 1)$$

Therefore since d is every positive divisor of n, the number of divisors of n will be given by $(\alpha_1 + 1)(\alpha_2 + 1)...(\alpha_k + 1)$.

Q02b Let m, n both be positive positive integer greater then 1, then by the Unique Factorization Theorem, n can be expressed as:

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

where p_1,p_2,...,p_k, $k \geq 1$ are a list of prime divisors of n and a_1,a_2,...,a_k are all non-negative integers. From the DFPF, we know that all of n's divisors (known as the integer c) are of the form:

$$d = p_1^{\beta_1} p_2^{\beta_2} \cdots p_k^{\beta_k}, \text{ where } 0 \leq \beta_i \leq \alpha_i \text{ for } i = 1, 2, ..., k$$

We know from DFPF that for each prime factor of d (known as p_i), that its exponent (β_i) must be bounded by: $0 \leq \beta_i \leq \alpha_i$. In other words this means that the for p_i the choice is between 0, 1, 2,...,α_i which means that for each p_i the total combinations it can have is $\alpha_i + 1$.

$$\text{Divisor (d) combinations for each } p_i = \alpha_i + 1$$

Let c_i corrisponds to the combinations of p_i for $1, 2, ..., k$, the total combinations of d will be given by:

$$\text{Total Combinations of d } = (c_1)(c_2)...(c_k)$$

Since we know $c_i = \alpha_i + 1$, this can be replaced by:

$$\text{Total Combinations of d } = (\alpha_1 + 1)(\alpha_2 + 1)...(\alpha_k + 1)$$

Therefore since d is every positive divisor of n, the number of divisors of n will be given by $(\alpha_1 + 1)(\alpha_2 + 1)...(\alpha_k + 1)$.