

$$\begin{aligned}
\binom{2n}{n} &= \frac{2n!}{n!(2n-n)!} \\
&= \frac{2n!}{n! * n!} \\
&= \frac{(2n) * 2n - 1 * (2n - 2) * \dots * (n + 2) * (n + 1)}{n!} \\
&= \prod_{k=1}^n \frac{n+k}{k} \\
&= \prod_{k=1}^n \left( \frac{n}{k} + \frac{k}{k} \right) \\
&= \prod_{k=1}^n \left( \frac{n}{k} + 1 \right)
\end{aligned}$$

We will now factor out the first term (when  $k = 1$ )

$$\begin{aligned}
\prod_{k=1}^n \left( \frac{n}{k} + 1 \right) &= \left( \frac{n}{1} + 1 \right) \prod_{k=2}^n \left( \frac{n}{k} + 1 \right) \\
&= (n + 1) \prod_{k=2}^n \left( \frac{n}{k} + 1 \right)
\end{aligned}$$

However if we have  $n = 1$  , we will get:

$$(n + 1) \prod_{k=2}^1 \left( \frac{n}{k} + 1 \right) = (n + 1) * 1$$

Therefore as long as  $n > 1$  , the binomial expression will have a factor of  $n + 1$ . Since it has this factor, the binomial will always be divisible by  $n + 1$