

Q02A If our we set $[a]$ to be $[7]$ our system of equations for \mathbb{Z}_{12} and multiply the first equation by 5 and the second by 3 we get:

$$\begin{cases} [35][x] + [15][y] & \equiv [5](1) \\ [6][x] + [15][y] & \equiv [-3](2) \end{cases}$$

Subtracting equation (1) by equation (2) we will get the new equation: $[29][y] = [8]$. Note that $[29] = [5]$ our equation will become:

$$[5][x] \equiv [8]$$

We know that $[5]$ and 12 are co-prime, and thus by INV (with integers) we know that $[5]$ will have a mathematical inverse, thus if we multiply both sides we will get:

$$[5]^{-1}[5][y] \equiv [8][5]^{-1}$$

By definition $[5]^{-1}[5] = 1$ and we know that $[5][5] = [1] \pmod{12}$ and thus the mathematical inverse of $[5]$ is $[5]$. Our equation thus becomes:

$$[x] \equiv [40] \equiv [4]$$

Plugging this into equation (2) we get that:

$$\begin{aligned} [6][x] + [15][y] & \equiv [-3] \\ [15][y] & \equiv [-3] - [6][4] \\ [3][y] & \equiv [-3] - [0] \\ [3][y] & \equiv [9] \end{aligned}$$

SFKMASKF We know this condition will be satisfied if $[y] = [3]$ and thus our solution set is given by: $[x] = [4]$ and $[y] = [3]$.

Q02B ...