

Q5a) This is false as if $n^5 - 1$ is even to $n^4 + n^3 + n^2 + n + 1$, then $n^5 - 1$ is divisible by $n^4 + n^3 + n^2 + n + 1$ then that would mean it would be divisible by itself! As well the proof doesn't take into account $n=1$, so $n^5 - 1$ would be divisible $(n=1)$ and $n^4 + n^3 + n^2 + n + 1$.

b) We will disprove this by counter example, if $x=2$ then

$$2^5 - 1 = (2-1)(2^4 + 2^3 + 2^2 + 2 + 1)$$

$$2^5 - 1 = (2^4 + 2^3 + 2^2 + 2 + 1)$$

$$= 31 \quad \text{which is prime}$$

\therefore the statement is false for when $n=2$ as that provides a prime

c) $n \geq 3$, \therefore the statement begins:

$\forall n \in \mathbb{Z}$, if $n \geq 3$, then $n^5 - 1$ is composite