

2nd) If $P \Rightarrow A$ is always $n \mid (12k+5)$ and $n \mid (18k-1)$ for some integer k
 $B \equiv n = 17$

for every $n > 1$
 \rightarrow

- then the original statement is equivalent to: $(A \Rightarrow B) \wedge (B \Rightarrow A)$

- If both $(A \Rightarrow B)$ and $(B \Rightarrow A)$ is true then the original statement is true.

Case [1]: $(A \Rightarrow B)$

- we will assume that for every $n > 1$ that $n \mid (12k+5)$ and $n \mid (18k-1)$ for some integer

- from DTC we

$$\therefore n \mid (12k+5)x + (18k-1)y \quad \text{for some integers } x, y \quad \leftarrow \text{from DTC}$$

$$\Rightarrow n \mid (12k+5)3 + (18k-1)2 \quad \leftarrow \text{pick } x=3 \text{ and } y=2$$

$$\Rightarrow n \mid 36k + 15 - 36k + 2 \quad \leftarrow \text{for every integer } k$$

$$\Rightarrow n \mid 17 \Rightarrow n = 17$$

\therefore In order for $n \mid (12k+5)x + (18k-1)y$ to be divisible for any integers x, y ,
 n must be 17. This means B is true so case [1] is true.

Case [2] $(B \Rightarrow A)$

- we will assume $n = 17$, we will show that a k exists such that

$$\Rightarrow n \mid (12k+5) \text{ and } n \mid (18k-1)$$

$$\Rightarrow 17 \mid (12k+5) \text{ and } 17 \mid (18k-1) \quad \leftarrow \text{Assume } B$$

$$\Rightarrow 17 \mid (12 \cdot 0 + 5) \text{ and } 17 \mid (18 \cdot 0 - 1) \quad \leftarrow \text{plug } k=0$$

$$\Rightarrow 17 \mid 17 \text{ and } 17 \mid 17$$

\therefore if $k=0$

\therefore a k exists such that A is true

\therefore since both the cases are true we know that $A \Leftrightarrow B$
 is true.