Q02A If our we set [a] to be [7] our system of equations for $\mathbb{Z}_{12}$ and multiply the first equation by 5 and the second by 3 we get:

$$\begin{cases} [35][x] + [15][y] & \equiv [5] (1) \\ [6][x] + [15][y] & \equiv [-3] (2) \end{cases}$$

Subtracting equation (1) by equation (2) we will get the new equation: [29][y] = [8]. Note that [29] = [5] our equation will become:

$$[5][x] \equiv [8]$$

We know that [5] and 12 are co-prime, and thus by INV (with integers) we know that [5] will have a mathimatical inverse, thus if we multiply both sides we will get:

$$[5]^{-1}[5][y] \equiv [8][5]^{-1}$$

By definition $[5]^{-1}[5] = 1$ and we know that $[5][5] = [1] \pmod{12}$ and thus the mathimatical inverse of [5] is [5]. Our equation thus becomes:

$$[x] \equiv [40] \equiv [4]$$

Plugging this into equation (2) we get that:

$$\begin{aligned} [6][x] + [15][y] &\equiv [-3] \\ [15][y] &\equiv [-3] - [6][4] \\ [3][y] &\equiv [-3] - [0] \\ [3][y] &\equiv [9] \end{aligned}$$

We know this condition will be satisfied if [y] = [3] and thus our solution set is given by both:

$$\{x \in \mathbb{Z} : x \equiv 4 \pmod{12}\}$$

$$\{y \in \mathbb{Z} : y \equiv 3 \pmod{12}\}$$

Q02B To start we will mutliply the first equation by 5 and the second by 3 so we will get:

$$\begin{cases} [3][a][x] + [15][y] & \equiv [5] (1) \\ [6][x] + [15][y] & \equiv [-3] (2) \end{cases}$$

substracting equation (1) by equation (2) we will get the new equation:

$$\begin{aligned} [3][a][x] - [6][x] &\equiv [8] \\ [3][x]([a] - [2]) &\equiv [8] \end{aligned}$$

Let [b] = [a] - [2]:

$$[3][x]([b]) \equiv [8]$$