Q05LDET 2 shows us that for a given a,b,c a x_0 exists such that:

$$x = x_0 + \frac{b}{d} * n$$

We also know that x' is bounded by:

$$0 \leq x' < \frac{b}{d}$$

$$0 \leq x_0 + \frac{b}{d} * n < \frac{b}{d}$$

if we divide by $\frac{b}{d}$, we get that

$$0 \leq \frac{(x_0)(d)}{b} + 1 * n < 1$$

$$-\frac{(x_0)(d)}{b} \leq n < 1 - \frac{(x_0)(d)}{b}$$

We know that since b is not zero (as b is positive) this implies that $-\frac{(x_0)(d)}{b}$ is a real number r, plugging this in we find that:

$$r \leq n < 1 + r$$

I will show that only one intenger n exists such that it is $n \in [r, r+1)$, by contradiction. Let the natural k represent the count of integers within the interval $[r, r+1)$:

Assume that $k \geq 2$. Since the smallest possible integer (n_0) that exists within the interval will always be $\lceil r \rceil$, we also know that the largest integer (n_{k-1}) will be equal to:

$$\text{largest integer} = \text{smallest integer} + (\text{amount of integers - 1})$$

$$n_{k-1} = n_0 + (k-1)$$

We also know that since $k \geq 2$ that:

$$n_0 + (k-1) \geq n_0 + ((2) - 1)$$

$$n_0 + (k-1) \geq n_0 + 1$$

Which thus implies:

$$n_{k-1} \geq n_0 + 1$$

$$n_{k-1} \geq \lceil r \rceil + 1$$

Since $n_{k-1} \in [r, r+1)$ this means that it is bounded by:

$$r \leq n_{k-1} < 1 + r$$

$$r \leq \lceil r \rceil + 1 \leq n_{k-1} < 1 + r$$

$$r \leq \lceil r \rceil + 1 < 1 + r$$

Splitting this into two equations we get that:

1. $r \leq \lceil r \rceil + 1$
2. $\lceil r \rceil + 1 < 1 + r$

We know that (1) is correct, but notice that equation (2) goes to:

$$\lceil r \rceil + 1 < 1 + r$$

$$\lceil r \rceil < r$$

This is a clear contradictoin as the $\lceil r \rceil$ is always greater then or equal to r. This contradiction happens whenever $k \geq 2$.
We know that since k is a natural number and $k \nleq 2$, that k must be 1. This means that there is only one possible n such that:

$$x' = x_0 + \frac{b}{d} * n$$

and by extension from LDET 2:

$$y' = y_0 - \frac{a}{d} * n$$

Therefore we have shown that for any positive a,b and integer c such that $\gcd(a, b)|c$ we have only one set of solutions x' and y' (which are therefore unqiue) such that:

$$ax' + by' = c \text{ and } x' < \frac{b}{\gcd(a, b)}$$

$$x_0 = \frac{(c - b(y_0))}{a}$$

$$x_0 = \frac{(c - b(y_0))}{a}$$

$$-x_0 = \frac{(c)gcd(a, b)}{ba} - \frac{(y_0)gcd(a, b)}{a}$$

$$(a(x_0) + b(y_0) = c$$

$$b(y_0) = c - (a(x_0)$$