Q05 To start we are given that:

$$\lim_{x\to 4} f(x) = 7$$

From our $\epsilon - \delta$ definition we can rewrite this as:

$$\text{if } 0 < |x-4| < \delta \text{ then } |f(x) - 7| < \epsilon$$

We can further expand out our equation for epsilon:

$$\begin{aligned} |f(x) - 7| &< \epsilon \\ 7 - \epsilon < f(x) &< 7 + \epsilon \\ 15 - \epsilon < f(x) + 8 &< 15 + \epsilon \\ |f(x) + 8| &< \epsilon + 15 \end{aligned}$$

So we could rewrite the given arbitrary equation as:

$$|f(x) + 8||f(x) - 7| < (15 + \epsilon)\epsilon$$

We know that $(15+\epsilon)\epsilon$ will result in a value that's a little larger then the original epsilon, so we can define a new epsilon (ϵ') that's equal to it such that:

$$\epsilon' = (15 + \epsilon)\epsilon$$

This results in the previous equation becoming:

$$\begin{aligned} |f(x) + 8||f(x) - 7| &< \epsilon' \\ |(f(x))^2 + f(x) - 56| &< \epsilon' \\ |(f(x))^2 + f(x) + 1 - 57| &< \epsilon' \end{aligned}$$

This is in the form $|f(x) - L| < \epsilon'$, so by the $\epsilon - \delta$ definition it will become:

$$\lim_{x\to 4} [(f(x))^2 + f(x) + 1] = 57$$