Q04A We know from BT we know that we can express $(1+i)^n$ and $(1-i)^n$ as:

$$
\begin{aligned}
(1+i)^n &= \sum_{k=0}^{n} \binom{n}{k} (i)^k \\
(1-i)^n &= \sum_{k=0}^{n} \binom{n}{k} (-i)^k
\end{aligned}
$$

Adding the two together we find that:

$$
\begin{aligned}
(1+i)^n + (1-i)^n &= \sum_{k=0}^{n} \binom{n}{k} (i)^k + \sum_{k=0}^{n} \binom{n}{k} (-i)^k \\
&= \sum_{k=0}^{n} \binom{n}{k} [-i^k + i^k]
\end{aligned}
$$

Notice that we can thus split this sigma into two cases, when k is even and when k is odd. Lets start when k is odd, we know that we can express k as $2d+1$ (where $d \in \mathbb{N}$), so our equation becomes:

$$
\begin{aligned}
\sum_{k=0}^{n} \binom{n}{k} [-i^k + i^k] &= \binom{n}{2d+1} [-i^{2d+1} + i^{2d+1}] \\
&= \binom{n}{2d+1} [(-i^2)^d(-i) + (i^2)^d(i)] \\
&= \binom{n}{2d+1} [(-1)^d(-i) + (-1)^d(i)] \\
&= \binom{n}{2d+1} [-(-1)^d(i) + (-1)^d(i)] \\
&= \binom{n}{2d+1} [0] \\
&= 0
\end{aligned}
$$

This means that for each odd k, the value of the summation will be unchanged.

In the second case we will let k be even, it can thus be expressed as k as $2d$ (where $d \in \mathbb{N}$), so our equation becomes:

$$\begin{aligned}
\sum_{k=0}^{n} \binom{n}{k} [-i^k + i^k] &= \binom{n}{2d} [-i^{2d} + i^{2d}] \\
&= \binom{n}{2d} [(-i^2)^d + (i^2)^d] \\
&= \binom{n}{2d} [(-1)^d + (-1)^d] \\
&= \binom{n}{2d} [2(-1)^d]
\end{aligned}$$

We thus know that only even values will impact the summation, the equation for all the even values is:

$$\sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n}{2k} [2(-1)^k]$$

And because odd values are redundent we find that:

$$(1+i)^n + (1-i)^n = 2 \sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n}{2k} (-1)^k.$$