

case [2]:  $\forall x \in \mathbb{Z}, (x \in B_n) \Rightarrow (x \in A_n)$ , let  $n$   
be an arbitrary integer  $\geq 3$ ,

$\therefore$  Assume that the hypothesis is correct such that  
 $n = \frac{b+1}{k}$  for some integer  $k$  and sub  $b$  in for  $a$  in [8]

$$\left[ \frac{b}{k} \right] a_n + 1 \in \mathbb{Z}$$

$$\Rightarrow b_n + 1 \in \mathbb{Z}$$

$$\Rightarrow b \frac{b+1}{k} + 1 \in \mathbb{Z}$$

$$\Rightarrow b \left( \frac{b}{k} + \frac{1}{k} \right) + 1 \in \mathbb{Z}$$

$$\Rightarrow \frac{b^2}{k} + \frac{b}{k} + 1 \in \mathbb{Z}$$

$\uparrow$   
contradiction

-  $b$  and  $(b+1)$  are coprime so if  $a$   $k$  exists such  
that  $(b+1)$  can be divided into an integer  $n$ , the same  $k$   
can not divide  $b$  into an integer, so there's a contradiction.

$\therefore$  Since case [1] and case [2] is false,  $A_n$  is not a  
proper subset of  $B_n$  and  $B_n$  is not a subset of  $A_n$  so  $A_n$  and  
 $B_n$  must be disjoint  $\wedge$   
proper