

5a) Let  $k$  be an arbitrary integer;

$$\begin{aligned} \boxed{1} \quad & \text{Since } 13k + 2 = 2(5k - 1) + 3k + 4 \\ & \Rightarrow \gcd(13k + 2, 5k - 1) = \gcd(5k - 1, 3k + 4), \end{aligned}$$

$$\begin{aligned} \boxed{2} \quad & \text{Since } 5k - 1 = -1(3k + 4) + 2k - 5 \\ & \Rightarrow \gcd(5k - 1, 3k + 4) = \gcd(3k + 4, 2k - 5) \end{aligned}$$

$$\begin{aligned} \boxed{3} \quad & \text{Since } 3k + 4 = 1(2k - 5) + k + 9 \\ & \Rightarrow \gcd(3k + 4, 2k - 5) = \gcd(2k - 5, k + 9) \end{aligned}$$

$$\begin{aligned} \boxed{4} \quad & \text{Since } 2k - 5 = 1(k + 9) - k - 14 \\ & \Rightarrow \gcd(2k - 5, k + 9) = \gcd(k + 9, k - 14) \end{aligned}$$

$$\begin{aligned} \boxed{5} \quad & \text{Since } k + 9 = 1(k - 14) + 23 \\ & \Rightarrow \gcd(k + 9, k - 14) = \gcd(k - 14, 23) \end{aligned}$$

Since 23 is prime the only possible divisors are  $\{1, 23\}$

$\therefore$  the elements of the set  $\{g(k) : k \in \mathbb{Z}\}$  are  $\{1, 23\}$

b) Since the only divisor that's greater than 1 ( $g(k) \geq 1$ ) is 23, it means that;

$$g(k) = 23 \mid k - 74 \quad \leftarrow 23 \mid (k + 14) \quad \boxed{5} \text{ub}$$

$$\therefore a \geq 23 \quad \text{and} \quad b = -14$$