Q17. We know the equation we need to prove is of the form:

$$x \not\equiv [50] \pmod{79} \implies x^{11} \not\equiv [2] \pmod{79}$$

Proof by Contrapositive tells us that proving the equation's contrapositive will also prove the original equation. Thus we need to prove:

$$x^{11} \equiv [2] \pmod{79} \implies x \equiv [50] \pmod{79}$$

Lets start by assuming the hypothesis:

$$x^{11} \equiv [2] \pmod{79}$$

We know from (CP), that the equation to the seventh power will equal:

$$x^{77} \equiv [2]^7 \pmod{79}$$
$$x^{77} \equiv [128] \pmod{79}$$
$$x^{77} \equiv [49] \pmod{79}$$

We know that x is not divisible by 79, thus Fermat's Little Theorem (FLT) states:

$$x^{78} \equiv [1] \pmod{79}$$

This can thus be rewritten:

$$x^{77} \cdot x^1 = [1] \pmod{79}$$

Subbing in our value for x^{77} we get:

$$[49]x = [1] \pmod{79}$$

We know that $gcd(49, 79) = 1$ and that $1|1$, therefore by Linear Congruence Theorem (LCT) we know there will be solution x. We also know that the set of solutions will be of the form (where x_0 is a specific solution):

$$x \equiv [x_0] \pmod{79}$$

Therefore to find this specific solution (x_0), we will rewrite our equation as:

$$49x = 1 + 79z \text{ (For some } z \in \mathbb{Z})$$
$$49x - 79z = 1$$

If we let $b = -z$, we will thus get:

$$49x + 79b = 1$$

Applying Extended Euclidean Algorithm (EEA) to this equation we get the following table:

x	b	r	q
0	1	79	0
1	0	49	0
-1	1	30	1
2	-1	19	1
-3	2	11	1
5	-3	8	1
-8	5	3	2
21	-13	2	1
-29	18	1	1

Therefore our bottom equation tells us that our equation has a solution (x_0) when:

$$x_0 = -29$$

Thus from Linear Congruence Theorem (LCT) we know our solution for x will be for the form:

$$x \equiv [-29] \pmod{79}$$

This is equivalent to:

$$x \equiv [-29] + [79] \pmod{79}$$
$$x \equiv [50] \pmod{79}$$

This thus proves the contrapositive, and as a result we have proved the original statement.