Robert (Robbie) Knowles MATH 135 Fall 2020: WA08

Q01A We will start by simplifying, we know that from CISR that:

$$7007x \equiv 224x + 399(17) \equiv 224x \pmod{399}$$
$$-201 \equiv -201 + 399(1) \equiv 189 \pmod{399}$$

Thus our equation will become:

$$224x \equiv 189 \pmod{399}$$

Applying EEA to the corrisponding linear Diaphantine equation $224x + 399y = 189$:

x	y	r	q
0	1	399	0
1	0	224	0
-1	1	175	1
2	-1	49	1
-7	4	28	3
9	-5	21	1
-16	9	7	1
57	-32	0	3

LCT tells us that since d = gcd(7007,399) = 7 (from the certificate of correctness) and 7|189, that there must be an integer x that satisfies the equation:

$$7007x \equiv -210 \pmod{399}$$

We also know that when x = -16 that:

$$224(-16) \equiv -3584 + 399(9) \equiv 7 \pmod{399}$$

Also notice that this will means 7007(-16) (mod 399) = 7 as well, so any solution of 224x (mod 399) will apply to 7007x (mod 399). Since 7(27) = 189 (mod 399), if we multiple both sides by 27 we get that:

$$224(-432) \equiv -96768 + 399(243) \equiv 7(27) \pmod{399}$$

$$224(-432) \equiv -96768 + 399(243) \equiv 189 \pmod{399}$$

We know that the set of all solutions will be given by $x = x_0 \pmod{399}$ where x_0 is a particular solution. Thus:

$$x \equiv -432 \pmod{399} \equiv -432 + 399(2) \equiv 366 \pmod{399}$$

Therefore the set of solutions to the linear congruence are given by all integers x such that:

$$x \equiv 366 \pmod{399}$$

Q01B Our equation is in its simpliest form (as each term its less then 8645), thus we will start by applying EEA to the corrisponding linear Diaphantine equation

$$1323x + 8645y = 1155$$

x	y	r	q
0	1	8645	0
1	0	1323	0
-6	1	707	6
7	-1	616	1
-13	2	91	1
85	-13	70	6
-98	15	21	1
379	-58	7	3
-1235	189	0	3

LCT tells us that since d = gcd(1323,8645) = 7 (from the certificate of correctness) and 7|1155, that there must be an integer x that satisfies the equation:

$$1323x \equiv 1155 \pmod{8645}$$

We also know that when x = 379 that:

$$1323(379) \equiv 501417 - 8645(58) \equiv 7 \pmod{8645}$$

Since 7(165) = 1155 (mod 8645), if we multiple both sides by 165 we get that:

$$224(62535) \equiv -96768(165) + 399(9570) \equiv 7(165) \pmod{8645}$$

$$224(62535) \equiv 1155 \pmod{399}$$

We know that the set of all solutions will be given by $x = x_0(\bmod 8645)$ where x_0 is a particular solution. Thus:

$$x \equiv 62535 \pmod{8645} \equiv 2020 - 8645(7) \equiv 2020 \pmod{8645}$$

Therefore the set of solutions to the linear congruence are given by all integers x such that:

$$x \equiv 2020 \pmod{8645}$$