

Q3) to prove the inequality is true we will use induction.

① base case: $n=1$

$$\Rightarrow 1 - \sum_{i=1}^1 a_i \leq \prod_{j=1}^1 (1 - a_j)$$

$$\Rightarrow 1 - a_1 \leq 1 - a_1$$

$$\Rightarrow 1 \leq 1$$

② Inductive hypothesis: for any $k \in \mathbb{N}$, $1 - \sum_{i=1}^k a_i \leq \prod_{j=1}^k (1 - a_j)$

③ Inductive Step: prove $k+1$:

$$1 - \sum_{i=1}^{k+1} a_i \leq \prod_{j=1}^{k+1} (1 - a_j)$$

$$\Rightarrow 1 - \sum_{i=1}^k (1 - a_i) - a_{k+1} \leq (1 - a_{k+1}) \prod_{j=1}^k (1 - a_j)$$

$$\Rightarrow 1 - \sum_{i=1}^k (1 - a_i) - a_{k+1} \leq \prod_{j=1}^k (1 - a_j) - a_{k+1} \prod_{j=1}^k (1 - a_j)$$

because $1 - \sum_{i=1}^k (1 - a_i) \leq \prod_{j=1}^k (1 - a_j)$, make $1 - \sum_{i=1}^k (1 - a_i) = \prod_{j=1}^k (1 - a_j)$

$$\Rightarrow \prod_{j=1}^k (1 - a_j) - a_{k+1} \leq \prod_{j=1}^k (1 - a_j) - a_{k+1} \prod_{j=1}^k (1 - a_j)$$

$$-a_{k+1} \leq -a_{k+1} \prod_{j=1}^k (1 - a_j)$$

$$-1 \leq -\prod_{j=1}^k (1 - a_j)$$

since a_i is always less than 1, $\prod_{j=1}^k (1 - a_j) < 1$

∴ this step works for any $k+1$, we have proven the inequality