c) Let n be an arbitrary integer, n has 2 cases either (for some its odd $(n = 2k+1)$ or even $(n = 2k)$ if $n^3 - n + 7$ is odd integer k) then it will be true for all integer values of n

$\boxed{1}$ n is odd $(n = 2k+1)$:

$$n^3 - n + 7$$

$\Rightarrow (2k+1)^3 - (2k+1) + 7$ ← sub $2k+1$ in for n

$\Rightarrow (2k+1)(4k^2 + 4k + 1) - 2k - 1 + 7$

$\Rightarrow (8k^3 + 12k^2 + 6k + 1) - 2k + 6$

$\Rightarrow 2a + 12k^2 + 4k + 7$ ← let $a = 4k^3$ (an integer)

$\Rightarrow 2a + 2b + 4k + 7$ ← let $b = 6k^2$ (an integer)

$\Rightarrow 2(a + b + 2k + 3) + 1$

$\Rightarrow 2c + 1$ ← let $c = a + b + 2k + 3$ (an integer)

$\therefore$ if n is odd $n^3 - n + 7$ must be odd

$\boxed{2}$ n is even $(n = 2k)$

$$n^3 - n + 7$$

$\Rightarrow (2k)^3 - 2k + 7$ ← sub $n = 2k$ in

$\Rightarrow 2a - 2k + 7$ ← let $a = 4k^3$ (an integer)

$\Rightarrow 2(a - k + 3) + 1$

$\Rightarrow 2c + 1$ ← let $c = a - k + 3$ (an integer)

$\therefore$ if n is even $n^3 - n + 7$ must be odd

$\therefore$ Since the even case and the odd case is true, all integer values of $n^3 - n + 7$ must be odd