Q03a We will prove the results by using strong induction on n, where $P(n)$ is the statement:

$$a_{n+2} = 6a_n$$

Base Case: When n is 0, the statement $P(0)$ is given by:

$$x_2 \equiv 6x_0 \pmod{10}$$

This is from definition and satifies the general equation, thus proving $P(0)$. When n is 1, the statement $P(1)$ is given by:

$$x_3 \equiv 6x_1 \pmod{10}$$

This is also from definition and satifies the general equation thus proving $P(1)$. When n is 2, the statement $P(2)$ is given by:

$$\begin{aligned} x_4 &= x_3 + x_2 \pmod{19} \\ &= (6x_1) + (6x_0) \pmod{19} \\ &= 6(x_1 + x_0) \pmod{19} \\ &= 6(x_2) \pmod{19} \end{aligned}$$

We have thus shown that when n is 2, it satifies the general equation thus proving $P(1)$. Inductive Hypothesis: Let k be a positive integer such that ($k \geq 2$). Assume for all integers i = 1,2,3,...,k, $a_{i+2} = 6a_i$. We wish to prove that when P(k+1), we will get:

$$x_{(k+3} = 6k + 1$$

We know that from our starting definition P(k+1):

$$\begin{aligned} x_{(k+1)+2} &= x_{(k+1)+2-1} + x_{(k+1)+2-2} \pmod{19} \\ &= (6x_1) + (6x_0) \pmod{10} \\ &= 6(x_1 + x_0) \pmod{10} \\ &= 6(x_2) \pmod{10} \end{aligned}$$