

d) We can use proof by contrapositive. This means if

$(\neg B) \Rightarrow (\neg A)$  is true then  $A \Rightarrow B$  is true

$\therefore$  the statement becomes: if  $n$  is not even then  $n^3 - n + 7$  is not even

Which can further be simplified to: if  $n$  is odd then  $n^3 - n + 7$  is odd.

We assume  $n$  is odd.  $(2k+1)$  for some integer  $k$

$$n^3 - n + 7$$

$$\Rightarrow (2k+1)^3 - (2k+1) + 7 \leftarrow \text{Sub } 2k+1 \text{ in for } n$$

$$\Rightarrow (2k+1)(4k^2 + 4k + 1) - (2k+1) + 7$$

$$\Rightarrow (8k^3 + 12k^2 + 6k + 1) - (2k+1) + 7$$

$$\Rightarrow 8k^3 + 12k^2 + 4k + 7$$

$$\Rightarrow 2a + 12k^2 + 4k + 7 \leftarrow \text{let } a \text{ be } 4k^3 \text{ (an integer)}$$

$$\Rightarrow 2a + 2b + 4k + 7 \leftarrow \text{let } b \text{ be } 6k^2 \text{ (an integer)}$$

$$\Rightarrow 2(a+b+2k+3) + 1$$

$$\Rightarrow 2c + 1 \leftarrow \text{let } c \text{ be } (a+b+2k+3) \text{ (an integer)}$$

$\therefore$  The contrapositive is true so the original statement is true