Robert (Robbie) Knowles MATH 137 Fall 2020: WA08

Q01. Let c be an arbitrary real number such that $(c > 0)$ This means that we start with the equation:

$$\begin{aligned}f\sqrt{x}+\sqrt{y} &= \sqrt{c}\\ \sqrt{y} &= \sqrt{c}-\sqrt{x}\\ y &= (\sqrt{c}-\sqrt{x})^2\end{aligned}$$

This also impliies that:

$$f(x) = (\sqrt{c}-\sqrt{x})^2$$

If we take the implicit derivative of both sides we will find that:

$$\begin{aligned}\frac{d}{dx}y &= \frac{d}{dx}(\sqrt{c}-\sqrt{x})^2\\ \frac{dy}{dx} &= 2(\sqrt{c}-\sqrt{x}) \times \frac{d}{dx}[\sqrt{c}-\sqrt{x}]\\ \frac{dy}{dx} &= 2(\sqrt{c}-\sqrt{x}) \times [\frac{d}{dx}\sqrt{c}-\frac{d}{dx}\sqrt{x}]\end{aligned}$$

Notice that c is a constant and x is of the form $x^{(\frac{1}{2})}$ thus the derivative will become:

$$\begin{aligned}\frac{dy}{dx} &= 2(\sqrt{c}-\sqrt{x}) \times [0-\frac{1}{2}x^{\frac{-1}{2}}]\\ \frac{dy}{dx} &= 2(\sqrt{c}-\sqrt{x}) \times \frac{1}{-2x^{\frac{1}{2}}}\\ \frac{dy}{dx} &= -\frac{\sqrt{c}-\sqrt{x}}{\sqrt{x}}\end{aligned}$$

This can also be rewritten as:

$$f'(x) = -\frac{\sqrt{c}-\sqrt{x}}{\sqrt{x}}$$

Let x_0 and y_0 be an arbitrary solution to $\sqrt{x}+\sqrt{y}=\sqrt{c}$, therefore a tangent line will exist that goes through (x_0, y_0). We can thus apply the Linear Approximation at $x = a$, and we get:

$$L_a(x) = f(a) + f'(a)(x-a)$$

The next step we will split into two parts, finding the y intercept and finding the x intercept. Starting with the first we know that at the y intercept the value of x will be zero, thus our equation becomes:

$$
\begin{aligned}
L_a(x) &= f(a) + f'(a)(x-a) \\
L_a(0) &= f(a) + f'(a)(0-a) \\
&= (\sqrt{c}-\sqrt{a})^2 + f'(a)(-a) \\
&= (\sqrt{c}-\sqrt{a})^2 - \frac{\sqrt{c}-\sqrt{a}}{\sqrt{a}}(-a) \\
&= (\sqrt{c}^2 - 2\sqrt{a}\sqrt{c} + \sqrt{a}^2) - (-\sqrt{c}\sqrt{a} + \sqrt{a}\sqrt{a}) \\
&= c - 2\sqrt{a}\sqrt{c} + a + \sqrt{c}\sqrt{a} - a \\
&= c - \sqrt{a}\sqrt{c}
\end{aligned}
$$

Our x intercept will happen when $y = 0$, in other terms this is equivilent to $L_a(x) = 0$. This means our equation will become:

$$
\begin{aligned}
L_a(x) &= f(a) + f'(a)(x-a) \\
0 &= f(a) + f'(a)(x-a) \\
&= f(a) + f'(a)(x-a) \\
&= \frac{f(a)}{f'(a)} + (x-a) \\
&= \frac{(\sqrt{c}-\sqrt{a})^2}{-\frac{\sqrt{c}-\sqrt{a}}{\sqrt{a}}} + (x-a) \\
&= -\frac{\sqrt{a}(\sqrt{c}-\sqrt{a})^2}{\sqrt{c}-\sqrt{a}} + (x-a) \\
&= -\sqrt{a}(\sqrt{c}-\sqrt{a})) + (x-a) \\
&= -\sqrt{a}\sqrt{c} + \sqrt{(a)}\sqrt{a} + x - a \\
\sqrt{a}\sqrt{c} &= x
\end{aligned}
$$

Since x is the x intercept and $L_a(0)$ is the y intercept, the sum of the two will equal:

$$
\begin{aligned}
&= L_a(0) + x \\
&= \mathrm{c} - \sqrt{a}\sqrt{c} + \sqrt{a}\sqrt{c} \\
&= \mathrm{c}
\end{aligned}
$$

Thus showing that the x and y intercepts of the tangent will sum up to c.