

2) We will start by choosing Claim 2:

To prove that $S \subseteq T$, we will prove that $\boxed{1}$ T contains a value that S doesn't and $\boxed{2}$ every value of S is in T . To start we will use induction

base case: $n, m = 1$

$$S = \sum_{k=1}^n \frac{1}{k(k+1)} = \sum_{k=1}^1 \frac{1}{k(k+1)} = \frac{1}{1(2)} = \frac{1}{2}$$

$$T = \frac{m-1}{m} = \frac{1-1}{1} = 0$$

$S = \{1/2\}$ $T = \{0\}$

* $\boxed{1}$ is proven as $\{0\} \subseteq T$ but since S is an increasing series and the smallest value of S is $1/2$, $\{0\} \not\subseteq S$

Inductive Step: Let $i \in \mathbb{N}$, $\sum_{k=1}^i \frac{1}{k(k+1)} \in T$ or that

$$\sum_{k=1}^i \frac{1}{k(k+1)} = \frac{Z-1}{Z} \text{ for some natural number } Z, \text{ we will now show the } i+1 \text{ case holds:}$$

$$\text{NTP: } \sum_{k=1}^{i+1} \frac{1}{k(k+1)} = \frac{(Z+1)-1}{Z+1}$$

$$\text{LHS: } \sum_{k=1}^{i+1} \frac{1}{k} - \frac{1}{(k+1)}$$

$$\Rightarrow \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) \dots + \left(\frac{1}{i} - \frac{1}{i+1}\right) + \left(\frac{1}{i+1} - \frac{1}{i+2}\right)$$

- we will expand out the sigma notation and then simplify:

$$\Rightarrow \left(\frac{1}{1} - \frac{1}{(i+1)}\right) + \left(\frac{1}{2} - \frac{1}{(i+1)}\right) + \left(\frac{1}{3} - \frac{1}{(i+1)}\right) + \dots + \left(\frac{1}{i} - \frac{1}{(i+1)}\right) + \left(\frac{1}{i+1} - \frac{1}{(i+1)}\right)$$

$$\Rightarrow \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{i} - \frac{1}{i+1}\right) + \left(\frac{1}{i+1} - \frac{1}{i+2}\right)$$

$$\Rightarrow 1 - \frac{1}{i+2}$$

$$\Rightarrow \frac{i+1}{i+2}$$

$$\text{LHS: } \frac{i+z-1}{i+z}$$

$$\text{RHS: } \frac{(z+1)-1}{(z+1)}$$

- however since z is any arbitrary natural number, we can make $v \geq z+1$ such that

$$(i \in \mathbb{N}) \quad \text{LHS: } \frac{(i+z)-1}{(i+z)} \quad \text{RHS: } \frac{(v+z)-1}{(v+z)} \quad (v \in \mathbb{N})$$

- Since there will always be a natural number bigger than the last, $[2]$ is proven as any S value can be found in T

\therefore because of $[1]$ and $[2]$ S must be a proper subsequence of T so claim 2 is correct and proven