

Q3) a) Start with $b \neq 0$ proves the contrapositive, the statement below is
 If $c \mid (5a + 11b)$ and $c \mid (6a + 13b)$ then $c \mid b$

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We know that $c \mid a$ and $c \mid b$ then $c \mid (ax + by)$ for any integers x, y
 This is from D.C

$$\Rightarrow c \mid (5a + 11b)x + (6a + 13b)y \quad \leftarrow \text{from D.C}$$

$$\Rightarrow c \mid (5a + 11b)5 + (6a + 13b)(-5) \quad \leftarrow \text{pick } y=5 \text{ and } x=-5$$

$$\Rightarrow c \mid (30a - 30a + 66b - 65b)$$

$$\Rightarrow c \mid b \quad \leftarrow \text{from simplification}$$

b) the converse is:

if $c \nmid (5a + 11b)$ or $c \nmid (6a + 13b)$ then $c \nmid b$

c) the converse is false

d) We will disprove the universal quantifier with a counter example

If c and $b \geq 7$, and $a \geq 1$ we get;

$$= 7 \nmid (5 \cdot 1 + 7 \cdot 11) \text{ or } 7 \nmid (6 + 11) \quad \text{then } 7 \nmid 7$$

$$7 \nmid 82 \quad \text{or} \quad 7 \nmid 97 \quad 7 \nmid 7$$

$$\text{true} \quad \text{true} \quad \text{false}$$

\therefore Since one hypothesis is true but one conclusion is false, there exists a ^{choice} set of a, b, c where the statement is false. This means the converse is false.