Q1) to start we take the equation and simplify:

$$\Rightarrow \lim_{x \to 2} x^3 - 7x + 1 = -5$$

$$\Rightarrow \lim_{x \to 2} x^3 - 7x + 6 \qquad \leftarrow \text{when } x = 1,\ (1)^3 - 7 \cdot 1 + 6 = 0 \quad \therefore (x-1) \text{ is a factor}$$

$$\Rightarrow \lim_{x \to 2} (x-1)(x^2 + x - 6)$$

$$\Rightarrow \lim_{x \to 2} (x-1)(x+3)(x-2)$$

Since this is in the form $f(x) - L$ we can express it as $|f(x) - L|$:

$$\Rightarrow \lim_{x \to 2} |(x-1)(x+3)(x-2)|$$

$$\Rightarrow \lim_{x \to 2} |x-1||x+3||x-2|$$

Note $\lim_{x \to 2}$ is equivalent to $0 < |x-2| < \delta$ for any $\delta \in \mathbb{R}$, we can make $\delta \leq 1$ as if $0 < |x-2| \leq 1$ it will also work for $0 < |x-2| < z$ where $z > 1$. $\therefore 0 < |x-2| \leq 1$

$$\Rightarrow 0 < |x-2| < 1$$

$$\Rightarrow 1 < x < 3 \qquad (x\text{'s upper bound is } 3)$$

$$\therefore |x-1| < 2 \quad \text{and} \quad |x+3| < 6 \qquad (\text{if } x > 1)$$

$$\Rightarrow |x-1||x+3||x-2| < 2 \cdot 6\, |x-2|$$

$$12\, |x-2|$$

* make δ the min of $(1, \frac{\epsilon}{12})$ for some $\epsilon > 0$

$$\Rightarrow |x-2| < \frac{\epsilon}{12} \qquad (\text{as } \delta \leq 1)$$

$$\therefore |f(x) - L| = |x^3 - 7x + 6| = |x-1||x+3||x-2| < 2 \cdot 6 \cdot |x-2|$$

$$< 2 \cdot 6 \cdot \frac{\epsilon}{12}$$

$$< \epsilon \qquad (\text{for any } \epsilon > 0)$$

$\therefore$ since $0 < |x-2| < \delta$ and $|f(x) - L| < \epsilon$

the limit is proven!