18a. To start we know that the modulus of z_0 is

$$\begin{aligned} r_{z_0} &= \sqrt{(2)^2 + (2sqrt(3))^2} \\ &= \sqrt{4 + (4)(3)} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

We also know that the θ of z_0 is:

$$\begin{aligned} \theta_{z_0} &= \tan^{-1}(\frac{2\sqrt{3}}{2}) \\ \theta_{z_0} &= \tan^{-1}(\sqrt{3}) \\ \theta_{z_0} &= \frac{\pi}{3} \end{aligned}$$

Thus we know z_0 will be:

$$\begin{aligned} z_0 &= r(\cos\theta + \sin\theta i) \\ z_0 &= 4(\cos(\frac{\pi}{3}) + \sin(\frac{\pi}{3})i) \end{aligned}$$

Moving on to z_1 we know its modulus is:

$$\begin{aligned} r_{z_1} &= \sqrt{(-\frac{1}{\sqrt{2}})^2 + (-\frac{\sqrt{3}}{\sqrt{2}})^2} \\ &= \sqrt{\frac{1}{2} + \frac{3}{2}} \\ &= \sqrt{\frac{4}{2}} \\ &= \sqrt{2} \end{aligned}$$

We also know that the θ of z_1 is:

$$\begin{aligned} \theta_{z_1} &= \tan^{-1}(\frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{2}}) \\ \theta_{z_1} &= \tan^{-1}(\sqrt{3}) \\ \theta_{z_1} &= \frac{\pi}{3} \end{aligned}$$

We know from the signs that it is in third quadrant, thus it becomes:

$$\theta_{z_1} = \frac{4\pi}{3}$$

Thus we know z_1 will be:

$$z_1 = r(\cos\theta + \sin\theta i)$$
$$z_1 = \sqrt{2}(\cos(\frac{4\pi}{3}) + i\sin(\frac{4\pi}{3})i)$$

18b. To start we are given the equation:

$$z_n = (\cos(\frac{3\pi}{4}) + \sin(\frac{3\pi}{4})i)\frac{z_{n-1}}{|z_{n-2}|}$$

We can replace $\frac{1}{|z_{n-2}|}$ with $|z_{n-2}|^{-1}$ using Properties of Modulus (PM) and our equation becomes:

$$z_n = (\cos(\frac{3\pi}{4}) + \sin(\frac{3\pi}{4})i) \cdot z_{n-1} \cdot |z_{n-2}|^{-1}$$

However we are trying to solve for $|z_n|$ so our equation becomes:

$$|z_n| = |(\cos(\frac{3\pi}{4}) + \sin(\frac{3\pi}{4})i) \cdot z_{n-1} \cdot |z_{n-2}|^{-1}|$$

We can use PM to distribute out the modulus so we get:

$$|z_n| = |(\cos(\frac{3\pi}{4}) + \sin(\frac{3\pi}{4})i)| \cdot |z_{n-1}| \cdot ||z_{n-2}|^{-1}|$$

Notice that $|z| = ||z||$, as the imaginary component will not exist after the first modulus, so you will get (for some real number r:

$$|z| = r$$

Thus:

$$||z|| = \sqrt{r^2 + 0^2}$$
$$||z|| = r$$

So our equation can be rewritten as:

$$|z_n| = |(\cos(\frac{3\pi}{4}) + \sin(\frac{3\pi}{4})i)| \cdot |z_{n-1}| \cdot |z_{n-2}|^{-1}$$

We also know that $|(\cos(\frac{3\pi}{4}) + \sin(\frac{3\pi}{4})i)| = 1$ as its in polar form and it's $r = 1$, so:

$$|z_n| = 1 \cdot |z_{n-1}| \cdot |z_{n-2}|^{-1}$$
$$|z_n| = \frac{|z_{n-1}|}{|z_{n-2}|}$$

By finding the shortest non repeating sequence within z, we can find the positive integer p such that for any positive integer n:

$$|z_{n+p}| = |z_n|$$