MATH 135 Fall 2020: Written Assignment 7 (WA07)
Due at 11:55 PM EST on Friday, November 13th, 2020
Covers the contents of Lessons 6.7, 6.8, 7.1 and 7.2

Q01. (5 marks) For each of the following LDE's, either prove that there are no solutions, or find all possible solutions.

(a) (2 marks) $1771x + 8050y = 23$

(b) (3 marks) $1197x - 5145y = -42$

Q02. (5 marks)

(a) (2 marks) Prove that the number of positive divisors of an integer $n > 1$ with unique prime factorization

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

is given by the product $(\alpha_1 + 1)(\alpha_2 + 1) \cdots (\alpha_k + 1)$.

(b) (3 marks) We say that a function $f(n)$ is *multiplicative* if $f(mn) = f(m)f(n)$ for all coprime positive integers m and n. Let $d(n)$ denote the number of positive divisors of n. Prove that $d(n)$ is multiplicative.

Q03. (5 marks) Let n be a positive integer. We say that a positive integer k is a *perfect n-th power* if there exists a positive integer a such that $k = a^n$.

Prove that for every positive integer k and all *coprime* positive integers m and n, if k is both a perfect m-th power and a perfect n-th power, then it is a perfect (mn)-th power.

Q04. (5 marks) It is known that, for all integers n and k such that $1 \leq k \leq n$,

$$\frac{n}{\gcd(n,k)} \mid \binom{n}{k}$$

You may use this fact, without proof, in your solution.

Let n be a positive integer and let s be the largest non-negative integer such that $2^s \mid n$. Prove that $2^s \mid \binom{n}{k}$ for every odd integer k such that $1 \leq k \leq n$.

Q05. (5 marks) Let a and b be positive integers. Let c be an integer such that $\gcd(a,b) \mid c$. Prove that there exists a unique (integer) solution (x', y') to the linear Diophantine equation $ax + by = c$ such that $0 \leq x' < \frac{b}{\gcd(a,b)}$.