135 WA 04

Q1) We will prove this via contradiction, we will start by assuming that $a^2 - 6$ is a perfect square for some $a \in \mathbb{N}$, this can be expressed as:

$$\exists a, z \in \quad a^2 - 6 = z^2$$

This can be simplified as follows:

$$\Rightarrow a^2 - z^2 = 6 \quad \{1, 2, 3, 6\} \text{ or } 1, 6$$
$$\Rightarrow (a+z)(a-z) \quad \text{or} \quad 6 = 1 \cdot 6$$

* Since 6's factors are $\{1, 2, 3, 6\}$, we have two cases:

Case $\boxed{1}$: 6 is the multiple of 2, 3	Case $\boxed{2}$: 6 is the multiple of 1, 6
* $(a+z) > (a-z)$ as $a, z \in \mathbb{N}$	* $(a+z) > (a-z)$ as $a, z \in \mathbb{N}$
$\therefore (a+z) = 3$ and $(a-z) = 2$	$\therefore (a+z) = 6$ and $(a-z) = 1$
$(a+z) = 3$	$(a+z) = 6$
$-\ (a-z) = 2$	$-\ (a-z) = 1$
$2z = 1$	$2z = 5$
$z = 0.5$	$z = 2.5$

Case $\boxed{1}$ and Case $\boxed{2}$ have contradictions as z is defined as a natural number but here it is equal to 0.5 and 2.5 for any value of a. This means that our assumption is false that for some $a \in \mathbb{N}$ then $a^2 - 6$ is a perfect square.

$\therefore$ Since the negation is false, for every $a \in \mathbb{N}$, $a^2 - 6$ is not a perfect square.

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$$\exists a, z \in \quad a^2 - 6 = z^2$$

This can be simplified as follows:

$$\Rightarrow a^2 - z^2 = 6$$
$$\Rightarrow (a+z)(a-z) = 6$$

* Since 6's factors are $\{1, 2, 3, 6\}$, we have two cases

Case $\boxed{1}$: 6 is the multiple of 2, 3

* $(a+z) > (a-z)$ as $a, z \in \mathbb{N}$

$\therefore (a+z) = 3$ and $(a-z) = 2$

$$\begin{aligned} &(a+z) = 3 \\ -\ &(a-z) = 2 \\ \hline &2z = 1 \\ &z = 0.5 \end{aligned}$$

Case $\boxed{2}$: 6 is the multiple of 1, 6

* $(a+z) > (a-z)$ as $a, z \in \mathbb{N}$

$\therefore (a+z) = 6$ and $(a-z) = 1$

$$\begin{aligned} &(a+z) = 6 \\ -\ &(a-z) = 1 \\ \hline &2z = 5 \\ &z = 2.5 \end{aligned}$$

Case $\boxed{1}$ and Case $\boxed{2}$ have contradictions as z is defined as a natural number but here it is equal to 0.5 and 2.5 for any value of a. This means that our assumption is false that for some $a \in \mathbb{N}$ then $a^2 - 6$ is a perfect square.

$\therefore$ Since the negation is false, for every $a \in \mathbb{N}$, $a^2 - 6$ is not a perfect square.