

Case [2]: $\forall x \in \mathbb{Z}, (x \in B_n) \Rightarrow (x \in A_n)$, let n
be an arbitrary integer ≥ 3 ,

\therefore Assume that the hypothesis is correct such that

$n = \frac{b+1}{k}$ for some integer k and sub b in for a in [B]

$$(a + x) \in [B] \quad a_{n+1} \in \mathbb{Z}$$

$$\Rightarrow b_{n+1} \in \mathbb{Z}$$

$$\Rightarrow b \frac{b+1}{k} + 1 \in \mathbb{Z}$$

$$\Rightarrow b(b/k + 1/k) + 1 \in \mathbb{Z}$$

$$\Rightarrow b^2/k + b/k + 1 \in \mathbb{Z}$$

\uparrow
contradiction

- b and $(b+1)$ are coprime so if a k exists such
that $(b+1)$ can be divided into an integer n , the same k
can not divide b into an integer, so there's a contradiction.

\therefore Since Case [1] and Case [2] is false, A_n is not a
subset of B_n and B_n is not a subset of A_n so A_n and
 B_n must be disjoint