Q02A If our we set [a] to be [7] our system of equations and multiply the first equation by 5 and the second by 3 we get:

$$\begin{cases} [35][x] + [15][y] & = [5] (1) \\ [6][x] + [15][y] & = [-3] (2) \end{cases}$$

Subtracting equation (1) by equation (2) we will get the new equation: [29][y] = [8]. Note that [29] = [5] our equation will become:

$$[5][x] = [8]$$

We know that [5] and 12 are co-prime, and thus by INV (with integers) we know that [5] will have a mathimatical inverse, thus if we multiply both sides we will get:

$$[5]^{-1}[5][y] = [8][5]^{-1}$$

By definition $[5]^{-1}[5] = 1$ and we know that [5][5] = [1] (mod 12) and thus the mathimatical inverse of [5] is [5]. Our equation thus becomes:

$$[x] = [40] = [4]$$

Plugging this into equation (2) we get that:

$$\begin{aligned} [6][x] + [15][y] &= [-3] \\ [15][y] &= [-3] - [6][4] \\ [3][y] &= [-3] - [0] \\ [3][y] &= [9]] \end{aligned}$$

We know this condition will be satisfied if [y] = [3] and thus our solution is: [x] = [4] and [y] = [3]

Q01B Visually we know that $\mathbb{Z}_7$ is a field, if we look at the multiplication table, each possible congruence class [a] has a corrisponding congruence class $[b]^{-1}$ such that:

$$[a][b] = 1$$

This happens because 7 is a prime and [a] is co-prime to 7. This means that $d = \gcd([a], 7]) = 1$ and by definition of MAT since $d|1$ there must be a solution [b] for each [a] that solves the above equality (which means [a] will have a multiplictive inverse).

On the other hand 8 is not prime and thus not all [a]'s are co-prime to 8. If [a] is not coprime to 7 this would result in $d = \gcd([a], 8] \neq 1$ and thus MAT could not apply as $d \nmid 1$, which means for all [b] of that a:

$$[a][b] \neq 1$$

Which means that [a] has no multiplicitive inverse. As an illistutive example lets consider [a] = [2] the multiplictive table will give us:

$\cdot$	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
[2]	[0]	[2]	[4]	[6]	[0]	[2]	[4]	[6]

We can thus see that [1] is never a result and thus [a] will never have a multiplictive inverse.