

It: If we have $2 \mid y^2(x^2+3)$ [A] either x is odd (case 1a) or y is even (case 1b) as only in those cases is [A] true for all values of the other value.

* Case 1 is true so we move on to case 2!

Case 2: Assume x is odd or y is even, let $\exists k \in \mathbb{Z}$

Case 2a: x is odd

$$\Rightarrow y^2(x^2+3) \mid y^2+6$$

$$\Rightarrow y^2((2k+1)^2+3) \mid y^2+6$$

$$\Rightarrow y^2(4k^2+4k+1+3) \mid y^2+6$$

$$\Rightarrow y^2(4k^2+4k+4) \mid y^2+6$$

$$\Rightarrow 2y^2(2k^2+2k+2) \mid y^2+6$$

$$\exists a_2 \in \mathbb{Z}, a_2 = \frac{1}{2}y^2(2k^2+2k+2) \mid y^2+6$$

$$\Rightarrow 2a_2 \mid y^2+6$$

$$\Rightarrow 2 \mid 2a_2$$

Case 2b: y is even

$$\Rightarrow y^2+6 \mid x^2+3$$

$$\Rightarrow (2k)^2+6 \mid x^2+3$$

$$\Rightarrow 4k^2+6 \mid x^2+3$$

$$\Rightarrow 2(2k^2+3) \mid x^2+3$$

$$\exists b_2 \in \mathbb{Z}, b_2 = (2k^2+3) \mid x^2+3$$

$$\Rightarrow 2b_2 \mid x^2+3$$

$$\Rightarrow 2 \mid 2b_2$$

We know that in case 2a and case 2b, that $y^2+6 \mid x^2+3$ is divisible by 2. Thus we have proved [2]

Since [1] and [2] is true, the original statement will be true for all x or y integers.