1) $\lim_{n \to \infty} a_n = \infty$ is logically equivalent to:

for every $M > 0$, there exsists an $N \in \mathbb{N}$ so that if $n \geq N$ then:

$$a_n > M$$

The reciprical must hold:

$\Rightarrow \quad \frac{1}{a_n} < \frac{1}{M}$

$\Rightarrow \quad \frac{1}{a_n} - 0 = \frac{1}{a_n} < \frac{1}{M}$

Since a_n is positive: $\left|\frac{1}{a_n} - 0\right| = \frac{1}{a_n} - 0$

$\Rightarrow \quad \left|\frac{1}{a_n} - 0\right| = \frac{1}{a_n} - 0 = \frac{1}{a_n} < \frac{1}{M}$

$\Rightarrow \quad \left|\frac{1}{a_n} - 0\right| = \frac{1}{a_n} < \frac{1}{M}$

If we choose $\epsilon = \frac{1}{M}$, ϵ will have the same range as M or that $\epsilon > 0$

$\Rightarrow \quad \left|\frac{1}{a_n} - 0\right| = \frac{1}{a_n} < \epsilon$

$\Rightarrow$

This means that:

$\Rightarrow \quad \frac{1}{\epsilon} < a_n$

Which will be true if:

$\Rightarrow \quad \frac{1}{\epsilon} < a_N$