Q1) To start we will let c, d be an arbitrary real number, we will also split the piece wise function into five parts, note that if $\boxed{1}, \boxed{2}, \boxed{3}, \boxed{4}, \boxed{5}$ is continuous for the same c, d then the piece wise function is continuous everywhere. for that c, d

$\boxed{1}$ for $x < 2$, $f_1(x) = cx + 4d$,

- $f_1(x)$ is a polynomial, so theres no possible value of x ~~x~~ such that $f_1(x)$ doesnt exist.
- Since c and d are real coefficients in the form $\overset{a \in \mathbb{R}}{\downarrow}\ a_c x + a_d$, there is no value of c and such that $f_1(x)$ doesnt exist.
- We know from the "limit of polynomials" that, $\forall a \in \mathbb{R}$: $\lim_{x \to a} f_1(x) = f_1(a)$,

$\therefore$ Since $\forall x \in \mathbb{R}$, $x < 2$, $\forall c, d \in \mathbb{R}$, $f_1(x)$ exsists and $\forall a \in \mathbb{R}$ $\lim_{x \to a} f_1(x) = f_1(a)$, then $f_1(x)$ must be continuous for any real c and d! $\leftarrow$ (from formal definition of continuity)

$\boxed{2}$ For $2 < x < 3$, $f_2(x) = x^2 + 4$,

- $f_2(x)$ is a polynomial, so theres no possible values of x such that $f_2(x)$ doesnt exsist.
- We know since $f_2(x)$ is a polynomial, that the "limit of polynomials" holds and thus $\forall a \in \mathbb{R}$, $\lim_{x \to a} f_2(x) = f_2(a)$

$\therefore$ $\forall x \in \mathbb{R}$, $2 < x < 3$, $f_2(x)$ exists and $\forall a \in \mathbb{R}$, $\lim_{x \to a} f_2(x) = f_2(a)$, thus from the formal definition of continuity $f_2(x)$ will be continuous for any real c, d

$\boxed{3}$ for $x > 3$, $f_3(x) = dx^2 + \frac{2}{c}x + 1$ $\quad (c \neq 0)$

- For every $c, d, x \in \mathbb{R}$, if $c \neq 0$, $f_3(x)$ will need to exsist because there will be no possible discontinuities as $f_3(x)$ is a simple polynomial
- Since $F_3(x)$ is a polynomial in the form $a_1x^2 + a_2x + 1$ where $a \in \mathbb{R}$ (true as long as $c \neq 0$) the "limit of polynomials" holds such that:

$\lim_{x \to a} f_3(x) = f_3(a)$

$\therefore$ as long as $c \neq 0$, any other value of c, d, x will result in $f_3(x)$ being continous, as it will exsist at every point and $\forall a \in \mathbb{R}$ $\lim_{x \to a} f_3(x) \to f_3(a)$

$\Rightarrow \boxed{1}, \boxed{2}, \boxed{3}$ show that for any x, ($x \neq 2$ or $x \neq 3$) and $c \neq 0$ that $f(x)$ will be continous. $\boxed{4}$ and $\boxed{5}$ will show that $\exists c, d \in \mathbb{R}$, $c \neq 0$, $x = 2$ or $x = 3$ that $f(x)$ is continous

$\boxed{4}$ NTP: $f(x)$ is continous at 2

$\lim_{x \to 2} f(x) = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\Rightarrow f(2^-) = f(2^+)$

$\Rightarrow c \cdot (2) + 4d = (2)^2 + 4$

$\Rightarrow 2c + 4d = 8$

$\Rightarrow c = 4 - 2d \quad \boxed{A}$

$\boxed{5}$ NTP: $f(x)$ is continous at 3

$\lim_{x \to 3} f(x) = \lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$

$\Rightarrow f(3^+) = f(3^-)$

$\Rightarrow d(3)^2 + \frac{2}{c}(3) + 1 = (3)^2 + 4$

$\Rightarrow 9d + \frac{6}{c} = 12$

$\Rightarrow 9dc - 12c + 6 = 0 \quad \boxed{B}$

The

$\boxed{4}$ and $\boxed{5}$ need to be continous for the same c, d. So we sub $\boxed{A} \to \boxed{B}$

$\Rightarrow 9d(4 - 2d) - 12(4 - 2d) + 6 = 0 \quad \leftarrow d \neq 2$ as $6/c = 6/(4 - 2d)$

$\Rightarrow 36d - 18d^2 - 48 + 24d + 6 = 0$

$\Rightarrow -18d^2 + 60d - 42 = 0$

$\Rightarrow -6(3d^2 - 10d + 7) = 0$

$a = \frac{-b \pm \sqrt{b^2-4ac}}{2 \cdot 3} \Rightarrow \frac{10 \pm \sqrt{100-4 \cdot 3 \cdot 7}}{6} \Rightarrow \frac{10 \pm \sqrt{16}}{6} \Rightarrow \frac{10 \pm 4}{6} = 1 \text{ or } \frac{14}{6}$

for $\boxed{4}$ and $\boxed{5}$, $a = 1$ or $\frac{14}{6}$, we will find the corresponding c:

case $\boxed{6}$ $a = 1$

$c = 4 - 2a$

$c = 4 - 2$

$c = 2$

case $\boxed{7}$ $a = \frac{14}{6}$

$c = 4 - 2a$

$c = \frac{12}{3} - \frac{14}{3}$

$c = -2/3$

$\therefore$ when $a = 1$ and $c = 2$, or $a = \frac{14}{6}$ and $c = -2/3$, $\boxed{4}$ and $\boxed{5}$ will be true

or $f(x)$ will be continuous at 2 and 3.

$\therefore$ $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$ is only true $\forall x \in \mathbb{R}$, if $a = 1$ and $c = 2$ or $a = \frac{14}{6}$ and $c = -2/3$,

In other words $f(x)$ will be continuous everywhere if $a = 1$ and $c = 2$ or

$a = 14/6$ and $c = -2/3$.