

Q9. For all natural numbers let q and t , there exists integers r, s such that:

$$18qu - 35tv = 1$$

Assuming the hypothesis, we can rearrange to get that:

$$(9q)(2u) - (7v)(5t) = 1$$

Therefore from this equation we can know some integers $(x_0, y_0 \in \mathbb{Z})$ exist such that:

$$x_0 = 2u \text{ and } y_0 = -5t$$

So our hypothesis equation will become:

$$(9q)x_0 + (7v)y_0 = 1$$

We can thus apply CCT, which tells us that since integers x_0, y_0 exist:

$$\gcd(9q, 7v) = 1$$

If we go back to our hypothesis equation, we know some integers $(x_1, y_1 \in \mathbb{Z})$ exist such that:

$$x_1 = 9q \text{ and } y_1 = -7v$$

So our hypothesis equation will become:

$$x_1(2u) + y_1(5t) = 1$$

We can thus apply CCT, which tells us that since integers x_1, y_1 exist:

$$\gcd(5t, 2u) = 1$$

Therefore for all natural numbers q and t , there exists integers r, s such that:

$$18qu - 35tv = 1$$

We thus know that:

$$\gcd(9q, 7v) = 1 \text{ and } \gcd(5t, 2u) = 1$$

$$\gcd(9q, 7v) = \gcd(5t, 2u)$$