Q05 We will start by letting $\epsilon_1 > 0$ so that if $0 < |x+4| < \delta$, then $|f(x) - 7| < \epsilon$. ASSUME WE CAN COMBINE 1 and 57....

$$f(x)^2 + f(x) + 1 - 57 \implies$$

$$f(x)^2 + f(x) - 56 < \implies$$

$$(f(x)+8)(f(x)-7) < (f(x)+8)\epsilon$$

Crazy shity area.... Let $\epsilon_2 > 0$ so that if $0 < |f(x) - 7| < \delta_2$, then:

$$f(x)^2 + f(x) + 1 - 57 < \epsilon$$

We will let delta have a minimum value of 1 such that:

$$0 < |f(x) - 7| < 1$$

$$7 < f(x) < 8$$

So thus we know that $7 < f(x) < 8$ are the only values we need to be concered with, thus we can find a fixed $\delta_2 \leq 1$:

$$|f(x) + 8| < 8 + 8 = 16$$

Return

$$x^2 + x + 1 = x^2 + x - 2 + 3 = (x+2)(x-1) + 3$$

$$|(x+2)(x-1) + 3|$$

NTP: $x^2 + x + 1 = e$

$$0 <= |x+2| < delta$$

$$0 <= |x+2| < 1$$

$$-3 <= x < -1$$

$$-3(x-1) + 3 = e$$

$$3(-(x-1) + 1) = e$$

$$3(x)$$