

Q02a Let n be a positive integer (greater than 1), then by the Unique Factorization Theorem, n can be expressed as:

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

where p_1, p_2, \dots, p_k , $k \geq 1$ are a list of prime divisors of n and $\alpha_1, \alpha_2, \dots, \alpha_k$ are all non-negative integers. From the DFPF, we know that all of n 's divisors (known as the integer d) are of the form:

$$d = p_1^{\beta_1} p_2^{\beta_2} \cdots p_k^{\beta_k}, \text{ where } 0 \leq \beta_i \leq \alpha_i \text{ for } i = 1, 2, \dots, k$$

We know from DFPF that for each prime factor of d (known as p_i), that its exponent (β_i) must be bounded by: $0 \leq \beta_i \leq \alpha_i$. In other words this means that for p_i the choice of its exponent is between 0, 1, 2, ..., α_i , which means the total combinations it can have is $\alpha_i + 1$.

Divisor (d) combinations for each $p_i = \alpha_i + 1$

Let c_i corresponds to the combinations of p_i for $1, 2, \dots, k$, the total combinations of d will be given by:

$$\text{Total Combinations of } d = (c_1)(c_2) \dots (c_k)$$

Since we know $c_i = \alpha_i + 1$, this can be replaced by:

$$\text{Total Combinations of } d = (\alpha_1 + 1)(\alpha_2 + 1) \dots (\alpha_k + 1)$$

Therefore since d is every positive divisor of n , the number of divisors of n will be given by $(\alpha_1 + 1)(\alpha_2 + 1) \dots (\alpha_k + 1)$.