

Q1A. For S to be a subset of T it would imply that every element of S belongs to T. To show this is the case, I will demonstrate that every solution to S is also solution to T.

To start we know that the solution set of S is given by (where for some $s \in \mathbb{Z}$):

$$c = 8s \pmod{12}$$

Congruent To Remainder (CTR), tells us that:

$$12|(c - 8s)$$

From definition of divisibility we can rewrite this as (for some $t \in \mathbb{Z}$):

$$\begin{aligned} 12t &= c - 8s \\ 12t + 8s &= c \\ 2(6t + 4s) &= c \end{aligned}$$

Therefore we will let some integer $n = 6t + 4s$, we thus find that:

$$2n = c$$

By definition we find that:

$$2|c$$

Which shows that every solution to S is a solution to T, and thus S is a subset of T.

Q1B. Lets look at the element $e = 2$ in set T, we know that:

$$2|2$$

Thus we know that $e = 2$ is a element of T. We know that the solutions (c) to S are of the form (for some integer s):

$$c = [8][s] \pmod{12}$$

Notice that $0 \leq [s] \leq 11$, thus we can construct a table that gives all possible values of c:

$[s]$	0	1	2	3	4	5	6	7	8	9	10	11
$[s][8] \pmod{12}$	0	8	4	0	8	4	0	8	4	0	8	4

Therefore we notice that $e = 2$ never occurs as a solution for any value of s. Therefore we know that the set S does not contain $e = 2$.

More over we can also prove that it doesn't exist within S using Linear Congruence Theorem (LCT). The equation that we will check is:

$$2 = 8s \pmod{12}$$

We know this will have a solution if and only if $d|2$, where $d = \gcd(8, 12) = 4$, however this is false as:

$$4 \nmid 2$$

Thus we have proved via both methods that $e = 2$ does not exist within S. But does exist within T.