

FIVE STAR. ★★★★★

5) We will start by assuming that any integer  $n$  can be expressed as

$\boxed{1} 3k-1$ , or  $\boxed{2} 3k$  or,  $\boxed{3} 3k+1$ . We will prove that  $\boxed{2}$  can be a power of 3.

base case  $k=$  Inductive Proof of  $|Z|$  being a power of 3

(a) base case:  $k = 1$

$3k \Rightarrow 3(1) \equiv 3^1 \leftarrow \text{a power of } 3$

FIVE STAR. ★★★★★

\* Note  $k = 3k$  and  $k = a_0 3^0 + a_1 3^1 + \dots + a_s 3^s$  (For some  $s$  and  $a \in \{-1, 0, 1\}$ )

(b) inductive hypothesis: for any  $k, z \geq k$  such that  $3z$  is a power of 3

③ Inductive Step ( $n \geq 1$ ):

$3(z+1) \geq 3z+3$  ←  $3z$  is a power of 3 so replace with definition

(iii)  $3 \leq s \Rightarrow 3(\alpha_0 z^0 + \alpha_1 z^1 + \alpha_2 z^2 + \dots + \alpha_{s-1} z^{s-1} + \alpha_s z^s)$  ( $\alpha \in [-1, \alpha_0]$  and solve  $I \mapsto S$ )

$$(n \text{ Polyn. in } z) \Rightarrow a_0 z^1 + a_1 z^2 + a_2 z^3 + \dots + a_{s-1} z^s + a_s z^{s+1} + z^1$$

$$\Rightarrow (1+a_0)z^1 + a_1 z^2 + a_2 z^3 + \dots + a_{s-1} z^s + a_s z^{s+1}$$

∴ for any  $a, b$ , where  $0 \leq b \leq 5$ , we have 4 cases:

$(1 + \alpha_D)$  has the interval  $[0, 2]$

Case 1:  $a_b = -1$

Case 2:  $a_b = 0$  or  $a_b + 1$   
( $a_b = -1$ )

Case 3:  $a_b = 1$  or  $a_{b+1}$   
( $a_b \geq 0$ )

\* In this case  $\alpha_0$  rebar  
berms - 1

\* in this case  $a_b$  becomes

1. In this case ab becomes

FIVE STAR. ★★★★★

Case 4:  $a_b \geq 1$  and  $a_b + 1$

$$\begin{aligned} (a_b + 1) 3^b &= (a_b) 3^{b+1} - a_b 3^b \\ &= 3^{b+1} - 3^b \end{aligned}$$



$a_{s+1} \in [-1, 0, 1]$  each  $h_i 3^i$  will be in the interval  $[-1, 0, 1]$ ,  
 So for all  $3(K+1)$ :

$$3(K+1) = n_0 3^1 + n_1 3^2 + n_2 3^3 + \dots + n_s 3^{s+1} + n_{s+1} 3^{s+1}$$

(For some  $s \in \mathbb{Z}$  and  $n_i \in [-1, 0, 1]$ ) (Number of cases)

$\therefore$  We have shown that if a number is of the form  $3K$  it is a power of 3, or

**[1] Proving  $3K-1$ :**

$$\begin{aligned} 3K-1 &\Rightarrow 3(a_0 3^0 + a_1 3^1 + a_2 3^2 + \dots + a_{s-1} 3^{s-1} + a_s 3^s) - 1 \text{ (For some } s, a_i \in [-1, 0, 1]) \\ &\Rightarrow (a_0 3^1 + a_1 3^2 + a_2 3^3 + \dots + a_{s-1} 3^{s-1} + a_s 3^{s+1}) - 3^0 \\ &\Rightarrow -1 3^0 + a_0 3^1 + a_1 3^2 + a_2 3^3 + \dots + a_{s-1} 3^s + a_s 3^{s+1} \end{aligned}$$

- Since for any  $a_i$ , the interval of  $a_i \in [-1, 0, 1]$  it is a power of 3

**[2] Proving  $3K+1$**

$$\begin{aligned} 3K+1 &\Rightarrow 3(a_0 3^0 + a_1 3^1 + a_2 3^2 + \dots + a_s 3^s) + 1 \text{ (For some } s, a_i \in [-1, 0, 1]) \\ &\Rightarrow a_0 3^1 + a_1 3^2 + a_2 3^3 + \dots + a_s 3^{s+1} + 3^0 \\ &\Rightarrow 3^0 + a_0 3^1 + a_1 3^2 + a_2 3^3 + \dots + a_s 3^{s+1} \end{aligned}$$

- Since for any  $a_i$  the interval of  $a_i \in [-1, 0, 1]$  then any  $3K+1$  is a power of 3

Since every number falls into **[1]**, **[2]** or **[3]**, and since **[1]** and **[2]** or **[3]** is a power of 3, then every positive number is a power of 3