Robert (Robbie) Knowles MATH 135 Fall 2020: WA09

Q01A We know that the possible congruence classes [a] of $\mathbb{Z}_7$ are:

$$[a] \equiv [0], [1], [2], [3], [4], [5], [6]$$

These congruence classes would have the following addition table:

+	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[0]	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[1]	[1]	[2]	[3]	[4]	[5]	[6]	[0]
[2]	[2]	[3]	[4]	[5]	[6]	[0]	[1]
[3]	[3]	[4]	[5]	[6]	[0]	[1]	[2]
[4]	[4]	[5]	[6]	[0]	[1]	[2]	[3]
[5]	[5]	[6]	[0]	[1]	[2]	[3]	[4]
[6]	[6]	[0]	[1]	[2]	[3]	[4]	[5]

They would also have the following multiplication tables:

·	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[2]	[0]	[2]	[4]	[6]	[1]	[3]	[5]
[3]	[0]	[3]	[6]	[2]	[5]	[1]	[4]
[4]	[0]	[4]	[1]	[5]	[2]	[6]	[3]
[5]	[0]	[5]	[3]	[1]	[6]	[4]	[2]
[6]	[0]	[6]	[5]	[4]	[3]	[2]	[1]

Q01B Visually we know that $\mathbb{Z}_7$ is a field, if we look at the multiplication table, each possible congruence class [a] has a corrisponding congruence class $[b]^{-1}$ such that:

$$[a][b] \equiv 1$$

This happens because 7 is a prime and [a] is co-prime to 7. This means that $d = \gcd([a], 7]) = 1$ and by definition of MAT since $d|1$ there must be a solution [b] for each [a] that solves the above equality (which means [a] will have a multiplictive inverse). This means that $\mathbb{Z}_7$ is a field

On the other hand if we take $\mathbb{Z}_8$, we know that 8 is not prime and thus not all [a]'s are co-prime to 8. If [a] is not coprime to 8 this would result in $d = \gcd([a], [8]) \neq 1$ and thus MAT could not apply as $d \nmid 1$, which means for all possible [b] values of that given [a]:

$$[a][b] \not\equiv 1$$

Which means that [a] has no multiplicitive inverse. As an illistutive example lets consider [a] = [2] the multiplictive table will give us:

$\cdot$	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
[2]	[0]	[2]	[4]	[6]	[0]	[2]	[4]	[6]

We can thus see that [1] is never a result and thus [a] will never have a multiplictive inverse in $\mathbb{Z}_8$, thus it is not a field.