Q 8) Let $\forall x, y \in \mathbb{Z}$ such that:

$$2 \mid (x^2+3)(y^2+6) \iff (2 \nmid x) \vee (2 \mid y)$$

* This is logically equivilent too: [1] and [2]:

[1] $2 \mid (x^2+3)(y^2+6) \Rightarrow (2 \nmid x) \vee (2 \mid y)$

[2] $(2 \nmid x) \vee (2 \mid x) \Rightarrow 2 \mid (x^2+3)(y^2+6)$

Case [1]: Assume $2 \mid (x^2+3)(y^2+6)$

$\Rightarrow 2 \mid (x^2y^2 + 6x^2 + 3y^2 + 18)$

$\Rightarrow 2 \mid y^2(x^2+3) + 6x^2 + 18$

$\Rightarrow 2 \mid y^2(x^2+3) + 2(3x^2+9)$

$\Rightarrow 2a = y^2(x^2+3) + 2(3x^2+9) \leftarrow (\exists a \in \mathbb{Z}$

$\Rightarrow 2a = y^2(x^2+3) + 2b \leftarrow (\exists b \in \mathbb{Z}, b = 3x^2+9)$

$\Rightarrow 2a - 2b = y^2(x^2+3)$

$\Rightarrow 2(a-b) = y^2(x^2+3)$

$\Rightarrow 2c = y^2(x^2+3) \leftarrow (\exists c \in \mathbb{Z}, c = a-b)$

$\Rightarrow 2 \mid y^2(x^2+3)$ [A]

to find when this is true we will do 4 cases ($\exists k \in \mathbb{Z}$) on

Case 1a: x is odd	Case 1b: x is even	Case 1c: y is odd	Case 1d: y is even:
$\Rightarrow x = 2k+1$	$\Rightarrow x = 2k$	$\Rightarrow y = 2k+1$	$\Rightarrow y = 2k$
[A] $\Rightarrow 2 \mid y^2(x^2+3)$	[A] $\Rightarrow 2 \mid y^2(x^2+3)$	$\Rightarrow 2 \mid y^2(x^2+3)$	$\Rightarrow 2 \mid y^2(x^2+3)$
$\Rightarrow 2 \mid y^2(4k^2+4k+1+3)$	$\Rightarrow 2 \mid y^2(4k^2+3)$	$\Rightarrow 2 \mid (4k^2+4k+1)(x^2+3)$	$\Rightarrow 2 \mid 4k^2(x^2+3)$
$\Rightarrow 2 \mid 2(y^2(2k^2+2k+2))$	$\Rightarrow$ Not true $\forall$ cases	$\Rightarrow$ Not true $\forall$ cases	$\Rightarrow 2 \mid 2(2k^2(x^2+3))$
$\exists a_1 \in \mathbb{Z}, a_1 = (2k^2+2k+2)y^2$		$\exists b_1 \in \mathbb{Z}, b_1 = 2k^2(x^2+3)$	
$\Rightarrow 2 \mid 2a_1$		$\Rightarrow 2 \mid 2b_1$	
$\Rightarrow$ True for all cases		$\Rightarrow$ true for all cases	