Q02A We will disprove by counter example, consider when $a = -2$ and $b = 4$, Note that:

1. $\min(a, b) = \min(-2, 4) = -2$.
2. $\gcd(a, b) = \gcd(-2, 4) = 2$.

However since we know that $-2 < 2$, this would imply that for $a = -2$ and $b = 4$, $\min(a, b) < \gcd(a, b)$. Therefore since this violates the universality of the original statement, the origonal statement is disproved.

Q02B We will disprove by counter example, consider when $a = 0$ and $b = 2$, Note that:

1. $\min(|a|, |b|) = \min(|0|, |2|) = \min(0, 2) = 0$.
2. $\gcd(a, b) = \gcd(0, 2) = 2$.

However since we know that $0 < 2$, this would imply that for $a = 0$ and $b = 2$, $\min(|a|, |b|) < \gcd(a, b)$. Therefore since this violates the universality of the original statement, the origonal statement is disproved.

Q02C We will prove by using case analysis, note for all cases $a, b \neq 0$ and a,b are integers. **The first case will be when a = b:**

1. $\min(|a|, |b|) = a$
2. $\gcd(a, b) = a$

We thus have that $\min(|a|, |b|) = \gcd(a, b)$ when $a = b$, proving the statement for the first case. **The second case will be when $a > b$:**

1. $\min(|a|, |b|) = b$
2. Let d be an integer such that $\gcd(a, b) = d$

Since d must be a common divisor of a and b, d is bounded by:

$$d \leq b < a$$

As d is the $\gcd(a, b)$ and b is the $\min(|a|, |b|)$ we have that:

$$d \leq b < a$$
$$gcd(a, b) \leq \min(|a|, |b|) < a$$

Therefore when $a > b$, we will have $gcd(a, b) \leq \min(|a|, |b|)$ proving the statement for the second case. **The third case will be when $a < b$:**

1. $\min(|a|, |b|) = a$
2. Let d be an integer such that $\gcd(a, b) = d$

Since d must be a common divisor of a and b, d is bounded by:

$$d \leq a < b$$

As d is the $\gcd(a, b)$ and a is the $\min(|a|, |b|)$ we have that:

$$d \leq a < b$$
$$gcd(a, b) \leq \min(|a|, |b|) < b$$

Therefore when $a < b$, we will have $gcd(a, b) \leq \min(|a|, |b|)$ proving the statement for the third case.

Since the three cases cover all the possible values of a,b and they are are true for each case, it must be that for all non-zero integers a and b, $\gcd(a, b) \leq \min(|a|, |b|)$.