Q13) Let s and t be an arbitrary integer with $s \leq t$ and

$2\binom{t}{s-1} = \binom{t}{s}$:

$$\Rightarrow 2 \cdot \frac{t!}{(t-s+1)!\,(s-1)!} = \frac{t!}{s!\,(t-s)!}$$

$$\Rightarrow 2 \cdot \frac{t!}{(t-s+1)\,(t-s)!\,(s-1)!} = \frac{t!}{s\,(s-1)!\,(t-s)!}$$

$$\Rightarrow \frac{2}{(t-s+1)} = \frac{1}{s}$$ ← cancel out common $t!$, $(s-1)!$ and $(t-s)!$

$$\Rightarrow 2s = t-s+1$$

$$\Rightarrow 3s = t+1$$

∴ Since s can be any integer, this is our definition of divisibility so we can re arrange:

$$3s = t+1 \equiv 3 \mid (t+1)$$

∴ we have shown for all integers s and t, that when $2\binom{t}{s-1} = \binom{t}{s}$, then $3 \mid (t+1)$