"How does the Temperature of a Wire Affect its Velocity Through a Medium?"

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Words: 4000 (approximately)

Introduction:

Ice cream cakes are a big tradition in my family. While I find them much tastier than their solid counterpart, they are much more difficult to cut. In fact, my parents will always put the knife in hot water first, as they state that it "makes it much easier to cut". I took this for fact, and I only started to wonder why this is when my friend showed me a YouTube video[1]. In this video, a super-heated knife is used to cut everyday items with more ease then its unheated counterpart.

From these two examples, I came up with a hypothesis that the hotter cutting tool, the faster it will cut. Before this paper derives a more specific research question to help prove/disprove the hypothesis, relevant background information will be explored to see if this hypothesis has been tested in the real world.

Background Information:

Heated blades are used in the real world, but not in the way our hypothesis assumes. Heated blades are only used in some specialized cases like for cauterizing wounds[2] and trimming thermoplastic[3]. The vast majority of papers I found on heat blades, did not talk about the blades themselves but rather how to remove heat from the blades. This is because hotter blades are less structurally stable, as a wire of length l would elongate by Δl when temperature rises by ΔT.

Figure 1: Expansion of a Wire Due to an Increase in Temperature

The formula for change in length due to change in temperature is given by formula 1 [1]:

$$\Delta l = \alpha \Delta T l \quad [1]$$

Where α is the thermal coefficient with units (1/℃) which describe the extent to which a material expands when it is heated. This is relevant to our experiment, for if the wire is bound at both sides, it will experience stress (as shown by Figure 2).

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1 https://www.youtube.com/watch?v=hjSheQ7LgJ4
2 https://onlinelibrary.wiley.com/doi/abs/10.1002/lsm.1900020113
3 http://www.freepatentsonline.com/4679474.html

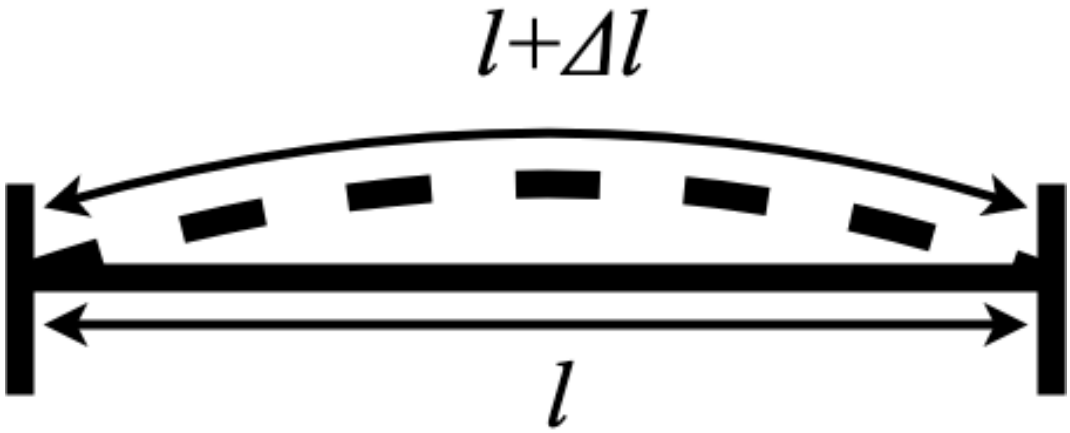


Figure 2: Structural Failure of Wire as it Expands due to Temperature Change

This is consistent with other papers and scientific literature, that state: “High cutting temperature… reduces tool life and also impairs the product quality”[4]. As seen in [1], the increase in temperature reduces the quality of the wire as it expands, which will make it structurally weaker and cut/move at a slower rate. In summary, both the scientific literature and background theory suggest that our hypothesis is incorrect and that the temperature of the wire is inversely proportionate to the speed at which it cuts.

Research Question

In order to test our hypothesis and determine what effect temperature has on the cutting speed of a blade, this paper will ask how change in temperature of a Nichrome wire will impact the speed at which it cuts through Styrofoam of a constant density.

To answer this research question, this paper will use the following experiment: we will start by attaching a nichrome wire to a weight and hook it up to a circuit. We then vary the resistance in that circuit to change the temperature of the wire. We will then place it on top of a Styrofoam cube and filming the apparatus, we can measure how long it takes to cut through, and determine how heat effects the wire’s velocity.

Independent variable: Temperature of the wire, with a range of $700°C - 900°C$.
How we differ the independent variable: Temperature will be controlled by changing the resistance and current of series circuit, as:

$$Power\ (P) = Voltage\ (V)\ \cdot\ Current\ (I)\ \ [2]$$

Where we will assume power (P) represents thermal energy dissipated. Voltage (V) represents energy dissipated per charge and current (I) which represents amount of charge. By applying Ohm’s law to [2] we get:

$$Power = Current\ (I)^2\ \cdot\ Resistance\ (R)\ \ [3]$$

Since power (P) represents thermal energy dissipated, which is proportionate to temperature, so:

$$Temperature \propto Current\ (I)^2 \cdot\ Resistance\ (R)\ \ [4]$$

[4] https://www.sciencedirect.com/science/article/abs/pii/S089069550600229X

Effect on Dependant Variable: Similar to our hypothesis, the expected result of a temperature increase is a subsequent increase in the velocity of the wire.
Limitations of our method to vary the independent variable: The equation [4] we use for finding temperature change has 2 major assumption. In real life, power is converted into many different types of energy, but for the sake of calculation, we assume the power will only contribute to changes in thermal energy and temperature. The second assumption we make in [4] is that Ohm's law holds true even though our resistor is over 700°C.

Dependant variable: Velocity of the wire (ms^{-1})
How we determine the velocity: If we know the positions of the wire, and the amount of time that passed between each point, we can average that rate of change of displacement to find average velocity. This is shown in the following equation:

$$\vec{v}_{average} = \frac{\sum_0^n \frac{\overrightarrow{D_n}}{t}}{(n-1)} \quad [5]$$

Where $\overrightarrow{D_n}$ represents the displacement since the last point, *t* represents the time between points and *n* represents the number of data points.
Why we use this method: While the velocity could have been found by dividing change in distance by change in time, this method should be more accurate as we use many more then just 2 data points. If we so choose in our evaluation, it will also allow us to distinguish speeds between different time quarters, whereas the other method for average velocity would not.
Limitations of our method of measuring the dependant variable: This method is limited by the number of frames in the camera, as because we are finding the summation of many data points, we will have more uncertainty then if we only took a reading at the start and end.

Controlled Variables

Length of the Nichrome Wire: The nichrome has a constant starting length of 0.0315*m*.
Why we control it: We control it because a longer wire requires more current to reach the same temperature as a smaller wire
How we control it: As seen by [1], a change in temperature will cause the wire to expand. To minimize this, we allow the wire to cool in the air for 2 minutes after each experiment

Density of Cutting Medium: The Styrofoam used has a constant density of 50kgm^{-3}.
Why we control it: If the density of the cutting medium changes, even if the wire is at constant temperature, the velocity of the wire will change.
How we control it: By only using 4-inch-tall and 1-inch-wide slices from the same Styrofoam sheet, the density of each of the slices should remain constant.

U-Shaped Weight used to House the Wire: The weight used has a constant mass of 1*kg*.
Why we control it: We need to use acceleration due to gravity to have the wire move through the Styrofoam. If the mass of the weight changes, we will not be able establish if changes in velocity are due to changes in the mass of the weight or changes in temperature of the wire.
How we control it: We will use the exact same U-shaped mass, and check after every test if its mass changes. If the mass of the U-Shaped weight does change then we increase it.

Calibration of Tools used*:* Height of camera set to 1.3m and Ammeter set to 40A.

Why we calibrate: If they camera angle changes, it will change the relative height of the wire which will skew our data. If the Ammeter is on a different setting the precision and accuracy of our readings will change.

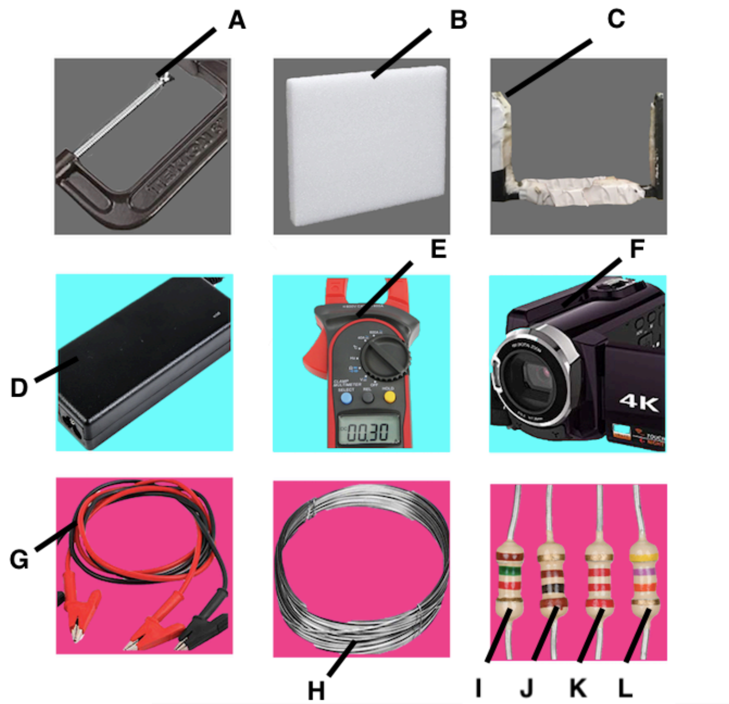
List of Apparatus:

Structural Materials:
- 4 x Clamps (A)
- U-Shaped 1kg weight I
- 4-inch-tall Styrofoam (B)

Electronics:
- 12V power supply (D)
- Clamp ammeter I
- Camera (F)

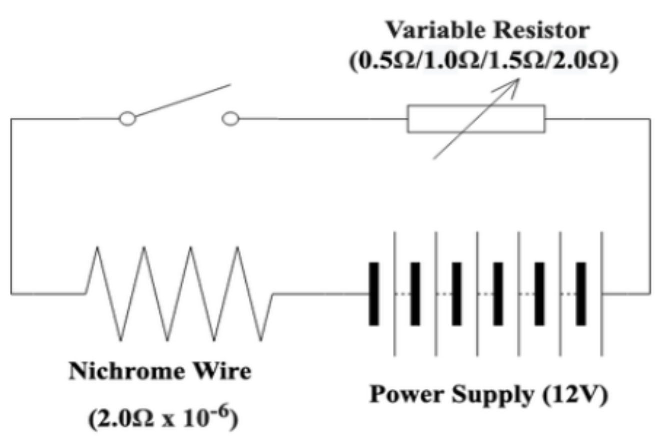
Circuit Equipment:
- 8 x Wires with Clips (G)
- 24 AWG nichrome wire (H)
- 0.5Ω resistor (I)
- 1Ω resistor (J)
- 1.5Ω resistor (K)
- 2Ω resistor (L)

Not Pictured:
- Respirator mask
- 2-meter plank of wood
- 10-inch ruler
- Leather gloves
- Water pitcher



* Resistor Color Code does not match Actual Resistance

Figure 3: Series Circuit Used to Heat the Nichrome Wire.



Method:

Safety: We will be dealing with a wire reaching temperatures exceeding 900 °C. A wire this hot poses a multitude of threats: fumes (resulting from deposition of Styrofoam), burns (from the wire surface), electric shock (from the circuit). To minimize the threat of inhalation, I made sure to wear a respirator and I undertook the experiment outside. To minimize the burning threat, I made sure to wear leather gloves and I had a pitcher of water on hand. To neutralize the threat of electric shock, I had a switch to cut power. On top of this, I did this experiment in eye sight of my family just in case anything went wrong.

Preparation Phase up: Before we begin with our actual experiment, we set up our apparatus for testing.

1) Clamp the plank down to a flat surface such that you have an overhang of roughly one meter.
2) Adjust the ruler and place tape on it such that it is flush with the board and standing straight.
3) Attach the nichrome wire to the u-shaped weight and add two wires on either end.
4) Attach one of those wires to one end of the switch (make sure it is open), and the other to the ground.
5) Plug in the power supply and put one wire in the positive 12V end, set the portable clamp ammeter to the 40A setting.

Experimental Phase: Now that everything is set up, we can repeat this phase to get data.
A video showing the experimental phase is available at: https://bit.ly/2W2isNG

6) Attach one of the resistors (0.5Ω, 1Ω, 1.5Ω, 2Ω) depending on the test. These should be attached to the two open wires.
7) Place the wire and weight on the Styrofoam, start the camera and make sure that everything is in frame. We use a camera rather than just a regular timer as it allows us to accurate view the position of the wire every 60^{th} of a second.
8) Flick the switch and watch as the wire cuts through the Styrofoam.
9) When the experiment is over, use the ammeter to find the current going through the wire and then wait 2 minutes. We wait 2 minutes to allow the wire to contract again to help reduce random error.
10) Keeping repeating the experimental phase until you have 5 trials for each of the 4 tests. While we could do only one trial, this will allow us to average all the trials to reduce random error.

Data Collection:

Qualitative Data:

The first thing I noticed is that the larger the resistance in the circuit, the less smoke would appear as the Styrofoam is cut. For resistances greater than 1.5 Ohms the blade took much longer to start then in comparison to the lower resistances. As well as the resistance gets lower, the device would skew more in contrast to that of the lower resistances. This would contribute to systematic error as some of the energy would be wasted on rotational energy.

Quantitative Data:

After each video I would go frame by frame and use the ruler to determine the position of the wire at that time. After going through all 20 videos, I had 20 tables (4 tables for each of the 5 trails). I complied these 20 tables into 4 averaged tables, which you can see the first 17 values of underneath, along with a graph (Figure 4) which shows all of averaged values of velocity for each trial:

0.5 Ω Average Velocity		1 Ω Average Velocity		1.5 Ω Average Velocity		2 Ω Average Velocity	
Position (±0.5mm)	Frames (± 1)	Position (±0.5mm)	Frames (± 1)	Position (±0.5mm)	Frames (± 1)	Position (±0.5mm)	Frames (± 1)
210.0	0	210.0	0	210.0	0	210.0	0
210.0	1	210.0	1	210.0	1	210.0	1
210.0	2	210.0	2	210.0	2	210.0	2
210.0	3	209.8	3	210.0	3	210.0	3
209.8	4	209.8	4	210.0	4	210.0	4
209.4	5	209.6	5	209.8	5	209.8	5
209.0	6	209.4	6	209.6	6	209.7	6
208.6	7	209.1	7	209.4	7	209.6	7
208.2	8	208.7	8	209.2	8	209.4	8
207.6	9	208.4	9	208.9	9	209.4	9
206.8	10	208.0	10	208.6	10	209.1	10
206.2	11	207.6	11	208.2	11	208.8	11
205.4	12	207.3	12	207.8	12	208.5	12
204.4	13	206.8	13	207.5	13	208.1	13
203.4	14	206.2	14	207.0	14	207.8	14
202.0	15	205.4	15	206.5	15	207.3	15
200.8	16	204.4	16	205.8	16	206.6	16
199.4	17	202.9	17	204.9	17	206.1	17

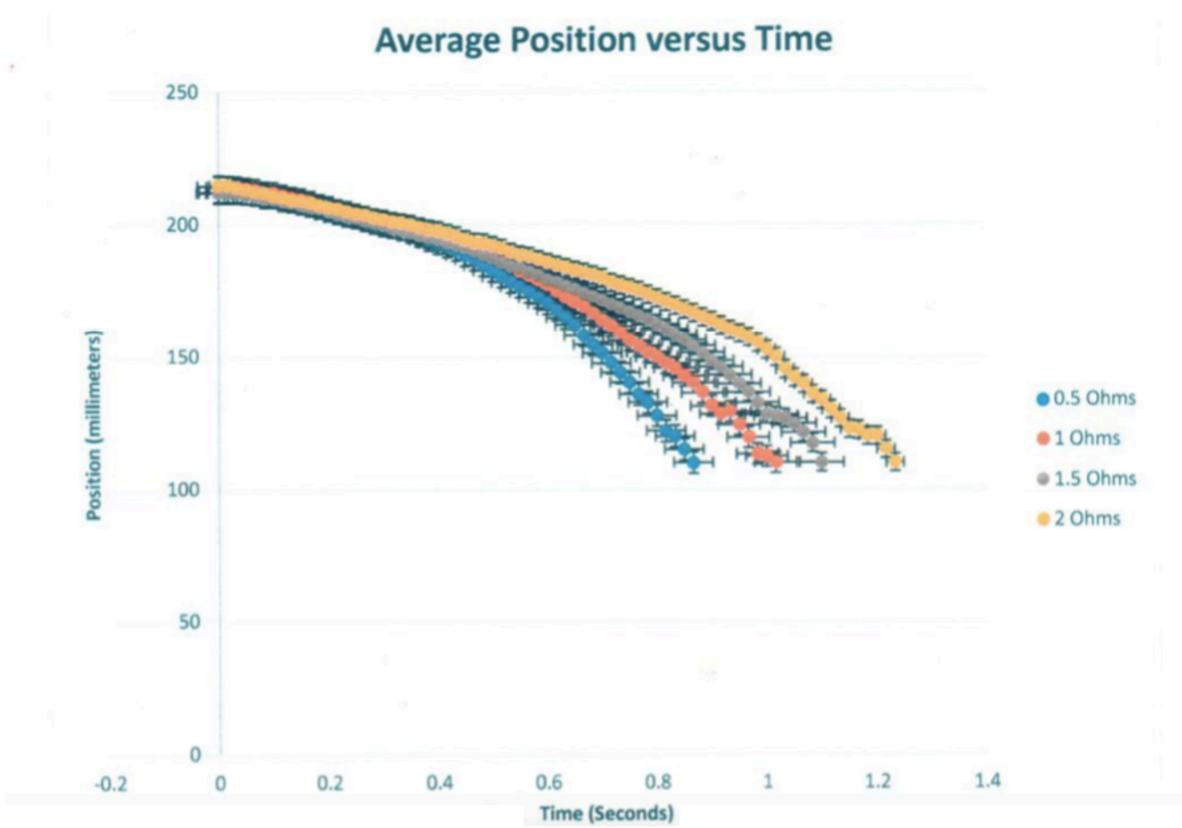


Figure 4: A Graph Showing the Average Position Across all 4 Tests.

As stated previously, each of these averages are from 5 trials of the same resistance. For the sake of visualization, I included Figure 5 which shows the first 10 frames (for the third trial of the 0.5Ω test) and a graph showing the values for all 51 frames of that test:

Position (± 0.5mm)	Frames (± 1)
210.0	0
210.0	1
210.0	2
210.0	3
210.0	4
209.0	5
209.0	6
208.0	7
207.0	8
207.0	9
206.0	10

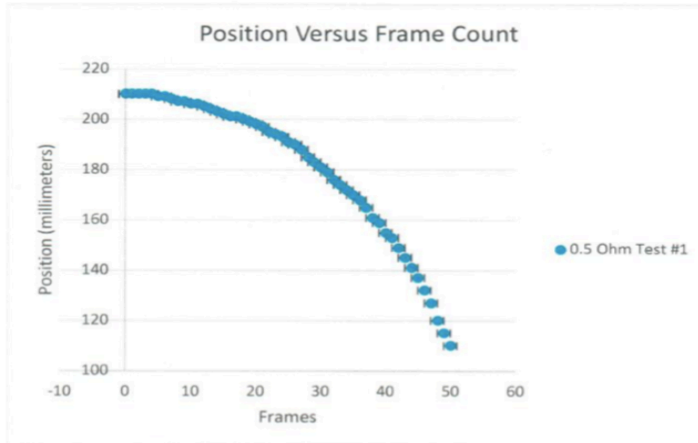


Figure 5: Sample Raw Data and Full Graph for the 0.5Ω Test, Third Trail.

Uncertainty of Figure 5 and Figure 6: The value for positional uncertainty was ± 0.5mm. This is because the smallest value the ruler could show is 1mm, and because its analog we must further divide it by two to arrive at our uncertainty. The frames however are recorded digitally and thus they have an uncertainty of ± 1 frame or ± 0.0017 seconds.

As mentioned in Step 9 of our method, we also need to figure out the current going through the wire using a portable ammeter. Because, in order solve for the temperature of the wire, we will need the current going through the wire as seen by [4]. Figure 6 shows the average current for each of the tests I did.

	0.50 Ω	1.00 Ω	1.50 Ω	2.00 Ω
Uncertainty (± 0.01A)	8.32A	7.98A	7.31A	6.26A

Figure 6: A Table Representing How Average Current Changes with Different Resistances.

Uncertainty of Figure 6: 0.01A was picked as the uncertainty, as it that is the smallest value that the ammeter can show and the ammeter is an analog measuring device.

Processing the Data

To solve for our independent and dependant variable we need to need to do 2 calculations:

1) Use the average positions to find average velocity across all 4 tests.
2) Use the current to solve for temperature.

701. **Solving for Average Velocity:**

We can now use our equation [5] to solve for average velocity. While I could have only used two points and found the average velocity by doing distance change by time change, our equation [5] should give more efficient results. To make this more clear, I will do a sample calculation with 3 data points, so:

$$\vec{v}_{average} = \frac{\sum_0^n \frac{\overrightarrow{D_n}}{t}}{(n-1)} \quad [5]$$

Thus becomes:

$$\vec{v}_{average} = \frac{\frac{(position\ 1 - position\ 2)}{1\ frame} + \frac{(position\ 2 - position\ 3)}{1\ frame}}{2}$$

Sample Calculation: The following table shows the first 2-3 frames of data for the 1Ω test.

Frame 1.0 ± 1.0	Frame 2.0 ± 1.0	Frame 3.0 ± 1.0
214.4 ± 1.0 mm	214.2 ± 1.0 mm	213.9 ± 1.0 mm

$$\vec{v}_{average} = \frac{\frac{(214.4\ \pm\ 1.0\text{mm} - 214.2\ \pm\ 1.0\text{mm})}{1.0\ \pm\ 1.0\ frame} + \frac{(214.2\ \pm\ 1.0\text{mm} - 213.9\ \pm\ 1.0\text{mm})}{1.0\ \pm\ 1.0\ frame}}{2}$$

However, since a frame is just a 60th of a second, we will just convert it into seconds. Note that because of the way that VLC calculates frame position, we assume an uncertainty of a frame.

$$\vec{v}_{average} = \frac{\frac{(0.2\ \pm\ 2.0\text{mm})}{0.017\ \pm\ 0.017s} + \frac{(0.3\ \pm\ 2.0\text{mm})}{0.017\ \pm\ 0.017s}}{2}$$

If we convert all the absolute uncertainties into relative uncertainties, we will get the following.

$$\vec{v}_{average} = \frac{\frac{(0.2\text{mm}\ \pm\ 100\%)}{0.017s\ \pm\ 100\%} + \frac{(0.3mm\ \pm\ 66.7\%)}{0.017s\ \pm\ 100\%}}{2}$$

$$\vec{v}_{average} = \frac{(11.77mms^{-1}\ \pm 200.0\%) + (17.65mms^{-1}\ \pm 166.7\%)}{2}$$

$$\vec{v}_{average} = 5.89mms^{-1}\ \pm 100.\% + \ 8.82mms^{-1}\ \pm 83.3\%$$

$$\vec{v}_{average} = 14.71mms^{-1}\ \pm 183.3\%$$

Converting percent uncertainty into absolute uncertainty and millimetres into centimetres.

$$\vec{v}_{average} = 0.0147 \pm 0.0270ms^{-1}$$

By using excel, I did the exact same thing except across all 4 tests with every single data point. Figure 7 demonstrates the average velocity for all 4 tests.

Test 1 with 0.5 Ω	Test 2 with 1.0 Ω	Test 3 with 1.5 Ω	Test 4 with 2Ω
1.163 ± 1.374ms^{-1}	0.922 ± 1.102ms^{-1}	0.702 ± 0.894ms^{-1}	0.533 ± 0.764ms^{-1}

Figure 7: Average Velocity Across all 4 Tests.

2. Solving for Temperature:

In order to solve for temperature, many labs will use infrared cameras to get accurate readings. I do not have access to an infrared camera and, so the next best thing would be to use our temperature equation that we derived before:

$$Temperature \propto Current\ (I)^2\ * Resistance\ (R)\ \ [4]$$

While we have to make the assumption that Power is equal to heat, I believe that this equation will give us the most accurate results. However, because resistance is constant, we can get:

$$Temperature = n\ \cdot Current\ (I)^2\ \ [4a]$$

Where *n* is a constant. To solve for n, the formula was rearranged to get:

$$n = \frac{Temperature}{Current\ (I)^2}\ \ [4b]$$

The wire I was using was rated at 24 AWG, which stands for American Wire Gage and it is a measure of a wires' cross-sectional area. By using the AWG and a conversion table[5], I found that a wire with a current of 8.76A should have a temperature of 982 °C.

$$n = \frac{982℃}{8.76A^2} = 12.8\ ℃A^{-2}$$

This means that our new equation for Temperature is:

$$Temperature = 12.8\ ℃A^{-2}\ \cdot Current\ (I)^2\ \ [4c]$$

By taking our table of values for current (as seen below), we can now use this equation to convert each one into its corresponding temperature.

	0.50 Ω	1.00 Ω	1.50 Ω	2.00 Ω
Uncertainty (± 0.01A)	8.32A	7.98A	7.31A	6.26A

Sample Calculation: Solving for temperature for the 0.5 Ω test.

$Temperature = 12.8\ ℃A^{-2}\ \cdot Current\ (I)^2\ \ [4c]$

$Temperature = 12.8℃A^{-2}\ \cdot (8.32\ \pm\ 0.01A)^2$

$Temperature = 12.8℃A^{-2}\ \cdot (8.32A^2\ \pm\ 1.2\%)^2$

$Temperature = 12.8℃A^{-2}\ \cdot (69.22A^2\ \pm\ 2.4\%)$

$Temperature = 886℃\ \pm\ 2.4\%$

$Temperature = 886℃\ \pm\ 21℃)$

The following table shows the exact result of the same calculation but for the four other tests.

0.5 Ω	1.0 Ω	1.5 Ω	2.0 Ω
$886.0\ \pm\ 21℃$	$815.7\ \pm\ 20℃$	$684.3\ \pm\ 17℃$	$501.9\ \pm\ 13℃$

Figure 8: Temperature for Wire for each Trials

[5] https://web.archive.org/web/20120920075813/http://www.pelicanwire.com/category/formulas-resistance/

Evaluation of Data:

By combining both of the tables above into a graph, we are able to draw a correlation between temperature and velocity of the wire. Figure 9 shows the relationship between average velocity and Temperature for all 4 trials.

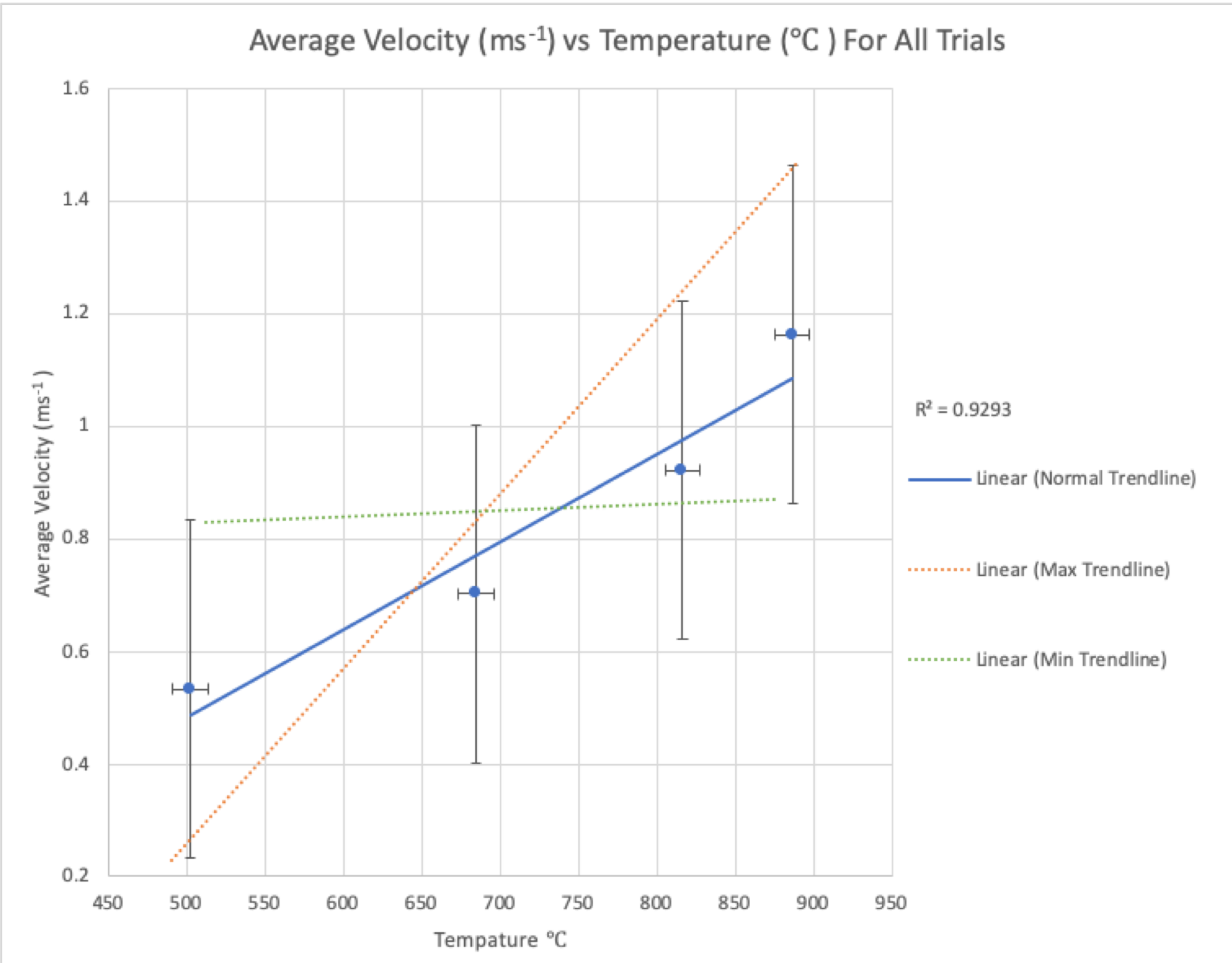


Figure 9: Graph Showing the Relation Between Temperature and Average Velocity.

Figure 9 shows us that there is a positive correlation between the temperature of the wire and the velocity of the wire. This correlation is positive as both the minimum trendline and the maximum trendline have a positive gradient. Specifically, the minimum gradient is $0.0002 ms^{-1} °C^{-1}$. and the maximum gradient is $0.0049 ms^{-1} °C^{-1}$. This means that for every single possible outcome within our error, there is a positive correlation between temperature and velocity. This huge fluctuation in the two gradients is due to our very large uncertainty in finding average velocity. However, the trendline that fits best has an R^2 of 0.9101 has a gradient of $0.0026 ms^{-1} °C$, we are assuming this correlation is linear, but with such high error it is hard to tell.

Conclusion:

Our findings strongly support our hypothesis that the temperature of the wire has a positive correlation with the velocity of the wire. Our raw data supports this as tests with lower temperatures had less data points as they reached the bottom faster. It is also supported strongly by our qualitative observations as wires that were less hot visibly accelerated slower. Finally Figure 8 and our evaluation support this hypothesis as even our minimum trend line has a positive correlation. Therefore, we can conclude that a 24AWG Nichrome Wire being pulled by a weight will speed up through Styrofoam if heated (above 700 ℃).

As evident by our very specific conclusion, a big limitation of our research is that our findings are not universal. If the object that the blade is trying to cut changes, heating the object may not have any affect. As well above certain temperatures there may not be a impact when changing the temperature. As will be discussed in the final section, certain improvements can be made to the experiment to allow us to draw more relevant conclusions.

Errors and Assumptions in our Model:

We made quite a few assumptions in our model which partly result in the very large error seen in figure 8. The major assumption that we made is that the relationship between current and temperature remains constant as the temperature raises. When we found n in equation [4c], that assumes that the nichrome wire is ohmic and thus follows Ohms Law. Ohms law states that resistance (R) is equal to voltage (V) divided by current (I) and thus we used this law to solve for power and thus temperature, as we assumed the nichrome wire was ohmic. In reality, it's very unlikely that at temperatures such above 700 the wire would be ohmic, this would add to systematic error, skewing change in temperature in the downwards direction. The next assumption we made is that the temperature of the wire would not be changing, when in reality, wind and the air would dissipate some of the heat. This would also skew change in temperature in the downwards direction, downplaying the affect that current has. Finally, we assume that the wire is traveling straight down the Styrofoam, when in reality it often skews a bit to the side. This contributes to random error as the temperature can remain constant but the velocity that the wire travels at can change.

A limitation of our method is that we only took values for temperature in a very small range. As seen from Figure 8, it appears that if we took more data points, we might have had a hyperbolic function. This can be seen as our 0.5Ω test is quite a bit above our 1Ω, possibly indicating that at higher temperatures, changes in temperature result in greater changes in the velocity of the wire.

Improvements:

As stated in our conclusion, our results are not very precise and very specific. Precision can be solved if we had a higher frame per second camera as our random error when trying to calculate the position would be much less. As well if we were able to do the test in a vacuum less heat from the wire would be able to dissipate which should help the accuracy of the method. Finally, the reliability of the results could improve if we could find a material that's density is more consistent then Styrofoam.

However, none of those changes help with how specific our findings are. To solve this, I would try to get a larger range of temperature data. That is because a 350℃ range isn't large enough to accurately see the relationship. As well I would try changing the material we are cutting, it's very possible that Styrofoam has a special property that allows this correlation for temperature and velocity to exist. But if we are able to test different materials maybe we can learn if this relationship is constant no matter the mass we are cutting.

Future Work and Exploration:

We proved that our hypothesis was correct and that there is a positive relationship between the heat of the nichrome wire and the speed in which it cuts. To extend on our improvements and the limitations that we have just described, I would propose that a new research question should examine how the temperature of the wire changes its resistance. By solving this research question, we could learn more about the assumptions that this paper made and thus can gage how accurate our results were.

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