Network Balance and Transitivity

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Network Analysis Reading Group - UT Austin

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Roadmap

- Heider's balance theory
- Balance theory in social network analysis
- Transitivity
- Application
- Linking balance and transitivity

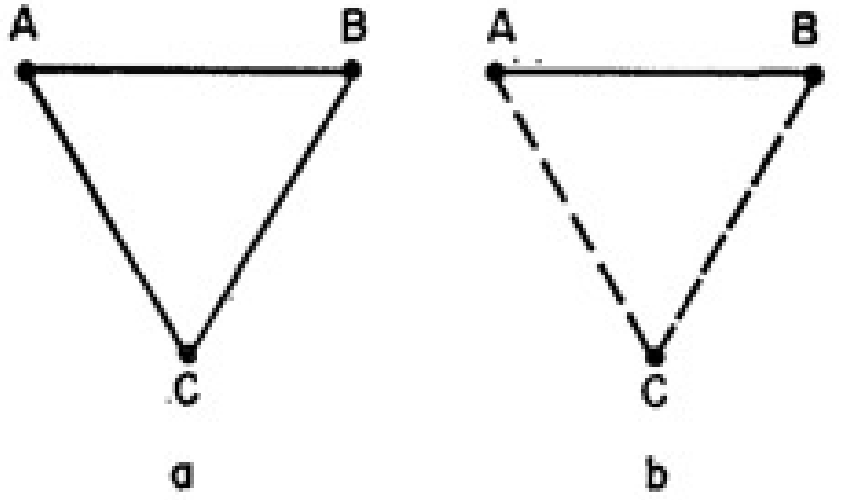
Heider's Balance Theory

- Two major types of relations
  - Attitudes (liking or evaluating)
    - P**L**O vs. P$\sim$**L**O
  - More specific relations, such as similarity, possession, causality, proximity, or belonging
    - P**U**X vs. P$\sim$**U**X
- A *balanced state* is defined in terms of certain combinations of these relations

Heider's Balance Theory

- Heider's definition of a *balanced state* is stated separately for two and for three entities:
  - Two entities: A balanced state exists if the relation between them is positive (or negative) in all respects
  - Three entities: A balanced state exists if all three relations are positive in all respects, or if two are negative and one is positive
- Heider's theory asserts that there is a tendency for units to achieve a balanced state

Depicting Balance



Balance in Social Network Analysis

- Cartwright and Harary (1956) extend the concept of balance to larger networks, where:
  - A **cycle** is a sequence of links that begins and ends at the same node
  - For signed graphs, the sign of a cycle is given by the product of the signs of the links that comprise the cycle
    - Here, the balance of a signed graph depends on the signs of its cycles
    - And, a signed graph is balanced if every cycle is positive
- This definition allows us to assess balance for social networks with any number of actors

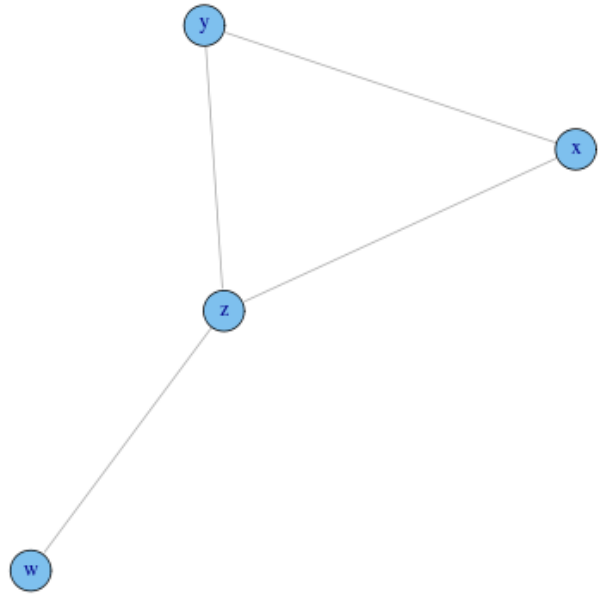
Transitivity in Networks

- Transitivity refers to the extent to which the relation that relates two nodes in a network that are connected by an edge is transitive
- Perfect transitivity implies that if x is connected (through an edge) to y, and y is connected to z, then x is also connected to z

Quantifying Transitivity in Networks

- We can quantify transitivity in an undirected network as follows:
  - Here, a **loop of length three** is a sequence of nodes x, y, z, x such that (x, y), (y, z), and (z, x) are edges of the graph
  - A **path of length two** is a sequence of nodes x, y, z such that (x, y) and (y, z) are edges
  - Note: Paths and loops are sequences (x, y, z, x is different from y, z, x, y)
- The **transitivity coefficient** T of a network is the ratio of the number of loops of length three and the number of paths of length two
  - $T = 1$ implies perfect transitivity, a network whose components are all cliques

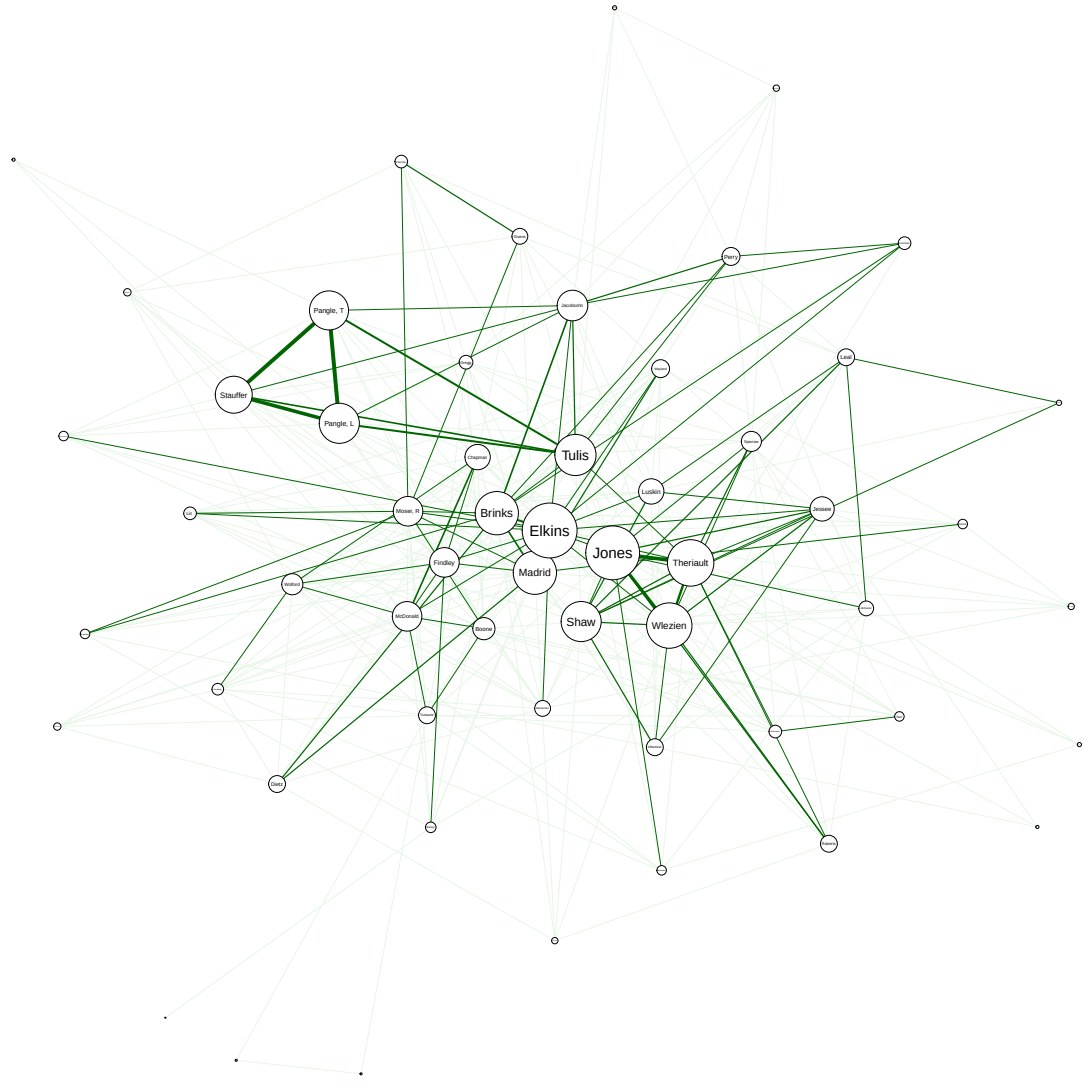
Quantifying Transitivity in Networks



- Here, we have 6 loops of length three and 10 paths of length two, so $T = 6/10 = 0.6$

Application

- Using our network of faculty on dissertation committees and the command `transitivity(igraph_obj)`, our $T = 0.42$



Linking Balance and Transitivity

- Balance is a theory of network relationships
- Transitivity is a property of networks