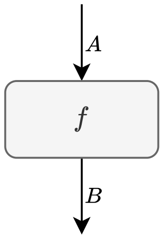
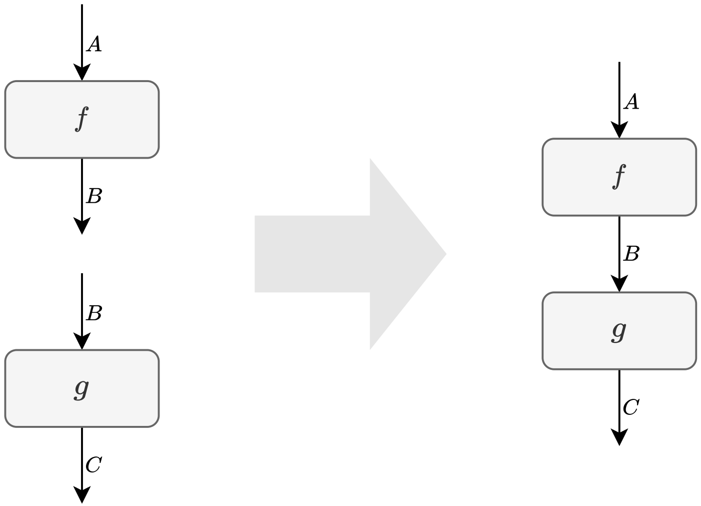
Transformation



A transformation f
maps any input of type A
to the output of type B.

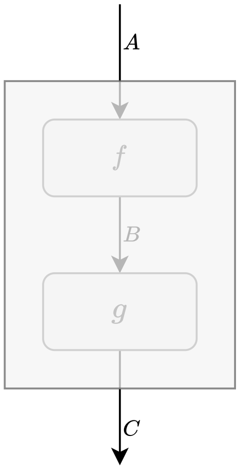
Composition



$\mathrm{chain\_of}(f, g)$

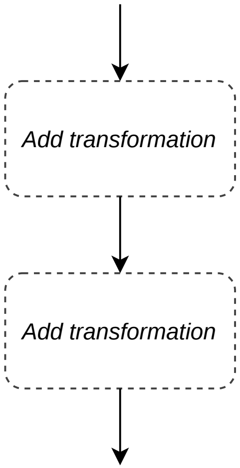
Transformations with compatible input and output
can be composed.

Composition is a Transformation



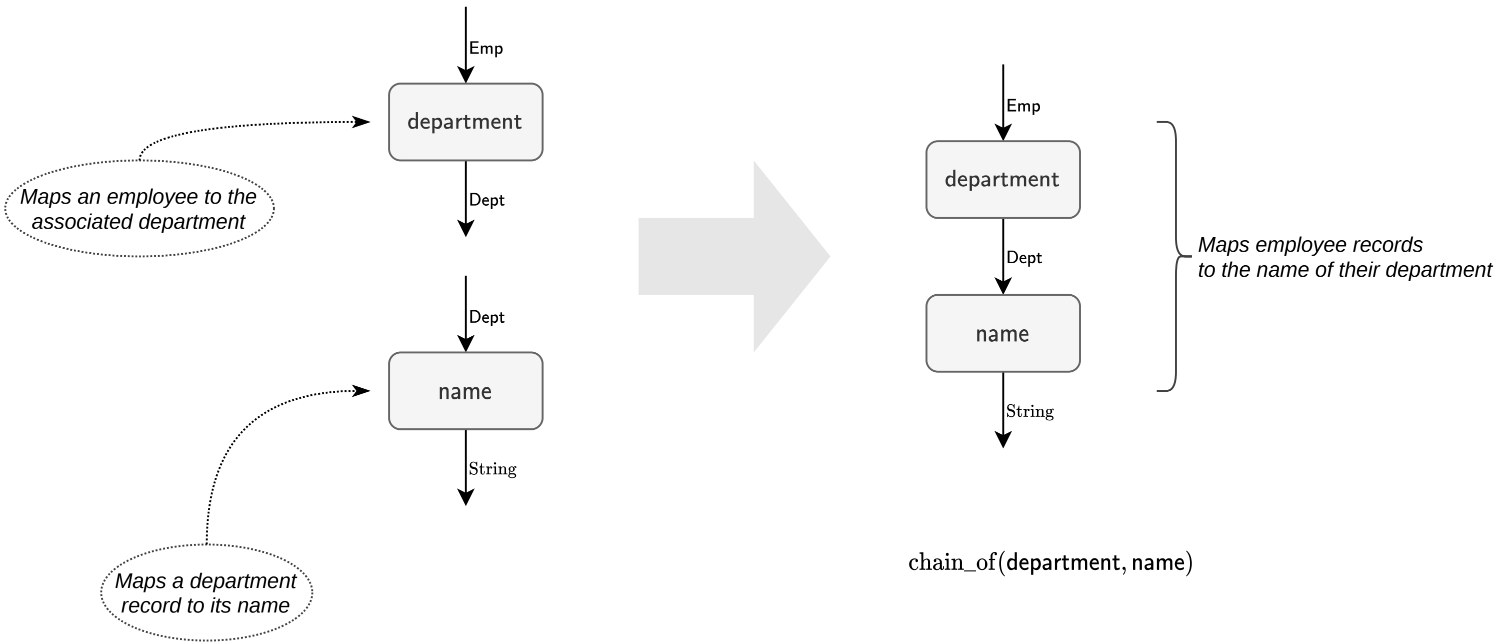
Crucially, composition of transformations is again a transformation.

Composition Combinator



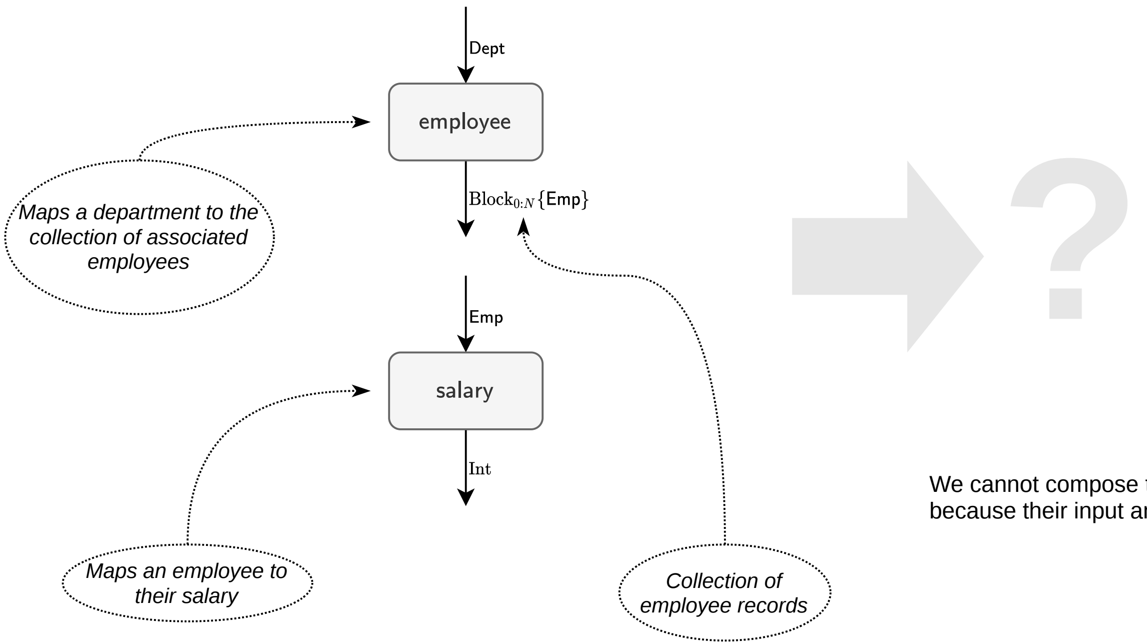
Composition $\mathrm{chain\_of}(\square, \square)$
is a transformation combinator
with two arguments.

Example: Composition



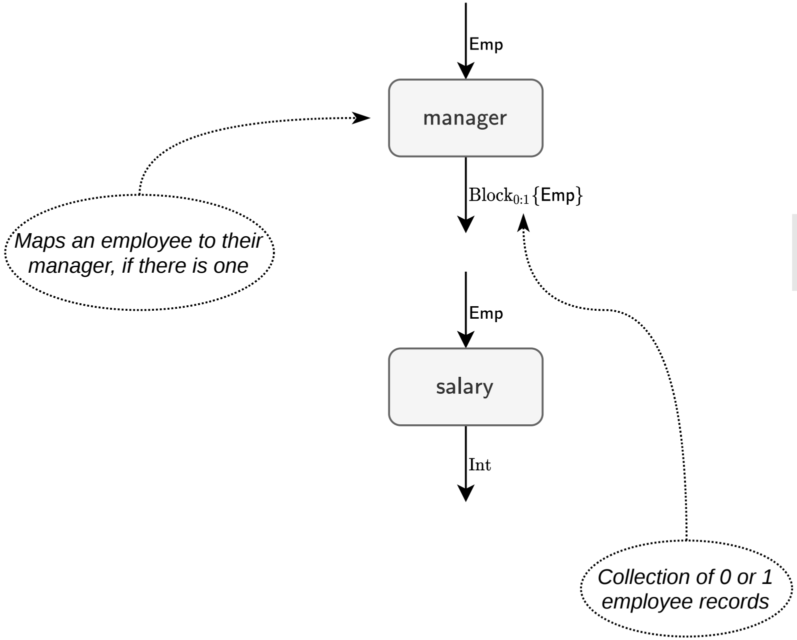
chain_of(**department**, **name**)

Counter-example: Plural Component



We cannot compose these transformations because their input and output do not quite match.

Counter-example: Optional Component

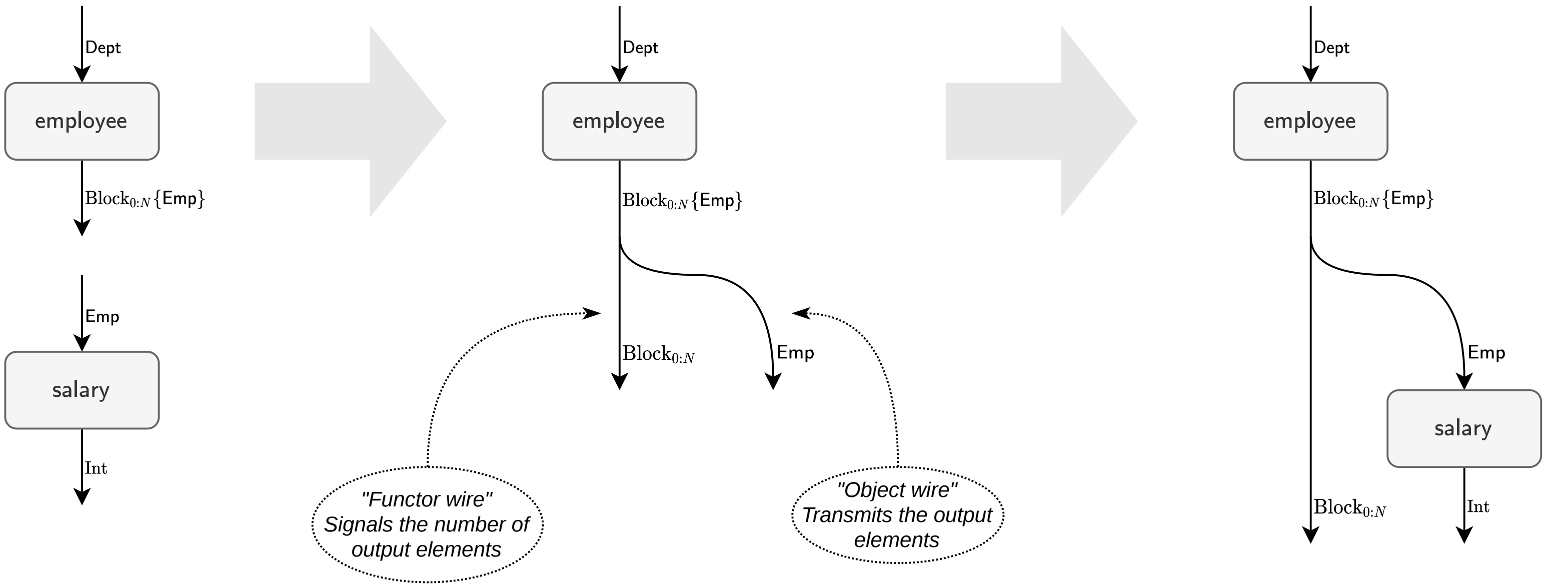


Even so, the input and the output share a common component,
which suggests there should be a way to compose these transformations.

Idea: Unbundle the Wire

Unbundle the composite wire.

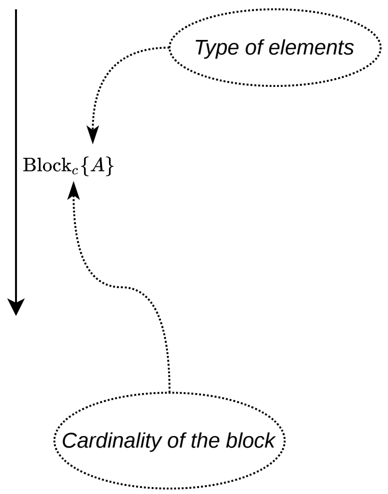
Compose using the object wire.



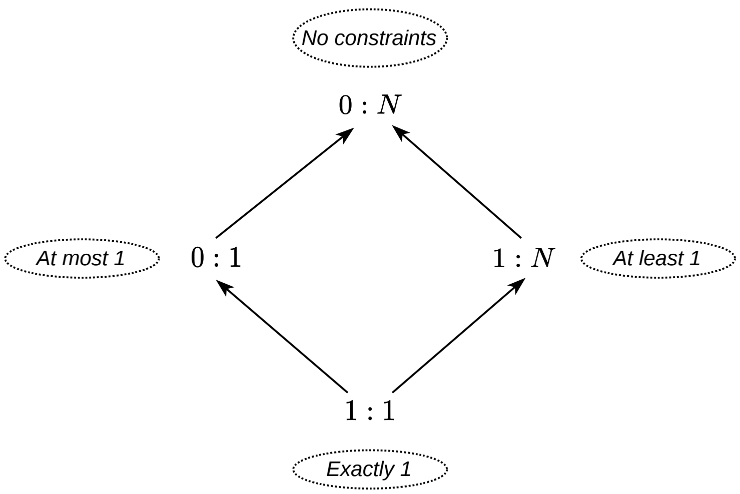
Attaching a transformation to the object wire indicates that the transformation is applied to all element of the collection.

Block Type

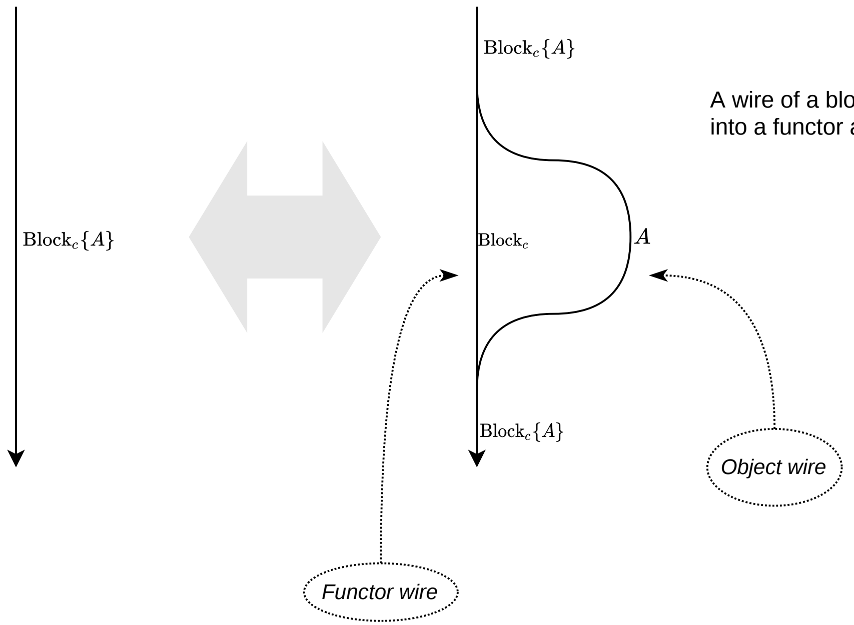
A block is a collection
of homogeneous elements.



Cardinality is a constraint
on the number of elements in a block.

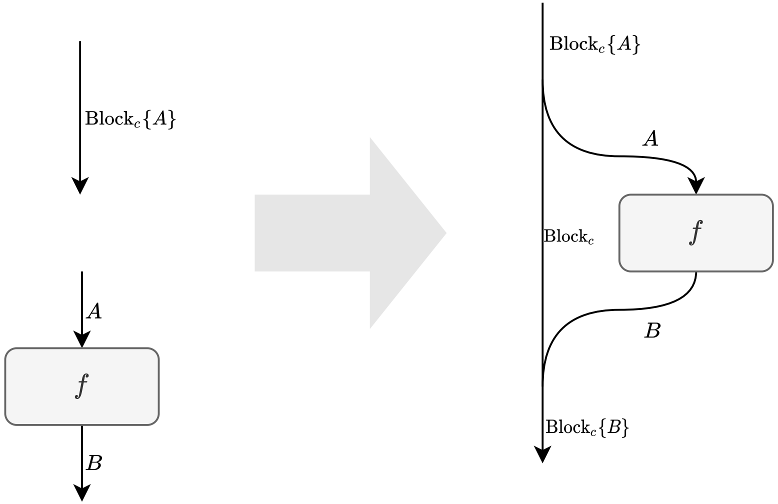


Unbundling



A wire of a block type can be unbundled into a functor and an object components.

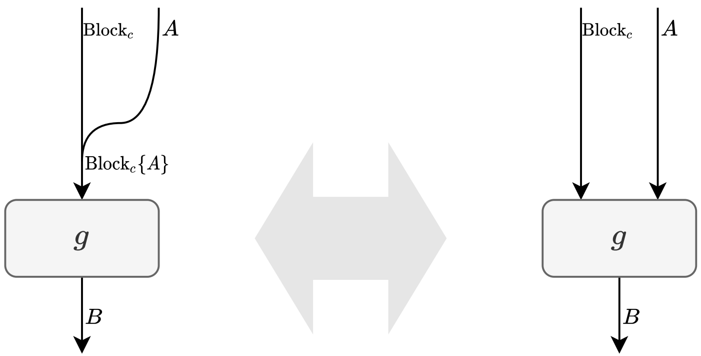
Object Transformation



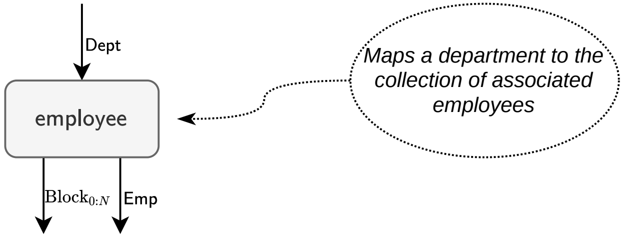
$\mathrm{with\_elements}(f)$

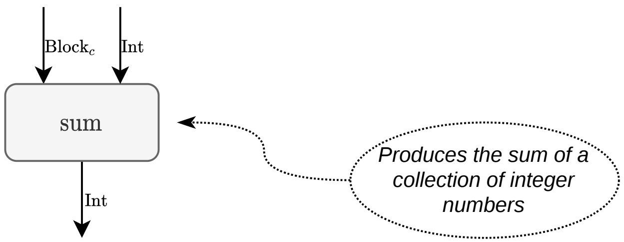
Any compatible transformation
can be applied to the object wire,
which indicates that the transformation
is applied to every element of the block.

Multiwired Transformations

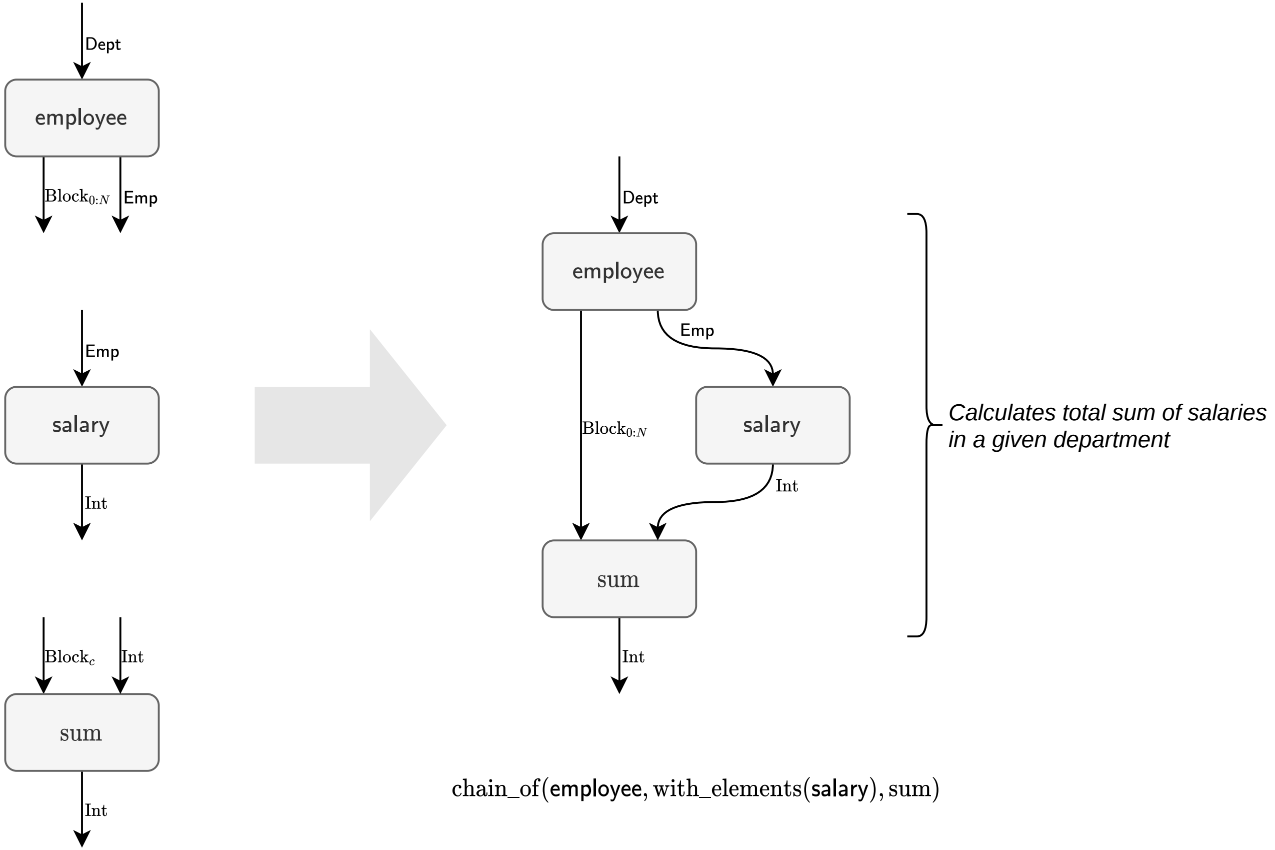


Example: Multiwired Transformations



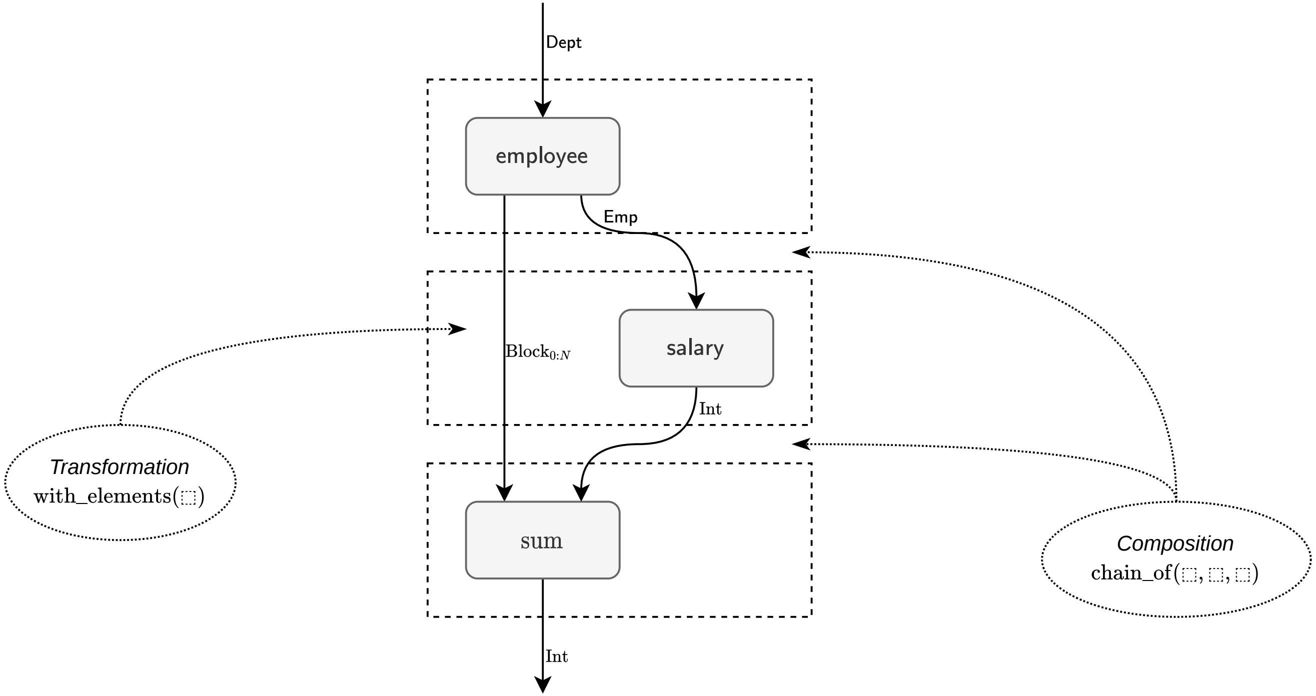


Example: Multiwired Composition



$$\mathrm{chain\_of}(\textsf{employee}, \mathrm{with\_elements}(\textsf{salary}), \mathrm{sum})$$

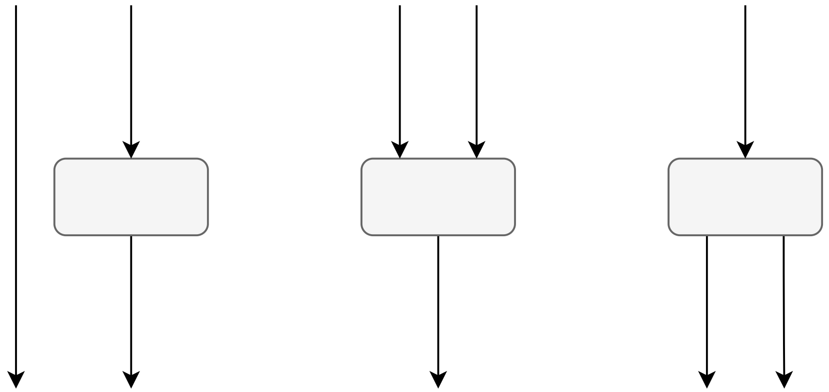
Example: Multiwired Composition Details



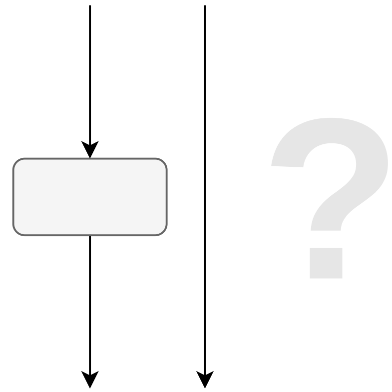
chain_of(**employee**, with_elements(**salary**), sum)

Challenge: Functor Transformation

So far we have seen different combinations of transformations and wires: transformations that are applied to the object wire, as well as transformations that consume or produce both functor and object wires.

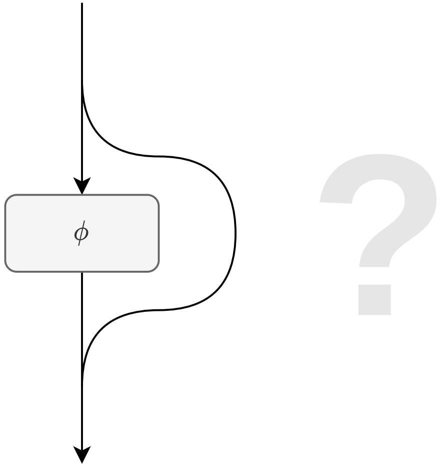


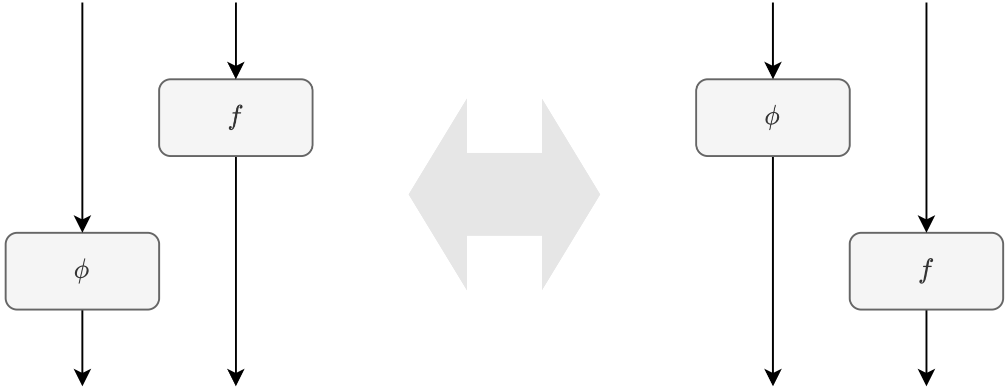
What we miss is a notion of a transformation applied to the functor wire.



Functor Transformation

Since a functor transformation does not act on the object wire,
it cannot interfere with any transformations applied to the object wire.

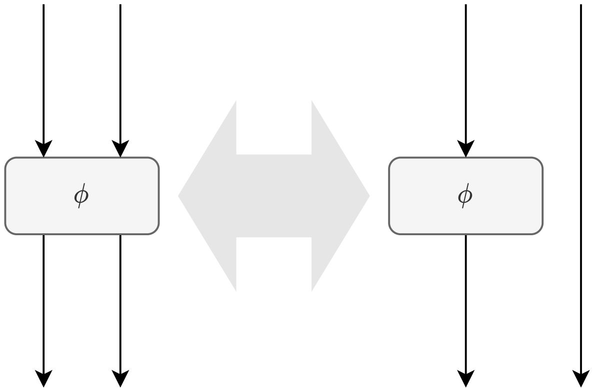




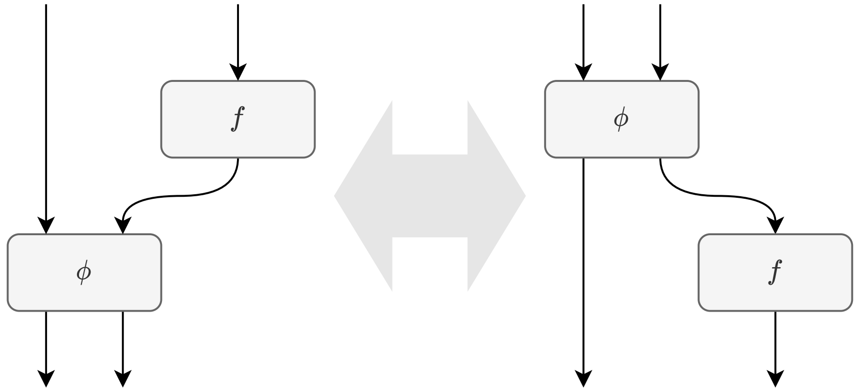
How to define a functor transformation?

This property can be taken as a definition of a functor transformation.

Definition: Functor Transformation



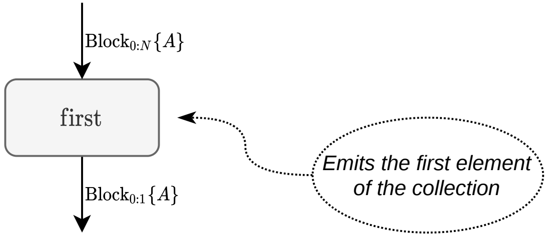
when for any f



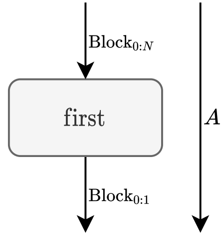
Such transformations are also called *natural*.

Branch of mathematics that studies natural transformations is called the *category theory*.

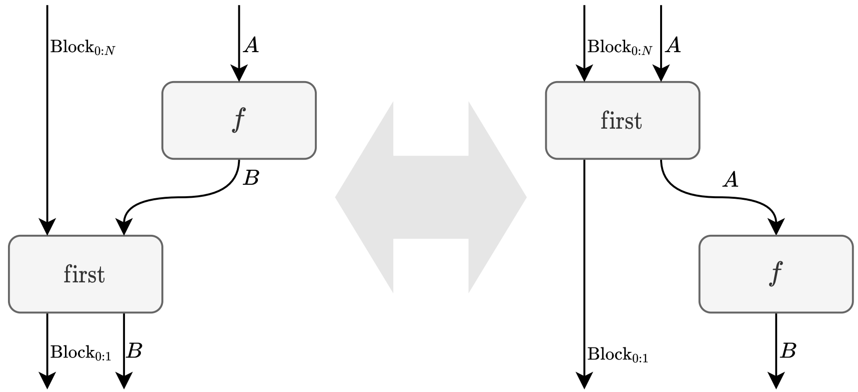
Example: Functor Transformation



This is, in fact, a natural transformation.



Indeed, for any $f : A \to B$,



$$\text{chain\_of}(\text{with\_elements}(f), \text{first}) \equiv \text{chain\_of}(\text{first}, \text{with\_elements}(f))$$