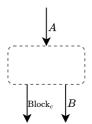
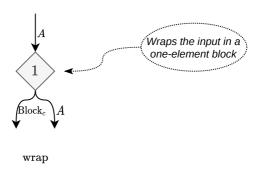
## **Monadic Interface**



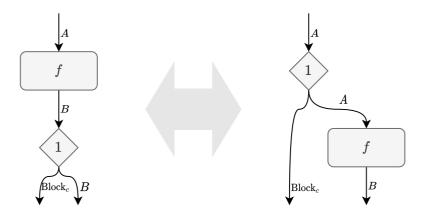
#### **Unit Transformation**



Unit is a natural transformation.



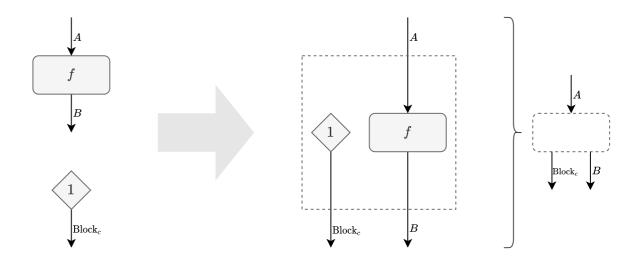
Indeed, for any f:A o B,



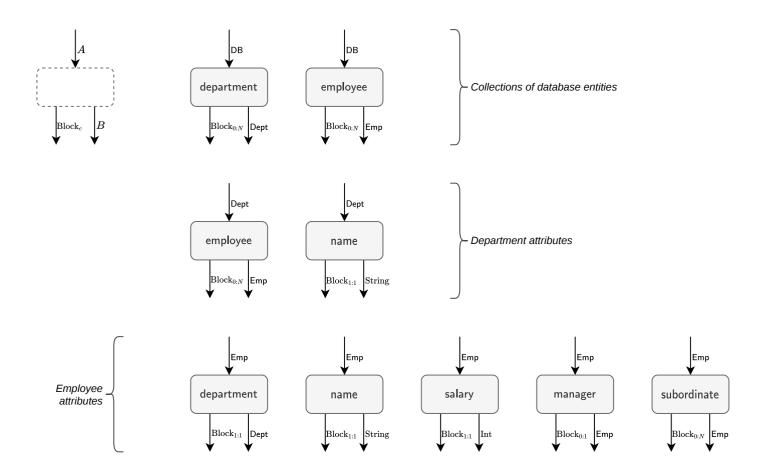
 $\operatorname{chain\_of}(f, \operatorname{wrap}) \equiv \operatorname{chain\_of}(\operatorname{wrap}, \operatorname{with\_elements}(f))$ 

# **Using Unit Transformation**

Unit transformation can adapt any regular transformation to the monadic interface.

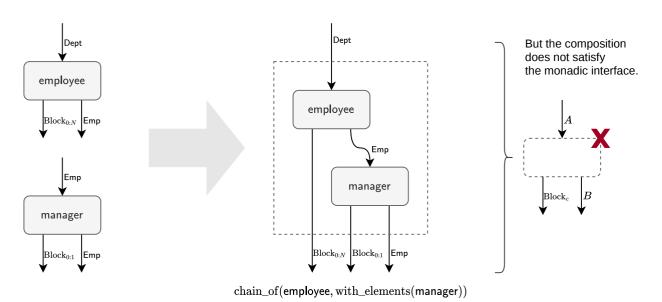


#### **Monadic Interface: Examples**

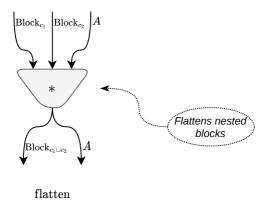


### **Composition Challenge**

Two monadic transformations can be composed.



### **Multiplication Transformation**

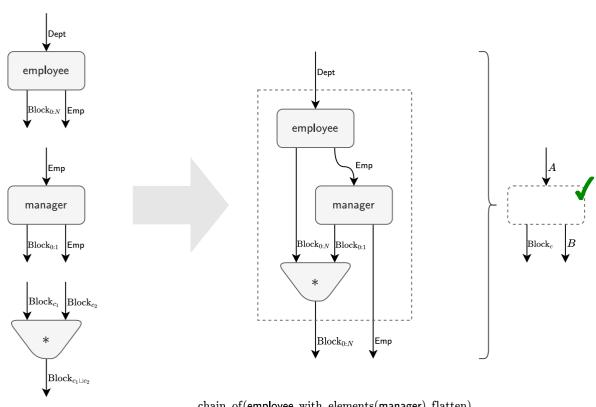


It is easy to check that the multiplication is a natural transformation.



### **Using Multiplication Transformation**

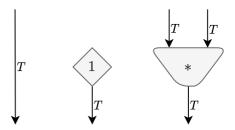
Multiplication can be used to combine two monadic transformations into a new monadic transformation.



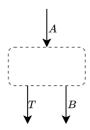
chain\_of(employee, with\_elements(manager), flatten)

#### Monad

In general, a *monad* is a functor wire which posesses two special natural transformations: unit and multiplication.

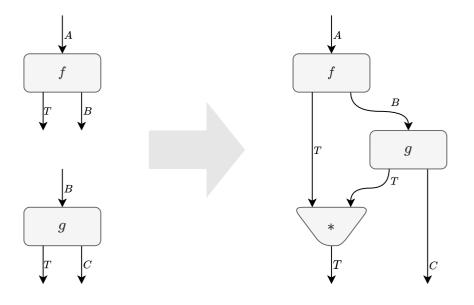


Monadic transformation is a transformation with the following shape.

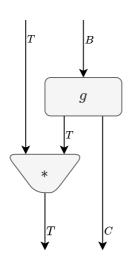


### **Monadic Composition**

Using multiplication, two compatible monadic transformations can be composed to form a new monadic transformation.



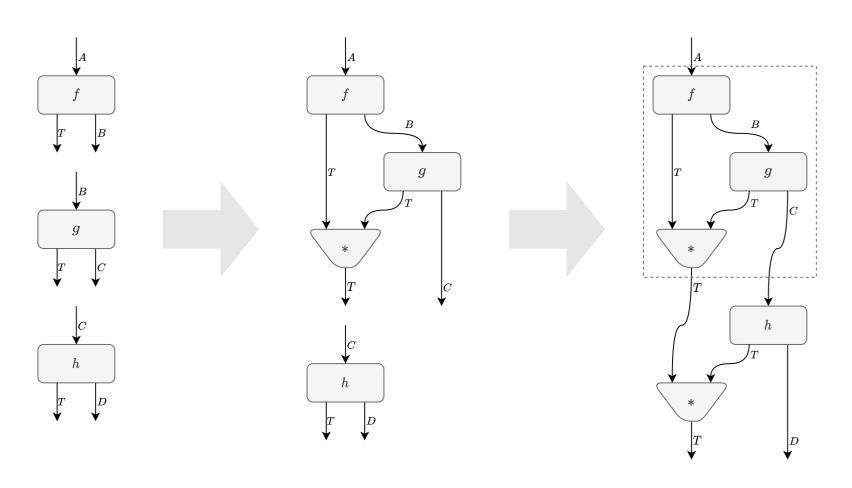
Aside: this is known as the operation  $\operatorname{bind}(g)$ .



For this definition to be coherent, unit and multiplication must satisfy certain properties.

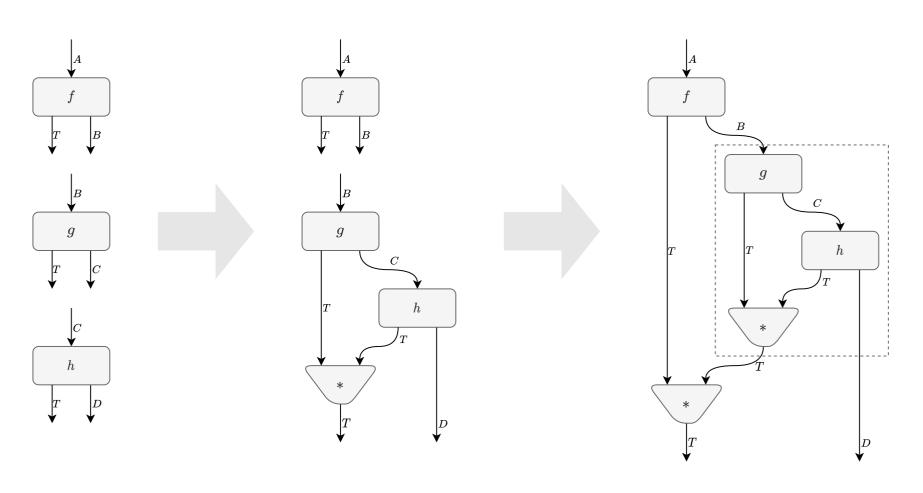
### **Associativity of Monadic Composition 1**

Given three compatible monadic transformations, they could be composed in two distinct ways.



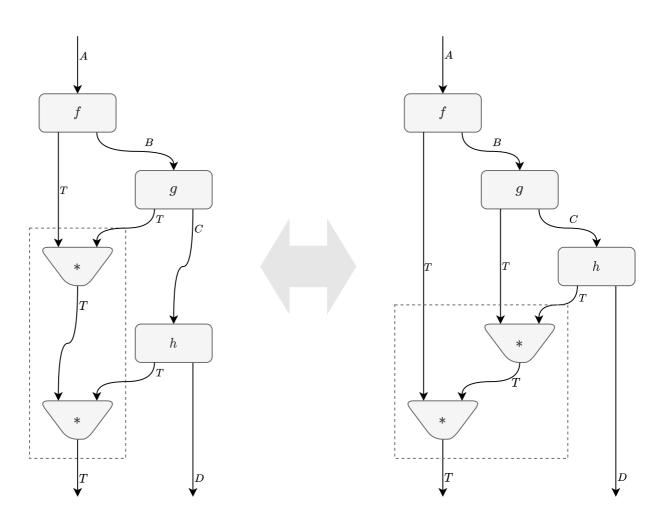
### **Associativity of Monadic Composition 2**

Given three compatible monadic transformations, they could be composed in two distinct ways.



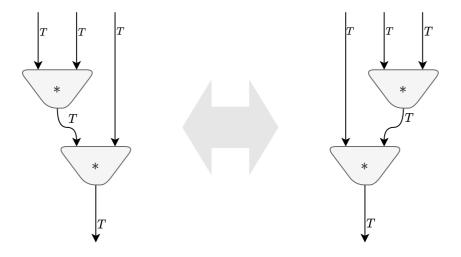
### **Associativity of Monadic Composition 3**

For monadic composition to be associative, these two transformations must be equivalent for any f, g, and h.



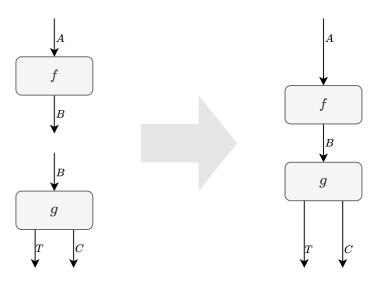
### **Monad Law I**

Monadic multiplication must satisfy the following equivalence condition.



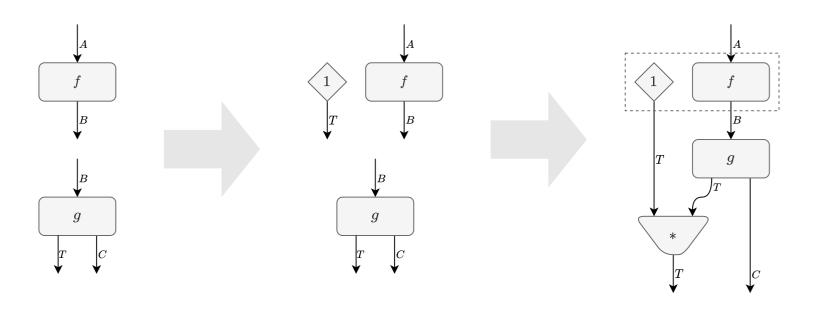
### **Composition of Regular and Monadic Transformations 1**

Given one regular and one monadic transformation, they could be composed in two distinct ways.



### **Composition of Regular and Monadic Transformations 2**

Given one regular and one monadic transformation, they could be composed in two distinct ways.



### **Composition of Regular and Monadic Transformations 3**

We demand that these two transformations must be equivalent for any f and g.

