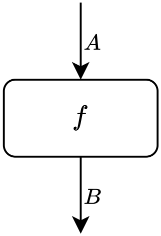
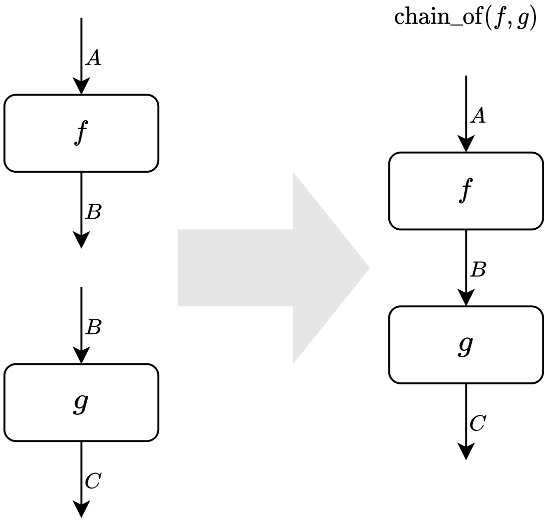
1. Transformation



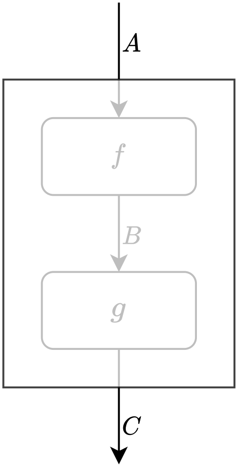
A transformation f
with input of type A
and output of type B

2. Composition



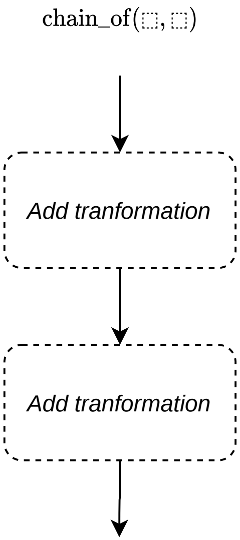
Transformations with
compatible input and output
can be composed

3. Composition is a Transformation



Trivially (but crucially),
a composition of transformations
is again a transformation

4. Composition Combinator



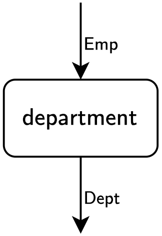
$\mathrm{chain\_of}(\square, \square)$

Add tranformation

Add tranformation

Composition is a transformation combinator
with two placeholders

5. Example: Components of a Composition

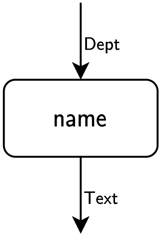


Emp

department

Dept

Maps an employee to
the associated department



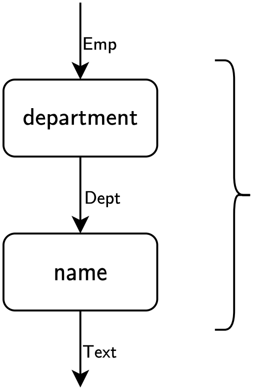
Dept

name

Text

Maps a department record
to its name

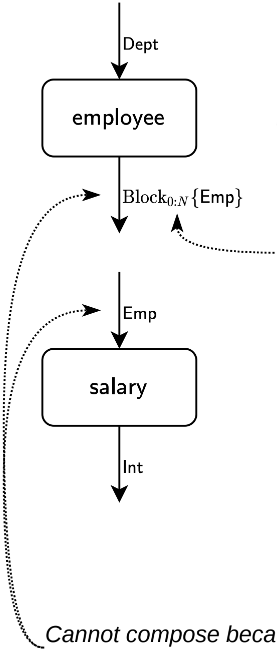
6. Example: Composition



Maps an employee record
to the name of their department

$\text{chain\_of}(\mathsf{department}, \mathsf{name})$

7. Counter-example: Plural Component



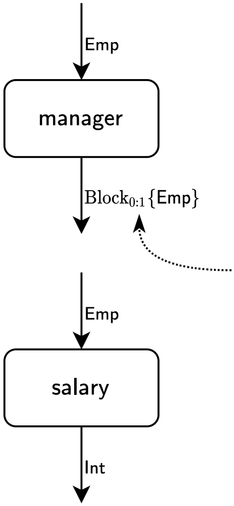
Maps a department to the collection
of associated employees

Collection of employee records

Maps an employee to their salary

*Cannot compose because
input and output do not quite match*

8. Counter-example: Optional Component



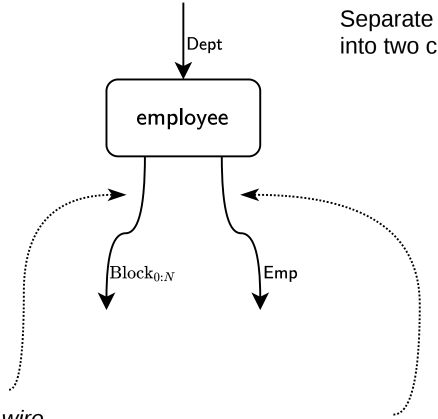
Maps an employee to their manager,
if there is one

Collection of 0 or 1 employee records

Can we represent composition of these transformations
with an intuitive diagrammatic notation?

9. Idea: Unbundle the Wire

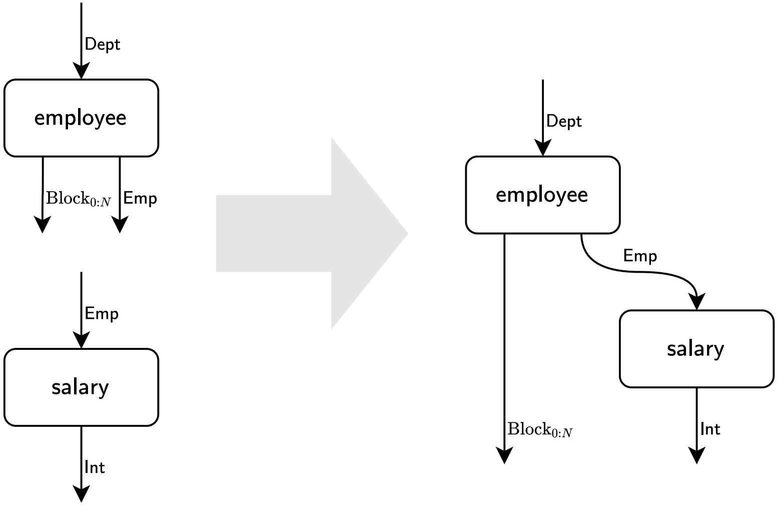
Separate the output wire
into two components



Functor wire
Intuitively, it signals the number
of output elements

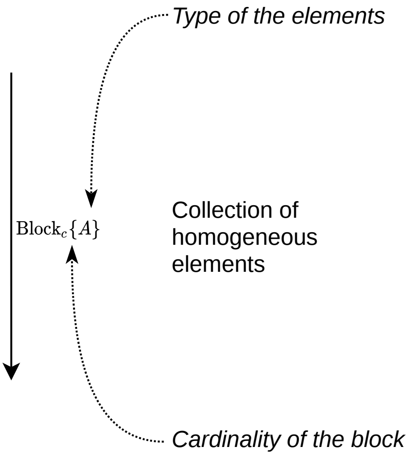
Object wire
Transmits the output elements

10. Idea: Compose Using the Object Wire



Attaching a transformation to the object wire
indicates that the transformation is applied
to each element of the collection

11. Block Type



Type of the elements

$\mathrm{Block}_c\{A\}$

Collection of homogeneous elements

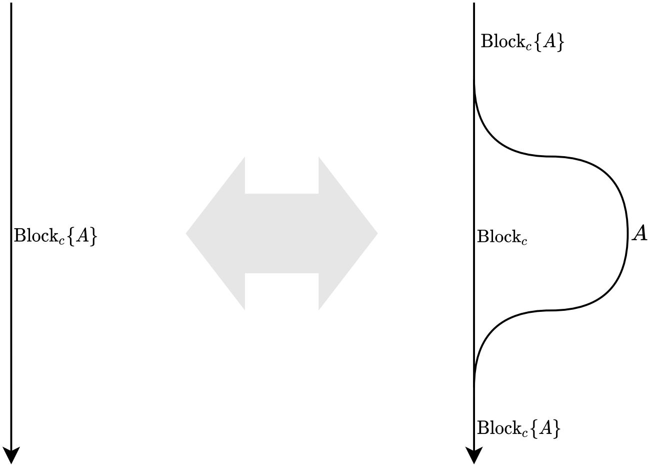
Cardinality of the block

12. Cardinality

Cardinality is a constraint on the number of elements in a block

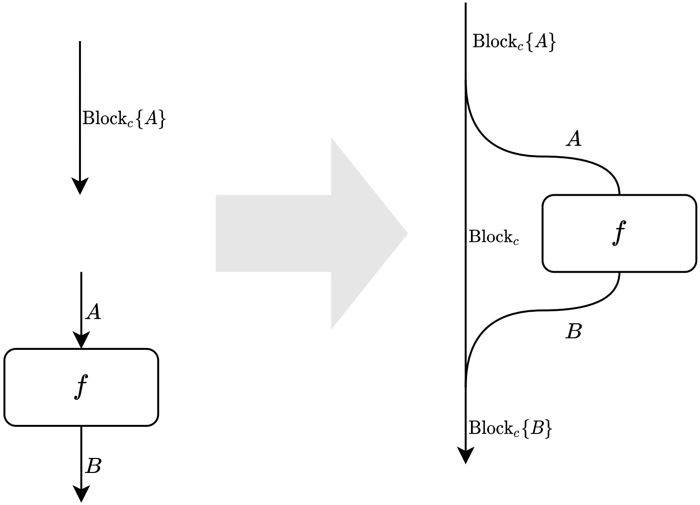
$$\begin{tikzcd}
 & & 0 : N & \textit{No constraints} & \\
\textit{At most 1} & 0 : 1 \arrow[ur, dotted] & & 1 : N \arrow[ul, dotted] & \textit{At least 1} \\
 & & 1 : 1 \arrow[ul, dotted] \arrow[ur, dotted] & \textit{Exactly 1} &
\end{tikzcd}$$

13. Unbundling



We can unbundle a wire of a block type into a functor and object components

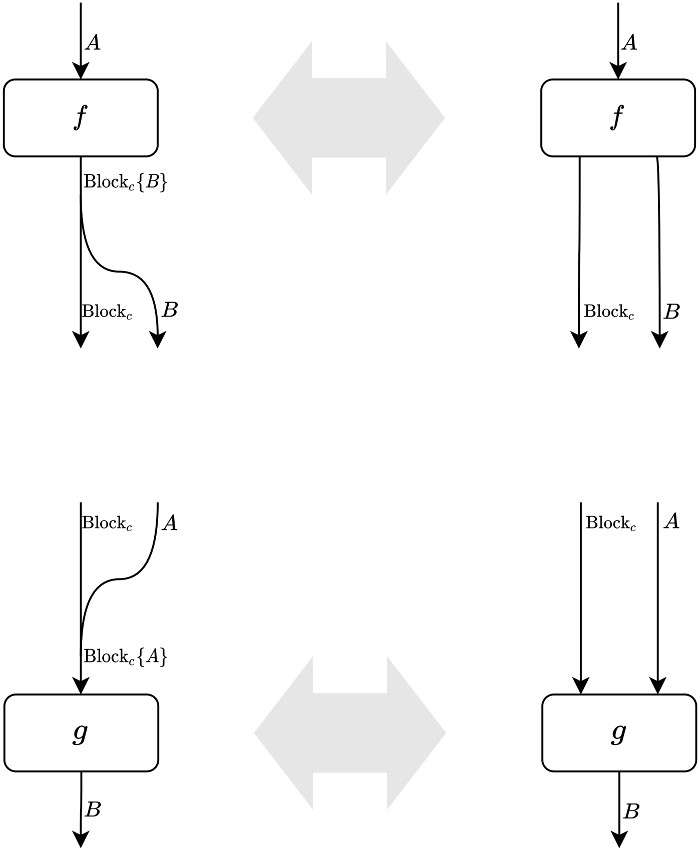
14. Object Transformation



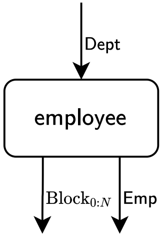
$\mathrm{with\_elements}(f)$

Then any compatible transformation can be applied to the object wire, which indicates that the transformation is applied to every element of the block

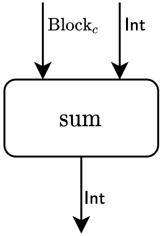
13. Multiwired transformations



14. Example: Multiwired Transformations

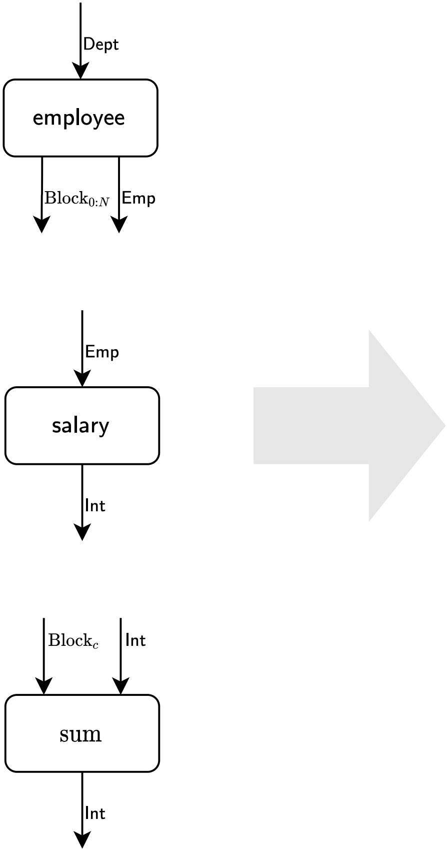


Maps a department to the collection of associated employees

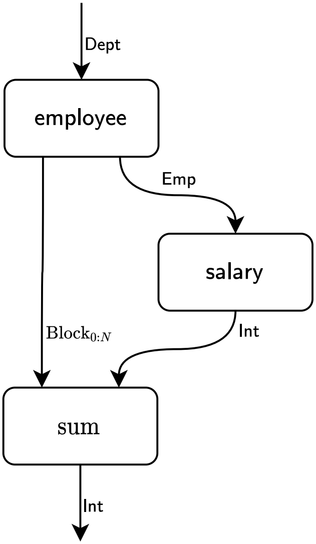


Produces the sum of a collection of integers

14. Example: Multiwired Composition

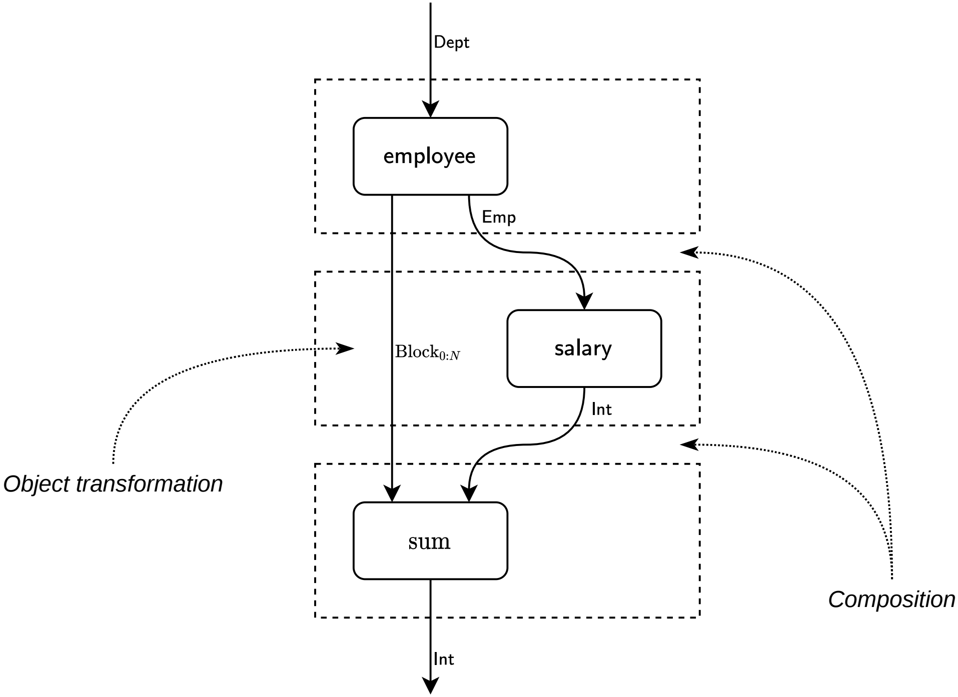


Total sum of salaries in a given department



$\mathrm{chain\_of}(\textsf{employee}, \mathrm{with\_elements}(\textsf{salary}), \mathrm{sum})$

15. Example: Details



$\mathrm{chain\_of}(\mathsf{employee}, \mathrm{with\_elements}(\mathsf{salary}), \mathrm{sum})$