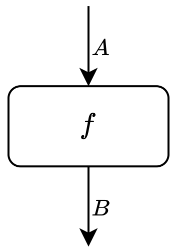
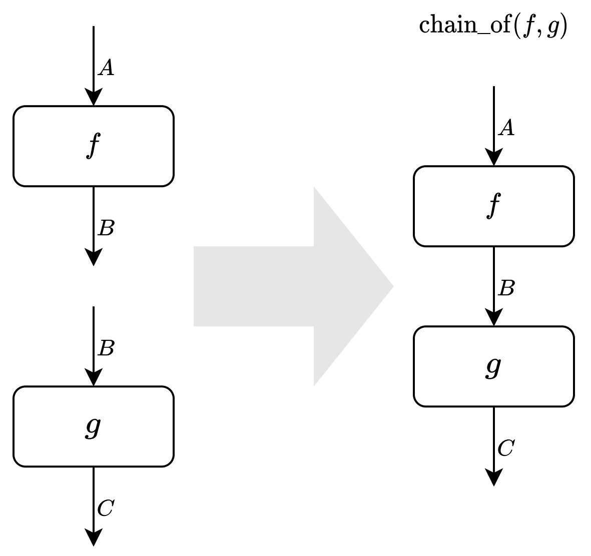
1. Transformation



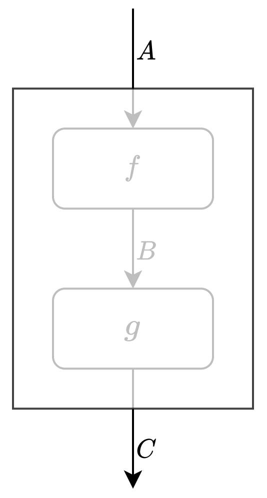
A transformation f
with input of type A
and output of type B

2. Composition



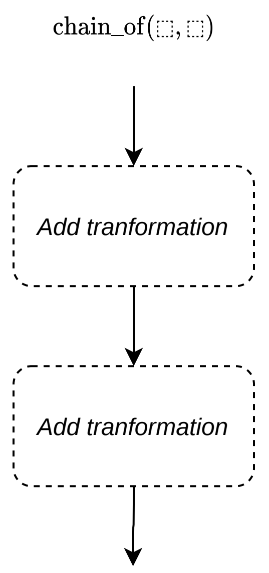
Transformations with
compatible input and output
can be composed

3. Composition is a Transformation



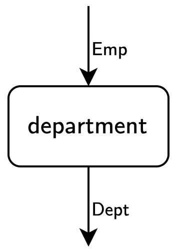
Trivially (but crucially),
a composition of transformations
is again a transformation

4. Composition Combinator

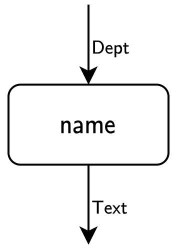


Composition is a transformation combinator
with two placeholders

5. Example: Components of a Composition

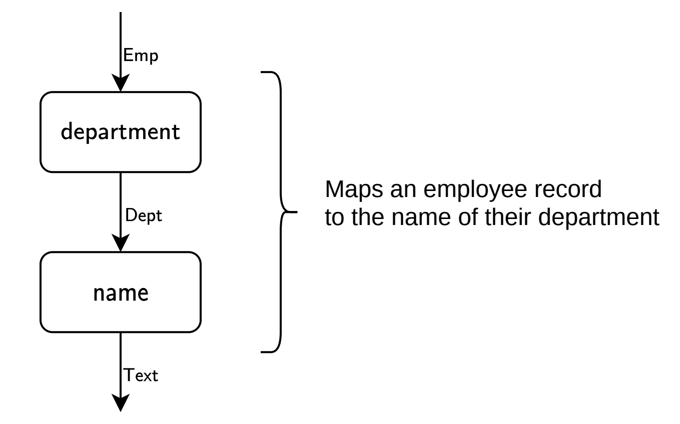


Maps an employee to
the associated department



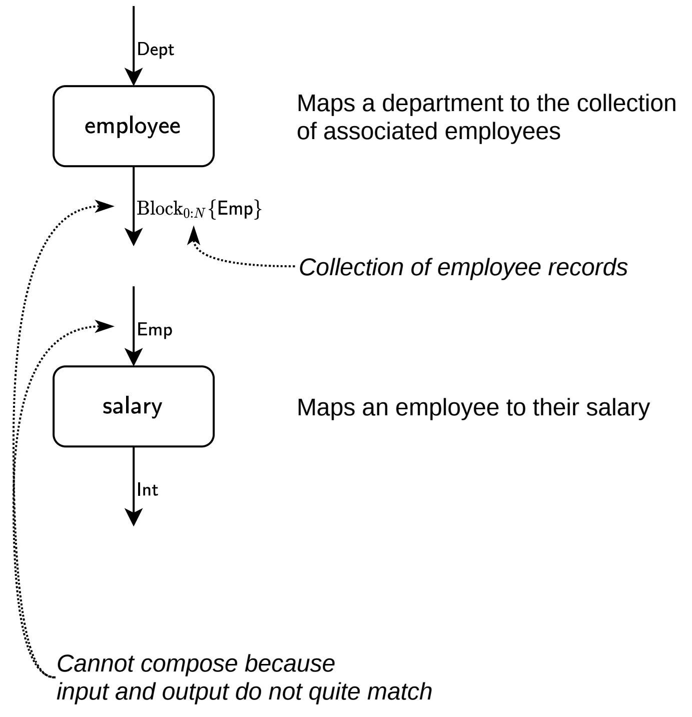
Maps a department record
to its name

6. Example: Composition

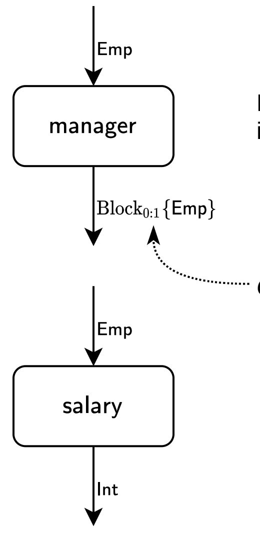


$\mathrm{chain\_of}(\textsf{department}, \textsf{name})$

7. Counter-example: Plural Component


Dept
employee
Maps a department to the collection of associated employees
$\mathrm{Block}_{0:N}\{\mathsf{Emp}\}$
Collection of employee records
Emp
salary
Maps an employee to their salary
Int
Cannot compose because input and output do not quite match

8. Counter-example: Optional Component

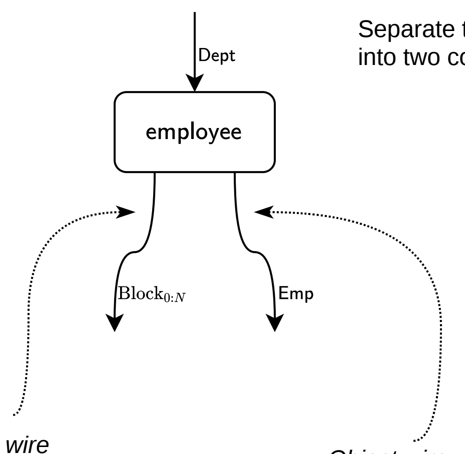


Maps an employee to their manager,
if there is one

Collection of 0 or 1 employee records

Can we represent composition of these transformations
with an intuitive diagrammatic notation?

9. Idea: Unbundle the Wire

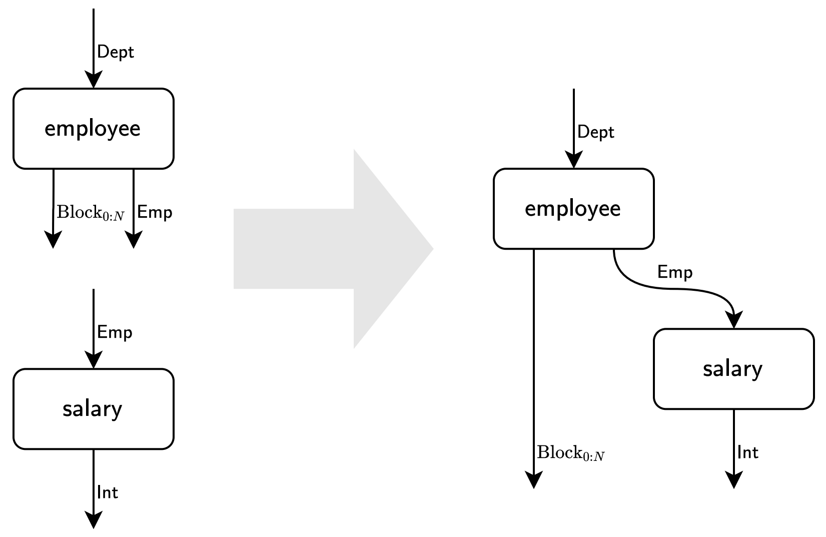


Separate the output wire
into two components

Functor wire
Intuitively, it signals the number
of output elements

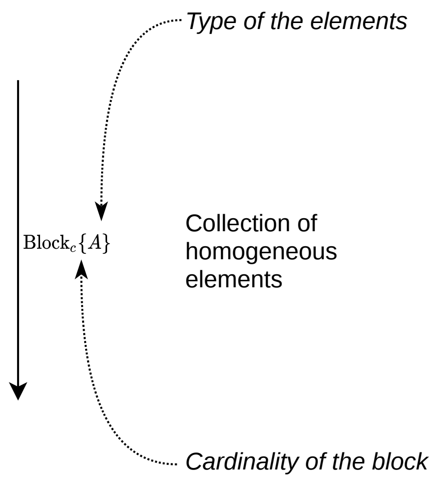
Object wire
Transmits the output elements

10. Idea: Compose Using the Object Wire



Attaching a transformation to the object wire
indicates that the transformation is applied
to each element of the collection

11. Block Type

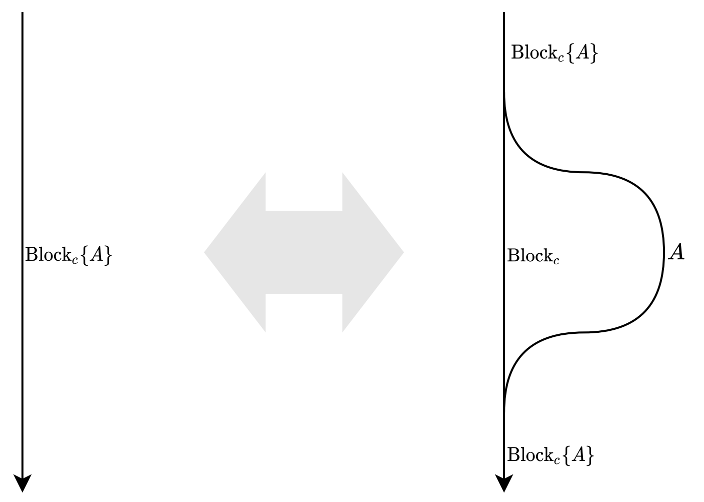


12. Cardinality

Cardinality is a constraint on the number of elements in a block

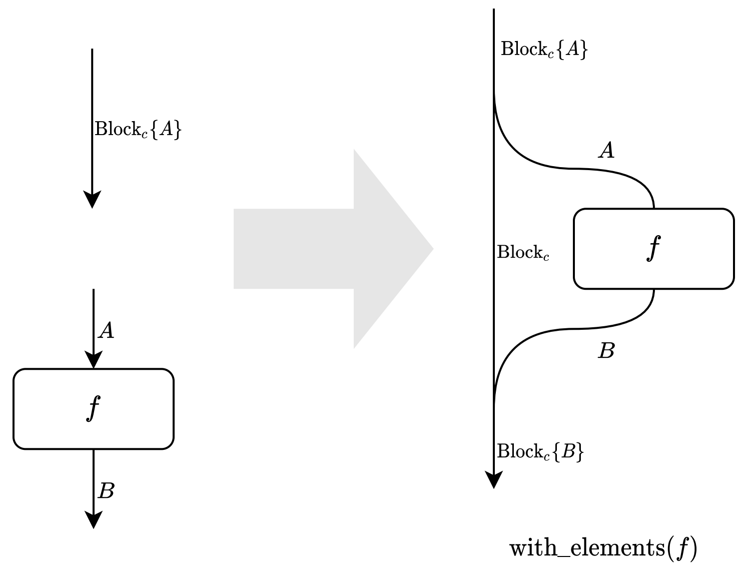
$$\begin{tikzcd}
 & & 0 : N & \textit{No constraints} & \\
\textit{At most 1} & 0 : 1 \arrow[ur, dotted] & & 1 : N \arrow[ul, dotted] & \textit{At least 1} \\
 & & 1 : 1 \arrow[ul, dotted] \arrow[ur, dotted] & \textit{Exactly 1} &
\end{tikzcd}$$

13. Unbundling



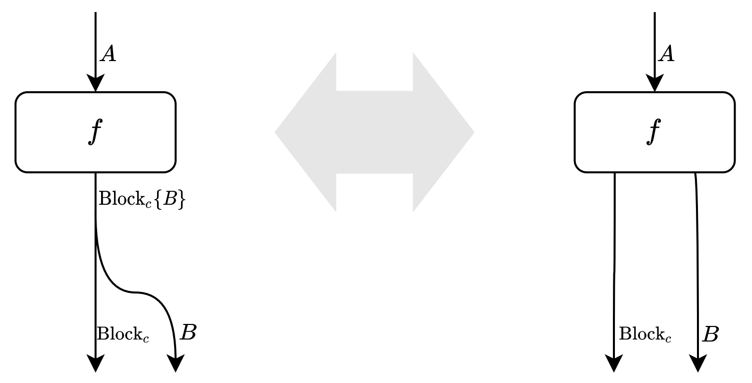
We can unbundle a wire of a block type into a functor and object components

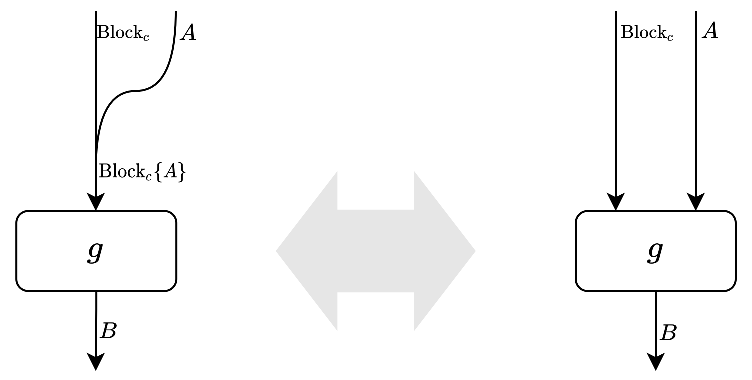
14. Object Transformation



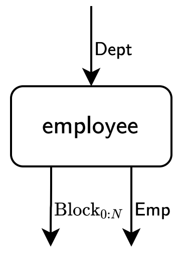
Then any compatible transformation can be applied to the object wire, which indicates that the transformation is applied to every element of the block

13. Multiwired transformations

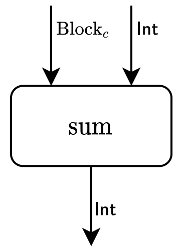




14. Example: Multiwired Transformations

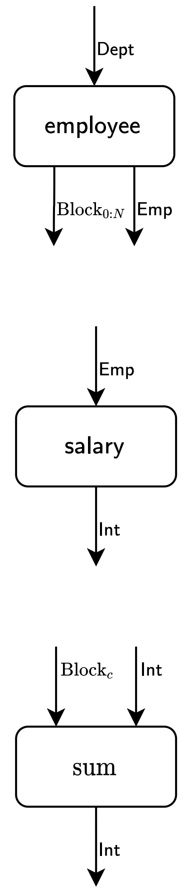


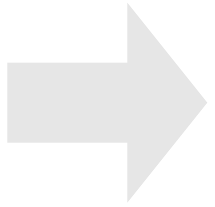
Maps a department to the collection
of associated employees



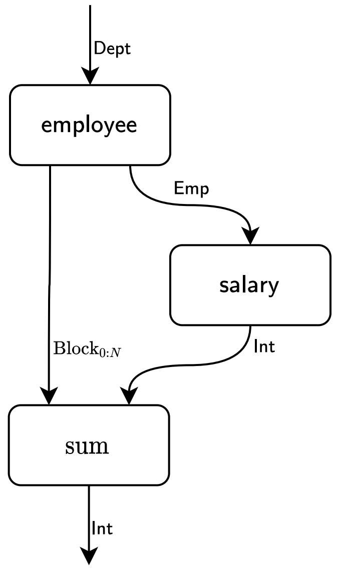
Produces the sum of a collection of integers

14. Example: Multiwired Composition



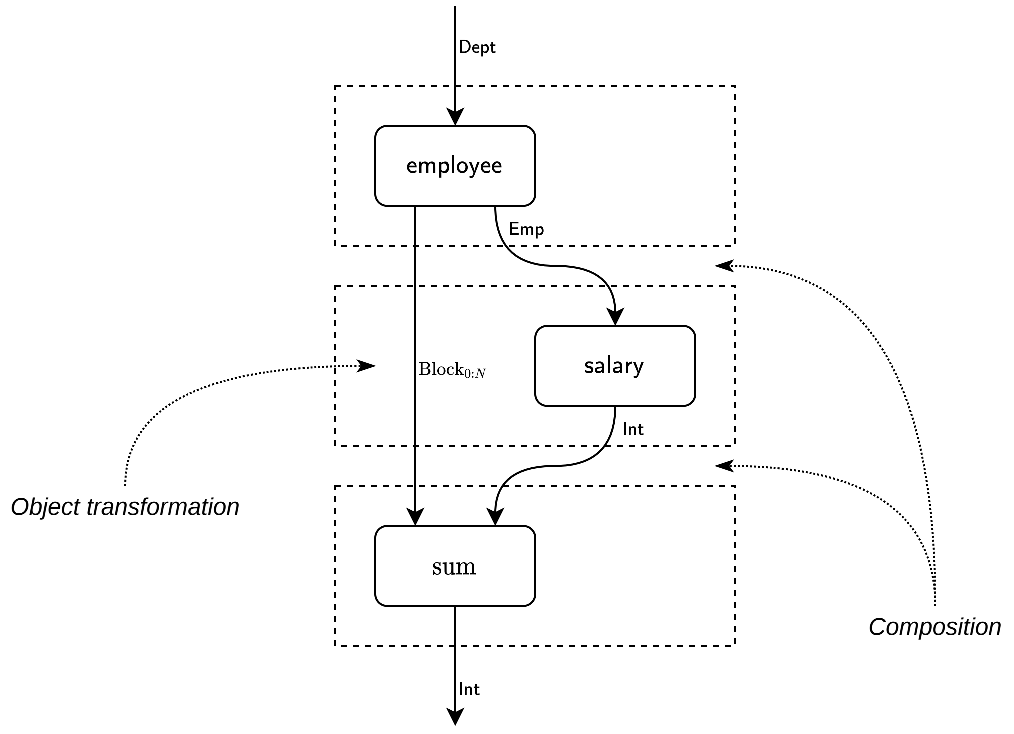


Total sum of salaries in a given department



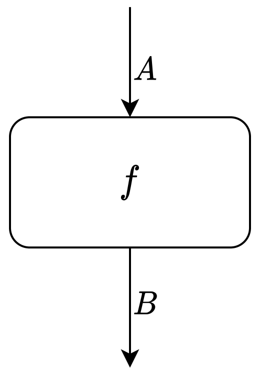
$\mathrm{chain\_of}(\mathsf{employee}, \mathrm{with\_elements}(\mathsf{salary}), \mathrm{sum})$

15. Example: Details



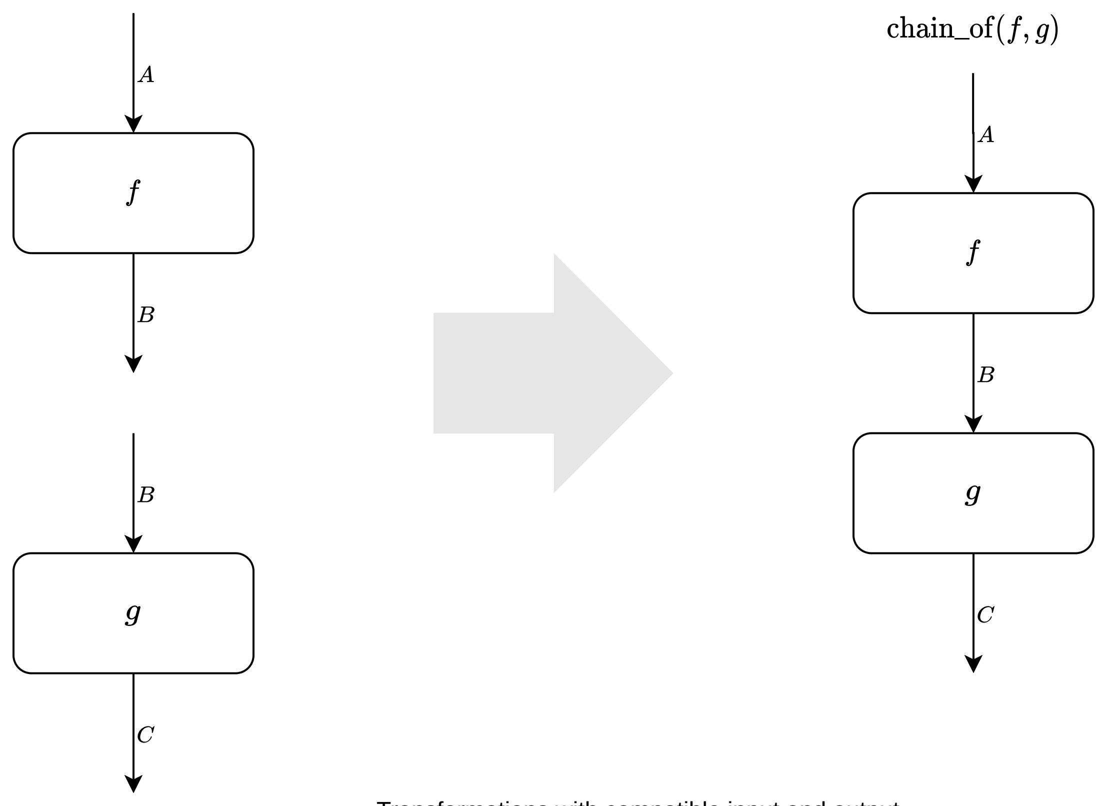
$\text{chain\_of}(\mathsf{employee}, \text{with\_elements}(\mathsf{salary}), \text{sum})$

Transformation



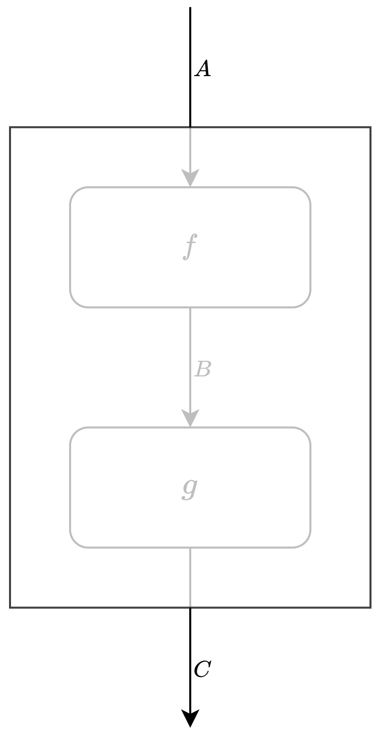
A transformation f
maps input of type A
to the output of type B.

Composition



Transformations with compatible input and output can be composed.

Composition is a Transformation



Crucially, a composition of transformations is again a transformation.