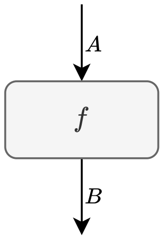
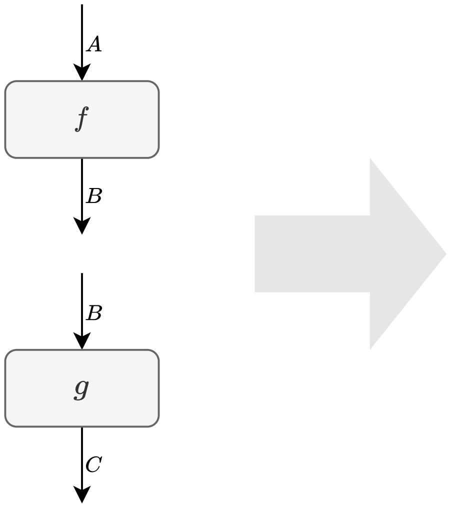
Transformation



A transformation f
maps any input of type A
to the output of type B.

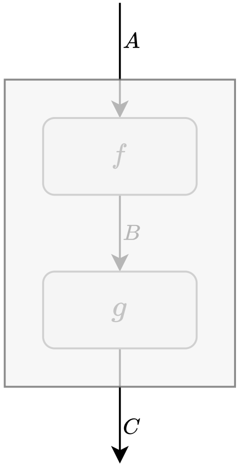
Composition



$\mathrm{chain\_of}(f, g)$

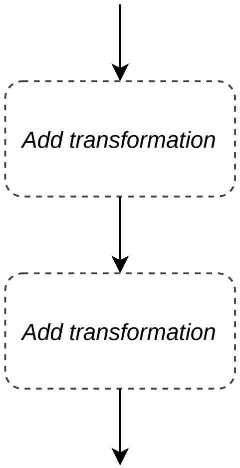
Transformations with compatible input and output can be composed.

Composition is a Transformation



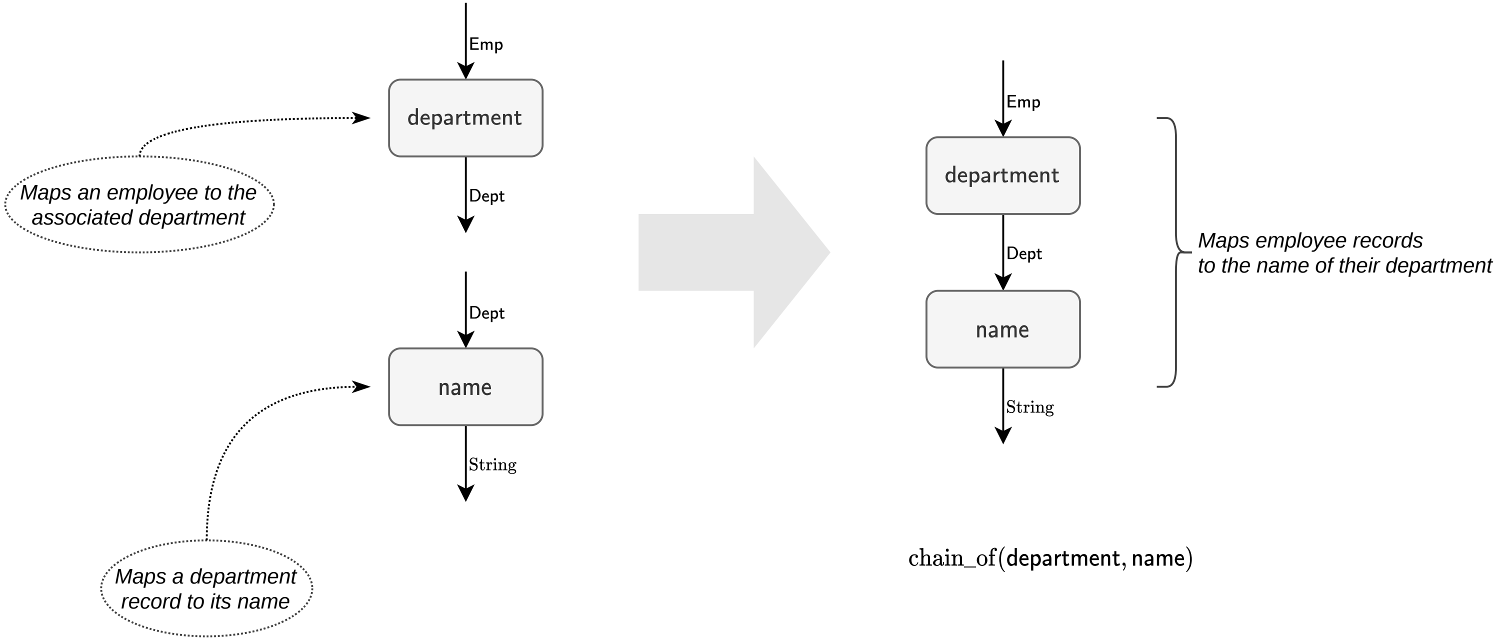
Crucially, composition of transformations is again a transformation.

Composition Combinator

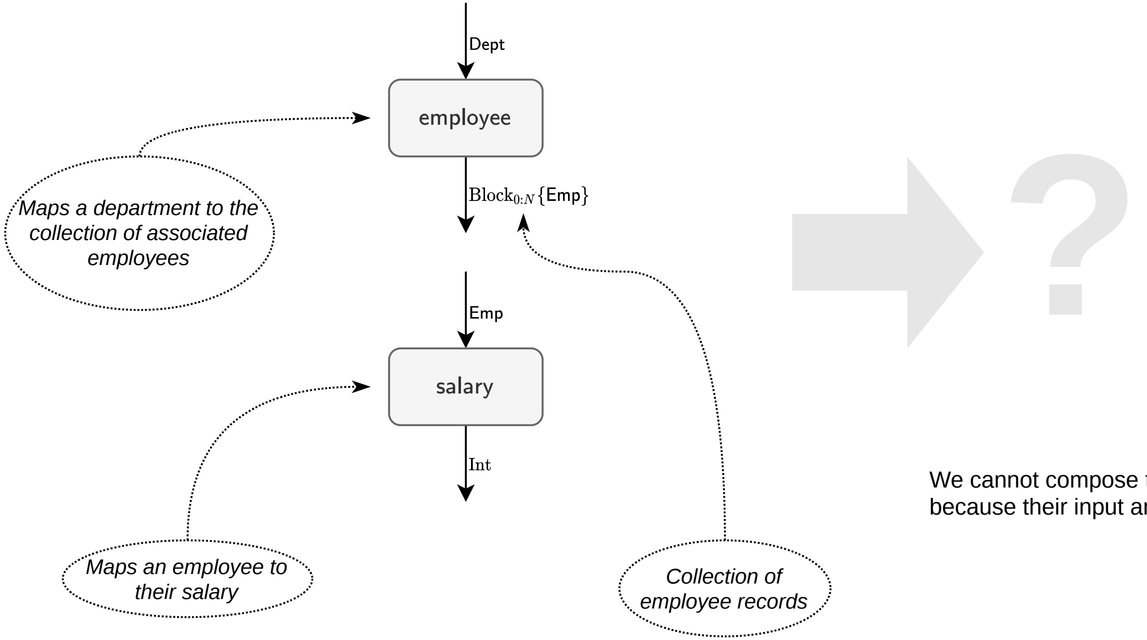


Composition $\mathrm{chain\_of}(\square, \square)$
is a transformation combinator
with two arguments.

Example: Composition

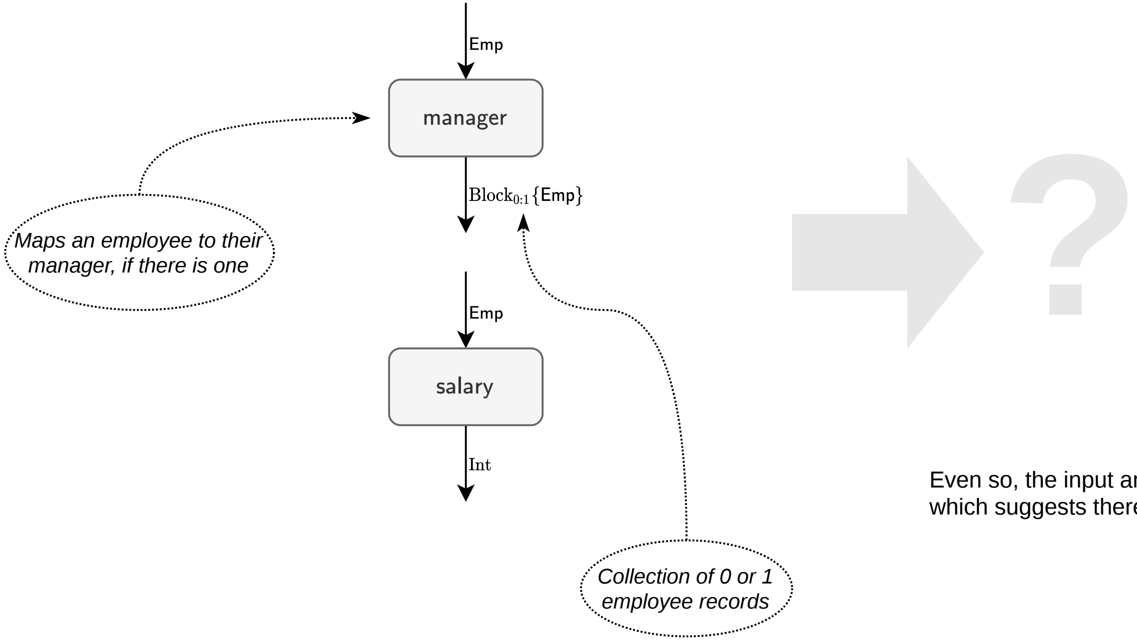


Counter-example: Plural Component



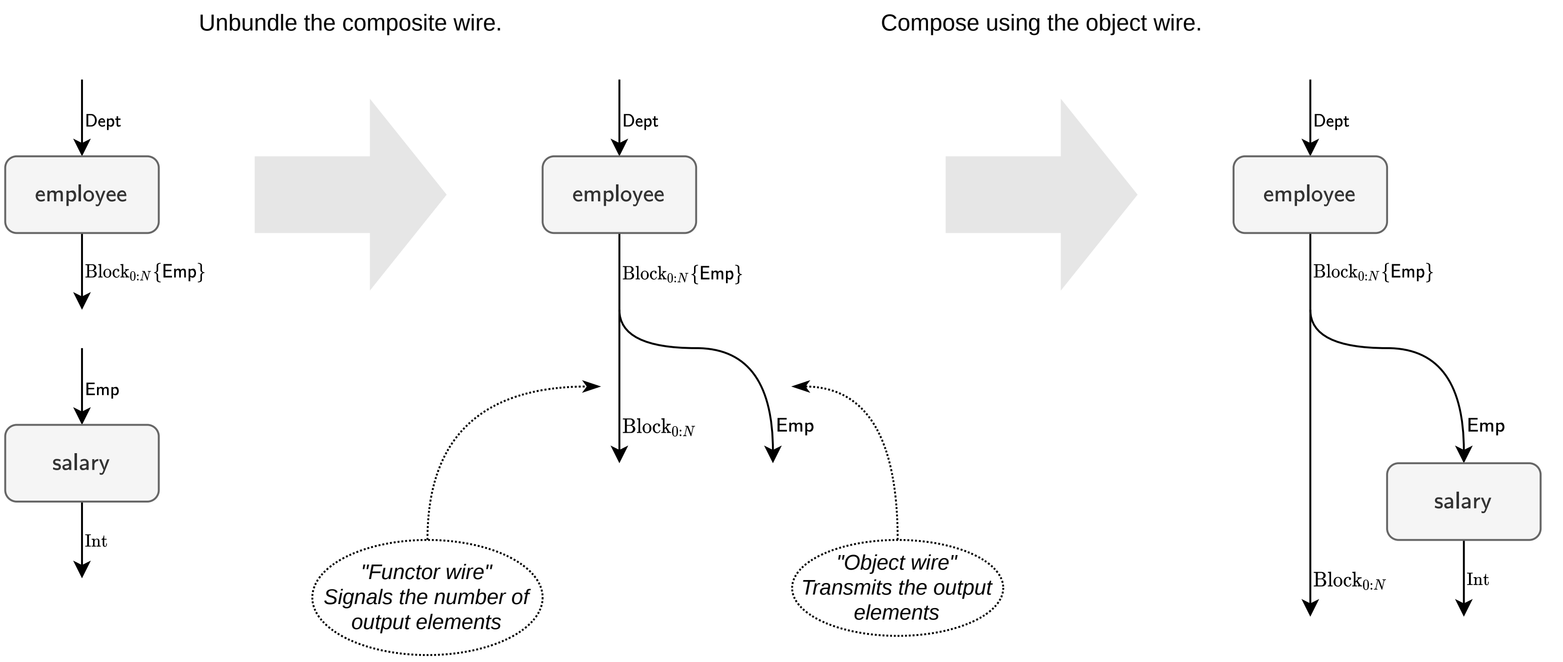
We cannot compose these transformations because their input and output do not quite match.

Counter-example: Optional Component



Even so, the input and the output share a common component,
which suggests there should be a way to compose these transformations.

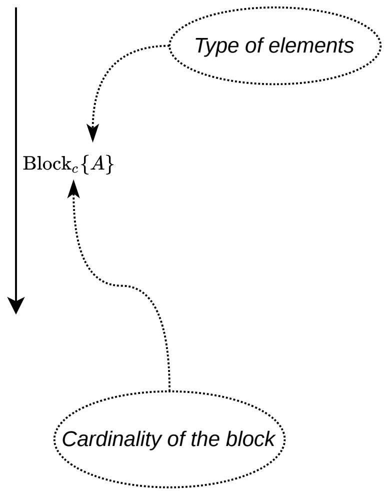
Idea: Unbundle the Wire



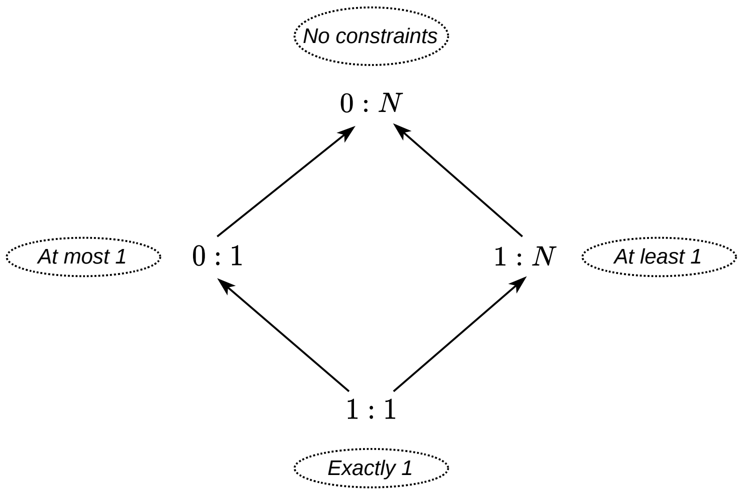
Attaching a transformation to the object wire indicates that the transformation is applied to all element of the collection.

Block Type

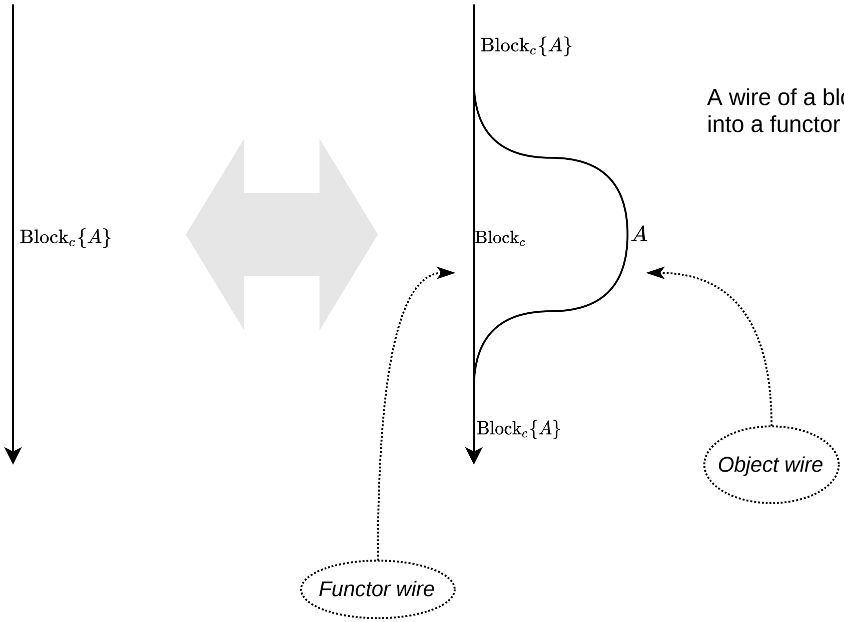
A block is a collection
of homogeneous elements.



Cardinality is a constraint
on the number of elements in a block.

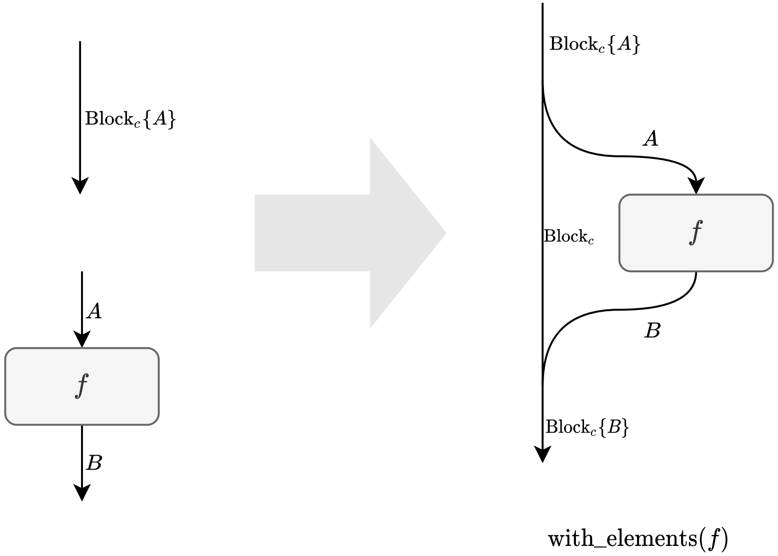


Unbundling



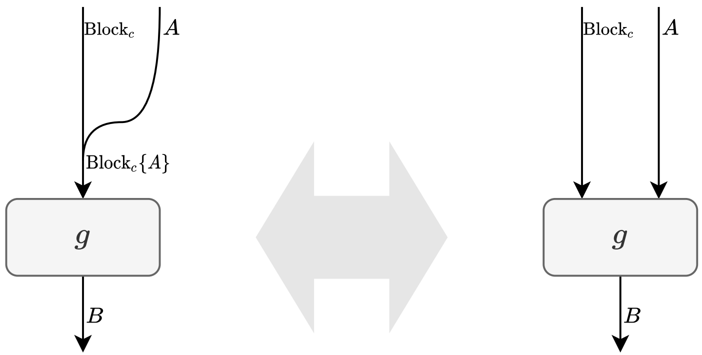
A wire of a block type can be unbundled into a functor and an object components.

Object Transformation

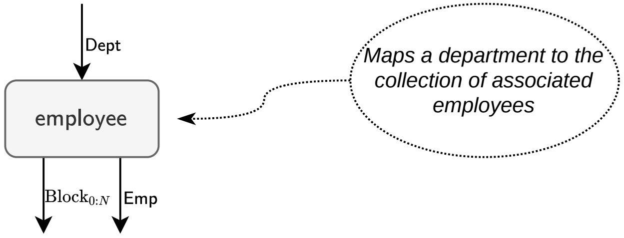


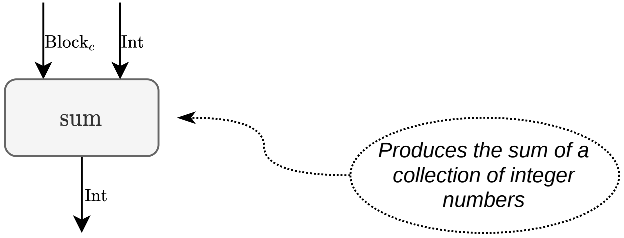
Any compatible transformation
can be applied to the object wire,
which indicates that the transformation
is applied to every element of the block.

Multiwired Transformations

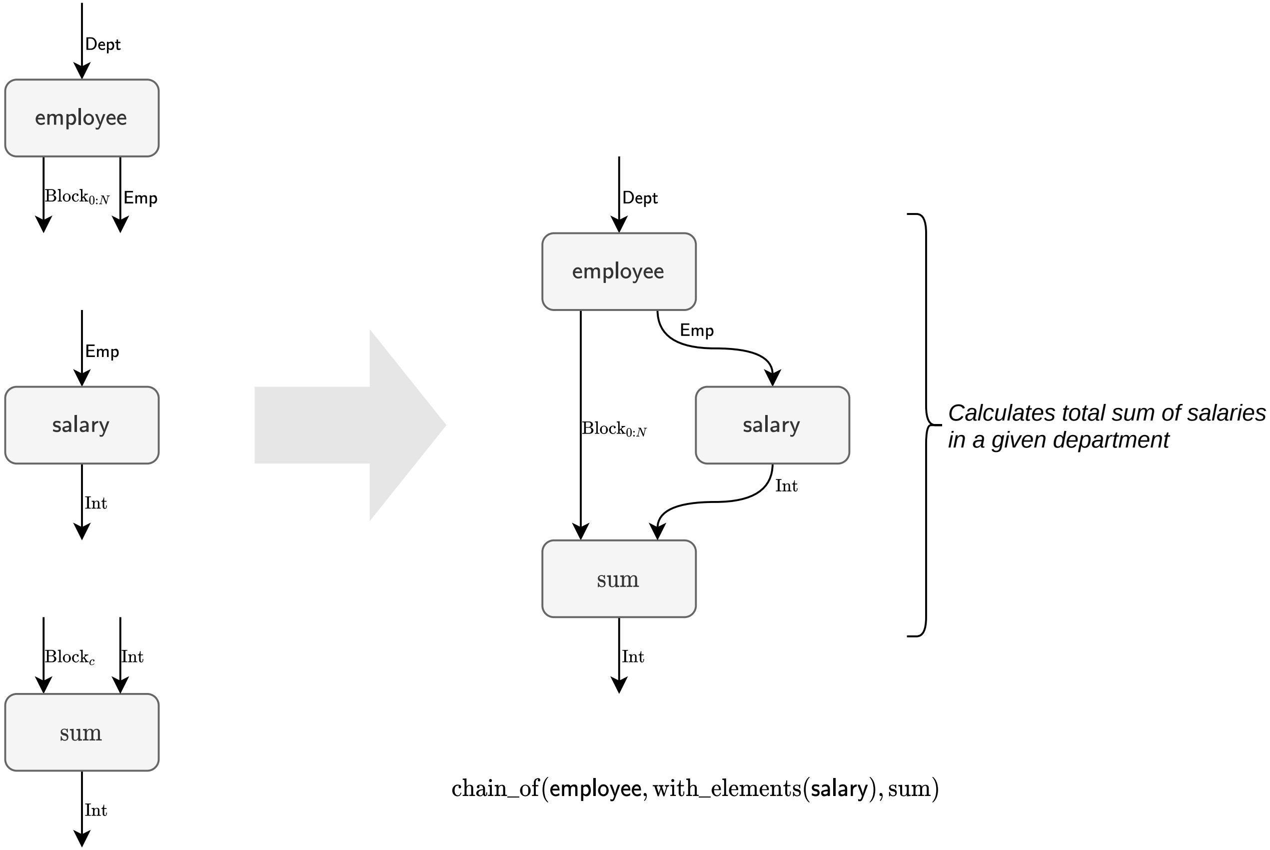


Example: Multiwired Transformations



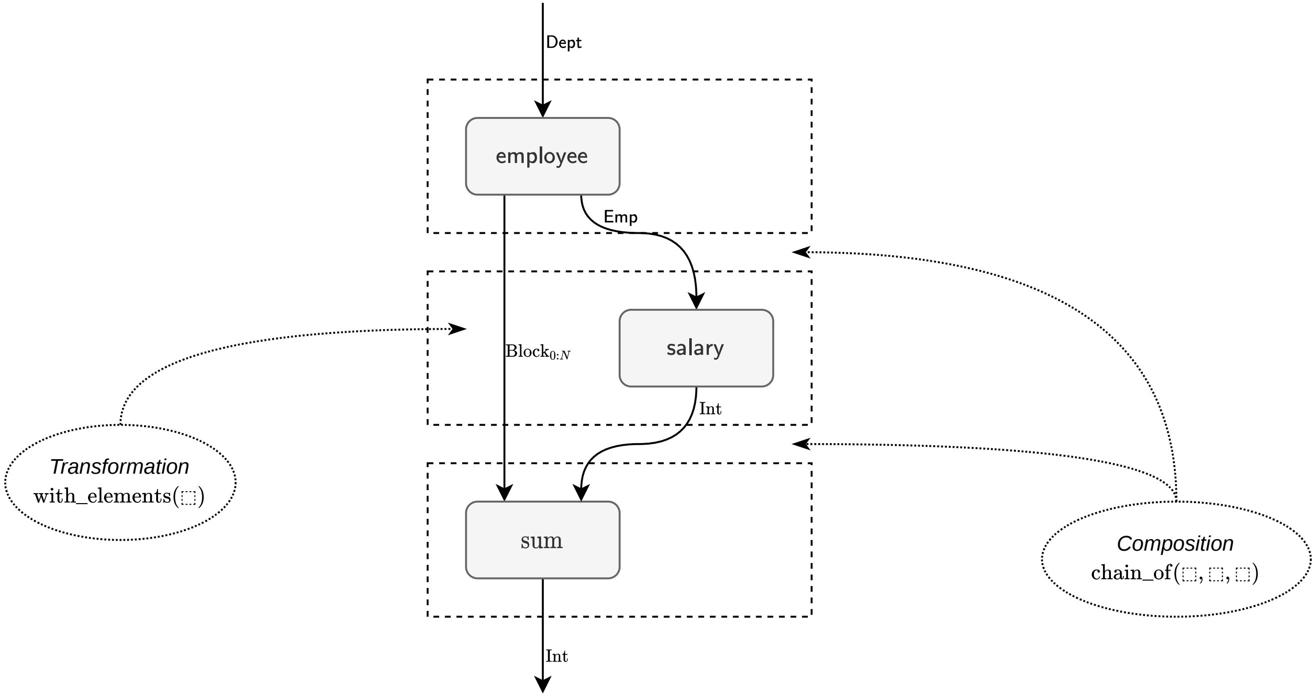


Example: Multiwired Composition



$$\mathrm{chain\_of}(\textsf{employee}, \mathrm{with\_elements}(\textsf{salary}), \mathrm{sum})$$

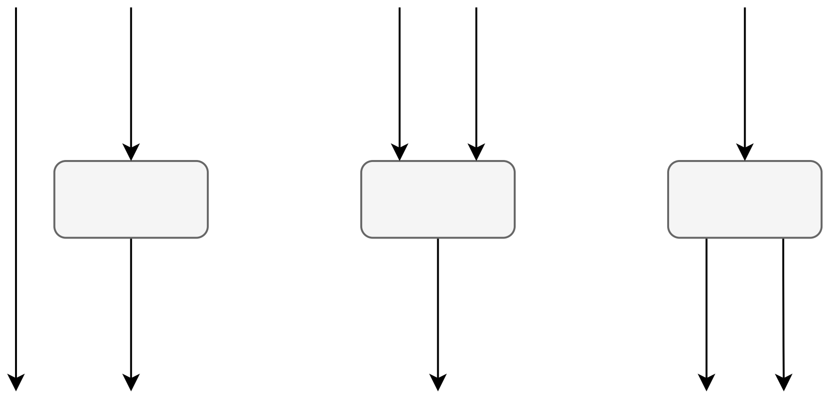
Example: Multiwired Composition Details



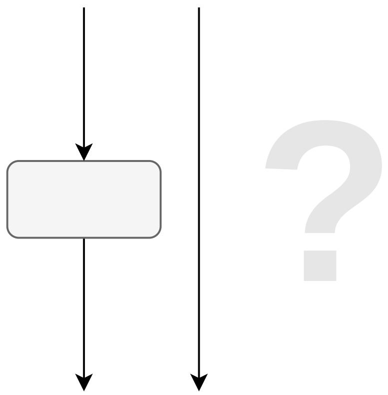
$\mathrm{chain\_of}(\textsf{employee}, \mathrm{with\_elements}(\textsf{salary}), \mathrm{sum})$

Challenge: Functor Transformation

So far we have seen different combinations of transformations and wires: transformations that are applied to the object wire, as well as transformations that consume or produce both functor and object wires.

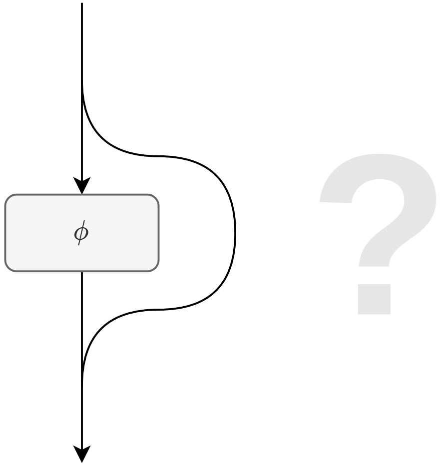


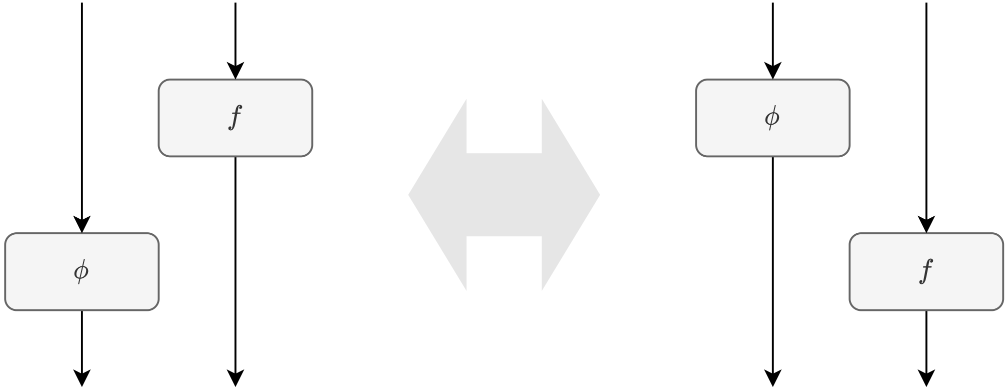
What we miss is a notion of a transformation applied to the functor wire.



Functor Transformation

Since a functor transformation does not act on the object wire,
it cannot interfere with any transformations applied to the object wire.

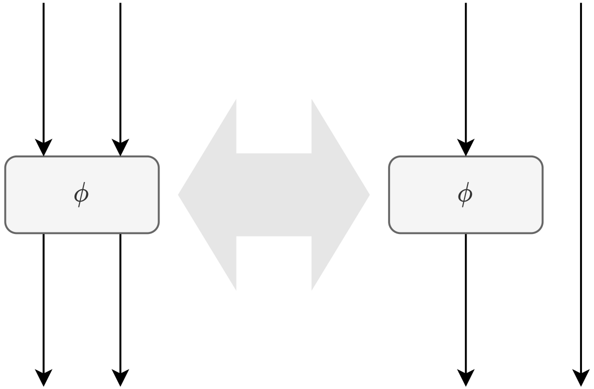




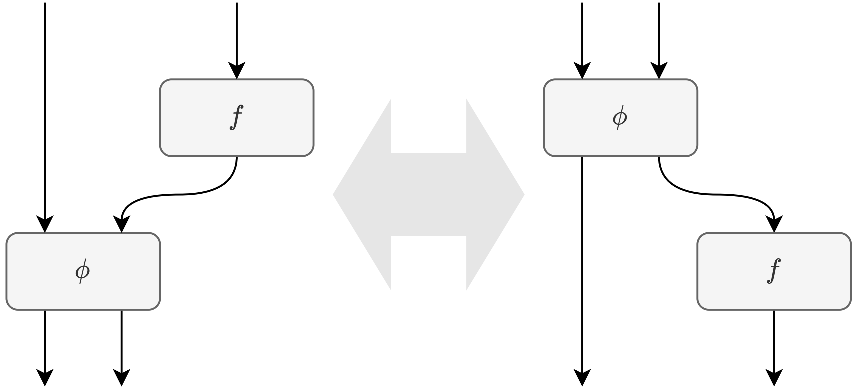
How to define a functor transformation?

This property can be taken as a definition of a functor transformation.

Definition: Functor Transformation



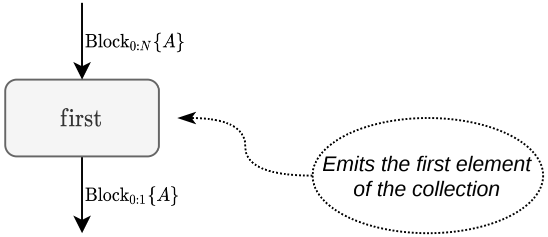
when for any f



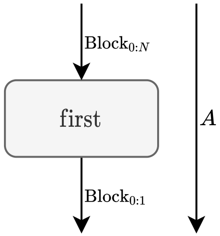
Such transformations are also called *natural*.

Branch of mathematics that studies natural transformations
is called the *category theory*.

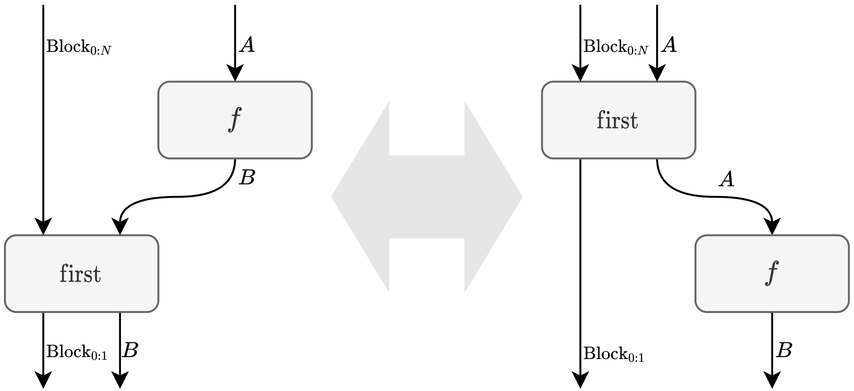
Example: Functor Transformation



This is, in fact, a natural transformation.



Indeed, for any $f : A \to B$,



$$\text{chain\_of}(\text{with\_elements}(f), \text{first}) \equiv \text{chain\_of}(\text{first}, \text{with\_elements}(f))$$