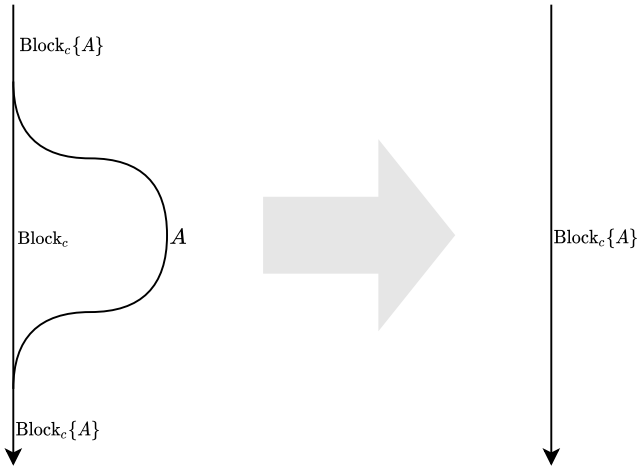
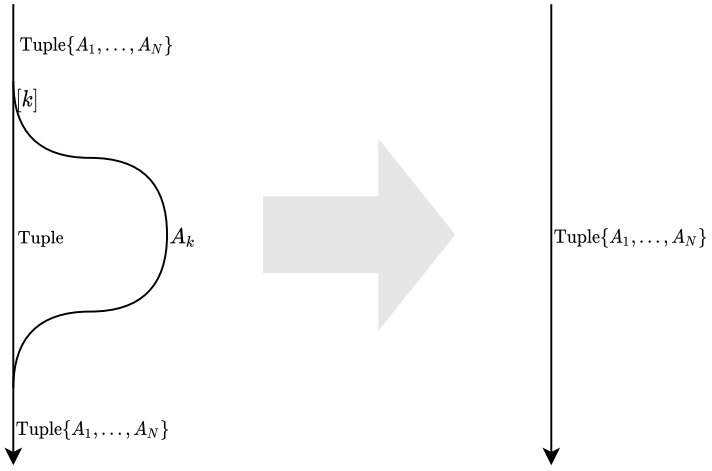
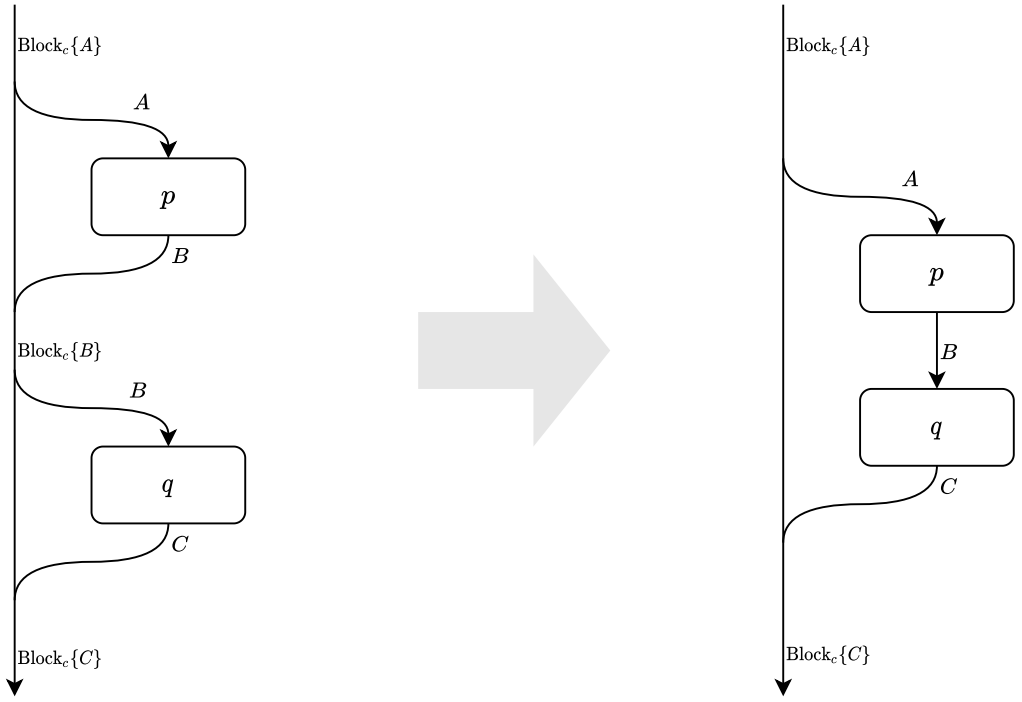
$\text{with\_elements}(\text{pass}()) \Rightarrow \text{pass}()$



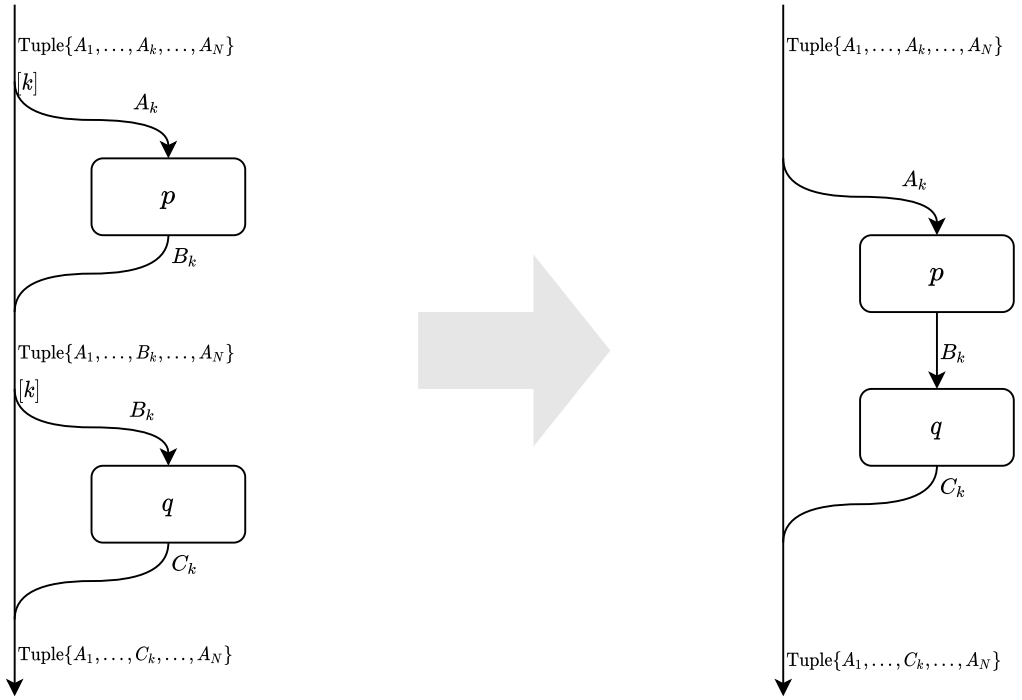
$\text{with\_column}(k, \text{pass}()) \Rightarrow \text{pass}()$



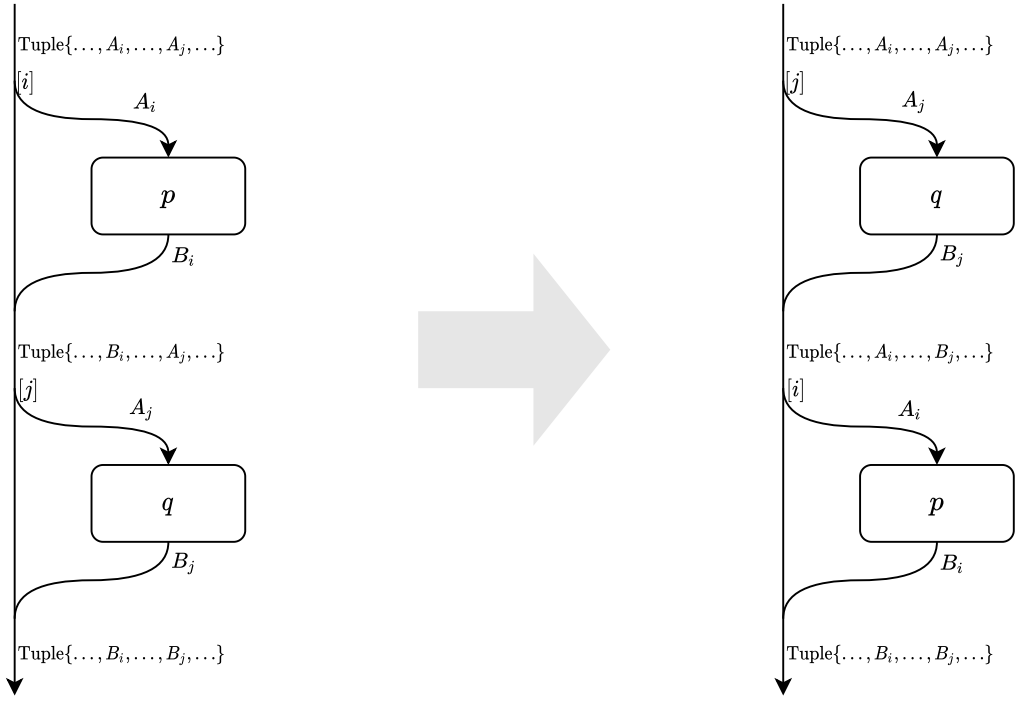
$\text{chain\_of}(\text{with\_elements}(p), \text{with\_elements}(q)) \Rightarrow \text{with\_elements}(\text{chain\_of}(p, q))$



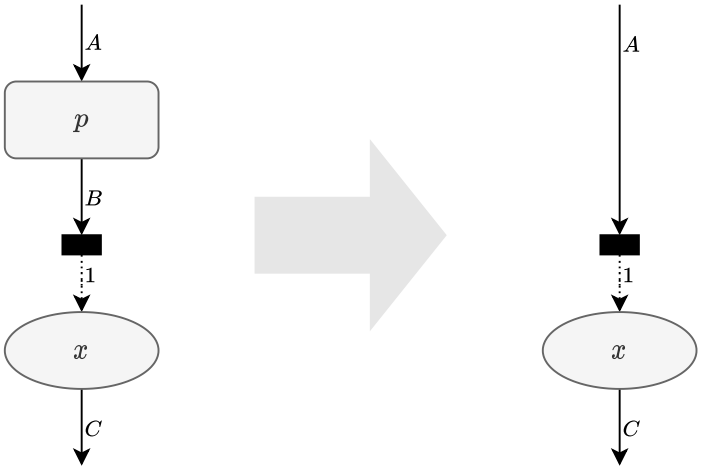
$$\text{chain\_of}(\text{with\_column}(k, p), \text{with\_column}(k, q)) \Rightarrow \text{with\_column}(k, \text{chain\_of}(p, q))$$

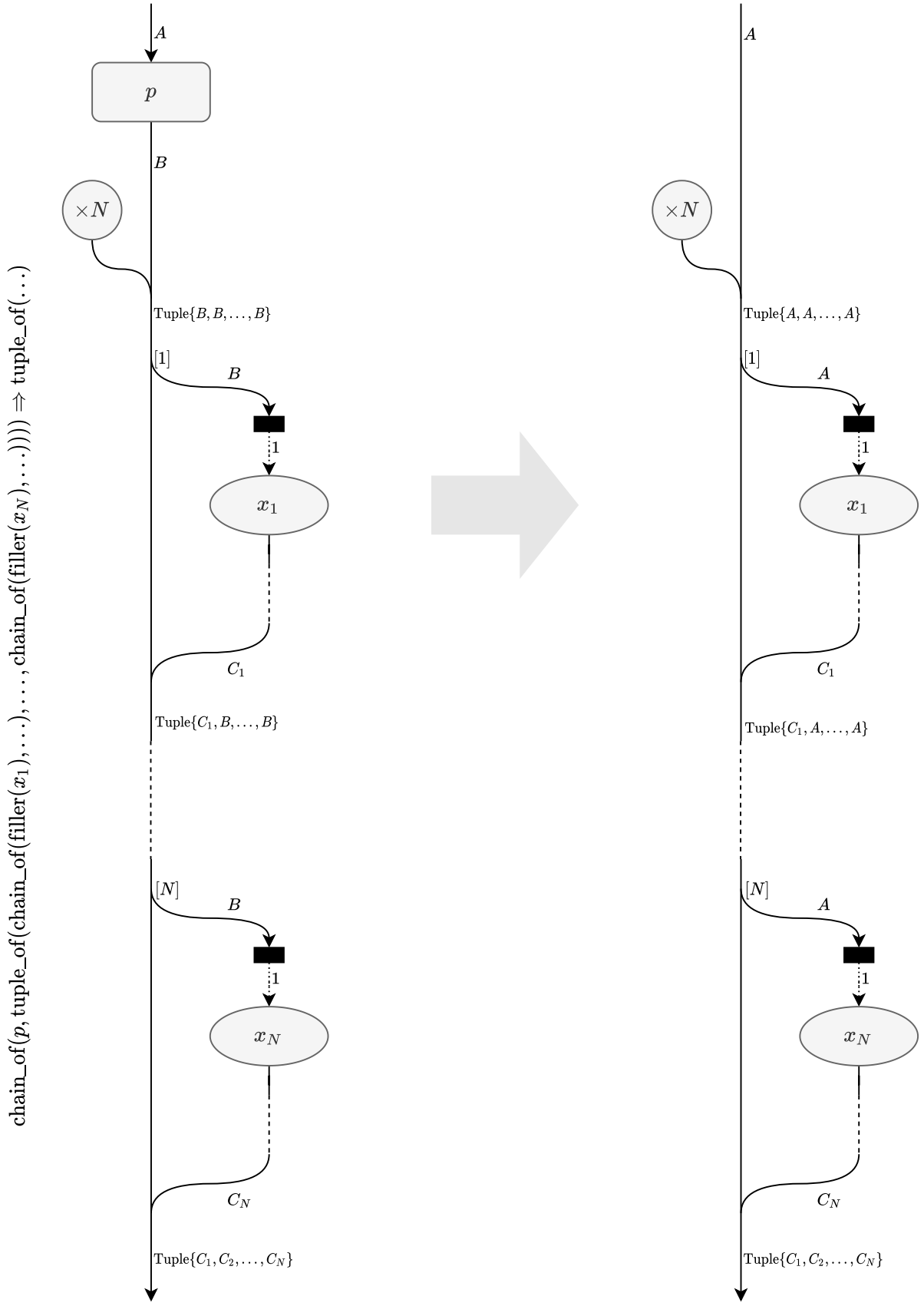


$$\text{chain\_of}(\text{with\_column}(i, p), \text{with\_column}(j, q)) \Rightarrow \text{chain\_of}(\text{with\_column}(j, p), \text{with\_column}(i, q)) \qquad i \neq j$$

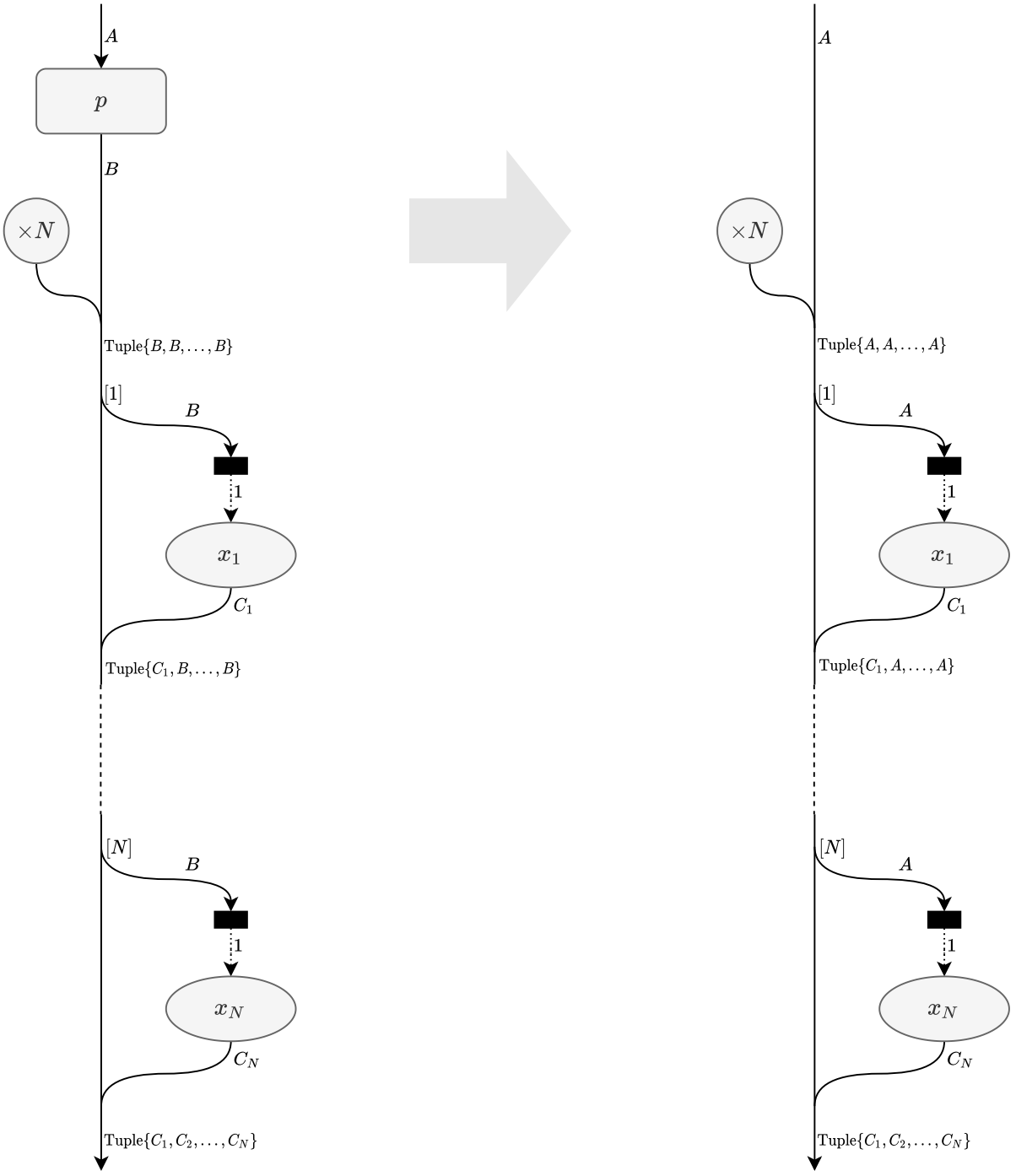


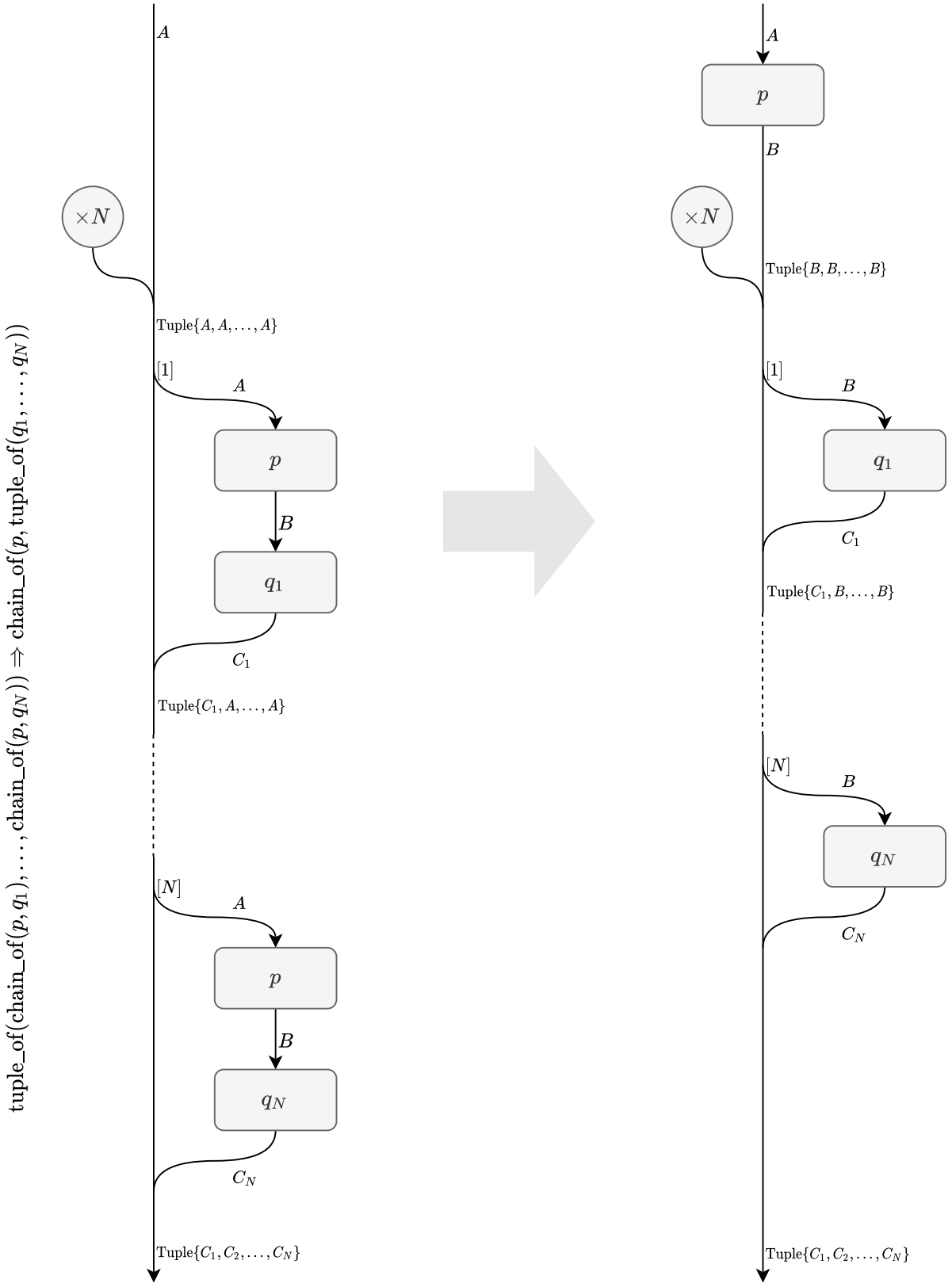
$\text{chain\_of}(p, \text{filler}(x)) \Rightarrow \text{filler}(x)$




chain_of(p, tuple_of(chain_of(filler(x1), ...), ..., chain_of(filler(xN), ...))) ⇒ tuple_of(...)
A
p
B
×N
Tuple{B, B, ..., B}
[1]
B
1
x1
C1
Tuple{C1, B, ..., B}
[N]
B
1
xN
CN
Tuple{C1, C2, ..., CN}
A
×N
Tuple{A, A, ..., A}
[1]
A
1
x1
C1
Tuple{C1, A, ..., A}
[N]
A
1
xN
CN
Tuple{C1, C2, ..., CN}

$\text{chain\_of}(p, \text{tuple\_of}(\text{filler}(x_1), \ldots, \text{filler}(x_N))) \Rightarrow \text{tuple\_of}(\text{filler}(x_1), \ldots, \text{filler}(x_N))$




tuple_of(chain_of(p, q1), . . . , chain_of(p, qN)) ⇒ chain_of(p, tuple_of(q1, . . . , qN))
A
×N
Tuple{A, A, . . . , A}
[1]
A
p
B
q1
C1
Tuple{C1, A, . . . , A}
[N]
A
p
B
qN
CN
Tuple{C1, C2, . . . , CN}
A
p
B
×N
Tuple{B, B, . . . , B}
[1]
B
q1
C1
Tuple{C1, B, . . . , B}
[N]
B
qN
CN
Tuple{C1, C2, . . . , CN}

$\text{chain\_of}(\text{tuple\_of}(p_1, \ldots, p_N), \text{column}(k)) \Rightarrow p_k$