Transformation

$$A$$

$$f$$

$$B$$

A transformation f
maps any input of type A
to the output of type B.

Composition

$$\text{chain\_of}(f, g)$$

Transformations with compatible input and output
can be composed.

Composition is a Transformation

A

f

B

g

C

Crucially, composition of transformations is again a transformation.

Composition Combinator

Add transformation

Add transformation

Composition $\mathrm{chain\_of}(\square, \square)$
is a transformation combinator
with two arguments.

Example: Composition

Emp

department

Dept

Maps an employee to the associated department

Dept

name

String

Maps a department record to its name

Emp

department

Dept

name

String

Maps employee records to the name of their department

$\mathrm{chain\_of(department, name)}$

Counter-example: Plural Component

Dept

employee

Maps a department to the collection of associated employees

$\text{Block}_{0:N}\{\text{Emp}\}$

Emp

salary

Int

Maps an employee to their salary

Collection of employee records

We cannot compose these transformations because their input and output do not quite match.

Counter-example: Optional Component

Emp

manager

$Block_{0:1}\{Emp\}$

Maps an employee to their manager, if there is one

Emp

salary

Int

Collection of 0 or 1 employee records

Even so, the input and the output share a common component,
which suggests there should be a way to compose these transformations.

Idea: Unbundle the Wire

Unbundle the composite wire.

Compose using the object wire.

Dept

employee

$Block_{0:N}\{Emp\}$

Emp

salary

Int

Dept

employee

$Block_{0:N}\{Emp\}$

$Block_{0:N}$

Emp

"Functor wire"
Signals the number of
output elements

"Object wire"
Transmits the output
elements

Dept

employee

$Block_{0:N}\{Emp\}$

Emp

salary

$Block_{0:N}$

Int

Attaching a transformation to the object wire
indicates that the transformation is applied
to all element of the collection.

Block Type

A block is a collection
of homogeneous elements.

Cardinality is a constraint
on the number of elements in a block.

Type of elements

$\mathrm{Block}_c\{A\}$

Cardinality of the block

No constraints

$$0 : N$$

At most 1 $0 : 1$ $1 : N$ *At least 1*

$$1 : 1$$

Exactly 1

Unbundling

$\mathrm{Block}_c\{A\}$

$\mathrm{Block}_c\{A\}$

$\mathrm{Block}_c\{A\}$

Block_c

A

$\mathrm{Block}_c\{A\}$

A wire of a block type can be unbundled
into a functor and an object components.

Object wire

Functor wire

Object Transformation

$\mathrm{Block}_c\{A\}$

A

f

B

$\mathrm{Block}_c\{A\}$

A

Block_c f

B

$\mathrm{Block}_c\{B\}$

$\mathrm{with\_elements}(f)$

Any compatible transformation
can be applied to the object wire,
which indicates that the transformation
is applied to every element of the block.

Multiwired Transformations

Example: Multiwired Transformations

Dept

employee

Maps a department to the collection of associated employees

$Block_{0:N}$ Emp

$Block_c$ Int

sum

Produces the sum of a collection of integer numbers

Int

Example: Multiwired Composition

Dept

employee

$\text{Block}_{0:N}$ | Emp

Emp

salary

Int

Block_c | Int

sum

Int

Dept

employee

Emp

$\text{Block}_{0:N}$

salary

Int

sum

Int

Calculates total sum of salaries in a given department

$\text{chain\_of}(\textbf{employee}, \text{with\_elements}(\textbf{salary}), \text{sum})$

Example: Multiwired Composition Details

$$\text{chain\_of}(\mathbf{employee}, \text{with\_elements}(\mathbf{salary}), \text{sum})$$

Challenge: Functor Transformation

So far we have seen different combinations of transformations and wires: transformations that are applied to the object wire, as well as transformations that consume or produce both functor and object wires.

What we miss is a notion of a transformation applied to the functor wire.

Functor Transformation

Since a functor transformation does not act on the object wire,
it cannot interfere with any transformations applied to the object wire.

How to define a functor transformation?

This property can be taken as a definition of a functor transformation.
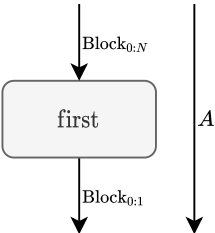
Definition: Functor Transformation

when for any f

Such transformations are also called *natural*.

Branch of mathematics that studies natural transformations
is called the *category theory*.

Example: Functor Transformation

$\text{Block}_{0:N}\{A\}$

first

$\text{Block}_{0:1}\{A\}$

Emits the first element of the collection

This is, in fact, a natural transformation.

$\text{Block}_{0:N}$

first

A

$\text{Block}_{0:1}$

Indeed, for any $f : A \to B$,

$\text{Block}_{0:N}$

A

f

B

first

$\text{Block}_{0:1}$ B

$\text{Block}_{0:N}$ A

first

A

f

$\text{Block}_{0:1}$ B

$$\text{chain\_of}(\text{with\_elements}(f), \text{first}) \equiv \text{chain\_of}(\text{first}, \text{with\_elements}(f))$$