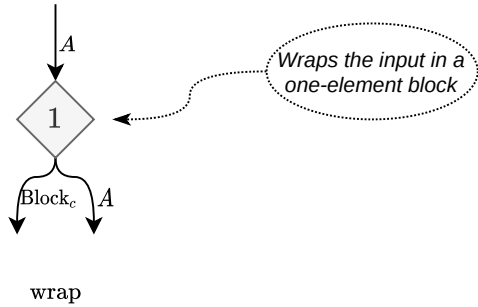


Monadic Interface



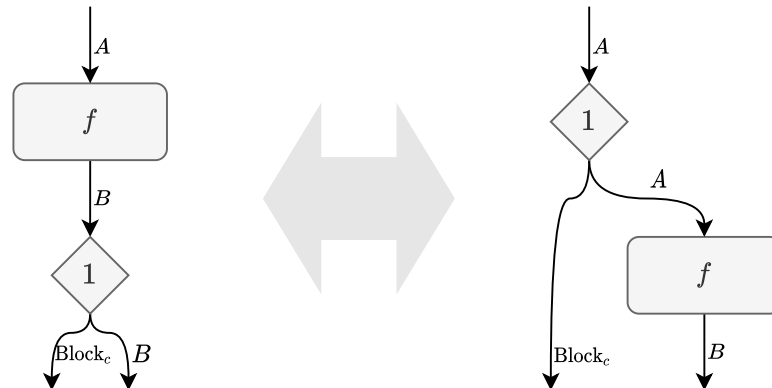
Unit Transformation



Unit is a natural transformation.



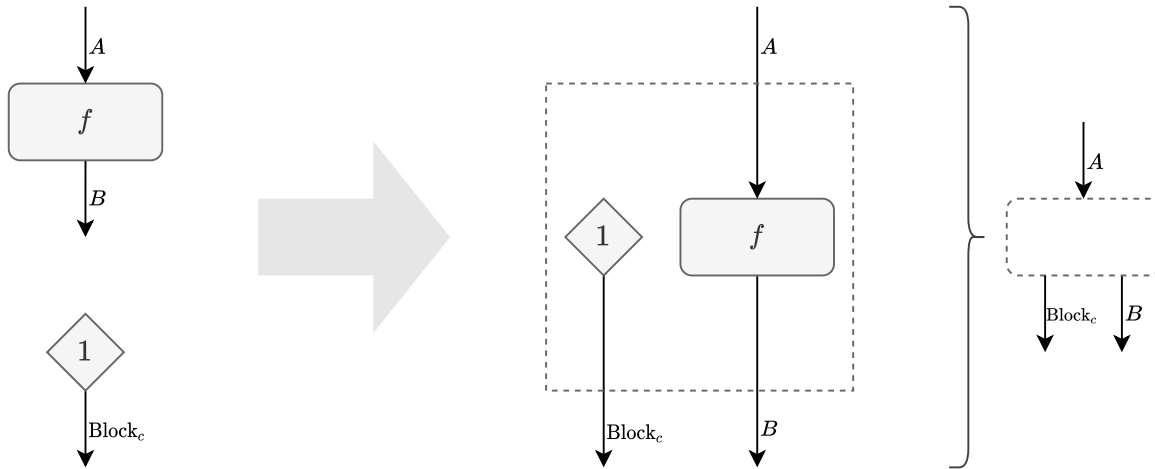
Indeed, for any $f : A \rightarrow B$,



$$\text{chain_of}(f, \text{wrap}) \equiv \text{chain_of}(\text{wrap}, \text{with_elements}(f))$$

Using Unit Transformation

Unit transformation can adapt any regular transformation to the monadic interface.



Monadic Interface: Examples



Composition Challenge

Two monadic transformations
can be composed.



`chain_of(employee, with_elements(manager))`

But the composition
does not satisfy
the monadic interface.



Multiplication Transformation



It is easy to check that the multiplication is a natural transformation.



Using Multiplication Transformation

Multiplication can be used to combine two monadic transformations into a new monadic transformation.

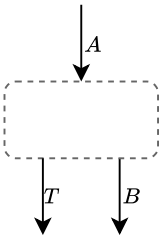


Monad

In general, a *monad* is a functor wire which possesses two special natural transformations: unit and multiplication.



Monadic transformation is a transformation with the following shape.

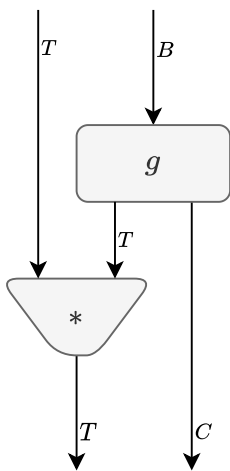


Monadic Composition

Using multiplication, two compatible monadic transformations can be composed to form a new monadic transformation.



Aside: this is known as the operation $\text{bind}(g)$.



For this definition to be coherent, unit and multiplication must satisfy certain properties.

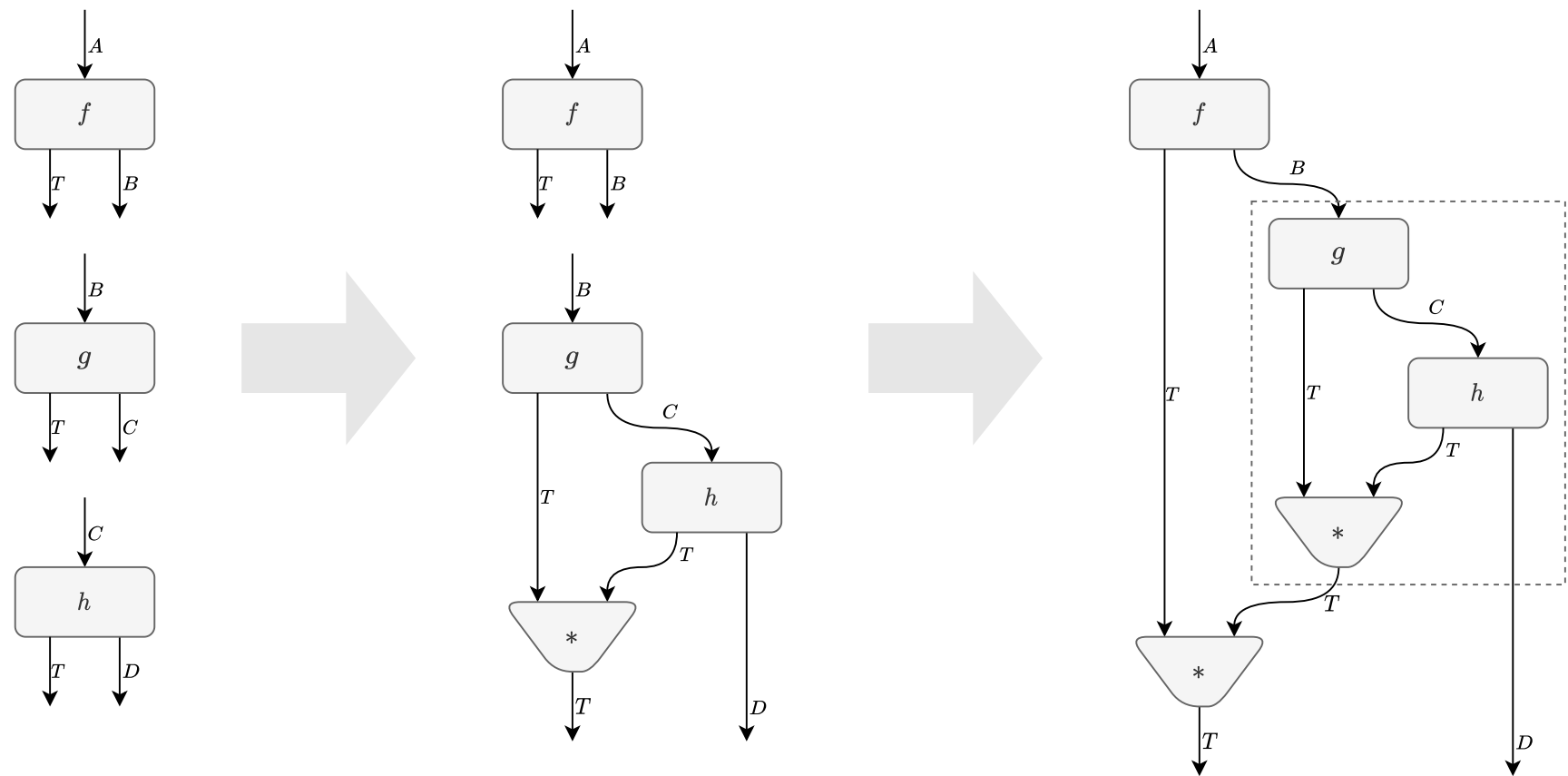
Associativity of Monadic Composition 1

Given three compatible monadic transformations, they could be composed in two distinct ways.



Associativity of Monadic Composition 2

Given three compatible monadic transformations, they could be composed in two distinct ways.



Associativity of Monadic Composition 3

For monadic composition to be associative, these two transformations must be equivalent for any f , g , and h .



Monad Law I

Monadic multiplication must satisfy the following equivalence condition.



Composition of Regular and Monadic Transformations 1

Given one regular and one monadic transformation, they could be composed in two distinct ways.



Composition of Regular and Monadic Transformations 2

Given one regular and one monadic transformation, they could be composed in two distinct ways.



Composition of Regular and Monadic Transformations 3

We demand that these two transformations must be equivalent for any f and g .



Monad Law II & III

Monadic unit and multiplication must satisfy the following equivalence condition.



Composing monadic and regular transformations in a different order, we arrive to a symmetric condition.

