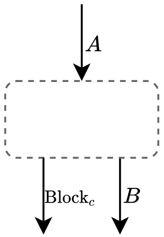
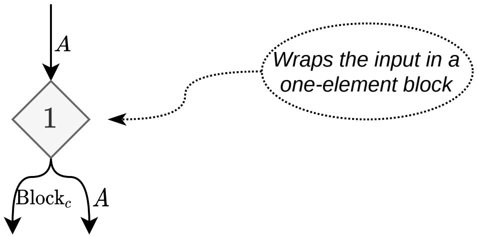
Monadic Interface



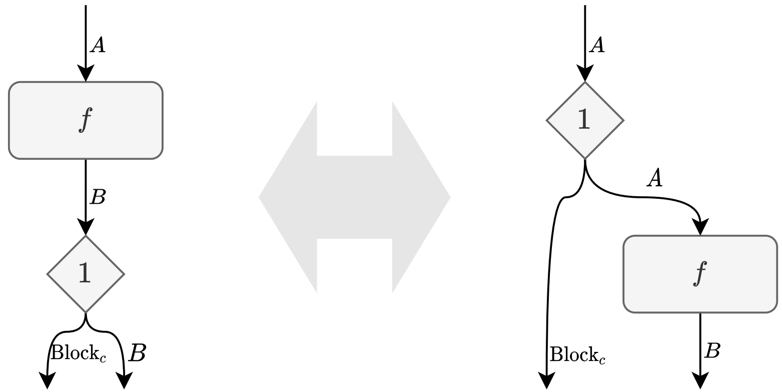
Unit Transformation



wrap

Unit is a natural transformation.

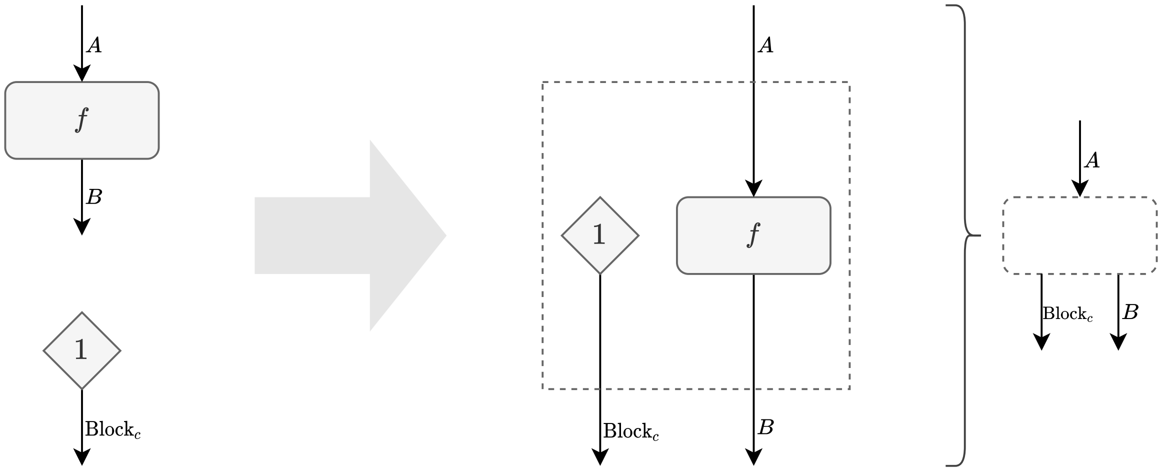
Indeed, for any $f : A \to B$,



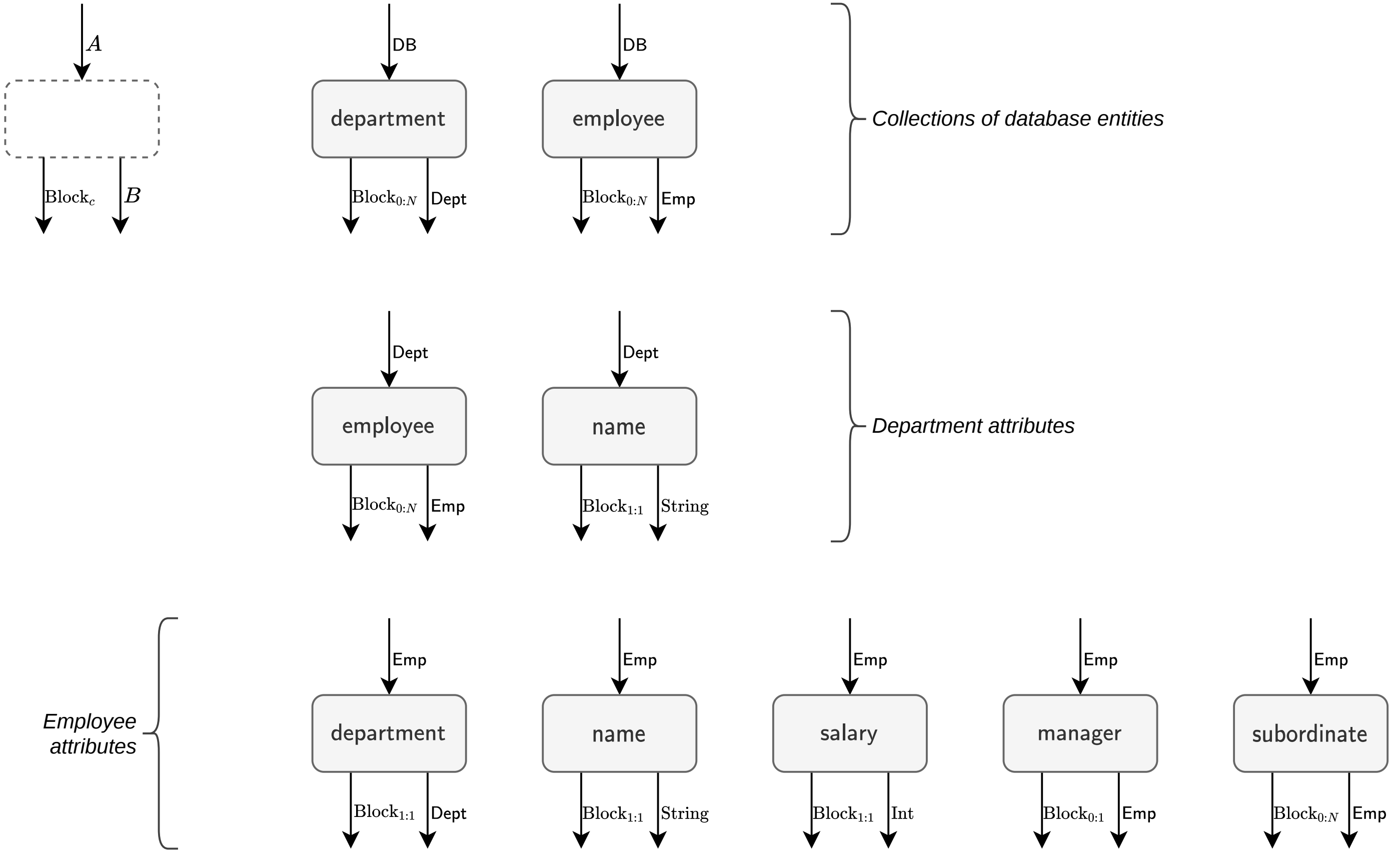
$$\mathrm{chain\_of}(f, \mathrm{wrap}) \equiv \mathrm{chain\_of}(\mathrm{wrap}, \mathrm{with\_elements}(f))$$

Using Unit Transformation

Unit transformation can adapt any regular transformation
to the monadic interface.

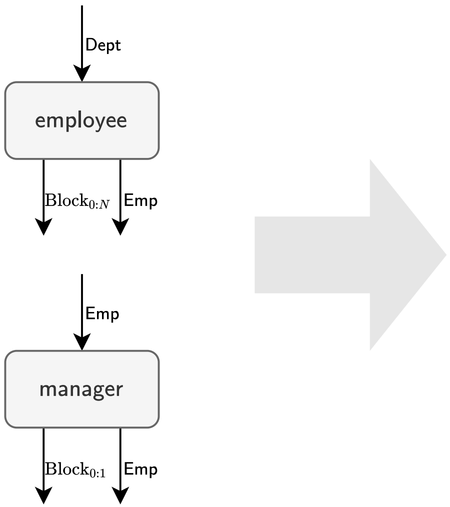


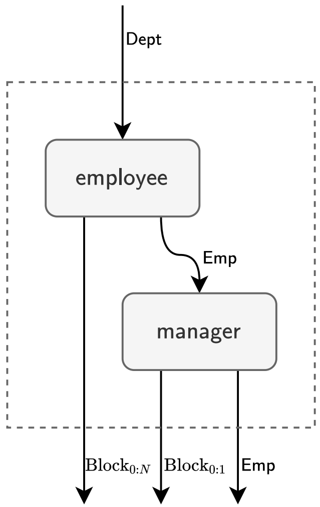
Monadic Interface: Examples



Composition Challenge

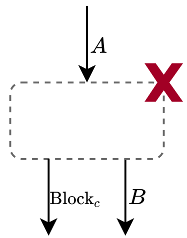
Two monadic transformations
can be composed.



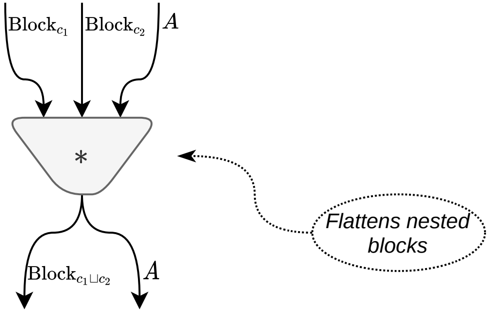


$\text{chain\_of}(\textsf{employee}, \text{with\_elements}(\textsf{manager}))$

But the composition
does not satisfy
the monadic interface.



Multiplication Transformation

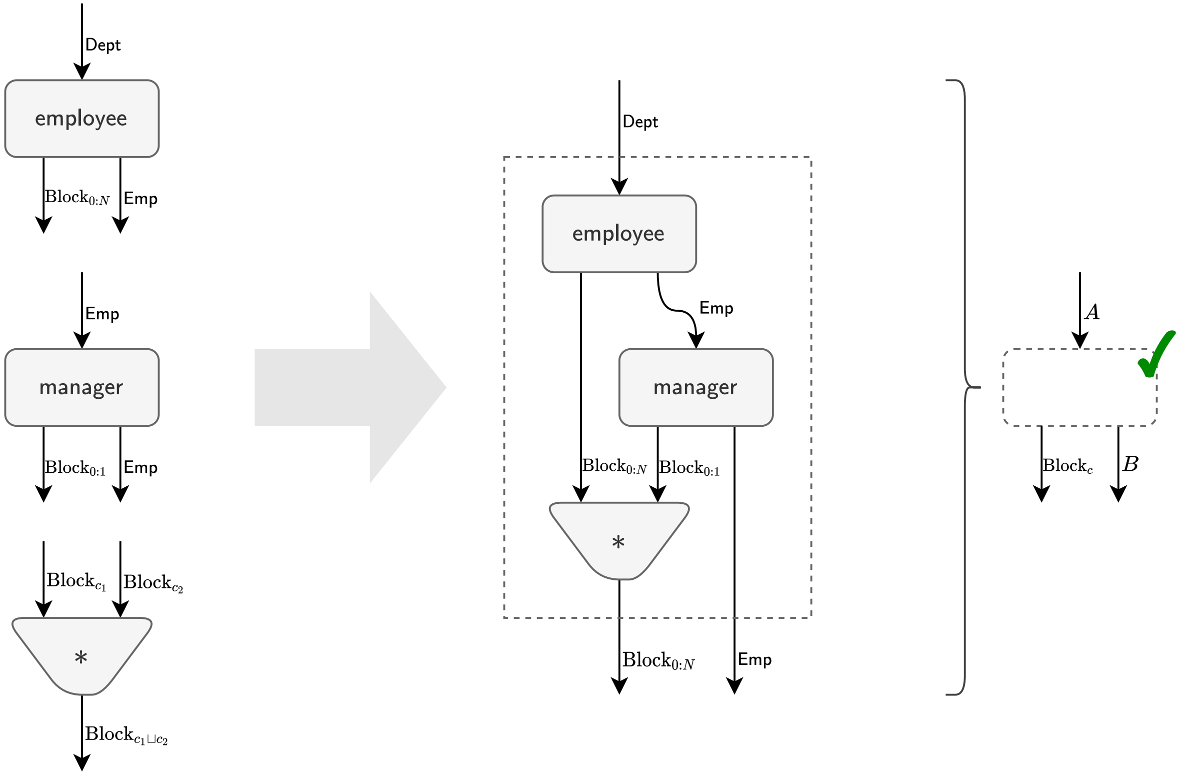


flatten

It is easy to check that the multiplication is a natural transformation.

Using Multiplication Transformation

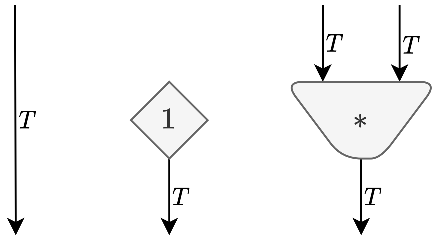
Multiplication can be used to combine two monadic transformations into a new monadic transformation.



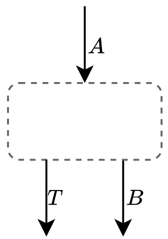
chain_of(**employee**, with_elements(**manager**), flatten)

Monad

In general, a *monad* is a functor wire which posesses
two special natural transformations: unit and multiplication.

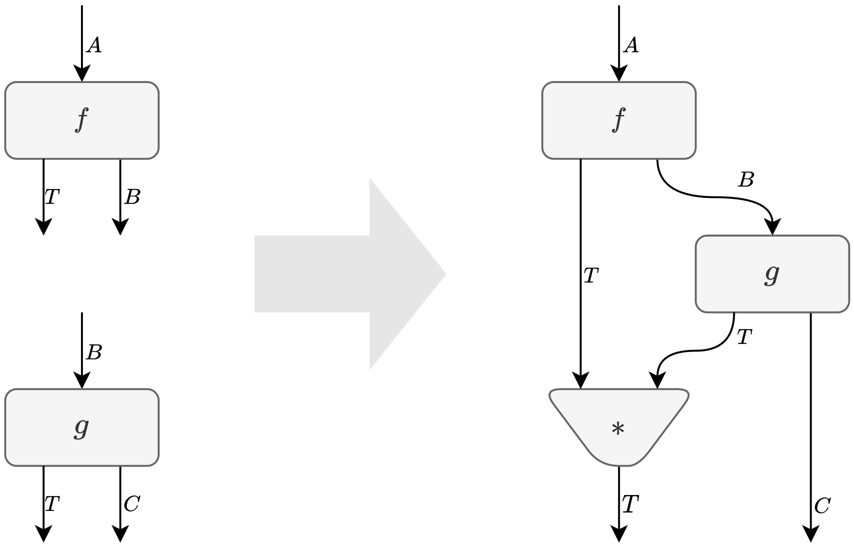


Monadic transformation is a transformation
with the following shape.

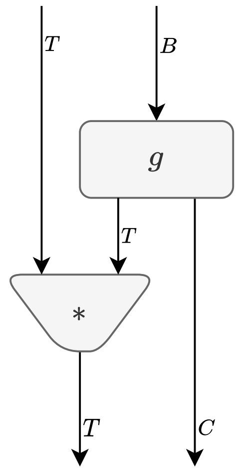


Monadic Composition

Using multiplication, two compatible monadic transformations
can be composed to form a new monadic transformation.



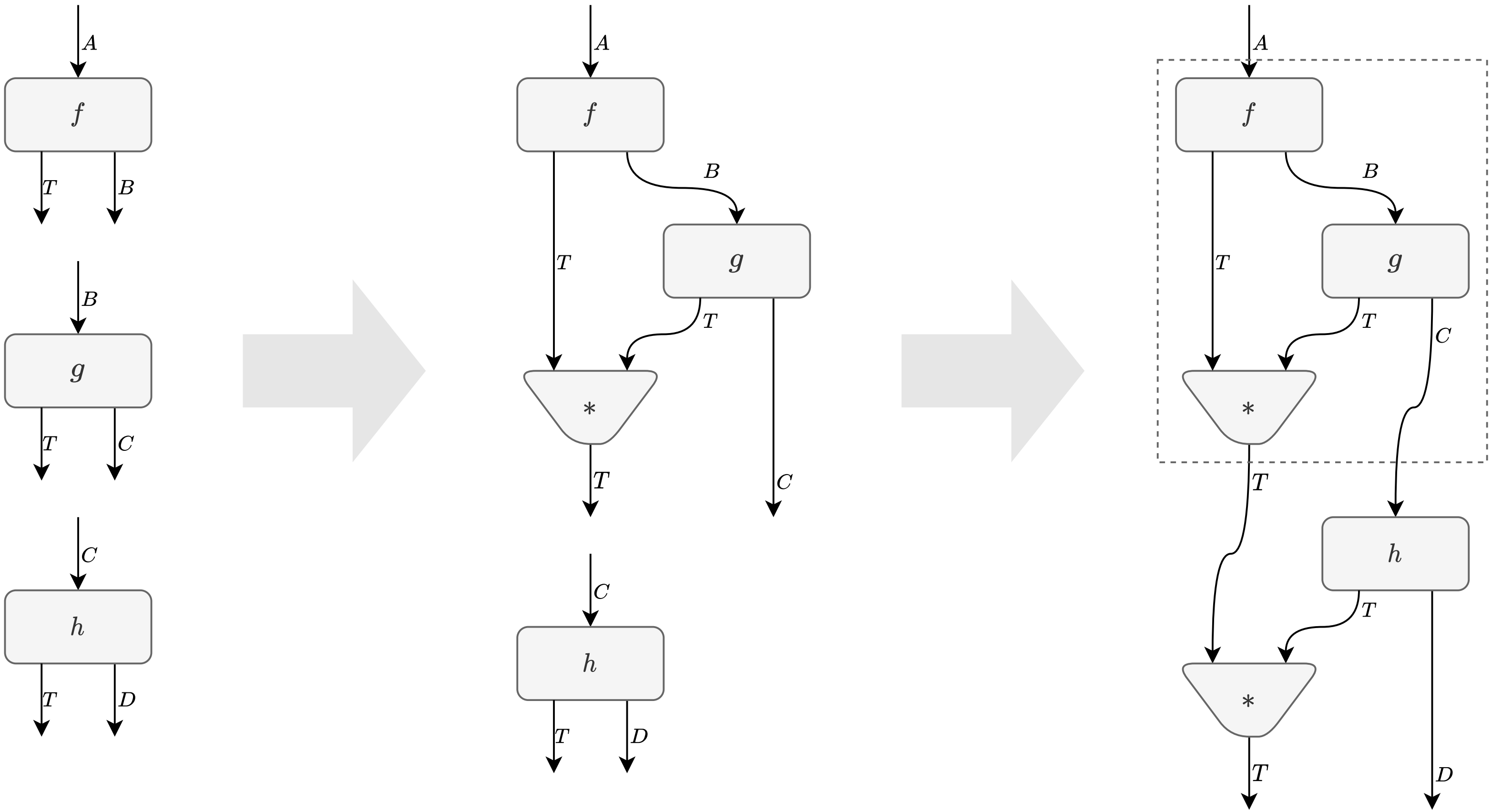
Aside: this is known as
the operation $\mathrm{bind}(g)$.



For this definition to be coherent,
unit and multiplication must satisfy certain properties.

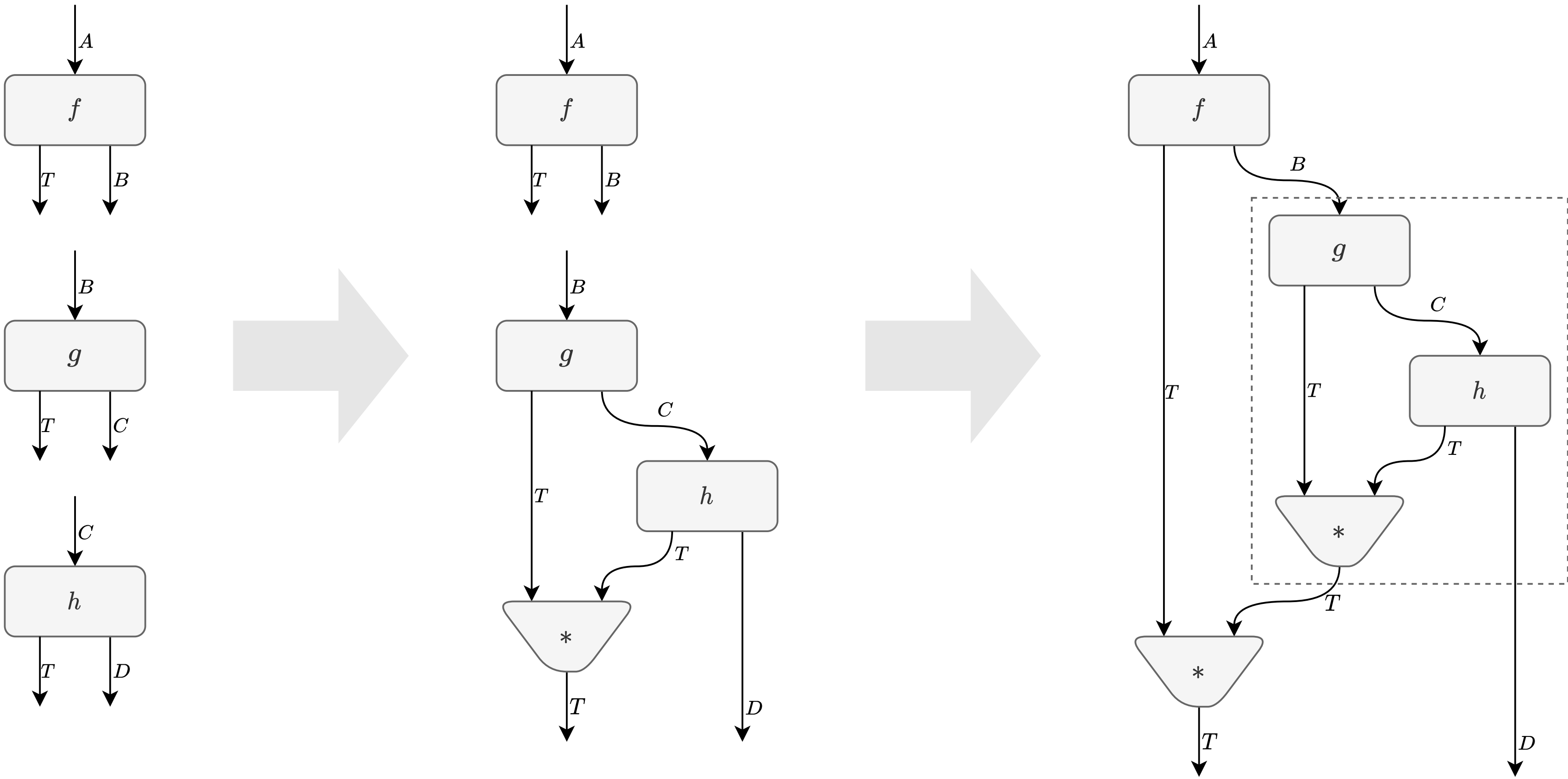
Associativity of Monadic Composition 1

Given three compatible monadic transformations,
they could be composed in two distinct ways.



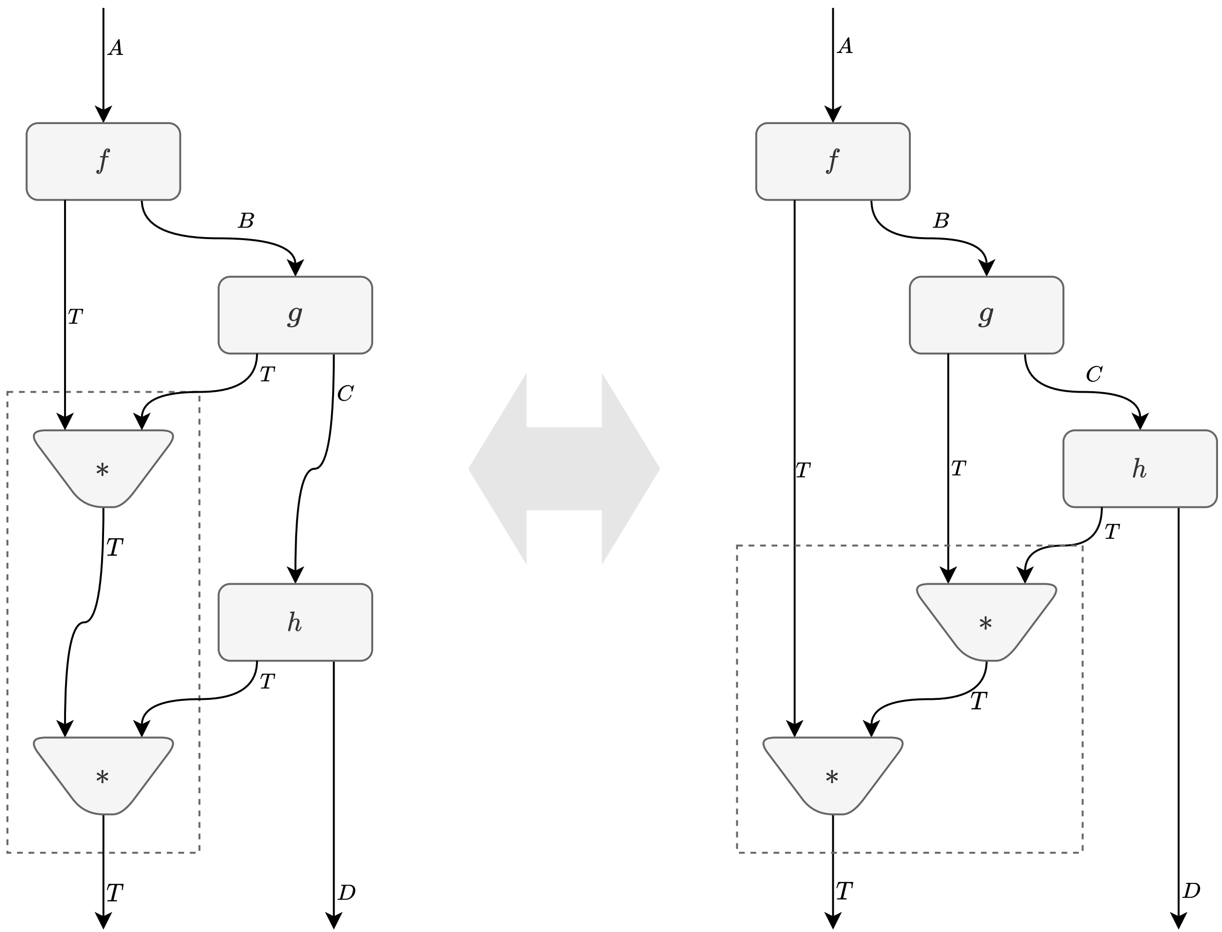
Associativity of Monadic Composition 2

Given three compatible monadic transformations,
they could be composed in two distinct ways.



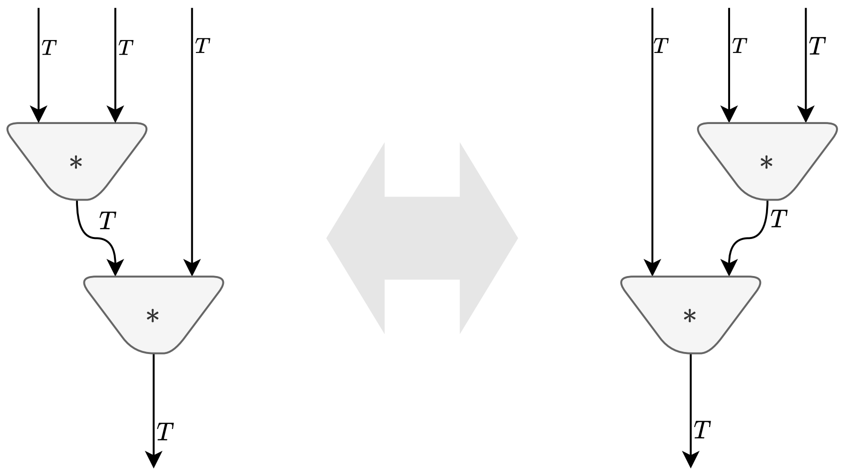
Associativity of Monadic Composition 3

For monadic composition to be associative, these two transformations must be equivalent for any f, g, and h.



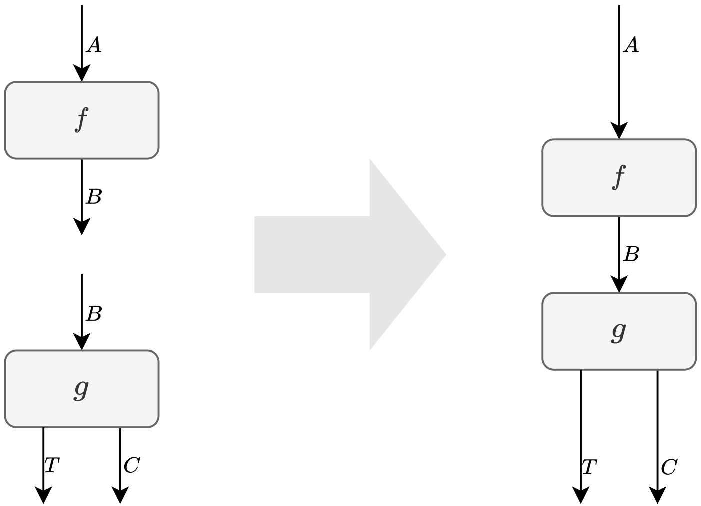
Monad Law I

Monadic multiplication must satisfy the following equivalence condition.



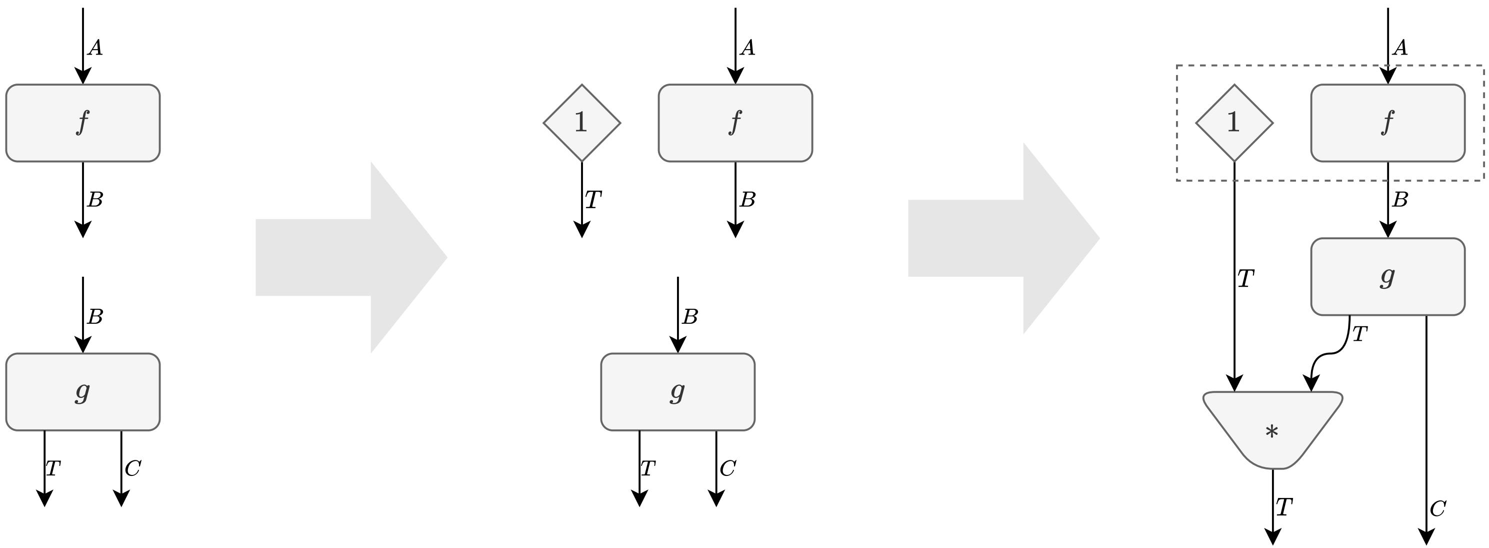
Composition of Regular and Monadic Transformations 1

Given one regular and one monadic transformation,
they could be composed in two distinct ways.



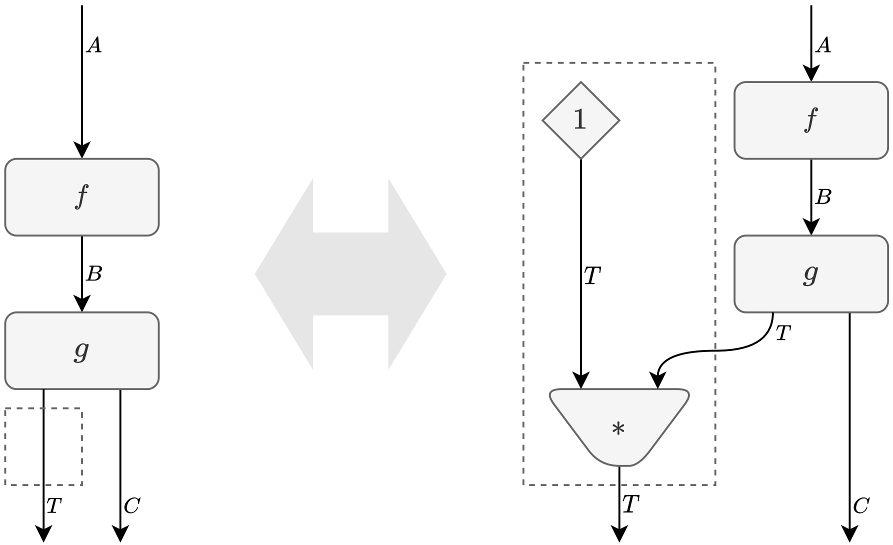
Composition of Regular and Monadic Transformations 2

Given one regular and one monadic transformation,
they could be composed in two distinct ways.



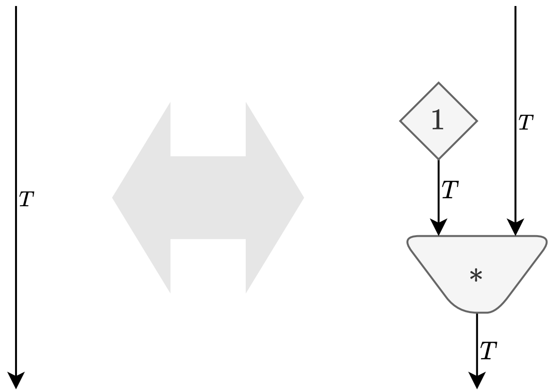
Composition of Regular and Monadic Transformations 3

We demand that these two transformations must be equivalent for any f and g.



Monad Law II & III

Monadic unit and multiplication must satisfy the following equivalence condition.



Composing monadic and regular transformations in a different order, we arrive to a symmetric condition.