

# Monadic Interface



# Unit Transformation



Unit is a natural transformation.



Indeed, for any  $f : A \rightarrow B$ ,



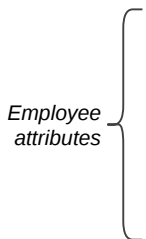
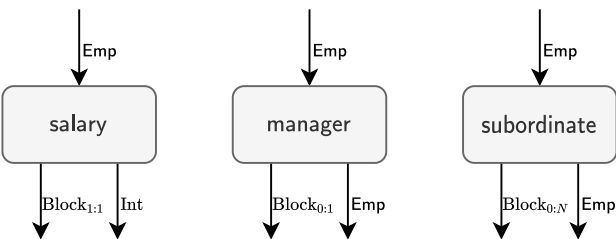
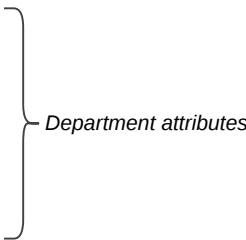
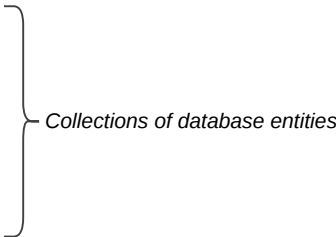
$$\text{chain\_of}(f, \text{wrap}) \equiv \text{chain\_of}(\text{wrap}, \text{with\_elements}(f))$$

# Using Unit Transformation

Unit transformation can adapt any regular transformation to the monadic interface.



# Monadic Interface: Examples



# Composition Challenge

Two monadic transformations can be composed.



`chain_of(employee, with_elements(manager))`

But the composition does not satisfy the monadic interface.



# Multiplication Transformation

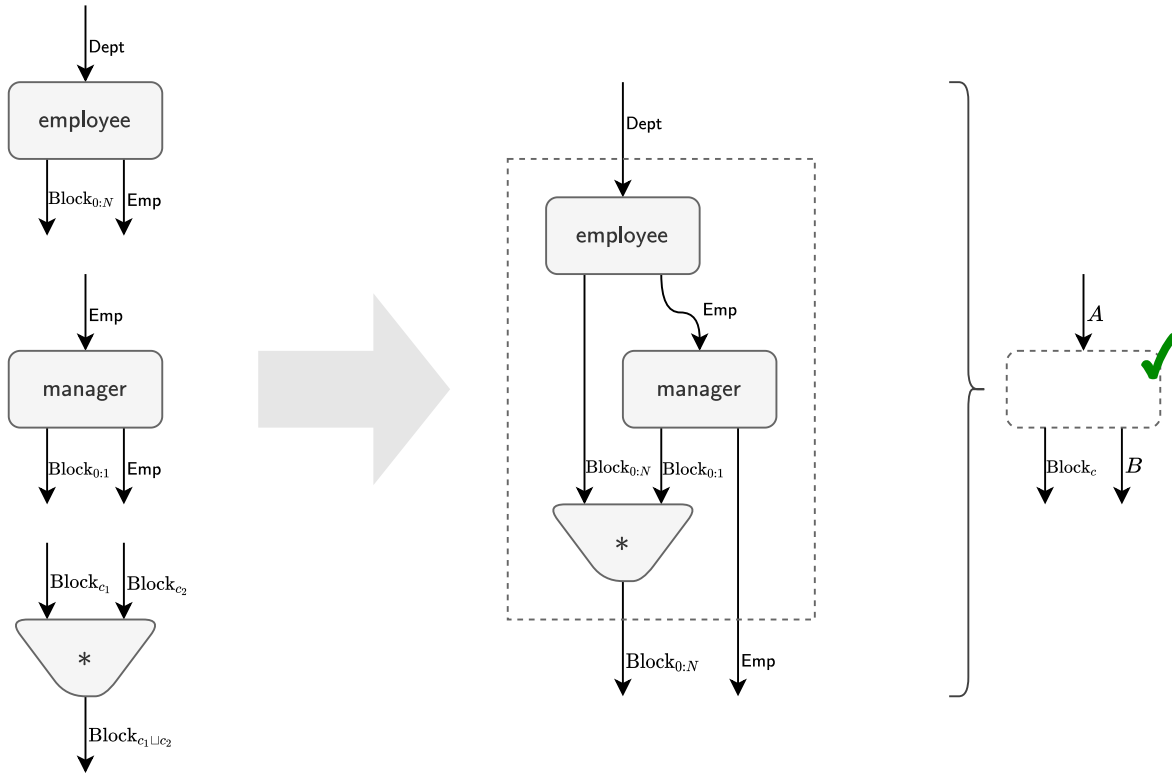


It is easy to check that the multiplication is a natural transformation.



# Using Multiplication Transformation

Multiplication can be used to combine two monadic transformations into a new monadic transformation.



`chain_of(employee, with_elements(manager), flatten)`

# Monad

In general, a *monad* is a functor wire which possesses two special natural transformations: unit and multiplication.



Monadic transformation is a transformation with the following shape.





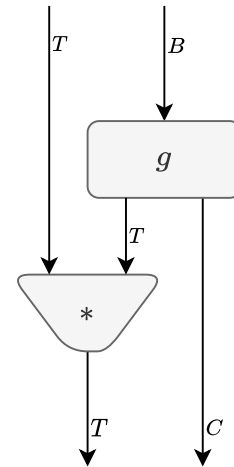
# Monadic Composition

Using multiplication, two compatible monadic transformations can be composed to form a new monadic transformation.



For this definition to be coherent, unit and multiplication must satisfy certain properties.

Aside: this is known as the operation  $\text{bind}(g)$ .



# Associativity of Monadic Composition 1

Given three compatible monadic transformations, they could be composed in two distinct ways.



# Associativity of Monadic Composition 2

Given three compatible monadic transformations, they could be composed in two distinct ways.



# Associativity of Monadic Composition 3

For monadic composition to be associative, these two transformations must be equivalent for any  $f$ ,  $g$ , and  $h$ .



# Monad Law I

Monadic multiplication must satisfy the following equivalence condition.



# Composition of Regular and Monadic Transformations 1

Given one regular and one monadic transformation, they could be composed in two distinct ways.



# Composition of Regular and Monadic Transformations 2

Given one regular and one monadic transformation, they could be composed in two distinct ways.



# Composition of Regular and Monadic Transformations 3

We demand that these two transformations must be equivalent for any  $f$  and  $g$ .





# Monad Law II & III

Monadic unit and multiplication must satisfy the following equivalence condition.



Composing monadic and regular transformations in a different order, we arrive to a symmetric condition.

