$$\text{with\_elements}(\text{pass}()) \Leftrightarrow \text{pass}()$$

$\text{Block}_c\{A\}$

Block_c A

$\text{Block}_c\{A\}$

$\text{Block}_c\{A\}$

$$\text{with\_column}(k, \text{pass}()) \Leftrightarrow \text{pass}()$$

$\text{Tuple}\{A_1, \ldots, A_N\}$

$[k]$

Tuple A_k

$\text{Tuple}\{A_1, \ldots, A_N\}$

$\text{Tuple}\{A_1, \ldots, A_N\}$

$$\mathrm{chain\_of}(\mathrm{with\_elements}(p), \mathrm{with\_elements}(q)) \Leftrightarrow \mathrm{with\_elements}(\mathrm{chain\_of}(p, q))$$

$\mathrm{Block}_c\{A\}$

A

p

B

$\mathrm{Block}_c\{B\}$

B

q

C

$\mathrm{Block}_c\{C\}$

$\mathrm{Block}_c\{A\}$

A

p

B

q

C

$\mathrm{Block}_c\{C\}$
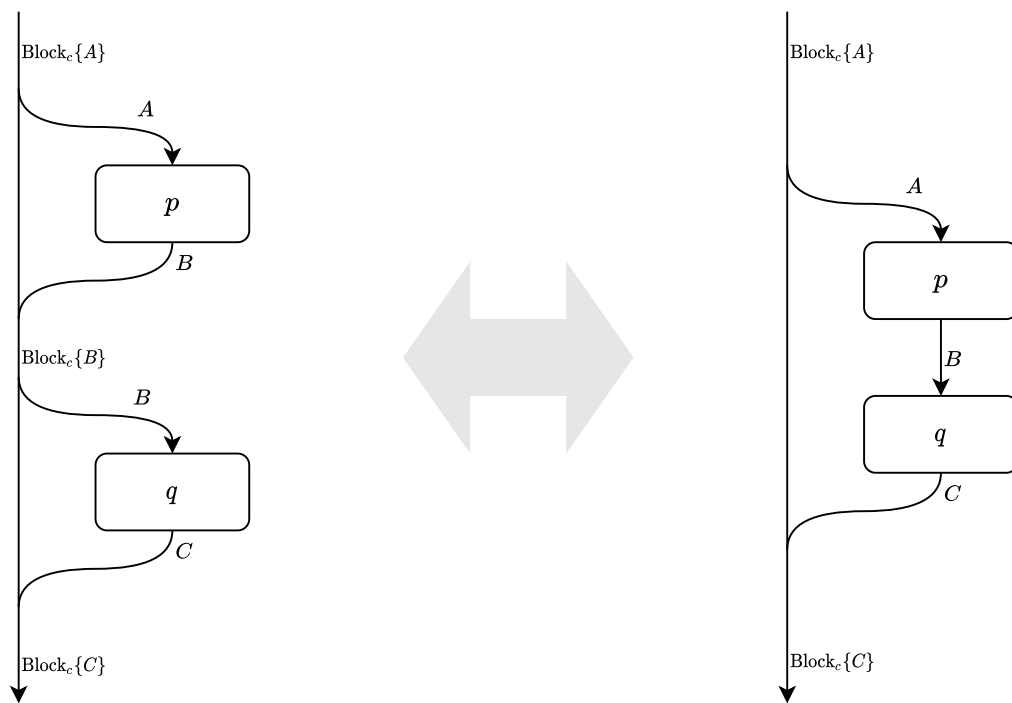
$$\text{chain\_of}(\text{with\_column}(k, p), \text{with\_column}(k, q)) \Leftrightarrow \text{with\_column}(k, \text{chain\_of}(p, q))$$

Tuple$\{A_1, \ldots, A_k, \ldots, A_N\}$

$[k]$

A_k

p

B_k

Tuple$\{A_1, \ldots, B_k, \ldots, A_N\}$

$[k]$

B_k

q

C_k

Tuple$\{A_1, \ldots, C_k, \ldots, A_N\}$

Tuple$\{A_1, \ldots, A_k, \ldots, A_N\}$

$[k]$

A_k

p

B_k

q

C_k

Tuple$\{A_1, \ldots, C_k, \ldots, A_N\}$
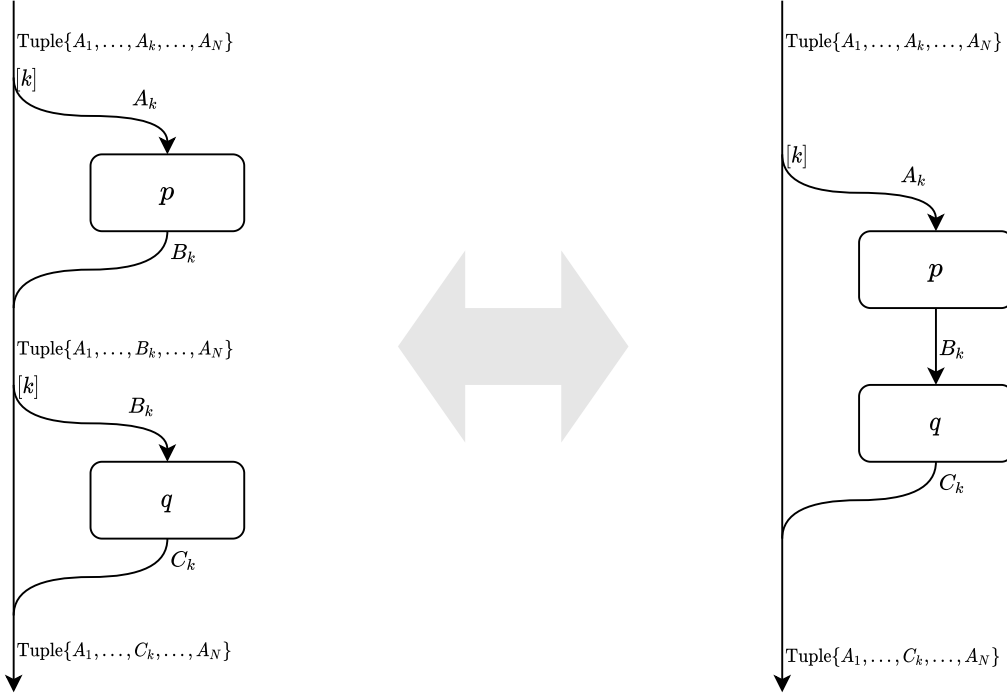
$$\text{chain\_of}(\text{with\_column}(i, p), \text{with\_column}(j, q)) \Leftrightarrow \text{chain\_of}(\text{with\_column}(j, q), \text{with\_column}(i, p)) \qquad i \neq j$$

Tuple$\{\ldots, A_i, \ldots, A_j, \ldots\}$

$[i]$

A_i

p

B_i

Tuple$\{\ldots, B_i, \ldots, A_j, \ldots\}$

$[j]$

A_j

q

B_j

Tuple$\{\ldots, B_i, \ldots, B_j, \ldots\}$

Tuple$\{\ldots, A_i, \ldots, A_j, \ldots\}$

$[j]$

A_j

q

B_j

Tuple$\{\ldots, A_i, \ldots, B_j, \ldots\}$

$[i]$

A_i

p

B_i

Tuple$\{\ldots, B_i, \ldots, B_j, \ldots\}$
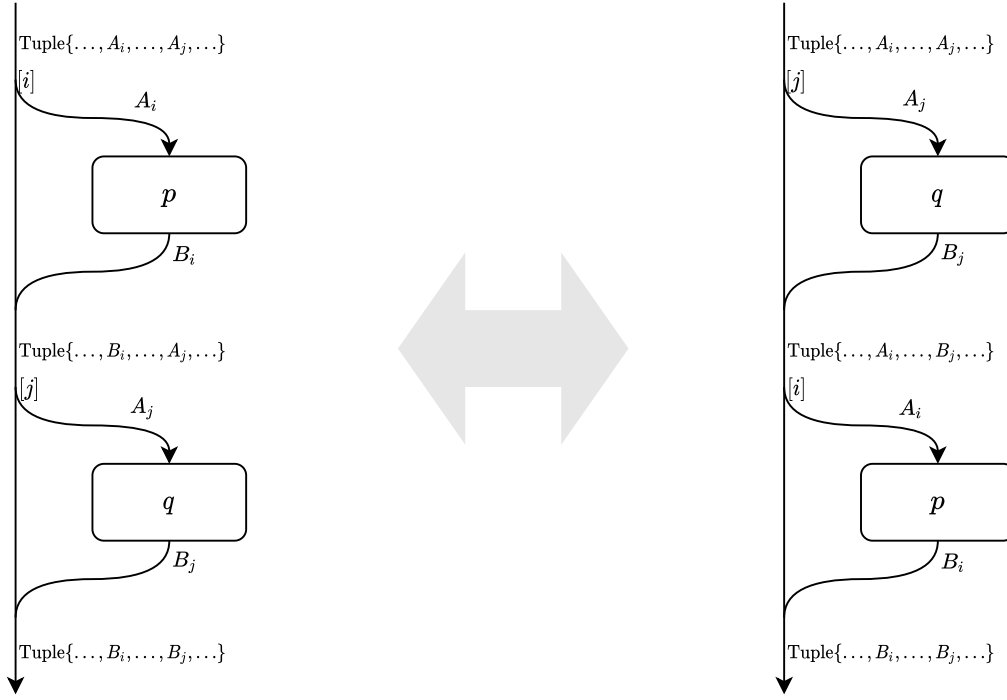
$$\text{chain\_of}(p, \text{filler}(x)) \Leftrightarrow \text{filler}(x)$$

$$A$$

$$p$$

$$B$$

$$1$$

$$x$$

$$C$$

$$A$$

$$1$$

$$x$$

$$C$$

$$\text{tuple\_of}(p, \ldots, p) \Leftrightarrow \text{chain\_of}(p, \text{tuple\_of}(\text{pass}(), \ldots, \text{pass}()))$$

$$\text{chain\_of}(\text{tuple\_of}(p_1, \ldots, p_N), \text{column}(k)) \Leftrightarrow p_k$$

A

$\times N$

$\text{Tuple}\{A, A, \ldots, A\}$

$[1]$

A

p_1

B_1

$\text{Tuple}\{B_1, A, \ldots, A\}$

$[N]$

A

p_N

B_N

$\text{Tuple}\{B_1, B_2, \ldots, B_N\}$

$[k]$

B_k

A

p_k

B_k