$\text{with\_elements}(\text{pass}()) \Rightarrow \text{pass}()$

$\text{Block}_c\{A\}$

Block_c A

$\text{Block}_c\{A\}$

$\text{Block}_c\{A\}$

$\text{with\_column}(k, \text{pass}()) \Rightarrow \text{pass}()$

$\text{Tuple}\{A_1, \ldots, A_N\}$

$[k]$

Tuple A_k

$\text{Tuple}\{A_1, \ldots, A_N\}$

$\text{Tuple}\{A_1, \ldots, A_N\}$
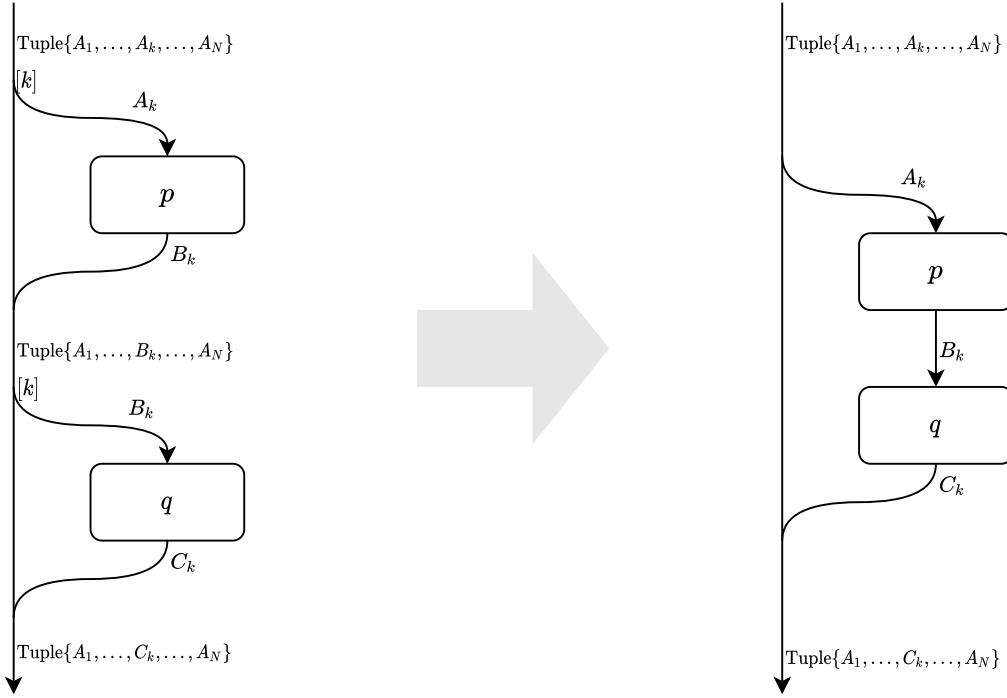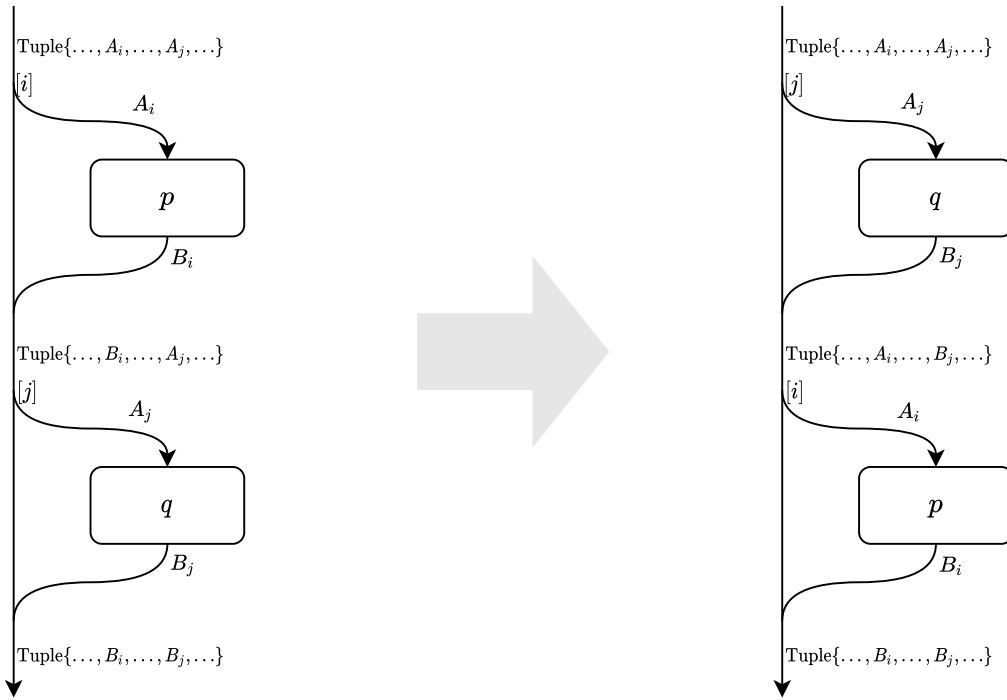
$$\text{chain\_of}(\text{with\_elements}(p), \text{with\_elements}(q)) \Rightarrow \text{with\_elements}(\text{chain\_of}(p, q))$$

$\text{Block}_c\{A\}$

A

p

B

$\text{Block}_c\{B\}$

B

q

C

$\text{Block}_c\{C\}$

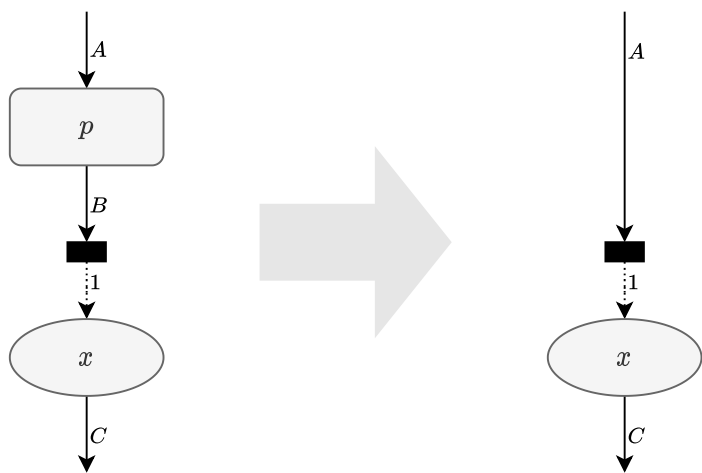$\text{Block}_c\{A\}$

A

p

B

q

C

$\text{Block}_c\{C\}$
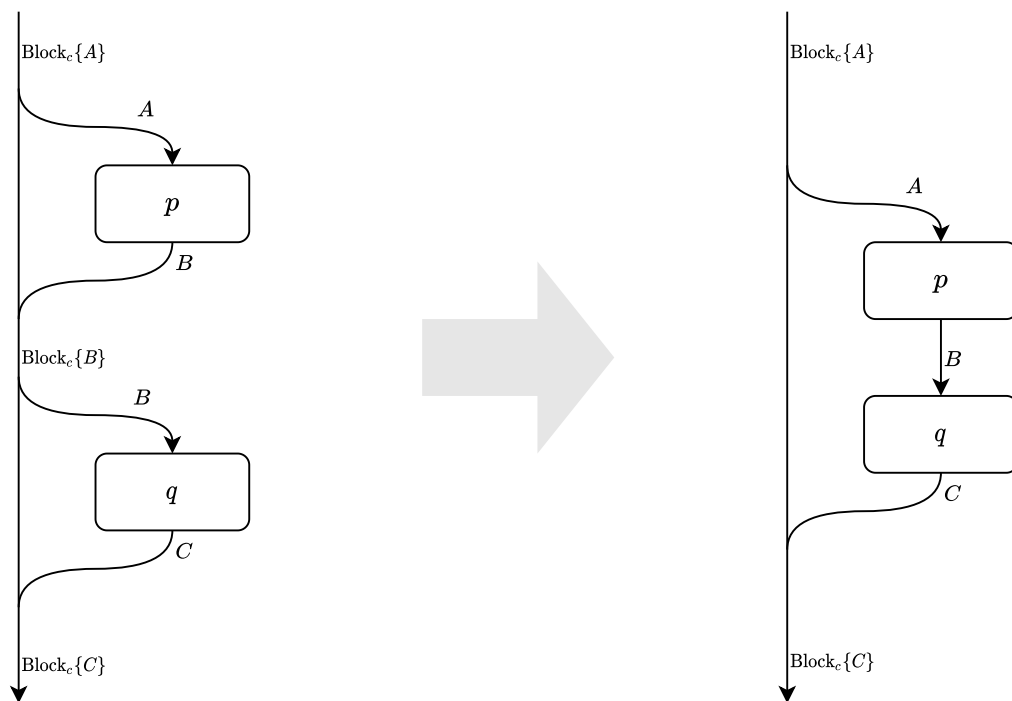
$$\text{chain\_of}(\text{with\_column}(k, p), \text{with\_column}(k, q)) \Rightarrow \text{with\_column}(k, \text{chain\_of}(p, q))$$

$\text{Tuple}\{A_1, \ldots, A_k, \ldots, A_N\}$

$[k]$

A_k

p

B_k

$\text{Tuple}\{A_1, \ldots, B_k, \ldots, A_N\}$

$[k]$

B_k

q

C_k

$\text{Tuple}\{A_1, \ldots, C_k, \ldots, A_N\}$

$\text{Tuple}\{A_1, \ldots, A_k, \ldots, A_N\}$

A_k

p

B_k

q

C_k
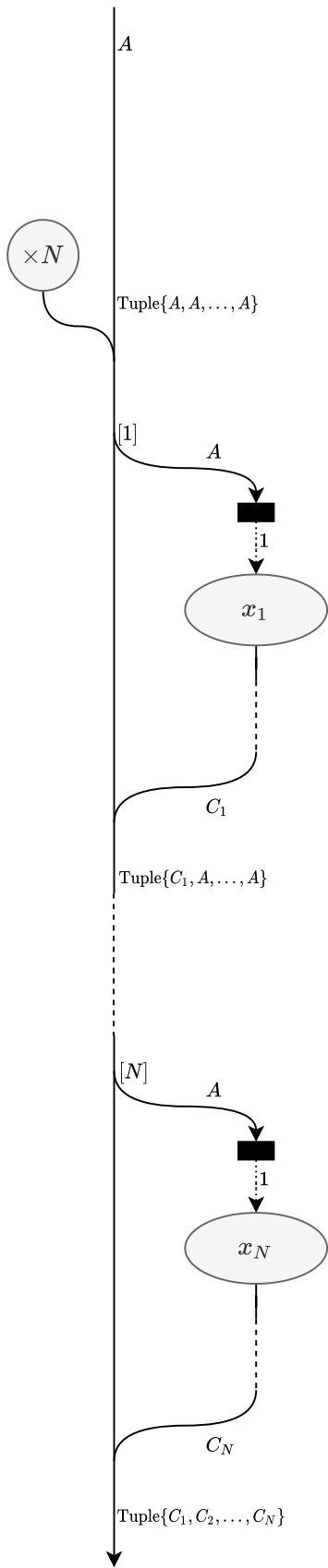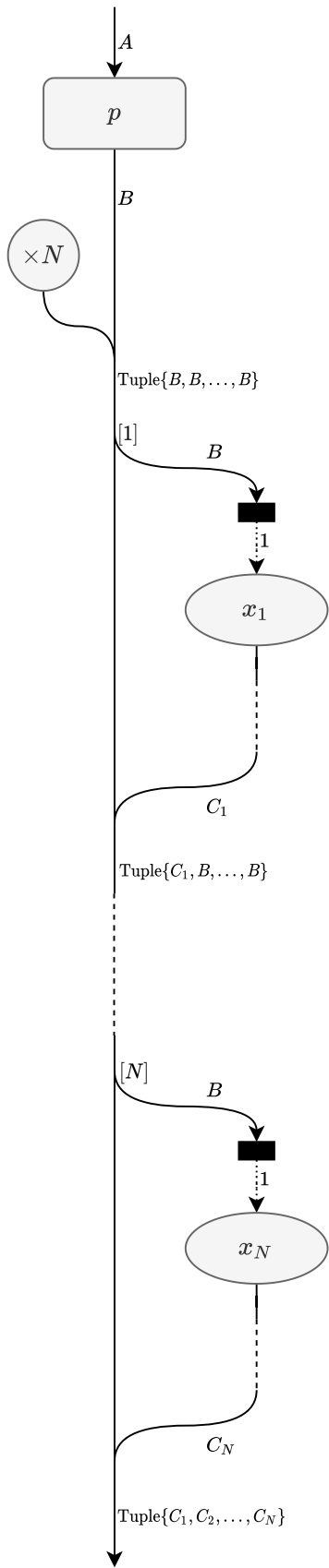
$\text{Tuple}\{A_1, \ldots, C_k, \ldots, A_N\}$

$$\text{chain\_of}(\text{with\_column}(i, p), \text{with\_column}(j, q)) \Rightarrow \text{chain\_of}(\text{with\_column}(j, p), \text{with\_column}(i, q)) \qquad i \neq j$$

$\text{Tuple}\{\ldots, A_i, \ldots, A_j, \ldots\}$

$[i]$

A_i

p

B_i

$\text{Tuple}\{\ldots, B_i, \ldots, A_j, \ldots\}$

$[j]$

A_j

q

B_j

$\text{Tuple}\{\ldots, B_i, \ldots, B_j, \ldots\}$

$\text{Tuple}\{\ldots, A_i, \ldots, A_j, \ldots\}$

$[j]$

A_j

q

B_j

$\text{Tuple}\{\ldots, A_i, \ldots, B_j, \ldots\}$

$[i]$

A_i

p

B_i

$\text{Tuple}\{\ldots, B_i, \ldots, B_j, \ldots\}$

$$\text{chain\_of}(p, \text{filler}(x)) \Rightarrow \text{filler}(x)$$

A

p

B

1

x

C

A

1

x

C

$\mathrm{chain\_of}(p, \mathrm{tuple\_of}(\mathrm{chain\_of}(\mathrm{chain\_of}(\mathrm{filler}(x_1),\dots),\dots,\mathrm{chain\_of}(\mathrm{filler}(x_N),\dots),\dots))) \Rightarrow \mathrm{tuple\_of}(\dots)$

$\text{tuple\_of}(\text{chain\_of}(p, q_1), \dots, \text{chain\_of}(p, q_N)) \Rightarrow \text{chain\_of}(p, \text{tuple\_of}(q_1, \dots, q_N))$

$\times N$

A

Tuple$\{A, A, \dots, A\}$

[1] A

p

B

q_1

C_1

Tuple$\{C_1, A, \dots, A\}$

[N] A

p

B

q_N

C_N

Tuple$\{C_1, C_2, \dots, C_N\}$

A

p

B

$\times N$

Tuple$\{B, B, \dots, B\}$

[1] B

q_1

C_1

Tuple$\{C_1, B, \dots, B\}$

[N] B

q_N

C_N

Tuple$\{C_1, C_2, \dots, C_N\}$
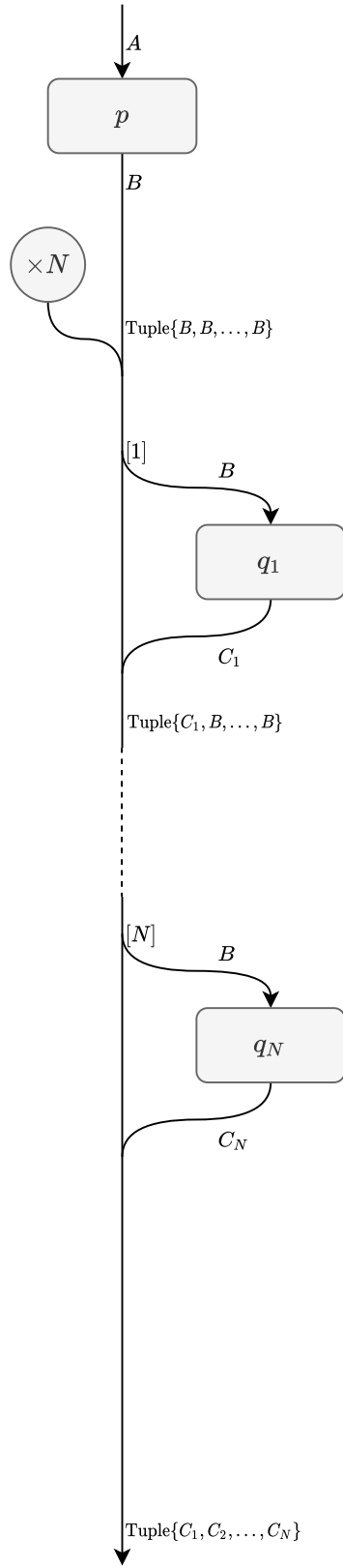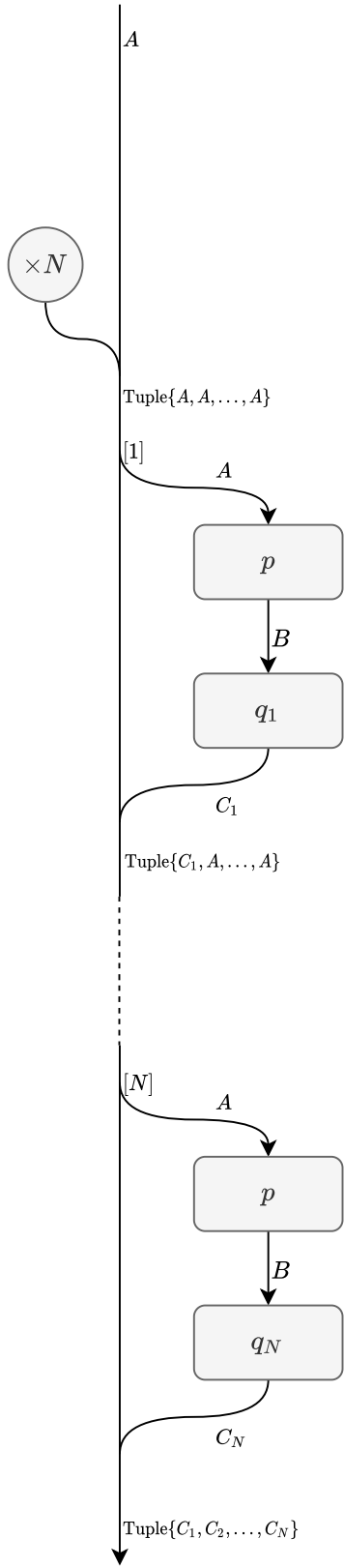
$$\mathrm{chain\_of}(\mathrm{tuple\_of}(p_1, \ldots, p_N), \mathrm{column}(k)) \Rightarrow p_k$$

A

$\times N$

$\mathrm{Tuple}\{A, A, \ldots, A\}$

[1]

A

p_1

B_1

$\mathrm{Tuple}\{B_1, A, \ldots, A\}$

[N]

A

p_N

B_N

$\mathrm{Tuple}\{B_1, B_2, \ldots, B_N\}$

[k]

B_k

A

p_k

B_k