Parsing Expression Grammars

Overview & an Equivalent Computational Model

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Based on MFCS 2024 paper
(joint work with Nikita Chudinov)
September 26, 2024

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Motivation, Definition, Examples

Practical Motivation

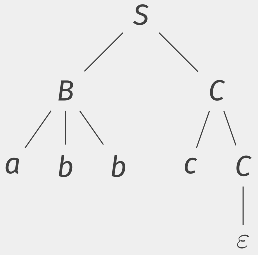
Classical Parsing Approaches

- Top-Down: LL(k)
  easy to design, but the power is limited
- Bottom-Up: LR(k)
  powerful (generate all DCFLs), but hard to design

Parsing Expression Grammars

- PEGs can be considered as a generalization of LL-grammars
- PEGs are more powerful than LR-grammars, but there is no (currently?) direct translation LR-grammars to PEGs
- PEGs now being used in compilers
  Python replaced LL(1)-parser by PEG
- PEGs are popular for solving parsing problems

PEGs as LL(k)-generalization



a	b	b	c	$\triangleleft$

LL(1)-Grammar:

$$S \rightarrow BC$$
$$B \rightarrow abb \mid b$$
$$C \rightarrow cC \mid \varepsilon$$

PEGs as LL(k)-generalization

S

LL(1)-Grammar:

$$S \to BC$$
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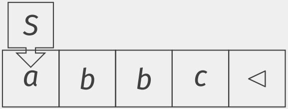

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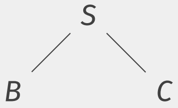

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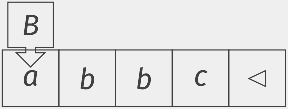

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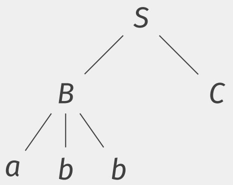

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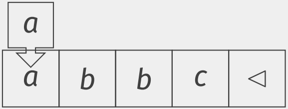

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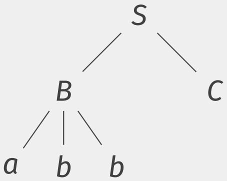

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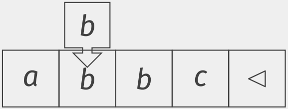

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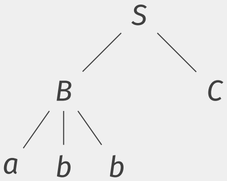

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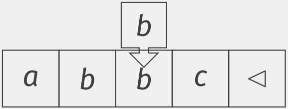

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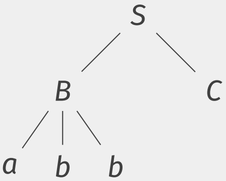

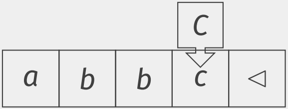

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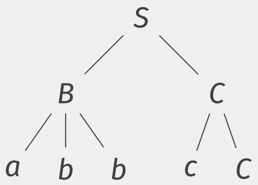

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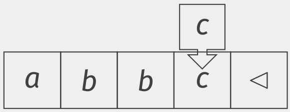

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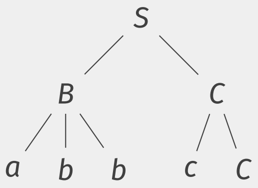

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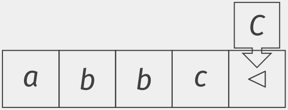

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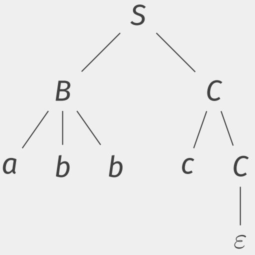

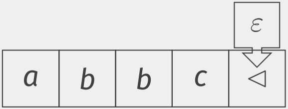

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PEGs as LL(k)-generalization

CFG

$$S \to AB \mid BC$$
$$A \to aA \mid a$$
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PEG

$$S \leftarrow AB \mathbin{/} BC$$
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PEGs as LL(k)-generalization

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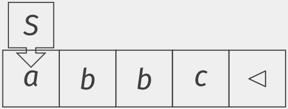

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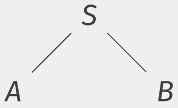

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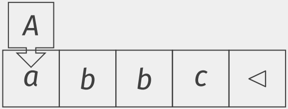

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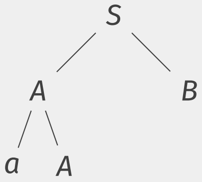

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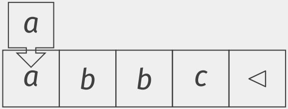

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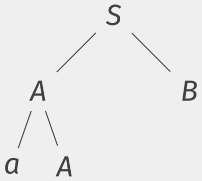

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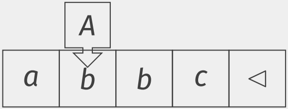

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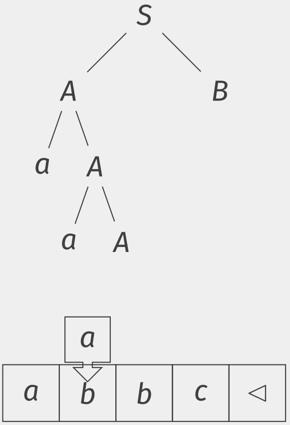

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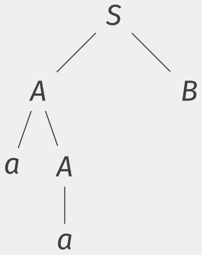

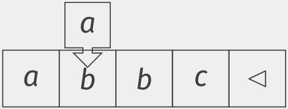

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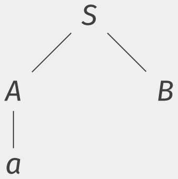

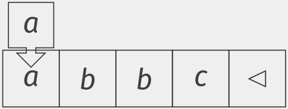

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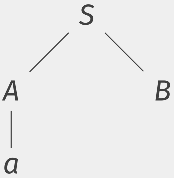

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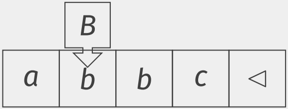

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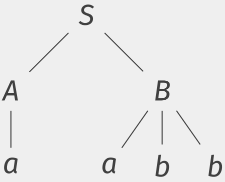

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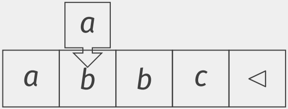

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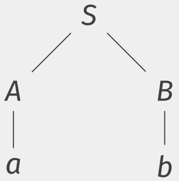

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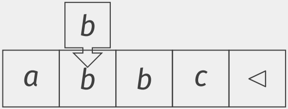

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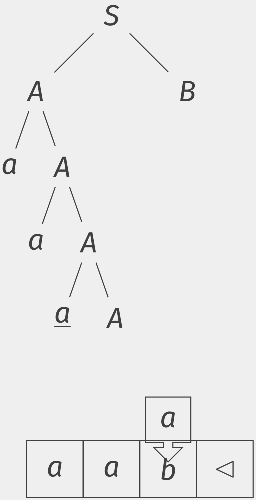

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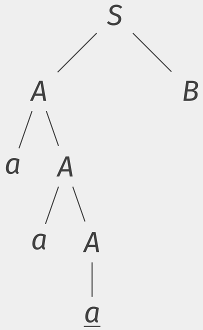

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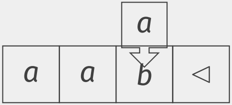

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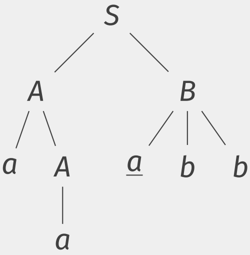

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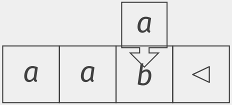

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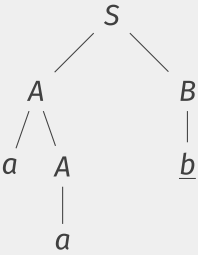

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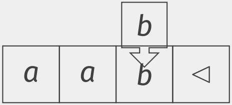
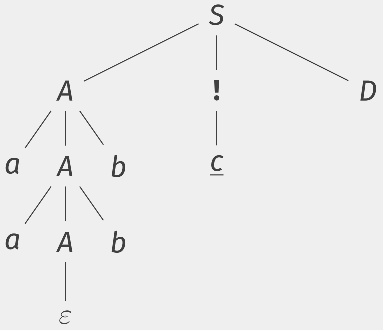


PEGS EXAMPLES: OPERATOR !



PEG:

$$S \leftarrow A(!c)D \;/\; A'B$$
$$A \leftarrow aAb \;/\; \varepsilon$$
$$A' \leftarrow aA' \;/\; \varepsilon$$
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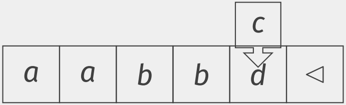
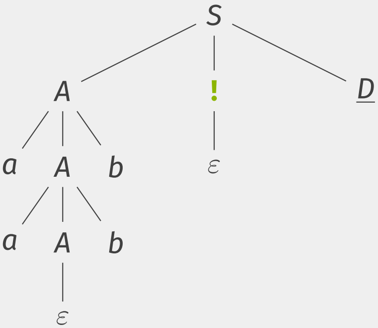
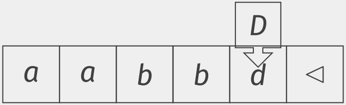


$$a^n b^n d^* \cup a^* b^n c^n$$

PEGs Examples: operator !





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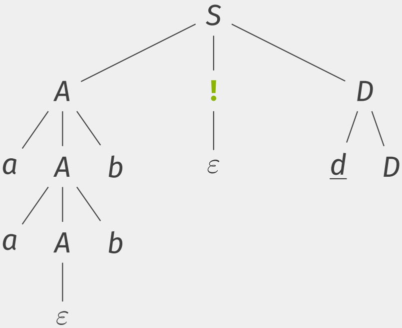

PEG:

$$
\begin{aligned}
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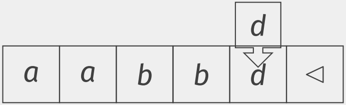
\end{aligned}
$$



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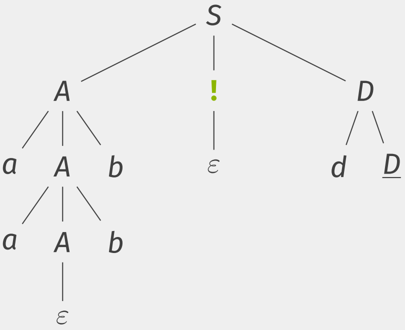

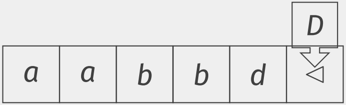

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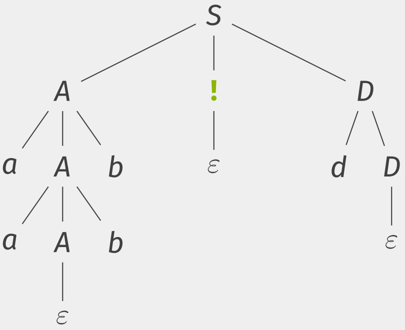

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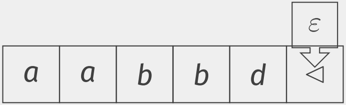

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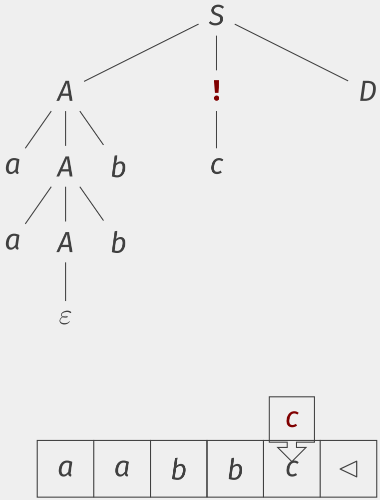

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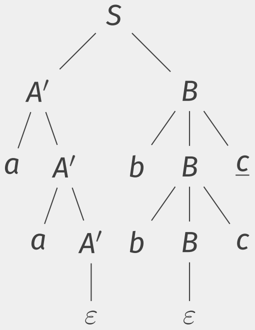
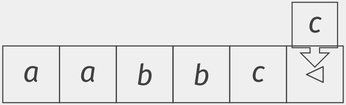
$$a^n b^n d^* \cup a^* b^n c^n$$

PEGs Examples: operator !





PEG:

$$S \leftarrow A(!c)D \;/\; A'B$$
$$A \leftarrow aAb \;/\; \varepsilon$$
$$A' \leftarrow aA' \;/\; \varepsilon$$
$$B \leftarrow bBc \;/\; \varepsilon$$
$$D \leftarrow dD \;/\; \varepsilon$$

$$a^n b^n d^* \cup a^* b^n c^n$$

PEGs Examples: operator &

& is a syntactic sugar

$$\&X = \mathbf{!}(\mathbf{!}X)$$

- if X yields to fail then $\mathbf{!}X$ yields to ε
- if X does not yield to fail then $\&X$ yields to ε

$$G : S \leftarrow (\&(Ac))BC \quad A \leftarrow aAb \,/\, \varepsilon \quad B \leftarrow aB \,/\, a \quad C \leftarrow bCc \,/\, \varepsilon$$

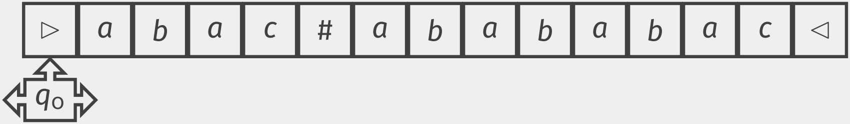
$$L(G) = \{a^n b^n c^n \mid n \geq 1\}$$

Discussion

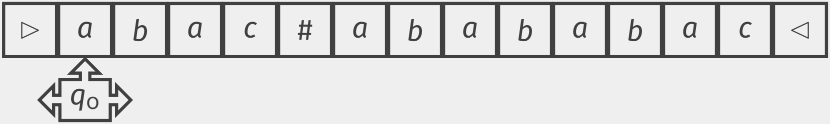
- "Concatenation" in PEGs is not a real concatenation!
  - Conjecture: PELs are not closed over concatenation
- PELs are closed over Boolean operations: $\Gamma_{\mathrm{Bool}}(\mathrm{PEL}) = \mathrm{PEL}$
- $\mathrm{PEL} \setminus \mathrm{CFL} \neq \varnothing$
- *Open question:* $\mathrm{CFL} \setminus \mathrm{PEL} \stackrel{?}{=} \varnothing$
  - $\Omega(n^{1+\varepsilon})$ bound on CFLs parsing implies $\mathrm{CFL} \setminus \mathrm{PEL} \neq \varnothing$

Pointer Pushdown Automata

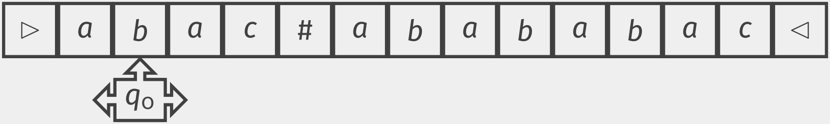
Pointer Pushdown Automata



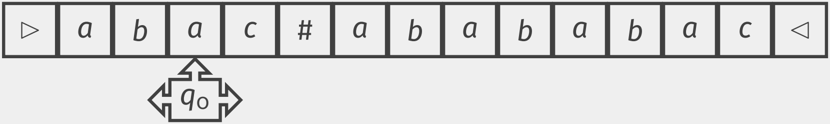
Pointer Pushdown Automata



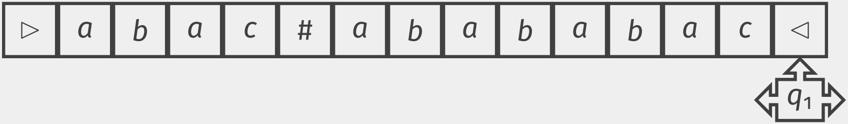
Pointer Pushdown Automata



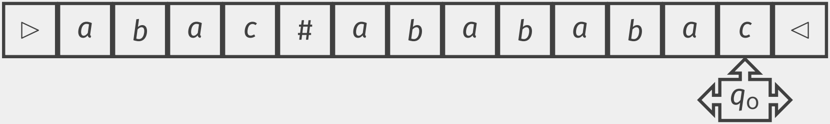
Pointer Pushdown Automata



Pointer Pushdown Automata



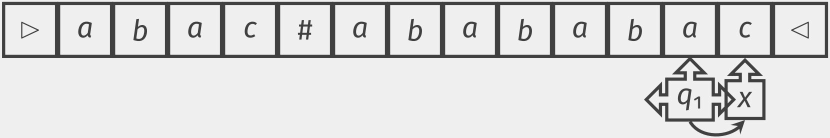
Pointer Pushdown Automata



Stack Operations:

- push(x) $\leftarrow$

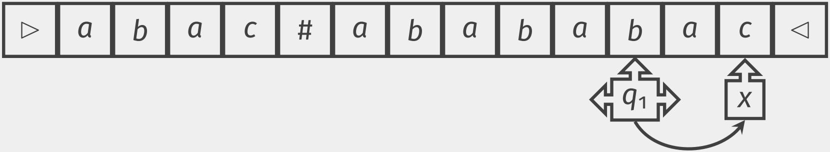
Pointer Pushdown Automata



Stack Operations:

- push(x) $\leftarrow$

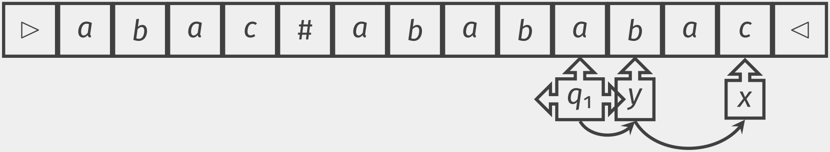
Pointer Pushdown Automata



Stack Operations:

- push(x) $\leftarrow$
- push(y) $\leftarrow$

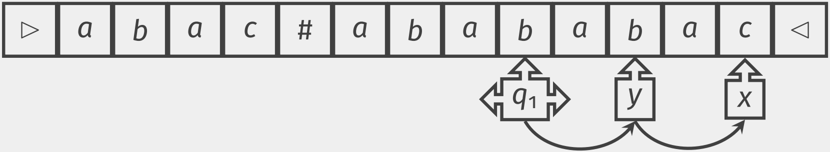
Pointer Pushdown Automata



Stack Operations:

- push(x) $\leftarrow$
- push(y) $\leftarrow$

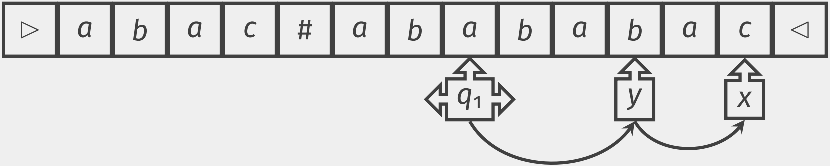
Pointer Pushdown Automata



Stack Operations:

- push(x) $\leftarrow$
- push(y) $\leftarrow$

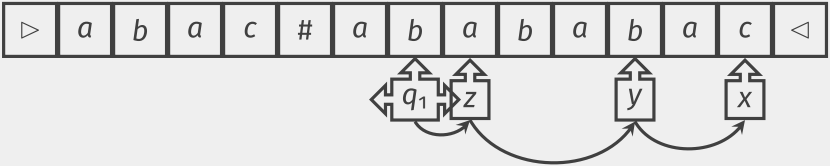
Pointer Pushdown Automata



Stack Operations:

- push(x) $\leftarrow$
- push(y) $\leftarrow$
- push(z) $\leftarrow$

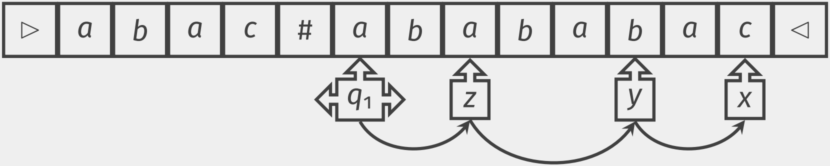
Pointer Pushdown Automata



Stack Operations:

- push(x) $\leftarrow$
- push(y) $\leftarrow$
- push(z) $\leftarrow$

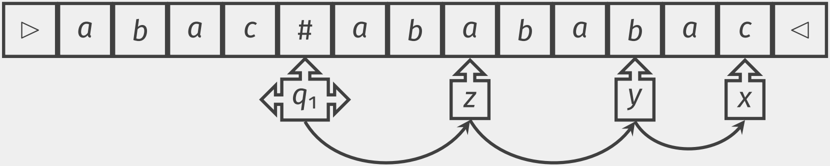
Pointer Pushdown Automata



Stack Operations:

- push(x) $\leftarrow$
- push(y) $\leftarrow$
- push(z) $\leftarrow$

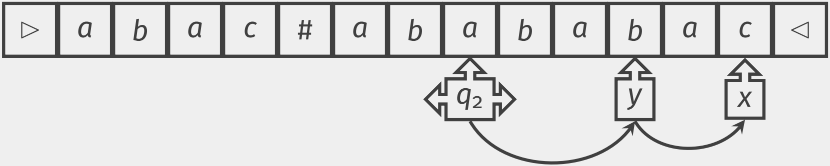
Pointer Pushdown Automata



Stack Operations:

- push(x) $\leftarrow$
- push(y) $\leftarrow$
- push(z) $\leftarrow$
- pop(z) $\uparrow$

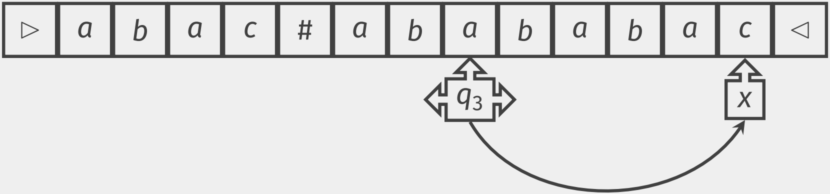
Pointer Pushdown Automata



Stack Operations:

- push(x) $\leftarrow$
- push(y) $\leftarrow$
- push(z) $\leftarrow$
- pop(z) $\uparrow$
- pop(y) $\downarrow$

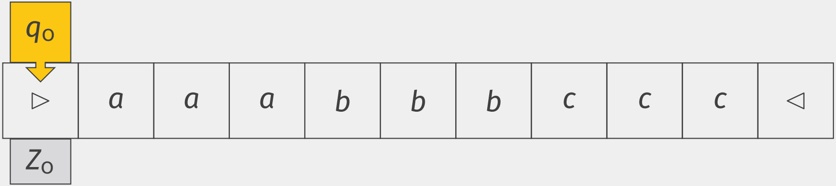
Pointer Pushdown Automata



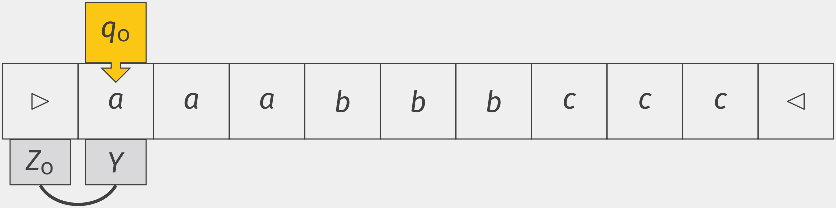
Stack Operations:

- push(x) $\leftarrow$
- push(y) $\leftarrow$
- push(z) $\leftarrow$
- pop(z) $\uparrow$
- pop(y) $\downarrow$

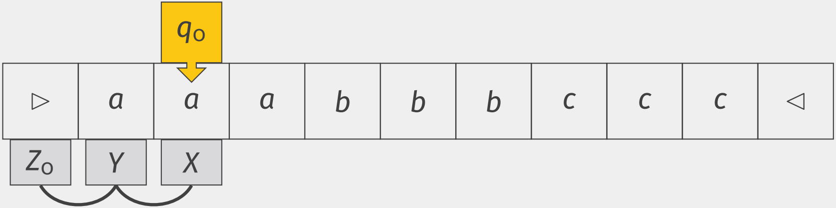
$a^n b^n c^n$



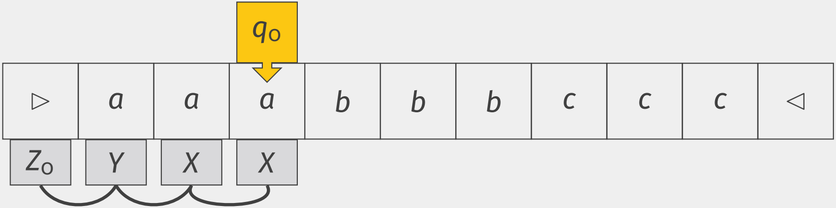
$a^n b^n c^n$



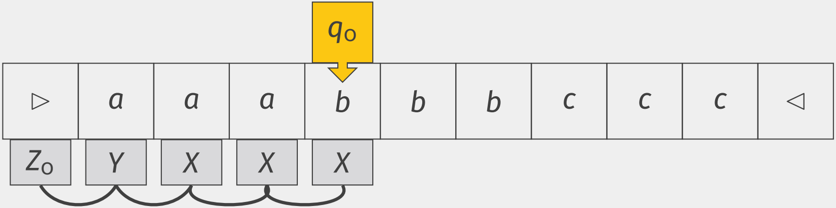
$a^n b^n c^n$



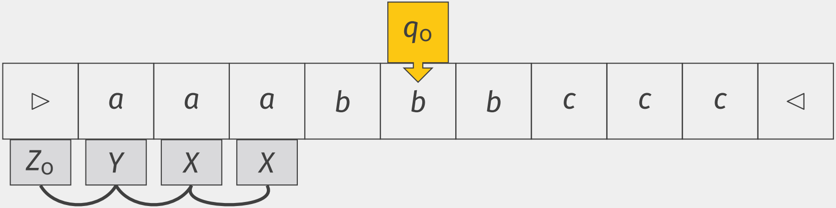
$a^n b^n c^n$



$a^n b^n c^n$



$a^n b^n c^n$



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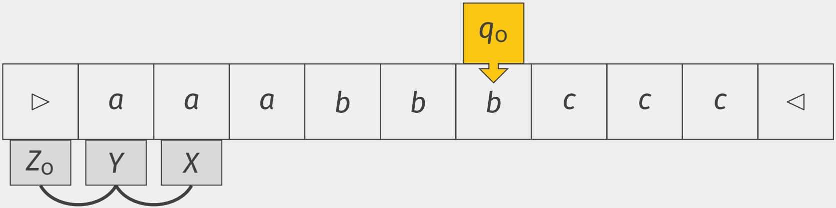

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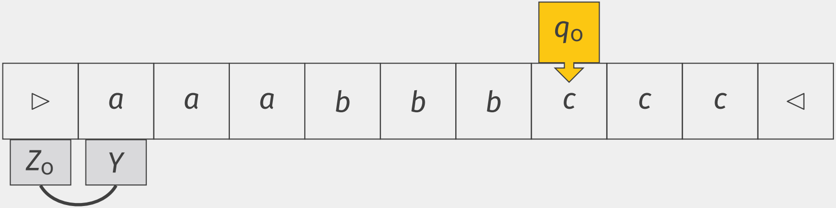

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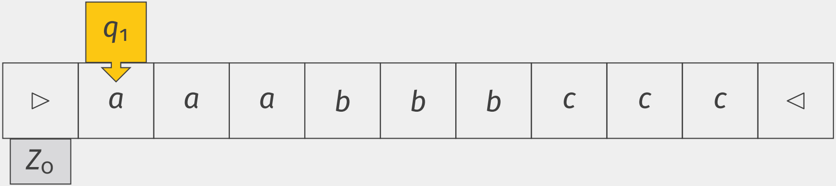
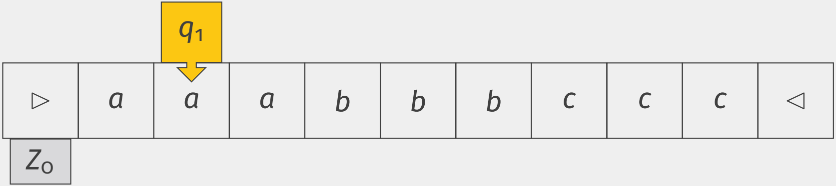


$a^n b^n c^n$



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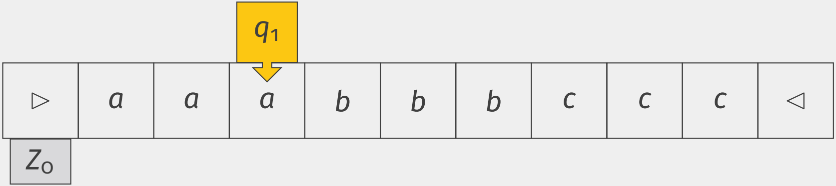

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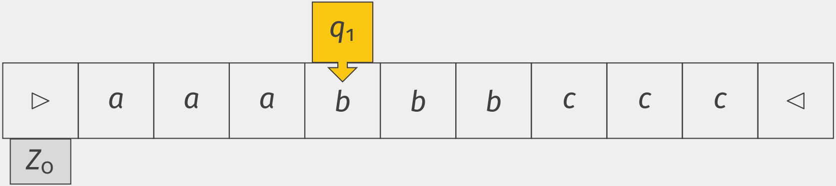

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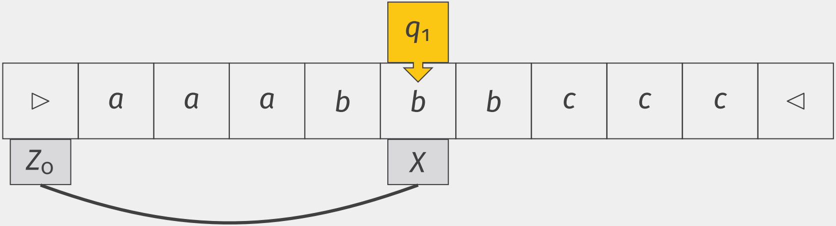

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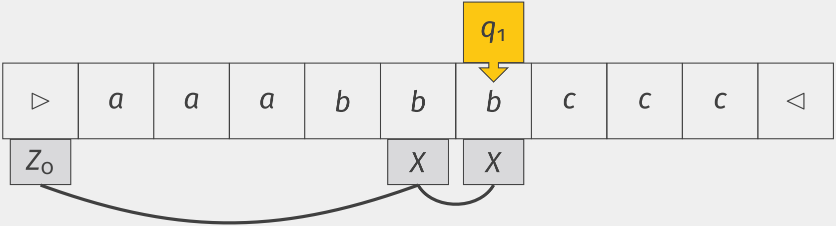
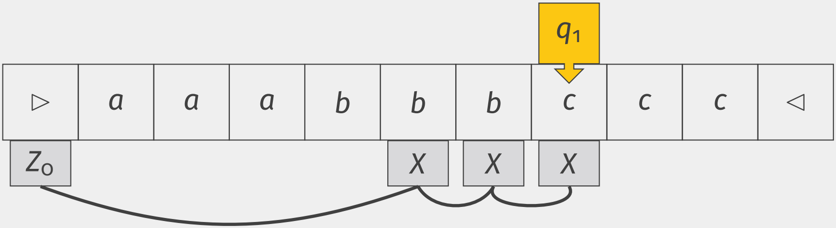
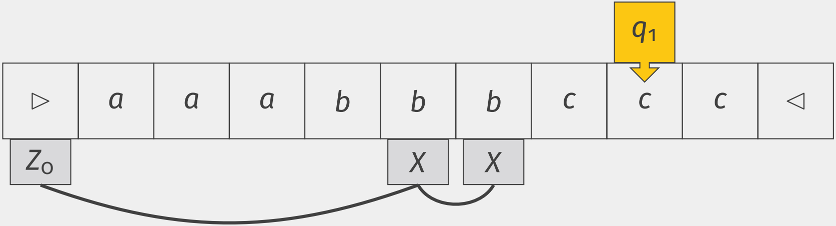


$a^n b^n c^n$



$a^n b^n c^n$



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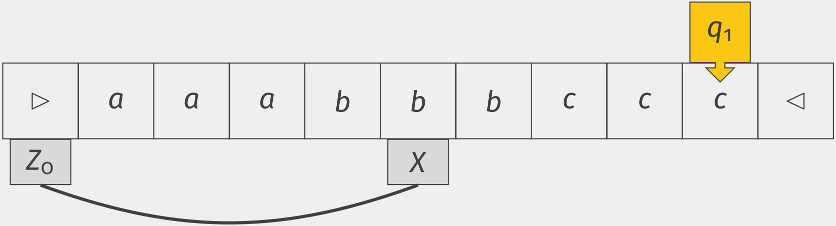
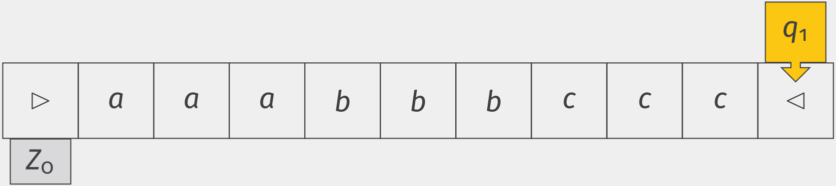
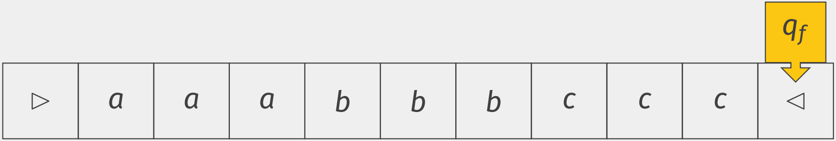


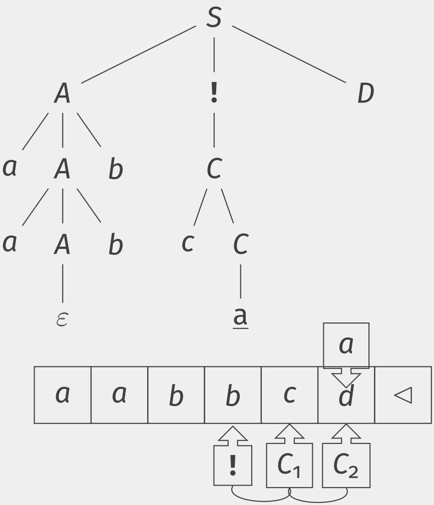
$a^n b^n c^n$



$a^n b^n c^n$



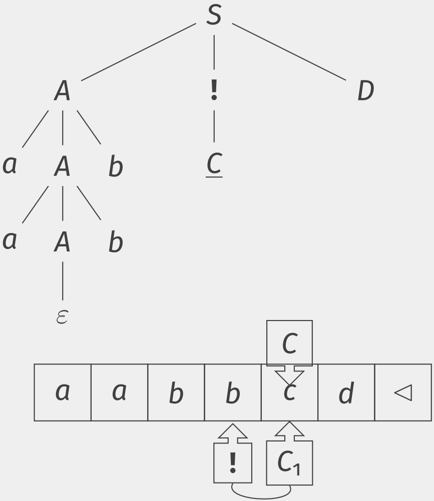
PEG → DPPDA (IDEA)



PEG:

$$S \leftarrow A(!C)D \,/\, A'B$$
$$A \leftarrow aAb \,/\, \varepsilon$$
$$A' \leftarrow aA' \,/\, \varepsilon$$
$$B \leftarrow bBc \,/\, \varepsilon$$
$$C \leftarrow cC \,/\, a$$
$$D \leftarrow dD \,/\, \varepsilon$$

PEG $\to$ DPPDA (IDEA)



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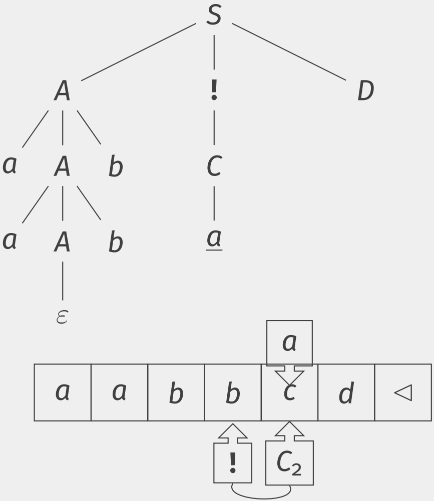
PEG $\to$ DPPDA (IDEA)



PEG:

$$S \leftarrow A(!C)D \,/\, A'B$$
$$A \leftarrow aAb \,/\, \varepsilon$$
$$A' \leftarrow aA' \,/\, \varepsilon$$
$$B \leftarrow bBc \,/\, \varepsilon$$
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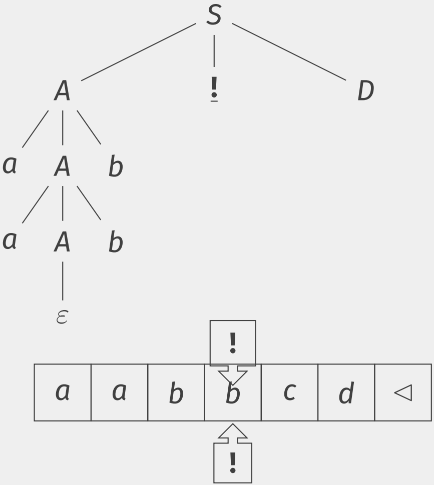
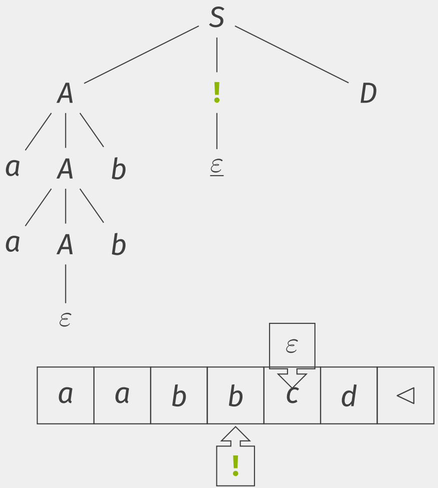
PEG $\rightarrow$ DPPDA (idea)



PEG:

$$S \leftarrow A(\mathbf{!}C)D \;/\; A'B$$
$$A \leftarrow aAb \;/\; \varepsilon$$
$$A' \leftarrow aA' \;/\; \varepsilon$$
$$B \leftarrow bBc \;/\; \varepsilon$$
$$C \leftarrow cC \;/\; a$$
$$D \leftarrow dD \;/\; \varepsilon$$

PEG → DPPDA (IDEA)



PEG:

$$S \leftarrow A(!C)D \mathbin{/} A'B$$
$$A \leftarrow aAb \mathbin{/} \varepsilon$$
$$A' \leftarrow aA' \mathbin{/} \varepsilon$$
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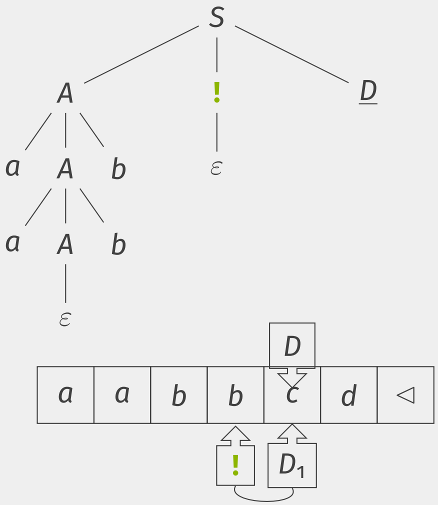
PEG → DPPDA (idea)



PEG:

$$S \leftarrow A(!C)D \,/\, A'B$$
$$A \leftarrow aAb \,/\, \varepsilon$$
$$A' \leftarrow aA' \,/\, \varepsilon$$
$$B \leftarrow bBc \,/\, \varepsilon$$
$$C \leftarrow cC \,/\, a$$
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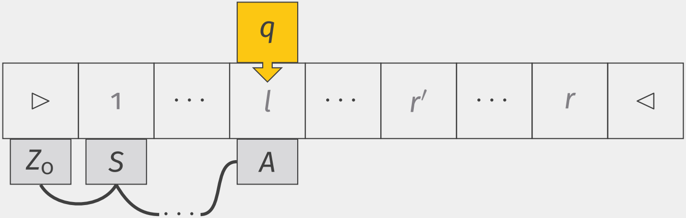
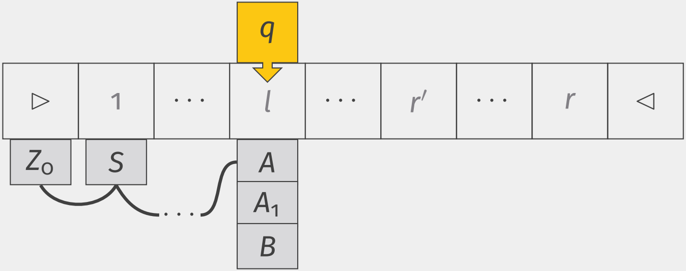
PEG $\to$ DPPDA (PART OF CONSTRUCTION)

$$A \leftarrow BC$$



PEG $\to$ DPPDA (PART OF CONSTRUCTION)

$$A \leftarrow BC$$



PEG $\to$ DPPDA (part of construction)

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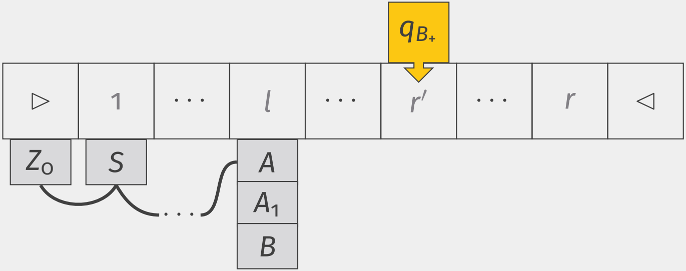

PEG $\to$ DPPDA (part of construction)

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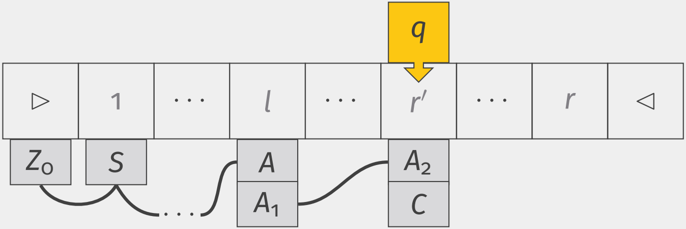
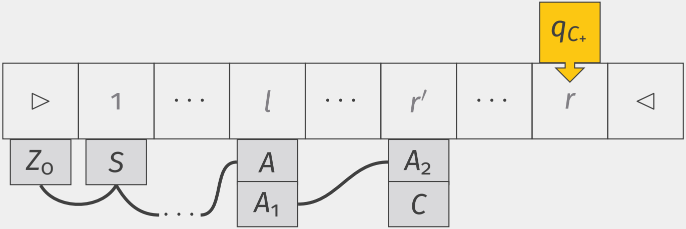


PEG $\rightarrow$ DPPDA (PART OF CONSTRUCTION)

$A \leftarrow BC$



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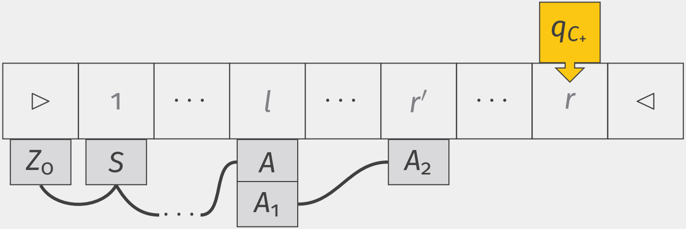

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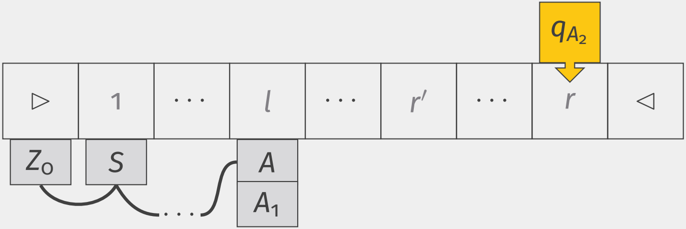
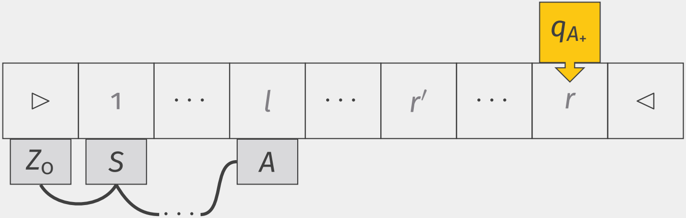
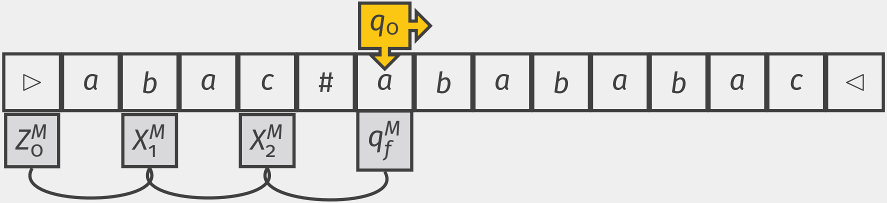


PEG $\to$ DPPDA (PART OF CONSTRUCTION)

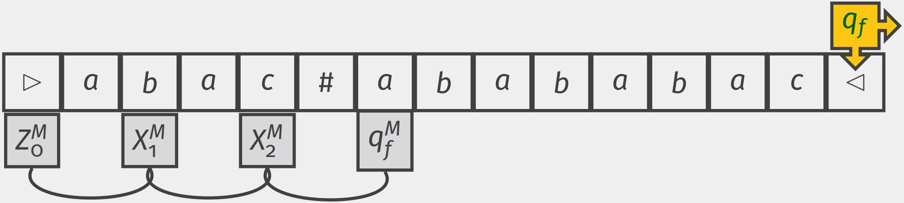
$$A \leftarrow BC$$



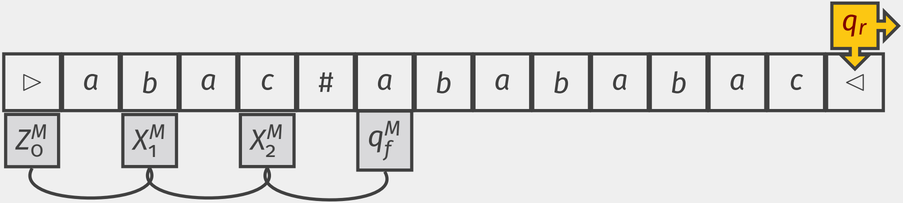
DCFL · PEL = PEL



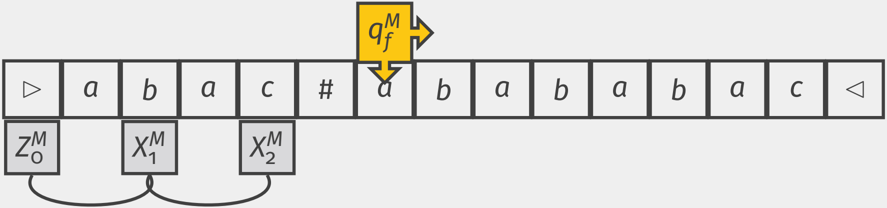
DCFL · PEL = PEL



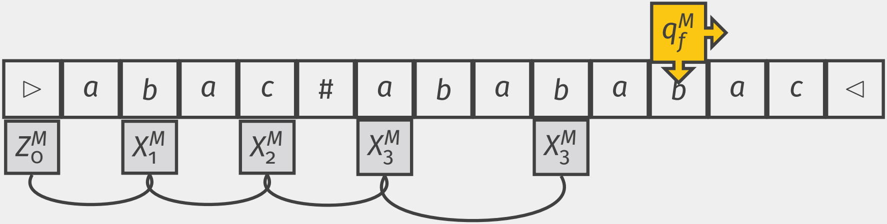
DCFL · PEL = PEL



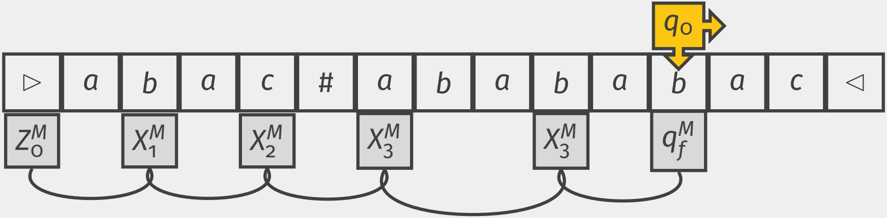
DCFL · PEL = PEL



DCFL · PEL = PEL



DCFL · PEL = PEL



Theorem

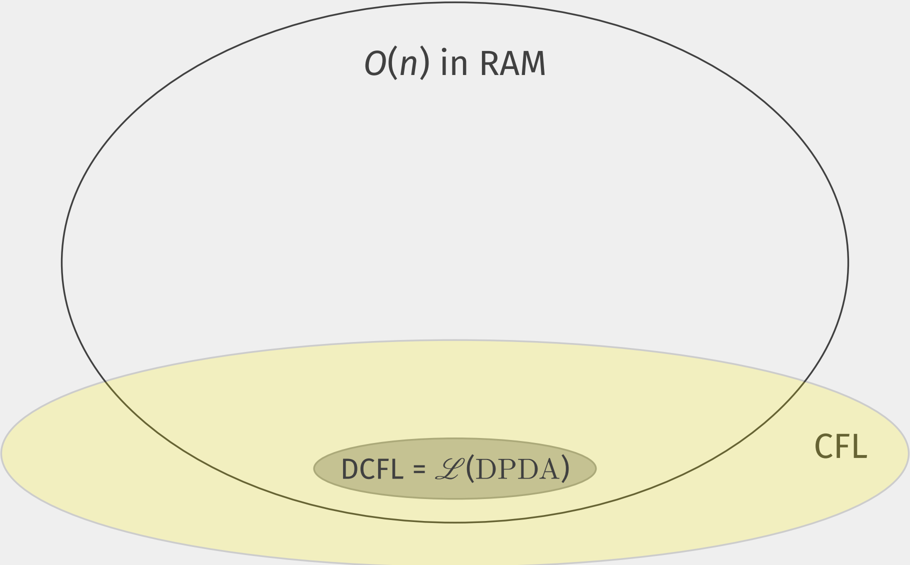
$\Gamma_{\mathrm{REG}}(\mathrm{DCFL}) \cdot \mathrm{PEL} = \mathrm{PEL}$

$$\psi : a(a|bc^*)^*b \mapsto L_a(L_a|L_bL_c^*)^*L_b, \text{ where } L_a, L_b, L_c \in \mathrm{DCFL}$$

$$\Gamma_{\mathrm{REG}}(\mathrm{DCFL}) = \bigcup_{\psi,R} \psi(R)$$

Overview of Results on Computational Complexity

Results on Complexity [1/2]



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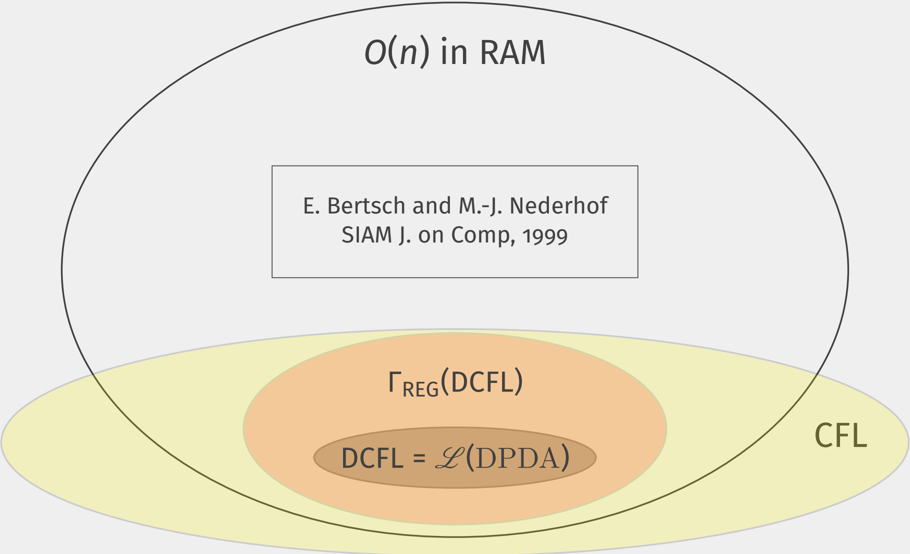

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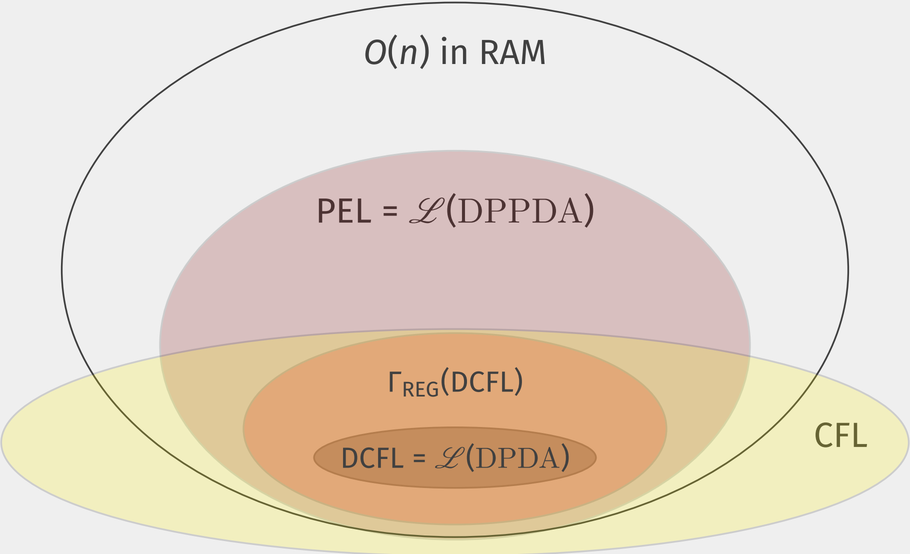

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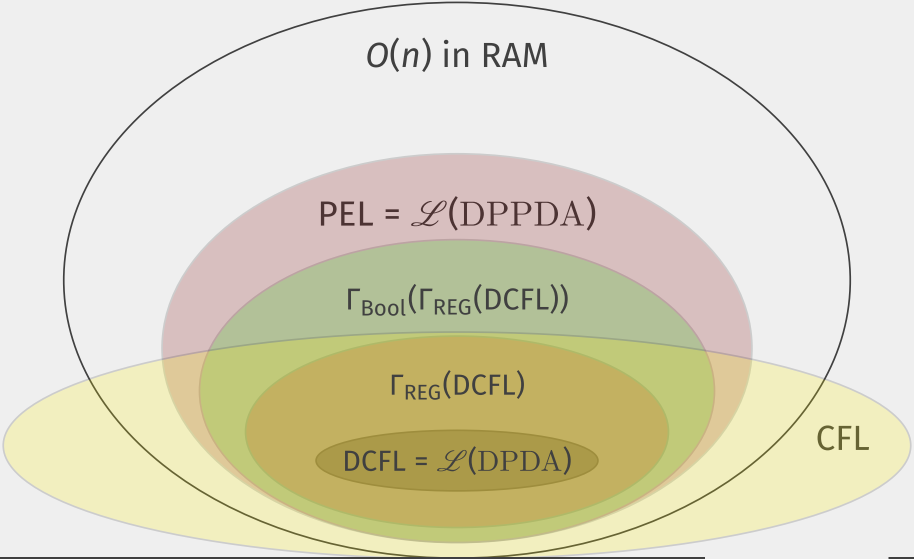
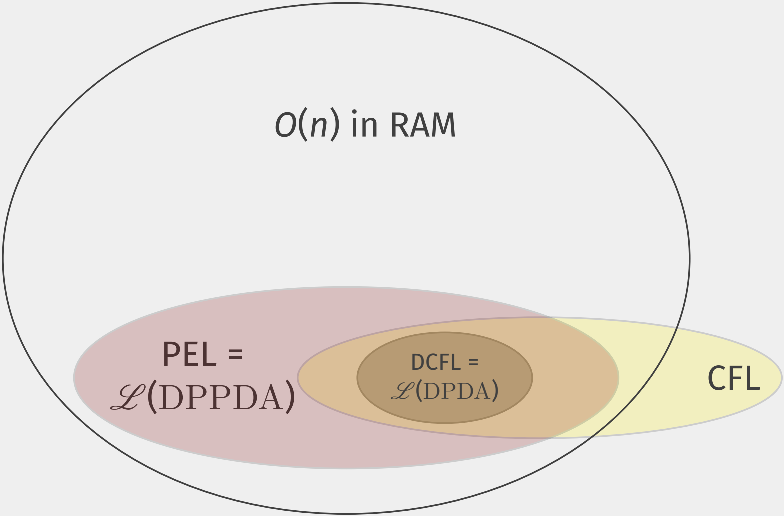
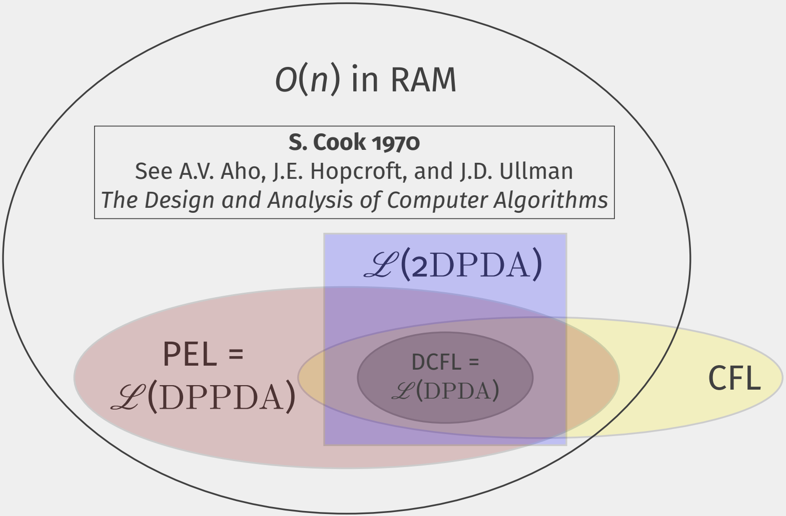


Results on Complexity [2/2]



RESULTS ON COMPLEXITY [2/2]



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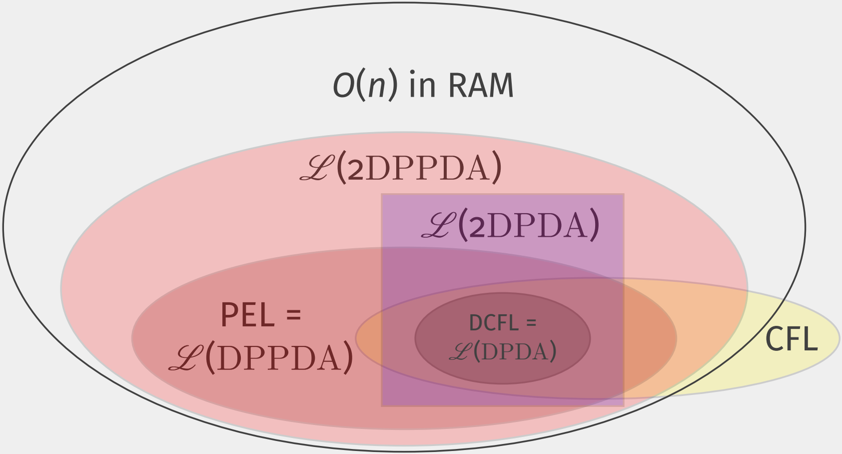

Results and Conclusion

Overview of Previous Results

- PEGs is an upgrade of Top-Down Parsing Languages (TDPLs) introduced by A. Birman and J. Ullman in the 1960-s
  - DCFL $\subseteq$ TDPL, $a^n b^n c^n \in$ TDPL
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- Python replaced LL-parser by PEG (PEP 617, 2020)
- B. Loff, N. Moreira, and R. Reis presented the first computational model for PEGs (DLT, 2018)
  - $a^{2^n} \in$ PEL and palindromes of length 2^n in PEL
  - Structural Results: there is no pumping lemma for PEL

Our Results

- New computational model: Pushdown Pointer Automata
  - $\mathscr{L}(\mathrm{DPPDA})$ = PEL
  - Linear-time simulation algorithm for 2-DPPDA (in RAM), modification of S. Cook algorithm for 2-DPDA
  - Clarification of PEL place among the formal languages
  - Simplicity: now the inclusion DCFL $\subseteq$ PEL is trivial
- **Thm.** $\Gamma_{\mathrm{REG}}(\mathrm{DCFL}) \cdot \mathrm{PEL} = \mathrm{PEL}$
  - **Corollary:** $\Gamma_{\mathrm{REG}}(\mathrm{DCFL}) \in \mathrm{PEL} \Rightarrow \Gamma_{\mathrm{Bool}}(\Gamma_{\mathrm{REG}}(\mathrm{DCFL})) \in \mathrm{PEL} \Rightarrow$ $\Rightarrow \Gamma_{\mathrm{Bool}}(\Gamma_{\mathrm{REG}}(\mathrm{DCFL}))$ is $O(n)$-recognizable in RAM.
    - It is a simplification and upgrade of the previously known result: $\Gamma_{\mathrm{REG}}(\mathrm{DCFL})$ is $O(n)$-recognizable [E. Bertsch and M.-J. Nederhof, SIAM J. on Comp, 1999]
- We hope that our model will rase interest to PELs in TCS community

Discussion

- The relations between following models are unknown:
  - DPPDA vs 2-DPDA
  - 2-DPDA vs 2-DPPDA
  - 2-DPDA vs 2-NPDA
  - 2-NPDA vs 2-NPPDA
- It is unknown whether CFL $\overset{?}{\subseteq} \mathscr{L}(2\text{-}\mathrm{DPPDA})$

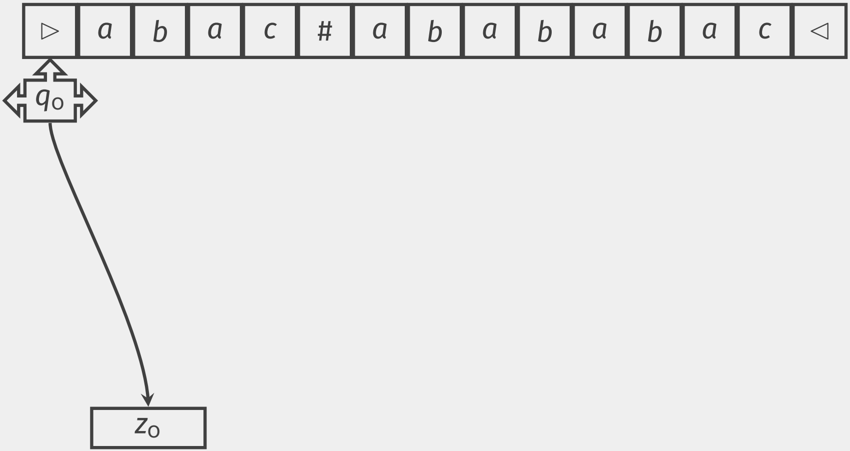
News

Now practically-motivated class PEL is among these classes

Remark

Many questions for 2-NDPA from Rupak Majumdar's talk are relevant for 2-NPPDA

KMP-algorithm + 2DPDA



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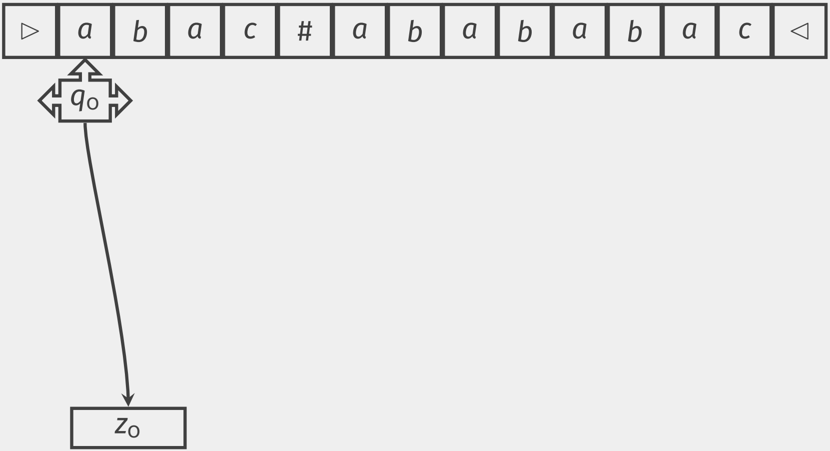

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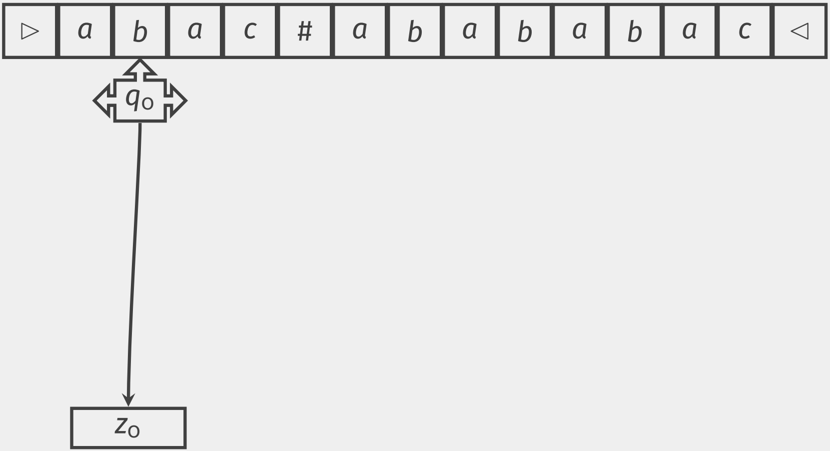

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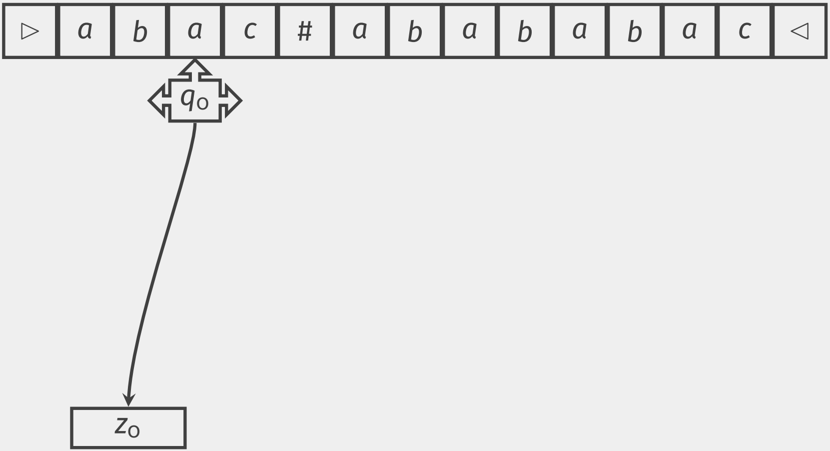

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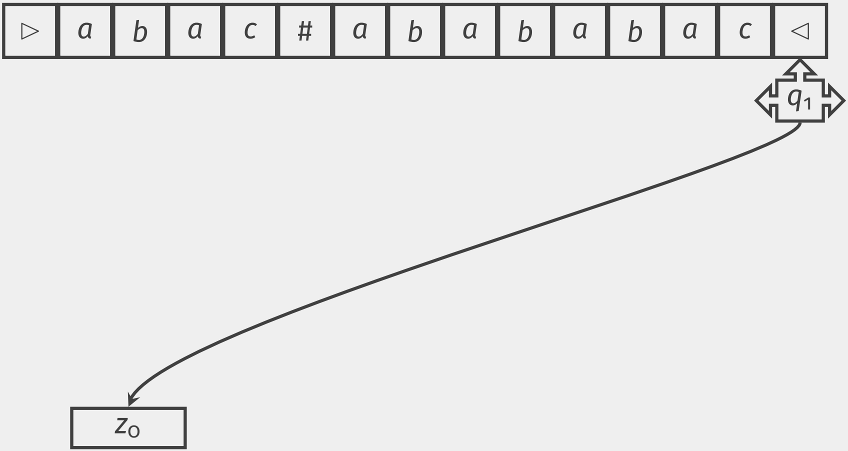

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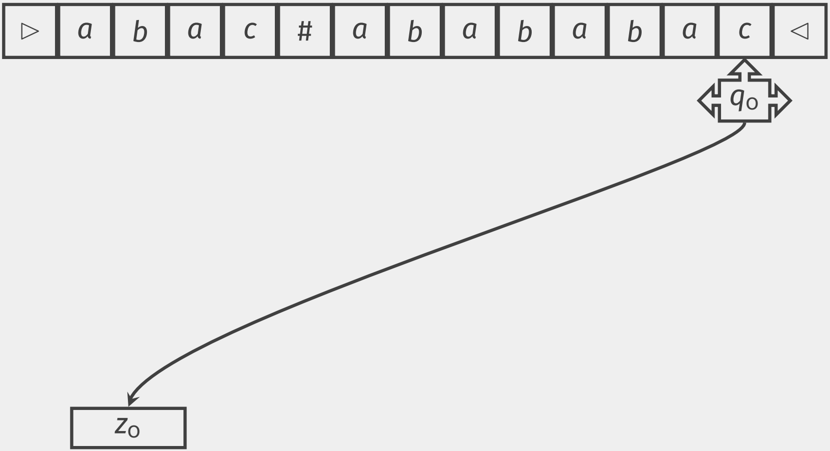

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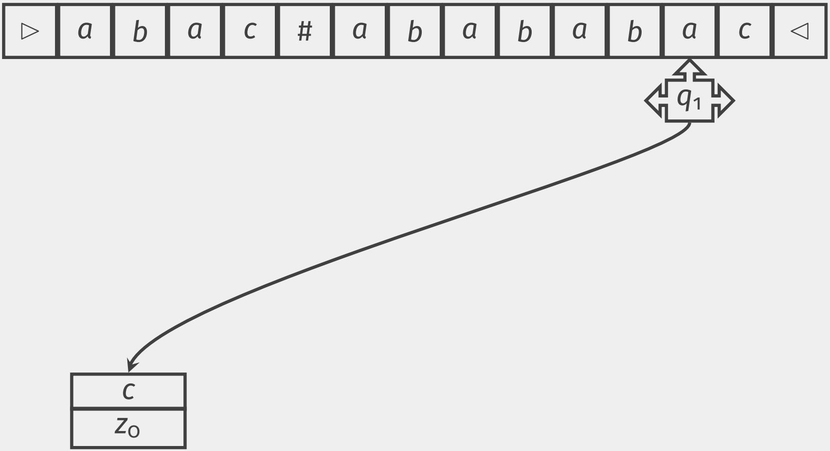

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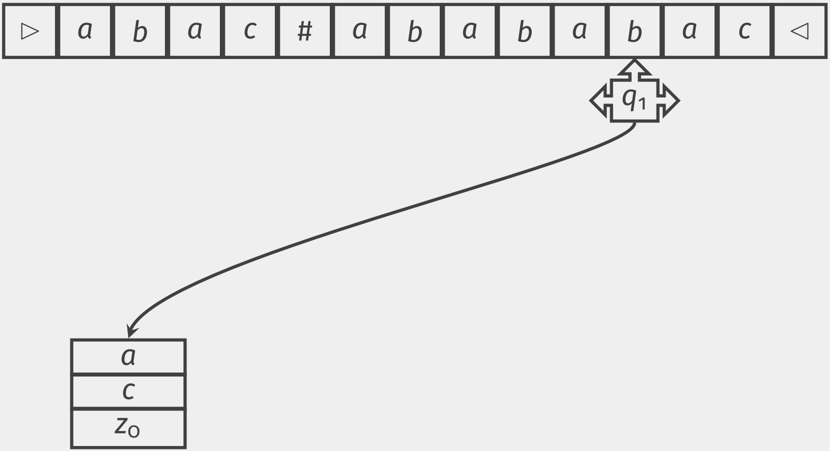

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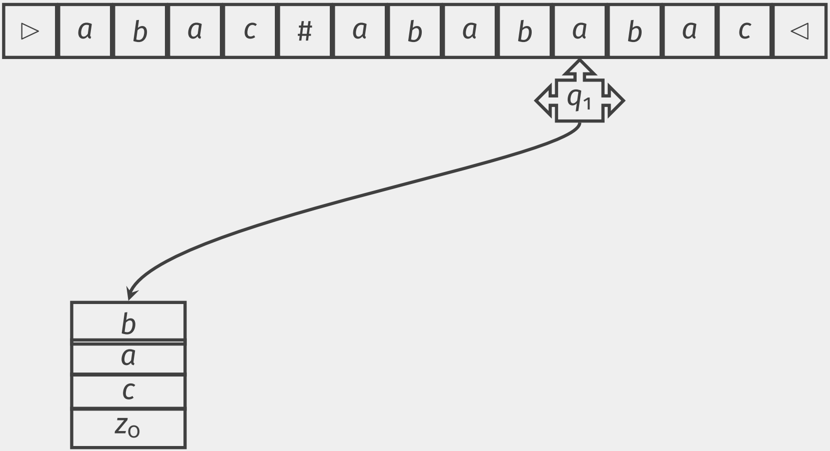

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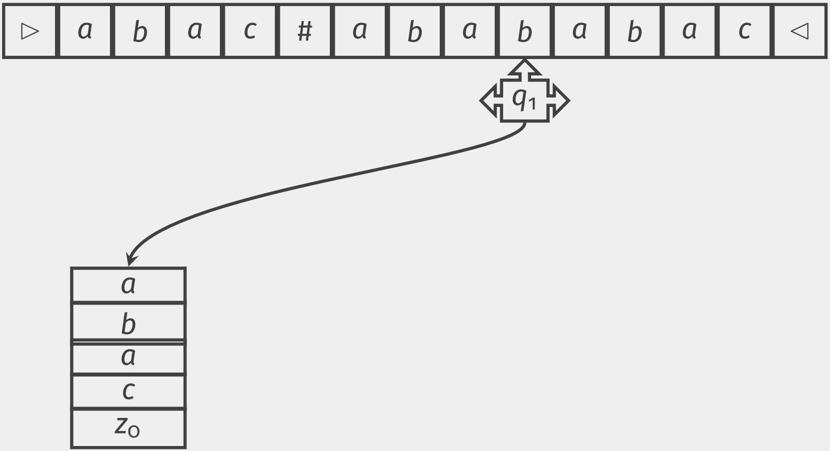

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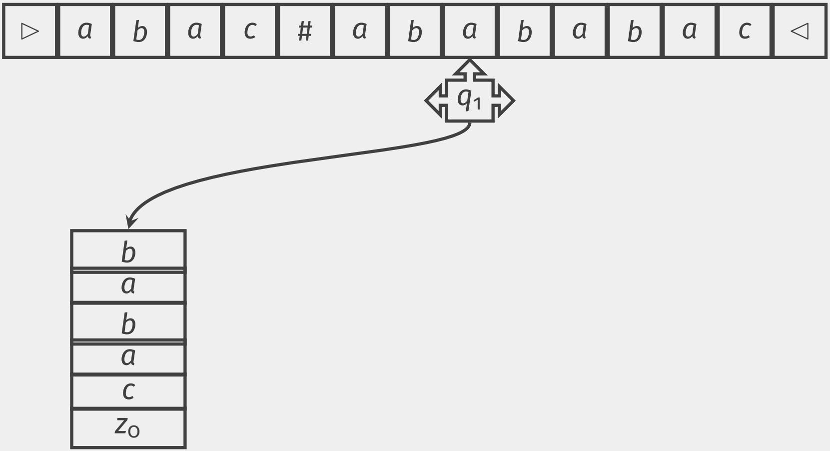

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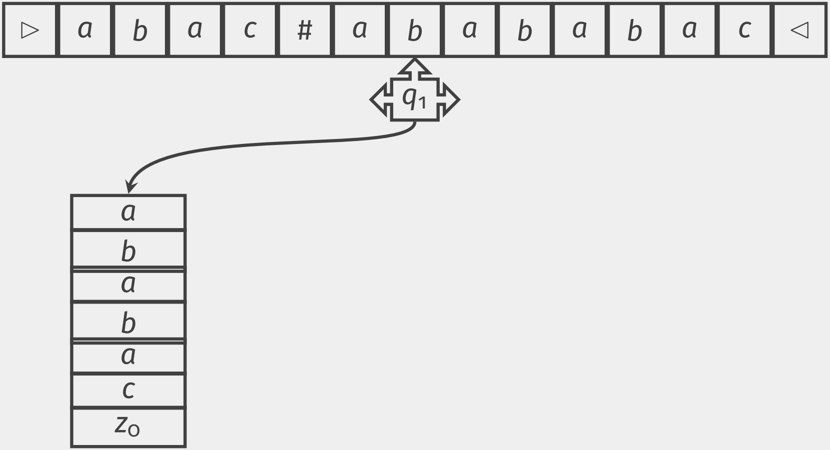

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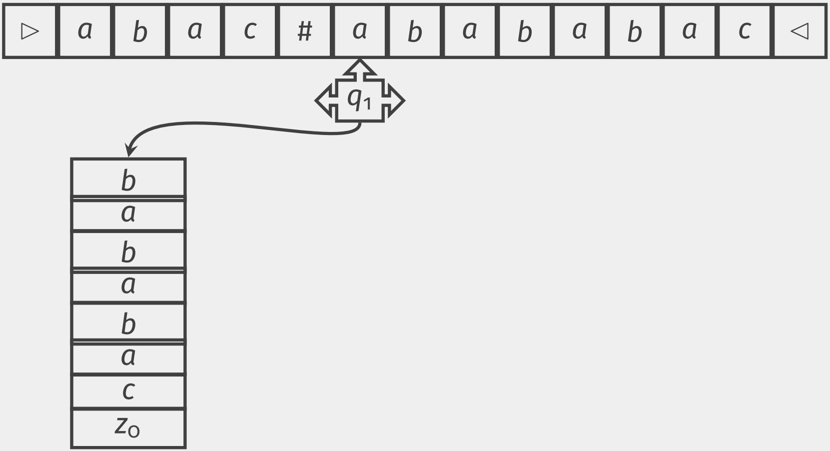

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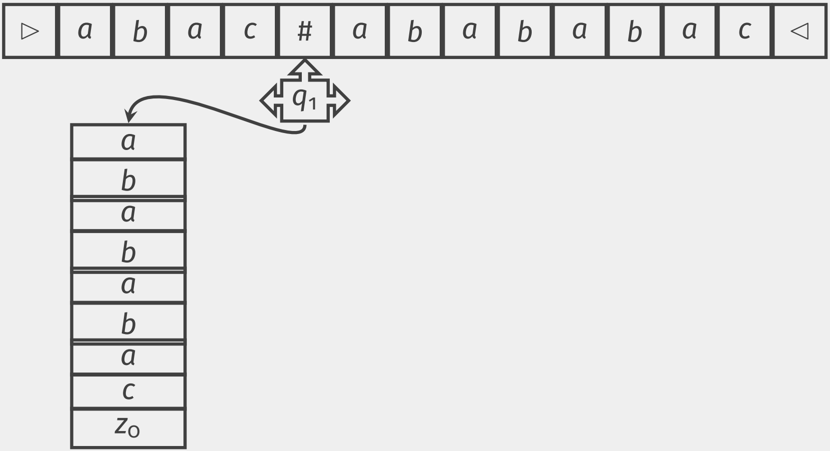

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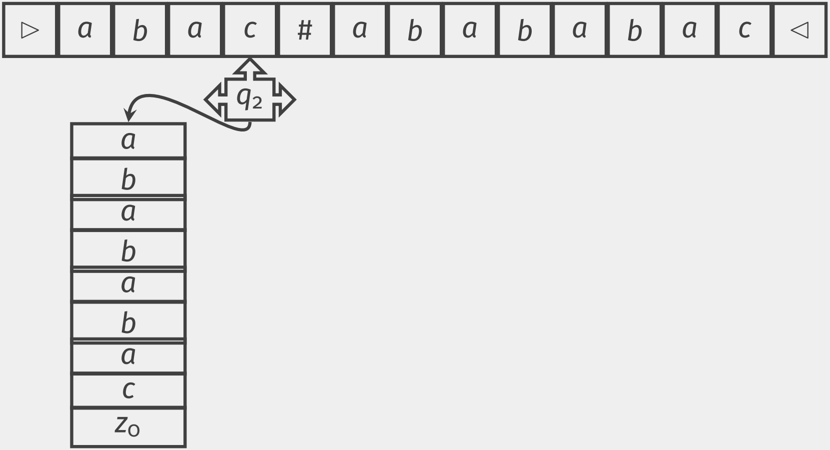

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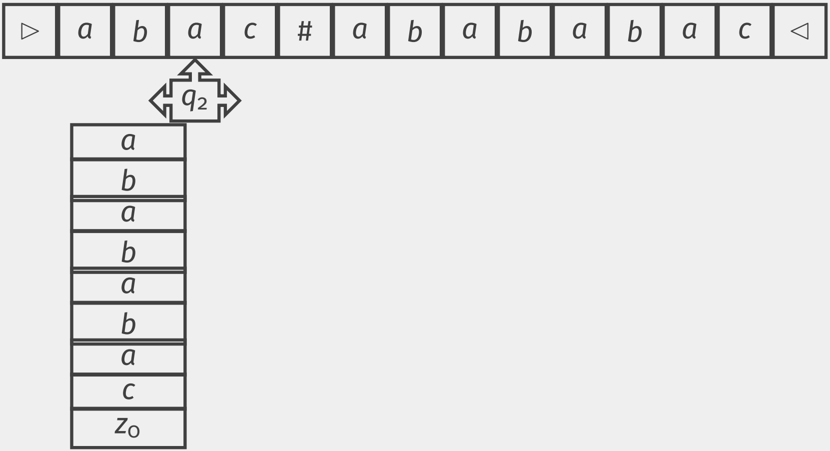

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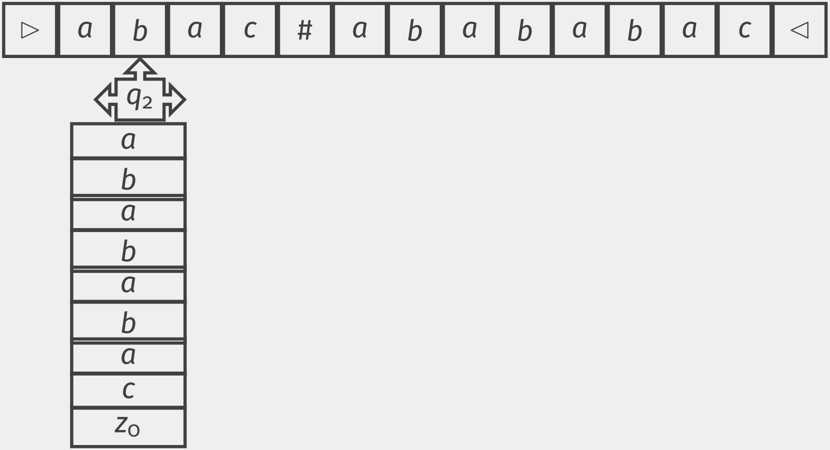

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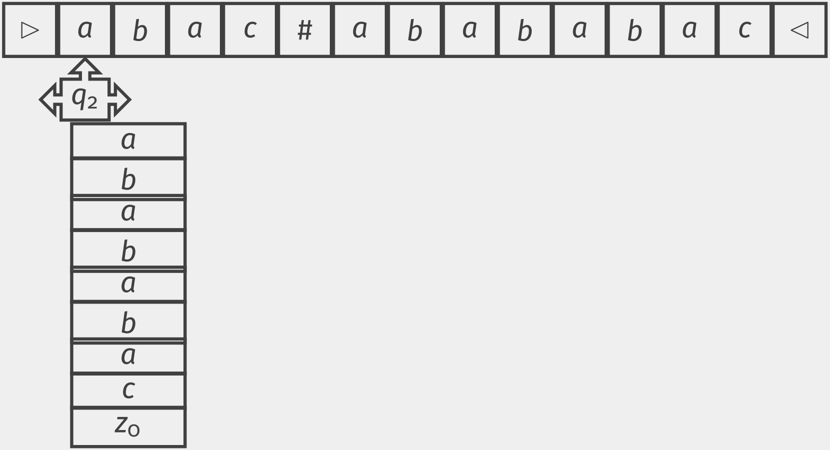

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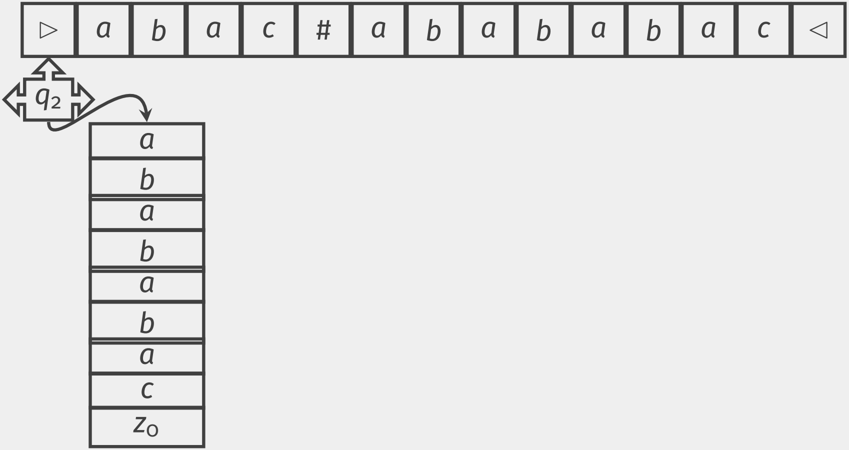

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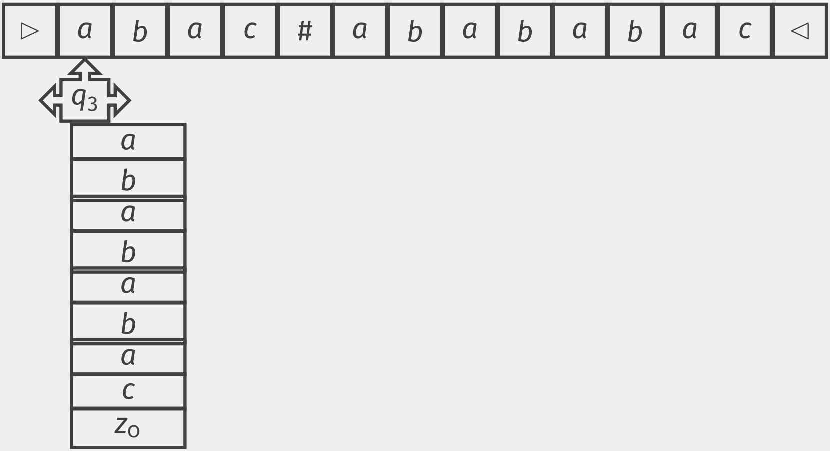

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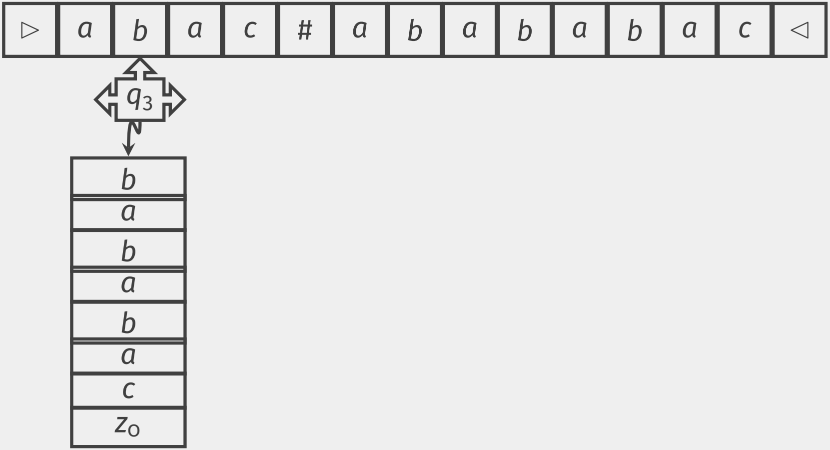

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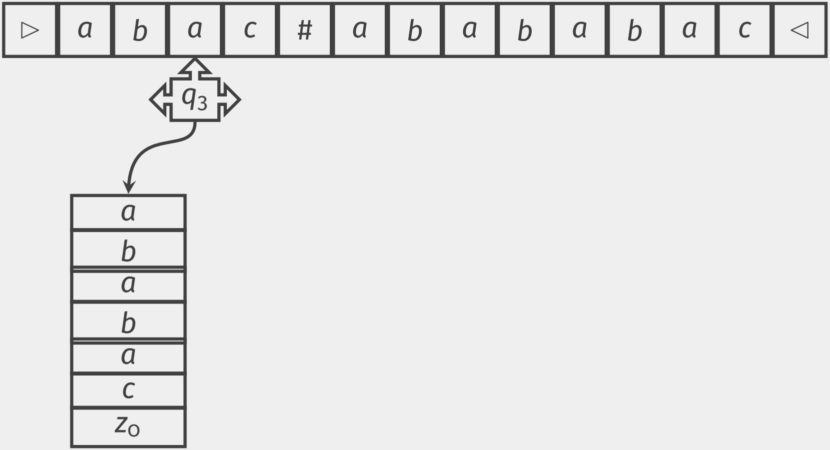

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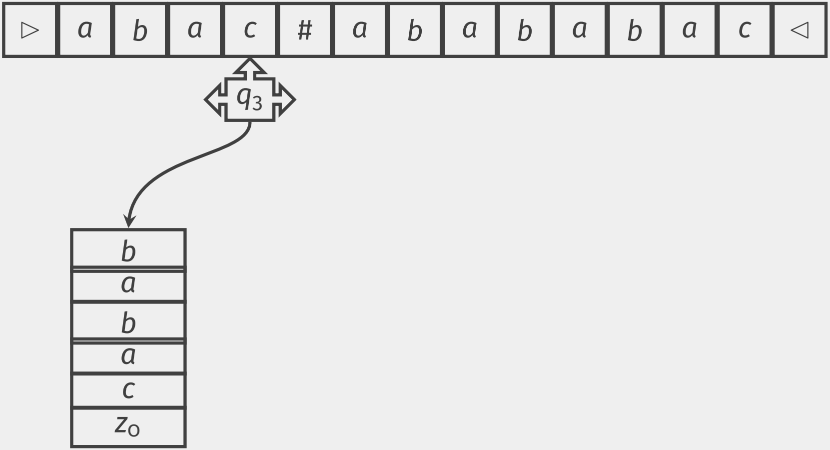

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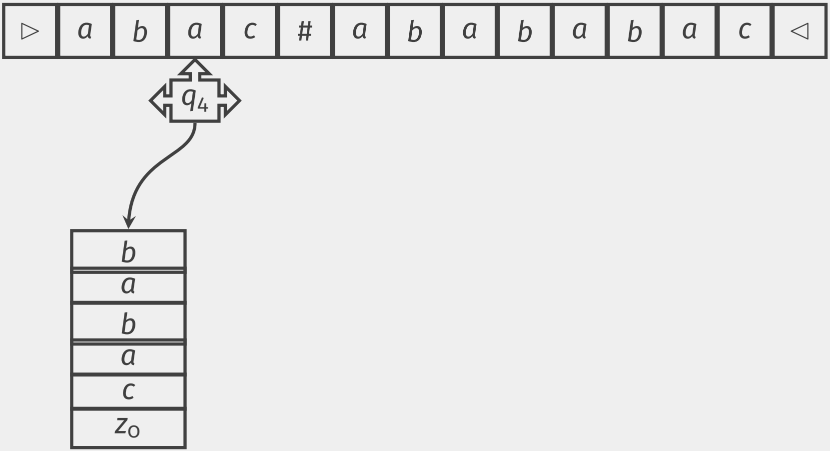

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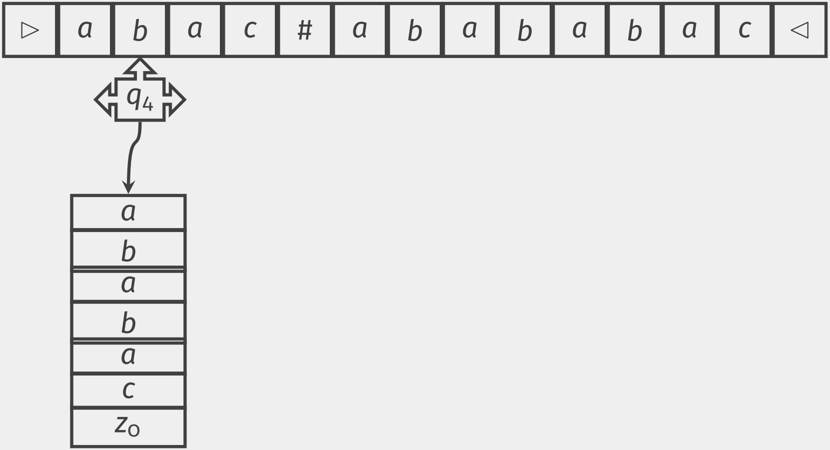

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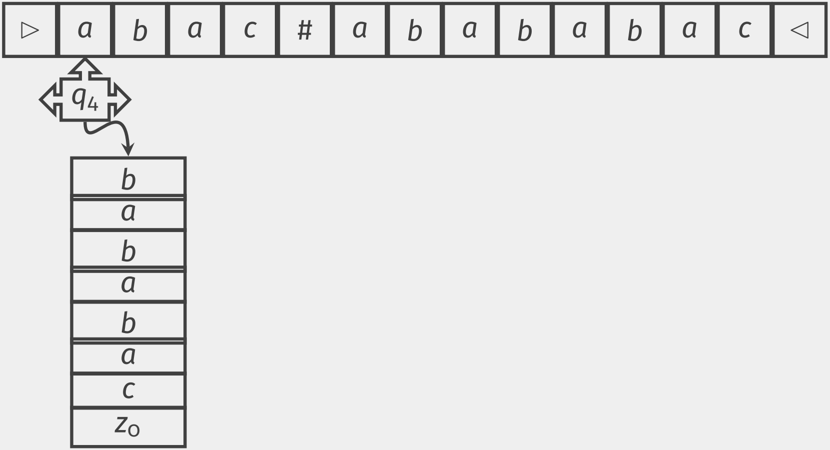

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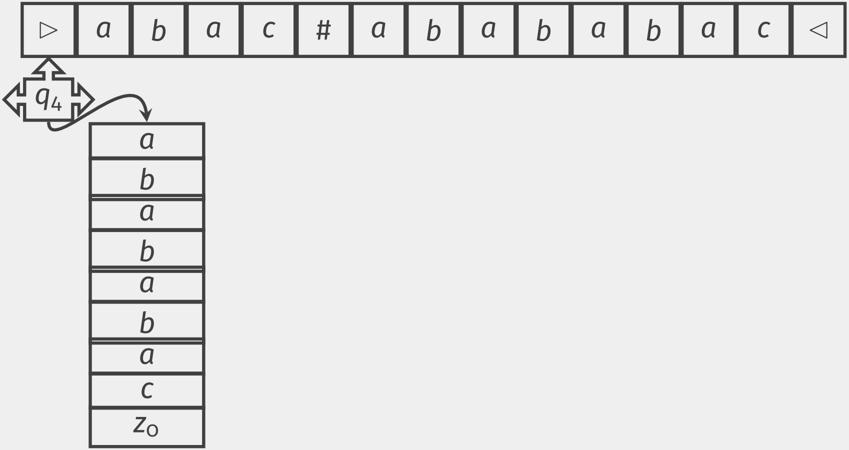

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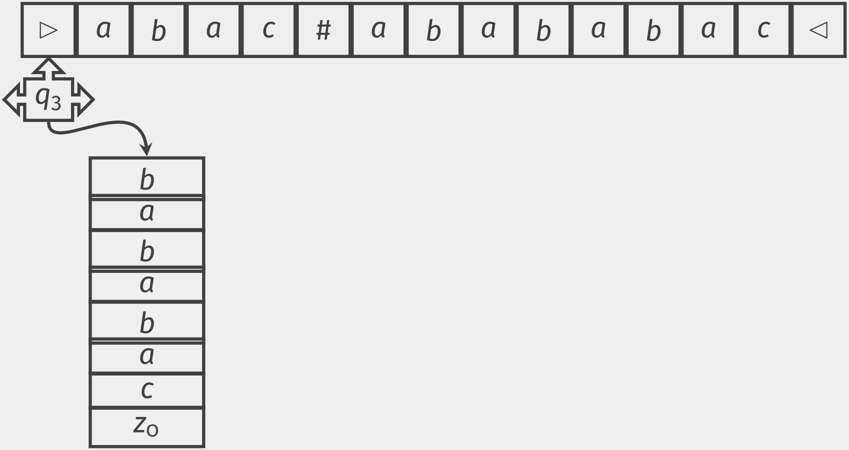

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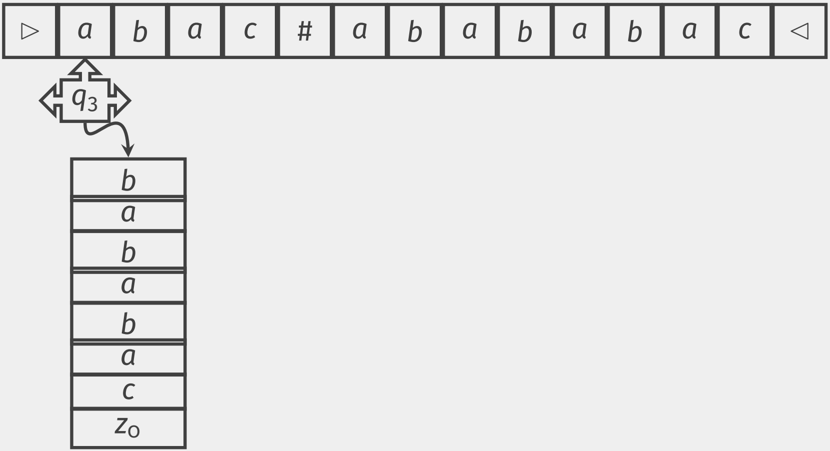

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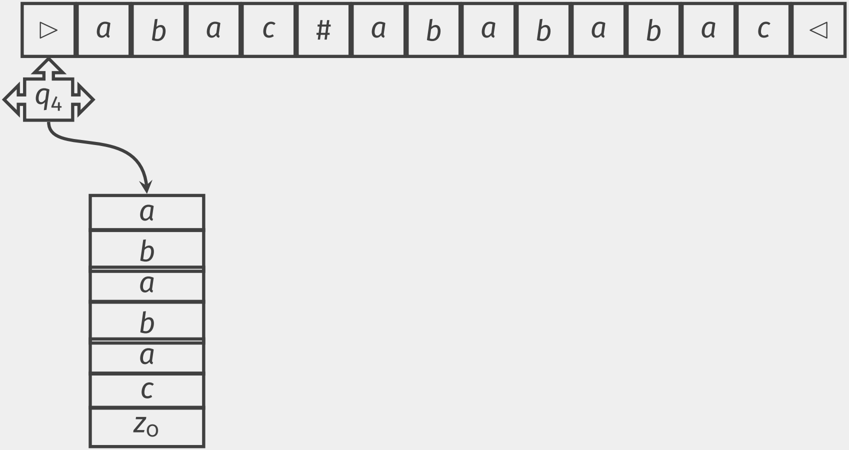

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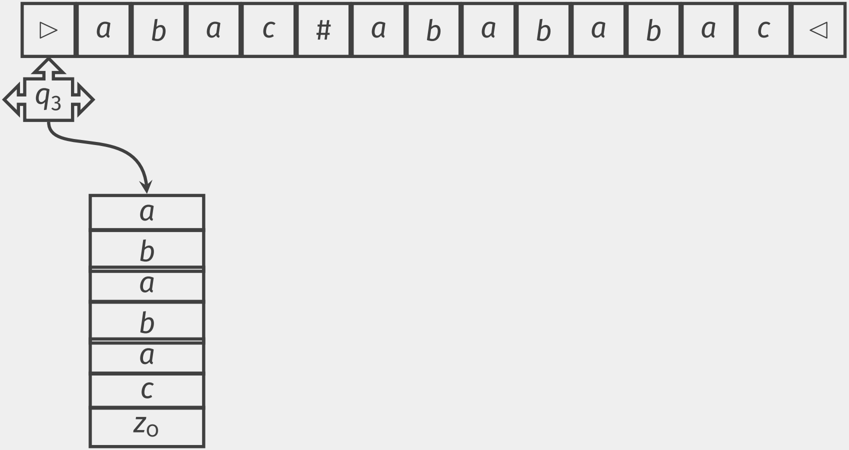

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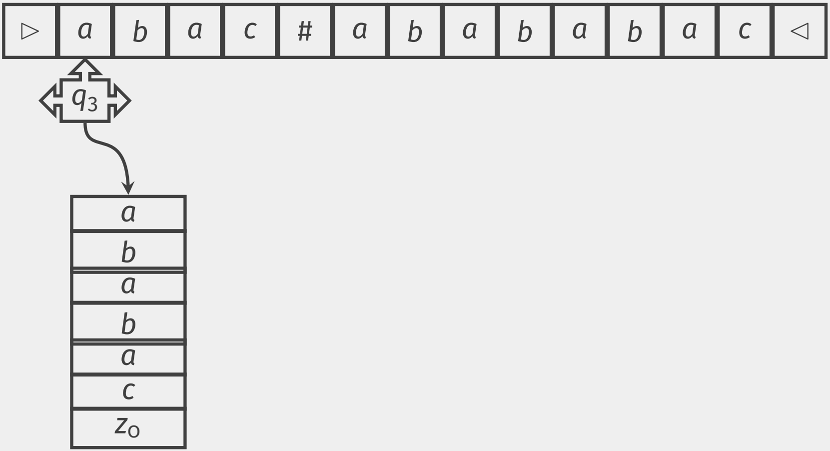

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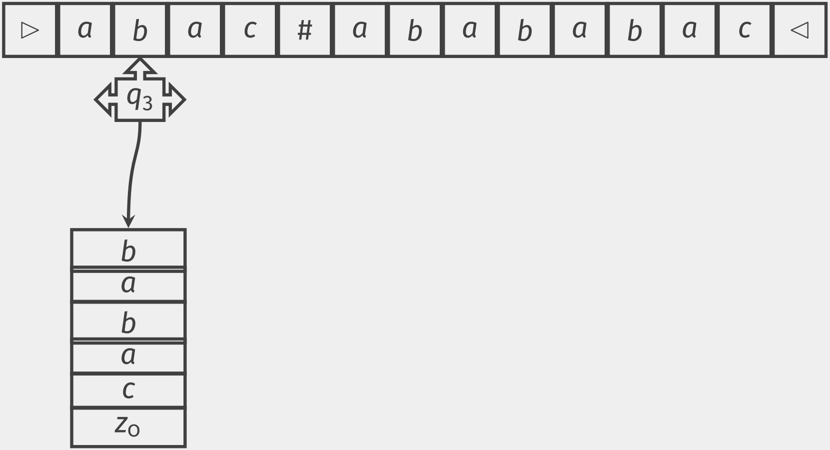

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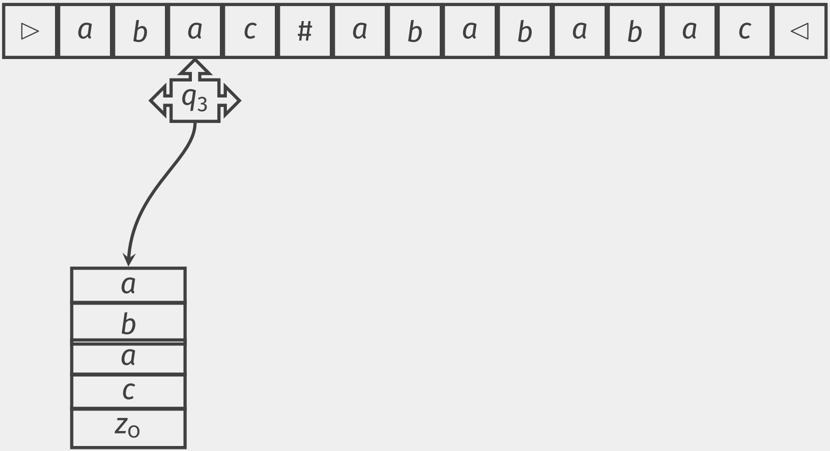

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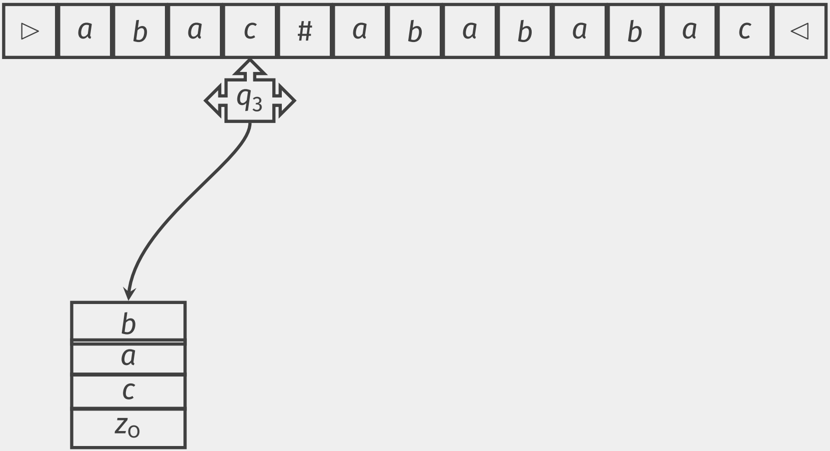

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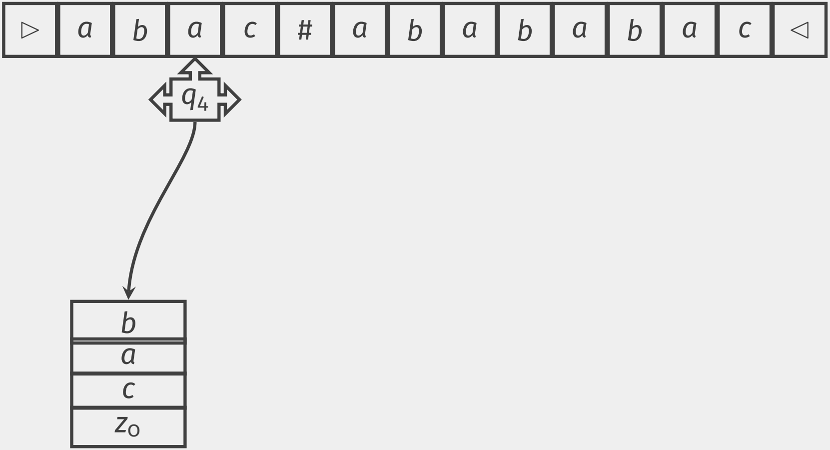

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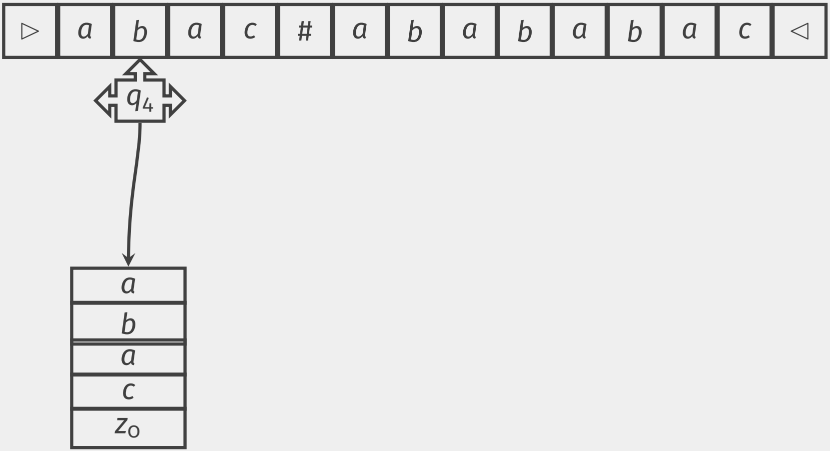

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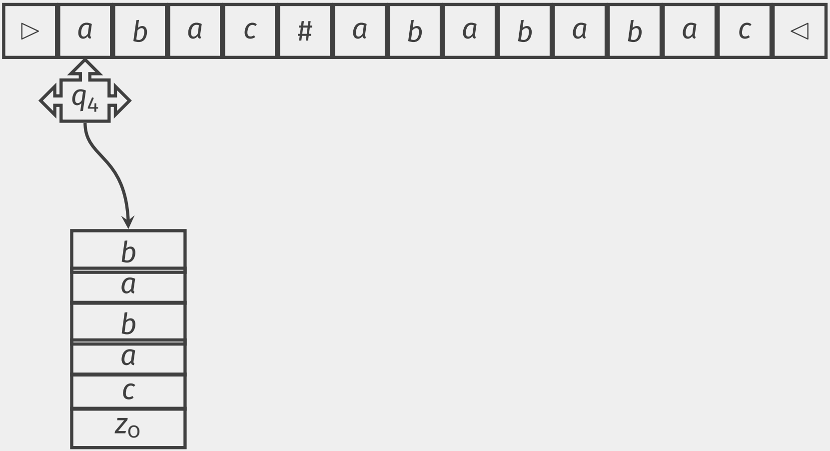

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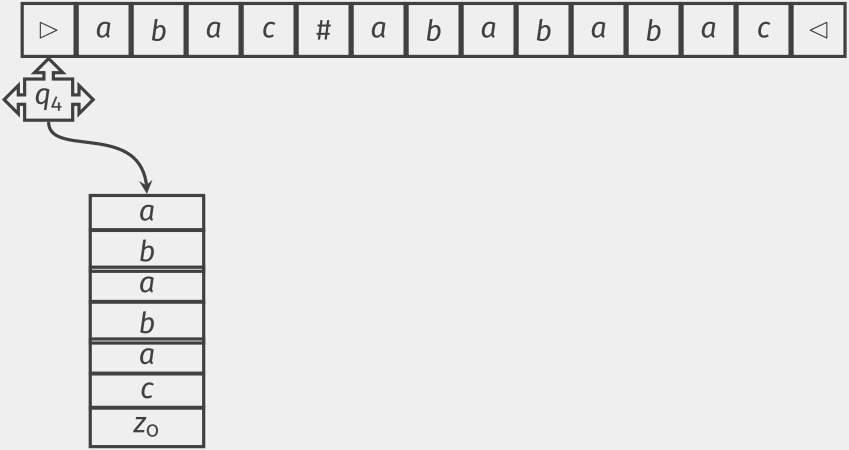

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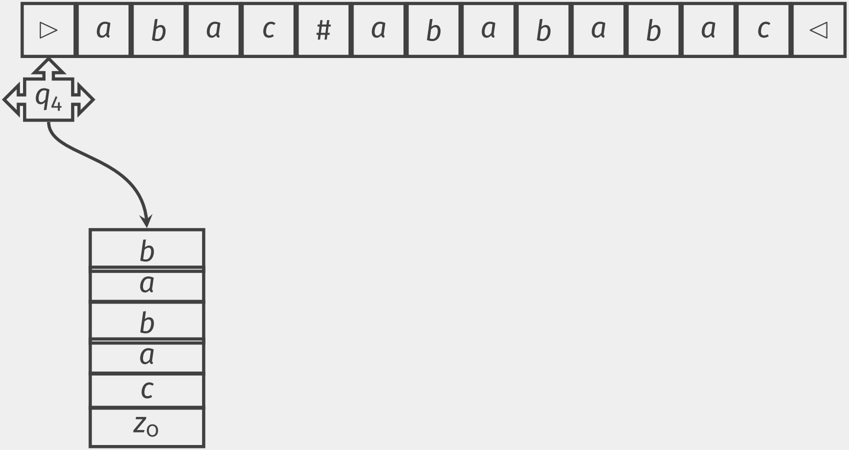

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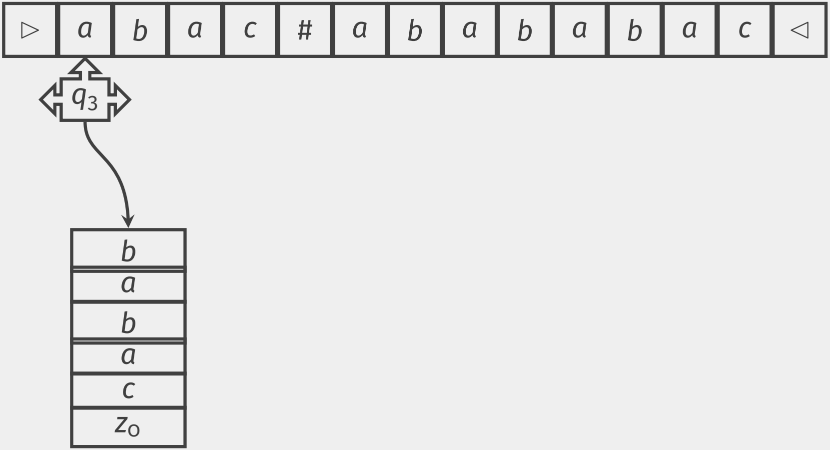

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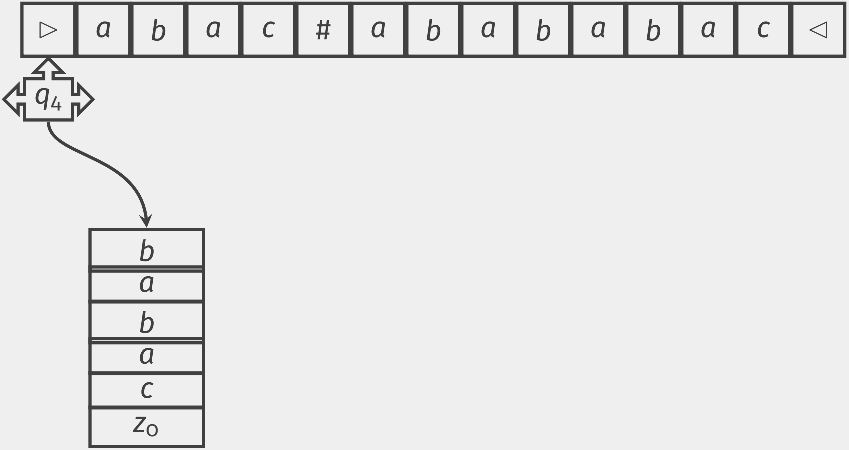

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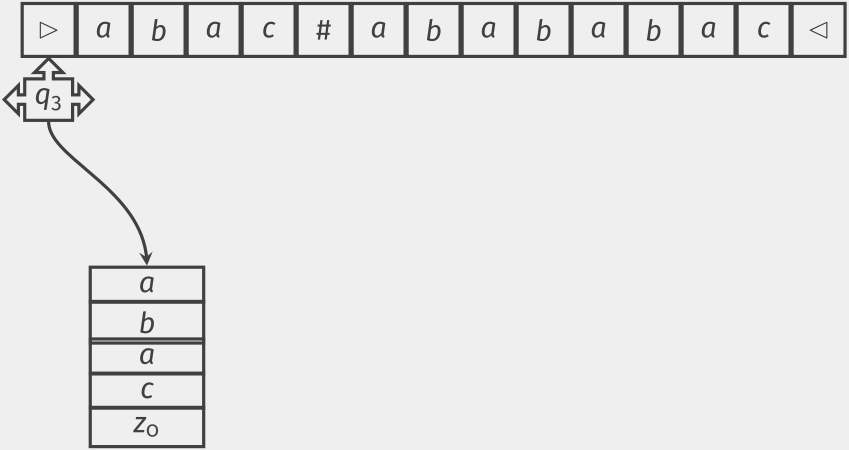

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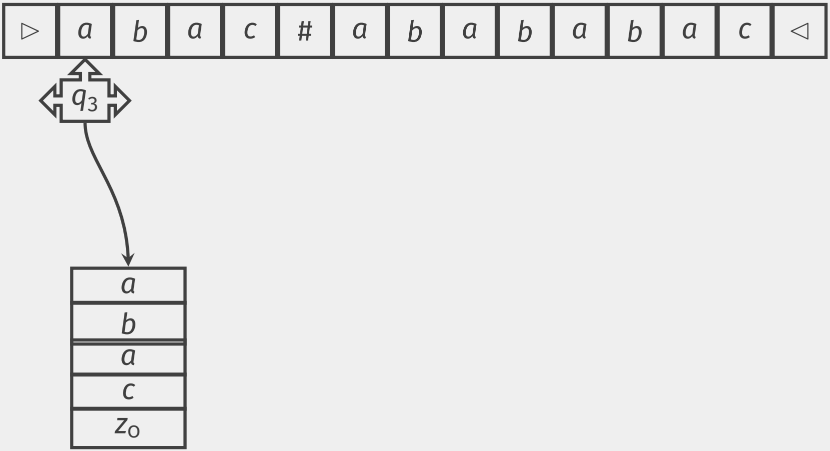

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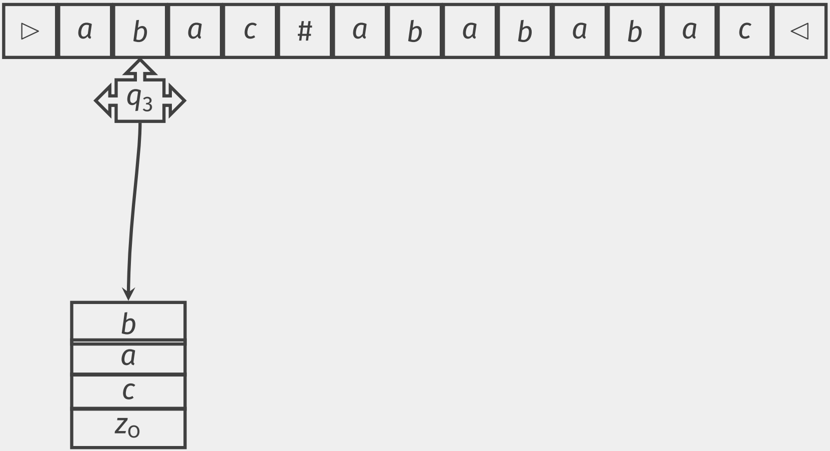

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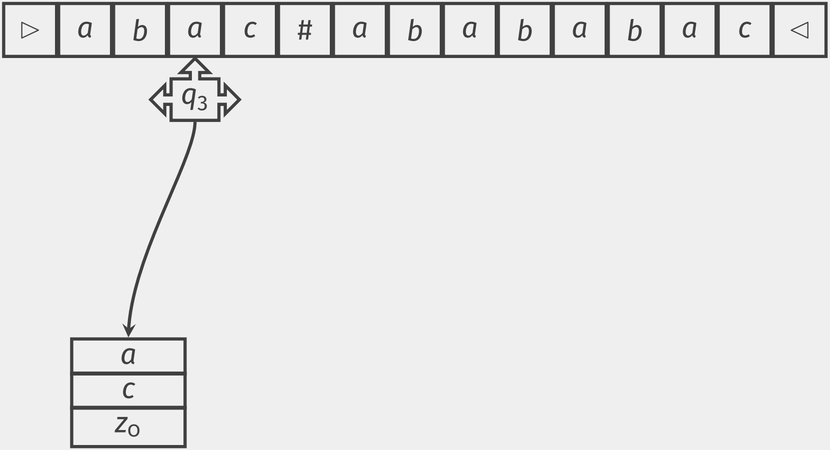

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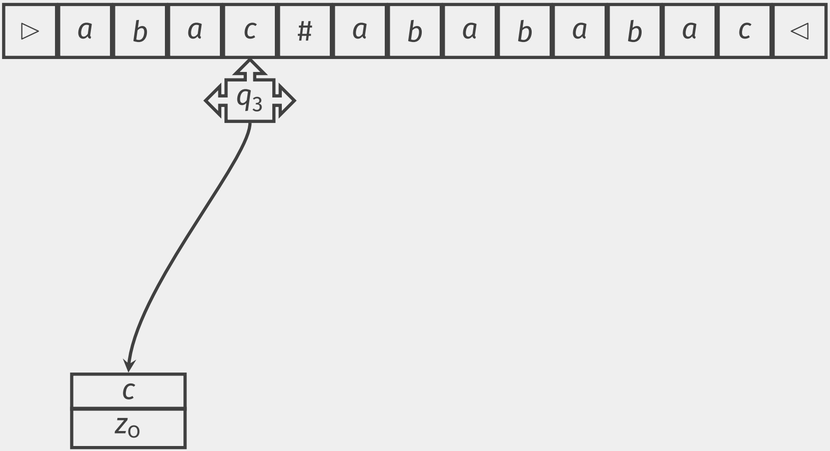

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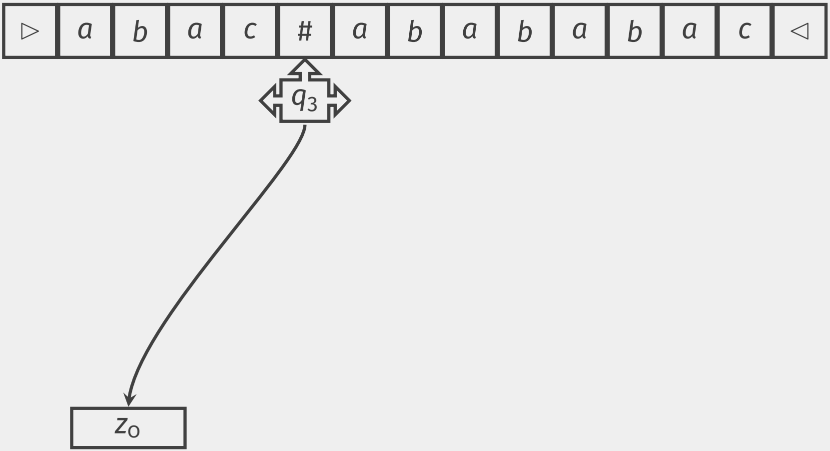
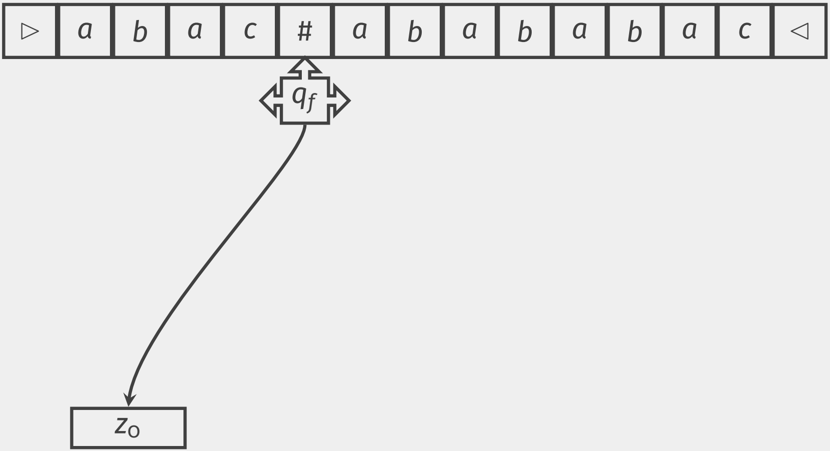


KMP-ALGORITHM + 2DPDA



Cook's Theorem

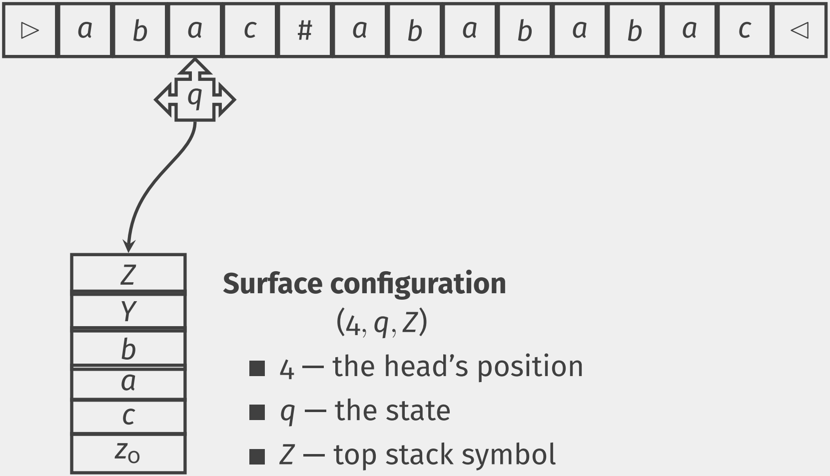
Theorem (Cook 1972)

A 2-DPDA-recognizable language is recognizable in linear time (in the RAM model).

Cook also provided a linear-time simulation algorithm.

- KMP algorithm has been investigated by Knuth by this simulation (and independently discovered by Morris without it).
- LR-parsers are 1-DPDA, so Cook's results show that there is an option of linear time parsing for wider class than DCFL.

Proof of Cook's Theorem



Surface configuration

$$(4, q, Z)$$

- 4 — the head's position
- q — the state
- Z — top stack symbol

Terminator for (i, q, Z) is (j, p, Z) s.t. Z is poped.

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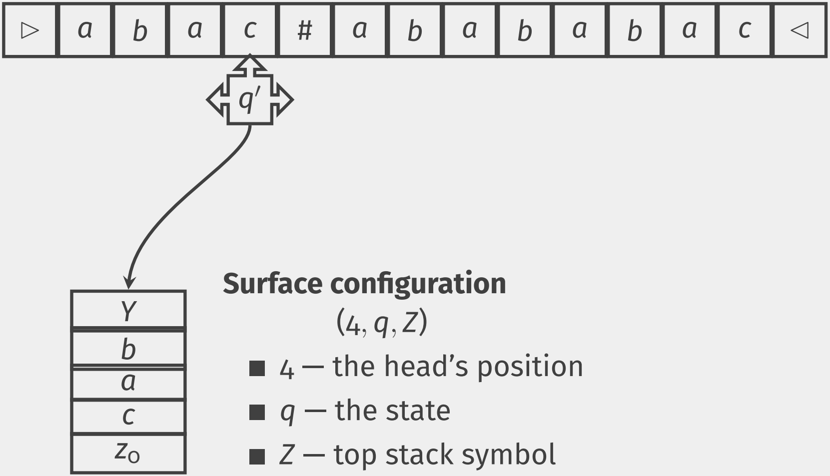

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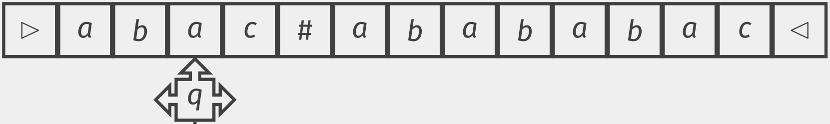
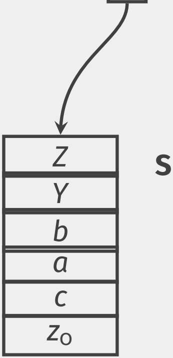
Proof of Cook's Theorem





Surface configuration

$$(4, q, Z)$$

- 4 — the head's position
- q — the state
- Z — top stack symbol

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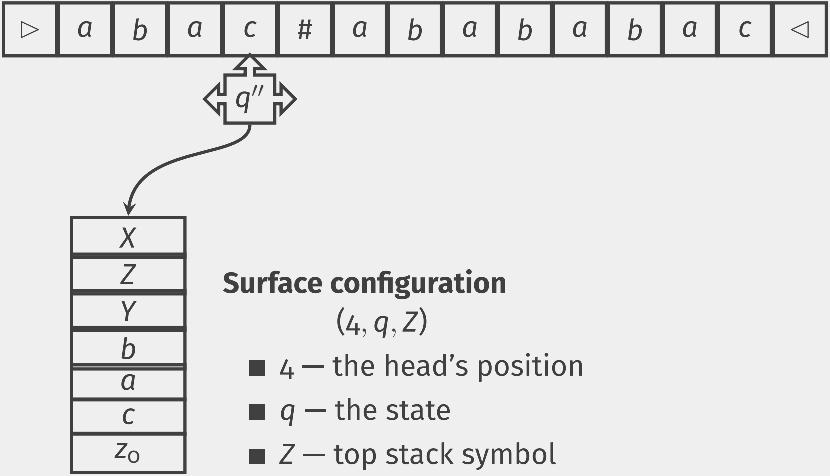

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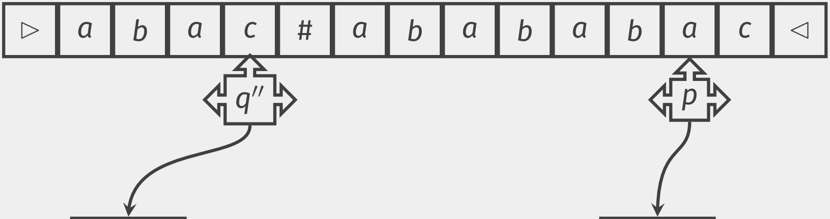

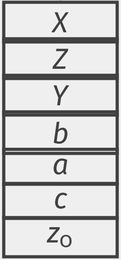

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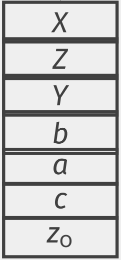

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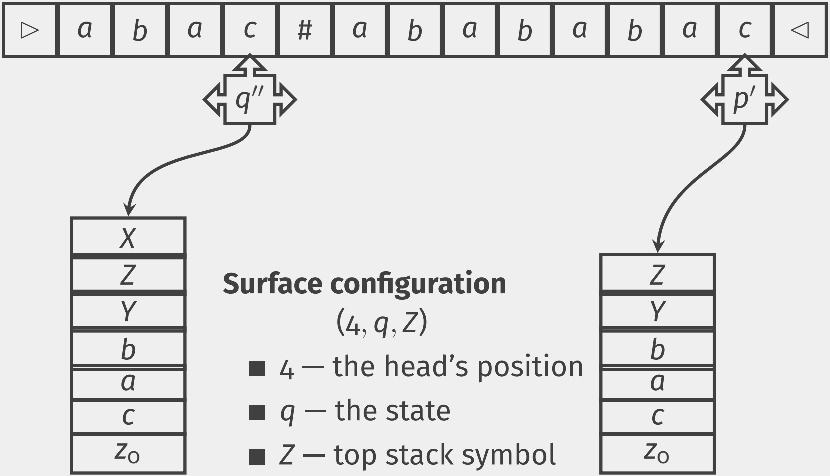

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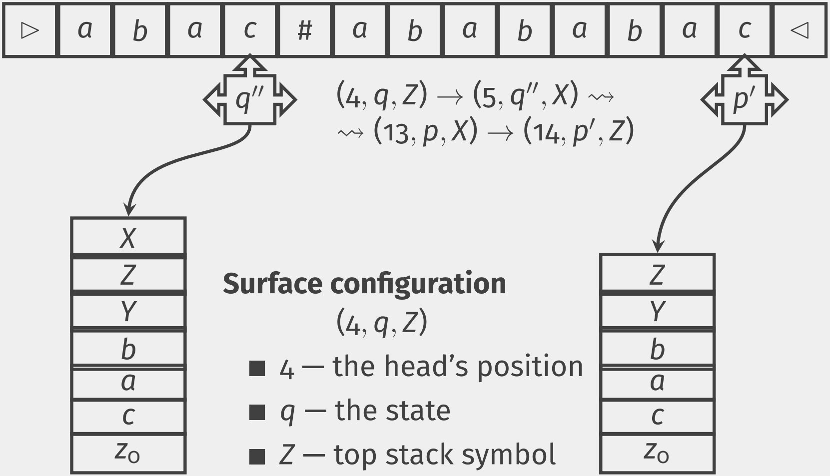

$(4, q, Z) \to (5, q'', X) \leadsto$
$\leadsto (13, p, X) \to (14, p', Z)$

Surface configuration

$(4, q, Z)$

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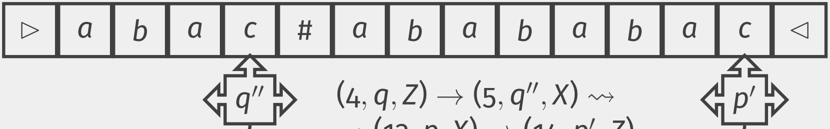
Terminator for (i, q, Z) is (j, p, Z) s.t. Z is poped.

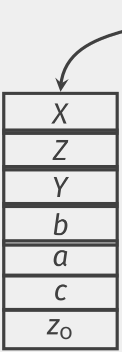
Proof of Cook's Theorem



$(4, q, Z) \to (5, q'', X) \rightsquigarrow$
$\rightsquigarrow (13, p, X) \to (14, p', Z)$

$T(5, q'', X) = (13, p, X)$

$T(4, q, Z) = T(14, p', Z)$



Surface configuration

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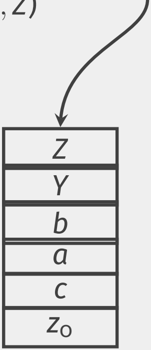

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