PROBLEM 1

Draw the three dimensional Mohr's circle for the following states of stress.

$$[\sigma]=\begin{bmatrix} 20 & 10 & -10 \\ 10 & 30 & 0 \\ -10 & 0 & 50 \end{bmatrix} MPa$$

$$[\sigma]=\begin{bmatrix} 40 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix} MPa$$

$$[\sigma]=\begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix} MPa$$

SOLUTION

Starting by finding the principal stress of the first state of stress

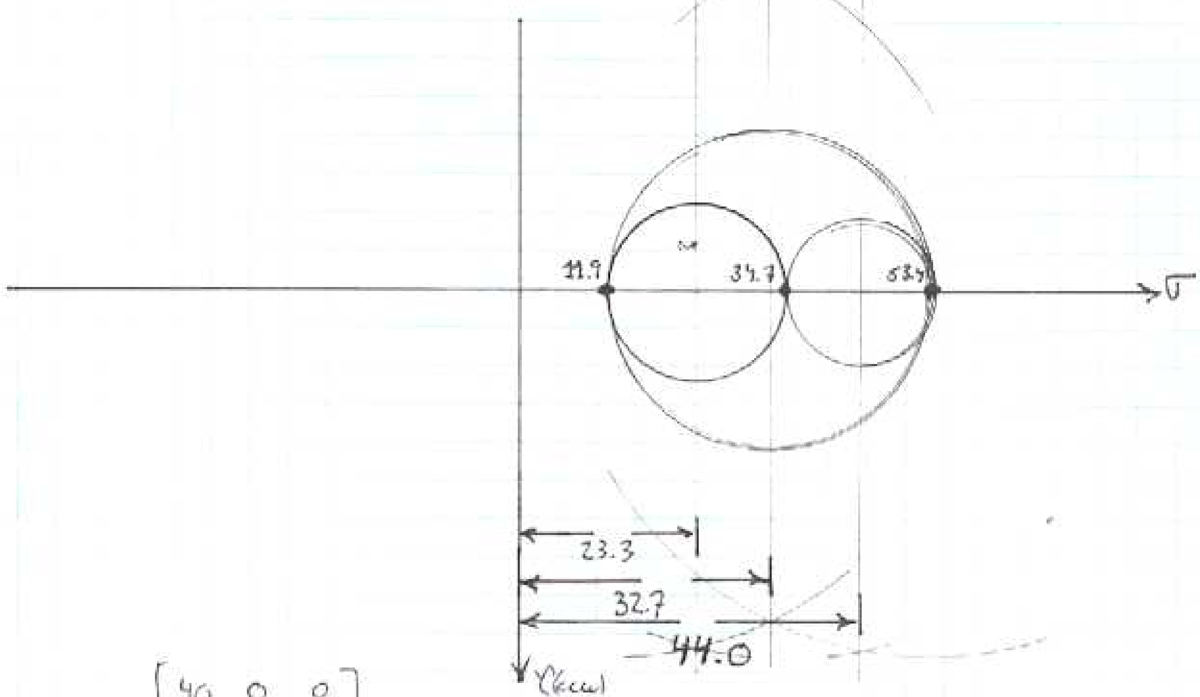
```
>> Stress01=[20 10 -10; 10 30 0; -10 0 50]


Stress01 =


   20   10  -10

   10   30    0

  -10    0   50


>> [M1,V1]=eig(Stress01)


M1 =


  -0.8535    0.4103   -0.3213
```

0.4706	0.8716	-0.1371
-0.2238	0.2683	0.9370

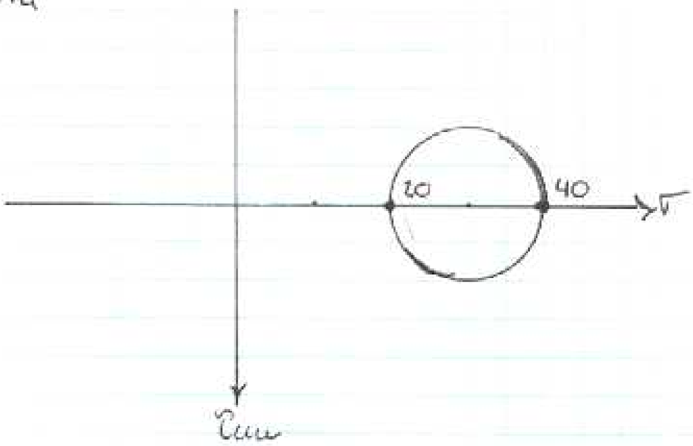
V1 =

11.8639	0	0
0	34.7068	0
0	0	53.4292

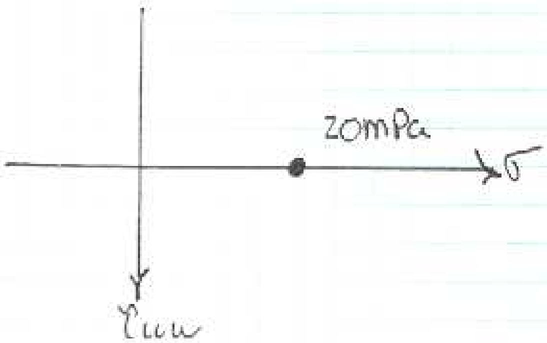
MOHR'S CIRCLE FOR $[\sigma] = \begin{bmatrix} 20 & 10 & -10 \\ 10 & 30 & 0 \\ -10 & 0 & 50 \end{bmatrix}$ MPa $\Rightarrow [\sigma_p] = \begin{bmatrix} 11.9 & 0 & 0 \\ 0 & 34.7 & 0 \\ 0 & 0 & 53.4 \end{bmatrix}$ MPa



MOHR'S CIRCLE FOR $[\sigma] = \begin{bmatrix} 40 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$ MPa



MOHR'S CIRCLE FOR $[\sigma] = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$ MPa



PROBLEM 2:

Given the state of stress

$$[\sigma]=\begin{bmatrix} 20 & 12 & -15 \\ 12 & 0 & 10 \\ -15 & 10 & 6 \end{bmatrix} MPa$$

a. **Determine the stress invariants.**
b. **Determine the principal stresses.**
c. **Determine the direction cosines to each of the principal stresses.**
d. **Determine the transformation matrix from the original state of stress to the principal state of stress and prove that it is the transformation matrix by using it to transform the original state of stress.**
e. **Determine the state of stress defined by rotating x,y through an angle of 30° counterclockwise about the z axis.**
f. **Determine the maximum shear stress for this state of stress.**
g. **Determine the transformation matrix that needs to be used to transform the original state of stress to a state of tress that contains the maximum shear stress on two of the faces and a principal state of stress on the third.**

SOLUTION

```
>> STRE=[20 12 -15; 12 0 10; -15 10 6]

STRE =

    20   12  -15
    12    0   10
   -15   10    6
```

a. STRESS INVARIANTS

```
>> poly(STRE)

ans =

  1.0e+003 *

    0.0010   -0.0260   -0.3490    6.4640
```

b. PRINCIPAL STRESS

```
>> [V,P]=eig(STRE)

P =

  -16.9786         0         0
         0   12.4852         0
         0         0   30.4934
```

c. DIRECTION COSINES

V =

-0.4555 0.1894 -0.8699

0.6683 0.7183 -0.1935

-0.5882 0.6695 0.4537

-16.9786: (-0.4555 0.6683 -0.5882)

12.4852: (0.1894 0.7183 0.6695)

30.4934: (-0.8699 -0.1935 0.4537)

d. TRANSFORMATION MATRIX TO PRINCIPAL STATE OF STRESS

>> T=V'

T =

-0.4555 0.6683 -0.5882

0.1894 0.7183 0.6695

-0.8699 -0.1935 0.4537

e. TRANSFORM STRESS STATE 30 DEGREES CCW

```
>> T2=[.866 .5 0; -.5 .866 0; 0 0 1]

T2 =

    0.8660    0.5000         0
   -0.5000    0.8660         0
         0         0    1.0000

>> STRE30z=T2*STRE*T2'

STRE30z =

   25.3911   -2.6605   -7.9900
   -2.6605   -5.3920   16.1600
   -7.9900   16.1600    6.0000
```

f. MAXIMUM SHEAR STRESS

```
>> (P(3,3)-P(1,1))/2

ans =

   23.7360
```

g. THE TRANSFORMATION MATRIX TO THE MAXIMUM SHEAR ON TWO SURFACES AND PRINCIPAL STRESS ON THE THIRD

```
>> T3=[.7071 0 .7071; 0 1 0; -.7071 0 .7071]

T3 =

    0.7071         0    0.7071
         0    1.0000         0
   -0.7071         0    0.7071

>> STRE3=T3*T*STRE*T'*T3'

STRE3 =

    6.7573         0   23.7355
   -0.0000   12.4852    0.0000
   23.7355   -0.0000    6.7573
```