

THE TORQUE IN THE CROSS-SECTION IS GIVEN BY

$$T = 2 \cdot \left[\frac{\pi}{2} \cdot (0.225\text{m})^2 \cdot q_1 + (0.2\text{m} \cdot 0.3\text{m} + 0.2\text{m} \cdot 0.15\text{m}) \cdot q_2 + \frac{1}{2} \cdot 0.3\text{m} \cdot 0.1\text{m} \cdot q_3 \right]$$

$$\underline{12(10^3)\text{N}\cdot\text{m} = 0.1590\text{m}^2 \cdot q_1 + 0.180\text{m}^2 \cdot q_2 + 0.120\text{m}^2 \cdot q_3} \quad (1)$$

SINCE THE ANGLE OF TWIST IN EACH SECTION IS THE SAME, THREE MORE EQUATIONS CAN BE WRITTEN. STARTING WITH THE FIRST CELL

CELL 1:

$$\frac{\Phi}{L} = \frac{1}{2 \cdot G \cdot A_1} \cdot \left[q_1 \cdot \frac{\pi \cdot 0.225\text{m}}{0.002\text{m}} + (q_1 - q_2) \cdot \frac{0.450\text{m}}{0.003\text{m}} \right]$$

$$\frac{\Phi}{L} \cdot 2 \cdot 80(10^9) \frac{\text{N}}{\text{m}^2} \cdot \frac{\pi}{2} (0.225\text{m})^2 = 117.8 \cdot q_1 + (q_1 - q_2) \cdot 150$$

$$\underline{12.72(10^9)\text{N} \frac{\Phi}{L} = 267.8 \cdot q_1 - 150 \cdot q_2} \quad (2)$$

CELL 2:

$$\frac{\Phi}{L} = \frac{1}{2 \cdot 80(10^9) \frac{\text{N}}{\text{m}^2} \cdot (0.2\text{m} \cdot 0.3\text{m} + 0.2\text{m} \cdot 0.15\text{m})} \left[(q_2 - q_1) \frac{0.45\text{m}}{0.003\text{m}} + 2 \cdot q_2 \cdot \frac{0.2136\text{m}}{0.006\text{m}} + (q_2 - q_3) \cdot \frac{0.300\text{m}}{0.003\text{m}} \right]$$

$$\underline{14.40(10^9)\text{N} \cdot \frac{\Phi}{L} = -150 \cdot q_1 + 321 \cdot q_2 - 100 \cdot q_3} \quad (3)$$

CELL 3:

$$\frac{\Phi}{L} = \frac{1}{2 \cdot 80(10^9) \frac{\text{N}}{\text{m}^2} \cdot \frac{1}{2} \cdot 0.3\text{m} \cdot 0.1\text{m}} \cdot \left[(q_3 - q_1) \frac{0.300\text{m}}{0.003\text{m}} + 2 \cdot \frac{0.4\text{m}}{0.006\text{m}} \cdot q_3 \right]$$

$$\underline{9.60(10^9)\text{N} \cdot \frac{\Phi}{L} = -100 \cdot q_2 + 242.4 \cdot q_3} \quad (4)$$

EQUATIONS (1)-(4) CAN NOW BE REWRITTEN IN A FORM THAT ~~ALLOWS~~ ENABLES AN ALGEBRAIC SOLUTION

$$\begin{aligned} 12 \times 10^3 \text{N}\cdot\text{m} &= 0.1590 \cdot \text{m}^2 \cdot q_1 + 0.180 \text{m}^2 \cdot q_2 + 0.120 \text{m}^2 \cdot q_3 \\ 0 &= 267.8 \cdot q_1 - 150 \cdot q_2 + 0 \cdot q_3 - 12.72(10^9)\text{N} \cdot \frac{\Phi}{L} \\ 0 &= -150 \cdot q_1 + 321 \cdot q_2 - 100 \cdot q_3 - 14.40(10^9)\text{N} \cdot \frac{\Phi}{L} \\ 0 &= 0 \cdot q_1 - 100 \cdot q_2 + 242.4 \cdot q_3 - 9.60(10^9)\text{N} \cdot \frac{\Phi}{L} \end{aligned}$$