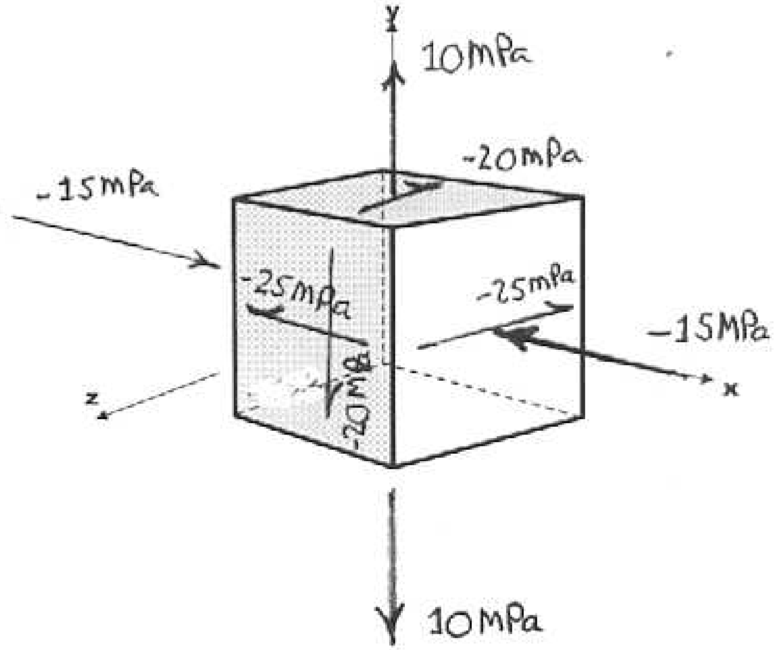
NAME: SOLUTION

PROBLEM 1: At a point in a loaded member, the stresses relative to a x-y-z coordinate system are given by:

$$[\sigma]_{xyz} = \begin{bmatrix} -15 & 0 & -25 \\ 0 & 10 & -20 \\ -25 & -20 & 0 \end{bmatrix} MPa$$

1a. Draw the state of stress on the cube shown and determine the principal stresses for this state of stress and their directions cosines. (If you use MATLAB or Excel to perform calculations, be sure to print out the Command Window or spreadsheet that contains the commands you used to perform the calculations.)



```
>> Sigma=[-15 0 -25; 0 10 -20; -25 -20 0]

Sigma =

  -15    0  -25
    0   10  -20
  -25  -20    0
```

ORIGINAL STATE OF STRESS IN MPa

```
>> [EV,PS]=eig(Sigma)

EV =
```

	1	2	3
x	-0.7232	-0.5855	-0.3663
y	-0.2706	0.7281	-0.6298
z	-0.6355	0.3563	0.6850

DIRECTION COSINES

```
PS =

  -36.9676        0        0
         0   0.2130        0
         0        0  31.7546
```

PRINCIPAL STRESSES IN MPa

1b. What angles (in degrees) do each of the principal stresses make with the x, y, and z axes?

σ_1: θ_{1x}= 136.3 θ_{1y}= 105.7 θ_{1z}= 129.5

σ_2: θ_{2x}= 125.8 θ_{2y}= 43.3 θ_{2z}= 69.1

σ_3: θ_{3x}= 111.5 θ_{3y}= 129.0 θ_{3z}= 46.8

EV =

-0.7232	-0.5855	-0.3663
-0.2706	0.7281	-0.6298
-0.6355	0.3563	0.6850

```
>> acos(EV)*180/pi
```

ans =

	1	2	3
x	136.3170	125.8414	111.4864
y	105.6995	43.2698	129.0325
z	129.4536	69.1260	46.7636

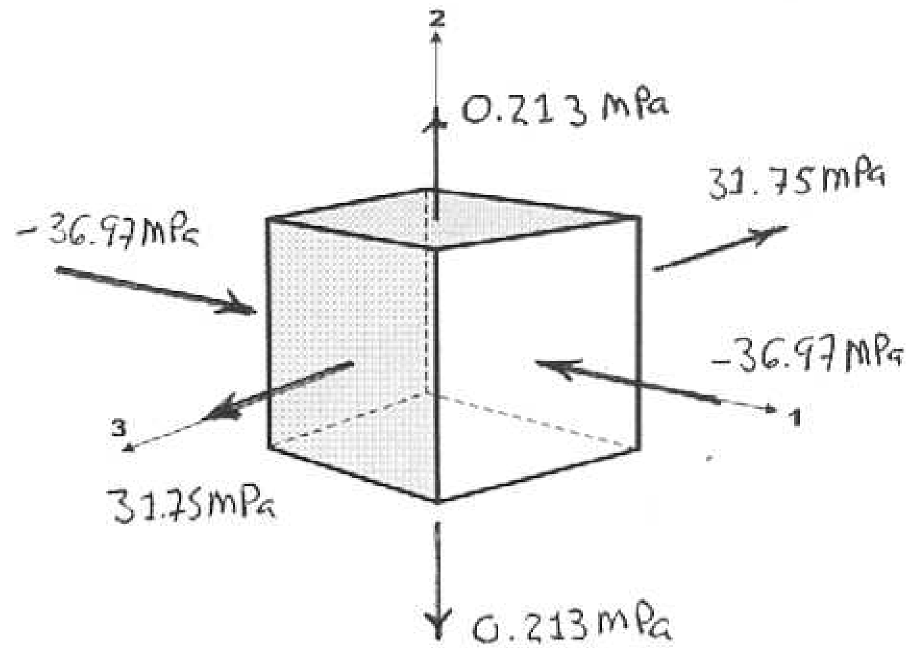
Checking the dot product between direction cosine vectors

```
>> dot(EV(:,1),EV(:,2))

ans =

 -1.1102e-016

>> dot(EV(:,1),EV(:,3))

ans =

 -2.7756e-016

>> dot(EV(:,2),EV(:,3))

ans =

  1.9429e-016
```

ALL THREE DOT PRODUCTS ARE ESSENTIALLY ZERO, MEANING THAT THE THREE PRINCIPAL AXES ARE ORTHOGONAL.

(2a)

1c. Determine the transformation matrix from the original state of stress to the principal state of stress and prove that it is the transformation matrix by using it to transform the original state of stress to the principal state. Illustrate the stress cube the represents the principal state of stress.



```
>> T=EV'

T =

  -0.7232  -0.2706  -0.6355
  -0.5855   0.7281   0.3563
  -0.3663  -0.6298   0.6850

>> SigPrincipal=T*Sigma*T'

SigPrincipal =

 -36.9676  -0.0000  -0.0000
  -0.0000   0.2130   0.0000
  -0.0000   0.0000  31.7546
```

IN MPa

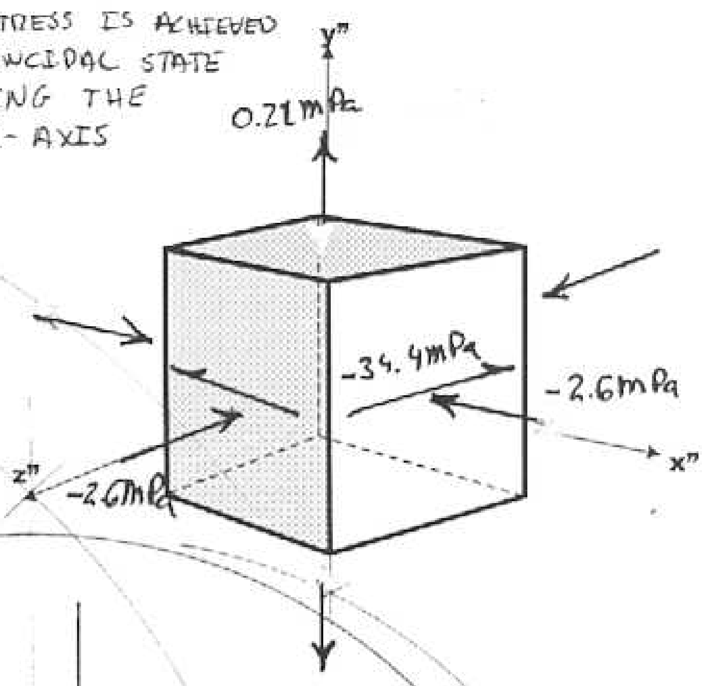
(3a)

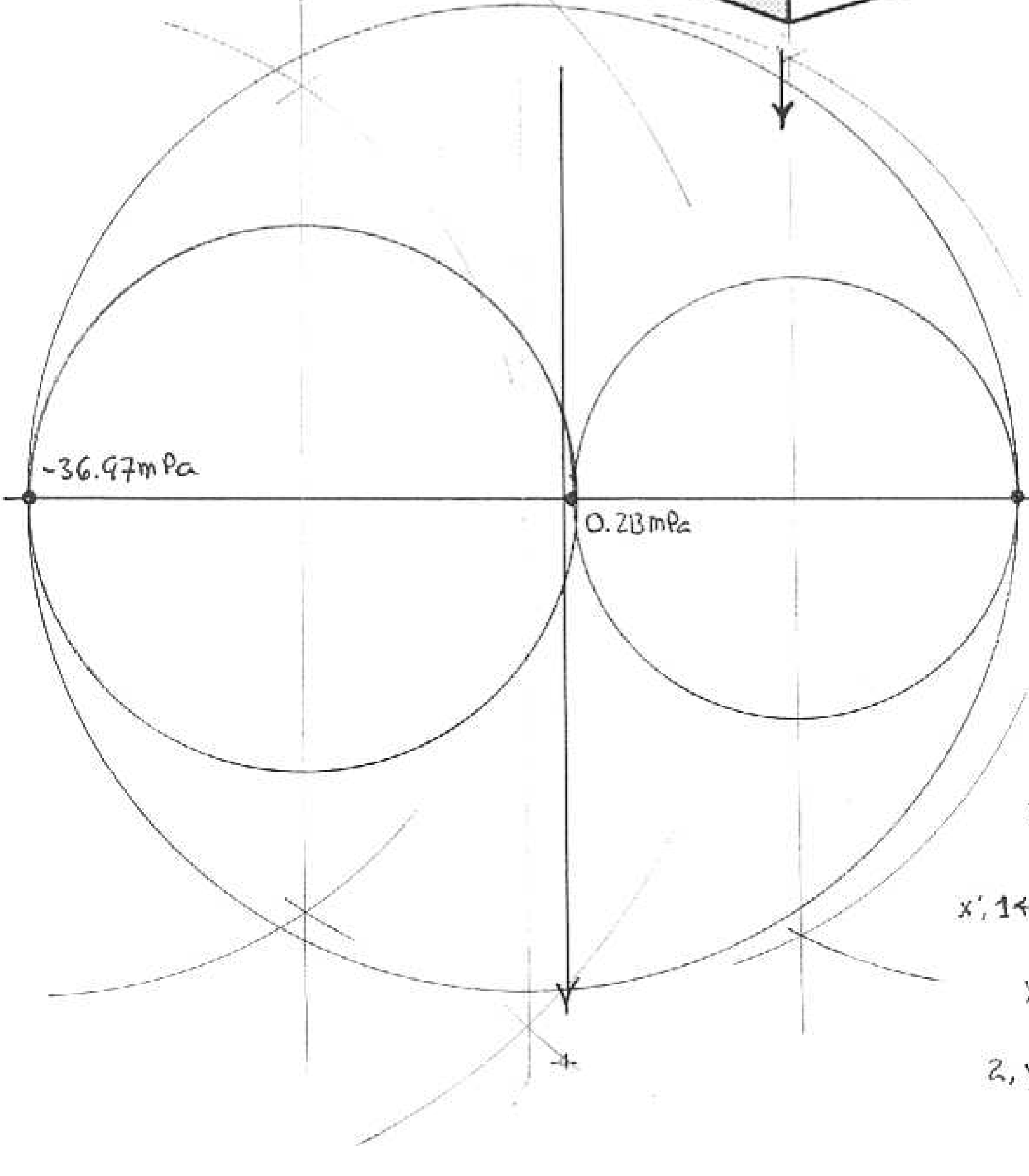
1c. What is the absolute maximum shear stress and the normal stresses that accompany it? Illustrate the stress cube in this state.

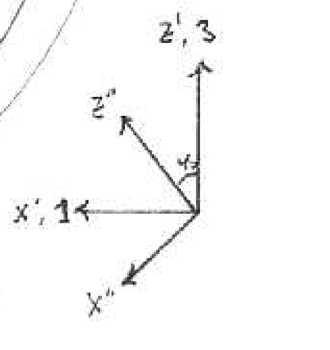
THE ABSOLUTE MAXIMUM SHEAR STRESS IS ACHIEVED BY FIRST ROTATING TO THE PRINCIPAL STATE OF STRESS AND THEN ROTATING THE CUBE 45° ABOUT THE 2-AXIS

$$[T_2] = \begin{bmatrix} \cos 45 & \cos 90 & \cos 135 \\ \cos 90 & \cos 0 & \cos 90 \\ \cos 45 & \cos 90 & \cos 45 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7071 & 0 & -0.7071 \\ 0 & 1 & 0 \\ 0.7071 & 0 & 0.7071 \end{bmatrix}$$







2, y' OUT OF PAPER

```
>> T2=[.7071 0 -.7071; 0 1 0; .7071 0 .7071]

T2 =

   0.7071         0   -0.7071
        0    1.0000         0
   0.7071         0    0.7071

>> T3=T2*T

T3 =

  -0.2524    0.2540   -0.9337
  -0.5855    0.7281    0.3563
  -0.7704   -0.6366    0.0350

>> MaxShear=T3*Sigma*T3'

MaxShear =

  -2.6064   -0.0000  -34.3604
  -0.0000    0.2130    0.0000
 -34.3604         0   -2.6064
```

IN MPa

(4a)

1d. What angles does the absolute shear stress make with the x, y, and z coordinates

τ_{max}: θ_x= 104.6 θ_y= 75.3 θ_z= 159.0 x''

140.4 129.5 88.0 z''

```
>> acos(T3)*180/pi

ans =

  104.6171   75.2875  159.0191
  125.8414   43.2698   69.1260
  140.3855  129.5418   87.9918
```

(5a)

1e. Determine the stress invariants for the states of stress for the original stress state, the principal stress state, and the state of stress that contains the maximum shear stress. What is the relationship between the invariants in these three states of stress?

THE INVARIANTS FOR THE THREE STRESS STATES ARE THE SAME (INVARIANT)

$I_1 = 5$ MPa

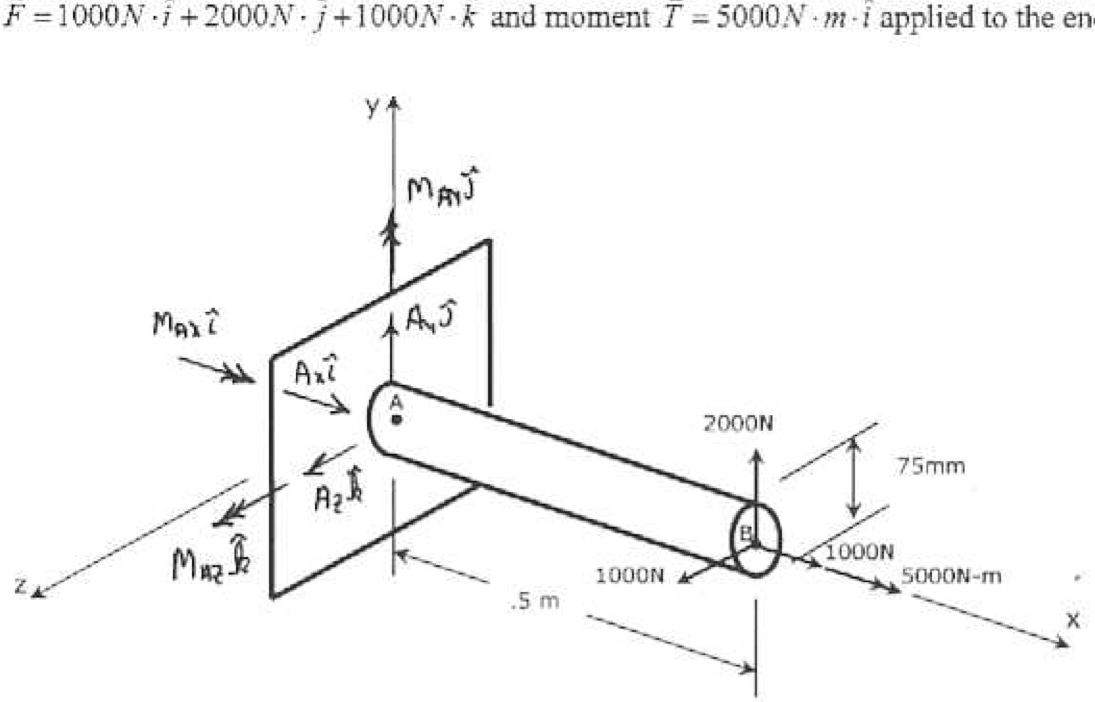
$I_2 = -1175$ MPa^2

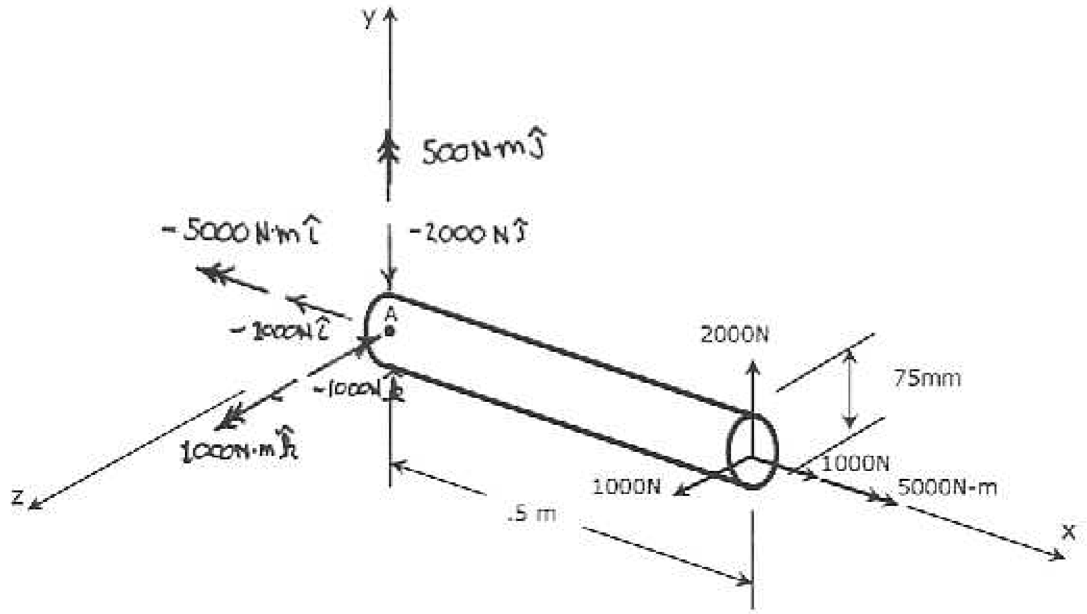
$I_3 = 250$ MPa^3

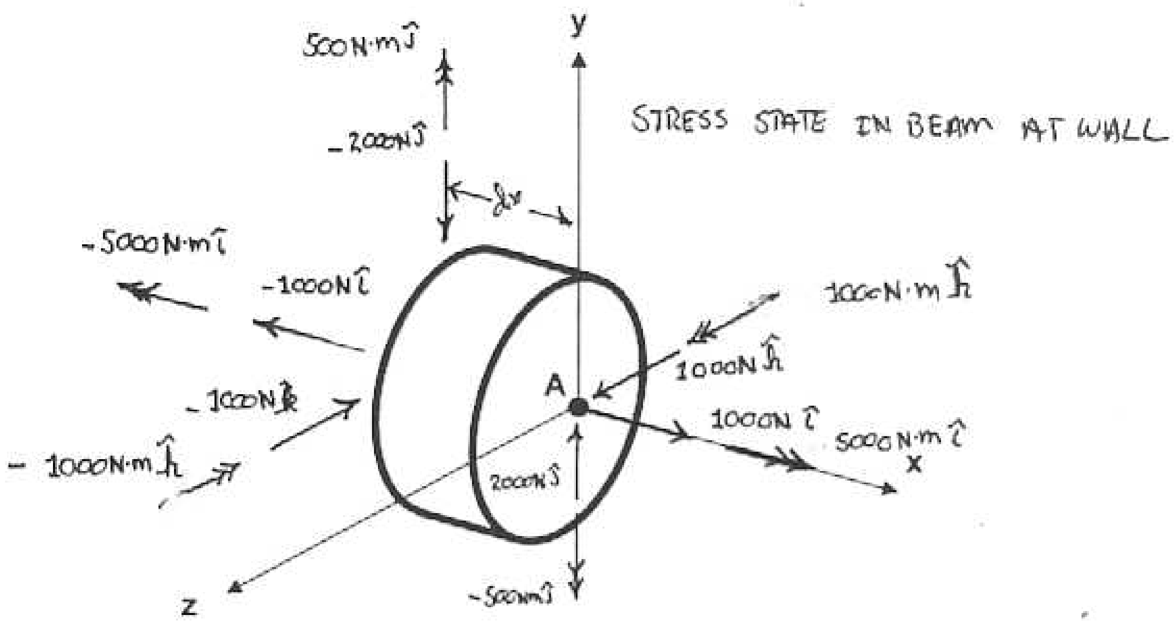
```
>> poly(Sigma)

ans =

  1.0e+003 *

   0.0010    0.0050   -1.1750    0.2500


>> poly(SigPrincipal)

ans =

  1.0e+003 *

   0.0010    0.0050   -1.1750    0.2500


>> poly(MaxShear)

ans =

  1.0e+003 *

   0.0010    0.0050   -1.1750    0.2500
```

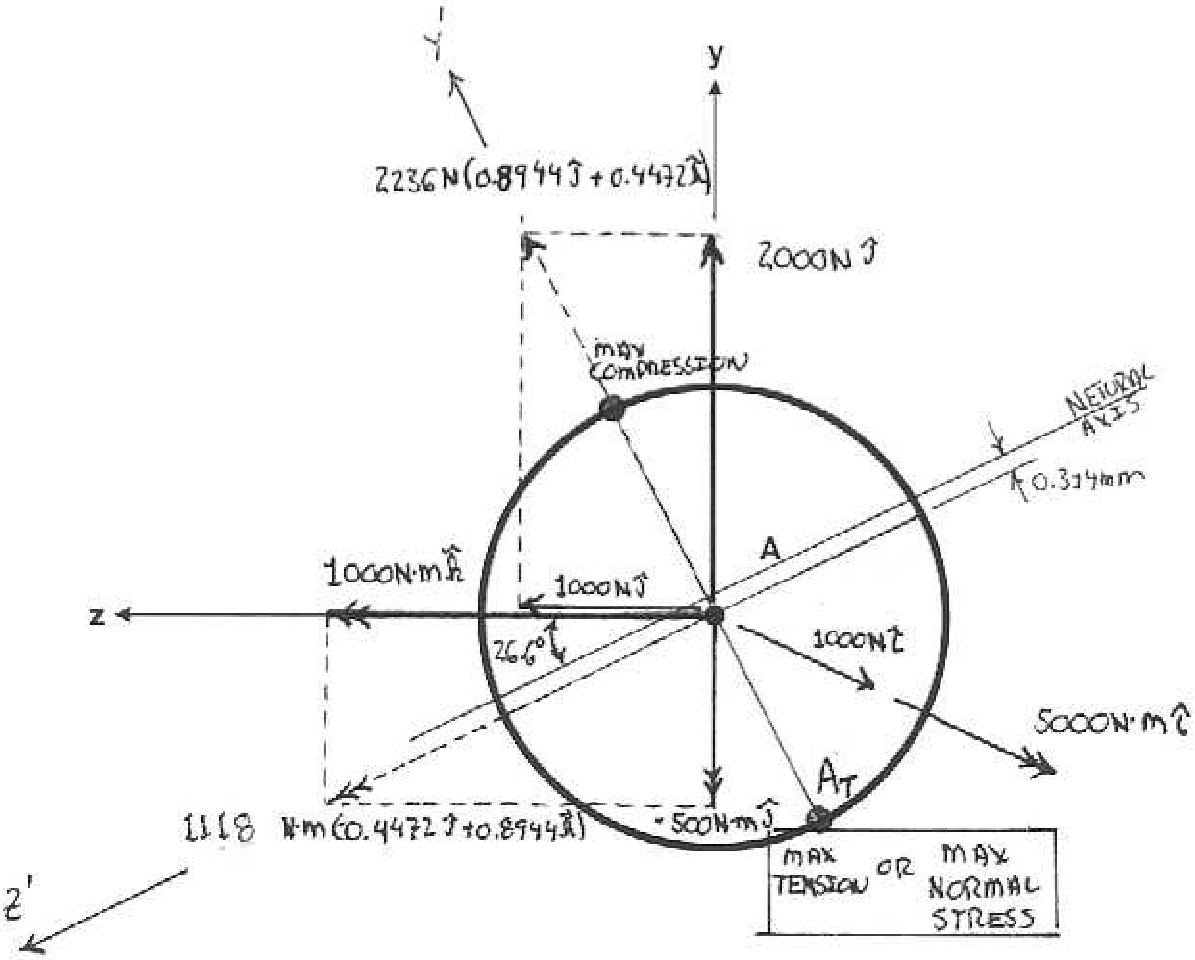
(6a)

PROBLEM 2: The circular beam shown in the figure below is fixed into the wall at A and has a force $\bar{F} = 1000N \cdot \hat{i} + 2000N \cdot \hat{j} + 1000N \cdot \hat{k}$ and moment $\bar{T} = 5000N \cdot m \cdot \hat{i}$ applied to the end at B.






STRESS STATE IN BEAM AT WALL
500N·m ĵ
-2000N ĵ
-5000N·m î
-1000N î
-1000N k̂
-1000N·m k̂
1000N·m k̂
1000N k̂
A
1000N î
5000N·m î
2000N ĵ
-500Nm ĵ
x
y
z


y'
y
2236N(0.8944ĵ+0.4472k̂)
2000N ĵ
MAX COMPRESSION
NEUTRAL AXIS
0.374mm
A
1000N·m k̂
1000N ĵ
z
26.6°
1000N î
5000N·m î
A_T
1118 N·m(0.4472ĵ+0.8944k̂)
-500N·m ĵ
MAX TENSION OR MAX NORMAL STRESS
z'

2a. Using the bottom figure on Page 5, complete the free body diagram of the circular beam. Using the top figure on page 6 illustrate the resultant internal forces and moments in the beam at the wall.

$$\Sigma F_x = 0 = A_x + 1000\text{N} \quad \Rightarrow \quad \underline{\underline{A_x = -1000\text{N}}}$$

$$\Sigma F_y = 0 = A_y + 2000\text{N} \quad \Rightarrow \quad \underline{\underline{A_y = -2000\text{N}}}$$

$$\Sigma F_z = 0 = A_z + 1000\text{N} \quad \Rightarrow \quad \underline{\underline{A_z = -1000\text{N}}}$$

$$\Sigma \vec{M}_{@A} = \vec{0} = \vec{M} + 5000\text{N}\cdot\text{m}\,\hat{\imath} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0.5\text{m} & 0 & 0 \\ 1000\text{N} & 2000\text{N} & 1000\text{N} \end{vmatrix}$$

$$= \vec{M}_A + 5000\text{N}\cdot\text{m}\,\hat{\imath} + [0\hat{\imath} - (0.5\text{m})(1000\text{N})\hat{\jmath} + (0.5\text{m})(2000\text{N})\hat{k}]$$

$$= M_{Ax}\hat{\imath} + M_{Ay}\hat{\jmath} + M_{Az}\hat{k} + 5000\text{N}\cdot\text{m}\,\hat{\imath} - 500\text{N}\cdot\text{m}\,\hat{\jmath} + 1000\text{N}\cdot\text{m}\,\hat{k}$$

$$\Rightarrow \underline{\underline{M_{Ax} = -5000\text{ N}\cdot\text{m}}}$$

$$\underline{\underline{M_{Ay} = 500\text{ N}\cdot\text{m}}}$$

$$\underline{\underline{M_{Az} = -1000\text{ N}\cdot\text{m}}}$$

2b. Using the lower diagram on Page 6 illustrate the natural axis, include the angle it makes with respect to the y or z axes, and identify the location where the normal stress will be maximum.

RESULTANT FORCE IN Y-Z PLANE

$$F_R = \sqrt{(1000N)^2 + (2000N)^2} = \underline{\underline{2236N}}$$

$$\hat{e}_{RF} = \frac{2000N}{2236N}\hat{j} + \frac{1000N}{2236N}\hat{k} = \underline{\underline{0.8944\hat{j} + 0.4472\hat{k}}}$$

$$\vec{F}_R = \underline{\underline{2236N(0.8944\hat{j} + 0.4472\hat{k}}}$$

RESULTANT MOMENT IN THE Y-Z PLANE

$$M_R = \sqrt{(-500N\cdot m)^2 + (1000N\cdot m)^2} = \underline{\underline{1118\,N\cdot m}}$$

$$\hat{e}_{Rm} = -\frac{500N\cdot m}{1118N\cdot m}\hat{j} + \frac{1000N\cdot m}{1118N\cdot m}\hat{k} = \underline{\underline{-0.4472\hat{j} + 0.8944\hat{k}}}$$

$$\vec{M}_R = \underline{\underline{1118N\cdot m\,(-0.4472\hat{j} + 0.8944\hat{k})}}$$

LOCATION OF THE NETURAL AXIS

$$\sigma_x = 0 = \frac{1000N}{\pi\left(\frac{0.075m}{2}\right)^2} - \frac{(1118N\cdot m)\cdot r}{\frac{\pi(0.075m)^4}{64}}$$

$$r = \frac{1000N}{\left(\frac{0.075m}{2}\right)^2}\cdot\frac{(0.075m)^4}{64\cdot(1118N\cdot m)} = \underline{\underline{0.0003145m}}$$

ANGLE NETURAL AXIS MAKES WITH Z AXIS

$$\theta_z = \cos^{-1} 0.8944 = \underline{\underline{26.57^\circ}}$$

2c. Draw the complete stress cube for the location of the maximum normal stress on this surface.

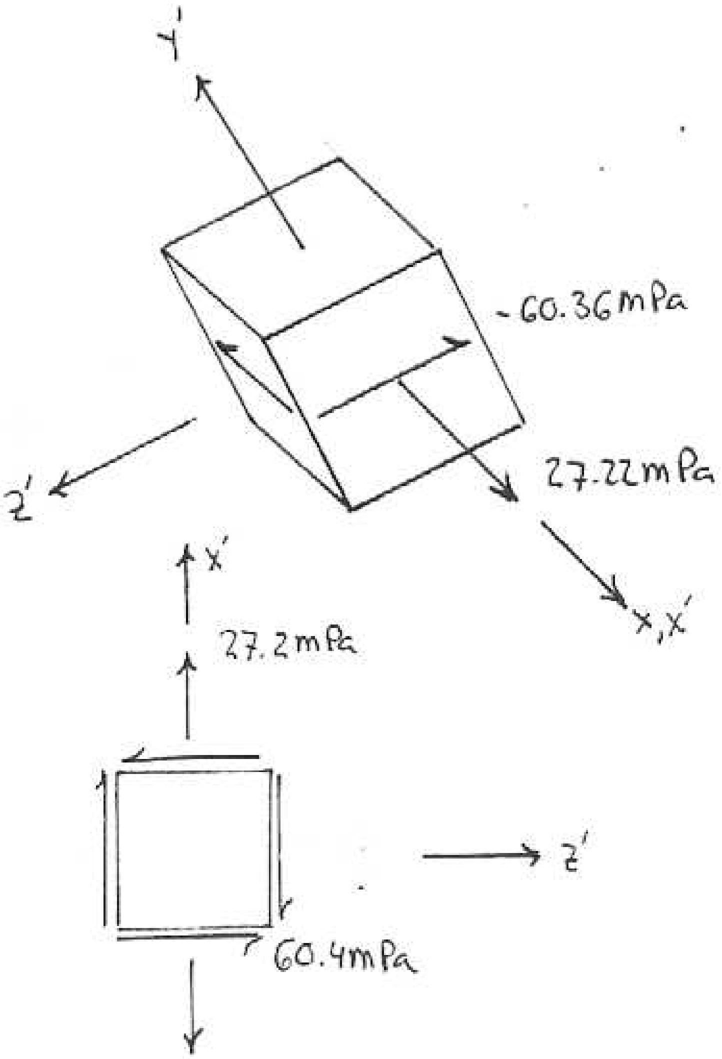
NORMAL STRESS AT A_T

$$\sigma_x = \frac{1000N}{\pi\left(\frac{0.075m}{2}\right)^2} + \frac{(1118N\cdot m)\cdot\left(\frac{0.075m}{2}\right)}{\frac{\pi\cdot(0.075m)^4}{64}}$$

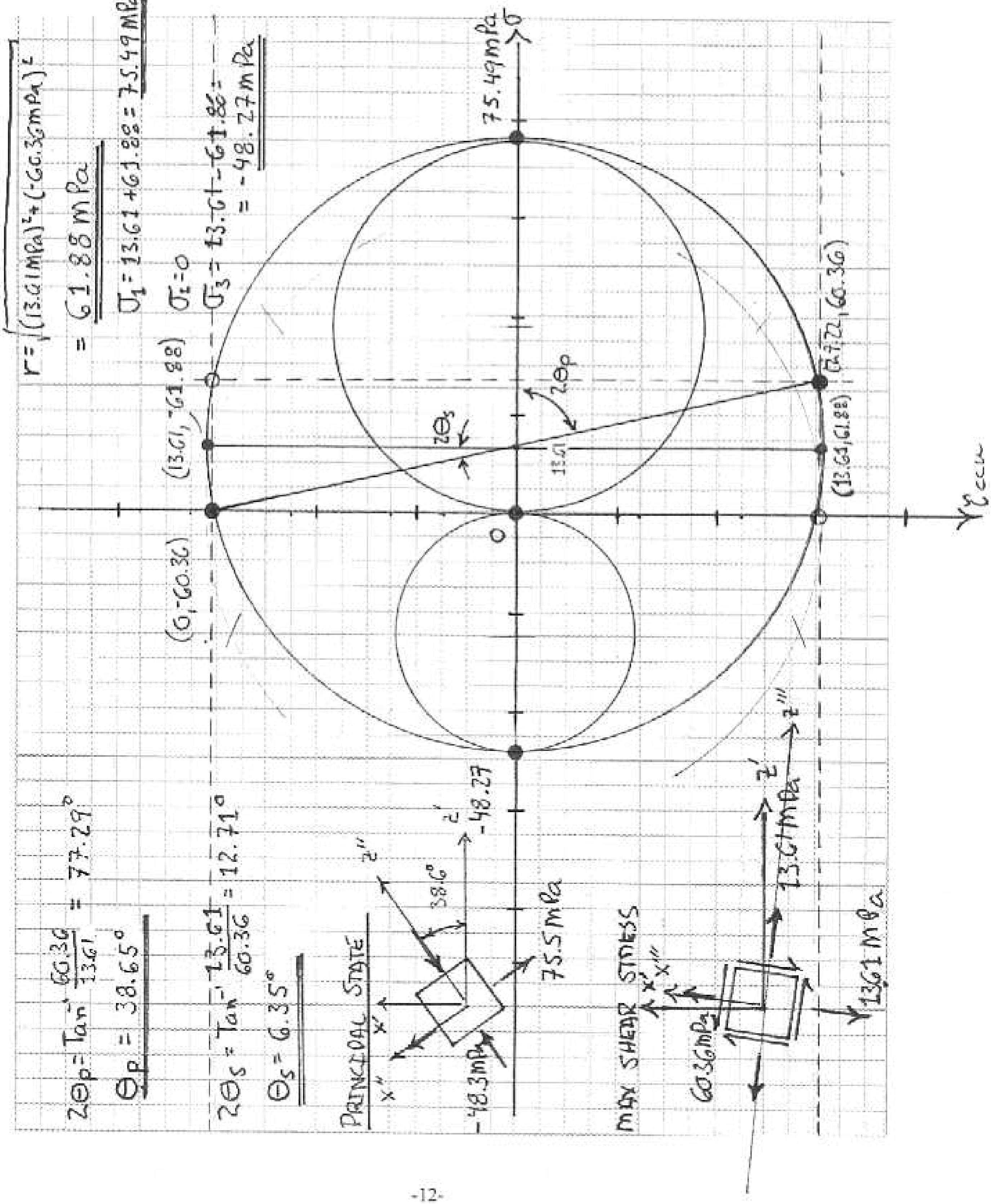
$$= 226.4(10^3)Pa + 27.00(10^6)Pa = 27.22(10^6)Pa = \underline{\underline{27.22MPa}}$$

THE SHEAR STRESS AT A_T IS ONLY DUE TO THE TORQUE SINCE F_R IS PERPENDICULAR TO THE NEUTRAL AXIS

$$\tau_{y'z'} = \frac{-5000N\cdot m\left(\frac{0.075m}{2}\right)}{\frac{\pi}{32}(0.075m)^4} = -60.36(10^6)Pa = \underline{\underline{-60.36MPa}}$$



2d. Using the state of stress illustrated in the previous part, draw Mohr's circle and determine the principle stresses, maximum shear stress, normal stress corresponding to the maximum shear and the angles these states of stress make with the original states of stress. Draw a stress cube that illustrates both states of stress relative to the original state of stress.



NAME: ______________________________

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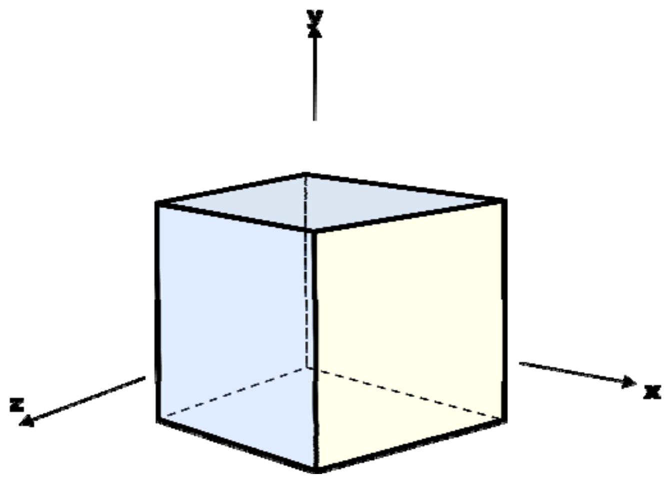

1b. What angles (in degrees) do each of the principal stresses make with the x, y, and z axes?

σ_1: θ_{1x}= θ_{1y}= θ_{1z}=

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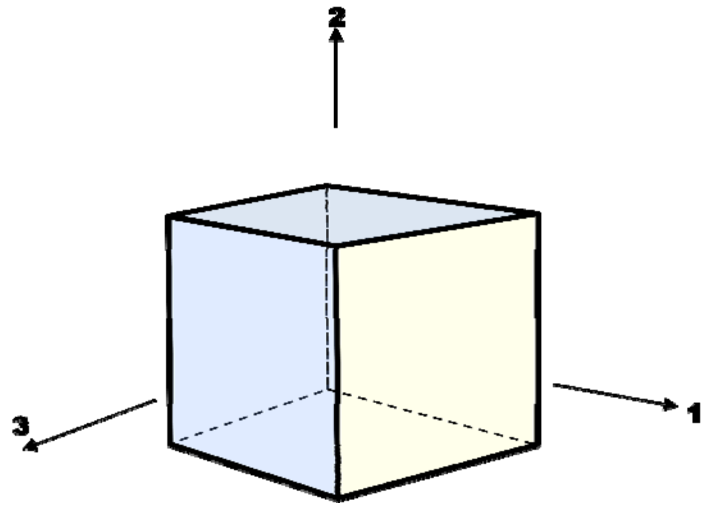

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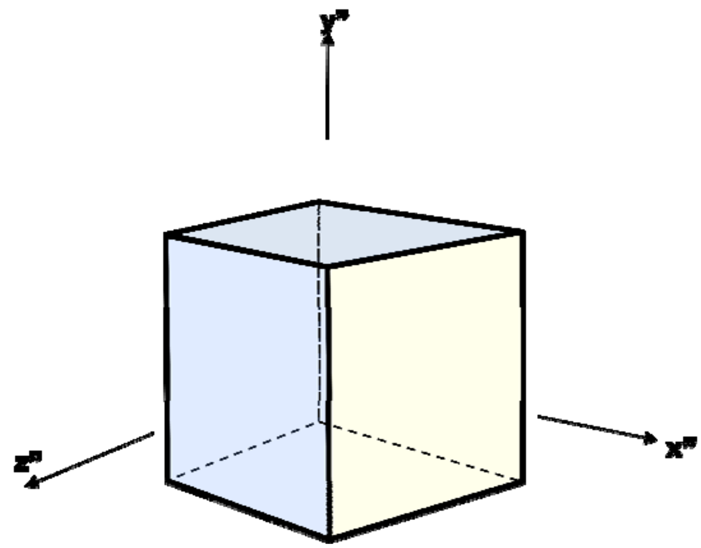

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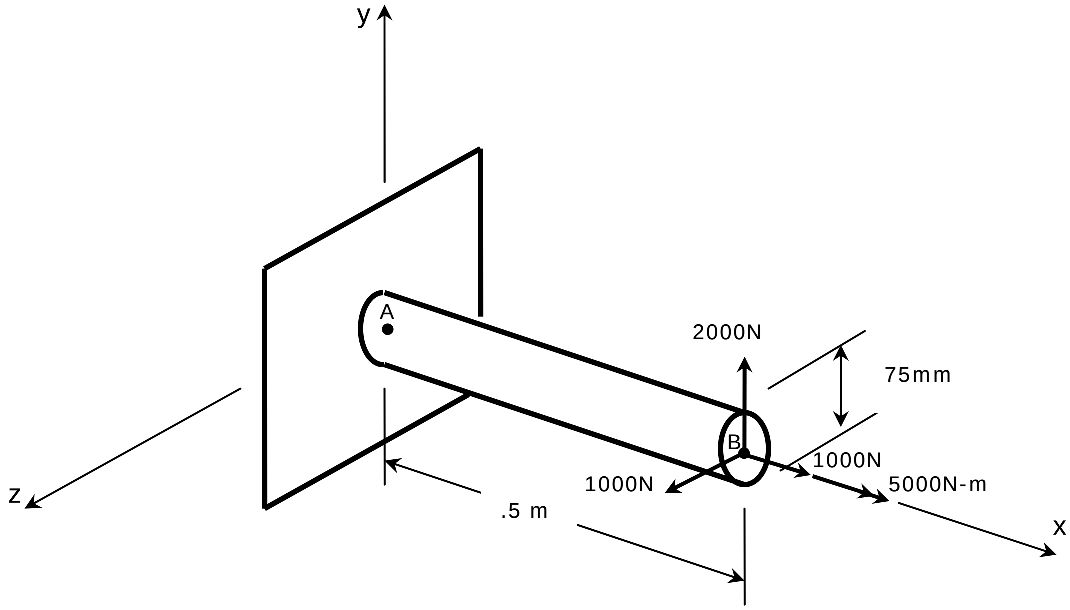
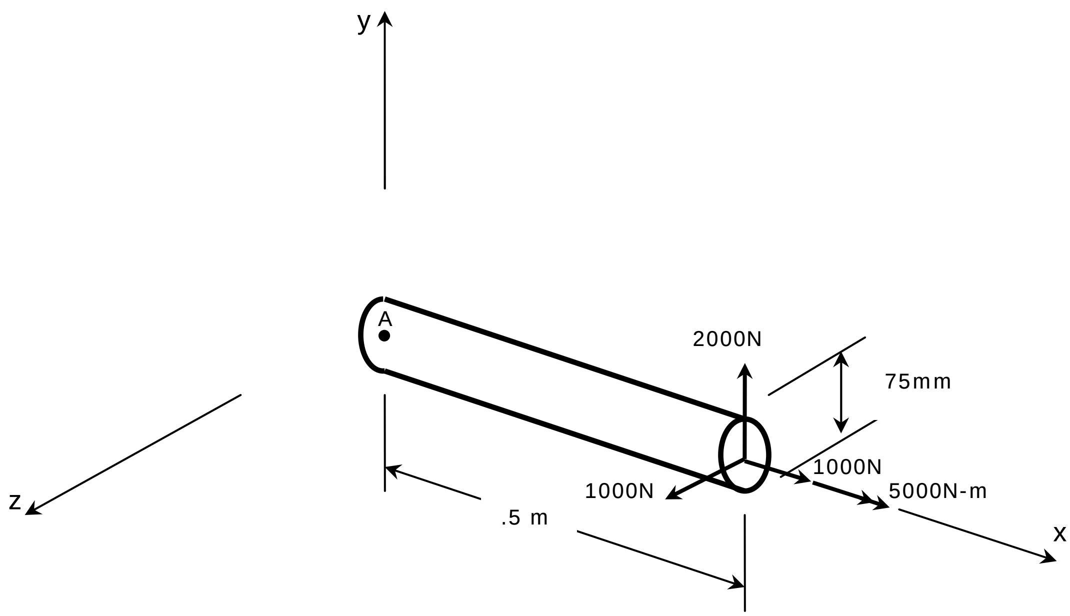
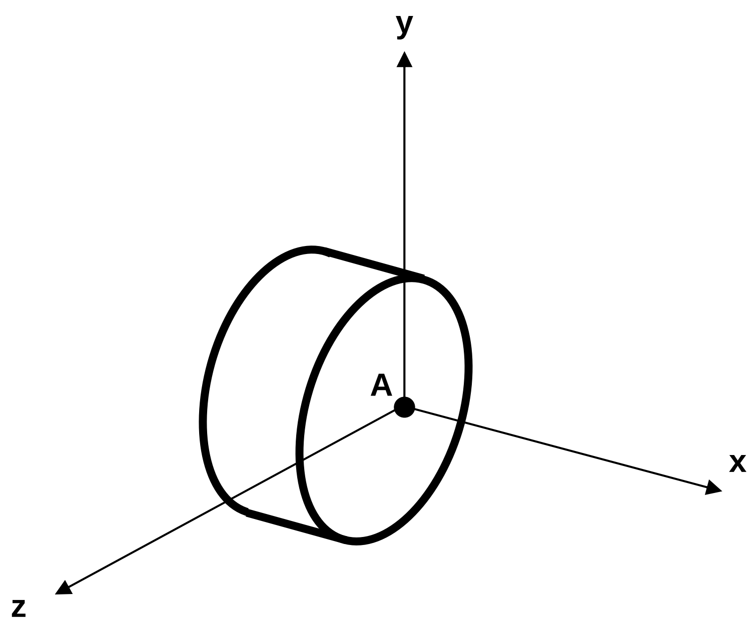
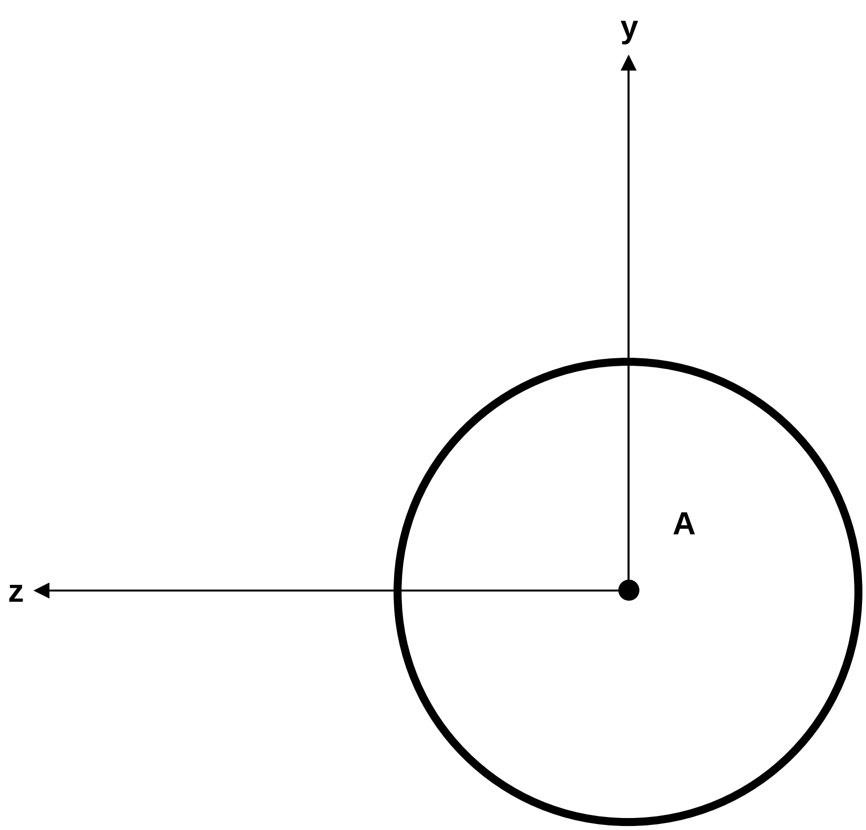





y
A
x
z


y
A
z

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