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1. I **must** work alone in writing out the solutions to the problems in this exam.
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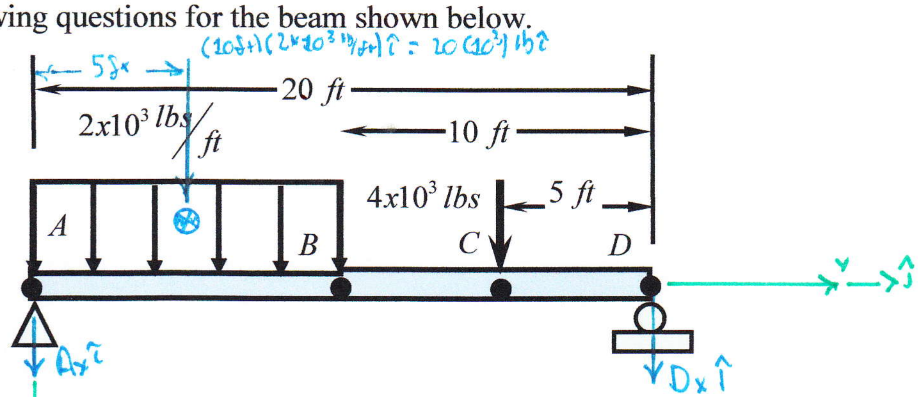
3. I **cannot** use any electronic resources to assist me in the solution to the questions on this exam except for my calculator to only performing appropriate calculations on the exam.
4. I **can** use one page - single sided - of notes during the exam. This one page of notes **cannot** contain any solutions to problems. **I must staple this page to the back of my exam at the end of the exam.**

Signature: ______________________

Print Name: SOLUTION

Exam Date: 13 MAY 2016

PROBLEM 1: Answer the following questions for the beam shown below.



1a. Determine the reactions at A and D, and using the provided beam bending tables determine the deflections at B andC. Identify and Illustrate the beams used in the solution on the figure provided below.

$$\Sigma F_x = 0 = A_x + 20(10^3)\,lb + 4(10^3)\,lb + D_x \Rightarrow A_x + D_x = -24(10^3)\,lb$$

$$\Sigma M_{z@A} = 0 = -(5ft)(20\times10^3)\,lb - (15ft)\cdot(4\times10^3)\,lb - 20ft\cdot D_x$$

$$\Rightarrow D_x = -\frac{(5ft)\cdot(20\times10^3 lb) + (15ft)\cdot(4\times10^3 lb)}{20ft} = \underline{\underline{-8(10^3)\,lb}}$$

$$\Rightarrow A_x - 8\times10^3\,lb = -24\times10^3\,lb \quad \Rightarrow \quad \underline{\underline{A_x = -16\times10^3\,lb}}$$

DEFLECTION AT B

$$\delta_B = \frac{-5\cdot(2\times10^3\frac{lb}{ft})\cdot(20ft)^4}{768\,EI} - \frac{(4\times10^3 lb)\cdot 5ft\cdot 10ft}{6\cdot 20ft\cdot EI}\cdot\left[(20ft)^2-(5ft)^2-(10ft)^2\right] = \frac{-2.083(10^6)\,lb\cdot ft^3}{EI} - \frac{458(10^3)\,lb\cdot ft^3}{EI}$$

$$= \frac{-2.542(10^6)\,lb\cdot ft^3}{EI} \Rightarrow \text{FOR THE ABOVE COORDINATE SYSTEM} \Rightarrow \boxed{u_B = \frac{2.542(10^6)\,lb\cdot ft^3}{EI}}$$

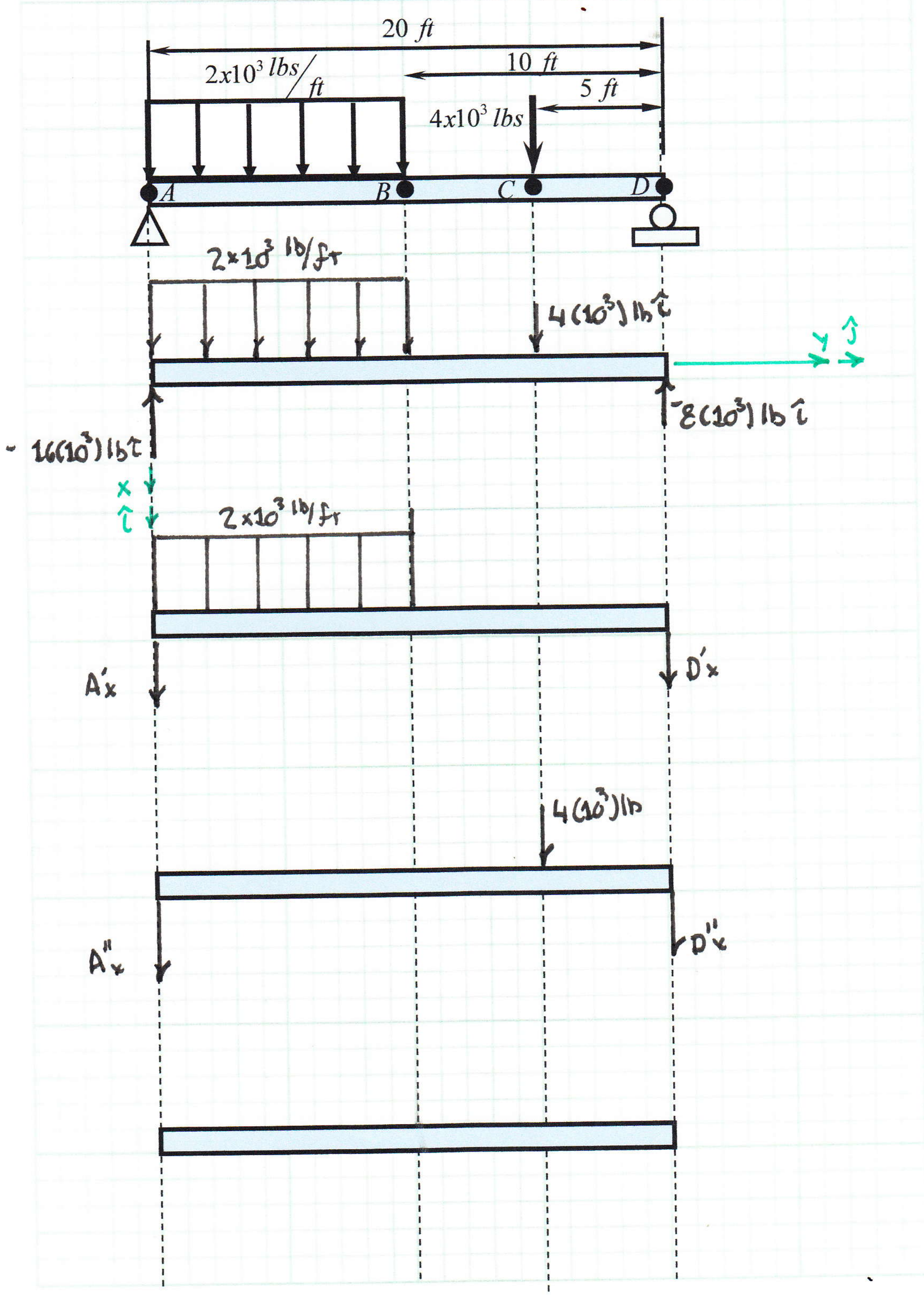
DEFLECTION AT C

$$\delta_C = -\frac{(2\times10^3\frac{lb}{ft})\cdot 20ft}{384\cdot EI}\cdot\left[8\cdot(15ft)^3 - 24\cdot 20ft\cdot(15ft)^2 + 17\cdot(20ft)^2\cdot(15ft) - (20ft)^3\right]$$

$$- \frac{4\times10^3 lb\cdot 5ft\cdot 15ft}{6\cdot 20ft\cdot EI}\cdot\left[(20ft)^2-(5ft)^2-(15ft)^2\right]$$

$$= \frac{-1.354(10^6)\,lb\cdot ft^3}{EI} - \frac{375.0(10^3)\,lb\cdot ft^3}{EI} = -\frac{1.729(10^6)\,lb\cdot ft^3}{EI}$$

$$\Rightarrow \text{FOR THE ABOVE COORDINATE SYSTEM} \Rightarrow \boxed{u_C = \frac{1.729(10^6)\,lb\cdot ft^3}{EI}}$$


20 ft
10 ft
5 ft
2x10^3 lbs/ft
4x10^3 lbs
A
B
C
D
2×10^3 lb/ft
4(10^3) lb î
y ĵ
-8(10^3) lb î
-16(10^3) lb î
x î
2×10^3 lb/ft
A'x
D'x
4(10^3) lb
A''x
D''x

1b. Write a general expression for the load, shear, bending moment, curvature, and deflection of the beam using singularity functions. Make sure to calculate all constants.

$$q(y) = -16.0\,(10^3)\,\text{lb}\,\langle y-0\rangle_{-1} + 2\cdot 10^3\,\tfrac{\text{lb}}{\text{ft}}\langle y-0\rangle^0 - 2\cdot 10^3\,\tfrac{\text{lb}}{\text{ft}}\langle y-10\,\text{ft}\rangle^0$$
$$+\,4\cdot 10^3\,\text{lb}\,\langle y-15\,\text{ft}\rangle_{-1} - 8\cdot 10^3\,\text{lb}\,\langle y-20\,\text{ft}\rangle_{-1}$$

$$V(y) = -\int q(y)\,dy = +16.0\,(10^3)\,\text{lb}\,\langle y-0\rangle^0 - 2\cdot 10^3\,\tfrac{\text{lb}}{\text{ft}}\langle y-0\rangle^1 + 2\cdot 10^3\,\tfrac{\text{lb}}{\text{ft}}\langle y-10\,\text{ft}\rangle^1$$
$$-\,4\cdot 10^3\,\text{lb}\,\langle y-15\,\text{ft}\rangle^0 + 8\cdot 10^3\,\text{lb}\,\langle y-20\,\text{ft}\rangle^0$$

$$M(y) = \int V(y)\,dy = 16.0\,(10^3)\,\text{lb}\,\langle y-0\rangle^1 - \frac{2\cdot 10^3\,\frac{\text{lb}}{\text{ft}}}{2}\langle y-0\rangle^2 + \frac{2\cdot 10^3\,\frac{\text{lb}}{\text{ft}}}{2}\langle y-10\,\text{ft}\rangle^2$$
$$-\,4\cdot 10^3\,\text{lb}\,\langle y-15\,\text{ft}\rangle^1 + 8\cdot 10^3\,\text{lb}\,\langle y-20\,\text{ft}\rangle^1$$

$$\theta(y) = -\int \frac{M}{EI}\,dy = -\frac{16.0\cdot 10^3\,\text{lb}}{2\cdot EI}\langle y-0\rangle^2 + \frac{2\cdot 10^3\,\frac{\text{lb}}{\text{ft}}}{6\,EI}\langle y-0\rangle^3 - \frac{2\cdot 10^3\,\frac{\text{lb}}{\text{ft}}}{6\,EI}\langle y-10\,\text{ft}\rangle^3$$
$$+\,\frac{4\cdot 10^3\,\text{lb}}{2\cdot EI}\langle y-15\,\text{ft}\rangle^2 - \frac{8\cdot 10^3\,\text{lb}}{2\cdot EI}\langle y-20\,\text{ft}\rangle^2 + \cancel{C_1}$$

$$\frac{437.5\,(10^3)\,\text{lb}\cdot\text{ft}^2}{EI}$$

$$u(y) = \int \theta(y)\,dy = -\frac{16\cdot 10^3\,\text{lb}}{6\,EI}\langle y-0\rangle^3 + \frac{2\cdot 10^3\,\frac{\text{lb}}{\text{ft}}}{24\,EI}\langle y-0\rangle^4 - \frac{2\cdot 10^3\,\frac{\text{lb}}{\text{ft}}}{24\,EI}\langle y-10\,\text{ft}\rangle^4$$
$$+\,\frac{4\cdot 10^3\,\text{lb}}{6\,EI}\langle y-15\,\text{ft}\rangle^3 - \frac{8\cdot 10^3\,\text{lb}}{6\cdot EI}\langle y-20\,\text{ft}\rangle^3 + \cancel{C_1}\cdot y + \cancel{C_2}$$

$$\frac{437.5\,(10^3)\,\text{lb}\cdot\text{ft}^2}{EI} \qquad 0$$

USING THE BOUNDARY CONDITION $u(0)=0$

$$u(0) = 0 = -\frac{16\cdot 10^3\,\text{lb}}{6EI}\cdot 0 + \frac{2\cdot 10^3\,\frac{\text{lb}}{\text{ft}}}{24\,EI}\cdot 0 + C_1\cdot 0 + C_2 \implies \boxed{C_2 = 0}$$

USING THE BOUNDARY CONDITION $u(20\,\text{ft})=0$

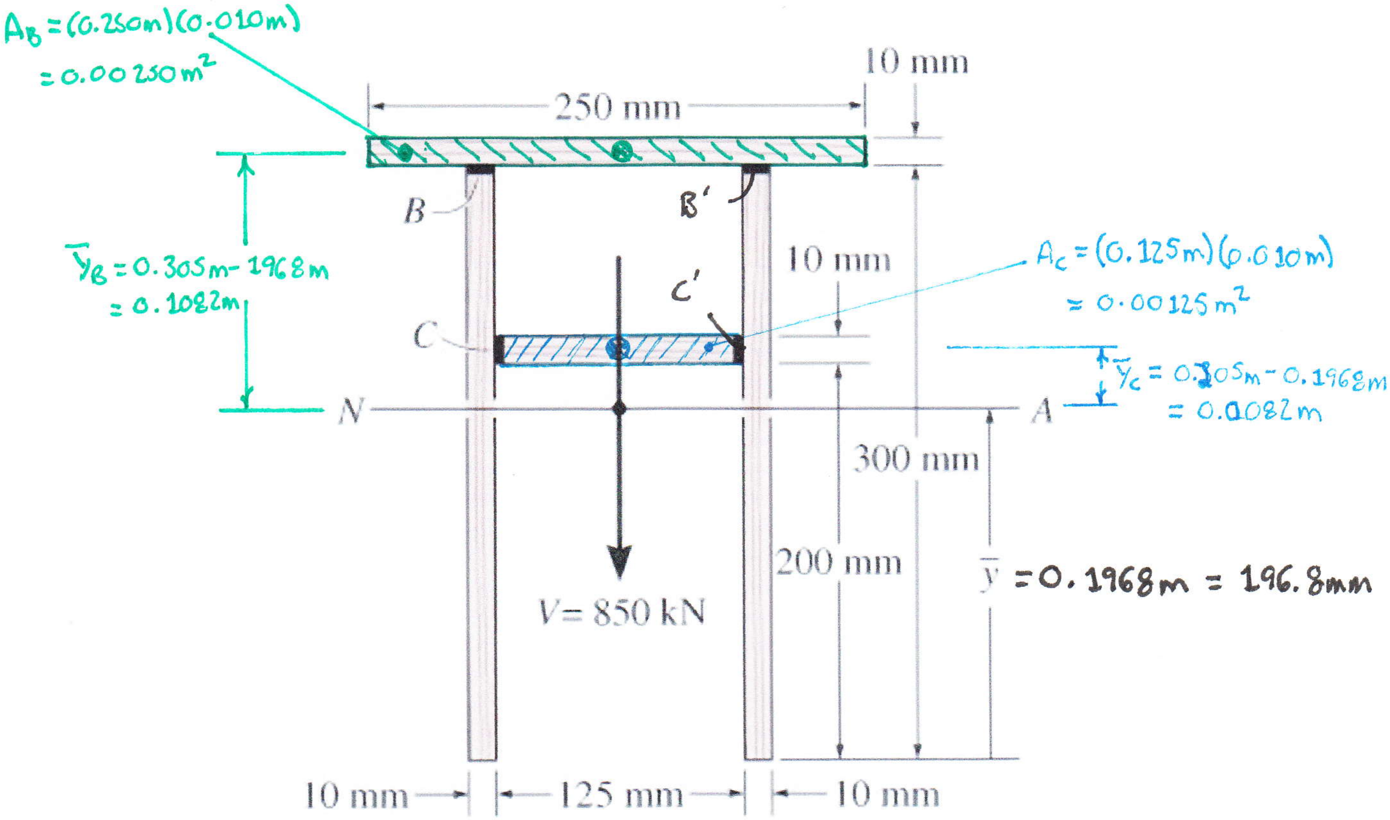
$$u(20\,\text{ft}) = 0 = -\frac{16\cdot 10^3\,\text{lb}}{6\,EI}\cdot(20\,\text{ft})^3 + \frac{2\cdot 10^3\,\frac{\text{lb}}{\text{ft}}}{24\,EI}(20\,\text{ft})^4 - \frac{2\cdot 10^3\,\frac{\text{lb}}{\text{ft}}}{24\,EI}(10\,\text{ft})^4$$
$$+\,\frac{4\cdot 10^3\,\text{lb}}{6EI}(5\,\text{ft})^3 - \frac{8\cdot 10^3\,\text{lb}}{6\,EI}\cdot(0)^3 + 20\,\text{ft}\cdot C_1$$

$$\implies \boxed{C_1 = \frac{437.5\cdot 10^3\,\text{lb}\cdot\text{ft}^2}{EI}}$$

$C_1 =$

PROBLEM 2: The beam below is constructed from four boards nailed together at B and C. This board is subjected to a shear of V=850 kN as shown.

$$\bar{y} = 0.1968\ m$$
$$I = 87.52(10^{-6})\ m^4$$



2b: Calculate the shear flow through B and C.

For B & B'

$$q_B = \frac{V \cdot Q_B}{I} = \frac{850(10^3)N \cdot 0.1082m \cdot 0.0025m^2}{87.52(10^{-6})\,m^4}$$

$$= \boxed{2.627(10^6)\frac{N}{m}}$$

For C & C'

$$q_C = \frac{V Q_C}{I} = \frac{850(10^3)N \cdot 0.0082m \cdot 0.00125m^2}{87.52(10^{-6})m^4}$$

$$= \boxed{99.55(10^3)\frac{N}{m}}$$

2c: If each nail can carry 5 kN, what nail spacing is needed at B and C.

$F_{NAIL} = S \cdot q^*$

$F_{NAIL} = 5\ kN$

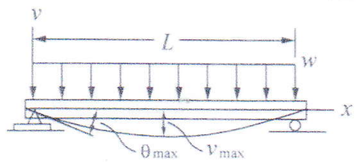
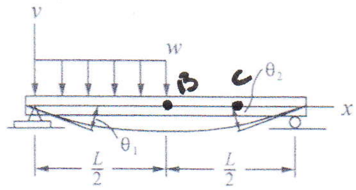
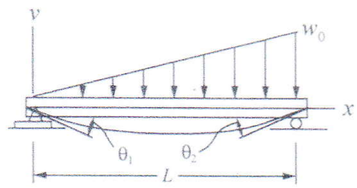
$S \equiv$ SPACING

$q^* \equiv$ shear flow THROUGH SECTION INTERFACE

- IN THIS PROBLEM, FOR BOTH CASES $q/2$

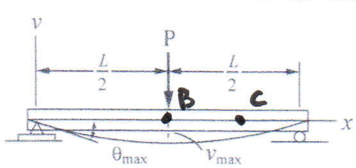
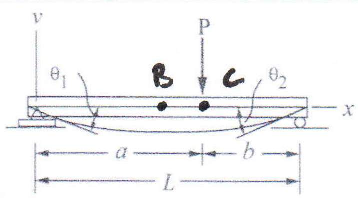
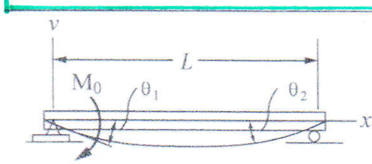
$$S_B = \frac{F_{NAIL}}{q_B/2} = \frac{2 \cdot 5(10^3)\,N}{2.627(10^6)\,N/m} = 3.8(10^{-3})\,m = \boxed{3.8mm}$$

$$S_C = \frac{F_{NAIL}}{q_C/2} = \frac{2 \cdot 5(10^3)\,N}{99.55(10^3)\,N/m} = 0.1005\,m = \boxed{100mm}$$

	$\theta_{max} = \frac{-wL^3}{24EI}$	$v_{max} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI}(x^3 - 3Lx^2 + L^3)$
	$\theta_1 = \frac{-3wL^3}{128EI}$ $\theta_2 = \frac{7wL^3}{384EI}$	$v\big\vert_{x=L/2} = \frac{-5wL^4}{768EI}$ $v_{max} = -0.006563\frac{wL^4}{EI}$ at $x = 0.4598L$	$v = \frac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \frac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \frac{-7w_0L^3}{360EI}$ $\theta_2 = \frac{w_0L^3}{45EI}$	$v_{max} = -0.00652\frac{w_0L^4}{EI}$ at $x = 0.5193$	$v = \frac{-w_0x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

Hibbeler, R.C., *Mechanics of Materials*, 4th ed., Prentice Hall, 2000.

Simply Supported Beam Slopes and Deflections

BEAM	SLOPE	DEFLECTION	ELASTIC CURVE
	$\theta_{max} = \frac{-PL^2}{16EI}$	$v_{max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \frac{-Pab(L+b)}{6EIL}$ $\theta_2 = \frac{Pab(L+a)}{6EIL}$	$v\big\vert_{x=a} = \frac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \frac{-M_0L}{3EI}$ $\theta_2 = \frac{M_0L}{6EI}$	$v_{max} = \frac{-M_0L^2}{\sqrt{243}EI}$	$v = \frac{-M_0x}{6EIL}(x^2 - 3Lx + 2L^2)$

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Exam Date: ____

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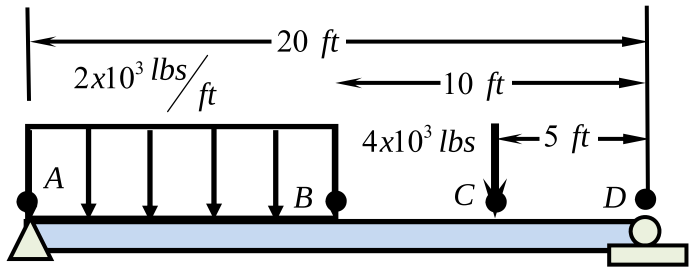

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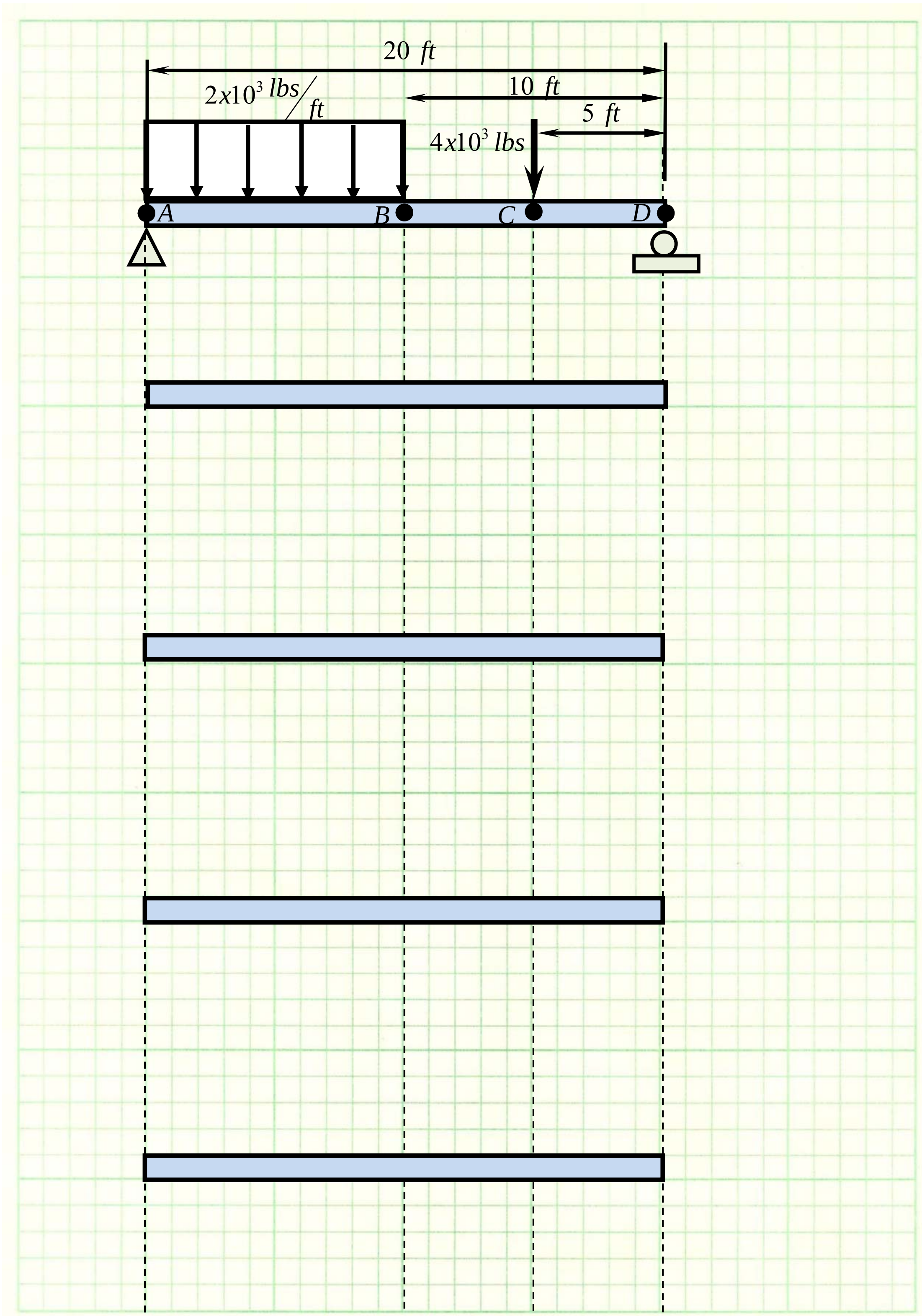
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2x10³ lbs/ft
10 ft
5 ft
4x10³ lbs
A
B
C
D

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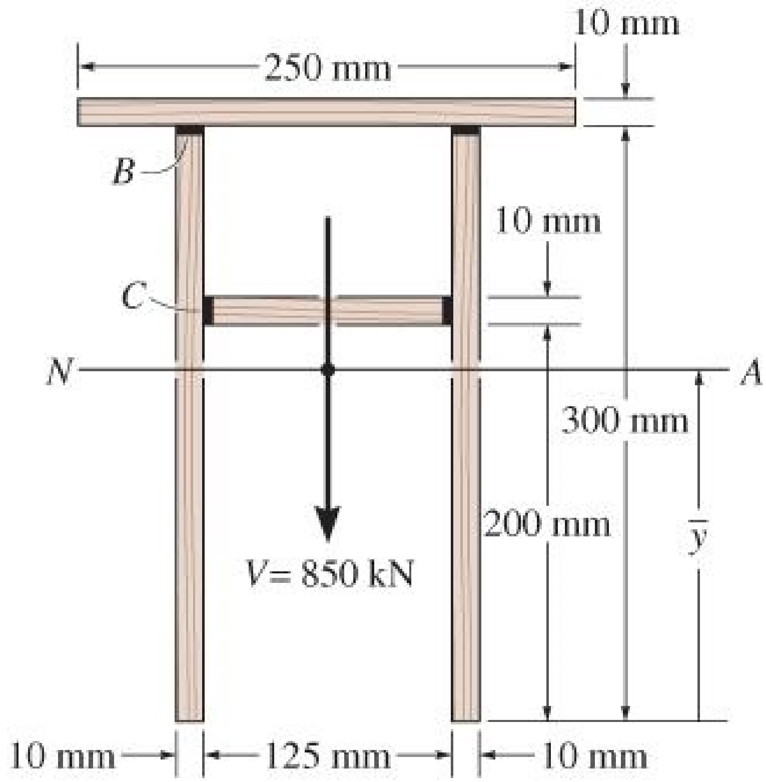
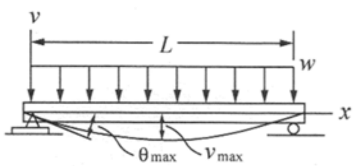
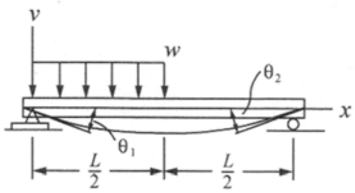
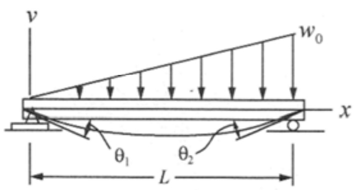


2b: Calculate the shear flow through B and C.

2c: If each nail can carry 5 kN, what nail spacing is needed at B and C.

	$\theta_{max} = \dfrac{-wL^3}{24EI}$	$v_{max} = \dfrac{-5wL^4}{384EI}$	$v = \dfrac{-wx}{24EI}(x^3 - 3Lx^2 + L^3)$
	$\theta_1 = \dfrac{-3wL^3}{128EI}$ $\theta_2 = \dfrac{7wL^3}{384EI}$	$v\Big\rvert_{x=L/2} = \dfrac{-5wL^4}{768EI}$ $v_{max} = -0.006563\dfrac{wL^4}{EI}$ at $x = 0.4598L$	$v = \dfrac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \dfrac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \dfrac{-7w_0L^3}{360EI}$ $\theta_2 = \dfrac{w_0L^3}{45EI}$	$v_{max} = -0.00652\dfrac{w_0L^4}{EI}$ at $x = 0.5193$	$v = \dfrac{-w_0x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

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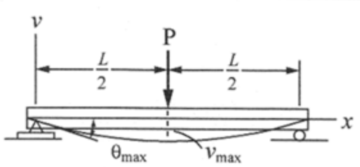
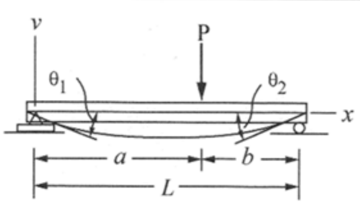
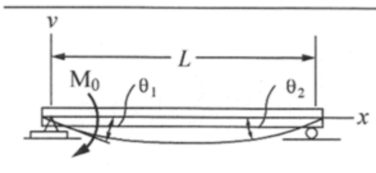
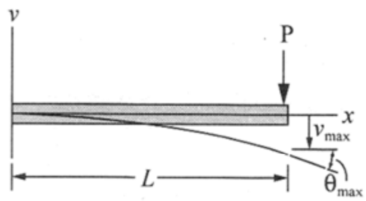
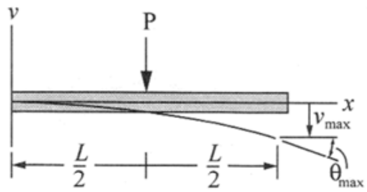
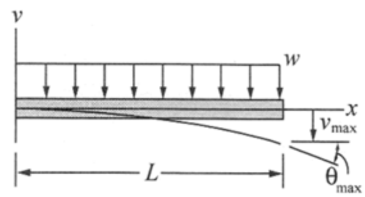
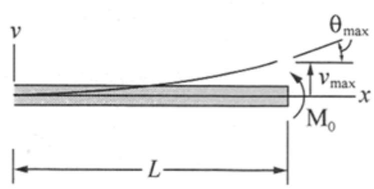
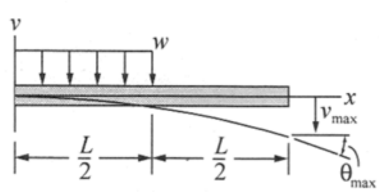
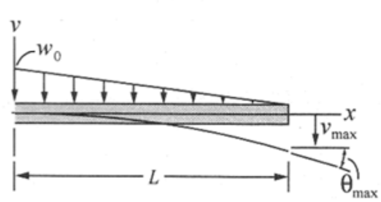
Simply Supported Beam Slopes and Deflections

BEAM	SLOPE	DEFLECTION	ELASTIC CURVE
	$\theta_{max} = \dfrac{-PL^2}{16EI}$	$v_{max} = \dfrac{-PL^3}{48EI}$	$v = \dfrac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \dfrac{-Pab(L+b)}{6EIL}$ $\theta_2 = \dfrac{Pab(L+a)}{6EIL}$	$v\Big\rvert_{x=a} = \dfrac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \dfrac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \dfrac{-M_0L}{3EI}$ $\theta_2 = \dfrac{M_0L}{6EI}$	$v_{max} = \dfrac{-M_0L^2}{\sqrt{243}EI}$	$v = \dfrac{-M_0x}{6EIL}(x^2 - 3Lx + 2L^2)$

Cantilevered Beam Slopes and Deflections

BEAM	SLOPE	DEFLECTION	ELASTIC CURVE
	$\theta_{max} = \dfrac{-PL^2}{2EI}$	$v_{max} = \dfrac{-PL^3}{3EI}$	$v = \dfrac{-Px^2}{6EI}(3L-x)$
	$\theta_{max} = \dfrac{-PL^2}{8EI}$	$v_{max} = \dfrac{-5PL^3}{48EI}$	$v = \dfrac{-Px^2}{6EI}\left(\frac{3}{2}L-x\right) \quad 0 \le x \le L/2$ $v = \dfrac{-PL^2}{24EI}\left(3x-\frac{1}{2}L\right) \quad L/2 \le x \le L$
	$\theta_{max} = \dfrac{-wL^3}{6EI}$	$v_{max} = \dfrac{-wL^4}{8EI}$	$v = \dfrac{-wx^2}{24EI}(x^2-4Lx+6L^2)$
	$\theta_{max} = \dfrac{M_0L}{EI}$	$v_{max} = \dfrac{M_0L^2}{2EI}$	$v = \dfrac{M_0x^2}{2EI}$
	$\theta_{max} = \dfrac{-wL^3}{48EI}$	$v_{max} = \dfrac{-7wL^4}{384EI}$	$v = \dfrac{-wx^2}{24EI}\left(x^2-2Lx+\frac{3}{2}L^2\right)$ $0 \le x \le L/2$ $v = \dfrac{-wL^3}{192EI}(4x-L/2)$ $L/2 \le x \le L$
	$\theta_{max} = \dfrac{-w_0L^3}{24EI}$	$v_{max} = \dfrac{-w_0L^4}{30EI}$	$v = \dfrac{-w_0x^2}{120EIL}(10L^3-10L^2x+5Lx^2-x^3)$

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