HOMEWORK SOLUTION
ESC 30: MECHANICS OF DEFORMABLE SOLIDS
ASSIGNMENT #7

PROBLEM 11.154 Knowing that a factor of safety of 2.8 is required and using Euler's formula, determine the largest load P which may be applied to the structure shown when $\theta = 30°$. Use $E = 29(10^6)$ psi and consider only buckling in the plane of the structure.

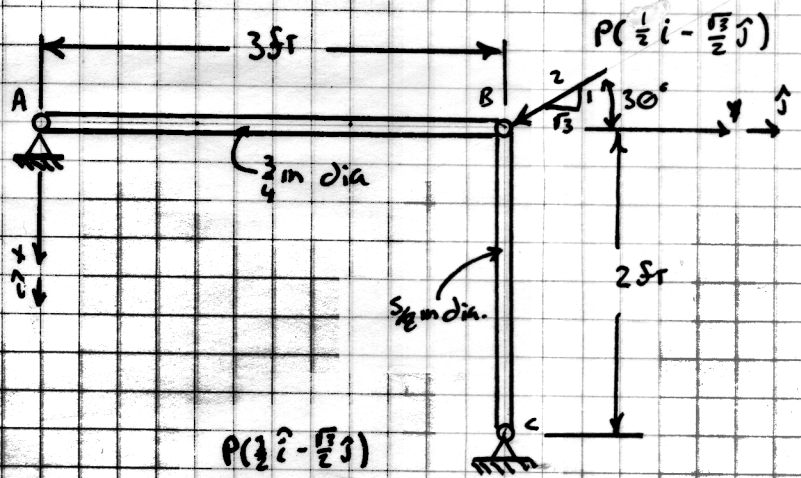
A) GIVEN:

A.1) CONSTRAINTS:
(1) structure shown
(2) safety factor of 2.8
(3) load angle with horizontal plane $\theta = 30°$
(4) Modulus of elasticity for the material $E = 29(10^6)$ psi

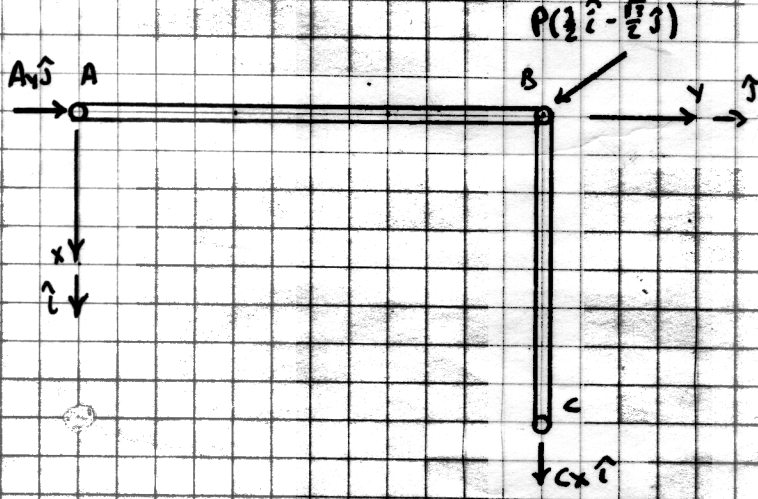
A.2) ASSUMPTIONS:
(1) homogeneous material
(2) columns are straight
(3) failure occurs due to buckling
(4) $P_{cr} < \sigma_y$ for the material

B) FIND:
(1) largest value of P.

C) SKETCH:



D) FREE BODY DIAGRAM:



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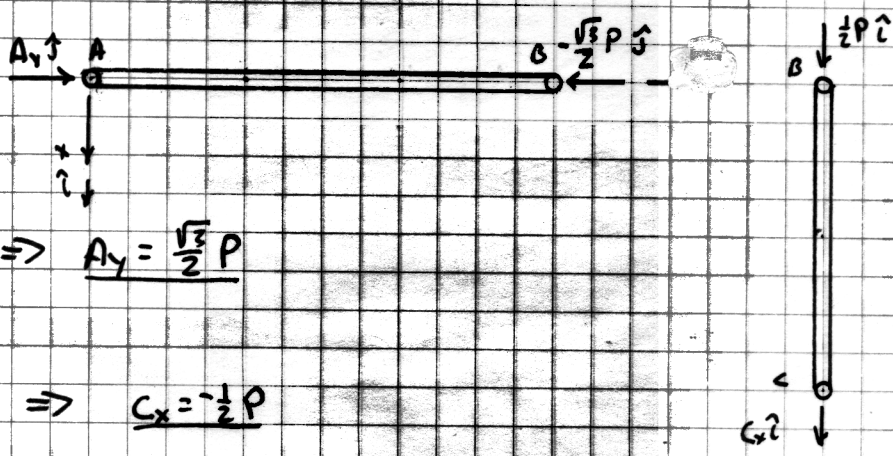
E) EQUILIBRIUM:

Members AB and BC are both two-force members since all connections are pinned; therefore, no moments can be reacted out at the constraints. This eliminating the possibility of transverse loading in the beam. Thus the beam can be broken up to determine the internal loads.



Member AB

$$\sum F_y = 0 = A_y - \frac{\sqrt{3}}{2}P \Rightarrow \underline{A_y = \frac{\sqrt{3}}{2}P}$$

Member BC

$$\sum F_x = 0 = C_x + \frac{1}{2}P \Rightarrow \underline{C_x = -\frac{1}{2}P}$$

Both members AB and BC have compressive loads

F) MECHANICS:

Starting with Member AB

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{1}{2}\cdot\frac{3}{4}\text{in}\right)^4 = 0.01553\,\text{in}^4$$

$$P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 (29)(10^6)\,\text{psi}\cdot(0.01553\,\text{in}^4)}{(36\,\text{in})^2} = 3430\,\text{lb}$$

$$F_{AB} = \frac{P_{cr}}{S.F.} = \frac{3430\,\text{lb}}{2.8} = 1225\,\text{lb}$$

$$F_{AB} = \frac{\sqrt{3}}{2}P = 1225\,\text{lb} \Rightarrow P = \frac{2}{\sqrt{3}}\cdot 1225\,\text{lb} = \underline{1414\,\text{lb}}$$ (J)

Now considering member BC

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{1}{2}\cdot\frac{5}{8}\text{in}\right)^4 = 0.00749\ \text{in}^4$$

$$P_{cr} = \frac{\pi^2 E \cdot I}{L^2} = \frac{\pi^2 \cdot (29)(10^6)\ \text{psi} \cdot (0.00749\ \text{in}^4)}{(24\ \text{in})^2} = 3722\text{lb}$$

$$F_{BC} = \frac{P_{cr}}{SF} = \frac{3722\text{lb}}{2.8} = 1329\text{lb}$$

$$F_{BC} = \frac{1}{2}P = 1329\text{lb} \Rightarrow \underline{P = 2659\text{lb}} \quad (2)$$

(1) and (2) are estimates for P. we take the lower value since the higher value (member BC) will cause member AB to buckle

$$\boxed{P = 1415\text{lb}}$$

G) OTHER CONSIDERATIONS:

H) REVIEW: ✓

I) SUMMARY: This problem demonstrates that when designing an engineer must consider all the failure modes in a structure and then make sure that the loading will prevent all these failures from occuring ✓