PROBLEM 9.4-10 DERIVE THE EQUATIONS OF THE DEFLECTION CURVE FOR A SIMPLE BEAM AB SUPPORTING A TRIANGULARLY DISTRIBUTED LOAD OF MAXIMUM INTENSITY q_0 ACTING ON THE RIGHT-HAND HALF OF THE BEAM. ALSO, DETERMINE THE ANGLES OF ROTATION θ_A AND θ_B AT THE ENDS AND THE DEFLECTION δ_C AT THE MIDPOINT.

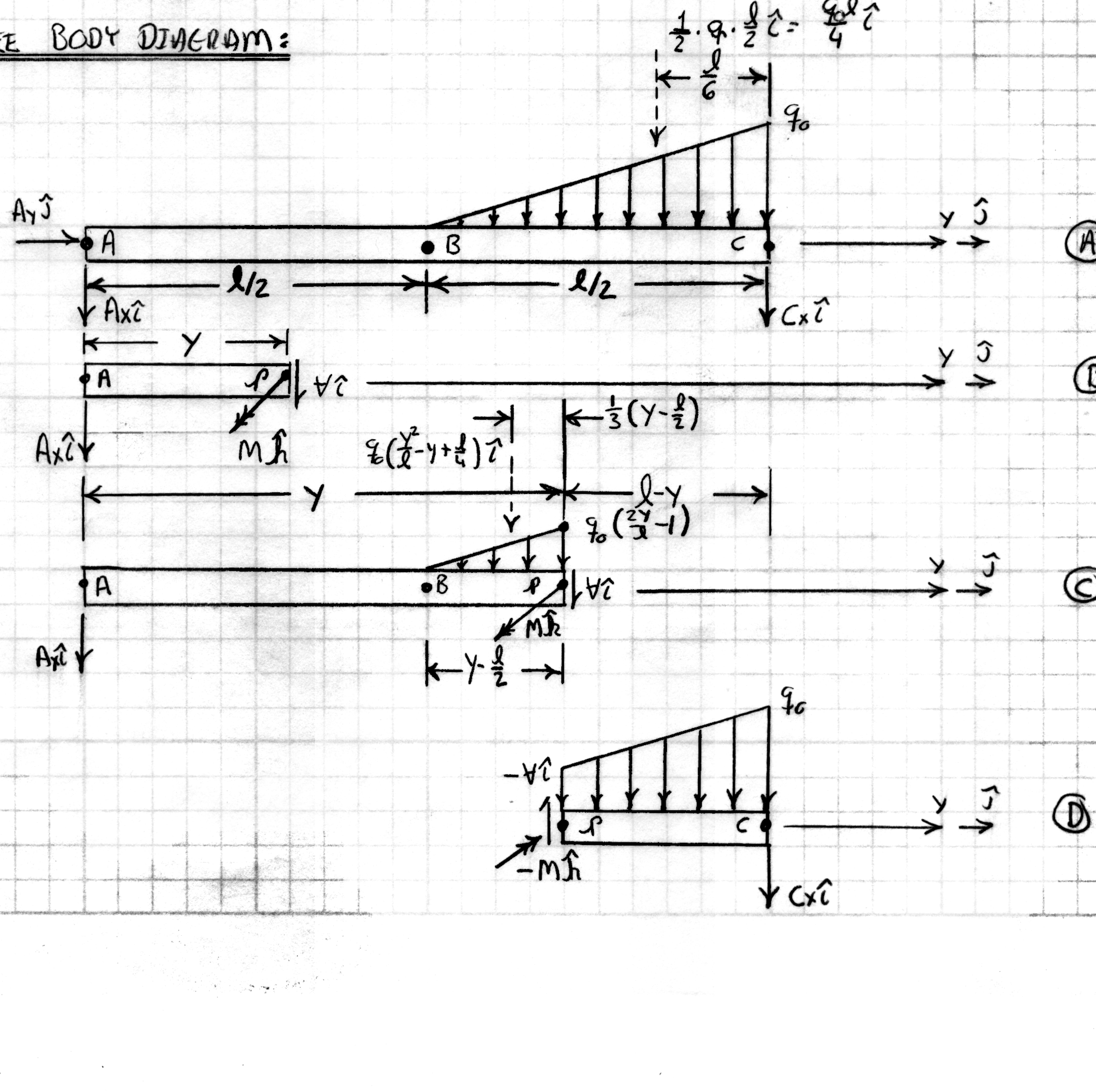
GIVEN:

CONSTRAINTS
1) BEAM WITH PIN SUPPORT AT LEFT HAND SIDE AND ROLLER SUPPORT AT RIGHT-HAND SIDE.
2) TRIANGULAR LOAD STARTING AT MID SPAN AND INCREASING LINEARLY TO THE RIGHT HAND SUPPORT

ASSUMPTIONS
1) DEFLECTIONS ARE SMALL
2) MATERIAL RESPONDS IN A LINEAR ELASTIC MANNER

FIND:
1) ANGLES OF ROTATION AT THE END POINTS
2) DEFLECTION AT THE MIDPOINT

FREE BODY DIAGRAM:



STATICS:

FIRST LET'S DETERMINE THE REACTIONS AT A & C USING (A)

$$\Sigma F_x = 0 = A_x + C_x + \frac{q_0 l}{4} = 0 \Rightarrow A_x + C_x = -\frac{q_0 l}{4} \quad (1)$$

$$\Sigma M_z|_{at\,C} = 0 = A_x \cdot l + \frac{l}{6} \cdot \frac{q_0 l}{4} \Rightarrow A_x = -\frac{q_0 l}{24} \quad (2)$$

$$(2) \rightarrow (1) \Rightarrow C_x = -\frac{q_0 l}{4} \cdot \frac{6}{6} + \frac{q_0 l}{24} = -\frac{5 q_0 l}{24} \quad (3)$$

BEFORE THE SOLUTION PROCEEDS AN EQUATION FOR THE DISTRIBUTED LOAD MUST BE DEVELOPED

$$q = m \cdot y + b$$

$$m = \frac{\text{RISE}}{\text{RUN}} = \frac{q_0}{l/2} = \frac{2 q_0}{l} \quad ; \quad b = -q_0 \text{ (DUE TO SYMMETRY)}$$

$$q = \frac{2 \cdot q_0}{l} \cdot y - q_0 = q_0 \left(\frac{2y}{l} - 1\right) \quad (4)$$

STARTING WITH THE FREE BODY DIAGRAM (B) THE EQUATIONS FOR THE INTERNAL EQUILIBRIUM FOR THE REGION $0 < y < l/2$

$$\Sigma F_x = 0 = -\frac{q_0 l}{24} + V \Rightarrow \underline{V = \frac{q_0 l}{24}} \quad (5)$$

$$\Sigma M_z|_{at\,p} = 0 = -\frac{q_0 l}{24} \cdot y + M \Rightarrow \underline{M = \frac{q_0 l}{24} \cdot y} \quad (6)$$

NOW LET'S CONSIDER THE REGION $\frac{l}{2} < y < l$. THE EQUIVALENT FORCE THAT IS A RESULT OF THE DISTRIBUTED LOAD IS

$$\frac{1}{2} \cdot \left(y - \frac{l}{2}\right) q_0 \left(\frac{2 \cdot y}{l} - 1\right) = \frac{q_0}{2}\left(\frac{2y^2}{l} - y - y + \frac{l}{2}\right) = q_0 \left(\frac{y^2}{l} - y + \frac{l}{4}\right) \quad (7)$$

NOW CONSIDERING THE EQUILIBRIUM OF (C)

$$\Sigma F_x = 0 = -\frac{q_0 l}{24} + q_0 \left(\frac{y^2}{l} - y + \frac{l}{4}\right) + V$$

$$\Rightarrow V = \frac{q_0 l}{24} - q_0 \left(\frac{y^2}{l} - y + \frac{l \cdot 6}{4 \cdot 6}\right) = \underline{-q_0 \left(\frac{y^2}{l} - y + \frac{5 \cdot l}{24}\right)} \quad (8)$$

$$\Sigma M_z|_{p} = 0 = M - \frac{q_0 l}{24} \cdot y + q_0 \left(\frac{y^2}{l} - y + \frac{l}{4}\right) \cdot \frac{1}{3}\left(y - \frac{l}{2}\right)$$

$$\Rightarrow M = \frac{q_0 l}{24} y - q_0 \left(\frac{y^2}{l} - y + \frac{l}{4}\right) \cdot \frac{1}{3} \cdot \left(y - \frac{l}{2}\right)$$

$$= \frac{q_0 l}{24} \cdot y - \frac{q_0}{3}\left(\frac{y^3}{l} - \cancel{2} y^2 + \frac{y l}{4} - \frac{y^2}{2} + \frac{y \cdot l^2}{2 \cdot 2} - \frac{l^2}{8}\right)$$

$$M = \frac{q_0 \cdot l}{24} \cdot y - \frac{q_0}{3}\left(\frac{y^3}{l} - \frac{3}{2}y^2 + \frac{3 \cdot y \cdot l}{4} - \frac{l^2}{8}\right) = \frac{q_0 l \cdot y}{24} - q_0\left(\frac{y^3}{3l} - \frac{y^2}{2} + \frac{6yl}{64} - \frac{l^2}{24}\right)$$

$$M = -q_0\left(\frac{y^3}{3l} - \frac{y^2}{2} + \frac{5 \cdot y \cdot l}{24} - \frac{l^2}{24}\right) = q_0\left(\frac{-y^3}{3l} + \frac{y^2}{2} - \frac{5}{24} y \cdot l + \frac{l^2}{24}\right) \quad (9)$$

JUST AS A CHECK, THE DERIVATIVE OF (9) DOES EQUAL (8).

MECHANICS:

NOW THAT EQUATIONS FOR THE BENDING MOMENTS IN BOTH SECTIONS OF THE BEAM HAVE BEEN DEVELOPED, EQUATIONS FOR THE CURVATURE AND DEFLECTION IN BOTH OF THESE SECTIONS CAN NOW BE DETERMINED

$0 < y < l/2$

$$\frac{d^2u}{dy^2} = -\frac{M}{EI} = \frac{-q_0 \cdot l \cdot y}{24 \cdot E \cdot I}$$

$$\theta = \frac{du}{dy} = \frac{-q_0 \cdot l}{24 \cdot EI} \int y \cdot dy = \frac{-q_0 \cdot l}{24 \cdot EI} \frac{y^2}{2} + C_1 = \frac{-q_0 l}{48 \cdot EI} y^2 + C_1 \quad (10)$$

$$u = \int \left(\frac{-q_0 l}{48 EI} y^2 + C_1\right) dy = \frac{-q_0 \cdot l}{48 \cdot EI} \frac{y^3}{3} + C_1 \cdot y + C_2$$

$$= \frac{-q_0 l}{144 \cdot EI} y^3 + C_1 y + C_2 \quad (11)$$

THE CONSTANTS C_1 & C_2 in (10) AND (11) MUST BE DETERMINED THROUGH BOUNDARY CONDITIONS. IN THE REGION UNDER CONSIDERATION THE ONLY BOUNDARY CONDITION AVAILABLE AT THIS TIME IS

$$u(0) = 0$$

SUBSTITUTING THIS INTO (11)

$$u(0) = 0 = \frac{-q_0 \cdot l}{144 EI} \cdot (0)^3 + C_1(0) + C_2 = \underline{C_2 = 0}$$

THUS THE CURVATURE AND DEFLECTION IN THE REGION FROM $0 < y < l/2$ BECOME

$$\theta = -\frac{q_0 l}{48 EI} y^2 + C_1 \quad (10)$$

$$u = -\frac{q_0 l}{144 EI} y^3 + C_1 y \quad (11)$$

THE CONSTANT C_3 WILL BE DETERMINED THROUGH THE CONTINUITY CONDITIONS BETWEEN THE TWO SECTIONS OF THE BEAM. NOW LETS CONSIDER THE OTHER REGION

$\ell/2 < y < \ell$

$$\frac{d^2u}{dy^2} = -\frac{M}{EI} = \frac{q_0}{EI}\left(\frac{y^3}{3\ell} - \frac{y^2}{2} + \frac{5\ell}{24}y - \frac{\ell^2}{24}\right)$$

$$\theta = \frac{du}{dy} = \frac{q_0}{EI}\int\left(\frac{y^3}{3\ell} - \frac{y^2}{2} + \frac{5\ell}{24}\cdot y - \frac{\ell^2}{24}\right)dy$$

$$= \frac{q_0}{EI}\left(\frac{y^4}{12\cdot\ell} - \frac{y^3}{6} + \frac{5\ell}{48}y^2 - \frac{\ell^2}{24}y\right) + C_3 \qquad (12)$$

$$u = \frac{q_0}{E\cdot I}\left(\frac{y^5}{60\cdot\ell} - \frac{y^4}{24} + \frac{5\ell}{144}y^3 - \frac{\ell^2}{48}y^2\right) + C_3 y + C_4 \qquad (13)$$

THE BOUNDARY CONDITION IN THIS SECTION OF THE BEAM THAT IS AVAILABLE AT THIS TIME IS

$$u(\ell) = 0$$

$$u(\ell) = 0 = \frac{q_0}{EI}\left(\frac{\ell^4}{60}\cdot\frac{12}{12} - \frac{\ell^4}{24}\frac{30}{30} + \frac{5\ell^4}{144}\frac{5}{5} - \frac{\ell^4}{48}\frac{15}{15}\right) + C_3\ell + C_4$$

$$= \frac{-8}{720}\frac{q_0\cdot\ell^4}{EI} + C_3\ell + C_4$$

$$\Rightarrow C_4 = \frac{8}{720}\frac{q_0\cdot\ell^4}{EI} - C_3\ell \qquad (14)$$

FROM THIS RESULT (13) CAN BE REWRITTEN

$$u = \frac{q_0}{E\cdot I}\left(\frac{y^5}{60\cdot\ell} - \frac{y^4}{24} + \frac{5\ell}{144}y^3 - \frac{\ell^2}{48}y^2\right) + C_3 y + \frac{8}{720}\frac{q_0\ell^4}{EI} - C_3\cdot\ell$$

$$= \frac{q_0}{E\cdot I}\left(\frac{y^5}{60\cdot\ell} - \frac{y^4}{24} + \frac{5\ell}{144}y^3 - \frac{\ell^2}{48}y^2\right) - C_3(\ell - y) + \frac{8}{720}\frac{q_0\ell^4}{EI} \qquad (15)$$

TO COMPLETE THE SOLUTION TO THIS PROBLEM THE CURVATURE AND DISPLACEMENT AT THE INTERFACE BETWEEN THESE TWO REGIONS MUST BE CONSIDERED. THE VALUE OF THESE QUANTITIES AT THIS POINT FOR THE TWO REGIONS MUST BE THE SAME IN ORDER TO SATISFY CONTINUITY.

STARTING BY CONSIDERING THE VALUE OF THE CURVATURES AT $l/2$ FROM (10) AND (12)

$$(10) \rightarrow \Theta\left(\frac{l}{2}\right) = -\frac{q_0 \cdot l}{48 \cdot EI}\left(\frac{l}{2}\right)^2 + C_1 = \frac{-q_0 \cdot l^3}{192 \cdot EI} + C_1$$

$$(12) \rightarrow \Theta\left(\frac{l}{2}\right) = \frac{q}{EI}\left[\frac{(l/2)^4}{12 \cdot l} - \frac{(l/2)^3}{6} + \frac{5l}{48}\cdot\left(\frac{l}{2}\right)^2 - \frac{l^2}{24}\left(\frac{l}{2}\right)\right] + C_3$$

$$= \frac{q_0}{EI}\left(\frac{l^3}{192} - \frac{4 \cdot l^3}{4 \cdot 48} + \frac{5 l^3}{192} - \frac{4 \cdot l^3}{4 \cdot 48}\right) + C_3$$

$$= \frac{-2 q_0 l^3}{192 EI} + C_3$$

$$\frac{-q_0 l^3}{192 EI} + C_1 = \frac{-2 q_0 l^3}{192 EI} + C_3$$

$$C_1 - C_3 = \frac{-q_0 \cdot l^3}{192 \cdot E \cdot I} \Rightarrow C_1 = \frac{-q_0 l^3}{192 EI} + C_3 \qquad (16)$$

NOW THE DEFLECTIONS AT $\frac{l}{2}$ FOR BOTH REGIONS CAN BE DETERMINED USING (11) AND (15)

$$(11) \rightarrow u\left(\frac{l}{2}\right) = -\frac{q_0 \cdot l}{144 EI}\left(\frac{l}{2}\right)^3 + C_1\left(\frac{l}{2}\right)$$

$$= -\frac{q_0 \cdot l^4}{1152 \cdot EI} + \frac{C_1 l}{2}$$

$$(15) \rightarrow u\left(\frac{l}{2}\right) = \frac{q_0}{EI}\left(\frac{(l/2)^5}{60 l} - \frac{(l/2)^4}{24} + \frac{5 \cdot l}{144}\left(\frac{l}{2}\right)^3 - \frac{l^2}{48}\left(\frac{l}{2}\right)^2\right) - C_3 \cdot \left(l - \frac{l}{2}\right) + \frac{8}{720}\frac{q_0 l^4}{EI}$$

$$= \frac{q_0}{EI}\left(\frac{l^4 \cdot 3}{1920 \cdot 3} - \frac{l^4 \cdot 15}{384 \cdot 15} + \frac{5 \cdot l^4 \cdot 5}{1152 \cdot 5} - \frac{l^4 \cdot 30}{192 \cdot 30}\right) - \frac{C_3 \cdot l}{2} + \frac{8}{720}\frac{q_0 l^4}{EI}\cdot\frac{8}{8}$$

$$= \frac{-q_0 l^4 \cdot 17}{5760 EI} - \frac{C_3 l}{2} + \frac{64}{5760}\frac{q_0 l^4}{EI} = \frac{47}{5760}\frac{q_0 \cdot l^4}{E \cdot I} - \frac{C_3 \cdot l}{2}$$

$$\therefore -\frac{q_0 \cdot l^4}{1152 EI} + \frac{C_1 l}{2} = \frac{47}{5760}\frac{q_0 \cdot l^4}{EI} - \frac{C_3 \cdot l}{2}$$

$$\frac{l}{2}(C_1 + C_3) = \frac{52}{5760}\frac{q_0 l^4}{EI} \qquad (17)$$

$$(16) \rightarrow (17) \Rightarrow \frac{l}{2}\left[\frac{-q_0 l^3}{192EI} + C_3 + C_3\right] = \frac{52}{5760}\frac{q_0 l^4}{EI}$$

$$\frac{-q_0 l^4}{384EI}\cdot\frac{15}{15} + C_3 l = \frac{52}{5760}\frac{q_0 l^4}{EI}$$

$$\Rightarrow C_3 = \frac{67}{5760}\frac{q_0 l^3}{EI} \qquad (18)$$

$$(16) \rightarrow \quad C_1 = \frac{-q_0 l^3}{192EI}\cdot\frac{30}{30} + \frac{67}{5760}\frac{q_0 l^3}{EI} = \frac{37}{5760}\frac{q_0 l^3}{EI} \qquad (19)$$

NOW THE CURVATURE AND DEFLECTION IN THE REGION FROM $0 < y < l/2$ CAN BE WRITTEN

$$(10) \rightarrow \quad \theta = \frac{120}{120}\cdot\frac{-q_0 \cdot l}{48 \cdot EI}\cdot y^2 + \frac{37}{5760}\frac{q_0 l^3}{EI} = \frac{q_0 \cdot l}{5760EI}\left(-120y^2 + 37l^2\right) \qquad (20)$$

$$(11) \rightarrow \quad u = \frac{40}{40}\cdot\frac{-q_0 l}{144EI}\cdot y^3 + \frac{37}{5760}\frac{q_0 l^3}{EI}\cdot y$$

$$= \frac{q_0 \cdot l}{5760 \cdot EI}\left(-40 \cdot y^3 + 37 \cdot l^2 \cdot y\right) \qquad (21)$$

THUS AT THE LEFT MOST SUPPORT THE ROTATION AND DEFLECTION ARE

$$(20) \rightarrow \theta(0) = \frac{q_0 \cdot l}{5760EI}\left(-120(0)^2 + 37l^2\right) = \boxed{\frac{37 q_0 l^3}{5760EI}}$$

$$(21) \rightarrow u(0) = \frac{q_0 \cdot l}{5760EI}\left(-40 \cdot (0)^3 + 37 \cdot l^2(0)\right) = 0$$ THIS AGREES WITH THE BOUNDARY CONDITION

NOW THE DEFLECTION AND CURVATURE AT $l/2$ CAN BE CALCULATED

$$(20) \rightarrow \theta\left(\frac{l}{2}\right) = \frac{q_0 \cdot l}{5760EI}\left(-120\left(\frac{l}{2}\right)^2 + 37l^2\right) = \frac{7}{5760}\frac{q_0 l^3}{EI} \qquad (22)$$

$$(21) \rightarrow u\left(\frac{l}{2}\right) = \frac{q_0 \cdot l}{5760EI}\left(-40\left(\frac{l}{2}\right)^3 + 37l^2\left(\frac{l}{2}\right)\right) = \frac{27 q_0 \cdot l^4}{11520EI} = \boxed{\frac{3}{1280}\frac{q_0 l^4}{EI}} \qquad (23)$$

NOW THE CURVATURE AND DEFLECTION IN THE REGION $l/2 < y < l$ CAN BE WRITTEN USING THE RESULTS IN (18) AND (19)

$$(12) \rightarrow \theta = \frac{q_0}{E \cdot I}\left(\frac{y^4}{12 \cdot l} - \frac{y^3}{6} + \frac{5}{48} l \cdot y^2 - \frac{l^2}{24} y\right) + \frac{67}{5760}\frac{q_0 \cdot l^3}{EI}$$

$$= \frac{q_0}{5760 \cdot E \cdot I}\left(\frac{480 \cdot y^4}{l} - 960 \cdot y^3 + 600 l \cdot y^2 - 240 \cdot l^2 \cdot y + 67 l^3\right) \quad (24)$$

$$(15) \rightarrow u = \frac{q_0}{EI}\left(\frac{y^5}{60 l} - \frac{y^4}{24} + \frac{5}{144} l \cdot y^3 - \frac{l^2 \cdot y^2}{48}\right) - \frac{67}{5760}\frac{q_0 l^3}{EI}(l - y) + \frac{8}{720}\frac{q_0 \cdot l^4}{EI}$$

$$= \frac{q_0}{EI}\left(\frac{y^5}{60 l} - \frac{y^4}{24} + \frac{5 \cdot l \cdot y^3}{144} - \frac{l^2 \cdot y^2}{48}\right) - \frac{67}{5760}\frac{q_0 l^4}{EI} + \frac{67}{5760}\frac{q_0 l^3 y}{EI} + \frac{8}{720}\frac{q_0 l^4}{EI}$$

$$= \frac{q_0}{5760 \cdot EI}\left(\frac{96 \cdot y^5}{l} - 240 \cdot y^4 + 200 \cdot l \cdot y^3 - 120 \cdot l^2 \cdot y^2 + 67 l^3 y - 3 l^4\right) \quad (25)$$

THE CURVATURE AND DEFLECTION AT THE FAR RIGHT SUPPORT ARE

$$(24) \rightarrow \theta(l) = \frac{q_0}{5760 \cdot EI}\left(480 l^3 - 960 l^3 + 600 l^3 - 240 l^3 + 67 l^3\right) = \boxed{\frac{-53}{5760}\frac{q_0 \cdot l^3}{EI}}$$

$$(25) \rightarrow u(l) = \frac{q_0}{5760 EI}\left(96 l^4 - 240 l^4 + 200 l^4 - 120 l^4 + 67 l^4 - 3 l^4\right) = 0$$ Checks with Boundary Cond.

NOW CONSIDER THE DEFLECTION AND CURVATURE AT MID SPAN

$$(24) \rightarrow \theta\left(\frac{l}{2}\right) = \frac{q_0}{5760 EI}\left(\frac{\overset{30}{480}\left(\frac{l}{2}\right)^4}{l} - 960\left(\frac{l}{2}\right)^3 + 600 l\left(\frac{l}{2}\right)^2 - 240 l^2\left(\frac{l}{2}\right) + 67 l^3\right)$$

$$= \frac{7 q_0 l^3}{5760 EI}$$ THIS VALUE IS THE SAME AS (22) THE CONTINUITY CONDITION IS SATISFIED.

$$(25) \rightarrow u\left(\frac{l}{2}\right) = \frac{q_0}{5760 EI}\left(\frac{96\left(\frac{l}{2}\right)^5}{l} - 240 \cdot \left(\frac{l}{2}\right)^4 + 200 l \cdot \left(\frac{l}{2}\right)^3 - 120 l^2\left(\frac{l}{2}\right)^2 + 67 l^3\left(\frac{l}{2}\right) - 3 l^4\right)$$

$$= \frac{27 q_0 l^4}{11{,}520 \cdot EI} = \boxed{\frac{3}{1280}\frac{q_0 l^4}{EI}}$$ THIS VALUE IS THE SAME AS (23)

NOW LET'S DRAW THE V, M, θ, AND u DIAGRAMS FOR THIS BEAM.

HOMEWORK SOLUTION
ESC23: MECHANICS III
ASSIGNMENT #8

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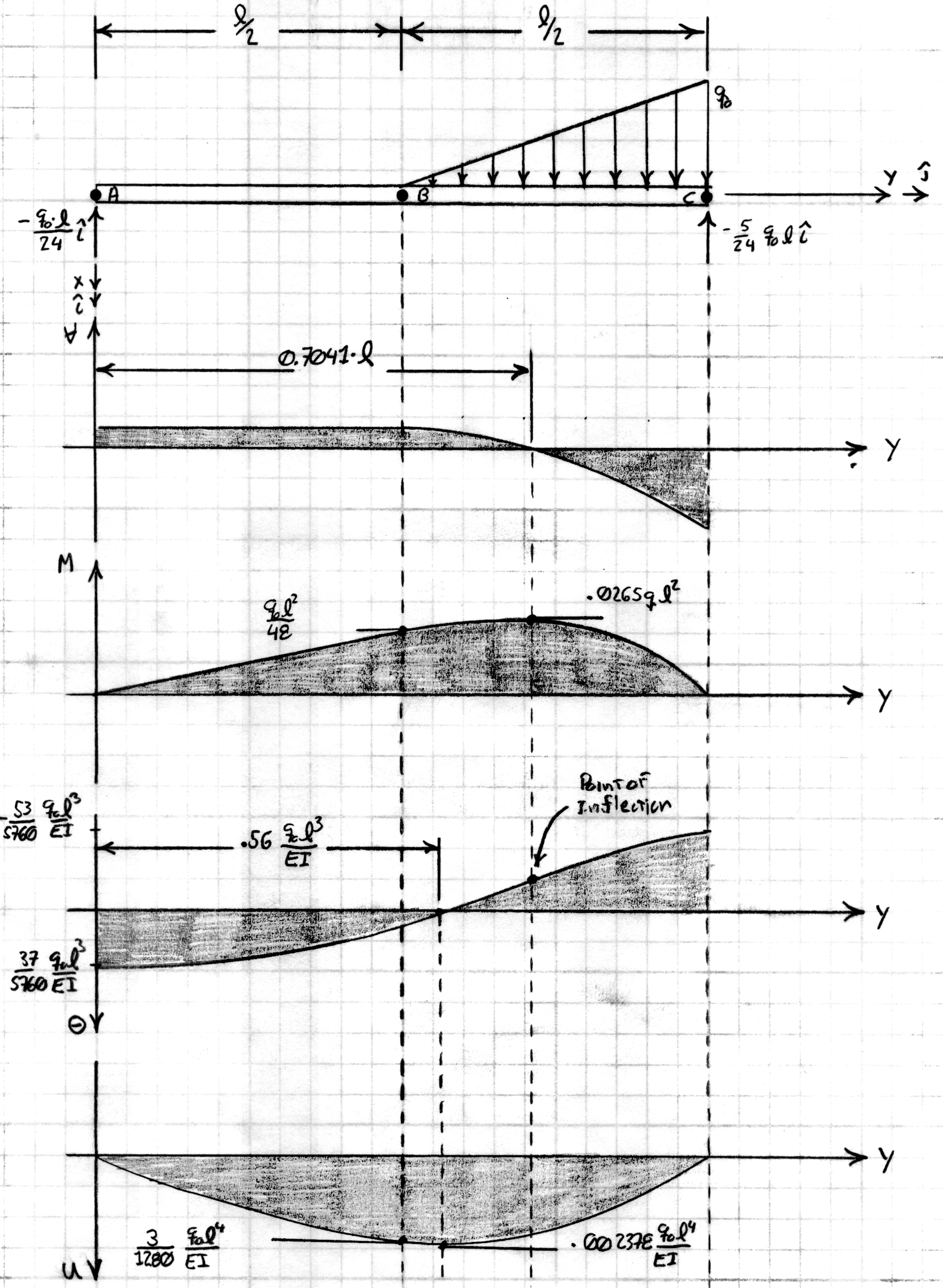

DETERMINING THE LOCATION WHERE $V = 0$

$$⑧ \rightarrow \quad V = 0 = -q_0\left(\frac{y^2}{l} - y + \frac{5}{24} l\right)$$

$$0 = y^2 - yl + \frac{5}{24} l^2 = y^2 - yl + \left(-\frac{l}{2}\right)^2 - \left(-\frac{l}{2}\right)^2 + \frac{5}{24} l^2$$

$$0 = \left(y - \frac{l}{2}\right)^2 - \frac{l^2}{24} \quad \Rightarrow \quad y = \frac{l}{2} \pm \sqrt{l^2/24}$$

SINCE WE ARE ONLY CONSIDERING THE DOMAIN BETWEEN $l/2 < y < l$

$$y = \frac{l}{2} + l\sqrt{1/24} = 0.7041 \cdot l$$

THE MAXIMUM VALUE OF THE MOMENT OCCURS AT $0.7041l$ AND EQUALS

$$⑨ \rightarrow \quad M(0.7041l) = q_0\left(-\frac{(0.7041l)^3}{3l} + \frac{(.7041l)^2}{2} - \frac{5}{24}(.7041l) \cdot l + \frac{l^2}{24}\right)$$

$$= 0.0265\, q_0 l^2$$

THE LOCATION WHERE THE CURVATURE IS 0 NEEDS TO BE LOCATED. THIS IS WHERE THE DEFLECTION IS MAXIMUM. TO DO THIS WE NEED TO CHECK FOR INTERCEPTS IN BOTH REGIONS OF THE BEAM

$$⑩ \rightarrow \quad \frac{q_0 l}{5760EI}\left(-120 \cdot y^2 + 37l^2\right) = 0$$

$$-120y^2 + 37l^2 = 0 \quad \Rightarrow \quad y^2 = \frac{37}{120} l^2 \quad \Rightarrow \quad y = \pm 0.555l$$

SINCE THIS IS OUTSIDE THE DOMAIN FOR EQUATION ⑩ WE WILL NOW CONSIDER THE OTHER REGION OF THE BEAM

$$㉔ \rightarrow \quad 0 = \frac{q_0}{5760EI}\left(\frac{480 \cdot y^4}{l} - 960y^3 + 600ly^2 - 240l^2 \cdot y + 67l^3\right)$$

$$= \frac{q_0}{5760EIl}\left(480 \cdot y^4 - 960l \cdot y^3 + 600l^2y^2 - 240l^3 \cdot y + 67l^4\right)$$

$$y \cong .56\, l$$

THE MAXIMUM DEFLECTION AT $.56l$ CAN NOW BE CALCULATED

$$(25) \rightarrow u(.56\ell) = \frac{q_0}{5760EI}\left(\frac{96\cdot y^5}{\ell} - 240\cdot y^4 + 200\cdot \ell\cdot y^3 - 120\ell^2\cdot y^2 + 67\ell^3\cdot y - 3\ell^4\right)$$

$$= \frac{q_0}{5760\ell EI}\left(96y^5 - 240\ell\cdot y^4 + 200\ell^2 y^3 - 120\ell^3 y^2 + 67\ell^4\cdot y - 3\ell^5\right)$$

$$= \frac{q_0}{5760\ell EI}\left(96(.56\ell)^5 - 240\cdot\ell\cdot(.56\ell)^4 + 200\ell^2(.56\ell)^3 - 120\ell^3(.56\ell)^2 + 67\ell^4(.56\ell) - 3\ell^5\right)$$

$$= 0.002378\,\frac{q_0\ell^4}{EI}$$

SUMMARY:

THE SOLUTION PRESENTED GOES BEYOND THE QUESTION ASKED; HOWEVER, I FELT THAT IT WAS NECESSARY TO PRESENT THE COMPLETE SOLUTION. IT IS IMPORTANT TO SEE THE CONNECTION BETWEEN THE V, M, θ, AND u DIAGRAMS. IT IS IMPORTANT TO TAKE NOTE OF THE DEFLECTION DIAGRAM AND SEE THAT THE MAXIMUM DEFLECTION OCCURS AT 0.56ℓ AND NOT AT MIDSPAN. DEFLECTION IS OFTEN USED AS A CRITERION FOR FAILURE; HOWEVER, THE MAXIMUM STRESS IN THE BEAM MUST ALSO BE CONSIDERED. THE STRESS AT $.7041\ell$ LOOKS TO BE THE MAXIMUM SINCE THIS IS WHERE THE BENDING MOMENT IS MAXIMUM. ONE SHOULD ALSO CHECK THE LOCATION OF MAXIMUM SHEAR STRESS TO MAKE SURE THIS IS NOT CAUSING AN UNACCEPTABLE PROBLEM.