

THE ACCELERATION ANALYSIS STARTS BY TAKING THE DERIVATIVE OF (26)

$$\ddot{r}_2 \cdot \ddot{\theta}_2 \cdot \hat{e}_{\theta 2} + \ddot{r}_2 \cdot \dot{\theta}_2^2 \cdot \hat{e}_{r2} + \ddot{r}_3 \cdot \hat{e}_{r3} + \ddot{r}_3 \cdot \dot{\theta}_3^2 \cdot \hat{e}_{\theta 3} + \ddot{r}_3 \cdot \dot{\theta}_3 \cdot \hat{e}_{\theta 3} + \ddot{r}_4 \cdot \dot{\theta}_4^2 \cdot \hat{e}_{r4} - \ddot{r}_4 \cdot \dot{\theta}_4 \cdot \hat{e}_{\theta 4} = \ddot{r}_4 \cdot \dot{\theta}_4 \cdot \hat{e}_{\theta 4} - \ddot{r}_4 \cdot \dot{\theta}_4^2 \cdot \hat{e}_{r4} \quad (33)$$

FROM TAKING THE DERIVATIVE OF (27)

$$\ddot{\theta}_4 = \ddot{\theta}_3 \quad (34)$$

SUBSTITUTING (27) AND (34) INTO (33)

$$\ddot{r}_2 \cdot \ddot{\theta}_2 \cdot \hat{e}_{\theta 2} - \ddot{r}_2 \cdot \dot{\theta}_2^2 \cdot \hat{e}_{r2} + \ddot{r}_3 \cdot \hat{e}_{r3} + \ddot{r}_3 \cdot \dot{\theta}_3^2 \cdot \hat{e}_{\theta 3} + 2 \cdot \dot{r}_3 \cdot \dot{\theta}_3 \cdot \hat{e}_{\theta 3} + \ddot{r}_3 \cdot \ddot{\theta}_3 \cdot \hat{e}_{r3} - \ddot{r}_3 \cdot \dot{\theta}_3^2 \cdot \hat{e}_{\theta 3} = \ddot{r}_4 \cdot \ddot{\theta}_3 \cdot \hat{e}_{\theta 4} - \ddot{r}_4 \cdot \dot{\theta}_3^2 \cdot \hat{e}_{r4} \quad (35)$$

IN (35) THE ONLY TWO UNKNOWNS ARE \ddot{r}_3 AND $\ddot{\theta}_3$. SINCE (35) IS A VECTOR EQUATION FOR A PLANNER SYSTEM, IT REPRESENTS TWO EQUATIONS. THE SOLUTION FOR THE TWO UNKNOWN STARTS BY COMBINING TERMS IN (35)

$$\ddot{r}_2 \cdot \ddot{\theta}_2 \cdot \hat{e}_{\theta 2} - \ddot{r}_2 \cdot \dot{\theta}_2^2 \cdot \hat{e}_{r2} + \ddot{r}_3 \cdot \hat{e}_{r3} + 2 \cdot \dot{r}_3 \cdot \dot{\theta}_3 \cdot \hat{e}_{\theta 3} + \ddot{r}_3 \cdot (\ddot{r}_3 \cdot \hat{e}_{\theta 3} - \ddot{r}_4 \cdot \hat{e}_{\theta 4}) - \ddot{r}_3 \cdot \dot{\theta}_3^2 \cdot \hat{e}_{r3} + \ddot{r}_4 \cdot \dot{\theta}_3^2 \cdot \hat{e}_{r4} = 0 \quad (36)$$

DOTTING (36) WITH \hat{e}

$$-\ddot{r}_2 \cdot \ddot{\theta}_2 \cdot \sin \theta_2 - \ddot{r}_2 \cdot \dot{\theta}_2^2 \cdot \cos \theta_2 + \ddot{r}_3 \cdot \cos \theta_3 - 2 \cdot \dot{r}_3 \cdot \dot{\theta}_3 \cdot \sin \theta_3 - \ddot{\theta}_3 \cdot (\ddot{r}_3 \cdot \sin \theta_3 - \ddot{r}_4 \cdot \sin \theta_4) - \ddot{r}_3 \cdot \dot{\theta}_3^2 \cdot \cos \theta_3 + \ddot{r}_4 \cdot \dot{\theta}_3^2 \cdot \cos \theta_4 = 0$$

SOLVING FOR \ddot{r}_3

$$\ddot{r}_3 = \frac{\ddot{r}_2 \cdot \ddot{\theta}_2 \cdot \sin \theta_2 + \ddot{r}_2 \cdot \dot{\theta}_2^2 \cdot \cos \theta_2 + 2 \cdot \dot{r}_3 \cdot \dot{\theta}_3 \cdot \sin \theta_3 + \ddot{\theta}_3 \cdot (\ddot{r}_3 \cdot \sin \theta_3 - \ddot{r}_4 \cdot \sin \theta_4) + \ddot{r}_3 \cdot \dot{\theta}_3^2 \cdot \cos \theta_3 - \ddot{r}_4 \cdot \dot{\theta}_3^2 \cdot \cos \theta_4}{\cos \theta_3} \quad (37)$$

DOTTING (36) WITH \hat{j}

$$\ddot{r}_2 \cdot \ddot{\theta}_2 \cdot \cos \theta_2 - \ddot{r}_2 \cdot \dot{\theta}_2^2 \cdot \sin \theta_2 + \ddot{r}_3 \cdot \sin \theta_3 + 2 \cdot \dot{r}_3 \cdot \dot{\theta}_3 \cdot \cos \theta_3 + \ddot{\theta}_3 \cdot (\ddot{r}_3 \cdot \cos \theta_3 - \ddot{r}_4 \cdot \cos \theta_4) - \ddot{r}_3 \cdot \dot{\theta}_3^2 \cdot \sin \theta_3 + \ddot{r}_4 \cdot \dot{\theta}_3^2 \cdot \sin \theta_4 = 0 \quad (38)$$

SUBSTITUTING (37) INTO (38)

$$\begin{aligned} & r_2 \ddot{\theta}_2 \cos \theta_2 \cos \theta_3 - r_2 \dot{\theta}_2^2 \sin \theta_2 \cos \theta_3 + r_2 \ddot{\theta}_2 \sin \theta_2 \sin \theta_3 + r_2 \dot{\theta}_2^2 \cos \theta_2 \sin \theta_3 + 2 \cdot r_3 \dot{\theta}_2 \dot{\theta}_3 \sin^2 \theta_3 \\ & + r_3 \ddot{\theta}_3 \sin^2 \theta_3 - r_4 \dot{\theta}_3^2 \sin \theta_3 \sin \theta_4 + r_3 \dot{\theta}_3^2 \cos \theta_3 \sin \theta_4 - r_4 \dot{\theta}_3^2 \cos \theta_3 \sin \theta_4 + 2 \cdot r_3 \dot{\theta}_3 \dot{\theta}_4 \cos^2 \theta_3 \\ & + r_3 \ddot{\theta}_3 \cos \theta_3 - r_4 \ddot{\theta}_3 \cos \theta_3 \cos \theta_4 - r_3 \dot{\theta}_3^2 \sin \theta_3 \cos \theta_4 + r_4 \dot{\theta}_3^2 \sin \theta_4 \cos \theta_3 = 0 \end{aligned}$$

$$\begin{aligned} & r_2 \ddot{\theta}_2 (\cos \theta_3 \cos \theta_2 + \sin \theta_3 \sin \theta_2) + r_2 \dot{\theta}_2^2 (\sin \theta_3 \cos \theta_2 - \cos \theta_3 \sin \theta_2) + 2 \cdot r_3 \dot{\theta}_2 \dot{\theta}_3 (\sin^2 \theta_3 + \cos^2 \theta_3) \\ & + r_3 \ddot{\theta}_3 (\sin^2 \theta_3 + \cos^2 \theta_3) - r_4 \ddot{\theta}_3 (\cos \theta_3 \cos \theta_4 + \sin \theta_3 \sin \theta_4) + r_3 \dot{\theta}_3^2 (\sin \theta_3 \cos \theta_4 - \cos \theta_3 \sin \theta_4) \\ & - r_4 \dot{\theta}_3^2 (\sin \theta_3 \cos \theta_4 - \cos \theta_3 \sin \theta_4) = 0 \end{aligned}$$

$$r_2 \ddot{\theta}_2 \cos (\theta_3 - \theta_2) + r_2 \dot{\theta}_2^2 \sin (\theta_3 - \theta_2) + 2 \cdot r_3 \dot{\theta}_2 \dot{\theta}_3 + r_3 \ddot{\theta}_3 \cos (\theta_3 - \theta_4) + r_3 \dot{\theta}_3^2 \sin (\theta_3 - \theta_4) - r_4 \ddot{\theta}_3 \cos (\theta_3 - \theta_4) = 0$$

$$r_2 \ddot{\theta}_2 \cos (\theta_3 - \theta_2) + r_2 \dot{\theta}_2^2 \sin (\theta_3 - \theta_2) + 2 \cdot r_3 \dot{\theta}_2 \dot{\theta}_3 + [r_3 - r_4 \cos (\theta_3 - \theta_4)] \ddot{\theta}_3 - r_4 \dot{\theta}_3^2 \sin (\theta_3 - \theta_4) = 0$$

$$\ddot{\theta}_3 = - \frac{r_2 \ddot{\theta}_2 \cos (\theta_3 - \theta_2) + r_2 \dot{\theta}_2^2 \sin (\theta_3 - \theta_2) + 2 \cdot r_3 \dot{\theta}_2 \dot{\theta}_3 - r_4 \dot{\theta}_3^2 \sin (\theta_3 - \theta_4)}{r_3 - r_4 \cos (\theta_3 - \theta_4)}$$