FOR THE TWO-DIMENSIONAL TRUSS STRUCTURE SHOWN, DETERMINE THE NODAL DEFLECTIONS AND THE STRESS AND NODAL FORCES OF EACH ELEMENT. THE SUPPORT REACTION FORCES R_{1x}, R_{1y}, AND R_{3x} ARE ALSO INDICATED IN THE FIGURE NEXT TO THE SUPPORTS. MEMBERS (1) AND (2) EACH HAVE A LENGTH OF 2m. EACH MEMBER HAS A CROSS-SECTIONAL ARE OF $80 mm^2$ AND A MODULUS OF ELASTICITY OF 200 GPa. IGNOR THE POSSIBILITY OF BUCKLING IN COMPRESSION MEMBERS.

GIVEN:

CONSTRAINTS

1. LENGTH OF (1) AND (2) IS 2m.
2. CROSS-SECTION OF EACH MEMBER $80 mm^2$
3. MODULUS OF ELASTICITY FOR EACH MEMBER IS 200GPa
4. REACTIONS AT NODE 1 ARE RESTRICTED FROM TRANSLATION IN X ANY Y
5. REACTION AT NODE 3 RESTRICTED FROM TRANSLATION IN X ONLY
6. LOADS OF 2 kN IN THE X AND 3 kN IN THE Y ARE APPLIED AT NODE 2

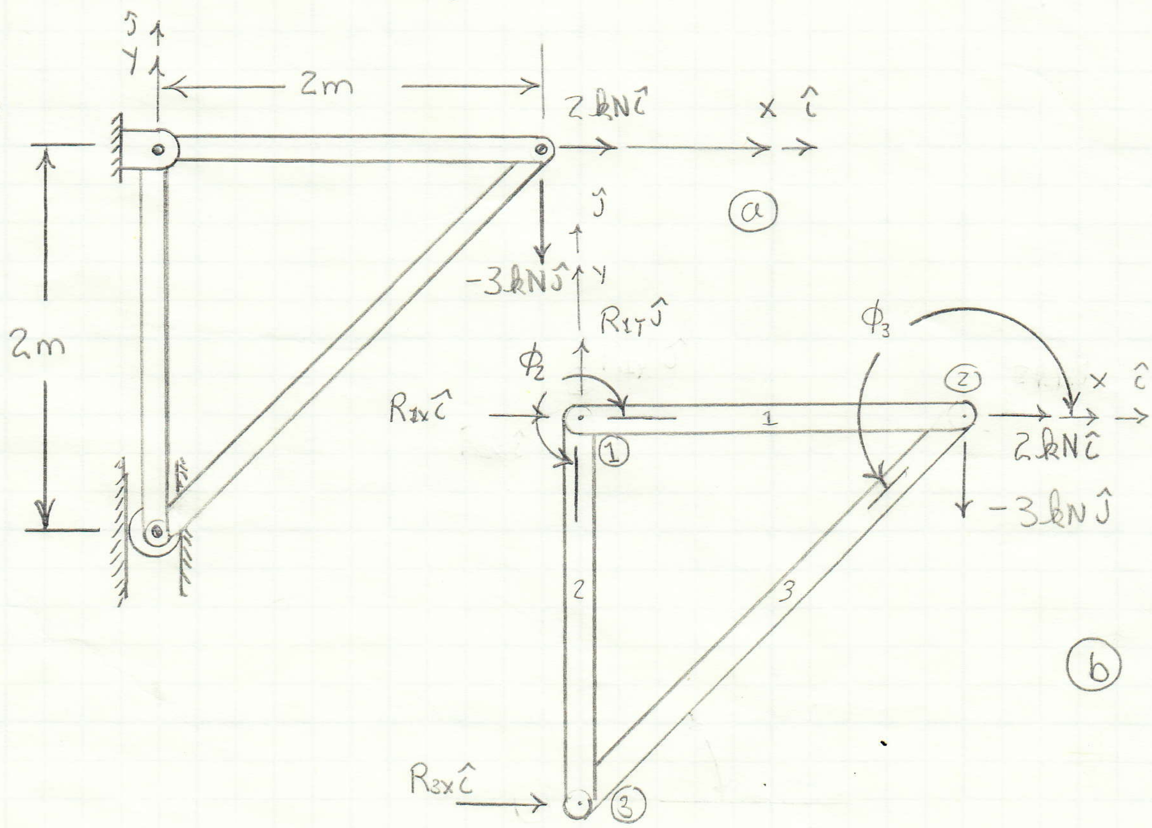
ASSUMPTIONS

1. DISPLACEMENTS ARE SMALL IN ALL MEMBERS
2. NONE OF THE MEMBERS BUCKLE

FIND:

1. DETERMINE ALL NODAL DEFLECTIONS
2. DETERMINE THE STRESS IN EACH ELEMENT
3. DETERMINE THE NODAL FORCES

DIAGRAM:



FINITE ELEMENT SOLUTION:

FIGURE (b) SHOWS THIS STRUCTURE TO HAVE 3 ELEMENTS. THE STIFFNESS FOR EACH OF THE ELEMENTS IS GIVEN BY

$$k_1 = \frac{A_1 \cdot E_1}{L_1} = \frac{80(10^{-6})m^2 \cdot 200(10^9)\frac{N}{m^2}}{2m} = 8(10^6)\frac{N}{m} \qquad (1)$$

$$k_2 = \frac{A_2 \cdot E_2}{L_2} = \frac{80(10^{-6})m^2 \cdot 200(10^9)\frac{N}{m^2}}{2m} = 8(10^6)\frac{N}{m} \qquad (2)$$

$$k_3 = \frac{A_3 \cdot E_3}{L_3} = \frac{80(10^{-6})m^2 \cdot 200(10^9)\frac{N}{m^2}}{2.828m} = 5.658(10^6)\frac{N}{m} \qquad (3)$$

NOW CALCULATING THE ANGLE EACH ELEMENT MAKES WITH THE POSITIVE HORIZONTAL AXES. THE CONVENTION THAT WILL BE USED HERE IS THAT THE LOWER NODE NUMBER IS DEFINED AS "i" AND THE HIGHER NODE NUMBER AS "j".

$$\phi_1 = \tan^{-1}\left(\frac{Y_2 - Y_1}{X_2 - X_1}\right) = \tan^{-1}\left(\frac{0-0}{2-0}\right) = 0 \qquad (4)$$

$$\phi_2 = \tan^{-1}\left(\frac{Y_3 - Y_1}{X_3 - X_1}\right) = \tan^{-1}\left(\frac{-2-0}{0-0}\right) = 270^\circ \qquad (5)$$

$$\phi_3 = \tan^{-1}\left(\frac{Y_3 - Y_2}{X_3 - X_2}\right) = \tan^{-1}\left(\frac{-2-0}{0-2}\right) = 225^\circ \qquad (6)$$

NOTE: THE TANGENT FUNCTION IS POSITIVE IN THE FIRST AND THIRD QUADRENTS, AND NEGATIVE IN THE SECOND AND FOURTH QUADRENTS. TO UNIQUELY DETERMINE THE CORRECT ANGLE, TYPICALLY INVERSE SINE AND COSINE FUNCTIONS ARE USED. ANOTHER WAY THIS IS ACHIEVED IS TO CAREFULLY KEEP TRACT OF THE SIGNS IN THE NUMERATOR AND DENOMINATOR OF THE FRACTION INSIDE THE TANGENT FUNCTION.

THE TRANSFORMATION MATRIX FOR EACH ELEMENT CAN NOW BE CALCULATED KNOWING THAT $l = \cos\phi$ AND $m = \sin\phi$

$$[T]_1 = \begin{bmatrix} l_1 & m_1 & 0 & 0 \\ 0 & 0 & l_1 & m_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (7)$$

$$[T]_2 = \begin{bmatrix} l_2 & m_2 & 0 & 0 \\ 0 & 0 & l_2 & m_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad (8)$$

$$[T]_3 = \begin{bmatrix} l_3 & m_3 & 0 & 0 \\ 0 & 0 & l_3 & m_3 \end{bmatrix} = \begin{bmatrix} -.7071 & -.7071 & 0 & 0 \\ 0 & 0 & -.7071 & -.7071 \end{bmatrix} \qquad (9)$$

THE LOCAL STIFFNESS MATRICES TRANSFORMED INTO THE GLOBAL SYSTEM ARE GIVEN BY, STARTING WITH ELEMENT 1.

$$[k_1]_L = \frac{A_1 \cdot E_1}{L_1} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k_1]_G = [T_1]^T \cdot [k_1]_L \cdot [T_1]$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot 8(10^6)\frac{N}{m} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= 8(10^6)\frac{N}{m} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

$$\begin{Bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \end{Bmatrix} = 8(10^6)\frac{N}{m} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

THE ABOVE EQUATION CAN BE WRITTEN IN TERMS OF ALL THE SYSTEM PARAMETERS AS

$$\begin{Bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \end{Bmatrix} = 8(10^6)\frac{N}{m} \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \tag{11}$$

NOW CONSIDERING ELEMENT 2

$$[k_2]_L = \frac{A_2 \cdot E_2}{L_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 8(10^6) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k_2]_G = [T_2]^T \cdot [k_2]_L \cdot [T_2]$$

$$= \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} 8(10^6)\frac{N}{m} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$= 8(10^6)\frac{N}{m} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} f_{x1} \\ f_{y1} \\ f_{x3} \\ f_{y3} \end{Bmatrix} = 8(10^6)\frac{N}{m} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{Bmatrix}$$

THE ABOVE EQUATION CAN NOW BE WRITTEN IN TERMS OF ALL THE SYSTEM PARAMETERS

$$\begin{Bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \end{Bmatrix} = 8(10^6)\frac{N}{m} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \quad (12)$$

NOW CONSIDERING ELEMENT 3

$$[k_3]_L = \frac{A_3 \cdot E_3}{L_3} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 5.658(10^6)\frac{N}{m} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k_3]_G = [T_3]^T \cdot [k_3]_L \cdot [T_3]$$

$$= \begin{bmatrix} -.7071 & 0 \\ -.7071 & 0 \\ 0 & -.7071 \\ 0 & -.7071 \end{bmatrix} 5.658(10^6)\frac{N}{m} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -.7071 & -.7071 & 0 & 0 \\ 0 & 0 & -.7071 & -.7071 \end{bmatrix}$$

$$= 2.828(10^6)\frac{N}{m} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \end{Bmatrix} = 2.828(10^6)\frac{N}{m} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

$$\begin{Bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \end{Bmatrix} = 2.828(10^6)\frac{N}{m} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & -1 & -1 & 1 & 1 \\ 0 & 0 & -1 & -1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \quad (13)$$

NOW (11) - (13) CAN BE ADDED TOGETHER TO FORM THE SYSTEM STIFFNESS MATRIX.

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \end{Bmatrix} = \begin{Bmatrix} (f_{x1})_1 + (f_{x1})_2 + (f_{x1})_3 \\ (f_{y1})_1 + (f_{y1})_2 + (f_{y1})_3 \\ (f_{x2})_1 + (f_{x2})_2 + (f_{x2})_3 \\ (f_{y2})_1 + (f_{y2})_2 + (f_{y2})_3 \\ (f_{x3})_1 + (f_{x3})_2 + (f_{x3})_3 \\ (f_{y3})_1 + (f_{y3})_2 + (f_{y3})_3 \end{Bmatrix} = (10^6)\tfrac{N}{m} \begin{bmatrix} 8 & 0 & -8 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & -8 \\ -8 & 0 & 10.828 & 2.828 & -2.828 & -2.828 \\ 0 & 0 & 2.828 & 2.828 & -2.828 & -2.828 \\ 0 & 0 & -2.828 & -2.828 & 2.828 & 2.828 \\ 0 & -8 & -2.828 & -2.828 & 2.828 & 10.828 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \quad (14)$$

$F_{x1}, F_{y1}, F_{x2}, F_{y2}, F_{x3}, F_{y3}$ ARE THE EXTERNAL LOADS APPLIED AT NODES (1), (2), AND (3) IN THE "X" AND "Y" DIRECTIONS. FOR THIS PROBLEM WE CAN WRITE

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \end{Bmatrix} = \begin{Bmatrix} R_{x1} \\ R_{y1} \\ 2\,kN \\ -3\,kN \\ R_{x3} \\ 0 \end{Bmatrix}$$

THE CONSTRAINT REACTIONS R_{x1}, R_{y1}, AND R_{x3} MUST BE CALCULATED. THE KNOWN DISPLACEMENTS IN THIS PROBLEM ARE

$$\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ u_2 \\ v_2 \\ 0 \\ v_3 \end{Bmatrix}$$

(14) CAN NOW BE WRITTEN

$$\begin{Bmatrix} R_{x1} \\ R_{y1} \\ 2\,kN \\ -3\,kN \\ R_{x3} \\ 0 \end{Bmatrix} = (10^6)\tfrac{N}{m} \begin{bmatrix} 8 & 0 & -8 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & -8 \\ -8 & 0 & 10.828 & 2.828 & -2.828 & -2.828 \\ 0 & 0 & 2.828 & 2.828 & -2.828 & -2.828 \\ 0 & 0 & -2.828 & -2.828 & 2.828 & 2.828 \\ 0 & -8 & -2.828 & -2.828 & 2.828 & 10.828 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_2 \\ v_2 \\ 0 \\ v_3 \end{Bmatrix} \quad (15)$$

NOW THE ROWS ASSOCIATED WITH KNOWN DISPLACEMENTS ARE PARTITIONED OUT

$$R_{x1} = -8(10^6)\cdot\tfrac{N}{m}\cdot u_2 \quad (16)$$

$$R_{y1} = -8(10^6)\tfrac{N}{m}\cdot v_3 \quad (17)$$

$$R_{x3} = -2.828(10^6)\cdot\tfrac{N}{m}\cdot u_2 - 2.828(10^6)\tfrac{N}{m} v_2 + 2.828(10^6)\cdot\tfrac{N}{m} v_3 \quad (18)$$

IN THE REMAINING MATRIX, THE COLUMNS ON THE RIGHT-HAND SIDE OF (15) ASSOCIATED WITH KNOWN DISPLACEMENTS ARE TAKEN TO THE LEFT-HAND SIDE OF THE EQUATION

$$\begin{Bmatrix} 2\,kN \\ -3\,kN \\ 0 \end{Bmatrix} = (10^{6})\tfrac{N}{m} \begin{bmatrix} 10.828 & 2.828 & -2.828 \\ 2.828 & 2.828 & -2.828 \\ -2.828 & -2.828 & 10.828 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ v_3 \end{Bmatrix} \qquad (19)$$

SOLVING (19) FOR THE DISPLACEMENTS

$$\begin{Bmatrix} u_2 \\ v_2 \\ v_3 \end{Bmatrix} = (10^{-9})\tfrac{m}{N} \begin{bmatrix} 125 & -125 & 0 \\ -125 & 603.5 & 125 \\ 0 & 125 & 125 \end{bmatrix} \begin{Bmatrix} 2(10^3) \\ -3(10^3) \\ 0 \end{Bmatrix} = \boxed{\begin{Bmatrix} 625(10^{-6})\,m \\ -2061(10^{-6})\,m \\ -375(10^{-6})\,m \end{Bmatrix}} \qquad (20)$$

USING THE RESULTS FROM (20) IN EQUATIONS (16) - (18) TO SOLVE FOR THE UNKNOWN REACTIONS.

$$(16) \rightarrow R_{x1} = -8(10^{6})\tfrac{N}{m} \cdot 625(10^{-6})\,m = \boxed{-5\,kN} \qquad (21)$$

$$(17) \rightarrow R_{Y1} = -8(10^{6})\tfrac{N}{m} \cdot {-375}(10^{-6})\,m = \boxed{3\,kN} \qquad (22)$$

$$(18) \rightarrow R_{x3} = -2.828(10^{6})\tfrac{N}{m} \cdot 625(10^{-6})\,m - 2.828(10^{6})\tfrac{N}{m} \cdot {-2061}(10^{-6})\,m + 2.828(10^{6})\tfrac{N}{m} \cdot {-375}(10^{-6})\,m = \boxed{3\,kN} \qquad (23)$$

THE NEXT STEP IS TO CALCULATE THE ELEMENT STRESSES. THESE ARE CALCULATED IN THE LOCAL COORDINATE SYSTEM. FROM THE LECTURE NOTES

$$(\sigma_x)_e = \frac{E_e}{L_e} \cdot [-1 \quad 1] \cdot \begin{Bmatrix} u_i' \\ u_j' \end{Bmatrix}_e$$

$$\begin{Bmatrix} u_i' \\ u_j' \end{Bmatrix} = [T] \cdot \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{Bmatrix}$$

$$\sigma_x = \frac{E}{L_e} \cdot [-1 \quad 1] \cdot [T] \cdot \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{Bmatrix} \qquad (24)$$

(24) CAN NOW BE APPLIED TO EACH ELEMENT.

ELEMENT 1

$$(\sigma)_1 = \frac{200(10^9)\frac{N}{m^2}}{2\,m} \cdot [-1 \quad 1] \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ 0 \\ 625(10^{-6})\,m \\ 2061(10^{-6})\,m \end{Bmatrix} = 62.5\,MPa \qquad (25)$$

ELEMENT 2

$$(\sigma)_2 = \frac{200(10^9)\frac{N}{m^2}}{2\,m} \cdot [-1 \quad 1] \cdot \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -375(10^{-6})\,m \end{Bmatrix} = 37.5\,MPa \qquad (26)$$

ELEMENT 3

$$(\sigma)_3 = \frac{200(10^9)\frac{N}{m^2}}{2.828\,m} \cdot [-1 \quad 1] \cdot \begin{bmatrix} -.7071 & -.7071 & 0 & 0 \\ 0 & 0 & -.7071 & -.7071 \end{bmatrix} \cdot \begin{Bmatrix} 625(10^{-6})\,m \\ 2061(10^{-6})\,m \\ 0 \\ -375(10^{-6})\,m \end{Bmatrix} = -53.03\,MPa \qquad (27)$$

IT IS ALSO INSTRUCTIVE TO CALCULATE ELEMENT NODAL FORCES. FROM THE LECTURE

$$\begin{Bmatrix} f'_{xi} \\ f'_{xj} \end{Bmatrix} = \frac{A_1 \cdot E_1}{L_1} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{Bmatrix} u'_i \\ u'_j \end{Bmatrix}$$

$$= \frac{A_1 \cdot E_1}{L_1} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot [T] \cdot \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{Bmatrix} \qquad (28)$$

(28) CAN NOW BE APPLIED TO EACH ELEMENT.

ELEMENT 1

$$\begin{Bmatrix} f'_{x1} \\ f'_{x2} \end{Bmatrix}_1 = 8(10^6)\frac{N}{m} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ 0 \\ 625(10^{-6})m \\ 2061(10^{-6})m \end{Bmatrix}$$

$$= \boxed{\begin{Bmatrix} -5\,kN \\ 5\,kN \end{Bmatrix}} \qquad (29)$$

ELEMENT 2

$$\begin{Bmatrix} f'_{x1} \\ f'_{x3} \end{Bmatrix}_2 = 8(10^6)\frac{N}{m} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -375(10^{-6})m \end{Bmatrix}$$

$$= \boxed{\begin{Bmatrix} -3\,kN \\ 3\,kN \end{Bmatrix}} \qquad (30)$$

ELEMENT 3

$$\begin{Bmatrix} f'_{x2} \\ f'_{x3} \end{Bmatrix}_3 = 5.658(10^6)\frac{N}{m} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -.7071 & -.7071 & 0 & 0 \\ 0 & 0 & -.7071 & -.7071 \end{bmatrix} \cdot \begin{Bmatrix} 625(10^{-6})m \\ 2061(10^{-6})m \\ 0 \\ -375(10^{-6})m \end{Bmatrix}$$

$$= \boxed{\begin{Bmatrix} 4.243\,kN \\ -4.243\,kN \end{Bmatrix}} \qquad (31)$$

NOW LET'S CONSIDER THIS SAME PROBLEM USING STATICS. FROM (6)

$$\Sigma F_x = 0 = R_{1x} + R_{3x} + 2\,kN \Rightarrow R_{1x} + R_{3x} = -2\,kN \quad (32)$$

$$\Sigma F_y = 0 = R_{1y} - 3\,kN \Rightarrow \boxed{R_{1y} = 3\,kN} \text{ (THIS CHECKS WITH (22))} \quad (33)$$

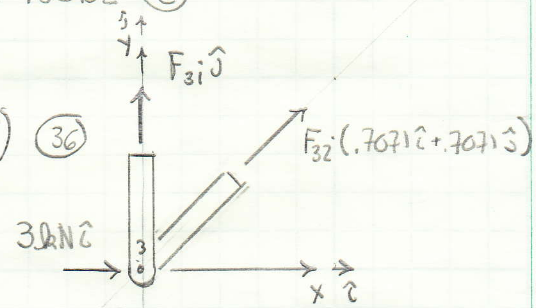
$$\Sigma M_{z/O1} = 0 = -(2m)\cdot(3\,kN) + (2m)\cdot(R_{3x})$$

$$\Rightarrow R_{3x} = \frac{2m \cdot 3\,kN}{2m} = \boxed{3\,kN = R_{3x}} \text{ (THIS CHECKS WITH (23))} \quad (34)$$

SUBSTITUTING (34) INTO (32)

$$R_{1x} + 3\,kN = -2\,kN \Rightarrow \boxed{R_{1x} = -5\,kN} \text{ (THIS CHECKS WITH (21))} \quad (35)$$

NOW LETS CONSIDER THE LOADS INTERNAL TO EACH ELEMENT OF THE TRUSS. USING METHOD OF JOINTS, STARTING WITH NODE (3)



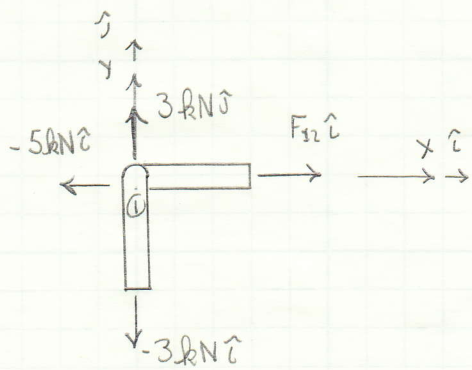
$$\Sigma F_x = 0 = 3\,kN + .7071 \cdot F_{32}$$

$$\Rightarrow F_{32} = -\frac{3\,kN}{.7071} = \boxed{-4.243\,kN} \text{ (THIS CHECKS WITH (31))} \quad (36)$$

$$\Sigma F_y = 0 = F_{31} + .7071 \cdot F_{32}$$

$$\Rightarrow F_{31} = -.7071 \cdot (-4.243\,kN) = \boxed{3\,kN} \text{ (THIS CHECKS WITH (30))} \quad (37)$$

NOW CONSIDER NODE (1)



$$\Sigma F_x = 0 = -5\,kN + F_{12}$$

$$\Rightarrow \boxed{F_{12} = 5\,kN} \text{ (THIS CHECKS WITH (29))}$$