PROBLEM 5.10 | THE CROSS SECTION OF THE ALUMINUM TUBE SHOWN TRANSMITS A TORSIONAL MOMENT OF 5000 lb·in. LET E = 10 Msi AND ν = 0.33. THE SECTION, DIMENSIONED IN INCHES, IS SYMMETRIC WITH RESPECT TO THE y AND z AXES. THE 1.0 in DIMENSIONS ARE BETWEEN WALL CENTERLINES, AND THE CENTER LINES OF THE 0.25 IN VERTICAL WALLS ALIGN WITH THE CENTERLINES OF THE ENDS OF THE CIRCULAR WALLS. DETERMINE THE AVERAGE SHEAR STRESS IN EACH WALL AND THE ANGLE OF TWIST OVER A LENGTH OF 24 in.

GIVEN:

1. TORSIONAL MOMENT OF 5000 lb·in
2. MATERIAL WITH E = 10 Msi AND ν = 0.33
3. LENGTH OF THE TUBE IS 24 in
4. CROSS-SECTION SHOWN

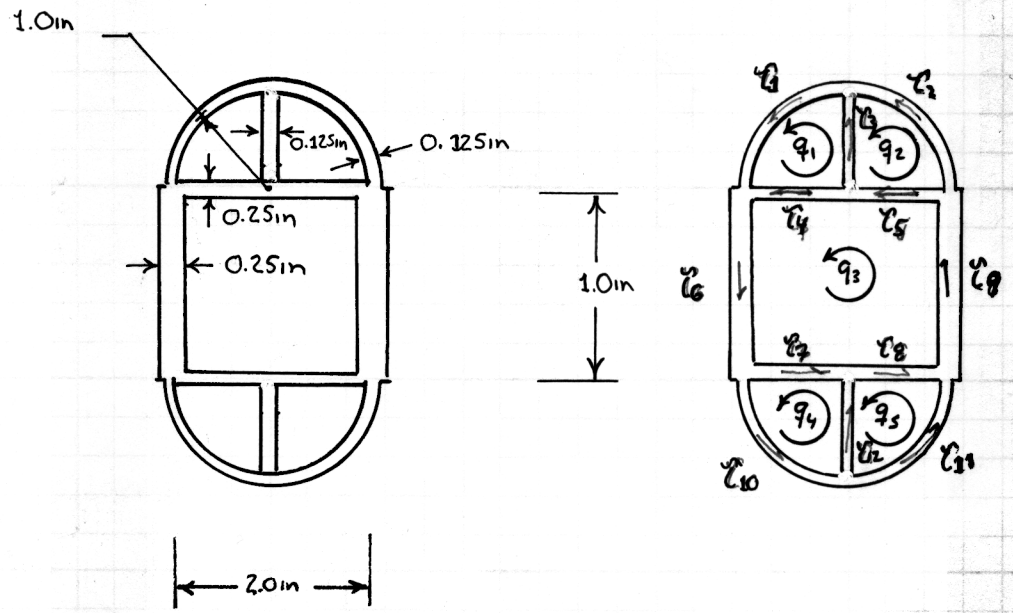
ASSUMPTIONS:

1. LINEAR ELASTIC BEHAVIOR
2. SMALL DEFORMATIONS

FIND:

1. DETERMINE THE AVERAGE SHEAR STRESS IN EACH WALL
2. DETERMINE THE ANGLE OF TWIST OVER THE 24 in LENGTH.

FIGURE:



SOLUTION:

THE TORQUE IN THIS CROSS-SECTION IS WRITTEN IN TERMS OF THE TORQUE CONTRIBUTIONS OF EACH CELL

$$T = \sum T_i = 2 \cdot \sum q_i \cdot A_i$$

$$= 2 \cdot [q_1 \cdot \tfrac{1}{4} \cdot \pi \cdot (1.0in)^2 + q_2 \cdot \tfrac{1}{4} \cdot \pi \cdot (1.0in)^2 + q_3 \cdot (1.0in)(2.0in) + q_4 \cdot \tfrac{1}{4} \cdot \pi \cdot (1.0in)^2 + q_5 \cdot \tfrac{1}{4} \cdot \pi \cdot (1.0in)^2]$$

$$5000 lb \cdot in = 1.571 in^2 \cdot q_1 + 1.571 in^2 \cdot q_2 + 4.0 in^2 \cdot q_3 + 1.571 in^2 \cdot q_4 + 1.571 in^2 \cdot q_5 \quad (1)$$

IT IS ALSO ASSUMED THAT THE ANGLE OF TWIST IS THE SAME FOR EACH SUB-SECTION OF THE CROSS-SECTION.

$$\phi = \frac{L}{2 \cdot G \cdot A_i} \left(\oint \frac{q}{t} \cdot ds \right)_i \Rightarrow \frac{2 \cdot G \cdot A_i}{L} \cdot \phi = \left(\oint \frac{q}{t} ds \right)_i$$

THE ABOVE EQUATION CAN NOW BE APPLIED TO EACH SECTION SEPERATELY. FIRST THE SHEAR MODULUS CAN BE CALCULATED

$$G = \frac{E}{2(1+\nu)} = \frac{10(10^6) lb/in^2}{2(1+0.33)} = 3.759(10^6) lb/in^2$$

NOW THE FIVE EQUATIONS CAN BE WRITTEN. FOR SECTION 1

$$\frac{2 \cdot 3.759(10^6) lb/in^2 \cdot \frac{1}{4} \cdot \pi \cdot (1.0in)^2}{24in} \cdot \phi = \frac{\frac{1}{4} \cdot 2 \cdot \pi \cdot (1in)}{0.125in} q_1 + \frac{1.0in}{0.125in}(q_1 - q_3) + \frac{1.0in}{0.125in}(q_1 - q_2)$$

(12.57, 4, 8)

$$246.0 \times 10^3 \frac{lb}{in} \cdot \phi = 24.57 \cdot q_1 + 4 \cdot (q_1 - q_3) + 8 \cdot (q_1 - q_2)$$

$$0 = 24.57 \cdot q_1 - 8 \cdot q_2 - 4 \cdot q_3 - 246.0 \times 10^3 \frac{lb}{in} \phi \quad (2)$$

FOR SECTION 2

$$\frac{2 \cdot 3.759(10^6) lb/in^2 \cdot \frac{1}{4} \cdot \pi \cdot (1.0in)^2}{24in} \cdot \phi = \frac{\frac{1}{4} \cdot 2 \cdot \pi \cdot (1in)}{0.125in} \cdot q_2 + \frac{1.0in}{0.125in}(q_2 - q_1) + \frac{1.0in}{0.125in}(q_2 - q_3)$$

(12.57, 8, 4)

$$246 \times 10^3 \frac{lb}{in} \cdot \phi = 24.57 \cdot q_2 + 8 \cdot (q_2 - q_1) + 4 \cdot (q_2 - q_3)$$

$$0 = -8 \cdot q_1 + 24.57 \cdot q_2 - 4 \cdot q_3 - 246.0 \times 10^3 \frac{lb}{in} \phi \quad (3)$$

FOR SECTION 3.

$$\frac{2\cdot 3.759(10^6)\,lb/in^2\cdot 2in\cdot 1in}{24in}\cdot\phi = \frac{2\cdot 1.0in}{0.25in}\cdot q_3 + \frac{1.0in}{0.25in}(q_3-q_1) + \frac{1.0in}{0.25in}(q_3-q_2) + \frac{1.0in}{0.25in}(q_3-q_4) + \frac{1.0in}{0.25in}(q_3-q_5)$$

$$625.0(10^3)\,\frac{lb}{in}\cdot\phi = -4.0\cdot q_1 - 4.0 q_2 + 24\cdot q_3 - 4.0\cdot q_4 - 4.0\cdot q_5 \qquad (4)$$

SECTION 4.

$$\frac{2\cdot 3.759\times 10^6\,\frac{lb}{in^2}\cdot\frac{1}{4}\cdot\pi\cdot(1.0in)^2}{24in}\cdot\phi = \frac{\frac{1}{4}\cdot 2\cdot\pi\cdot(1in)}{0.125in}\cdot q_4 + \frac{1.0in}{0.125in}(q_4-q_5) + \frac{1.0in}{0.25in}(q_4-q_3)$$

$$246.0\times 10^3\,\frac{lb}{in}\cdot\phi = 24.57\cdot q_4 - 4.0\cdot q_3 - 8.0\cdot q_5 \qquad (5)$$

SECTION 5.

$$\frac{2\cdot 3.759\times 10^6\,\frac{lb}{in^2}\cdot\frac{1}{4}\cdot\pi\cdot(1.0in)^2}{24in}\cdot\phi = \frac{\frac{1}{4}\cdot 2\cdot\pi\cdot(1in)}{0.125in}\cdot q_5 + \frac{1.0in}{0.25in}\cdot(q_5-q_3) + \frac{1.0}{0.125in}(q_5-q_4)$$

$$246.0\times 10^3\,\frac{lb}{in}\cdot\phi = 24.57\cdot q_5 - 4.0\cdot q_3 - 8.0\cdot q_4 \qquad (6)$$

EQUATIONS (1)-(6) CONTAIN SIX UNKNOWNS (q_1, q_2, q_3, q_4, q_5, & ϕ). THESE SIX EQUATIONS CAN NOW WRITTEN IN MATRIX FORM AND SOLVED SIMULTANEOUSLY.

$$\begin{Bmatrix} 5000\,lb\cdot in \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 1.571in^2 & 1.571in^2 & 2.0in^2 & 1.571in^2 & 1.571in^2 & 0 \\ 24.57 & -8.0 & -4.0 & 0 & 0 & -246\times10^3\,\frac{lb}{in} \\ -8.0 & 24.57 & -4.0 & 0 & 0 & -246\times10^3\,\frac{lb}{in} \\ -4.0 & -4.0 & 24.0 & -4.0 & -4.0 & -625\times10^3\,\frac{lb}{in} \\ 0 & 0 & -4.0 & 24.57 & -8.0 & -246\times10^3\,\frac{lb}{in} \\ 0 & 0 & -4.0 & -8.0 & 24.57 & -246\times10^3\,\frac{lb}{in} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ \phi \end{Bmatrix}$$

USING MATLAB

$$\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ \phi \end{Bmatrix} = \begin{Bmatrix} 382.1\,\frac{lb}{in} \\ 382.1\,lb/in \\ 649.8\,lb/in \\ 382.1\,lb/in \\ 382.1\,lb/in \\ 0.0152 \end{Bmatrix}$$

$$\tau_1 = \tau_2 = \tau_{10} = \tau_{11} = \frac{382.1\,\frac{lb}{in}}{0.125in} = \boxed{3.06\,ksi}$$

$$\tau_3 = \tau_{12} = \frac{(q_1-q_2)}{0.125in} = \frac{(q_4-q_5)}{0.125in} = \frac{(382.1\,lb/in - 382.1\,lb/in)}{0.125in} = \boxed{0}$$

$$\tau_4 = \tau_5 = \tau_7 = \tau_8 = \frac{(q_3-q_1)}{0.25in} = \frac{(q_3-q_2)}{0.25in} = \frac{(q_3-q_4)}{0.25in} = \frac{(q_3-q_5)}{0.25in}$$

$$= \frac{(649.8\,\frac{lb}{in} - 382.1\,\frac{lb}{in})}{0.25in} = \boxed{1.07\,ksi}$$

$$\tau_6 = \tau_9 = \frac{q_3}{0.25in} = \frac{649.8\,lb/in}{0.25in} = \boxed{2.60\,ksi}$$

$$\phi = \boxed{0.0152\,rad \cdot \frac{180^\circ}{\pi} = 0.87^\circ}$$

SUMMARY: CARE MUST BE TAKEN TO INSURE PROPER UNITS.