PROBLEM 3 | THE BEAM ABCD IS LOADED BY A FORCE W = 27 kN BY THE ARRANGEMENT SHOWN IN THE FIGURE. THE CABLE PASSES OVER A SMALL FRICTIONLESS PULLEY AT "B" AND IS ATTACHED AT "E" TO THE VERTICAL ARM. CALCULATE THE AXIAL FORCE N, SHEAR FORCE V, AND BENDING MOMENT M AT SECTION "C", WHICH IS JUST TO THE LEFT OF THE VERTICAL ARM.



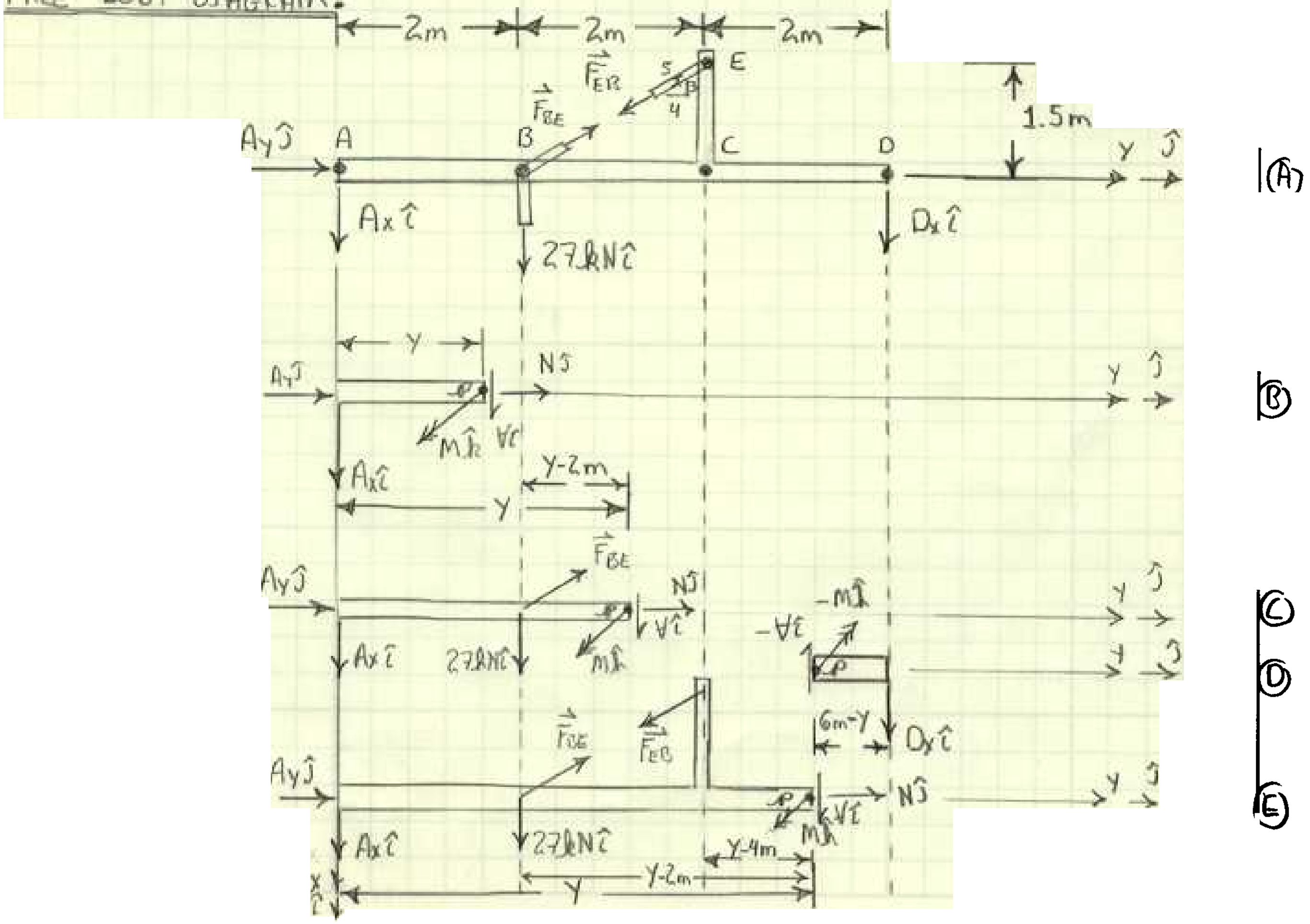
GIVEN:

CONSTRAINTS
1) BEAM SHOWN WITH 27 kN WEIGHT ATTACHED
2) PIN JOINT AT "A" AND ROLLER SUPPORT AT "D"
3) CABLE ATTACHING POINTS B AND E TO THE WEIGHT

ASSUMPTIONS
1) BEAM AND CABLE MATERIAL ARE LINEAR-ELASTIC
2) THE ROLLER AT "B" IS FRICTIONLESS
3) ALL DEFLECTIONS ARE SMALL

FIND:
1) THE AXIAL FORCE, SHEAR FORCE, AND BENDING MOMENT AT SECTION "C", JUST TO THE LEFT OF THE VERTICAL ARM

FREE BODY DIAGRAM:



## MECHANICS:

THE SOLUTION TO THIS PROBLEM STARTS BY DETERMINING THE REACTIONS AT "A" AND "D". IMPOSING EQUILIBRIUM ON THE STRUCTURE IN (A) STARTS BY DEFINING $\vec{F}_{BE}$ AND $F_{EB}$. SINCE THE PULLEY IS FRICTIONLESS AT B,

$$F_{BE} = F_{EB} = 27\ kN \qquad (1)$$

FROM NEWTON'S THIRD LAW

$$\vec{F}_{BE} = -\vec{F}_{EB} \qquad (2)$$

SINCE THE STRING CAN BE A TWO FORCE MEMBER

$$\vec{F}_{BE} = -\vec{F}_{EB} \quad ) = 27\ kN\ (-.6\,\hat{\imath} + .8\,\hat{\jmath})$$

$$\underline{\vec{F}_{BE} = -16.2\ kN\,\hat{\imath} + 21.6\ kN\,\hat{\jmath}} \qquad (3)$$

$$\underline{\vec{F}_{EB} = 16.2\ kN\,\hat{\imath} - 21.6\ kN\,\hat{\jmath}} \qquad (4)$$

IMPOSING EQUILIBRIUM

$$\Sigma F_x = \emptyset = A_x + D_x + 27\ kN - 16.2\ kN + 16.2\ kN$$

$$\Rightarrow \underline{A_x + D_x = -27\ kN} \qquad (5)$$

$$\Sigma F_y = \emptyset = A_y + 21.6\ kN - 21.6\ kN \quad \Rightarrow \qquad (6)$$

$$\Sigma \vec{M}/_{at\,A} = \vec{\emptyset} = \vec{r}_{AB} \times \vec{W} \qquad + \vec{r}_{AD} \times \vec{D}_x$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} \\ \emptyset & 2m \\ 27kN & \emptyset \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \emptyset & 2m & \emptyset \\ -16.2kN & 21.6kN & \emptyset \end{vmatrix}$$

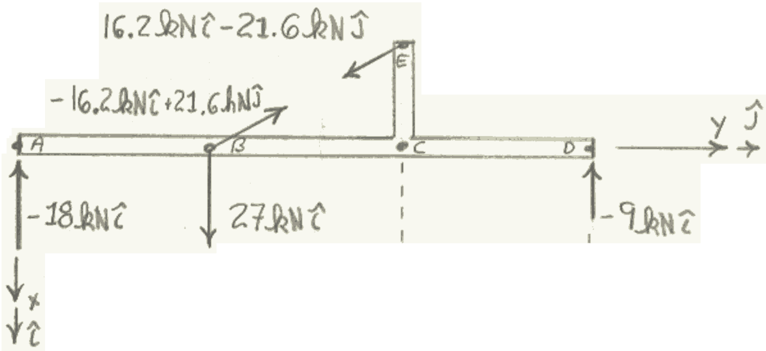
$$+ \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1.5m & 4m & \emptyset \\ 16.2kN & -21.6kN & \emptyset \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \emptyset & 6m & \emptyset \\ D_x & \emptyset & \emptyset \end{vmatrix}$$

$$= -(2m)\cdot(27kN)\hat{k} - (2m)\cdot(-16.2kN)\hat{k} + [(-1.5m)(-21.6kN) - (4m)(16.2kN)]\hat{k}$$
$$- 6m \cdot D_x$$

DOTTING THE ABOVE EQUATION WITH $\hat{k}$

$$\underline{D_x = -9.00\ kN} \rightarrow (5) \Rightarrow \underline{A_x = -18\ kN} \qquad (7)$$

Now lets consider the internal loads in each section




$0<y<2m$ (B)

$\Sigma F_x = 0 = -18\,kN + V$

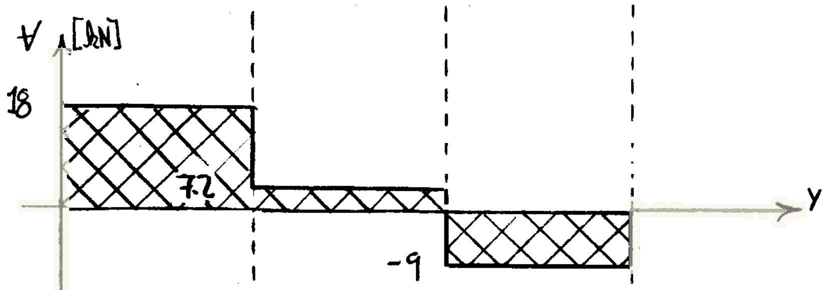
$\Rightarrow \underline{V = 18\,kN}$

$\Sigma F_y = 0 = N$

$\Sigma M_z|_p = 0 = M - 18\,kN\cdot y$

$\Rightarrow \underline{M = 18\,kN\cdot y}$

$M(0) = 0\ ;\ M(2m) = 36\,kN\cdot m$



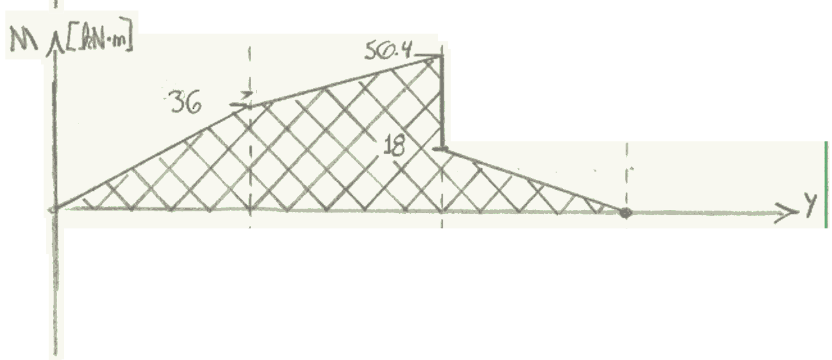

$2m<y<4m$ (C)

$\Sigma F_x = 0 = -18\,kN + 27\,kN - 16.2\,kN + V$

$\Rightarrow \boxed{V = 7.2\,kN}$

$\Sigma F_y: 0 = 21.6\,kN + N$

$\Rightarrow \boxed{N = -21.6\,kN}$

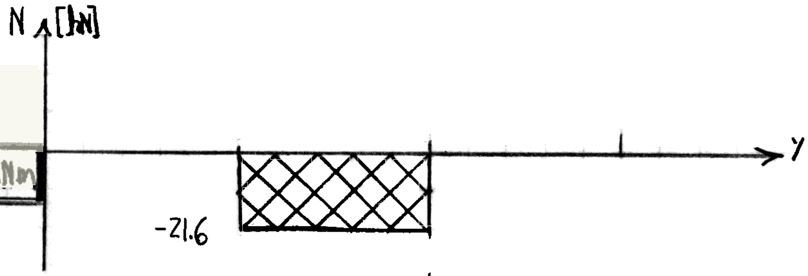



$\Sigma M_z|_p = 0 = -18\,kN\cdot y + 27\,kN(y-2m) - 16.2\,kN\cdot(y-2m) + M$

$\Rightarrow M = 18\,kN\cdot y - 10.8\,kN(y-2m)$

$= \underline{7.2\,kN\cdot y + 21.6\,kN\cdot m}$

$M(2m) = 36\,kN\cdot m$ $\boxed{M(4m) = 50.4\,kN\cdot m}$




$4m<y<6m$ (E)

$\Sigma F_x = 0 = -18\,kN + 27\,kN - 16.2\,kN + 16.2\,kN + V \Rightarrow \underline{V = 9.0\,kN}$

$\Sigma F_y = 0 = 21.6\,kN - 21.6\,kN + N \Rightarrow \underline{N = 0}$

$\Sigma M_z|_p = 0 = -18\,kN\cdot y + 27\,kN\cdot(y-2m) - 16.2\,kN\,(y-2m) + 16.2\,kN\cdot(y-4m) + 21.6\,kN\cdot 1.5m + M$

$M = +18\,kN\cdot y - 10.8\,kN(y-2m) - 16.2\,kN\cdot(y-4m) - 32.4\,kN\cdot m$

$= \underline{-9\,kN\cdot y + 54.0\,kN\cdot m}$

$M(4m) = 18.0\,kN\cdot m,\quad M(6m) = 0$

THE REGION $4m < y < 6m$ CAN ALSO BE EVALUATED USING (D)

$$\Sigma F_x = \emptyset = -V - 9\,kN \Rightarrow \underline{\underline{V = -9\,kN}}$$

$$\Sigma F_y = \emptyset = N \Rightarrow \underline{\underline{N = \emptyset}}$$

$$\Sigma M_{z/p} = \emptyset = M + 9\,kN(6m - y) \Rightarrow \underline{\underline{M = -9\,kN \cdot y + 54.0\,kN \cdot m}}$$

## SUMMARY

THE VERTICAL EXTENSION IN THIS BEAM HAS THE SAME EFFECT AS A COUPLE AT C. THIS ACCOUNTS FOR THE DISCONTINUITY IN THE M-CURVE AT C. I HAVE INCLUDED THE COMPLETE "V" AND "M" DIAGRAMS ALONG WITH THE DEVELOPMENT OF THE EQUATIONS FOR THESE CURVES.