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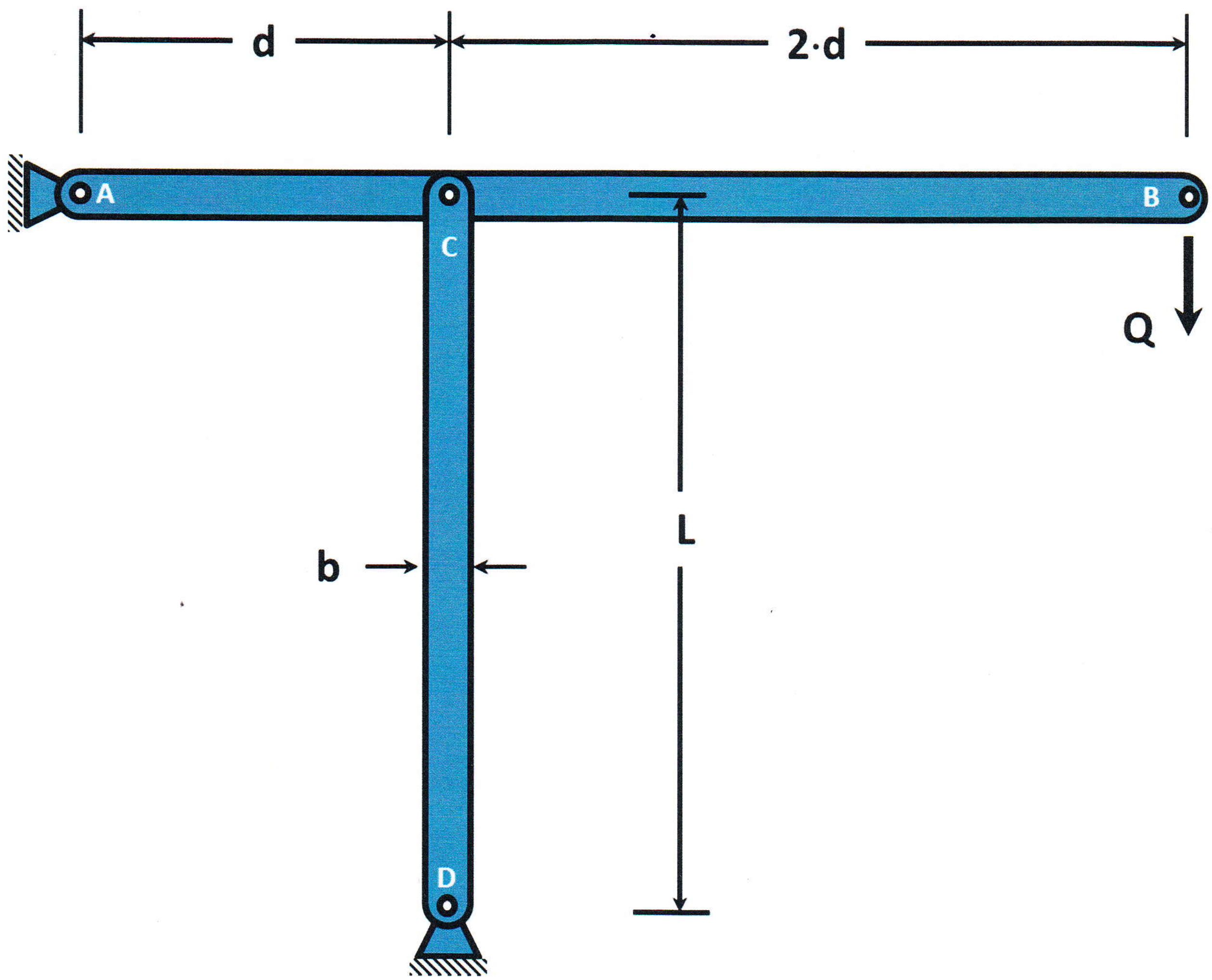
Signature: ______________________

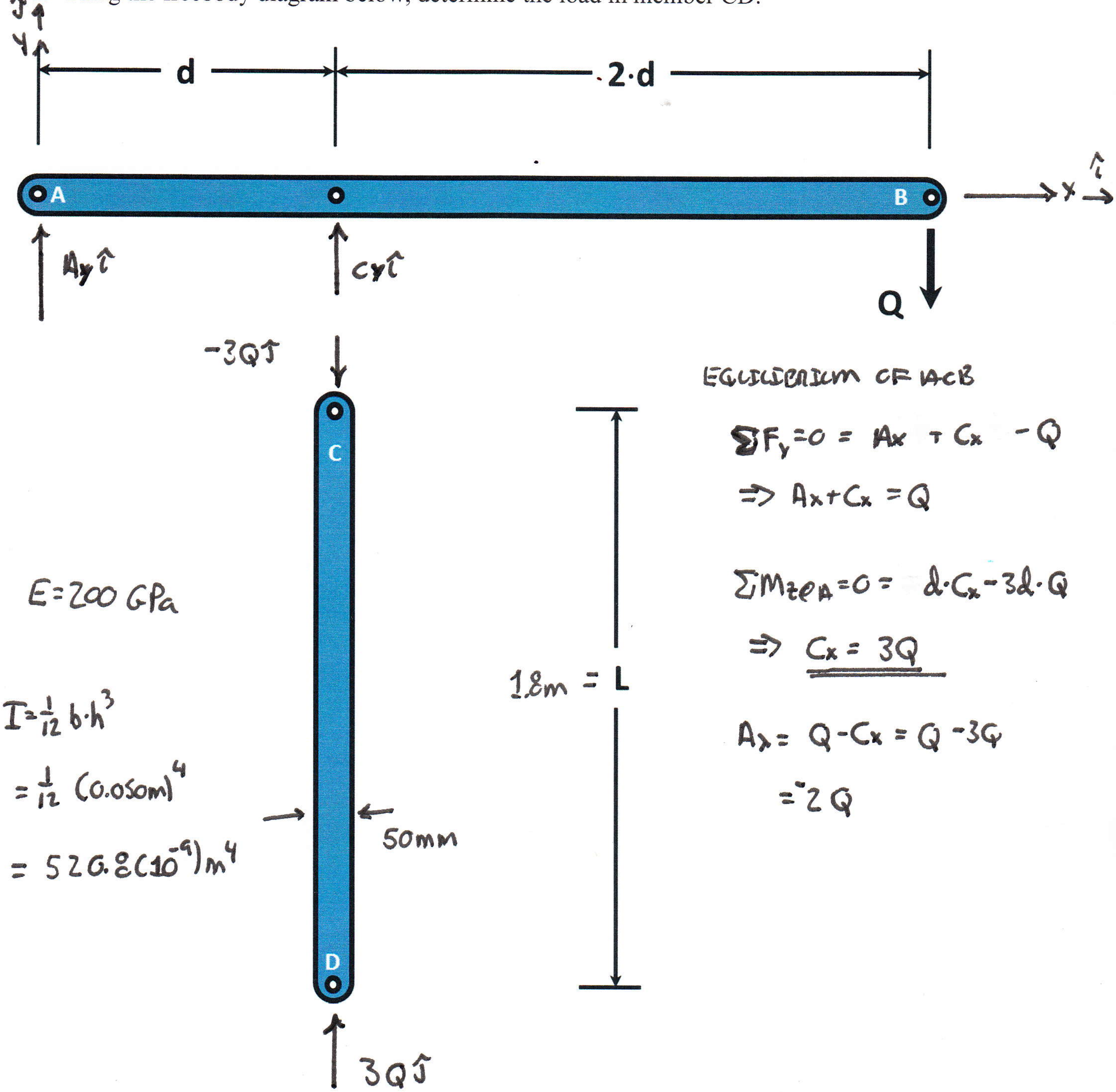
Print Name: SOLUTION

Exam Date: 7 JUNE 2016

PROBLEM 1: A horizontal beam AB is supported by a pinned-end column CD, as shown in the figure. The column CD is a solid steel bar (E=200 GPa) of square cross section having length 1.8m and side dimensions b=50mm ($I = \frac{1}{12} \cdot b \cdot h$).



1a. Using the freebody diagram below, determine the load in member CD.



EQUILIBRIUM OF ACB

$\Sigma F_y = 0 = A_x + C_x - Q$

$\Rightarrow A_x + C_x = Q$

$\Sigma M_{z@A} = 0 = d \cdot C_x - 3d \cdot Q$

$\Rightarrow \underline{C_x = 3Q}$

$A_x = Q - C_x = Q - 3Q$

$= -2Q$

$1.8m = L$

$E = 200\ GPa$

$I = \frac{1}{12} b \cdot h^3$

$= \frac{1}{12} (0.050m)^4$

$= 520.8(10^{-9})m^4$

50mm

$-3Q\hat{j}$

$3Q\hat{j}$

$A_y\hat{i}$

$C_y\hat{i}$

1b. Based upon the critical load of the column, determine the allowable load Q with respect to buckling.

FOR THE PINNED-PINNED CASE

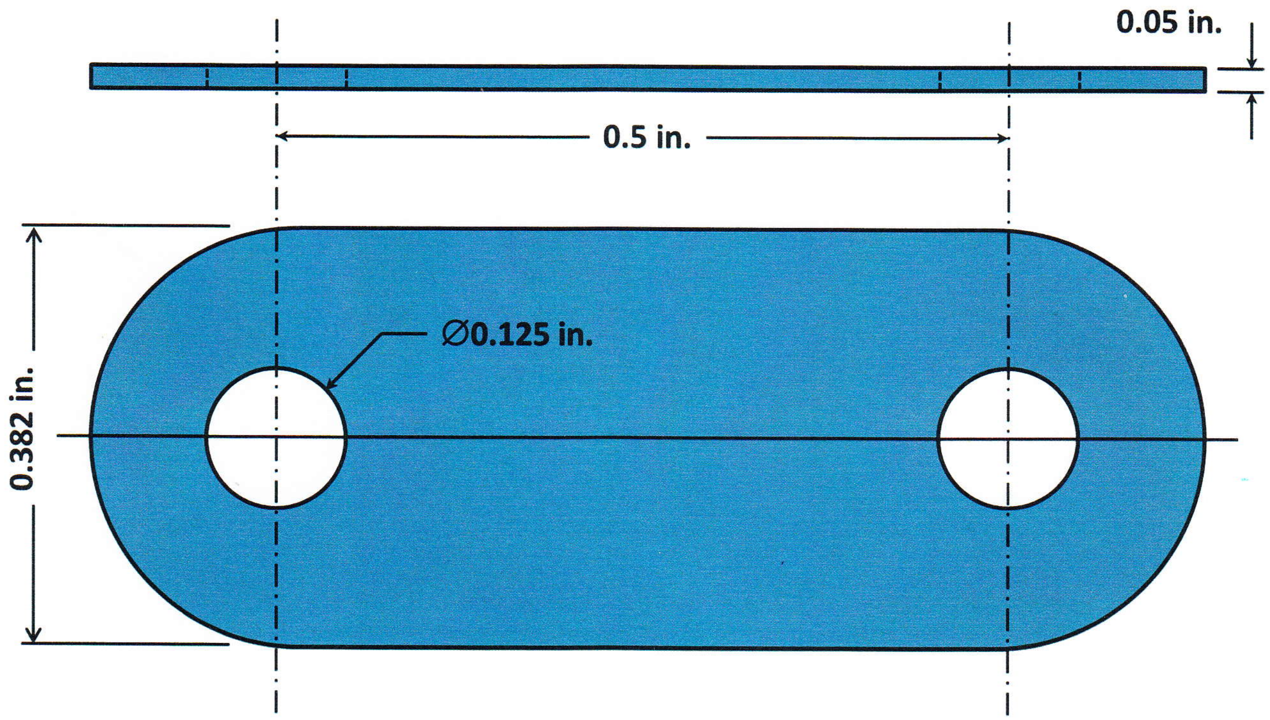
$$P_{CR} = \frac{\pi^2}{L^2} \cdot E \cdot I$$

$$3Q = \frac{\pi^2}{(1.8m)^2} \cdot 200(10^9)\frac{N}{m^2} \cdot 520.8(10^{-9})m^4$$

$$Q = \frac{\pi^2}{3 \cdot (1.8m)^2} \cdot 200(10^9)\frac{N}{m^2} \cdot 520.8(10^{-9})m^4$$

$$= \boxed{105.8(10^3)\,N}$$

PROBLEM 2: Below is a ½-in pitch roller chain plate, as used on a bicycle chain. Since a roller chain cannot transmit compression, the link is loaded in repeated axial tension (load fluctuates between 0 and a maximum force as the link goes from the slack side to the tight side of the chain) by pins that go through the two holes shown.



2a: Using the graph paper on the next page, draw the Modified Goodman diagram for this problem. The link is made of carbon steel, heat-treated to give S_u=140 ksi and S_y=110 ksi. All surfaces are comparable to the "machined" category. Use the conservative estimate of Notch Sensitivity q=1.0.

$$S_e' = 0.5 \cdot S_{UT} = 0.5 \cdot (140\,\text{ksi}) = 70\,\text{ksi}$$

$$S_e = (0.69) \cdot (1) \cdot (0.85) \cdot 70\,\text{ksi} = 41.1\,\text{ksi}$$

↳ LOAD FACTOR (AXIAL LOADING)
↳ SIZE EFFECT (AXIAL LOADING)
↳ SURFACE FINISH (SEE CHART)

FROM THE FIGURE

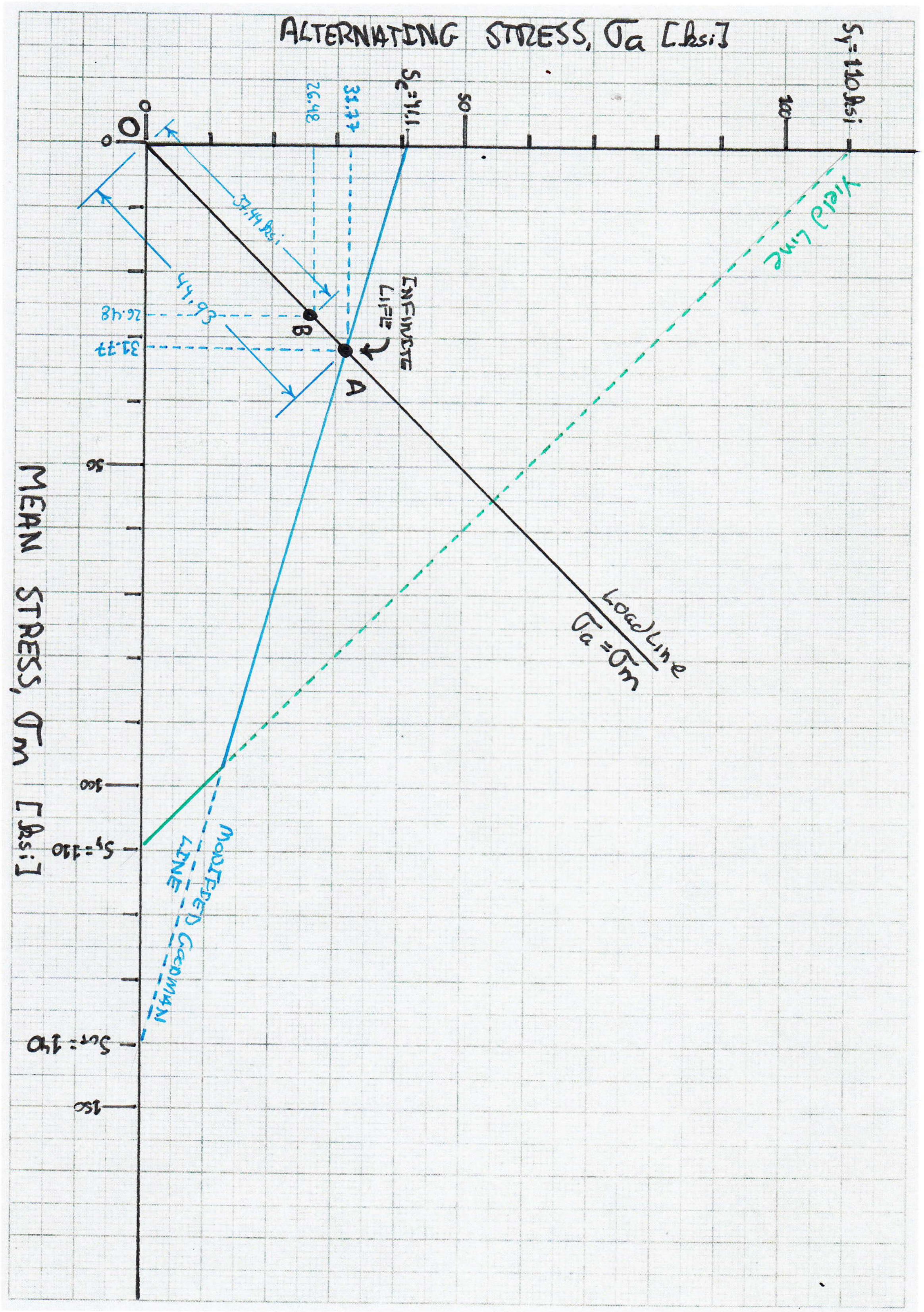
K_t = STATIC STRESS CONCENTRATION FACTOR $\left(\text{FOR } \frac{d}{D} = \frac{0.125}{0.382} = 0.327\right)$

$= 2.3$

K_f = FATIGUE STRESS CONCENTRATION FACTOR ($q=1$)

$= 1 + q \cdot (K_t - 1) = 1 + 1 \cdot (2.3 - 1)$

$= \underline{\underline{2.3}}$ (TO BE USED ON THE NOMINAL STRESS)


ALTERNATING STRESS, σa [ksi]
MEAN STRESS, σm [ksi]
Se=41.1
Sy=110 ksi
Sy=110
Sut=140
26.48
31.77
37.44 ksi
44.93
INFINITE LIFE
A
B
Load Line
σa = σm
Yield Line
MODIFIED GOODMAN LINE

2b. Estimate the maximum tensile force to prevent yielding for static loading and the maximum tensile force for a static factor of safety of 2.

THE MAXIMUM STRESS WILL OCCUR AT THE PIN HOLES

$$\sigma = K_T \cdot \frac{F}{A} = 2.3 \cdot \frac{F}{(0.05in)\cdot(0.382in - 0.125in)} = 110\,ksi$$

$$\Rightarrow F = \frac{110(10^3)\frac{lb}{in^2}\cdot(0.05in)\cdot(0.382in - 0.125in)}{2.3} = \boxed{615\,lb}$$ STATIC LOAD TO PREVENT YIELD

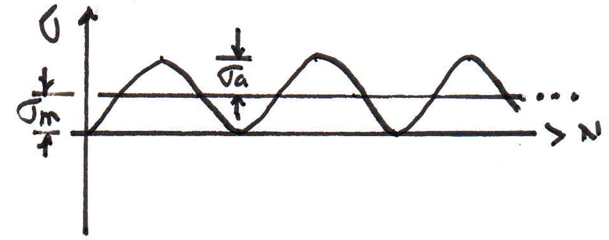
STATIC LOADING TO PREVENT YIELDING USING A FACTOR OF SAFETY OF 2

$$\sigma = K_T \cdot \frac{F}{A} \cdot n = \frac{2.3 \cdot F \cdot 2}{(0.05in)\cdot(0.382in - 0.125in)} = 110\,ksi$$

$$\Rightarrow F = \frac{110(10^3)\frac{lb}{in^2}\cdot(0.05in)\cdot(0.382in - 0.125in)}{2.3 \cdot 2} = \boxed{307\,lb}$$ STATIC LOAD TO PREVENT YIELD USING F.S. = 2

2c. Estimate the maximum tensile force that will give infinite fatigue life. Draw the fatigue stresses on the Modified Goodman Diagram in part 2a. What is the tensile force for a fatigue factor of safety of 1.2?

THE LOADS ~~ARE~~ GO FROM 0 TO MAXIMUM



$$\left.\begin{array}{l}\sigma_a = \dfrac{\sigma}{2}\\[2ex] \sigma_m = \dfrac{\sigma}{2}\end{array}\right\} \rightarrow \text{THE LOAD LINE EQUATION} \rightarrow \underline{\underline{\sigma_a = \sigma_m}}$$

THE EQUATION OF THE GOODMAN LINE

$$\sigma_a = m\cdot\sigma_m + b = -0.2936\cdot\sigma_m + 41.1\text{ ksi}$$

$$b = 41.1\text{ ksi } (S_e)$$

$$m = \frac{-41.1\text{ ksi}}{140\text{ ksi}} = \frac{-S_e}{S_{UT}} = -0.2936$$

POINT A IS THE INTERSECTION OF THE LOAD LINE AND THE MODIFIED GOODMAN LINE. POINT A'S LOCATION IS

$$\sigma_m^{(A)} = \sigma_a^{(A)} = -0.2936\cdot\sigma_m^{(A)} + 41.1\text{ ksi}$$

$$1.2936\cdot\sigma_m^{(A)} = 41.1\text{ ksi} \Rightarrow \sigma_m^{(A)} = \sigma_A^{(A)} = \frac{41.1\text{ ksi}}{1.2936} = \underline{\underline{31.77\text{ ksi}}}$$

OA ALONG THE LOAD LINE IS

$$OA = \sqrt{2\cdot(31.77\text{ ksi})^2} = 44.93\text{ ksi}$$

THE MAXIMUM TENSILE FORCE FOR INFINITE LIFE

$$\frac{\sigma}{2} = \sigma_A = \sigma_m \Rightarrow \sigma = 2\sigma_A = K_f\cdot\frac{F}{A}$$

$$\Rightarrow F = \frac{2\cdot\sigma_A\cdot A}{K_f} = \frac{2\cdot(31.77\times10^3\,\frac{\text{lb}}{\text{in}^2})\cdot(0.382\text{in}-0.125\text{in})(.05\text{in})}{2.3} = \boxed{355\text{ lb}}$$ MAX LOAD FOR INFINITE LIFE

CALCULATING THE MAXIMUM LOAD FOR INFINITE LIFE AND A FACTOR OF SAFETY OF 1.2

$$F.S. = n = \frac{OA}{OB} \Rightarrow OB = \frac{OA}{n} = \frac{44.93\,ksi}{1.2} = 37.44\,ksi$$

Now $\sigma_a^{(B)}$ AND $\sigma_m^{(B)}$ CAN BE CALCULATED

$$\sigma_a^{(B)} = \sigma_m^{(B)} = \sqrt{\frac{(OB)^2}{2}} = \sqrt{\frac{(37.44\,ksi)^2}{2}} = 26.48\,ksi$$

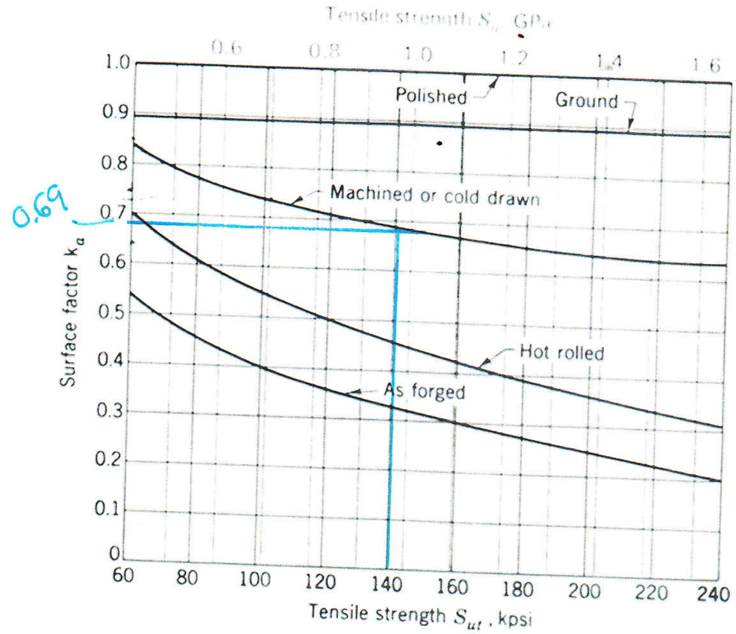
THE MAXIMUM TENSILE FORCE

$$\frac{\sigma}{2} = \sigma_a^{(B)} = \sigma_m^{(B)} \Rightarrow \sigma = 2 \cdot \sigma_a^{(B)} = 2 \cdot \sigma_m^{(B)} = K_f \cdot \frac{F}{A}$$

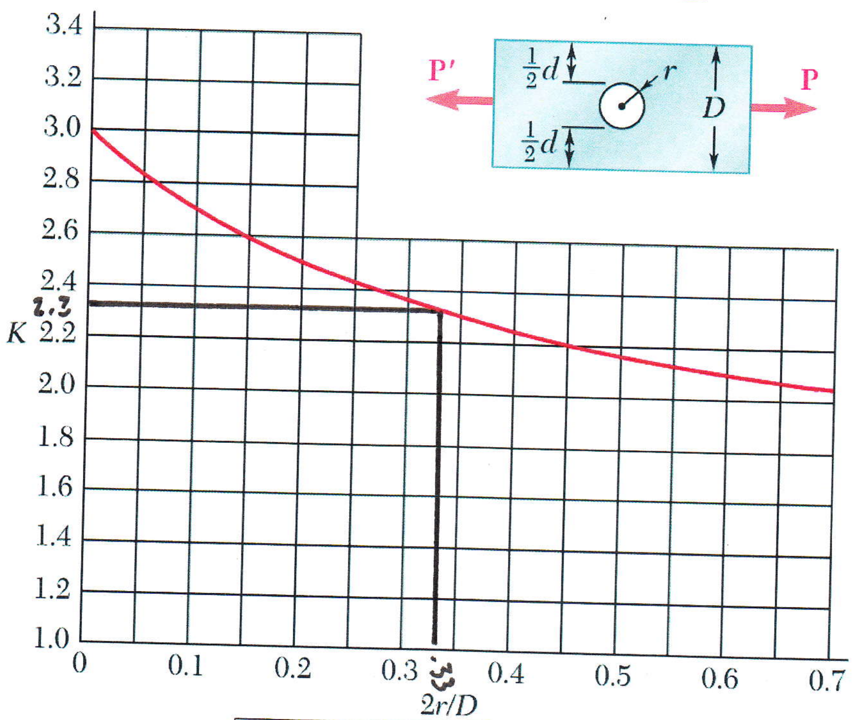
$$\Rightarrow F = \frac{2 \cdot \sigma_a^{(B)} \cdot A}{K_f} = \frac{2 \cdot (26.48 \times 10^3 \frac{lb}{in^2}) \cdot (0.05)(0.382\,in - 0.125\,in)}{2.3}$$

$$= \boxed{296\ lb}$$

MAX LOAD FOR INFINITE LIFE AND A FACTOR OF SAFETY OF 1.2



Fatigue Surface Finish Factor Chart



Static Stress Concentration Factor

$$\frac{d}{D} = \frac{0.125}{0.382} = 0.327$$

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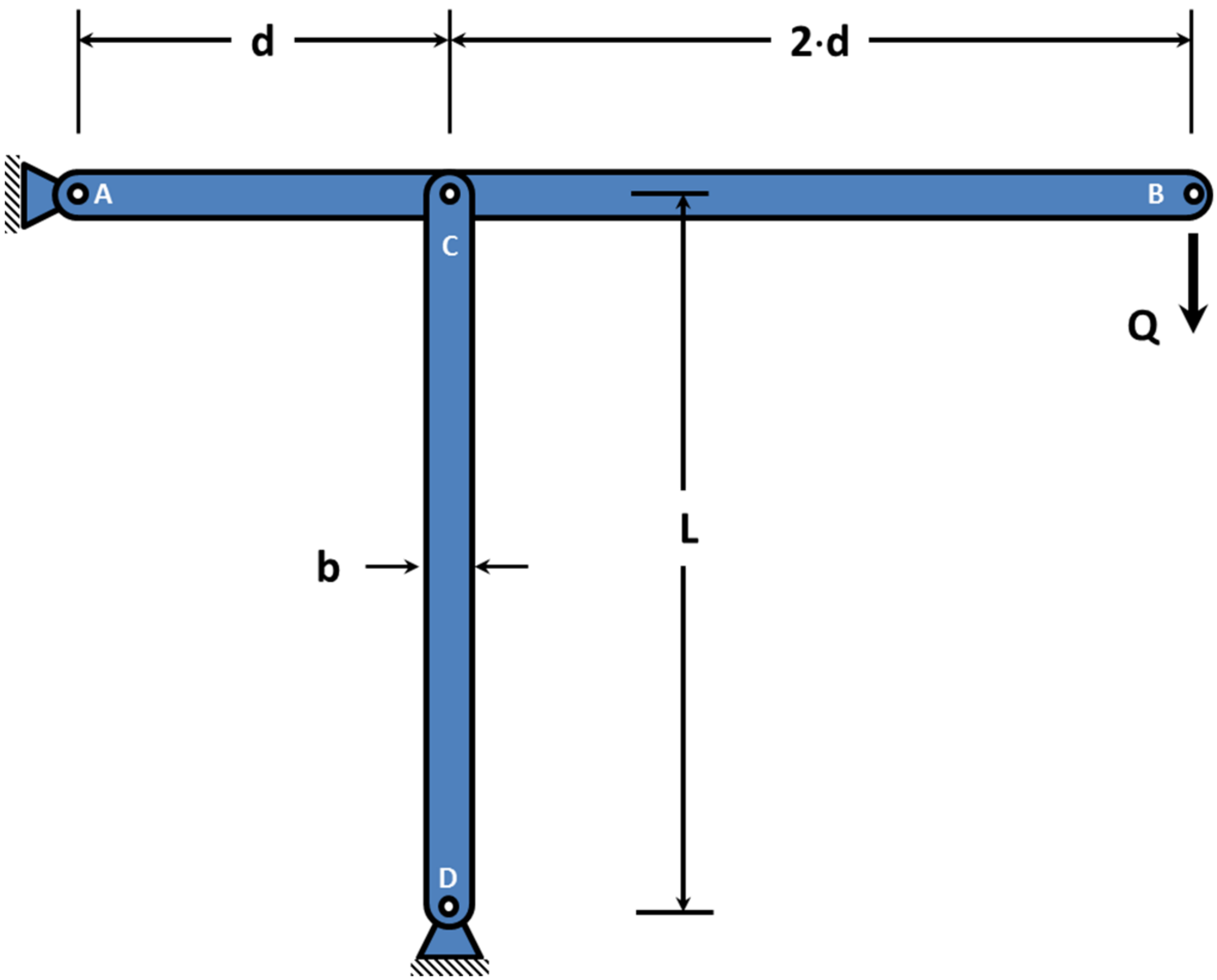

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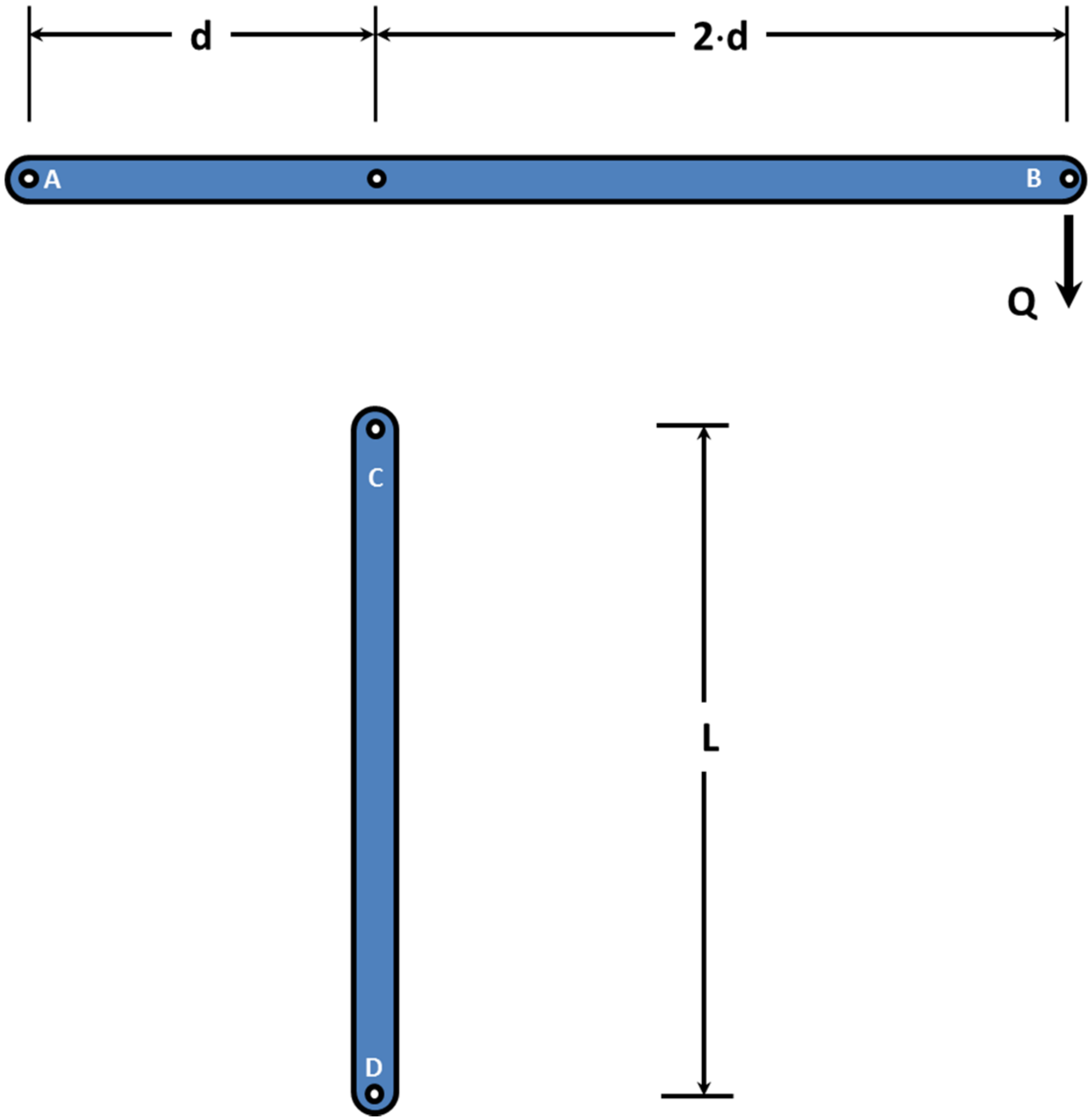

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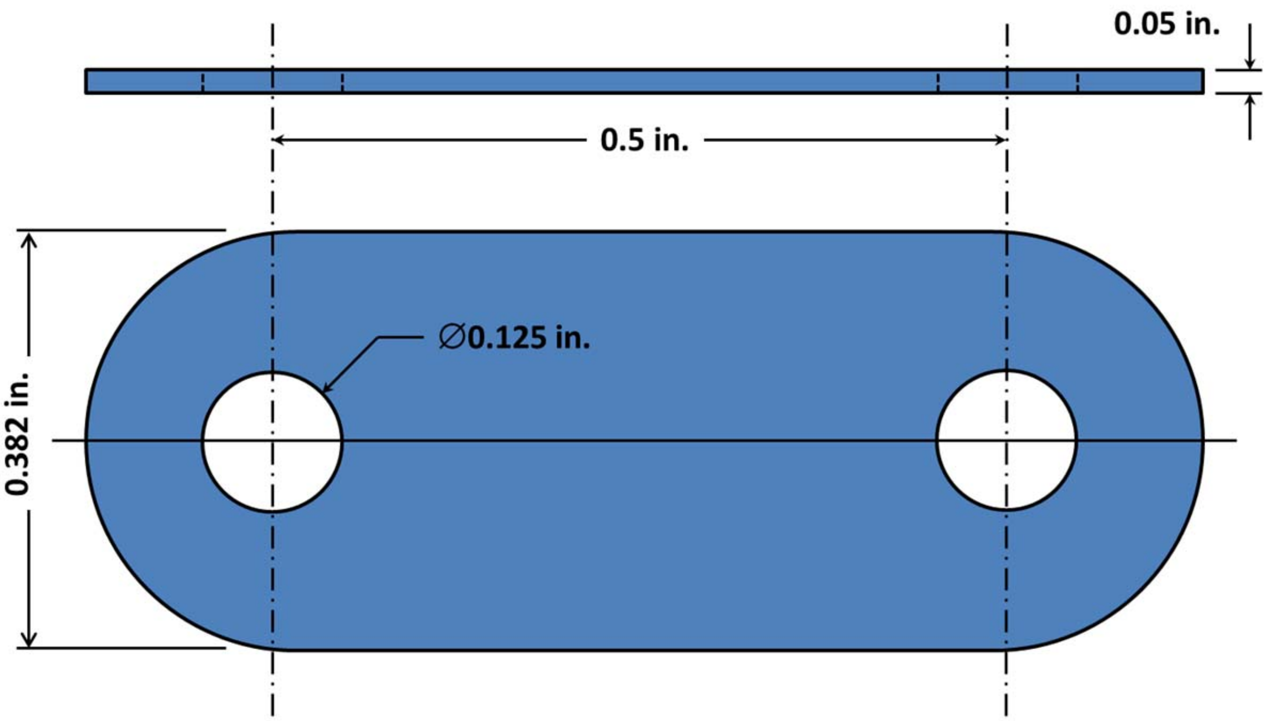

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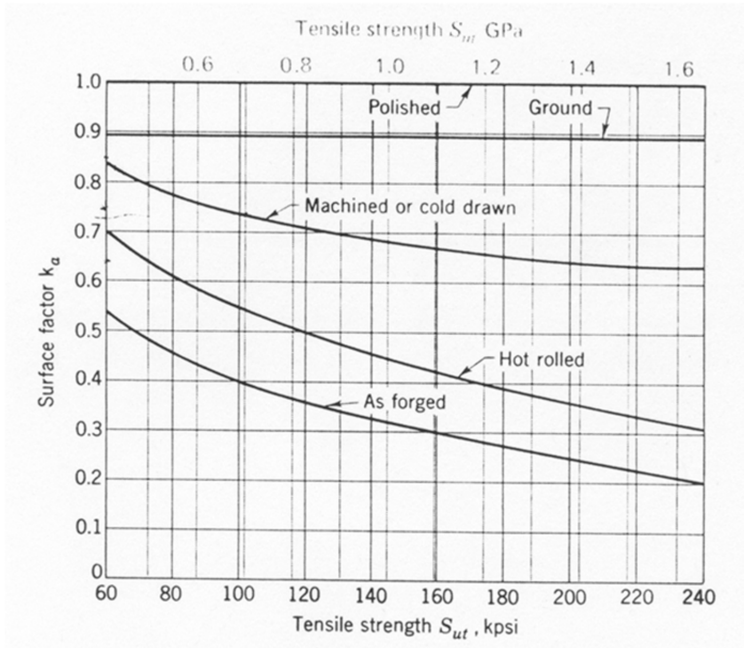

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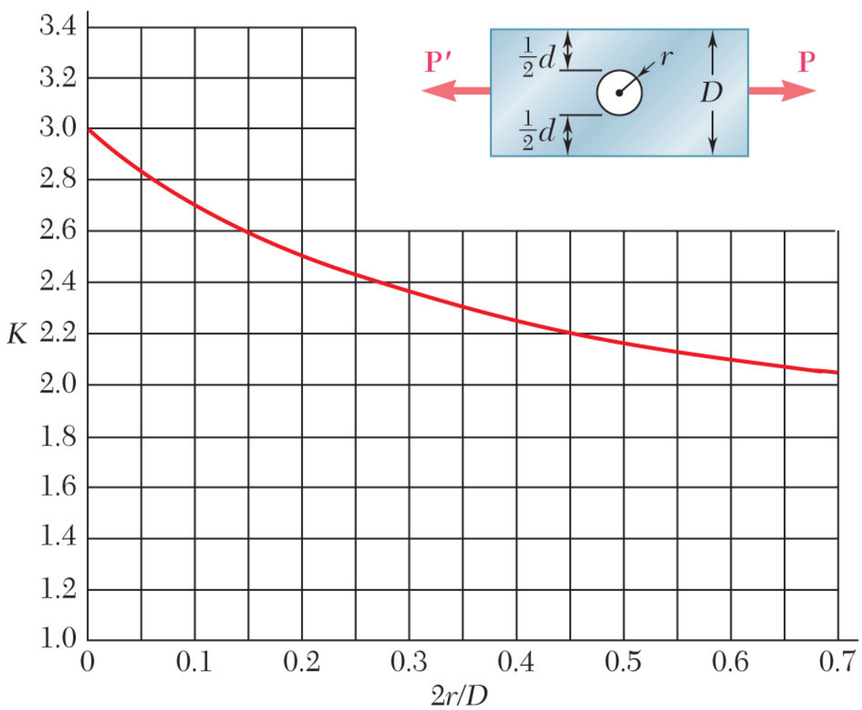

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