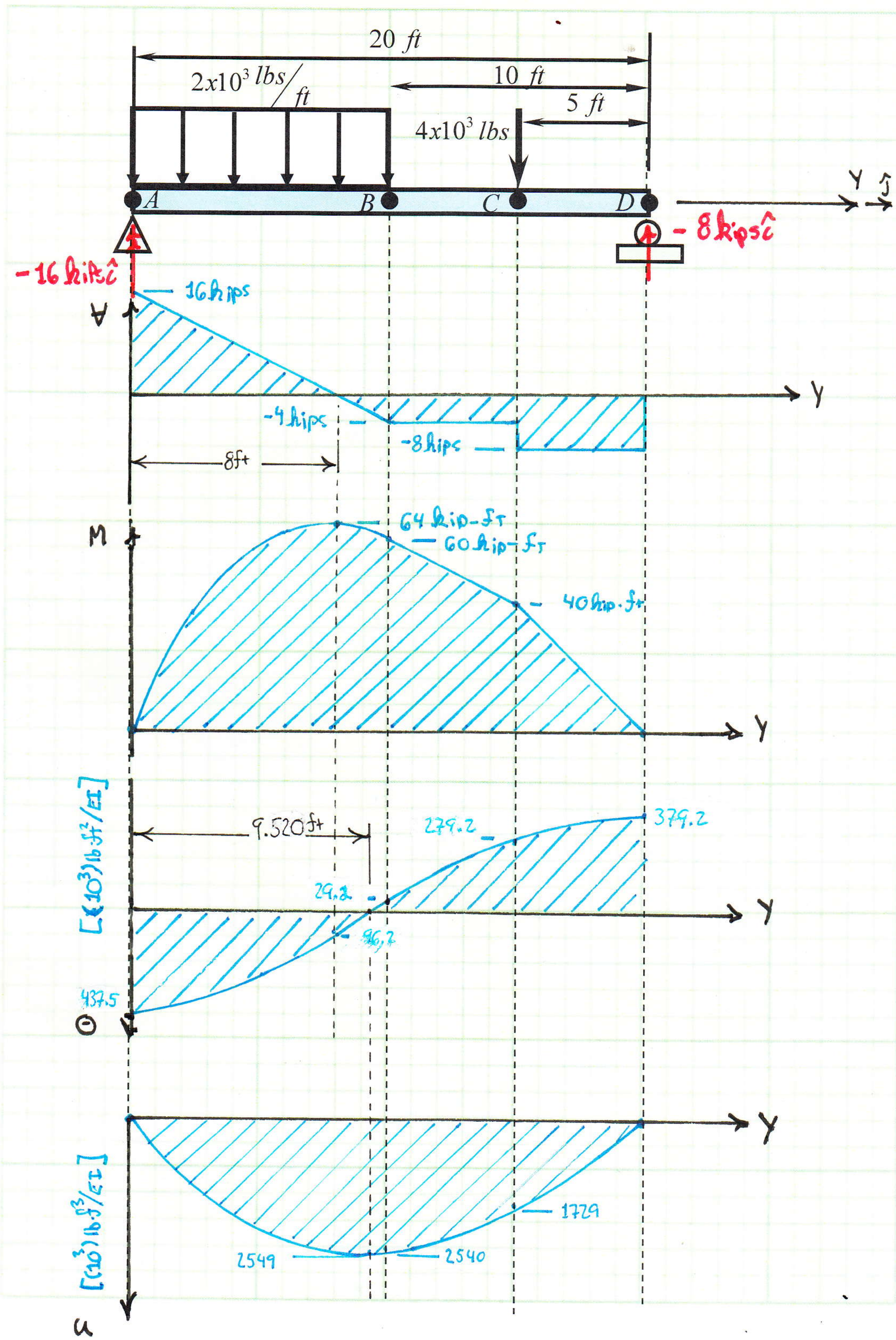
SPRING EXAM II BONUS

MER311: ADVANCED MECHANICS

RBB



FOR THE BEAM IN THE EXAM

$$q(y) = -16(10^3)\,lb\langle y-0\rangle_{-1} + 2(10^3)\tfrac{lb}{ft}\langle y-0\rangle^0 - 2(10^3)\tfrac{lb}{ft}\langle y-10ft\rangle^0 + 4(10^3)\,lb\langle y-15ft\rangle_{-1} - 8(10^3)\,lb\langle y-20ft\rangle_{-1} \quad (1)$$

$$V(y) = +16(10^3)\,lb\langle y-0\rangle^0 - 2(10^3)\tfrac{lb}{ft}\langle y-0\rangle^1 + 2(10^3)\tfrac{lb}{ft}\langle y-10ft\rangle^1 - 4(10^3)\,lb\langle y-15ft\rangle^0 + 8(10^3)\,lb\langle y-20ft\rangle^0 \quad (2)$$

$$M(y) = 16(10^3)\,lb\langle y-0\rangle^1 - 1(10^3)\tfrac{lb}{ft}\langle y-0\rangle^2 + 1(10^3)\tfrac{lb}{ft}\langle y-10ft\rangle^2 - 4(10^3)\,lb\langle y-15ft\rangle^1 + 8(10^3)\,lb\langle y-20ft\rangle^1 \quad (3)$$

$$\Theta(y) = -\frac{8(10^3)\,lb}{EI}\langle y-0\rangle^2 + \frac{1(10^3)\tfrac{lb}{ft}}{3\cdot EI}\langle y-0\rangle^3 - \frac{1(10^3)\tfrac{lb}{ft}}{3\cdot EI}\langle y-10ft\rangle^3 + \frac{2(10^3)\,lb}{EI}\langle y-15ft\rangle^2 - \frac{4(10^3)\,lb}{EI}\langle y-20ft\rangle^2 + \frac{435.5(10^3)\,lb\cdot ft^2}{EI} \quad (4)$$

$$u(y) = \frac{-8(10^3)\,lb}{3\cdot EI}\langle y-0\rangle^3 + \frac{1(10^3)\tfrac{lb}{ft}}{12\cdot EI}\langle y-0\rangle^4 - \frac{1(10^3)\tfrac{lb}{ft}}{12EI}\langle y-10ft\rangle^4 + \frac{2(10^3)\,lb}{3\cdot EI}\langle y-15ft\rangle^3 - \frac{4(10^3)\,lb}{3\cdot EI}\langle y-20ft\rangle^3 + \frac{435.5(10^3)\,lb\cdot ft^2}{EI}\cdot y \quad (5)$$

THE DIAGRAMS FOR EACH OF THESE FUNCTIONS CAN BE DRAWN. CRITICAL VALUES ON THESE DIAGRAMS WILL BE CALCULATED USING (2)-(5).

STARTING WITH THE SHEAR FORCE DIAGRAM, FROM (2)

$$V(0) = 16(10^3)\,lb\,(0-0)^0 = \underline{\underline{16(10^3)\,lb}} \quad (6)$$

$$V(10ft) = 16(10^3)\,lb\cdot(10ft-0)^0 - 2(10^3)\tfrac{lb}{ft}(10ft-0)^1 + 2(10^3)\tfrac{lb}{ft}\cancelto{0}{(10ft-10ft)}$$

$$= 16(10^3)\,lb - 2(10^3)\tfrac{lb}{ft}\cdot 10ft = \underline{\underline{-4(10^3)\,lb}} \quad (7)$$

SINCE THE TWO EXTREMES OF THIS REGION HAVE OPPOSITE SIGNS AND IN THIS REGION THE SHEAR FORCE CHANGES LINEARLY, THE SHEAR FORCE WILL GO TO ZERO IN THIS REGION. THE DISTANCE ALONG THE BEAM WHERE THIS WILL OCCUR NEEDS TO BE LOCATED.

IN THE REGION $0\,ft < y < 10\,ft$, FROM (2)

$$V(y) = 16(10^3)\,lb - 2(10^3)\,\tfrac{lb}{ft}\cdot y = 0$$

$$\Rightarrow\ y = \frac{16(10^3)\,lb}{2(10^3)\,lb/ft} = \underline{\underline{8\,ft}} \qquad (8)$$

CONTINUING ALONG THE BEAM TO FIND VALUES OF SHEAR FORCE AT CRITICAL LOCATIONS

$$V(15ft) = 16(10^3)\,lb\cdot(\cancel{15ft})^{\overset{1}{0}} - 2(10^3)\,\tfrac{lb}{ft}\cdot(15ft) + 2(10^3)\,\tfrac{lb}{ft}\,(15ft-10ft)^1$$
$$- 4(10^3)\,lb\,(\cancel{15ft-15ft})^{\overset{0}{0}}$$

$$V(15ft)^+ = 16(10^3)\,lb - 30(10^3)\,lb + 10(10^3)\,lb - 4(10^3)\,lb$$
$$= \underline{\underline{-8(10^3)\,lb}} \quad (9a)$$

$$V(15ft)^- = 16(10^3)\,lb - 30(10^3)\,lb + 10(10^3)\,lb$$
$$= \underline{\underline{-4(10^3)\,lb}} \quad (9b)$$

THE DIFFERENCE BETWEEN (9a) AND (9b) IS THE $-4(10^3)\,lb$ POINT LOAD APPLIED AT 15 ft. THERE IS A DISCONTINUITY AT THIS POINT INDICATED BY THE $\langle y-15ft\rangle^0$ SINGULARITY FUNCTION. THEREFORE THE FUNCTION AT $15ft^+$ AND $15ft^-$ ARE CONSIDERED.

FINALLY, FROM (2)

$$V(20ft) = 16(10^3)\,lb\cdot(\cancel{20ft})^{\overset{1}{0}} - 2(10^3)\,\tfrac{lb}{ft}\,(20ft) + 2(10^3)\,\tfrac{lb}{ft}\cdot(10ft)$$
$$- 4(10^3)\,lb\cdot(\cancel{5ft})^{\overset{1}{0}} + 8(10^3)\,lb\cdot(\cancel{0})^{\overset{1}{0}}$$

$$V(20ft)^+ = 16(10^3)\,lb - 40(10^3)\,lb + 20(10^3)\,lb - 4(10^3)\,lb + 8(10^3)\,lb$$
$$= \underline{\underline{0\,lb}} \quad (10a)$$

$$V(20ft)^- = 16(10^3)\,lb - 40(10^3)\,lb + 20(10^3)\,lb - 4(10^3)\,lb$$
$$= \underline{\underline{-8(10^3)\,lb}} \quad (10b)$$

HERE AGAIN THE LAST TERM INDICATES A DISCONTINUITY IN THE CURVE THAT REQUIRES THE EVALUATION JUST PRIOR TO THE DISCONTINUITY AND JUST AFTER IT.

NOW THE <u>BENDING MOMENT DIAGRAM</u> CRITICAL VALUES ARE COMPUTED, USING (3)

$$M(0) = 16(10^3)\,\text{lb}\cdot(0\,\text{ft})^1 = \underline{\underline{0}} \quad (11)$$

$$M(8\,\text{ft}) = M_{max} = 16(10^3)\,\text{lb}\cdot(8\,\text{ft}) - 1(10^3)\tfrac{\text{lb}}{\text{ft}}\cdot(8\,\text{ft})^2$$

$$= \underline{\underline{64(10^3)\,\text{lb}\cdot\text{ft}}} \quad (12)$$

$$M(10\,\text{ft}) = 16(10^3)\,\text{lb}\cdot(10\,\text{ft}) - 1(10^3)\tfrac{\text{lb}}{\text{ft}}\cdot(10\,\text{ft})^2 + 1(10^3)\cdot(0)^2$$

$$= \underline{\underline{60(10^3)\,\text{lb}\cdot\text{ft}}} \quad (13)$$

$$M(15\,\text{ft}) = 16(10^3)\,\text{lb}\cdot(15\,\text{ft}) - 1(10^3)\tfrac{\text{lb}}{\text{ft}}\cdot(15\,\text{ft})^2 + 1(10^3)\tfrac{\text{lb}}{\text{ft}}\cdot(5\,\text{ft})^2 - 4(10^3)\,\text{lb}(0)$$

$$= \underline{\underline{40(10^3)\,\text{lb}\cdot\text{ft}}} \quad (14)$$

$$M(20\,\text{ft}) = 16(10^3)\,\text{lb}\cdot(20\,\text{ft}) - 1(10^3)\tfrac{\text{lb}}{\text{ft}}(20\,\text{ft})^2 + 1(10^3)\tfrac{\text{lb}}{\text{ft}}(10\,\text{ft})^2$$

$$- 4(10^3)\,\text{lb}\,(5\,\text{ft}) + 8(10^3)\,\text{lb}\,(0)^1 = \underline{\underline{0}} \quad (15)$$

NOW THE CRITICAL VALUES OF THE <u>SLOPE OF THE DEFLECTION CURVE</u> ARE COMPUTED, USING (4)

$$\theta(0) = \frac{1}{EI}\left[-8(10^3)\,\text{lb}\cdot(0)^2 + \tfrac{1}{3}(10^3)\cdot(0)^3 + 437.5(10^3)\,\text{lb}\cdot\text{ft}^2\right]$$

$$= \underline{\underline{\frac{437.5(10^3)\,\text{lb}\cdot\text{ft}^2}{EI}}} \quad (16)$$

$$\theta(10\,\text{ft}) = \frac{1}{EI}\left[-8(10^3)\,\text{lb}\cdot(10\,\text{ft})^2 + \tfrac{1}{3}(10^3)\tfrac{\text{lb}}{\text{ft}}(10\,\text{ft})^3 - \tfrac{1}{3}(10^3)\tfrac{\text{lb}}{\text{ft}}\cdot(0)^3 + 437.5(10^3)\,\text{lb}\cdot\text{ft}^2\right.$$

$$= \underline{\underline{-\frac{29.17(10^3)\,\text{lb}\cdot\text{ft}^2}{EI}}} \quad (17)$$

SINCE (16) AND (17) HAVE DIFFERENT SIGNS, AT SOME POINT IN THIS REGION THE CURVATURE FUNCTION EQUALS ZERO. THIS POINT NEEDS TO BE DETERMINED BECAUSE THIS IS THE LOCATION WHERE THE DEFLECTION WILL BE A MAX OR MIN.

FOR THE REGION $0 < y < 10 ft$, THE GENERAL EXPRESSION FOR (4) REDUCES TO

$$\Theta(y) = \frac{1}{EI}\left[-8(10^3)lb \cdot y^2 + \frac{1}{3}(10^3)\frac{lb}{ft} \cdot y^3 + 437.5(10^3)lb \cdot ft^2\right]$$

THE ROOTS OF THIS EQUATION ARE FOUND WHEN $\Theta(y) = 0$

$$0 = \frac{1}{3}(10^3)\frac{lb}{ft} \cdot y^3 - 8(10^3)lb \cdot y^2 + 437.5(10^3)lb \cdot ft^2$$

THE MATLAB FUNCTION ROOTS, IS USED TO FIND ~~SOLUTIONS~~ THE ROOTS OF THE ABOVE POLYNOMIAL. THIS EQUATION EQUALS 0 WHEN

$$y = [\,21.04 ft,\ 9.520 ft,\ -6.552 ft\,]$$

THE FIRST AND LAST ROOTS ARE OUTSIDE THE DOMAIN OF THIS REGION; THEREFORE,

$$\Theta(y = 9.520\ ft) = 0 \qquad (18)$$

CONTINUING TO FIND VALUES OF THE ELASTIC CURVES CURVATURE, RETURNING TO (4) FOR $y = 15 ft$

$$\Theta(15 ft) = \frac{1}{EI}\left[-8(10^3)lb \cdot (15 ft)^2 + \frac{1}{3}(10^3)\frac{lb}{ft}(15 ft)^3 - \frac{1}{3}(10^3)\frac{lb}{ft}(5 ft)^3 + 2(10^3)lb \cdot (0 ft)^2 + 437.5(10^3)lb \cdot ft^2\right]$$

$$= -\frac{279.2\,(10^3)\,lb \cdot ft^2}{EI} \qquad (19)$$

$$\Theta(20 ft) = \frac{1}{EI}\left[-8(10^3)lb \cdot (20 ft)^2 + \frac{1}{3}(10^3)\frac{lb}{ft} \cdot (20 ft)^3 - \frac{1}{3}(10^3)\frac{lb}{ft} \cdot (10 ft)^3 + 2(10^3)lb \cdot (5 ft)^2 - 4(10^3)lb \cdot (0)^2 + 437.5(10^3)lb\,ft^2\right]$$

$$= \frac{-379.2\,(10^3)\,lb \cdot ft^2}{EI} \qquad (20)$$

IT IS ALSO HELPFUL TO KNOW THE VALUE OF $\Theta(y)$ WHERE THE CONCAVITY OF Θ CHANGES, $y = 8 ft$

$$\Theta(8 ft) = \frac{1}{EI}\left[-8(10^3)lb \cdot (8 ft)^2 + \frac{1}{3}(10^3)\frac{lb}{ft} \cdot (8 ft)^3 + 437.5(10^3)lb\,ft^2\right]$$

$$= \frac{98.2\ lb \cdot ft^2}{EI} \qquad (21)$$

THE CRITICAL VALUES OF THE DEFLECTION CURVE ARE NOW COMPUTED TO ASSIST IN THE DRAWING OF THE ELASTIC CURVE. USING THE GENERAL FORM OF THE ELASTIC CURVE IN (S)

$$u(0) = \frac{1}{EI}\left[-\frac{8}{3}(10^3)\,lb\cdot(0)^3 + \frac{1}{12}(10^3)\frac{lb}{ft}\cdot(0)^4 + 437.5(10^3)\,lb\cdot ft^2\cdot(0)\right]$$

$$= \underline{\underline{0}} \qquad (22)$$

$$u(9.520\,ft) = \frac{1}{EI}\left[-\frac{8}{3}(10^3)\,lb\cdot(9.520\,ft)^3 + \frac{1}{12}(10^3)\frac{lb}{ft}\cdot(9.520\,ft)^4 + 437.5(10^3)\,lb\cdot ft^2\cdot(9.520\,ft)\right]$$

$$= \underline{\underline{\frac{2549\,(10^3)\,lb\cdot ft^3}{EI}}} \qquad (23)$$

THIS IS THE MAXIMUM BEAM DEFLECTION

$$u(10\,ft) = \frac{1}{EI}\left[-\frac{8}{3}(10^3)\,lb\cdot(10\,ft)^3 + \frac{1}{12}(10^3)\frac{lb}{ft}\cdot(10\,ft)^4 - \frac{1}{12}(10^3)\frac{lb}{ft}\cdot(0)^4 + 437.5(10^3)\,lb\cdot ft^2\cdot(10\,ft)\right]$$

$$= \underline{\underline{\frac{2540\,(10^3)\,lb\cdot ft^3}{EI}}} \qquad (24)$$

$$u(15\,ft) = \frac{1}{EI}\left[-\frac{8}{3}(10^3)\,lb\cdot(15\,ft)^3 + \frac{1}{12}(10^3)\frac{lb}{ft}\cdot(15\,ft)^4 - \frac{1}{12}(10^3)\frac{lb}{ft}\cdot(5\,ft)^4 + \frac{2}{3}(10^3)\,lb\cdot(0)^3 + 437.5(10^3)\,lb\cdot ft^2\cdot(15\,ft)\right]$$

$$= \underline{\underline{\frac{1729\,(10^3)\,lb\cdot ft^3}{EI}}} \qquad (25)$$

$$u(20\,ft) = \frac{1}{EI}\left[-\frac{8}{3}(10^3)\,lb\cdot(20\,ft)^3 + \frac{1}{12}(10^3)\frac{lb}{ft}\cdot(20\,ft)^4 - \frac{1}{12}(10^3)\frac{lb}{ft}\cdot(10\,ft)^4 + \frac{2}{3}(10^3)\,lb\cdot(5\,ft)^3 - \frac{4}{3}(10^3)\,lb\cdot(0)^3 + 437.5(10^3)\,lb\cdot ft^2\cdot(20\,ft)\right]$$

$$= \underline{\underline{0}}$$