FATIGUE

- Finite Life/Strength
- Cumulative Fatigue Damage

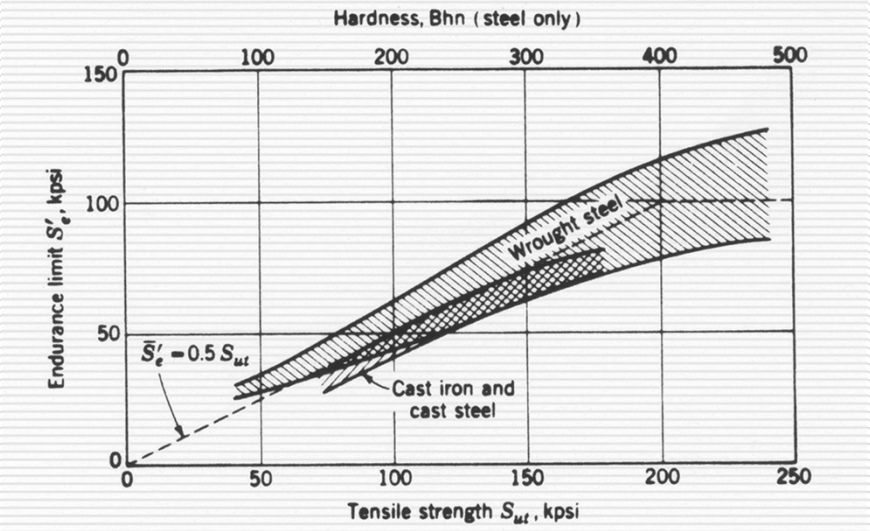
Endurance Limit and Strength

$S_{UT} \equiv$ *Ultimate Tensile Strength*

$\bar{S}'_e \equiv$ *Mean Endurance Limit of Test Specimen*

$\bar{S}_e \equiv$ *Mean Endurance Limit of Structural Element*

$S'_f \equiv$ *Fatigue Strength of Test Specimen*



Rules of Thumb

Steel

English Units

$$\bar{S}_e' = 0.5 \cdot S_{UT} \quad when \quad S_{UT} \leq 200 ksi$$

$$\bar{S}_e' = 100 ksi \quad when \quad S_{UT} > 200 ksi$$

Metric Units

$$\bar{S}_e' = 0.5 \cdot S_{UT} \quad when \quad S_{UT} \leq 1400 MPa$$

$$\bar{S}_e' = 700 MPa \quad when \quad S_{UT} > 1400 MPa$$

Rules of Thumb

Cast Iron

$$\bar{S}_e' = 0.4 \cdot S_{UT}$$

Aluminum and Magnesium

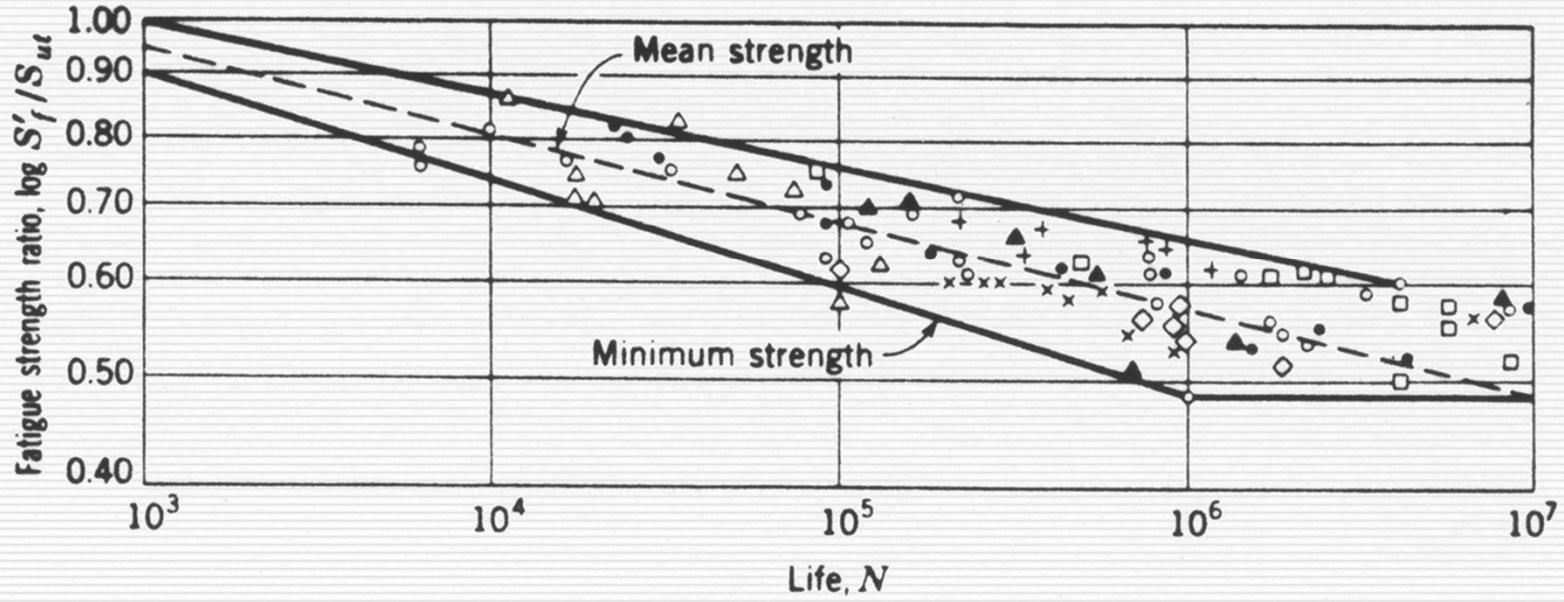
$$\bar{S}_e' \approx 0.3 \; to \; 0.4 \cdot S_{UT}$$

Values

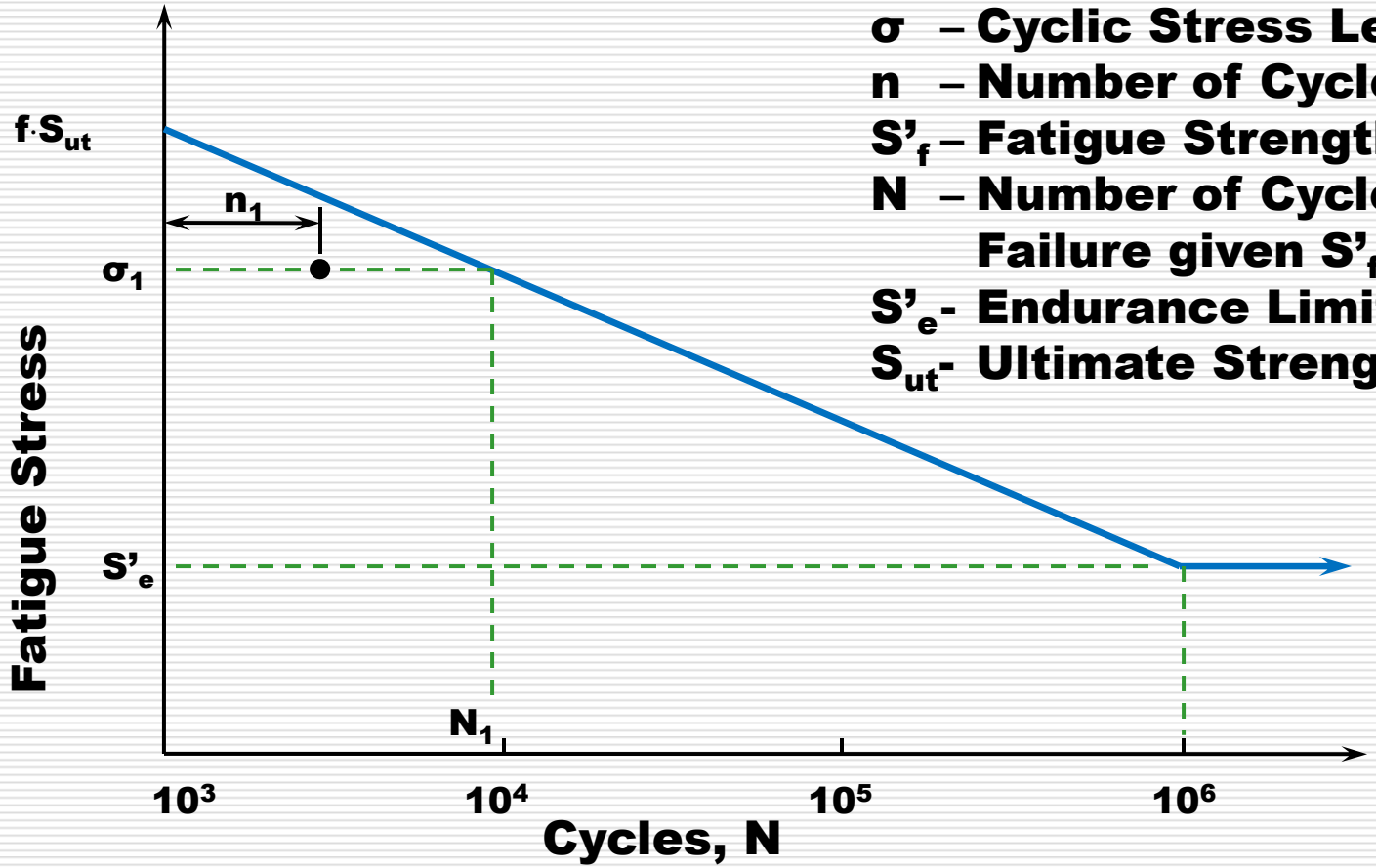
(Reported by F.B. Stulen, H.N. Cummings, and W.C. Schulte, Preventing Fatigue Failures, Part 5, *Machine Design*, vol. 33, P. 161, 22 June 1961)

Material UNS No. (Alloys are heat treated, hot worked, specimens smooth, subjected to long life rotating beam test)	Tensile Strength, S_{ut}		Endurance Limit, S'_E		Standard Deviation	
	MPa	ksi	MPa	ksi	ksi	%
G43400 Steel	965 1310 1580 1790	140 190 230 260	489 586 620 668	71 85 90 97	3.5 6.7 5.3 6.3	4.9 7.8 5.9 6.5
G43500 Steel	2070	300	689	100	4.4	4.4
R50001-series Titanium Alloy	1000	145	579	84	5.4	6.4
A97076 Aluminum Alloy	524	76	186	27	1.6	6.0
C63000 Aluminum Bronze	806	117	331	48	4.5	9.4
C17200 Beryllium Copper	1210	175	248	36	2.7	7.5

Finite Life, S'_f, The S-N Curve



Cumulative Damage Log-Log Plot



σ – Cyclic Stress Level
n – Number of Cycles
S'_f – Fatigue Strength at N
N – Number of Cycles to Failure given S'_f
S'_e - Endurance Limit
S_{ut} - Ultimate Strength

Equation for the S-N Curve

S-N Curve Equation

$$\log S'_f = -m \cdot \log n + b$$

where

$$m = \frac{1}{3} \cdot \log\left(\frac{f \cdot S_{ut}}{S'_e} \right)$$

$$b = \log\left(\frac{(f \cdot S_{ut})^2}{S'_e} \right)$$

f is often taken as 0.9

S-N Curve Calculations

In the range $10^3 \le y \le 10^6$

Given N, S'_f is

$$S' = \frac{10^b}{N^m}$$

Given S'_f, N is

$$N = \frac{10^{b/m}}{S'^{1/m}_f}$$

Example

The endurance limit of a steel member is 112MPa and the tensile strength is 385MPa. What fatigue strength corresponds to a life of $70(10^3)$ cycles.

Cumulative Damage

- **Multiple Stress Levels**
  - σ_1 for n_1 cycles
  - σ_2 for n_2 cycles
  - Etc.
- **Palmgren-Miner's Rule**

$$\frac{n_1}{N_1}+\frac{n_2}{N_2}+\cdots+\frac{n_n}{N_n}=C \quad \Rightarrow \quad \sum\frac{n_i}{N_i}=1$$

  - n-nubmer of cycles of stress σ applied
  - N-life corresponding to σ
  - C-determined experimentally
    - 0.7<C<2.2
    - Typically C=1

Example

Steel, S_{ut}=80ksi
3000 cycles at 60ksi
How many cycles remain at 50ksi and 40ksi?