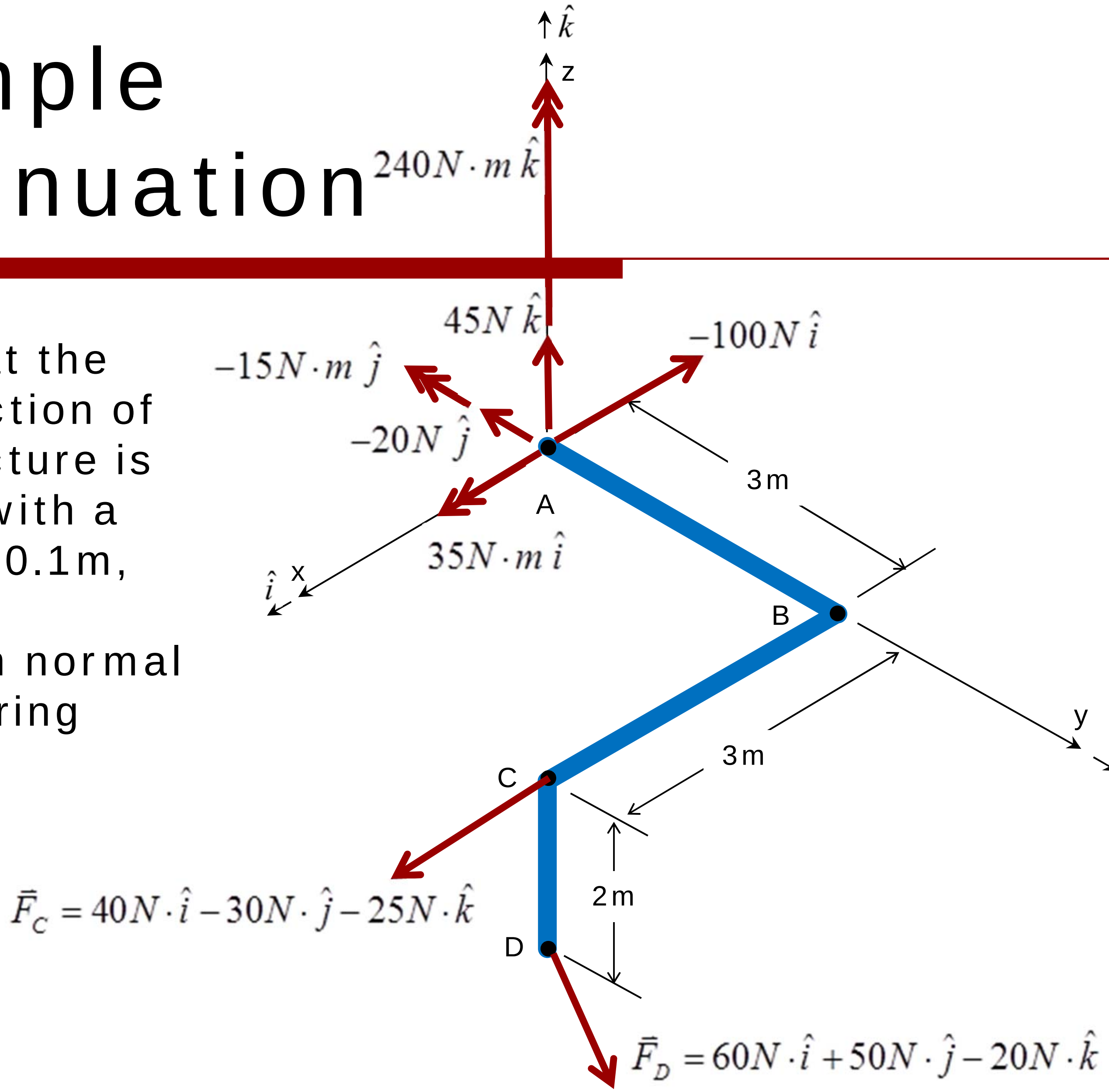
MER311: Advanced Strength of Materials

LECTURE OUTLINE

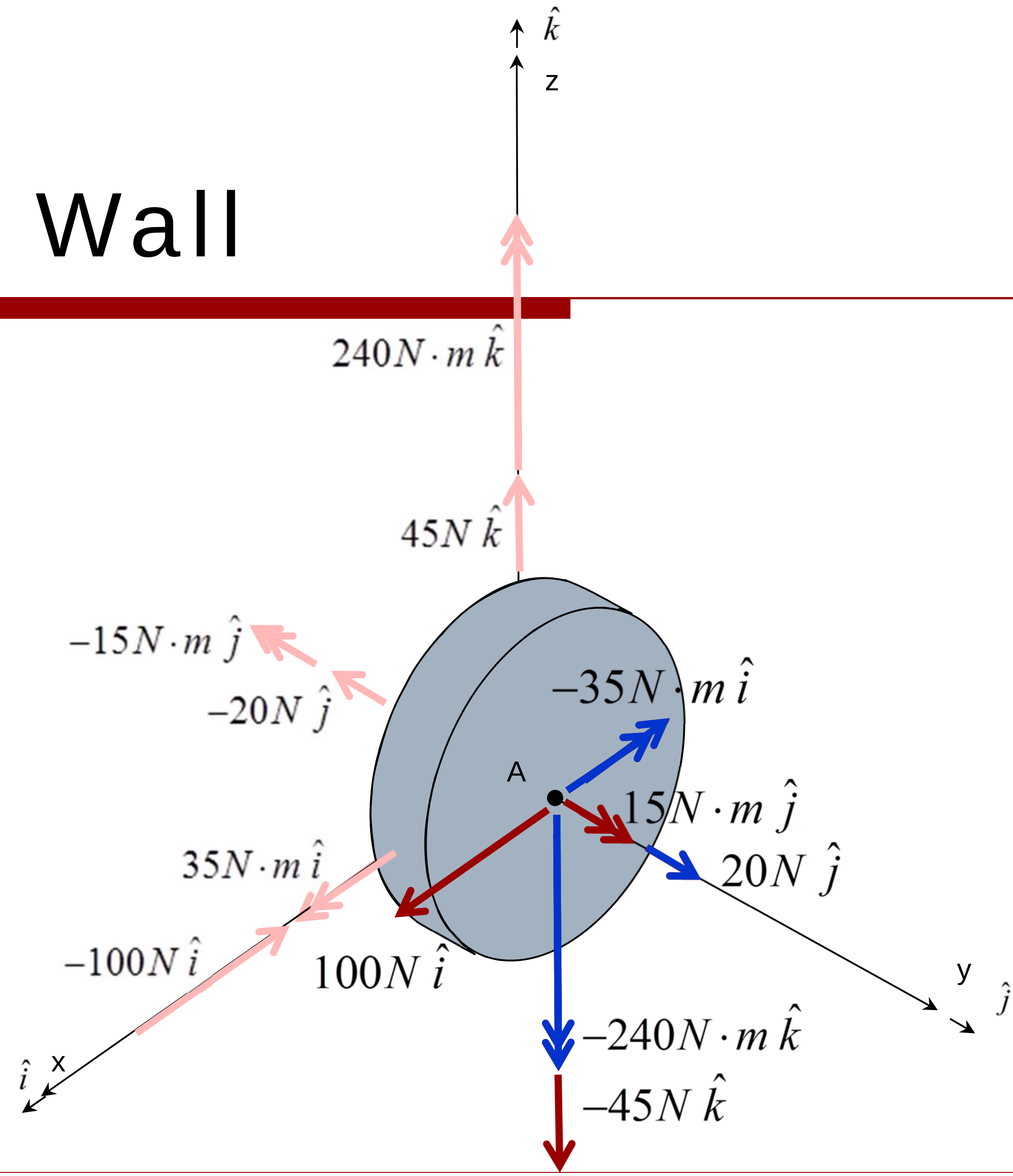
- State of Stress
- Stress Tensor
- Equilibrium

Example Continuation

Given that the cross-section of this structure is circular with a radius of 0.1m, find the maximum normal and shearing stresses.



At the Wall



$\hat{k}$
z
$240N\cdot m\,\hat{k}$
$45N\,\hat{k}$
$-15N\cdot m\,\hat{j}$
$-20N\,\hat{j}$
$-35N\cdot m\,\hat{i}$
A
$15N\cdot m\,\hat{j}$
$20N\,\hat{j}$
$35N\cdot m\,\hat{i}$
$-100N\,\hat{i}$
$100N\,\hat{i}$
y
$\hat{j}$
$-240N\cdot m\,\hat{k}$
$-45N\,\hat{k}$
$\hat{i}$
x

Calculating the State of Stress Using Strength of Materials

$$\sigma_y = \frac{F_y}{A_y} \pm \frac{M_x \cdot z}{I_{xx}} \pm \frac{M_z \cdot x}{I_{zz}}$$

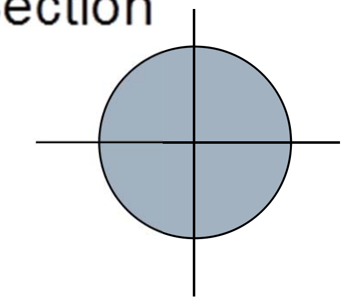
$$\tau_{yx} = \frac{V_x \cdot Q}{I_{zz} \cdot t_z}$$

$$\tau_{zx} = \frac{V_z \cdot Q}{I_{xx} \cdot t_x}$$

$$\tau_{yr} = \frac{T_y \cdot r}{J}$$

For a Circular Cross-Section

$$A = \pi \cdot r^2$$



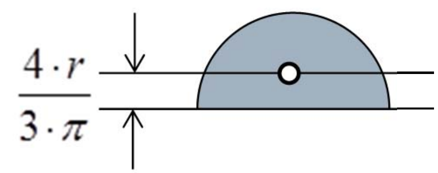
$$I = \frac{\pi \cdot r^4}{4} = \frac{\pi \cdot d^4}{64}$$

$$J = \frac{\pi \cdot r^4}{2} = \frac{\pi \cdot d^4}{32}$$

For a Semi-Circular Cross-Section

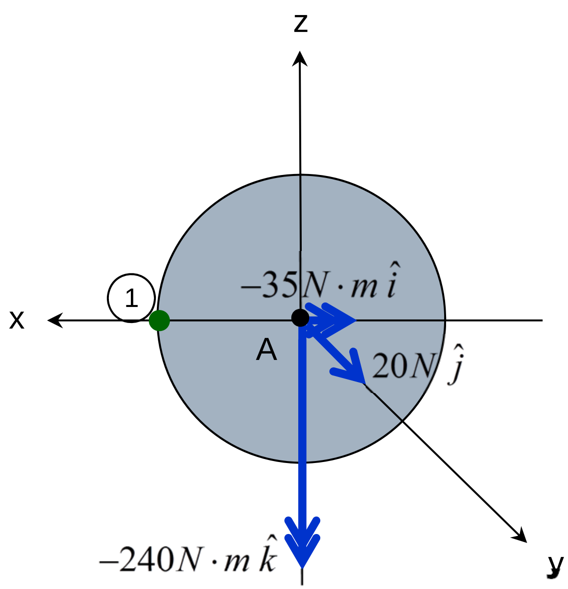
$$A = \frac{\pi \cdot r^2}{2}$$

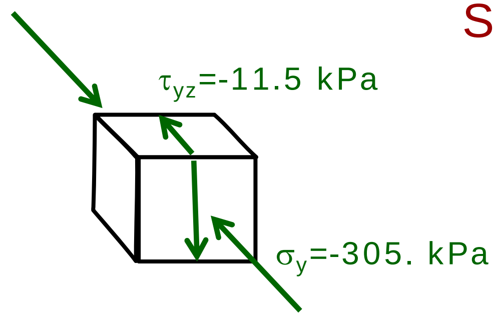
$$I = 0.110 \cdot r^4$$



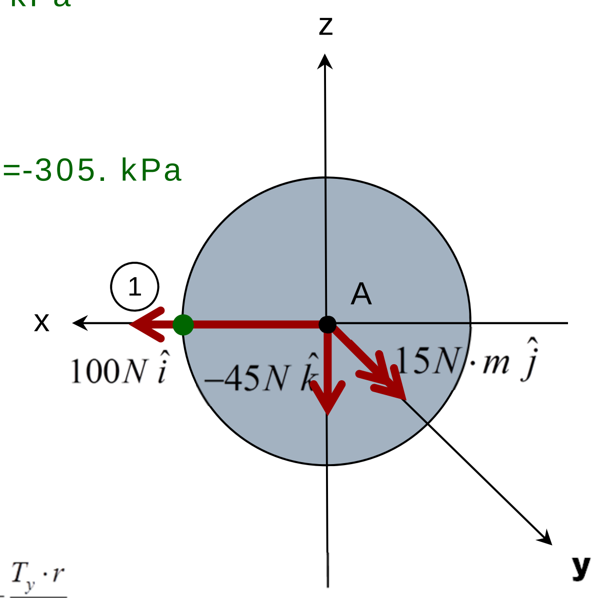
Forces and Couples Related to Normal and Shear Stress @ 1

Normal Stress





Shear Stress



$$\sigma_{y1} = \frac{F_y}{A_y} - \frac{M_x\cdot z}{I_{xx}} + \frac{M_z\cdot x}{I_{zz}}$$

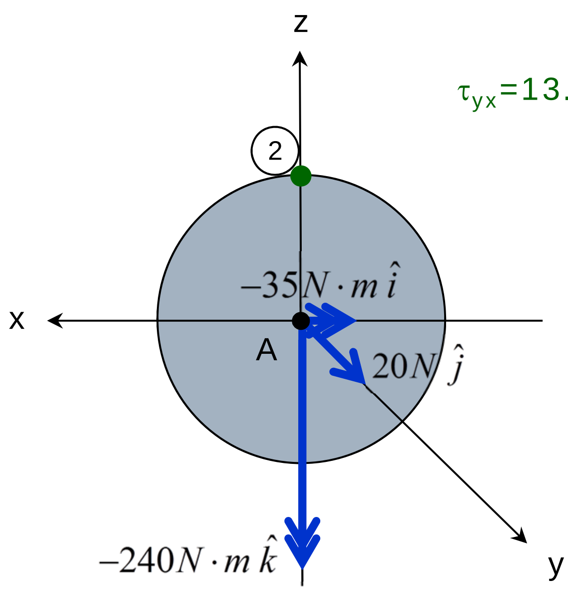
$$= \frac{20N}{\pi\cdot(0.1m)^2} - \frac{(-35N\cdot m)\cdot(0m)}{\pi\cdot(0.1m)^4/4} + \frac{(-240N\cdot m)\cdot(0.1\text{m})}{\pi\cdot(0.1m)^4/4} = -304.9\left(10^3\right)\tfrac{N}{m^2}$$

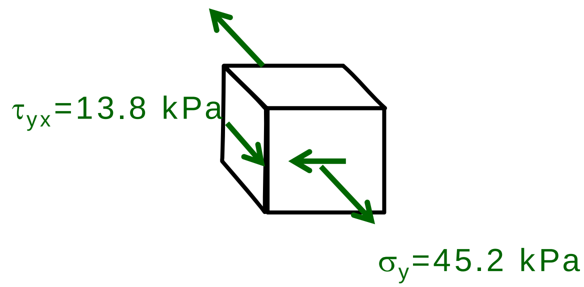
$$\tau_{zx} = \frac{V_z\cdot Q}{I_{xx}\cdot t_x} - \frac{T_y\cdot r}{J}$$

$$= \frac{(-45N)\cdot\left(\frac{4}{3\cdot\pi}\right)\cdot(0.1m)\cdot\frac{\pi\cdot(0.1m)^2}{2}}{\left(\pi\cdot(0.1m)^4/4\right)\cdot(0.2m)} - \frac{(15N\cdot m)\cdot(0.1\text{m})}{\pi\cdot(0.1m)^4/4} = -11.46\left(10^3\right)\tfrac{N}{m^2}$$

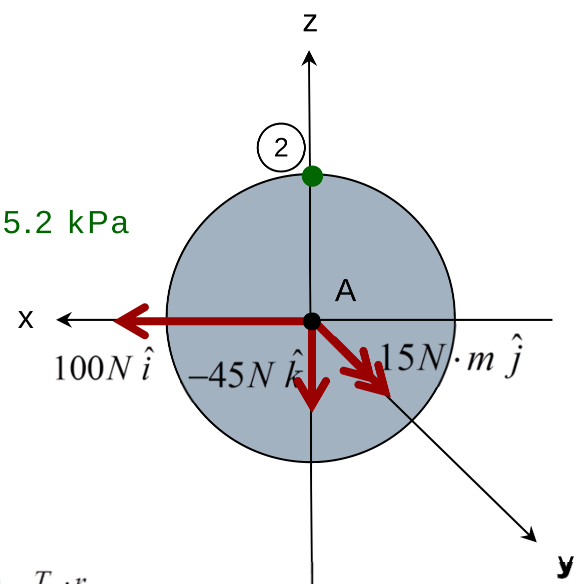
Forces and Couples Related to Normal and Shear Stress @ 2

Normal Stress





Shear Stress



$$\sigma_{y2} = \frac{F_y}{A_y} - \frac{M_x \cdot z}{I_{xx}} + \frac{M_z \cdot x}{I_{zz}}$$

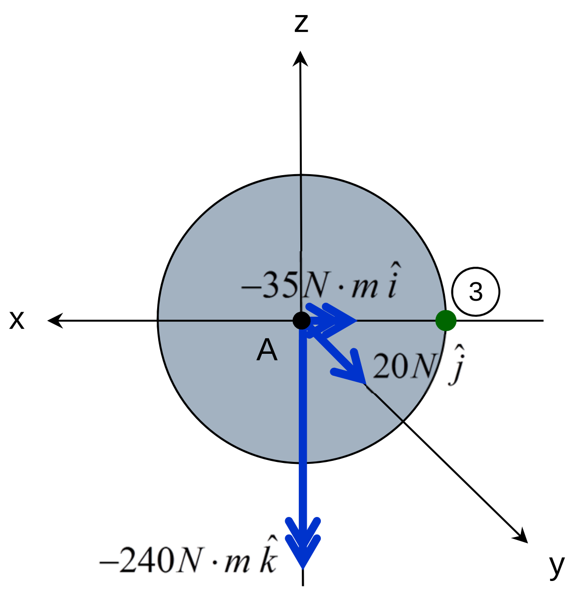
$$= \frac{20N}{\pi \cdot (0.1m)^2} - \frac{(-35N \cdot m)\cdot(0.1m)}{\pi \cdot (0.1m)^4 / 4} + \frac{(-240N \cdot m)\cdot(0\,\mathrm{m})}{\pi \cdot (0.1m)^4 / 4} = 45.20\left(10^3\right)\tfrac{N}{m^2}$$

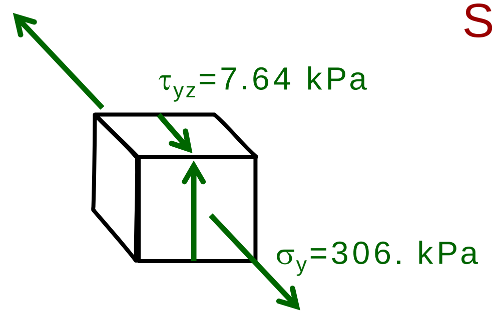
$$\tau_{zx2} = \frac{V_z \cdot Q}{I_{xx} \cdot t_x} + \frac{T_y \cdot r}{J}$$

$$= \frac{(100N)\cdot\left(\frac{4}{3\cdot\pi}\right)\cdot(0.1m)\cdot\frac{\pi\cdot(0.1m)^2}{2}}{\left(\pi\cdot(0.1m)^4 / 4\right)\cdot(0.2m)} + \frac{(15N \cdot m)\cdot(0.1\,\mathrm{m})}{\pi \cdot (0.1m)^4 / 4} = 13.79\left(10^3\right)\tfrac{N}{m^2}$$

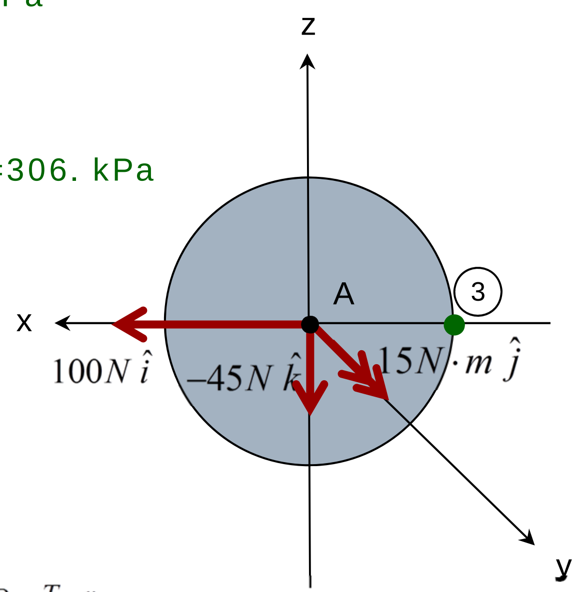
Forces and Couples Related to Normal and Shear Stress @ 3

Normal Stress





Shear Stress



$$\sigma_{y3} = \frac{F_y}{A_y} - \frac{M_x\cdot z}{I_{xx}} + \frac{M_z\cdot x}{I_{zz}}$$

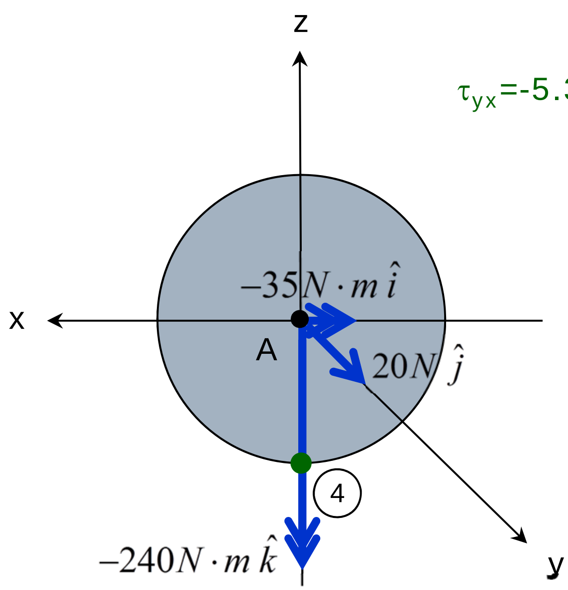
$$= \frac{20N}{\pi\cdot(0.1m)^2} - \frac{(-35N\cdot m)\cdot(0m)}{\pi\cdot(0.1m)^4/4} + \frac{(-240N\cdot m)\cdot(-0.1\text{m})}{\pi\cdot(0.1m)^4/4} = 306.2\left(10^3\right)\frac{N}{m^2}$$

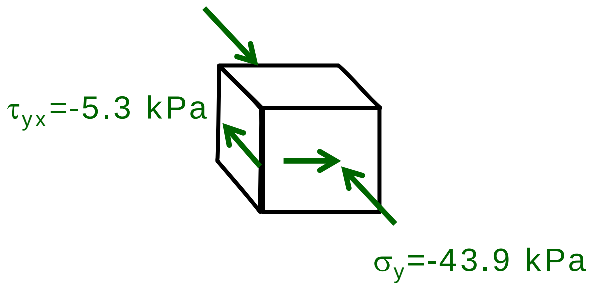
$$\tau_{zx3} = \frac{V_z\cdot Q}{I_{xx}\cdot t_x} - \frac{T_y\cdot r}{J}$$

$$= \frac{(-45N)\cdot\left(\frac{4}{3\cdot\pi}\right)\cdot(0.1m)\cdot\frac{\pi\cdot(0.1m)^2}{2}}{\left(\pi\cdot(0.1m)^4/4\right)\cdot(0.2m)} + \frac{(15N\cdot m)\cdot(0.1\text{m})}{\pi\cdot(0.1m)^4/4} = 7.64\left(10^3\right)\frac{N}{m^2}$$

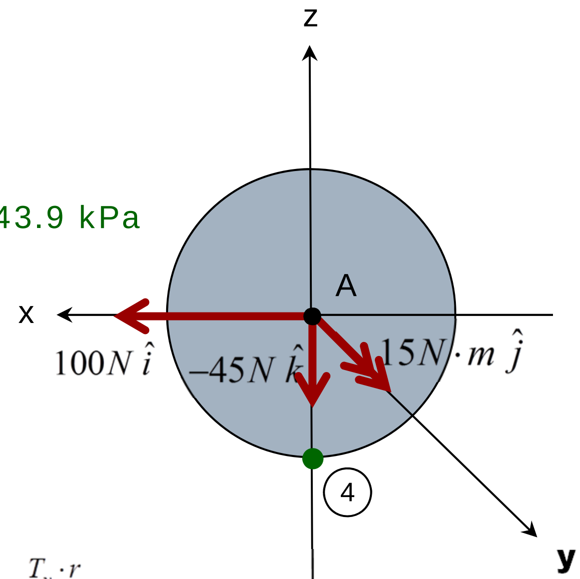
Forces and Couples Related to Normal and Shear Stress @ 4

Normal Stress





Shear Stress



$$\sigma_{y4} = \frac{F_y}{A_y} - \frac{M_x\cdot z}{I_{xx}} + \frac{M_z\cdot x}{I_{zz}}$$

$$= \frac{20N}{\pi\cdot(0.1m)^2} - \frac{(-35N\cdot m)\cdot(-0.1m)}{\pi\cdot(0.1m)^4/4} + \frac{(-240N\cdot m)\cdot(0\,\mathrm{m})}{\pi\cdot(0.1m)^4/4} = -43.92\left(10^3\right)\tfrac{N}{m^2}$$

$$\tau_{zx4} = \frac{V_z\cdot Q}{I_{xx}\cdot t_x} + \frac{T_y\cdot r}{J}$$

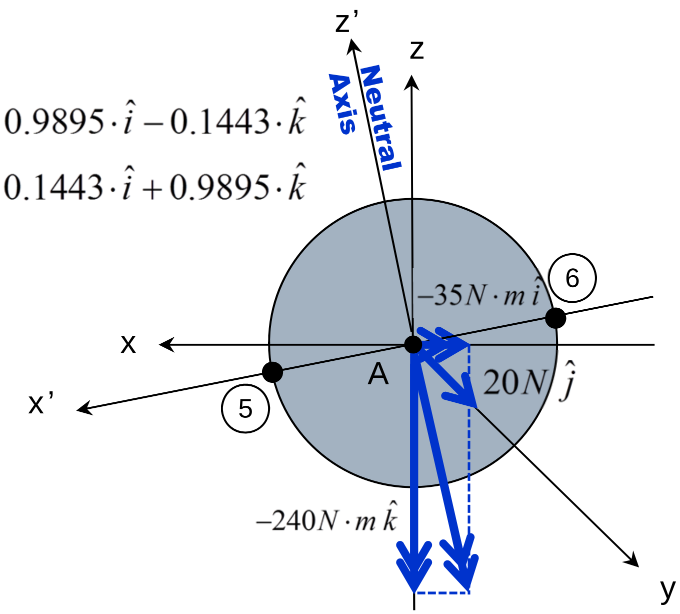
$$= \frac{(100N)\cdot\left(\frac{4}{3\cdot\pi}\right)\cdot(0.1m)\cdot\frac{\pi\cdot(0.1m)^2}{2}}{\left(\pi\cdot(0.1m)^4/4\right)\cdot(0.2m)} - \frac{(15N\cdot m)\cdot(0.1\,\mathrm{m})}{\pi\cdot(0.1m)^4/4} = -5.306\left(10^3\right)\tfrac{N}{m^2}$$

For the Circular X-Section Maximum Stresses

Normal Stress

$$\hat{e}_{x'} = 0.9895 \cdot \hat{i} - 0.1443 \cdot \hat{k}$$

$$\hat{e}_{z'} = 0.1443 \cdot \hat{i} + 0.9895 \cdot \hat{k}$$



$$-242.5N \cdot m \cdot \hat{e}_{z'}$$

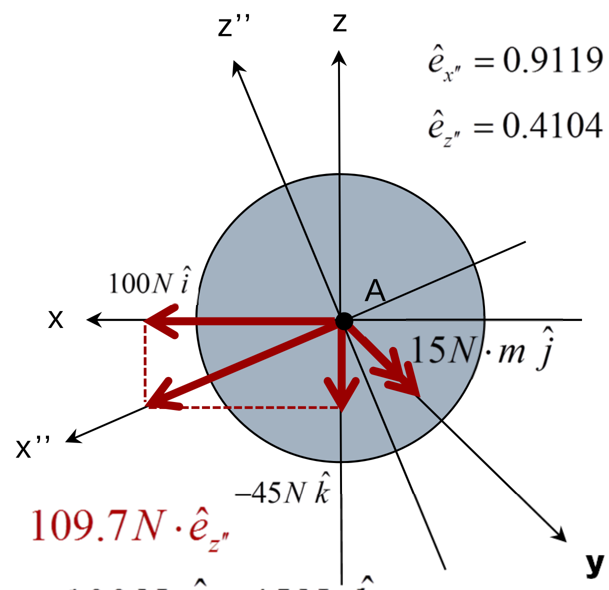
$$-35N \cdot m \cdot \hat{i} - 240N \cdot m \cdot \hat{k} =$$

$$242.5N \cdot m \cdot \left(-0.1443 \cdot \hat{i} - 0.9895 \cdot \hat{k}\right) =$$

Shear Stress

$$\hat{e}_{x''} = 0.9119 \cdot \hat{i} - 0.4104 \cdot \hat{k}$$

$$\hat{e}_{z''} = 0.4104 \cdot \hat{i} + 0.9119 \cdot \hat{k}$$



$$109.7N \cdot \hat{e}_{z''}$$

$$= 100N \cdot \hat{i} - 45N \cdot \hat{k}$$

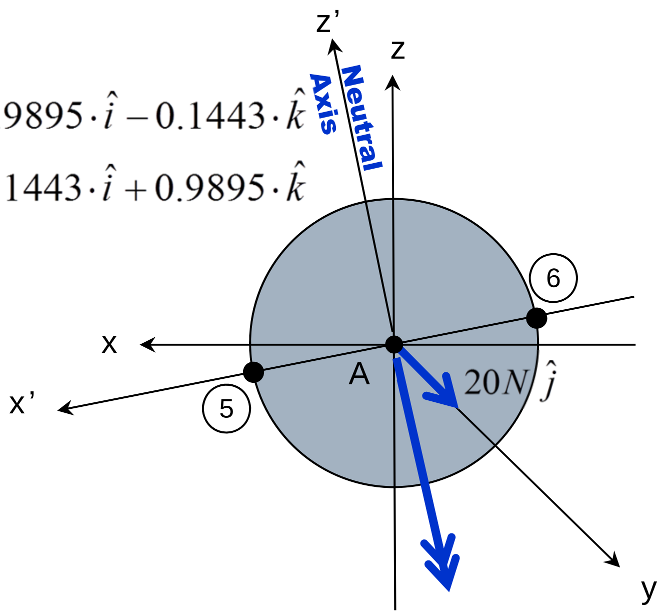
$$= 109.7N \cdot \left(0.9119 \cdot \hat{i} - 0.4104 \cdot \hat{k}\right)$$

Only a Component of the Shear Stress Important

Normal Stress

$$\hat{e}_{x'} = 0.9895\cdot\hat{i} - 0.1443\cdot\hat{k}$$

$$\hat{e}_{z'} = 0.1443\cdot\hat{i} + 0.9895\cdot\hat{k}$$



$$-242.5N\cdot m\cdot\hat{e}_{z'}$$

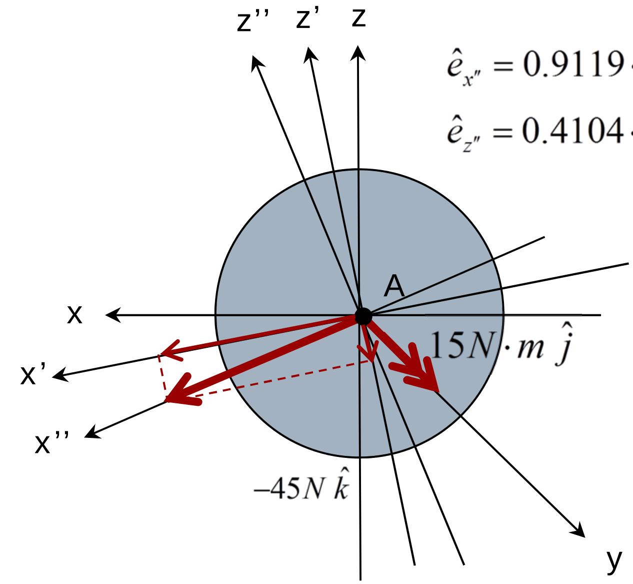
$$-35N\cdot m\cdot\hat{i} - 240N\cdot m\cdot\hat{k} =$$

$$242.5N\cdot m\cdot\left(-0.1443\cdot\hat{i} - 0.9895\cdot\hat{k}\right) =$$

Shear Stress

$$\hat{e}_{x''} = 0.9119\cdot\hat{i} - 0.4104\cdot\hat{k}$$

$$\hat{e}_{z''} = 0.4104\cdot\hat{i} + 0.9119\cdot\hat{k}$$

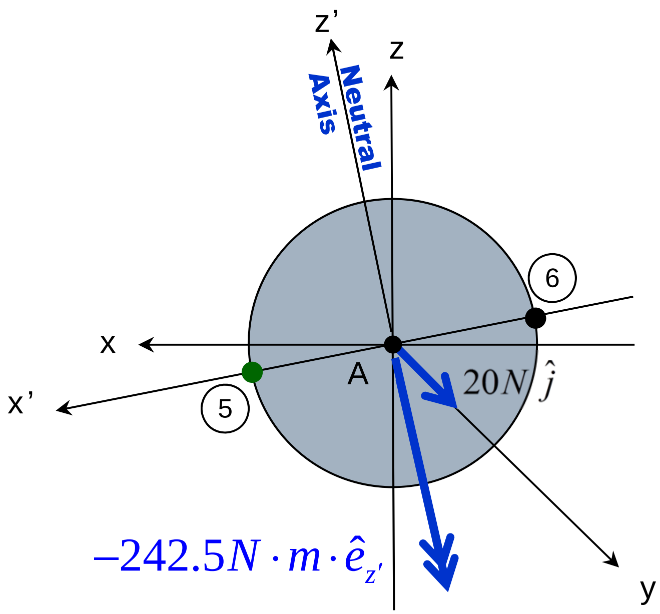


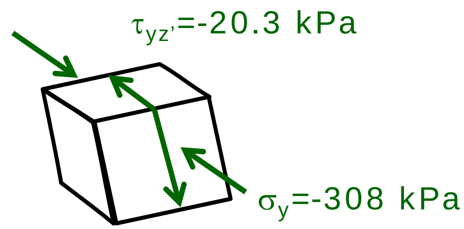
$$109.7N\cdot\hat{e}_{z''} = 100N\cdot\hat{i} - 45N\cdot\hat{k}$$

$$= 105.4N\cdot\hat{e}_{x'} - 30.1N\cdot\hat{e}_{z'}$$

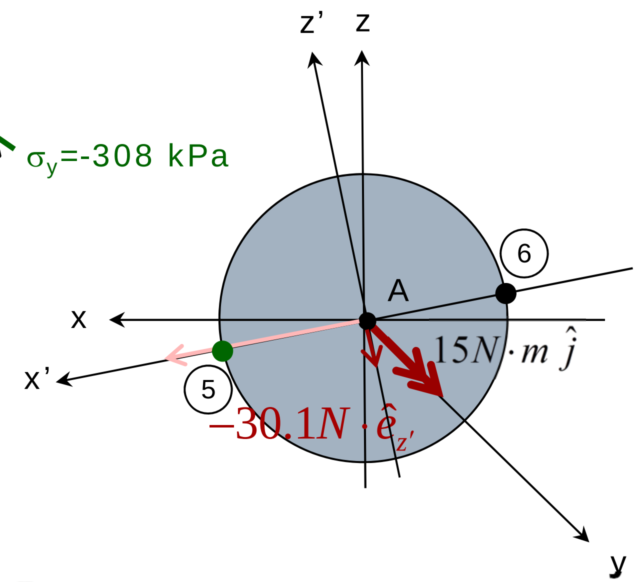
Forces and Couples Related to Normal and Shear Stress @ 5

Normal Stress





Shear Stress



$$\sigma_{y5}=\frac{F_y}{A_y}+\frac{M_{z'}\cdot x'}{I_{z'z'}}$$

$$=\frac{20N}{\pi\cdot(0.1m)^2}+\frac{(-242.5N\cdot m)\cdot(0.1\mathrm{m})}{\pi\cdot(0.1m)^4/4}=-308.1\left(10^3\right)\tfrac{N}{m^2}$$

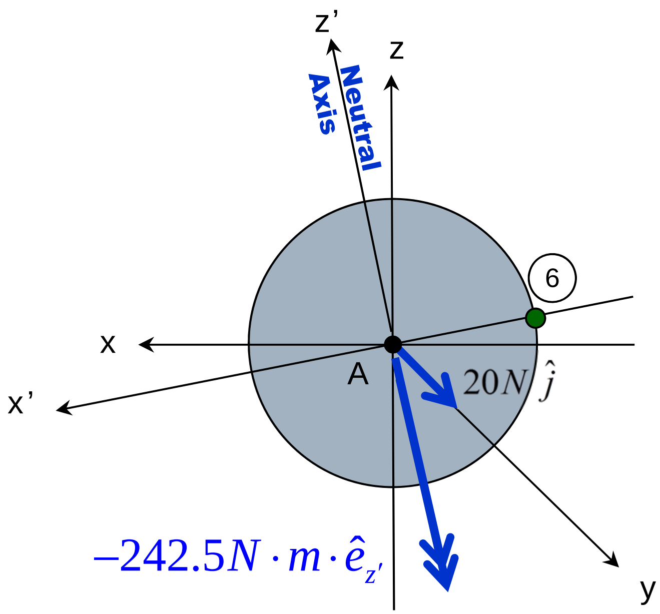
$$\tau_{yz'5}=\frac{V_{z'}\cdot Q}{I_{x'x'}\cdot t_{x'}}-\frac{T_y\cdot r}{J}$$

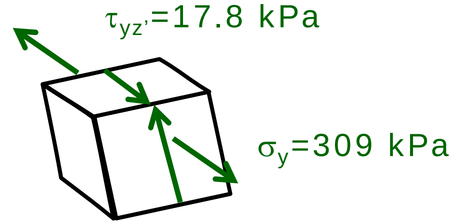
$$=\frac{(-30.09N)\cdot\left(\frac{4}{3\cdot\pi}\right)\cdot(0.1m)\cdot\frac{\pi\cdot(0.1m)^2}{2}}{\left(\pi\cdot(0.1m)^4/4\right)\cdot(0.2m)}-\frac{(15N\cdot m)\cdot(0.1\mathrm{m})}{\pi\cdot(0.1m)^4/4}=-20.34\left(10^3\right)\tfrac{N}{m^2}$$

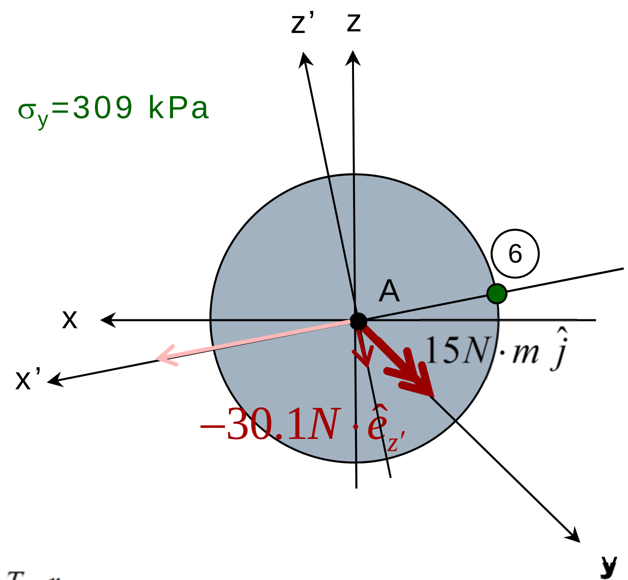
Forces and Couples Related to Normal and Shear Stress @ 6

Normal Stress

Shear Stress







$$\sigma_{y6}=\frac{F_y}{A_y}+\frac{M_{z'}\cdot x'}{I_{z'z'}}$$

$$=\frac{20N}{\pi\cdot(0.1m)^2}+\frac{(-242.5N\cdot m)\cdot(-0.1\text{m})}{\pi\cdot(0.1m)^4/4}=309.4\left(10^3\right)\tfrac{N}{m^2}$$

$$\tau_{yz'6}=\frac{V_{z'}\cdot Q}{I_{x'x'}\cdot t_{x'}}-\frac{T_y\cdot r}{J}$$

$$=\frac{(-30.09N)\cdot\left(\frac{4}{3\cdot\pi}\right)\cdot(0.1m)\cdot\frac{\pi\cdot(0.1m)^2}{2}}{\left(\pi\cdot(0.1m)^4/4\right)\cdot(0.2m)}+\frac{(15N\cdot m)\cdot(0.1\text{m})}{\pi\cdot(0.1m)^4/4}=17.82\left(10^3\right)\tfrac{N}{m^2}$$