PROBLEM 4 THE BEAM ABCDE SHOWN BELOW HAS SIMPLE SUPPORTS AT A, C, AND E; AND A HINGE (OR PIN) AT D. A LOAD OF 4 kN ACTS AT THE END OF THE BRACKET THAT EXTENDS FROM THE BEAM AT B, AND A LOAD OF 2 kN ACTS AT THE MIDPOINT OF MEMBER DE. DRAW THE SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR THIS BEAM.

GIVEN:

1) A 6m MEMBER IS PIN CONNECTED TO A 2m MEMBER
2) THE 6 m MEMBER IS SIMPLY SUPPORTED AT 0m AND 4m
3) THE 2m MEMBER IS SIMPLY SUPPORTED 2m FROM THE PIN CONNECTION
4) A 4kN LOAD IS APPLIED TO THE END OF A BRACKET LOCATED AT 2m, WITH A HORIZONTAL DIMENSION OF -1m

ASSUMPTIONS:

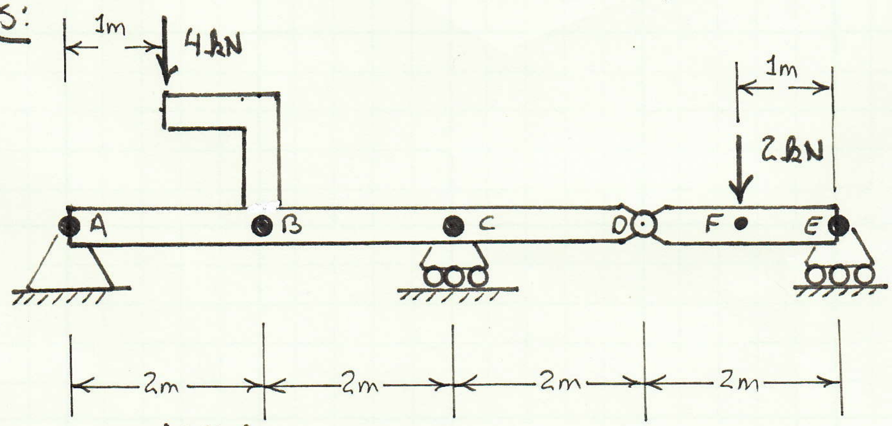
1) ALL COMPONENTS START OUT STRAIGHT
2) LINEAR - ELASTIC DEFORMATION
3) SMALL DEFLECTIONS.
4) THE PIN JOINT IS FRICTIONLESS.

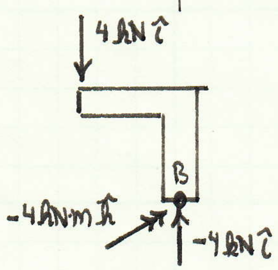
FIND:

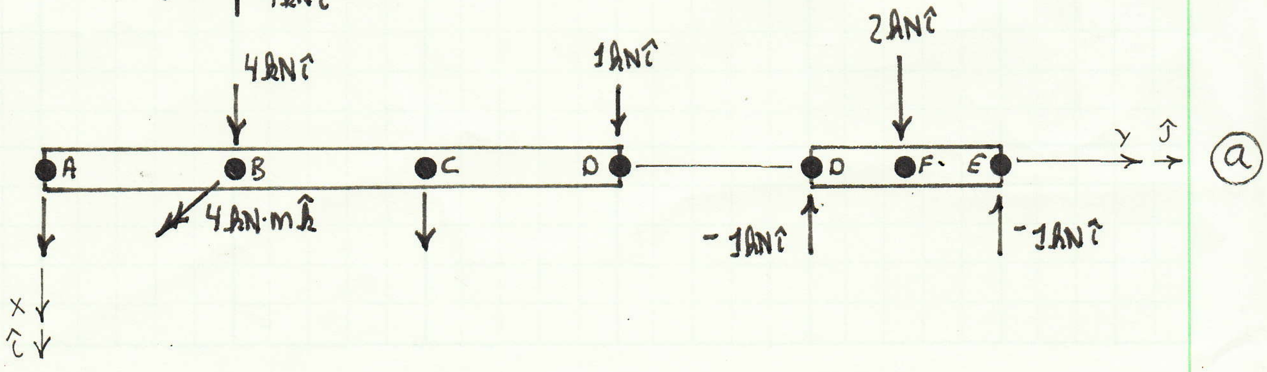
1. SHEAR FORCE DIAGRAM
2. BENDING MOMENT DIAGRAM

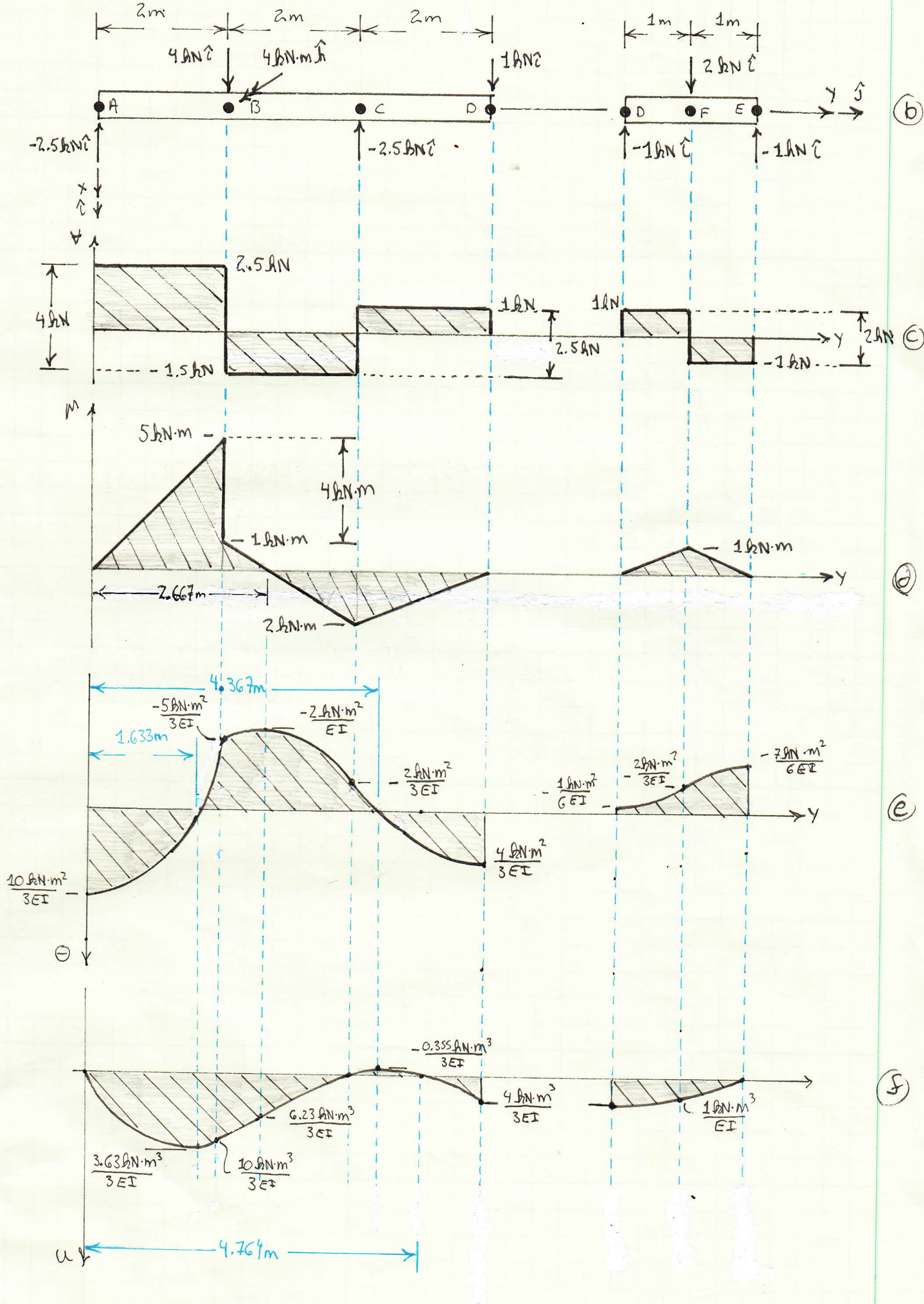
SUPPLEMENTAL QUESTIONS
3. ELASTIC CURVE CURVATURE
4. ELASTIC CURVE DEFLECTION

FIGURES:








2m
2m
2m
1m
1m
4 kN î
4 kN·m k̂
1 kN î
2 kN î
A
B
C
D
D
F
E
y ĵ
-2.5 kN î
-2.5 kN î
-1 kN î
-1 kN î
x î
(b)
V
2.5 kN
4 kN
1 kN
1 kN
2.5 kN
2 kN
-1.5 kN
-1 kN
y
(c)
M
5 kN·m
4 kN·m
1 kN·m
1 kN·m
2.667m
2 kN·m
y
(d)
4.367m
1.633m
-5 kN·m²/3EI
-2 kN·m²/EI
-2 kN·m²/3EI
1 kN·m²/6EI
2 kN·m²/3EI
-7 kN·m²/6EI
10 kN·m²/3EI
4 kN·m²/3EI
θ
y
(e)
-0.355 kN·m³/3EI
4 kN·m³/3EI
6.23 kN·m³/3EI
3.63 kN·m³/3EI
10 kN·m³/3EI
1 kN·m³/EI
4.764m
u
(f)

SINGULARITY FUNCTION SOLUTION

STARTING WITH BE BEAM $0<y<6m$ IN FIGURE (b)

$$q(y) = -2.5kN\langle y-0\rangle_{-1} + 4kN\langle y-2m\rangle_{-1} + 4kN\cdot m\langle y-2m\rangle_{-2} - 2.5kN\langle y-4m\rangle_{-1} + 1kN\langle y-6m\rangle_{-1} \quad (76)$$

$$V = \int -q(y)\,dy = 2.5kN\langle y-0\rangle^0 - 4kN\langle y-2m\rangle^0 - 4kN\cdot m\langle y-2m\rangle_{-1} + 2.5kN\langle y-4m\rangle^0 - 1kN\langle y-6m\rangle^0 \quad (77)$$

$$M = \int V\,dy = 2.5kN\cdot\langle y-0\rangle^1 - 4kN\langle y-2m\rangle^1 - 4kN\cdot m\langle y-2m\rangle^0 + 2.5kN\langle y-4m\rangle^1 - 1kN\langle y-6m\rangle^1 \quad (78)$$

$$\theta = \int -\frac{M}{EI}\,dy = -\frac{2.5kN}{2EI}\langle y-0\rangle^2 + \frac{2kN}{EI}\langle y-2m\rangle^2 + \frac{4kN\cdot m}{EI}\langle y-2m\rangle^1 - \frac{2.5kN}{2EI}\langle y-4m\rangle^2 + \frac{1kN}{2EI}\langle y-6m\rangle^2 + \cancel{C_1}^{\frac{10kN\cdot m^2}{3EI}} \quad (79)$$

$$u = \int \theta\,dy = \frac{-2.5kN}{6EI}\langle y-0\rangle^3 + \frac{2kN}{3EI}\langle y-2m\rangle^3 + \frac{2kN\cdot m}{EI}\langle y-2m\rangle^2 - \frac{2.5kN}{6EI}\langle y-4m\rangle^3 + \frac{1kN}{6EI}\langle y-6m\rangle^3 + \cancel{C_1}^{\frac{10kN\cdot m^2}{3EI}}\cdot y + \cancel{C_2}^{0} \quad (80)$$

USING THE BOUNDARY CONDITION $u(0m)=0$

$$u(0) = 0 = \frac{-2.5kN}{6EI}(0)^3 + C_1\cdot(0) + C_2 \Rightarrow \boxed{C_2 = 0}$$

$$(80) \rightarrow u = -\frac{2.5kN}{6EI}\langle y-0\rangle^3 + \frac{2kN}{3EI}\langle y-2m\rangle^3 + \frac{2kN\cdot m}{EI}\langle y-2m\rangle^2 - \frac{2.5kN}{6EI}\langle y-4m\rangle^3 + \frac{1kN}{6EI}\langle y-6m\rangle^3 + C_1 y$$

Using the Second Boundary Condition $u(4m)=0$

$$u(4m) = 0 = -\frac{2.5kN}{6EI}(4m)^3 + \frac{2kN}{3EI}(2m)^3 + \frac{2kN\cdot m}{EI}(2m)^2 - \frac{2.5kN}{6EI}(0)^3 + C_1(4m)$$

$$= -\frac{80kN\cdot m^3}{3EI} + \frac{16kN}{3EI} + \frac{8kN\cdot m^3}{EI} + C_1\cdot(4m)$$

$$\Rightarrow C_1 = \frac{40kN\cdot m^3}{4m\cdot 3EI} = \boxed{\frac{10kN\cdot m^2}{3EI} = C_1} \quad (81)$$

SUBSTITUTING (81) INTO (79) AND (80)

$$(79) \rightarrow \quad \theta = -\frac{2.5\,kN}{2\cdot EI}\langle y-0\rangle^2 + \frac{2\,kN}{EI}\langle y-2m\rangle^2 + \frac{4\,kN\cdot m}{EI}\cdot\langle y-2m\rangle^1 - \frac{2.5\,kN}{2\cdot EI}\cdot\langle y-4m\rangle^2 + \frac{1\,kN}{2EI}\cdot\langle y-6m\rangle^2 + \frac{10\,kN\cdot m^2}{3\cdot EI} \qquad (82)$$

$$(80) \rightarrow \quad u = -\frac{2.5\,kN}{6EI}\cdot\langle y-0\rangle^3 + \frac{2\,kN}{3\cdot EI}\langle y-2m\rangle^3 + \frac{2\,kN\cdot m}{EI}\cdot\langle y-2m\rangle^2 - \frac{2.5\,kN}{6EI}\cdot\langle y-4m\rangle^3 + \frac{1\,kN}{6EI}\langle y-6m\rangle^3 + \frac{10\,kN\cdot m^2}{3\cdot EI}\cdot y \qquad (83)$$

THE CRITICAL VALUES OF V, M, Q, AND u IN EACH SECTION OF ABCD CAN NOW BE DETERMINED.

REGION AB: $0 \le y \le 2m$

$$(77) \rightarrow \quad V(0) = 2.5\,kN\cdot(0)^0 = 0\,kN,\ 2.5\,kN \qquad (84)$$

$$(77) \rightarrow \quad V(2m) = 2.5\,kN\cdot(2m)^0 - 4\,kN\cdot(0)^0 = 2.5\,kN,\ -1.5\,kN \qquad (85)$$

$$(78) \rightarrow \quad M(0) = 2.5\,kN\cdot(0) = 0 \qquad (86)$$

$$(78) \rightarrow \quad M(2m) = 2.5\,kN\cdot(2m) - 4\,kN\cdot(0m) - 4\,kN\cdot m\cdot(0)^0 = 5.0\,kN\cdot m,\ 1.0\,kN\cdot m \qquad (87)$$

$$(79) \rightarrow \quad \theta(0) = -\frac{2.5\,kN}{2\cdot EI}\cdot(0)^2 + \frac{10\,kN\cdot m^2}{3\cdot EI} = \frac{10\,kN\cdot m^2}{3\cdot EI} \qquad (88)$$

$$(79) \rightarrow \quad \theta(2m) = -\frac{2.5\,kN}{2\cdot EI}\cdot(2m)^2 + \frac{2\,kN}{EI}\cdot(0)^2 + \frac{4\,kN\cdot m}{EI}\cdot(0) + \frac{10\,kN\cdot m^2}{3EI} = -\frac{5\,kN\cdot m^2}{3\cdot EI} \qquad (89)$$

THE SIGN OF θ CHANGES IN THIS REGION WHICH INDICATES THAT THE A MAX/MIN VALUE OF u EXISTS IN THIS REGION. THE ROOTS OF $\theta(y)$ IN THIS REGION NEED TO BE FOUND. STARTING WITH (82), SETTING IT EQUAL TO ZERO, AND CONSIDERING $0 < y < 2m$ (NOTE THE "=" IS NOT PRESENT)

$$(82) \rightarrow \quad 0 = -\frac{2.5\,kN}{2\cdot EI}\cdot y^2 + \frac{10\,kN\cdot m^2}{3EI} \Rightarrow y^2 = \frac{10\,kN\cdot m^2}{3EI}\cdot\frac{2EI}{2.5\,kN} = \frac{20}{7.5}m^2$$

$$\Rightarrow y = \pm\sqrt{\frac{20m^2}{7.5}} = \pm 1.633m \Rightarrow y = 1.633m \qquad (90)$$

THE NEGATIVE ROOT IS OUTSIDE THE DOMAIN OF THE REGION.

(80) $\rightarrow$ $$u(0) = -\frac{2.5\,kN}{6\cdot EI}\cdot(0)^3 + \frac{10\,kN\cdot m^2}{3\cdot EI}\cdot(0) = 0 \quad (91)$$

(80) $\rightarrow$ $$u(2m) = -\frac{2.5\,kN}{6EI}(2m)^3 + \frac{2\,kN}{3EI}\cdot(0)^3 + \frac{2\,kN\cdot m}{EI}(0)^2 + \frac{10\,kN\cdot m^2}{3EI}\cdot(2m)$$

$$= \frac{10\,kN\cdot m^2}{3EI} \quad (92)$$

(80) $\rightarrow$ $$u(1.633m) = -\frac{2.5\,kN}{6\cdot EI}\cdot(1.633m)^3 + \frac{10\,kN\cdot m^2}{3\,EI}\cdot(1.633m) =$$

$$= \frac{3.629\,kN\cdot m^3}{EI} \quad (93)$$

MAXIMUM VALUE OF u IN THIS REGION

REGION BC: $2m \leq y \leq 4m$

(85) $\rightarrow$ $V(2m) = 2.5\,kN,\ -1.5\,kN$

(77) $\rightarrow$ $V(4m) = 2.5\,kN\cdot(4m)^0 - 4\,kN\cdot(2m)^0 - \cancel{4\,kN\cdot m\,\langle 2m\rangle_{-1}} + 2.5\,kN\,(0)^0$

$= -1.5\,kN,\ \underline{1\,kN} \quad (94)$

(87) $\rightarrow$ $M(2m) = 5.0\,kN\cdot m,\ 1.0\,kN\cdot m$

(78) $\rightarrow$ $M(4m) = 2.5\,kN\cdot(4m) - 4\,kN\cdot(2m) - 4\,kN\cdot m\cdot(2m)^0 + 2.5\,kN\cdot(0)^1$

$= \underline{-2.0\,kN\cdot m} \quad (95)$

THE SIGN OF M CHANGES IN THIS REGION INDICATING A MAX/MIN θ IN THIS REGION. THE ROOTS OF M IN THIS REGION NEED TO BE DETERMINED SO THE MAX/MIN VALUE OF θ CAN BE DETERMINED. STARTING WITH (78), SETTING IT EQUAL TO ZERO, AND CONSIDERING THE REGION $2m < y < 4m$ (NOT THE "=" IS ABSENT).

$$M(y) = 0 = 2.5\,kN\cdot y - 4\,kN\cdot y + 8\,kN\cdot m - 4\,kN\cdot m$$

$$= -1.5\,kN\cdot y + 4\,kN\cdot m$$

$$\Rightarrow\ y = \frac{4\,kN\cdot m}{1.5\,N} = \frac{8}{3}\,m \quad (96)$$

CONTINUING BY CALCULATING THE CRITICAL VALUES OF θ IN THIS REGION

(89) → $$\Theta(2m) = -\frac{5\,kN\cdot m^2}{3\cdot EI}$$

(82) → $$\Theta(4m) = -\frac{2.5\,kN}{2EI}\cdot(4m)^2 + \frac{2\,kN}{EI}\cdot(2m)^2 + \frac{4\,kN\cdot m}{EI}\cdot(2m) - \frac{2.5\,kN}{2EI}\cdot(0)^2 + \frac{10\,kN\cdot m^2}{3\cdot EI} = -\frac{4\,kN\cdot m^2}{6EI}$$

$$= -\frac{2\,kN\cdot m^2}{3\,EI} \quad (97)$$

(82) → $$\Theta\left(\tfrac{8}{3}m\right) = -\frac{2.5\,kN}{2\cdot EI}\cdot\left(\tfrac{8}{3}m\right)^2 + \frac{2\,kN}{EI}\left(\tfrac{8}{3}m-2m\right)^2 + \frac{4\,kN\cdot m}{EI}\cdot\left(\tfrac{8}{3}m-2m\right)^1 + \frac{10\,kN\cdot m^2}{3\cdot EI}$$

$$= -\frac{2.5\,kN}{\cancel{2}\cdot EI}\cdot\frac{\overset{32}{\cancel{64}}m^2}{9} + \frac{2\,kN}{EI}\cdot\frac{4m^2}{9} + \frac{4\,kN\cdot m}{EI}\cdot\frac{2m}{3}\cdot\frac{3}{3} + \frac{10\,kN\cdot m^2}{3\cdot EI}$$

$$= -\frac{18\,kN\cdot m^2}{9EI} = -\frac{2.0\,kN\cdot m^2}{EI} \quad (98)$$

(92) → $$u(2m) = \frac{10\,kN\cdot m^2}{3\cdot EI}$$

(83) → $$u(4m) = -\frac{2.5\,kN}{6\cdot EI}(4m)^3 + \frac{2\,kN}{3\cdot EI}(2m)^3 + \frac{2\,kN\cdot m}{EI}\cdot(2m)^2 - \frac{2.5\,kN}{6EI}(0)^3 + \frac{10\,kN\cdot m^2}{3EI}\cdot(4m)$$

$$= 0 \quad (99)$$

THE FINAL SECTION OF MEMBER ABCD CAN NOW BE CONSIDERED

SECTION CD: $4m \leq y \leq 6m$

(94) → $$V(4m) = -1.5\,kN,\ 1\,kN$$

(77) → $$V(6m) = 2.5\,kN(6m)^0 - 4\,kN(4m)^0 - 4\,kN\cdot m\,\cancel{\langle 4m\rangle}_{-1} + 2.5\,kN(2m)^0 - 1\,kN(0)^0$$

$$= 1\,kN,\ 0\,kN \quad (100)$$

(95) $\rightarrow$ $M(4m) = -2.0\,kN\cdot m$

(78) $\rightarrow$ $M(6m) = 2.5\,kN\cdot(6m) - 4\,kN\cdot(4m) - 4\,kN\cdot m(4m)^0 + 2.5\,kN(2m) - 1\,kN(0)$

$= 0\,kN\cdot m$ (101)

(97) $\rightarrow$ $\Theta(4m) = -\dfrac{2\,kN\cdot m^2}{3\cdot EI}$

(82) $\rightarrow$ $\Theta(6m) = -\dfrac{2.5\,kN}{2\cdot EI}\cdot(6m)^2 + \dfrac{2\,kN}{EI}\cdot(4m)^2 + \dfrac{4\,kN\cdot m}{EI}\cdot(4m) - \dfrac{2.5\,kN}{2\cdot EI}(2m)^2 + \dfrac{1\,kN}{2EI}\cdot(0)^2 + \dfrac{10}{3}\dfrac{kN\cdot m^2}{EI}$

$= \dfrac{8\,kN\cdot m^2}{6\,EI} = \dfrac{4}{3}\dfrac{kN\cdot m^2}{EI}$ (102)

THE CHANGE IN SIGN BETWEEN (97) & (102) INDICATES THAT A ROOT EXISTS IN THIS REGION THAT LOCATES A MIN/MAX VALUE OF U. FOR $4m<y<6m$

(82) $\rightarrow$ $\Theta(y) = 0 = -\dfrac{2.5\,kN}{2EI}\cdot y^2 + \dfrac{2\,kN}{EI}\cdot(y-2m)^2 + \dfrac{4\,kN\cdot m}{EI}(y-2m) - \dfrac{2.5\,kN}{2\cdot EI}(y-4m)^2 + \dfrac{10\,kN\cdot m^2}{3\,EI}$

$0 = -\dfrac{2.5}{2}\cdot y^2 + 2\cdot(y^2 - 4m\cdot y + 4m) + 4m\cdot y - 8m^2 - \dfrac{2.5}{2}\cdot(y^2 - 8m\cdot y + 16m^2) + \dfrac{10}{3}m^2$

$0 = y^2\left(-\dfrac{2.5}{2} + 2 - \dfrac{2.5}{2}\right) + y\left(-2\cdot 4m + 4m + \dfrac{2.5}{2}\cdot 8m\right) + \left(2\cdot 4m - 8m^2 - \dfrac{2.5}{2}\cdot 16m^2 + \dfrac{10}{3}m^2\right)$

$0 = -\dfrac{1}{2}y^2 + 6m\cdot y - \dfrac{50}{3}m^2$

$\Rightarrow 0 = y^2 - 12m\cdot y + \dfrac{100}{3}m^2 = \underbrace{y^2 - 12m\cdot y + (-6m)^2}_{(y-6m)^2} - (-6m)^2 + \dfrac{100}{3}m^2$

$0 = (y-6m)^2 - 36m^2 + \dfrac{100}{3}m^2 = (y-6m)^2 - \dfrac{108m^2}{3} + \dfrac{100m^2}{3} = (y-6m)^2 - \dfrac{8}{3}m^2$

$\Rightarrow (y-6m)^2 = \dfrac{8}{3}m^2 \Rightarrow y = 6m \pm \sqrt{\dfrac{8}{3}m^2} = 7.633m,\ 4.367m$ (103)

ONLY THE SECOND VALUE ($y = 4.367$) IS IN THE REGION UNDER CONSIDERATION.

CALCULATING THE DISPLACEMENTS u AT POINTS OF INTEREST

(99) $\rightarrow$ $u(4m) = 0$

(83) $\rightarrow$ $u(6m) = -\dfrac{2.5\,kN}{6EI}\cdot(6m)^3 + \dfrac{2\,kN}{3\,EI}\cdot(4m)^3 + \dfrac{2\,kN\cdot m}{EI}\cdot(4m)^2 - \dfrac{2.5\,kN}{6EI}(2m)^3 + \dfrac{1\,kN}{6EI}(0m)^3 + \dfrac{10\,kN\cdot m^2}{3\,EI}\cdot 6m = \dfrac{8}{6}\dfrac{kN\cdot m^3}{EI} = \dfrac{4\,kN\cdot m^3}{3\,EI}$ (104)

SUBSTITUTING THE ROOT IN THIS REGION, (103), INTO (83) TO DETERMINE THE MAXIMUM DEFLECTION IN THE REGION

$$(83) \rightarrow \quad u(4.367m) = -\frac{2.5kN}{6EI}\cdot(4.367m)^3 + \frac{2kN}{3EI}(2.367m)^3 + \frac{2kN\cdot m}{EI}(2.367m)^2 - \frac{2.5kN}{6EI}(0.367m)^3 + \frac{10kN\cdot m^2}{3EI}\cdot 4.367m$$

$$= -\frac{0.7093\,kN\cdot m^3}{6EI} = -\frac{0.1182\,kN\cdot m^3}{EI} = -\frac{0.355\,kN\cdot m^3}{3EI} \quad (105)$$

ANOTHER DISPLACEMENT THAT IS HELPFULL IN PLOTTING THE DISPLACEMENT OF THE ELASTIC CURVE IS $y = 2.667m$. THIS IS WHERE THE CONCAVITY OF THE ELASTIC CURVE CHANGES.

$$(83) \rightarrow \quad u(2.667m) = -\frac{2.5kN}{6EI}\cdot(2.667m)^3 + \frac{2kN}{3EI}\cdot(0.667m)^3 + \frac{2kN\cdot m}{EI}(0.667m)^2 + \frac{10kN\cdot m^2}{3EI}\cdot(2.667m) = \frac{6.23\,kN\cdot m^3}{3EI} \quad (106)$$

THE SIGN CHANGE IN THE REGION $4m<y<6m$ BETWEEN VALUES OF u AT $y = 2m, 4.367m$, AND $6m$ ((99), (105), & (104)) INDICATE ROOTS EXIST IN THIS REGION. $4m<y<6m$

$$(83) \rightarrow \quad u(y) = 0 = -\frac{2.5kN}{6\cancel{EI}}\cdot y^3 + \frac{2kN}{3\cancel{EI}}(y-2m)^3 + \frac{2kN\cdot m}{\cancel{EI}}(y-2m)^2 - \frac{2.5kN}{6\cancel{EI}}(y-4m)^3 + \frac{10kN\cdot m^2}{3\cancel{EI}}\cdot y$$

$$\Rightarrow 0 = -\frac{2.5}{6}\cdot y^3 + \frac{2}{3}(y^2 - 4m\cdot y + 4m^2)(y-2m)$$

$$+ 2m(y^2 - 4m\cdot y + 4m^2) - \frac{2.5}{6}(y^2 - 8m\cdot y + 16m^2)(y-4m) + \frac{10}{3}m^2\cdot y$$

$$0 = -\frac{2.5}{6}\cdot y^3 + \frac{2}{3}(y^3 - 4m\cdot y^2 + 4m^2\cdot y - 2m\cdot y^2 + 8m^2 y - 8m^3)$$

$$+ 2m\cdot y^2 - 8m^2\cdot y + 8m^3 - \frac{2.5}{6}(y^3 - 8m\cdot y^2 + 16m^2 y - 4m\cdot y^2 + 32m^2 y - 64m^3)$$

$$+ \frac{10}{3}m^2\cdot y$$

$$0 = -\frac{2.5}{6}\cdot y^3 + \frac{2}{3}y^3 - \frac{12m}{3}\cdot y^2 + \frac{24m^2}{3}\cdot y - \frac{16}{3}m^3 + 2m\cdot y^2 - 8m^2\cdot y + 8m^3$$

$$- \frac{2.5}{6}y^3 + \frac{30}{6}m\cdot y^2 - \frac{120}{6}m^2\cdot y + \frac{160}{6}m^3 + \frac{10}{3}m^2\cdot y$$

$$0 = -\frac{1}{6}\cdot y^3 + \frac{18}{6}m\cdot y^2 - \frac{100}{6}m^2\cdot y + \frac{176m^3}{6}$$

$$0 = y^3 - 18m\cdot y^2 + 100m^2\cdot y - 176\,m^3$$

MATLAB'S FUNCTION "ROOTS" CAN BE USED TO SOLVE THIS CUBIC EQUATION'S ROOTS. THE ROOTS ARE FOUND TO BE

$$y = \underline{4.0m},\ \underline{4.7639m},\ 9.2361\,m \qquad (107)$$

THE LAST ROOT IN (107) ($y = 9.2361m$) IS OUTSIDE THE REGION BEING CONSIDERED AND THEREFORE IS NOT RELEVANT. THE FIRST ROOT IN (107) ($y = 4.0m$) IS THE BOUNDARY CONSTRAINT. THE ROOT THAT IS MOST RELEVANT IN FINDING WHERE $u=0$ BETWEEN $4m < y < 6m$ IS the second

$$\underline{\underline{y = 4.7639m}}$$

NOW THE BEAM SEGMENT <u>DFE</u> THAT SPANS FROM <u>$6m < y < 8m$</u> CAN BE EVALUATED. STARTING WITH THE SINGULARITY FUNCTION REPRESENTATION OF THE LOAD

$$q(y) = -1kN\langle y-6m\rangle_{-1} + 2kN\langle y-7m\rangle_{-1} - 1kN\langle y-8m\rangle_{-1} \qquad (108)$$

V, M, Θ, AND u ARE FOUND THROUGH DIRECT INTEGRATION

$$V(y) = \int -q(y)\,dy = \int\left[1kN\langle y-6m\rangle_{-1} - 2kN\langle y-7m\rangle_{-1} + 1kN\langle y-8m\rangle_{-1}\right]dy$$

$$= 1kN\langle y-6m\rangle^0 - 2kN\langle y-7m\rangle^0 + 1kN\langle y-8m\rangle^0 \qquad (109)$$

$$M(y) = \int V(y)\,dy = 1kN\langle y-6m\rangle^1 - 2kN\langle y-7m\rangle^1 + 1kN\langle y-8m\rangle^1 \qquad (110)$$

$$\Theta(y) = \int -\frac{M}{EI}\,dy = \int\left[-\frac{1kN}{EI}\langle y-6m\rangle^1 + \frac{2kN}{EI}\langle y-7m\rangle^1 - \frac{1kN}{EI}\langle y-8m\rangle^1\right]dy$$

$$= -\frac{1kN}{2\cdot EI}\langle y-6m\rangle^2 + \frac{1kN}{EI}\langle y-7m\rangle^2 - \frac{1kN}{2EI}\langle y-8m\rangle^2 + \cancelto{-\frac{1kN\cdot m^2}{6\cdot EI}}{C_3} \qquad (111)$$

$$u(y) = \int \Theta(y)\,dy =$$

$$= -\frac{1kN}{6EI}\langle y-6m\rangle^3 + \frac{1kN}{3EI}\langle y-7m\rangle^3 - \frac{1kN}{6EI}\langle y-8m\rangle^3 + \cancelto{-\frac{1kN\,m^2}{6EI}}{C_3}\cdot y + \cancelto{\frac{7kN\,m^3}{3EI}}{C_4} \qquad (112)$$

THE CONSTANTS C_3 & C_4 IN (111) & (112) NEED TO BE DETERMINED FROM BOUNDARY CONDITIONS. FOR THE DFE BEAM SEGMENT ($6m \leq y \leq 8m$), THE TWO BOUNDARY CONDITIONS ARE

$$(104) \rightarrow u(6m) = \frac{4\,kN \cdot m^2}{3EI} \quad (113)$$

$$u(8m) = 0m \quad (114)$$

[THE VALUE FOR $\Theta(6m)$ FROM (102) IS NOT A CONTINUITY CONDITION THAT CAN BE USED TO EVALUATE THE CONSTANTS C_3 & C_4, BECAUSE THE PIN AT D DOES NOT CONSTRAIN ROTATIONS OF ABCD OR DFE AT $y=6m$]

SUBSTITUTING (113) AND (114) INTO (112)

$$u(6m) = \frac{4\,kN \cdot m^2}{3 \cdot EI} = -\frac{1\,kN}{6EI} \cdot (0)^3 + 6m \cdot C_3 + C_4$$

$$\Rightarrow \frac{4\,kN \cdot m^3}{3 \cdot EI} = 6m \cdot C_3 + C_4 \quad (115)$$

$$u(8m) = 0 = -\frac{1\,kN}{6EI}(2m)^3 + \frac{1\,kN}{3EI}(1m)^3 - \frac{1\,kN}{6EI}(0m)^3 + 8m \cdot C_3 + C_4$$

$$0 = -\frac{6\,kN \cdot m^3}{6 \cdot EI} + 8m \cdot C_3 + C_4 = -\frac{1\,kN \cdot m^3}{EI} + 8m \cdot C_3 + C_4$$

$$\Rightarrow \frac{1\,kN\,m^3}{EI} = 8m \cdot C_3 + C_4$$

$$\Rightarrow C_4 = \frac{1\,kN \cdot m^3}{EI} - 8m \cdot C_3 \quad (116)$$

$$(116) \rightarrow (115) \rightarrow \frac{4\,kN \cdot m^3}{3 \cdot EI} = 6m \cdot C_3 + \frac{1\,kN \cdot m^3}{EI} - 8m \cdot C_3$$

$$\frac{1\,kN \cdot m^3}{3EI} = -2m \cdot C_3 \Rightarrow C_3 = -\frac{1\,kN \cdot m^2}{6 \cdot EI} \quad (117)$$

$$(117) \rightarrow (116) \rightarrow C_4 = \frac{1\,kN \cdot m^3}{EI} + 8m \cdot \frac{1\,kN \cdot m^2}{6EI} = \frac{7\,kN \cdot m^2}{3EI} \quad (118)$$

SUBSTITUTING (117) AND (118) INTO (111) AND (112)

$$(111) \rightarrow \quad \Theta(y) = -\frac{1\,kN}{2EI}\cdot\langle y-6m\rangle^2 + \frac{1\,kN}{EI}\langle y-7m\rangle^2 - \frac{1\,kN}{2EI}\langle y-8m\rangle^2 - \frac{1\,kN\cdot m^2}{6\,EI} \quad (119)$$

$$(112) \rightarrow \quad u(y) = -\frac{1\,kN}{6EI}\cdot\langle y-6m\rangle^3 + \frac{1\,kN}{3EI}\cdot\langle y-7m\rangle^3 - \frac{1\,kN}{6EI}\cdot\langle y-8m\rangle^3 - \frac{1\,kN\cdot m^2}{6EI}\cdot y + \frac{7\,kN\cdot m^3}{3EI} \quad (120)$$

CRITICAL VALUES OF V, M, Θ, AND u IN EACH REGION OF DFE CAN NOW BE DETERMINED.

REGION DF: $6m \leq y \leq 7m$

$$(109) \rightarrow \quad V(6m) = 1\,kN\,(0)^0 = 0\,kN,\ 1\,kN \quad (121)$$

$$(109) \rightarrow \quad V(7m) = 1\,kN\,(1m)^0 - 2\,kN\,(0)^0 = 1\,kN,\ -1\,kN \quad (122)$$

$$(110) \rightarrow \quad M(6m) = 1\,kN\,(0)^1 = 0 \quad (123)$$

$$(110) \rightarrow \quad M(7m) = 1\,kN\,(1m)^1 - 2\,kN\,(0)^1 = 1\,kN\cdot m \quad (124)$$

$$(119) \rightarrow \quad \Theta(6m) = -\frac{1\,kN}{2EI}(0)^2 - \frac{1\,kN\cdot m^2}{6\,EI} = -\frac{1\,kN\cdot m^2}{6\,EI} \quad (125)$$

$$(119) \rightarrow \quad \Theta(7m) = -\frac{1\,kN}{2EI}\cdot(1m)^2 + \frac{1\,kN}{EI}(0)^2 - \frac{1\,kN\,m^2}{6EI} = -\frac{4\,kN\cdot m^2}{6\cdot EI} \quad (126)$$

$$(120) \rightarrow \quad u(6m) = -\frac{1\,kN}{6EI}(0)^3 - \frac{1\,kN\cdot m^2}{6\,EI}\cdot(6m) + \frac{7\,kN\cdot m^3}{3\,EI} = \frac{8\,kN\cdot m^3}{6\,EI} = \frac{4\,kN\cdot m^3}{3EI} \quad (127)$$

$$(120) \rightarrow \quad u(7m) = -\frac{1\,kN}{6EI}\cdot(1m)^3 + \frac{1\,kN}{3EI}(0)^3 - \frac{1\,kN\cdot m^2}{6EI}\cdot(7m) + \frac{7\,kN\cdot m^3}{3EI} = \frac{6\,kN\cdot m^3}{6EI}$$

$$= \frac{1\,kN\cdot m^3}{EI} \quad (128)$$

REGION FE: $7m \leq y \leq 8m$

$$(122) \rightarrow \quad V(7m) = 1\,kN,\ -1\,kN$$

$$(109) \rightarrow \quad V(8m) = 1\,kN\,(2m)^0 - 2\,kN\cdot(1m)^0 + 1\,kN\cdot(0)^0 = -1\,kN,\ 0 \quad (129)$$

(124) → $M(7m) = 1\,kN\cdot m$

(110) → $M(8m) = 1\,kN(2m) - 2\,kN(1m) + 1\,kN\cdot(0) = 0$ (130)

(126) → $\Theta(7m) = -\dfrac{4\,kN\cdot m^2}{6EI} = -\dfrac{2\,kN\cdot m^2}{3EI}$

(111) → $\Theta(8m) = -\dfrac{1\,kN}{2EI}(2m)^2 + \dfrac{1\,kN}{EI}(1m)^2 - \dfrac{1\,kN}{2\cdot EI}(0m)^2 - \dfrac{1\,kN\cdot m^2}{6EI} = \dfrac{-7\,kN\cdot m^2}{6EI}$ (131)

(128) → $u(7m) = \dfrac{1\,kN\cdot m^3}{EI}$

(112) → $u(8m) = -\dfrac{1\,kN}{6EI}(2m)^3 + \dfrac{1\,kN}{3EI}(1m)^3 - \dfrac{1\,kN}{6EI}(0m)^3 - \dfrac{1\,kN\cdot m^2}{6EI}\cdot 8m + \dfrac{7\,kN\,m^3}{3EI}$

$= 0$ (132)

SINGULARITY FUNCTION SUMMARY

THE VALUES FOR V, M, Θ, AND U CALCULATED USING SINGULARITY FUNCTIONS ARE SUMMARIZED IN FIGURES (c)-(f). THESE VALUES EXACTLY MATCH THE VALUES COMPUTED USING THE DIRECT INTEGRATION APPROACH.