Name: Solution

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Problem 1. A 20 mm diameter steel rod of S_y=350 MPa is loaded as a column with pinned ends. If it is sufficiently short, it can carry a limiting load of $S_y \bullet A$=110 kN.

1a. Using the figure provided, draw the P/A and L/ρ locations for the location on Euler's curve where Euler's Equation is no longer valid. Also, draw the Johnson (Parabolic) curve on this figure being sure to label the critical P/A and L/ρ locations. Where

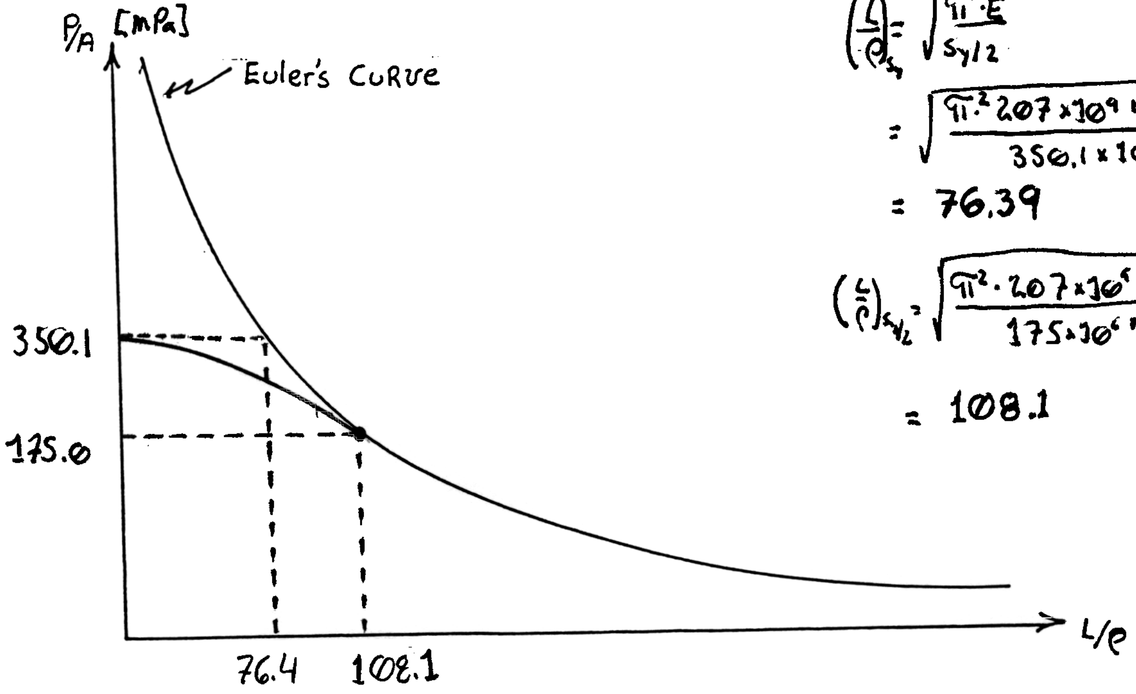
E=207 GPa $\rho = \sqrt{I/A} = d/4 = 5\text{mm}$

$A = \pi \frac{D^2}{4} = \pi \frac{(20\times10^{-3} m)^2}{4}$
$= 0.3142 (10^{-3}) m^2$

$S_y = 350.1 (10^6) Pa$
350.1 MPa

$\left(\frac{L}{\rho}\right)_{S_y} = \sqrt{\frac{\pi^2 E}{S_y/2}}$
$= \sqrt{\frac{\pi^2\, 207\times10^9\, N/m^2}{350.1\times10^6\, N/m^2}}$
$= 76.39$

$\left(\frac{L}{\rho}\right)_{S_y/2} = \sqrt{\frac{\pi^2 \cdot 207\times10^9\, N/m^2}{175\times10^6\, N/m^2}}$
$= 108.1$



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1b. How long can the rod be and still carry 10% of the limiting load?

$.10 \cdot 350.1\ mPa = 35.01\ mPa$

$$\left(\frac{L}{\rho}\right) = \sqrt{\frac{\pi^2 \cdot 207 \cdot 10^{9} N/m^2}{35.01 \times 10^{6}\ N/m^2}} = 241.6 \Rightarrow L = 241.6 \cdot 5\times10^{-3} m$$

$$= 1.208\ m$$

$$\boxed{1.21\ m}$$

1c. How long can the rod be and still carry 90% of the limiting load?

$$.90 \cdot 350.1 \text{ MPa} = 315.1 \text{ MPa}$$

$$\left(\frac{L}{\rho}\right) = \sqrt{\frac{\pi^2 \cdot 207\times10^9 \text{ N/m}^2}{315.1\times10^6 \text{ N/m}^2}} = 80.52$$

This places the beam in an intermediate range. Johnson's equation is required.

$$\frac{P_{cr}}{A} = S_y - \frac{S_y^2}{4\cdot\pi^2\cdot E}\left(\frac{l}{\rho}\right)^2$$

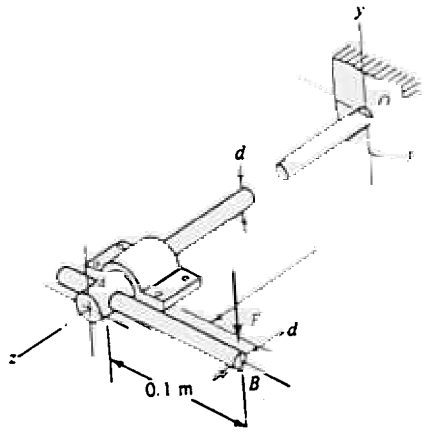
$$315.1\times10^6 \text{ N/m}^2 = 350.1\times10^6 \text{ N/m}^2 - \frac{(350.1\times10^6 \text{ N/m}^2)^2}{4\times\pi^2\cdot207\times10^9 \text{ N/m}^2}\cdot\left(\frac{l}{\rho}\right)^2$$

$$\frac{l}{\rho} = 48.31 \Rightarrow l = 48.31 \cdot 5\times10^{-3}\text{ m} = 0.2416 \text{ m} = \boxed{0.242 \text{ m}}$$

Problem 2. Illustrated below is a torsion-bar spring OA having a diameter d=12mm. The actuating cantilever AB also has d=12mm. Both parts are made of carbon steel.

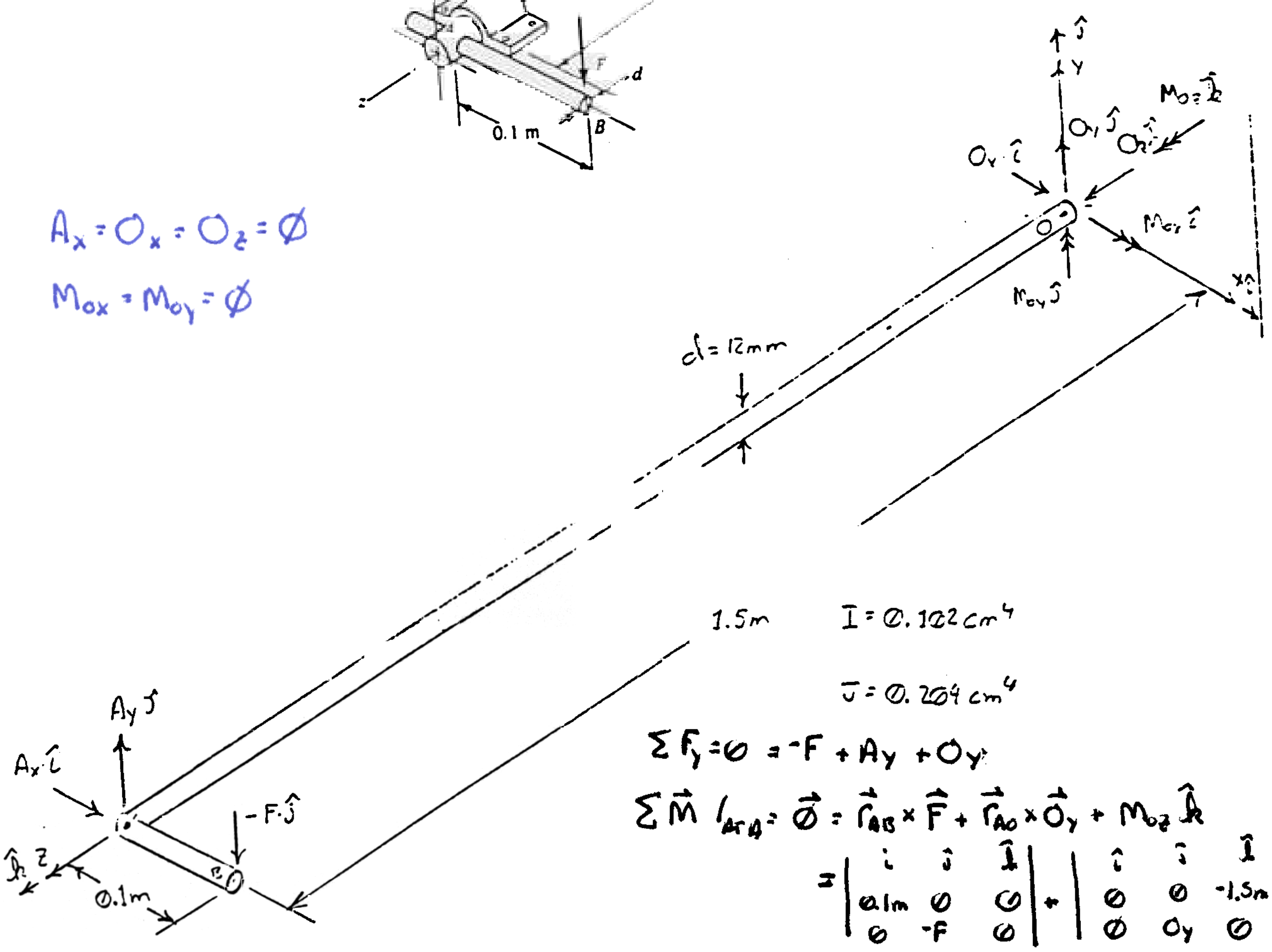
G=79.3 GPa, E=207 GPa, I=0.102cm^4, J=0.204cm^4

Using Castigliono's Theorem, find the spring force k corresponding to a force acting at B.



$A_x = O_x = O_z = \emptyset$

$M_{ox} = M_{oy} = \emptyset$



$I = 0.102\,cm^4$

$J = 0.204\,cm^4$

$$\Sigma F_y = 0 = -F + A_y + O_y$$

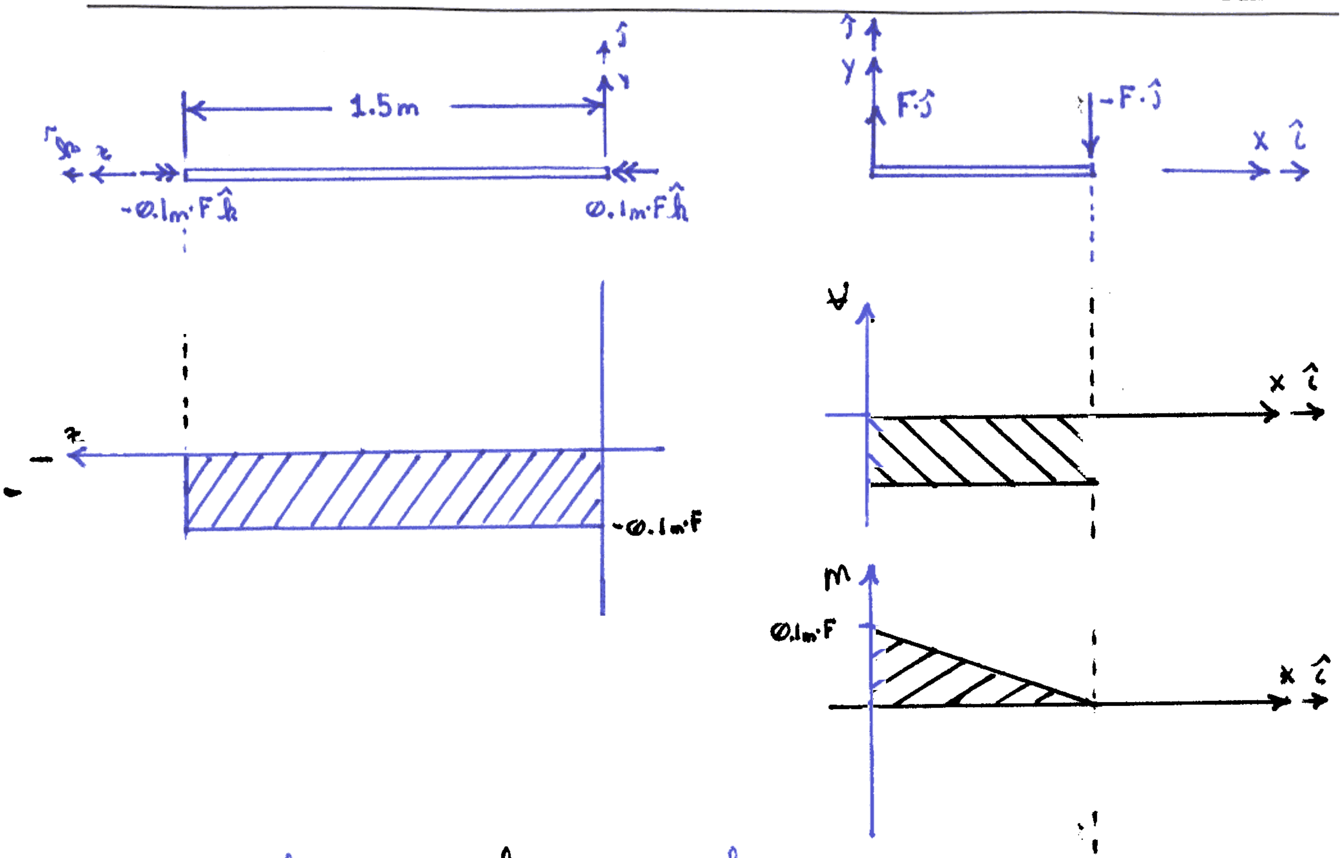
$$\Sigma \vec{M}/_{A} = \vec{0} = \vec{r}_{AB} \times \vec{F} + \vec{r}_{AO} \times \vec{O}_y + M_{oz}\hat{k}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.1m & 0 & 0 \\ 0 & -F & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -1.5m \\ 0 & O_y & 0 \end{vmatrix} + M_{oz}\hat{k}$$

$$= -0.1m \cdot F\hat{k} + 1.5m \cdot O_y\hat{i} + M_{oz}\hat{k}$$

$$\Sigma M_x/_A = 0 = 1.5m \cdot O_y \Rightarrow \boxed{O_y = 0} \Rightarrow \boxed{A_y = F}$$

$$\Sigma M_z/_A = 0 = -0.1m \cdot F + M_{oz} \Rightarrow \boxed{M_{oz} = 0.1m \cdot F}$$



$$U = \int_0^{l_2} \frac{T^2}{2 \cdot G \cdot J}\, dz + \int_0^{l_1} \frac{M^2}{2 \cdot E \cdot I}\, dx = \int_0^{l_2} \frac{(1+\nu)\cdot T^2}{E \cdot J}\, dz + \int_0^{l_1} \frac{M^2}{2 \cdot E \cdot I}\, dx$$

$$= \int_0^{1.5m} \frac{0.01\ m^2 \cdot F^2}{2\cdot(79.3\times 10^9\ N/m^2)\cdot(0.204\times 10^{-8} m^4)}\, dz + \int_0^{0.1m} \frac{(-x+0.1m)^2 \cdot F^2}{2\cdot(207\times 10^9\ N/m^2)\cdot(0.102\times 10^{-8} m^4)}\, dx$$

$$= \int_0^{1.5m} \frac{0.01\ m^2 \cdot F^2}{323.5\ N\cdot m^2}\, dz + \int_0^{0.1m} \frac{(-x+0.1m)^2}{422.3\ N\cdot m^2}\, dx$$

$$\frac{\partial U}{\partial F} = \int_0^{1.5m} \frac{0.02\ m^2 \cdot F}{323.5\ N\cdot m^2}\, dz + \int_0^{0.1m} \frac{(2x^2 - 0.4\ m\cdot x + 0.02 m^2)\cdot F}{422.3\ N\cdot m^2}\, dx$$

$$= \frac{0.02 m^2 \cdot F \cdot z}{323.5\ N\cdot m^2}\Bigg|_0^{1.5m} + \frac{F}{422.3\ N\cdot m^2}\left(\frac{2}{3}x^3 - 0.2m\cdot x^2 + 0.02 m^2\, x\right)\Bigg|_0^{0.1m}$$

$$\delta = \frac{\partial U}{\partial F} = 94.31(10^{-6})\frac{m}{N}\cdot F \quad \Rightarrow \quad F = 10.60\times 10^3 \frac{N}{m}\cdot \delta$$

$$k = \frac{F}{\delta} = 10.6\,(10^3)\frac{N}{m} = \boxed{10.6\ \frac{kN}{m}}$$

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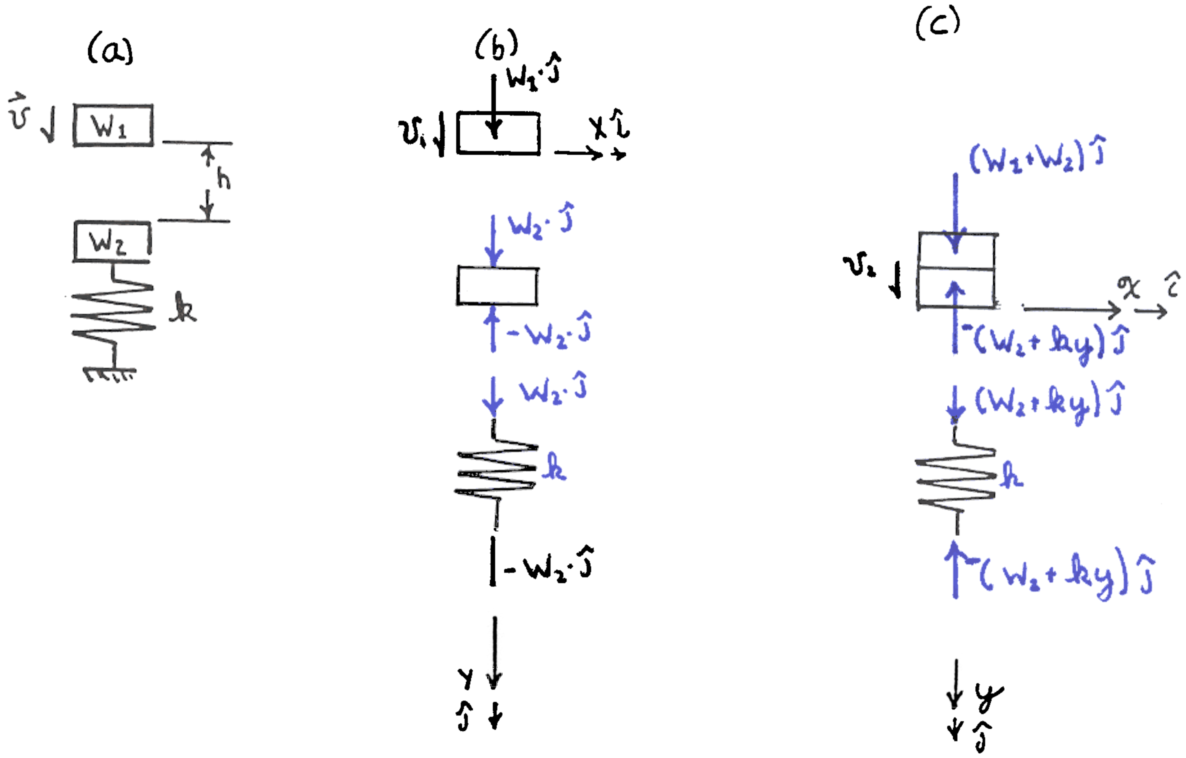
Problem 3. As shown in the figure, the weight W_1 strikes W_2 from a height h.

3a. Figure b is an incomplete free-body-diagram of the system prior to W_1 impacting W_2 and Figure c is an incomplete free-body-diagram of the system after W_1 has impacted W_2. Complete these free-body-diagrams.



3b. Impose equilibrium on the free-body-diagram found in Figure c and determine the differential equation that represents the motion of the system. What are the initial conditions for this system after W_1 and W_2 have impacted.

$$\sum \vec{F} = m\vec{a}$$

Dotting the above equation with $\hat{j}$

$$(W_1+W_2) - (W_2 + k\cdot y) = m\cdot\ddot{y} = \frac{W_1+W_2}{g}\cdot\ddot{y}$$

$$\boxed{W_1 - k\cdot y = \frac{W_1+W_2}{g}\ddot{y}}$$

The initial conditions are

$$t=0, \quad y=0$$

$$t=0, \quad \dot{y} = v^2 = \frac{W_1}{W_1+W_2}\cdot v_1 = \frac{W_1}{W_1+W_2}\cdot\sqrt{2\cdot g\cdot h}$$