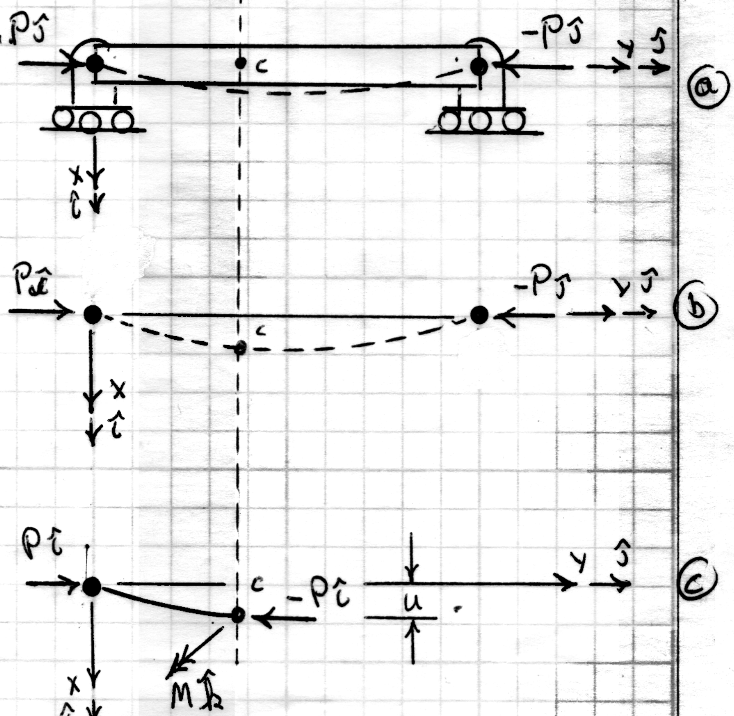
DETERMINE THE CRITICAL LOAD FOR THIS BEAM SUBJECTED TO BUCKLING LOADS

FIGURE (b) SHOWS THE BEAM IN EQUILIBRIUM. NOW LETS CONSIDER A RANDOM POINT ON THE BEAM INDICATED AS POINT C. FIGURE (c) REPRESENTS THE INTERNAL EQUILIBRIUM AT C. STARTING BY DETERMINING AN EXPRESSION FOR THE INTERNAL MOMENT



$$\sum M_{z}/_{@c} = 0 = M - P \cdot u$$

$$\Rightarrow M = P \cdot u \qquad (1)$$

FROM STRENGTH OF MATERIALS. THE RELATIONSHIP BETWEEN THE INTERNAL MOMENT AND THE DEFLECTION IS GIVEN BY

$$\frac{\partial^2 u}{\partial y^2} = -\frac{M}{E \cdot I} = \frac{-P \cdot u}{E \cdot I}$$

$$\frac{\partial^2 u}{\partial y^2} + \frac{P}{EI} \cdot u = 0 \qquad (2)$$

EQUATION (2) IS A HOMOGENEOUS DIFFERENTIAL EQUATION. THE SOLUTION IS ASSUMED TO TAKE THE FORM

$$u = A_n \cdot e^{S_n \cdot y}$$

$$u' = \partial u/\partial y = A_n \cdot S_n \cdot e^{S_n \cdot y} \qquad (3)$$

$$u'' = \partial^2 u/\partial y^2 = A_n \cdot S_n^2 \cdot e^{S_n \cdot y}$$

SUBSTITUTING THE RESULTS IN (3) INTO (2)

$$A_n \cdot S_n^2 e^{S_n \cdot y} + \frac{P}{EI} \cdot A_n \cdot e^{S_n \cdot y} = 0$$

$$A_n \cdot e^{S_n \cdot y} \left(S_n^2 + \frac{P}{EI}\right) = 0$$

ALLOWING $A_n = 0$ WILL YIELD THE TRIVIAL SOLUTION, $e^{S_n \cdot y}$ CAN NEVER EQUAL ZERO, THEREFORE THE ABOVE EQUATION IS SATISFIED ONLY IF

$$S_n^2 + \frac{P}{EI} = 0 \Rightarrow S_n = \pm\sqrt{-\frac{P}{EI}} = \pm i\sqrt{\frac{P}{EI}} \qquad (4)$$

SUBSTITUTING THIS RESULT BACK INTO (3)

$$u = A_0 \cdot e^{i\sqrt{\frac{P}{EI}} \cdot y} + A_1 \cdot e^{-i\sqrt{\frac{P}{EI}} \cdot y}$$

KNOWING $e^{\theta} = \cos\theta + i \cdot \sin\theta$ AND $e^{i\theta} = \cos\theta - i\sin\theta$, u CAN NOW BE WRITTEN

$$u = A_0\left(\cos\sqrt{\tfrac{P}{EI}}\,y + i\sin\sqrt{\tfrac{P}{EI}}\,y\right) + A_1\left(\cos\sqrt{\tfrac{P}{EI}}\,y - i\cdot\sin\sqrt{\tfrac{P}{EI}}\,y\right)$$

$$= (A_0 + A_1)\cdot\cos\sqrt{\tfrac{P}{EI}}\cdot y + (A_0 - A_1)\,i\cdot\sin\sqrt{\tfrac{P}{EI}}\cdot y$$

$$= C_0\cdot\cos\sqrt{\tfrac{P}{EI}}\cdot y + C_1\cdot\sin\sqrt{\tfrac{P}{EI}}\cdot y \qquad (5)$$

THE CONSTANTS C_0 AND C_1 ARE DETERMINED USING BOUNDARY CONDITIONS. THE FIRST BOUNDARY CONDITION IS AT $y=0$

$$u(0) = 0 = C_0\cdot\cancelto{1}{\cos\sqrt{\tfrac{P}{EI}}\cdot 0} + C_1\cdot\cancelto{0}{\sin\sqrt{\tfrac{P}{EI}}\cdot 0} \Rightarrow C_0 = 0 \qquad (6)$$

SUBSTITUTING (6) INTO (5)

$$u = C_1\cdot\sin\sqrt{\tfrac{P}{EI}}\cdot y$$

THE SECOND BOUNDARY CONDITION OCCURS AT $y=L$

$$u(L) = 0 = C_1\cdot\sin\sqrt{\tfrac{P}{EI}}\cdot L$$

SINCE SETTING $C_1 = 0$ WOULD YIELD THE TRIVIAL SOLUTION, WHERE $\sin\sqrt{\tfrac{P}{EI}}\,L = 0$ MUST BE FOUND

$$\sqrt{\tfrac{P}{EI}}\cdot L = n\pi \quad ; n = 0, 1, 2, 3, \ldots$$

NOW P_{CR} WILL OCCUR AT THE LOWEST LOAD LEVEL

$$\boxed{P_{CR} = \frac{\pi^2}{L^2}\,E\cdot I}$$