NAME: Solution

PROBLEM #1: Consider the mechanism shown in the figure below. The triangular wedge, coupler CDB is attached to two sliders at B and C. The joints at B and C are full joints. Both sliders are constrained to move along the wall frictionlessly. Point B is being forced to move at a constant velocity of 6.10 m/s to the left. For the position shown the loop closure equation is as follows:

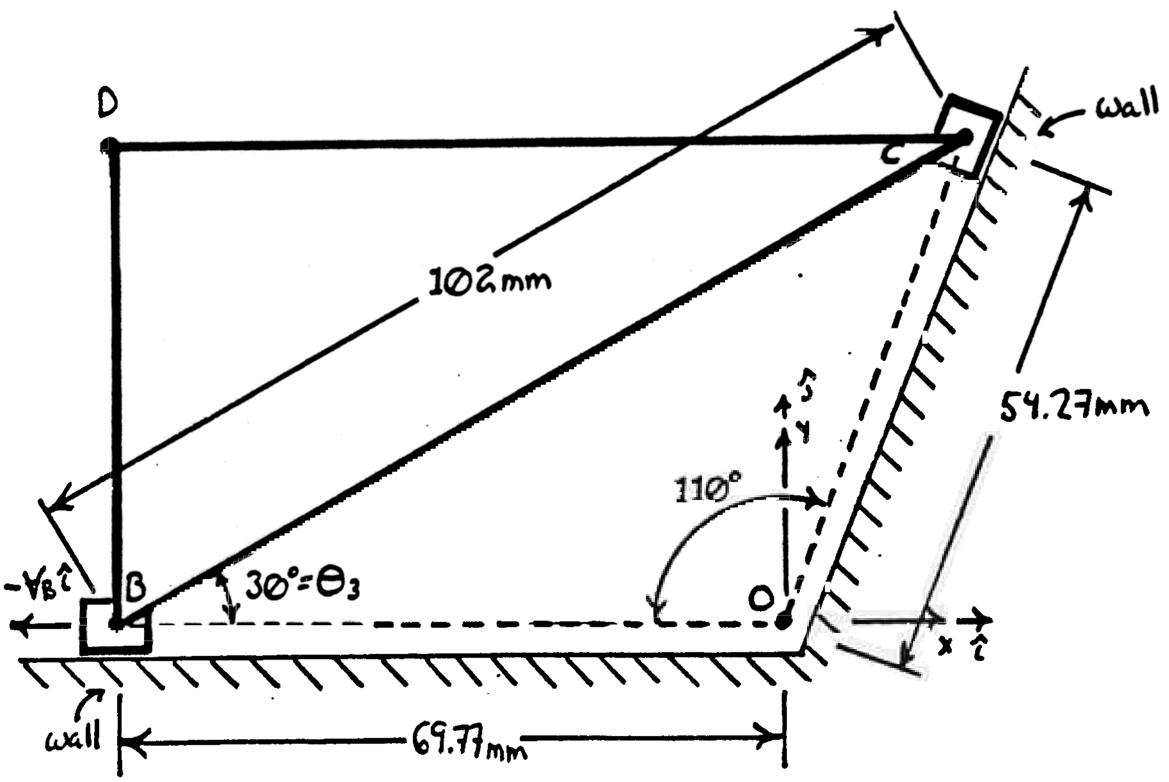
$$\vec{R}_{BO} + \vec{R}_{OC} = \vec{R}_{BC}$$

where

$$\vec{R}_{BO} = R_{BO} \cdot e^{j\theta_1} = 69.77\text{mm} \cdot e^{j0^\circ}$$
$$= R_{BO} \cdot \hat{e}_{BO} = 69.77\text{mm} \cdot \hat{i}$$

$$\vec{R}_{OC} = R_{OC} \cdot e^{j\theta_2} = 54.77\text{mm} \cdot e^{j70^\circ} = 18.56\text{mm} + j \cdot 51.0\text{mm}$$
$$= R_{OC} \cdot \hat{e}_{OC} = 54.77\text{mm} \cdot (0.3420 \cdot \hat{i} + 0.9397 \cdot \hat{j})$$

$$\vec{R}_{BC} = R_{BC} \cdot e^{j\theta_3} = 102\text{mm} \cdot e^{j30^\circ} = 88.33\text{mm} + j \cdot 51.0\text{mm}$$
$$= R_{BC} \cdot \hat{e}_{BC} = 102\text{mm} \cdot (0.8660 \cdot \hat{i} + 0.5 \cdot \hat{j})$$



1a) From a velocity analysis it was found that $\dot{\theta}_3 = -73.4\ \frac{1}{s}$ and $\dot{R}_{OC} = -5.396\ \frac{m}{s}$. differentiate the loop closure equation twice and determine $\ddot{R}_{OC}$ using one of the analyitical approaches.

Complex Method

$$R_{BO}e^{j\theta_1} + R_{OC}e^{j\theta_2} = R_{BC}e^{j\theta_3}$$

$$\dot{R}_{BO}e^{j\theta_1} + R_{BO}j\cancel{\dot{\theta}_1}^{0}e^{j\theta_1} + \dot{R}_{OC}e^{j\theta_2} + R_{OC}j\cancel{\dot{\theta}_2}^{0}e^{j\theta_2} = \cancel{\dot{R}_{BC}}^{0}e^{j\theta_3} + R_{BC}j\dot{\theta}_3e^{j\theta_3}$$

$$\dot{R}_{BO}e^{j\theta_1} + \dot{R}_{OC}e^{j\theta_2} = R_{BC}j\dot{\theta}_3e^{j\theta_3}$$

10

$$\cancel{\ddot{R}_{BO}}^{0}e^{j\theta_1} + \dot{R}_{BO}j\cancel{\dot{\theta}_1}^{0}e^{j\theta_1} + \ddot{R}_{OC}e^{j\theta_2} + \dot{R}_{OC}j\cancel{\dot{\theta}_2}^{0}e^{j\theta_2} = \cancel{\dot{R}_{BC}}^{0}j\dot{\theta}_3e^{j\theta_3} + R_{BC}j\ddot{\theta}_3e^{j\theta_3} - R_{BC}\dot{\theta}_3^2e^{j\theta_3}$$

$$\ddot{R}_{OC}e^{j\theta_2} = R_{BC}j\ddot{\theta}_3e^{j\theta_3} - R_{BC}\dot{\theta}_3^2e^{j\theta_3}$$

$$\ddot{R}_{OC}(\cos\theta_2 + j\sin\theta_2) = R_{BC}j\ddot{\theta}_3(\cos\theta_3 + j\sin\theta_3) - R_{BC}\dot{\theta}_3^2(\cos\theta_3 + \sin\theta_3)$$

$$\ddot{R}_{OC}\cos\theta_2 = -R_{BC}\ddot{\theta}_3\sin\theta_3 - R_{BC}\dot{\theta}_3^2\cos\theta_3$$

$$\ddot{R}_{OC}\sin\theta_2 = R_{BC}\ddot{\theta}_3\cos\theta_3 - R_{BC}\dot{\theta}_3^2\sin\theta_3$$

10

$$\ddot{R}_{OC}\cos\theta_2\cos\theta_3 = -R_{BC}\ddot{\theta}_3\sin\theta_3\cos\theta_3 - R_{BC}\dot{\theta}_3^2\cos\theta_3\cos\theta_3$$

$$+\ \ddot{R}_{OC}\sin\theta_2\sin\theta_3 = R_{BC}\ddot{\theta}_3\cos\theta_3\sin\theta_3 - R_{BC}\dot{\theta}_3^2\sin\theta_3\sin\theta_3$$

$$\ddot{R}_{OC}\cos(\theta_2-\theta_3) = -R_{BC}\dot{\theta}_3^2$$

$$\ddot{R}_{OC} = \frac{-R_{BC}\dot{\theta}_3^2}{\cos(\theta_2-\theta_3} = \frac{(102mm)(-73.4\frac{1}{s})^2}{\cos(70°-30°)} = -7\ 7.4(10^3)\frac{mm}{s^2} = 717.4\frac{M}{s^2}$$

Vector Method

$$R_{BO}\hat{e}_{BO} + R_{OC}\hat{e}_{OC} = R_{BC}\hat{e}_{BC}$$

$$\dot{R}_{BO}\hat{e}_{BO} + R_{BC}\cancel{\dot{\hat{e}}_{BO}}^{0} + \dot{R}_{OC}\hat{e}_{OC} + R_{OC}\cancel{\dot{\hat{e}}_{OC}}^{0} = \cancel{\dot{R}_{BC}}^{0}\hat{e}_{BC} + R_{BC}\dot{\hat{e}}_{BC}$$

$$\dot{R}_{BO}\hat{e}_{BC} + \dot{R}_{OC}\hat{e}_{OC} = R_{BC}\dot{\theta}_3(\hat{k}\times\hat{e}_{BC})$$

$$\cancel{\ddot{R}_{BC}}^{0}\hat{e}_{BO} + \dot{R}_{BC}\cancel{\dot{\hat{e}}_{BO}}^{0} + \ddot{R}_{OC}\hat{e}_{OC} + \dot{R}_{OC}\cancel{\dot{\hat{e}}_{OC}}^{0} = \cancel{\dot{R}_{BC}}^{0}\dot{\theta}_3(\hat{k}\times\hat{e}_{BC}) + R_{BC}\ddot{\theta}_3(\hat{k}\times\hat{e}_{BC})$$
$$+ R_{BC}\dot{\theta}_3^2(\hat{k}\times(\hat{k}\times\hat{e}_{BC}))$$

$$\ddot{R}_{OC}\hat{e}_{OC} = R_{BC}\ddot{\theta}_3(\hat{k}\times\hat{e}_{BC}) + R_{BC}\dot{\theta}_3^2(\hat{k}\times(\hat{k}\times\hat{e}_{BC}))$$

Dotting with $\hat{e}_{BC}$

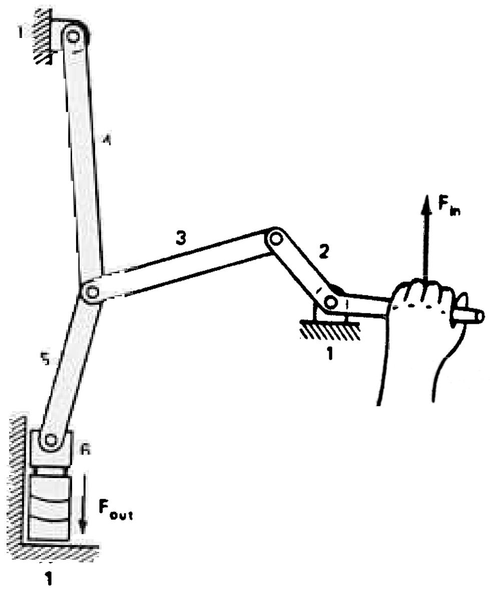
$$\ddot{R}_{OC}\;\hat{e}_{BC}\cdot\hat{e}_{OC} = R_{BC}\ddot{\theta}_3\overbrace{\hat{e}_{BC}\cdot(\hat{k}\times\hat{e}_{BC})}^{0} + R_{BC}\dot{\theta}_3^2\overbrace{\hat{e}_{BC}\cdot(\hat{k}\times(\hat{k}\times\hat{e}_{BC}))}^{-1}$$

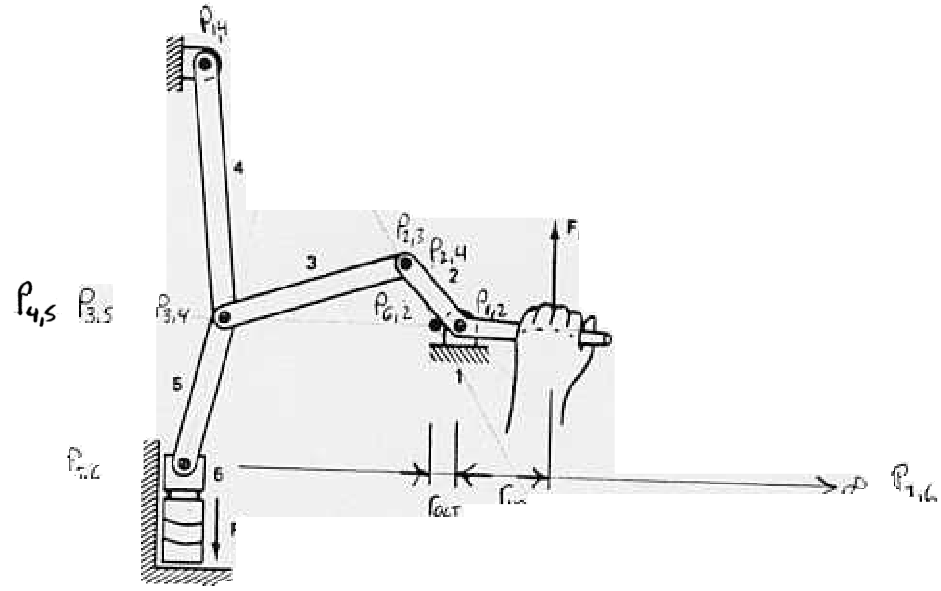
$$\ddot{R}_{OC} = R_{BC}\,\dot{\theta}_3^2\,\frac{\hat{e}_{BC}\cdot(\hat{k}\times(\hat{k}\times\hat{e}_{BC}))}{\hat{e}_{BC}\cdot\hat{e}_{OC}}$$

$$\hat{e}_{BC}\cdot\hat{e}_{OC} = (.866\hat{i} + .5\hat{j})\cdot(.3420\hat{i} + .9397\hat{j}) = 0.7660$$

$$\ddot{R}_{OC} = 102\text{mm}\,(-73.4\tfrac{1}{s})^2\,\frac{-1}{.7660} = -717.4(10^3)\frac{\text{mm}}{s} = -717.4\frac{\text{m}}{s^2}$$

PROBLEM 2: For the mechanism shown, determine the mechanical advantage.





Knowing that

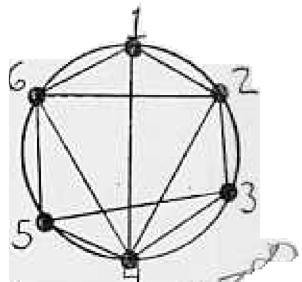
P P_{out} $\Rightarrow$ $T_{in}\,\omega_{in}$ F_{i} V r_{out} $\cancel{V}_{out}$ T_{out} ω_{out}

Because of the constraints lin 6 T can stated that V_6 $V_{P_{6,2}}$

V_{in} r_{in} ω_2 $\rightarrow$ F_{in} V_{in} F_{out} V_{ou}

V_{out} r_{out} ω_2 $\quad$ F_{i} r_{in} $\cancel{\omega_2}$ F_{out} r_{out} $\cancel{\omega}$

$$MA \quad \frac{F_{out}}{F} \quad \frac{r_{in}}{r_{out}} \quad \frac{15}{4} \quad \boxed{3.75}$$



F

MA

$\frac{r_2}{r_6}$

PROBLEM 3: For a cam with the following characteristics:

a. rise ¾ in with constant acceleration in 90°,
b. rise ¾ in with constant deceleration in 90°,
c. dwell 30°,
d. fall ¾ in with constant acceleration in 60°,
e. fall ¾ in with constant deceleration in 60°, and
d. dwell 30°.

Determine the boundary conditions and order of the polynomial for each section of the cam.

Region #1 $0° < \theta < 90°$, $\beta = 90° = \frac{\pi}{2}$ rad

$S_1 = C_0 + C_1\left(\frac{\theta_1}{\beta_1}\right) + C_2\left(\frac{\theta_1}{\beta_1}\right)^2$

B.C: $\theta_1 = 0$, $S_1(0) = 0$ in, $v_1(0) = 0$ in/rad
$= \frac{\pi}{2}$, $S_1\left(\frac{\pi}{2}\right) = 3/4''$

Region 6 $330 < \theta < 360$
$0 < \theta_6 < 30°$
$\beta = \pi/6 = 30°$

$S_6 = 0$

Region #2 $90° < \theta < 180°$, $0 < \theta_2 < \pi/2$, $\beta_2 = \pi/2$

$S_2 = C_0 + C_1\left(\frac{\theta_2}{\beta_2}\right) + C_2\left(\frac{\theta_2}{\beta_2}\right)^2$

B.C. $\theta_2 = 0$, $S_2(0) = 0.75$ in
$\theta_2 = \pi/2$, $S_2\left(\frac{\pi}{2}\right) = 1.5$ in, $v\left(\frac{\pi}{2}\right) = 0$ in/rad

Region #3 $180° < \theta < 210°$ $0 < \theta_3 < \pi/6$, $\beta_3 = \pi/6$

$S_3 = 1.5$ in

Region #4 $210° < \theta < 270°$, $0 < \theta_4 < \frac{\pi}{3}$, $\beta_4 = \pi/3$

$S_4 = C_0 + C_1\left(\frac{\theta_4}{\beta_4}\right) + C_2\left(\frac{\theta_4}{\beta_4}\right)^2$

B.C.: $\theta_4 = 0$, $S_4(0) = 1.5$ in, $v_4(0) = 0$ in/rad
$\theta_4 = \frac{\pi}{3}$, $S_4\left(\frac{\pi}{3}\right) = 0.75$ in

Region #5 $270 < \theta < 330°$, $0 < \theta_5 < \frac{\pi}{3}$, $\beta_5 = \pi/3$

$S_5 = C_0 + C_1\left(\frac{\theta_5}{\beta_5}\right) + C_2\left(\frac{\theta_5}{\beta_5}\right)^2$

B.C.: $\theta_5 = 0$, $S_5(0) = .75$ in

$S_5(\pi/3) = 0$ in, $v_5(\pi/3) = 0$ in/rad

EXTRA CREDIT: Draw the SVAJ diagram for this cam. Note, it is not necessary to solve the polynomial for each section of the cam.