

PROBLEM STATEMENT: | A HORIZONTAL BEAM OF LENGTH  $3a$  IS SHOWN BELOW. THIS BEAM HAS A PIN SUPPORT AT THE LEFT END OF THE BEAM AND A ROLLER SUPPORT  $2a$  FROM THE LEFT END OF THE BEAM. A  $3P$  LOAD IS APPLIED  $a$  FROM THE LEFT END OF THE BEAM AND A LOAD  $P$  IS APPLIED AT THE RIGHT END OF THE BEAM. USE FIRST THE DIRECT INTEGRATION METHOD AND THEN USING SINGULARITY FUNCTIONS DETERMINE EXPRESSIONS FOR THE SHEAR FORCE, BENDING MOMENT, CURVATURE, AND DISPLACEMENT IN THIS BEAM. DRAW THE SHEAR FORCE, BENDING MOMENT, CURVATURE, AND DISPLACEMENT DIAGRAMS; AND LABELING ALL CRITICAL VALUES AND THEIR LOCATIONS.

GIVEN:

1. A BEAM OF LENGTH  $3a$  ( $EI$ )
2. A TRANSVERSE LOAD OF  $3P$   $a$  FROM THE LEFT END OF THE BEAM
3. A TRANSVERSE LOAD OF  $P$  AT THE RIGHT MOST END OF THE BEAM
4. A PIN CONSTRAINT AT THE LEFT MOST END OF THE BEAM
5. A ROLLER SUPPORT  $2a$  FROM THE LEFT END OF THE BEAM

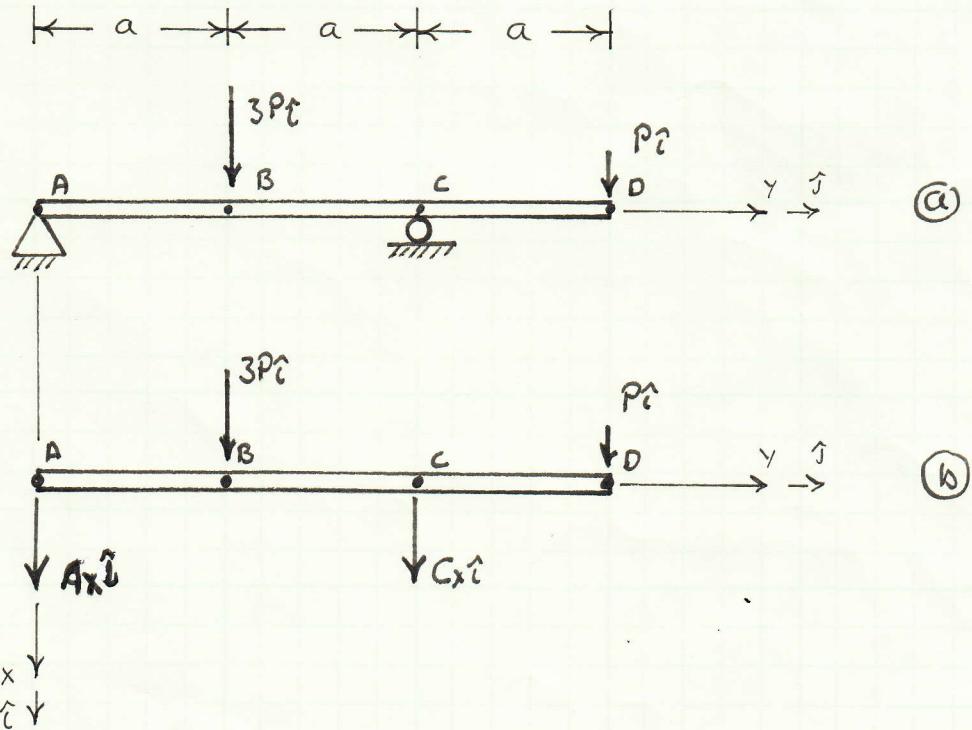
ASSUMPTIONS:

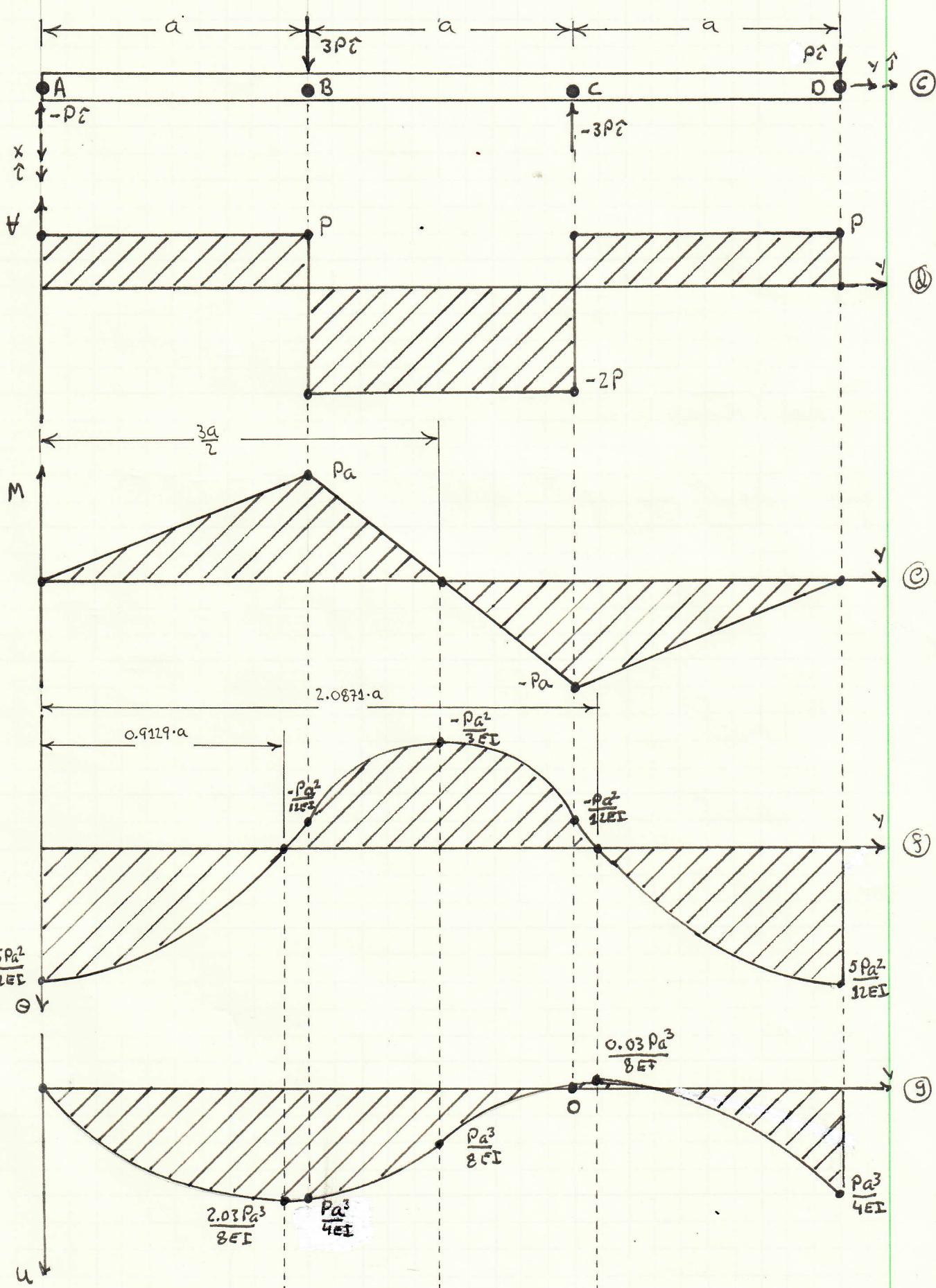
1. THE BEAM IS INITIALLY STRAIGHT
2. THE MATERIAL IS LINEAR ELASTIC
3. ALL DISPLACEMENTS ARE SMALL.

FIND:

1. USING DIRECT INTEGRATION DETERMINE EXPRESSIONS FOR  $V, M, \Theta, u$
2. USING SINGULARITY FUNCTIONS DETERMINE EXPRESSIONS FOR  $V, M, \Theta, u$
3. DRAW THE  $V, M, \Theta, u$  DIAGRAMS.

FIGURE:





Solution

STARTING WITH THE FREE BODY DIAGRAM IN (b), APPLYING EQUILIBRIUM TO SOLVE FOR THE UNKNOWNS  $A_x$  AND  $C_x$

$$\sum F_x = 0 = A_x + 3P + C_x + P \Rightarrow \underline{A_x + C_x = -4P} \quad (1)$$

$$\begin{aligned} \sum M_{z@A} = 0 & \Rightarrow -a \cdot 3P - C_x \cdot 2a - 3a \cdot P \\ \Rightarrow 2a \cdot C_x & = -6 \cdot Pa \Rightarrow \underline{C_x = -3P} \end{aligned} \quad (2)$$

SUBSTITUTING (2) INTO (1)

$$A_x - 3P = -4P \Rightarrow \underline{A_x = -P} \quad (3)$$

THE ABOVE RESULTS ARE SUMMARIZED IN THE SOLUTION DIAGRAM (c).

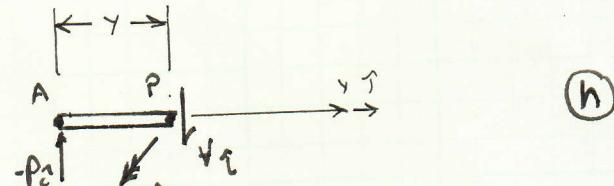
EXPRESSIONS FOR THE SHEAR FORCE AND BENDING MOMENT IN EACH SECTION OF THIS BEAM CAN NOW BE DETERMINED.

REGION AB:  $0 \leq y \leq a$

THE FREE BODY DIAGRAM FOR THIS REGION IS SHOWN IN (h)

$$\begin{aligned} \sum F_x = 0 & = -P + V \\ \Rightarrow \underline{V = P} \quad (4) \end{aligned}$$

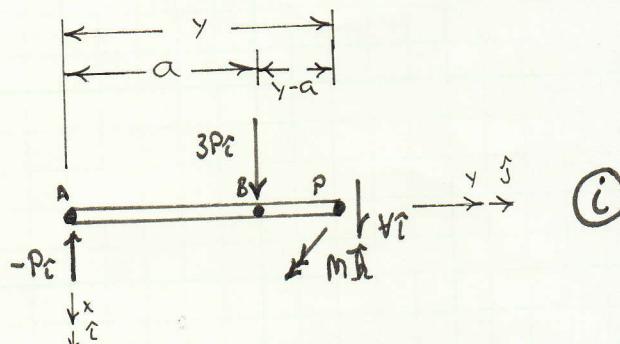
$$\begin{aligned} \sum M_{z@P} = 0 & = M - P \cdot y \\ \Rightarrow \underline{M = P \cdot y} \quad (5) \end{aligned}$$



REGION BC:  $a \leq y \leq 2a$

THE FREE BODY DIAGRAM FOR THIS REGION IS SHOWN IN (i)

$$\sum F_x = 0 = V + 3P - P \Rightarrow \underline{V = -2P} \quad (6)$$



$$\sum M_{z@P} = 0 = M + 3P \cdot (y-a) - y \cdot P$$

$$M = -3P \cdot y + 3Pa + y \cdot P = \underline{-2P \cdot y + 3Pa} \quad (7)$$

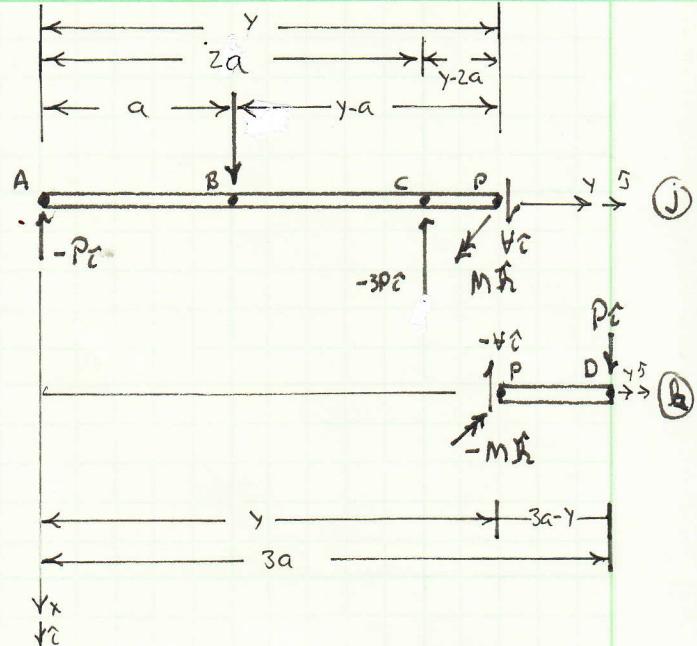
REGION CD:  $2a \leq y \leq 3a$

EITHER THE FREEBODY DIAGRAM IN ③ OR ④ CAN BE USED TO DETERMINE THE INTERNAL SHEAR AND BENDING MOMENT IN THIS REGION OF THE BEAM. ④ WILL BE USED HERE

$$\sum F_y = 0 = -A + P \Rightarrow A = P \quad ⑧$$

$$\sum M_{zep} = 0 = -M - P(3a - y)$$

$$\Rightarrow M = -3a \cdot P + P_y \quad ⑨$$



⑤, ⑦, & ⑨ CAN NOW BE USED TO DEVELOP EXPRESSIONS FOR THE CURVATURE Θ AND DEFLECTION U OF THE ELASTIC CURVE.

REGION AB:  $0 \leq y \leq a$

STARTING WITH ⑤

$$\Theta_{AB} = \int \frac{-M}{EI} dy = \int -\frac{P \cdot y}{EI} dy = -\frac{P \cdot y^2}{2 \cdot EI} + C_1 \quad ⑩$$

$$U_{AB} = \int \Theta_{AB} dy = \int \left[ -\frac{P \cdot y^2}{2 \cdot EI} + C_1 \right] dy = -\frac{P \cdot y^3}{6 \cdot EI} + C_1 \cdot y + C_2 \quad ⑪$$

IN THIS REGION THERE IS ONE BOUNDARY CONDITION,  $U_{AB}(0) = 0$

$$⑪ \rightarrow U_{AB}(0) = 0 = -\frac{P \cdot (0)^3}{6 \cdot EI} + C_1 \cdot (0) + C_2 \Rightarrow C_2 = 0$$

⑫ CAN NOW BE REWRITTEN

$$U_{AB} = -\frac{P \cdot y^3}{6 \cdot EI} + C_1 \cdot y \quad ⑫$$

⑩ AND ⑫ ARE NOW USED TO CREATE THE CONTINUITY CONDITIONS THAT FORM THE INITIAL CONDITIONS FOR THE NEXT REGION.

$$⑩ \rightarrow \Theta_{AB}(a) = -\frac{P \cdot a^2}{2 \cdot EI} + C_1 \quad ⑬$$

$$⑫ \rightarrow U_{AB}(a) = -\frac{P \cdot a^3}{6 \cdot EI} + C_1 \cdot a \quad ⑭$$

REGION BC:  $a \leq y \leq 2a$

STARTING WITH ⑦

$$\Theta_{BC} = \int -\frac{M}{EI} dy = \int -\left[ \frac{-2P.y}{EI} + \frac{3Pa}{EI} \right] dy = \frac{2 \cdot P \cdot y^2}{2EI} - \frac{3Pa}{EI} y + C_3 \\ = \frac{P \cdot y^2}{EI} - \frac{3Pa}{EI} y + C_3 \quad (15)$$

$$U_{BC} = \int \Theta_{BC} dy = \int \left[ \frac{P \cdot y^2}{EI} - \frac{3Pa}{EI} y + C_3 \right] dy \\ = \frac{P \cdot y^3}{3EI} - \frac{3 \cdot P \cdot a}{2EI} y^2 + C_3 \cdot y + C_4 \quad (16)$$

THE TWO CONTINUITY CONDITIONS ⑬ AND ⑭ NOW NEED TO BE EQUATED TO ⑯ AND ⑰ AT THE START OF THIS REGION.

$$\Theta_{AB}(a) = \Theta_{BC}(a)$$

$$⑬ + ⑯ \rightarrow -\frac{P \cdot a^2}{2EI} + C_1 = \frac{P \cdot a^2}{EI} - \frac{3P \cdot a^2}{EI} + C_3$$

$$C_3 = -\frac{P \cdot a^2}{2 \cdot EI} + C_1 + \frac{2 \cdot 3Pa^2}{2 \cdot EI} = \frac{3 \cdot P \cdot a^2}{2 \cdot EI} + C_1 \quad (17)$$

⑮ CAN NOW BE BACK SUBSTITUTED INTO ⑯ AND ⑰

$$⑯ \rightarrow \Theta_{BC} = \frac{P \cdot y^2}{EI} - \frac{3Pa}{EI} y + \frac{3 \cdot P \cdot a^2}{2 \cdot EI} + C_1 \quad (18)$$

$$⑰ \rightarrow U_{BC} = \frac{P \cdot y^3}{3 \cdot EI} - \frac{3 \cdot P \cdot a}{2 \cdot EI} y^2 + \left[ \frac{3 \cdot P \cdot a^2}{2 \cdot EI} + C_1 \right] y + C_4 \quad (19)$$

$$U_{AB}(a) = U_{BC}(a)$$

$$⑮ + ⑯ \rightarrow -\frac{P \cdot a^3}{6EI} + C_1 \cdot a = \frac{P \cdot a^3}{3 \cdot EI} - \frac{3Pa^3}{2EI} + \frac{3 \cdot P \cdot a^3}{2 \cdot EI} + C_1 \cdot a + C_4$$

$$\Rightarrow C_4 = -\frac{P \cdot a^3}{6EI} + C_1 \cancel{a} - \frac{P \cdot a^3}{2 \cdot 3EI} + \frac{3 \cdot 3Pa^3}{3 \cdot 2EI} - \frac{3 \cdot 3Pa^3}{3 \cdot 2EI} - C_1 \cancel{a} = -\frac{3 \cdot Pa^3}{6EI}$$

$$= -\frac{Pa^3}{2EI} \quad (20)$$

(20) IS NOW BACK SUBSTITUTED INTO (19)

$$(19) \rightarrow U_{BC} = \frac{P \cdot y^3}{3 \cdot EI} - \frac{3 \cdot Pa}{2 \cdot EI} \cdot y^2 + \frac{3 \cdot Pa^2}{2 \cdot EI} \cdot y + C_1 \cdot y - \frac{Pa^3}{2 \cdot EI} \quad (21)$$

THE BOUNDARY CONDITION IN THIS REGION IS  $U_{BC}(2a) = 0$ , THEREFORE

$$(21) \rightarrow U_{BC}(2a) = \frac{P \cdot (2a)^3}{3 \cdot EI} - \frac{3 \cdot Pa \cdot (2a)^2}{2 \cdot EI} + \frac{3 \cdot Pa^2 \cdot (2a)}{2 \cdot EI} + C_1 \cdot (2a) - \frac{Pa^3}{2 \cdot EI} = 0$$

$$2a \cdot C_1 = - \frac{P \cdot (2a)^3}{3 \cdot EI} + \frac{3 \cdot Pa \cdot (2a)^2}{2 \cdot EI} - \frac{3 \cdot Pa^2 \cdot (2a)}{2 \cdot EI} + \frac{Pa^3}{2 \cdot EI}$$

$$C_1 = - \frac{P \cdot (2a)^2}{3 \cdot EI} + \frac{3 \cdot Pa \cdot (2a)}{2 \cdot EI} - \frac{3 \cdot Pa^2}{2 \cdot EI} + \frac{Pa^2}{4 \cdot EI}$$

$$= - \frac{4 \cdot a^2 \cdot P}{3 \cdot EI} + \frac{3 \cdot Pa^2}{8 \cdot EI} - \frac{3 \cdot Pa^2}{2 \cdot EI} + \frac{a^2}{4 \cdot EI} = \frac{5 \cdot Pa^2}{12 \cdot EI} \quad (22)$$

(22) CAN NOW BE BACK SUBSTITUTED INTO (10), (12), (18), & (21)

REGION AB:  $0 \leq y \leq a$

$$(10) \rightarrow \Theta_{AB} = - \frac{P \cdot y^2}{2 \cdot EI} + \frac{5 \cdot Pa^2}{12 \cdot EI} \quad (23)$$

$$(12) \rightarrow U_{AB} = - \frac{P \cdot y^3}{6 \cdot EI} + \frac{5 \cdot Pa^2}{12 \cdot EI} \cdot y \quad (24)$$

REGION BC:  $a \leq y \leq 2a$

$$(18) \rightarrow \Theta_{BC} = \frac{P \cdot y^2}{EI} - \frac{3 \cdot Pa}{EI} \cdot y + \frac{3 \cdot Pa^2}{2 \cdot EI} + \frac{5 \cdot Pa^2}{12 \cdot EI} = \frac{P \cdot y^2}{EI} - \frac{3 \cdot Pa}{EI} \cdot y + \frac{23 \cdot Pa^2}{12 \cdot EI} \quad (25)$$

$$(21) \rightarrow U_{BC} = \frac{P \cdot y^3}{3 \cdot EI} - \frac{3 \cdot Pa}{2 \cdot EI} \cdot y^2 + \frac{3 \cdot Pa^2}{2 \cdot EI} \cdot y + \frac{5 \cdot Pa^2}{12 \cdot EI} \cdot y - \frac{Pa^3}{2 \cdot EI}$$

$$= \frac{P \cdot y^3}{3 \cdot EI} - \frac{3 \cdot Pa}{2 \cdot EI} \cdot y^2 + \frac{23 \cdot Pa^2}{12} \cdot y - \frac{Pa^3}{2 \cdot EI} \quad (26)$$

(23) - (26) CAN NOW BE USED TO CALCULATE CRITICAL VALUES OF  $\Theta$  AND  $U$  IN THESE TWO REGIONS.

REGION AB:  $0 \leq y \leq a$ 

$$(27) \rightarrow \Theta_{AB}(0) = -\frac{P \cdot (0)^2}{2 \cdot EI} + \frac{5 \cdot Pa^2}{12EI} = \frac{5 \cdot Pa^2}{12EI} \quad (27)$$

$$(28) \rightarrow U_{AB}(0) = -\frac{P \cdot (0)^3}{6 \cdot EI} + \frac{5 \cdot Pa^2}{12EI} \cdot (0) = 0 \quad (28)$$

$$(29) \rightarrow \Theta_{AB}(a) = -\frac{P \cdot a^2}{2EI} + \frac{5 \cdot Pa^2}{12EI} = -\frac{P \cdot a^2}{12EI} \quad (29)$$

$$(30) \rightarrow U_{AB}(a) = -\frac{P \cdot a^3}{6EI} + \frac{5 \cdot Pa^3}{12EI} = \frac{3 \cdot Pa^3}{12EI} = \frac{P \cdot a^3}{4EI} \quad (30)$$

SINCE THERE IS A SIGN CHANGE IN  $\Theta_{AB}$  BETWEEN  $\Theta_{AB}(0)$  (27) AND  $\Theta_{AB}(a)$  (29), THE FUNCTION  $\Theta_{AB}$  MUST GO TO ZERO IN THIS REGION. THE LOCATION OF THE ROOTS OF  $\Theta_{AB}$  IN THIS REGION ARE IMPORTANT BECAUSE THEY ARE LOCATIONS OF MAX/MINS IN  $U_{AB}$ .

$$\Theta_{AB} = 0 = -\frac{P \cdot y^2}{2EI} + \frac{5 \cdot Pa^2}{12EI}$$

$$\Rightarrow y^2 = \frac{5}{6}a^2$$

$$\Rightarrow y = \pm \sqrt{\frac{5}{6}} \cdot a = -0.9129 \cdot a, \underline{0.9129 \cdot a}$$

ONLY THE POSITIVE ROOT IS IN THE DOMAIN OF THE REGION.

$$\Theta_{AB}(0.9129 \cdot a) = -\frac{P \cdot (0.9129 \cdot a)^2}{2 \cdot EI} + \frac{5 \cdot Pa^2}{12 \cdot EI} = 0$$

$$U(0.9129 \cdot a) = -\frac{P \cdot (0.9129 \cdot a)^3}{6 \cdot EI} + \frac{5 \cdot Pa^2}{12EI} \cdot (0.9129 \cdot a)$$

$$= 0.2536 \cdot \frac{Pa^3}{EI} \quad (31)$$

REGION BC:  $a \leq y \leq 2a$

$$\Theta_{BC}(a) = \frac{P \cdot a^2}{EI} - \frac{3 \cdot Pa^2}{EI} + \frac{23 \cdot Pa^2}{12EI} = -\frac{Pa^2}{12EI} \quad (32)$$

$$\begin{aligned} u_{BC}(a) &= \frac{P \cdot a^3}{3 \cdot EI} - \cancel{\frac{3 \cdot Pa^3}{2EI}} + \cancel{\frac{3 \cdot Pa^3}{2EI}} + \frac{5 \cdot Pa^3}{12 \cdot EI} - \frac{P \cdot a^3}{2EI} \\ &= \frac{3 \cdot Pa^3}{12 \cdot EI} = \frac{Pa^3}{4EI} \quad (33) \end{aligned}$$

$$\Theta_{BC}(2a) = \frac{P \cdot (2a)^2}{EI} - \frac{3 \cdot Pa}{EI} \cdot (2a) + \frac{23 \cdot Pa^2}{12EI} = -\frac{Pa^2}{12EI} \quad (33)$$

$$\begin{aligned} u_{BC}(2a) &= \frac{P \cdot (2a)^3}{3EI} - \frac{3 \cdot Pa}{2EI} \cdot (2a)^2 + \frac{3 \cdot Pa^2}{2 \cdot EI} (2a) + \frac{5}{12} \frac{Pa^2}{EI} (2a) - \frac{Pa^3}{2EI} \\ &= 0 \quad (34) \end{aligned}$$

EXPRESSIONS FOR  $\Theta$  AND  $u$  IN THE FINAL REGION OF THE BEAM CAN NOW BE DETERMINED. (33) AND (34) ARE CONTINUITY CONDITIONS FOR THIS REGION

REGION CD:  $2a \leq y \leq 3a$

STARTING WITH (9)

$$\begin{aligned} \Theta_{CD} &= - \int \frac{M}{EI} dy = \int - \left[ -\frac{3a \cdot P}{EI} + \frac{P \cdot y}{EI} \right] dy = \int \left[ \frac{3a \cdot P}{EI} - \frac{P \cdot y}{EI} \right] dy \\ &= \frac{3 \cdot a \cdot P \cdot y}{EI} - \frac{P \cdot y^2}{2 \cdot EI} + C_5 \quad (35) \end{aligned}$$

USING THE CONTINUITY CONDITION (33)

$$\Theta_{BC}(2a) = \Theta_{CD}(2a)$$

$$-\frac{Pa^2}{12EI} = \frac{3 \cdot Pa}{EI} (2a) - \frac{P \cdot (2a)^2}{2EI} + C_5$$

$$\Rightarrow C_5 = -\frac{P \cdot a^2}{12EI} - \frac{6 \cdot Pa^2}{EI} + \frac{2 \cdot Pa^2}{EI} = -\frac{49 \cdot Pa^2}{12EI} \quad (36)$$

(36) INTO (35)

$$\Theta_{CD} = -\frac{P \cdot y^2}{2 \cdot EI} + \frac{3 \cdot Pa}{EI} \cdot y - \frac{49 Pa^2}{12 EI} \quad (37)$$

CONTINUING

$$\begin{aligned} u_{CD} &= \int \Theta_{CD} \cdot dy = \int \left[ -\frac{P \cdot y^2}{2 \cdot EI} + \frac{3 \cdot Pa}{EI} \cdot y - \frac{49 Pa^2}{12 EI} \right] dy \\ &= -\frac{P \cdot y^3}{6 \cdot EI} + \frac{3 \cdot Pa}{2 \cdot EI} \cdot y^2 - \frac{49 \cdot Pa^2}{12 EI} \cdot y + C_6 \end{aligned} \quad (38)$$

THE CONSTANT IS DETERMINED FROM THE CONTINUITY CONDITION (34)

$$u_{BC}(2a) = u_{CD}(2a)$$

$$0 = -\frac{P \cdot (2a)^3}{6 \cdot EI} + \frac{3 \cdot Pa \cdot (2a)^2}{2 \cdot EI} - \frac{49 \cdot Pa^2 \cdot (2a)}{12 EI} + C_6$$

$$\Rightarrow C_6 = \frac{8 \cdot Pa^3}{6 EI} - \frac{12 \cdot Pa^3}{2 EI} + \frac{98 \cdot Pa^3}{12 EI}$$

$$= \frac{8}{6} \frac{Pa^3}{EI} - \frac{36}{6} \frac{Pa^3}{EI} + \frac{49}{6} \frac{Pa^3}{EI} = \frac{21}{6} \frac{Pa^3}{EI} \quad (39)$$

SUBSTITUTING (39) INTO (38)

$$(38) \rightarrow u_{CD} = -\frac{P \cdot y^3}{6 \cdot EI} + \frac{3 \cdot Pa}{2 \cdot EI} \cdot y^2 - \frac{49 \cdot Pa^2}{12 EI} \cdot y + \frac{21}{6} \frac{Pa^3}{EI} \quad (40)$$

(37) AND (40) CAN NOW BE USED TO CALCULATE CRITICAL VALUES OF  $\Theta_{CD}$  AND  $u_{CD}$  IN THIS REGION

REGION CD:  $2a \leq y \leq 3a$

$$(37) \rightarrow \Theta_{CD}(2a) = -\frac{P \cdot (2a)^2}{2 \cdot EI} + \frac{3 \cdot Pa}{EI} \cdot (2a) - \frac{49 Pa^2}{12 EI} = -\frac{Pa^2}{12 EI} \quad (41)$$

$$(40) \rightarrow u_{CD}(2a) = -\frac{6 \cdot (2a)^3}{6 \cdot EI} + \frac{3 \cdot Pa}{2 \cdot EI} \cdot (2a)^2 - \frac{49 \cdot Pa^2 \cdot (2a)}{12 EI} + \frac{21 Pa^3}{6 EI}$$

$$= 0 \quad (42)$$

$$\textcircled{37} \rightarrow \Theta_{CD}(3a) = -\frac{P \cdot (3a)^2}{2 \cdot EI} + \frac{3Pa}{EI} \cdot (3a) - \frac{49 \cdot Pa^2}{12EI} = \frac{5Pa^2}{12EI} \quad \textcircled{43}$$

$$\textcircled{40} \rightarrow U_{CD}(3a) = -\frac{P \cdot (3a)^3}{6 \cdot EI} + \frac{3 \cdot Pa \cdot (3a)^2}{2EI} - \frac{49 \cdot Pa^2 \cdot (3a)}{12EI} + \frac{21 \cdot Pa^3}{6EI}$$

$$= \frac{3Pa^3}{12EI} = \frac{Pa^3}{4EI} \quad \textcircled{44}$$

SINCE THERE IS A CHANGE OF SIGN IN  $\Theta_{CD}(2a)$   $\textcircled{41}$  AND  $\Theta_{CD}(3a)$   $\textcircled{43}$ ,  $\Theta_{CD}$  BECOMES 0 IN THIS REGION WHICH INDICATES A MAX/MIN IN U. THE LOCATION WHERE  $\Theta_{CD} = 0$  IS FOUND BY SETTING  $\textcircled{37}$  EQUAL TO 0 AND SOLVING FOR Y

$$\textcircled{37} \rightarrow \Theta_{CD} = 0 = -\frac{P \cdot y^2}{2EI} + \frac{3Pa}{EI} \cdot y - \frac{49 \cdot Pa^2}{12EI}$$

$$0 = \frac{1}{2} \cdot y^2 - 3 \cdot ya + \frac{49}{12} a^2$$

$$0 = y^2 - 6ya + \frac{49}{6} a^2 = y^2 - 6a \cdot y + \left(\frac{6a}{2}\right)^2 - \left(-\frac{6a}{2}\right)^2 + \frac{49}{6} a^2$$

$$(y - 3a)^2$$

$$0 = (y - 3a)^2 - \frac{36}{4} a^2 + \frac{49}{6} a^2 = (y - 3a)^2 - \frac{20}{24} a^2$$

$$(y - 3a)^2 = \frac{5}{6} a^2$$

$$\Rightarrow y = 3a \pm \sqrt{\frac{5}{6}} \cdot a = 3a \pm 0.9129 \cdot a$$

$$= \underline{2.087 \cdot a}, 3.9129 \cdot a \quad \textcircled{45}$$

THE SECOND VALUE IS OUTSIDE THE DOMAIN OF THE PROBLEM, SO THE ROOT OF THIS EQUATION IS THE FIRST. THE VALUE OF  $U_{CD}$  AT THIS POINT MUST BE CALCULATED USING  $\textcircled{40}$

$$\textcircled{40} \rightarrow U_{CD}(2.087 \cdot a) = -\frac{P \cdot (2.087 \cdot a)^3}{6 \cdot EI} + \frac{3Pa \cdot (2.087a)^2}{2EI} - \frac{49 \cdot Pa^2 (2.087a)}{12EI} + \frac{21 \cdot Pa^3}{6EI}$$

$$= -0.00358 \cdot \frac{Pa^3}{EI} \quad \textcircled{46}$$

THE RESULTS OF THE CALCULATIONS OF  $\Theta$  AND U IN EACH REGION OF THIS BEAM ARE SUMMARIZED IN  $\textcircled{5}$  AND  $\textcircled{9}$ .