**PROBLEM 2.1** THE STATE OF STRESS AT A POINT IN A BODY RELATIVE TO THE XYZ COORDINATE SYSTEM IS GIVEN BY

$$[\sigma] = \begin{bmatrix} 0 & -30 & 25 \\ -30 & -40 & -15 \\ 25 & -15 & 10 \end{bmatrix} \text{MPa} \qquad (1)$$

DETERMINE THE STRESS MATRIX RELATIVE TO A COORDINATE SYSTEM DEFINED BY FIRST ROTATING THE XYZ COORDINATE SYSTEM 45° ABOUT THE X-AXIS, THEN ROTATING -45° ABOUT THE NEW Z AXIS.

## GIVEN:

CONSTRAINTS

1. STATE OF STRESS IN (1)
2. A 45° ROTATION ABOUT THE X-AXIS AND THEN A -45° ROTATION ABOUT THE NEW Z AXIS

ASSUMPTIONS

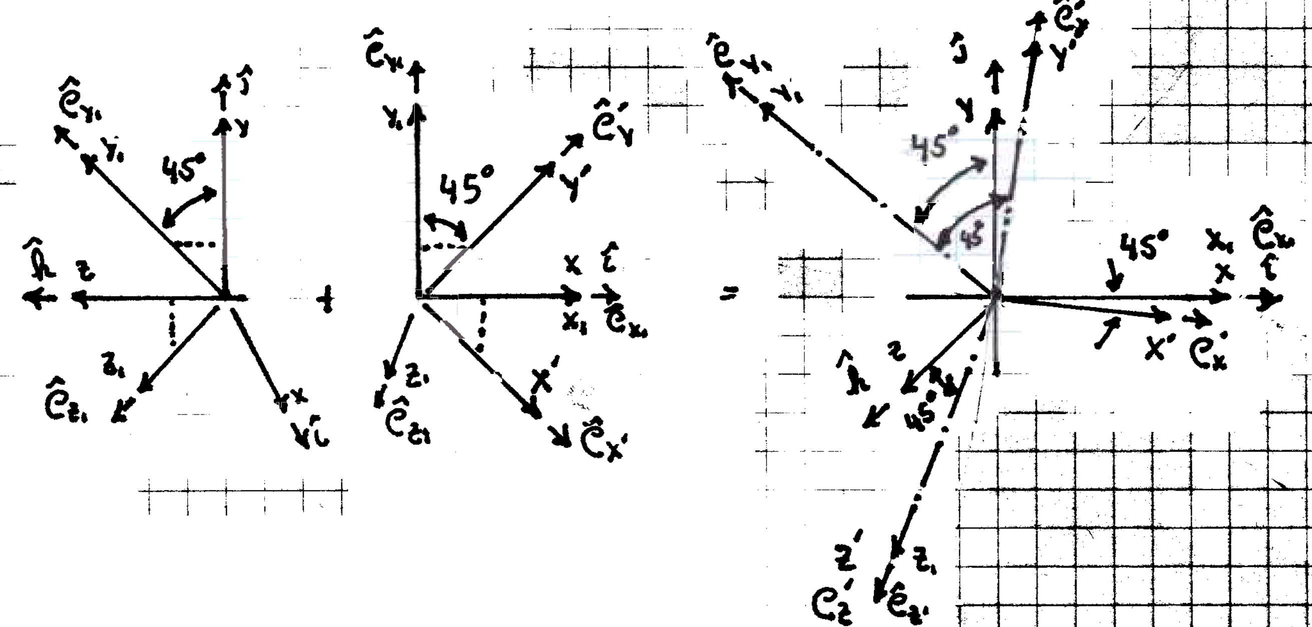
1. (1) REPRESENTS THE STATE OF STRESS AT A POINT IN THE STRUCTURE

## FIND:

1. DETERMINE THE STATE OF STRESS IN THE NEW CONFIGURATION OF THE CUBE.

## SOLUTION:

BEFORE THE TRANSFORMATION OF THE STRESS CAN BE PERFORMED, THE TRANSFORMATION MATRIX MUST BE CONSTRUCTED.




(A)

STARTING BY TRANSFORMING THE xyz COORDINATES TO $x_1 y_1 z_1$ COORDINATES.

$$\begin{Bmatrix} x_1 \\ y_1 \\ z_1 \end{Bmatrix} = \begin{bmatrix} & 0 & 0 \\ 0 & .7071 & .7071 \\ 0 & -.7071 & .7071 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{bmatrix} .7071 & -.7071 & 0 \\ .7071 & .7071 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ z_1 \end{Bmatrix}$$

$$\begin{Bmatrix} x' \\ \cdot \\ z' \end{Bmatrix} = \begin{bmatrix} .7071 & -.7071 & 0 \\ & .7071 & 0 \\ & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & .7071 & .7071 \\ 0 & -.7071 & .7071 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \underbrace{\begin{bmatrix} 0.7071 & -.5 & -.5 \\ 0.7071 & .5 & .5 \\ 0 & -0.7071 & 0.7071 \end{bmatrix}}_{[T]} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \qquad (2)$$

NOW THE TRANSFORMED STATE OF STRESS CAN BE CALCULATED.

$$[\sigma]_{x'y'z'} = [T]\cdot[\sigma]_{xyz}\cdot[T]'$$

$$[\sigma]_{x'y'z'} = \begin{bmatrix} .7071 & -.5 & -.5 \\ .7071 & .5 & .5 \\ 0 & -.7071 & .7071 \end{bmatrix} \cdot \begin{bmatrix} 0 & -30 & 25 \\ -30 & -40 & -15 \\ 25 & -15 & 10 \end{bmatrix} \text{MPa} \begin{bmatrix} .7071 & .7071 & 0 \\ -.5 & .5 & -.7071 \\ -.5 & .5 & .7071 \end{bmatrix}$$

$$= \begin{bmatrix} -11.46 & 15.00 & 9.82 \\ 15.00 & -18.53 & 45.18 \\ 9.82 & 45.18 & 0 \end{bmatrix}$$

SUMMARY:

THE TRANSFORMATION MATRIX IS FORMED BY ROTATING ABOUT THE AXES IN TWO STEPS. THIS PROCESS YIELDS THE DIRECTION COSINES FOR THE COORDINATES UNDER CONSIDERATION. ONCE THE TRANSFORMATION MATRIX IS FORMED, MATLAB OR EXCEL CAN BE USED TO PERFORM THE MATRIX MULTIPLICATIONS.

```
>> S=[0 -30 25; -30 -40 -15; 25 -15 10]

S =

     0   -30    25
   -30   -40   -15
    25   -15    10

>> T1=[1 0 0; 0 .7071 .7071; 0 -.7071 .7071]

T1 =

    1.0000         0         0
         0    0.7071    0.7071
         0   -0.7071    0.7071

>> T2=[.7071 -.7071 0; .7071 .7071 0; 0 0 1]

T2 =

    0.7071   -0.7071         0
    0.7071    0.7071         0
         0         0    1.0000
```

```
>> TT=T2*T1

TT =

    0.7071   -0.5000   -0.5000
    0.7071    0.5000    0.5000
         0   -0.7071    0.7071

>> ST=TT*S*TT'

ST =

  -11.4640   14.9994    9.8223
   14.9994  -18.5349   45.1766
    9.8223   45.1766   -0.0000

>>
```