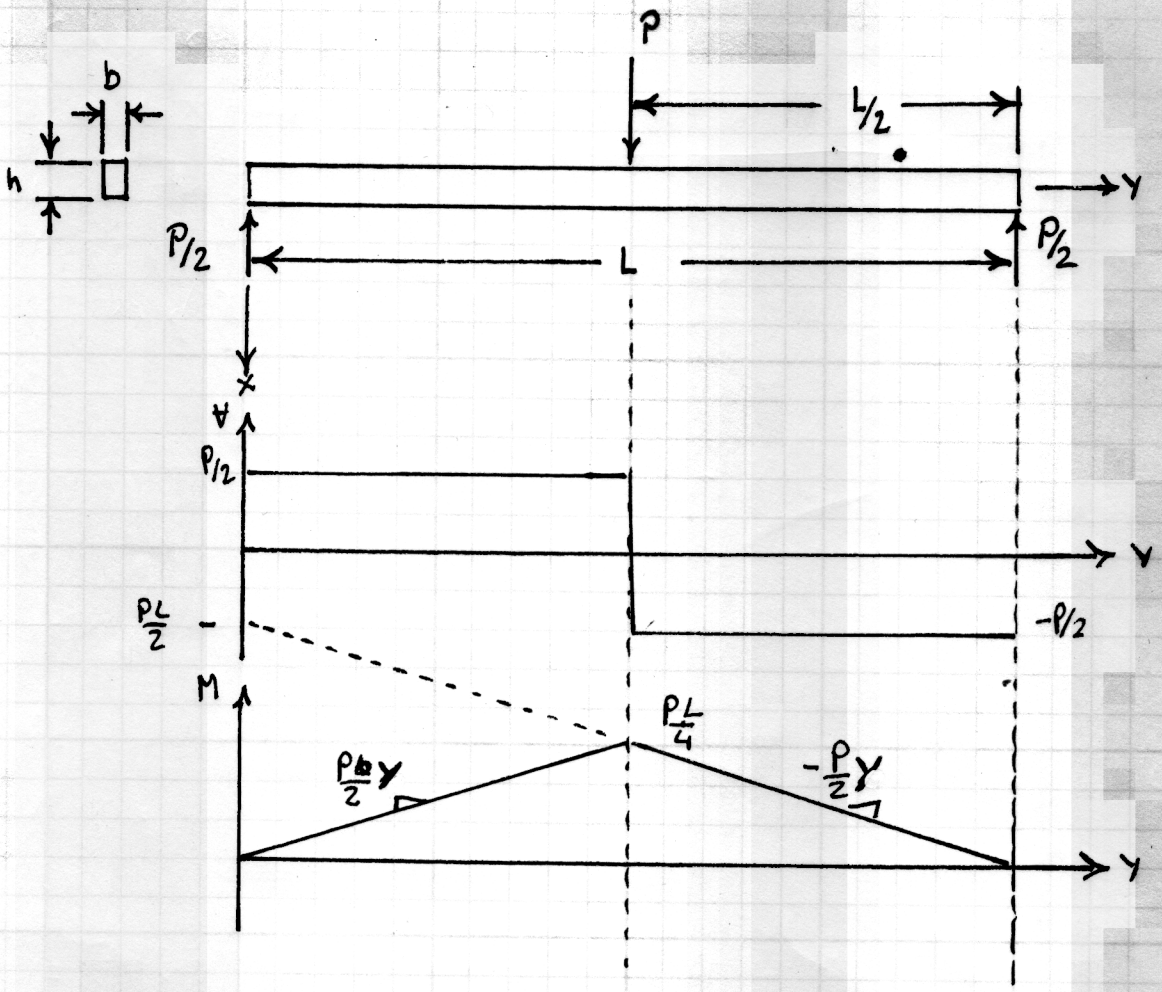

THE ELASTIC ENERGY DUE TO BENDING IS GIVEN BY

$$U = \int_0^L \frac{M^2}{2EI}\, dy$$

$0 < y < L/2$: $M_1 = \frac{P \cdot y}{2}$

$L/2 < y < L$: $M_2 = -\frac{P \cdot y}{2} + \frac{PL}{2}$

$$U = \int_0^{L/2} \frac{\left(\frac{Py}{2}\right)^2}{2EI}\, dy + \int_{L/2}^{L} \frac{\left(-\frac{Py}{2} + \frac{PL}{2}\right)^2}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{L/2} \frac{P^2y^2}{4}\, dy + \int_{L/2}^{L}\left(\frac{P^2y^2}{4} - \frac{P^2Ly}{2} + \frac{P^2L^2}{4}\right) dy\right]$$

$$= \frac{P^2}{2EI}\left[\frac{y^3}{12}\Big|_0^{L/2} + \left(\frac{y^3}{12} - \frac{Ly^2}{4} + \frac{L^2y}{4}\right)\Big|_{L/2}^{L}\right]$$

$$= \frac{P^2}{2EI} \cdot \left[\frac{L^3}{96} + \frac{L^3}{12}\frac{8}{8} - \frac{L^3}{4}\cdot\frac{24}{24} + \frac{L^3}{4}\cdot\frac{24}{24} - \frac{L^3}{96} + \frac{L^3}{16}\frac{6}{6} - \frac{L^3}{8}\frac{12}{12}\right] = \frac{P^2L^3}{96EI}$$

$$\delta = \frac{\partial U}{\partial P} = \frac{2PL^3}{96EI} = \frac{PL^3}{48EI}$$