PROBLEM 6.8-8 || THE CROSS SECTION OF A SLIT RECTANGULAR TUBE OF CONSTANT THICKNESS IS SHOWN IN THE FIGURE. DERIVE THE FOLLOWING FORMULA FOR THE DISTANCE e FROM THE CENTERLINE OF THE WALL OF THE TUBE TO THE SHEAR CENTER S:

$$e = \frac{b\cdot(2\cdot h + 3\cdot b)}{2\cdot(h + 3\cdot b)}$$

GIVEN:

CONSTRAINTS

1) SLIT RECTANGULAR TUBE OF HEIGHT h AND WIDTH b

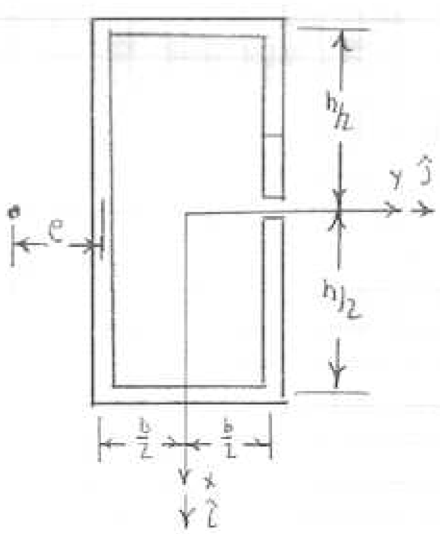
ASSUMPTIONS

1) LINEAR/ELASTIC MATERIAL RESPONSE
2) SMALL DEFLECTIONS

FIND:

1) LOCATION OF THE SHEAR CENTER

DIAGRAM:

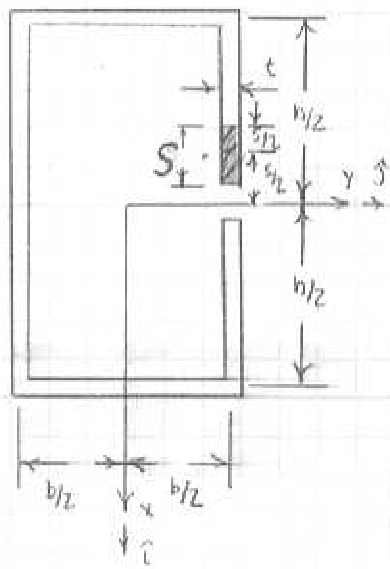


MECHANICS

A LOAD V IS BEING APPLIED TO THE END OF THIS BEAM. THE QUESTION IS WHERE SHOULD V BE APPLIED IN ORDER TO AVOID TWISTING IN THE CROSS SECTION. THE SOLUTION STARTS BY DETERMINING THE SHEAR STRESS DISTRIBUTION IN THIS BEAM. STARTING AT THE SPLIT

$$\tau = \frac{V \cdot Q}{I\, t}$$

$$I = 2 \cdot \left(\tfrac{1}{12} \cdot b \cdot t^3 + t \cdot b \left(\tfrac{h}{2}\right)^2 \right) + 2\left(\tfrac{1}{12} t \cdot h^3 \right)$$
$$= 2 \cdot \left(\tfrac{1}{12} \cdot b \cdot t^3 + t \cdot b \left(\tfrac{h}{2}\right)^2 + \tfrac{1}{12} \cdot t \cdot h^3 \right)$$



THE t^3 IN THE FIRST TERM WILL CAUSE THIS TERM TO BE MUCH SMALLER THAN THE OTHER TWO TERMS SINCE WE ARE CONSIDERING THE WALLS OF THE CROSS SECTION TO BE THIN. THUS THE MOMENT OF INERTIA CAN BE WRITTEN

$$I = 2\left(\tfrac{1}{12} \cdot t \cdot h^3 + t \cdot b \cdot \left(\tfrac{h}{2}\right)^2 \right)$$
$$= \underline{\frac{t\, h^2}{6} (h + 3b)} \qquad (1)$$

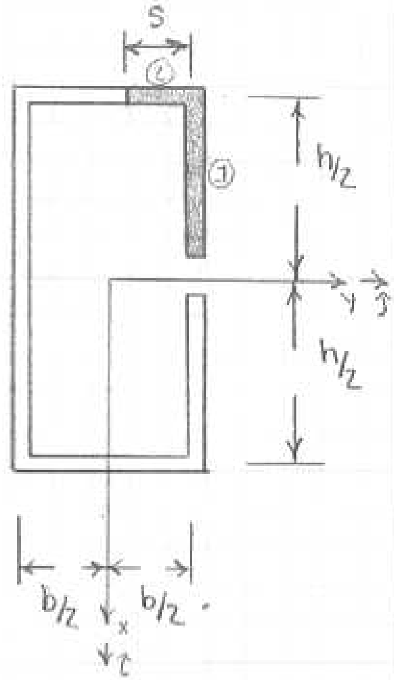
NOW Q NEEDS TO BE DETERMINED, FOR THE SHADED SECTION SHOWN, THE PATH VARIABLE IS S, S GOES FROM 0 AT THE NEUTRAL AXIS WHERE THE SPLIT IS TO h/2.

$$Q = \bar{x} \cdot A = \frac{s}{2} \cdot s \cdot t = \frac{s^2 t}{2} \qquad (2)$$

THUS THE SHEAR STRESS FUNCTION IN THIS PORTION OF THE BEAM TAKES THE FORM

$$\tau = \frac{V \cdot Q}{I \cdot t} = \frac{V \cdot \frac{s^2 t}{2}}{\frac{t \cdot h^2}{6}(h+3b) \cdot t} = \underline{\underline{\frac{3 \cdot V}{t \cdot h^2 (h+3b)} \cdot s^2}} \qquad (3)$$

$$\tau(0) = 0$$
$$\tau(h/2) = \frac{3 \cdot V}{t \cdot h^2 (h+3b)} \cdot \frac{h^2}{4} = \frac{3}{4} \frac{V}{t \cdot (h+3b)}$$



FOR THE TOP FLANGE OF THE BEAM V, I, AND t ALL STAY THE SAME IN THE CALCULATION OF THE SHEAR STRESS. Q MUST BE RECALCULATED

$$Q = \Sigma \bar{x}_i \cdot A_i$$
$$= \frac{h}{4} \cdot t \cdot \frac{h}{2} + \frac{h}{2} \cdot t \cdot s$$
$$= \frac{h^2 \cdot t}{8} + \frac{h\,t}{2} s = \frac{h \cdot t}{2}\left(\frac{h}{4} + s\right) \qquad (4)$$

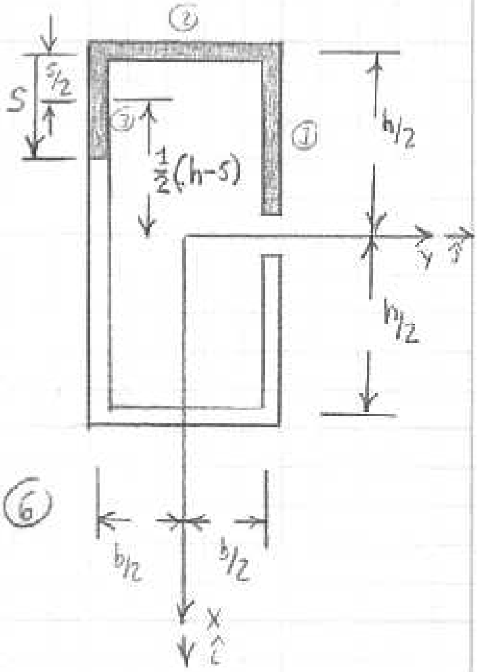
THUS THE SHEAR STRESS IN THIS SECTION OF THE BEAM TAKES THE FORM

$$\tau = \frac{V \cdot Q}{I \cdot t} = \frac{V \cdot \frac{h \cdot t}{2}\left(\frac{h}{4} + s\right)}{\frac{t \cdot h^2}{6}(h+3b) \cdot t}$$

$$= \frac{3 \cdot V}{t \cdot h \cdot (h+3b)}\left(\frac{h}{4} + s\right) = \frac{3}{4} \frac{V}{t \cdot h(h+3b)} \cdot (h + 4s) \qquad (5)$$

$$\tau(0) = \frac{3}{4} \frac{V}{t \cdot (h+3b)}$$ (SAME AS THE ENDING POINT FOR THE OPEN PORTION OF THE WEB)

$$\tau(b) = \frac{3}{4} \frac{V(h+4b)}{t \cdot h(h+3b)}$$



NOW WE MUST CONSIDER THE LEFT-HAND PORTION OF THE WEB. FOR THE CALCULATION OF THE SHEAR STRESS V, I, AND t STILL ARE THE SAME. Q NEEDS TO BE CALCULATED

$$Q = \Sigma \bar{x}_i \cdot A$$
$$= \frac{h}{4} \cdot t \cdot \frac{h}{2} + \frac{h}{2} \cdot t \cdot b + \frac{1}{2}(h-s) \cdot s \cdot t$$
$$= \frac{h^2 \cdot t}{8} + \frac{h \cdot b \cdot t}{2} + \frac{h \cdot t}{2} s - \frac{t}{2} \cdot s^2$$
$$= \frac{t}{8}\left[h^2 + 4h \cdot b + 4hs - 4s^2\right] \qquad (6)$$

THE SHEAR STRESS CAN NOW BE WRITTEN

$$\tau = \frac{V \cdot Q}{I \cdot t} = \frac{V \cdot \frac{t}{8} \cdot [h^2 + 4 \cdot h \cdot b + 4 \cdot h \cdot s - 4 s^2]}{\frac{t \cdot h^2}{6}(h + 3b) \cdot t}$$

$$= \frac{3}{4} \cdot V \cdot \frac{h^2 + 4 \cdot h \cdot b + 4 \cdot h \cdot s - 4 s^2}{t \cdot h^2 (h + 3b)} \qquad (7)$$

$$\tau(0) = \frac{3}{4} \cdot V \cdot \frac{h^2 + 4 \cdot h \cdot b}{t \cdot h^2 (h + 3b)} = \frac{3}{4} \cdot \frac{V \cdot h \cdot (h + 4b)}{t \cdot h^2 \cdot (h + 3b)} = \frac{3}{4} \frac{V}{t \cdot h} \frac{(h + 4b)}{(h + 3b)}$$

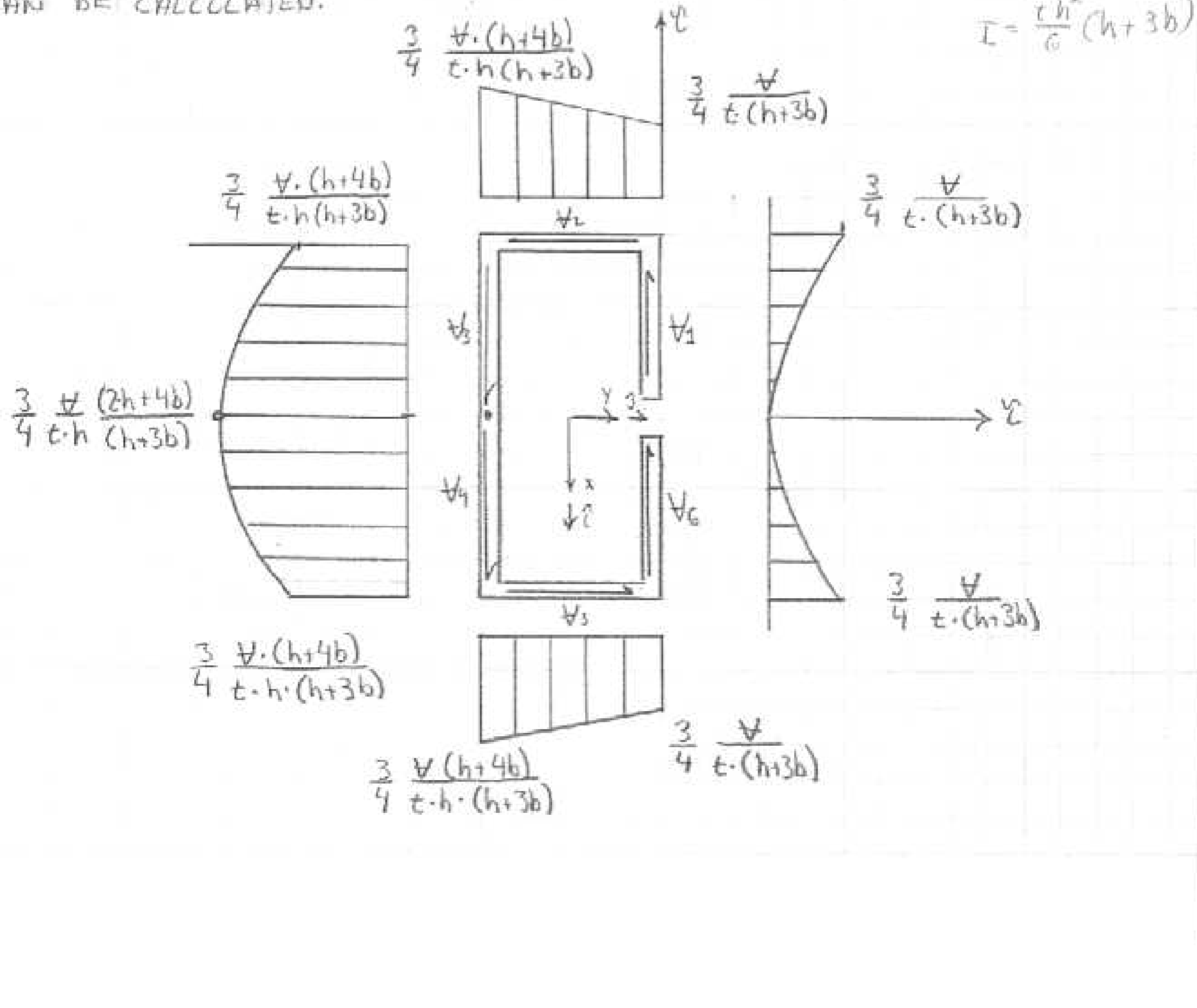
$$\tau\left(\frac{h}{2}\right) = \frac{3}{4} \cdot V \frac{h^2 + 4 \cdot h \cdot b + 4 \cdot h \cdot \left(\frac{h}{2}\right) - 4\left(\frac{h}{2}\right)^2}{t \cdot h^2 (h + 3b)}$$

$$= \frac{3}{4} V \cdot \frac{h^2 + 4 \cdot h \cdot b + 2 \cdot h^2 - h^2}{t \cdot h^2 (h + 3b)} = \frac{3}{4} V \cdot \frac{2 \cdot h^2 + 4 \cdot h \cdot b}{t \cdot h^2 \cdot (h + 3b)}$$

$$= \frac{3}{4} \frac{V}{t \cdot h} \frac{(2 \cdot h + 4b)}{(h + 3b)}$$

NOW THE SHEAR STRESS DISTRIBUTION AROUND THE CROSS SECTION CAN BE CALCULATED.

$I = \frac{t h^2}{6}(h + 3b)$



NOW THE SHEAR FORCES GENERATED BY THE SHEAR STRESS IN EACH SEGMENT OF THE BEAM NEED TO BE CALCULATED. V_3 AND V_4 DO NOT NEED TO BE CALCULATED BECAUSE WE ARE GOING TO SUM MOMENTS ABOUT THE CENTER OF THIS SEGMENT OF THE BEAM.

$$V_1 = \int \frac{3 \cdot V}{t \cdot h^2 (h+3b)} \cdot s^2 \cdot t \cdot ds = \frac{3 V}{h^2 \cdot (h+3b)} \int_0^{h/2} s^2 \cdot ds$$

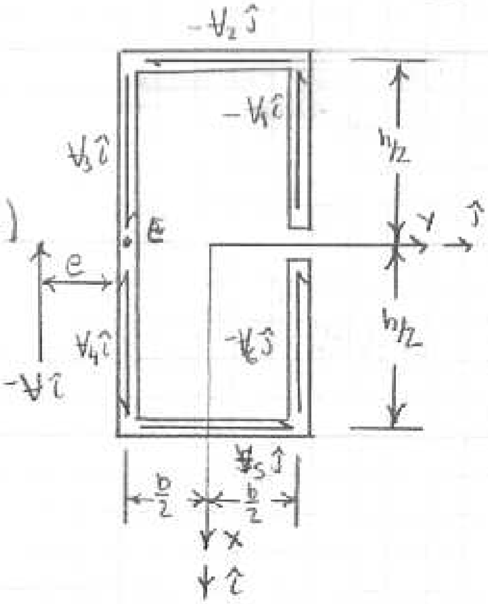
$$= \frac{3 \cdot V}{h^2 \cdot (h+3b)} \left. \frac{s^3}{3} \right|_0^{h/2} = \frac{V \; \frac{h^3}{8}}{h^2 \cdot (h+3b)} = \underline{\frac{V \cdot h}{8 \cdot (h+3b)}} = V_6 \quad (7)$$

$$V_2 = V_5 = \int \frac{3}{4} \cdot \frac{V}{t \cdot h \cdot (h+3b)} (h+4s) \cdot t \cdot ds = \frac{3}{4} \frac{V}{h \cdot (h+3b)} \int_0^b (h+4s)\, ds$$

$$= \frac{3}{4} \frac{V}{h \cdot (h+3b)} \left. (h \cdot s + 2 \cdot s^2) \right|_0^b$$

$$= \frac{3}{4} \frac{V \cdot b}{h \cdot (h+3b)} (h + 2b) \quad (8)$$

TO DETERMINE THE DISTANCE e THE MOMENTS ARE SUMMED ABOUT E



$$\sum M_{z/at\, E} = 0 = -V \cdot e + V_2 \cdot h + b \cdot (V_1 + V_6)$$

$$e = \frac{V_2 \cdot h + b \cdot (V_1 + V_6)}{V} = \frac{V_2 \cdot h + 2 \cdot b \cdot V_1}{V}$$

$$= \frac{\frac{3}{4} \cdot \frac{V \cdot b \cdot (h+2b)}{h \cdot (h+3b)} \cdot h + 2 \cdot b \cdot \frac{V \cdot h}{{}_4 \cancel{8} \cdot (h+3h)}}{V}$$

$$= \frac{3}{4} \cdot \frac{b \cdot (h+2b)}{(h+3b)} + \frac{1}{4} \frac{b \cdot h}{(h+3b)}$$

$$= \frac{3bh + 6b^2 + bh}{4(h+3b)} = \frac{4 \cdot b \cdot h + 6 b^2}{4(h+3b)} = \boxed{\frac{b \cdot (2h + 3b)}{2 \cdot (h+3b)}}$$

<u>SUMMARY:</u> NOTE THE DIRECTION OF V IN THE FINAL FREE BODY DIAGRAM. REMEMBER THIS IS THE V THAT COUNTER ACTS THE V ON THE SURFACE OF THE CROSS-SECTION SHOWN. THE FORCES ON THE CROSS-SECTION ADD UP TO V IN THE OPPOSITE DIRECTION SHOWN TO KEEP THE ELEMENT IN EQUILIBRIUM.