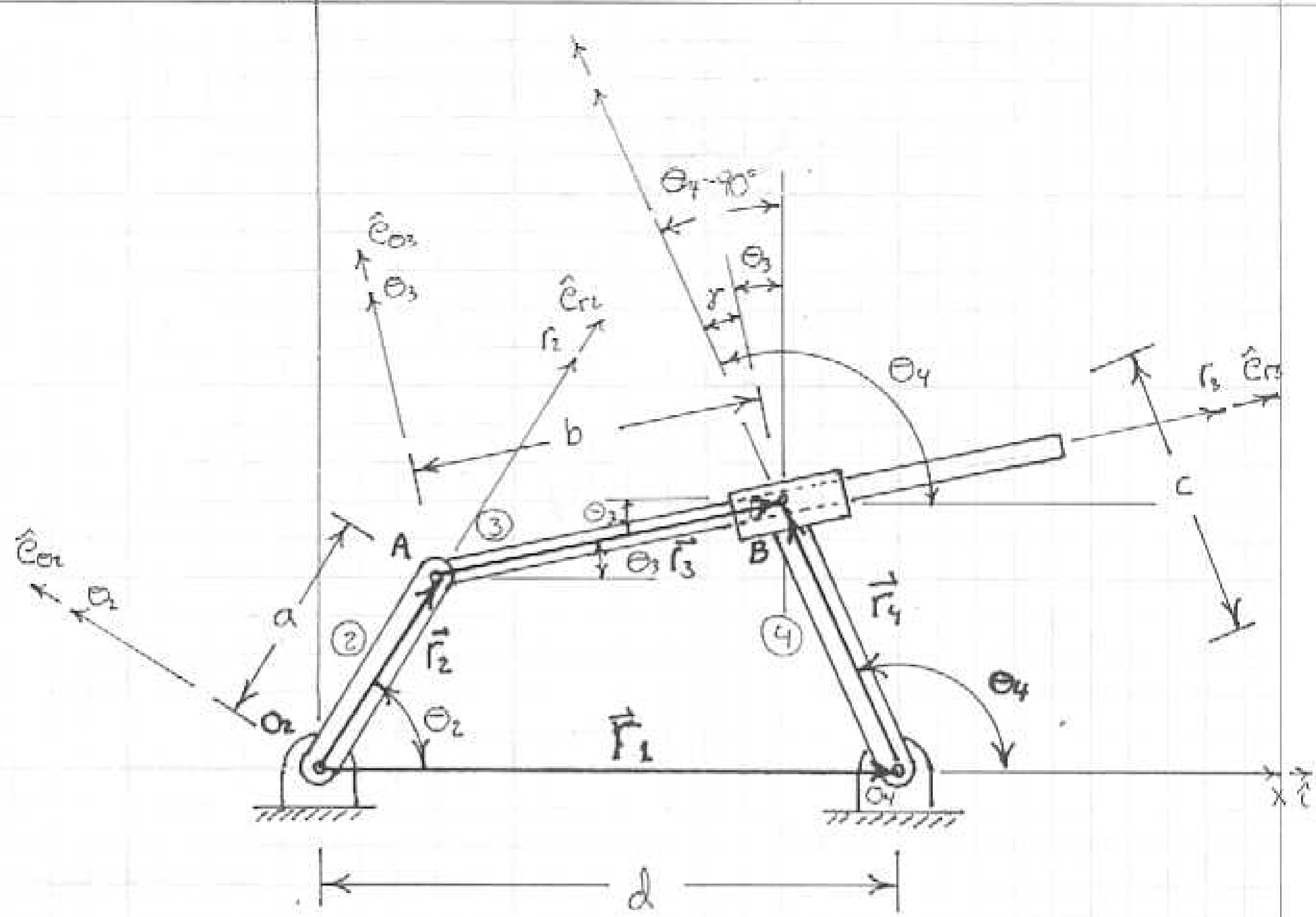
INVERTED SLIDER-CRANK LINKAGE



FOR THE INVERTED SLIDER CRANK THE FIXED ANGLE γ DEFINES THE OFFSET BETWEEN THE SLIDER AND LINK 4. THIS RESULTS IN A RELATIONSHIP BETWEEN θ_3 AND θ_4

$$\theta_4 - 90° = \theta_3 + \gamma$$

$$\boxed{\theta_4 = \theta_3 + 90° + \gamma = \theta_3 + \xi \quad \text{WHERE } \xi = 90° + \gamma} \qquad (1)$$

NOTE THAT γ IS A CONSTANT. THE LOOP THAT DEFINES THE KINEMATICS OF THIS PROBLEM CAN BE WRITTEN.

$$\vec{r}_2 + \vec{r}_3 = \vec{r}_1 + \vec{r}_4 \qquad (2)$$

$$\vec{r}_2 = r_2 \cdot \hat{e}_{r2} \ (= a \cdot \hat{e}_{r2}) = r_2(\cos\theta_2 \hat{\imath} + \sin\theta_2 \hat{\jmath}) \qquad (3)$$

$$\vec{r}_3 = r_3 \cdot \hat{e}_{r3} \ (= b \cdot \hat{e}_{r3}) = r_3(\cos\theta_3 \hat{\imath} + \sin\theta_3 \hat{\jmath}) \qquad (4)$$

$$\vec{r}_1 = d \cdot \hat{\imath} \qquad (5)$$

$$\vec{r}_4 = r_4 \cdot \hat{e}_{r4} \ (= c \cdot \hat{e}_{r4}) = r_4 \cdot (\cos\theta_4 \hat{\imath} + \sin\theta_4 \hat{\jmath}) \qquad (6)$$

THE LENGTH $r_3 = b$ VARIES AS THE LINKAGE MOVES. THUS $r_3 = b$ IS A VARIABLE THAT MOST BE SOLVED FOR. THIS CREATES AN ADDITIONAL UNKNOWN THAT NEEDS TO BE DETERMINED, $\theta_4, \theta_3, r_3 = b$. HOWEVER, (1) IS A THIRD EQUATION THAT CAN BE USED TO DETERMINE THESE VARIABLES GIVEN θ_2, a, c, d.

STARTING WITH THE LOOP EQUATION (2) AND SUBSTITUTING (3)-(6)

$$r_2\cdot(\cos\theta_2\hat{i}+\sin\theta\hat{j})+r_3\cdot(\cos\theta_3\hat{i}+\sin\theta_3\hat{j})=d\hat{i}+r_4\cdot(\cos\theta_4\hat{i}+\sin\theta_4\hat{j}) \quad (7)$$

DOTTING WITH $\hat{i}$

$$r_2\cdot\cos\theta_2+r_3\cdot\cos\theta_3=d+r_4\cdot\cos\theta_4$$

$$a\cdot\cos\theta_2+r_3\cdot\cos\theta_3=d+c\cdot\cos\theta_4 \quad (8)$$

DOTTING WITH $\hat{j}$

$$r_2\cdot\sin\theta_2+r_3\cdot\sin\theta_3=r_4\cdot\sin\theta_4$$

$$a\cdot\sin\theta_2+r_3\cdot\sin\theta_3=c\cdot\sin\theta_4 \quad (9)$$

EQUATIONS (1), (8), (9) ARE THREE EQUATIONS THAT CONTAIN THREE UNKNOWNS: r_3; θ_3, θ_4. SOLVING (9) FOR r_3

$$\boxed{r_3=\frac{c\cdot\sin\theta_4-a\cdot\sin\theta_2}{\sin\theta_3}} \quad (10)$$

SUBSTITUTING (10) INTO (8)

$$a\cdot\cos\theta_2+\left(\frac{c\cdot\sin\theta_4-a\cdot\sin\theta_2}{\sin\theta_3}\right)\cdot\cos\theta_3=d+c\cdot\cos\theta_4$$

$$a\cdot\cos\theta_2+\frac{c\cdot\sin\theta_4\cos\theta_3-a\cdot\sin\theta_2\cdot\cos\theta_3}{\sin\theta_3}-c\cdot\cos\theta_4-d=0 \quad (11)$$

SUBSTITUTING (1) INTO (11)

$$a\cdot\cos\theta_2+\frac{c\cdot\sin\theta_4\cdot\cos(\theta_4-\gamma)-a\cdot\sin\theta_2\cdot\cos(\theta_4-\gamma)}{\sin(\theta_4-\gamma)}-c\cdot\cos\theta_4-d=0$$

$$a\cdot\cos\theta_2+\frac{c\cdot\sin\theta_4(\cos\theta_4\cos\gamma+\sin\theta_4\cdot\sin\gamma)-a\cdot\sin\theta_2(\cos\theta_4\cos\gamma+\sin\theta_4\cdot\sin\gamma)}{\sin\theta_4\cos\gamma-\cos\theta_4\sin\gamma}$$

$$-c\cdot\cos\theta_4-d=0$$

$$\frac{c\cdot\sin\theta_4\cdot\cos\theta_4\cdot\cos\gamma+c\cdot\sin\theta_4\cdot\sin\theta_4\cdot\sin\gamma-a\cdot\sin\theta_2\cdot\cos\theta_4\cdot\cos\gamma-a\cdot\sin\theta_2\cdot\sin\theta_4\cdot\sin\gamma}{\sin\theta_4\cos\gamma-\cos\theta_4\sin\gamma}$$

$$=c\cos\theta_4+d-a\cdot\cos\theta_2$$

$$c\cdot\cos\gamma\cdot\sin\theta_4\cdot\cos\theta_4+c\cdot\sin\gamma\cdot\sin^2\theta_4-a\cdot\cos\gamma\cdot\sin\theta_2\cos\theta_4-a\cdot\sin\gamma\cdot\sin\theta_2\sin\theta_4$$
$$=(c\cdot\cos\theta_4+d-a\cdot\cos\theta_2)\cdot(\sin\theta_4\cdot\cos\gamma-\cos\theta_4\cdot\sin\gamma)$$

$$c\cdot\cos\zeta\cdot\sin\theta_4\cdot\cos\theta_4 + c\cdot\sin\zeta\cdot\sin^2\theta_4 - a\cdot\cos\zeta\cdot\sin\theta_2\cdot\cos\theta_4 - a\cdot\sin\zeta\cdot\sin\theta_2\cdot\sin\theta_4$$
$$= c\cdot\cos\theta_4\cdot\sin\theta_4\cdot\cos\zeta - c\cdot\cos^2\theta_4\cdot\sin\zeta + d\cdot\sin\theta_4\cdot\cos\zeta - d\cdot\cos\theta_4\cdot\sin\zeta$$
$$- a\cdot\cos\theta_2\cdot\sin\theta_4\cdot\cos\zeta + a\cdot\cos\theta_2\cos\theta_4\cdot\sin\zeta$$

$$c\cdot\cos\zeta\cdot\sin\theta_4\cdot\cos\theta_4 + c\cdot\sin\zeta\cdot\sin^2\theta_4 - a\cdot\cos\zeta\cdot\sin\theta_2\cdot\cos\theta_4 - a\cdot\sin\zeta\cdot\sin\theta_2\cdot\sin\theta_4$$
$$- c\cdot\cos\theta_4\cdot\sin\theta_4\cdot\cos\zeta + c\cdot\cos^2\theta_4\cdot\sin\zeta - d\cdot\sin\theta_4\cdot\cos\zeta + d\cdot\cos\theta_4\cdot\sin\zeta$$
$$+ a\cdot\cos\theta_2\cdot\sin\theta_4\cdot\cos\zeta - a\cdot\cos\theta_2\cdot\cos\theta_4\cdot\sin\zeta = 0$$

$$c\cdot\sin\zeta\cdot(\sin^2\theta_4+\cos^2\theta_4) - a\cdot(\sin\theta_2\cdot\cos\zeta + \sin\zeta\cdot\cos\theta_2)\cdot\cos\theta_4$$
$$+ a\cdot(\cos\theta_2\cdot\cos\zeta - \sin\theta_2\cdot\sin\zeta)\cdot\sin\theta_4 - d\cdot\cos\zeta\cdot\sin\theta_4 + d\sin\zeta\cdot\cos\theta_4 = 0$$

$$c\cdot\sin\zeta - a\cdot\sin(\theta_2+\zeta)\cdot\cos\theta_4 + a\cdot\cos(\theta_2+\zeta)\cdot\sin\theta_4 - d\cdot\cos\zeta\cdot\sin\theta_4$$
$$+ d\cdot\sin\zeta\cdot\cos\theta_4 = 0$$

$$[a\cdot\cos(\theta_2+\zeta) - d\cdot\cos\zeta]\cdot\sin\theta_4 + [-a\cdot\sin(\theta_2+\zeta) + d\cdot\sin\zeta]\cdot\cos\theta_4$$
$$+ c\cdot\sin\zeta = 0$$

$$\boxed{\begin{aligned} &K_1\cdot\sin\theta_4 + K_2\cdot\cos\theta_4 + K_3 = 0 && (12)\\ &\qquad K_1 = a\cdot\cos(\theta_2+\zeta) - d\cdot\cos\zeta && (13)\\ &\qquad K_2 = -a\cdot\sin(\theta_2+\zeta) + d\cdot\sin\zeta && (14)\\ &\qquad K_3 = c\cdot\sin\zeta && (15)\end{aligned}}$$

USING THE TRIGOMETRIC IDENTITIES

$$\sin 2\cdot\alpha = \frac{2\cdot\tan\alpha}{1+\tan^2\alpha} \Rightarrow \sin\theta = \frac{2\cdot\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \quad (16)$$

$$\cos 2\cdot\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha} \Rightarrow \cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \quad (17)$$

SUBSTITUTING (16) AND (17) INTO (12)

$$K_1\cdot\frac{2\cdot\tan\frac{\theta_4}{2}}{1+\tan^2\frac{\theta_4}{2}} + K_2\cdot\frac{1-\tan^2\frac{\theta_4}{2}}{1+\tan^2\frac{\theta_4}{2}} + K_3 = 0$$

$$2\cdot K_1\cdot\tan\frac{\theta_4}{2} + K_2 - K_2\cdot\tan^2\frac{\theta_4}{2} + K_3 + K_3\cdot\tan^2\frac{\theta_4}{2} = 0$$

$$(K_3-K_2)\cdot\tan^2\frac{\theta_4}{2} + 2\cdot K_1\cdot\tan\frac{\theta_4}{2} + (K_3+K_2) = 0$$

$$\tan^2\frac{\theta_4}{2} + \frac{2\cdot K_1}{K_3-K_2}\cdot\tan\frac{\theta_4}{2} + \frac{K_3+K_2}{K_3-K_2} = 0$$

$$\tan^2\frac{\theta_4}{2} + \frac{2\cdot K_1}{K_3-K_2}\cdot\tan\frac{\theta_4}{2} + \left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_1}{K_3-K_2}\right)^2 + \frac{K_3+K_2}{K_3-K_2} = 0$$

$$\left(\tan\frac{\theta_4}{2} + \frac{K_1}{K_3-K_2}\right)^2 = \left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_3+K_2}{K_3-K_2}\right)$$

$$\tan\frac{\theta_4}{2} = -\frac{K_1}{K_3-K_2} \pm \sqrt{\left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_3+K_2}{K_3-K_2}\right)}$$

$$\tan\frac{\theta_4}{2} = -\frac{K_1}{K_3-K_2} \pm \sqrt{\left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_3+K_2}{K_3-K_2}\right)\left(\frac{K_3-K_2}{K_3-K_2}\right)}$$

$$\tan\frac{\theta_4}{2} = -\frac{K_1}{K_3-K_2} \pm \sqrt{\frac{K_1^2 - (K_3^2 - K_2^2)}{(K_3-K_2)^2}}$$

$$\tan\frac{\theta_4}{2} = \frac{-K_1 \pm \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_3-K_2}$$

$$\boxed{\theta_4 = 2\cdot\tan^{-1}\left[\frac{-K_1 \pm \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_3-K_2}\right]} \quad (18)$$

THE SOLUTION OF THE INVERTED SLIDER CRANK LINKAGE STARTS WITH THE DEFINATION OF THE LINKAGE PARAMETERS

GIVEN: a, c, d, θ_2, & γ

THE PARAMETERS THAT NEED TO BE DETERMINED INCLUDE

FIND: b, θ_3, & θ_4

THESE PARAMETERS ARE FOUND USING (1), (10), (13), (14), (15) AND (18)

INVERTED SLIDER CRANK VELOCITY ANALYSIS

THE VECTOR LOOP EQUATION FOR THE INVERTED SLIDER CRANK CAN BE WRITTEN

$$r_2 \cdot \hat{e}_{r2} + r_3 \cdot \hat{e}_{r3} = r_1 \cdot \hat{i} + r_4 \cdot \hat{e}_{r4} \qquad (19)$$

WHERE

$$\hat{e}_{r2} = \cos\theta_2 \cdot \hat{i} + \sin\theta_2 \hat{j} \qquad (20)$$
$$\hat{e}_{\theta 2} = -\sin\theta_2 \hat{i} + \cos\theta_2 \hat{j} \qquad (21)$$

$$\hat{e}_{r3} = \cos\theta_3 \hat{i} + \sin\theta_3 \hat{j} \qquad (22)$$
$$\hat{e}_{\theta 3} = -\sin\theta_3 \hat{i} + \cos\theta_3 \hat{j} \qquad (23)$$

$$\hat{e}_{r4} = \cos\theta_4 \hat{i} + \sin\theta_4 \hat{j} \qquad (24)$$
$$\hat{e}_{\theta 4} = -\sin\theta_4 \hat{i} + \cos\theta_4 \hat{j} \qquad (25)$$

THE SOLUTION TO FINDING THE VELOCITY COMPONENTS OF THE INVERTED SLIDER CRANK STARTS WITH TAKING THE DERIVATIVE OF (19) WITH RESPECT TO TIME.

$$\cancel{\dot{r}_2}^{\,0} \cdot \hat{e}_{r2} + r_2 \cdot \underbrace{\dot{\hat{e}}_{r2}}_{\dot{\theta}_2 \hat{k} \times \hat{e}_{r2}} + \dot{r}_3 \cdot \hat{e}_{r3} + r_3 \cdot \underbrace{\dot{\hat{e}}_{r3}}_{\dot{\theta}_3 \hat{k} \times \hat{e}_{r3}} = \cancel{\dot{r}_1}^{\,0} \hat{i} + \cancel{\dot{r}_4}^{\,0} \cdot \hat{e}_{r4} + r_4 \cdot \underbrace{\dot{\hat{e}}_{r4}}_{\dot{\theta}_4 \hat{k} \times \hat{e}_{r4}}$$

$$r_2 \cdot \dot{\theta}_2 \hat{e}_{\theta 2} + \dot{r}_3 \cdot \hat{e}_{r3} + r_3 \dot{\theta}_3 \cdot \hat{e}_{\theta 3} = r_4 \dot{\theta}_4 \cdot \hat{e}_{\theta 4} \qquad (26)$$

THE UNKNOWNS IN EQUATION (26) ARE $\dot{\theta}_3$, $\dot{\theta}_4$, AND $\dot{r}_3$. $\dot{\theta}_2$ IS TYPICALLY GIVEN FOR A INVERTED SLIDER CRANK LINKAGE. EQUATION (26) CAN ALSO BE REPRESENTED BY TWO SCALAR EQUATIONS. A THIRD EQUATION IS NEEDED TO SOLVE ALL THREE VELOCITY UNKNOWNS. THE THIRD EQUATION IS FOUND BY TAKING THE DERIVATIVE OF (1)

$$\boxed{\dot{\theta}_4 = \dot{\theta}_3} \qquad (27)$$

SUBSTITUTING THE UNIT VECTORS (20) - (25) INTO (26)

$$r_2 \cdot \dot{\theta}_2 \cdot (-\sin\theta_2 \hat{i} + \cos\theta_2 \hat{j}) + \dot{r}_3 \cdot (\cos\theta_3 \hat{i} + \sin\theta_3 \hat{j})$$
$$+ r_3 \cdot \dot{\theta}_3 \cdot (-\sin\theta_3 \hat{i} + \cos\theta_3 \hat{j}) = r_4 \dot{\theta}_4 (-\sin\theta_4 \hat{i} + \cos\theta_4 \hat{j}) \qquad (28)$$

DOTTING (28) WITH $\hat{i}$

$$-r_2 \cdot \dot{\theta}_2 \cdot \sin\theta_2 + \dot{r}_3 \cdot \cos\theta_3 - r_3 \cdot \dot{\theta}_3 \cdot \sin\theta_3 = -r_4 \cdot \dot{\theta}_4 \cdot \sin\theta_4$$

$$r_2 \cdot \dot{\theta}_2 \cdot \sin\theta_2 - \dot{r}_3 \cdot \cos\theta_3 + r_3 \cdot \dot{\theta}_3 \cdot \sin\theta_3 = r_4 \cdot \dot{\theta}_4 \cdot \sin\theta_4 \qquad (29)$$

DOTTING (28) WITH $\hat{j}$

$$r_2 \cdot \dot{\theta}_2 \cdot \cos\theta_2 + \dot{r}_3 \cdot \sin\theta_3 + r_3 \cdot \dot{\theta}_3 \cdot \cos\theta_3 = r_4 \cdot \dot{\theta}_4 \cdot \cos\theta_4 \qquad (30)$$

SOLVING (29) FOR $\dot{r}_3$

$$\boxed{\dot{r}_3 = \frac{r_2 \cdot \dot{\theta}_2 \cdot \sin\theta_2 + r_3 \cdot \dot{\theta}_3 \cdot \sin\theta_3 - r_4 \cdot \dot{\theta}_4 \cdot \sin\theta_4}{\cos\theta_3}} \qquad (31)$$

SUBSTITUTING (31) INTO (30)

$$r_2 \cdot \dot{\theta}_2 \cdot \cos\theta_2 + \sin\theta_3 \cdot \frac{r_2 \cdot \dot{\theta}_2 \cdot \sin\theta_2 + r_3 \cdot \dot{\theta}_3 \cdot \sin\theta_3 - r_4 \cdot \dot{\theta}_4 \cdot \sin\theta_4}{\cos\theta_3}$$
$$+ r_3 \cdot \dot{\theta}_3 \cdot \cos\theta_3 = r_4 \dot{\theta}_4 \cdot \cos\theta_4$$

$$r_2 \cdot \dot{\theta}_2 \cdot \cos\theta_2 \cdot \cos\theta_3 + r_2 \cdot \dot{\theta}_2 \cdot \sin\theta_2 \cdot \sin\theta_3 + r_3 \cdot \dot{\theta}_3 \cdot \sin^2\theta_3 - r_4 \cdot \dot{\theta}_4 \cdot \sin\theta_3 \cdot \sin\theta_4$$
$$+ r_3 \cdot \dot{\theta}_3 \cos^2\theta_3 = r_4 \cdot \dot{\theta}_4 \cdot \cos\theta_4 \cdot \cos\theta_3$$

$$r_2 \cdot \dot{\theta}_2 \cdot \cos(\theta_2 - \theta_3) + r_3 \cdot \dot{\theta}_3 - r_4 \cdot \dot{\theta}_4 \cdot \cos(\theta_4 - \theta_3) = 0$$

SUBSTITUTING (27) INTO THE ABOVE EQUATION

$$r_3 \cdot \dot{\theta}_2 \cdot \cos(\theta_2 - \theta_3) + r_3 \cdot \dot{\theta}_3 - r_4 \cdot \dot{\theta}_3 \cdot \cos(\theta_4 - \theta_3) = 0$$

$$r_2 \cdot \dot{\theta}_2 \cdot \cos(\theta_2 - \theta_3) + \dot{\theta}_3 \cdot [r_3 - r_4 \cdot \cos(\theta_4 - \theta_3)] = 0$$

$$\boxed{\dot{\theta}_3 = -\frac{r_2 \cdot \dot{\theta}_2 \cdot \cos(\theta_2 - \theta_3)}{r_3 - r_4 \cdot \cos(\theta_4 - \theta_3)}} \qquad (32)$$