PROBLEM 7.33 | DETERMINE THE MAXIMUM TRESCA AND VON MISES STRESSES FOR THE GIVEN STRESS MATRIX. COMPARE THE VON MISES STRESS WITH TWO TIMES THE TRESCA STRESS.

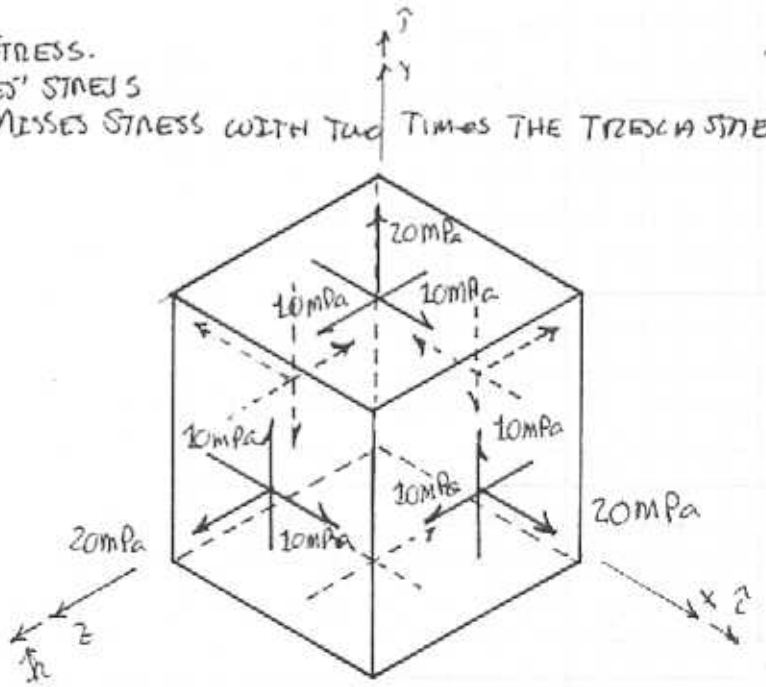
$$\sigma = \begin{bmatrix} 20 & 10 & 10 \\ 10 & 20 & 10 \\ 10 & 10 & 20 \end{bmatrix} \text{MPa}$$

GIVEN:
1. $\sigma_x = \sigma_y = \sigma_z = 20\ \text{MPa}$
2. $\tau_{yz} = \tau_{xz} = \tau_{xy} = 10\text{MPa}$

ASSUMPTIONS:
1. LINEAR ELASTIC MATERIAL
2. SMALL DEFORMATIONS

FIND:
1. MAXIMUM TRESCA STRESS.
2. MAXIMUM VON MISES' STRESS
3. COMPARE THE VON MISES STRESS WITH TWO TIMES THE TRESCA STRESS

FIGURE



(a)

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>> S=[20 10 10; 10 20 10; 10 10 20]

S =
   20    10    10
   10    20    10
   10    10    20

>> [EV,PS]=eig(S)

EV =
   0.7071    0.4082    0.5774
  -0.7071    0.4082    0.5774
        0   -0.8165    0.5774

PS =
   10     0     0
    0    10     0
    0     0    40

>> EV'*S*EV

ans =
  10.0000    0.0000    0.0000
   0.0000   10.0000   -0.0000
            0   -0.0000   40.0000
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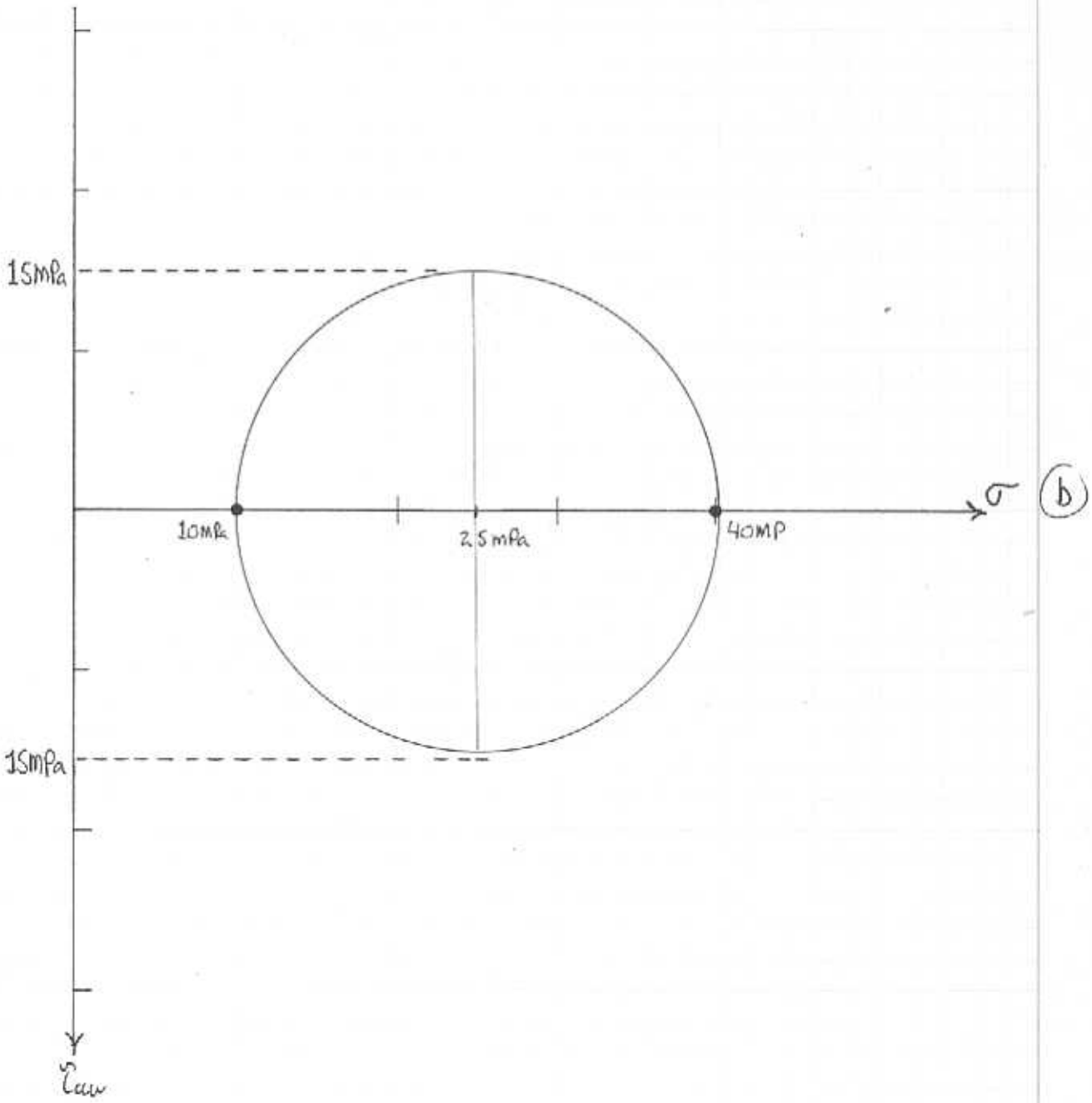
THE PRINCIPAL STRESSES ARE GIVEN BY

$$\sigma_1 = 40\,\text{MPa}, \quad \sigma_2 = \sigma_3 = 10\,\text{MPa} \qquad (1)$$

A QUICK DRAWING OF MOHR'S CIRCLE WILL HELP TO VISUALIZE THIS STRESS STATE



THE VON MISES STRESS IS CALCULATED FROM THE PRINCIPAL STRESSES

$$\sigma_{vm} = \sqrt{0.5\cdot[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}$$

$$= \sqrt{0.5[(10-10)^2+(10-40)^2+(10-40)^2} = \sqrt{0.5\cdot 2\cdot(10-40)^2} = \sqrt{(30\text{MPa})^2}$$

$$= \boxed{30\text{MPa}} \quad (2)$$

THE MAXIMUM TRESCA STRESS IS SEEN IN (B) AS

$$\sigma_T = \boxed{15\text{MPa}} \quad (3)$$

FROM (2) AND (3)

$$\boxed{\sigma_m = 2\cdot\sigma_T}$$

Summary

MOHR'S CIRCLE IS NOT REQUIRED FOR THIS PROBLEM BUT IT DOES HELP TO VISUALIZE THE STRESS FIELD. NOTE THAT THE THREE CIRCLES THAT MAKE UP MOHRS CIRCLE REDUCE TO A SINGLE CIRCLE IN THIS PROBLEM.