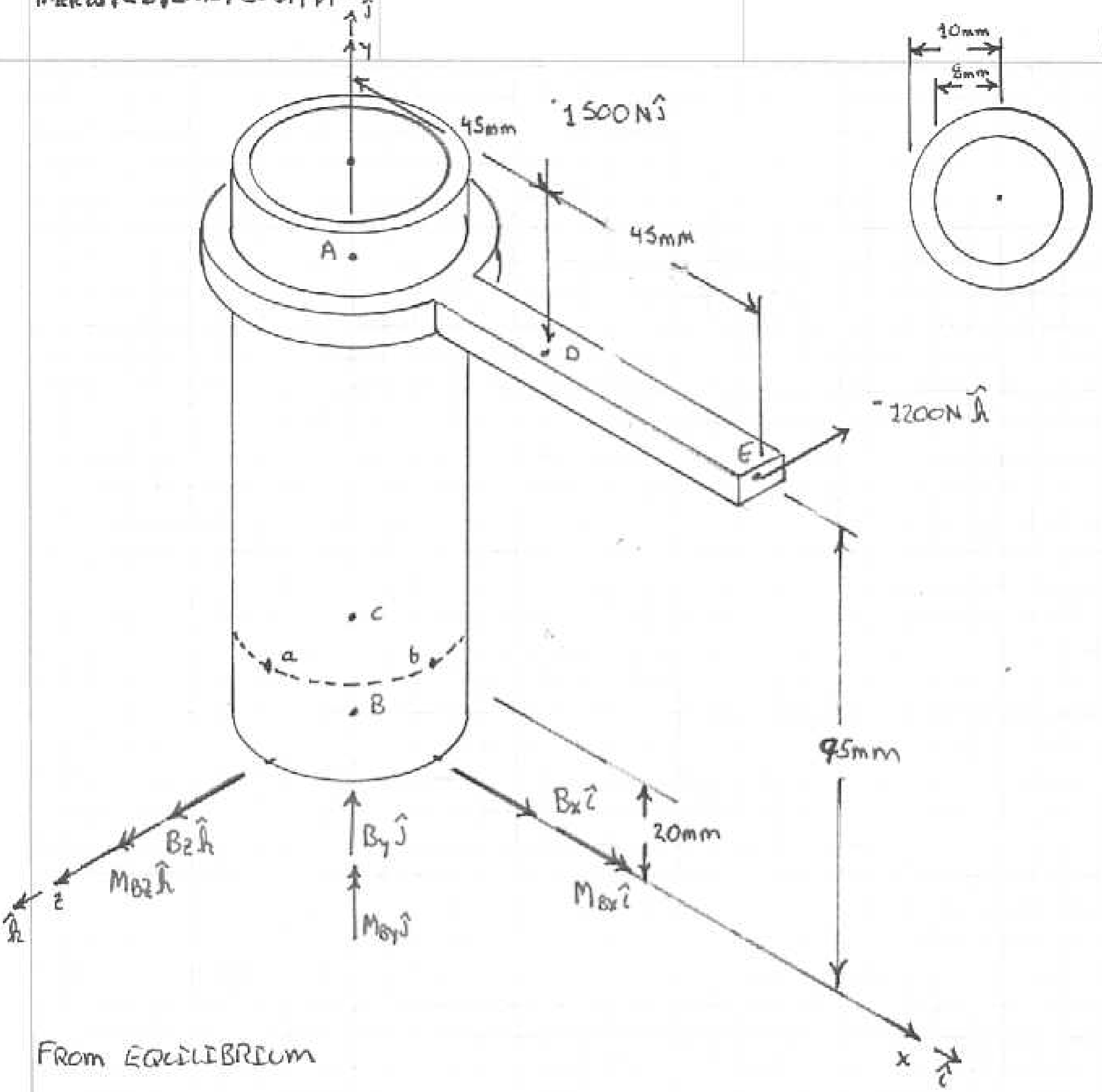


(a)

FROM EQUILIBRIUM

$$\Sigma F_x = 0 = \underline{\underline{B_x}}$$

$$\Sigma F_y = 0 = -1500\text{N} + B_y \Rightarrow \underline{\underline{B_y = 1500\text{N}}}$$

$$\Sigma F_z = 0 = 1200\text{N} + B_z \Rightarrow \underline{\underline{B_z = 1200\text{N}}}$$

$$\Sigma \vec{M}_{@B} = \vec{0} = \vec{M}_B + \vec{r}_{BD} \times \vec{F}_D + \vec{r}_{BE} \times \vec{F}_E$$

$$\vec{0} = \vec{M}_B + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 45\text{mm} & 95\text{mm} & 0 \\ 0 & -1500\text{N} & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 90\text{mm} & 95\text{mm} & 0 \\ 0 & 0 & -1200\text{N} \end{vmatrix}$$

$$\vec{0} = \vec{M}_B + [(45\text{mm})(-1500\text{N})]\hat{k} + [(95\text{mm})(-1200\text{N})]\hat{i} - [(90\text{mm})(-1200\text{N})]\hat{j}$$

$$\vec{0} = \vec{M}_B - 114\ \text{N·m}\ \hat{i} + 108\ \text{N·m}\ \hat{j} - 67.5\ \text{N·m}\ \hat{k}$$

DOTTING WITH $\hat{i}$

$$0 = M_{Bx} - 114\ N\cdot m \quad \Rightarrow \quad \underline{\underline{M_{Bx} = 114\ N\cdot m}}$$

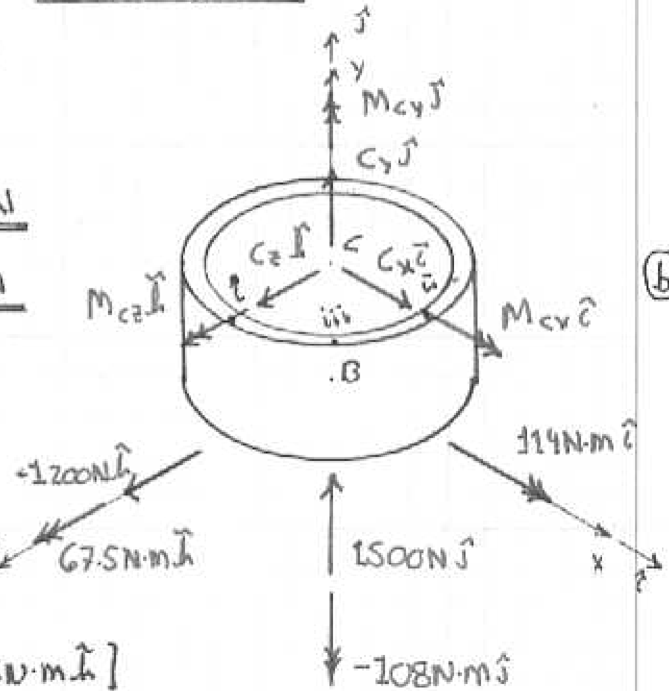
DOTTING WITH $\hat{j}$

$$0 = M_{By} + 108\ N\cdot m \quad \Rightarrow \quad \underline{\underline{M_{By} = -108\ N\cdot m}}$$

DOTTING WITH $\hat{k}$

$$0 = M_{Bz} - 67.5\ N\cdot m \quad \Rightarrow \quad \underline{\underline{M_{Bz} = 67.5\ N\cdot m}}$$

CONSIDERING THE CROSS SECTION AT C



$$\Sigma F_x = 0 = \underline{\underline{C_x}}$$

$$\Sigma F_y = 0 = C_y + 1500N \Rightarrow \underline{\underline{C_y = -1500N}}$$

$$\Sigma F_z = 0 = C_z + 1200N \Rightarrow \underline{\underline{C_z = -1200N}}$$ (b)

$$\Sigma \vec{M}_{@C} = \vec{0} = \vec{M}_C + \vec{M}_B + \vec{r}_{CB} \times \vec{F}_B$$

$$\vec{0} = \vec{M}_C + \vec{M}_B + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -0.02m & 0 \\ 0 & 1500N & 1200N \end{vmatrix}$$

$$\vec{0} = \vec{M}_C + [114N\cdot m\,\hat{i} - 108N\cdot m\,\hat{j} + 67.5N\cdot m\,\hat{k}]$$

$$+ [(-0.02m)(1200N)]\hat{i}$$

$$\vec{0} = \vec{M}_C + 90N\cdot m\,\hat{i} - 108N\cdot m\,\hat{j} + 67.5N\cdot m\,\hat{k}$$

DOTTING WITH

$$0 = M_{Cx} + 90N\cdot m \Rightarrow \underline{\underline{M_{Cx} = -90N\cdot m}}$$

$$0 = M_{Cy} - 108N\cdot m \Rightarrow \underline{\underline{M_{Cy} = 108N\cdot m}}$$

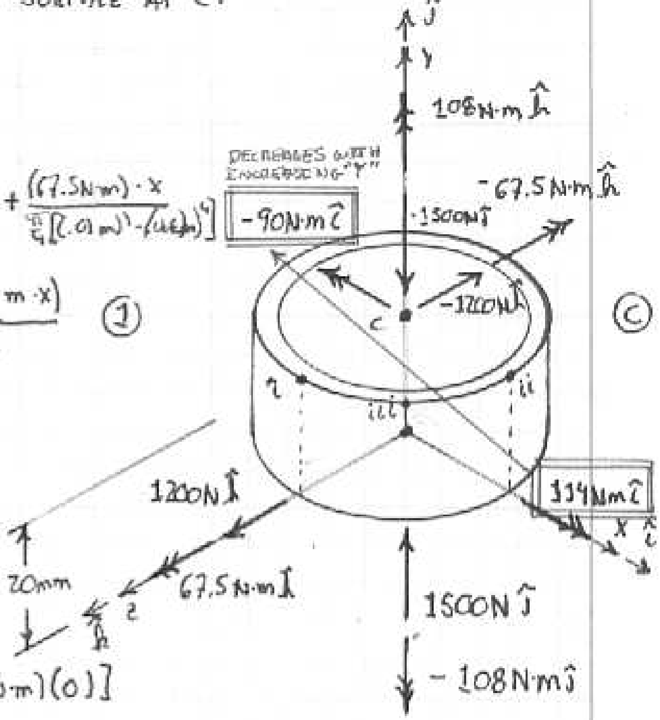
$$0 = M_{Cz} + 67.5N\cdot m \Rightarrow \underline{\underline{M_{Cz} = -67.5N\cdot m}}$$

NOW THE STATE OF STRESS IN THIS CROSS-SECTION CAN BE COMPUTED. STARTING WITH THE NORMAL STRESS FOR THE SURFACE AT C.

$$\sigma_y = \frac{F_y}{A} - \frac{M_x \cdot z}{I_{xx}} + \frac{M_z \cdot x}{I_{zz}}$$

$$= \frac{-1500N}{\pi[(.01m)^2-(.008m)^2]} - \frac{(-90N\cdot m)\cdot z}{\frac{\pi}{4}[(.01m)^4-(.008m)^4]} + \frac{(67.5N\cdot m)\cdot x}{\frac{\pi}{4}[(.01m)^4-(.008m)^4]}$$

$$= \frac{-1500N}{\pi[(.01m)^2-(.008m)^2]} + \frac{4\,(90N\cdot m\cdot z - 67.5N\cdot m\cdot x)}{\pi[(.01m)^4-(.008m)^4]} \qquad (1)$$



FOR THE THREE POINTS OF INTEREST THE NORMAL STRESS CAN NOW BE COMPUTED.

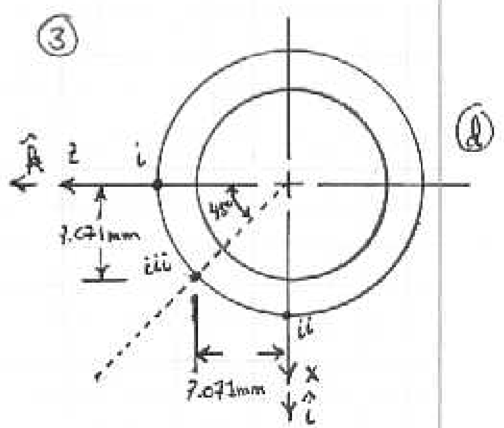
<u>POINT i</u>

$$\sigma_{y,i} = -13.26(10^6)\frac{N}{m^2} + 215.7(10^6)\frac{1}{m^4}\cdot[(90N\cdot m)\cdot(.01m) - (67.5N\cdot m)(0)]$$

$$= 180.8(10^6)\frac{N}{m^2} = \underline{180.8\,MPa} \qquad (2)$$

$$\sigma_{y,ii} = -13.26(10^6)\frac{N}{m^2} + 215.7(10^6)\frac{1}{m^4}[(90N\cdot m)\cdot(0) - (67.5N\cdot m)(.01m)]$$

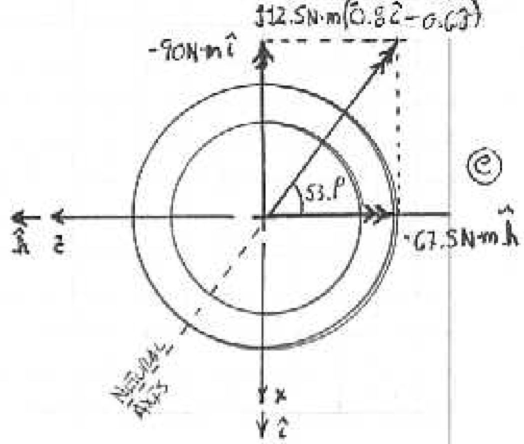
$$= -158.9(10^6)\frac{N}{m^2} = \underline{-158.9\,MPa} \qquad (3)$$



$$\sigma_{y,iii} = -13.26(10^6)\frac{N}{m^2} + 215.7(10^6)\frac{1}{m^4}[(90N\cdot m)\cdot(.007071m) - (67.5N\cdot m)(.007071m)]$$

$$= 21.06(10^6)\frac{N}{m^2} = \underline{21.06\,MPa} \qquad (4)$$

THE SHEARING STRESS AT THE THREE POINTS UNDER CONSIDERATION HAVE TWO COMPONENTS. ONE COMPONENT RESULTS FROM THE APPLIED TORQUE AND THE OTHER FROM THE SHEAR FORCE DIRECTED ALONG THE Z-AXIS.

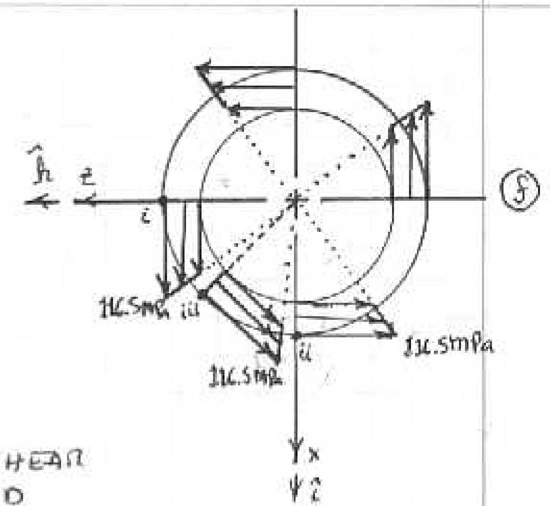


THE SHEAR STRESS RESULTING FROM THE TORQUE IN THE "Y" DIRECTION IS CALCULATED BELOW

$$\tau = \frac{T_y \cdot r}{J} = \frac{(108N\cdot m)\cdot(0.01m)}{\frac{1}{2}\pi[(.01m)^4-(.008m)^4]}$$

$$\tau = \frac{2\cdot(108N\cdot m)\cdot(0.01m)}{\pi[(.01m)^4-(.008m)^4]}$$

$$= 116.5(10^6)\frac{N}{m^2} = \underline{116.5\ MPa} \qquad (5)$$



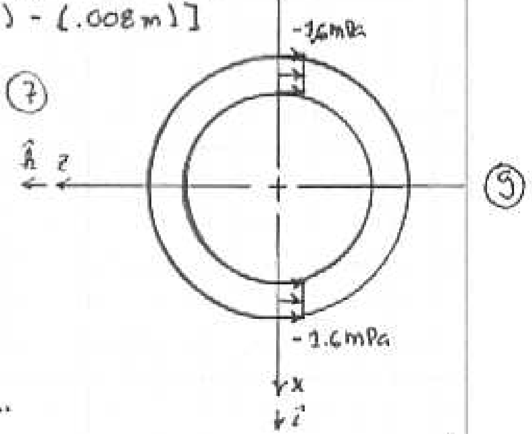
THE MAGNITUDE OF THIS SHEAR STRESS IS ONLY DEPENDENT ON RADIUS, NOT ON LOCATION AROUND THE CIRCUMFERENCE.

THE SHEARING STRESS THAT RESULTS FROM THE SHEAR FORCE IN THE "Z" DIRECTION MUST BE COMPUTED SEPARATELY FOR THE THREE POINTS OF INTEREST.

$$\tau_{yz,i} = \frac{V\cdot Q}{I\cdot t} = \frac{(-1200N)(0)}{\frac{\pi}{4}[(.01m)^4-(.008m)^4](0)} = \underline{\underline{0}} \qquad (6)$$

$$\tau_{yz,ii} = \frac{V\cdot Q}{I\cdot t} = \frac{(-1200N)\frac{4}{3\pi}[(.01m)-(.008m)]\cdot\frac{\pi}{4}[(.01m)^2-(.008m)^2]}{\frac{\pi}{4}[(.01m)^4-(.008m)^4]\cdot 2[(.01m)-(.008m)]}$$

$$= -1.553(10^6)\frac{N}{m^2} = \underline{\underline{-1.553\ MPa}} \qquad (7)$$

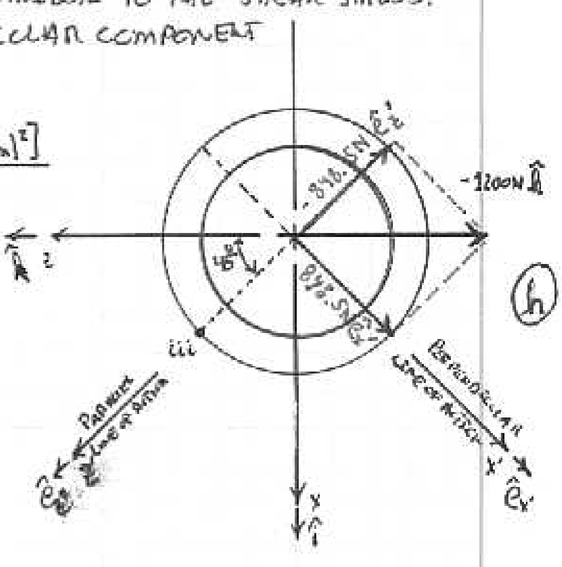


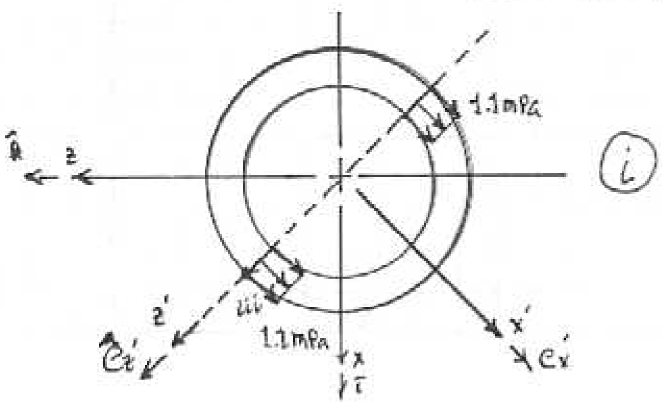
TO CALCULATE THE SHEAR STRESS RESULTING FROM THE SHEAR FORCE IN THE "Z" DIRECTION AT iii, THE Z-DIRECTION SHEAR FORCE NEEDS TO BE BROKEN INTO COMPONENTS PARALLEL TO THE LINE OF ACTION FROM THE CENTER OF THE CIRCLE TO POINT iii AND A COMPONENT THAT IS PERPENDICULAR.

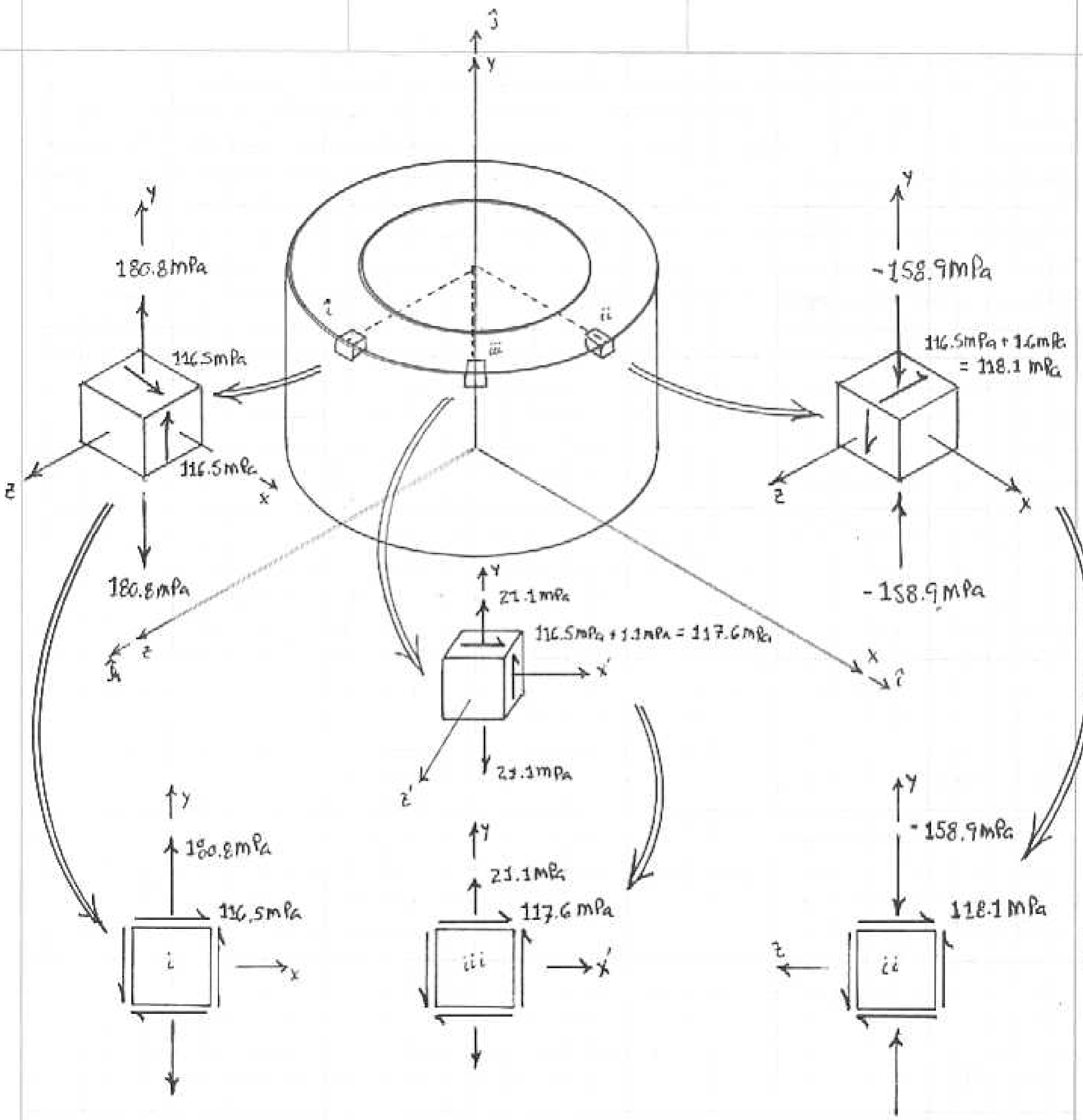
THE COMPONENT OF THE SHEAR FORCE PARALLEL TO THE LINE OF ACTION FROM THE CENTER OF THE SECTION OUT TO iii (z'-direction, $\hat{e}_{z'}$) DOES NOT CONTRIBUTE TO THE SHEAR STRESS. THE SHEAR STRESS THAT RESULTS FROM THE PERPENDICULAR COMPONENT (x'-DIRECTION) IS CALCULATED AS FOLLOWS

$$\tau_{yx',iii} = \frac{V\cdot Q}{I\cdot t} = \frac{(848.5N)\frac{4}{3\pi}[(.01m)-(.008m)]\cdot\frac{\pi}{4}[(.01m)^2-(.008m)^2]}{\frac{\pi}{4}[(.01m)^4-(.008m)^4]\cdot 2\cdot[(.01m)-(.008m)]}$$

$$= 1.098(10^6)\frac{N}{m^2} = \underline{1.098\ MPa} \qquad (8)$$





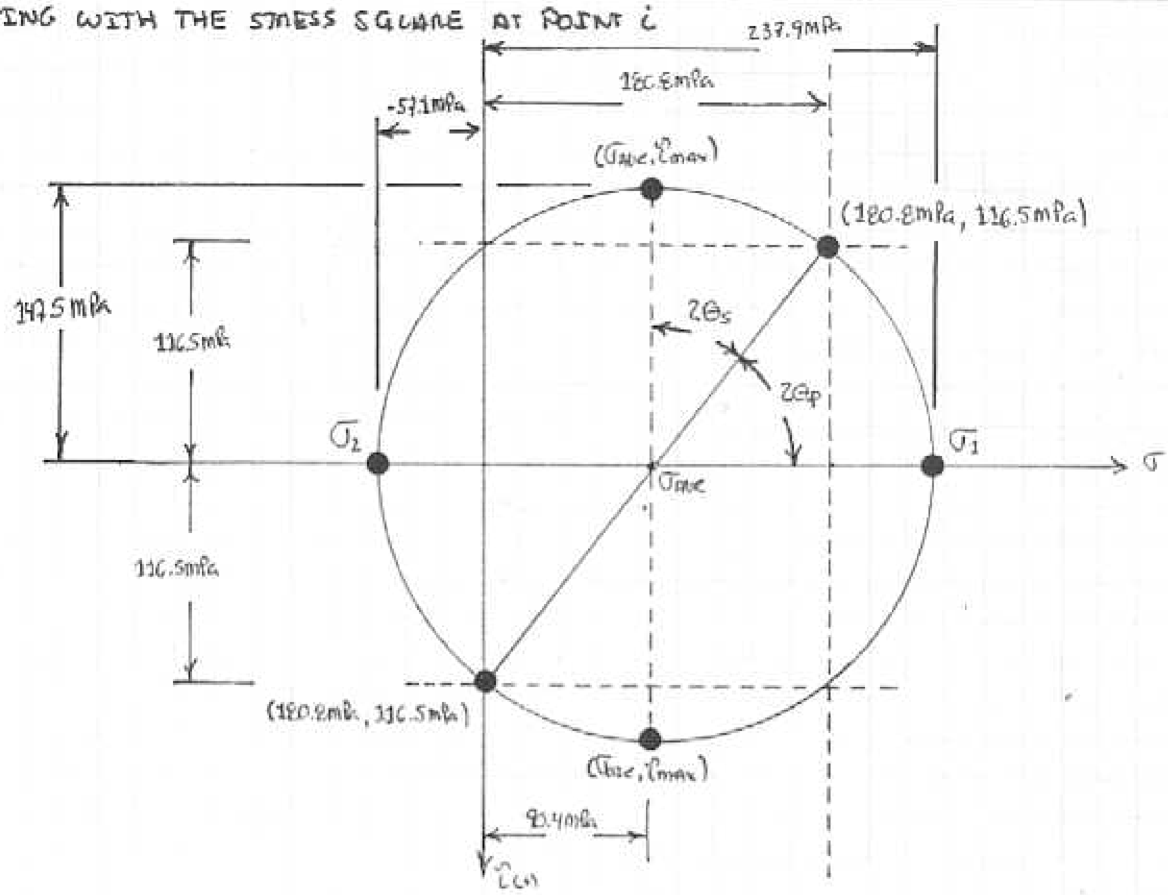


(j)

(k)

FIGURE (j) ILLUSTRATES THE STRESS CUBE ORIENTATION AT LOCATIONS i, ii, & iii. THE STRESS STATE ON THESE STRESS CUBES ARE ALSO ILLUSTRATED IN THREE DIMENSIONS IN FIGURE (j). BECAUSE THERE IS NO NORMAL OR SHEARING STRESS ON ONE FACE FOR EACH OF THESE STRESS CUBES, THEY CAN BE SIMPLIFIED TO TWO DIMENSIONAL STRESS SQUARES SHOWN IN FIGURE (k). THE THREE STRESS CUBES CAN NOW BE USED TO DETERMINE THE PRINCIPAL STATE OF STRESS AND THE STATE OF MAXIMUM SHEARING STRESS USING MOHR'S CIRCLE.

STARTING WITH THE STRESS SQUARE AT POINT i



$$r = \sqrt{(180.8\text{MPa} - 90.4\text{MPa})^2 + (116.5\text{MPa})^2} = 147.5\text{MPa}$$

$$\sigma_1 = \sigma_{AVE} + r = 90.4\text{MPa} + 147.5\text{MPa} = 237.9\text{ MPa}$$

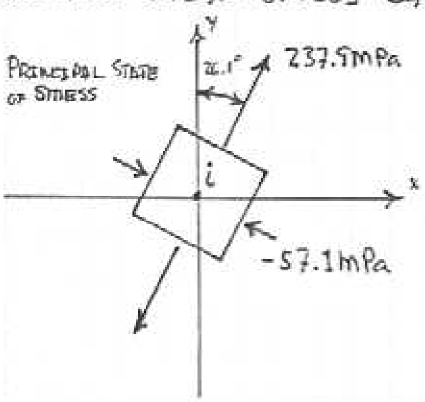
$$\sigma_2 = \sigma_{AVE} - r = 90.4\text{MPa} - 147.5\text{MPa} = -57.1\text{MPa}$$

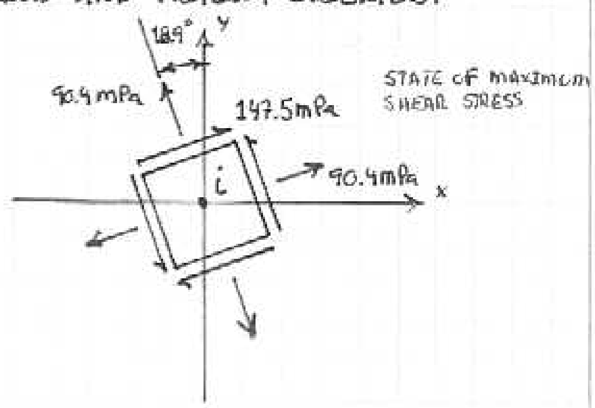
$$\tau_{max} = r = 147.5\text{ MPa}$$

$$2\theta_p = \tan^{-1}\frac{116.5\text{MPa}}{180.8\text{MPa} - 90.4\text{MPa}} = 52.1° \Rightarrow \theta_p = 26.1°$$

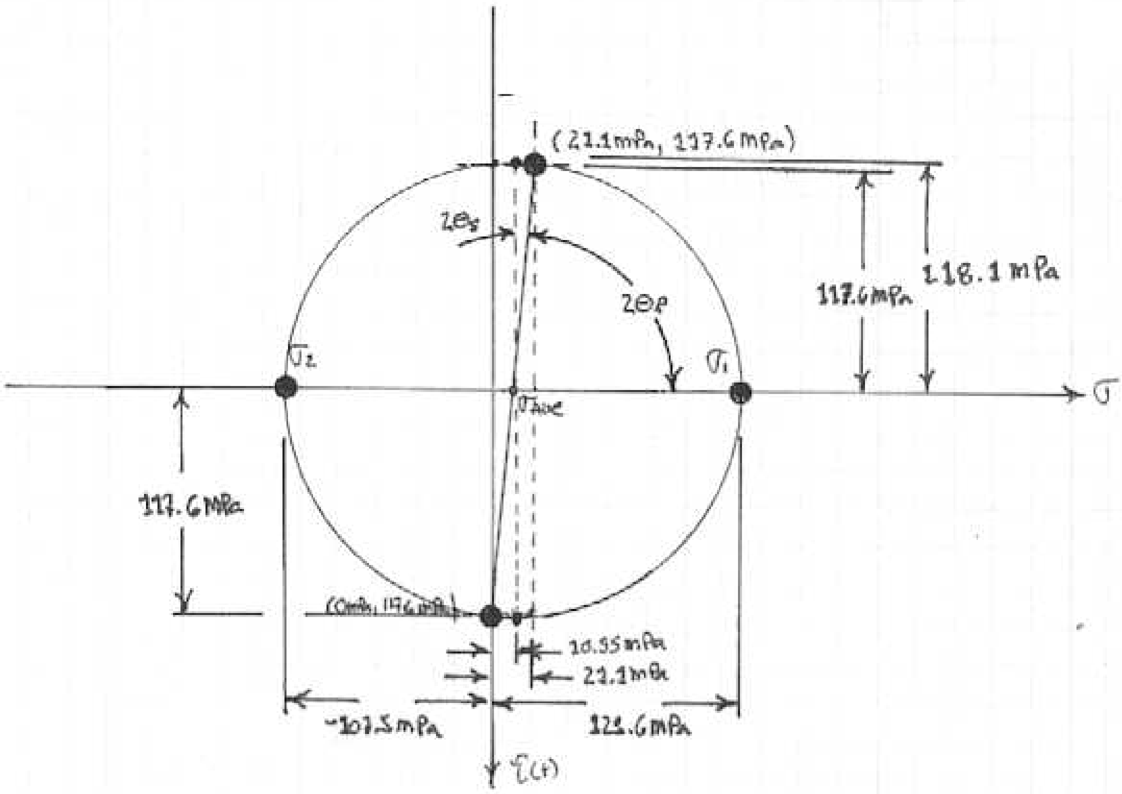
$$2\theta_s = \tan^{-1}\frac{180.8\text{MPa} - 90.4\text{MPa}}{116.5\text{MPa}} = 37.8° \Rightarrow \theta_s = 18.9°$$

NOW THE STRESS SQUARES THAT REPRESENT THE PRINCIPAL STATE OF STRESS AND THE STATE OF MAXIMUM SHEAR STRESS CAN BE DRAWN AND PROPERLY ORIENTED.





NOW POINT *iii* WILL BE EVALUATED.



$$r = \sqrt{(117.6\,mPa)^2 + (21.1\,mPa - 10.55\,mPa)^2} = 118.1\,mPa$$

$$\sigma_1 = \sigma_{AVE} + r = 10.55\,mPa + 118.1\,mPa = 128.6\,mPa$$

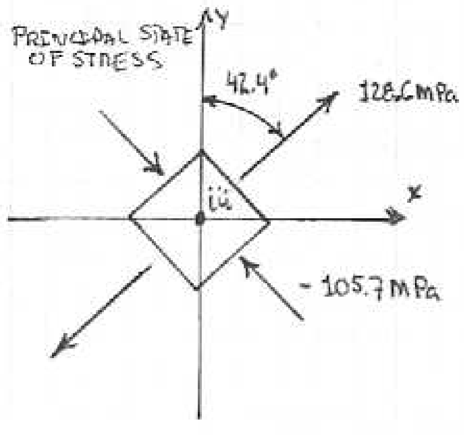
$$\sigma_2 = \sigma_{AVE} - r = 10.55\,mPa - 118.1\,mPa = -107.5\,mPa$$

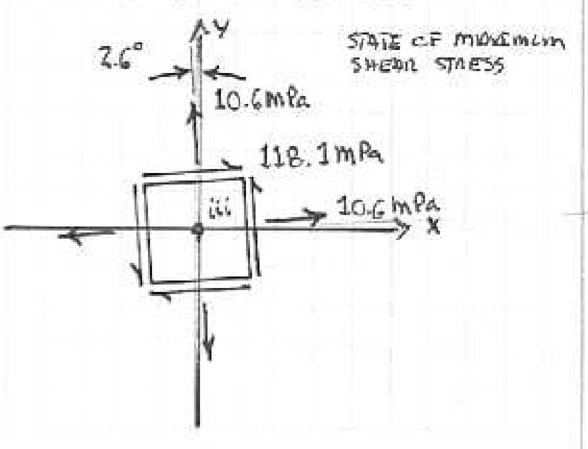
$$\tau_{max} = r = 118.1\,mPa$$

$$2\theta_p = \tan^{-1}\frac{117.6\,mPa}{21.1\,mPa - 10.55\,mPa} = 84.9^\circ \Rightarrow \theta_p = 42.4^\circ$$

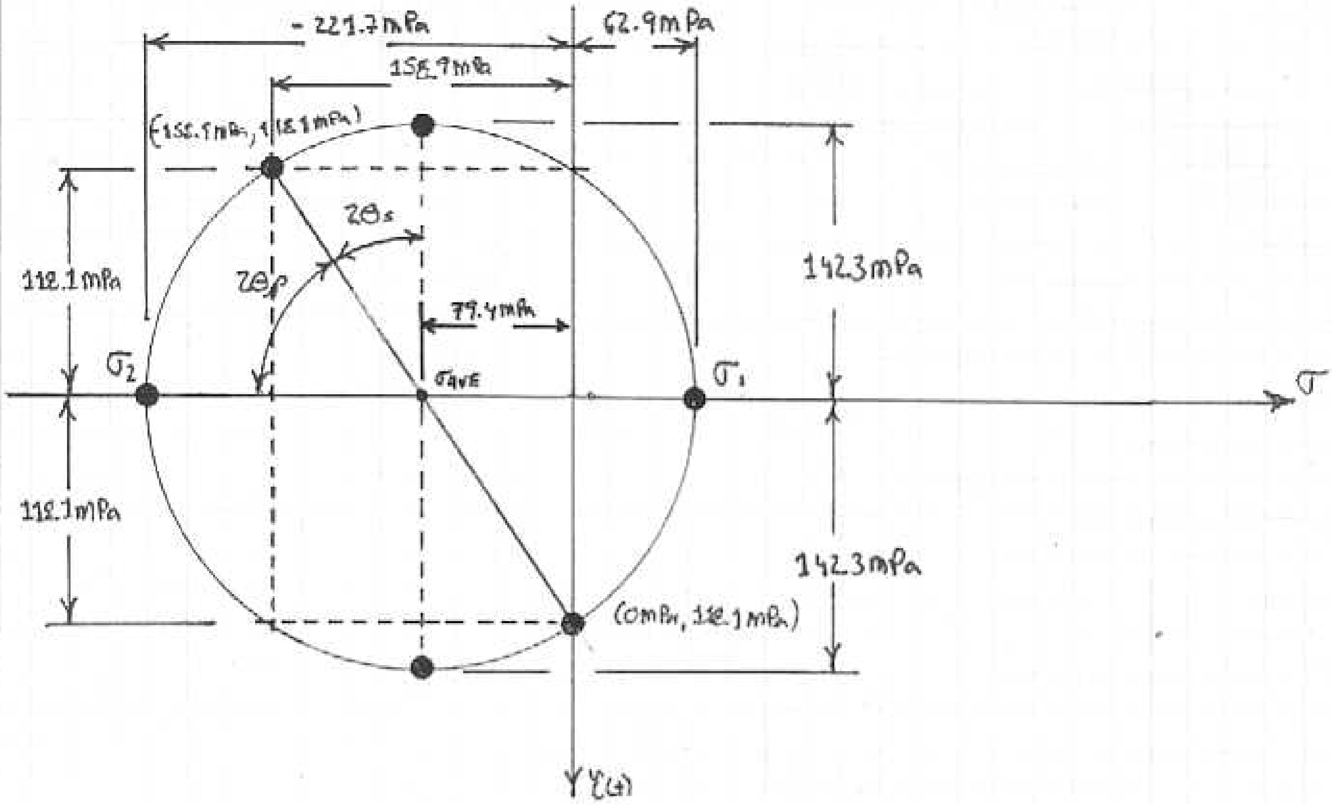
$$2\theta_s = \tan^{-1}\frac{21.1\,mPa - 10.55\,mPa}{117.6\,mPa} = 5.1^\circ \Rightarrow \theta_s = 2.6^\circ$$

NOW THE STRESS SQUARES THAT REPRESENT THE PRINCIPAL STATE OF STRESS AND THE STATE OF MAXIMUM SHEAR STRESS CAN BE DRAWN AND PROPERLY ORIENTED.





NOW POINT *ii* WILL BE EVALUATED



$$r = \sqrt{(158.9\,\text{mPa} - 79.4\,\text{mPa})^2 + (118.1\,\text{mPa})^2} = 142.3\,\text{mPa}$$

$$\sigma_1 = \sigma_{AVE} + r = -79.4\,\text{mPa} + 142.3\,\text{mPa} = 62.9\,\text{mPa}$$

$$\sigma_2 = \sigma_{AVE} - r = -79.4\,\text{mPa} - 142.3\,\text{mPa} = -221.7\,\text{MPa}$$

$$\tau_{max} = r = 142.3\,\text{mPa}$$

$$2\theta_p = \tan^{-1}\frac{118.1\,\text{mPa}}{158.9\,\text{mPa} - 79.4\,\text{mPa}} = 56.0^\circ \Rightarrow \theta_p = 28.0^\circ$$

$$2\theta_s = \tan^{-1}\frac{158.9\,\text{mPa} - 79.4\,\text{mPa}}{118.1\,\text{mPa}} = 34.0 \Rightarrow \theta_s = 17.0^\circ$$

NOW THE STRESS SQUARES THAT REPRESENT THE PRINCIPAL STATE OF STRESS AND THE STATE OF MAXIMUM SHEAR STRESS CAN BE DRAWN.