HOMEWORK SOLUTION
ESCO23: MECHANICS III
ASSIGNMENT # 10

HIBBELER, 4TH
SOLUTION BY BUCINELL

PROBLEM 7.75 THE BEAM IS FABRICATED FROM FOUR BOARDS NAILED TOGETHER AS SHOWN. DETERMINE THE SHEAR FORCE EACH NAIL ALONG THE SIDE C AND THE TOP D MUST RESIST IF THE NAILS ARE UNIFORMLY SPACED AT $s = 3$ in. THE BEAM IS SUBJECTED TO A SHEAR OF 4.5 kips.

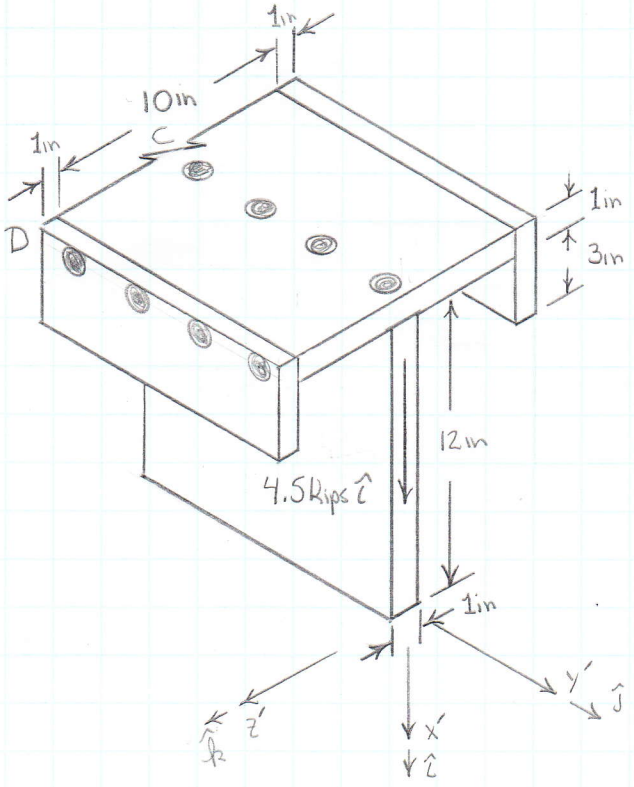
GIVEN:

CONSTRAINTS
1. T- BEAM CONSTRUCTED FROM FOUR BOARDS NAILED TOGETHER
2. EACH NAIL SPACED 3 INCHES APART
3. THE BEAM IS SUBJECTED TO A SHEAR FORCE OF 4.5 kips.

ASSUMPTIONS
1. LINEAR ELASTIC RESPONSE IN THE BEAM
2. DEFLECTIONS ARE SMALL
3. NO FRICTION BETWEEN THE BOARDS (i.e., THE NAILS CARRY ALL OF THE LOAD

FIND:
1. THE SHEAR FORCE IN EACH NAIL ALONG SIDE C
2. THE SHEAR FORCE IN EACH NAIL ALONG SIDE D

FREE BODY DIAGRAM:



SOLUTION:

THE SOLUTION STARTS WITH THE DETERMINATION OF THE CENTROID FOR THIS CROSS-SECTION. THEN THE MOMENT OF INERTIA ABOUT THE CENTROID CAN BE DETERMINED. THE $x'y'z'$ SYSTEM WILL BE USED AS THE REFRENCE FRAME FOR FINDING THE CENTROID. SINCE THE GEOMETRY IS SYMETRIC ABOUT THE x' AXIS, IT IS NOT NECESSARY TO CALCULATE THE LOCATION OF THE $\hat{j}$ DIRECTION CENTROID. FINDING THE $\hat{i}$ DIRECTION CENTROID

$$\bar{X} \cdot A = \Sigma \bar{X}_i \cdot A_i$$

$$\bar{X} = \frac{2\cdot(11\text{in})\cdot(1\text{in})\cdot(4\text{in}) + (12.5\text{in})\cdot(10\text{in})\cdot(1\text{in}) + (6\text{in})\cdot(1\text{in})\cdot(12\text{in})}{2\cdot(1\text{in})\cdot(4\text{in}) + (10\text{in})\cdot(1\text{in}) + (1\text{in})\cdot(12\text{in})}$$

$$\bar{X} = \underline{9.5\text{in}} \qquad (1)$$

NOW THE MOMENT OF INERTIA ABOUT THE CENTROID CAN BE CALCULATED WITH THE USE OF THE PARALLEL AXIS THEROM

$$I_{zz} = \left[\frac{1}{12}(1\text{in})\cdot(12\text{in})^3 + (1\text{in})(12\text{in})(3.5\text{in})^2\right]$$
$$+ 2\left[\frac{1}{12}\cdot(1\text{in})\cdot(4\text{in})^3 + (1\text{in})\cdot(4\text{in})(1.5\text{in})^2\right]$$
$$+ \left[\frac{1}{12}(10\text{in})(1\text{in})^3 + (10\text{in})\cdot(1\text{in})\cdot(3\text{in})^2\right] = \underline{410.5\text{in}^4} \qquad (2)$$

NOW THE SHEAR FLOW ALONG THE NAILS AT C CAN BE CALCOCATED.

$$q_c = \frac{V\cdot Q}{I}$$

THE ONLY VALUE THAT NEEDS TO BE CALCULATED IS Q. Q IS CALCULATED WITH RESPECT TO THE CENTROIDAL COORDINATE SYSTEM (i.e., xyz)

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PROB 7.75 PG 3 OF 3
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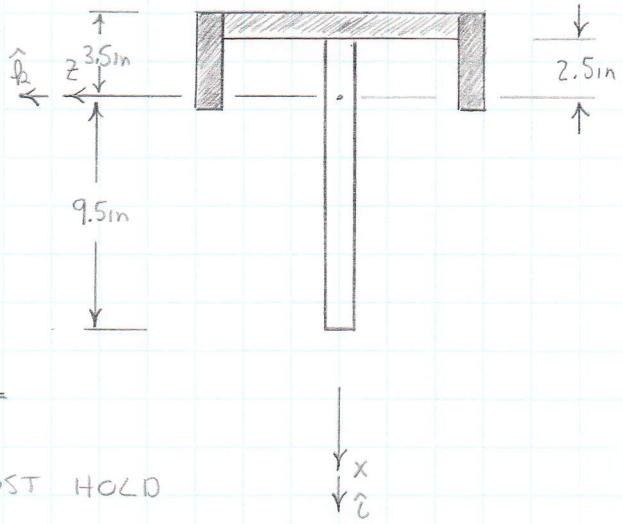
Q_c IS CALCULATED FOR THE SHADED REGION

$$Q_c = \Sigma \bar{x} \cdot A$$

$$= 2[(1.5in)\cdot(4in)\cdot(1in)] + (3in)\cdot(12in)\cdot(1in)]$$

$$= \underline{48in^3}$$

$$q_c = \frac{(4.5(10^3)lb)\cdot(48in^3)}{410.5in^4} = \underline{526.2\ \frac{lb}{in}}$$



NOW THE SHEAR FORCE THAT EACH NAIL MUST HOLD CAN BE CALCULATED

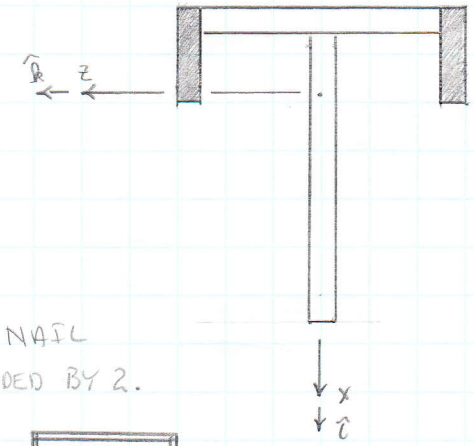
$$F_c = q_c \cdot s = 526.2\ \frac{lb}{in}\cdot 3in = 1579\ lb = \boxed{1580\ lb}$$

NOW CONSIDED THE NAILS ALONG D

$$Q_D = \Sigma \bar{x} A$$

$$= 2[(1.5in)(1in)(4in)] = \underline{12in^3}$$

$$q_D = \frac{(4.5(10^3)lb)\cdot(12in^3)}{410.5in^4} = \underline{131.6\ \frac{lb}{in}}$$



NOW CALCULATING THE SHEAR FORCE THAT EACH NAIL MUST HOLD. SINCE THERE ARE TWO SIDES, q_D IS DIVIDED BY 2.

$$F_D = \tfrac{1}{2} q_D \cdot s = \tfrac{1}{2}(131.6\ \tfrac{lb}{in})\cdot 3in = 197.3\ lb = \boxed{197\ lb}$$

Summary:

THE SOLUTION REQUIRES THAT Q IS FOUND BY CUTTING THE BEAM AT THE INTERFACE WHERE THE NAILS ARE USED TO FASTEN THE BOARDS TOGETHER. FOR THE SECOND CASE, q IS DIVIDED BY TWO BECAUSE THERE ARE TWO INTERFACES.