

**PROBLEM 5-E** | DESIGN A LINKAGE TO CARRY THE BODY SHOWN BELOW THROUGH THE TWO POSITIONS  $P_1$  AND  $P_2$  AT THE ANGLES SHOWN IN THE FIGURE. USE ANALYTICAL SYNTHESIS WITHOUT REGARD TO THE FIXED POINTS SHOWN.

**GIVEN:**

1.  $P_1$  &  $P_2$  IN ORIENTATION SHOWN BELOW

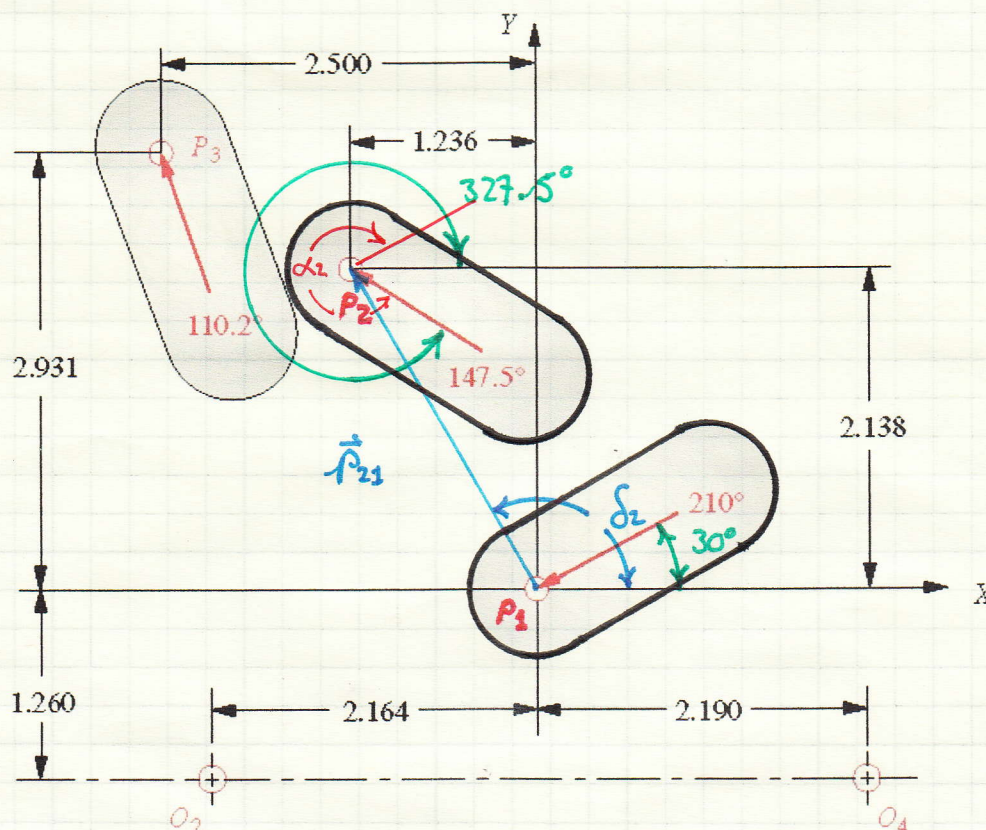
**ASSUMPTIONS:**

1. PLANAR MOTION

**FIND:**

1. SYNTHESIZE THE MECHANISM THAT WILL CREATE THE MOTION BELOW

**FIGURE:**



(a)



SOLUTION:

TWO POSITION SYNTHESIS HAS TWO APPROACHES. THE INFORMATION GIVEN IN THIS PROBLEM STATEMENT CORRESPONDS TO APPROACH A IN WHICH  $p_{12}$ ,  $\delta_2$ , AND  $\alpha_2$  ARE GIVEN. THESE PARAMETERS ARE ILLUSTRATED IN (a). THE VALUES ARE CALCULATED AS FOLLOWS.

$$p_{12} = \sqrt{(-1.236\text{in})^2 + (2.138\text{in})^2} = 2.4696\text{in} \quad (1)$$

$$\delta_2 = \tan^{-1} \frac{2.138\text{in}}{-1.236\text{in}} = 120.0^\circ \quad (2)$$

$$\alpha_2 = 327.5^\circ - 30^\circ = 297.5^\circ \quad (3)$$

THE BOOK SUGGESTS THE FOLLOWING FREE CHOICES

$$z = 1075\text{in} \quad (4) \quad s = 1.24'' \quad (7)$$

$$\phi = 204.4^\circ \quad (5) \quad \psi = 74^\circ \quad (8)$$

$$\beta_2 = -27^\circ \quad (6) \quad \gamma_2 = -40^\circ \quad (9)$$

$z$  &  $\phi$  DEFINE ONE SIDE OF THE COUPLER LINK AND  $s$  &  $\psi$  DEFINE THE OTHER.  $\beta_2$  AND  $\gamma_2$  DEFINE THE TRAVEL OF LINKS 2 AND 3.

THE SOLUTION TO THIS SYNTHESIS USING ANALYTICAL METHODS IS FOUND ON THE NEXT PAGE. THE PAGE THAT FOLLOWS THAT IS THE CALCULATION OF THE DRIVE DYAD FOR THIS MECHANISM, AND THE LAST PAGE SHOWS THE SYNTHESIZED MECHANISM.

SUMMARY:

THE USE OF AN ANALYTICAL SOLUTION RELIES ON CORRECTLY CALCULATING THE INPUT PARAMETERS. FIGURES ARE OFTEN NECESSARY TO PROPERLY IDENTIFY THE INPUT PARAMETERS. EVEN AFTER THE GIVEN PARAMETERS ARE IDENTIFIED THERE ARE STILL MORE VARIABLES THAN EQUATIONS. THE FREE CHOICES IN THIS PROBLEM ARE GIVEN IN THE PROBLEM STATEMENT. THESE CHOICES SPECIFY THE GEOMETRY OF THE COUPLER LINK AND THE ANGLE THE OTHER TWO LINKS TRAVEL THROUGH

# APPROACH B FIRST DYAD

| GIVEN: | CHOSEN:  | FIND:   |
|--------|----------|---------|
| P12    | 2.4696 z | 1.075 w |
| 82     | 120 ϕ    | 204.4 θ |
| α2     | 297.5 β2 | -27 W1x |
|        |          | W1y     |
|        |          | W1      |
|        |          | W2      |
|        |          | Z1      |
|        |          | Z2      |

|    | x-coord | y-coord |
|----|---------|---------|
| O2 | 2.443   | 3.808   |
| A1 | 0.979   | 0.444   |
| A2 | -0.389  | 1.475   |
| P1 | 0.000   | 0.000   |
| P2 | -1.235  | 2.139   |

# APPROACH B SECOND DYAD

| GIVEN: | CHOSEN:  | FIND:   |
|--------|----------|---------|
| P12    | 2.4696 s | 1.24 u  |
| 82     | 120 ψ    | 74 σ    |
| α2     | 297.5 γ2 | -40 U1x |
|        |          | U1y     |
|        |          | U1      |
|        |          | U2      |
|        |          | S1      |
|        |          | S2      |

|    | x-coord | y-coord |
|----|---------|---------|
| O4 | 2.840   | 3.246   |
| B1 | -0.342  | -1.192  |
| B2 | -2.450  | 1.892   |
| P1 | 0.000   | 0.000   |
| P2 | -1.235  | 2.139   |

$$\begin{bmatrix} -0.108993476 & 0.4539905 \\ -0.4539905 & -0.108993476 \end{bmatrix} \begin{Bmatrix} W1x \\ W1y \end{Bmatrix} = \begin{bmatrix} -1.36783 \\ 1.031336 \end{bmatrix} \begin{matrix} \text{inverse} \\ -0.5 \quad -2.082649885 \\ 2.08264989 \quad -0.5 \end{matrix}$$

$$\begin{bmatrix} \cos(\beta_2)-1 & -\sin(\beta_2) \\ \sin(\beta_2) & \cos(\beta_2)-1 \end{bmatrix} \begin{Bmatrix} W1x \\ W1y \end{Bmatrix} = \begin{Bmatrix} p_{21} \cdot \cos(\delta_2) - z \cdot [\cos(\phi + \alpha_2) - \cos(\phi)] \\ p_{21} \cdot \sin(\delta_2) - z \cdot [\sin(\phi + \alpha_2) - \sin(\phi)] \end{Bmatrix}$$

$$\begin{bmatrix} \cos(\beta_2)-1 & -\sin(\beta_2) \\ \sin(\beta_2) & \cos(\beta_2)-1 \end{bmatrix} \begin{Bmatrix} W1x \\ W1y \end{Bmatrix} = \begin{Bmatrix} p_{21} \cdot \cos(\delta_2) - z \cdot [\cos \phi \cdot \cos \alpha_2 - \sin \phi \cdot \sin \alpha_2 - \cos(\phi)] \\ p_{21} \cdot \sin(\delta_2) - z \cdot [\sin \phi \cdot \cos \alpha_2 + \cos \phi \cdot \sin \alpha_2 - \sin(\phi)] \end{Bmatrix}$$

$$\begin{bmatrix} -0.23396 & 0.642788 \\ -0.64279 & -0.23396 \end{bmatrix} \begin{Bmatrix} U1x \\ U1y \end{Bmatrix} = \begin{bmatrix} -2.10812 \\ 3.083485 \end{bmatrix} \begin{matrix} \text{inverse} \\ -0.5 \quad -1.37374 \\ 1.373739 \quad -0.5 \end{matrix}$$

$$\begin{bmatrix} \cos(\gamma_2)-1 & -\sin(\gamma_2) \\ \sin(\gamma_2) & \cos(\gamma_2)-1 \end{bmatrix} \begin{Bmatrix} U1x \\ U1y \end{Bmatrix} = \begin{Bmatrix} p_{21} \cdot \cos(\delta_2) - s \cdot [\cos(\psi + \alpha_2) - \cos(\psi)] \\ p_{21} \cdot \sin(\delta_2) - s \cdot [\sin(\psi + \alpha_2) - \sin(\psi)] \end{Bmatrix}$$

$$\begin{bmatrix} \cos(\gamma_2)-1 & -\sin(\gamma_2) \\ \sin(\gamma_2) & \cos(\gamma_2)-1 \end{bmatrix} \begin{Bmatrix} U1x \\ U1y \end{Bmatrix} = \begin{Bmatrix} p_{21} \cdot \cos(\delta_2) - s \cdot [\cos \psi \cdot \cos \alpha_2 - \sin \psi \cdot \sin \alpha_2 - \cos \psi] \\ p_{21} \cdot \sin(\delta_2) - s \cdot [\sin \psi \cdot \cos \alpha_2 + \cos \psi \cdot \sin \alpha_2 - \sin \psi] \end{Bmatrix}$$

## SYNTHESIZED LINKAGE KINEMATICS

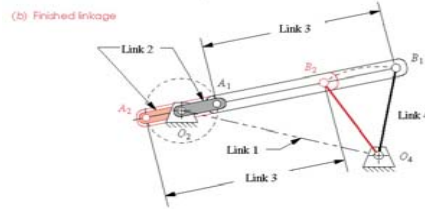
| Angular Velocity     | DL α4-1 ⇒ α2-1    | 10.000 1/s   |
|----------------------|-------------------|--------------|
| Angular Velocity     | DL α4-2 ⇒ α2-2    | 10.000 1/s*2 |
| ω3-1                 | 391.019 1/s       |              |
| ω3-2                 | 31.050 1/s        |              |
| ω4-1                 | 166.912 1/s       |              |
| ω4-2                 | 17.451 1/s        |              |
| Angular Acceleration | DL α4-1 ⇒ α2-1    | 0.000        |
| Angular Acceleration | DL α4-2 ⇒ α2-2    | 0.000        |
| α3-1                 | 1409201.880 1/s*2 |              |
| α3-2                 | 541.192 1/s*2     |              |
| α4-1                 | 545089.973 1/s*2  |              |
| α4-2                 | 320.551 1/s*2     |              |

| Kinematics    |           | x comp     | y comp      | mag        | angle  | Normal (r) | Perpendicular (θ) |
|---------------|-----------|------------|-------------|------------|--------|------------|-------------------|
|               |           | i          | j           | i          | j      | i          | j                 |
| Displacements | G = r1    | 0.397      | -0.563      | 0.689      | -54.8  | 0.5766     | -0.8170           |
| Displacements | U1 = r4-1 | -3.182     | -4.438      | 5.461      | -125.6 | -0.5827    | -0.8127           |
| Displacements | U2 = r4-2 | -5.290     | -1.354      | 5.461      | -165.6 | -0.9688    | -0.2480           |
| Displacements | W1 = r2-1 | -1.464     | -3.364      | 3.669      | -113.5 | -0.3990    | -0.9169           |
| Displacements | W2 = r2-2 | -2.832     | -2.333      | 3.669      | -140.5 | -0.7718    | -0.6359           |
| Displacements | V1 = r3-1 | -1.321     | -1.636      | 2.103      | -128.9 | -0.6281    | -0.7781           |
| Displacements | V2 = r3-2 | -2.061     | 0.416       | 2.103      | 168.6  | -0.9802    | 0.1979            |
| Displacements | Z1        | -0.979     | -0.444      | 1.075      | -155.6 | -0.9107    | -0.4131           |
| Displacements | Z2        | -0.846     | 0.663       | 1.075      | 141.9  | -0.7869    | 0.6170            |
| Displacements | S1        | 0.342      | 1.192       | 1.240      | 74.0   | 0.2756     | 0.9613            |
| Displacements | S2        | 1.215      | 0.247       | 1.240      | 11.5   | 0.9799     | 0.1994            |
| Velocity      | vA-1      | 3.3644E+01 | 4.5294E+02  | 4.5418E+02 | 85.8   | 0.0741     | 0.9973            |
| Velocity      | vA-2      | 2.3330E+01 | 1.0845E+03  | 1.0848E+03 | 88.8   | 0.0215     | 0.9998            |
| Velocity      | vB-1      | 7.4071E+02 | -5.3109E+02 | 9.1143E+02 | -35.6  | 0.8127     | -0.5827           |
| Velocity      | vB-2      | 2.3633E+01 | -9.2314E+01 | 9.5291E+01 | -75.6  | 0.2480     | -0.9688           |
| Velocity      | vP-1      | 2.0729E+02 | -3.9744E+02 | 4.4825E+02 | -62.5  | 0.4624     | -0.8866           |
| Velocity      | vP-2      | 2.7345E+00 | -5.4585E+01 | 5.4654E+01 | -87.1  | 0.0500     | -0.9987           |
| Acceleration  | aA-1      | 1.4640E+02 | 3.3644E+02  | 3.6691E+02 | 66.5   | 0.3990     | 0.9169            |
| Acceleration  | aA-2      | 2.6316E+02 | 2.3330E+02  | 3.6691E+02 | 39.5   | 0.7718     | 0.6359            |
| Acceleration  | aB-1      | 2.5076E+06 | -1.6108E+06 | 2.9804E+06 | -32.7  | 0.8414     | -0.5405           |
| Acceleration  | aB-2      | 2.0451E+03 | -1.2833E+03 | 2.4144E+03 | -32.1  | 0.8470     | -0.5315           |
| Acceleration  | aB-P      | 7.7564E+05 | -1.3114E+06 | 1.5236E+06 | -59.4  | 0.5091     | -0.8607           |
| Acceleration  | aB-P      | 7.3979E+02 | -8.6402E+02 | 1.1375E+03 | -49.4  | 0.6504     | -0.7596           |

# NON-QUICK-RETURN (From Two-Position Approach B Results)

|             | X-pos | Y-pos | mag  | angle | i       | j      |
|-------------|-------|-------|------|-------|---------|--------|
| 2P-O2 => O4 | 2.44  | 3.81  | 4.52 | 57.3  | 0.5399  | 0.8417 |
| 2P-A1       | 0.98  | 0.44  | 1.08 | 24.4  | 0.9107  | 0.4131 |
| 2P-A2       | -0.39 | 1.48  | 1.53 | 104.8 | -0.2548 | 0.9670 |

| Factors |      |                                  |
|---------|------|----------------------------------|
| P       | 0.5  | % dist up Link 4                 |
| K       | 2.5  | Length of Link 3+Link 2 wrt B1B2 |
| Link 1  | 2.47 |                                  |
| Link 2  | 0.43 |                                  |
| Link 3  | 1.71 |                                  |
| Link 4  | 1.83 | Grashof                          |



|                     |                |
|---------------------|----------------|
| $\dot{\theta}_2 =$  | 10.0000 1/s    |
| $\ddot{\theta}_2 =$ | 0.0000 1/s^2   |
| $\omega_{3-1}$      | -2.5000 1/s    |
| $\omega_{3-2}$      | 2.5000 1/s     |
| $\omega_{4-1}$      | 0.0000 1/s     |
| $\omega_{4-2}$      | 0.0000 1/s     |
| $\alpha_{3-1}$      | -7.5025 1/s^2  |
| $\alpha_{3-2}$      | -4.5015 1/s^2  |
| $\alpha_{4-1}$      | -30.0098 1/s^2 |
| $\alpha_{4-2}$      | 18.0059 1/s^2  |

|          | x comp | y comp | mag  | angle  | Normal ( r ) |         | Perpendicular ( $\theta$ ) |         |
|----------|--------|--------|------|--------|--------------|---------|----------------------------|---------|
|          |        |        |      |        | i            | j       | i                          | j       |
| rO4      | 2.44   | 3.81   | 4.52 | 57.3   | 0.5399       | 0.8417  | -0.8417                    | 0.5399  |
| rP2O2-A1 | -1.46  | -3.36  | 3.67 | -113.5 | -0.3990      | -0.9169 | 0.9169                     | -0.3990 |
| rP2O2-A2 | -2.83  | -2.33  | 3.67 | -140.5 | -0.7718      | -0.6359 | 0.6359                     | -0.7718 |
| rB1      | 1.71   | 2.13   | 2.73 | 51.2   | 0.6269       | 0.7791  | -0.7791                    | 0.6269  |
| rO4B1    | -0.73  | -1.68  | 1.83 | -113.5 | -0.3990      | -0.9169 | 0.9169                     | -0.3990 |
| rB2      | 1.03   | 2.64   | 2.83 | 68.8   | 0.3623       | 0.9320  | -0.9320                    | 0.3623  |
| rO4B2    | -1.42  | -1.17  | 1.83 | -140.5 | -0.7718      | -0.6359 | 0.6359                     | -0.7718 |
| rB1B2    | -0.68  | 0.52   | 0.86 | 143.0  | -0.7985      | 0.6020  | -0.6020                    | -0.7985 |
| rO2      | 0.00   | 3.42   | 3.42 | 90.0   | 0.0004       | 1.0000  | -1.0000                    | 0.0004  |
| rB1O2    | -1.71  | 1.29   | 2.14 | 143.0  | -0.7985      | 0.6020  | -0.6020                    | -0.7985 |
| rA1      | 0.34   | 3.16   | 3.18 | 83.8   | 0.1080       | 0.9941  | -0.9941                    | 0.1080  |
| rO2A1    | 0.34   | -0.26  | 0.43 | -37.0  | 0.7985       | -0.6020 | 0.6020                     | 0.7985  |
| rA2      | -0.34  | 3.67   | 3.69 | 95.3   | -0.0924      | 0.9957  | -0.9957                    | -0.0924 |
| rO2A2    | -0.34  | 0.26   | 0.43 | 143.0  | -0.7985      | 0.6020  | -0.6020                    | -0.7985 |
| rB1A1    | -1.37  | 1.03   | 1.71 | 143.0  | -0.7985      | 0.6020  | -0.6020                    | -0.7985 |
| rB2A2    | -1.37  | 1.03   | 1.71 | 143.0  | -0.7985      | 0.6020  | -0.6020                    | -0.7985 |
| rO4O2    | -2.44  | -0.39  | 2.47 | -170.9 | -0.9873      | -0.1589 | 0.1589                     | -0.9873 |

| Kinematics |        |        |       |           | Normal ( r ) |           | Perpendicular ( $\theta$ ) |           |
|------------|--------|--------|-------|-----------|--------------|-----------|----------------------------|-----------|
|            | x comp | y comp | mag   | angle     | i            | j         | i                          | j         |
| r1         | 2.44   | 0.39   | 2.47  | 9.1       | 0.9873       | 0.1589    | -0.1589                    | 0.9873    |
| r4-1       | -0.73  | -1.68  | 1.83  | -113.5    | -0.3990      | -0.9169   | 0.9169                     | -0.3990   |
| r4-2       | -1.42  | -1.17  | 1.83  | -140.5    | -0.7718      | -0.6359   | 0.6359                     | -0.7718   |
| r2-1       | 0.34   | -0.26  | 0.43  | -37.0     | 0.7985       | -0.6020   | 0.6020                     | 0.7985    |
| r2-2       | -0.34  | 0.26   | 0.43  | 143.0     | -0.7985      | 0.6020    | -0.6020                    | -0.7985   |
| r3-1       | 1.37   | -1.03  | 1.71  | -37.0     | 0.7985       | -0.6020   | 0.6020                     | 0.7985    |
| r3-2       | 1.37   | -1.03  | 1.71  | -37.0     | 0.7985       | -0.6020   | 0.6020                     | 0.7985    |
| vA-1       | 2.58   | 3.42   | 4.28  | 53.0      | 0.6020       | 0.7985    | -0.7985                    | 0.6020    |
| vA-2       | -2.58  | -3.42  | 4.28  | -127.0    | -0.6020      | -0.7985   | 0.7985                     | -0.6020   |
| vB-1       | 0.00   | 0.00   | 0.00  | undefined | undefined    | undefined | undefined                  | undefined |
| vB-2       | 0.00   | 0.00   | 0.00  | undefined | undefined    | undefined | undefined                  | undefined |
| aA-1       | -34.20 | 25.78  | 42.83 | 143.0     | -0.7985      | 0.6020    | -0.6020                    | -0.7985   |
| aA-2       | 34.20  | -25.78 | 42.83 | -37.0     | 0.7985       | -0.6020   | 0.6020                     | 0.7985    |
| aB-1       | -50.48 | 21.97  | 55.05 | 156.5     | -0.9169      | 0.3990    | -0.3990                    | -0.9169   |
| aB-2       | 21.00  | -25.49 | 33.03 | -50.5     | 0.6359       | -0.7718   | 0.7718                     | 0.6359    |



