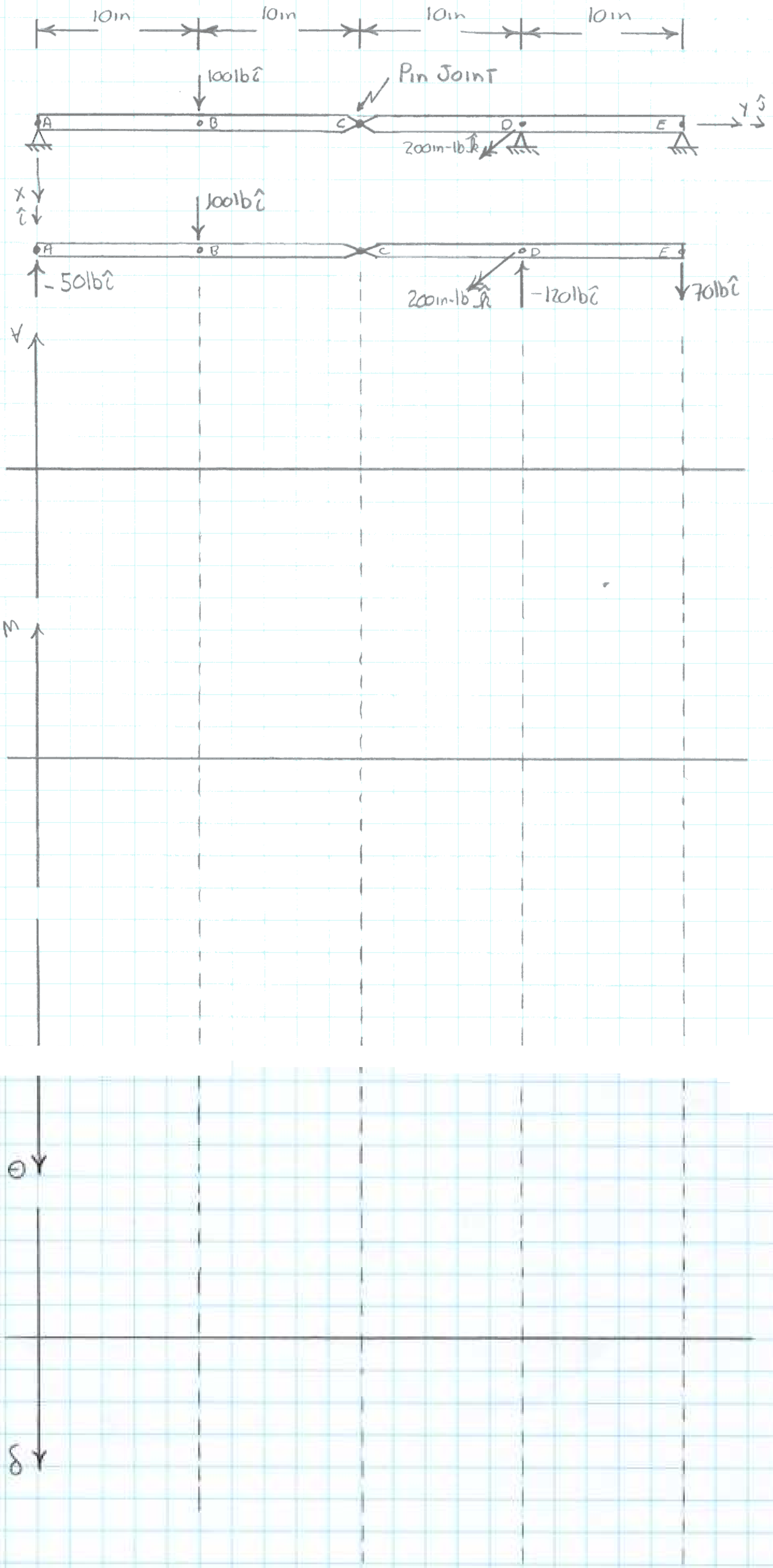
NAME: SOLUTION

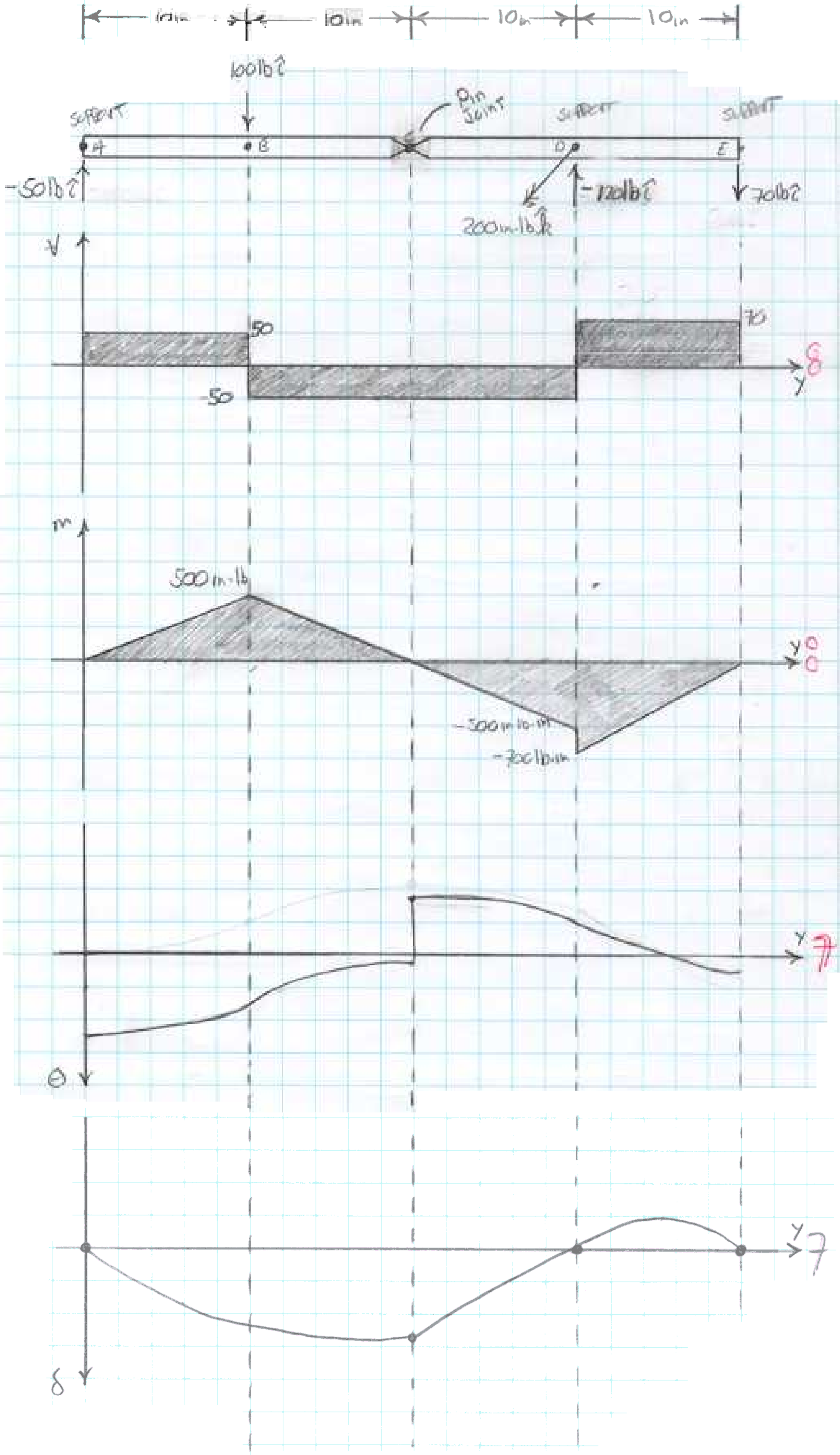
Vert.

PROBLEM 1: The compound beam shown on the next page is supported by pin joints at A, D, and E. A 100 lb load in the horizontal direction is applied at B and a 200 in-lb couple is applied at D. The two halves of the beam (ABC and CDE) are joined at C by a pin joint. EI for the beam is given as 5.667×10^6 lb-in^2.

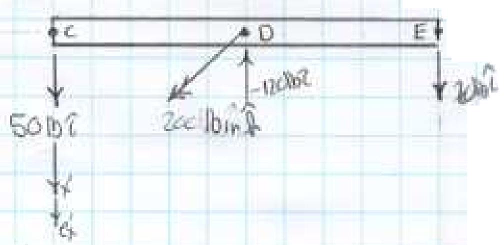
1a. Using the diagrams supplied on the next page, draw the shear, moment, curvature, and deflection diagrams.



OPS FORM 33051


10in
10in
10in
10in
100lb î
Pin Joint
SUPPORT
SUPPORT
SUPPORT
A
B
D
E
-50lb î
-120lb î
70lb î
200in-lb k̂
V
50
-50
70
m
500in-lb
-500in-lb
-700lb-in
θ
δ
y

1b. Using section CDE of the beam, determine the deflection of the beam at C.



$q(y') = 50\,lb\langle y'-0\rangle_{-1} - 120\,lb\langle y'-10\rangle_{-1} + 200\,lb\cdot in\langle y-10\rangle_{-2} + 70\,lb\langle y-20in\rangle_{-1}$

$V(y') = -50\langle y'-0\rangle^{0} + 120\,lb\langle y'-10in\rangle^{0} + 200\,lb\cdot in\langle y-10in\rangle_{-1} - 70\,lb\langle y-20in\rangle^{0}$

$M(y') = -50\langle y'-0\rangle^{1} + 120\,lb\langle y'-10in\rangle^{1} - 200\,lb\cdot in\langle y-10in\rangle^{0} - 70\,lb\langle y-20in\rangle^{1}$

$\theta(y') = \frac{1}{EI}\left[+25\langle y'-0\rangle^{2} - 60\,lb\langle y'-10in\rangle^{2} + 200\,lb\cdot in\langle y-10in\rangle^{1} + 35\,lb\langle y-20in\rangle^{2} + C_1\right]$

$\delta(y') = \frac{1}{EI}\left[+\frac{25\,lb}{3}\langle y'-0\rangle^{3} - 20\,lb\langle y'-10in\rangle^{3} + 100\,lb\cdot in\langle y-10in\rangle^{2} + \frac{35\,lb}{3}\langle y-20in\rangle^{2} + C_1\cdot y' + C_2\right]$

$\frac{lb}{in^2}\,in^4$

BC

$\delta(10in) = 0$
$\delta(20in) = 0$

$\delta(10in) = 0 = +\frac{25\,lb}{3}(10in)^3 - C_1\cdot 10in + C_2 = +8{,}333\,lb\cdot in^3 + 10in\cdot C_1 + C_2$ $\qquad \frac{lb}{in^2}\,in^4$

$\Rightarrow -8{,}333\,lb\cdot in^3 = 10in\,C_1 + C_2 \quad (1) \quad \Rightarrow \quad C_2 = -8{,}333\,lb\cdot in^3 - 10in\cdot C_1$

$\delta(20in) = 0 = +\frac{25\,lb}{3}(20in)^3 - 20\,lb\,(10in)^3 + 100\,lb\cdot in\,(10in)^2 + C_1\cdot 20in + C_2$

$-56{,}667\,lb\cdot in^3 = 20in\cdot C_1 + C_2$

$-56667\,lb\cdot in^3 = 20in\,C_1 - 8333\,lb\,in - 10\,C_1 \quad \Rightarrow \quad C_1 = -4833\,lb\,in^2$

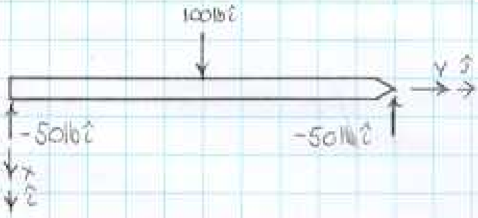
$C_2 = -8333\,lb\cdot in^3 - 10in\cdot 4833\,lb\cdot in^2 = -56670\,lb\cdot in^3$

$\delta = \frac{1}{EI}\left[-\frac{25\,lb}{3}\langle y'-0\rangle^{3} + 20\,lb\langle y'-10in\rangle^{3} - 100\,lb\,in\langle y-10in\rangle^{2} - \frac{35\,lb}{3}\langle y-20in\rangle^{2} - 4833\,lb\cdot in^2\cdot y' - 56670\,lb\,in^3\right]$

$\delta(0) = \frac{1}{EI}\left[56670\,lb\cdot in^3\right] = \boxed{.01\,in}$ $\qquad EI = 5.667(10^6)\,lb\cdot in^2$

$\theta(y') = \frac{1}{EI}\left[25\,lb\langle y'-0\rangle^{2} - 60\,lb\langle y'-10in\rangle^{2} + 200\,lb\cdot in\langle y-10in\rangle^{1} + 35\,lb\langle y-20in\rangle^{2} - 4833\,lb\cdot in^2\right]$

$\theta(0) = \frac{1}{5.667(10^6)\,lb\,in^2}\left(-4833\,lb\,in^2\right) = -0.852(10^{-3})\,rad$



$$q(y) = -50\text{lb}\langle y-0\rangle_{-1} + 100\text{lb}\langle y-10\text{in}\rangle_{-1} - 50\text{lb}\langle y-20\text{lb}\rangle_{-1}$$

$$V(y) = 50\text{lb}\langle y-0\rangle^0 - 100\text{lb}\langle y-10\text{in}\rangle^0 + 50\text{lb}\langle y-20\text{in}\rangle^0$$

$$M(y) = 50\text{lb}\langle y-0\rangle^1 - 100\text{lb}\langle y-10\text{in}\rangle^1 + 50\text{lb}\langle y-20\text{in}\rangle^1$$

$$\Theta(y) = \frac{1}{EI}\left[-25\text{lb}\langle y-0\rangle^2 + 50\text{lb}\langle y-10\text{in}\rangle^2 - 25\text{lb}\langle y-20\text{in}\rangle^2 + C_1\right]$$

$$\delta(y) = \frac{1}{EI}\left[\frac{-25\text{lb}}{3}\langle y-0\rangle^3 + \frac{50\text{lb}}{3}\langle y-10\text{in}\rangle^3 - \frac{25\text{lb}}{3}\langle y-20\text{in}\rangle^3 + C_1\cdot y + C_2\right]$$

$\underline{y=0}$

$$\delta(0) = 0 = \frac{1}{EI}\left[C_2\right] \Rightarrow C_2 = 0$$

$\underline{y=20\text{in}}$

$$\delta(20\text{in}) = .01\text{in} = \frac{1}{5.667(10^6)\text{lb}\cdot\text{in}^2}\left[-\frac{25\text{lb}}{3}(20\text{in})^3 + \frac{50\text{lb}}{3}(10\text{in})^3 - \frac{25\text{lb}}{3}(0)^3 + C_1\cdot 20\text{in}\right]$$

$$56.670(10^3)\,\text{lb}\cdot\text{in}^3 = -\frac{25\text{lb}}{3}(20\text{in})^3 + \frac{50\text{lb}}{3}(10\text{in})^3 + C_1\cdot 20\text{in}$$

$$C_1 = \left[56.67(10^3)\text{lb}\cdot\text{in}^3 + \frac{25\text{lb}}{3}(20\text{in})^3 - \frac{50\text{lb}}{3}(10\text{in})^3\right]\frac{1}{20\text{in}} = 5.333(10^3)\,\text{lb}\cdot\text{in}^2$$

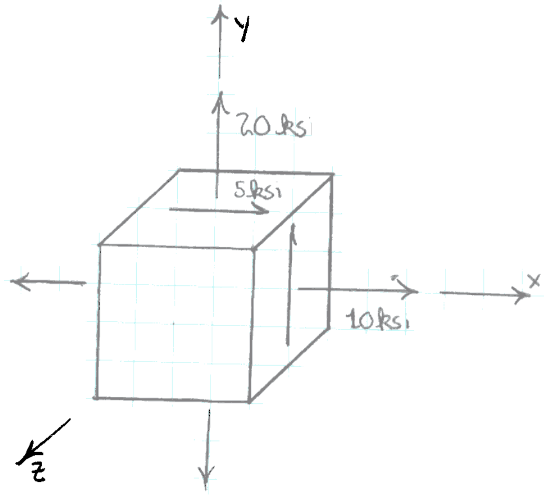
$$\Theta(y) = \frac{1}{5.667(10^6)\text{lb}\cdot\text{in}^2}\left[-25\text{lb}\langle y-0\rangle^2 + 50\text{lb}\langle y-10\text{in}\rangle^2 - 25\text{lb}\langle y-20\text{in}\rangle^2 + 5.333(10^3)\,\text{lb}\cdot\text{in}^2\right]$$

$$\delta(y) = \frac{1}{5.667(10^6)\text{lb}\cdot\text{in}^2}\left[-\frac{25\text{lb}}{3}\langle y-0\rangle^3 + \frac{50\text{lb}}{3}\langle y-10\text{in}\rangle^3 - \frac{25\text{lb}}{3}\langle y-20\text{in}\rangle^3 + 5.333(10^3)\,\text{lb}\cdot\text{in}^2\cdot y\right]$$

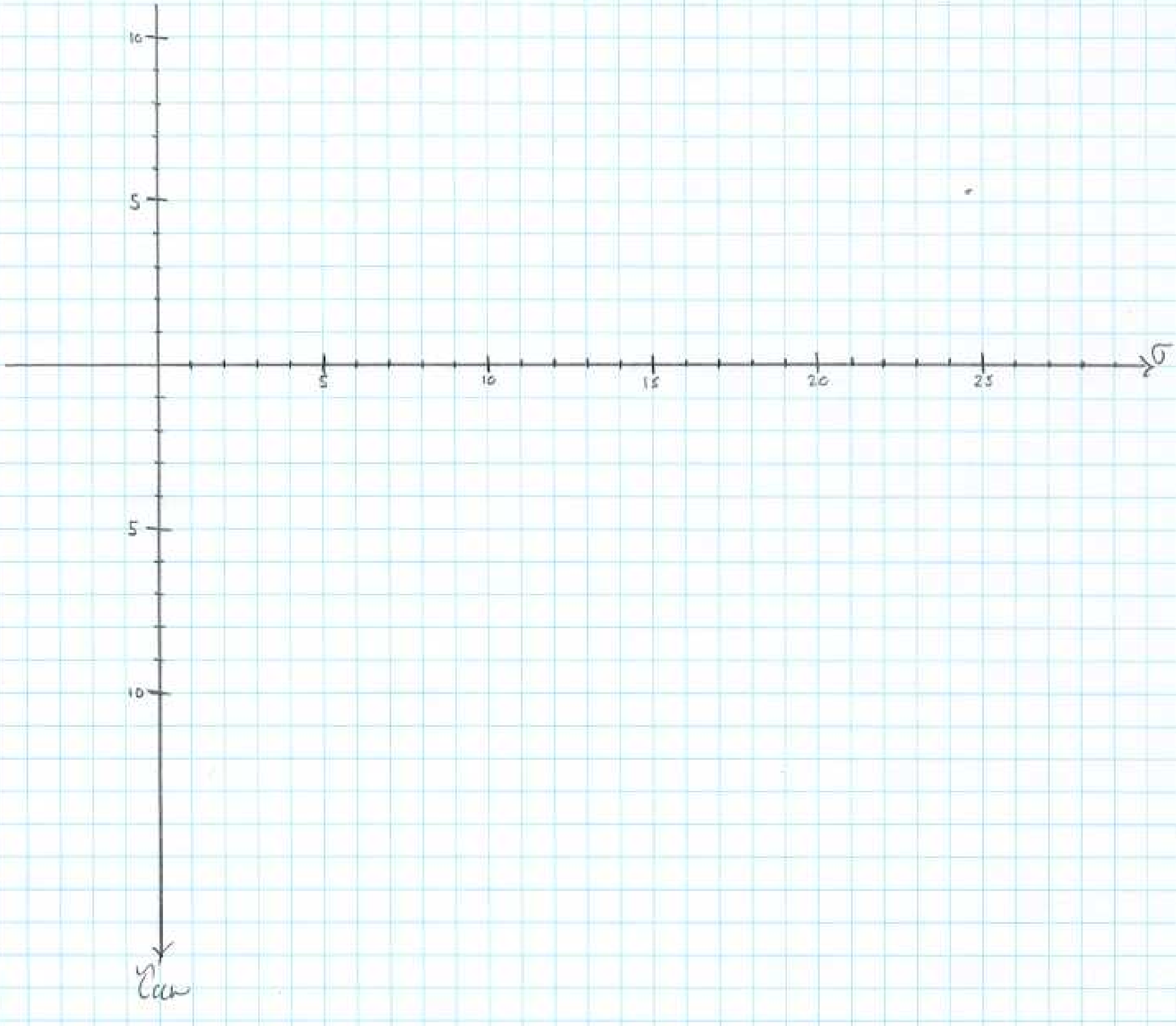
$$\delta(20\text{in}) = 0.01\text{in}$$

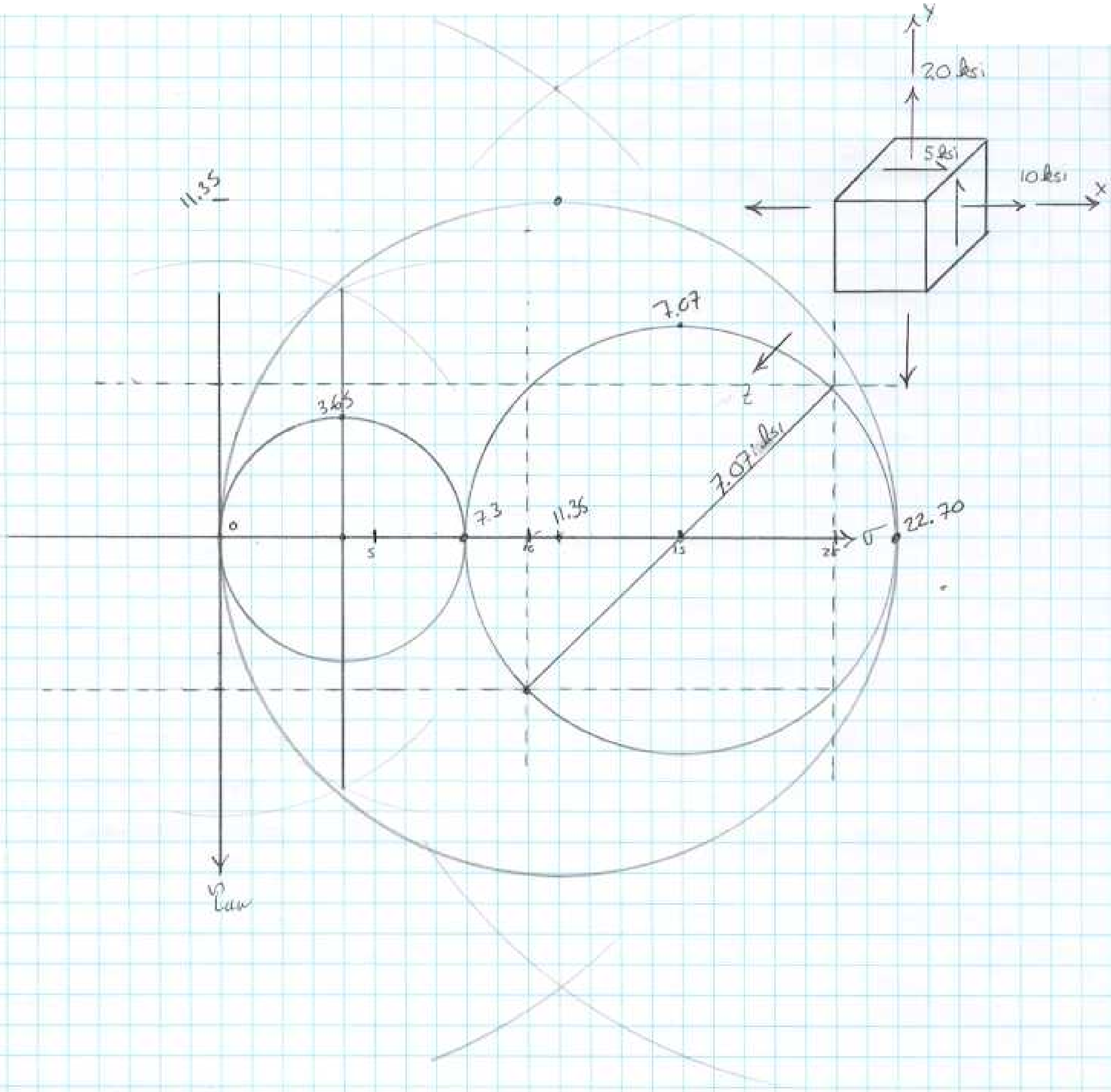
$$\Theta(20\text{in}) = 0.05876(10^{-3})\text{ rad}$$

PROBLEM 3: For the state of stress shown, draw Mohr's circle and identify all of the principal stresses and maximum shearing stresses (be sure to identify any normal stresses that accompany the maximum shearing stresses). If the material that is under consideration is made from a material that is known to be ductile and has an ultimate tensile stress of 22 ksi, will the material survive when subjected to the stress state given? Explain?




C
D
E


10
5
5
10
5
10
15
20
25
σ
τcw



y
20 ksi
5 ksi
10 ksi
x
11.35
7.07
3.65
7.07 ksi
7.3
11.35
0
5
10
15
20
22.70
σ
τ