PROBLEM 3-20 | THE STATE OF STRESS AT A POINT IS

$$\sigma_x = -6\text{ ksi} \qquad \tau_{xy} = 9\text{ ksi}$$
$$\sigma_y = 18\text{ ksi} \qquad \tau_{yz} = 6\text{ ksi}$$
$$\sigma_z = -12\text{ ksi} \qquad \tau_{zx} = -15\text{ ksi}$$

DETERMINE THE PRINCIPAL STRESSES, DRAW A COMPLETE MOHR'S CIRCLE DIAGRAM, LABELING ALL POINTS OF INTEREST, AND REPORT THE MAXIMUM SHEAR STRESS FOR THIS CASE.

GIVEN:

1. THE STATE OF STRESS $[\sigma] = \begin{bmatrix} -6 & 9 & -15 \\ 9 & 18 & 6 \\ -15 & 6 & -12 \end{bmatrix}$

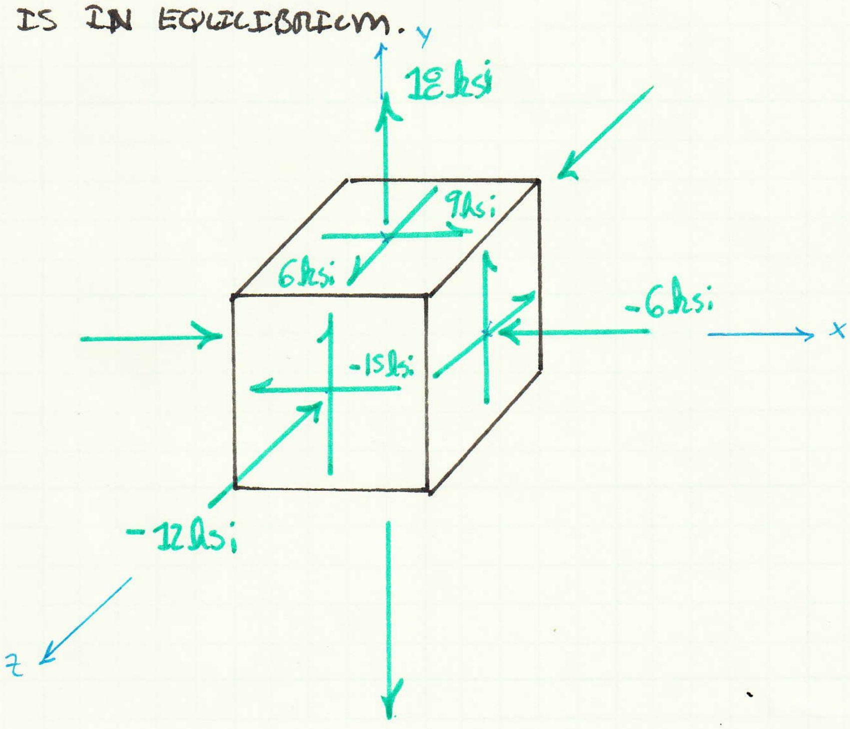
FIND:

1. DETERMINE THE PRINCIPAL STRESSES
2. DRAW A COMPLETE MOHR'S CIRCLE
3. DETERMINE THE MAXIMUM SHEAR STRESS
4. DETERMINE THE STRESS INVARIANTS
5. DETERMINE THE PRINCIPAL STRESS TRANSFORMATION MATRIX

ASSUMPTIONS:

1. THE MATERIAL IS IN EQUILIBRIUM.

FIGURE:



```
>> S

S =

    -6     9   -15
     9    18     6
   -15     6   -12
```

← ORIGIONAL STRESS TENSOR

```
>> [DIS,PS]=eig(S)

DIS =

    0.6341   -0.7143   -0.2961
   -0.2269    0.1942   -0.9544
    0.7392    0.6723   -0.0389
```

← THESE ARE THE PRINCIPAL STRESS EIGEN VECTORS. THIS MATRIX IS THE TRANSPOSE OF THE TRANSFORMATION MATRIX THAT TAKES THE ORIGIONAL STATE OF STRESS TO THE PRINCIPAL STATE OF STRESS

```
PS =

  -26.7075         0         0
         0    5.6706         0
         0         0   21.0369
```

$-26.7075 = \sigma_3$, $5.6706 = \sigma_2$, $21.0369 = \sigma_1$

← THE PRINCIPAL STRESS MATRIX

```
>> T=DIS'

T =

    0.6341   -0.2269    0.7392
   -0.7143    0.1942    0.6723
   -0.2961   -0.9544   -0.0389
```

← THE PRINCIPAL STRESS TRANSFORMATION MATRIX FOR THE ORIGIONAL STATE OF STRESS

```
>> T*S*T'

ans =

  -26.7075   -0.0000   -0.0000
   -0.0000    5.6706   -0.0000
   -0.0000   -0.0000   21.0369
```

DEMONSTRATING THAT THE TRANSFORMATION MATRIX DOES TRANSFORM THE ORIGIONAL STATE OF STRESS TO THE PRINCIPAL STATE OF STRESS.

```
>> poly(S)

ans =

   1.0e+03 *

    0.0010    0.0000   -0.5940    3.1860

>>
```

$$\sigma^3 + \underbrace{0}_{I_1} \cdot \sigma^2 - \underbrace{594}_{I_2}\sigma + \underbrace{3186}_{I_3} = 0$$

THE THREE STRESS INVARIANTS

$I_1 = 0$ ksi

$I_2 = 594$ ksi^2

$I_3 = 3186$ ksi^3

HOMEWORK SOLUTION
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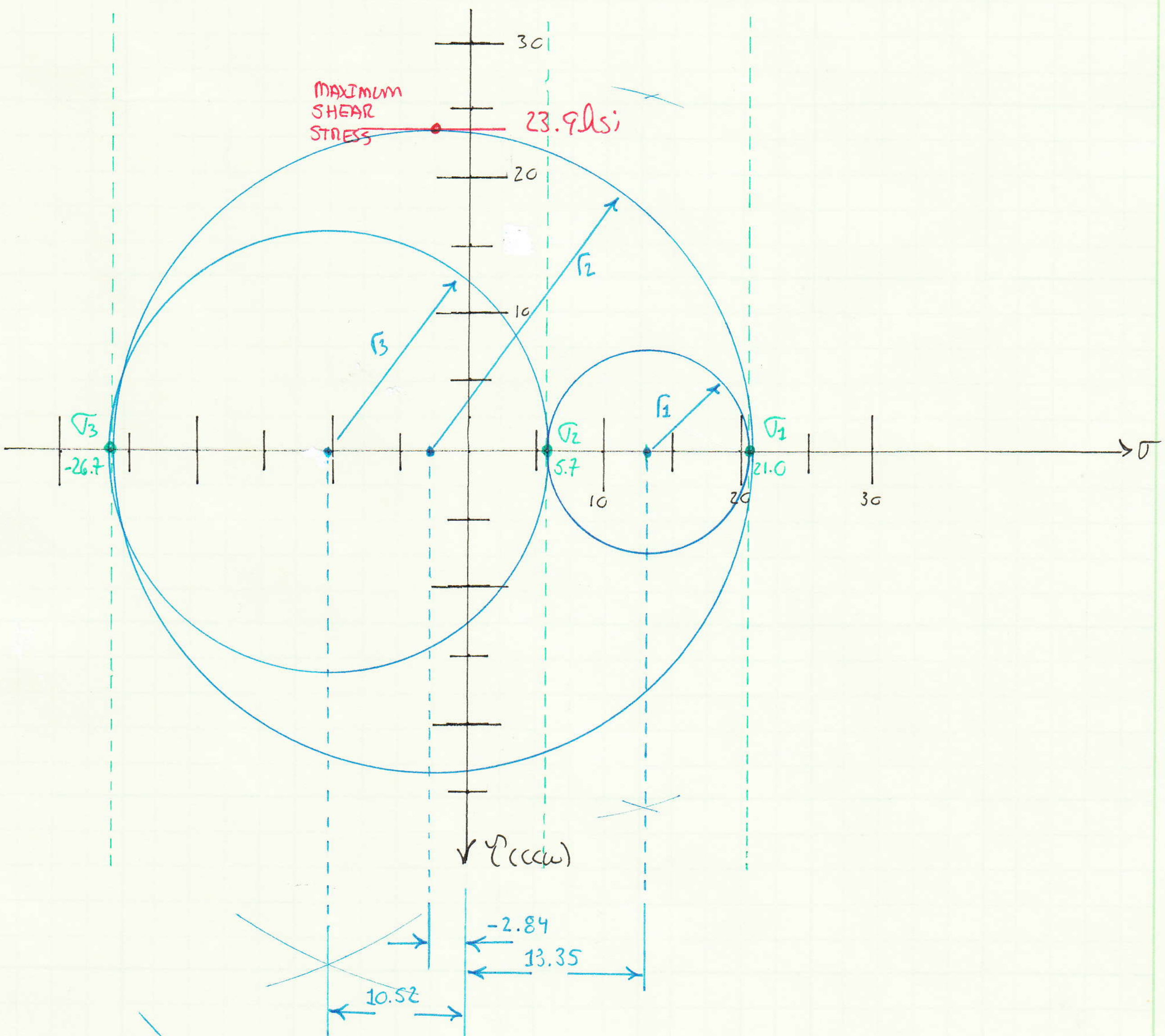
MOHR'S CIRCLE CAN NOT BE DRAWN USING THE ORIGINAL STATE OF STRESS BECAUSE ALL FACES HAVE SHEAR STRESS ON THEM. THEREFORE, THE PRINCIPAL STRESS MATRIX WILL BE USED TO DRAW MOHR'S CIRCLE.

$$r_1 = \tfrac{1}{2}(21.034\,\text{ksi} - 5.671\,\text{ksi}) = 7.68\,\text{ksi}$$

$$\tau_{max} = r_2 = \tfrac{1}{2}(21.034\,\text{ksi} - (-26.708\,\text{ksi})) = \boxed{23.87\,\text{ksi}}$$

$$r_3 = \tfrac{1}{2}(5.671\,\text{ksi} - (-26.708\,\text{ksi})) = 16.19\,\text{ksi}$$



SUMMARY:

MATLAB MAKES THE DETERMINATION OF THE EIGEN VALUES (PRINCIPAL STRESSES) AND THE CORRESPONDING EIGEN VECTORS (TRANSFORMATION MATRIX) VERY EASY. THERE IS NO CURRENT BUILT IN FUNCTION IN EXCEL THAT WILL PERFORM THESE OPERATIONS.