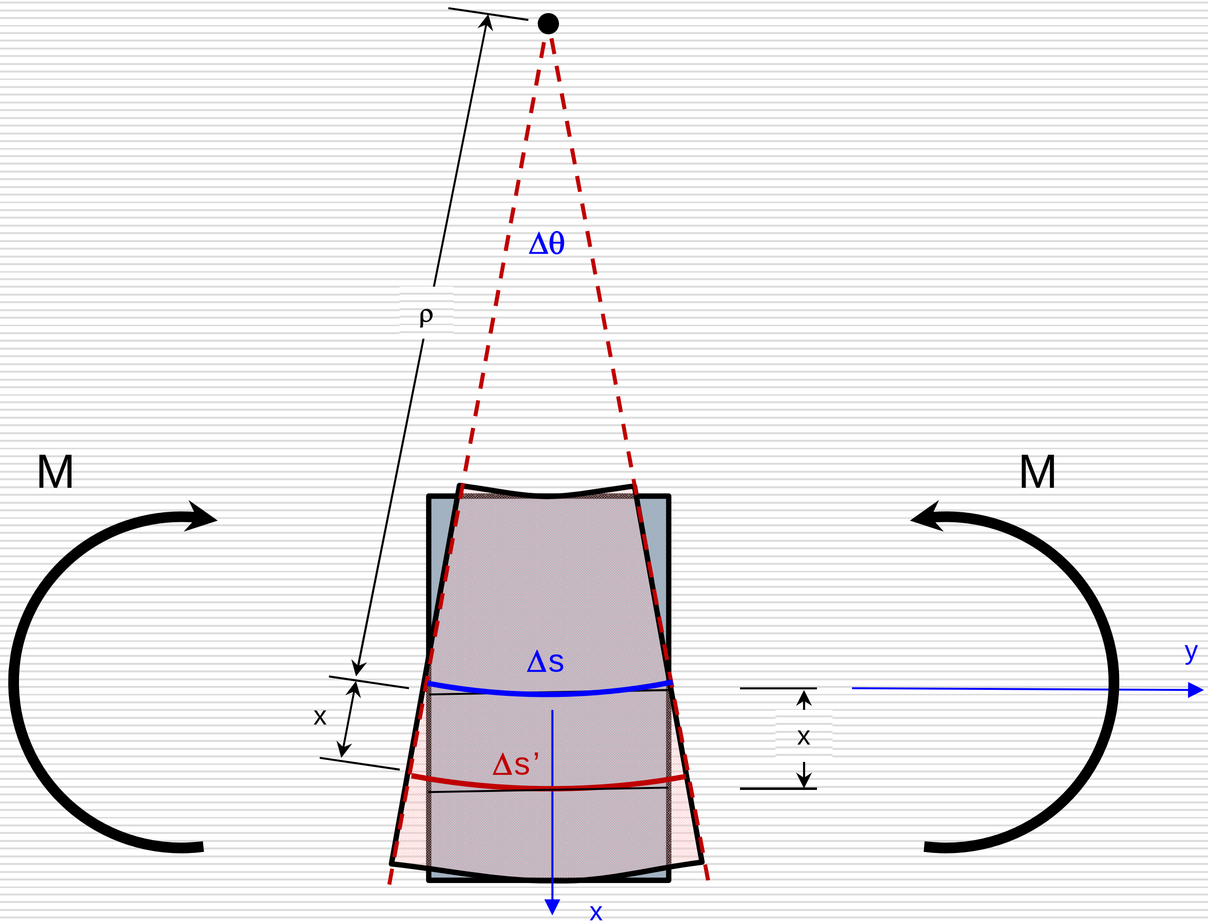
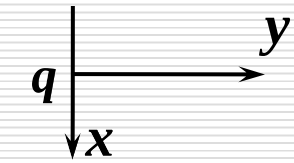
# Deflection of Beams Through Direct Integration

- Moment-Curvature Relationship
- Moment-Deflection Relationship
- q-V-M-$\theta$-u

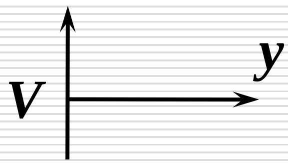
# Defining the Moment Deflection Relationship



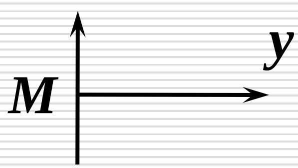
# Linear-Elastic Response



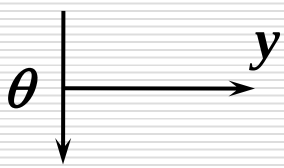

$$-q = {dV}\big/{dy} = -\frac{d^2}{dy^2}\left(E\cdot I\cdot\frac{d^2u}{dy^2}\right) = -\left(E\cdot I\cdot u''\right)'' \quad \Rightarrow \quad \underset{E\cdot I}{Constant} \Rightarrow q(y) = -E\cdot I\cdot {d^4u}\big/{dy^4}$$



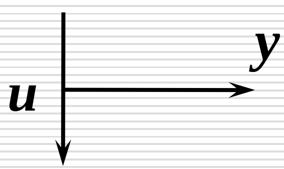

$$V = {dM}\big/{dy} = -\frac{d}{dy}\left(E\cdot I\cdot\frac{d^2u}{dy^2}\right) = -\left(E\cdot I\cdot u''\right)' \quad \Rightarrow \quad \underset{E\cdot I}{Constant} \Rightarrow V(y) = -E\cdot I\cdot {d^3u}\big/{dy^3}$$




$$M = -E\cdot I\cdot {d^2u}\big/{dy^2} = -E\cdot I\cdot u'' \quad \Rightarrow \quad \underset{E\cdot I}{Constant} \Rightarrow M(y) = -E\cdot I\cdot {d^2u}\big/{dy^2}$$

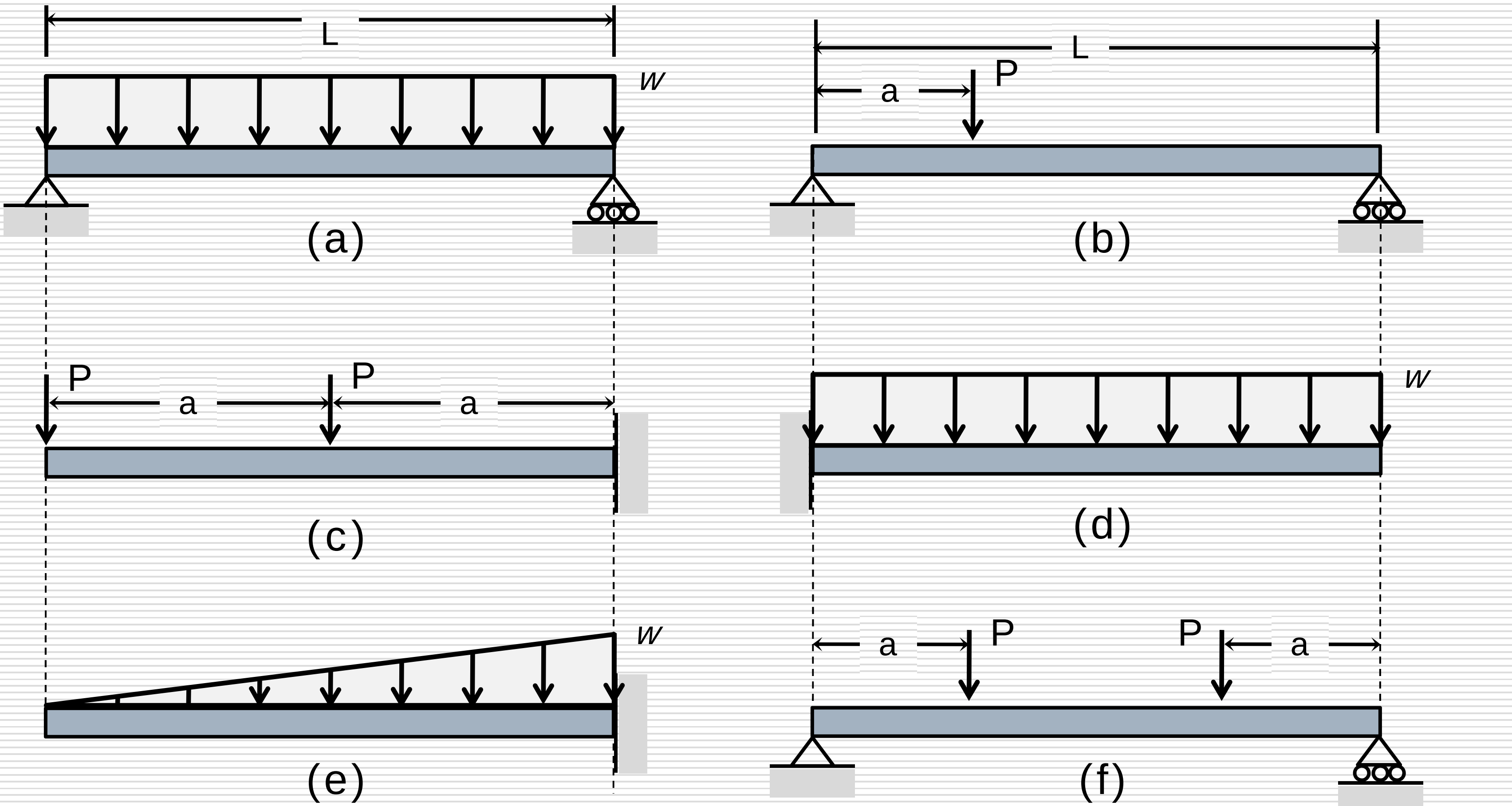



$$\theta \equiv {du}\big/{dy} = u' \equiv Slope\ \ of\ \ the\ \ Elastic\ \ Curve$$

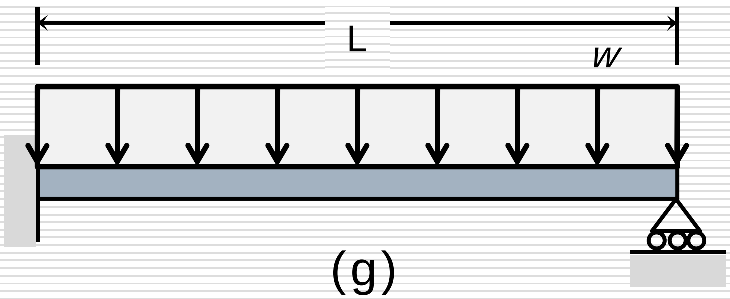



$$u \equiv Deflection\ \ of\ \ the\ \ Elastic\ \ Curve$$

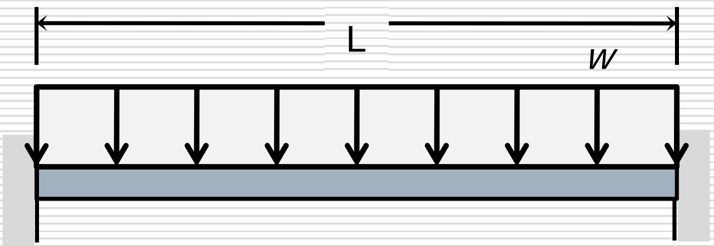
# Beams



# Beams – Statically Indeterminate




(g)




(h)

# Example

$$0 < y < 2a$$

$$V = -\frac{P}{2}$$

$$M = -\frac{P \cdot y}{2}$$

$$2a < y < 3a$$

$$V = P$$

$$M = -(3 \cdot a - y) \cdot P$$