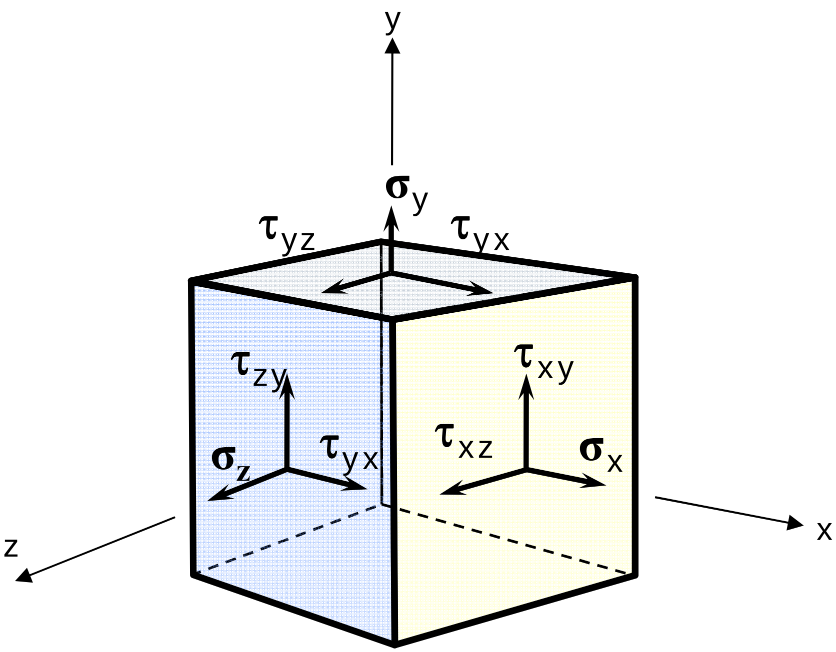
MER311: Advanced Strength of Materials

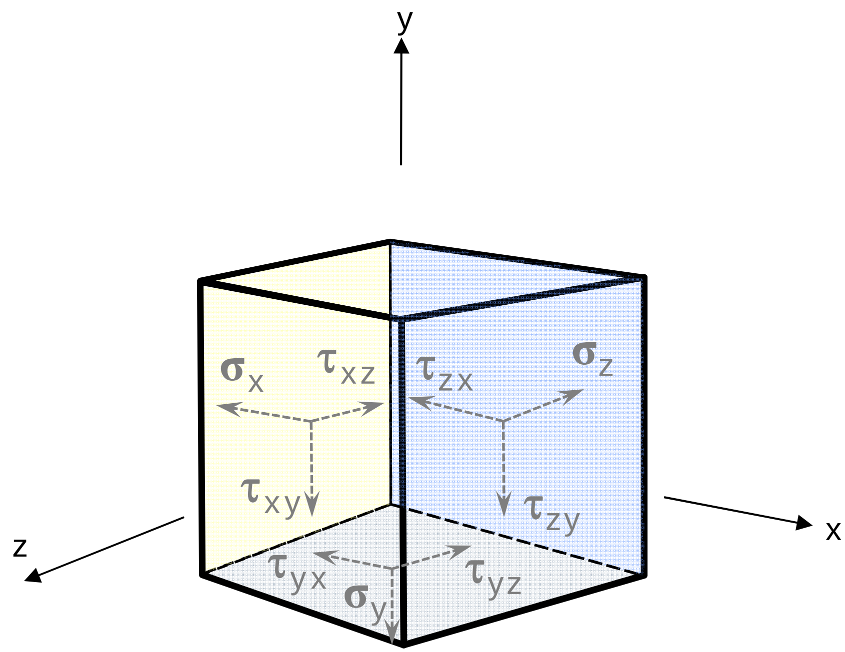
LECTURE OUTLINE

- Three Dimensional Transformations
- Intro to MatLab

Stress at a Point
Shown in the Tensile (+) Direction

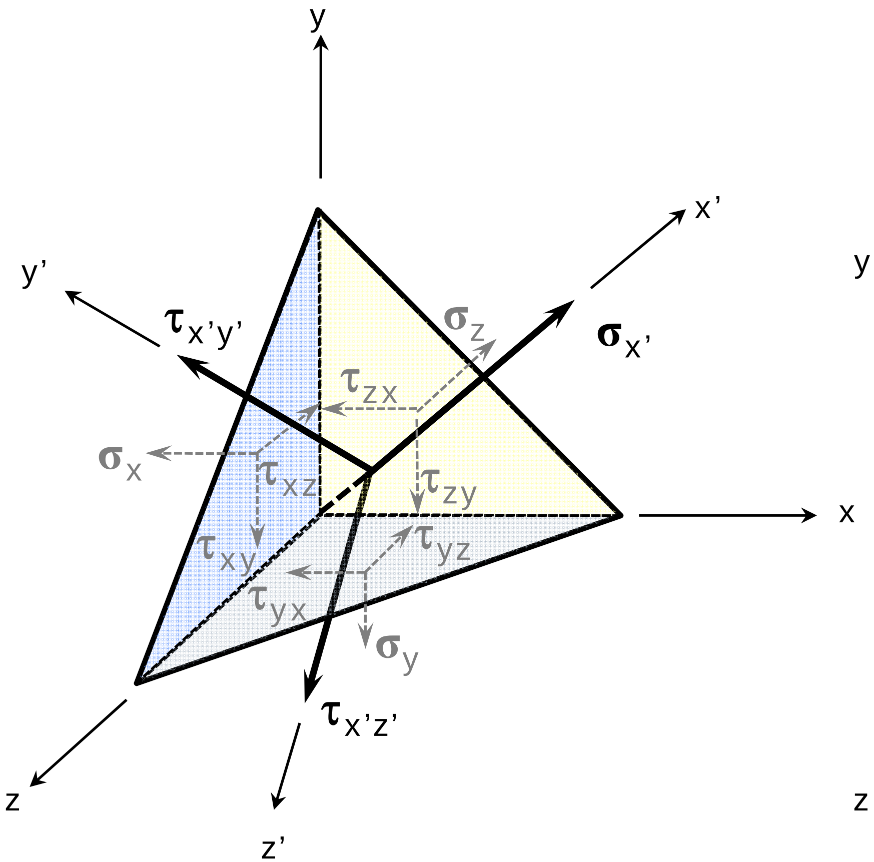


Surfaces with a Positive Directed Area Normal

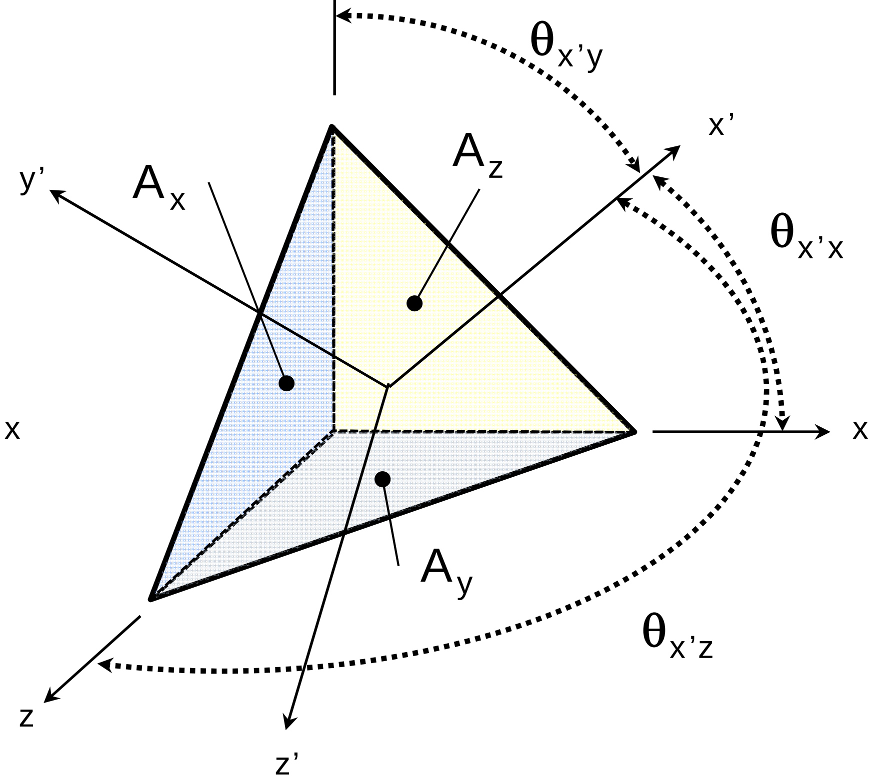


Surfaces with a Negative Directed Area Normal

Transforming Stress in Three Dimensions



A is the area of the inclined surface



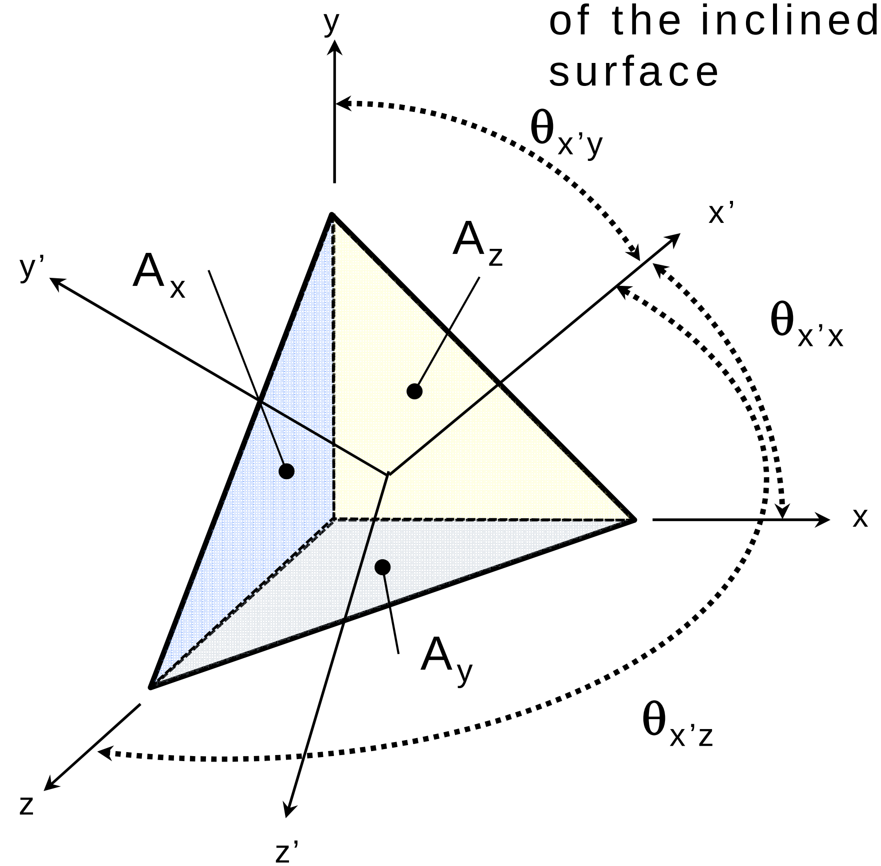
Direction Cosines Between Coordinate Axes

$$A_x = A \cdot \cos\theta_{x',x} = A \cdot n_{x',x}$$

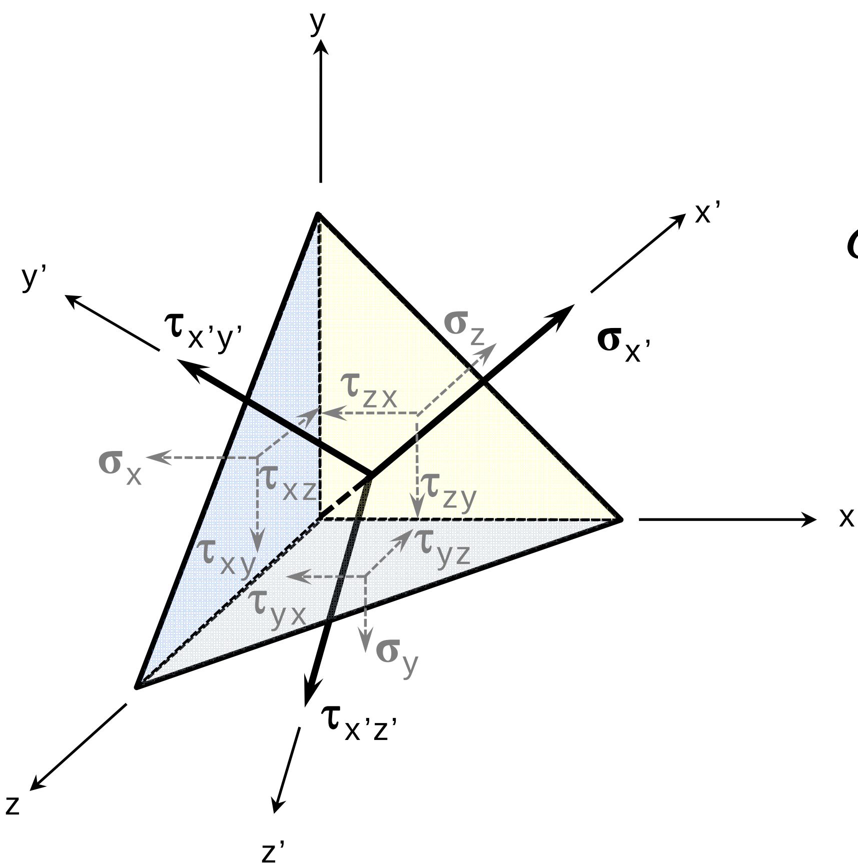
$$A_y = A \cdot \cos\theta_{x',y} = A \cdot n_{x',y}$$

$$A_z = A \cdot \cos\theta_{x',z} = A \cdot n_{x',z}$$

A is the area of the inclined surface



Resultant Stress Found From Equilibrium Equation



$$\sigma_{x'} = \sigma_x \cdot n_{x'x}^2 + \sigma_y \cdot n_{x'y}^2 + \sigma_z \cdot n_{x'z}^2$$
$$+ 2 \cdot \tau_{xy} \cdot n_{x'x} \cdot n_{x'y}$$
$$+ 2 \cdot \tau_{yz} \cdot n_{x'y} \cdot n_{x'z}$$
$$+ 2 \cdot \tau_{zx} \cdot n_{x'z} \cdot n_{x'x}$$

Transformation Equations

$$\sigma_{x'} = \sigma_x \cdot n_{x'x}^2 + \sigma_y \cdot n_{x'y}^2 + \sigma_z \cdot n_{x'z}^2 + 2 \cdot \tau_{xy} \cdot n_{x'x} \cdot n_{x'y} + 2 \cdot \tau_{yz} \cdot n_{x'y} \cdot n_{x'z} + 2 \cdot \tau_{zx} \cdot n_{x'z} \cdot n_{x'x}$$

$$\sigma_{y'} = \sigma_x \cdot n_{y'x}^2 + \sigma_y \cdot n_{y'y}^2 + \sigma_z \cdot n_{y'z}^2 + 2 \cdot \tau_{xy} \cdot n_{y'x} \cdot n_{y'y} + 2 \cdot \tau_{yz} \cdot n_{y'y} \cdot n_{y'z} + 2 \cdot \tau_{zx} \cdot n_{y'z} \cdot n_{y'x}$$

$$\sigma_{z'} = \sigma_x \cdot n_{z'x}^2 + \sigma_y \cdot n_{z'y}^2 + \sigma_z \cdot n_{z'z}^2 + 2 \cdot \tau_{xy} \cdot n_{z'x} \cdot n_{z'y} + 2 \cdot \tau_{yz} \cdot n_{z'y} \cdot n_{z'z} + 2 \cdot \tau_{zx} \cdot n_{z'z} \cdot n_{z'x}$$

$$\begin{aligned}\tau_{x'y'} = {} & \sigma_x \cdot n_{x'x} \cdot n_{y'x} + \sigma_y \cdot n_{x'y} \cdot n_{y'y} + \sigma_z \cdot n_{x'z} \cdot n_{y'z} + \tau_{xy} \cdot (n_{x'x} \cdot n_{y'y} + n_{x'y} \cdot n_{y'x}) \\ & + \tau_{yz} \cdot (n_{x'y} \cdot n_{y'z} + n_{x'z} \cdot n_{y'y}) + \tau_{zx} \cdot (n_{x'x} \cdot n_{y'z} + n_{x'z} \cdot n_{y'x})\end{aligned}$$

$$\begin{aligned}\tau_{z'x'} = {} & \sigma_x \cdot n_{x'x} \cdot n_{z'x} + \sigma_y \cdot n_{x'y} \cdot n_{z'y} + \sigma_z \cdot n_{x'z} \cdot n_{z'z} + \tau_{xy} \cdot (n_{x'x} \cdot n_{z'y} + n_{x'y} \cdot n_{z'x}) \\ & + \tau_{yz} \cdot (n_{x'y} \cdot n_{z'z} + n_{x'z} \cdot n_{z'y}) + \tau_{zx} \cdot (n_{x'x} \cdot n_{z'z} + n_{x'z} \cdot n_{z'x})\end{aligned}$$

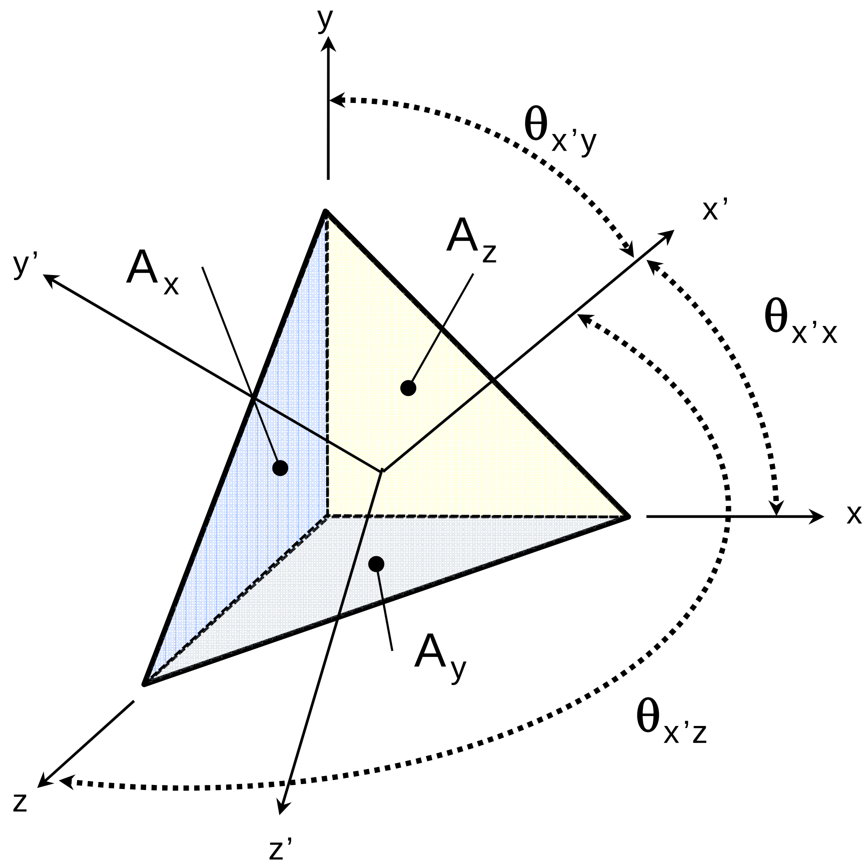
$$\begin{aligned}\tau_{y'z'} = {} & \sigma_x \cdot n_{y'x} \cdot n_{z'x} + \sigma_y \cdot n_{y'y} \cdot n_{z'y} + \sigma_z \cdot n_{y'z} \cdot n_{z'z} + \tau_{xy} \cdot (n_{y'x} \cdot n_{z'y} + n_{y'y} \cdot n_{z'x}) \\ & + \tau_{yz} \cdot (n_{y'y} \cdot n_{z'z} + n_{y'z} \cdot n_{z'y}) + \tau_{zx} \cdot (n_{y'x} \cdot n_{z'z} + n_{y'z} \cdot n_{z'x})\end{aligned}$$

Inverting the Stress Tensor

$$[\sigma]_{x'y'z'} = [T]\cdot[\sigma]_{xyz}\cdot[T]^T$$

$$T = \begin{bmatrix} n_{x',x} & n_{x',y} & n_{x',z} \\ n_{y',x} & n_{y',y} & n_{y',z} \\ n_{z',x} & n_{z',y} & n_{z',z} \end{bmatrix}$$

$$n_{i'j} = \mathbf{cos}\left(\theta_{i'j}\right)$$



EXAMPLE: Transformation of Axes

Write the transformation Matrix for the following:

- First a positive 45° about z axis
- Second a positive 30° about the new x' axis

$$T = \begin{bmatrix} n_{x',x} & n_{x',y} & n_{x',z} \\ n_{y',x} & n_{y',y} & n_{y',z} \\ n_{z',x} & n_{z',y} & n_{z',z} \end{bmatrix}$$

$$\boldsymbol{T1} = \begin{bmatrix} \cos 45 & \cos 45 & \cos 90 \\ \cos 135 & \cos 45 & \cos 90 \\ \cos 90 & \cos 90 & \cos 0 \end{bmatrix} \qquad \boldsymbol{T2} = \begin{bmatrix} \cos 0 & \cos 90 & \cos 90 \\ \cos 90 & \cos 30 & \cos 60 \\ \cos 90 & \cos 120 & \cos 30 \end{bmatrix}$$

SOLUTION: Transformation of Axes

Write the transformation Matrix for the following:
-First a positive 45° about z axis

$$T1 = \begin{bmatrix} 0.7017 & 0.7017 & 0 \\ -0.7017 & 0.7017 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Second a positive 30° about the new x' axis

$$T2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & 0.5 \\ 0 & -0.5 & 0.866 \end{bmatrix} \qquad T2 * T1 = \begin{bmatrix} 0.7071 & 0.7071 & 0 \\ -0.6124 & 0.6124 & 0.5 \\ 0.3536 & -0.3536 & 0.866 \end{bmatrix}$$

EXAMPLE:
Stress Transformation

The stress tensor at a point in a machine element with respect to the inertial coordinate system is

$$[\sigma]=\begin{bmatrix}50 & 10 & 0\\ 10 & 20 & 40\\ 0 & 40 & 30\end{bmatrix}MPa$$

Determine the state of stress if the stress element is rotated 45^o counterclockwise about the z axis followed by 30° about the new x’ axis.

SOLUTION: Stress Transformation

Stress Transformation

$$[\sigma]_{x'y'z'} = [T]\cdot[\sigma]_{xyz}\cdot[T]^T = \begin{bmatrix} 45.0 & 1.15 & 32.0 \\ 1.15 & 50.7 & 16.3 \\ 32.0 & 16.3 & 4.25 \end{bmatrix} MPa$$

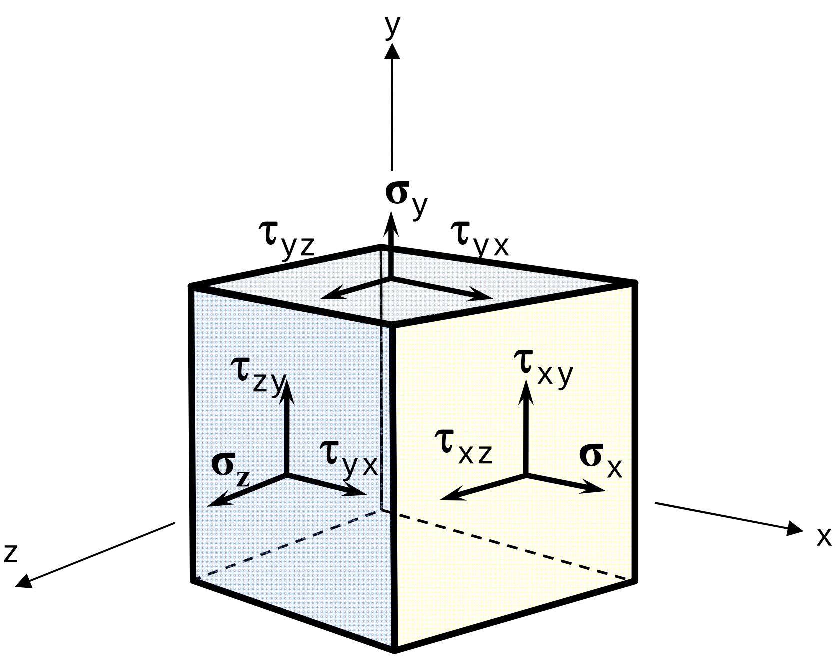
Transformation Matrix

$$T = T2 * T1 = \begin{bmatrix} 0.7071 & 0.7071 & 0 \\ -0.6124 & 0.6124 & 0.5 \\ 0.3536 & -0.3536 & 0.866 \end{bmatrix}$$

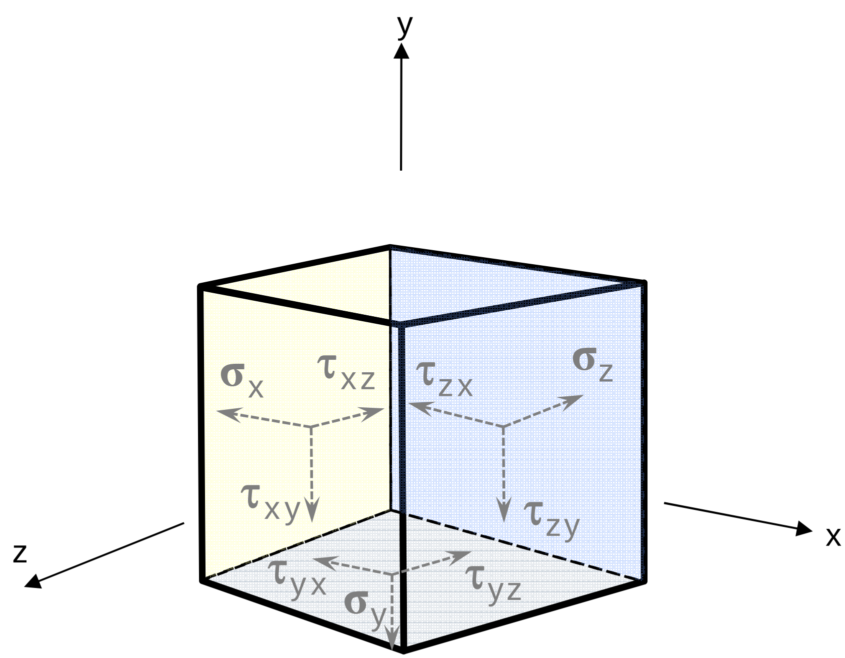
Transpose of Transformation Matrix

$$T^T = \begin{bmatrix} 0.7071 & -0.6124 & 0.3536 \\ 0.7071 & 0.6124 & -0.3536 \\ 0 & 0.5 & 0.866 \end{bmatrix}$$

Stress at a Point
Shown in the Tensile (+) Direction



Surfaces with a Positive Directed Area Normal



Surfaces with a Negative Directed Area Normal