NAME: SOLUTION

PROBLEM 1: The frame structure shown is fixed from translation at D and is fixed from translation in the y-direction at A. A couple loading M is applied to the structure at C.

(ALL COUPLES ARE DIRECTED OUT OF THE PAPER)

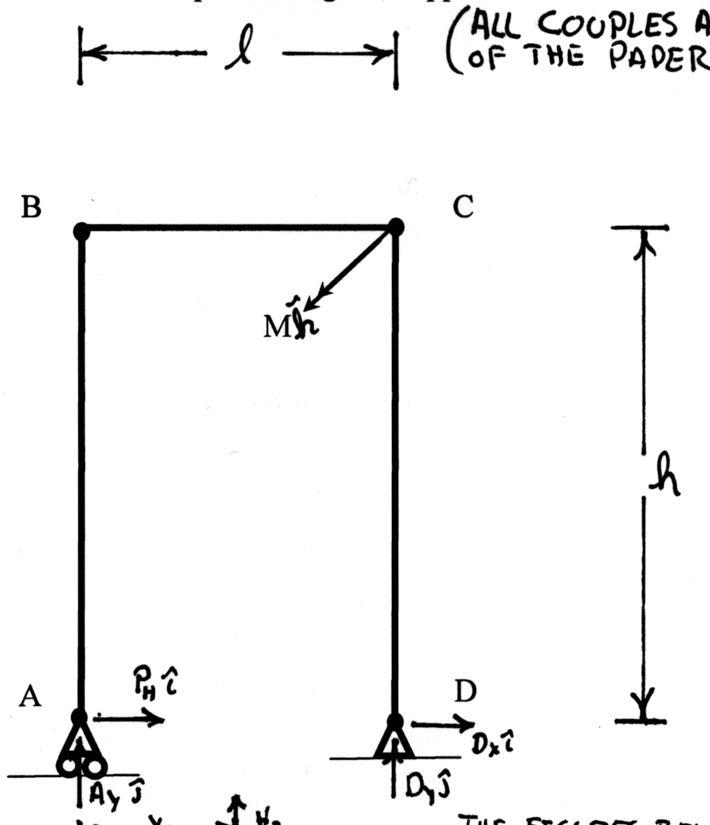
A PHANTOM LOAD P_H IS ADDED AT POINT A TO FACILITATE THE CALCULATION OF THE DEFLECTION IN THE HORIZONTAL DIRECTION AT THIS POINT.

$\sum F_x = 0 = P_H + D_x$
$\Rightarrow \underline{D_x = -P_H}$ ①

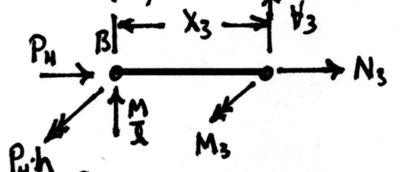
$\sum F_y = 0 = A_y + D_y$
$\Rightarrow \underline{D_y = -A_y}$ ②

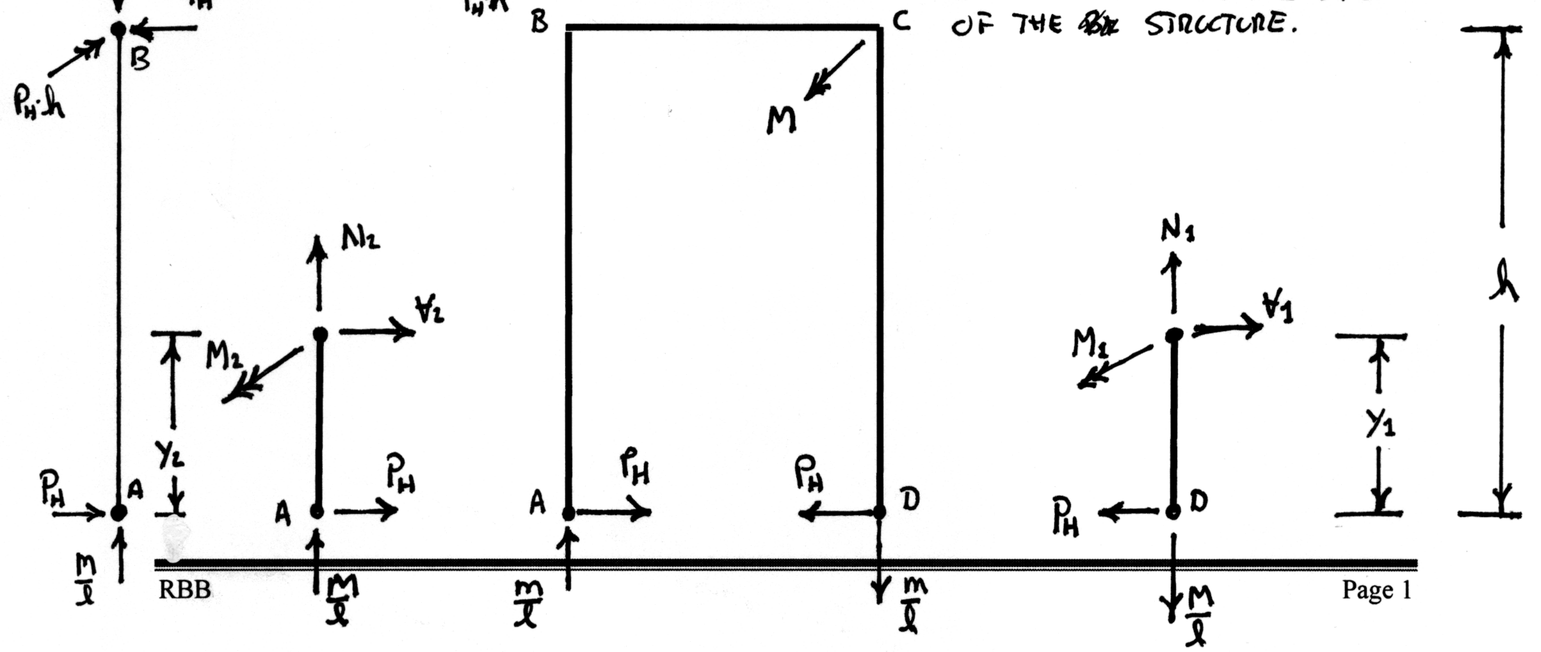
$\sum M_{z@D} = 0 = M - A_y \cdot l$
$\Rightarrow A_y = M/l$ ③

$\Rightarrow D_y = -M/l$ ④



THE FIGURES BELOW ARE INTENDED TO ASSIST IN THE DETERMINATION OF EXPRESSIONS FOR THE NORMAL FORCE AND MOMENTS IN THE VARIOUS SECTIONS OF THE STRUCTURE.





1a. Considering the bending and normal loads only, determine an expression for the horizontal deflection of point A of this structure when subjected to the load M.

EXPRESSIONS FOR THE NORMAL FORCE AND MOMENT IN EACH SECTION OF THE FRAME MUST BE DETERMINED. IN THE FRAME FROM D TO C

$$N_1 = M/\ell \quad (1)$$

$$M_1 = P_H \cdot Y_1 \quad (2)$$

IN THE FRAME FROM A TO B

$$N_2 = -M/\ell \quad (3)$$

$$M_2 = -P_H \cdot Y_2 \quad (4)$$

IN THE FRAME FROM B TO C

$$N_3 = -P_H \quad (5)$$

$$M_3 = \frac{M}{\ell} \cdot X_3 - P_H \cdot h \quad (6)$$

AN EXPRESSION FOR THE TOTAL STRAIN ENERGY IN THE FRAME CAN NOW BE WRITTEN

$$U = \frac{(M/\ell)^2 \cdot h}{2 \cdot A \cdot E} + \int_0^h \frac{(P_H \cdot Y_1)^2}{2 \cdot EI} \cdot dY_1 + \frac{(-M/\ell)^2 \cdot h}{2 \cdot A \cdot E} + \int_0^h \frac{(-P_H \cdot Y_2)^2}{2 \cdot E \cdot I} \cdot dY_2$$
$$+ \frac{(-P_H)^2 \cdot \ell}{2 \cdot A \cdot E} + \int_0^\ell \frac{(\frac{M}{\ell} \cdot X_3 - P_H \cdot h)^2}{2 \cdot E \cdot I} \cdot dX_3$$

$$= \frac{M^2 \cdot h}{2 \cdot \ell^2 \cdot A \cdot E} + \int_0^h \frac{P_H^2 \cdot Y_1^2}{2 \cdot E \cdot I} \cdot dY_1 + \frac{M^2 \cdot h}{2 \cdot \ell^2 \cdot A \cdot E} + \int_0^h \frac{P_H^2 \cdot Y_2^2}{2 \cdot E \cdot I} \cdot dY_2$$
$$+ \frac{P_H^2 \cdot \ell}{2 \cdot A \cdot E} + \int_0^\ell \frac{(\frac{M^2}{\ell^2} \cdot X_3^2 - 2 \cdot \frac{M \cdot P_H \cdot h}{\ell} X_3 + P_H^2 \cdot h^2)}{2 \cdot E \cdot I} \, dX_3$$

$$= \frac{M^2 \cdot h}{2 \cdot \ell^2 \cdot A \cdot E} + \int_0^h \frac{P_H^2 \cdot Y_1^2}{2 \cdot E \cdot I} \cdot dY_1 + \frac{M^2 \cdot h}{2 \cdot \ell^2 \cdot EA} + \int_0^h \frac{P_H^2 \cdot Y_2^2}{2 \cdot E \cdot I} \cdot dY_2 + \frac{P_H^2 \cdot \ell}{2 \cdot A \cdot E} + \int_0^\ell \left(\frac{M^2}{2 \cdot \ell^2 \cdot EI} \cdot X_3^2 - \frac{M \cdot P_H \cdot h}{\ell \cdot E \cdot I} \cdot X_3 + \frac{P_H^2 \cdot h^2}{2E \cdot I} \right) dX_3$$

$$= \frac{M^2 \cdot h}{2 \cdot \ell^2 \cdot A \cdot E} + \frac{P_H^2 h^3}{6 \cdot E \cdot I} + \frac{M^2 \cdot h}{2 \cdot \ell^2 \cdot EA} + \frac{P_H^2 \cdot h^3}{6 \cdot E \cdot I} + \frac{P_H^2 \cdot \ell}{2 \cdot A \cdot E} + \frac{M^2 \cdot \ell}{6 \cdot EI} - \frac{M \cdot P_H \cdot h \cdot \ell}{2 \cdot E \cdot I} + \frac{P_H^2 \cdot h^2 \cdot \ell}{2 \cdot E \cdot I}$$

$$= \frac{M^2 \cdot h}{\ell^2 \cdot A \cdot E} + \frac{P_H^2 \cdot h^3}{3 \cdot E \cdot I} + \frac{P_H^2 \cdot \ell}{2 \cdot A \cdot E} + \frac{P_H^2 \cdot h^2 \cdot \ell}{2 \cdot E \cdot I} - \frac{M \cdot P_H \cdot h \cdot \ell}{2EI} + \frac{M^2 \cdot \ell}{6EI}$$

$$U = \left(\frac{h}{\ell^2 \cdot A \cdot E} + \frac{\ell}{6 \cdot E \cdot I}\right) M^2 - \frac{h \cdot \ell}{2 \cdot E \cdot I} \cdot M \cdot P_H + \left(\frac{h^3}{3 \cdot E \cdot I} + \frac{\ell}{2 \cdot A E} + \frac{h^2 \cdot \ell}{2 \cdot E \cdot I}\right) \cdot P_H^2 \qquad (7)$$

THE EXPRESSION FOR THE HORIZONTAL DEFLECTION OF THE FRAME AT A IS FOUND BY TAKING THE PARTIAL DERIVATIVE OF (7) WITH RESPECT TO P_H AND THEN SETTING THE PHANTOM LOAD $P_H = 0$.

$$\delta_{AH} = \frac{\partial U}{\partial P_H} = -\frac{h \cdot \ell}{2 \cdot E \cdot I} \cdot M + 2\left(\frac{h^3}{3 \cdot E \cdot I} + \frac{\ell}{2 \cdot A \cdot E} + \frac{h^2 \cdot \ell}{2 E I}\right) \cancelto{0}{P_H}$$

$$\boxed{\delta_{AH} = -\frac{h \cdot \ell \cdot M}{2 E I}}$$

THE NEGATIVE SIGN INDICATES THAT THE HORIZONTAL DEFLECTION AT A IS IN THE OPPOSITE DIRECTION OF P_H.

1b. Determine an expression for the rotation of the corner C that results from the frame being loaded by M.

THE EXPRESSION FOR THE ROTATION OF THE FRAME AT C IS FOUND BY TAKING THE PARTIAL DERIVATIVE OF 7 WITH RESPECT TO M. P_H AGAIN IS SET TO ZERO BECAUSE IT DOES NOT EXIST.

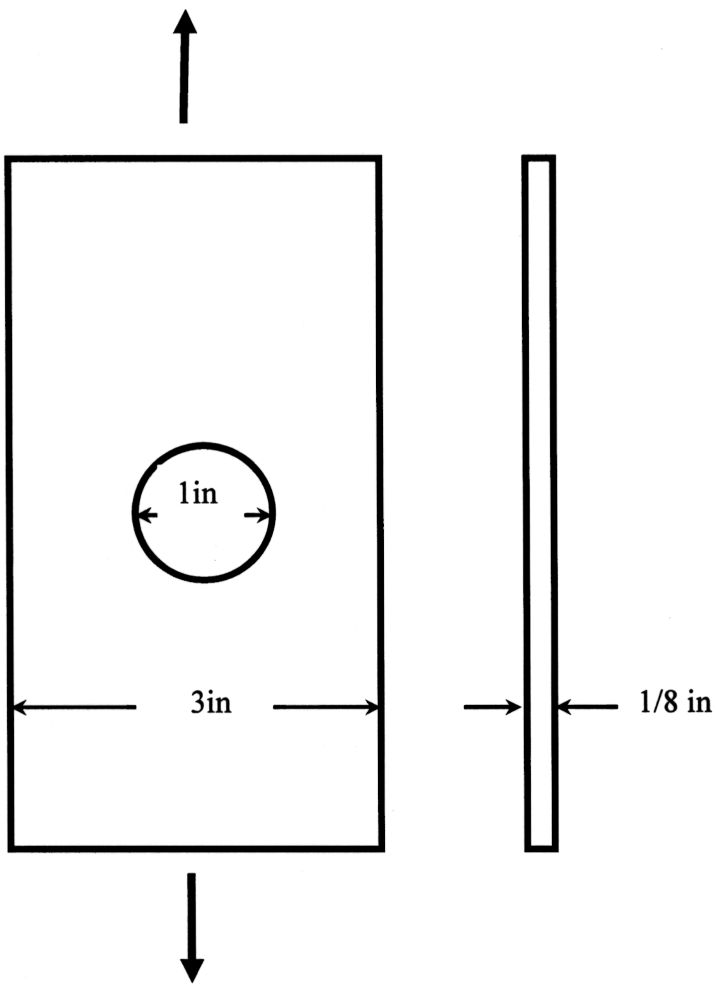
$$\delta_M = \Delta\theta_c = \frac{\partial U}{\partial M} = 2\left(\frac{h}{l^2 \cdot A \cdot E} + \frac{l}{6 \cdot E \cdot I}\right) \cdot M - \frac{h \cdot l}{2 \cdot E \cdot I} \cdot \cancel{P_H}^{0}$$

$$= \boxed{2\left(\frac{h}{l^2 A \cdot E} + \frac{l}{6 \cdot E \cdot I}\right) \cdot M}$$

PROBLEM 2: The figure below shows a part made of 1/8th in. thick 7075 aluminum. (S_u=82ksi and S_y=70ksi). The part is axially loaded.

$S_e' = 0.4(82 \text{ ksi}) = 32.8 \text{ ksi}$

2a. Draw the Goodman diagram that includes both the positive and negative mean stress domains on the paper provided.

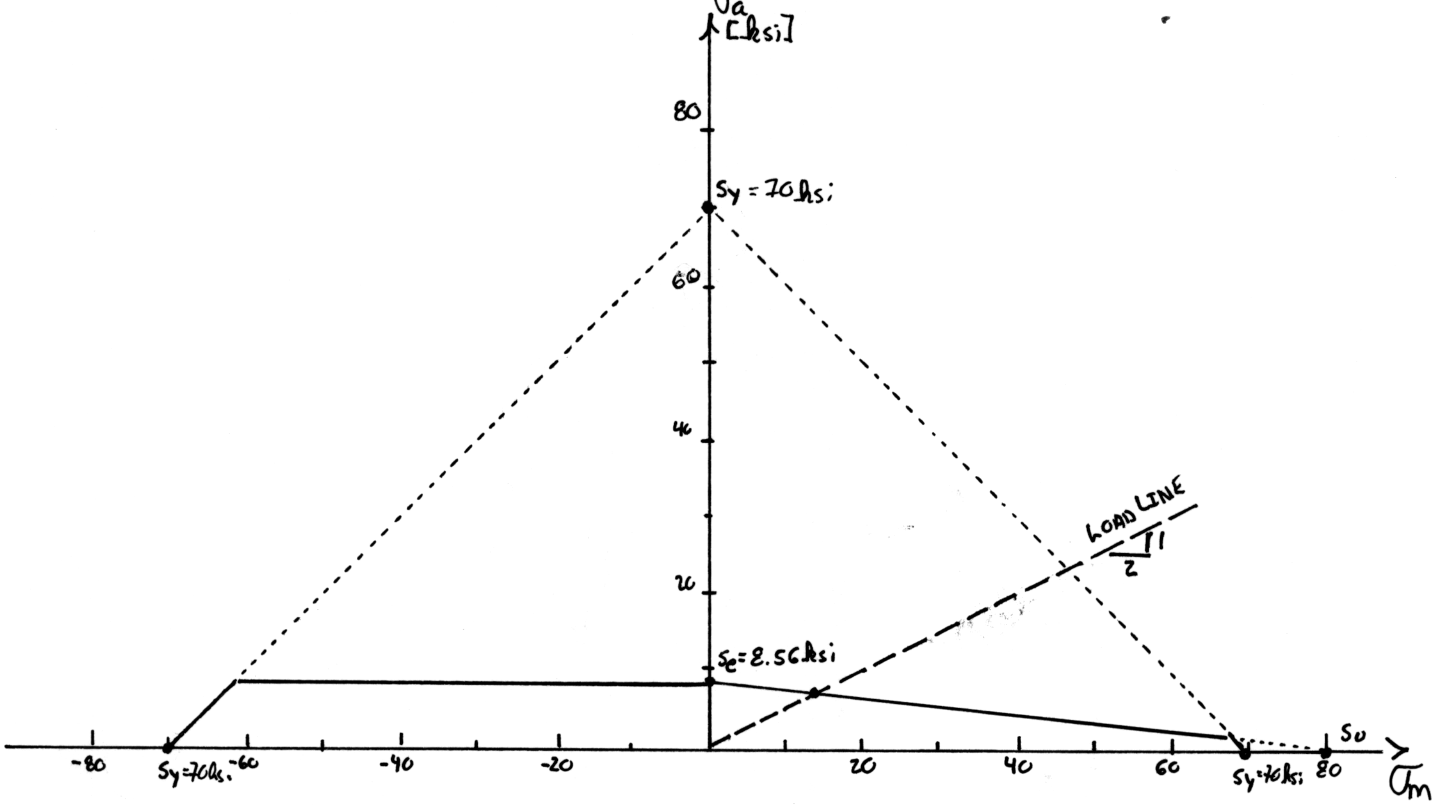


BEFORE THE GOODMAN DIAGRAM CAN BE CONSTRUCTED, THE ENDURANCE LIMIT FOR THIS COMPONENT MUST BE COMPUTED.

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S_e'$$

- $k_a = 0.8$ (FINISH IS ASSUMED TO BE MACHINED)
- $k_b = 0.75$ (SIZE FACTOR)
- $k_c = 1.0$ (RELIABILITY)
- $k_d = 1.0$ (TEMPERATURE)
- $k_e = 1/K_f = 1/2.3 = 0.4348$
  - $K_t = 2.3$ (APPENDIX F.1, BUDYNAS 2ND)
    - $d/w = 1in/3in = 0.333$
  - $K_f = 1 + q(K_t - 1) = 1 + 1(2.3 - 1) = 2.3$
    - $q = 1.0$ (EXTRAPOLATING THE NOTES LECTURE 15 PG 22)

$$S_e = (0.8)(0.75)(1.0)(1.0)(0.4348) \cdot 32.8\,ksi = \underline{\underline{8.56\,ksi}}$$



2b. If the part is load such that the mean stress is two times the stress amplitude, what is the maximum values of the mean and amplitude stress if the part is to be designed for infinite life?

THE EQUATION OF THE GOODMAN LINE IS WRITTEN

$$\sigma_{a,G} = m \cdot \sigma_{m,G} + b = -\frac{8.56\text{ ksi}}{80\text{ ksi}} \cdot \sigma_m + 8.56\text{ ksi}$$

$$= -0.107 \cdot \sigma_{m,G} + 8.56\text{ ksi} \qquad (1)$$

THE EQUATION FOR THE LOADING LINE

$$\sigma_{a,L} = m \cdot \sigma_{m,L} + b = \frac{\sigma_a}{2 \cdot \sigma_a} \cdot \sigma_{m,L} + 0$$

$$= 0.5 \cdot \sigma_{m,L} \qquad (2)$$

THE LOCATION OF WHERE THESE TWO LINES INTERSECT IS FOUND BY SETTING THE σ_a's EQUAL AND SOLVING FOR σ_m

$$-0.107 \cdot \sigma_m + 8.56\text{ ksi} = 0.5\,\sigma_m$$

$$\sigma_m = \frac{8.56\text{ ksi}}{0.5 + 0.107} = \boxed{14.1\text{ ksi}} \qquad (3)$$

SUBSTITUTING THIS VALUE INTO (2)

$$\sigma_a = \boxed{7.05\text{ ksi}} \qquad (4)$$

2c. If a factor of safety of 2 is required for the design described in 2b, what are the maximum values of the mean and amplitude stresses?

FOR A DESIGN FACTOR OF 2

$$\sigma_m = \frac{14.1\,ksi}{2} = \boxed{7.05\,ksi}$$

$$\sigma_a = \frac{7.05\,ksi}{2} = \boxed{3.53\,ksi}$$