Buckling – Chap 3

- Definition
- Four Cases
- Example problems

Types of Compression

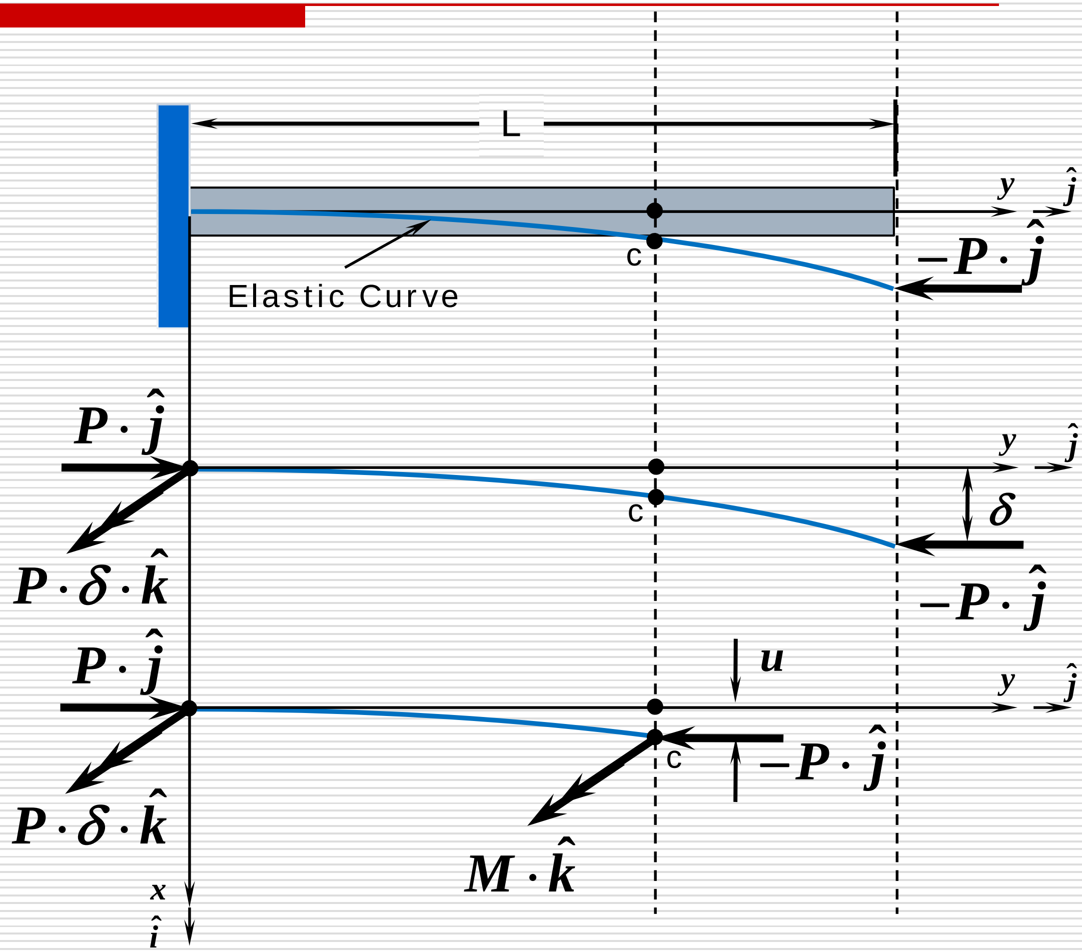
- Members are short in compression with their cross-sectional dimensions
  - Lateral deflections produced by axial or eccentric loads are disregarded
- Members are long in compression with their cross-sectional dimensions
  - Lateral deflections can not be disregarded

Causes for Small Eccentricities of the Load

- Lack of material homogeneity
- Impossible to make a straight column
- Inability to apply the load exactly along the geometric axis
- Non-uniform initial stresses
- Vibration

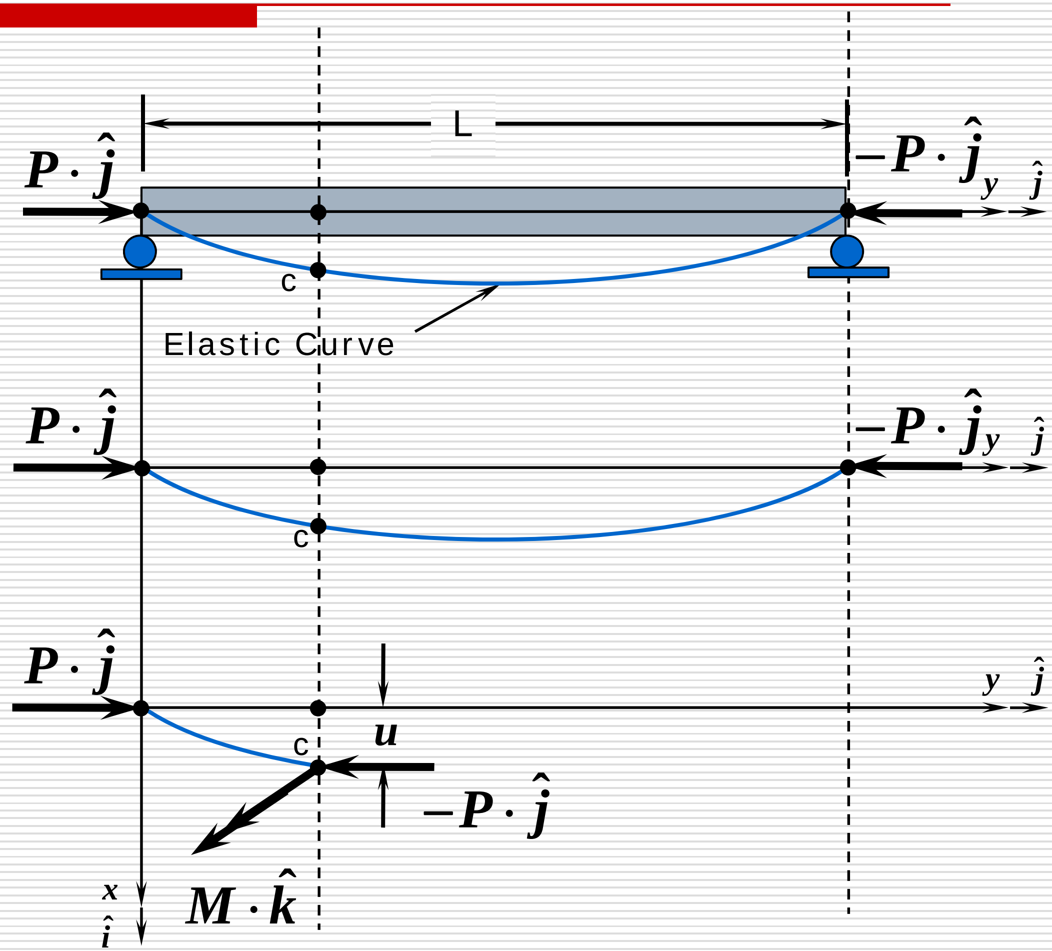
Fixed-Free

$$P_{cr} = \frac{\pi^2}{4 \cdot L^2} \cdot E \cdot I$$



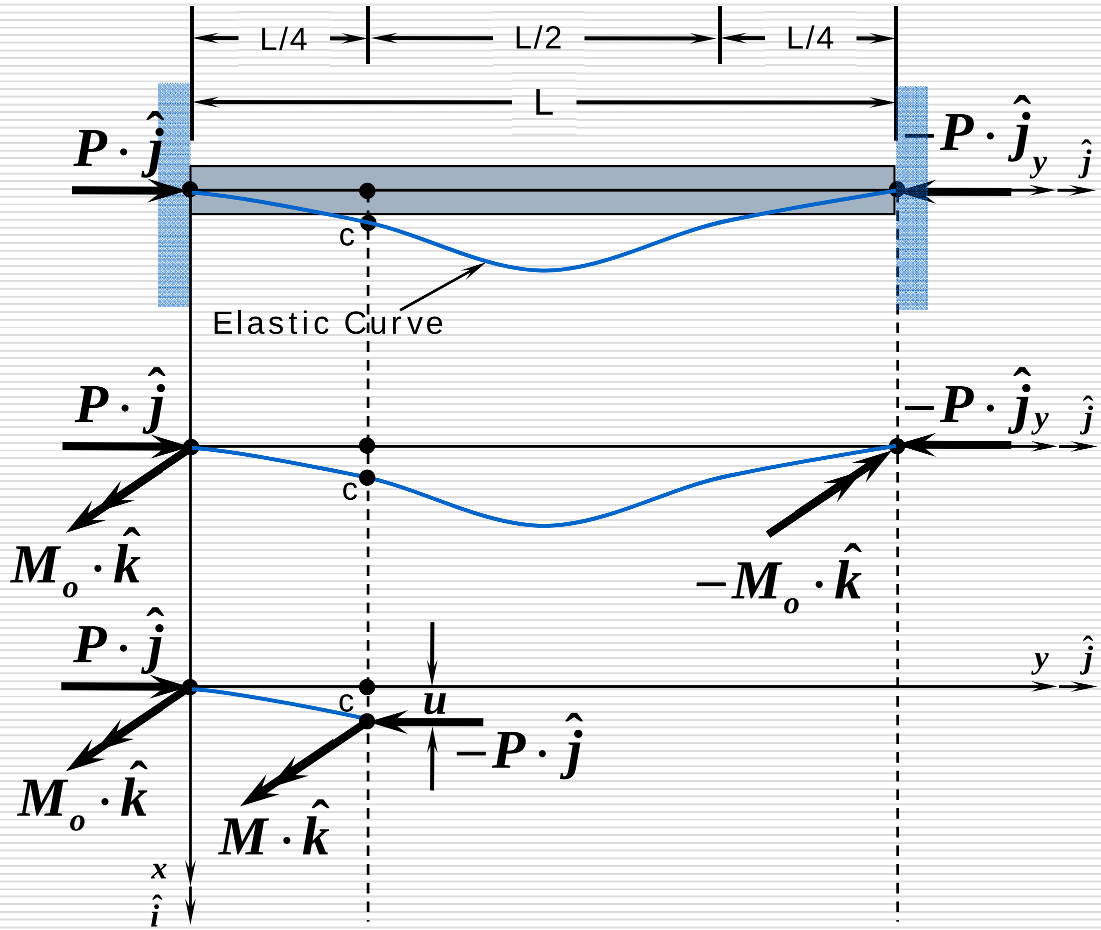
Pinned-Pinned

$$P_{cr} = \frac{\pi^2}{L^2} \cdot E \cdot I$$



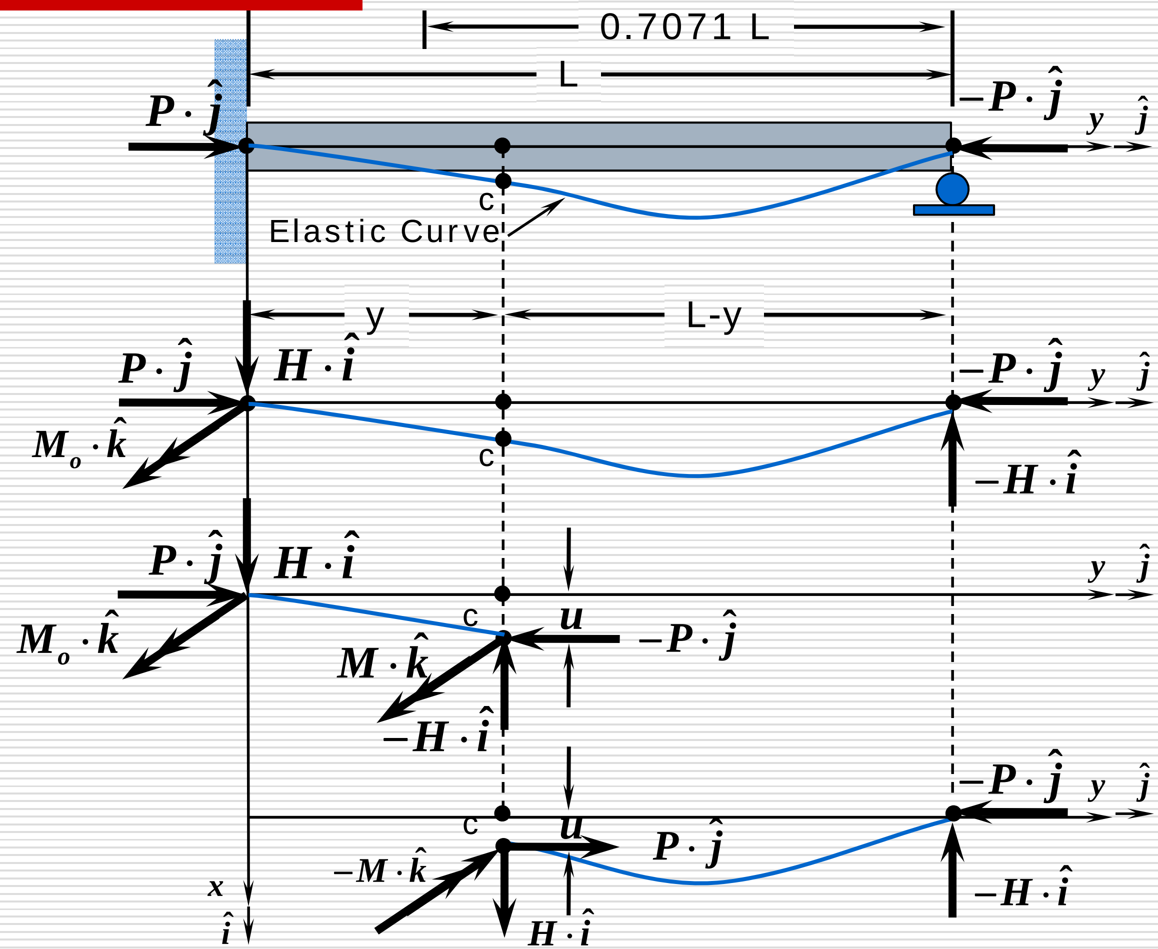
Fixed-Fixed

$$P_{cr} = \frac{4 \cdot \pi^2}{L^2} \cdot E \cdot I$$

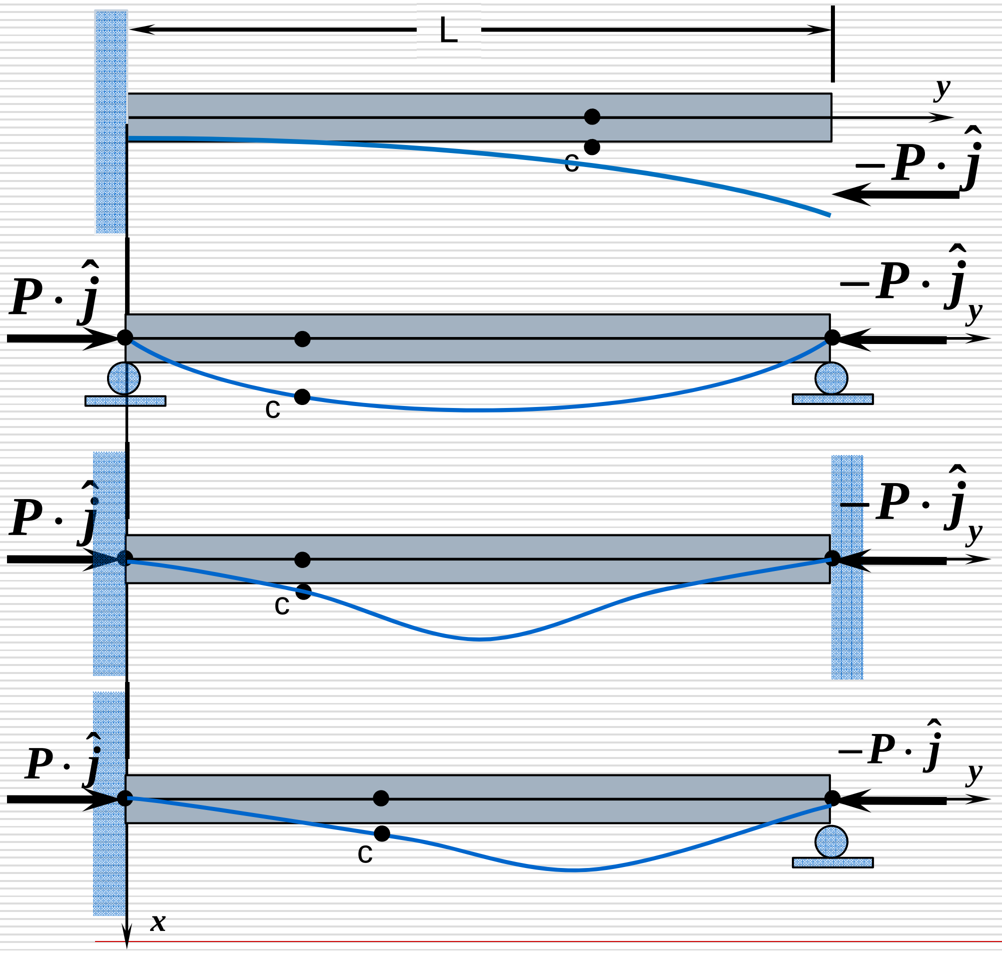


Fixed-Pinned

$$P_{cr} = \frac{\pi^2}{0.7^2 L^2} \cdot E \cdot I$$



Buckling



Fixed-Free
K=2.0
K_c=2.0
K_r=2.0

Pinned-Pinned
K=1.0
K_c=1.0
K_r=1.0

Fixed-Fixed
K=0.5
K_c=1.0
K_r=0.83

Fixed-Pinned
K=0.7
K_c=1.0
K_r=0.83

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I}{(K \cdot L)^2}$$

K-Theoretical
K_c-Conservative
K_r-Recommended

Slenderness Ratio

- Parameter used to quantify the affect of the lateral deflections
- The ratio of a columns length to the radius of gyration of its cross-sectional area with respect to the principal axis about which the column tends to bend, L/

$$I = \int r^2 \cdot dA = \rho^2 \cdot A$$

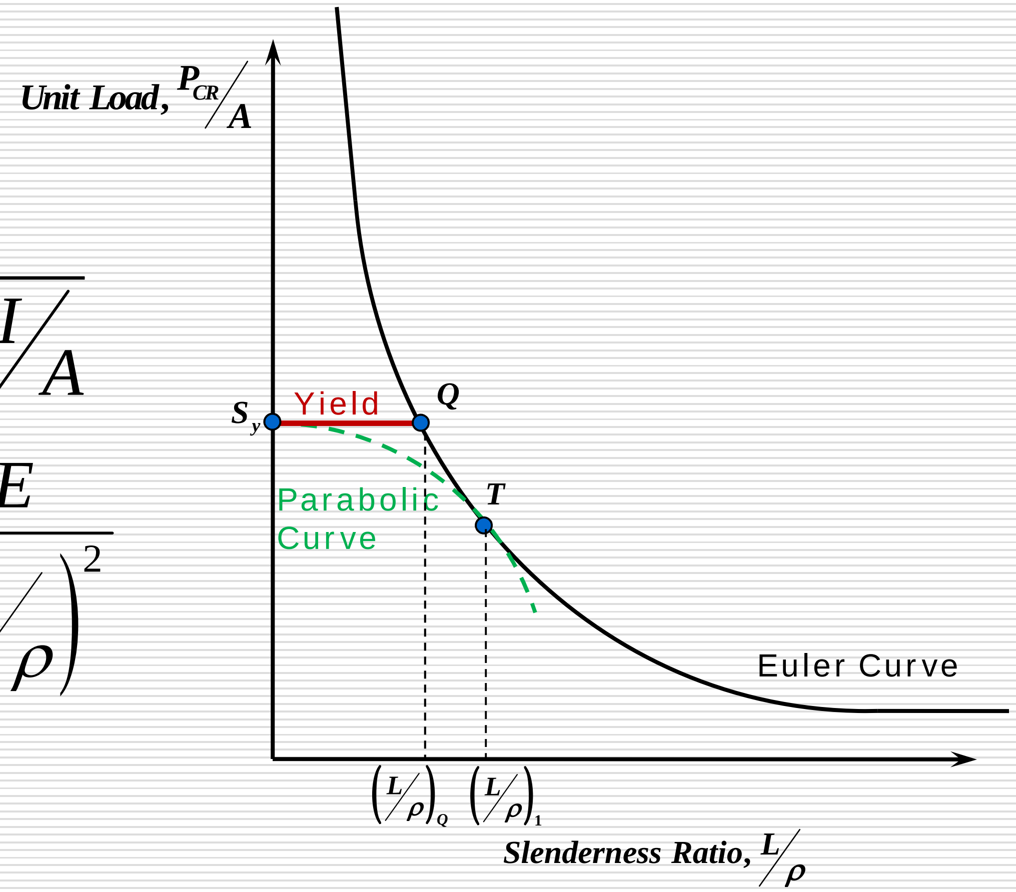
$$\rho = \sqrt{I/A}$$

Column Design

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I}{K^2 \cdot L^2}$$

$$\frac{P_{cr}}{A} = \frac{\pi^2 \cdot E \cdot I}{K^2 \cdot A \cdot L^2} \Rightarrow \rho = \sqrt{I/A}$$

$$\frac{P_{cr}}{A} = \frac{\pi^2 \cdot E \cdot \rho^2}{K^2 \cdot L^2} = \frac{\pi^2 \cdot E}{K^2 \cdot \left(L/\rho\right)^2}$$



Intermediate Length Columns

$$\rho = \sqrt{I/A} \qquad \frac{P_{cr}}{A} = \frac{\pi^2 \cdot E}{K^2 \cdot \left(L/\rho\right)^2}$$

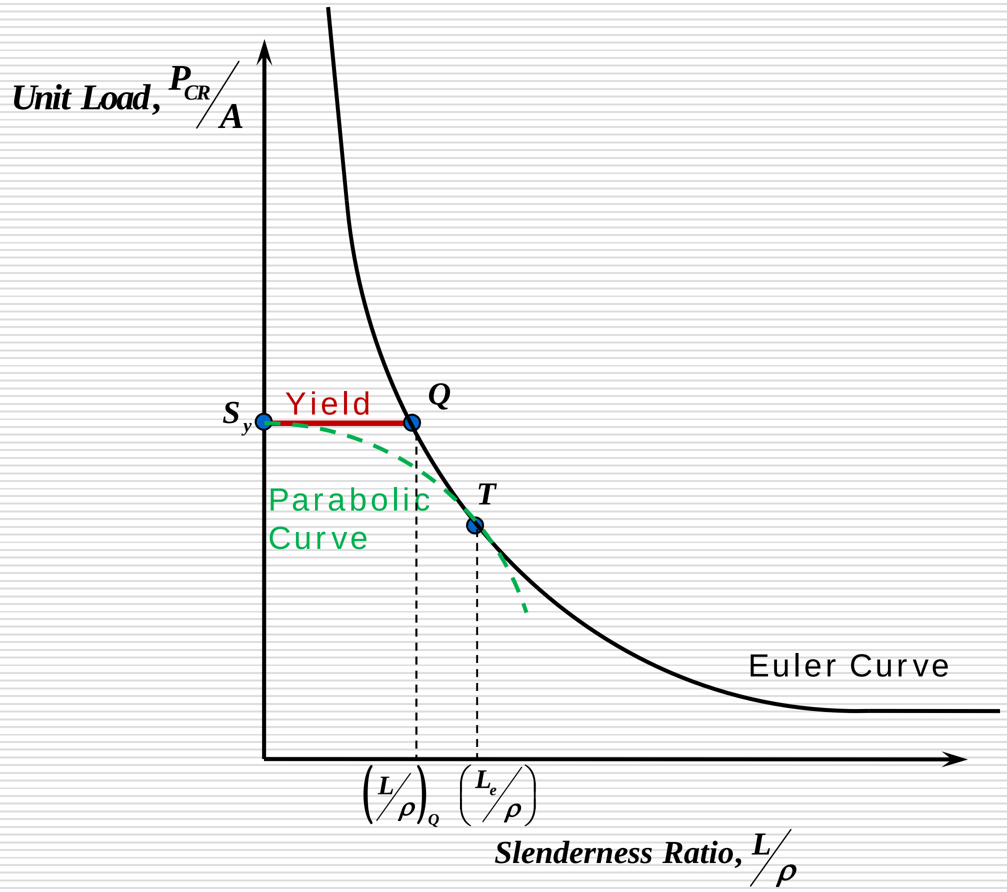
Intermediate-Lengths

$$\frac{P_{CR}}{A} = a - \left(\frac{L}{\rho}\right)^2 \cdot b$$

$$a = S_y$$

$$b = \left(\frac{S_y}{2 \cdot \pi}\right)^2 \cdot \frac{K^2}{E}$$

$$\frac{P_{CR}}{A} = S_y - \left(\frac{S_y}{2 \cdot \pi} \cdot \frac{L}{\rho}\right)^2 \cdot \frac{K^2}{E}$$



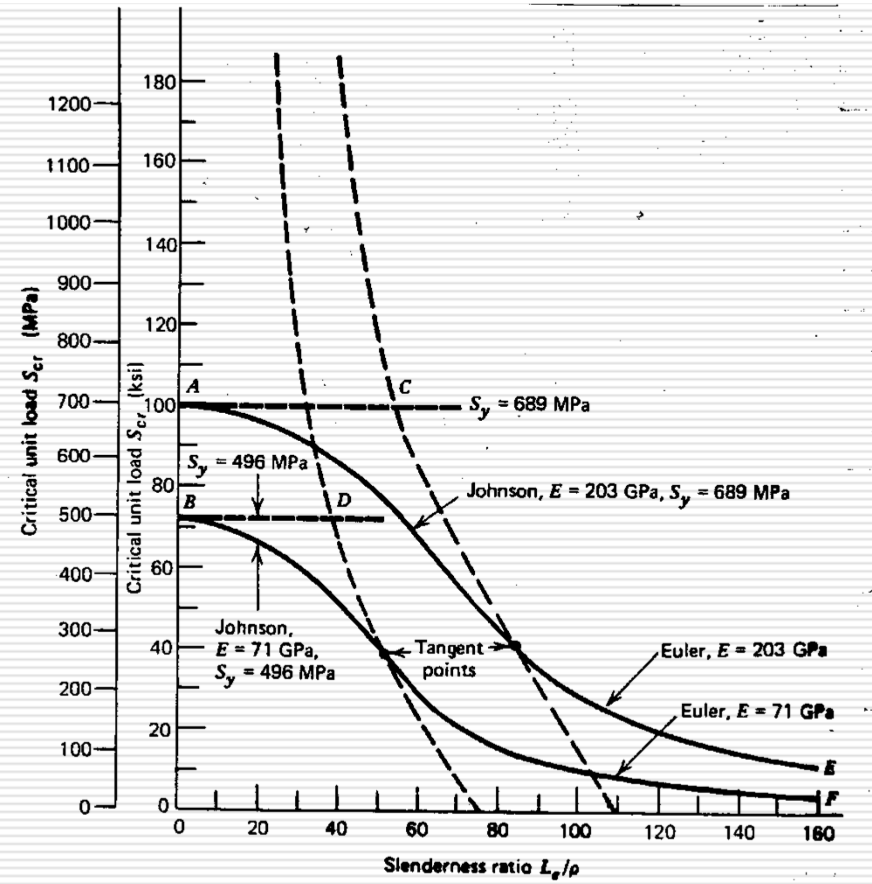
Column Design

$$\frac{P_{CR}}{A} = S_y - \frac{S_y^2}{4 \cdot \pi^2 \cdot E} \cdot \left(\frac{L_e}{\rho} \right)^2$$

Tangent Points

$$\frac{P_{CR}}{A} = \frac{S_y}{2}$$

$$\frac{L_e}{\rho} = \left(\frac{2 \cdot \pi^2 \cdot E}{S_y} \right)^{1/2}$$



Example

An industrial machine requires a solid, round piston connecting rod 200mm long (between pinned ends) that is subjected to a maximum compressive force of 80 kN. Using a safety factor of 2.5, what diameter is required if aluminum is used, having properties of S_y=496MPa, E=71GPa