PROBLEM 5-14 | DESIGN A LINKAGE TO CARRY THE BODY IN THE FIGURE BELOW THROUGH THE THREE POSITIONS P_1, P_2 & P_3 AT THE ANGLES SHOWN IN THE FIGURE. USE ANALYTICAL SYNTHESIS WITHOUT REGARD TO THE FIX PIVOTS SHOWN.

GIVEN:

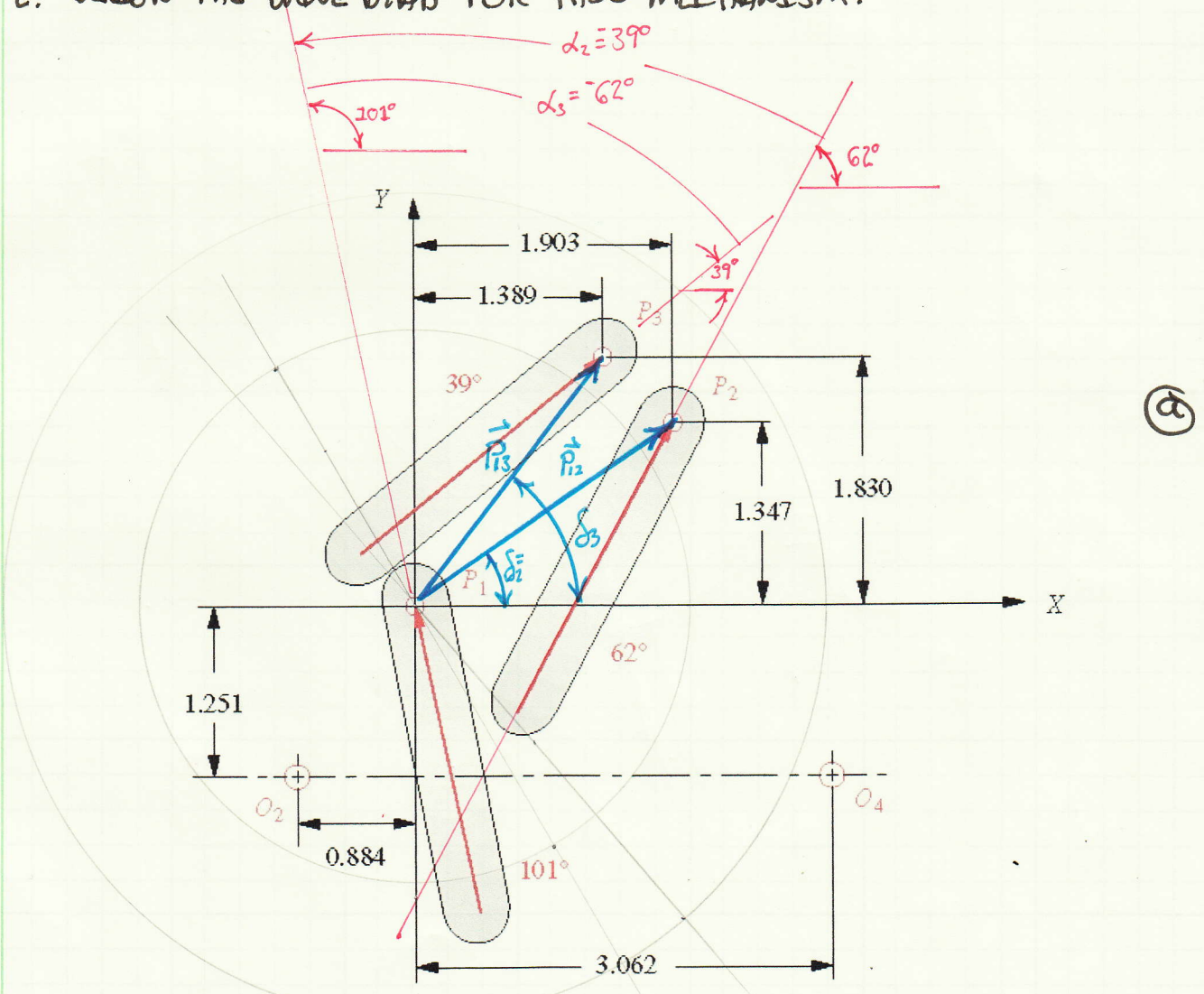
1. THE THREE POSITIONS OF A SURFACE ON THE COUPLER LINK.
2. P_1 @ (0in, 0in) ORIENTATED AT 101° TO THE +X AXIS
3. P_2 @ (1.903in, 1.347in) ORIENTATED AT 62° TO THE +X AXIS
4. P_3 @ (1.389in, 1.830in) ORIENTATED AT 39° TO THE +X AXIS

ASSUMPTIONS:

1. PLANAR MOTION OF THE LINKS
2. ALL ELEMENTS ARE RIGID

FIND:

1. SYNTHESIZE A MECHANISM THAT WILL TAKE POINT P THROUGH THE DESIRED POSITIONS IN THE ORIENTATION SHOWN.
2. DESIGN THE DRIVE DYAD FOR THIS MECHANISM.



SOLUTION:

THREE POSITION ANALYSIS REQUIRES $\rho_{12}, \rho_{13}, \delta_2, \delta_3, \alpha_2, \alpha_3$ BE GIVEN BY THE DEFINATION OF THE PROBLEM AN REQUIRES THE DESIGNER TO CHOOSE VALUES FOR $\beta_2, \beta_3, \gamma_2$ & γ_3,

FROM FIGURE ① $\rho_{12}, \rho_{13}, \delta_2, \delta_3, \alpha_2$ & α_3 CAN BE CALCULATED AS FOLLOWS.

$$\rho_{12} = \sqrt{(1.903in - 0in)^2 + (1.347in - 0in)^2} = \underline{2.331in} \quad (1)$$

$$\rho_{31} = \sqrt{(1.389in - 0in)^2 + (1.830in - 0in)^2} = \underline{2.297in} \quad (2)$$

$$\delta_2 = \tan^{-1}\frac{1.347in}{1.903in} = \underline{35.29^\circ} \quad (3)$$

$$\delta_3 = \tan^{-1}\frac{1.830in}{1.389in} = \underline{52.80^\circ} \quad (4)$$

$$\alpha_2 = 62^\circ - 101^\circ = \underline{-39^\circ} \quad (5)$$

$$\alpha_3 = 39^\circ - 101^\circ = \underline{-62^\circ} \quad (6)$$

THE FREE CHOICES ARE,

$\beta_2 = 40^\circ$ $\qquad$ $\gamma_2 = 0^\circ$

$\beta_3 = 75^\circ$ $\qquad$ $\gamma_3 = 30^\circ$

ON THE FOLLOWING PAGES ARE THE ANALYTICAL SYNTHESIS FOR THIS MECHANISM AND A DRIVE DYAD THAT IS SYNTHESISED TO DRIVE O_2-A.

SUMMARY:

THE 3 POSITION SYNTHESIS PROBLEM IS MORE RESTRICTIVE IN TERMS OF FREE CHOICES. OFTEN IT IS NECESSARY TO KNOW ADDITIONAL QUALITATIVE INFORMATION TO MAKE THESE CHOICES PROPERLY. FOR THIS PROBLEM γ_2 IS ZERO, THIS MEANS THAT IN THE SECOND POSITION O_4-B IS ORIENTATED EXACTLY THE SAME AS IN THE FIRST POSITION. THIS DOES NOT MEAN THAT O_4-B DOES NOT MOVE. THIS TYPE OF SYNTHESIS ONLY GUARANTEES THAT THE MECHANISM CAN BE CONSTRUCTED IN THESES THREE POSITION. IT DOES NOT DETERMINE IF IT CAN MOVE TO THE POSITIONS WITHOUT THE NEED FOR DISASSEMBLY.

FIRST DYAD

GIVEN:		CHOSEN:		FIND:	
P12	2.33	β2	40.00	w	3.285
P13	2.30	β3	75.00	θ	-18.846
δ2	35.29			z	3.214
δ3	52.80			ϕ	84.695
α2	-39.00			W1x	3.109
α3	-62.00			W1y	-1.061
				Z1x	0.297
				Z1y	3.200

	x-coord	y-coord.
O2	-3.407	-2.139
A1	-0.297	-3.200
A2	-0.342	-0.954
A3	-1.577	0.590
P1	0.000	0.000
P2	1.903	1.347
P3	1.389	1.830

-0.2340	-0.6428	-0.2229	0.6293	W1x	=	1.9027
0.6428	-0.2340	-0.6293	-0.2229	W1y		1.3467
-0.7412	-0.9659	-0.5305	0.8829	Z1x		1.3888
0.9659	-0.7412	-0.8829	-0.5305	Z1y		1.8296

$$\begin{bmatrix} \cos\beta_2 - 1 & -\sin\beta_2 & \cos\alpha_2 - 1 & -\sin\alpha_2 \\ \sin\beta_2 & \cos\beta_2 - 1 & \sin\alpha_2 & \cos\alpha_2 - 1 \\ \cos\beta_3 - 1 & -\sin\beta_3 & \cos\alpha_3 - 1 & -\sin\alpha_3 \\ \sin\beta_3 & \cos\beta_3 - 1 & \sin\alpha_3 & \cos\alpha_3 - 1 \end{bmatrix} \cdot \begin{Bmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{Bmatrix} = \begin{Bmatrix} p_{21} \cdot \cos\delta_2 \\ p_{21} \cdot \sin\delta_2 \\ p_{31} \cdot \cos\delta_3 \\ p_{31} \cdot \sin\delta_3 \end{Bmatrix}$$

KINEMATIC ANALYSIS - CRITICAL POSITIONS					Normal (r)		Perpendicular (θ)	
	x-coord	y-coord.	mag	angle	i	j	i	j
O2	-3.407	-2.139	4.022	-147.9	-0.8469	-0.5318	0.5318	-0.8469
A1	-0.297	-3.200	3.214	-95.3	-0.0925	-0.9957	0.9957	-0.0925
A2	-0.342	-0.954	1.013	-109.8	-0.3380	-0.9411	0.9411	-0.3380
A3	-1.577	0.590	1.683	159.5	-0.9367	0.3502	-0.3502	-0.9367
P1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
P2	1.903	1.347	2.331	35.3	0.8162	0.5777	-0.5777	0.8162
P3	1.389	1.830	2.297	52.8	0.6046	0.7965	-0.7965	0.6046

SECOND DYAD

GIVEN:		CHOSEN:		FIND:	
P12	2.33	γ2	0.00	u	3.747
P13	2.30	γ3	30.00	σ	63.737
δ2	35.29			s	3.492
δ3	52.80			ψ	144.790
α2	-39.00			U1x	1.658
α3	-62.00			U1y	3.360
				S1x	-2.853
				S1y	2.013

	x-coord	y-coord.
O4	1.195	-5.373
B1	2.853	-2.013
B2	2.853	-2.013
B3	0.951	-1.634
P1	0.000	0.000
P2	1.903	1.347
P3	1.389	1.830

0.0000	0.0000	-0.2229	0.6293	U1x	=	1.9027
0.0000	0.0000	-0.6293	-0.2229	U1y		1.3467
-0.1340	-0.5000	-0.5305	0.8829	S1x		1.3888
0.5000	-0.1340	-0.8829	-0.5305	S1y		1.8296

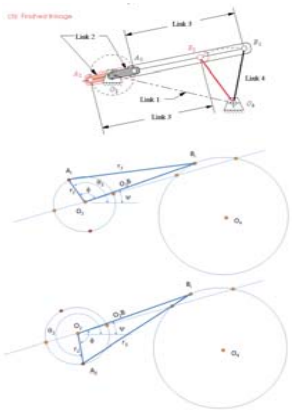
$$\begin{bmatrix} \cos\gamma_2 - 1 & -\sin\gamma_2 & \cos\alpha_2 - 1 & -\sin\alpha_2 \\ \sin\gamma_2 & \cos\gamma_2 - 1 & \sin\alpha_2 & \cos\alpha_2 - 1 \\ \cos\gamma_3 - 1 & -\sin\gamma_3 & \cos\alpha_3 - 1 & -\sin\alpha_3 \\ \sin\gamma_3 & \cos\gamma_3 - 1 & \sin\alpha_3 & \cos\alpha_3 - 1 \end{bmatrix} \cdot \begin{Bmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{Bmatrix} = \begin{Bmatrix} p_{21} \cdot \cos\delta_2 \\ p_{21} \cdot \sin\delta_2 \\ p_{31} \cdot \cos\delta_3 \\ p_{31} \cdot \sin\delta_3 \end{Bmatrix}$$

KINEMATIC ANALYSIS - CRITICAL POSITIONS					Normal (r)		Perpendicular (θ)	
	x-coord	y-coord.	mag	angle	i	j	i	j
O4	1.195	-5.373	5.504	-77.5	0.2170	-0.9762	0.9762	0.2170
B1	2.853	-2.013	3.492	-35.2	0.8170	-0.5766	0.5766	0.8170
B2	2.853	-2.013	3.492	-35.2	0.8170	-0.5766	0.5766	0.8170
B3	0.951	-1.634	1.891	-59.8	0.5028	-0.8644	0.8644	0.5028
P1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
P2	1.903	1.347	2.331	35.3	0.8162	0.5777	-0.5777	0.8162
P3	1.389	1.830	2.297	52.8	0.6046	0.7965	-0.7965	0.6046

NON-QUICK-RETURN (From Three Position Results)

	X-pos	Y-pos	mag	angle	i	j
3P-O2 => O4	-3.407	-2.139	4.022	-147.9	-0.8469	-0.5318
3P-A1	-0.297	-3.200	3.214	-95.3	-0.0925	-0.9957
3P-A2	-0.342	-0.954	1.013	-109.8	-0.3380	-0.9411
3P-A3	-1.577	0.590	1.683	159.5	-0.9367	0.3502

Factors		
P	0.5	% dist up Link 4
K	2.5	Length of Link 3+Link 2 wrt B1B2
Link 1	4.207	
Link 2	1.000	
Link 3	4.000	
Link 4	1.643	Grashof



ϕ=	86.0
ψ=	-66.4
θ2i=	19.6
θ2ii=	207.5

$$r_3^2 = r_2^2 + (O_2B)^2 - 2 \cdot r_2 \cdot (O_2B) \cdot \cos\phi$$

$$\phi = \cos^{-1} \frac{r_2^2 + (O_2B)^2 - r_3^2}{2 \cdot r_2 \cdot (O_2B)}$$

$$\psi = \tan^{-1} \frac{B_{1y} - O_{2y}}{B_{1x} - O_{2x}}$$

$$\theta_{2i} = \phi + \psi$$

$$\theta_{2ii} = 360 + \psi - \phi$$

$\dot{\theta}_2$ =	1.047 1/s
$\ddot{\theta}_2$ =	0.000 1/s^2
ω3-1	-0.262 1/s
ω3-i	0.007 1/s
ω3-ii	0.030 1/s
ω3-2	0.262 1/s
ω4-1	0.000 1/s
ω4-i	0.641 1/s
ω4-ii	-0.655 1/s
ω4-2	0.000 1/s
α3-1	0.263 1/s^2
α3-i	-0.108 1/s^2
α3-ii	0.469 1/s^2
α3-2	0.158 1/s^2
α4-1	1.051 1/s^2
α4-i	-0.037 1/s^2
α4-ii	-0.259 1/s^2
α4-2	-0.631 1/s^2

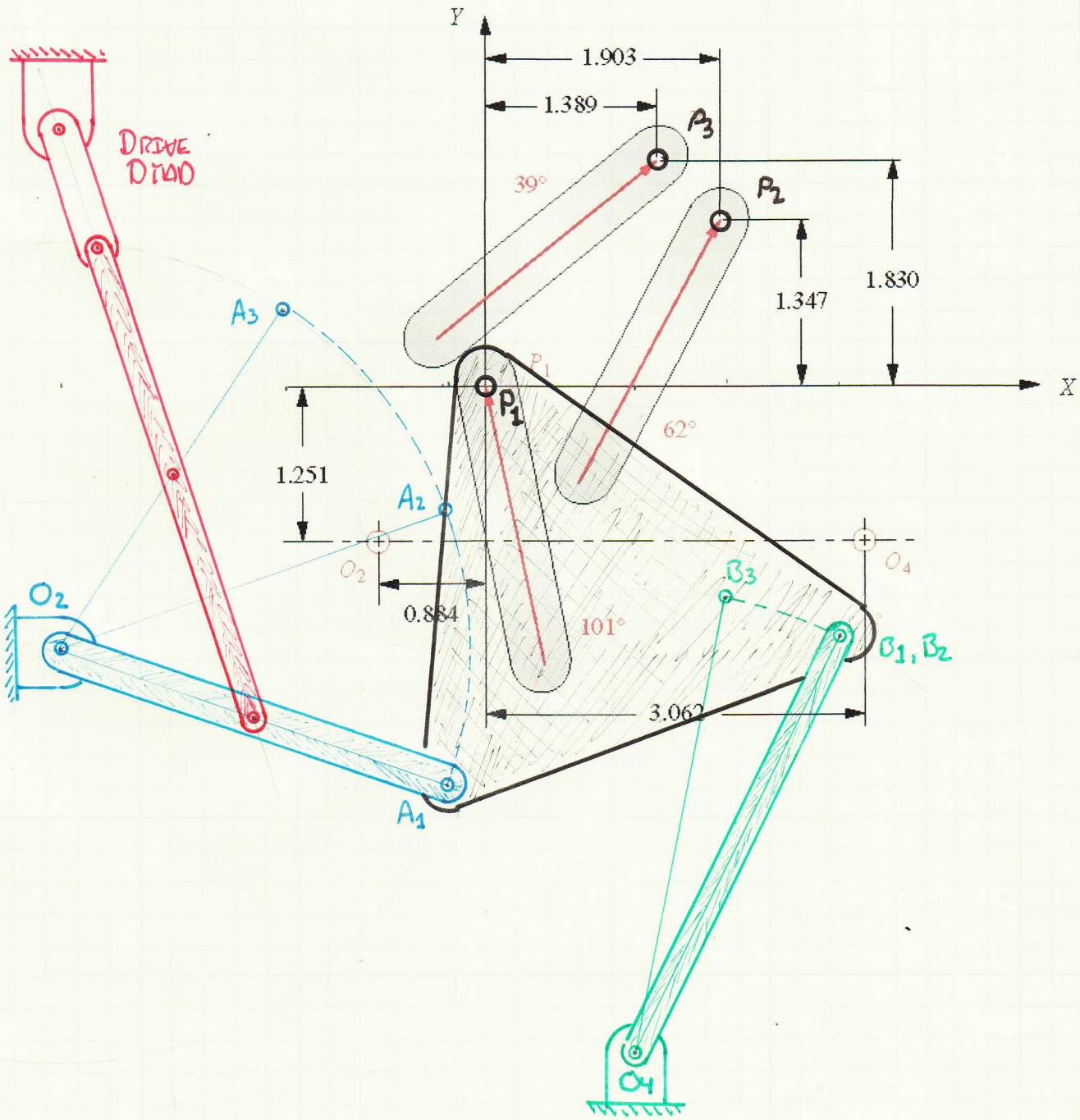
					Normal (r)		Perpendicular (θ)	
	x comp	y comp	mag	angle	i	j	i	j
rO4	**-3.407**	**-2.139**	**4.022**	**-147.9**	**-0.8469**	**-0.5318**	**0.5318**	**-0.8469**
rP3O2-A1	3.109	-1.061	3.285	-18.8	0.9464	-0.3230	0.3230	0.9464
rP3O2-A2	3.064	1.186	3.285	21.2	0.9326	0.3609	-0.3609	0.9326
rP3O2-A3	1.830	2.729	3.285	56.2	0.5570	0.8305	-0.8305	0.5570
rB1	**-1.852**	**-2.670**	**3.249**	**-124.7**	**-0.5699**	**-0.8217**	**0.8217**	**-0.5699**
rO4B1	1.555	-0.531	1.643	-18.8	0.9464	-0.3230	0.3230	0.9464
rB2	**-2.492**	**-0.775**	**2.609**	**-162.7**	**-0.9549**	**-0.2970**	**0.2970**	**-0.9549**
rO4B2	0.915	1.364	1.643	56.2	0.5570	0.8305	-0.8305	0.5570
rBi	**-1.874**	**-1.546**	**2.430**	**-140.5**	**-0.7714**	**-0.6364**	**0.6364**	**-0.7714**
rO4Bi	1.532	0.593	1.643	21.2	0.9326	0.3609	-0.3609	0.9326
rB1B2	-0.640	1.895	2.000	108.7	-0.3198	0.9475	-0.9475	-0.3198
rO2	**-3.451**	**2.068**	**4.023**	**149.1**	**-0.8578**	**0.5139**	**-0.5139**	**-0.8578**
rB102	-1.599	4.737	5.000	108.7	-0.3198	0.9475	-0.9475	-0.3198
rBi02	-1.577	3.614	3.943	113.6	-0.3999	0.9166	-0.9166	-0.3999
rB202	-0.960	2.842	3.000	108.7	-0.3198	0.9475	-0.9475	-0.3198
rA1	**-3.131**	**1.120**	**3.326**	**160.3**	**-0.9416**	**0.3368**	**-0.3368**	**-0.9416**
rO2A1	0.320	-0.947	1.000	-71.3	0.3198	-0.9475	0.9475	0.3198
rA2	**-3.771**	**3.015**	**4.828**	**141.4**	**-0.7810**	**0.6245**	**-0.6245**	**-0.7810**
rO2A2	-0.320	0.947	1.000	108.7	-0.3198	0.9475	-0.9475	-0.3198
rAi	-2.509	2.403	3.474	136.2	-0.7222	0.6917	-0.6917	-0.7222
rO2Ai	0.942	0.335	1.000	19.6	0.9421	0.3354	-0.3354	0.9421
rAii	-4.338	1.605	4.625	159.7	-0.9378	0.3471	-0.3471	-0.9378
rO2Aii	-0.887	-0.462	1.000	-152.5	-0.8867	-0.4624	0.4624	-0.8867
rB1A1	-1.279	3.790	4.000	108.7	-0.3198	0.9475	-0.9475	-0.3198
rBiAi	-0.635	3.949	4.000	99.1	-0.1586	0.9873	-0.9873	-0.1586
rBiAii	-2.463	3.152	4.000	128.0	-0.6158	0.7879	-0.7879	-0.6158
rB2A2	-1.279	3.790	4.000	108.7	-0.3198	0.9475	-0.9475	-0.3198
rO4O2	-0.045	4.207	4.207	90.6	-0.0106	0.9999	-0.9999	-0.0106

Kinematics					Normal (r)		Perpendicular (θ)	
	x comp	y comp	mag	angle	i	j	i	j
r1	0.045	-4.207	4.207	-89.4	0.0106	-0.9999	0.9999	0.0106
r4-1	1.555	-0.531	1.643	-18.8	0.9464	-0.3230	0.3230	0.9464
r4-i	1.532	0.593	1.643	21.2	0.9326	0.3609	-0.3609	0.9326
r4-2	0.915	1.364	1.643	56.2	0.5570	0.8305	-0.8305	0.5570
r2-1	0.320	-0.947	1.000	-71.3	0.3198	-0.9475	0.9475	0.3198
r2-i	0.942	0.335	1.000	19.6	0.9421	0.3354	-0.3354	0.9421
r2-ii	-0.887	-0.462	1.000	-152.5	-0.8867	-0.4624	0.4624	-0.8867
r2-2	-0.320	0.947	1.000	108.7	-0.3198	0.9475	-0.9475	-0.3198
r3-1	1.279	-3.790	4.000	-71.3	0.3198	-0.9475	0.9475	0.3198
r3-i	-0.635	3.949	4.000	99.1	-0.1586	0.9873	-0.9873	-0.1586
r3-ii	-2.463	3.152	4.000	128.0	-0.6158	0.7879	-0.7879	-0.6158
r3-2	1.279	-3.790	4.000	-71.3	0.3198	-0.9475	0.9475	0.3198
vA-1	0.992	0.335	1.047	18.7	0.9475	0.3198	-0.3198	0.9475
vA-i	-0.351	0.986	1.047	109.6	-0.3354	0.9421	-0.9421	-0.3354
vA-ii	0.484	-0.928	1.047	-62.5	0.4624	-0.8867	0.8867	0.4624
vA-2	-0.992	-0.335	1.047	-161.3	-0.9475	-0.3198	0.3198	-0.9475
vB-1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
vB-i	-0.380	0.982	1.053	111.2	-0.3609	0.9326	-0.9326	-0.3609
vB-ii	0.388	-1.003	1.076	-68.8	0.3609	-0.9326	0.9326	0.3609
vB-2	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
aA-1	-0.351	1.039	1.096	108.7	-0.3198	0.9475	-0.9475	-0.3198
aA-i	-1.033	-0.368	1.096	-160.4	-0.9421	-0.3354	0.3354	-0.9421
aA-ii	0.972	0.507	1.096	27.5	0.8867	0.4624	-0.4624	0.8867
aA-2	0.351	-1.039	1.096	-71.3	0.3198	-0.9475	0.9475	0.3198
aB-1	0.558	1.635	1.727	71.2	0.3230	0.9464	-0.9464	0.3230
aB-i	-0.607	-0.300	0.677	-153.7	-0.8968	-0.4424	0.4424	-0.8968
aB-ii	-0.504	-0.651	0.823	-127.7	-0.6117	-0.7911	0.7911	-0.6117
aB-2	0.861	-0.577	1.036	-33.8	0.8305	-0.5570	0.5570	0.8305

SYNTHESIZED LINKAGE KINEMATICS

Input from Drive Link	DLω4-1 => ω2-1	0.0000 1/s
	DLω4-2i => ω2-2i	0.6408 1/s
	DLω4-2ii => ω2-2ii	-0.6549 1/s
	DLω4-3 => ω2-3	0.0000 1/s
	DLα4-1 => α2-1	1.0514 1/s^2
	DLα4-2i => α2-2i	-0.0367 1/s^2
	DLα4-2ii => α2-2ii	-0.2590 1/s^2
	DLα4-3 => α2-3	-0.6309 1/s^2
Angular Velocity	ω3-1	0.0000 1/s
	ω3-2i	-0.4273 1/s
	ω3-2ii	0.4367 1/s
	ω3-3	0.0000 1/s
	ω4-1	0.0000 1/s
	ω4-2i	0.3609 1/s
	ω4-2ii	-0.3688 1/s
	ω4-3	0.0000 1/s
Angular Acceleration	α3-1	-1.4898 1/s^2
	α3-2i	0.2014 1/s^2
	α3-2ii	0.3576 1/s^2
	α3-3	0.5318 1/s^2
	α4-1	-0.8585 1/s^2
	α4-2i	0.4074 1/s^2
	α4-2i	0.3012 1/s^2
	α4-3	-0.7767 1/s^2

		x comp	y comp	mag	angle	Normal (r) i	Normal (r) j	Perpendicular (θ) i	Perpendicular (θ) j
Displacements	r1	4.601	-3.234	5.624	-35.1	0.8181	-0.5750	0.5750	0.8181
	r4-1	1.658	3.360	3.747	63.7	0.4425	0.8968	-0.8968	0.4425
	r4-2i	1.658	3.360	3.747	63.7	0.4425	0.8968	-0.8968	0.4425
	r4-2ii	1.658	3.360	3.747	63.7	0.4425	0.8968	-0.8968	0.4425
	r4-3	-0.244	3.739	3.747	93.7	-0.0652	0.9979	-0.9979	-0.0652
	r2-1	3.109	-1.061	3.285	-18.8	0.9464	-0.3230	0.3230	0.9464
	r2-2i	3.064	1.186	3.285	21.2	0.9326	0.3609	-0.3609	0.9326
	r2-2ii	3.064	1.186	3.285	21.2	0.9326	0.3609	-0.3609	0.9326
	r2-3	1.830	2.729	3.285	56.2	0.5570	0.8305	-0.8305	0.5570
	r3-1	3.150	1.187	3.366	20.7	0.9357	0.3527	-0.3527	0.9357
	r3-2i	3.195	-1.060	3.366	-18.3	0.9492	-0.3148	0.3148	0.9492
	r3-2ii	3.195	-1.060	3.366	-18.3	0.9492	-0.3148	0.3148	0.9492
	r3-3	2.527	-2.224	3.366	-41.3	0.7507	-0.6606	0.6606	0.7507
	rAP-1	0.297	3.200	3.214	84.7	0.0925	0.9957	-0.9957	0.0925
	rAP-2i	2.245	2.300	3.214	45.7	0.6985	0.7156	-0.7156	0.6985
	rAP-2ii	2.245	2.300	3.214	45.7	0.6985	0.7156	-0.7156	0.6985
	rAP-3	2.965	1.240	3.214	22.7	0.9226	0.3858	-0.3858	0.9226
Velocities	vA-1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
	vA-2i	-0.760	1.964	2.105	111.2	-0.3609	0.9326	-0.9326	-0.3609
	vA-2ii	0.776	-2.007	2.152	-68.8	0.3609	-0.9326	0.9326	0.3609
	vA-3	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
	vB-1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
	vB-2i	-1.213	0.598	1.352	153.7	-0.8968	0.4425	-0.4425	-0.8968
	vB-2ii	1.239	-0.611	1.382	-26.3	0.8968	-0.4425	0.4425	0.8968
	vB-3	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
	vP-1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
	vP-2i	0.223	1.004	1.029	77.5	0.2168	0.9762	-0.9762	0.2168
	vP-2ii	-0.228	-1.026	1.051	-102.5	-0.2168	-0.9762	0.9762	-0.2168
	vP-3	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
Accelerations	aA-1	1.116	3.269	3.454	71.2	0.3230	0.9464	-0.9464	0.3230
	aA-2i	-1.215	-0.599	1.355	-153.7	-0.8968	-0.4424	0.4424	-0.8968
	aA-2ii	-1.007	-1.302	1.646	-127.7	-0.6117	-0.7911	0.7911	-0.6117
	aA-3	1.721	-1.154	2.073	-33.8	0.8305	-0.5570	0.5570	0.8305
	aB-1	2.885	-1.423	3.217	-26.3	0.8968	-0.4425	0.4425	0.8968
	aB-2i	-1.585	0.238	1.602	171.5	-0.9889	0.1485	-0.1485	-0.9889
	aB-2ii	-1.237	0.042	1.238	178.0	-0.9994	0.0342	-0.0342	-0.9994
	aB-3	2.904	0.190	2.910	3.7	0.9979	0.0652	-0.0652	0.9979
	aP-1	5.884	2.826	6.528	25.7	0.9014	0.4330	-0.4330	0.9014
	aP-2i	-2.088	-0.567	2.164	-164.8	-0.9651	-0.2620	0.2620	-0.9651
	aP-2ii	-2.258	-0.938	2.445	-157.4	-0.9235	-0.3837	0.3837	-0.9235
	aP-3	1.062	0.423	1.143	21.7	0.9291	0.3699	-0.3699	0.9291


Y
X
1.903
1.389
1.830
1.347
1.251
0.884
3.062
39°
62°
101°
P1
P2
P3
A1
A2
A3
O2
O4
B1, B2
B3
DRIVE
DYAD