PROBLEM 3-20a THE STRESS STATE AT A POINT IS $\sigma_x = -6$ ksi, $\sigma_y = 18$ ksi, $\sigma_z = -12$ ksi, $\tau_{xy} = 9$ ksi, $\tau_{yz} = 6$ ksi, $\tau_{zx} = -15$ ksi. TRANSFORM THIS STATE OF STRESS 1) A POSITIVE 30° ABOUT THE X-AXIS, 2) 45° ABOUT THE NEW Z AXIS, AND FINALLY 3) 60° ABOUT THE NEW Y-AXIS

GIVEN

1. THE STATE OF STRESS $\begin{bmatrix} -6 & 9 & -15 \\ 9 & 18 & 6 \\ -15 & 6 & -12 \end{bmatrix}$ ksi

2. TRANSFORMATION: i) 30° x, ii) 45° ABOUT NEW z, iii) 60° ABOUT NEW y

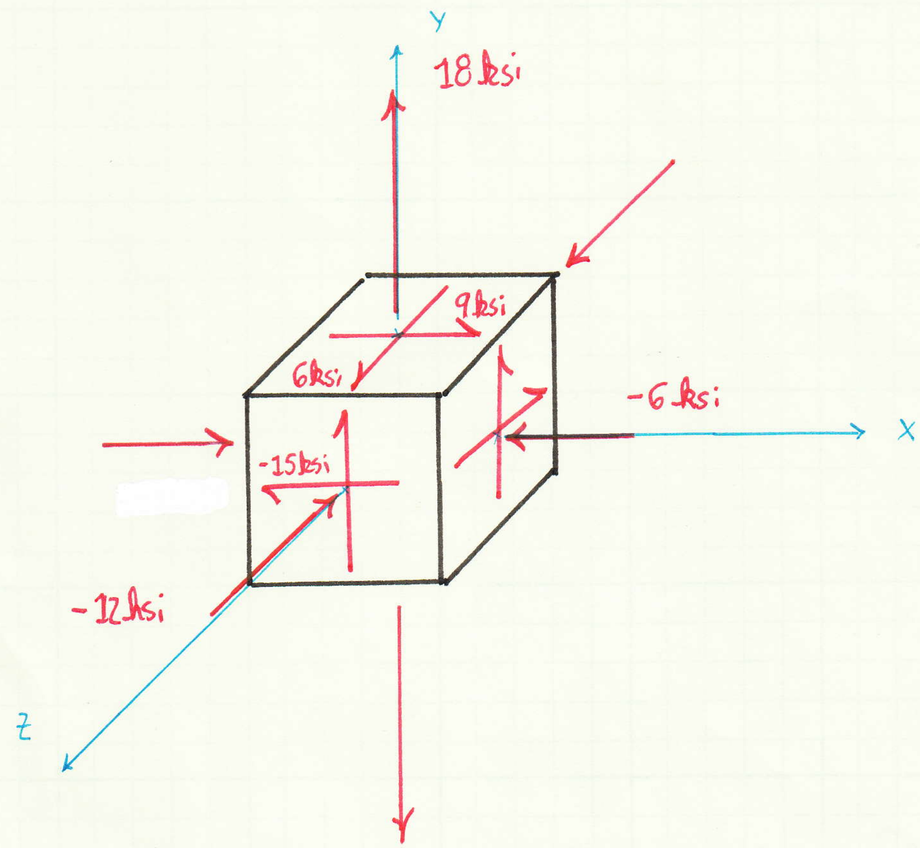
ASSUMPTIONS:

1. THE MATERIAL IS IN EQUILIBRIUM

FIND:

1. TRANSFORMED STATE OF STRESS.

FIGURE:



a

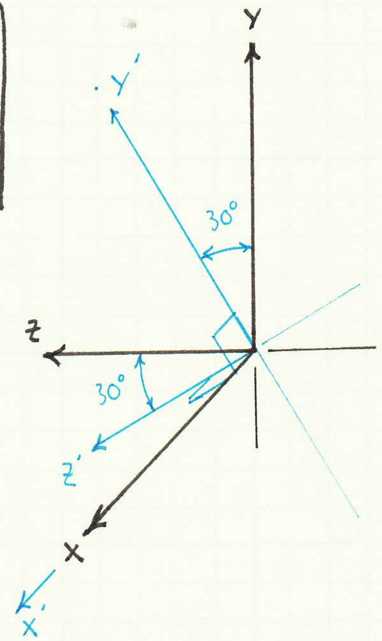
SOLUTION:

WRITING THE TRANSFORMATION MATRIX FOR THE INITIAL 30° ROTATION ABOUT THE X-AXIS. FIGURE (b) ILLUSTRATES THE INITIAL TRANSFORMATION

$$[T1] = \begin{bmatrix} \cos\theta_{x'x} & \cos\theta_{x'y} & \cos\theta_{x'z} \\ \cos\theta_{y'x} & \cos\theta_{y'y} & \cos\theta_{y'z} \\ \cos\theta_{z'x} & \cos\theta_{z'y} & \cos\theta_{z'z} \end{bmatrix}$$

$$= \begin{bmatrix} \cos 0 & \cos 90 & \cos 90 \\ \cos 90 & \cos 30 & \cos 60 \\ \cos 90 & \cos 120 & \cos 30 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & 0.5 \\ 0 & -0.5 & 0.866 \end{bmatrix} \quad (1)$$



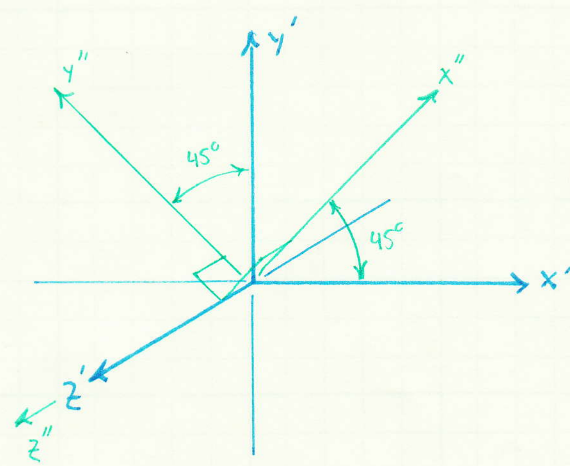
(b)

WRITING THE SECOND TRANSFORM FOR A 45° ROTATION ABOUT THE NEW Z AXIS (Z'). FIGURE (c) ILLUSTRATES THIS TRANSFORMATION.

$$[T2] = \begin{bmatrix} \cos\theta_{x''x'} & \cos\theta_{x''y'} & \cos\theta_{x''z'} \\ \cos\theta_{y''x'} & \cos\theta_{y''y'} & \cos\theta_{y''z'} \\ \cos\theta_{z''x'} & \cos\theta_{z''y'} & \cos\theta_{z''z'} \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45^\circ & \cos 45^\circ & \cos 90 \\ \cos 135^\circ & \cos 45^\circ & \cos 90 \\ \cos 90 & \cos 90 & \cos 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7071 & 0.7071 & 0 \\ -0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$



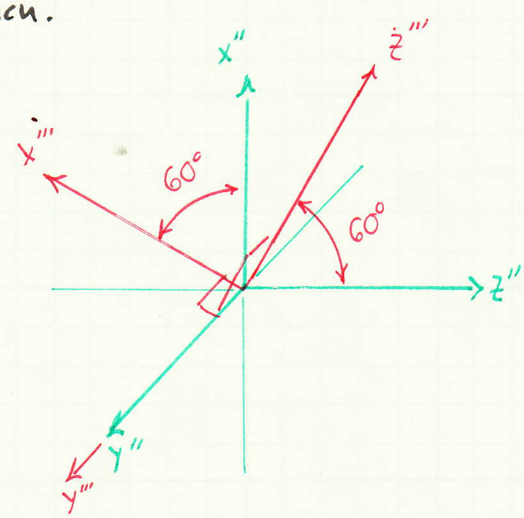
(c)

THE THIRD AND FINAL TRANSFORMATION IS 60° ABOUT THE NEW Y-AXIS (y''). FIGURE (d) ILLUSTRATES THIS TRANSFORMATION.

$$[T3] = \begin{bmatrix} \cos\theta_{x'''x''} & \cos\theta_{x'''y''} & \cos\theta_{x'''z''} \\ \cos\theta_{y'''x''} & \cos\theta_{y'''y''} & \cos\theta_{y'''z''} \\ \cos\theta_{z'''x''} & \cos\theta_{z'''y''} & \cos\theta_{z'''z''} \end{bmatrix}$$

$$= \begin{bmatrix} \cos 60 & \cos 90 & \cos 150 \\ \cos 90 & \cos 0 & \cos 90 \\ \cos 30 & \cos 90 & \cos 60 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & -0.866 \\ 0 & 1 & 0 \\ 0.866 & 0 & 0.5 \end{bmatrix} \quad (3)$$



THE TOTAL TRANSFORMATION CAN BE WRITTEN AS A SINGLE TRANSFORMATION MATRIX, USING (1), (2) & (3)

$$[TT] = [T3][T2][T1] \quad (4)$$

THE MATHEMATICAL OPERATIONS USED TO PEFORM THE OPERATION SUGGESTED IN (4) ALONG WITH THE CALCULATION OF THE TRANSFORMED STRESS STATE ARE PERFORMED USING MATLAB AND INCLUDED ON THE NEXT PAGE.

<u>SUMMARY:</u>

BECAUSE ROTATIONS ARE NOT VECTORS, THE ORDER OF THE TRANSFORMATION IS VERY IMPORTANT. THE MATLAB RESULT ON THE NEXT PAGE FIRST COMBINES ~~THE~~ THE THREE TRANSFORMATIONS INTO

```
>> S=[-6 9 -15; 9 18 6; -15 6 -12]

S =
```

$$\begin{bmatrix} -6 & 9 & -15 \\ 9 & 18 & 6 \\ -15 & 6 & -12 \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} = [\sigma] = \sigma_{xyz}$$

```
>> T1=[1 0 0; 0 .866 .5; 0 -.5 .866]

T1 =

    1.0000         0         0
         0    0.8660    0.5000
         0   -0.5000    0.8660

>> T2=[.7071 .7071 0; -.7071 .7071 0; 0 0 1]

T2 =

    0.7071    0.7071         0
   -0.7071    0.7071         0
         0         0    1.0000

>> T3=[.5 0 -.866; 0 1 0 ; .866 0 .5]

T3 =

    0.5000         0   -0.8660
         0    1.0000         0
    0.8660         0    0.5000

>> TT=T3*T2*T1

TT =

    0.3535    0.7392   -0.5732
   -0.7071    0.6123    0.3535
    0.6123    0.2803    0.7392

>> St=TT*S*TT'

St =
```

$$\begin{bmatrix} 10.8417 & 0.8312 & 16.1392 \\ 0.8312 & 4.5535 & 12.0454 \\ 16.1392 & 12.0454 & -15.3955 \end{bmatrix} = \begin{bmatrix} \sigma_{x'''} & \tau_{x'''y'''} & \tau_{x'''z'''} \\ \tau_{x'''y'''} & \sigma_{y'''} & \tau_{y'''z'''} \\ \tau_{x'''z'''} & \tau_{y'''z'''} & \sigma_{z'''} \end{bmatrix} = [\sigma'''] = \sigma_{x'''y'''z'''}$$

```
>>
```