PROBLEM 8.5-18 A horizontal bracket ABC consists of two perpendicular arms AB and BC, the later having a length of 0.4m. Arm AB has a solid circular cross-section with diameter equal to 60mm. At point C a load $P_1 = 2.02$ kN acts vertically and a load $P_2 = 3.07$ kN acts horizontally, parallel to arm AB. Considering only the forces P_1 and P_2, calculate the maximum tensile stress σ_t, the maximum compressive stress σ_c, and the maximum in-plane shear stress τ_{max} at point P, which is located at support A on the side of the bracket at midheight.

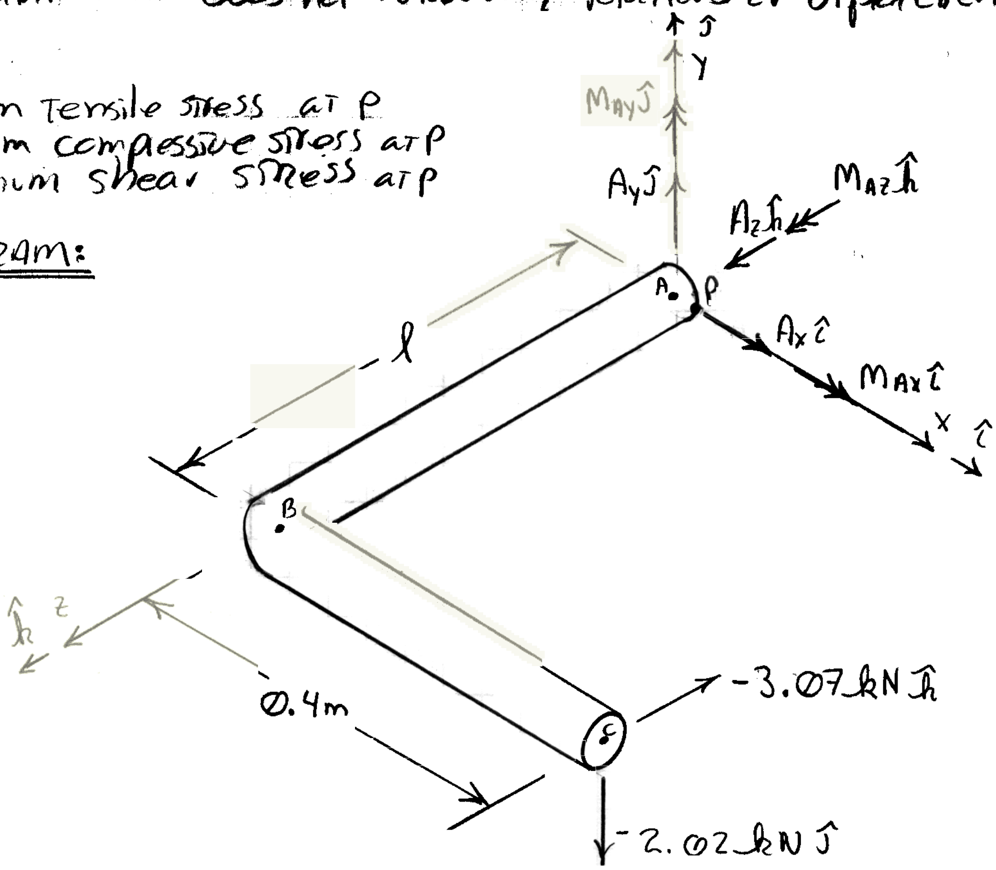
GIVEN:

1) CONSTRAINTS
   - Bracket that is fixed into a wall
   - Loads are applied in a cantilever manner
2) ASSUMPTIONS
   - Linear elastic material response
   - The constraint at A does not allow any rotations or displacements

FIND:

1) The maximum tensile stress at P
2) The maximum compressive stress at P
3) The maximum shear stress at P

FREE BODY DIAGRAM:



STATICS:

The problem starts by solving for the reactions at A. Imposing Equilibrium

$$\Sigma F_x = 0 = A_x \qquad (1)$$

$$\Sigma F_y = 0 = A_y - 2.02\ kN \Rightarrow A_y = 2.02\ kN \qquad (2)$$

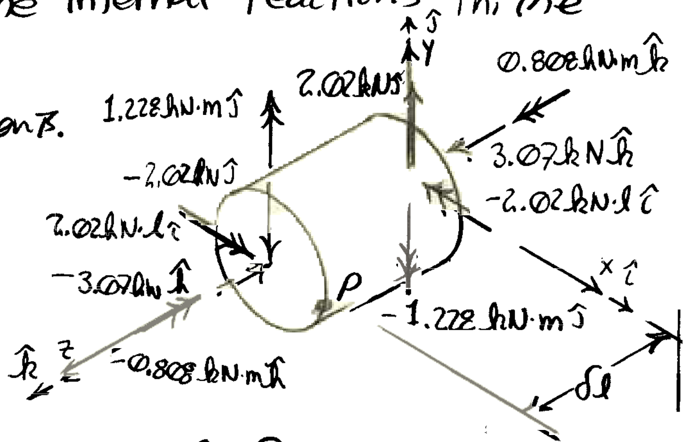
$$\Sigma F_z = 0 = A_z - 3.07\ kN \Rightarrow A_z = 3.07\ kN \qquad (3)$$

$$\Sigma M_x/_{at\ A} = 0 = M_{Ax} + 2.02\ kN\cdot\ell \Rightarrow M_{Ax} = -2.02\ kN\cdot\ell \qquad (4)$$

$$\Sigma M_y/_{at\ A} = 0 = M_{Ay} + (0.4m)(3.07\ kN) \Rightarrow M_{Ay} = -1.228\ kN\cdot m \qquad (5)$$

$$\Sigma M_z/_{at\ A} = 0 = M_{Az} - (0.4m)(2.02\ kN) \Rightarrow M_{Az} = 0.808\ kN\cdot m \qquad (6)$$

We have just found the reactions at A. To determine the stresses at P we need to determine the internal reactions in the beam just off the wall. The Free body diagram to the right Illustrates these forces and moments. Now we need to consider the stresses each of these forces and moments cause at point P.



MECHANICS:

The force in the z direction along with the moments in the x and y directions give rise to normal stress in the z direction as follows

$$\sigma_z = \frac{F_z}{A} + \frac{M_x \cdot y}{I_{xx}} - \frac{M_y \cdot x}{I_{yy}} \qquad (7)$$

Since point P is on the x axis ($y = 0$) the second term in (7) equals zero. The normal stress at point P is

$$\sigma_z = \frac{-3.07\ kN}{\pi(0.03m)^2} - \frac{(1.228\ kN\cdot m)(0.03m)}{\frac{\pi(0.06m)^4}{64}} = -59.0(10^3)\ \frac{kN}{m^2}$$

$$\sigma_z = -59.0\ MPa \qquad (8)$$

The forces in the x and y directions along with the couple in the z direction all give rise to shearing stresses. Starting with the forces, there is no x directed force therefore the shear stress that results from forces is given by

$$\tau_{yz} = \frac{V \cdot Q}{I \cdot t} = \frac{(2.02\,kN)\left(\frac{4\cdot(0.03m)}{3\pi}\right)\cdot\frac{\pi}{2}(0.03m)^2}{\frac{\pi\cdot(0.06m)^4}{64}\cdot 0.06m}$$

$$= 952.6\ \frac{kN}{m^2}$$

$$= \underline{0.9526\ MPa}$$

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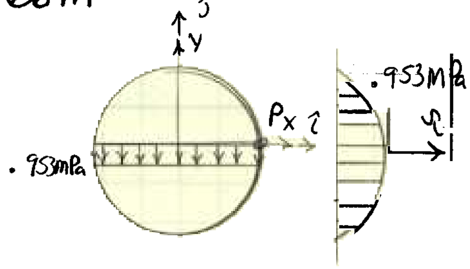

The shearing stress generated by the couple in the z direction is given by

$$\tau = \frac{T\cdot r}{J} = \frac{(0.808\,kN\cdot m)(0.03m)}{\frac{\pi\cdot(0.06m)^4}{32}} = \underline{19.05\,MPa}$$

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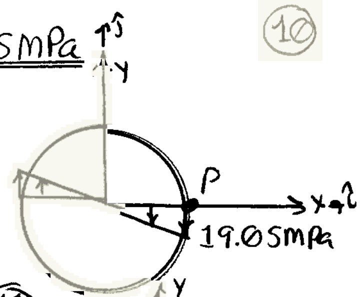

From the diagrams we see that the total shear stress at P is given by

$$\tau_{yz} = 0.9526\,MPa + 19.05\,MPa = 20.0\,MPa$$

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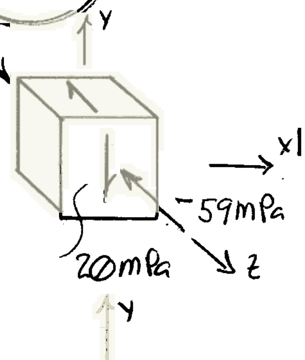
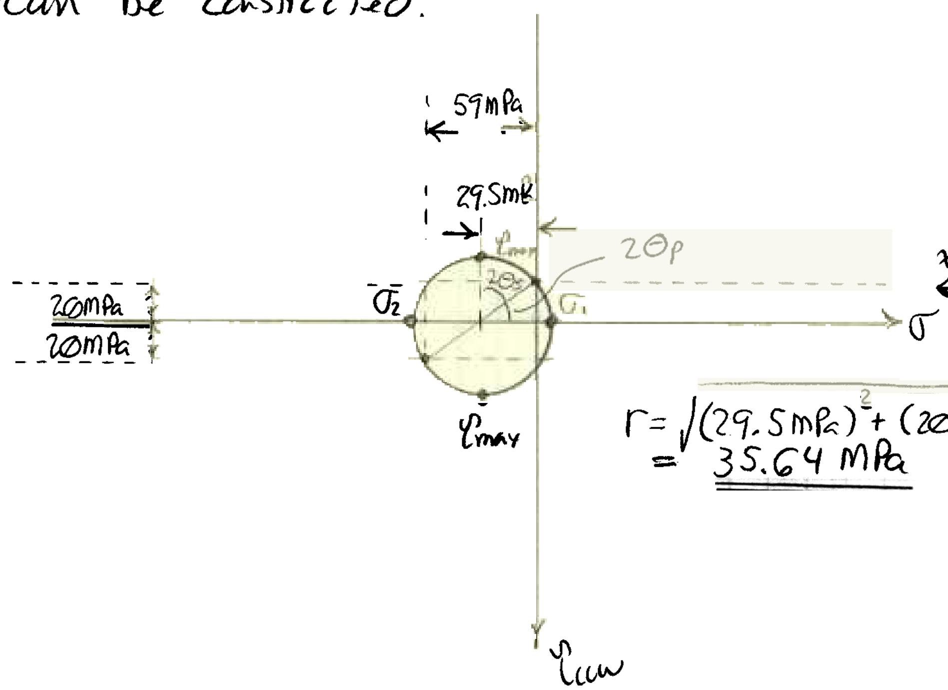
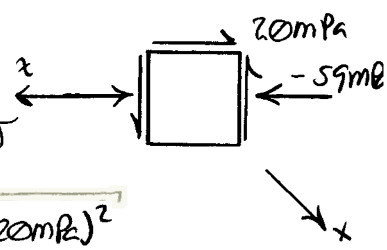


The stress element at this point can now be constructed using 8 and 11. Now Mohr's circle can be constructed.





$$r = \sqrt{(29.5\,MPa)^2 + (20\,MPa)^2}$$
$$= \underline{\underline{35.64\,MPa}}$$

From the circle the maximum tensile stress is given by

$$\sigma_1 = \sigma_z = \sigma_t = -29.5\,mPa + 35.64\,mPa = 6.141\,mPa$$

$$\boxed{\sigma_t = 6.14\,mPa}$$

The maximum compressive stress is given by

$$\sigma_2 = \sigma_z = \sigma_c = -29.5\,mPa - 35.64\,mPa = -65.14\,mPc$$

$$\boxed{\sigma_c = -65.1\,MPa}$$

And the maximum shear stress is given by

$$\boxed{\tau_{yz} = 35.6\,MPa}$$

SUMMARY:

The length of the bracket from A to B was not required in the solution to this problem because of where P was located. No matter what the length from A to B the component of normal stress that was effected was z since P was located along the centroidal axis parallel to this load. Once the stress were determined from the various components of forces and moments the stress element at point P was constructed and Mohr's circle was drawn. This enabled the principal stresses and shearing strains to be determined.