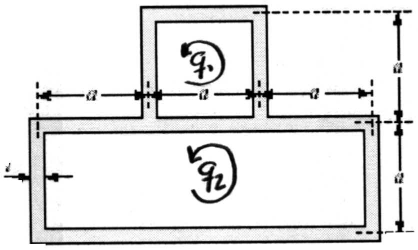
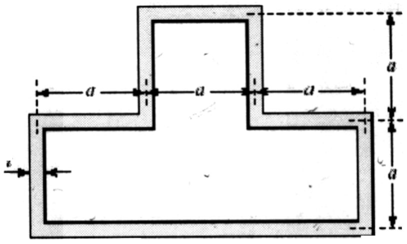
NAME: Solution

Problem 1: For the two beam sections shown, calculate the shear stress in each of the members of both sections given that a torque T is applied to the structures.



(a)



(b)

FOR

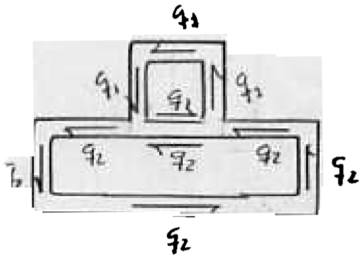
$$q_b = \frac{T}{2\cdot A} = \frac{T}{2\cdot(a^2+3a^2)} = \frac{T}{8a^2} \Rightarrow \boxed{\tau_b = \frac{q_b}{t} = \frac{T}{8\cdot t\cdot a^2}} \qquad (1)$$

$$\Theta_B = \frac{(1+\nu)\cdot T\cdot L\cdot 10\cdot a}{2\cdot E\cdot (4\cdot a^2)^2\, t} = \frac{(1+\nu)\cdot T\cdot L}{E\cdot a^3\cdot t}\cdot\frac{5}{16} \qquad (2)$$

FOR (a) IT IS NECESSARY TO COMPUTE THE q'S IN EACH SEGMENT FIRST. STARTING WITH THE CALCULATION OF THE TORQUE

$$T = 2[q_1\cdot a^2 + q_2\cdot 3a^2] = 2\cdot q_1\cdot a^2 + 6\cdot q_2\cdot a^2 \qquad (3)$$

NOW CALCULATING THE ANGLE OF TWIST FOR EACH SEGMENT



$$\Theta = \frac{(1+\nu)\cdot L}{E\,a^2}\left[\frac{3\cdot q_1\cdot a}{t} + \frac{a(q_1-q_2)}{t}\right]$$

$$\Theta = \frac{(1+\nu)\cdot L}{E\cdot a\cdot t}[4\cdot q_1 - q_2]$$

$$\frac{\Theta\cdot E\cdot a\cdot t}{(1+\nu)\cdot L} = 4\cdot q_1 - q_2 = C \qquad (4)$$

$$\frac{(1+\nu)\cdot L}{3E a^2}\left[\frac{7\cdot q_2\cdot a}{t} + \frac{a(q_2-q_1)}{t}\right] = \frac{(1+\nu)\cdot L}{3E\cdot a\cdot t}[8\cdot q_2 - q_1]$$

$$\frac{\Theta\cdot E\cdot a\cdot t}{(1+\nu)\cdot L} = \frac{8q_2 - q_1}{3} = C \qquad (5)$$

EQUATING (4) & (5)

$$4q_1 - q_2 = \frac{8q_2 - q_1}{3} \Rightarrow 12q_1 - 3q_2 = 8q_2 - q_1 \Rightarrow 13q_1 = 11q_2$$

$$\underline{q_1 = \frac{11}{13}q_2} \qquad (6)$$

SUBSTITUTING THIS RESULT INTO (3)

$$T = 2 \cdot \frac{11}{13} \cdot q_2 \cdot a^2 + 6 \cdot q_2 \cdot a^2 = \left[\frac{22}{13} + \frac{78}{13}\right] q_2 \cdot a^2 = \frac{100}{13} \cdot q_2 \cdot a^2$$

$$\underline{\underline{q_2 = 0.13 \frac{T}{a^2}}} \quad (7)$$

SUBSTITUTING (7) INTO (6)

$$q_1 = \frac{11}{13} \cdot \frac{13}{100} \frac{T}{a^2} = \underline{\underline{0.11 \frac{T}{a^2}}} \quad (8)$$

HE ANGLE OF TWIST CAN NOW BE CALCULATED FROM (4)

$$\Theta = \frac{(1+\nu) \cdot L}{E \cdot a \cdot t} \left[4 \cdot 0.11 \frac{T}{a^2} - 0.13 \frac{T}{a^2} \right] = \underline{0.31 \cdot \frac{(1+\nu) \cdot L \cdot T}{E \cdot a^3 \cdot t}} \quad (9)$$

THE SHEAR STRESS IN THE UPPER 3 SEGMENTS CAN NOW BE CALCULATED

$$\tau_1 = \frac{q_1}{t} = \boxed{0.11 \frac{T}{a^2 \cdot t}} \quad (10)$$

THE SHEAR STRESS IN THE LOWER SEGMENTS

$$\tau_2 = \frac{q_2}{t} = \boxed{0.13 \frac{T}{a^2 \cdot t}} \quad (11)$$

AND THE SHEAR STRESS IN THE INTERSECTING SEGMENT

$$\tau_{12} = \frac{q_1 - q_2}{t} = (0.11 - 0.13) \frac{T}{a^2 \cdot t} = \boxed{-0.02 \frac{T}{a^2 t}}$$

Problem 2: Determine the torsional stiffness for both of these cross-sections. Which of the structures is stiffer?

DEFINING THE TORSIONAL STIFFNESS AS $K = \frac{T}{\theta}$

$$\frac{E \cdot a^3 \cdot t}{0.31 \cdot (1+\nu) \cdot L} = 3.226 \frac{E \cdot a^3 \cdot t}{(1+\nu) \cdot L} \qquad (1)$$

$$\frac{16}{5} \cdot \frac{E \cdot a^3 \cdot t}{(1+\nu) \cdot L} = 3.200 \frac{E \cdot a^3 \cdot t}{(1+\nu) \cdot L} \qquad (2)$$

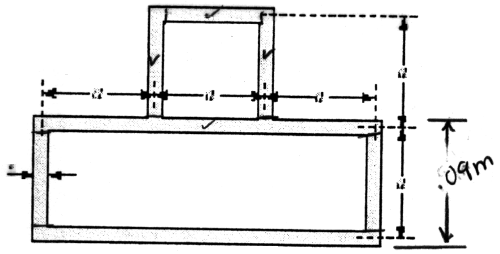
GIVEN $G = \frac{E}{2(1+\nu)}$

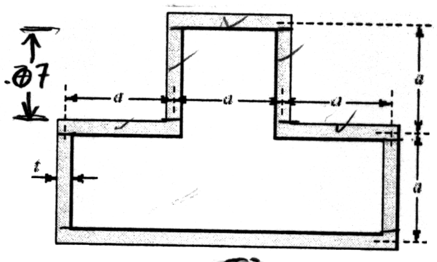
$$K_A = 6.452 \frac{G \cdot a^3 \cdot t}{L} \qquad (3)$$

$$K_B = 6.400 \frac{G \cdot a^3 \cdot t}{L} \qquad (4)$$

Problem 3: For the structures given in Problem 1, let a=80mm, t=10mm, G=80GPa, and T=4kN-m. Assume that in addition a horizontal moment is applied through the centroid directed to the right. Draw the stress cube and Mohr's circle for an element on the to segment of each of the cross-sections.

c





FOR CROSS SECTION @

$$\bar{Y}_a = \frac{(.07m)(.01m)(.165m) + 2\cdot(.08m)(.01m)(.13m) + (.25m)(.01m)(.085m) + 2(.07m)(.01m)(.045m) + (.25m)(.01m)(.005m)}{(.07m)(.01m) + 2(.08m)(.01m) + (.25)(.01m) + 2\cdot(.07m)(.01m) + (.25m)(.01m)}$$

$\underline{\underline{0.07029m}} = 70.29mm$ ①

$$I_a = \frac{1}{12}(.09m)(.08m)^3 + (.09m)(.08m)(.13m - .07029m)^2$$
$$\frac{1}{12}(.07m)(.07m)^3 - (.07m)(.07m)(.125m - .07029m)^2$$
$$+ \frac{1}{12}(0.25m)(.09m)^3 + (0.25m)(.09m)(.045m - .07029m)^2$$
$$- \frac{1}{12}(0.23m)(.07m)^3 - (0.23m)(.07m)(.045m - .07029m)^2$$
$= \underline{25.55(10^{-6})m^4}$ ②

FOR CROSS SECTION ⓑ

$$\bar{Y}_b = \frac{(.09m)(.01m)(.165m) + 2\cdot(.01)(.07m)(.125m) + 2\cdot(.09m)(.01m)(.085m) + 2(.01)(.07)(.045) + (.25m)(.01m)(.005m)}{(.09)(.01m) + 2\cdot(.01m)(.07m) + 2(.09m)(.01m) + 2(.01)(.07) + (.25m)(.01m)(}$$

$= \underline{\underline{0.06900m}} = 69.00mm$ ③

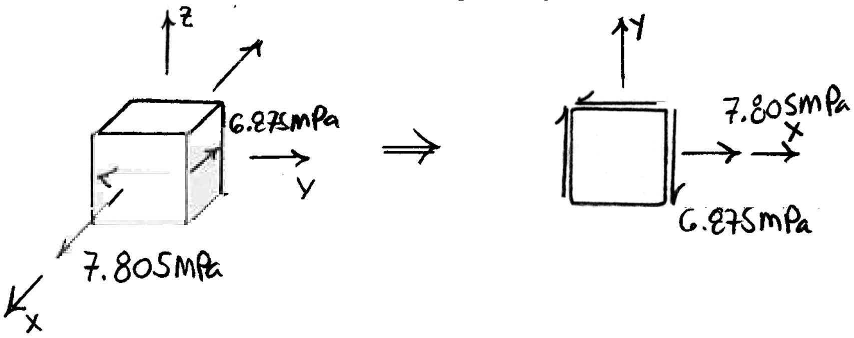
$$I_b = \frac{1}{12}(.09m)(.08m)^3 + (.09m)(.08m)(.13m - .0690m)^2$$
$$- \frac{1}{12}(.07m)(.08m)^3 - (.07m)(.08m)(.12m - .0690m)^2$$
$$+ \frac{1}{12}(.25m)(.09m)^3 + (.25m)(.09m)(.045m - .0690m)^2$$
$$\frac{1}{12}(.23m)(.07m)^3 - (.23m)(.07m)(.045m - .0690m)^2$$
$= \underline{\underline{25.38(10^{-6})m^4}}$ ④

THE NORMAL STRESS CAN NOW BE CALCULATED IN THE UPPER SEGMENT OF CROSS-SECTION (a)

$$\sigma_a = \frac{M\,c}{I} = \frac{2(10^3)\,N\cdot m\cdot(0.170-0.07029m)}{25.55(10^{-6})\,m^4} = 7.805(10^6)\frac{N}{m^2} = \underline{7.805\,MPa}\ (5)$$

THE SHEAR STRESS FOR THIS SECTION CAN ALSO BE CALCULATED

$$\tau_a = .11\cdot\frac{4(10^3)\,N\cdot m}{(.06m)^2\cdot(.01m)} = 6.875(10^6)\frac{N}{m^2} = \underline{6.875\,MPa}$$



THE NORMAL STRESS IN SECTION (B)

$$\sigma_B = \frac{M\cdot c}{I} = \frac{2(10^3)N\cdot m\cdot(0.170m-0.0690m)}{25.38(10^{-6})m^4}$$

$$= \underline{7.959\,MPa}$$

THE SHEAR STRESS IN SECTION (B)

$$\tau_B = \frac{4(10^3)\,N\cdot m}{8\cdot(.01m)(.06m)^2}$$

$$\underline{7.813\,MPa}$$