PROBLEM: FE L03 HW | GIVEN THE CONFIGURATION SHOWN IN THE FIGURE BELOW, DETERMINE THE NODAL FORCES AND DISPLACEMENTS.

GIVEN:

1. AN 80 lb LOAD APPLIED TO ELEMENT 1 OF THE SYSTEM
2. THE ELEMENT STIFFNESSES: $k_1 = 35$ lb/in, $k_2 = 35$ lb/in, $k_3 = 25$ lb/in, $k_4 = 30$ lb/in
3. ONE END OF ELEMENTS 2, 3, 4 ARE ATTACHED TO A WALL

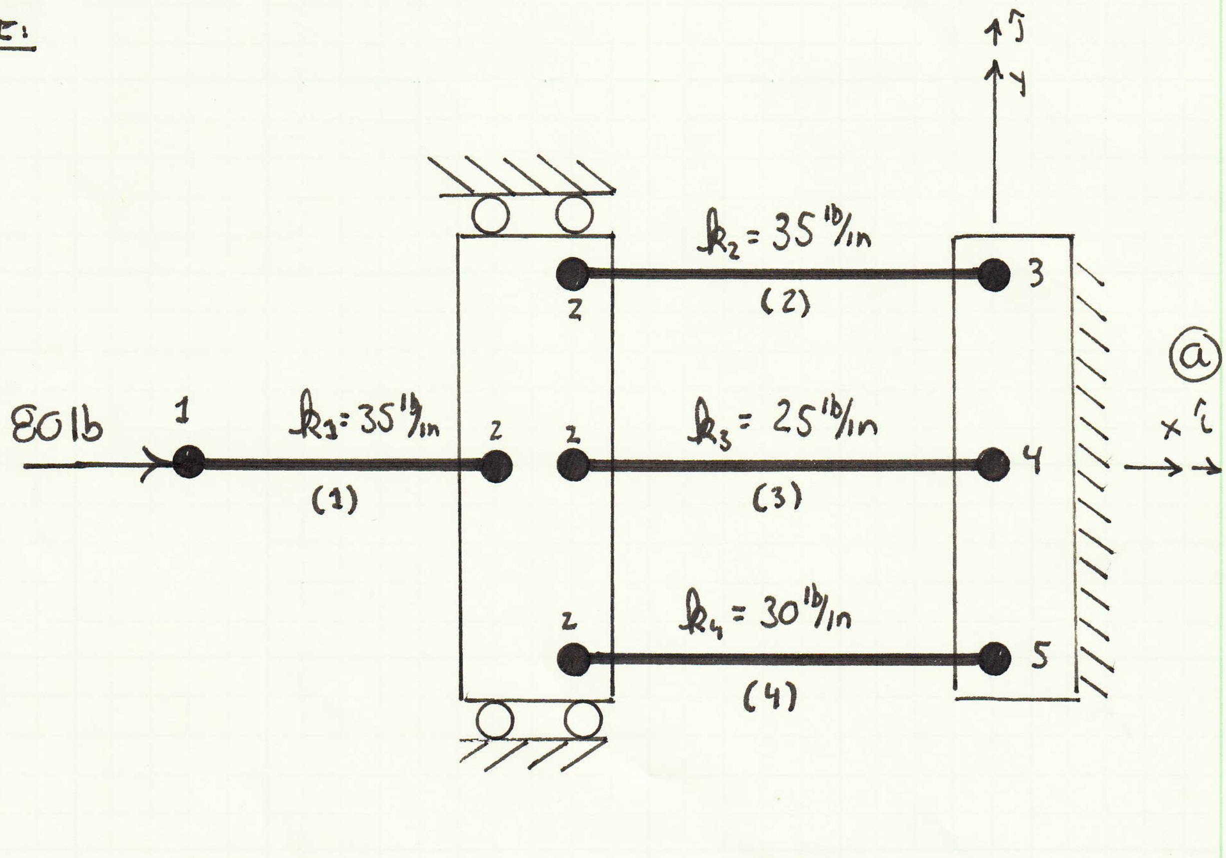
ASSUMPTIONS:

1. SMALL DISPLACEMENTS
2. LINEAR ELASTIC BEHAVIOR OF THE ELEMENTS
3. THE CENTER BODY THAT ALL ELEMENTS ARE ATTACHED TO
   - i. IS RIGID
   - ii. CAN NOT ROTATE
   - iii. CAN ONLY MOVE IN THE HORIZONTAL DIRECTION
   - iv. SLIDES HORIZONTALLY WITHOUT FRICTION.
4. ELEMENTS 1, 2, 3, & 4 DO NOT BUCKLE AND HAVE NO VERTICAL MOTION

FIND:

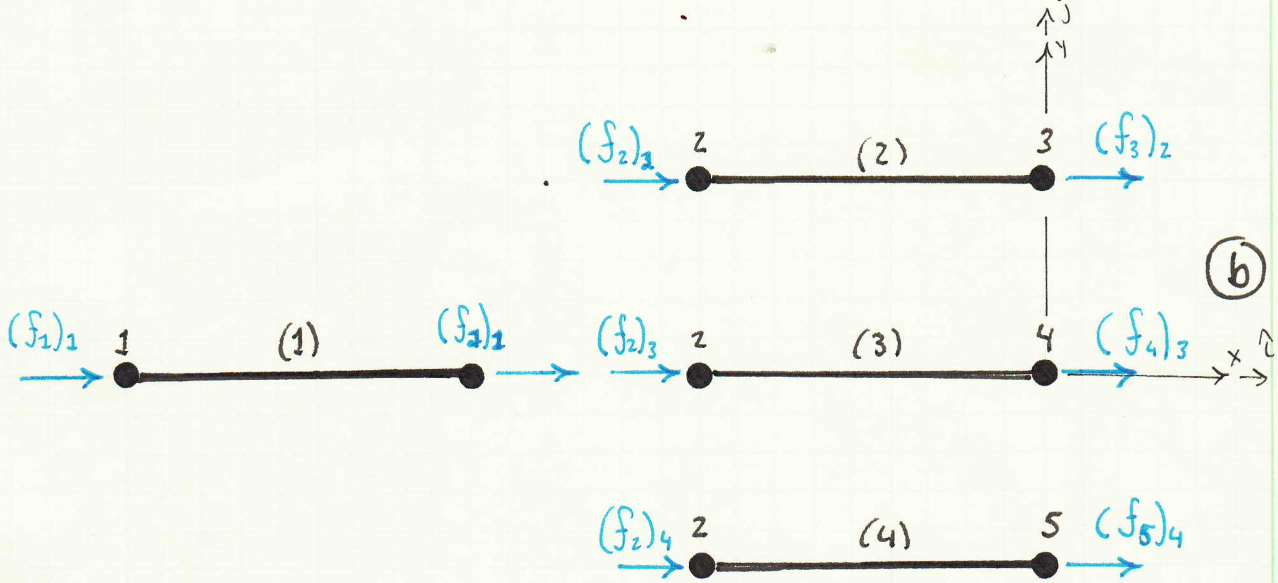
1. NODAL FORCES
2. NODAL DISPLACEMENTS.

FIGURE:



SOLUTION:

THE SOLUTION STARTS BY BREAKING DOWN THE STRUCTURE INTO THE FOUR ELEMENTS



THE STIFFNESS MATRICIES FOR EACH ELEMENT CAN NOW BE CONSTRUCTED

$$\begin{Bmatrix} (f_1)_1 \\ (f_2)_1 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad (1)$$

$$\begin{Bmatrix} (f_2)_2 \\ (f_3)_2 \end{Bmatrix} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} \quad (2)$$

$$\begin{Bmatrix} (f_2)_3 \\ (f_4)_3 \end{Bmatrix} = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_4 \end{Bmatrix} \quad (3)$$

$$\begin{Bmatrix} (f_2)_4 \\ (f_5)_4 \end{Bmatrix} = \begin{bmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_5 \end{Bmatrix} \quad (4)$$

CONSIDERING THE INTERNAL EQUILIBRIUM OF EACH NODE

$$F_1 = (f_1)_1 = 80\,\text{lb} \qquad (5)$$

$$F_2 = (f_2)_1 + (f_2)_2 + (f_2)_3 + (f_2)_4 = 0 \qquad (6)$$

$$F_3 = (f_3)_2 \qquad (7)$$

$$F_4 = (f_4)_3 \qquad (8)$$

$$F_5 = (f_5)_3 \qquad (9)$$

THE GLOBAL STIFFNESS MATRIX IS FORMED BY RELATING THE GLOBAL NODAL FORCES TO THE GLOBAL DISPLACEMENTS AT EACH NODE.

NODE 1; USING (5) & (1)

$$F_1 = (f_1)_1 = 80\,\text{lb} = k_1 \cdot u_1 - k_1 \cdot u_2 \qquad (10)$$

NODE 2, USING (6), (2), (3), & (4)

$$F_2 = (f_2)_1 + (f_2)_2 + (f_2)_3 + (f_2)_4 = 0 = -k_1 \cdot u_1 + k_1 \cdot u_2 + k_2 \cdot u_2 - k_2 \cdot u_3$$
$$k_3 u_2 - k_3 u_4 + k_4 \cdot u_2 - k_4 \cdot u_5$$

$$= 0 = -k_1 \cdot u_1 + (k_1 + k_2 + k_3 + k_4) u_2 - k_2 \cdot u_3 - k_3 \cdot u_4 - k_4 \cdot u_5 \qquad (11)$$

NODE 3, USING (7) & 2

$$F_3 = (f_3)_2 = -k_2 \cdot u_2 + k_2 \cdot u_3 \qquad (12)$$

NODE 4, USING (8) & (3)

$$F_4 = (f_4)_3 = -k_3 \cdot u_2 + k_3 \cdot u_4 \qquad (13)$$

NODE 5, USING (9) & (4)

$$F_5 = (f_5)_4 = -k_4 \cdot u_2 + k_4 \cdot u_5 \qquad (14)$$

THE GLOBAL STIFFNESS MATRIX IS NOW FORMED BY WRITING (10) - (14) IN MATRIX FORM.

$$\begin{Bmatrix} F_1 = 80\,lb \\ F_2 = 0 \\ F_3 \\ F_4 \\ F_5 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & (k_1+k_2+k_3+k_4) & -k_2 & -k_3 & -k_4 \\ 0 & -k_2 & k_2 & 0 & 0 \\ 0 & -k_3 & 0 & k_3 & 0 \\ 0 & -k_4 & 0 & 0 & k_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 = 0 \\ u_4 = 0 \\ u_5 = 0 \end{Bmatrix} \quad (15)$$

(15) NOW HAS TO BE PARTITIONED INTO KNOWN FORCES - UNKNOWN DISPLACEMENTS AND UNKNOWN FORCES - KNOWN DISPLACEMENTS.

KNOWN FORCES - UNKNOWN DISPLACEMENTS

$$\begin{Bmatrix} 80\,lb \\ 0\,lb \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & \boxed{\begin{matrix} 0 & 0 & 0 \\ -k_2 & -k_3 & -k_4 \end{matrix}} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \boxed{\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}} \end{Bmatrix}$$

$$\begin{Bmatrix} 80\,lb - 0\cdot 0 - 0\cdot 0 - 0\cdot 0 \\ 0\,lb - (-k_2)\cdot 0 - (-k_3)(0) - (-k_4)(0) \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1+k_2+k_3+k_4) \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\begin{Bmatrix} 80\,lb \\ 0\cdot lb \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1+k_2+k_3+k_4) \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \begin{bmatrix} 35\,lb/in & -35\,lb/in \\ -35\,lb/in & 125\,lb/in \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\begin{bmatrix} 35\,lb/in & -35\,lb/in \\ -35\,lb/in & 125\,lb/in \end{bmatrix}^{-1} \cdot \begin{Bmatrix} 80\,lb \\ 0\cdot lb \end{Bmatrix} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \underline{\underline{\begin{Bmatrix} 3.175\,in \\ 0.889\,in \end{Bmatrix}}} \quad (16)$$

NOW CONSIDER THE KNOWN DISPLACEMENTS – UNKNOWN FORCES. FROM (15) THIS PARTITION IS WRITTEN

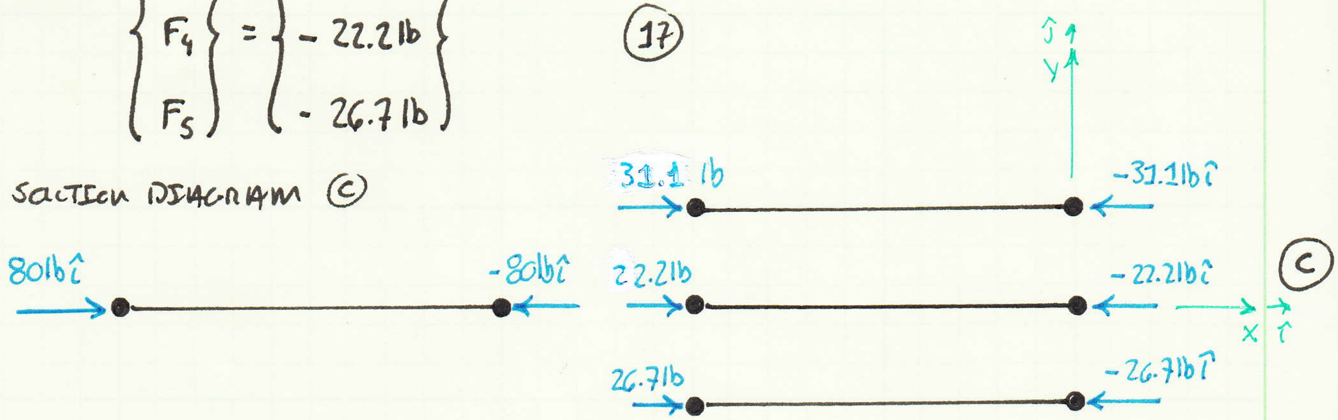
$$\begin{Bmatrix} F_3 \\ F_4 \\ F_5 \end{Bmatrix} = \begin{bmatrix} 0 & -k_2 & k_2 & 0 & 0 \\ 0 & -k_3 & 0 & k_3 & 0 \\ 0 & -k_4 & 0 & 0 & k_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

SUBSTITUTING IN THE RESULTS IN (16) AND THE VALUES FOR THE STIFFNESSES.

$$\begin{Bmatrix} F_3 \\ F_4 \\ F_5 \end{Bmatrix} = \begin{bmatrix} 0 & -35\,\text{lb/in} & 35\,\text{lb/in} & 0 & 0 \\ 0 & -25\,\text{lb/in} & 0 & 25\,\text{lb/in} & 0 \\ 0 & -30\,\text{lb/in} & 0 & 0 & 30\,\text{lb/in} \end{bmatrix} \begin{Bmatrix} 3.175\,\text{in} \\ 0.889\,\text{in} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} F_3 \\ F_4 \\ F_5 \end{Bmatrix} = \begin{Bmatrix} -31.1\,\text{lb} \\ -22.2\,\text{lb} \\ -26.7\,\text{lb} \end{Bmatrix} \qquad (17)$$

THE SOLUTION DIAGRAM (C)



SUMMARY:

THE SOLUTION DIAGRAM ILLUSTRATES THAT EQUILIBRIUM OF THE OVERALL STRUCTURE AND THE EQUILIBRIUM ON NODE 2 IS SATISFIED.