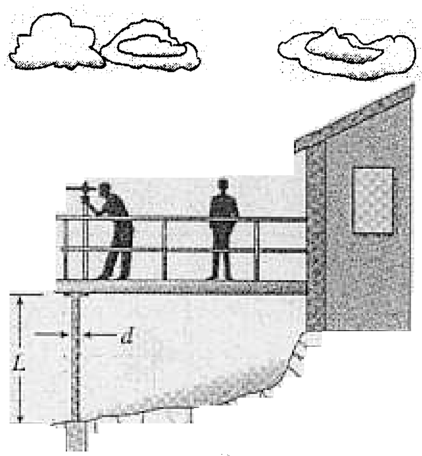
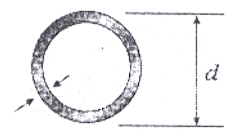
NAME: SOLUTION

Problem 1: A viewing platform in a wild-animal park is supported by a row of aluminum pipe columns having length L=3.25m and outer diameter d=100mm. The bases of the columns are set in concrete footings and the tops of the columns are supported laterally (pinned) by the platform. The columns are being designed to support compressive loads P=100kN. $E = 72GPa$, $\sigma_{yp} = 480MPa$



(a)



(b)

1a. Determine the minimum required thickness t of the columns if a factor of safety n=3 is required with respect to Euler buckling (P_{cr}=3P).

$$P_{CR} = \frac{\pi^2 \cdot E \cdot I}{(.699)^2 \cdot L^2}$$

$$I = \frac{\pi}{64}\left[(.1m)^4 - (.1m - 2\cdot t)^4\right]$$

$$P_{CR} = 3\cdot P = 3 \cdot 1000N$$

$$= \frac{\pi^2 \cdot 72\times10^9 \frac{N}{m^2}}{(.699)^2 (3.25m)^2} \cdot \frac{\pi}{64}\left[(.1m)^4 - (.1m - 2\cdot t)^4\right]$$

$$\frac{3\cdot 1000N \cdot (.699)^2 \cdot (3.25m)^2 \cdot 64}{\pi^3 \cdot 72\times10^9 \frac{N}{m^2}} = (.1m)^4 \quad (.1m - 2\cdot t)^4$$

$$(.1m - 2\cdot t)^4 = (.1m)^4 - \frac{3\cdot 1000N \cdot (.699)^2 \cdot (3.25m)^2 \cdot 64}{\pi^3 \cdot 72\times10^9 \frac{N}{m^2}}$$

$$t = \frac{.1m - \left((.1m)^4 - \frac{3\cdot 1000N \cdot (.699)^2 \cdot (3.25m)^2 \cdot 64}{\pi^3 \cdot 72\times10^9 N/m^2}\right)^{1/4}}{2}$$

$$= 0.006822m = \boxed{6.822mm}$$

1b. Calculate the compressive stress in the column under maximum loading. Is the beam in the Euler domain of buckling? Explain your answer.

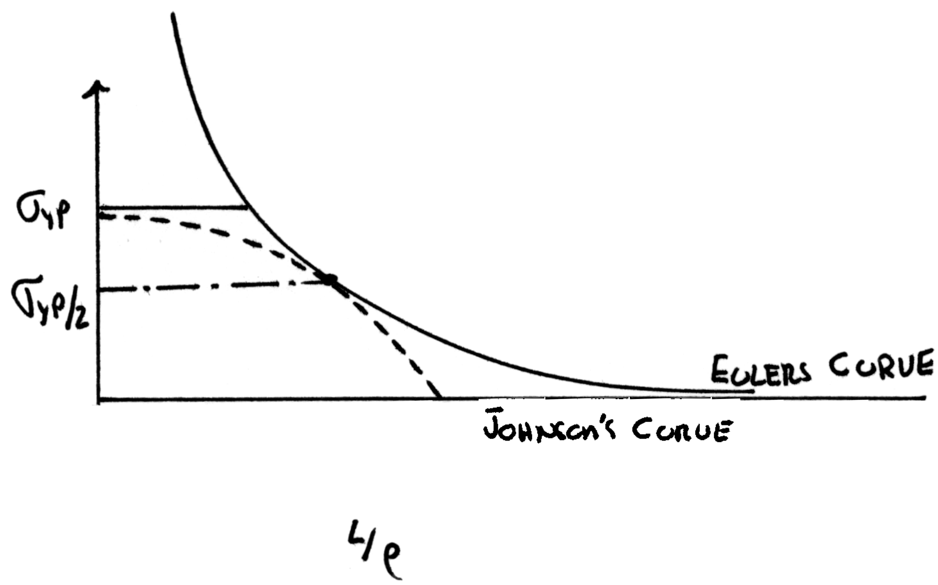
$$\sigma_{max} = \frac{3 \cdot 1000}{\frac{\pi}{4}\left[(.1m)^2 - (.1m - 2 \times .006822\,m)^2\right]}$$

$$= \underline{\underline{150.2\ MPa}}$$

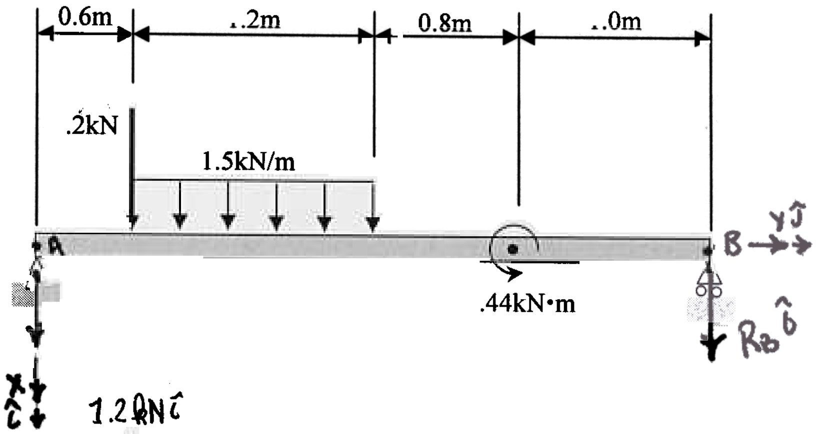
THE TANGENT POINT BETWEEN THE JOHNSON CURVE AND EULERS EQUATION IS AT

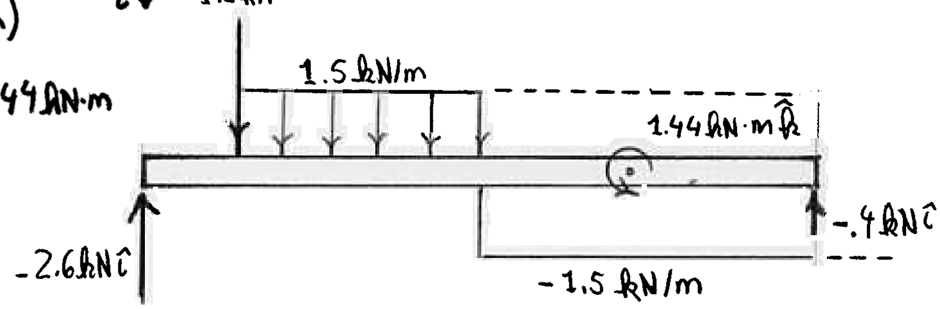
$$\frac{\sigma_{yp}}{2} = \frac{480\,MPa}{2} = 240\,MPa$$

SINCE THE TANGENT POINT IS ABOVE THE MAXIMUM STRESS SEEN BY THE STRUCTURE, EULERS EQUATION DOES APPLY



PROBLEM 2: Using singularity functions, write expressions for the shear, bending moment, curvature, and deflection of the beam shown. Be sure to calculate all integration constants.





$$\Sigma F_x = 0 = R_A + R_B + 1.2\,\text{kN} + (1.5\,\tfrac{\text{kN}}{\text{m}} \cdot 1.2\text{m})$$

$$R_A + R_B = -3\,\text{kN}$$

$$\Sigma M_{z/@A} = 0 = (-1.2\,\text{kN})(.6\text{m}) - (1.2\text{m})(1.5\,\tfrac{\text{kN}}{\text{m}})(1.2\text{m}) + 1.44\,\text{kN}\cdot\text{m} - (3.6\text{m})(R_B)$$

$$R_B = -.4\,\text{kN}$$

$$R_A = -2.6\,\text{kN}$$

NOW AN EQUATION FOR THE LOAD APPLIED TO THE BEAM CAN BE WRITTEN

$$= -2.6\text{kN}\langle y-0\rangle_{-1} + 1.2\text{kN}\langle y-.6\text{m}\rangle_{-1} + 1.5\,\tfrac{\text{kN}}{\text{m}}\langle y-.6\text{m}\rangle^0 - 1.5\,\tfrac{\text{kN}}{\text{m}}\langle y-1.8\text{m}\rangle^0 + 1.44\,\text{kN}\cdot\text{m}\langle y-2.6\rangle_{-2} - 0.4\,\text{kN}\langle y-3.6\text{m}\rangle_{-1} \qquad (1)$$

$$-\int q\,dy = 2.6\text{kN}\langle y-0\rangle^0 - 1.2\text{kN}\langle y-.6\text{m}\rangle^0 - 1.5\,\tfrac{\text{kN}}{\text{m}}\langle y-.6\text{m}\rangle^1 + 1.5\,\tfrac{\text{kN}}{\text{m}}\langle y-1.8\text{m}\rangle^1 - 1.44\,\text{kN}\cdot\text{m}\langle y-2.6\text{m}\rangle_{-1} + 0.4\,\text{kN}\langle y-3.6\text{m}\rangle^0 \qquad (2)$$

$$M = \int V\,dy = 2.6\,\text{kN}\langle y-0\rangle^1 - 1.2\,\text{kN}\langle y-.6\text{m}\rangle^1 - 0.75\,\tfrac{\text{kN}}{\text{m}}\langle y-.6\text{m}\rangle^2 + 0.75\,\tfrac{\text{kN}}{\text{m}}\langle y-1.8\text{m}\rangle^2 - 1.44\,\text{kN}\cdot\text{m}\langle y-2.6\text{m}\rangle^0 + 0.4\,\text{kN}\langle y-3.6\text{m}\rangle^1 \qquad (3)$$

$$\theta = -\int \frac{M}{EI}\,dy = \frac{1}{EI}\Big[-1.3\,\text{kN}\langle y-0\rangle^2 + 0.6\,\text{kN}\langle y-.6\text{m}\rangle^2 + 0.25\,\tfrac{\text{kN}}{\text{m}}\langle y-.6\text{m}\rangle^3 - 0.25\,\tfrac{\text{kN}}{\text{m}}\langle y-1.8\text{m}\rangle^3 + 1.44\,\text{kN}\cdot\text{m}\langle y-2.6\text{m}\rangle^1 - 0.2\,\text{kN}\langle y-3.6\text{m}\rangle^2 + C_1\Big] \qquad (4)$$

$$\int \Theta\, dy = \frac{1}{EI}\Big[-0.4333\,kN\langle y-0\rangle^3 + 0.2\,kN\langle y-.6m\rangle^3 + 0.0625\,\frac{kN}{m}\langle y-.6m\rangle^4$$
$$-0.0625\,\frac{kN}{m}\langle y-1.8m\rangle^4 + 0.72\,kN\cdot m\langle y-2.6m\rangle^2$$
$$-0.0667\,kN\langle y-3.6m\rangle^3 + C_1\cdot y + C_2\Big] \quad (5)$$

FIRST BOUNDARY CONDITION $u(0)=0$ IS APPLIED TO (5)

$$u(0)=0=\frac{1}{EI}\left[-0.4333\,kN\cdot(0)^3 + C_1(0) + C_2\right] \Rightarrow \underline{\underline{C_2=0}} \quad (6)$$

THE SECOND BOUNDARY CONDITION FOR THIS PROBLEM IS $u(3.6m)=0$
SUBSTITUTING THIS BOUNDARY CONDITION ALONG WITH (6) INTO (5)

$$u(3.6m)=0=\frac{1}{EI}\Big[-0.4333\,kN(3.6m)^3 + 0.2\,kN(3.0m)^3 + 0.0625\,\frac{kN}{m}(3.0m)^4$$
$$0.0625\,\frac{kN}{m}(1.8m)^4 + 0.72\,kN\cdot m\,(1.0m)^2 - 0.0667\,kN(0)^3$$
$$+ C_1\cdot 3.6m\Big]$$

$$\underline{\underline{C_1 = 2.692\,kN\cdot m^2}}$$

NOW (4) AND (5) CAN BE REWRITTEN

$$\frac{1}{EI}\Big[-1.3\,kN\langle y-0\rangle^2 + 0.6\,kN\langle y-.6m\rangle^2 + 0.25\,\frac{kN}{m}\langle y-.6m\rangle^3$$
$$-0.25\,\frac{kN}{m}\langle y-1.8m\rangle^3 + 1.44\,kN\cdot m\langle y-2.6m\rangle^1 - 0.2\,kN\langle y-3.6m\rangle^2$$
$$+ 2.692\,kN\cdot m^2\Big]$$

$$u = \frac{1}{EI}\Big[-0.4333\,kN\langle y-0\rangle^3 + 0.2\,kN\langle y-.6m\rangle^3 + 0.0625\,\frac{kN}{m}\langle y-.6m\rangle^4$$
$$-0.0625\,\frac{kN}{m}\langle y-1.8m\rangle^4 + 0.72\,kN\cdot m\langle y-2.6m\rangle^2$$
$$-0.0667\,kN\langle y-3.6m\rangle^3 + 2.692\,kN\cdot m^2\cdot y\Big]$$

PROBLEM 3: Sketch the shear, moment, curvature, and deflection diagrams.

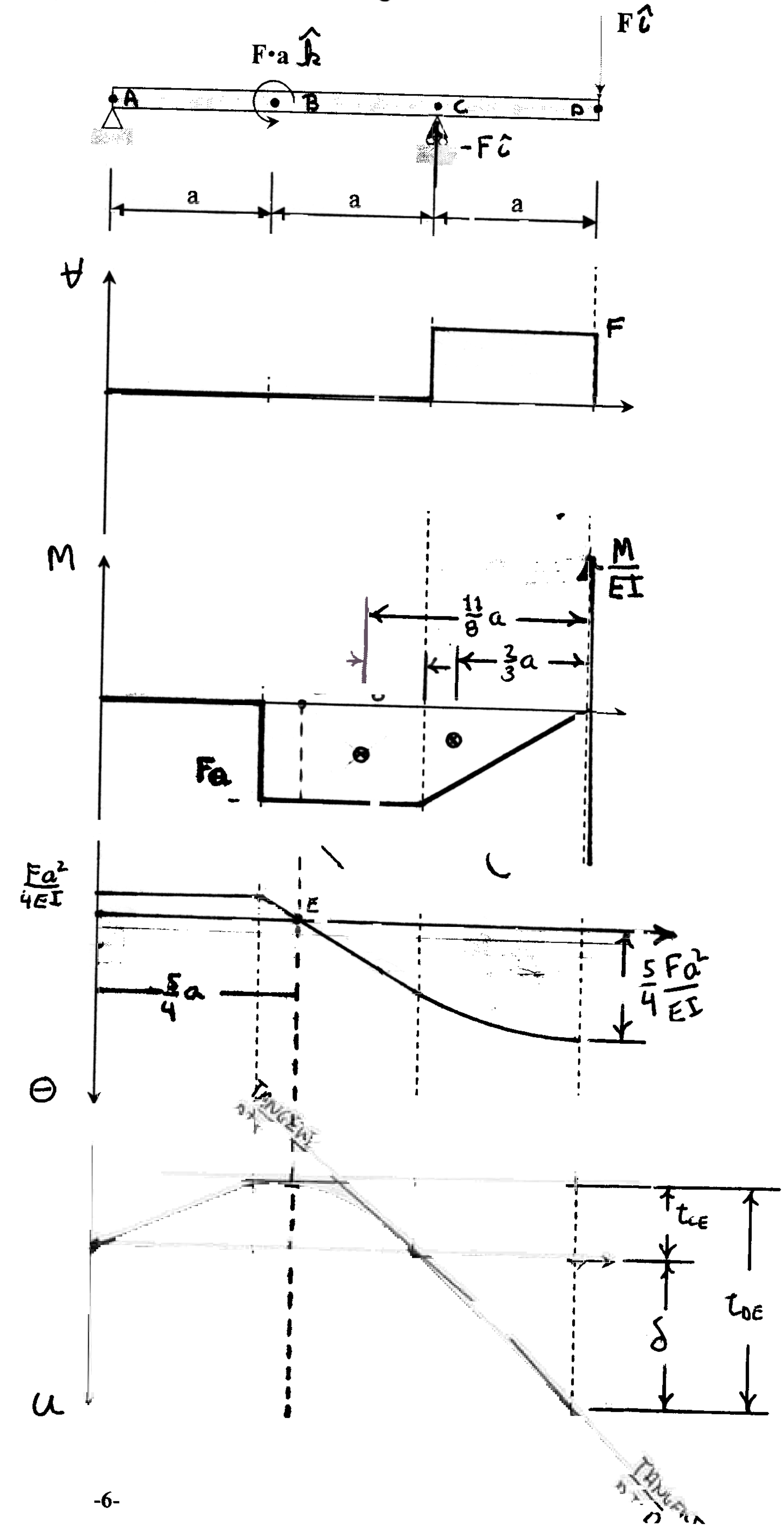
$\sum F_y = 0 = R_A + R_c + F = 0$

$R_A + R_c = -F$

$\sum M_{z/@c} = 0 = 2\cdot a \cdot R_A + F\cdot a - F\cdot a$

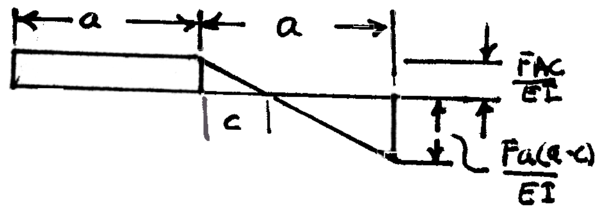
$\Rightarrow R_A = 0$

$R_c = -F$



FINDING THE POINT WHERE THE CURVATURE CURVE CROSSES ZERO REQUIRES AND UNDERSTANDING OF THE RELATIONSHIP BETWEEN THE CURVE

- WHERE θ EQUALS ZERO IS GOING TO BE WHERE u IS MAXIMUM
- THE RISE OF THE ELASTIC CURVE FROM "A" TO THE MAXIMUM HAS TO EQUAL THE FALL FROM THE MAXIMUM TO POINT "C".
- THIS MEANS THAT THE AREA UNDER THE θ CURVE BETWEEN POINT A AND THE INTERCEPT IS EQUAL TO THE AREA UNDER THE θ CURVE FROM THE INTERCEPT TO POINT C.



$\frac{F}{EI}\cancel{a}\cdot a\cdot c + \frac{1}{2}\frac{F}{EI}\cancel{a}\cdot c\cdot c = \frac{1}{2}(a-c)\frac{F}{EI}\cdot\cancel{a}(a-c)$

$a\cdot c + \frac{1}{2}\cdot c^2 = \frac{1}{2}(a-c)^2 = \frac{1}{2}(a^2 - 2ac + c^2)$

$2\cdot\cancel{a}\cdot c + \cancel{c^2} = a^2 - 2\cancel{a}\cdot c + \cancel{c^2}$

$= a$

$\frac{a}{4}$

BONUS (10pts): Using the moment area method, determine the deflection of the free end, the curvature of the free end, the curvature of the pinned end.

THE CURVATURE OF THE PINNED END IS FOUND BY DETERMINING THE AREA UNDER THE $\frac{M}{EI}$ CURVE FROM A TO WHERE THE CURVATURE DIAGRAM IS ZERO

$$\Theta_A = \frac{F\cdot a}{EI}\cdot\frac{a}{4} = \boxed{\frac{Fa^2}{4EI}}$$

THE CURVATURE OF THE ELASTIC CURVE AT THE FREE END IS FOUND BY FINDING THE AREA UNDER THE $\frac{M}{EI}$ DIAGRAM AND "D"

$$\Theta_D = \frac{F\cdot a}{EI}\cdot\frac{3\cdot a}{4} + \frac{1}{2}\frac{F\cdot a}{EI}\cdot a = \boxed{\frac{5}{4}\frac{F\cdot a^2}{EI}}$$

THE CALCULATION OF THE DEFLECTION OF THE BEAM AT D, δ, IS FOUND FROM

$$\delta = t_{DE} - t_{CE}$$

$$t_{CE} = \frac{3}{8}a\cdot\frac{F\cdot a}{EI}\cdot\frac{3}{4}a = \frac{9}{32}\cdot\frac{Fa^3}{EI}$$

$$t_{DE} = \frac{11}{8}a\cdot\frac{F\cdot a}{EI}\cdot\frac{3}{4}a + \frac{2}{3}a\cdot\frac{1}{2}\frac{F\cdot a}{EI}\cdot a = \frac{33}{32}\frac{F\cdot a^3}{EI} + \frac{2}{6}\frac{Fa^3}{EI}$$

$$= \frac{416}{192}\frac{Fa^3}{EI} = \frac{13}{6}\frac{Fa^3}{EI}$$

$$\delta = \frac{13}{6}\frac{Fa^3}{EI} - \frac{9}{32}\frac{F\cdot a^3}{EI} = \frac{416}{192}\frac{Fa^3}{EI} - \frac{15}{192}\frac{Fa^3}{EI} = \boxed{\frac{401}{192}\frac{Fa^3}{EI}}$$