

SOLUTION:

USING TRANSFORMATION EQUATIONS THE STRESSES ON THE ELEMENT
ROTATED 15° ABOUT THE Z AXIS ARE

$$\begin{aligned}\sigma_x' &= \sigma_x \cdot \cos^2 \theta + \sigma_y \cdot \sin^2 \theta + 2 \cdot \tau_{xy} \cdot \cos \theta \sin \theta \\ &= (40 \text{ MPa}) \cdot \cos^2(15^\circ) + (10 \text{ MPa}) \sin^2(15^\circ) + 2(0) \cdot \cos(15^\circ) \sin(15^\circ) \\ &= \boxed{38.0 \text{ MPa}}\end{aligned}$$

$$\begin{aligned}\sigma_y' &= \sigma_x \cdot \sin^2 \theta + \sigma_y \cdot \cos^2 \theta - 2 \cdot \tau_{xy} \cdot \sin \theta \cdot \cos \theta \\ &= (40 \text{ MPa}) \cdot \sin^2(15^\circ) + (10 \text{ MPa}) \cdot \cos^2(15^\circ) - 2 \cdot (0) \cdot \sin(15^\circ) \cos(15^\circ) \\ &= \boxed{12.0 \text{ MPa}}\end{aligned}$$

$$\begin{aligned}\tau_{x'y'} &= -(\sigma_x - \sigma_y) \cdot \sin \theta \cdot \cos \theta + \tau_{xy} \cdot (\cos^2 \theta - \sin^2 \theta) \\ &= -(40 \text{ MPa} - 10 \text{ MPa}) \cdot \sin(15^\circ) \cdot \cos(15^\circ) + (0) \cdot (\cos^2(15^\circ) - \sin^2(15^\circ)) \\ &= \boxed{7.5 \text{ MPa}}\end{aligned}$$

THE PRINCIPAL STRESSES, CALCULATED USING EQUATIONS, ARE

$$\begin{aligned}\sigma_{1,2} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{(40 \text{ MPa} + 10 \text{ MPa})}{2} \pm \sqrt{\left(\frac{40 \text{ MPa} - 10 \text{ MPa}}{2}\right)^2 + (0)^2} = \\ &= 25 \text{ MPa} \pm 15 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\sigma_1 &= 40 \text{ MPa} \\ \sigma_2 &= 10 \text{ MPa}\end{aligned}$$

$$\tau_{max} = \pm \sqrt{\left(\frac{40 \text{ MPa} - 10 \text{ MPa}}{2}\right)^2 + (0)^2} = \boxed{\pm 15 \text{ MPa}}$$