PROBLEM 5.14 FOR THE BEAM CROSS SECTION SHOWN, BENDING IS ABOUT THE Z-AXIS. ALL DIMENSIONS ARE IN INCHES, WHERE APPROPRIATE, ARE BETWEEN THE CENTERS OF THE 0.25-in WALLS. DETERMINE

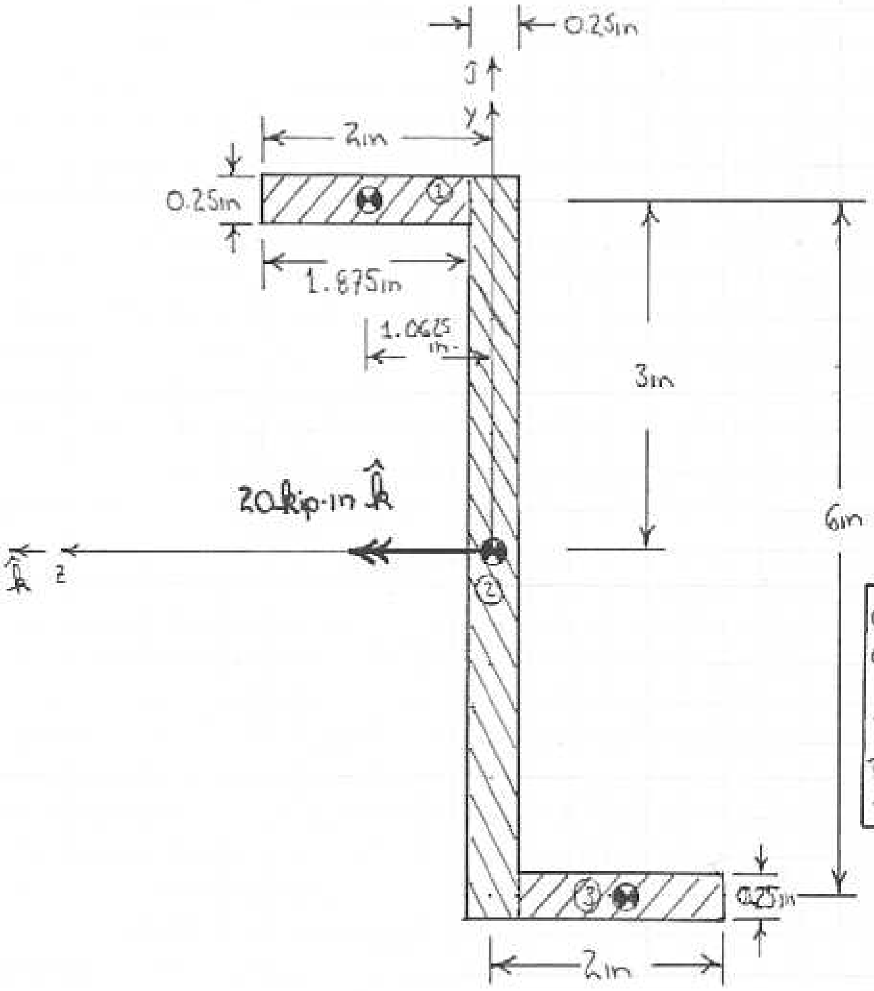
(a) THE SECOND-AREA MOMENTS I_{yy}, I_{zz}, AND I_{yz}
(b) THE ORIENTATION OF THE PRINCIPAL AXES OF THE SECOND-AREA MOMENTS
(c) THE VALUES OF THE PRINCIPAL SECOND-AREA MOMENTS I_m AND I_n
(d) THE LOCATION AND MAGNITUDES OF THE MAXIMUM TENSILE AND COMPRESSIVE BENDING STRESSES IF M_z = 20 kip-in

GIVEN:
1. CROSS-SECTION SHOWN
2. 20 kip·in BENDING MOMENT

ASSUMPTIONS:
1. MATERIAL IS LINEAR ELASTIC
2. ALL DEFORMATIONS ARE SMALL
3. THE 20 kip·in MOMENT IS THE ONLY LOAD ON THE STRUCTURE

FIND:
1. DETERMINE I_{yy}, I_{zz}, I_{yz}
2. DETERMINE THE ORIENTATION OF THE PRINCIPAL AXES OF THE SECOND-AREA MOMENTS.
3. DETERMINE THE PRINCIPAL SECOND-AREA MOMENTS I_m AND I_N
4. DETERMINE THE LOCATION ~~S~~ OF THE MAXIMUM TENSILE AND COMPRESSIVE STRESS

FIGURE:



(a)

GEOMETRY USED TO CALCULATE THE AREA-MOMENT PROPERTIES OF THIS CROSS-SECTION

SOLUTION:

USING THE GEOMETRY ILLUSTRATED IN (a), THE AREA-MOMENTS CAN BE CALCULATED FROM DEFINATION

$$I_{zz} = \tfrac{1}{12}(0.25in)(6.25in)^3 + 2\cdot\left[\tfrac{1}{12}(1.875in)(0.25in)^3 + (0.25in)(1.875in)(3in)^2\right]$$

$$= \boxed{13.529\,in^4} \quad (1)$$

$$I_{yy} = \tfrac{1}{12}(6.25in)(0.25in)^3 + 2\left[\tfrac{1}{12}(0.25in)(1.875in)^3 + (0.25in)(1.875in)(1.0625in)^2\right]$$

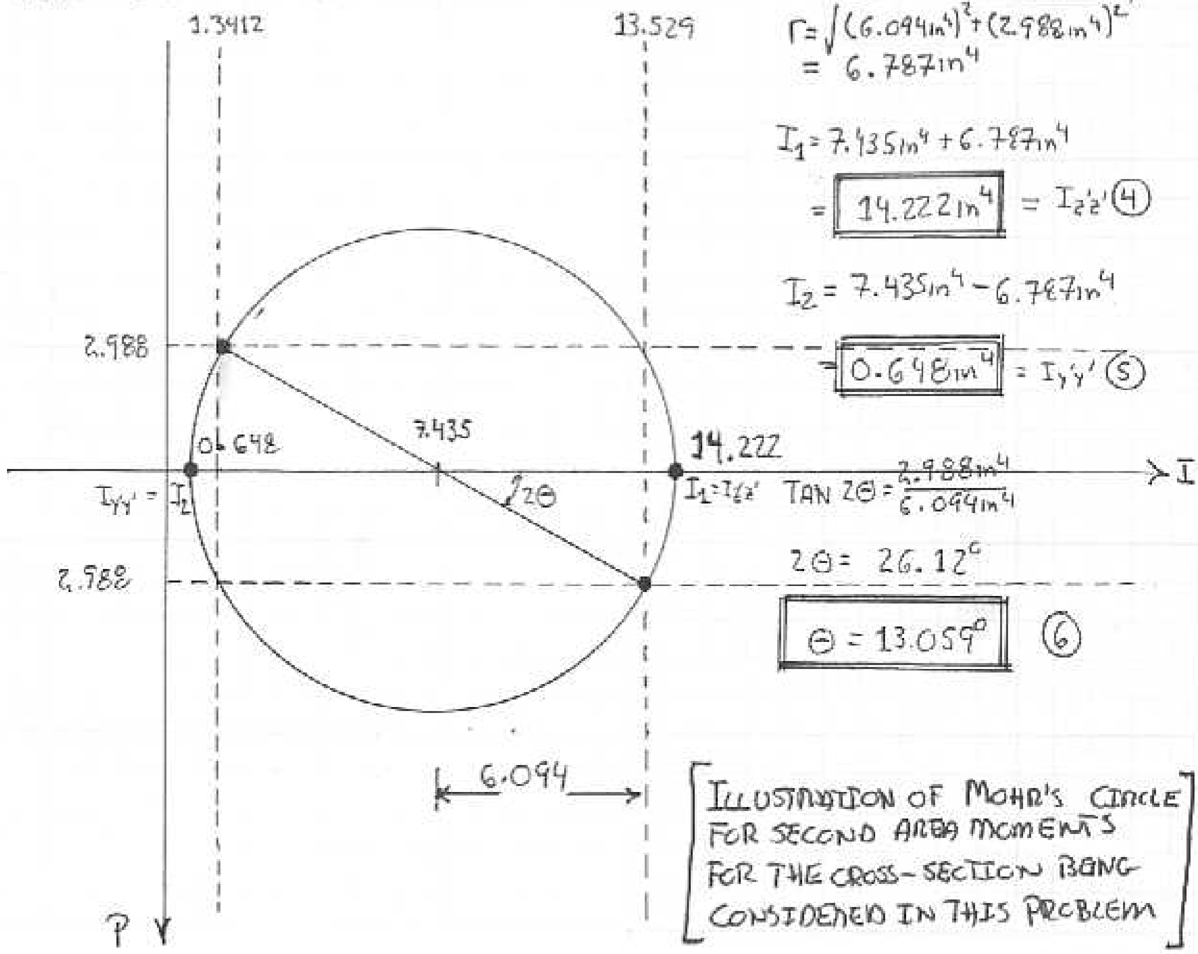
$$= \boxed{1.3412\,in^4} \quad (2)$$

$$I_{yz} = \left[\cancel{\textstyle\int z\cdot y\,dA}^{\,0} + A\cdot d_z\cdot d_y\right]_1 + \left[\textstyle\int z\cdot y\,dA + \cancel{A\cdot d_z\cdot d_y}^{\,0}\right]_2 + \left[\cancel{\textstyle\int z\cdot y\,dA}^{\,0} + A\cdot d_z\cdot d_y\right]_3$$

$$= (0.25in)(1.875in)(1.0625in)(3.0in) + (0.25in)(1.875in)(-1.0625in)(-3.0in)$$

$$= \boxed{2.988\,in^4} = P_{yz} \quad (3)$$

THE SECOND AREA MOMENTS ARE SECOND ORDER TENSORS, LOCATING THE PRINCIPAL VALUES IS FACILITATED WITH THE CONSTRUCTION OF MOHR'S CIRCLE FOR SECOND AREA MOMENTS.



$$r = \sqrt{(6.094in^4)^2 + (2.988in^4)^2} = 6.787in^4$$

$$I_1 = 7.435in^4 + 6.787in^4 = \boxed{14.222in^4} = I_{z'z'} \quad (4)$$

$$I_2 = 7.435in^4 - 6.787in^4 = \boxed{0.648in^4} = I_{y'y'} \quad (5)$$

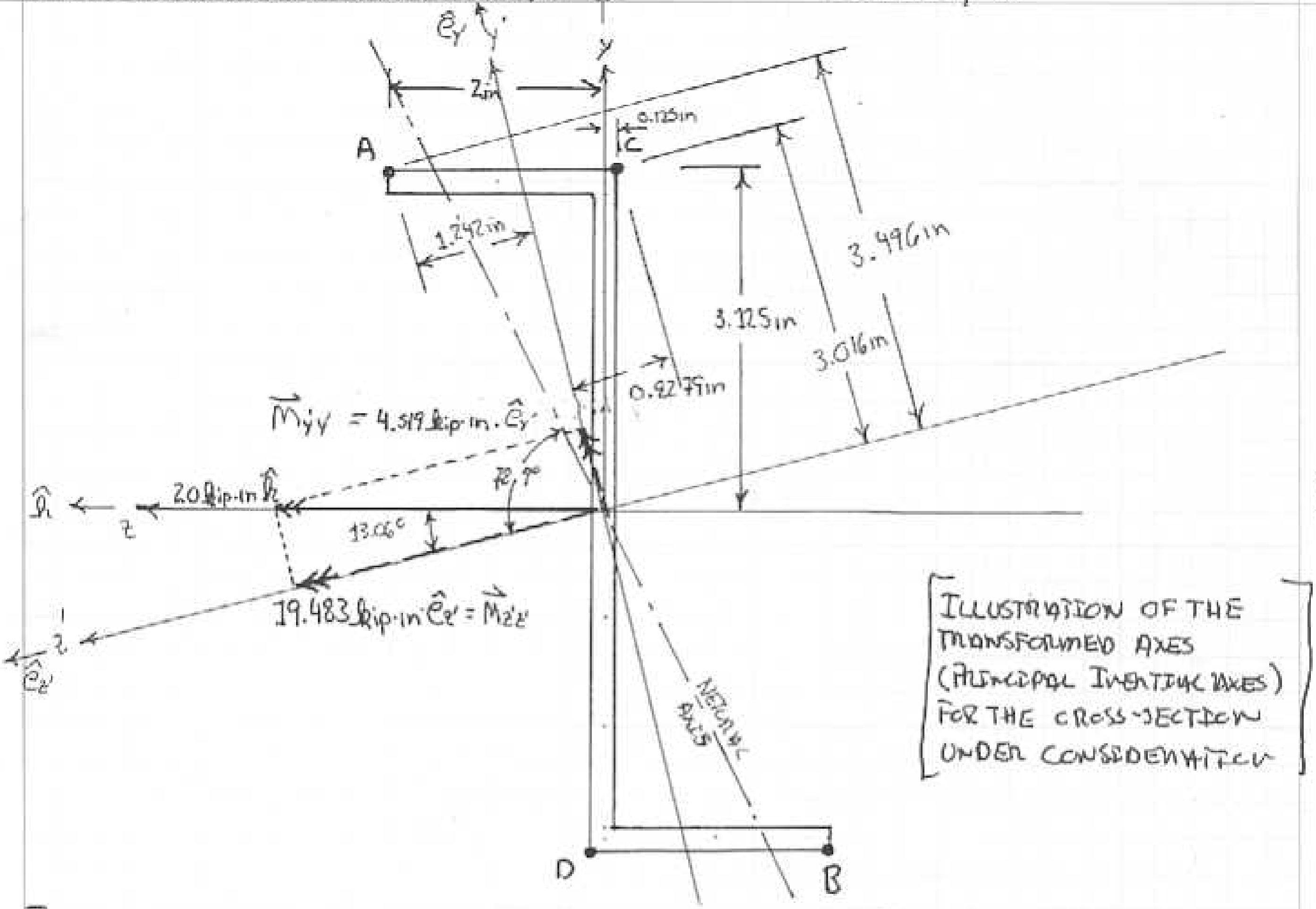
(b)

$$\tan 2\theta = \frac{2.988in^4}{6.094in^4}$$

$$2\theta = 26.12^\circ$$

$$\boxed{\theta = 13.059^\circ} \quad (6)$$

[ILLUSTRATION OF MOHR'S CIRCLE FOR SECOND AREA MOMENTS FOR THE CROSS-SECTION BEING CONSIDERED IN THIS PROBLEM]



(C)

THE STRESS IN THIS SECTION IS WRITTEN IN TERMS OF THE PRINCIPAL AXES

$$\sigma_x = \frac{M_{y'y'} \cdot z'}{I_{y'y'}} - \frac{M_{z'z'} \cdot y'}{I_{z'z'}} \qquad (7)$$

THE MOMENTS OF INERTIA IN (7) ARE FOUND IN (4) & (5). THE COMPONENTS OF THE APPLIED MOMENT ALONG THE PRINCIPAL AXES ARE

$$M_{z'z'} = 20\,\text{kip·in} \cdot \cos 13.06^\circ = 19.483\ \text{kip·in} \qquad (8)$$

$$M_{y'y'} = 20\,\text{kip·in} \cdot \sin 13.06^\circ = 4.519\ \text{kip·in} \qquad (9)$$

THE VALUE y' AND z' DIFFER FOR THE LOCATION OF THE MAXIMUM TENSILE AND COMPRESSIVE STRESSES. POINTS A, B, C, & D NEED TO BE CHECKED SINCE TWO OF THESE POINTS WILL CONTAIN THE MAXIMUM. THE LOCATION OF THESE POINTS ARE FIRST WRITTEN IN TERMS OF THE y-z COORDINATE SYSTEM AND THEN TRANSFORMED TO THE y'-z' COORDINATE SYSTEM.

	y	z
A:	3.125in	2in
B:	-3.125in	-2in
C:	3.125in	-0.125in
D:	-3.125in	0.125in

THE TRANSFORMATION FROM THE Y-Z COORDINATE SYSTEM TO THE Y'-Z' COORDINATE SYSTEM IS

$$\begin{Bmatrix} y'_A \\ z'_A \end{Bmatrix} = \begin{bmatrix} \cos 13.06° & \sin 13.06° \\ -\sin 13.06° & \cos 13.06° \end{bmatrix} \begin{Bmatrix} 3.125in \\ 2in \end{Bmatrix} = \begin{Bmatrix} 3.496in \\ 1.242in \end{Bmatrix} \quad (10)$$

$$\begin{Bmatrix} y'_B \\ z'_B \end{Bmatrix} = \begin{bmatrix} \cos 13.06° & \sin 13.06° \\ -\sin 13.06° & \cos 13.06° \end{bmatrix} \begin{Bmatrix} -3.125in \\ -2in \end{Bmatrix} = \begin{Bmatrix} -3.496in \\ -1.242in \end{Bmatrix} \quad (11)$$

$$\begin{Bmatrix} y'_C \\ z'_C \end{Bmatrix} = \begin{bmatrix} \cos 13.06° & \sin 13.06° \\ -\sin 13.06° & \cos 13.06° \end{bmatrix} \begin{Bmatrix} 3.125in \\ -0.125in \end{Bmatrix} = \begin{Bmatrix} 3.016in \\ -0.8279in \end{Bmatrix} \quad (12)$$

$$\begin{Bmatrix} y'_D \\ z'_D \end{Bmatrix} = \begin{bmatrix} \cos 13.06° & \sin 13.06° \\ -\sin 13.06° & \cos 13.06° \end{bmatrix} \begin{Bmatrix} -3.125in \\ 0.125in \end{Bmatrix} = \begin{Bmatrix} -3.016in \\ 0.8279in \end{Bmatrix} \quad (13)$$

(7) CAN NOW BE USED TO CALCULATE THE NORMAL STRESS AT A, B, C, & D

$$\sigma_{xA} = \frac{(4.519\times10^3 lb\cdot in)(1.242in)}{0.648in^4} - \frac{(19.438\times10^3 lb\cdot in)(3.496in)}{14.222in^4}$$

$$= 3.883\times10^3 \tfrac{lb}{in^2} = 3.88\,ksi \quad (14)$$

$$\sigma_{xB} = \frac{(4.519\times10^3 lb\cdot in)(-1.242in)}{0.648in^4} - \frac{(19.438\times10^3 lb\cdot in)(-3.496in)}{14.222in^4}$$

$$= -3.883\times10^3 \tfrac{lb}{in^2} = 3.88\,ksi \quad (15)$$

$$\sigma_{xC} = \frac{(4.519\times10^3 lb\cdot in)(-0.8279in)}{0.648in^4} - \frac{(19.438\times10^3 lb\cdot in)(3.016in)}{14.222in^4}$$

$$= -9.896\times10^3 \tfrac{lb}{in^2} = \boxed{-9.896\,ksi} \quad (16)$$

$$\sigma_{xD} = \frac{(4.519\times10^3 lb\cdot in)(0.8279in)}{0.648in^4} - \frac{(19.438\times10^3 lb\cdot in)(-3.016in)}{14.222in^4}$$

$$= 9.896\times10^3 \tfrac{lb}{in^2} = \boxed{9.896\,ksi} \quad (17)$$

LOCATING THE NETURAL AXIS WILL ASSIST IN EVALUATING THE LOCATION OF THE MAXIMUM TENSILE AND COMPRESSIVE STRESSES. THE LOCATION OF THE NETURAL AXIS IS FOUND BY SETTING (7) TO ZERO.

$$\sigma_x = 0 = \frac{M_{y'y'} \cdot z'}{I_{y'y'}} - \frac{M_{z'z'} \cdot y'}{I_{z'z'}}$$

$$0 = \frac{(4.519 \times 10^3 \,\text{lb/in}^2) \cdot z'}{0.648\,\text{in}^4} - \frac{(19.438 \times 10^3 \,\text{lb/in}^2) \cdot y'}{14.222\,\text{in}^4}$$

$$\tan\beta = \frac{y'}{z'} = \frac{(4.519 \times 10^3 \,\text{lb/in}^2)}{0.648\,\text{in}^4} \cdot \frac{14.222\,\text{in}^4}{19.438 \times 10^3 \,\text{lb/in}^2} = 5.102$$

$$\beta = \tan^{-1} 5.102 = \underline{\underline{78.9^\circ}} \quad \text{(FROM THE } z' \text{ AXIS)} \quad (18)$$

SUMMARY:

THE SOLUTION TO ASYMMETRIC CROSS-SECTION PROBLEMS REQUIRES THE TRANSFORMATION OF THE SECOND AREA MOMENTS OF INERTIA TO A PRINCIPAL SET OF AXES. THE LOCATION OF THE MAX TENSILE AND COMPRESSIVE STRESSES ARE FACILITATED BY EVALUATING THE LOCATION OF THE NETURAL AXES. THE RELATIVE MAGNITUDES OF (14) THROUGH (17) APPEAR TO BE CONSISTANT WITH THE CALCULATED LOCATION OF THE NETURAL AXES.