PROBLEM 5 Construct shear-force and bending-moment diagrams for the beam ABC loaded as shown in the figure. The cable passes over a small frictionless pulley at C and supports a weight W = 5.0 kN.

GIVEN:

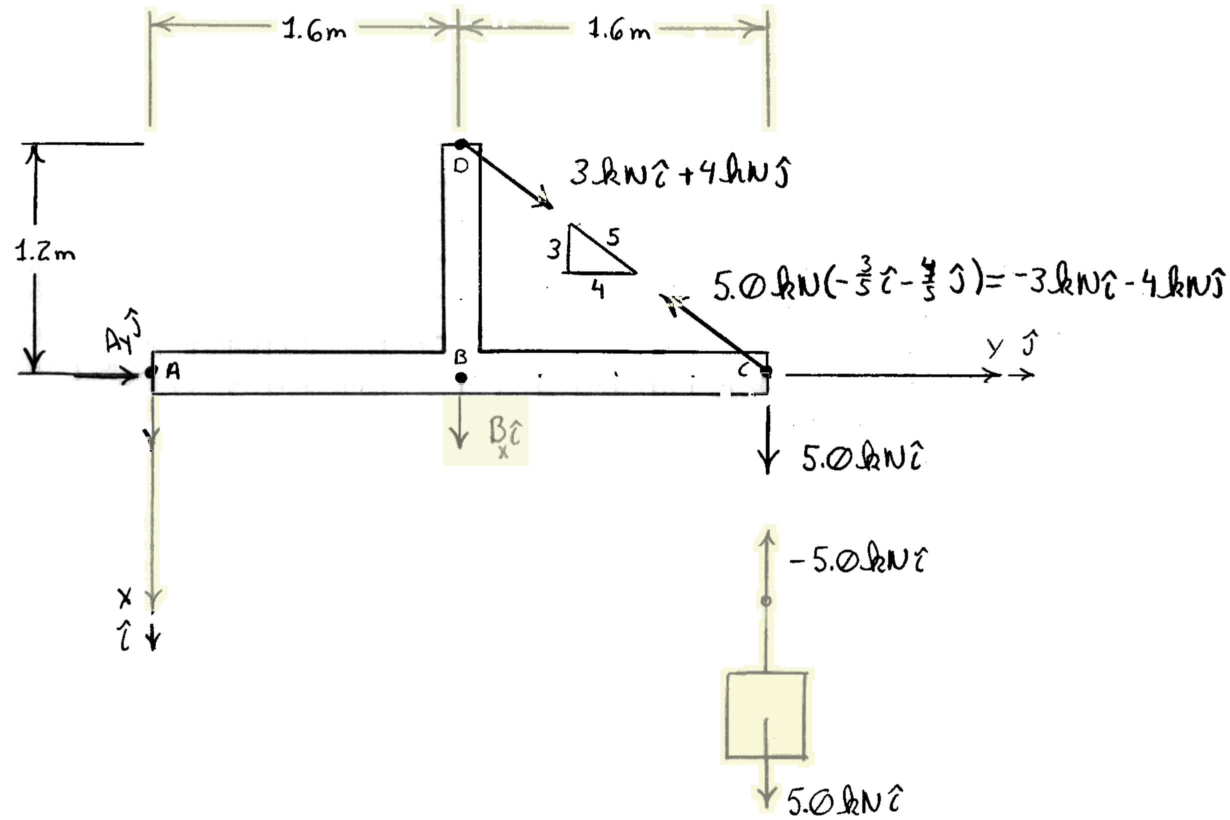
1) Constraint
- Beam subjected to loading shown
- Pin joints at A and B

2) Assumptions
- All deflections caused by the loading are small
- Pin joints do not provide any resistance to rotations
- Frictionless pulley at C does not provide any resistance to the cable running over it.

FIND:

1) shear-force and bending moment diagram for ABC

FREE BODY DIAGRAM:



STATICS:

We start by using equilibrium to determine the reactions at A and B

$$\Sigma F_x = \emptyset = A_x + B_x + 3kN - 3kN + 5kN \Rightarrow \underline{A_x + B_x = -5kN} \quad (1)$$

$$\Sigma F_y = \emptyset = A_y - 4kN + 4kN \Rightarrow \underline{A_y = \emptyset} \quad (2)$$

$$\Sigma \vec{M} = \emptyset = \vec{r}_{AD} \times \vec{F}_D + \vec{r}_{AB} \times \vec{B} + \vec{r}_{AC} \times \vec{F}_C$$

$\vec{r}_{AD} = -1.2m\,\hat{i} + 1.6m\,\hat{j}$; $\vec{F}_D = 3kN\,\hat{i} + 4kN\,\hat{j}$

$\vec{r}_{AB} = 1.6m\,\hat{j}$; $\vec{B} = B_x\,\hat{i}$

$\vec{r}_{AC} = 3.2m\,\hat{j}$; $\vec{F}_C = (-3kN\,\hat{i} - 4kN\,\hat{j}) + 5kN\,\hat{i}$

$= 2kN\,\hat{i} - 4kN\,\hat{j}$

$$\emptyset = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1.2m & 1.6m & \emptyset \\ 3kN & 4kN & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \emptyset & 1.6m & \emptyset \\ B_x & \emptyset & \emptyset \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \emptyset & 3.2m & \emptyset \\ 2kN & -4kN & \emptyset \end{vmatrix}$$

$$= [(-1.2m)(4kN) - (3kN)(1.6m)]\hat{k} + [-(B_x)\cdot(1.6m)]\hat{k} + [-(2kN)(3.2m)]\hat{k}$$

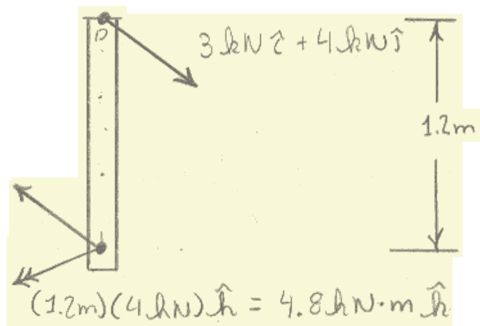
$$= [-9.6kN\cdot m - 1.6m\cdot B_x - 6.4kN\cdot m]\hat{k} = [16kN\cdot m - 1.6m\,B_x]\hat{k}$$

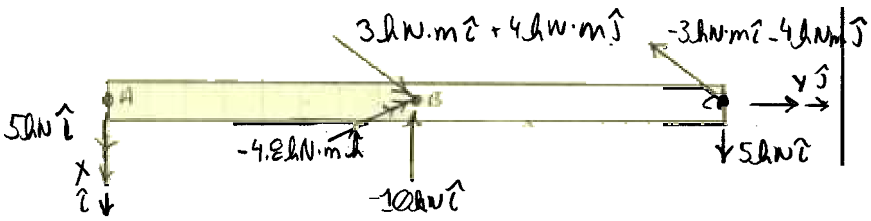
$$\Rightarrow \underline{B_x = -10kN} \quad (3)$$

From (1)

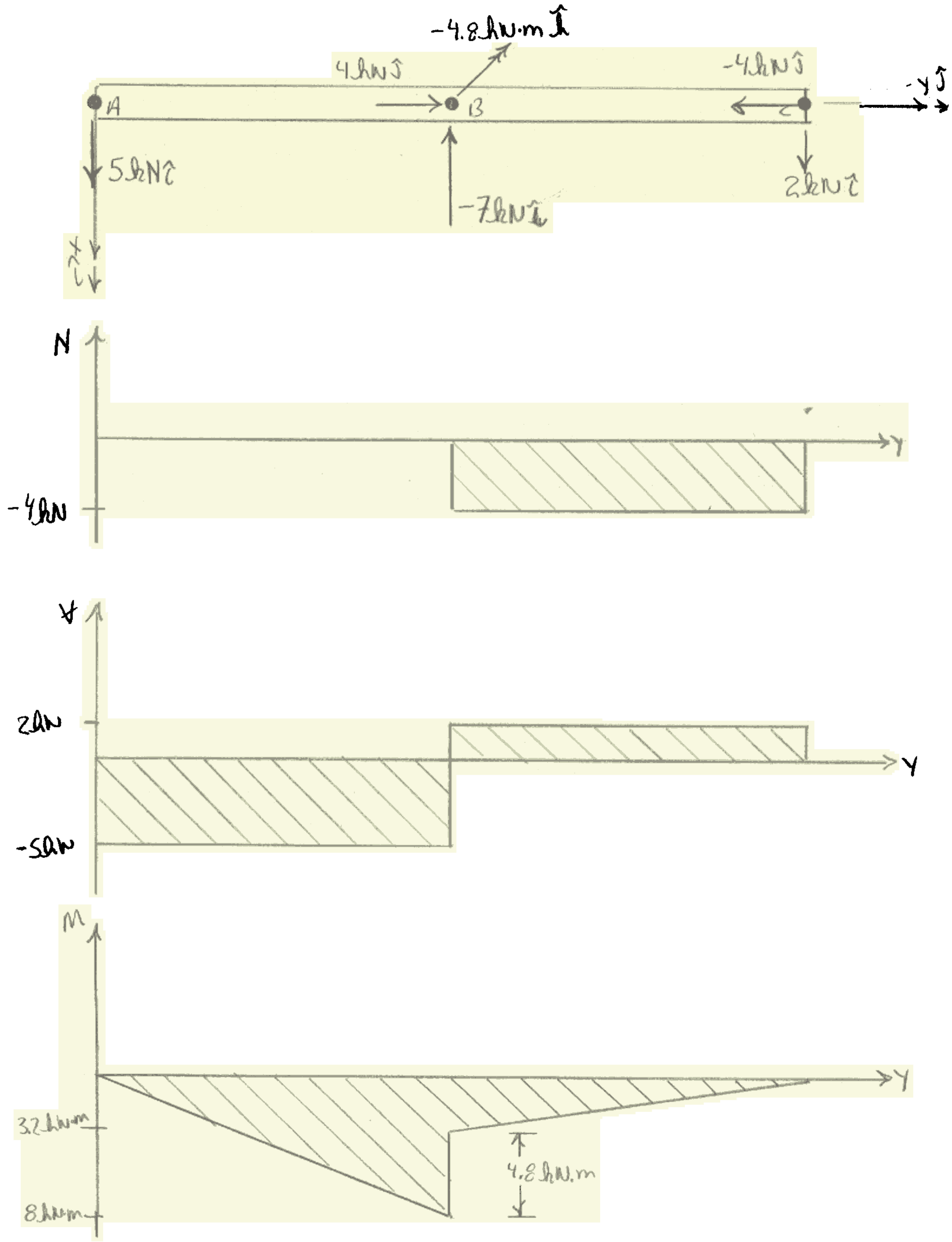
$$A_x - 10kN = -5kN \Rightarrow \underline{A_x = 5kN}$$

Now the solution diagram is drawn in a way that isolates member ABC. Therefore the effect of the vertical extention BD must be included as a force and moment





Now the shear force and bending moment diagrams can be drawn using the direct integration method.



Summary: This problem requires that you first understand that if a structure is in equilibrium the entire structure is in equilibrium. Then the normal force, shear force, and bending moment diagrams can be drawn knowing the loading and the differential relationships between the load, shear force, and bending moment.

PROB 5 SUPPLEMENT

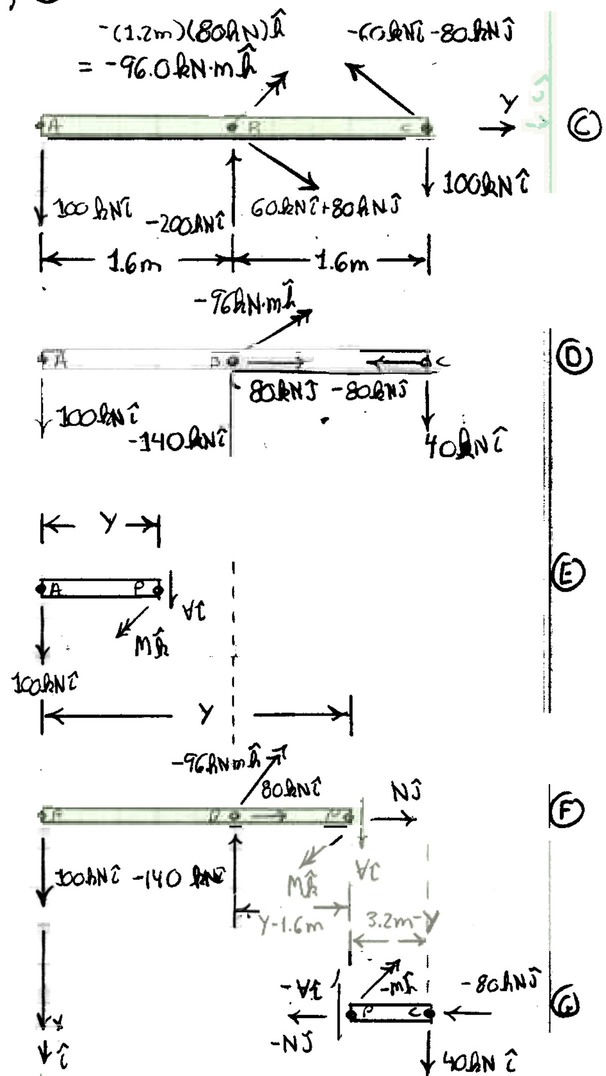
FROM (A) CONSIDERING EQUILIBRIUM

$$\Sigma F_x = 0 = A_x + B_x + 100\,kN \Rightarrow A_x + B_x = -100\,kN \quad (1)$$

$$\Sigma M_{z/eA} = 0 = -(1.6m)\cdot B_x - (3.2m)\cdot(100\,kN) \Rightarrow B_x = -\frac{3.2m}{1.6m}\cdot 100\,kN = -200\,kN \quad (2)$$

$$(2) \rightarrow (1) \Rightarrow A_x = 100\,kN \quad (3)$$

FROM (B), AN EQUIVALENT BEAM "ABC" IS CREATED, (C)
FIGURE (D) SHOWS THE SAME BEAM WITH THE NET FORCES SHOWN. NOW CONSIDER EXPRESSIONS FOR THE INTERNAL SHEAR AND BENDING MOMENT



$0 < y < 1.6m$ USING (E)

$$\Sigma F_x = 0 = 100\,kN + V \Rightarrow \boxed{V = -100\,kN}$$

$$\Sigma M_{z/ep} = 0 = M + 100\,kN\cdot y$$

$$\Rightarrow \boxed{M = -100\,kN\cdot y}$$

$1.6m < y < 3.2m$ USING (G)

$$\Sigma F_x = 0 = -V + 40\,kN \Rightarrow \boxed{V = 40\,kN}$$

$$\Sigma M_{z/ep} = 0 = -M - 40\,kN(3.2m - y)$$

$$\Rightarrow \boxed{M = 40\,kN\cdot y - 128.0\,N\cdot m}$$