PROBLEM 9 | WHAT ARE THE PRINCIPAL STRAINS AT POINT A CLOSE TO THE BASE. TAKE $E = 2\times10^{11}$ Pa AND $\nu = .3$.

GIVEN:

CONSTRAINTS

1. STEEL BAR WITH DIAMETER 75mm AND LENGTH .5m.
2. MATERIAL PROPERTIES OF STEEL ARE $E = 200\times10^9$ Pa AND $\nu = .3$
3. ROD IS IN A CANTILEVER CONFIGURATION
4. AT THE FREE-END OF THE BAR A 1000N LOAD IS APPLIED ALONG THE Z-AXIS, A 2000N LOAD IS APPLIED ALONG THE Y-AXIS, A 1000N LOAD IS APPLIED ALONG THE X AXIS, AND A 5000 N-m TORQUE IS APPLIED ALONG THE X-AXIS.

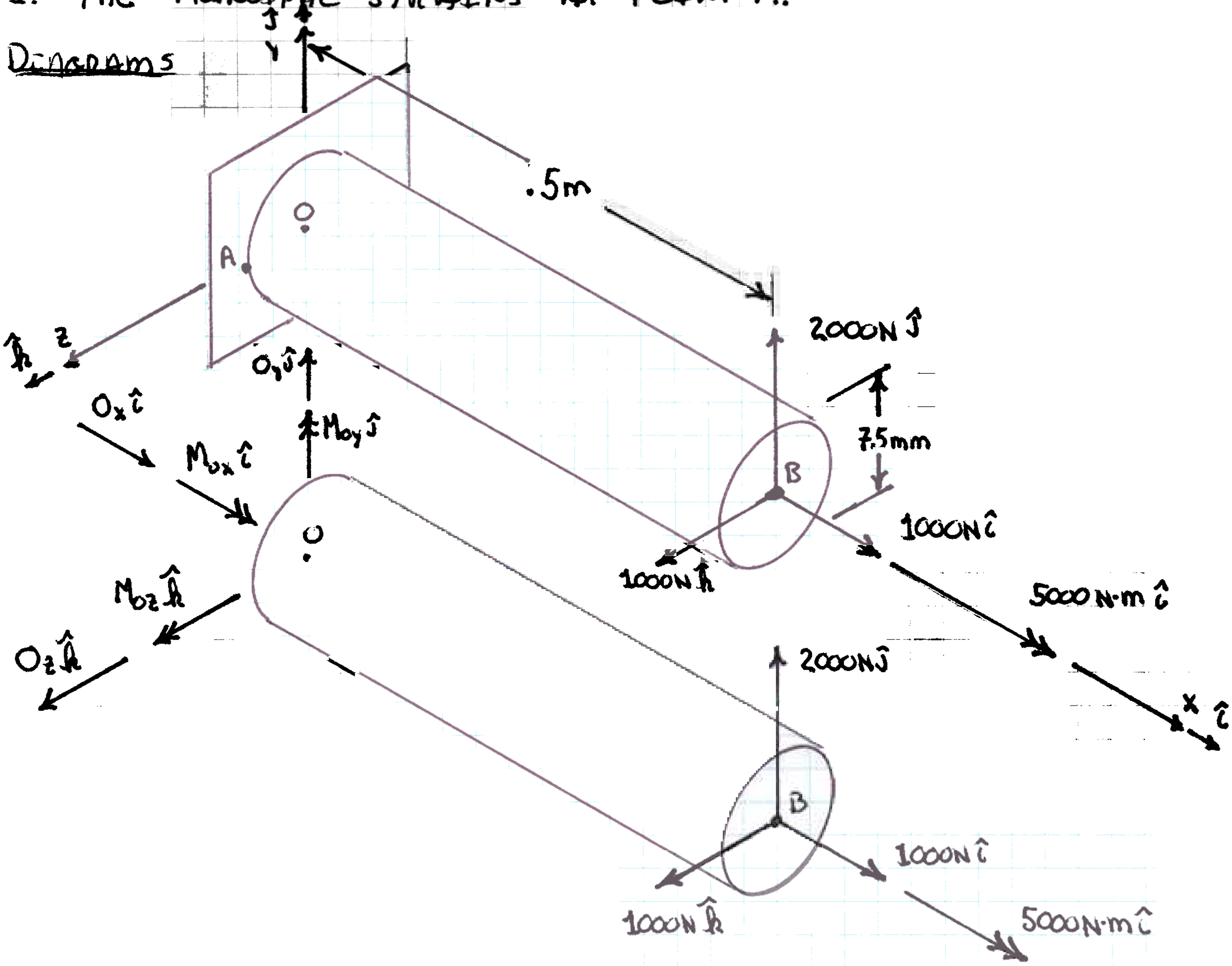
ASSUMPTIONS

1. THE WEIGHT OF THE BAR CAN BE NEGLECTED.
2. ALL DEFORMATIONS AND STRAINS ARE SMALL
3. THE MATERIAL IS LINEAR-ELASTIC

FIND:

1. THE PRINCIPAL STRAINS AT POINT A.

DIAGRAMS



SOLUTION:

CALCULATING THE REACTIONS AT THE WALL

$$\Sigma F_x = 0 = O_x + 1000\,N \Rightarrow O_x = -1000\,N$$

$$\Sigma F_y = 0 = O_y + 2000\,N \Rightarrow O_y = -2000\,N$$

$$\Sigma F_z = 0 = O_z + 1000\,N \Rightarrow O_z = -1000\,N$$

$$\Sigma \vec{M}_{@O} = \vec{0} = \vec{M}_O + 5000\,N{\cdot}m\,\hat{i} + \vec{r}_{OB} \times \vec{F}_B$$

$$= \vec{M}_O + 5000\,N{\cdot}m\,\hat{i} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ .5m & 0 & 0 \\ 1000N & 2000N & 1000N \end{vmatrix}$$

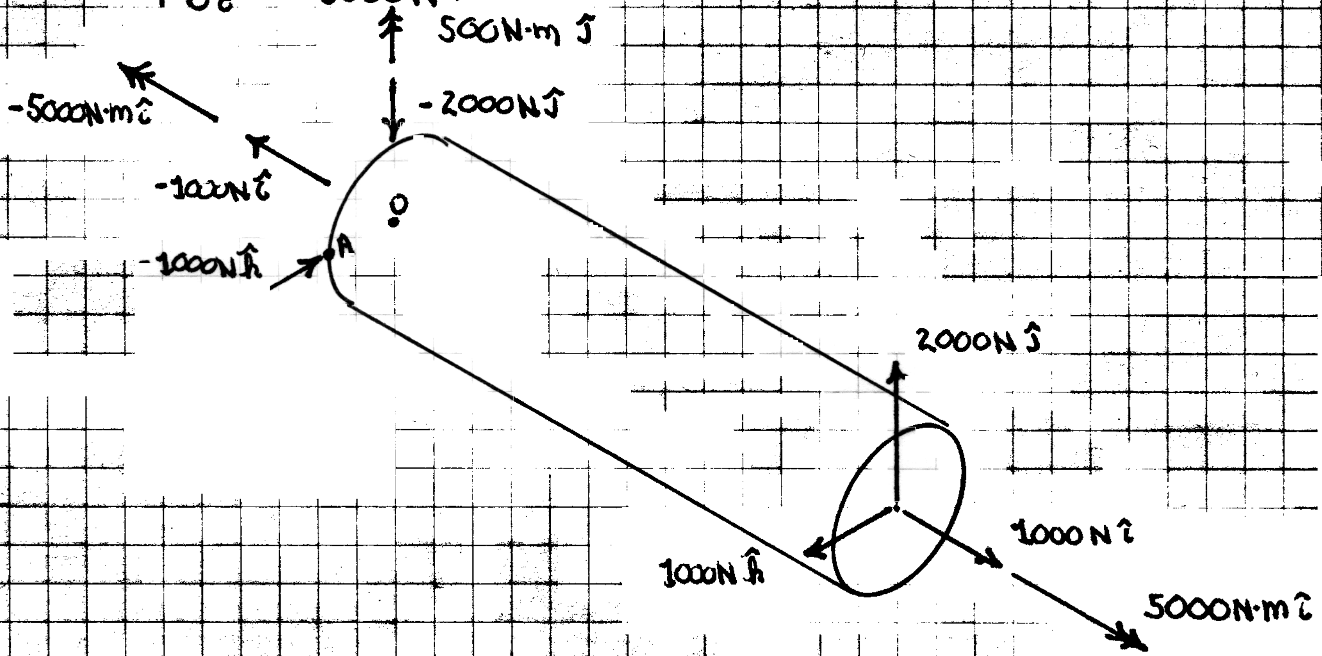
$$= \vec{M}_O + 5000\,N{\cdot}m\,\hat{i} + \left[0\hat{i} - (.5m)(1000N)\hat{j} + (.5m)(2000N)\hat{k}\right]$$

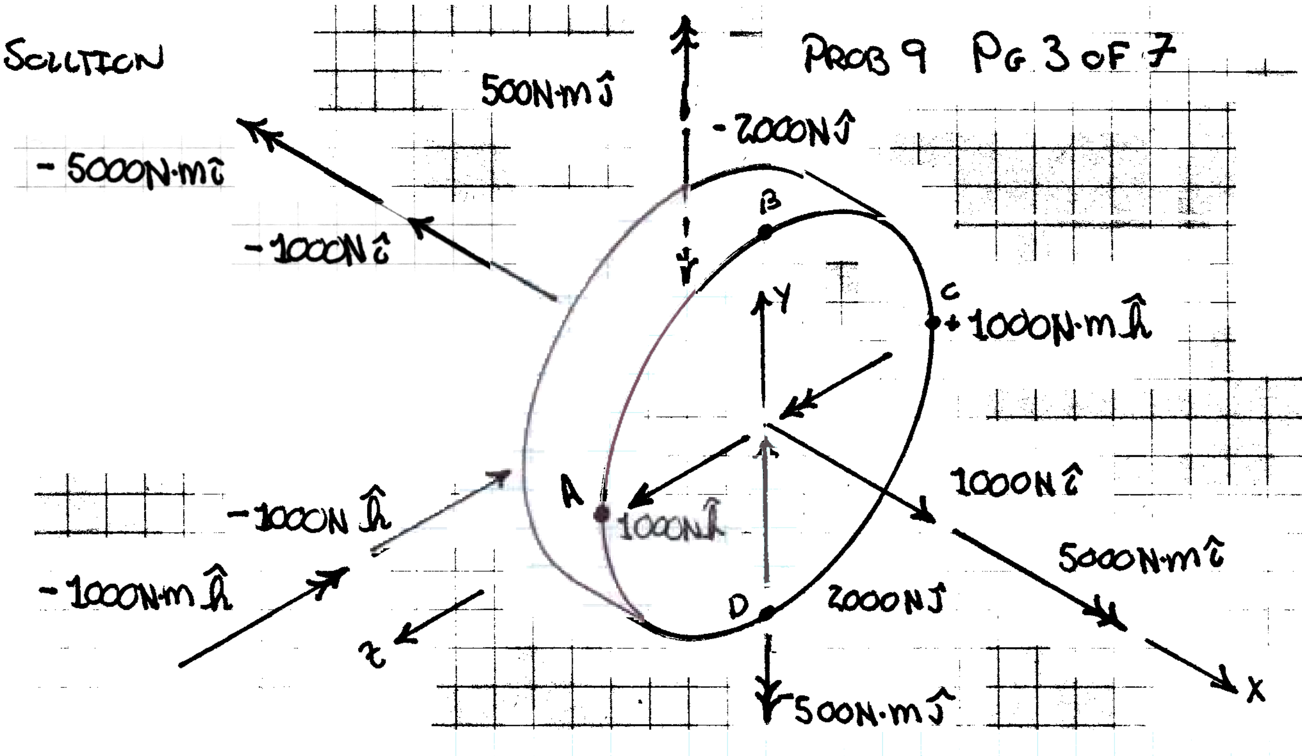
$$= M_{Ox}\hat{i} + M_{Oy}\hat{j} + M_{Oz}\hat{k} + 5000\,N{\cdot}m\,\hat{i} - 500\,N{\cdot}m\,\hat{j} + 1000\,Nm\,\hat{k}$$

$$\Rightarrow M_{Ox} = -5000\,N{\cdot}m$$

$$M_{Oy} = 500\,N{\cdot}m$$

$$M_{Oz} = -1000\,N{\cdot}m$$





SECTION OF ROD JUST OFF THE WALL

THE NORMAL STRESS AT POINT "A" CAN NOW BE CALCULATED.

$$\sigma_x = \frac{F_x}{A} + \frac{M_y \cdot z}{I_{yy}} = \frac{1000\text{N}}{\pi \cdot (.075\text{m})^2/4} + \frac{(-500\text{N}\cdot\text{m}) \cdot (.075\text{m}/2)}{\pi \frac{(.075\text{m})^4}{64}}$$

$$= \boxed{-11.84(10^6)\ \frac{\text{N}}{\text{m}^2}} \quad (1)$$

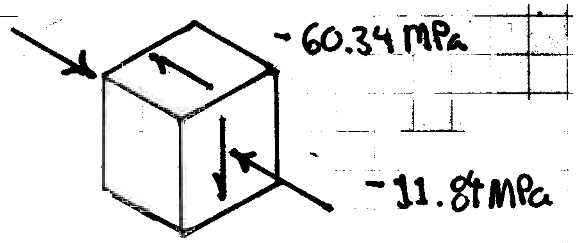
THE MOMENT IN THE z DIRECTION DOES NOT CONTRIBUTE TO THE NORMAL STRESS AT "A" BECAUSE "A" IS ON THE NEUTRAL AXIS FOR THE M_z MOMENT.

THE SHEAR STRESS AT "A" CAN NOW BE CALCULATED.

$$\tau_x = \frac{V_y Q}{I t} - \frac{T \cdot r}{J}$$

$$= \frac{(2000\text{N}) \cdot \frac{4}{3\pi} \cdot \left(\frac{.075\text{m}}{2}\right) \cdot \frac{\pi (.075\text{m})^2}{4 \cdot 2}}{\pi \cdot \frac{(.075\text{m})^4}{64} \cdot (.075\text{m})} - \frac{(5000\text{N}\cdot\text{m}) \cdot \left(\frac{.075\text{m}}{2}\right)}{\frac{\pi}{32}(.075\text{m})^4}$$

$$= \boxed{-60.34(10^6)\ \frac{\text{N}}{\text{m}^2}} \quad (2)$$



THE PRINCIPAL STRESSES ARE

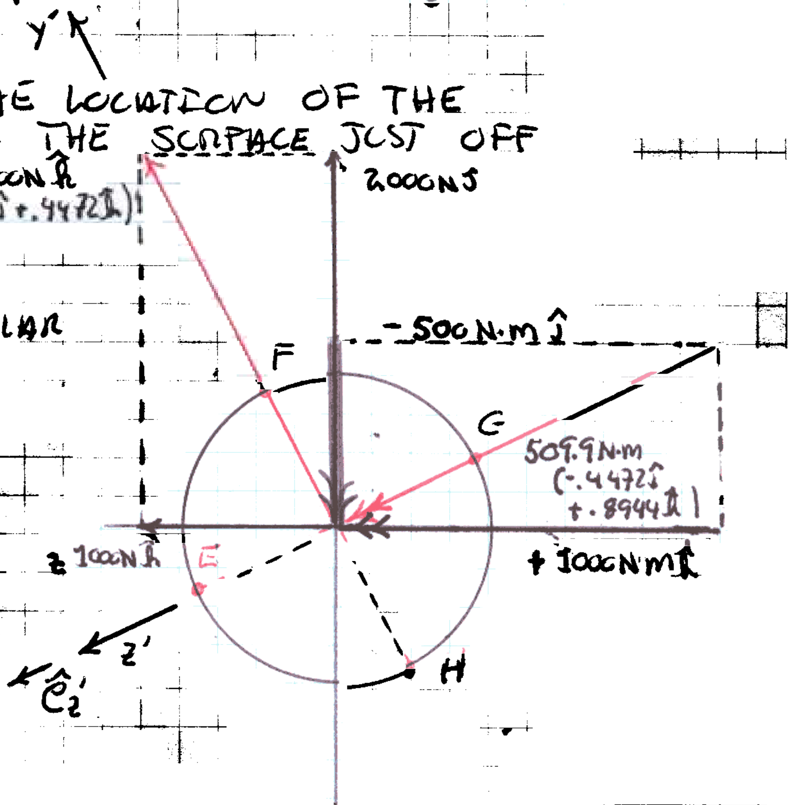
$\sigma_1 = 54.71$ MPa

$\tau_{max} = 60.63$ MPa

$\sigma_2 = 0$

$\sigma_3 = -66.55$ MPa

IT IS OF INTEREST TO FIND THE LOCATION OF THE MAXIMUM STRESSES. RECONSIDER THE SURFACE JUST OFF THE WALL



THIS PROBLEM IS UNIQUE IN THAT THE RESULTANT MOMENT IS PERPENDICULAR TO THE RESULTANT SHEAR FORCE. AS A RESULT

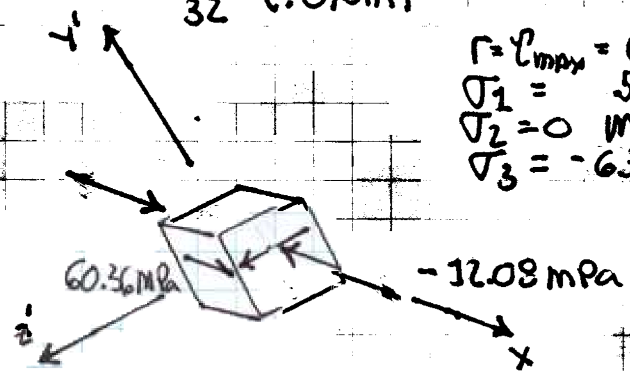
- THE LOCATION OF THE MAXIMUM AND MINIMUM BENDING STRESS WILL BE LOCATED AT F AND H
  - F IS COMPRESSIVE
  - H IS TENSILE
  - BOTH LOCATIONS HAVE NO SHEAR COMPONENT THAT RESULTS FROM THE RESULTANT SHEAR FORCE
- THE MAXIMUM SHEAR THAT RESULTS FROM THE RESULTANT SHEAR FORCE IS LOCATED AT E AND G
  - AT E AND G THE BENDING CONTRIBUTION TO THE NORMAL STRESS IS ZERO BECAUSE E AND G ARE ON THE NEUTRAL AXIS
  - BECAUSE OF THE DIRECTION OF THE TORQUE THE TWO SHEAR COMPONENTS WILL BE ADDATIVE AT G AND THE DIFFERENCE AT E.

POINT F

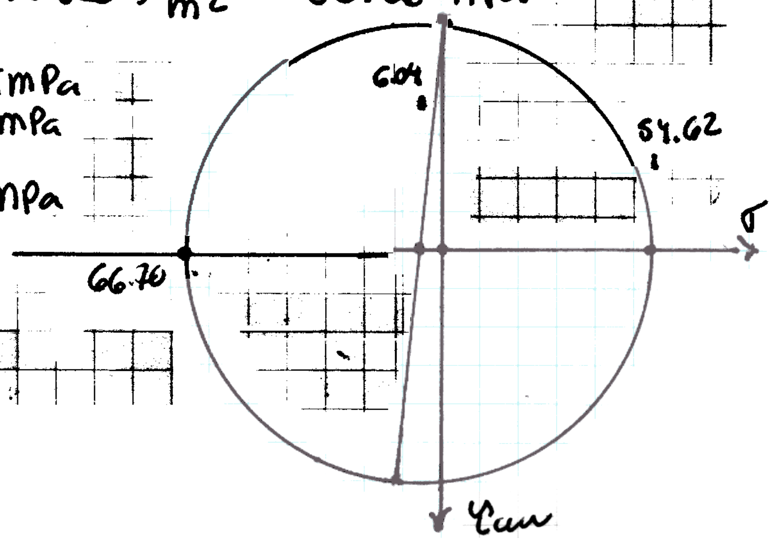
$$\sigma_x = \frac{1000N}{\pi\cdot(.075m)^2/4} - \frac{(509.9\,N\cdot m)\cdot(.075m/2)}{\pi\cdot\frac{(.075m)^4}{64}} = -12.08(10^6)\,\frac{N}{m^2}$$

$$= -12.08\ MPa$$

$$\tau_{xz'} = \frac{5000\,N\cdot m\cdot(.075m/2)}{\frac{\pi}{32}\,(.075m)^4} = 60.36(10^6)\,\frac{N}{m^2} = 60.36\ MPa$$



$r = \tau_{max} = 60.66\ MPa$
$\sigma_1 = 57.04\ MPa$
$\sigma_2 = 0\ MPa$
$\sigma_3 = -63.68\ MPa$

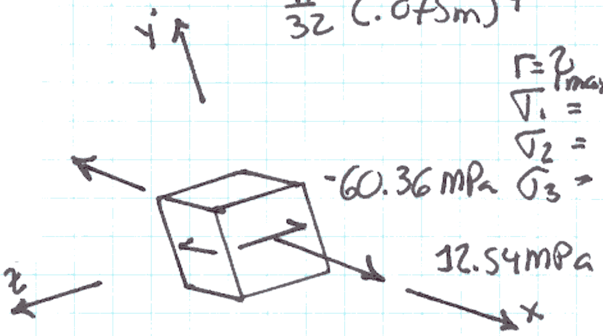


POINT H

$$\sigma_x \quad \frac{1000\,N}{\pi \cdot (.075m)^2/4} + \frac{(509.9\,N\cdot m)\cdot(.075m/2)}{\pi \cdot \frac{(.075m)^4}{64}} = 12.54(10^6)\frac{N}{m^2}$$

12.54 MPa

$$\tau_{xz'} = \frac{-5000\,N\cdot m \cdot (.075m/2)}{\frac{\pi}{32}(.075m)^4} = -60.36(10^6)\frac{N}{m^2} = -60.36\,MPa$$

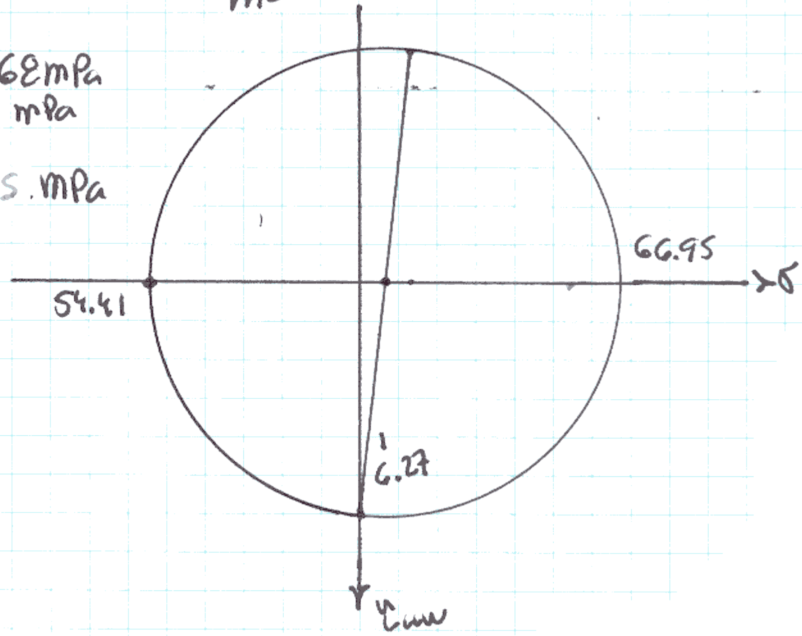


$r = \tau_{max} = 60.68\,mPa$

$\sigma_1 = 63.71\,mPa$

$\sigma_2 = 0$

$\sigma_3 = -57.55\,mPa$



POINT E

$$\sigma_x = \frac{1000\,N}{\pi \cdot (.075m)^2/4} = 0.226\,MPa$$

$$\tau_{xy'} = \frac{(2236\,N)\,\frac{4}{3\pi}\cdot\left(\frac{.075m}{2}\right)^2 \cdot \frac{\pi\cdot(.075m)^2}{4\cdot 2}}{\pi\cdot\frac{(.075m)^4}{64}\cdot(.075m)} - \frac{(5000\,N\cdot m)\cdot(.075m)}{\frac{\pi}{32}\cdot(.075m)^4}$$

$$= -57.80(10^6)\frac{N}{m^2}$$

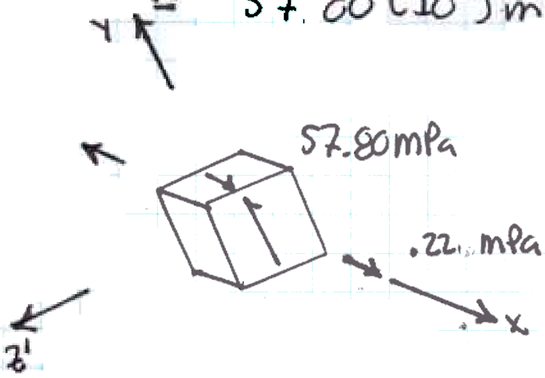
57.80 MPa

σ_1 58.02 mPa

σ_2 0

σ_3 -57.58 mPa

$\tau_{max} = 57.8\,MPa$

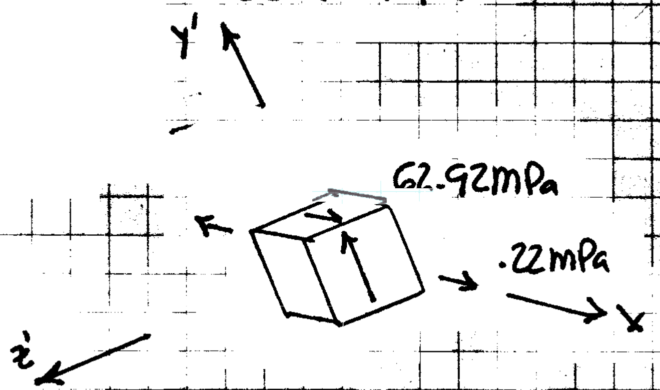


POINT G

$$\sigma_x = \frac{1000\,N}{\frac{\pi}{4}(.075m)^2} = 0.22\,MPa$$

$$\tau_{xy'} = \frac{(2236\,N)\,\frac{4}{3\pi}\cdot\left(\frac{.075\,m}{2}\right)^3\cdot\left(\frac{\pi\,(.075\,m)^2}{4\cdot 2}\right)}{\pi\cdot\frac{(.075m)^4}{64}\cdot(.075\,m)} + \frac{(5000\,N\cdot m)\cdot(.075m)}{\frac{\pi}{32}\cdot(.075\,m)^4}$$

$$= 62.92\ MPa$$



$\sigma_1 = 63.03\ MPa$

$\sigma_2 = 0\ MPa$

$\sigma_3 = 62.81\ MPa$

$\tau_{max} = 62.92\ MPa$

KNOWING THE PRINCIPAL STRESSES AT EACH POINT, THE PRINCIPAL STRAINS CAN BE CALCULATED FROM THE STRESS - STRAIN RELATIONS.

$$\begin{Bmatrix}\epsilon_1\\ \epsilon_2\\ \epsilon_3\end{Bmatrix} = \begin{bmatrix} 1/E & -\nu/E & -\nu/E \\ -\nu/E & 1/E & -\nu/E \\ -\nu/E & -\nu/E & 1/E \end{bmatrix}\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{Bmatrix} = \begin{bmatrix} 5.00(10^{-12})\frac{m^2}{N} & -1.50(10^{-12})\frac{m^2}{N} & -1.50(10^{-12})\frac{m^2}{N} \\ 1.50(10^{-12})\frac{m^2}{N} & 5.00(10^{-12})\frac{m^2}{N} & -1.50(10^{-12})\frac{m^2}{N} \\ 1.50(10^{-12})\frac{m^2}{N} & -1.50(10^{-12})\frac{m^2}{N} & 5.00(10^{-12})\frac{m^2}{N} \end{bmatrix}\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{Bmatrix}$$

POINT H

$$\begin{bmatrix} 5 & -1.5 & -1.5 \\ -1.5 & 5 & -1.5 \\ -1.5 & -1.5 & 5 \end{bmatrix}(10^{-12})\frac{m^2}{N}\begin{Bmatrix}54.71\\ 0\\ -66.55\end{Bmatrix}(10^6)\frac{N}{m^2} = \begin{Bmatrix}373.4\\ 17.8\\ -414.8\end{Bmatrix}\mu\epsilon = \begin{Bmatrix}\epsilon_1\\ \epsilon_2\\ \epsilon_3\end{Bmatrix}$$

POINT F

$$\begin{bmatrix} 5 & -1.5 & -1.5 \\ -1.5 & 5 & -1.5 \\ -1.5 & -1.5 & 5 \end{bmatrix}(10^{-12})\frac{m^2}{N}\begin{Bmatrix}57.04\\ 0\\ 63.68\end{Bmatrix}(10^6)\frac{N}{m^2} = \begin{Bmatrix}380.7\\ 10.0\\ 404.0\end{Bmatrix}\mu\epsilon = \begin{Bmatrix}\epsilon_1\\ \epsilon_2\\ \epsilon_3\end{Bmatrix}$$

POINT H

$$\begin{bmatrix} 5 & -1.5 & -1.5 \\ -1.5 & 5 & -1.5 \\ -1.5 & -1.5 & 5 \end{bmatrix} (10^{-12}) \frac{m^2}{N} \begin{Bmatrix} 63.71 \\ 0 \\ -57.55 \end{Bmatrix} (10^{6}) \frac{N}{m^2} = \begin{Bmatrix} 404.9 \\ -9.2 \\ 383.3 \end{Bmatrix} \mu\epsilon = \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{Bmatrix}$$

POINT E

$$\begin{bmatrix} 5 & -1.5 & -1.5 \\ -1.5 & 5 & -1.5 \\ -1.5 & -1.5 & 5 \end{bmatrix} (10^{-12}) \frac{m^2}{N} \begin{Bmatrix} 58.02 \\ 0 \\ -57.58 \end{Bmatrix} (10^{6}) \frac{N}{m^2} = \begin{Bmatrix} 376.5 \\ 0.7 \\ -374.9 \end{Bmatrix} \mu\epsilon = \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{Bmatrix}$$

POINT G

$$\begin{bmatrix} 5 & -1.5 & -1.5 \\ -1.5 & 5 & -1.5 \\ -1.5 & -1.5 & 5 \end{bmatrix} (10^{-12}) \frac{m^2}{N} \begin{Bmatrix} 63.03 \\ 0 \\ 62.81 \end{Bmatrix} (10^{6}) \frac{N}{m^2} = \begin{Bmatrix} 409.4 \\ -0.3 \\ -408.6 \end{Bmatrix} \mu\epsilon = \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{Bmatrix}$$

SUMMARY

THE SOLUTION PRESENTED HERE EXTENDS THE QUESTION ASKED. THIS SOLUTION ACTUALLY SEEKS OUT THE LOCATION OF THE MAXIMUM STRESS IN THIS STRUCTURE.