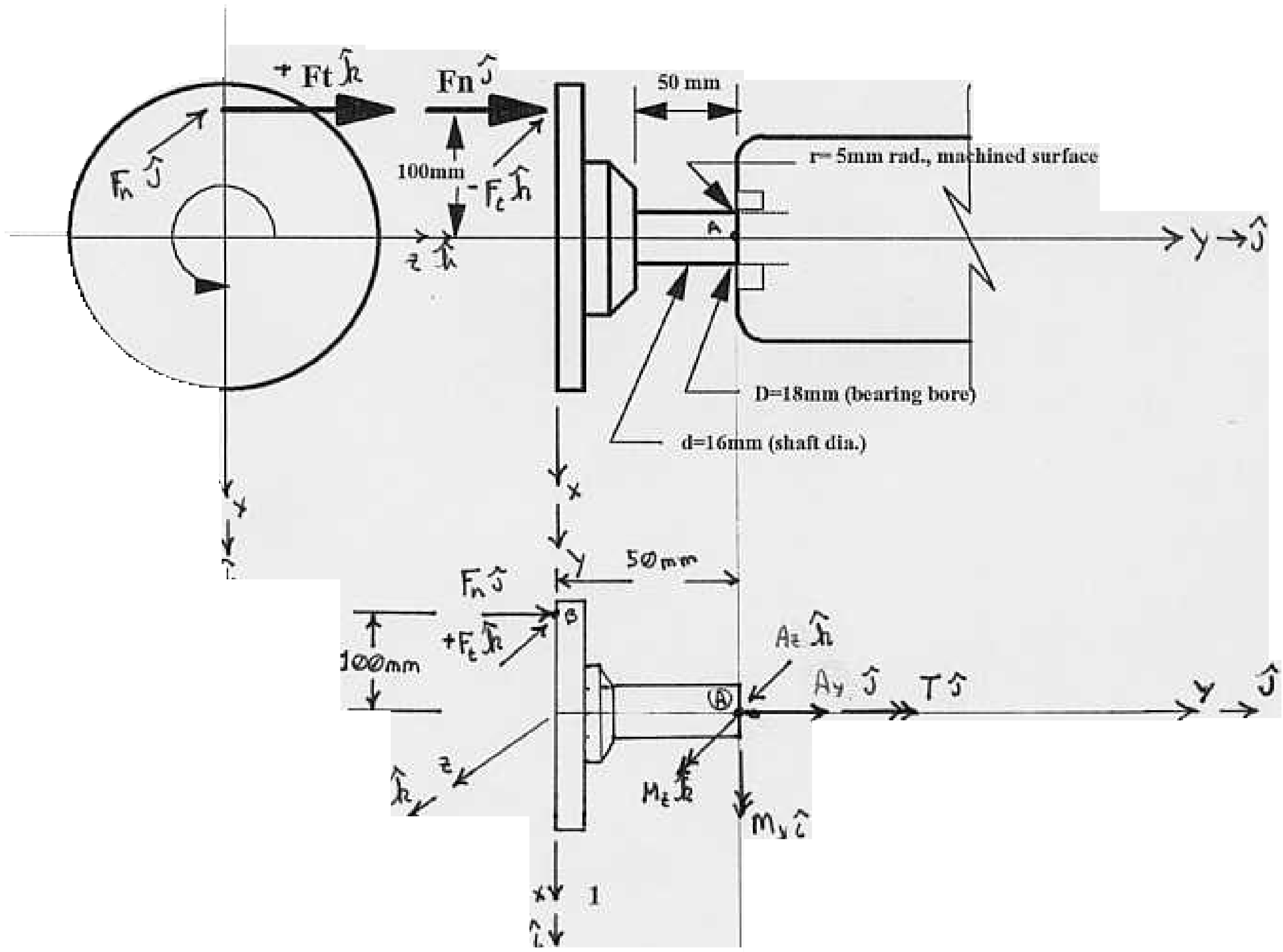
NAME: Solution

PROBLEM 1:(50 pts.) The figure below pertains to the shaft of a disk sander that is made of steel having S_u=900 MPa, and S_y=750 MPa. The most severe loading occurs when an object is held near the periphery of the disk (100-mm radius) with sufficient force to develop a friction torque of 12 N-m (which approaches the stall torque of the motor). Assume a coefficient of friction of 0.6 between the object and the disk. What is the safety factor with respect to eventual fatigue failure of the shaft?

Since we have a sufficient force ~~to~~ that approaches stal

$$T = \mu F_n d \Rightarrow F_n = \frac{T}{\mu d} = \frac{12000\ N\cdot mm}{0.6\ (100mm)} = \underline{200\ N = F_n}$$

$$\mu F_n = \underline{\underline{120\ N = F_t}}$$



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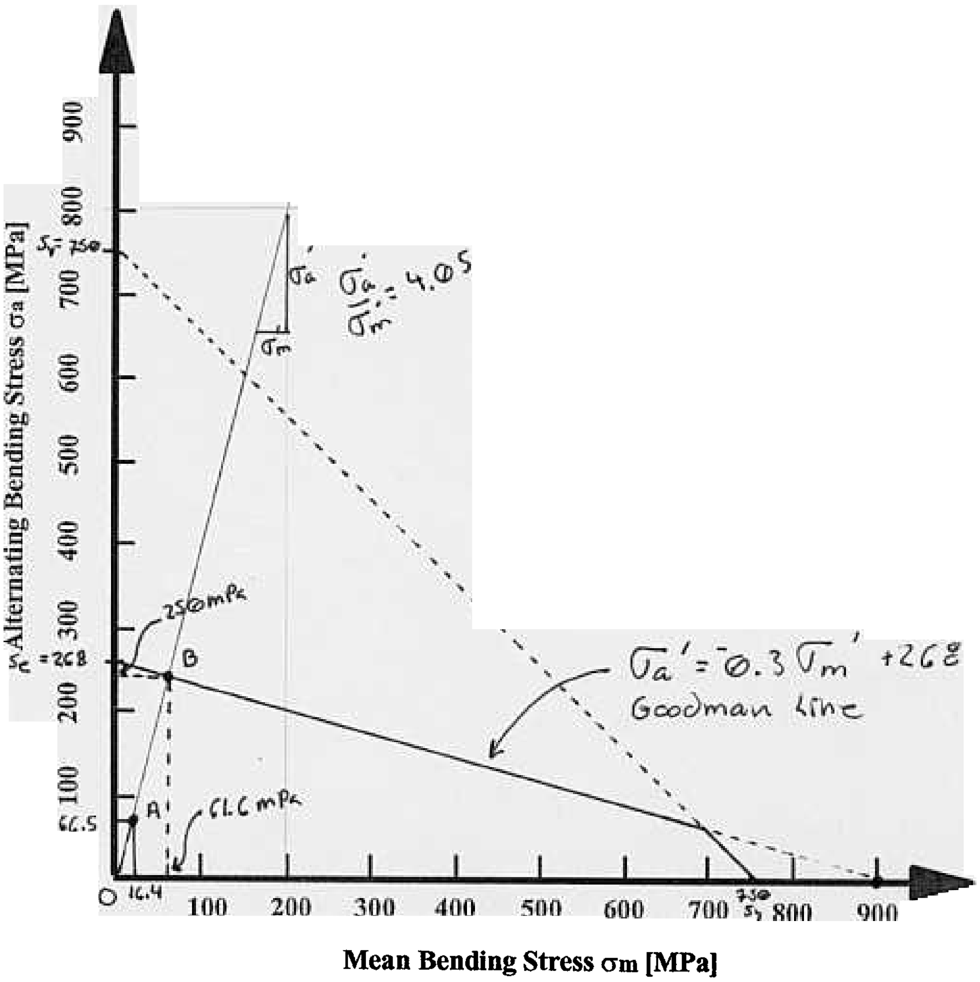


Draw the σ_a-σ_m diagram for this problem on the graph provided above.

Equilibrium

$$\sum F_x = 0 = 0$$
$$\sum F_y = 0 = A_y + F_n = A_y + 200\,N \Rightarrow \quad A_y = -200\,N$$
$$\sum F_z = 0 = A_z + F_t = A_z - 120\,N \Rightarrow \quad A_z = -120\,N$$
$$\sum \vec{M} = \vec{0} = M_x \hat{i} + M_z \hat{k} + T\hat{j} + \vec{r}_{AB} \times \vec{F}$$
$$\vec{r}_{AB} = -100\,mm\,\hat{i} - 50\,mm\,\hat{j}$$
$$F = F_n \hat{j} - F_t \hat{k} = [200\,\hat{j} - 120\,\hat{k}]\,N$$
$$\vec{0} = M_x \hat{i} + T\hat{j} + M_z \hat{k} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -100\,mm & -50\,mm & 0 \\ 0 & -200\,N & -120\,N \end{vmatrix}$$

$$\vec{0} = M_x\hat{i} + T\hat{j} + M_z\hat{k} + [(-50\,mm)(+120\,N)]\hat{i} - [(-100\,mm)(+120\,N)]\hat{j} + [(-100\,mm)(200\,N)]\hat{k}$$

$$= M_x\hat{i} + T\hat{j} + M_z\hat{k} + {}^{-}6\,000\,N\cdot mm\,\hat{i} + 12\,000\,N\cdot mm\,\hat{j} - 20\,000\,N\cdot mm\,\hat{k}$$

$$\Sigma\vec{M}|_{at\,A}\cdot\hat{i} = \Sigma M_x|_{at\,A} = 0 = M_x + 6\,000\,N\cdot mm \Rightarrow \underline{M_x = {}^{+}6\,000\,N\cdot mm}$$

$$\Sigma\vec{M}|_{at\,A}\cdot\hat{j} = \Sigma M_y|_{at\,A} = 0 = T - 12\,000\,N\cdot mm \Rightarrow \underline{T = {}^{-}12\,000\,N\cdot mm}$$

$$\Sigma M|_{at\,A}\cdot\hat{k} = \Sigma M_z|_{at\,A} = 0 = M_z - 20\,000\,N\cdot mm \Rightarrow \underline{M_z = 20\,000\,N\cdot mm}$$

10

Now we must consider what these forces and moments contribute to. A_y is a normal force that is constant throught the cycling

$$\sigma_m^* = \frac{A_y}{A} = \frac{-200\,N}{201\,mm^2} = 0.995\,\frac{N}{mm^2} = {}^{-}0.995\,MPa = \sigma_m^* \quad (1)$$

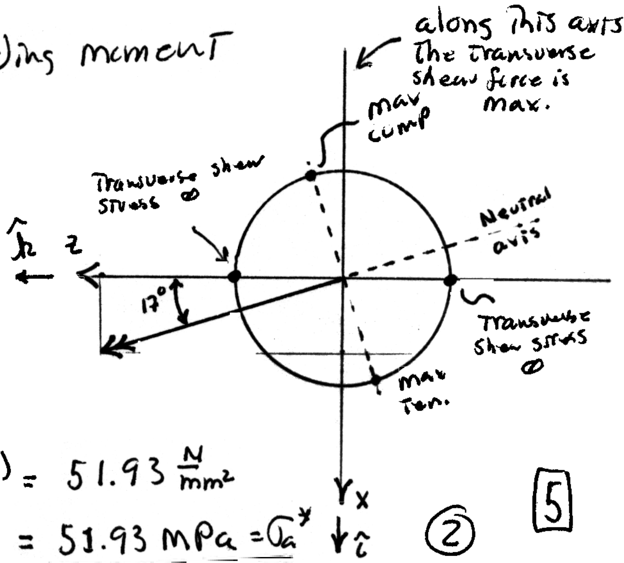
$\boxed{5}$

~~18~~

$$\pi\frac{d^2}{4} = 201\,mm^2$$

M_x and M_z combine to create a bending moment

$$M = \sqrt{(6000\,N\cdot mm)^2 + (20\,000\,N\cdot mm)^2}$$
$$= 20\,881\,N\cdot mm$$



This will contribute to the alternating Normal stress. Considering only tensile since this side is the most prone to fatigue failure.

$$\sigma_a^* = \frac{Mc}{I} = \frac{(20\,881\,N\cdot mm)(8\,mm)}{3217\,mm^4} = 51.93\,\frac{N}{mm^2}$$
$$= \underline{51.93\,MPa = \sigma_a^*} \quad (2)$$

$\boxed{5}$

$$\frac{\pi d^4}{64} = \frac{\pi(16\,mm)^4}{64} = 3217\,mm^4$$

T contributes to the mean shearing stress given by

$$\frac{Tr}{J} = \frac{(12\,000\,N\cdot mm)(8\,mm)}{6434\,mm^4} = 14.92\,\frac{N}{mm^2} = \underline{14.92\,MPa = \tau_m^*}$$

$\boxed{5}$

$$J = 2I = 6434\,mm^4$$

(sign is + since shear has no sign)

A_z contributes to the alternating part of the shear stress and is give by

$$\frac{VQ}{It} = \frac{(120\,N)(341.3\,mm^3)}{(3217\,mm^4)(16\,mm)} = 0.796\,\frac{N}{mm^2} = 0.796\,MPa = \tau_a^*$$

$$Q = \bar{y}A = \frac{4}{3}\frac{r}{\pi}\frac{\pi r^2}{2} = \frac{4}{3}\frac{r^3}{2} = 341.3\,mm^3$$

(This component will not be include since it's orientation at the max Normal stress does not align with τ_m^* thus making it even less of a contributor.)

15

Using the supplied tables for K_t and q we will now determine K_f's that are appropriate

$$\frac{r}{d} = \frac{5\,mm}{16\,mm} = 0.313\ ;\quad \frac{D}{d} = 1.125\ ;\ \text{and}\quad K_f = 1 + q(K_t - 1)$$

Therefore

	K_t	q	K_f
Bending	1.28	.9	1.09
Normal	1.3	.9	1.28
Torsion	1.09	.9	1.27

q & K_t come from Provided Tables $\boxed{5}$

Therefore

$$① \rightarrow \sigma_m = \sigma_m^* \cdot K_f^N = (-0.995\,MPa)(1.3) = \underline{-1.29\,MPa = \sigma_m}$$

$$② \rightarrow \sigma_a = \sigma_a^* \cdot K_f^M = (51.93\,MPa)(1.28) = \underline{\underline{66.5\,MPa = \sigma_a}}$$

$$③ \rightarrow \tau_m = \tau_m^* \cdot K_f^T = (14.92\,MPa)(1.09) = \underline{16.3\ MPa = \tau_m}$$ $\boxed{5}$

Now the von Misses stresses σ_a' and σ_m' can be calculated

$$\sigma_m' = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{(-1.29\,MPa)^2 + 3(16.3\,MPa)^2} = 16.4\ MPa$$ $\boxed{5}$

$$\sigma_a' = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{(66.5\,MPa)^2 + (0)^2} = 66.5\ MPa$$

Now we have to calculate S_e to plot a σ_a'-σ_m' diagram

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot \cancel{k_e} \cdot k_f \cdot 0.5\,S_u$$

$$S_e = (0.7)(0.85)(1)(1) \cdot 0.5\,(900\,MPa) = \underline{\underline{268\,MPa = S_e}}$$

k_e is not included here because K_f takes on different values for each type of loading $\boxed{5}$

Now the Figure on Pg 2 is completed.

$$\frac{\sigma_a'}{\sigma_m'} = \frac{66.5\,MPa}{16.4\,MPa} = 4.05$$

20

The equation of the Goodman line is given by

$$\sigma_a' = -\frac{S_e}{S_u}\sigma_m' + S_e = -0.30\,\sigma_m' + 268$$

and we know that O-A-B is given by $\sigma_a' = 4.05\,\sigma_m'$ setting σ_a' equal to find "B"

$$4.05\,\sigma_m' = -0.30\,\sigma_m' + 268 \implies \sigma_m'^{(B)} = 61.6\ MPa$$

$$\sigma_a'^{(B)} = 250\ MPa$$

Thus the safety factor is found by

$$SF = \frac{OB}{OA} = \frac{\sqrt{(61.6\,MPa)^2 + (250\,MPa)^2}}{\sqrt{(16.4\,MPa)^2 + (66.4\,MPa)^2}} = \boxed{4.0 = SF}$$

10

PROBLEM 2:(25 pts.) A simply supported rectangular beam 10 ft long is struck at the middle by a 100 lb weight falling from a height of 20 in. Determine the necessary cross-sectional area if the working stress is $\sigma_w = 10{,}000 \text{ lb/in}^2$, $E = 30(10^6) \text{ lb/in}^2$, and δ_{st} is neglected in comparison with h.

$$\sigma_{max} = \sqrt{\frac{Wv^2}{2g} \cdot \frac{18E}{L \cdot b \cdot h}}$$

$$A = b \cdot h = \frac{Wv^2}{2g} \frac{18E}{L\sigma_{max}^2} \quad \text{where } v = \sqrt{2gH}$$

$$\frac{W\,2gH}{2g} \frac{18E}{L\sigma_{max}^2}$$

$$\frac{(100\text{lb})(20\text{in})(18)(30)(10^6)\frac{\text{lb}}{\text{in}^2}}{(10\text{ft})\left(\frac{12\text{in}}{\text{ft}}\right)\left(10{,}000\frac{\text{lb}}{\text{in}^2}\right)^2}$$

$$\boxed{A = 90\text{in}^2}$$

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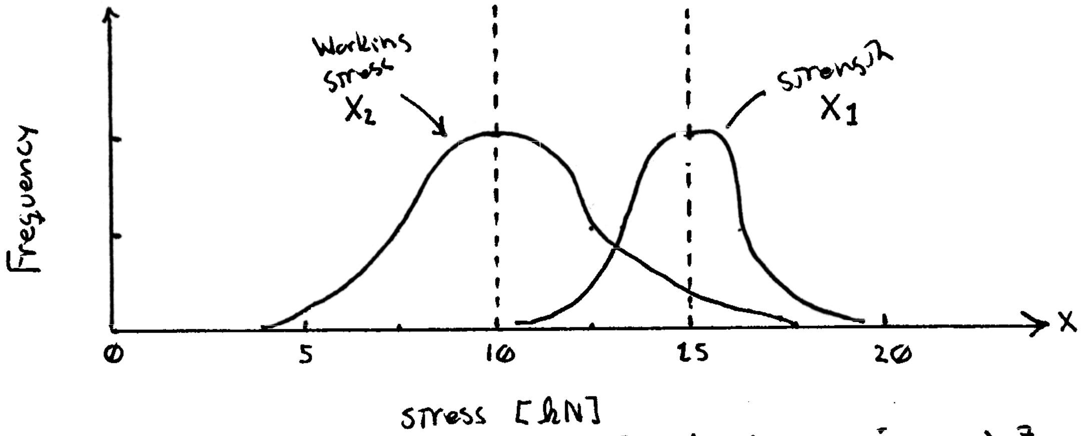
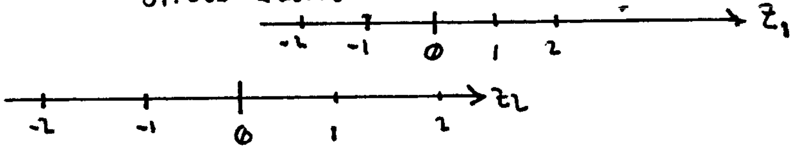
3

PROBLEM 3:(25 pts.) For the beam in Problem 2 the working stress is 10 ksi. The strength of the material is 15 ksi. If the maximum load encountered is normally distributed with a standard deviation of 2.5 ksi, and the beam strength is normally distributed with a standard deviation of 2.0 ksi, what failure percentage would be expected.



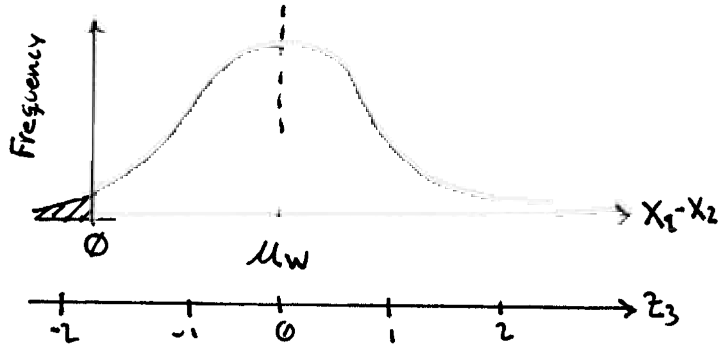


Defining the following random variables

material strength

x_2 Working stress

$X_1 - X_2$



I know that failures will occur when

$W < 0$

$X_1 - X_2 < 0$

Since X_1 and X_2 are normally distributed random variables I can write the mean of W as

$$\bar{W} = \bar{X}_1 - \bar{X}_2 = \mu_{x_1} - \mu_{x_2} = \underline{\underline{5 \text{ ksi} = \mu_W}}$$ 10

And the standard deviation as

$$\sigma_W = \sqrt{(2.0\text{ksi})^2 + (2.5\text{ksi})^2} = \underline{\underline{3.20 \text{ ksi} = \sigma_W}}$$

Now I find the standard normal random variable z_3 where $W = 0$

$$\frac{W - \mu_W}{\sigma} = \frac{0 - 5\ \text{ksi}}{3.2\ \text{ksi}} = -1.56 \quad \boxed{10}$$

going to the table provided

$$\text{failure} = 1 - 0.940 = 0.06$$

$$\boxed{\%\ \text{failure} = 6\%} \qquad \boxed{5}$$