MER42-Design III
Exam II

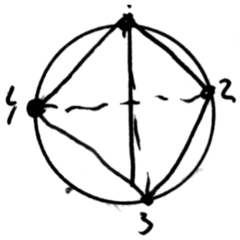
Union College
April 13, 1997

NAME: ____________________

Problem 1: Consider the mechanism shown in the figure below. The triangular wedge, coupler CDB, is attached to two sliders at B and C. The joints at B and C are full joints. Both sliders are constrained to move along the wall frictionlessly. Point B is being forced to move at a constant velocity of 6.10 m/s to the left. For the position shown the loop closure equation is as follows:

$$\vec{R}_{BO} + \vec{R}_{OC} = \vec{R}_{BC}$$

where

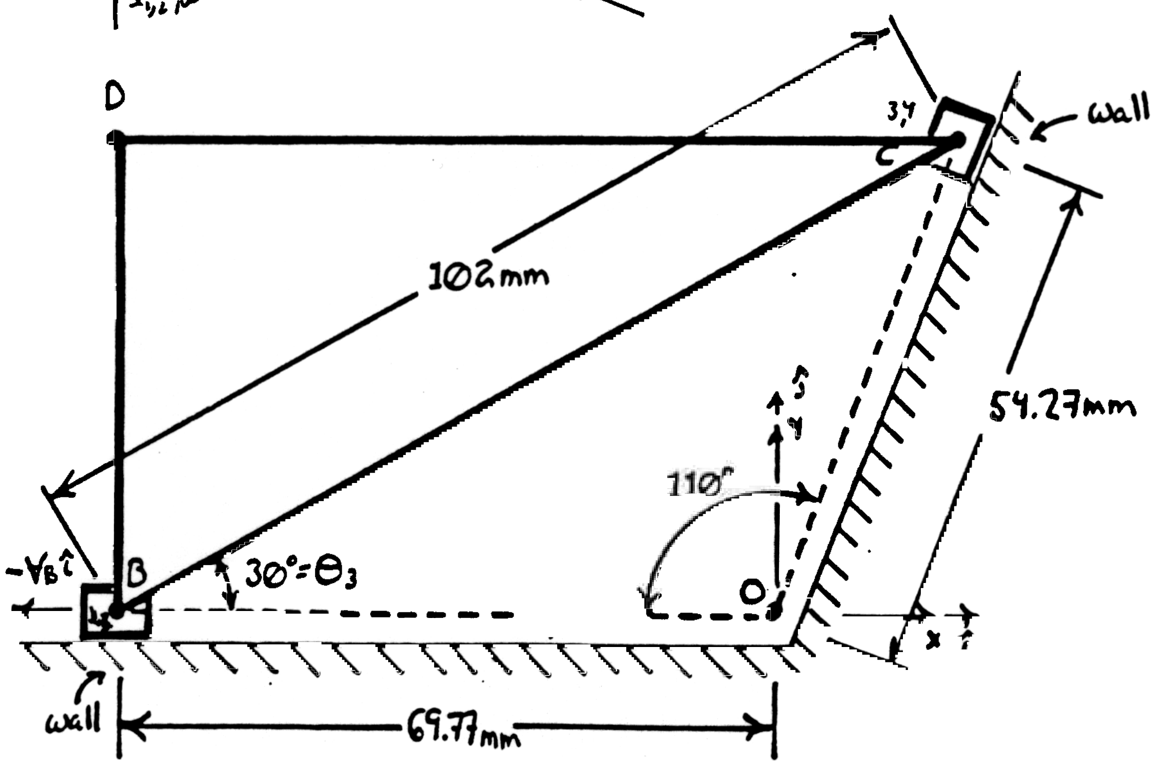


$$\vec{R}_{BO} = R_{BO}\cdot e^{j\cdot\theta_1} = 69.77\text{mm}\cdot e^{j\cdot 0^\circ}$$
$$= R_{BO}\cdot \hat{e}_{BO} = 69.77\text{mm}\cdot \hat{i}$$

$$\vec{R}_{OC} = R_{OC}\cdot e^{j\cdot\theta_2} = 54.77\text{mm}\cdot e^{j\cdot 70^\circ} = 18.56\text{mm} + j\cdot 51.0\text{mm}$$
$$(1,3) = R_{OC}\cdot \hat{e}_{OC} = 54.77\text{mm}\cdot(0.3420\cdot\hat{i} + 0.9397\cdot\hat{j})$$

$$\vec{R}_{BC} = R_{BC}\cdot e^{j\cdot\theta_3} = 102\text{mm}\cdot e^{j\cdot 30^\circ} = 88.33\text{mm} + j\cdot 51.0\text{mm}$$
$$= R_{BC}\cdot \hat{e}_{BC} = 102\text{mm}\cdot(0.8660\cdot\hat{i} + 0.5\cdot\hat{j}) \quad I_{1,4,\infty}$$

$I_{1,2,\infty}$



(using vectors)

$$R_{BO}\hat{e}_{BO} + R_{OC}\hat{e}_{OC} = R_{BC}\hat{e}_{BC}$$

$$\dot{R}_{BO}\hat{e}_{BO} + R_{BO}\cancel{\dot{\hat{e}}_{BO}}^{0} + \dot{R}_{OC}\hat{e}_{OC} + R_{OC}\cancel{\dot{\hat{e}}_{OC}}^{0} = \cancel{\dot{R}_{BC}}^{0}\hat{e}_{BC} + R_{BC}\dot{\hat{e}}_{BC}$$

$$\underset{\text{Known}}{\dot{R}_{BO}}\hat{e}_{BO} + \underset{\text{unknown}}{\dot{R}_{OC}}\hat{e}_{OC} = R_{BC}\dot{\hat{e}}_{BC} = \underset{\text{known}}{R_{BC}}\underset{\text{unknown}}{\dot{\Theta}_3}(\hat{k}\times\hat{e}_{BC})$$

Solving for $\dot{R}_{OC}$ by dotting both sides by $\hat{e}_{BC}$

$$\dot{R}_{BO}\hat{e}_{BC}\cdot\hat{e}_{BO} + \dot{R}_{OC}\hat{e}_{BC}\cdot\hat{e}_{OC} = R_{BC}\dot{\Theta}_3\underbrace{\hat{e}_{BC}\cdot(\hat{k}\times\hat{e}_{BC})}_{0}$$

$$\Rightarrow \dot{R}_{OC} = -\dot{R}_{BO}\frac{\hat{e}_{BC}\cdot\hat{e}_{BO}}{\hat{e}_{BC}\cdot\hat{e}_{OC}} = +6.10\tfrac{m}{s}\left[\frac{(.8660\hat{i}+0.5\hat{j})\cdot(\hat{i})}{(.8660\hat{i}+0.5\hat{j})\cdot(.3420\hat{i}+.9397\hat{j})}\right] = \boxed{6.896\tfrac{m}{s}}$$

Solving for $\dot{\Theta}_3$ by dotting both sides by $(\hat{k}\times\hat{e}_{OC})$

$$\dot{R}_{BO}\hat{e}_{BO}\cdot(\hat{k}\times\hat{e}_{OC}) + \dot{R}_{OC}\cancel{\hat{e}_{OC}\cdot(\hat{k}\times\hat{e}_{OC})}^{0} = R_{BC}\dot{\Theta}_3(\hat{k}\times\hat{e}_{BC})\cdot(\hat{k}\times\hat{e}_{OC})$$

$$\Rightarrow \dot{\Theta}_3 = -\frac{\dot{R}_{BO}}{R_{BC}}\frac{\hat{e}_{BO}\cdot(\hat{k}\times\hat{e}_{OC})}{(\hat{k}\times\hat{e}_{BC})\cdot(\hat{k}\times\hat{e}_{OC})}$$

$$\hat{k}\times\hat{e}_{OC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ .3420 & .9397 & 0\end{vmatrix} = .9397\hat{i} - .3420\hat{j}$$

$$\hat{k}\times\hat{e}_{BC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ .8660 & .5 & 0\end{vmatrix} = .5\hat{i} - .8660\hat{j}$$

$$\dot{\Theta}_3 = \frac{6.10\tfrac{m}{s}}{.102m}\cdot\frac{\hat{i}\cdot(.9397\hat{i} - .3420\hat{j})}{(.5\hat{i} - .8660\hat{j})\cdot(.9397\hat{i} - .3420\hat{j})} = \boxed{73.36\,1/s}$$

(Using Complex Approach)

$$R_{BO}e^{j\theta_2} + R_{OC}e^{j\theta_4} = R_{BC}e^{j\theta_3}$$

$$\dot{R}_{BO}e^{j\theta_2} + R_{BO}j\cancel{\dot{\theta}_2}^{\,0}e^{j\theta_2} + \dot{R}_{OC}e^{j\theta_4} + R_{OC}j\cancel{\dot{\theta}_4}^{\,0}e^{j\theta_4} = \cancel{\dot{R}_{BC}}^{\,0}e^{j\theta_3} + R_{BC}j\dot{\theta}_3e^{j\theta_3}$$

$$\underset{\text{known}}{\dot{R}_{BO}e^{j\theta_2}} + \underset{\text{unknown}}{\dot{R}_{OC}}e^{j\theta_4} = \underset{\text{known}}{R_{BC}}j\underset{\text{unknown}}{\dot{\theta}_3}e^{j\theta_3}$$

expanding using Euler's equation

$$\dot{R}_{BO}(\cos\theta_2 + j\sin\theta_2) + \dot{R}_{OC}(\cos\theta_4 + j\sin\theta_4) = R_{BC}j\dot{\theta}_3(\cos\theta_3 + j\sin\theta_3)$$

$$\dot{R}_{BO}\cos\theta_2 + \dot{R}_{OC}\cos\theta_4 = -R_{BC}\dot{\theta}_3\sin\theta_3$$

$$\dot{R}_{BO}\sin\theta_2 + \dot{R}_{OC}\sin\theta_4 = R_{BC}\dot{\theta}_3\cos\theta_3$$

Solving for $\dot{R}_{OC}$ by multiplying the real equation by $\cos\theta_3$ and the imaginary portion of the equation by $\sin\theta_3$

$$\dot{R}_{BO}\cos\theta_2\cos\theta_3 + \dot{R}_{OC}\cos\theta_4\cos\theta_3 = -R_{BC}\dot{\theta}_3\sin\theta_3\cos\theta_3$$

$$\dot{R}_{BO}\sin\theta_2\sin\theta_3 + \dot{R}_{OC}\sin\theta_4\sin\theta_3 = R_{BC}\dot{\theta}_3\cos\theta_3\sin\theta_3$$

$$\dot{R}_{BO}(\cos\theta_2\cos\theta_3 + \sin\theta_2\sin\theta_3) + \dot{R}_{OC}(\cos\theta_4\cos\theta_3 + \sin\theta_4\sin\theta_3) = 0$$

$$\dot{R}_{BO}\cos(\theta_3-\theta_2) + \dot{R}_{OC}\cos(\theta_4-\theta_3) = 0$$

$$\Rightarrow \dot{R}_{OC} = -\dot{R}_{BO}\frac{\cos(\theta_3-\theta_2)}{\cos(\theta_4-\theta_3)} = +(6.10\tfrac{m}{s})\cdot\frac{\cos(30^\circ-0^\circ)}{\cos(70^\circ-30^\circ)} = \boxed{6.896\tfrac{m}{s}}$$

Solving for $\dot{\theta}_3$ by multiplying the real equation by $\sin\theta_4$ and the imaginary portion of the equation by $\cos\theta_4$

$$\dot{R}_{BO}\cos\theta_2\sin\theta_4 + \dot{R}_{OC}\cos\theta_4\sin\theta_4 = -R_{BC}\dot{\theta}_3\sin\theta_3\sin\theta_4$$

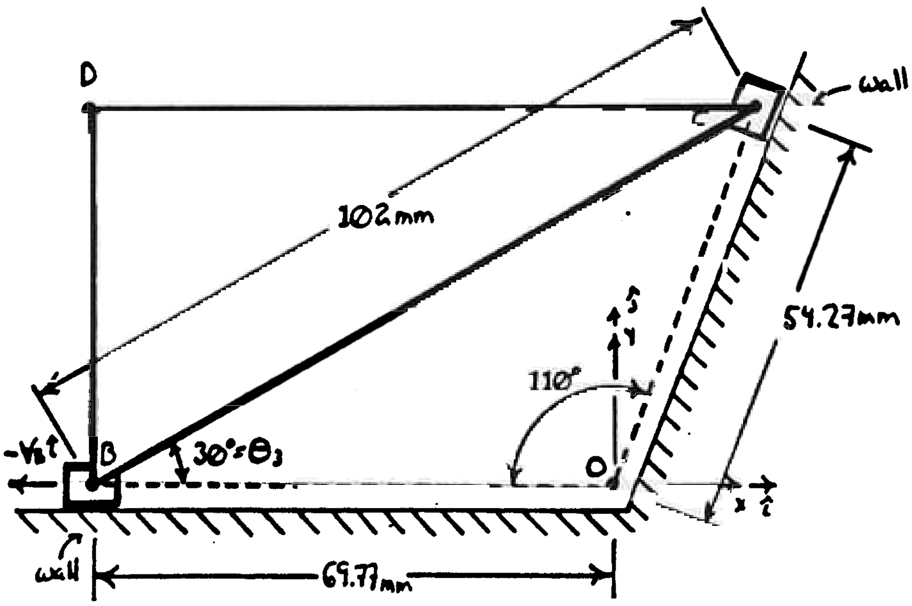
$$\left[\dot{R}_{BO}\sin\theta_2\cos\theta_4 + \dot{R}_{OC}\sin\theta_4\cos\theta_4 = R_{BC}\dot{\theta}_3\cos\theta_3\cos\theta_4\right]$$

$$\dot{R}_{BO}(\cos\theta_2\sin\theta_4 - \sin\theta_2\cos\theta_4) = -R_{BC}\dot{\theta}_3(\sin\theta_3\sin\theta_4 + \cos\theta_3\cos\theta_4)$$

$$\dot{R}_{BO}\sin(\theta_4-\theta_2) = -R_{BC}\dot{\theta}_3\cos(\theta_4-\theta_3)$$

$$\Rightarrow \dot{\theta}_3 = \frac{-\dot{R}_{BO}}{R_{BC}}\frac{\sin(\theta_4-\theta_2)}{\cos(\theta_4-\theta_3)} = \frac{+6.10}{.102m}\frac{\sin(70^\circ-0^\circ)}{\cos(70^\circ-30^\circ)} = \boxed{73.36\ 1/s}$$

1b. Using velocity polygons, determine the angular velocity of the triangular wedge coupler CDB ($\dot{\theta}_3$) and the velocity of point C ($\dot{R}_{OC}$).

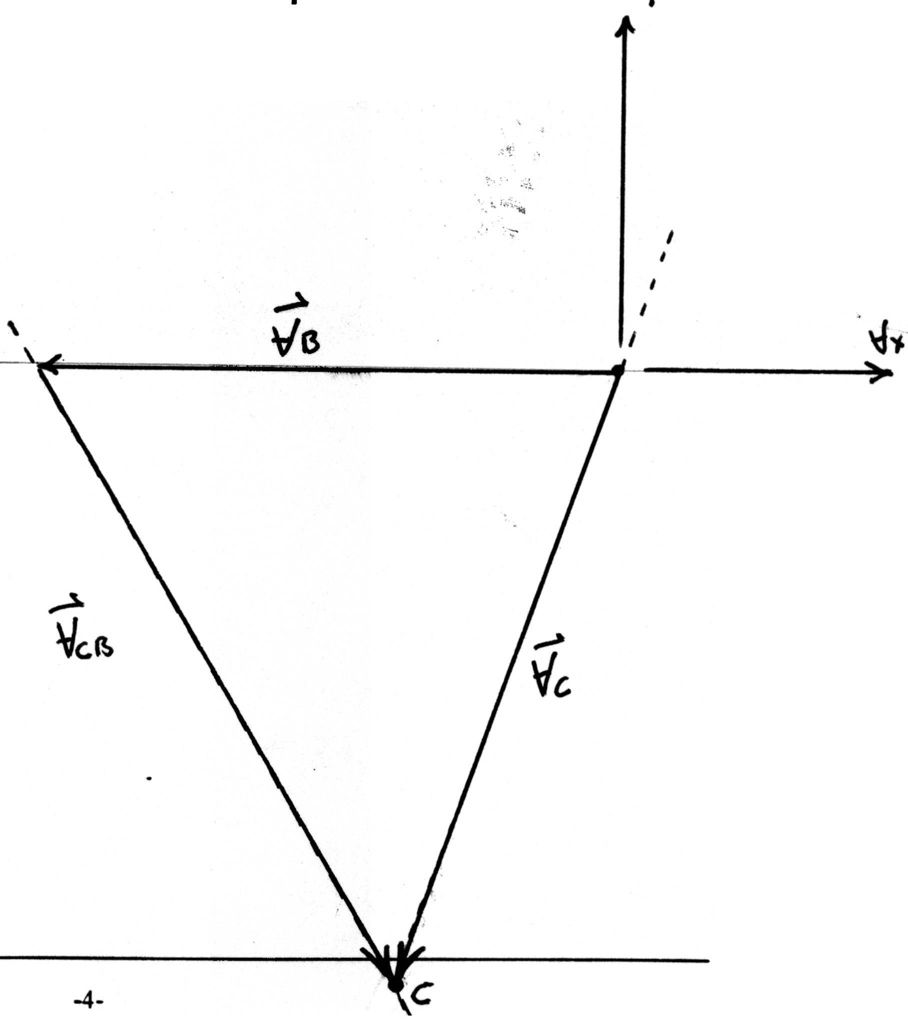


$$\vec{V}_C = \vec{V}_B + \vec{V}_{CB}$$

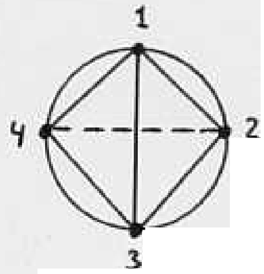
$$V_C = \boxed{6.9\ m/s}$$

$$V_{CB} = 7.5\ m/s = \omega_3 r$$

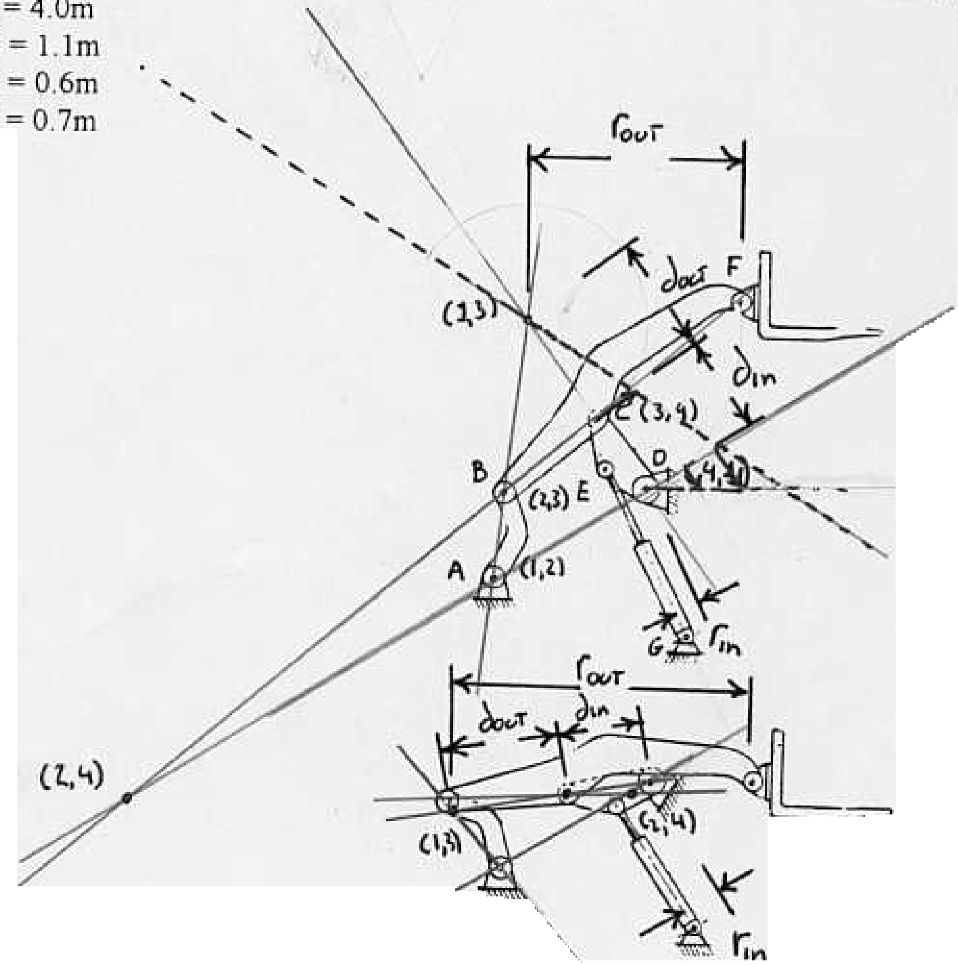
$$\omega_3 = \frac{7.5\ m/s}{.102m} = \boxed{73.5\ 1/s}$$

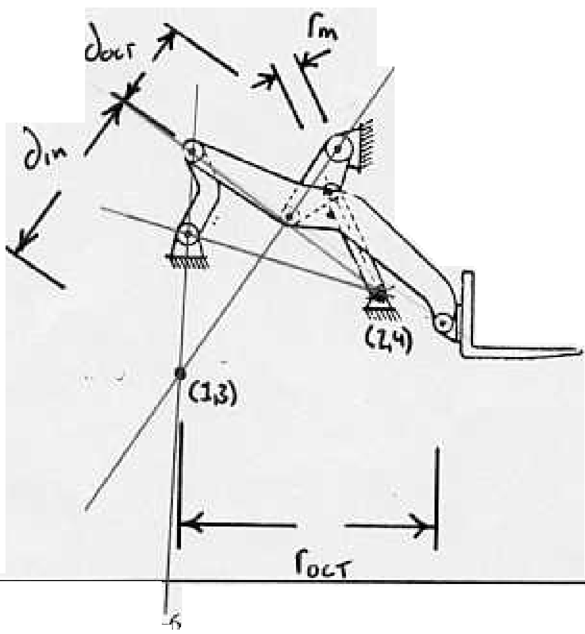


Problem 2: A no mast lift truck uses the mechanism shown below to lift its payload. The link dimensions are:



AB = 1.1m
BC = 1.6m
BF = 4.0m
CD = 1.1m
DE = 0.6m
CE = 0.7m





2a. Given that the input force is applied to the system through the hydraulic actuator attached at point E and that the output force is vertical at point F, determine the mechanical advantage for the linkage in the positions where the load is still on the ground and the position where the load is fully extended . **(BONUS:** Determine the mechanical advantage in the third position)

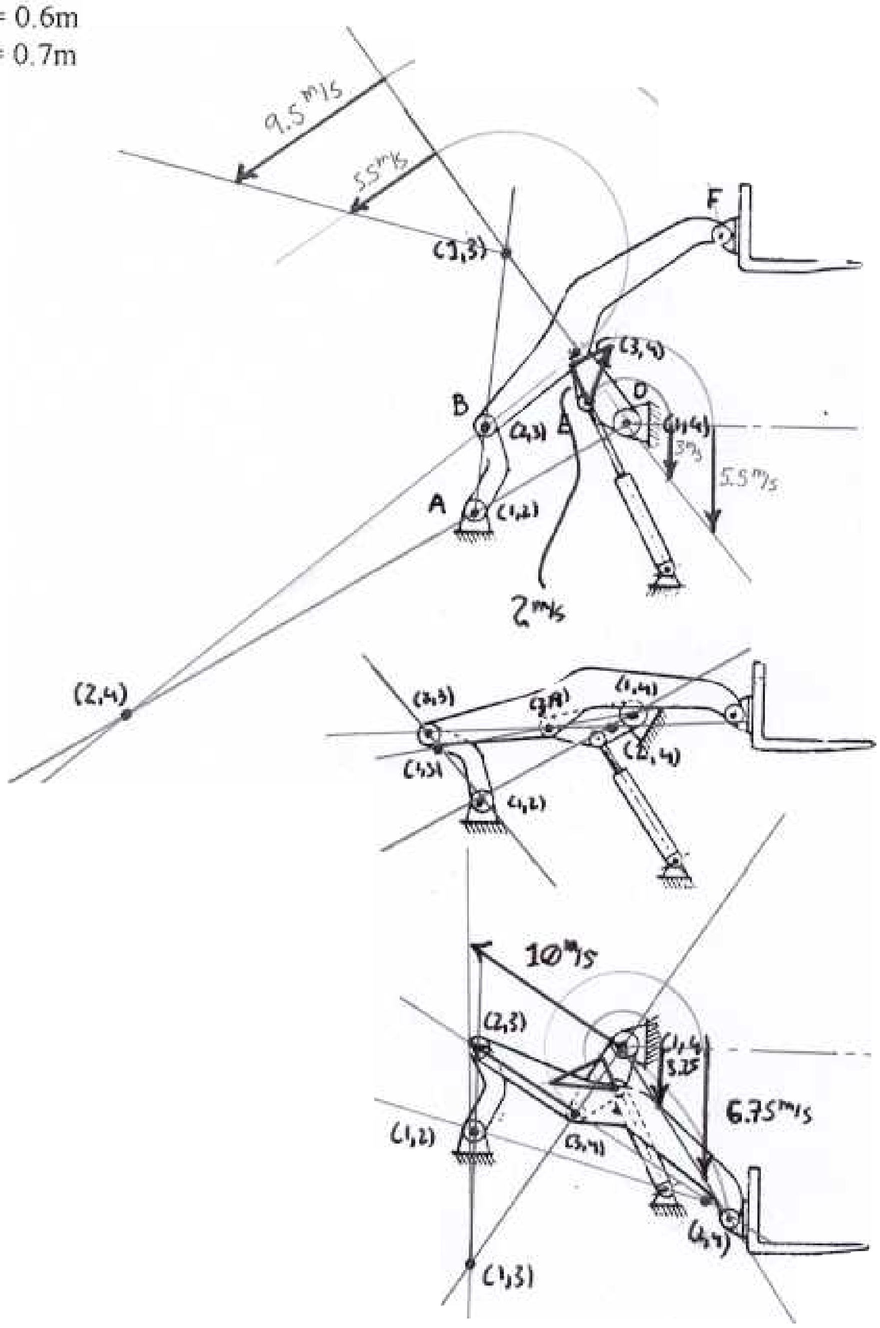
a) $MA = \frac{r_{in}}{r_{out}} \cdot \frac{d_{out}}{d_{in}} = \frac{.3}{2.8} \quad \frac{1.6}{1.2} = \boxed{\emptyset.132}$

b) $mA = \frac{.5}{3.9} \cdot \frac{1.5}{1.1} = \boxed{0.175}$

c) $mA = \frac{.3}{3.4} \cdot \frac{2.5}{1.1} = \boxed{\emptyset.2\emptyset}$

Problem 2: A no mast lift truck uses the mechanism shown below to lift its payload. The link dimensions are:

AB = 1.1m
BC = 1.6m
BF = 4.0m
CD = 1.1m
DE = 0.6m
CE = 0.7m



V_E = 3 m/s
V_F = 9.5 m/s

V_E = 3.25 m/s
V_F = 10 m/s

2b. If the hydraulic actuator extends at a constant rate of 2 m/s, using instant centers determine the initial velocity of the payload as it comes off the ground and in the fully extended positions. **(BONUS:** Determine the velocity of the payload in the third position.)

Graphical

$V_E = 3\ m/s$

$V_F = 9.5\ m/s$

b)

$V_E = 3.25\ m/s$

$V_F = 10\ m/s$