Name: Solution

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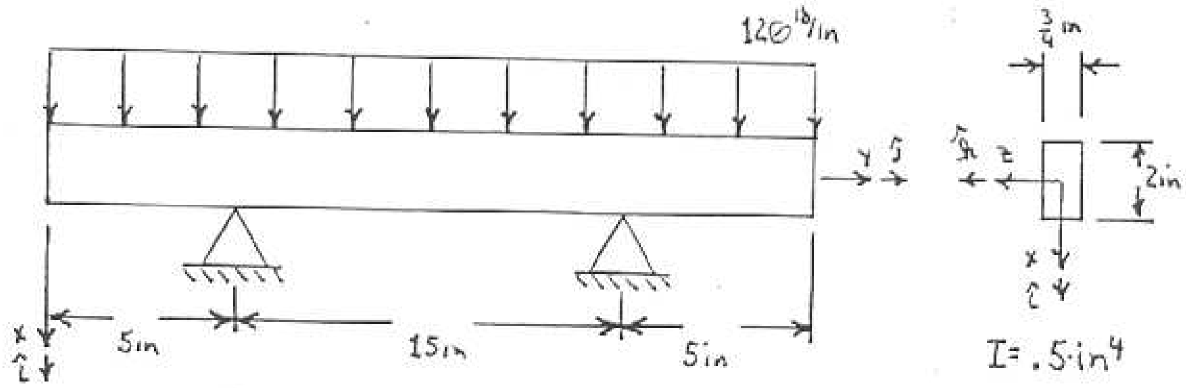
Problem 1. The beam shown below is subjected to the load shown. After a careful analysis of the external and internal loads, the following expressions were determined for the internal shear and bending moments in the regions specified.

in < y < in	5in < y < 12.5in
$V = -120\frac{lb}{in}\cdot y$	$V = 1500lb - 120\frac{lb}{in}\cdot y$
$M = -60\frac{lb}{in}\cdot y^2$	$M = -60\frac{lb}{in}\cdot y^2 + 1500lb\cdot y - 7500lb-in$

Determine expressions for the curvature and deflection of the beam in these regions.



Starting with the region from $5in < y < 12.5in$

$$\frac{d^2u}{dy^2} = -\frac{M}{EI} = \frac{60\frac{lb}{in}\cdot y^2 - 1500lb\cdot y + 7500\ lb\text{-}in}{(30\times10^6\ lb/in^2)\cdot(0.5in^4)}$$

$$= 4.0(10^{-6})\frac{1}{in^3}\cdot y^2 - 100(10^{-6})\frac{1}{in^2}\cdot y + 500(10^{-6})\frac{1}{in}$$

$$\theta = \frac{du}{dy} = 1.333(10^{-6})\frac{1}{in^3}\cdot y^3 - 50(10^{-6})\frac{1}{in^2}\cdot y^2 + 500(10^{-6})\frac{1}{in}\cdot y + C_1$$

$$u = 0.333(10^{-6})\frac{1}{in^3}\cdot y^4 - 16.67(10^{-6})\frac{1}{in^2}\cdot y^3 + 250(10^{-6})\frac{1}{in}\cdot y^2 + C_1\cdot y + C_2$$

The Boundary Conditions for this region

$u(5in) = 0$; $\frac{du}{dy}(12.5in) = 0$

Imposing Boundary conditions

$$\frac{du}{dy}(12.5in) = 0 = 1.041(10^{-3}) + C_1 \Rightarrow \underline{C_1 = -1.041(10^{-3})}$$

$$u(5in) = 0 = -830.6(10^{-6})in + C_2 \Rightarrow \underline{C_2 = 830.6(10^{-6})in}$$

Thus expressions for the curvature and deflection in the region $\underline{5in < x < 12.5in}$ are

$$\boxed{\begin{aligned} u(y) &= 0.333(10^{-6})\tfrac{1}{in^3}\cdot y^4 - 16.67(10^{-6})\tfrac{1}{in^2}\cdot y^3 + 250(10^{-6})\tfrac{1}{in}\cdot y^2 - 1.041(10^{-3})\cdot y + 830.6(10^{-6})in \\ \frac{du}{dy}(y) &= 1.333(10^{-6})\tfrac{1}{in^3}\cdot y^3 - 50(10^{-6})\tfrac{1}{in^2}\cdot y^2 + 500(10^{-6})\tfrac{1}{in}\cdot y - 1.041(10^{-3}) \end{aligned}}$$

Values of these functions at points of interest include

$u(5in) = 0\ in$ $\qquad \frac{du}{dy}(5in) = 375.6(10^{-6})$

$u(6.91in) = 8.33(10^{-6})in$ $\qquad \frac{du}{dy}(6.91in) = 466.4(10^{-6})$

$u(12.5in) = 2.452(10^{-3})in$ $\qquad \frac{du}{dy}(12.5in) = 0$

Now the curvature and deflection in the region from $0 < y < 5in$ can be derived

$$\frac{d^2u}{dy^2} = -\frac{M}{EI} = \frac{60\ ^{lb}/_{in}\cdot y^2}{(30\times10^6\ ^{lb}/_{in^2})\cdot(0.5in^4)} = 4.0\times10^{-6}\tfrac{1}{in^3}\cdot y^2$$

$$\Theta = \frac{du}{dy} = 1.33(10^{-6})\tfrac{1}{in^3}\cdot y^3 + C_1$$

$$u = 0.333(10^{-6})\tfrac{1}{in^3}\cdot y^4 + C_1\cdot y + C_2$$

Imposing the above stated B.C.'s

$$\frac{du}{dy}(5in) = 375.6(10^{-6}) = 166.6(10^{-6}) + C_1 \Rightarrow \underline{C_1 = 209(10^{-6})}$$

$$u(5in) = 0 = 1.253(10^{-3})in + C_2 \Rightarrow \underline{C_2 = -1.253(10^{-3})in}$$

Thus the curvature and deflection in the region $0 < y < 5in$ are

$$\boxed{\begin{aligned} u &= 0.333(10^{-6})\ ^1/_{in^3}\cdot y^4 + 209(10^{-6})\cdot y - 1.253(10^{-3})\ in \\ \frac{du}{dy} &= 1.33(10^{-6})\ ^1/_{in^3}\cdot y^3 + 209(10^{-6}) \end{aligned}}$$

Points of interest

$$u(0) = -1.253(10^{-3})in\ ;\ \frac{du}{dy}(0) = 209(10^{-6})$$

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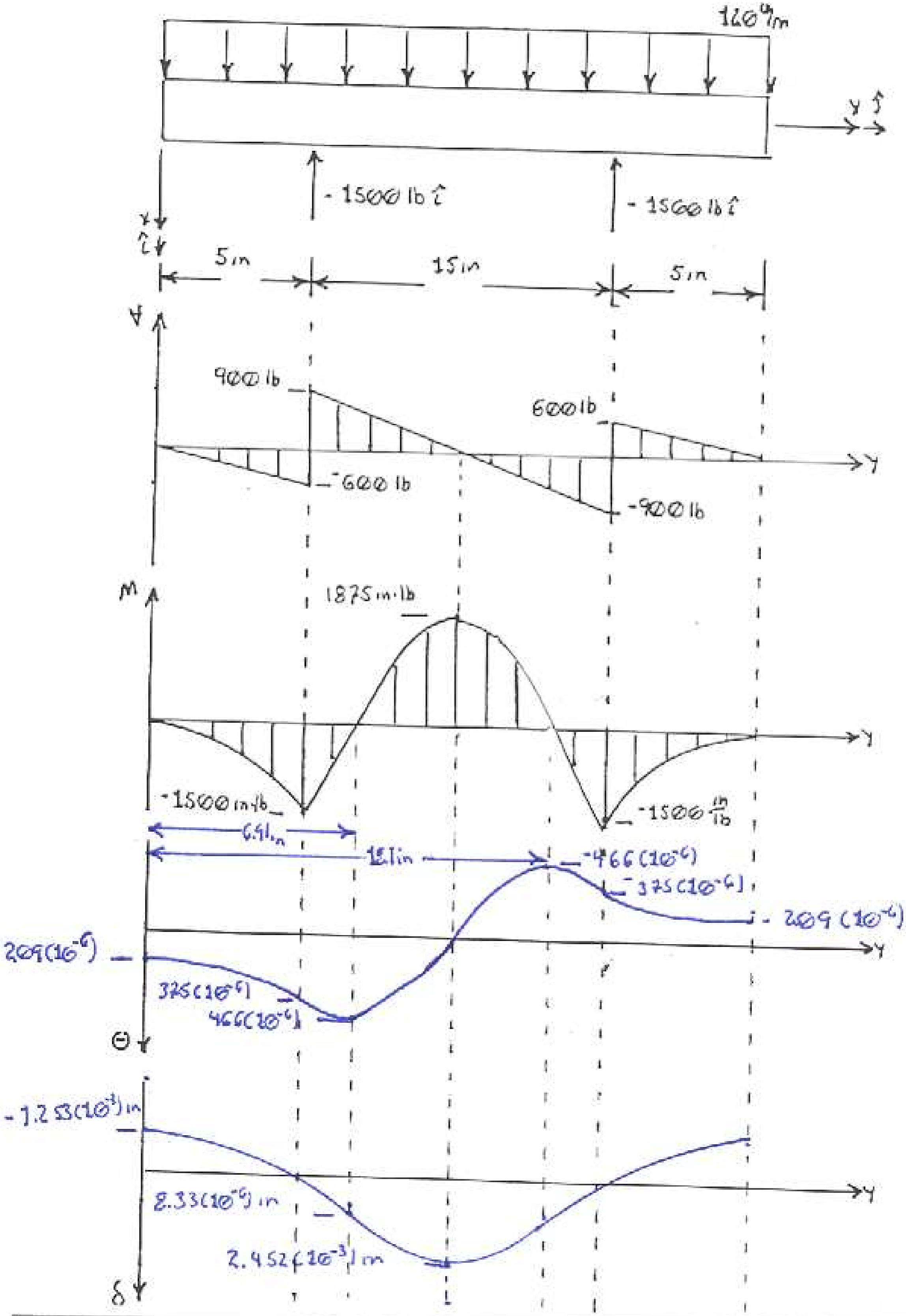
Problem 2. Draw the curvature and deflection diagrams for the beam in the previous problem using the figures below. Be sure to label all important points of interest.



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Problem 3. For a point just to the right of 5in along the length of the beam the value of the internal shear force and bending moment are:

V = 900 lb
M = -1500 lb-in.

For the cross-section shown determine the distribution of the normal and shear stress in the beam. Using the diagrams provided on the next page, draw the normal and shear stress distributions being sure to label all points of interest.

$$\sigma = \frac{Mc}{I} = \frac{-1500\,lb\cdot in \cdot 1\,in}{0.462\,in^4} = -3.25\,ksi$$

$$\tau_{zy} = \frac{VQ}{It} = \frac{(900\,lb)\cdot(0.9\,in)\cdot(.2\,in)\cdot s}{0.462\,in^4 \cdot .2\,in} = 1.75(10^3)\frac{lb}{in^3}\cdot s$$

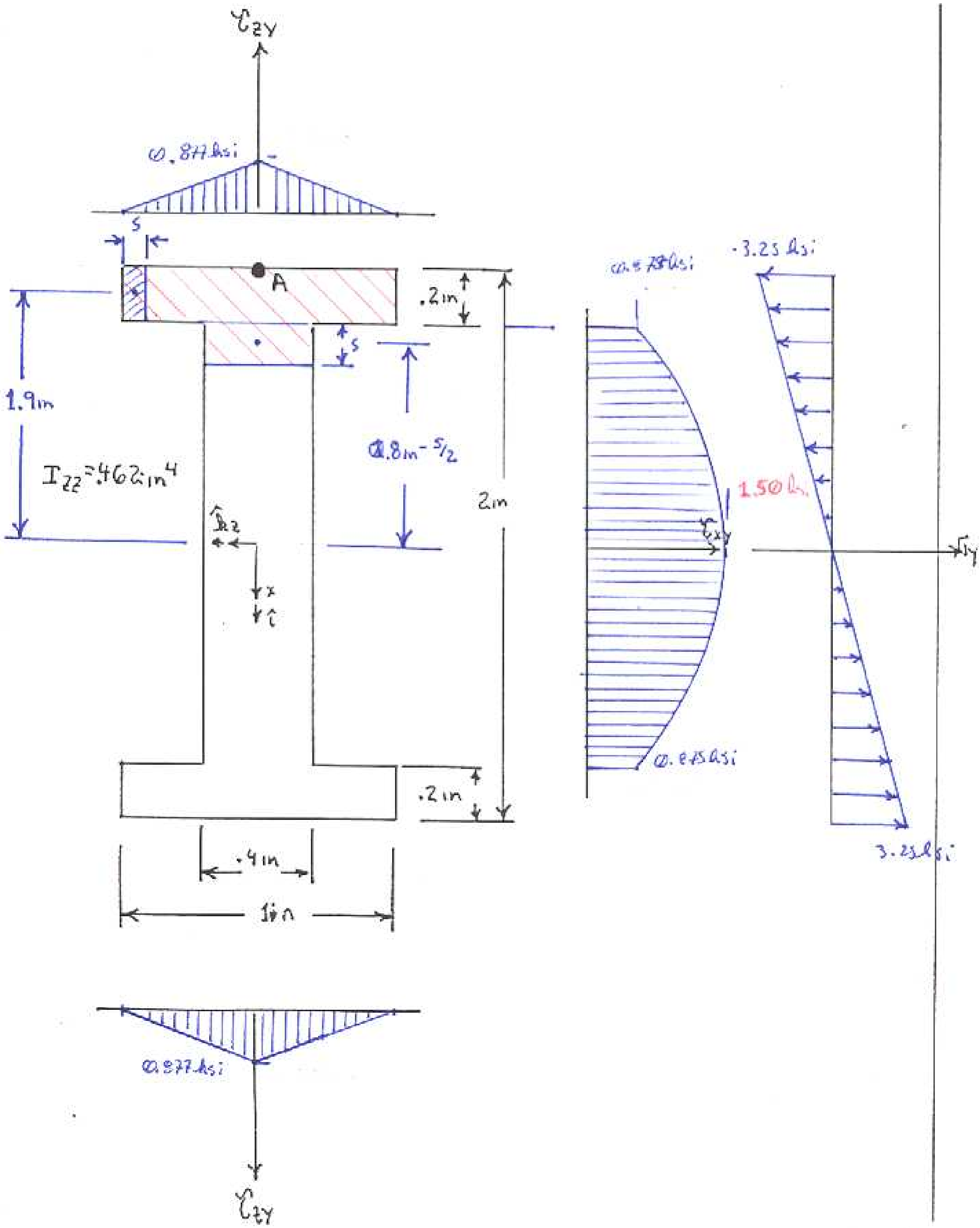
$$\tau_{zy}(0) = 0 \quad , \quad \tau_{yz}(.5) = 0.877(10^3)\,lb/in^2 = 0.877\,ksi$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(900\,lb)\cdot[(0.9\,in)(.2\,in)(1\,in) + (0.8\,in - s/2)(.4\,in)(s)]}{(0.462\,in^4)\cdot(.4\,in)}$$

$$= 876.6\frac{lb}{in^2} + 1.558(10^3)\frac{lb}{in^3}\cdot s - 974.0\frac{lb}{in^4}\cdot s^2$$

$$\tau_{xy}(0) = 0.875\,ksi$$

$$\tau_{xy}(.8) = 1.500\,ksi$$


A
.2in
s
1.9in
2in
.2in
.4in
1in
-3.25 ksi
1.50 ksi
3.25 ksi

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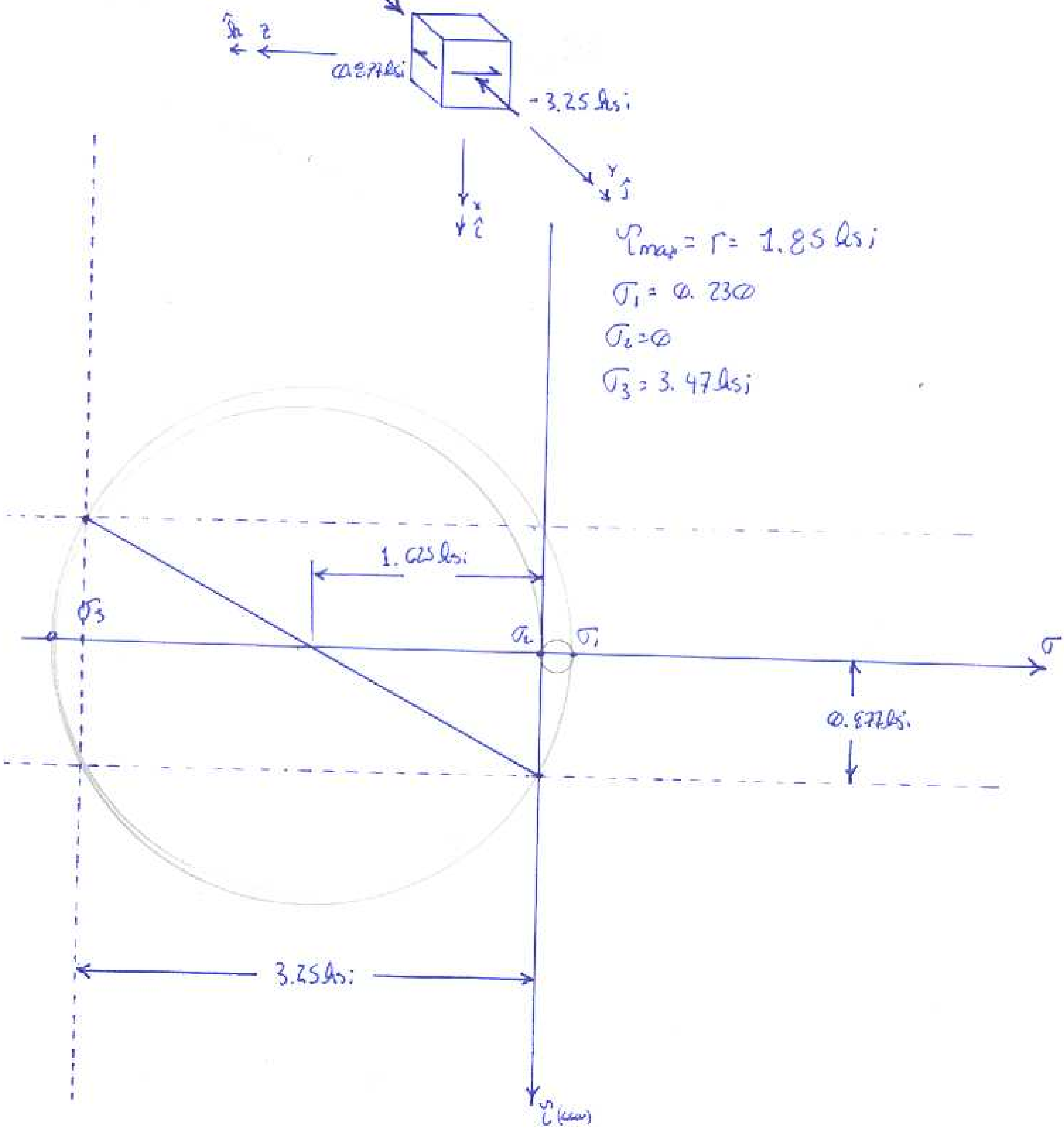
Problem 4. For Point A on the beam cross-section shown in the previous problem draw the three dimensional Mohr's circle and determine the principal stresses and the maximum shearing stress.



$\tau_{max} = r = 1.85$ ksi

$\sigma_1 = 0.230$

$\sigma_2 = 0$

$\sigma_3 = 3.47$ ksi

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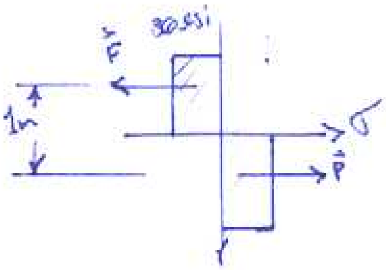
Problem 5. Assuming the original cross-section and that the material the beam is made from is a perfectly elastic-plastic material with a yield point of 30 ksi, what is the maximum bending moment that the beam can support assuming the entire cross-section has been plastically deformed?

$F = 30(10^3)\frac{lb}{in^2} \cdot \frac{3}{4}in \cdot 1in$

$= 22.5(10^3)\ lb$



$M = 22.5(10^3)\ in \cdot lb$