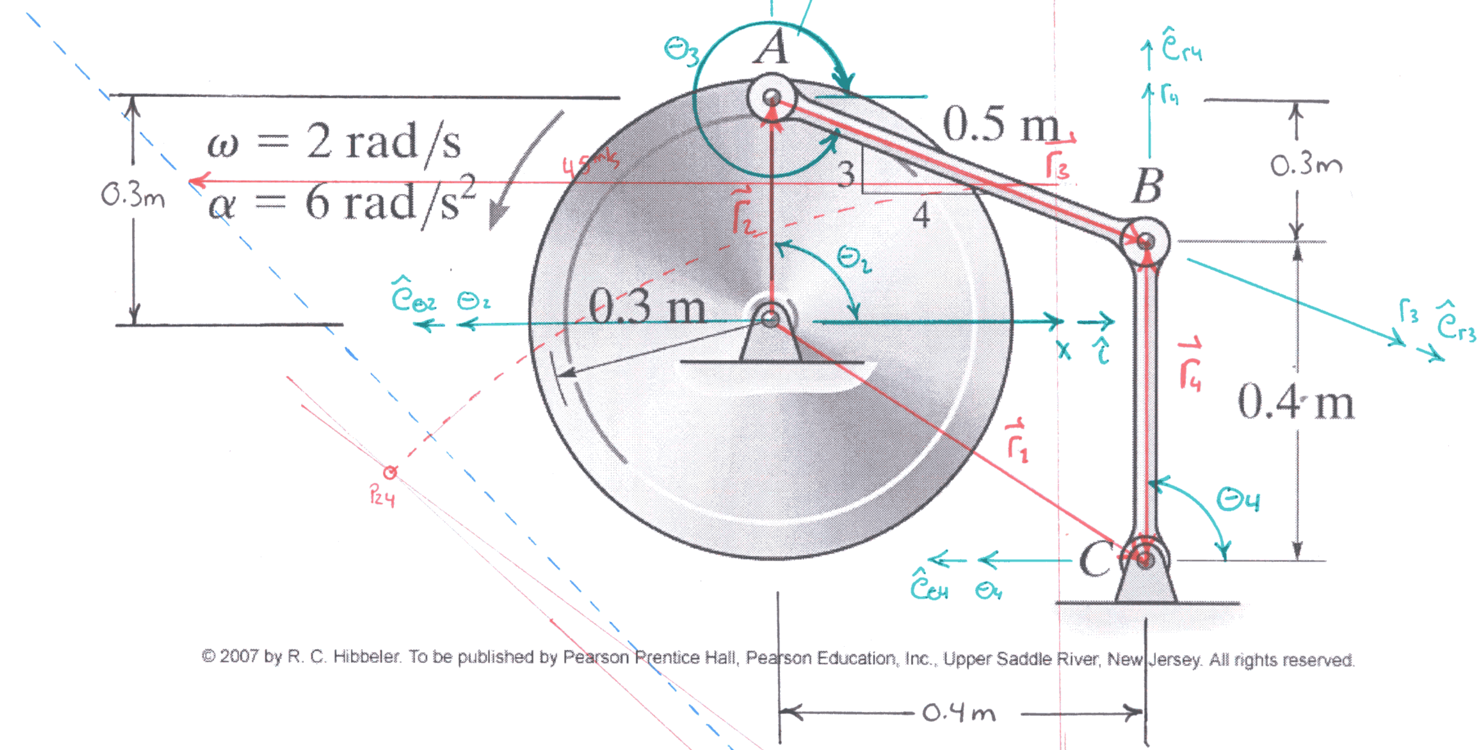
NAME: Solution

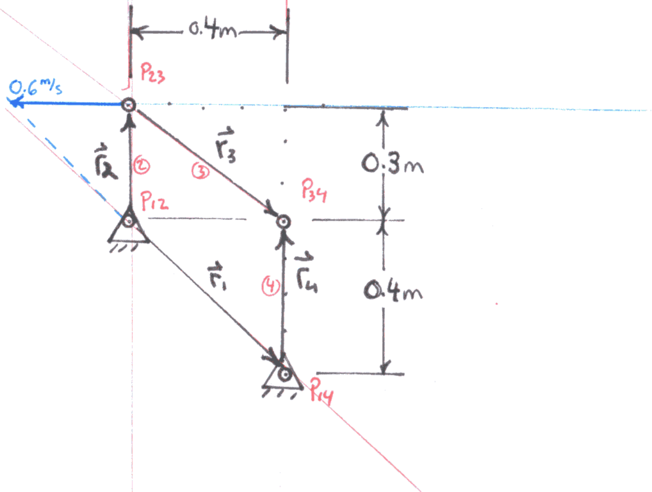
PROBLEM 1: The flywheel rotates with an angular velocity of ω=2 1/s and an angular acceleration α=6 1/s^2.



1a. Using analytical methods, determine the velocity of points A and B, and the angular velocities of links AB and BC for the configuration shown above. Make sure the answers are in vector form.

$$\vec{r}_2 + \vec{r}_3 = \vec{r}_1 + \vec{r}_4$$

$$\vec{r}_1 = r_1 \hat{e}_{r1} = .57m\,(0.7071\hat{\imath} - 0.7071\hat{\jmath}) = 0.4m\,\hat{\imath} - 0.4m\,\hat{\jmath} \quad (1)$$



$$\vec{r}_2 = r_2 \hat{e}_{r2} = 0.3m\,\hat{\jmath} \quad (2)$$

$$\vec{r}_3 = r_2 \hat{e}_{r3} = 0.5m\,(0.8\hat{\imath} - 0.6\hat{\jmath}) = 0.4m\,\hat{\imath} - 0.3m\,\hat{\jmath} \quad (3)$$

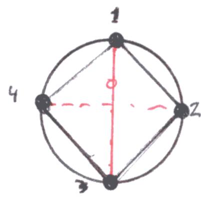
$$\vec{r}_4 = r_4 \hat{e}_{r4} = 0.4m\,\hat{\jmath} \quad (4)$$

$$\hat{e}_{r2} = \hat{\jmath} \qquad \hat{e}_{\theta 2} = -\hat{\imath} \quad (5)$$

$$\hat{e}_{r3} = 0.8\hat{\imath} - 0.6\hat{\jmath} \qquad \hat{e}_{\theta 3} = 0.6\hat{\imath} + 0.8\hat{\jmath} \quad (6)$$

$$\hat{e}_{r4} = \hat{\jmath} \qquad \hat{e}_{\theta 4} = -\hat{\imath} \quad (7)$$

$P_{13}\ \infty$



THE VELOCITY IS DETERMINED BY TAKING THE DERIVATIVE OF THE LOOP CLOSURE EQUATION

GROUND LINK

$$\vec{r}_2 + \vec{r}_3 = \vec{r}_1 + \vec{r}_4 \Rightarrow r_2\hat{e}_{r2} + r_3\hat{e}_{r3} = \vec{r}_1 + r_4\hat{e}_{r4}$$

$$\cancel{\dot{r}_2}\hat{e}_{r2} + r_2\dot{\hat{e}}_{r2} + \cancel{\dot{r}_3}\hat{e}_{r3} + r_3\dot{\hat{e}}_{r3} = \dot{\vec{r}}_1 + \cancel{\dot{r}_4}\hat{e}_{r4} + r_4\cdot\dot{\hat{e}}_{r4}$$

$$r_2\cdot(\dot{\theta}_2\hat{k}\times\hat{e}_{r2}) + r_3\cdot(\dot{\theta}_3\hat{k}\times\hat{e}_{r3}) = r_4(\dot{\theta}_4\hat{k}\times\hat{e}_{r4})$$

$$r_2\cdot\dot{\theta}_2\cdot\hat{e}_{\theta2} + r_3\cdot\dot{\theta}_3\cdot\hat{e}_{\theta3} = r_4\cdot\dot{\theta}_4\cdot\hat{e}_{\theta4} \qquad (8)$$

SUBSTITUTING IN THE UNIT VECTORS IN (5), (6), & (7)

$$-r_2\cdot\dot{\theta}_2\hat{\imath} + r_3\cdot\underbrace{\dot{\theta}_3}_{\text{UNKNOWN}}(0.6\hat{\imath}+0.8\hat{\jmath}) = -r_4\underbrace{\dot{\theta}_4}_{\text{UNKNOWN}}\hat{\imath}$$

DOTTING THE ABOVE EQUATION WITH $\hat{\imath}$ & $\hat{\jmath}$

$$-r_2\dot{\theta}_2 + r_3\cdot\dot{\theta}_3\cdot 0.6 = -r_4\dot{\theta}_4 \qquad (9)$$

$$0.8\, r_3\dot{\theta}_3 = 0 \Rightarrow \boxed{\dot{\theta}_3 = 0} \qquad (10)$$

SUBSTITUTING (10) INTO (9)

$$-r_2\cdot\dot{\theta}_2 = -r_4\dot{\theta}_4 \Rightarrow \dot{\theta}_4 = \frac{r_2}{r_4}\cdot\dot{\theta}_2 = \frac{0.3m}{0.4m}(2\,^1/_s) = \boxed{1.5\,^1/_s} \qquad (11)$$

$\vec{v}_A$ IS CALCULATED DIRECTLY FROM THE GIVENS

$$\vec{r}_A = r_2\cdot\hat{e}_{r2} \Rightarrow \dot{\vec{r}}_A = r_2\cdot\dot{\hat{e}}_{r2} = r_2\cdot\dot{\theta}_2\hat{e}_{\theta2} = \vec{v}_A \qquad (12)$$

$$\vec{v}_A = (0.3m)(2\,^1/_s)(-\hat{\jmath}) = \boxed{-0.6\,^m/_s\,\hat{\imath}} = \vec{v}_A$$

$\vec{v}_B$ IS CALCULATED USING (10) AND (11)

$$\vec{r}_B = r_2\hat{e}_{r2} + r_3\hat{e}_{r3} \Rightarrow \dot{\vec{r}}_B = \vec{v}_B = r_2\cdot\dot{\theta}_2\cdot\hat{e}_{\theta2} + r_3\cdot\cancel{\dot{\theta}_3}\cdot\hat{e}_{\theta3} \qquad (13)$$

$$= (0.3m)(2\,^1/_s)(-\hat{\jmath}) = \boxed{-0.6\,^m/_s\,\hat{\imath}}$$

OR

$$\vec{r}_B = r_4\hat{e}_{r4} \Rightarrow \dot{\vec{r}}_B = \cancel{\dot{r}_4}\hat{e}_{r4} + r_4\cdot\dot{\hat{e}}_{r4} = r_4\dot{\theta}_4\hat{e}_{\theta4}$$

$$= (0.4m)(1.5\,^1/_s)(-\hat{\imath}) = -0.6\,^m/_s\,\hat{\imath}$$

1b. Using analytical methods determine the accelerations of points A and B, and the angular accelerations of links AB and BC. Make sure the answers are in vector form.

THE ACCELERATION OF THE STRUCTURE IS CALCULATED BY TAKING THE DERIVATIVE OF (8)

$$\cancel{\dot r_2}\dot\theta_2\hat e_{\theta 2} + r_2\cdot\ddot\theta_2\cdot\hat e_{\theta 2} + r_2\cdot\dot\theta_2\dot{\hat e}_{\theta 2} + \cancel{\dot r_3}\cdot\dot\theta_3\cdot\hat e_{\theta 3} + r_3\cdot\ddot\theta_3\cdot\hat e_{\theta 3} + r_3\cdot\cancel{\dot\theta_3}\cdot\dot{\hat e}_{\theta 3}$$
$$= \cancel{\dot r_4}\dot\theta_4\hat e_{\theta 4} + r_4\cdot\ddot\theta_4\cdot\hat e_{\theta 4} + r_4\dot\theta_4\dot{\hat e}_{\theta 4}$$

$$r_2\cdot\ddot\theta_2\cdot\hat e_{\theta 2} - r_2\dot\theta_2^2\hat e_{r2} + r_3\cdot\underset{\text{UNKNOWN}}{\underline{\ddot\theta_3}}\cdot\hat e_{\theta 3} = r_4\cdot\underset{\text{UNKNOWN}}{\underline{\ddot\theta_4}}\hat e_{\theta 4} - r_4\dot\theta_4^2\hat e_{r4}$$

$$-r_2\ddot\theta_2\hat\imath - r_2\dot\theta_2^2\hat\jmath + r_3\underset{\text{UNKNOWN}}{\underline{\ddot\theta_3}}(.6\hat\imath + .8\hat\jmath) = -r_4\underset{\text{UNKNOWN}}{\underline{\ddot\theta_4}}\hat\imath - r_4\dot\theta_4^2\hat\jmath \qquad (14)$$

DOTTING (14) WITH $\hat\imath$ & $\hat\jmath$

$$-r_2\ddot\theta_2 + .6\cdot r_3\cdot\ddot\theta_3 = -r_4\ddot\theta_4 \qquad (15)$$
$$-r_2\dot\theta_2^2 + .8\,r_3\cdot\ddot\theta_3 = -r_4\dot\theta_4^2 \qquad (16)$$

FROM (16)

$$\ddot\theta_3 = \frac{r_2\dot\theta_2^2 - r_4\dot\theta_4^2}{.8\cdot r_3} = \frac{(0.3m)(2\,^1/_s)^2 - (.4m)(1.5\,^1/_s)^2}{.8\,(.5m)} = \boxed{0.750\ ^1/_{s^2}} \qquad (17)$$

FROM (15)

$$\ddot\theta_4 = \frac{r_2\ddot\theta_2 - .6\cdot r_3\cdot\ddot\theta_3}{r_4} = \frac{(0.3m)(6\,^1/_{s^2}) - (.6)(0.5m)(0.75\,^1/_{s^2})}{0.4m} = \boxed{3.948\ ^1/_{s^2}} \qquad (18)$$

THE ACCELERATION OF POINT "A" IS CALCULATED BY FIRST TAKING THE DERIVATIVE OF (12)

$$\vec a_A = \cancel{\dot r_2}\dot\theta_2\hat e_{\theta 2} + r_2\ddot\theta_2\hat e_{\theta 2} + r_2\dot\theta_2\dot{\hat e}_{\theta 2} = r_2\ddot\theta_2\hat e_{\theta 2} - r_2\dot\theta_2^2\hat e_{r2}$$
$$= (0.3m)(6\,^1/_{s^2})(-\hat\imath) - (0.3m)(2\,^1/_s)^2(\hat\jmath) = \boxed{-1.8\,^m/_{s^2}\,\hat\imath - 1.2\,^m/_{s^2}\,\hat\jmath}$$

THE ACCELERATION OF POINT "B" IS CALCULATED BY FIRST TAKING THE DERIVATIVE OF (13)

$$\vec a_B = \cancel{\dot r_2}\dot\theta_2\hat e_{\theta 2} + r_2\ddot\theta_2\hat e_{\theta 2} + r_2\dot\theta_2\dot{\hat e}_{\theta 2} + \cancel{\dot r_3}\cancel{\dot\theta_3}\hat e_{\theta 3} + r_3\cdot\ddot\theta_3\cdot\hat e_{\theta 3} + r_3\cancel{\dot\theta_3}\dot{\hat e}_{\theta 3}$$
$$= r_2\ddot\theta_2\hat e_{\theta 2} - r_2\dot\theta_2^2\hat e_{r2} + r_3\ddot\theta_3\hat e_{\theta 3}$$
$$= (0.3m)(6\,^1/_{s^2})(-\hat\imath) - (0.3m)(2\,^1/_s)^2(\hat\jmath) + (0.5m)(0.750\,^1/_{s^2})(0.6\hat\imath + 0.8\hat\jmath)$$
$$= -1.58\,^m/_{s^2}\,\hat\imath - 0.900\,^m/_{s^2}\,\hat\jmath$$

OR

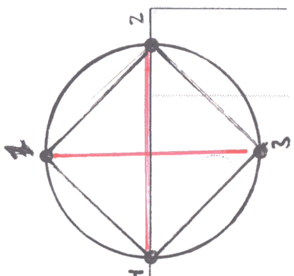
$$\vec a_B = \cancel{\dot r_4}\dot\theta_4\hat e_{\theta 4} + r_4\cdot\ddot\theta_4\hat e_{\theta 4} + r_4\dot\theta_4\dot{\hat e}_{\theta 4} = r_4\ddot\theta_4\hat e_{\theta 4} - r_4\dot\theta_4^2\hat e_{r4}$$
$$= (0.4m)(3.948\,^1/_{s^2})(-\hat\imath) - (0.4m)(1.5\,^1/_s)^2(\hat\jmath) =$$
$$= \boxed{-1.58\,^m/_{s^2}\,\hat\imath - 0.90\,^m/_{s^2}\,\hat\jmath}$$

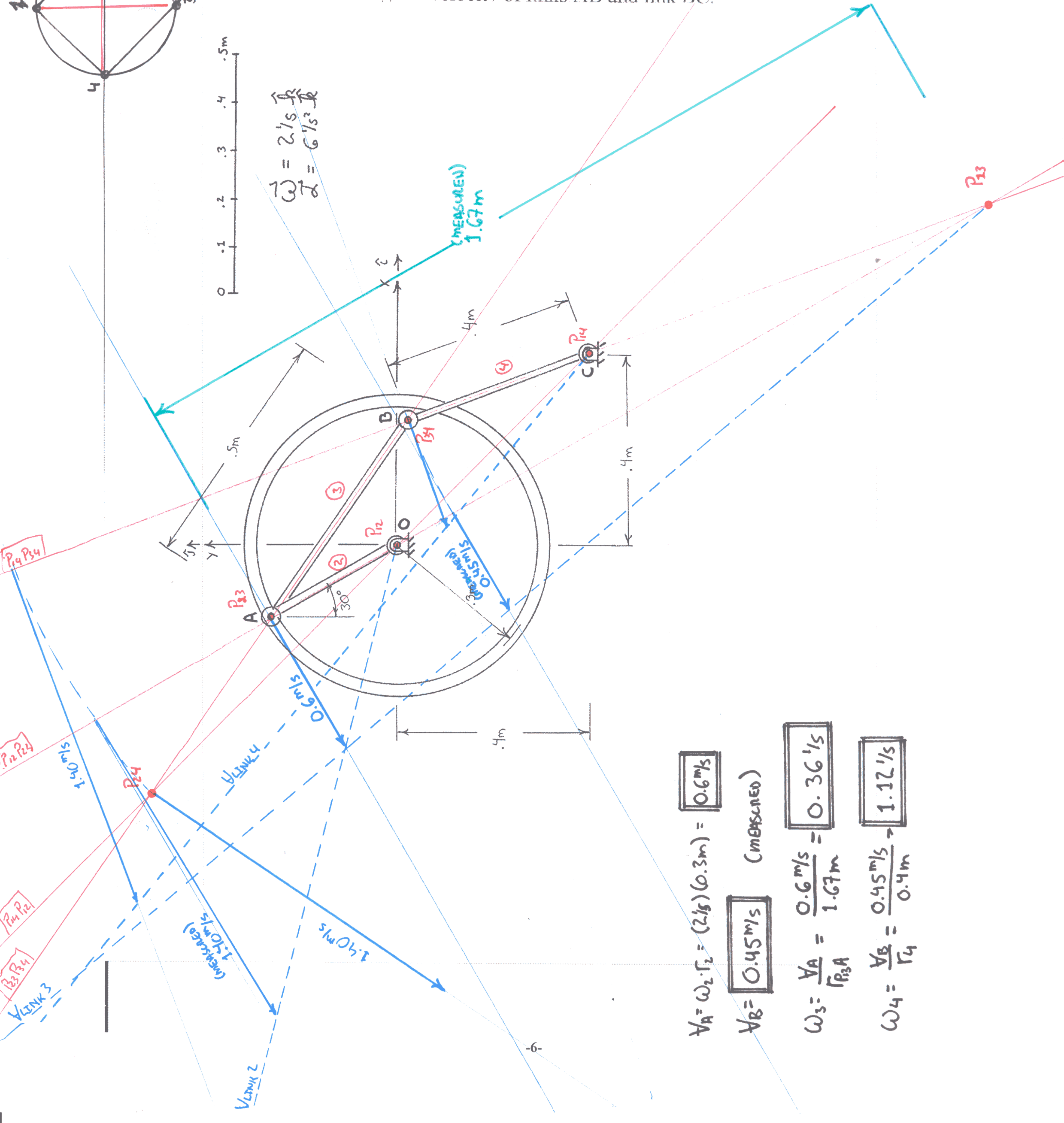
PROBLEM 2: The mechanism shown on the next page is rotating with a constant angular velocity of 2 rad/s ccw. Using instant centers to find the following:

2a. Locate and label all the instant centers associated with this mechanism.

2b. Determine the velocities of points A and B.

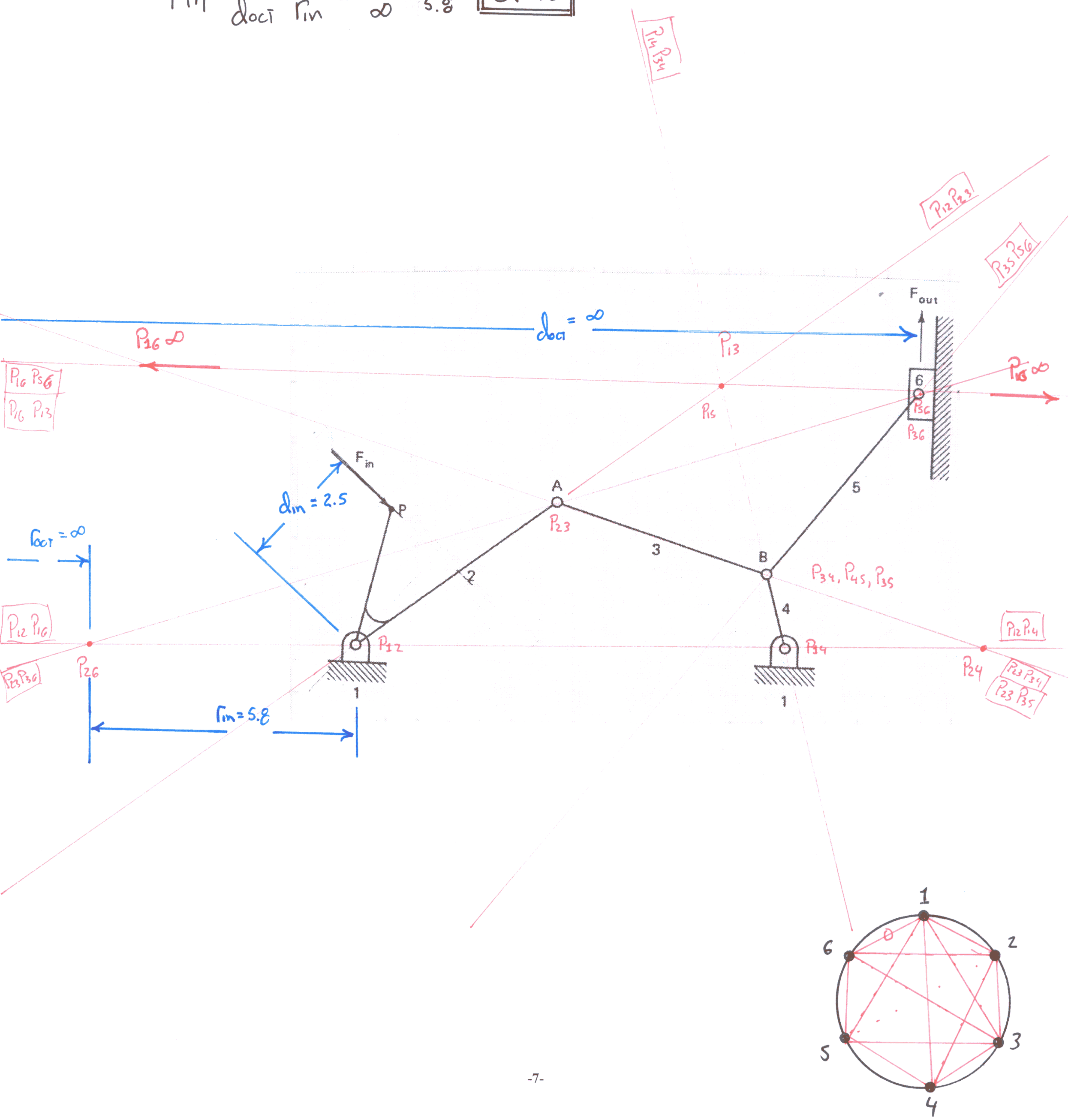
2c. Determine the angular velocity of links AB and link BC.





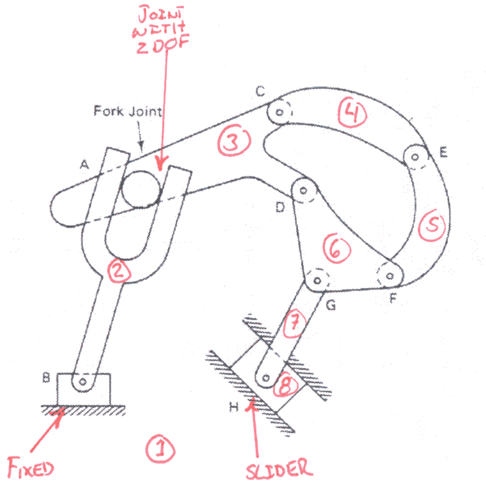
PROBLEM 3: Determine the Mechanical Advantage of the linkage shown.

$$MA = \frac{d_{in}}{d_{OUT}} \cdot \frac{r_{OUT}}{r_{in}} = \frac{2.5}{\infty} \cdot \frac{\infty}{5.8} = \boxed{0.43}$$



PROBLEM 4: Determine the Mobility of the following mechanisms.

4a.



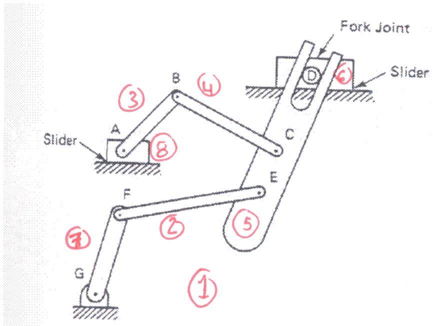
$$M = 3(L-1) - 2J_1 - J_2$$

WITH "B" BEING FIXED

$L = 8$, $J_1 = 8$, $J_2 = 1$

$$M = 3(8-1) - 2(8) - 1 = 21 - 17 = \boxed{4}$$

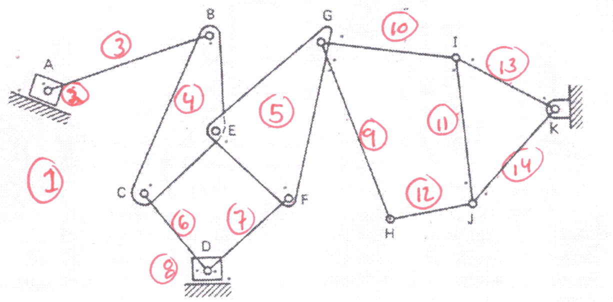
4b.



$L = 8$, $J_1 = 8$, $J_2 = 1$

$$M = 3(8-1) - 2(8) - 1 = 21 - 16 - 1 = \boxed{4}$$

4c.



$$M = 3(L-1) - 2 \cdot J_1 - J_2$$

$$= 3(14-1) - 2 \cdot 18 =$$

$$39 - 36 = \boxed{3}$$

NAME: ______________________

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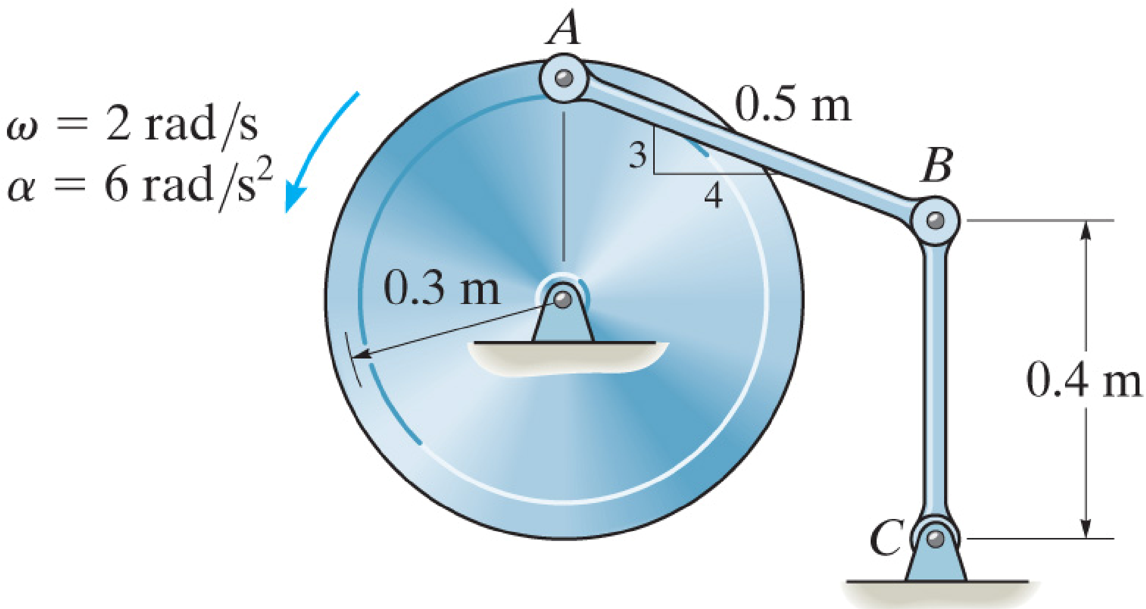

1a. Using analytical methods, determine the velocity of points A and B, and the angular velocities of links AB and BC for the configuration shown above. Make sure the answers are in vector form.

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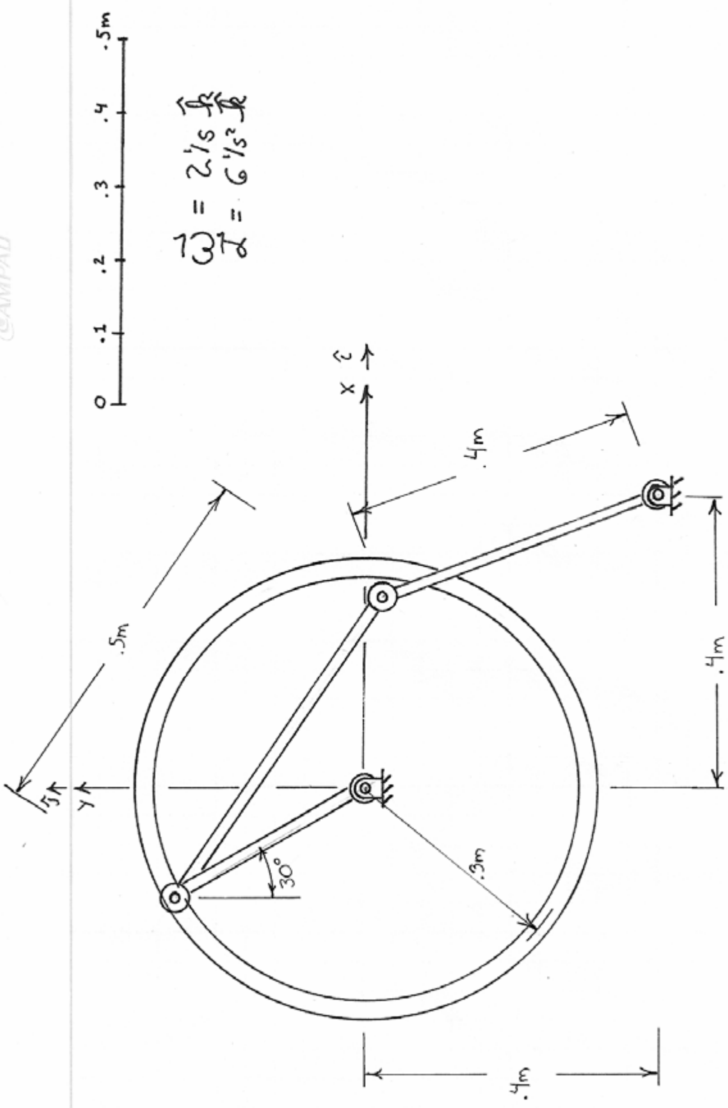
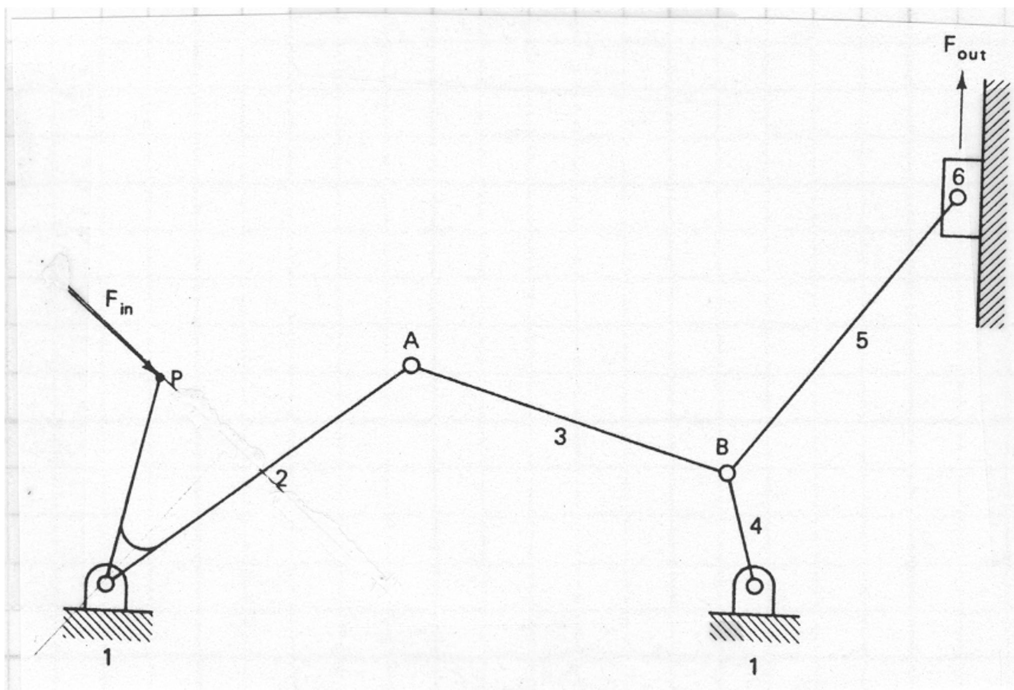


0 .1 .2 .3 .4 .5m
ω = 2 r/s
α = 6 r/s²
.5m
.4m
.4m
.4m
.3m
30°

PROBLEM 3: Determine the Mechanical Advantage of the linkage shown.



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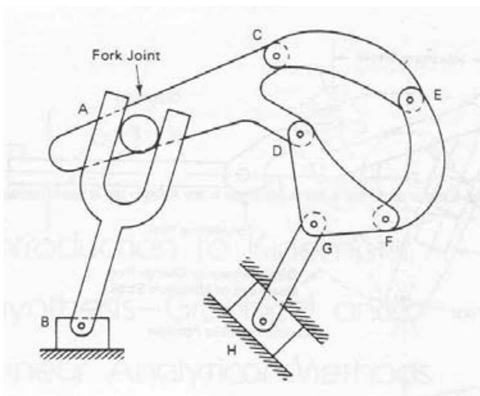
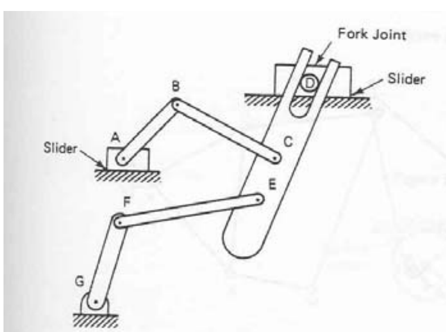


4b.



4c.