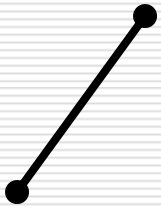
Introduction to Isomers and Linkage Synthesis

- Linkage Synthesis Overview
- Isomers

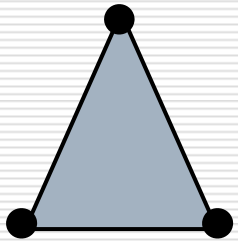
Number Synthesis

- Determining the number and order of links and joints necessary to produce motion of a particular mobility
- Link order
  - Number of Binary (B), Ternary (T), Quaternary (Q), Pentagonal (P), Hexagonal (H) links
  - L= B+T+Q+P+H
    ≡ total number of links

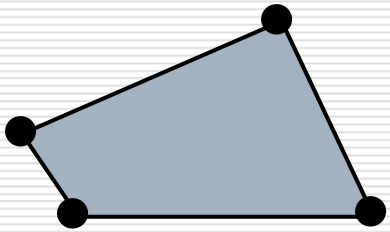
Link Order



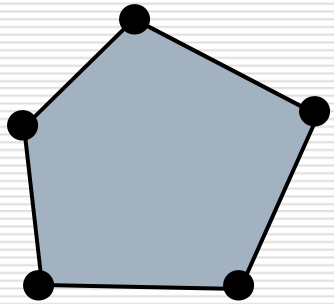
Binary



Ternary



Quaternary



Pentagonal

Isomer Joint Type Nomenclature

- Dashed Line: Imaginary Link
- R: Revolute Joint
  - Pin Joint
  - Hinged connection
- P/T: Lower Pair Sliding Joint
  - Prismatic Joint
  - Relative Translational Motion
- C: Center of Curvature
- B: Base Point

Grübler/Kutzbach Criterion

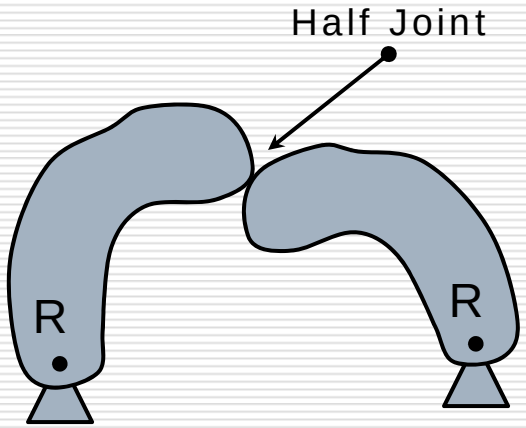
$$M = 3 \cdot (L - 1) - 2 \cdot j_1 \quad \text{(Grübler)}$$

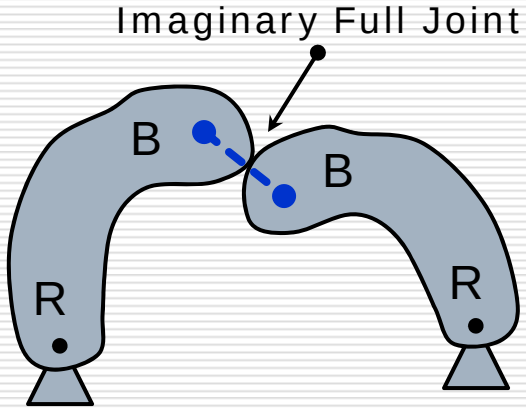
$$M = 3 \cdot (L - 1) - 2 \cdot j_1 - j_2 \quad \text{(Kutzback)}$$

- M=1
  - Mechanism can be driven by a single input direction
- M=2
  - Two separate input motions are necessary to produce constrained motion for the mechanism
  - Differential Mechanism
- M=0
  - Motion is impossible and the mechanism is a structure
- M=-1
  - Redundant constraint
  - Pre-Load

Number Synthesis Uses Grübler's Equation

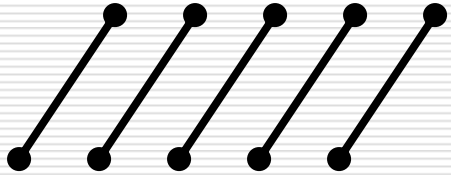
- Grübler's Criterion: M=3(L-1)-2J
  - A half joint is just a full joint with an imaginary link through the base points
  - Base points are the location of the center of curvatures
- For Linkages with Full Joints
  - L=B+T+Q+P+H+…
  - L-(M+3)=T+2Q+3P+4H+…
- DoF Must Be Uniformly Distributed





Uniformly Distributed DoF

Basic Links
To Work With

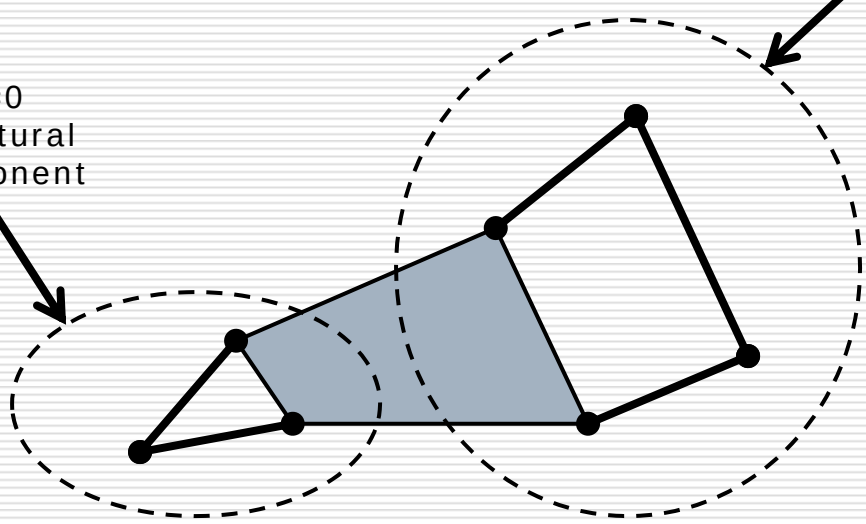


B=5

Q=1

M=+1
Four Bar
Linkage
Component

M=0
Structural
Component



An isomer of the link combination contributing nothing new for M=+1 synthesis use

Implication of Grübler's Equation

$$J = \frac{3}{2} \cdot L - \frac{(M+3)}{2}$$

- If all joints are full joints
  - An ODD number of DOF (M) requires an EVEN number of links (L)
  - An EVEN number of DOF (M) requires an ODD number of links (L)

Determining All Possible Combinations of Links for a DOF

- Total Number of Links in a Mechanism

$$L = B + T + Q + P + \cdots$$

- Total Number of Joints in a Mechanism
  - A node is a location in a link that can be used for a joint
  - 2 nodes are needed to make one joint

$$J = \frac{\text{total nodes}}{2} = \frac{2 \cdot B + 3 \cdot T + 4 \cdot Q + 5 \cdot P + \cdots}{2}$$

Equations Relevant For Number Synthesis

- Simultaneous Progressive Solution for up to Pentagonal Links

$$L = B + T + Q + P$$

$$T + 2 \cdot Q + 3 \cdot P = L - (M + 3)$$

- Total Number of Joints

$$J = \frac{3}{2} \cdot L - \frac{(M+3)}{2}$$

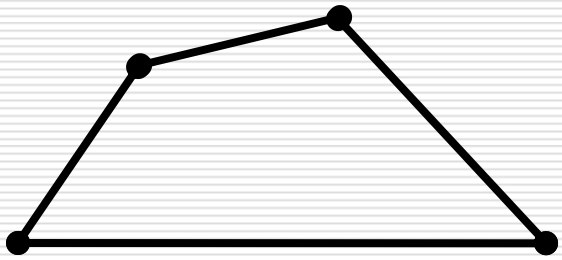
Consider F=+1
N must be even: 0, 2

- M=+1, L=0 $\Rightarrow$ Not Valid
  - $0 = B + T + Q + P$ Can Not have 0 Link Mechanism
  - $T + 2 \cdot Q + 3 \cdot P = -4$ T, Q, P can not be negative
- M=+1, L=2 $\Rightarrow$ Not Valid
  - $2 = B + T + Q + P$
  - $T + 2 \cdot Q + 3 \cdot P = -3$ T, Q, P can not be negative

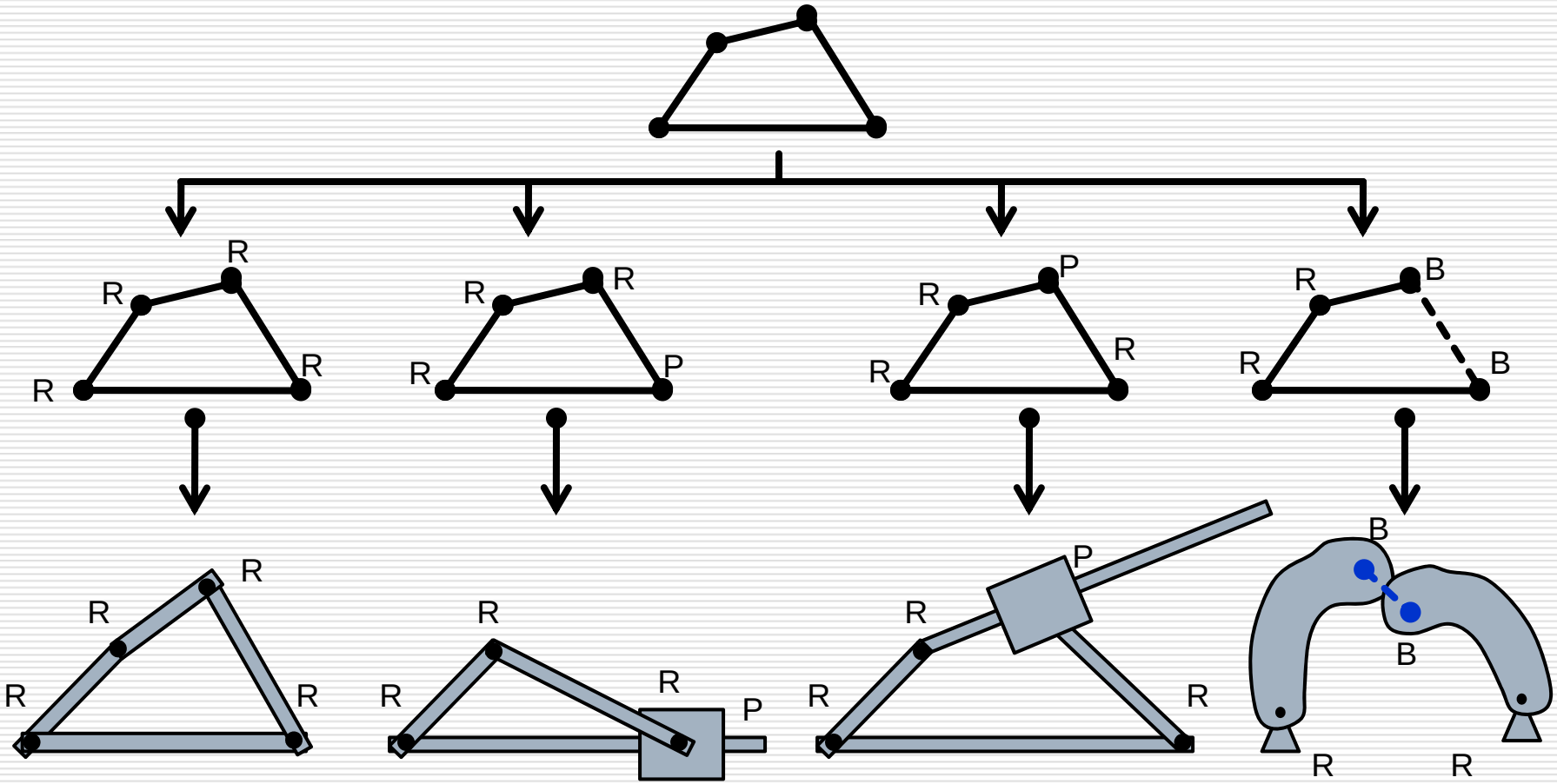
Considering F=+1 Continued
N must be even: 4

- M=+1, L=4
  - $4 = B + T + Q + P \quad \Rightarrow \quad \mathbf{4 = B}$

    $T + 2 \cdot Q + 3 \cdot P = 0$ T=Q=P=0
  - $J = \frac{3}{2} \cdot 4 - \frac{(1+3)}{2} = 4$



Synthesis of Mechanical Devices From The Isomer



Considering M=+1 Continued L must be even: 6

- M=+1, l=6
  - $L = B + T + Q + P \quad \Rightarrow \quad \mathbf{6 = B + T + Q + P}$

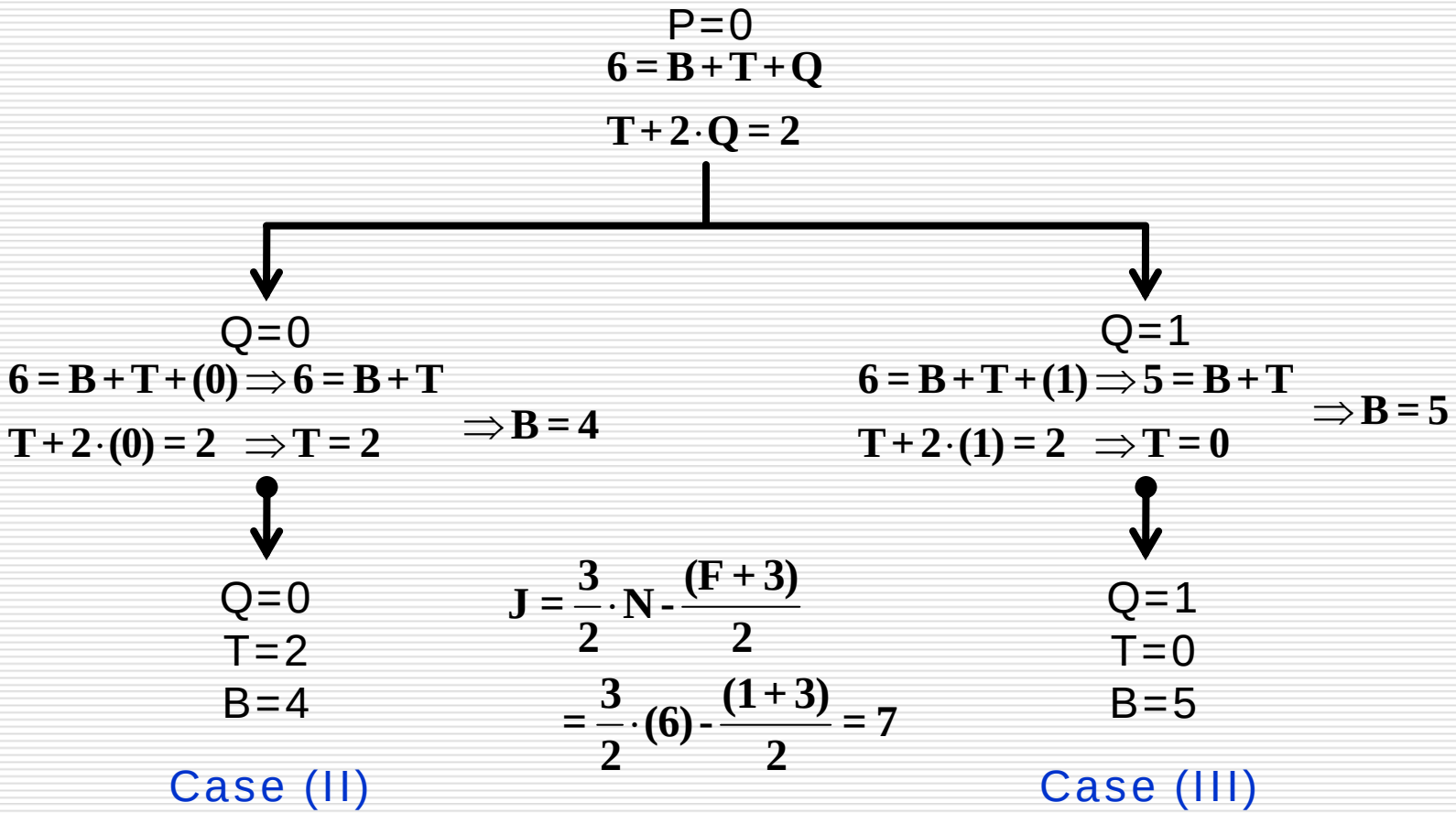
    $T + 2 \cdot Q + 3 \cdot P = M - (L + 3) \quad \Rightarrow \quad \mathbf{T + 2 \cdot Q + 3 \cdot P = 2}$

  - From the second equation P=0
    - T and Q are positive integers
    - The equations reduce to

      $$\mathbf{6 = B + T + Q}$$

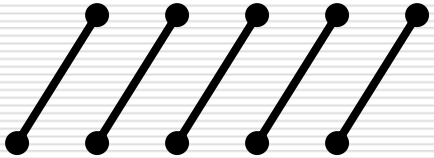
      $$\mathbf{T + 2 \cdot Q = 2}$$

Number Synthesis for M=+1, L=6, P=0

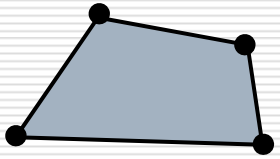


Case III – No Acceptable Isomers
M=+1, L=6, P=0, Q=1, T=0, B=5

Basic Links
To Work With

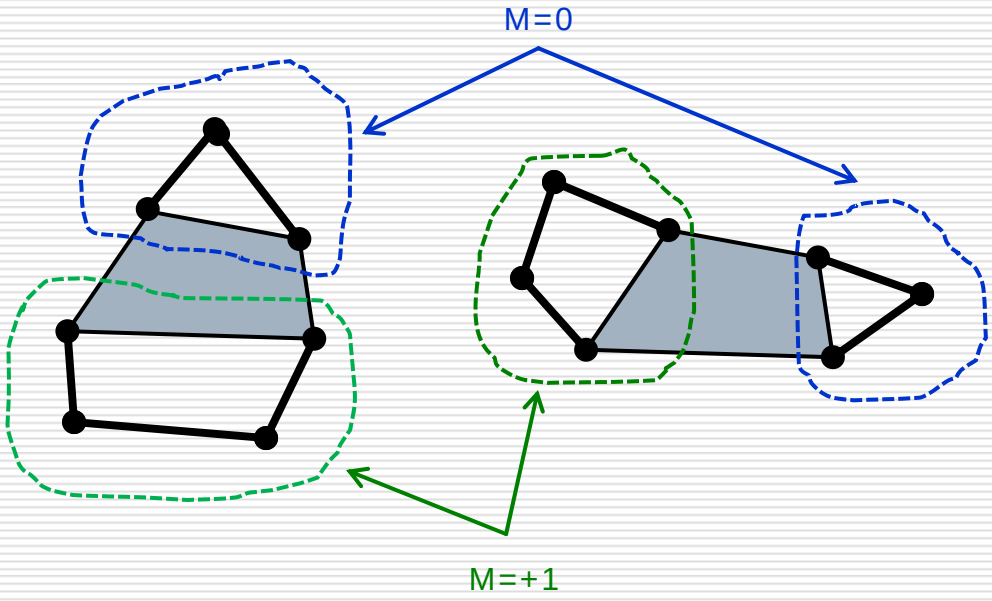


B=5



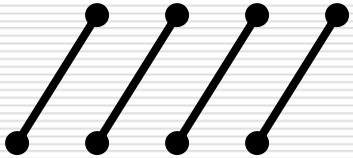
Q=1

Can Not Connect
Fifth Link and Have
Uniformly Distributed
M=+1

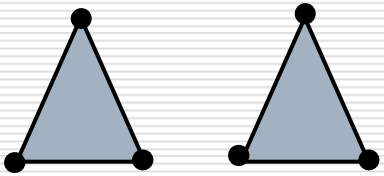


Case III – 2 Acceptable Isomers
M=+1, L=6, P=0, Q=0, T=2, B=4

Basic Links
To Work With

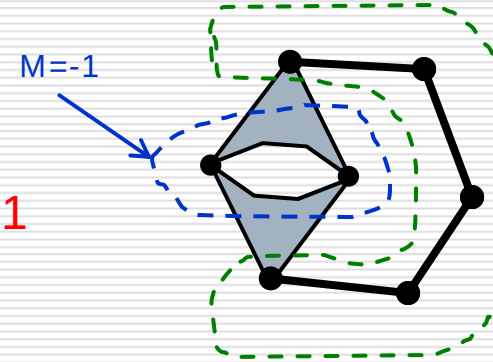


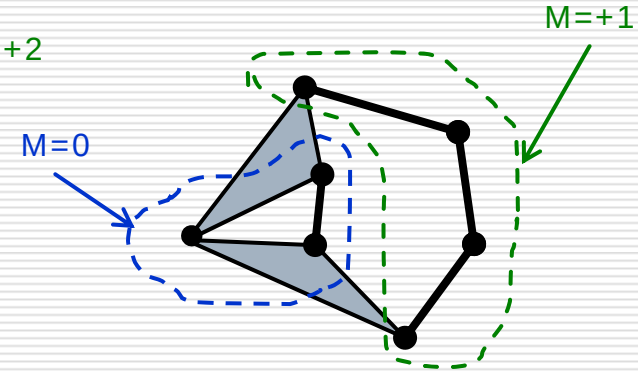
B=4



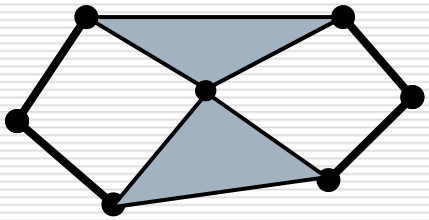
T=2

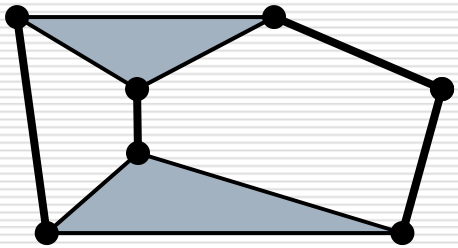
Unacceptable
Isomers for M=+1





Acceptable
Isomers for M=+1

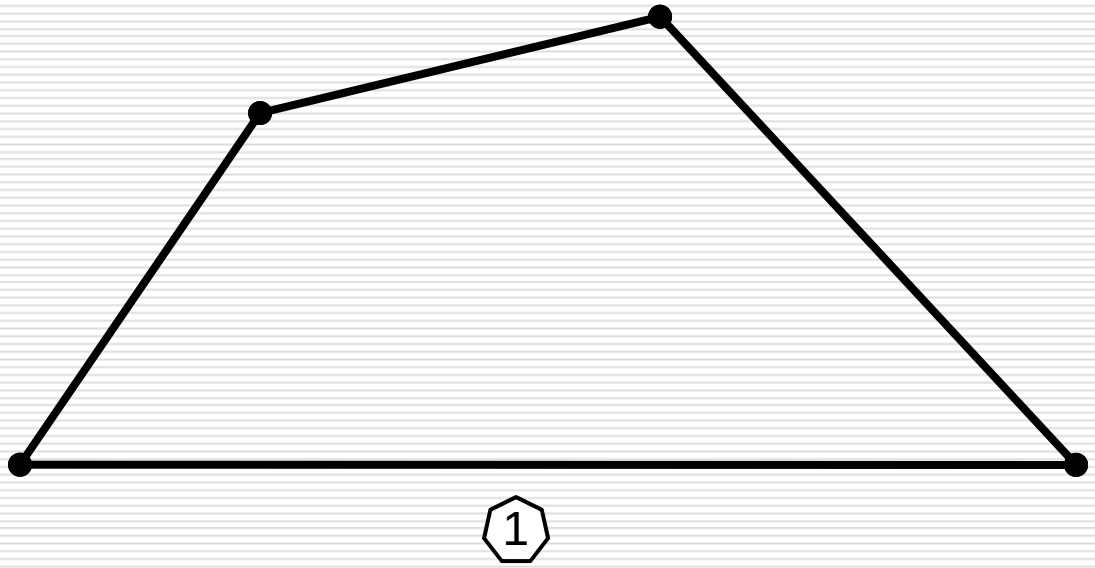




Link Combinations for Single Pin-Jointed Plane Linkages, M=+1

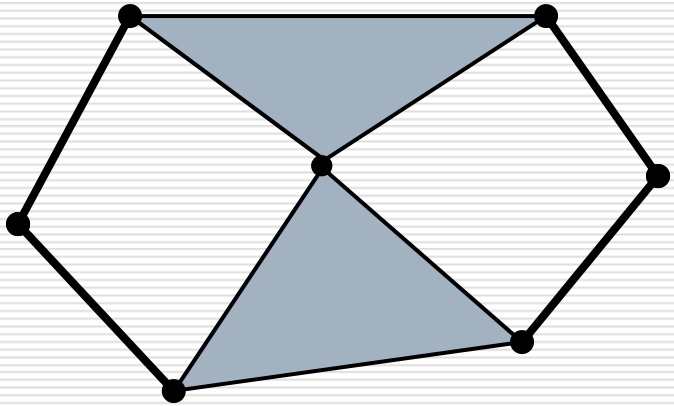
M	L	B	T	Q	P	Designation
+1	4	4	0	0	0	I
	6	4	2	0	0	II
	6	5	0	1	0	III
	8	4	4	0	0	IV
	8	5	2	1	0	V
	8	6	0	2	0	VI
	8	6	1	0	1	VII

M	L	B	T	Q	P	Designation
+1	4	4	0	0	0	I



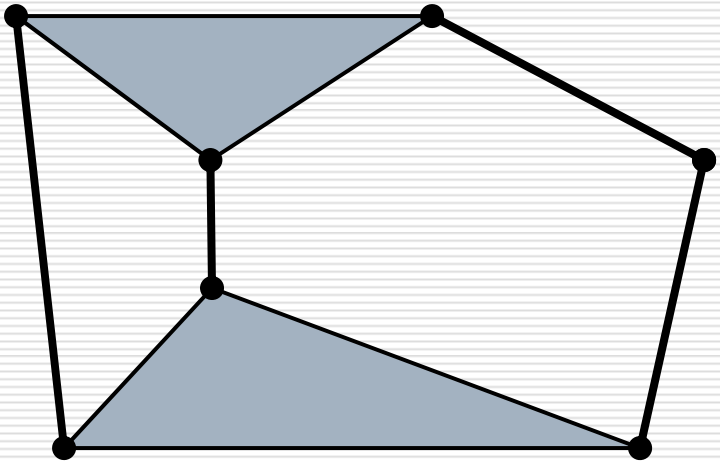
M	L	B	T	Q	P	Designation
+1	6	4	2	0	0	II

Watt's Linkage



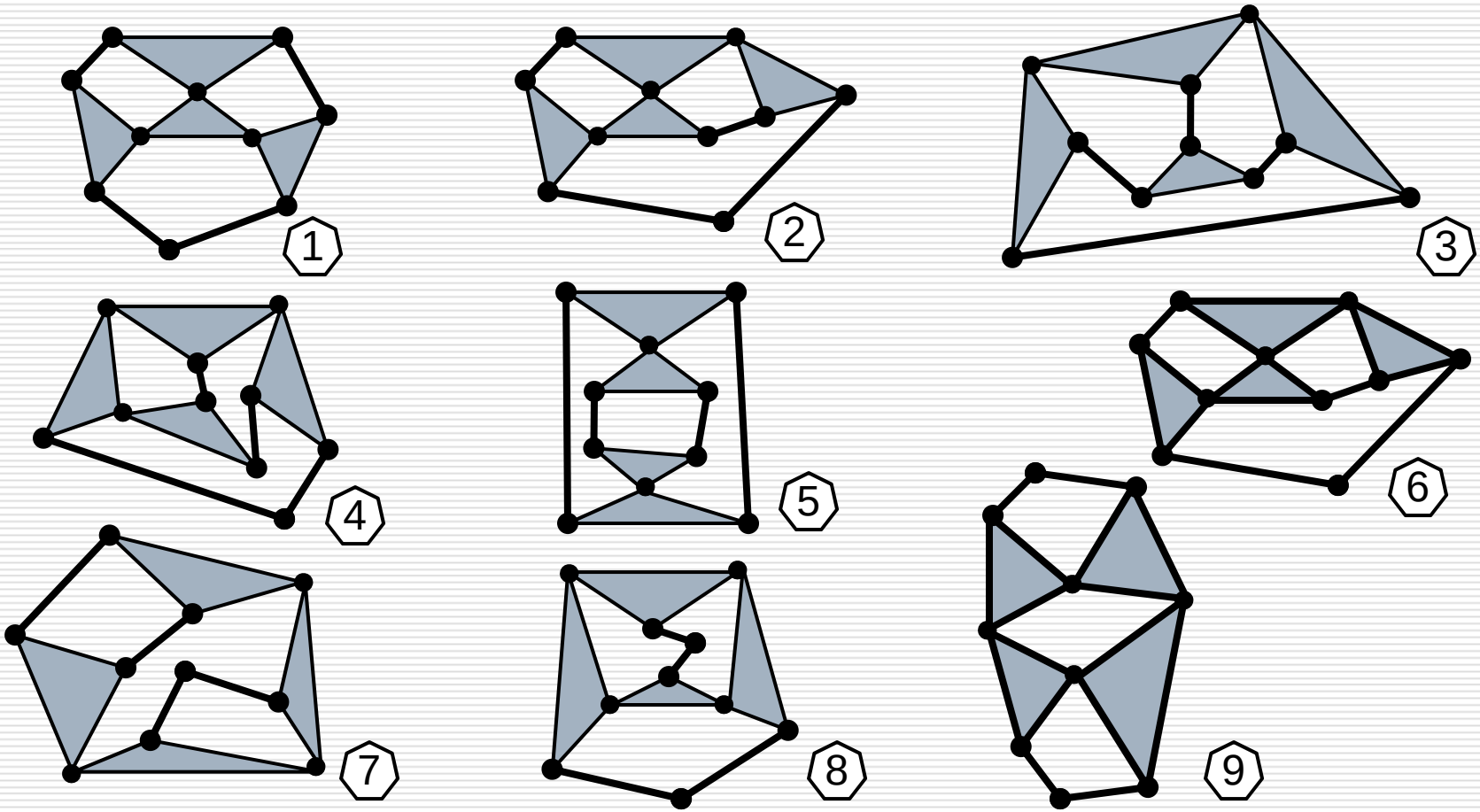
1

Stephenson's Linkage

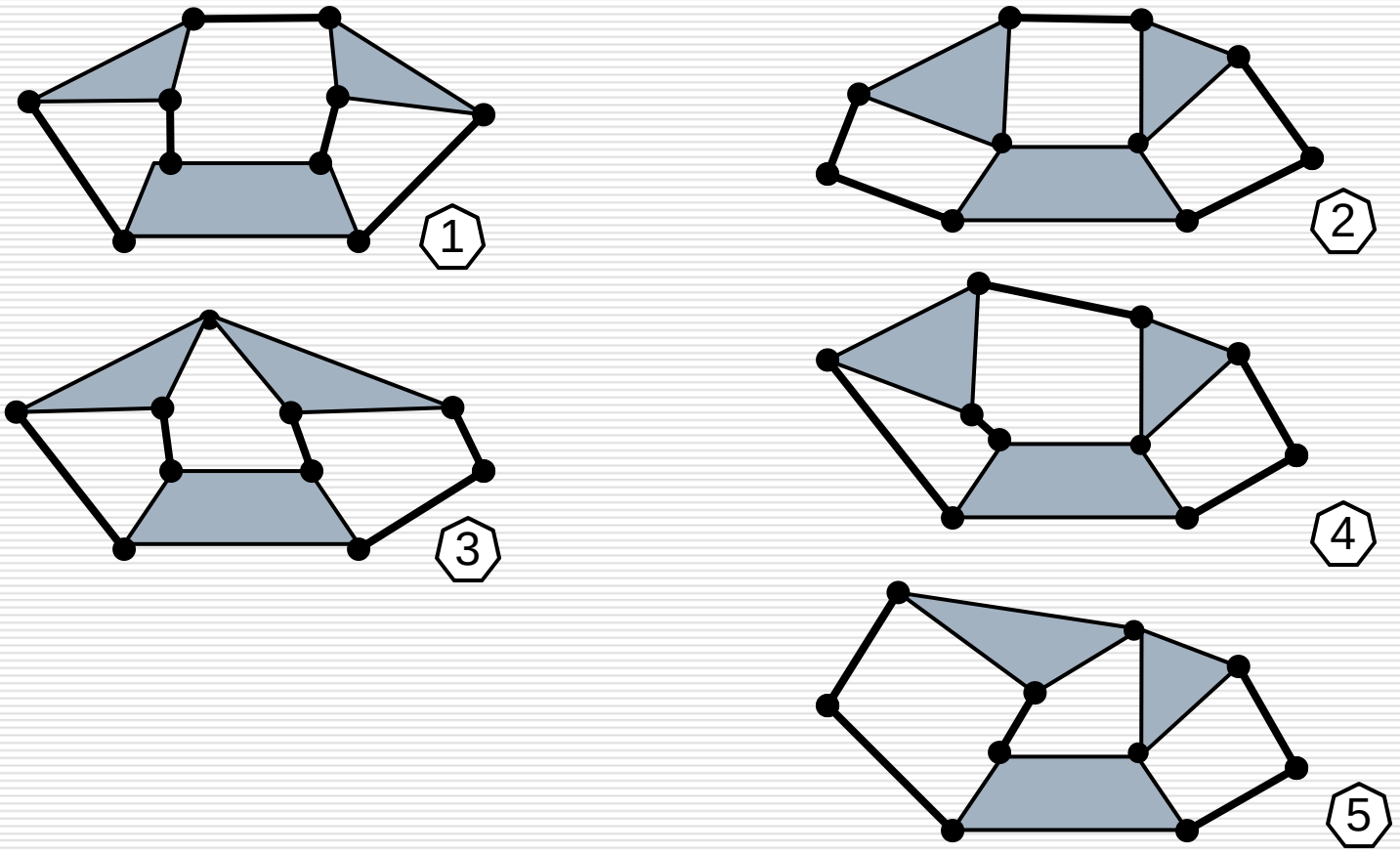


2

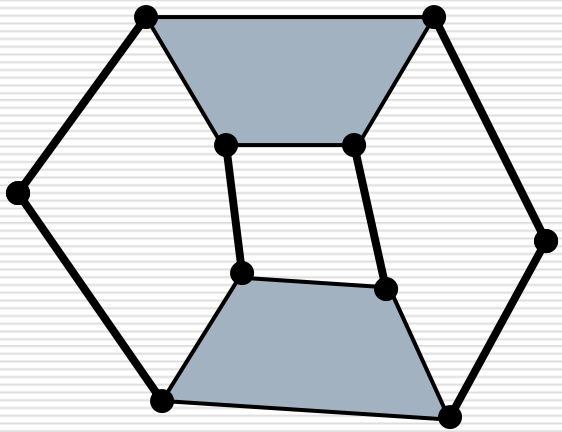
M	L	B	T	Q	P	Designation
+1	8	4	4	0	0	IV



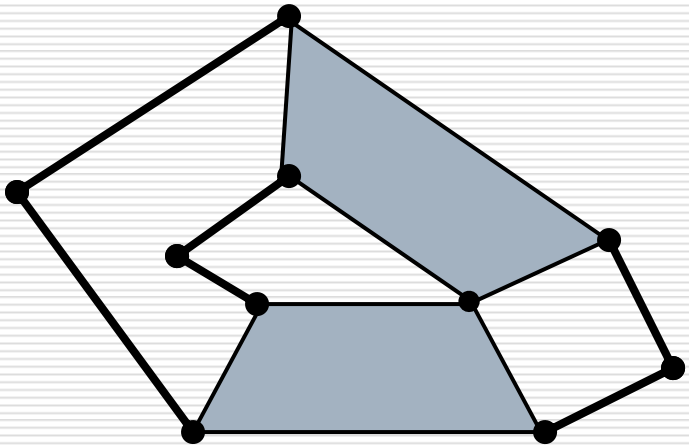
M	L	B	T	Q	P	Designation
+1	8	5	2	1	0	V



M	L	B	T	Q	P	Designation
+1	8	6	0	2	0	VI



1

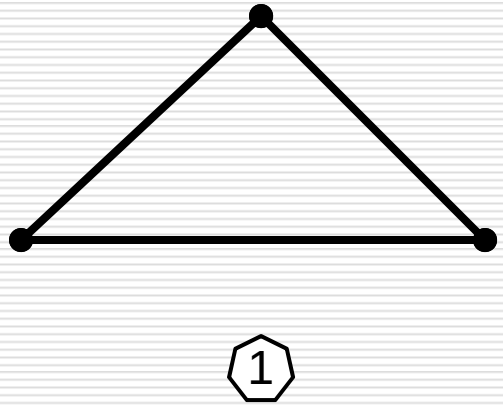


2

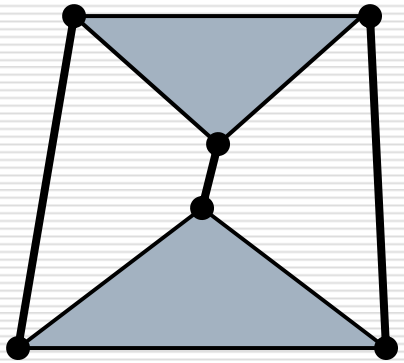
Link Combinations for Single Pin-Jointed Plane Linkages, M=0

M	L	B	T	Q	P	Designation
0	3	3	0	0	0	VIII
	5	3	2	0	0	IX
	5	4	0	1	0	X
	7	5	1	0	1	XI
	7	3	4	0	0	XII
	7	4	2	1	0	XIII
	7	5	0	2	0	XIV

M	L	B	T	Q	P	Designation
0	3	3	0	0	0	VIII



M	L	B	T	Q	P	Designation
0	5	3	2	0	0	IX



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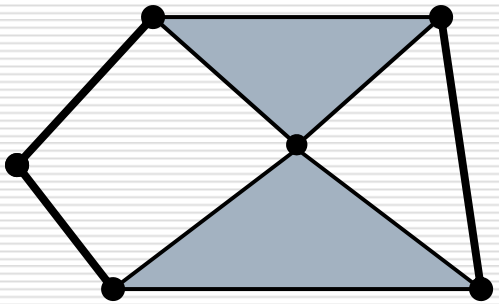
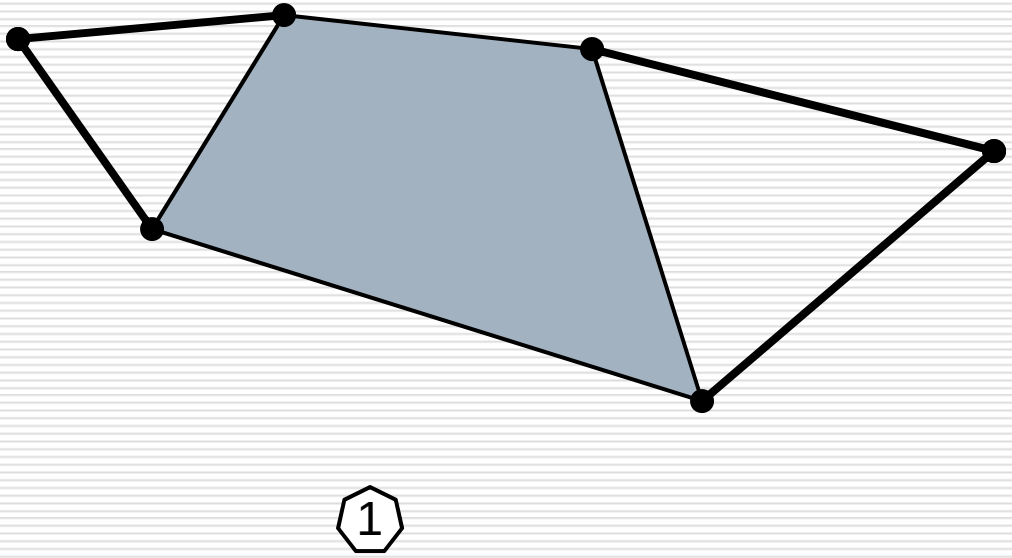
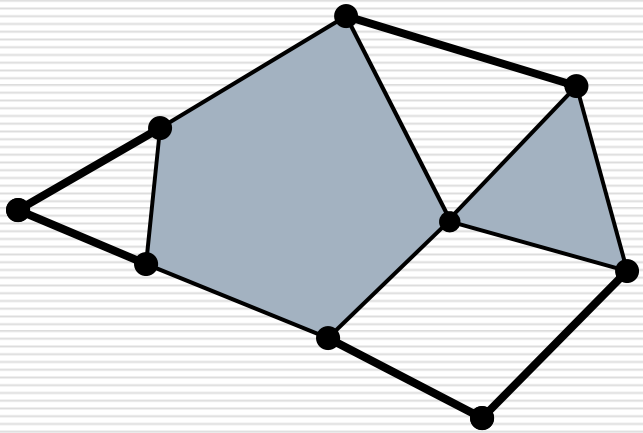


2

M	L	B	T	Q	P	Designation
0	5	4	0	1	0	X



M	L	B	T	Q	P	Designation
0	7	5	1	0	1	XI



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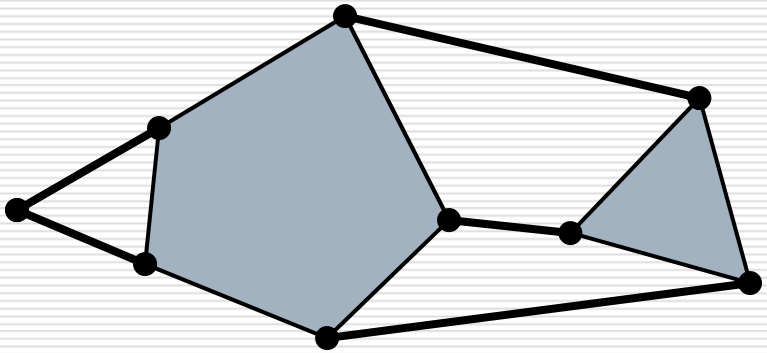
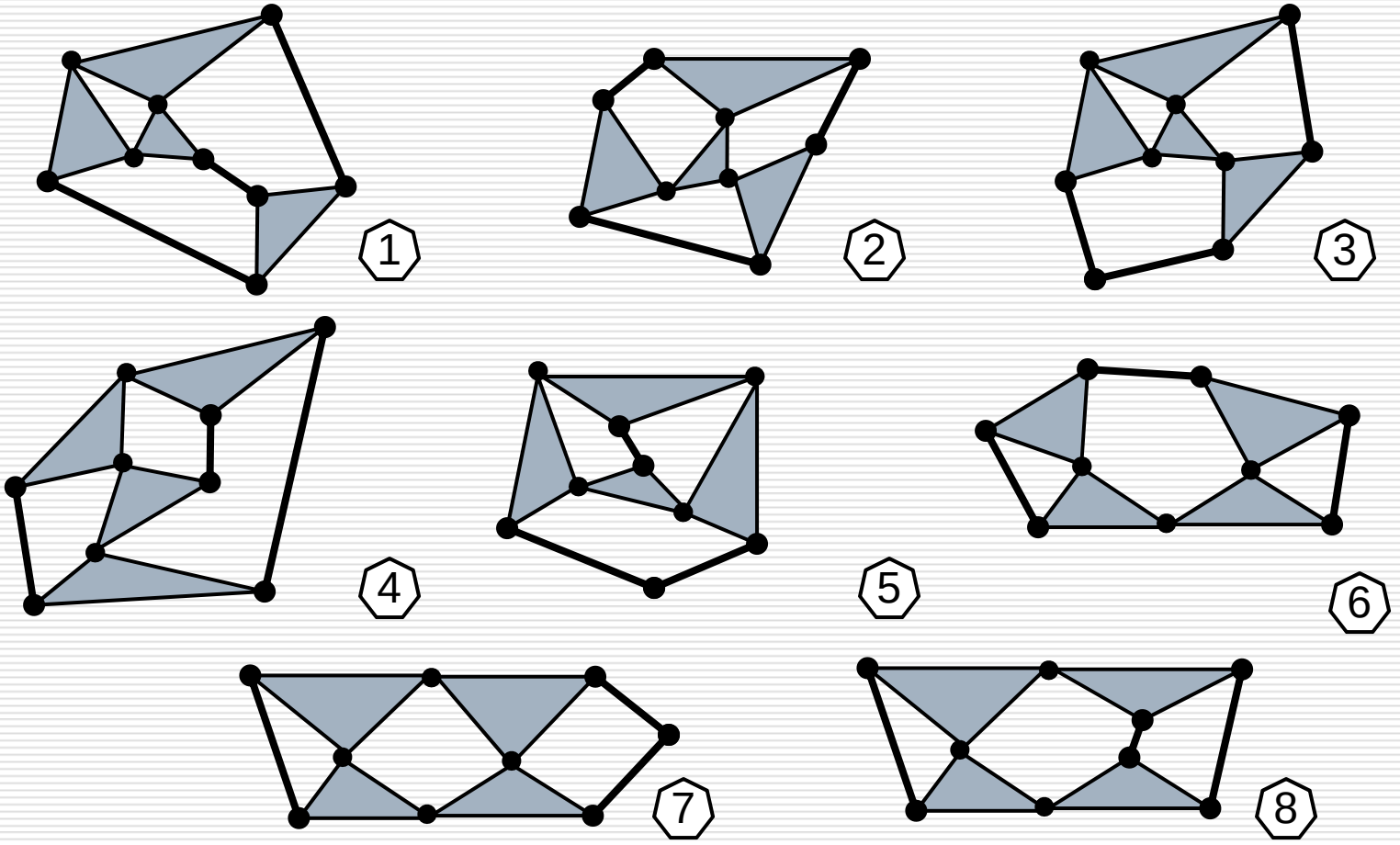
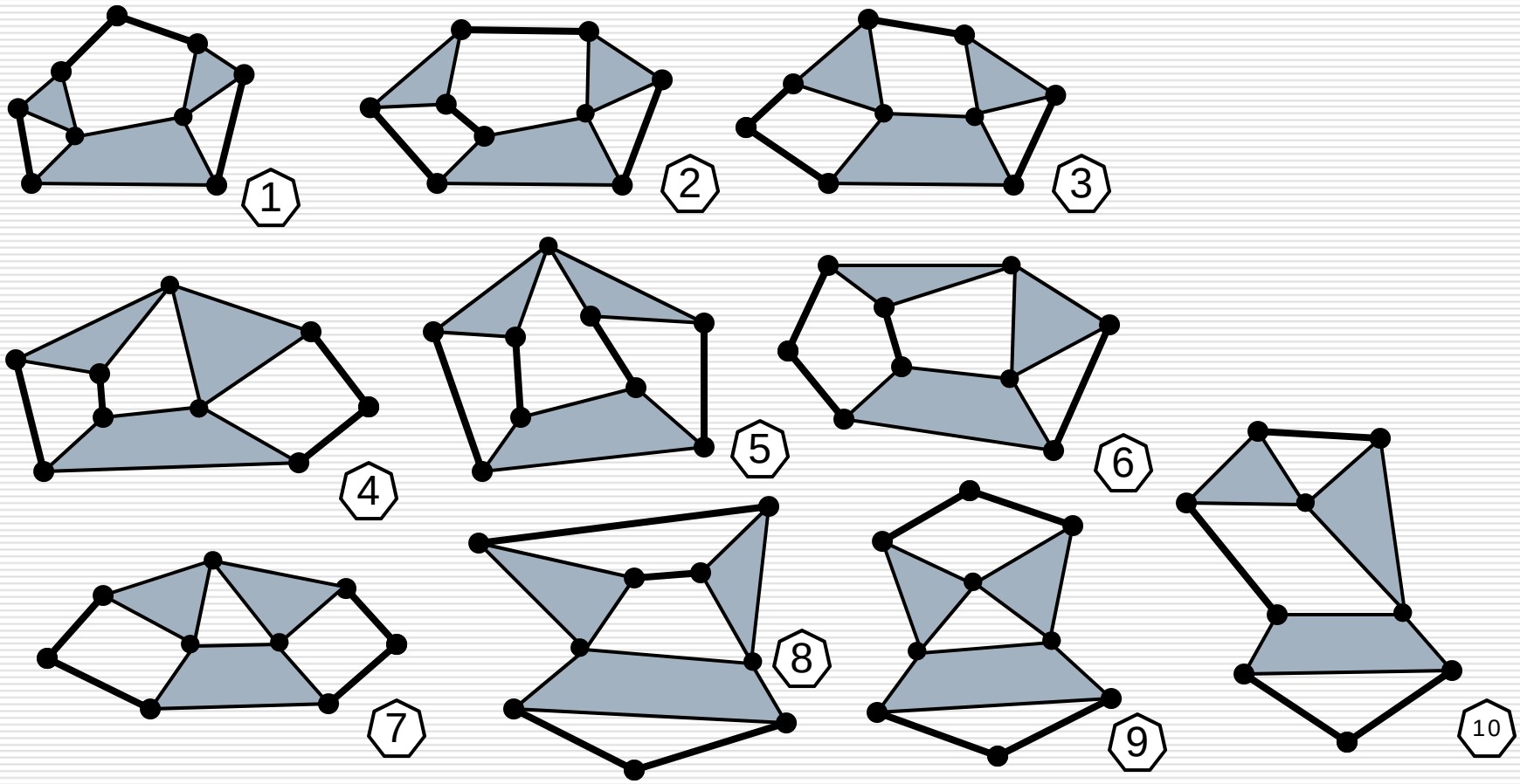


2

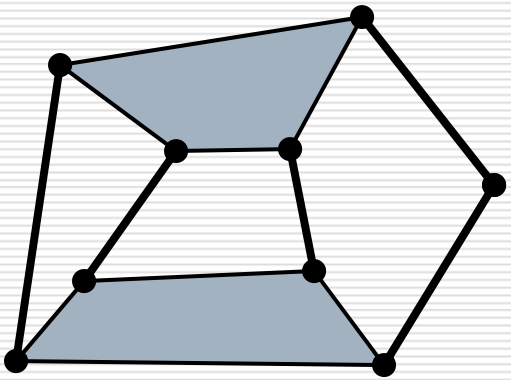
M	L	B	T	Q	P	Designation
0	7	3	4	0	0	XII



M	L	B	T	Q	P	Designation
0	7	4	2	1	0	XIII



M	L	B	T	Q	P	Designation
0	7	5	0	2	0	XIV



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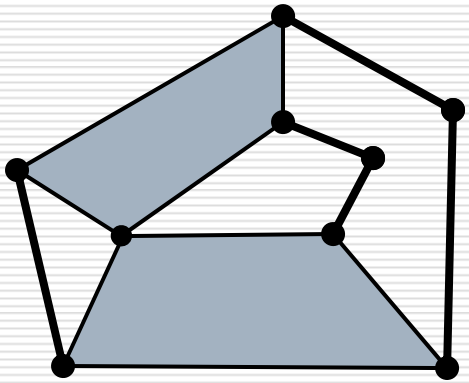

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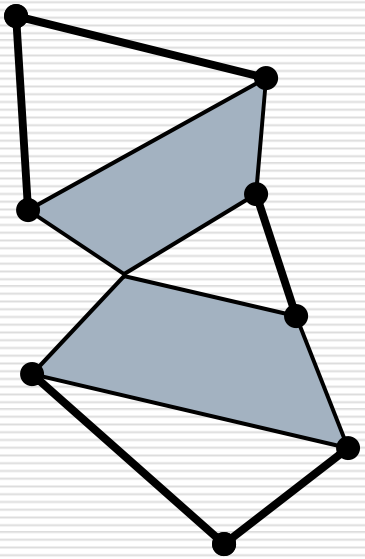


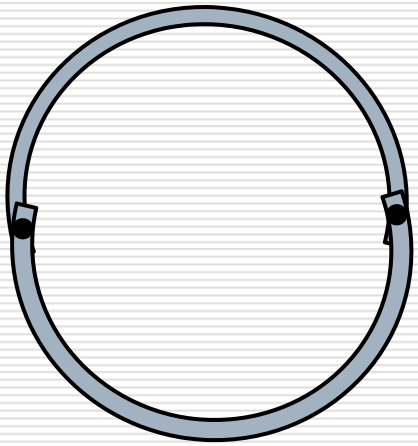
3

Link Combinations for Single Pin-Jointed Plane Linkages, M=-1

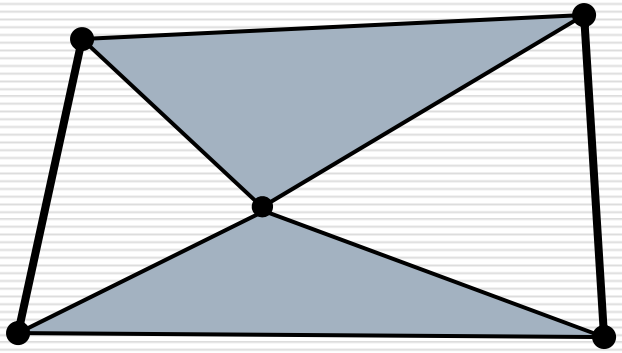
M	L	B	T	Q	P	Designation
-1	2	2	0	0	0	XV
	4	2	2	0	0	XVI
	4	3	0	1	0	XVII
	6	4	1	0	1	XVIII
	6	4	0	2	0	XIX
	6	3	2	1	0	XX
	6	2	4	0	0	XXI

M	L	B	T	Q	P	Designation
-1	8	6	0	0	2	XXII
	8	5	1	1	1	XXIII
	8	4	3	0	1	XXIV
	8	5	0	3	0	XXV
	8	4	2	2	0	XXVI
	8	3	4	1	0	XXVII
	8	2	6	0	0	XXVIII

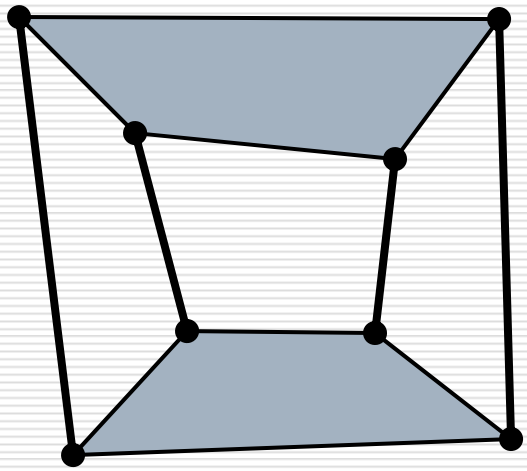
M	L	B	T	Q	P	Designation
-1	2	2	0	0	0	XV



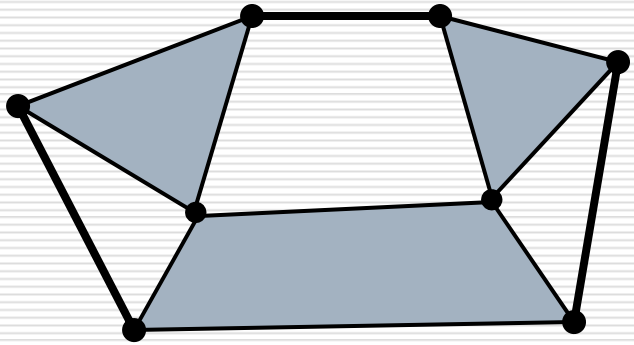
M	L	B	T	Q	P	Designation
-1	4	2	2	0	0	XVI



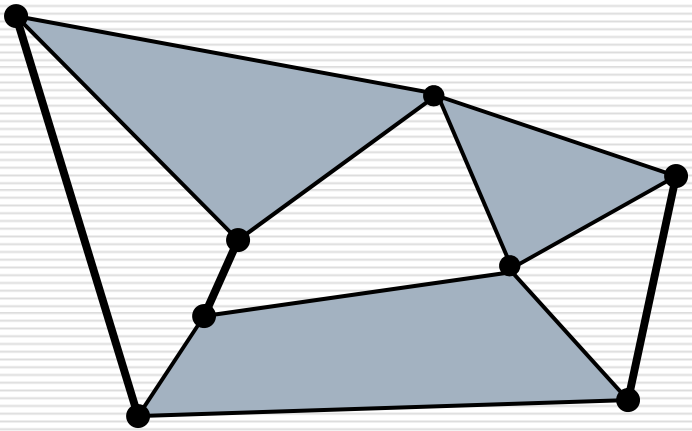
M	L	B	T	Q	P	Designation
-1	6	4	0	2	0	XIX



M	L	B	T	Q	P	Designation
-1	6	3	2	1	0	XX

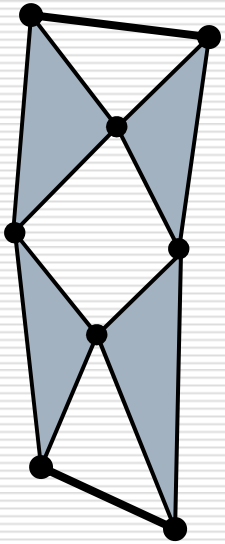


1

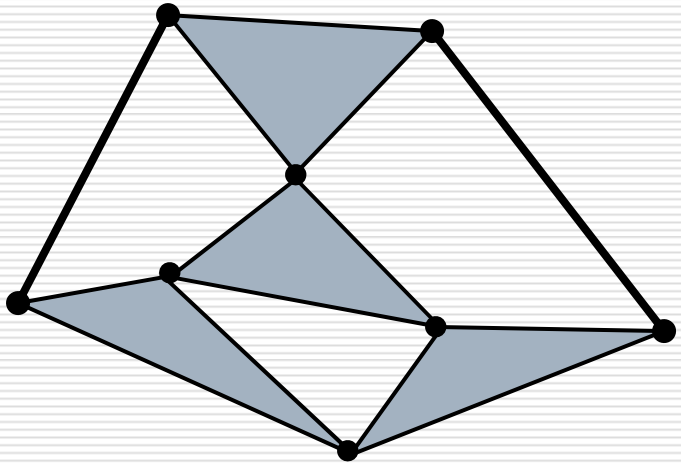


2

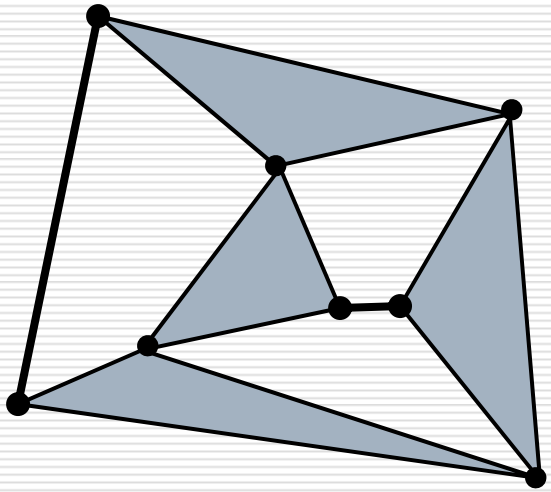
M	L	B	T	Q	P	Designation
-1	6	2	4	0	0	XXI



1

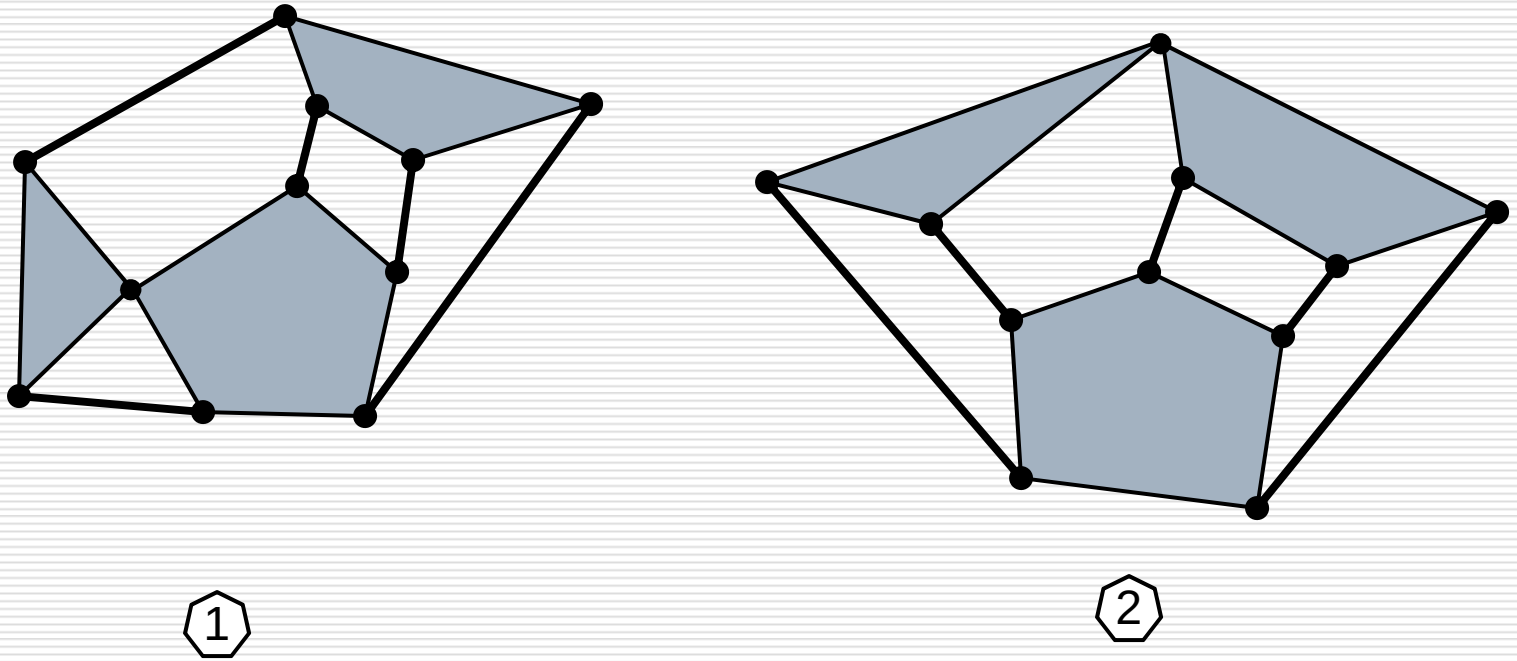


2

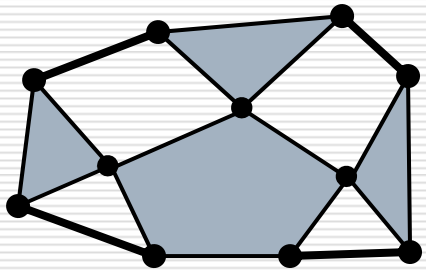


3

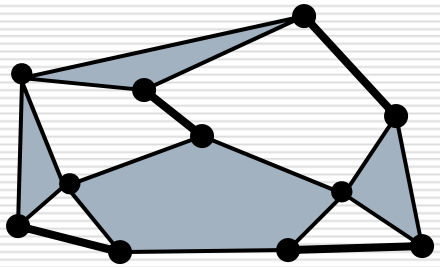
M	L	B	T	Q	P	Designation
-1	8	5	1	1	1	XXIII



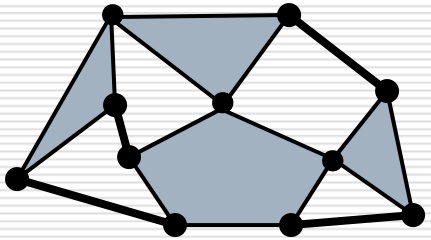
M	L	B	T	Q	P	Designation
-1	8	4	3	0	1	XXIV



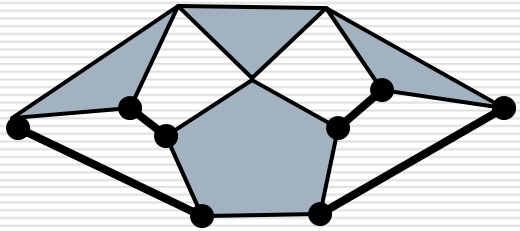
1



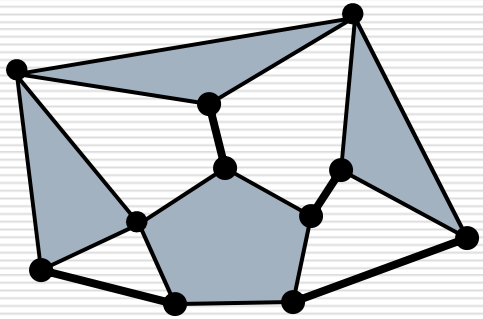
2



3

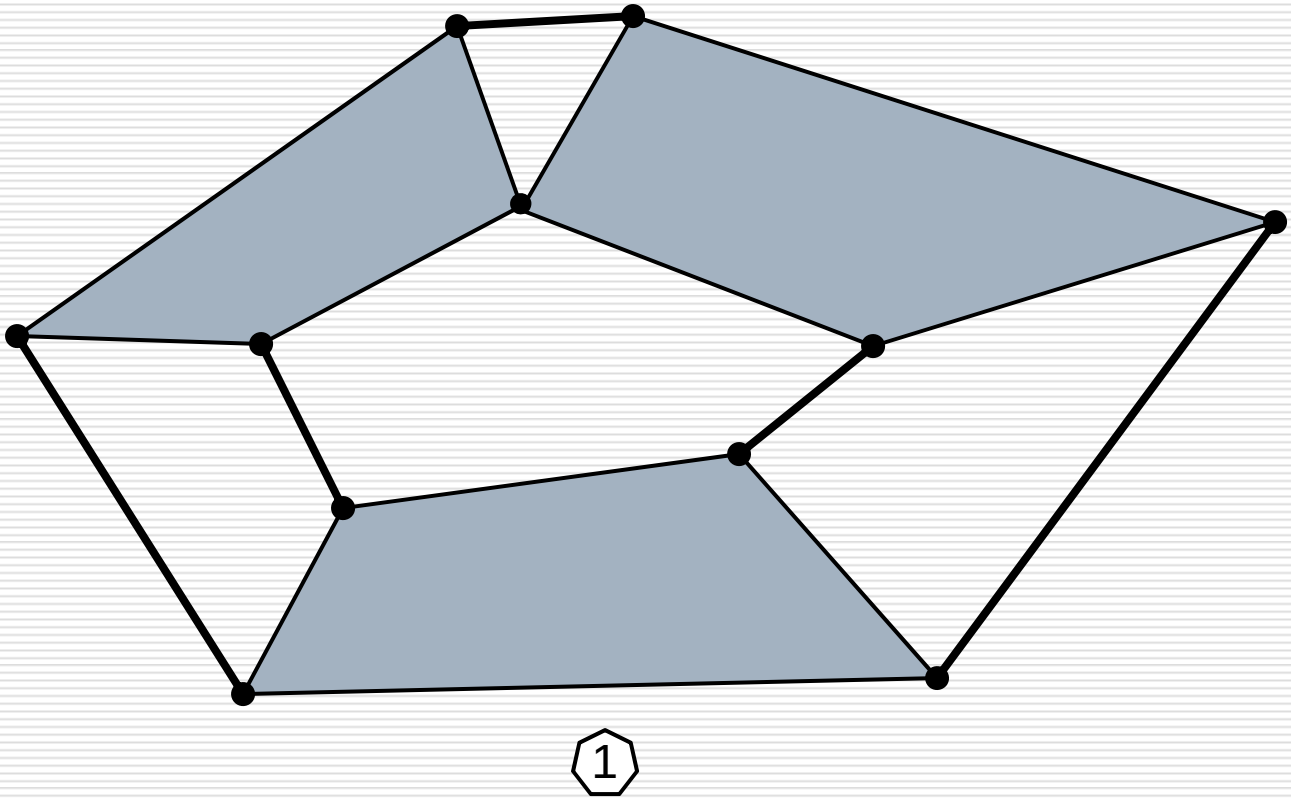


4



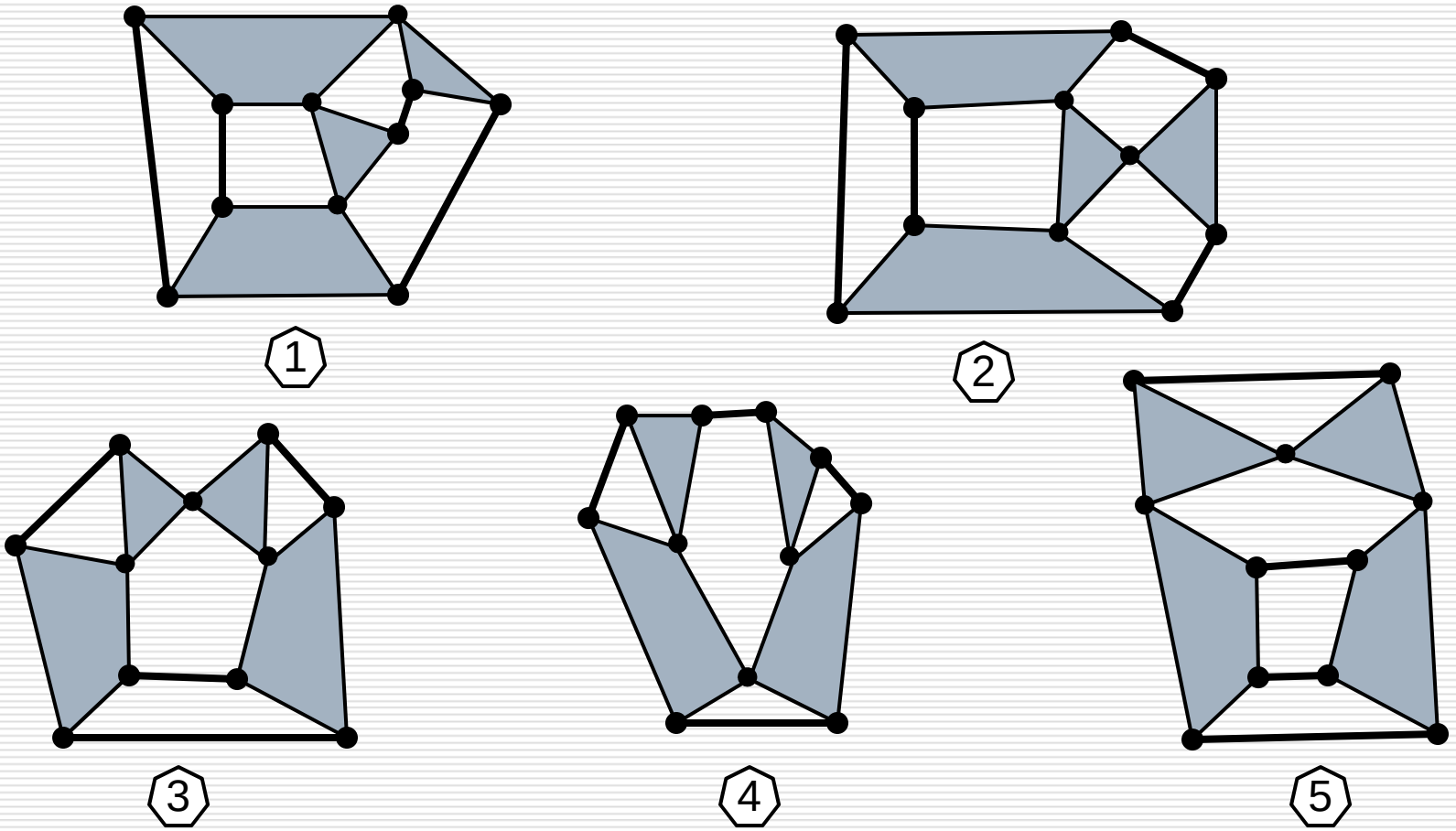
5

M	L	B	T	Q	P	Designation
-1	8	5	0	3	0	XXV



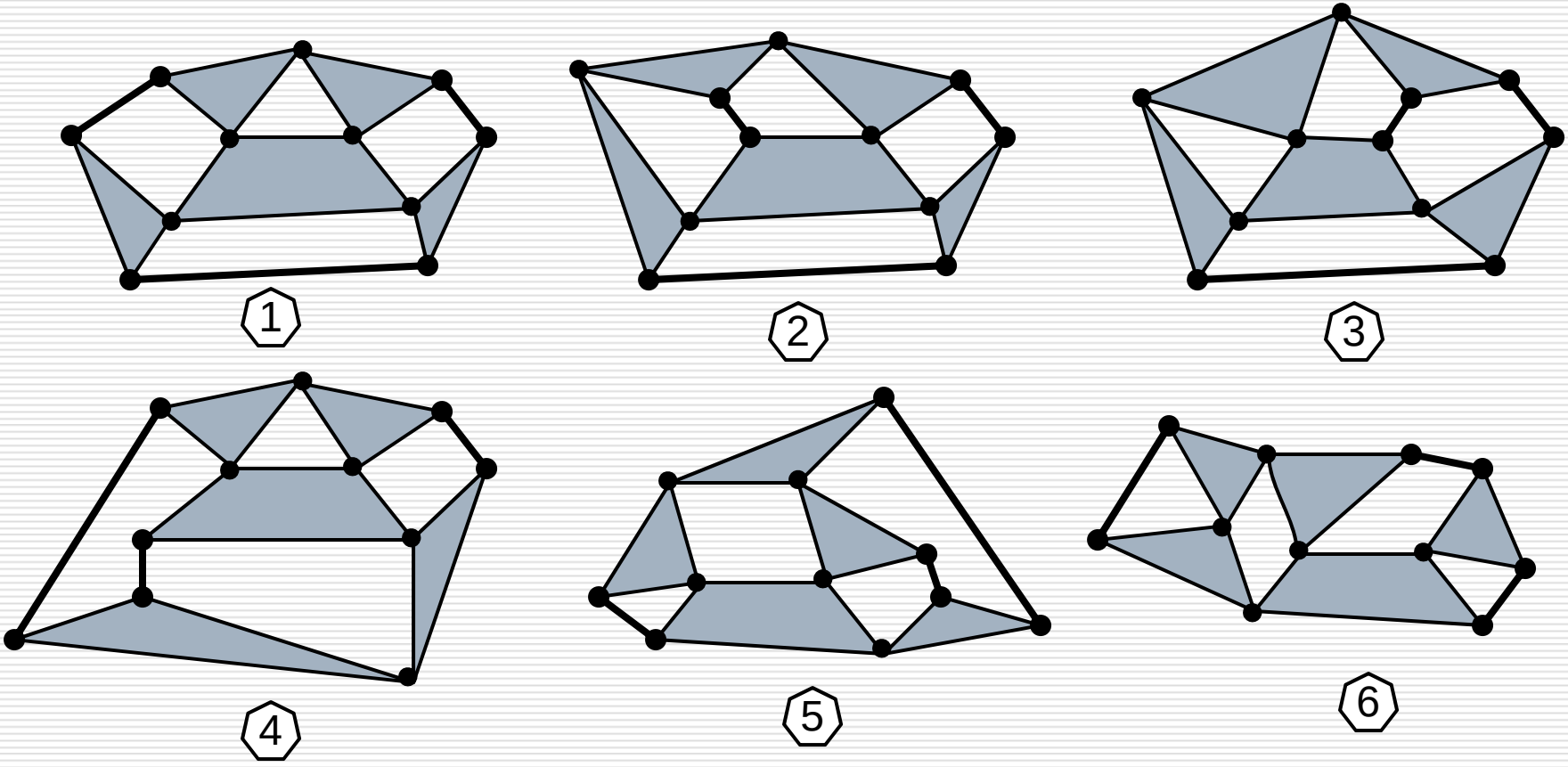
M	L	B	T	Q	P	Designation
-1	8	4	2	2	0	XXVI

(Plus 11 more not shown)



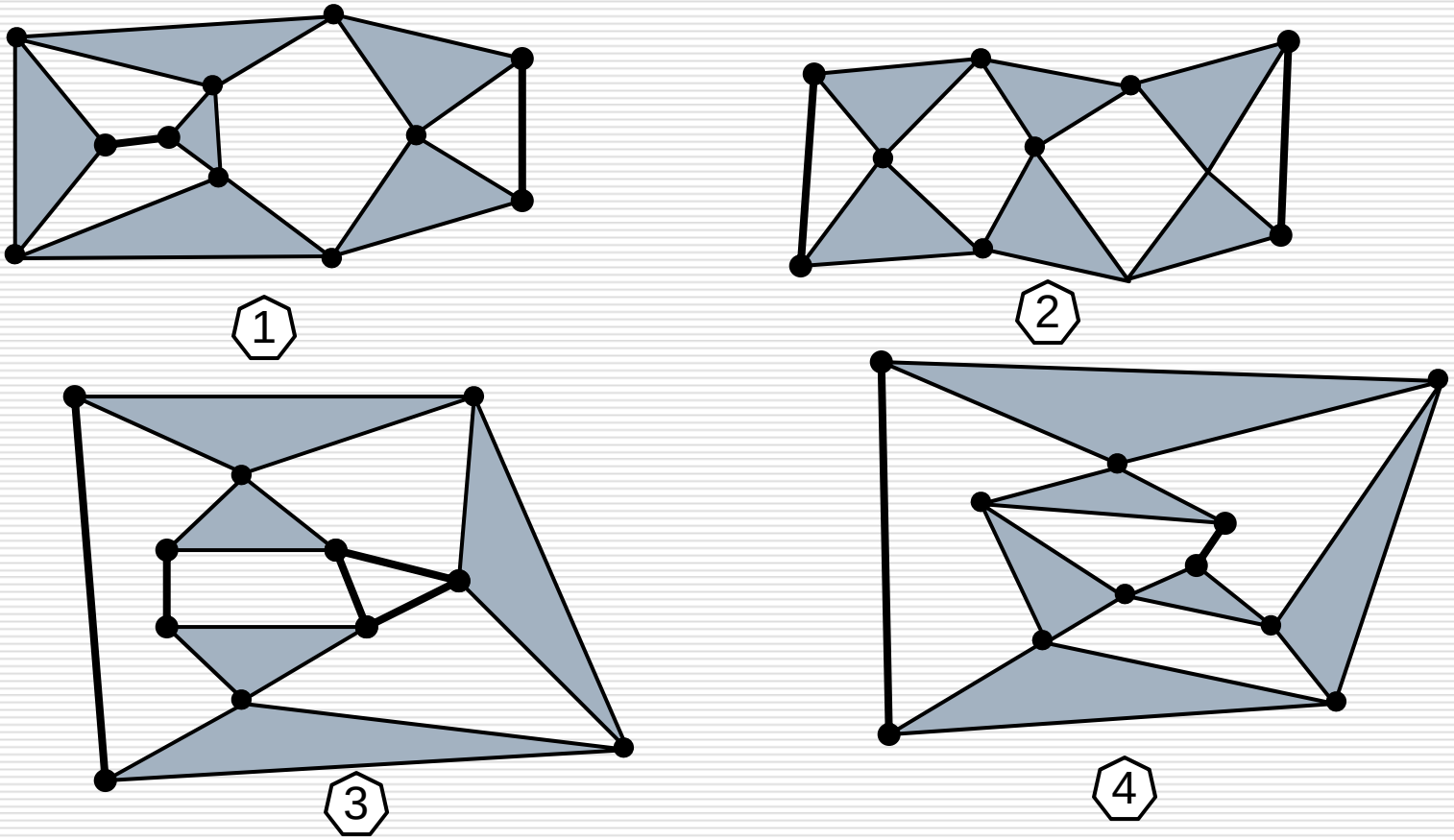
M	L	B	T	Q	P	Designation
-1	8	3	4	1	0	XXVII

(Plus 16 more not shown)



M	L	B	T	Q	P	Designation
-1	8	2	6	0	0	XXVIII

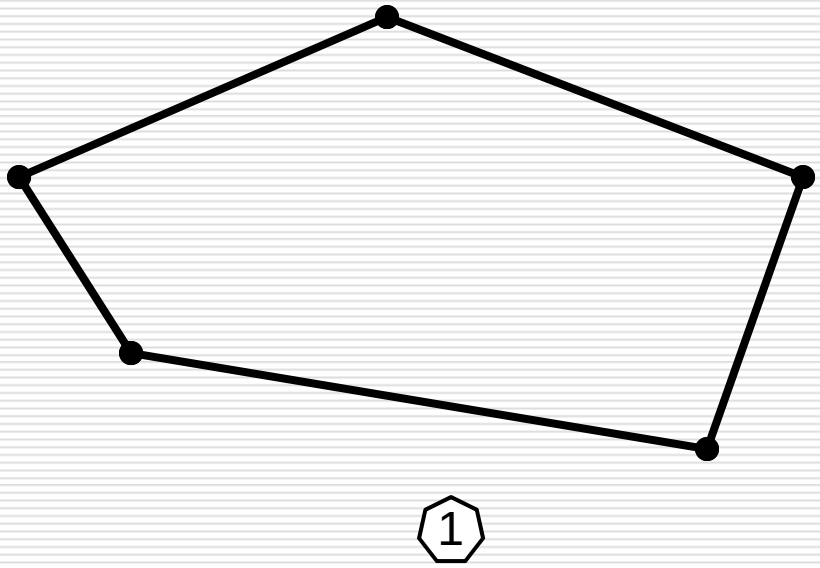
(Plus 11 more not shown)



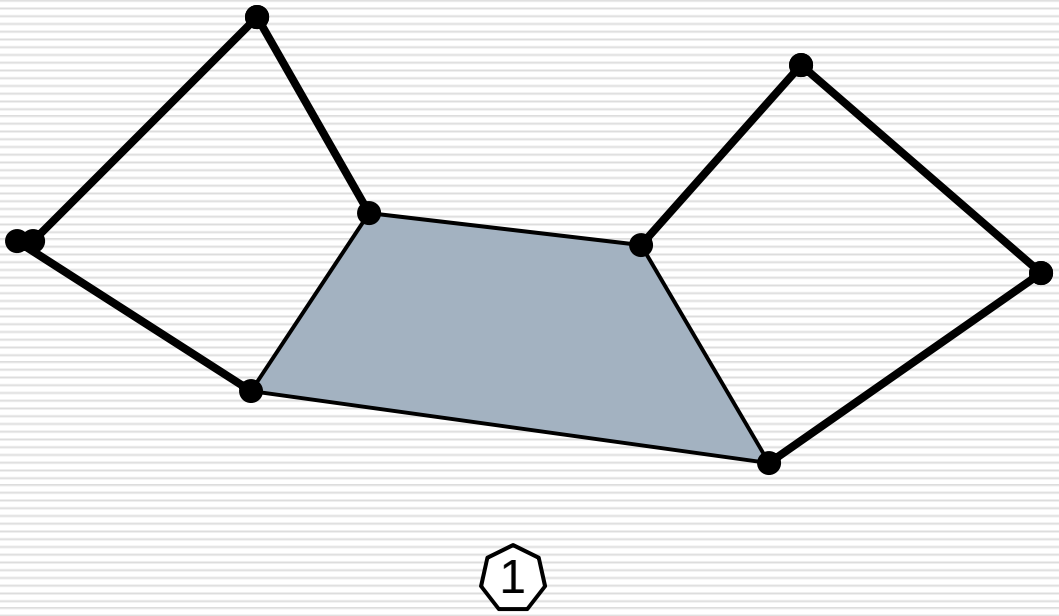
Link Combinations for Single Pin-Jointed Plane Linkages, M=+2

M	L	B	T	Q	P	Designation
2	5	5	0	0	0	XXIX
	7	6	0	1	0	XXX
	7	5	2	0	0	XXXI
	9	7	1	0	1	XXXII
	9	7	0	2	0	XXXIII
	9	6	2	1	0	XXXIV
	9	5	4	0	0	XXXV

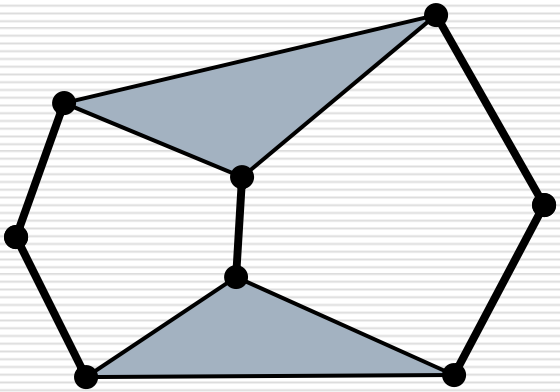
M	L	B	T	Q	P	Designation
2	5	5	0	0	0	XXIX



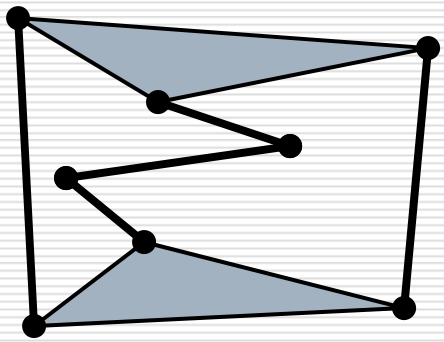
M	L	B	T	Q	P	Designation
2	7	6	0	1	0	XXX



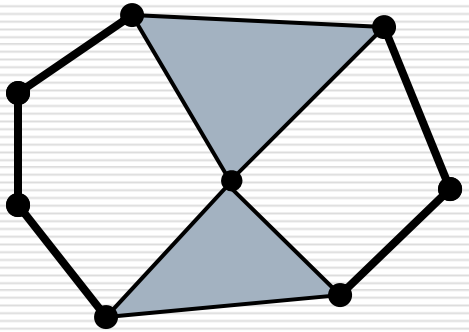
M	L	B	T	Q	P	Designation
2	7	5	2	0	0	XXXI



1

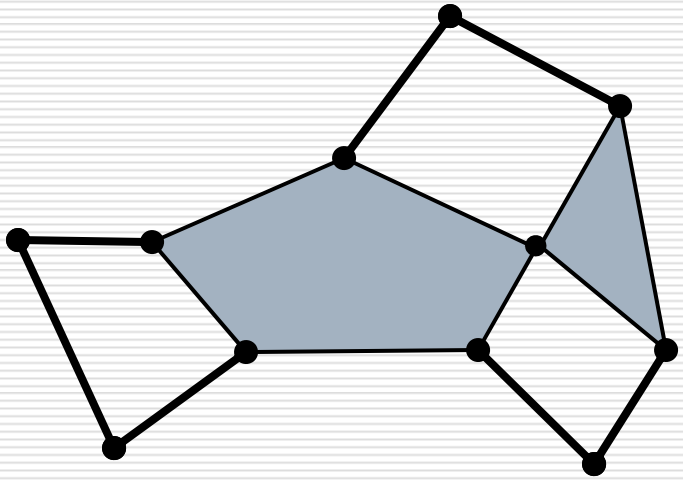


2

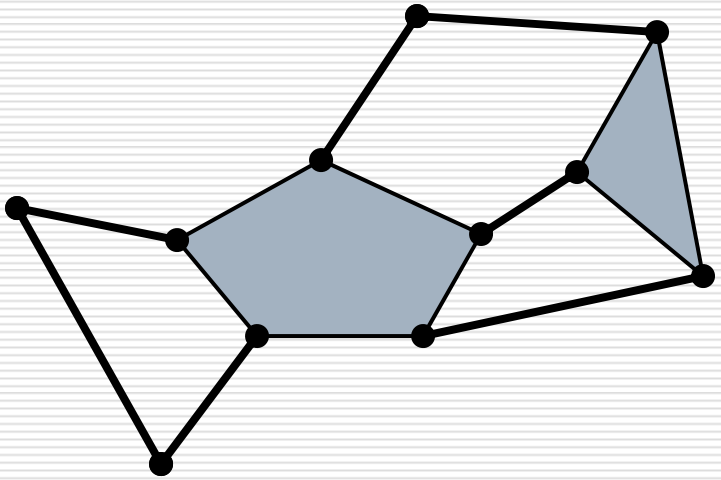


3

M	L	B	T	Q	P	Designation
2	9	7	1	0	1	XXXII

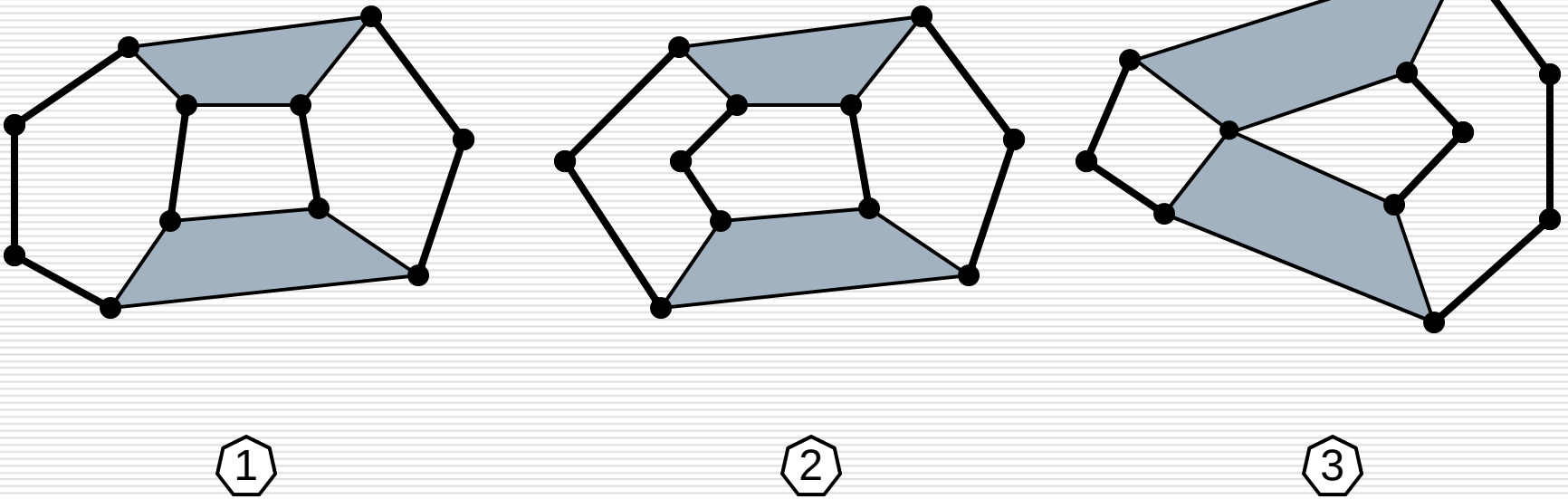


1



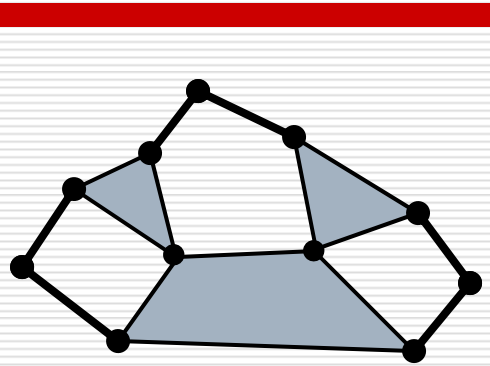
2

M	L	B	T	Q	P	Designation
2	9	7	0	2	0	XXXIII

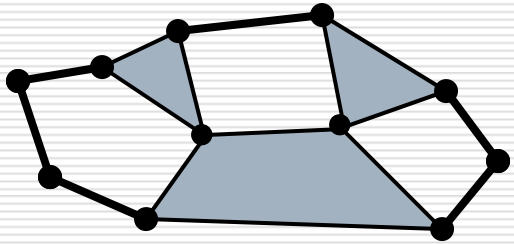


M	L	B	T	Q	P	Designation
2	9	6	2	1	0	XXXIV

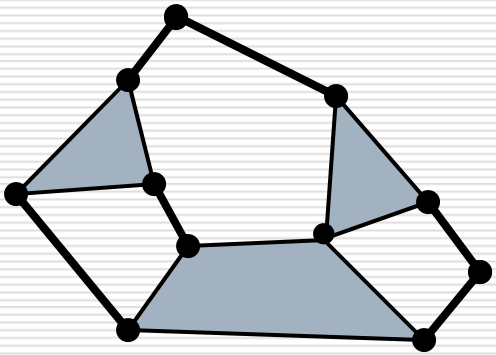
(Plus 10 more not shown)



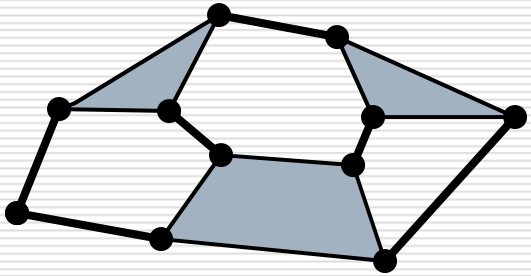
1



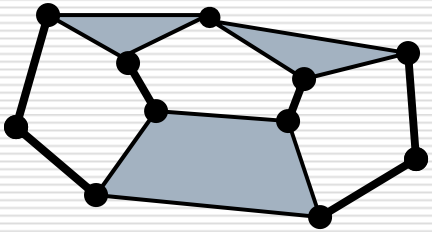
2



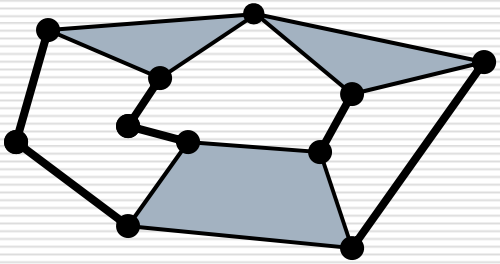
3



4



5



6

M	L	B	T	Q	P	Designation
2	9	5	4	0	0	XXXV

(Plus 11 more not shown)