PROBLEM 6-26 THE DEAD WEIGHT LOADING ALONG THE CENTERLINE OF THE AIRPLANE WING IS SHOWN. IF THE WING IS FIXED TO THE FUSELAGE AT A, DETERMINE THE REACTIONS AT A, AND THEN DRAW THE SHEAR AND BENDING MOMENT DIAGRAM.

GIVEN:

CONSTRAINT

1. DEAD LOADING ON AN AIRPLANE WING.

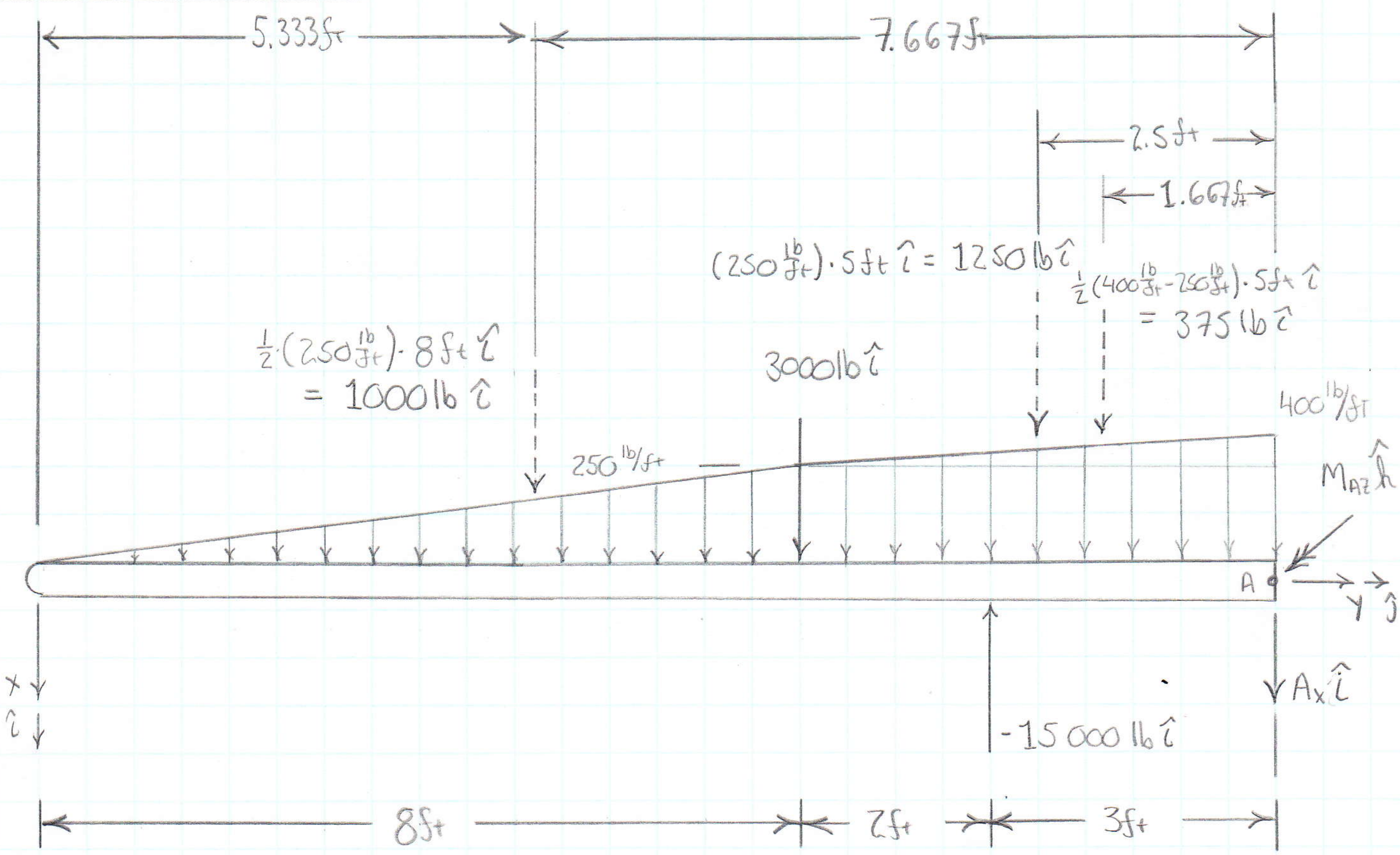
ASSUMPTIONS

1. SMALL DEFLECTIONS

FIND:

1. REACTION LOADS ON THE FUSELAGE.
2. DRAW SHEAR AND BENDING MOMENT DIAGRAM.

FREE BODY DIAGRAM:



SOLUTION:

THE SOLUTION STARTS WITH THE DETERMINATION OF THE REACTIONS AT A

$$\Sigma F_x = 0 = 1000\,lb + 3000\,lb + 1250\,lb - 15000\,lb + 375\,lb + A_x$$

$$\Rightarrow \quad \underline{A_x = 9375\,lb} \quad (1)$$

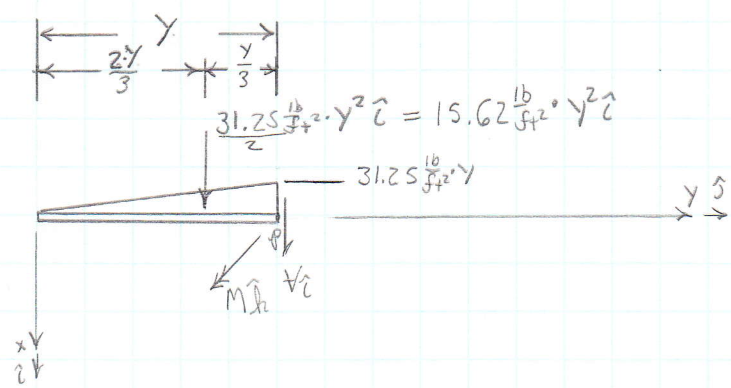
$$\Sigma M_{z/AT\,A} = 0 = 1000\,lb \cdot 7.667\,ft + 3000\,lb \cdot 5\,ft + 1250\,lb \cdot 2.5\,ft - 15000\,lb \cdot 3\,ft + 375\,lb \cdot 1.667\,ft + M_{zz}$$

$$\Rightarrow \quad \underline{M_{zz} = 18.58(10^3)\,ft \cdot lb} \quad (2)$$

NOW LETS CONSIDER THE INTERNAL LOADING OF THE BEAM FROM $\underline{0 < y < 8\,ft}$
TO START THE EQUATION FOR THE DESTRIBUTED LOAD NEEDS TO BE DETERMINED.

$$w(y) = \frac{250\frac{lb}{ft} - 0\frac{lb}{ft}}{8\,ft} \cdot y + \cancel{w_i}^{\,0}$$

$$\underline{w(y) = 31.25\,\frac{lb}{ft^2} \cdot y} \quad (3)$$



NOW DETERMINING V AND M IN THIS SECTION THROUGH EQUILIBRIUM

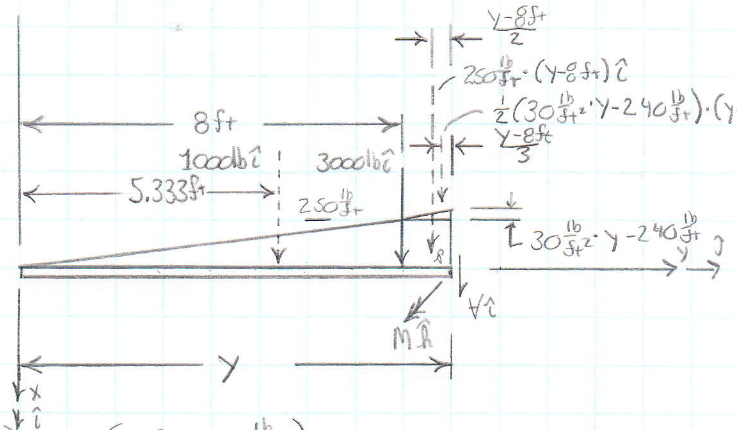
$$\Sigma F_x = 0 = 15.62\,\frac{lb}{ft^2} \cdot y^2 + V$$

$$\Rightarrow \quad \underline{V(y) = -15.62\,\frac{lb}{ft^2} \cdot y^2} \quad (4)$$

$$\Sigma M_{z/AT\,P} = 0 = M + 15.62\,\frac{lb}{ft^2} \cdot y^2 \cdot \frac{y}{3} \quad \Rightarrow \quad \underline{M(y) = -5.208\,\frac{lb}{ft^2} \cdot y^3} \quad (5)$$

NOW WE CONSIDER V AND M IN THE REGION FROM $\underline{8ft < y < 10ft}$

THE EQUATION FOR THE TRIANGULAR PORTION OF THE DISTRIBUTED LOAD NEEDS TO BE DETERMINED



$$w(y) = \frac{400\frac{lb}{ft} - 250\frac{lb}{ft}}{5ft}\cdot y + w_i$$

$$= 30\frac{lb}{ft^2}\cdot y + w_i$$

THE VALUE OF w_i IS DETERMINED BY SUBSTITUTING IN KNOWN VALUES $(y, w) = (8ft, 0\frac{lb}{ft})$

$$0 = 30\frac{lb}{ft^2}\cdot 8ft + w_i \implies w_i = -240\frac{lb}{ft}$$

$$\underline{w(y) = 30\frac{lb}{ft^2}\cdot y - 240\frac{lb}{ft}} \quad (6)$$

NOW DETERMINING V AND M FROM EQUILIBRIUM

$$\Sigma F_x = 0 = 1000lb + 3000lb + 250\frac{lb}{ft}\cdot(y-8ft) + \frac{1}{2}(30\frac{lb}{ft^2}\cdot y - 240\frac{lb}{ft})\cdot(y-8ft) + V$$

$$\implies -V = 4000lb + 250\frac{lb}{ft}\cdot y - 2000lb + 15\frac{lb}{ft^2}y^2 - 120\frac{lb}{ft}\cdot y - 120\frac{lb}{ft}y + 960lb$$
$$= 2960lb + 10\frac{lb}{ft}\cdot y + 15\frac{lb}{ft^2}\cdot y^2$$

$$\underline{V(y) = -15\frac{lb}{ft^2}\cdot y^2 - 10\frac{lb}{ft}\cdot y - 2960lb} \quad (7)$$

$$\Sigma M_{z/ATP} = 0 = M + 1000lb\cdot(y-5.333ft) + 3000lb\cdot(y-8ft)$$
$$+ 250\frac{lb}{ft}(y-8ft)\cdot\left(\frac{y-8ft}{2}\right) + \frac{1}{2}(30\frac{lb}{ft^2}\cdot y - 240\frac{lb}{ft})\cdot(y-8ft)\left(\frac{y-8ft}{3}\right)$$

$$-M = 1000lb\cdot y - 5333lb\cdot ft + 3000lb\cdot y - 24000lb\cdot ft + 125\frac{lb}{ft}(y^2 - 16ft\cdot y + 64ft^2)$$
$$+ (5\frac{lb}{ft^2}\cdot y - 40\frac{lb}{ft})(y^2 - 16ft\cdot y + 64ft^2)$$

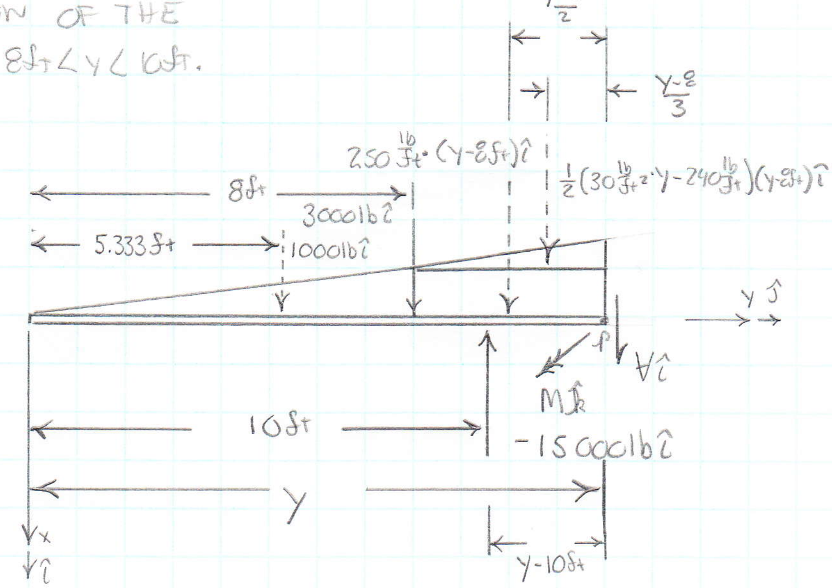
$$= 4000lb\cdot y - 29333lb\cdot ft + 125\frac{lb}{ft}\cdot y^2 - 2000lb\,y + 8000lb\cdot ft$$
$$+ 5\frac{lb}{ft^2}\cdot y^3 - 80\frac{lb}{ft}\cdot y^2 + 320lb\cdot y - 40\frac{lb}{ft}\cdot y^2 + 640lb\cdot y - 2560lb\cdot ft$$

$$= -23890lb\cdot ft + 2960lb\cdot y + 5\frac{lb}{ft}\cdot y^2 + 5\frac{lb}{ft^2}\cdot y^3$$

$$\underline{M(y) = -5\frac{lb}{ft^2}\cdot y^3 - 5\frac{lb}{ft}\cdot y^2 - 2960lb\cdot y + 23890lb\cdot ft} \quad (8)$$

NOW CONSIDER THE REGION FROM $10\,ft < y < 13\,ft$.
THE EQUATION FOR THE TRIANGULAR PORTION OF THE DISTRIBUTED LOAD IS THE SAME IN THIS PORTION OF THE BEAM AS IN THE REGION FROM $8\,ft < y < 10\,ft$.

$$w(y) = 30\tfrac{lb}{ft^2}\cdot y - 240\tfrac{lb}{ft} \quad (9)$$



NOW V AND M FOR THIS SECTION OF THE BEAM IS DETERMINED

$$\Sigma F_x = 0$$

$$\Rightarrow V(y) = -15\tfrac{lb}{ft^2}\cdot y^2 - 10\tfrac{lb}{ft}\,y - 2960\,lb + 15000\,lb$$

$$V(y) = -15\tfrac{lb}{ft^2}\cdot y^2 - 10\tfrac{lb}{ft}\cdot y + 12\,040\,lb \quad (10)$$

$$\Sigma M_{z/at\,P} = 0$$

$$\Rightarrow M(y) = -5\tfrac{lb}{ft^3}\cdot y^3 - 5\tfrac{lb}{ft}\cdot y^2 - 2960\,lb\cdot y + 23\,890\,lb\cdot ft + 15000\,lb\cdot(y - 10ft)$$
$$= -5\tfrac{lb}{ft^2}\cdot y^3 - 5\tfrac{lb}{ft}\cdot y^2 - 2960\,lb\cdot y + 23890\,lb\cdot ft + 15000\,lb\cdot y - 150\,000\,lb\,ft$$

$$M(y) = -5\tfrac{lb}{ft^3}\cdot y^3 - 5\tfrac{lb}{ft}\cdot y^2 + 12\,040\,lb\cdot y - 126\,100\,lb\cdot ft \quad (11)$$

NOW LETS CONSIDER THE END POINTS IN THESE REGIONS

$0 < y < 8\,ft$: $V(0) = 0$, $M(0) = 0$; $V(8ft) = -1000\,lb$, $M(8ft) = -2\,667\,lb\cdot ft$

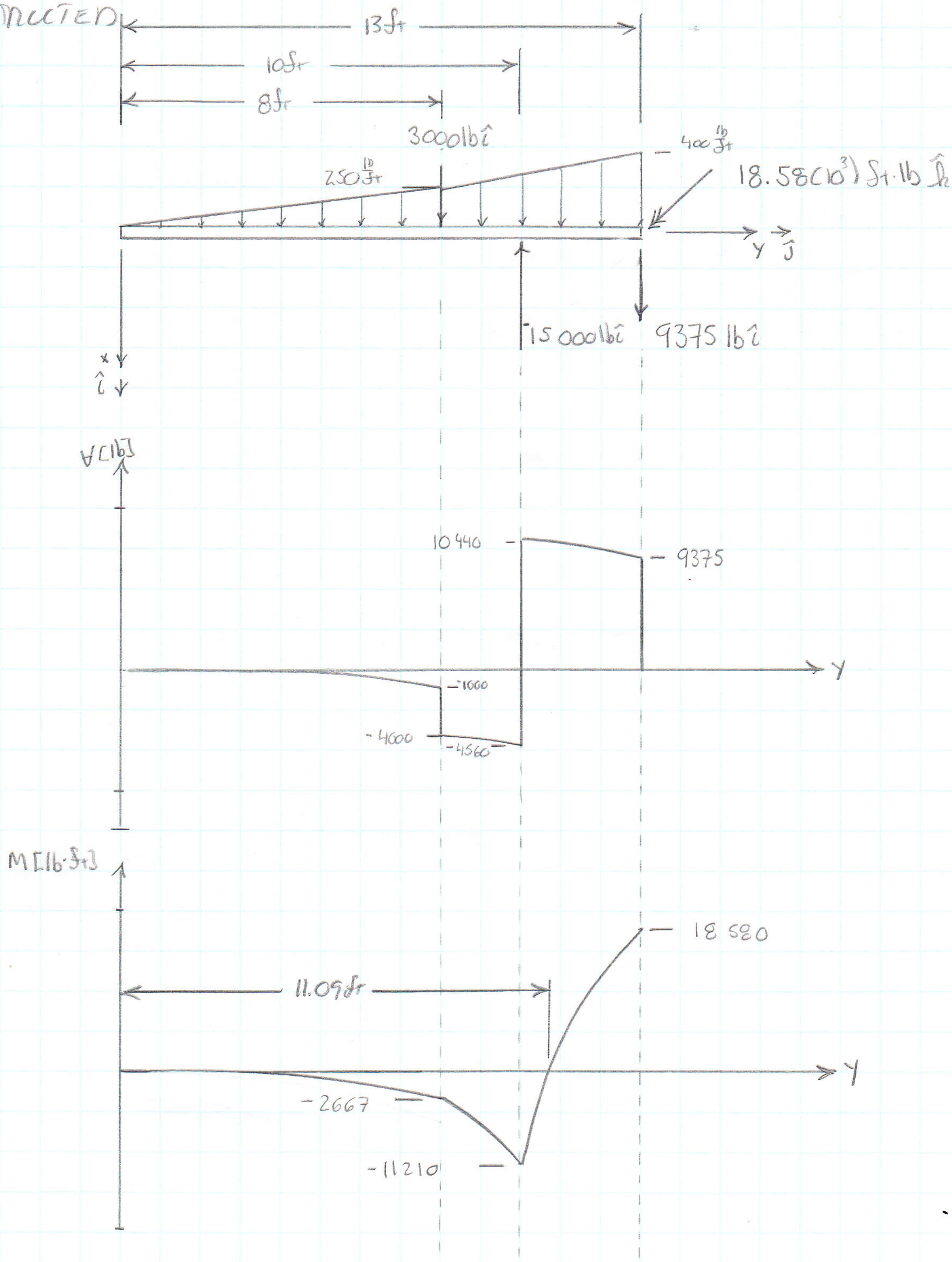
$8ft < y < 10ft$: $V(8ft) = -4000\,lb$, $M(8ft) = -2667\,lb\cdot ft$; $V(10ft) = -4\,560\,lb$, $M(10ft) = -11\,210\,lb\cdot ft$

$10ft < y < 13ft$: $V(10ft) = 10\,440\,lb$, $M(10ft) = -11\,210\,lb\cdot ft$; $V(13ft) = 9\,375\,lb$, $M(13ft) = 18.58\,(10^3)\,ft\cdot lb$

HOMEWORK SOLUTION
ESC023: MECHANICS III
ASSIGNMENT #7

PROB 6-26 PG 5 OF 6
HIBBELER 4TH
SOLUTION BY BUCKWELL

NOW THE SHEAR AND BENDING MOMENT DIAGRAMS CAN BE CONSTRUCTED



WE NEED TO DETERMINE WHERE THE MOMENT DIAGRAM CROSSES THE Y-AXES IN THE REGION 10 ft < y < 13 ft. FROM (8)

$$0 = -5\frac{lb}{ft^3}\cdot y^3 - 5\frac{lb}{ft}\cdot y^2 - 2960\,lb\cdot y + 23890\,lb\cdot ft$$

USING AN INTERPOLATION TECHNIQUE THE ONLY ROOT IN THE REGION 10 ft < y < 13 ft IS $y = 11.09\ ft$

SUMMARY

STARTING THE COORDINATE SYSTEM AT THE REACTIONS would HAVE MADE THIS PROBLEM SIGNIFICANTLY MORE DIFFICULT.