PROBLEM 4 | THE BEAM ABCDE SHOWN BELOW HAS SIMPLE SUPPORTS AT A, C, AND E; AND A HINGE (OR PIN) AT D. A LOAD OF 4 kN ACTS AT THE END OF THE BRACKET THAT EXTENDS FROM THE BEAM AT B, AND A LOAD OF 2 kN ACTS AT THE MIDPOINT OF MEMBER DE. DRAW THE SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR THIS BEAM.

GIVEN:

1) A 6m MEMBER IS PIN CONNECTED TO A 2m MEMBER
2) THE 6 m MEMBER IS SIMPLY SUPPORTED AT 0m AND 4m
3) THE 2m MEMBER IS SIMPLY SUPPORTED 2m FROM THE PIN CONNECTION
4) A 4 kN LOAD IS APPLIED TO THE END OF A BRACKET LOCATED AT 2m, WITH A HORIZONTAL DIMENSION OF -1m

ASSUMPTIONS:

1) ALL COMPONENTS START OUT STRAIGHT
2) LINEAR - ELASTIC DEFORMATION
3) SMALL DEFLECTIONS.
4) THE PIN JOINT IS FRICTIONLESS.

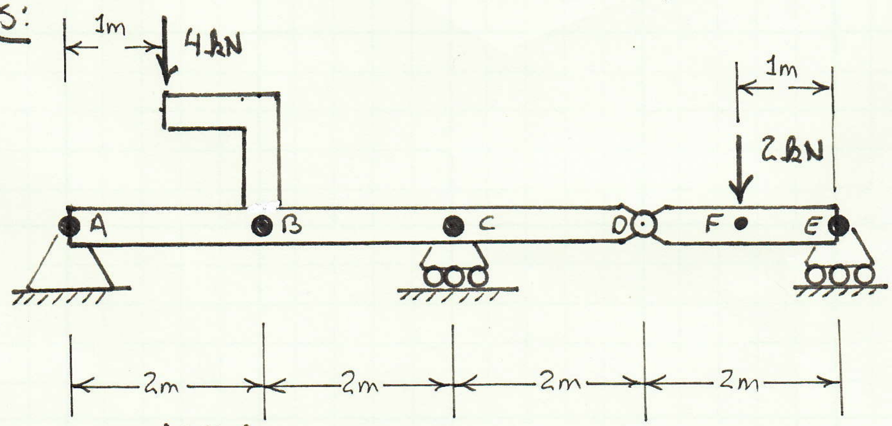
FIND:

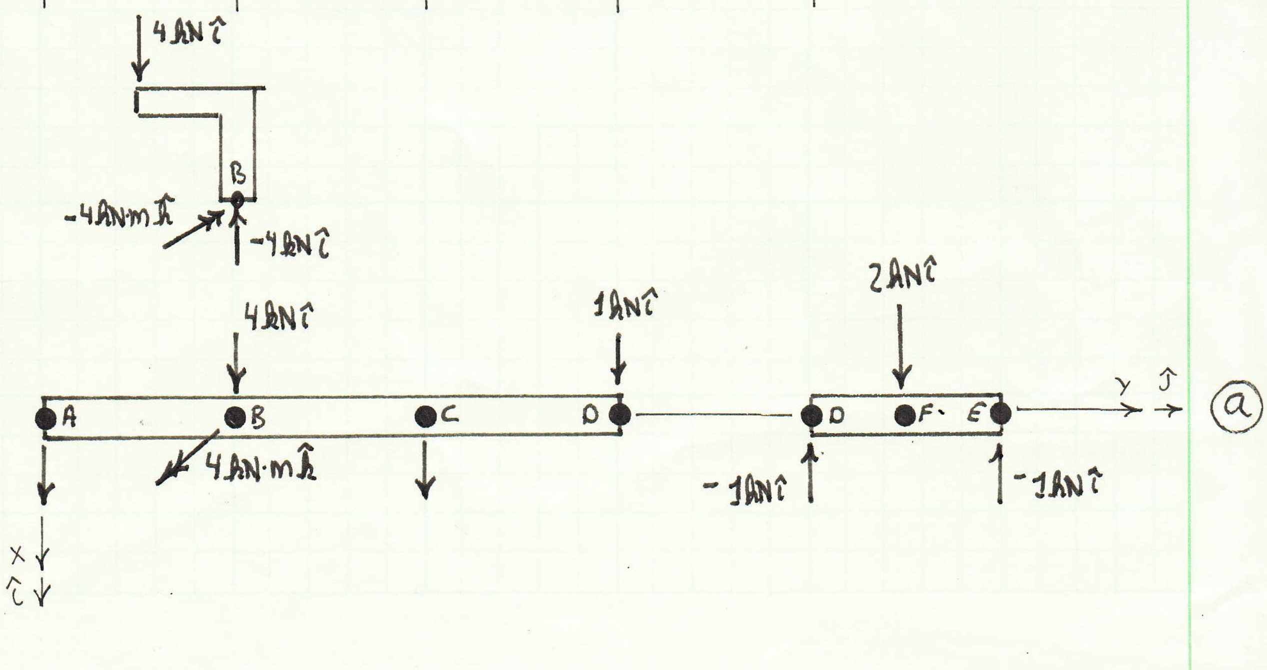
1. SHEAR FORCE DIAGRAM
2. BENDING MOMENT DIAGRAM

SUPPLEMENTAL QUESTIONS

3. ELASTIC CURVE CURVATURE
4. ELASTIC CURVE DEFLECTION

FIGURES:





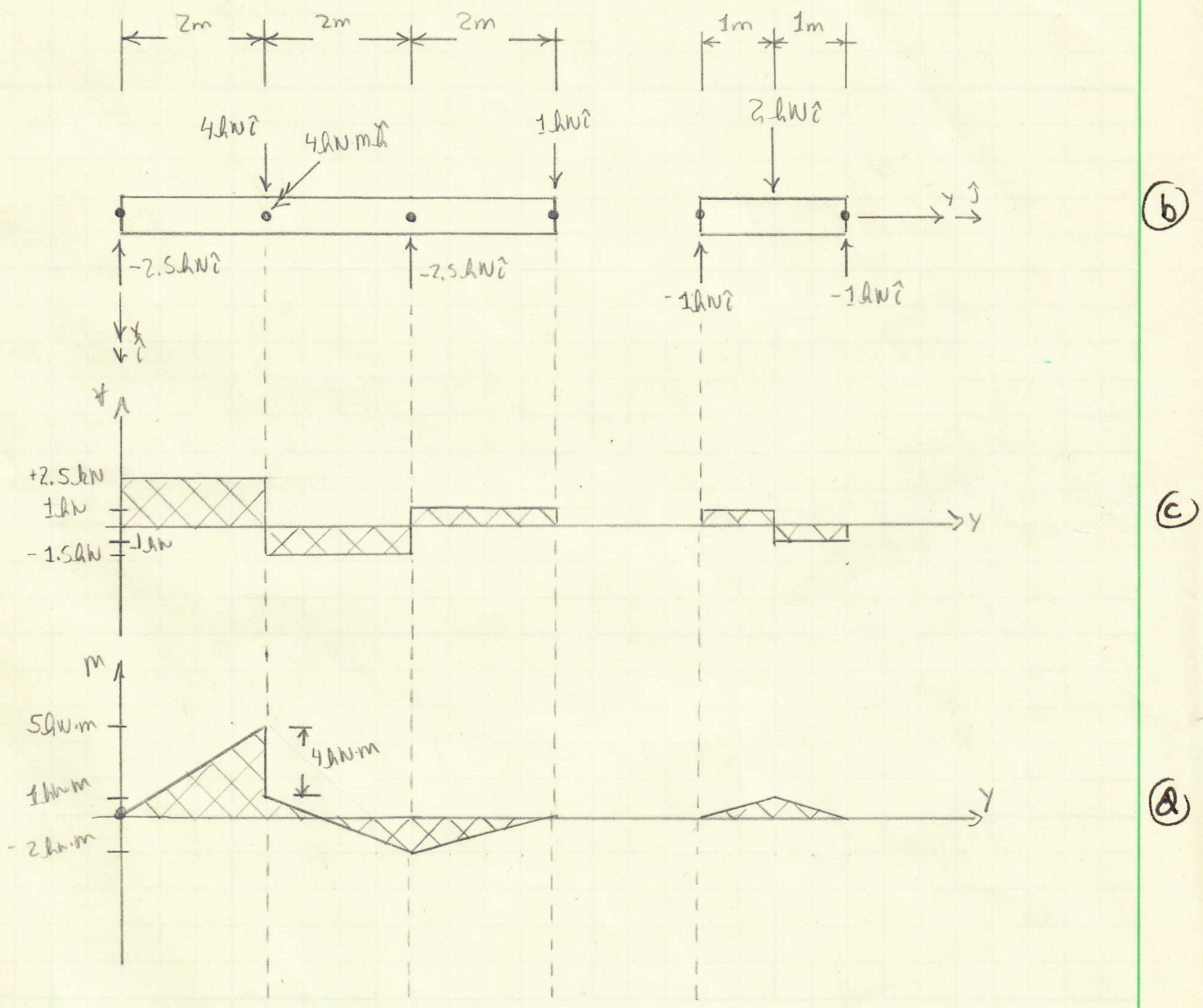
STATICS:

Starting with the solution to ABCD

$$\sum M_{z/A} = 0 = 4\,kN{\cdot}m - (2m)(4\,kN) - (4m)\cdot C_x - (6m)(1\,kN)$$

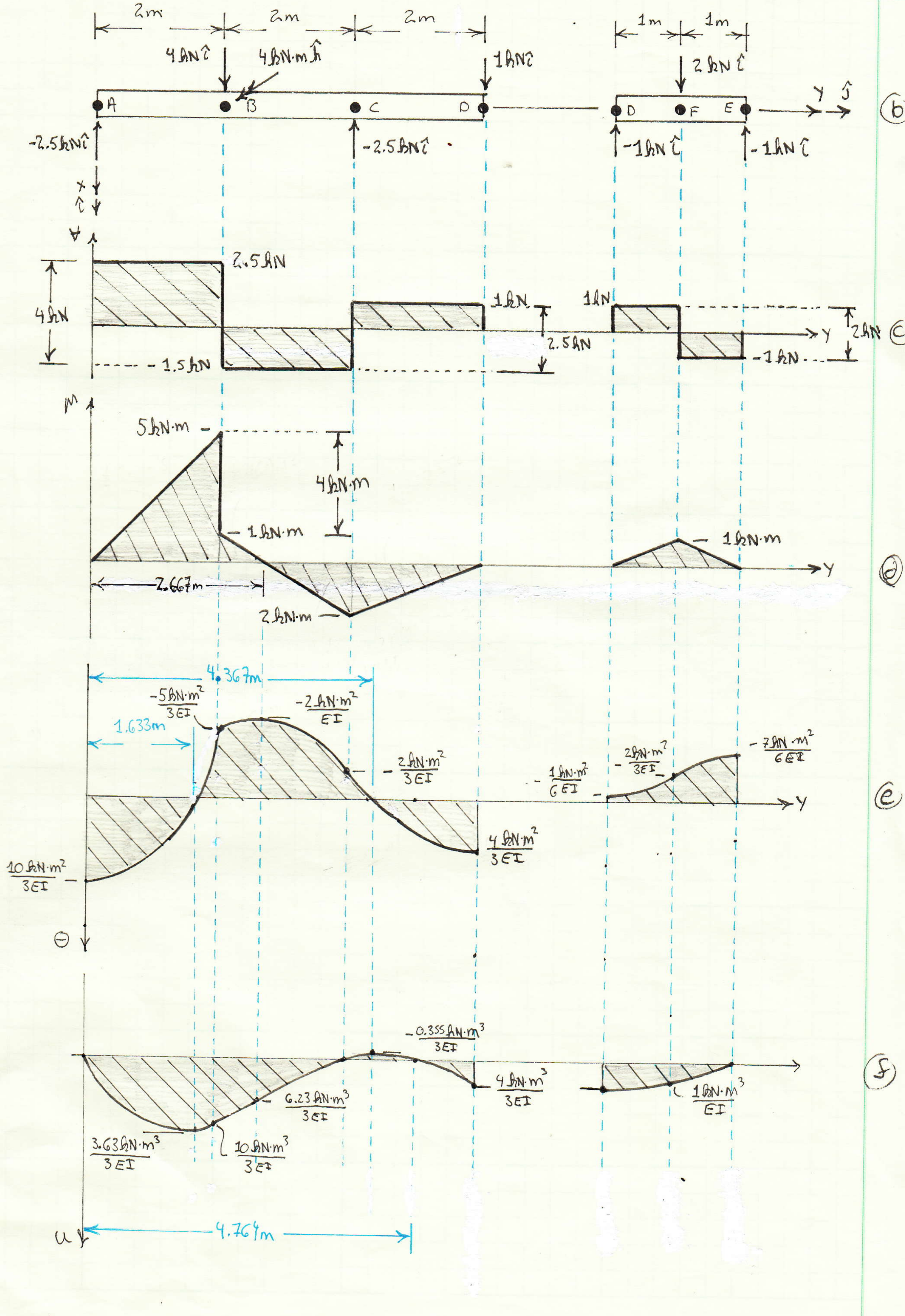
$$\Rightarrow \underline{C_x = -2.5\,kN}$$

$$\sum F_x = 0 = A_x + 4\,kN + C_x + 1\,kN \Rightarrow \underline{A_x = -2.5\,kN}$$



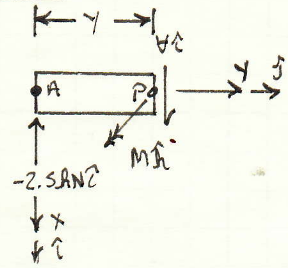
SUMMARY:

The equilibrium in this problem is trivial. The shear force and bending moment diagrams are drawn using direct integration.


2m
2m
2m
1m
1m
4 kN î
4 kN·m k̂
1 kN î
2 kN î
A
B
C
D
D
F
E
y ĵ
(b)
-2.5 kN î
-2.5 kN î
-1 kN î
-1 kN î
x î
V
2.5 kN
4 kN
1 kN
1 kN
1.5 kN
2.5 kN
y
2 kN
-1 kN
(c)
M
5 kN·m
4 kN·m
1 kN·m
1 kN·m
2.667 m
2 kN·m
y
(d)
4.367 m
1.633 m
-5 kN·m²/3EI
-2 kN·m²/EI
2 kN·m²/3EI
1 kN·m²/6EI
2 kN·m²/3EI
7 kN·m²/6EI
10 kN·m²/3EI
4 kN·m²/3EI
y
(e)
θ
-0.355 kN·m³/3EI
4 kN·m³/3EI
6.23 kN·m³/3EI
3.63 kN·m³/3EI
10 kN·m³/3EI
1 kN·m³/EI
(f)
u
4.764 m

SOLUTION USING DIRECT INTEGRATION

$0 < y < 2m$



$$\Sigma F_x = 0 = V - 2.5kN$$

$$\Rightarrow \underline{\underline{V = 2.5kN}} \quad (1)$$

$$\Sigma M_{z@p} = 0 = M - 2.5kN \cdot y$$

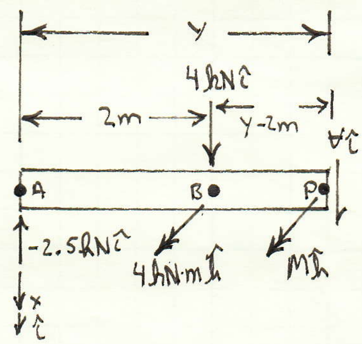
$$\Rightarrow \underline{\underline{M = 2.5kN \cdot y}} \quad (2)$$

SINCE M IS A FUNCTION OF y, THE VALUES OF M AT THE BOUNDARIES SHOULD BE CALCULATED. FROM (2)

$$M(0) = 0$$

$$M(2m) = 5kN \cdot m \quad (3)$$

$2m < y < 4m$



$$\Sigma F_x = 0 = V - 2.5kN + 4kN$$

$$\Rightarrow \underline{\underline{V = -1.5kN}} \quad (4)$$

$$\Sigma M_{z@p} = 0 = M + 4kN \cdot m + 4kN \cdot (y - 2m) - 2.5kN \cdot y$$

$$\Rightarrow M = -4kN \cdot m - 4kN \cdot y + 8kN \cdot m + 2.5kN \cdot y$$

$$= \underline{\underline{-1.5kN \cdot y + 4kN \cdot m}} \quad (5)$$

Since M IS A FUNCTION OF y, THE VALUES OF M AT THE BOUNDARIES OF THE REGION NEED TO BE CALCULATED. USING (5)

$$M(2m) = -1.5kN \cdot (2m) + 4kN \cdot m = 1kN \cdot m$$

$$M(4m) = -1.5kN \cdot (4m) + 4kN \cdot m = -2kN \cdot m \quad (6)$$

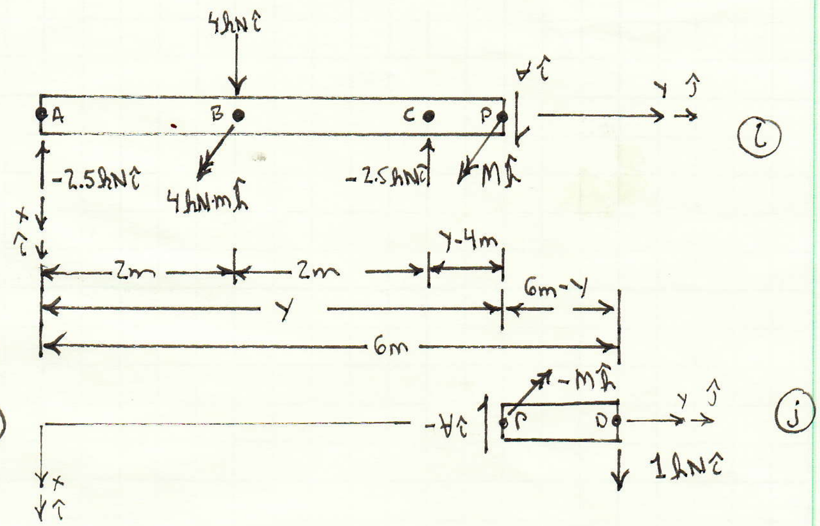
THE TWO REGIONAL BOUNDARY CONDITIONS HAVE OPPOSITE SIGN, INDICATING THAT THE MOMENT GOES TO ZERO IN THIS REGION. THIS POINT IS IMPORTANT AND MUST BE DETERMINED. FROM 5, SETTING M = 0

$$0 = -1.5kN \cdot y + 4kN \cdot m$$

$$\Rightarrow y = \frac{4kN \cdot m}{1.5kN} = 2.667m \quad (7)$$

$4m < y < 6m$

M & V WILL BE DETERMINED IN THIS REGION USING (j)



$$\Sigma F_x = 0 = -V + 1kN$$

$$\Rightarrow \underline{V = 1kN} \quad (8)$$

$$\Sigma M_{z@p} = 0 = -M - (6m - y) \cdot 1kN$$

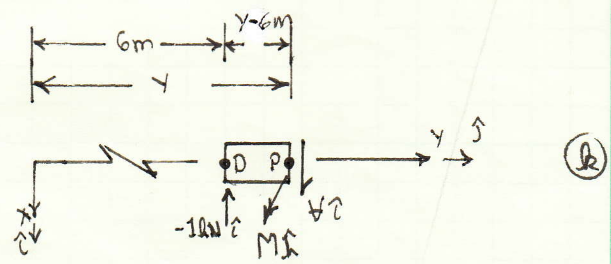
$$\Rightarrow \underline{M = 1kN \cdot y - 6kN \cdot m} \quad (9)$$

SINCE M IS A FUNCTION OF Y IN THIS REGION, THE VALUE OF M AT THE BOUNDARIES OF THIS REGION NEED TO BE CALCULATED. STARTING WITH (9)

$$M(4m) = 1kN(4m) - 6kN \cdot m = -2kN \cdot m$$
$$M(6m) = 1kN(6m) - 6kN \cdot m = 0 \quad (10)$$

NOW THE SECOND BEAM (AFTER THE PIVOT) FROM $6m < y < 8m$ CAN BE EVALUATED. THERE ARE TWO REGIONS TO THIS SEGMENT OF THE BEAM.

$6m < y < 7m$



$$\Sigma F_x = 0 = V - 1kN$$

$$\Rightarrow \underline{V = 1kN} \quad (11)$$

$$\Sigma M_{z@p} = 0 = M - 1kN(y - 6m)$$

$$\Rightarrow \underline{M = 1kN \cdot y - 6kN \cdot m} \quad (12)$$

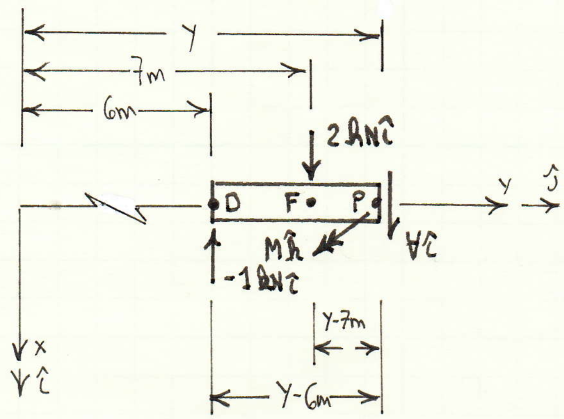
THE VALUES OF M AT THE BOUNDARIES OF THIS REGION NEEDS TO BE DETERMINED.

$$M(6m) = 1kN(6m) - 6kN \cdot m = 0$$
$$M(7m) = 1kN(7m) - 6kN \cdot m = 1kN \cdot m \quad (13)$$

7m < y < 8m

$$\Sigma F_x = 0 = \forall - 1\,kN + 2\,kN$$

$$\Rightarrow \underline{\forall = -1\,kN} \quad (14)$$



$$\Sigma M_{z@P} = 0 = M + 2\,kN(y-7m) - 1\,kN(y-6m)$$

$$\Rightarrow M = -2\,kN\cdot y + 14\,kN\cdot m + 1\,kN\cdot y - 6\,kN\cdot m$$

$$= -1\,kN\cdot y + 8\,kN\cdot m \quad (15)$$

THE VALUES OF M AT THE BOUNDARIES OF THIS REGION NEED TO BE CALCULATED, USING (15)

$$M(7m) = -1\,kN\cdot(7m) + 8\,kN\cdot m = 1\,kN\cdot m$$

$$M(8m) = -1\,kN\cdot(8m) + 8\,kN\cdot m = 0 \quad (16)$$

EXPRESSIONS FOR THE SHEAR FORCE AND BENDING MOMENT IN EACH REGION OF THE BE HAVE BEEN DETERMINED. THE BENDING MOMENT EXPRESSIONS CAN NOW BE INTEGRATED TO FORM EXPRESSIONS FOR THE SLOPE OF THE ELASTIC CURVE, Θ, AND THE DISPLACEMENT OF THE ELASTIC CURVE, u.

$$\Theta = \int -\frac{M}{EI}\,dy$$

$$u = \int \Theta \cdot dy$$

DIRECT INTEGRATION IS USED IN EACH REGION, BOUNDARY CONDITIONS ARE USED TO DETERMINE CONSTANTS, AND CONTINUITY CONDITIONS ARE USED TO TIE THE REGIONS TOGETHER.

$0<y<2m$ (REGION AB)

STARTING WITH M FROM (2)

$$\theta_{AB}(y) = \int \frac{-M}{EI} \cdot dy = \int -\frac{2.5\,kN \cdot y}{EI} dy = \int \frac{-5\,kN}{2 \cdot EI} \cdot y \cdot dy$$

$$= -\frac{5\,kN}{4EI} \cdot y^2 + C_1 \qquad (17)$$

$$u_{AB}(y) = \int \theta_{AB}(y) \cdot dy = \int \left[-\frac{5\,kN}{4 \cdot EI} \cdot y^2 + C_1 \right] dy$$

$$= -\frac{5\,kN}{12EI} \cdot y^3 + C_1 \cdot y + C_2 \qquad (18)$$

THE FIRST BOUNDARY CONDITION IS $u_{AB}(0)=0$, APPLYING THIS TO (18)

$$u_{AB}(0) = -\frac{5\,kN}{12 \cdot EI} \cdot (0)^3 + C_1(0) + C_2 = 0 \Rightarrow C_2 = 0$$

(18) CAN NOW BE REWRITTEN

$$u_{AB}(y) = -\frac{5\,kN}{12EI} \cdot y^3 + C_1 \cdot y \qquad (19)$$

THE VALUES OF (17) AND (19) AT THE END OF THIS REGION, $y=2m$, NEED TO BE CALCULATED BECAUSE THESE ARE THE CONTINUITY CONDITIONS THAT FORM THE INITIAL CONDITIONS FOR THE NEXT REGION.

$$(17) \rightarrow \theta_{AB}(2m) = -\frac{5\,kN}{4 \cdot EI} \cdot (2m)^2 + C_1 = -\frac{20\,kN \cdot m^2}{4EI} + C_1$$

$$= -\frac{5\,kN \cdot m^2}{EI} + C_1 \qquad (20)$$

$$(19) \rightarrow u_{AB}(2m) = -\frac{5\,kN}{12 \cdot EI} \cdot (2m)^3 + C_1 \cdot (2m) = -\frac{5\,kN \cdot 8m^3}{12 \cdot EI} + 2m \cdot C_1$$

$$= -\frac{10\,kN \cdot m^3}{3EI} + 2m \cdot C_1 \qquad (21)$$

$2m < y < 4m$ (REGION BC)

STARTING WITH (5)

$$\theta_{BC} = \int \frac{-M}{EI} dy = \int \left[\frac{1.5\,kN}{EI} y - \frac{4\,kN \cdot m}{EI} \right] dy = \int \left[\frac{3\,kN}{2 \cdot EI} y - \frac{4\,kN \cdot m}{EI} \right] dy$$

$$= \frac{3\,kN}{4\,EI} \cdot y^2 - \frac{4\,kN \cdot m}{EI} \cdot y + C_3 \qquad (22)$$

$$u_{BC} = \int \theta_{BC}\, dy = \int \left[\frac{3\,kN}{4EI} \cdot y^2 - \frac{4\,kN \cdot m}{EI} \cdot y + C_3 \right] dy$$

$$= \frac{3\,kN}{12 \cdot EI} \cdot y^3 - \frac{4\,kN \cdot m}{2 \cdot EI} \cdot y^2 + C_3 \cdot y + C_4$$

$$= \frac{1\,kN}{4 \cdot EI} \cdot y^3 - \frac{2\,kN \cdot m}{EI} \cdot y^2 + C_3 \cdot y + C_4 \qquad (23)$$

APPLYING THE CONTINUITY CONDITION IN (20), $\theta_{AB}(2m) = \theta_{BC}(2m)$ TO 22

$$\theta_{AB}(2m) = \theta_{BC}(2m)$$

$$-\frac{5\,kN \cdot m^2}{EI} + C_1 = \frac{3\,kN}{4EI} \cdot (2m)^2 - \frac{4\,kN \cdot m}{EI} \cdot (2m) + C_3$$

$$C_3 = -\frac{4 \cdot 5\,kN \cdot m^2}{4 \cdot EI} + C_1 - \frac{12\,kN \cdot m^2}{4\,EI} + \frac{4 \cdot 8\,kN \cdot m^2}{4 \cdot EI}$$

$$= C_1$$

SUBSTITUTING THIS RESULT INTO (22) & (23)

$$\theta_{BC} = \frac{3\,kN}{4\,EI} \cdot y^2 - \frac{4\,kN \cdot m}{EI} \cdot y + C_1 \qquad (24)$$

$$u_{BC} = \frac{1\,kN}{4 \cdot EI} \cdot y^3 - \frac{2\,kN \cdot m}{EI} \cdot y^2 + C_1 \cdot y + C_4 \qquad (25)$$

APPLYING THE SECOND CONTINUITY CONDITION IN THIS REGION, $u_{AB}(2m) = u_{BC}(2m)$, (21) AND (25)

$$u_{AB}(2m) = u_{BC}(2m)$$

$$-\frac{10\,kN\cdot m^3}{3\,EI} + \cancel{2m\cdot C_1} = \frac{1\,kN}{4\cdot EI}\cdot(2m)^3 - \frac{2\,kN\cdot m}{EI}\cdot(2m)^2 + \cancel{2m\cdot C_1} + C_4$$

$$C_4 = -\frac{10\,kN\cdot m^3}{3\,EI} - \frac{3\cdot 2\,kN\cdot m^3}{3\cdot EI} + \frac{3\cdot 8\,kN\cdot m^3}{3\cdot EI} = \frac{8\,kN\cdot m^3}{3\,EI}$$

THIS RESULT CAN NOW BE SUBSTITUTED INTO (25)

$$u_{BC} = \frac{1\,kN}{4\cdot EI}\cdot y^3 - \frac{2\,kN\cdot m}{EI}\cdot y^2 + C_1\cdot y + \frac{8\,kN\cdot m^3}{3\,EI} \qquad (26)$$

THE SECOND BOUNDARY CONDITION, $u_{BC}(4m) = 0$, CAN NOW BE APPLIED TO (26) AND C_1 IS DETERMINED.

$$u_{BC}(4m) = 0 = \frac{1\,kN}{4\cdot EI}\cdot(4m)^3 - \frac{2\,kN\cdot m}{EI}\cdot(4m)^2 + C_1\cdot(4m) + \frac{8\,kN\cdot m^3}{3\,EI}$$

$$C_1 = -\frac{64\,kN\cdot m^3}{4m\cdot 4\cdot EI} + \frac{32\,kN\cdot m^3}{4m\cdot EI} - \frac{8\,kN\cdot m^3}{4m\cdot 3\cdot EI}$$

$$= -\frac{3\cdot 4\,kN\cdot m^2}{3\cdot EI} + \frac{3\cdot 8\,kN\cdot m^2}{3\cdot EI} - \frac{2\cdot kN\cdot m^2}{3\cdot EI} = \frac{10\,kN\cdot m^2}{3\cdot EI} \qquad (27)$$

(27) CAN NOW BE SUBSTITUTED INTO (17), (19), (24), AND (26)

$0 < y < 2m$

$$(17) \rightarrow \quad \underline{\underline{\Theta_{AB} = -\frac{5\,kN}{4\cdot EI}\cdot y^2 + \frac{10\,kN\cdot m^2}{3\cdot EI}}} \qquad (28)$$

$$(19) \rightarrow \quad \underline{\underline{u_{AB} = -\frac{5\,kN}{12\,EI}\cdot y^3 + \frac{10\,kN\cdot m^2}{3\,EI}\cdot y}} \qquad (29)$$

$2m < y < 4m$

$$(24) \rightarrow \quad \underline{\underline{\Theta_{BC} = \frac{3\,kN}{4\cdot EI}\cdot y^2 - \frac{4\,kN\cdot m}{EI}\cdot y + \frac{10\,kN\cdot m^2}{3\,EI}}} \qquad (30)$$

$$(26) \rightarrow \quad \underline{\underline{u_{BC} = \frac{1\,kN}{4\cdot EI}\cdot y^3 - \frac{2\,kN\cdot m}{EI}\cdot y^2 + \frac{10\,kN\cdot m^2}{3\cdot EI}\cdot y + \frac{8\,kN\cdot m^3}{3\,EI}}} \qquad (31)$$

THE VALUES OF THE SLOPE AND DISPLACEMENT OF THE ELASTIC CURVE AT THE BOUNDARIES OF THESE TWO REGIONS CAN NOW BE COMPUTED

$0 < y < 2m$

$$(28) \rightarrow \Theta_{AB}(0) = -\frac{5\,kN}{4\cdot EI}\cdot(0)^2 + \frac{10\,kN\cdot m^3}{3EI} = \frac{10\,kN\cdot m^3}{3\cdot EI} \qquad (32)$$

$$(29) \rightarrow u_{AB}(0) = -\frac{5\,kN}{12EI}\cdot(0)^3 + \frac{10\,kN\cdot m^3}{3EI}\cdot(0) = 0 \qquad (33)$$

$$(28) \rightarrow \Theta_{AB}(2m) = -\frac{5\,kN}{4\cdot EI}(2m)^2 + \frac{10\,kN\cdot m^2}{3EI} = -\frac{5\,kNm^2}{3EI} \qquad (34)$$

$$(29) \rightarrow u_{AB}(2m) = -\frac{5\cdot kN}{12EI}\cdot(2m)^3 + \frac{10\,kN\cdot m^2}{3EI}(2m) = \frac{10\,kN\cdot m^3}{3\cdot EI} \qquad (35)$$

$2m < y < 4m$

$$(30) \rightarrow \Theta_{BC}(2m) = \frac{3\,kN}{4EI}\cdot(2m)^2 - \frac{4\,kN\cdot m}{EI}\cdot(2m) + \frac{10\,kN\cdot m^2}{3EI} = -\frac{5\,kN\cdot m^2}{3\,EI} \qquad (36)$$

$$(31) \rightarrow u_{BC}(2m) = \frac{1\,kN}{4\cdot EI}\cdot(2m)^3 - \frac{2\,kN\cdot m}{EI}\cdot(2m)^2 + \frac{10\,kN\cdot m^2}{3EI}\cdot(2m) + \frac{8\,kN\cdot m^3}{3EI}$$

$$= \frac{10\,kN\cdot m^3}{3EI} \qquad (37)$$

$$(30) \rightarrow \Theta_{BC}(4m) = \frac{3\,kN}{4EI}\cdot(4m)^2 - \frac{4\,kN\cdot m}{EI}\cdot(4m) + \frac{10\,kN\cdot m^2}{3EI} = -\frac{2\,kN\cdot m^2}{3EI} \qquad (38)$$

$$(31) \rightarrow u_{BC}(4m) = \frac{1\,kN}{4\cdot EI}(4m)^3 - \frac{2\,kN\cdot m}{EI}(4m)^2 + \frac{10\,kN\cdot m^2}{3EI}(4m) + \frac{8\,kN\cdot m^3}{3EI} = 0 \qquad (39)$$

(38) AND (39) ARE CONTINUITY CONDITIONS THAT ARE USED ~~IN~~ AS INITIAL CONDITIONS FOR THE NEXT REGION. ALSO NEEDED

THE BOUNDARY VALUES FOR THE SLOPE OF THE ELASTIC CURVE SHOW A CHANGE IN SIGN BETWEEN $0 < y < 2m$, REGION AB.

$$(32) \rightarrow \Theta_{AB}(0) = \frac{10 \, kN \cdot m^3}{3EI}$$

$$(34) \rightarrow \Theta_{AB}(2m) = \frac{5 \, kN \cdot m^3}{3EI}$$

THE LOCATION OF WHERE Θ_{AB} IS ZERO IS THE LOCATION OF WHERE U_{AB} IS A MAX/MIN. THE LOCATION OF THIS POINT IS FOUND BY SETTING (28) EQUAL TO 0 AND SOLVING FOR Y

$$(28) \rightarrow 0 = -\frac{5 \, kN}{4EI} \cdot y^2 + \frac{10 \, kN \cdot m^2}{3 \; EI}$$

$$y^2 = \frac{\overset{2}{\cancel{10}} \, kN \cdot m^2}{3 \cancel{EI}} \cdot \frac{4 \cancel{EI}}{\cancel{5} \, kN} = \frac{8}{3} m^2 \Rightarrow y = \pm \sqrt{\frac{8}{3} m^2} = \pm 1.633 m$$

SINCE THE NEGATIVE VALUE IS OUTSIDE THE DOMAIN OF THIS REGION, ONLY THE POSITIVE RESULT IS A VALID SOLUTION.

$$Y_{AB,\Theta=0} = \underline{\underline{1.633 m}} \qquad (40)$$

THE VALUE OF U_{AB} AT THIS POINT CAN NOW BE COMPUTED USING (29)

$$(29) \rightarrow U_{AB}(1.633m) = -\frac{5 \; kN}{12 \; EI} \cdot (1.633m)^3 + \frac{10 \, kN \cdot m^2}{3 \cdot EI} \cdot (1.633m)$$

$$= \underline{\underline{\frac{3.629 \, kN \cdot m^3}{EI}}} \qquad (41)$$

FROM THE MOMENT DIAGRAM (2), THE SLOPE OF THE ELASTIC CURVE Θ IS MAX/MIN IN REGION BC AT $y = 2.667m$. THE VALUE OF Θ_{BC} AT THIS LOCATION IS COMPUTED USING (30)

$$(30) \rightarrow \Theta_{BC} = \frac{3 \, kN}{4 \cdot EI} \cdot (2.667m)^2 - \frac{4 \, kN \cdot m}{EI} \cdot (2.667m) + \frac{10 \, kN \cdot m^2}{3 \, EI} = \underline{\underline{-\frac{2.00 \, kN \cdot m^2}{EI}}} \qquad (42)$$

THE FINAL SEGMENT OF THIS BEAM, REGION CD, FROM $4m < y < 6m$ CAN NOW BE EVALUATED.

$4m \leq y \leq 6m$

STARTING WITH THE EXPRESSION FOR THE MOMENT IN THIS REGION, (9)

$$\Theta_{CD} = \int \frac{-M}{EI} \cdot dy = \int \left[\frac{-1 kN \cdot y + 6 kN \cdot m}{EI} \right] dy$$

$$= -\frac{1 kN}{2EI} \cdot y^2 + \frac{6 kN \cdot m}{EI} \cdot y + C_5 \qquad (43)$$

THE CONTINUITY CONDITION (38) IS NOW USED TO EVALUATE C_5

$$\Theta_{BC}(4m) = \Theta_{CD}(4m)$$

$$-\frac{2 kN \cdot m^2}{3 \cdot EI} = -\frac{1 kN}{2EI} \cdot (4m)^2 + \frac{6 kN \cdot m}{EI} \cdot (4m) + C_5$$

$$\Rightarrow C_5 = -\frac{2 \cdot}{2 \cdot} \frac{2 kN \cdot m^2}{3 \cdot EI} + \frac{3 \cdot}{3 \cdot} \frac{1 kN \cdot 16 m^2}{2 \cdot EI} - \frac{6 \cdot}{6 \cdot} \frac{24 kN \cdot m^2}{EI} = -\frac{100 kN \cdot m^2}{6 EI} = -\frac{50 kN \cdot m^2}{3 EI}$$

SUBSTITUTING THIS RESULT INTO (43)

$$\Theta_{CD} = -\frac{1 kN}{2EI} \cdot y^2 + \frac{6 \cdot kN \cdot m}{EI} \cdot y - \frac{50 kN \cdot m^2}{3 \cdot EI} \qquad (44)$$

THE DISPLACEMENT IN THIS REGION CAN NOW BE FORMULATED

$$u_{CD} = \int \Theta_{CD} \, dy = \int \left[-\frac{1 kN}{2EI} \cdot y^2 + \frac{6 kN \cdot m}{EI} \cdot y - \frac{50 kN \cdot m^2}{3 \cdot EI} \right] dy$$

$$= -\frac{1 kN}{6EI} \cdot y^3 + \frac{3 kN \cdot m}{EI} \cdot y^2 - \frac{50 kN \cdot m^2}{3 \cdot EI} y + C_6 \qquad (45)$$

THE CONTINUITY CONDITION (39) IS NOW USED TO EVALUATE C_6

$$u_{BC}(4m) = u_{CD}(4m)$$

$$0 = -\frac{1 kN}{6EI} \cdot (4m)^3 + \frac{3 kN \cdot m}{EI} \cdot (4m)^2 - \frac{50 kN \cdot m^2}{3 \cdot EI} \cdot (4m) + C_6$$

$$C_6 = \frac{1 kN}{6EI} \cdot (4m)^3 - \frac{3 kN \cdot m}{EI} \cdot (4m)^2 + \frac{50 kN \cdot m^2}{3 \cdot EI} \cdot (4m) = \frac{88 kN \cdot m^2}{3 EI}$$

SUBSTITUTING THIS RESULT INTO (45)

$$u_{CD} = -\frac{1 kN}{6 \cdot EI} \cdot y^3 + \frac{3 kN \cdot m}{EI} \cdot y^2 - \frac{50 kN \cdot m^2}{3 \cdot EI} \cdot y + \frac{88 kN \cdot m^3}{3 EI} \qquad (46)$$

VALUES FOR θ AND u AT THE BOUNDARIES OF THIS REGION CAN NOW BE CALCULATED USING (44) AND (46)

$$(44) \rightarrow \quad \theta_{CD}(4m) = -\frac{1\,kN}{2EI}\cdot(4m)^2 + \frac{6\,kN\cdot m}{EI}\cdot(4m) - \frac{50\,kN\cdot m^2}{3EI} = -\frac{4\,kN\cdot m^2}{6\cdot EI} = -\frac{2\,kN\cdot m^2}{3\,EI} \quad (47)$$

$$(44) \rightarrow \quad \theta_{CD}(6m) = -\frac{1\,kN}{2EI}\cdot(6m)^2 + \frac{6\,kN\cdot m}{EI}\cdot(6m) - \frac{50\,kN\cdot m^2}{3\cdot EI} = \frac{8\,kN\cdot m^2}{6EI} = \frac{4\,kN\cdot m^2}{3EI} \quad (48)$$

THE CHANGE IN SIGN BETWEEN (47) & (48) INDICATES A ROOT EXISTS IN THIS REGION THAT WILL BE THE LOCATION OF A MAX/MIN u.

$$(44) \rightarrow \quad \theta_{CD}(y) = 0 = -\frac{1\,kN}{2\cdot EI}\cdot y^2 + \frac{6\,kN\cdot m}{EI}\cdot y - \frac{50\,kN\cdot m^2}{3EI}$$

$$\Rightarrow 0 = y^2 - 12m\cdot y + \frac{100}{3}\cdot m^2 = \underbrace{y^2 - 12m\cdot y + (-6m)^2}_{(y-6m)^2} - (-6m)^2 + \frac{100}{3}m^2$$

$$\Rightarrow (y-6m)^2 = \frac{36m^2\cdot 3}{3} - \frac{100}{3}m^2 = \frac{108m^2}{3} - \frac{100m^2}{3} = \frac{8}{3}m^2$$

$$\Rightarrow y = 6m \pm \sqrt{8/3\,m^2} = 7.633m,\ \underline{\underline{4.367m}} \quad (49)$$

ONLY THE 2ND VALUE FOR y ($=4.367m$) IS IN THE REGION BEING CONSIDERED. NOW THE DISPLACEMENTS OF THE BEAM AT POINTS OF INTEREST CAN BE CALCULATED

$$(46) \rightarrow \quad u_{CD}(4m) = -\frac{1\,kN}{6EI}\cdot(4m)^3 + \frac{3\,kN\cdot m}{EI}\cdot(4m)^2 - \frac{50\,kN\cdot m^2}{3EI}\cdot(4m) + \frac{88\,kN\cdot m^3}{3EI} = 0 \quad (50)$$

$$(46) \rightarrow \quad u_{CD}(6m) = -\frac{1\,kN}{6EI}(6m)^3 + \frac{3\,kN\cdot m}{EI}\cdot(6m)^2 - \frac{50\,kN\cdot m^2}{3EI}\cdot(6m) + \frac{88\,kN\cdot m^3}{3EI} =$$

$$= \frac{8\,kN\cdot m^3}{6\,EI} = \frac{4\,kN\cdot m^3}{3EI} \quad (51)$$

BECAUSE A PIN EXISTS IN THE BEAM AT $y=6m$, THE SLOPE OF THE TWO SEGMENTS OF THE BEAM AT THIS POINT ARE NOT CONSTRAINED TO BE EQUAL. THIS MEANS THAT (48) IS NOT A CONTINUITY CONDITION. THE PIN AT 6m DOES RESTRICT THE DEFLECTION AT $y=6m$, SO (51) IS A CONTINUITY CONDITION. ~~TH~~

THE MAXIMUM u IN THIS SEGMENT IS CALCULATED BY SUBSTITUTING (49) INTO (46)

$$(46) \rightarrow \quad u_{CD}(4.367m) = \frac{1\,kN}{6EI}\cdot(4.367m)^3 + \frac{6\cdot 3\,kN\cdot m}{6\cdot EI}\cdot(4.367m)^2 - \frac{2\cdot 50\,kN\cdot m^2}{2\cdot 3EI}\cdot(4.367m) + \frac{2\cdot 88\,kN\cdot m^3}{2\cdot 3EI}$$

$$= -\frac{0.7093\,kN\cdot m^3}{6\,EI} = -\frac{0.1182\,kN\cdot m^3}{EI} \quad (52)$$

THE SECOND SEGMENT OF THE BEAM, DEF, CAN NOW BE EVALUATED. THIS SEGMENT OF THE BEAM HAS TWO REGIONS, DF FROM $6m \le y \le 7m$ AND FE FROM $7m \le y \le 8m$.

REGION DF: $6m \le y \le 7m$

STARTING WITH THE EXPRESSION FOR THE MOMENT IN THIS REGION, (17)

$$\Theta_{DF} = \int -\frac{M}{EI}\cdot dy = \int -\left[\frac{1\,kN}{EI}\cdot y - \frac{6\,kN\cdot m}{EI}\right]dy = \int\left[-\frac{1\,kN}{EI}\cdot y + \frac{6\,kN\cdot m}{EI}\right]dy$$

$$= -\frac{1\,kN}{2EI}\cdot y^2 + \frac{6\,kN\cdot m}{EI}\cdot y + C_7 \qquad (53)$$

$$U_{DF} = \int \Theta_{DF}\cdot dy = \int\left[-\frac{1\,kN}{2EI}\cdot y^2 + \frac{6\,kN\cdot m}{EI}\cdot y + C_7\right]dy$$

$$= -\frac{1\,kN}{6\cdot EI}\,y^3 + \frac{6\,kN\cdot m}{2\cdot EI}\cdot y^2 + C_7\cdot y + C_8 = -\frac{1\,kN}{6\cdot EI}\cdot y^3 + \frac{3\,kN\cdot m}{EI}\cdot y^2 + C_7\cdot y + C_8$$

THE CONTINUITY CONDITION $U_{CD}(6m)$ (51) IS SET EQUAL TO $U_{DF}(6m)$ (54)

$$U_{CD}(6m) = U_{DF}(6m)$$

$$\frac{4\,kN\cdot m^3}{3\cdot EI} = -\frac{1\,kN}{6EI}\cdot(6m)^3 + \frac{3\,kN\cdot m}{EI}\cdot(6m)^2 + C_7\cdot(6m) + C_8$$

$$C_8 = \frac{2\cdot 4\,kN\cdot m^3}{2\cdot 3\cdot EI} + \frac{1\,kN}{6EI}\cdot(6m)^3 - \frac{6\cdot 3\,kN\cdot m}{6\cdot EI}\cdot(6m)^2 - C_7\cdot(6m)$$

$$= -\frac{212\,kN\cdot m^3}{3\cdot EI} + 6m\cdot C_7$$

THIS RESULT CAN NOW BE BACK SUBSTITUTED INTO (54)

$$U_{DF} = -\frac{1\,kN}{6\cdot EI}\cdot y^3 + \frac{6\,kN\cdot M}{2\cdot EF}\cdot y^2 + C_7\cdot y - \frac{212\,kN\cdot m^3}{3\cdot EF} - 6m\cdot C_7 \qquad (54)$$

THE VALUES FOR Θ_{DF} (53) AND U_{DF} (55) AT $y = 7m$ ARE CONTINUITY CONDITIONS FOR THE REGION FE, $7m \le y \le 8m$.

$$(53)\rightarrow \quad \Theta_{DF}(7m) = -\frac{1\,kN}{2\cdot EI}\cdot(7m)^2 + \frac{2\cdot 6\,kN\cdot m}{2\cdot EI}\cdot(7m) + C_7 = \frac{35\,kN\cdot m^2}{2\cdot EF} + C_7 \qquad (55)$$

$$(54)\rightarrow \quad U_{DF}(7m) = -\frac{1\,kN}{6EI}\cdot(7m)^3 + \frac{3\cdot 6\,kN\cdot m}{3\cdot 2\cdot EF}\cdot(7m)^2 + C_7\cdot(7m) - \frac{2\cdot 212\,kN\cdot m^3}{2\cdot 3EF} - 6m\cdot C_7$$

$$= \frac{115\,kN\cdot m^3}{6\cdot EI} + 1m\cdot C_7 \qquad (56)$$

REGION FE: $7m \le y \le 8m$

STARTING WITH THE EXPRESSION FOR M IN THIS REGION

$$\theta_{FE} = \int \frac{-M}{EI} \cdot dy = \int -\left[-\frac{1 kN}{EI} \cdot y + \frac{8 kN \cdot m}{EI}\right] dy = \int \left[\frac{1 kN}{EI} \cdot y - \frac{8 kN \cdot m}{EI}\right] dy$$

$$= \frac{1 kN}{2 \cdot EI} \cdot y^2 - \frac{8 kN \cdot m}{EI} \cdot y + C_9 \qquad (57)$$

USING (55) AND THE CONTINUITY CONDITION $\theta_{DF}(7m) = \theta_{FE}(7m)$

$$\frac{35 kN \cdot m^2}{2 \cdot EI} + C_7 = \frac{1 kN}{2 \cdot EI} \cdot (7m)^2 - \frac{8 kN \cdot m}{EI} \cdot 7m + C_9$$

$$\Rightarrow C_9 = \frac{35 kN \cdot m^2}{2 \cdot EI} + C_7 - \frac{49 kN \cdot m^2}{2EI} + \frac{56 kN m^2}{EI} = \frac{49 kN \cdot m^2}{EI} + C_7 \qquad (58)$$

(58) INTO (57)

$$\theta_{FE} = \frac{1 kN}{2 \cdot EI} \cdot y^2 - \frac{8 kN \cdot m}{EI} \cdot y + \frac{49 kN \cdot m^2}{EI} + C_7 \qquad (59)$$

$$u_{FE} = \int \theta_{FE} \cdot dy = \int \left[\frac{1 kN}{2EI} \cdot y^2 - \frac{8 kN \cdot m}{EI} \cdot y + \frac{49 kN \cdot m^2}{EI} + C_7\right] dy$$

$$= \frac{1 kN}{6 \cdot EI} \cdot y^3 - \frac{4 kN \cdot m}{EI} \cdot y^2 + \frac{49 kN \cdot m^2}{EI} y + C_7 \cdot y + C_{10} \qquad (60)$$

USING (56) AND THE CONTINUITY CONDITION $u_{DF}(7m) = u_{FE}(7m)$

$$\frac{115 kN \cdot m^3}{6 EI} + C_7 = \frac{1 kN}{6 \cdot EI} \cdot (7m)^3 - \frac{4 kN \cdot m}{EI} \cdot (7m)^2 + \frac{49 kN \cdot m^2}{EI} \cdot (7m) + C_7 \cdot 7m + C_{10}$$

$$C_{10} = \frac{115 kN \cdot m^3}{6 \cdot EI} + C_7 - \frac{1 kN}{6EI} \cdot (7m)^3 + \frac{4 kN \cdot m}{EI} (7m)^2 - \frac{49 kN \cdot m^2}{EI} \cdot 7m - 7m \cdot C_7$$

$$= -\frac{185 kN \cdot m^3}{EI} - 6m \cdot C_7 \qquad (61)$$

BACK SUBSTITUTING (61) INTO (60)

$$u_{FE} = \frac{1 kN}{6 \cdot EI} \cdot y^3 - \frac{4 kN \cdot m}{EI} \cdot y^2 + \frac{49 kN \cdot m^2}{EI} \cdot y - \frac{185 kN \cdot m^3}{EI} + C_7 \cdot y - 6m \cdot C_7 \qquad (62)$$

$$= \frac{1 kN}{6 \cdot EI} \cdot y^3 - \frac{4 kN \cdot m}{EI} \cdot y^2 + \frac{49 kN \cdot m^2}{EI} \cdot y - \frac{185 kN \cdot m^3}{EI} + C_7 (y - 6m)$$

THE FINAL BOUNDARY CONDITION CAN NOW BE APPLIED TO (62) AND AN EXPRESSION FOR C_7 DETERMINED

$$(62) \rightarrow \quad u_{FE}(8m) = 0 = \frac{1\,kN}{6EI}\cdot(8m)^3 - \frac{4\,kN\cdot m}{EI}\cdot(8m)^2 + \frac{49\,kN\cdot m^2}{EI}\cdot(8m) - \frac{185\,kN\cdot m^3}{EI} + C_7\cdot 2m$$

$$\Rightarrow \quad 2m\cdot C_7 = -\frac{1\,kN\cdot(8m)^3}{6EI} + \frac{4\,kN\cdot m\cdot(8m)^2}{EI} - \frac{49\,kN\cdot m^2\cdot(8m)}{EI} + \frac{185\,kN\cdot m^3}{EI} = -\frac{218\,kN\cdot m^3}{6EI}$$

$$\Rightarrow \quad C_7 = -\frac{109\,kN\cdot m^2}{6\cdot EI} \qquad (63)$$

(63) IS BACK SUBSTITUTED INTO (53), (54), (59), AND (62) SO FINAL EXPRESSIONS FOR θ AND u IN REGIONS DF AND FE CAN BE WRITTEN.

REGION DF: $6m < y \leq 7m$

$$(53) \rightarrow \quad \theta_{DF} = -\frac{1\,kN}{2EI}\cdot y^2 + \frac{6\,kN\cdot m}{EI}\cdot y - \frac{109\,kN\cdot m^2}{6EI} \qquad (64)$$

$$(54) \rightarrow \quad u_{DF} = -\frac{1\,kN}{6\cdot EI}\cdot y^3 + \frac{3\,kN\cdot m}{EI}\cdot y^2 - \frac{109\,kN\cdot m^2}{6EI}(y-6m) - \frac{212\,kN\cdot m^3}{3EI}$$

$$= -\frac{1\,kN}{6EI}\cdot y^3 + \frac{3\,kN\cdot m}{EI}\cdot y^2 - \frac{109\,kN\cdot m^2}{6\cdot EI}\cdot y + \frac{115\,kN\cdot m^3}{3EI} \qquad (65)$$

REGION FE: $7m \leq y \leq 8m$

$$(59) \rightarrow \quad \theta_{FE} = \frac{1\,kN}{2\cdot EI}\cdot y^2 - \frac{8\,kN\cdot m}{EI}\cdot y + \frac{49\,kN\cdot m^2}{EI} - \frac{109\,kN\cdot m^2}{6EI}$$

$$= \frac{1\,kN}{2\cdot EI}\cdot y^2 - \frac{8\,kN\cdot m}{EI}\cdot y + \frac{185\,kN\cdot m^2}{6EI} \qquad (66)$$

$$(62) \rightarrow \quad u_{FE} = \frac{1\,kN}{6\cdot EI}\cdot y^3 - \frac{4\,kN\cdot m}{EI}\cdot y^2 + \frac{49\,kN\cdot m^2}{EI}\cdot y - \frac{185\,kN\cdot m^3}{EI} - \frac{109\,kN\cdot m^2}{6\cdot EI}\cdot(y-6m)$$

$$= \frac{1\,kN}{6\cdot EI}\cdot y^3 - \frac{4\,kN\cdot m}{EI}\cdot y^2 + \frac{185\,kN\cdot m^2}{6EI}\cdot y - \frac{76\,kN\cdot m^3}{EI} \qquad (67)$$

THE VALUES FOR (64) — (67) AT THE ENDS AND CRITICAL POINTS IN THESE REGIONS NOW CAN BE CALCULATED.

REGION DF: $6m \le y \le 7m$

(64) → $$\theta_{DF}(6m) = -\frac{1\,kN}{2EI}\cdot(6m)^2 + \frac{6\,kN\cdot m}{EI}\cdot(6m) - \frac{109\,kN\cdot m^2}{6EI} = -\frac{1\,kN\cdot m^2}{6EI} \quad (68)$$

(65) → $$u_{DF}(6m) = -\frac{1\,kN}{6EI}\cdot(6m)^3 + \frac{3\,kN\cdot m}{EI}\cdot(6m)^2 - \frac{109\,kN\cdot m^2}{6EI}(6m) + \frac{115\,kN\cdot m^3}{3EI} = \frac{4\,kN\cdot m^3}{3EI} \quad (69)$$

(64) → $$\theta_{DF}(7m) = -\frac{1\,kN}{2EI}\cdot(7m)^2 + \frac{6\,kN\cdot m}{EI}\cdot(7m) - \frac{109\,kN\cdot m^2}{6EI} = -\frac{2\,kN\cdot m^2}{3EI} \quad (70)$$

(65) → $$u_{DF}(7m) = -\frac{1\,kN}{6EI}\cdot(7m)^3 + \frac{3\,kN\cdot m}{EI}\cdot(7m)^2 - \frac{109\,kN\cdot m^2}{6EI}\cdot(7m) + \frac{115\,kN\cdot m^3}{3EI} = \frac{1\,kN\cdot m^3}{EI} \quad (71)$$

REGION FE: $7m \le y \le 8m$

(66) → $$\theta_{FE}(7m) = \frac{1\,kN}{2\cdot EI}\cdot(7m)^2 - \frac{8\,kN\cdot m}{EI}\cdot(7m) + \frac{185\,kN\cdot m^2}{6EI} = -\frac{2\,kN\cdot m^2}{3EI} \quad (72)$$

(67) → $$u_{FE}(7m) = \frac{1\,kN}{6\cdot EI}\cdot(7m)^3 - \frac{4\,kN\cdot m}{EI}\cdot(7m)^2 + \frac{185\,kN\cdot m^2}{6EI}\cdot(7m) - \frac{76\,kN\cdot m^3}{EI} = \frac{1\,kN\cdot m^3}{EI} \quad (73)$$

(66) → $$\theta_{FE}(8m) = \frac{1\,kN}{2\cdot EI}\cdot(8m)^2 - \frac{8\,kN\cdot m}{EI}\cdot(8m) + \frac{185\,kN\cdot m^2}{6EI} = -\frac{7\,kN\cdot m^2}{6\cdot EI} \quad (74)$$

(67) → $$u_{FE}(8m) = \frac{1\,kN}{6EI}\cdot(8m)^3 - \frac{4\,kN\cdot m}{EI}\cdot(8m)^2 + \frac{185\,kN\cdot m^2}{6EI}\cdot(8m) - \frac{76\,kN\cdot m^3}{EI} = 0 \quad (75)$$

THE VALUES IN (68) — (75) ARE SUMMARIZED ON FIGURES (e) AND (f).

DIRECT INTEGRATION SUMMARY

THE DIRECT INTEGRATION APPROACH TO THE DETERMINATION OF θ AND u FOR THIS BEAM REQUIRES THE USE OF BOUNDARY CONDITIONS AND CONTINUITY CONDITION TO SOLVE FOR ALL THE CONSTANTS CREATED. THE MOST INTERESTING CONTINUITY CONDITION IS THE PIN JOINT BECAUSE IT DOES CREATE A CONTINUITY CONDITION FOR THE DISPLACEMENT, BUT NOT FOR THE CURVATURE BECAUSE THE PIN DOES NOT SUPPORT A MOMENT TRANSFER.

SINGULARITY FUNCTION SOLUTION

STARTING WITH BE BEAM $0<y<6m$ IN FIGURE ⓑ

$$q(y) = -2.5kN\langle y-0\rangle_{-1} + 4kN\langle y-2m\rangle_{-1} + 4kN\cdot m\langle y-2m\rangle_{-2} - 2.5kN\langle y-4m\rangle_{-1} + 1kN\langle y-6m\rangle_{-1} \qquad (76)$$

$$V = \int -q(y)\,dy = 2.5kN\langle y-0\rangle^0 - 4kN\langle y-2m\rangle^0 - 4kN\cdot m\langle y-2m\rangle_{-1} + 2.5kN\langle y-4m\rangle^0 - 1kN\langle y-6m\rangle^0 \qquad (77)$$

$$M = \int V\,dy = 2.5kN\cdot\langle y-0\rangle^1 - 4kN\langle y-2m\rangle^1 - 4kN\cdot m\langle y-2m\rangle^0 + 2.5kN\langle y-4m\rangle^1 - 1kN\langle y-6m\rangle^1 \qquad (78)$$

$$\theta = \int -\frac{M}{EI}\,dy = -\frac{2.5kN}{2EI}\langle y-0\rangle^2 + \frac{2kN}{EI}\langle y-2m\rangle^2 + \frac{4kN\cdot m}{EI}\langle y-2m\rangle^1 - \frac{2.5kN}{2EI}\langle y-4m\rangle^2 + \frac{1kN}{2EI}\langle y-6m\rangle^2 + \cancelto{\frac{10kN\cdot m^2}{3EI}}{C_1} \qquad (79)$$

$$u = \int \theta\,dy = \frac{-2.5kN}{6EI}\langle y-0\rangle^3 + \frac{2kN}{3EI}\langle y-2m\rangle^3 + \frac{2kN\cdot m}{EI}\langle y-2m\rangle^2 - \frac{2.5kN}{6EI}\langle y-4m\rangle^3 + \frac{1kN}{6EI}\langle y-6m\rangle^3 + \cancelto{\frac{10kN\cdot m^2}{3EI}}{C_1}\,y + \cancelto{0}{C_2} \qquad (80)$$

Using THE BOUNDARY CONDITION $u(0m)=0$

$$u(0) = 0 = \frac{-2.5kN}{6EI}(0)^3 + C_1\cdot(0) + C_2 \Rightarrow \boxed{C_2 = 0}$$

$$(80) \rightarrow \quad u = -\frac{2.5kN}{6EI}\langle y-0\rangle^3 + \frac{2kN}{3EI}\langle y-2m\rangle^3 + \frac{2kN\cdot m}{EI}\langle y-2m\rangle^2 - \frac{2.5kN}{6EI}\langle y-4m\rangle^3 + \frac{1kN}{6EI}\langle y-6m\rangle^3 + C_1 y$$

Using the Second Boundary Condition $u(4m)=0$

$$u(4m) = 0 = -\frac{2.5kN}{6EI}(4m)^3 + \frac{2kN}{3EI}(2m)^3 + \frac{2kN\cdot m}{EI}(2m)^2 - \frac{2.5kN}{6EI}(0)^3 + C_1(4m)$$

$$= -\frac{80kN\cdot m^3}{3EI} + \frac{16kN}{3EI} + \frac{8kN\cdot m^3}{EI} + C_1\cdot(4m)$$

$$\Rightarrow C_1 = \frac{40kN\cdot m^3}{4m\cdot 3EI} = \boxed{\frac{10kN\cdot m^2}{3EI} = C_1} \qquad (81)$$

SUBSTITUTING (81) INTO (79) AND (80)

$$(79) \rightarrow \quad \theta = -\frac{2.5\,kN}{2\cdot EI}\langle y-0\rangle^2 + \frac{2\,kN}{EI}\langle y-2m\rangle^2 + \frac{4\,kN\cdot m}{EI}\cdot\langle y-2m\rangle^1 - \frac{2.5\,kN}{2\cdot EI}\cdot\langle y-4m\rangle^2 + \frac{1\,kN}{2EI}\cdot\langle y-6m\rangle^2 + \frac{10\,kN\cdot m^2}{3\cdot EI} \qquad (82)$$

$$(80) \rightarrow \quad u = -\frac{2.5\,kN}{6EI}\cdot\langle y-0\rangle^3 + \frac{2\,kN}{3\cdot EI}\langle y-2m\rangle^3 + \frac{2\,kN\cdot m}{EI}\cdot\langle y-2m\rangle^2 - \frac{2.5\,kN}{6EI}\cdot\langle y-4m\rangle^3 + \frac{1\,kN}{6EI}\langle y-6m\rangle^3 + \frac{10\,kN\cdot m^2}{3\cdot EI}\cdot y \qquad (83)$$

THE CRITICAL VALUES OF V, M, θ, AND u IN EACH SECTION OF ABCD CAN NOW BE DETERMINED.

REGION AB: $0 \leq y \leq 2m$

$$(77) \rightarrow \quad V(0) = 2.5\,kN\cdot(0)^0 = 0\,kN,\ 2.5\,kN \qquad (84)$$

$$(77) \rightarrow \quad V(2m) = 2.5\,kN\cdot(2m)^0 - 4\,kN\cdot(0)^0 = 2.5\,kN,\ -1.5\,kN \qquad (85)$$

$$(78) \rightarrow \quad M(0) = 2.5\,kN\cdot(0) = 0 \qquad (86)$$

$$(78) \rightarrow \quad M(2m) = 2.5\,kN\cdot(2m) - 4\,kN\cdot(0m) - 4\,kN\cdot m\cdot(0)^0 = 5.0\,kN\cdot m,\ 1.0\,kN\cdot m \qquad (87)$$

$$(79) \rightarrow \quad \theta(0) = -\frac{2.5\,kN}{2\cdot EI}\cdot(0)^2 + \frac{10\,kN\cdot m^2}{3\cdot EI} = \underline{\frac{10\,kN\cdot m^2}{3\cdot EI}} \qquad (88)$$

$$(79) \rightarrow \quad \theta(2m) = -\frac{2.5\,kN}{2\cdot EI}\cdot(2m)^2 + \frac{2\,kN}{EI}\cdot(0)^2 + \frac{4\,kN\cdot m}{EI}\cdot(0) + \frac{10\,kN\cdot m^2}{3EI} = \underline{-\frac{5\,kN\cdot m^2}{3\cdot EI}} \qquad (89)$$

THE SIGN OF θ CHANGES IN THIS REGION WHICH INDICATES THAT THE A MAX/MIN VALUE OF u EXISTS IN THIS REGION. THE ROOTS OF $\theta(y)$ IN THIS REGION NEED TO BE FOUND. STARTING WITH (82), SETTING IT EQUAL TO ZERO, AND CONSIDERING $0 < y < 2m$ (NOTE THE "=" IS NOT PRESENT)

$$(82) \rightarrow \quad 0 = -\frac{2.5\,kN}{2\cdot EI}\cdot y^2 + \frac{10\,kN\cdot m^2}{3EI} \Rightarrow y^2 = \frac{10\,kN\cdot m^2}{3\cancel{EI}}\cdot\frac{2\cancel{EI}}{2.5\,kN} = \frac{20}{7.5}m^2$$

$$\Rightarrow y = \pm\sqrt{\frac{20m^2}{7.5}} = \pm 1.633m \Rightarrow y = 1.633m \qquad (90)$$

THE NEGATIVE ROOT IS OUTSIDE THE DOMAIN OF THE REGION.

80 $\rightarrow$ $$u(0) = -\frac{2.5\,kN}{6\cdot EI}\cdot(0)^3 + \frac{10\,kN\cdot m^2}{3\cdot EI}\cdot(0) = 0$$ 91

80 $\rightarrow$ $$u(2m) = -\frac{2.5\,kN}{6EI}(2m)^3 + \frac{2\,kN}{3EI}\cdot(0)^3 + \frac{2\,kN\cdot m}{EI}(0)^2 + \frac{10\,kN\cdot m^2}{3EI}\cdot(2m)$$

$$= \frac{10\,kN\cdot m^2}{3EI}$$ 92

80 $\rightarrow$ $$u(1.633m) = -\frac{2.5\,kN}{6\cdot EI}\cdot(1.633m)^3 + \frac{10\,kN\cdot m^2}{3\,EI}\cdot(1.633m) =$$

$$= \frac{3.629\,kN\cdot m^3}{EI}$$ 93

MAXIMUM VALUE OF u IN THIS REGION

REGION BC: $2m \leq y \leq 4m$

85 $\rightarrow$ $V(2m) = 2.5\,kN,\ -1.5\,kN$

77 $\rightarrow$ $$V(4m) = 2.5\,kN\cdot(4m)^0 - 4\,kN\cdot(2m)^0 - 4\,kN\cdot \cancel{m}\langle 2m\rangle_{-1} + 2.5\,kN\,(0)^0$$

$$= -1.5\,kN\ ,\ \underline{\underline{1\,kN}}$$ 94

87 $\rightarrow$ $M(2m) = 5.0\,kN\cdot m,\ \ 1.0\,kN\cdot m$

78 $\rightarrow$ $$M(4m) = 2.5\,kN\cdot(4m) - 4\,kN\cdot(2m) - 4\,kN\cdot m\cdot(2m)^0 + 2.5\,kN\cdot(0)^1$$

$$= \underline{\underline{-2.0\,kN\cdot m}}$$ 95

THE SIGN OF M CHANGES IN THIS REGION INDICATING A MAX/MIN θ IN THIS REGION. THE ROOTS OF M IN THIS REGION NEED TO BE DETERMINED SO THE MAX/MIN VALUE OF θ CAN BE DETERMINED. STARTING WITH 78, SETTING IT EQUAL TO ZERO, AND CONSIDERING THE REGION $2m < y < 4m$ (NOT THE "=" IS ABSENT).

$$M(y) = 0 = 2.5\,kN\cdot y - 4\,kN\cdot y + 8\,kN\cdot m - 4\,kN\cdot m$$

$$= -1.5\,kN\cdot y + 4\,kN\cdot m$$

$$\Rightarrow\ y = \frac{4\,kN\cdot m}{1.5\,N} = \frac{8}{3}\,m$$ 96

CONTINUING BY CALCULATING THE CRITICAL VALUES OF θ IN THIS REGION

(89) $\rightarrow$ $\Theta(2m) = -\dfrac{5\,kN\cdot m^2}{3\cdot EI}$

(82) $\rightarrow$ $\Theta(4m) = -\dfrac{2.5\,kN}{2EI}\cdot(4m)^2 + \dfrac{2\,kN}{EI}\cdot(2m)^2 + \dfrac{4\,kN\cdot m}{EI}\cdot(2m)$

$-\dfrac{2.5\,kN}{2EI}\cdot(0)^2 + \dfrac{10\,kN\cdot m^2}{3\cdot EI} = -\dfrac{4\,kN\cdot m^2}{6EI}$

$= -\dfrac{2\,kN\cdot m^2}{3\,EI}$ (97)

(82) $\rightarrow$ $\Theta(\tfrac{8}{3}m) = -\dfrac{2.5\,kN}{2\cdot EI}\cdot\left(\tfrac{8}{3}m\right)^2 + \dfrac{2\,kN}{EI}\left(\tfrac{8}{3}m - 2m\right)^2 + \dfrac{4\,kN\cdot m}{EI}\cdot\left(\tfrac{8}{3}m - 2m\right)^1$

$+\dfrac{10\,kN\cdot m^2}{3\cdot EI}$

$= -\dfrac{2.5\,kN}{\cancel{2}\cdot EI}\cdot\dfrac{\overset{32}{\cancel{64}}m^2}{9} + \dfrac{2\,kN}{EI}\cdot\dfrac{4m^2}{9} + \dfrac{4\,kN\cdot m}{EI}\cdot\dfrac{2m}{3}\cdot\dfrac{3}{3} + \dfrac{10\,kN\cdot m^2}{3\cdot EI}$

$= -\dfrac{18\,kN\cdot m^2}{9EI} = -\dfrac{2.0\,kN\cdot m^2}{EI}$ (98)

(92) $\rightarrow$ $u(2m) = \dfrac{10\,kN\cdot m^2}{3\cdot EI}$

(83) $\rightarrow$ $u(4m) = -\dfrac{2.5\,kN}{6\cdot EI}(4m)^3 + \dfrac{2\,kN}{3\cdot EI}(2m)^3 + \dfrac{2\,kN\cdot m}{EI}\cdot(2m)^2 - \dfrac{2.5\,kN}{6EI}(0)^3 + \dfrac{10\,kN\cdot m^2}{3EI}\cdot(4m)$

$= 0$ (99)

THE FINAL SECTION OF MEMBER ABCD CAN NOW BE CONSIDERED

<u>SECTION CD: $4m \leq y \leq 6m$</u>

(94) $\rightarrow$ $V(4m) = -1.5\,kN,\ 1\,kN$

(77) $\rightarrow$ $V(6m) = 2.5\,kN\,(6m)^0 - 4\,kN\,(4m)^0 - 4\,kN\cdot m\,\cancel{\langle 4m\rangle_{-1}} + 2.5\,kN\,(2m)^0 - 1\,kN\,(0)^0$

$= 1\,kN,\ 0\,kN$ (100)

(95) → $M(4m) = -2.0\,kN\cdot m$

(78) → $M(6m) = 2.5\,kN\cdot(6m) - 4\,kN\cdot(4m) - 4\,kN\cdot m(4m)^0 + 2.5\,kN(2m) - 1\,kN(0)$

$= 0\,kN\cdot m$ (101)

(97) → $\theta(4m) = -\dfrac{2\,kN\cdot m^2}{3\cdot EI}$

(82) → $\theta(6m) = -\dfrac{3\cdot 2.5\,kN}{3\cdot 2\cdot EI}\cdot(6m)^2 + \dfrac{6\cdot 2\,kN}{6\cdot EI}\cdot(4m)^2 + \dfrac{6\cdot 4\,kN\cdot m}{6\cdot EI}\cdot(4m)$
$-\dfrac{3\cdot 2.5\,kN}{3\cdot 2\cdot EI}(2m)^2 + \dfrac{1\,kN}{2EI}\cdot(0)^2 + \dfrac{2\cdot 10}{2\cdot 3}\dfrac{kN\cdot m^2}{EI}$

$= \dfrac{8\,kN\cdot m^2}{6\,EI} = \dfrac{4}{3}\dfrac{kN\cdot m^2}{EI}$ (102)

THE CHANGE IN SIGN BETWEEN (97) & (102) INDICATES THAT A ROOT EXISTS IN THIS REGION THAT LOCATES A MIN/MAX VALUE OF U. FOR $4m<y<6m$

(82) → $\theta(y) = 0 = -\dfrac{2.5\,kN}{2EI}\cdot y^2 + \dfrac{2\,kN}{EI}\cdot(y-2m)^2 + \dfrac{4\,kN\cdot m}{EI}(y-2m)$
$-\dfrac{2.5\,kN}{2\cdot EI}(y-4m)^2 + \dfrac{10\,kN\cdot m^2}{3\,EI}$

$0 = -\frac{2.5}{2}\cdot y^2 + 2\cdot(y^2 - 4m\cdot y + 4m) + 4m\cdot y - 8m^2 - \frac{2.5}{2}\cdot(y^2 - 8m\cdot y + 16m^2) + \frac{10}{3}m^2$

$0 = y^2\left(-\frac{2.5}{2} + 2 - \frac{2.5}{2}\right) + y\left(-2\cdot 4m + 4m + \frac{2.5}{2}\cdot 8m\right) + \left(2\cdot 4m - 8m^2 - \frac{2.5}{2}\cdot 16m^2 + \frac{10}{3}m^2\right)$

$0 = -\frac{1}{2}y^2 + 6m\cdot y - \frac{50}{3}m^2$

$\Rightarrow 0 = y^2 - 12m\cdot y + \frac{100}{3}m^2 = \underbrace{y^2 - 12m\cdot y + (-6m)^2}_{(y-6m)^2} - (-6m)^2 + \frac{100}{3}m^2$

$0 = (y-6m)^2 - 36m^2 + \frac{100}{3}m^2 = (y-6m)^2 - \frac{108m^2}{3} + \frac{100m^2}{3} = (y-6m)^2 - \frac{8}{3}m^2$

$\Rightarrow (y-6m)^2 = \frac{8}{3}m^2 \Rightarrow y = 6m \pm \sqrt{\frac{8}{3}m^2} = 7.633m,\ 4.367m$ (103)

ONLY THE SECOND VALUE ($y = 4.367$) IS IN THE REGION UNDER CONSIDERATION.

CALCULATING THE DISPLACEMENTS u AT POINTS OF INTEREST

(99) → $u(4m) = 0$

(83) → $u(6m) = -\dfrac{2.5\,kN}{6EI}\cdot(6m)^3 + \dfrac{2\,kN}{3\,EI}\cdot(4m)^3 + \dfrac{2\,kN\cdot m}{EI}\cdot(4m)^2 - \dfrac{2.5\,kN}{6EI}(2m)^3$
$+\dfrac{1\,kN}{6EI}(0m)^3 + \dfrac{10\,kN\cdot m^3}{3\,EI}\cdot 6m = \dfrac{8}{6}\dfrac{kN\cdot m^3}{EI} = \dfrac{4\,kN\cdot m^3}{3\,EI}$ (104)

SUBSTITUTING THE ROOT IN THIS REGION, (103), INTO (83) TO DETERMINE THE MAXIMUM DEFLECTION IN THE REGION

$$(83) \rightarrow \quad u(4.367m) = -\frac{2.5kN}{6EI}\cdot(4.367m)^3 + \frac{2kN}{3EI}(2.367m)^3 + \frac{2kN\cdot m}{EI}(2.367m)^2 - \frac{2.5kN}{6EI}(0.367m)^3 + \frac{10kN\cdot m^2}{3EI}\cdot 4.367m$$

$$= -\frac{0.7093\,kN\cdot m^3}{6EI} = -\frac{0.1182\,kN\cdot m^3}{EI} = -\frac{0.355\,kN\cdot m^3}{3EI} \quad (105)$$

ANOTHER DISPLACEMENT THAT IS HELPFULL IN PLOTTING THE DISPLACEMENT OF THE ELASTIC CURVE IS $y = 2.667m$. THIS IS WHERE THE CONCAVITY OF THE ELASTIC CURVE CHANGES.

$$(83) \rightarrow \quad u(2.667m) = -\frac{2.5kN}{6EI}\cdot(2.667m)^3 + \frac{2kN}{3EI}\cdot(0.667m)^3 + \frac{2kN\cdot m}{EI}(0.667m)^2 + \frac{10kN\cdot m^2}{3EI}\cdot(2.667m) = \frac{6.23\,kN\cdot m^3}{3EI} \quad (106)$$

THE SIGN CHANGE IN THE REGION $4m<y<6m$ BETWEEN VALUES OF u AT $y = 2m, 4.367m$, AND $6m$ ((99), (105), & (104)) INDICATE ROOTS EXIST IN THIS REGION. $4m<y<6m$

$$(83) \rightarrow \quad u(y) = 0 = -\frac{2.5\cancel{kN}}{6\cancel{EI}}\cdot y^3 + \frac{2\cancel{kN}}{3\cancel{EI}}(y-2m)^3 + \frac{2\cancel{kN}\cdot m}{\cancel{EI}}(y-2m)^2 - \frac{2.5\cancel{kN}}{6\cancel{EI}}(y-4m)^3 + \frac{10\cancel{kN}\cdot m^2}{3\cancel{EI}}\cdot y$$

$$\Rightarrow 0 = -\frac{2.5}{6}\cdot y^3 + \frac{2}{3}(y^2 - 4m\cdot y + 4m^2)(y-2m) + 2m(y^2 - 4m\cdot y + 4m^2) - \frac{2.5}{6}(y^2 - 8m\cdot y + 16m^2)(y-4m) + \frac{10}{3}m^2\cdot y$$

$$0 = -\frac{2.5}{6}\cdot y^3 + \frac{2}{3}(y^3 - 4m\cdot y^2 + 4m^2\cdot y - 2m\cdot y^2 + 8m^2 y - 8m^3) + 2m\cdot y^2 - 8m^2\cdot y + 8m^3 - \frac{2.5}{6}(y^3 - 8m\cdot y^2 + 16m^2 y - 4m\cdot y^2 + 32m^2 y - 64m^3) + \frac{10}{3}m^2\cdot y$$

$$0 = -\frac{2.5}{6}\cdot y^3 + \frac{2}{3}y^3 - \frac{12m}{3}\cdot y^2 + \frac{24m^2}{3}\cdot y - \frac{16}{3}m^3 + 2m\cdot y^2 - 8m^2\cdot y + 8m^3 - \frac{2.5}{6}y^3 + \frac{30}{6}m\cdot y^2 - \frac{120}{6}m^2\cdot y + \frac{160}{6}m^3 + \frac{10}{3}m^2\cdot y$$

$$0 = -\frac{1}{6}\cdot y^3 + \frac{18}{6}m\cdot y^2 - \frac{100}{6}m^2\cdot y + \frac{176m^3}{6}$$

$$0 = y^3 - 18m\cdot y^2 + 100m^2\cdot y - 176\,m^3$$

MATLAB'S FUNCTION "ROOTS" CAN BE USED TO SOLVE THIS CUBIC EQUATION'S ROOTS. THE ROOTS ARE FOUND TO BE

$$y = \underline{4.0m},\ \underline{4.7639m},\ 9.2361\,m \qquad (107)$$

THE LAST ROOT IN (107) ($y = 9.2361m$) IS OUTSIDE THE REGION BEING CONSIDERED AND THEREFORE IS NOT RELEVANT. THE FIRST ROOT IN (107) ($y = 4.0m$) IS THE BOUNDARY CONSTRAINT. THE ROOT THAT IS MOST RELEVANT IN FINDING WHERE $u = 0$ BETWEEN $4m < y < 6m$ IS the second

$$\underline{\underline{y = 4.7639m}}$$

NOW THE BEAM SEGMENT <u>DFE</u> THAT SPANS FROM <u>$6m < y < 8m$</u> CAN BE EVALUATED. STARTING WITH THE SINGULARITY FUNCTION REPRESENTATION OF THE LOAD

$$q(y) = -1kN\langle y-6m\rangle_{-1} + 2kN\langle y-7m\rangle_{-1} - 1kN\langle y-8m\rangle_{-1} \qquad (108)$$

V, M, Θ, AND u ARE FOUND THROUGH DIRECT INTEGRATION

$$V(y) = \int -q(y)\,dy = \int\left[1kN\langle y-6m\rangle_{-1} - 2kN\langle y-7m\rangle_{-1} + 1kN\langle y-8m\rangle_{-1}\right]dy$$

$$= 1kN\langle y-6m\rangle^0 - 2kN\langle y-7m\rangle^0 + 1kN\langle y-8m\rangle^0 \qquad (109)$$

$$M(y) = \int V(y)\,dy = 1kN\langle y-6m\rangle^1 - 2kN\langle y-7m\rangle^1 + 1kN\langle y-8m\rangle^1 \qquad (110)$$

$$\Theta(y) = \int -\frac{M}{EI}\,dy = \int\left[-\frac{1kN}{EI}\langle y-6m\rangle^1 + \frac{2kN}{EI}\langle y-7m\rangle^1 - \frac{1kN}{EI}\langle y-8m\rangle^1\right]dy$$

$$= -\frac{1kN}{2\cdot EI}\langle y-6m\rangle^2 + \frac{1kN}{EI}\langle y-7m\rangle^2 - \frac{1kN}{2EI}\langle y-8m\rangle^2 + \overset{-\frac{1kN\cdot m^2}{6\cdot EI}}{\cancel{C_3}} \qquad (111)$$

$$u(y) = \int \Theta(y)\,dy =$$

$$= -\frac{1kN}{6EI}\langle y-6m\rangle^3 + \frac{1kN}{3EI}\langle y-7m\rangle^3 - \frac{1kN}{6EI}\langle y-8m\rangle^3 + \overset{-\frac{1kN\,m^2}{6EI}}{\cancel{C_3}}\cdot y + \overset{\frac{7kN\,m^3}{3EI}}{\cancel{C_4}} \qquad (112)$$

THE CONSTANTS C_3 & C_4 IN (111) & (112) NEED TO BE DETERMINED FROM BOUNDARY CONDITIONS. FOR THE DFE BEAM SEGMENT ($6m \le y \le 8m$), THE TWO BOUNDARY CONDITIONS ARE

$$(104) \rightarrow u(6m) = \frac{4\,kN\cdot m^2}{3EI} \quad (113)$$

$$u(8m) = 0m \quad (114)$$

[THE VALUE FOR $\Theta(6m)$ FROM (102) IS <u>NOT</u> A CONTINUITY CONDITION THAT CAN BE USED TO EVALUATE THE CONSTANTS C_3 & C_4, BECAUSE ~~OF~~ THE PIN AT D DOES NOT CONSTRAIN ROTATIONS OF ABCD OR DFE AT $y = 6m$]

SUBSTITUTING (113) AND (114) INTO (112)

$$u(6m) = \frac{4\,kN\cdot m^2}{3\cdot EI} = -\frac{1\,kN}{6EI}\cdot(0)^3 + 6m\cdot C_3 + C_4$$

$$\Rightarrow \frac{4\,kN\cdot m^3}{3\cdot EI} = 6m\cdot C_3 + C_4 \quad (115)$$

$$u(8m) = 0 = -\frac{1\,kN}{6EI}(2m)^3 + \frac{1\,kN}{3EI}(1m)^3 - \frac{1\,kN}{6EI}(0m)^3 + 8m\cdot C_3 + C_4$$

$$0 = -\frac{6\,kN\cdot m^3}{6\cdot EI} + 8m\cdot C_3 + C_4 = -\frac{1\,kN\cdot m^3}{EI} + 8m\cdot C_3 + C_4$$

$$\Rightarrow \frac{1\,kN\cdot m^3}{EI} = 8m\cdot C_3 + C_4$$

$$\Rightarrow C_4 = \frac{1\,kN\cdot m^3}{EI} - 8m\cdot C_3 \quad (116)$$

$$(116) \rightarrow (115) \rightarrow \frac{4\,kN\cdot m^3}{3\cdot EI} = 6m\cdot C_3 + \frac{1\,kN\cdot m^3}{EI} - 8m\cdot C_3$$

$$\frac{1\,kN\cdot m^3}{3EI} = -2m\cdot C_3 \Rightarrow C_3 = -\frac{1\,kN\cdot m^2}{6\cdot EI} \quad (117)$$

$$(117) \rightarrow (116) \rightarrow C_4 = \frac{1\,kN\cdot m^3}{EI} + 8m\cdot\frac{1\,kN\cdot m^2}{6EI} = \frac{7\,kN\cdot m^2}{3EI} \quad (118)$$

Substituting (117) and (118) into (111) and (112)

(111) → $$\Theta(y) = -\frac{1\,kN}{2EI}\cdot\langle y-6m\rangle^2 + \frac{1\,kN}{EI}\langle y-7m\rangle^2 - \frac{1\,kN}{2EI}\langle y-8m\rangle^2 - \frac{1\,kN\cdot m^2}{6\,EI} \quad (119)$$

(112) → $$u(y) = -\frac{1\,kN}{6EI}\cdot\langle y-6m\rangle^3 + \frac{1\,kN}{3EI}\cdot\langle y-7m\rangle^3 - \frac{1\,kN}{6EI}\cdot\langle y-8m\rangle^3 - \frac{1\,kN\cdot m^2}{6EI}\cdot y + \frac{7\,kN\cdot m^3}{3EI} \quad (120)$$

Critical values of V, M, Θ, and u in each region of DFE can now be determined.

Region DF: $6m \leq y \leq 7m$

(109) → $$V(6m) = 1\,kN\,(0)^0 = 0\,kN,\ 1\,kN \quad (121)$$

(109) → $$V(7m) = 1\,kN\,(1m)^0 - 2\,kN\,(0)^0 = 1\,kN,\ -1\,kN \quad (122)$$

(110) → $$M(6m) = 1\,kN\,(0)^1 = 0 \quad (123)$$

(110) → $$M(7m) = 1\,kN\,(1m)^1 - 2\,kN\,(0)^1 = 1\,kN\cdot m \quad (124)$$

(119) → $$\Theta(6m) = -\frac{1\,kN}{2EI}(0)^2 - \frac{1\,kN\cdot m^2}{6\,EI} = -\frac{1\,kN\cdot m^2}{6\,EI} \quad (125)$$

(119) → $$\Theta(7m) = -\frac{1\,kN}{2EI}\cdot(1m)^2 + \frac{1\,kN}{EI}(0)^2 - \frac{1\,kN\,m^2}{6EI} = -\frac{4\,kN\cdot m^2}{6\cdot EI} \quad (126)$$

(120) → $$u(6m) = -\frac{1\,kN}{6EI}(0)^3 - \frac{1\,kN\cdot m^2}{6\,EI}\cdot(6m) + \frac{7\,kN\cdot m^3}{3\,EI} = \frac{8\,kN\cdot m^3}{6\,EI} = \frac{4\,kN\cdot m^3}{3EI} \quad (127)$$

(120) → $$u(7m) = -\frac{1\,kN}{6EI}\cdot(1m)^3 + \frac{1\,kN}{3EI}(0)^3 - \frac{1\,kN\cdot m^2}{6EI}\cdot(7m) + \frac{7\,kN\cdot m^3}{3EI} = \frac{6\,kN\cdot m^3}{6EI}$$

$$= \frac{1\,kN\cdot m^3}{EI} \quad (128)$$

Region FE: $7m \leq y \leq 8m$

(122) → $$V(7m) = 1\,kN,\ -1\,kN$$

(109) → $$V(8m) = 1\,kN\,(2m)^0 - 2\,kN\cdot(1m)^0 + 1\,kN\cdot(0)^0 = -1\,kN,\ 0 \quad (129)$$

(124) → $M(7m) = 1\,kN\cdot m$

(110) → $M(8m) = 1\,kN(2m) - 2\,kN(1m) + 1\,kN\cdot(0) = 0$ (130)

(126) → $\Theta(7m) = -\dfrac{4\,kN\cdot m^2}{6EI} = -\dfrac{2\,kN\cdot m^2}{3EI}$

(111) → $\Theta(8m) = -\dfrac{1\,kN}{2EI}(2m)^2 + \dfrac{1\,kN}{EI}(1m)^2 - \dfrac{1\,kN}{2\cdot EI}(0m)^2 - \dfrac{1\,kN\cdot m^2}{6EI} = \dfrac{-7\,kN\cdot m^2}{6EI}$ (131)

(128) → $u(7m) = \dfrac{1\,kN\cdot m^3}{EI}$

(112) → $u(8m) = -\dfrac{1\,kN}{6EI}(2m)^3 + \dfrac{1\,kN}{3EI}(1m)^3 - \dfrac{1\,kN}{6EI}(0m)^3 - \dfrac{1\,kN\cdot m^2}{6EI}\cdot 8m + \dfrac{7\,kN\,m^3}{3EI}$

$= 0$ (132)

SINGULARITY FUNCTION SUMMARY

THE VALUES FOR V, M, Θ, AND u CALCULATED USING SINGULARITY FUNCTIONS ARE SUMMARIZED IN FIGURES (c)-(f). THESE VALUES EXACTLY MATCH THE VALUES COMPUTED USING THE DIRECT INTEGRATION APPROACH.