PROBLEM 1.9 THE FIGURE DEPICTS A STRESS DISTRIBUTION ACROSS AN INTERNAL SURFACE OF A RECTANGULAR ROD OF HEIGHT 3IN AND DEPTH 1IN. ASSUMING THAT THE STRESS DISTRIBUTION DOES NOT VARY WITH RESPECT TO Z, DETERMINE THE NET FORCE IN THE X DIRECTION AND THE NET MOMENT ABOUT THE Z AXIS. THE STRESS DISTRIBUTION IS $\sigma_x = 2000y + 500psi$

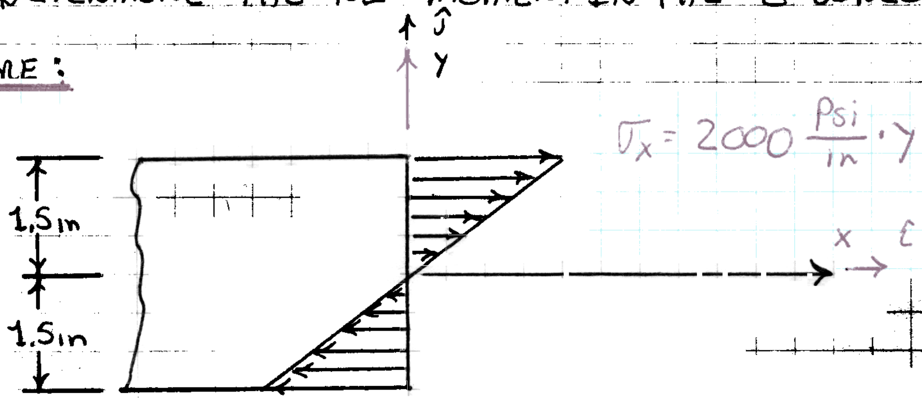
GIVEN:

CONSTRAINTS
1. A RECTANGULAR ROD 3IN HIGH BY 1IN DEPTH.
2. THE STRESS DISTRIBUTION IS GIVEN BY $\sigma_x = 2000 \frac{psi}{in} \cdot y + 500 psi$

ASSUMPTION
1. THE DEFORMATION IS SMALL.

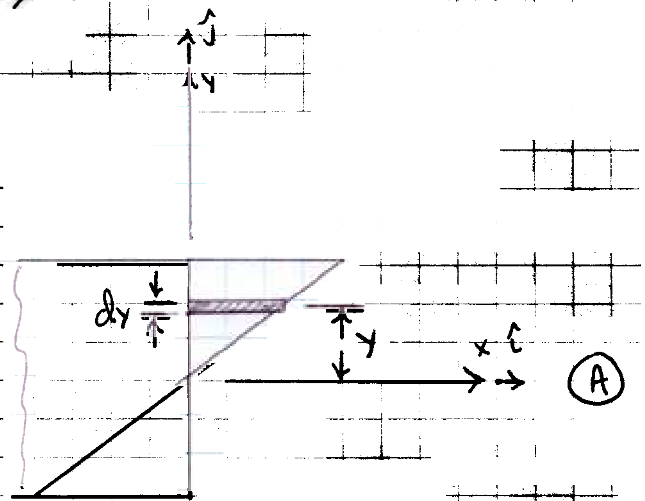
FIND:
1. DETERMINE THE NET FORCE IN THE X-DIRECTION
2. DETERMINE THE NET MOMENT IN THE Z-DIRECTION.

FIGURE:



SOLUTION:

INTEGRATING THE STRESS FIELD WITH RESPECT TO "y" WILL RESULT IN THE NET FORCE IN THE X DIRECTION



$$F_x = \int \sigma_x \cdot dA \cdot (1in)$$

$$= \int_{-1.5in}^{1.5in} \left(2000 \frac{psi}{in} \cdot y + 500psi\right) dy \cdot (1in)$$

$$= \left[2000 \frac{psi}{in} \cdot \frac{y^2}{2} + 500psi \cdot y\right]_{-1.5in}^{1.5in} \cdot (1in)$$

$$= \left[1000 \frac{psi}{in} \cdot (1.5in)^2 + 500psi \cdot (1.5in) - 1000 \frac{psi}{in} (-1.5in)^2 - 500psi \cdot (-1.5in)\right](1in)$$

$$= \boxed{1500 lb}$$

FROM FIGURE (A) A SECOND INTEGRAL IS FORMED TO CALCULATE THE MOMENT ABOUT THE Z-AXIS.

$$M_z = \int y \cdot \sigma_x \cdot dy \cdot 1in = \int_{-1.5in}^{1.5in} y \cdot \left[2000 \tfrac{psi}{in} \cdot y + 500psi\right] dy \cdot 1in$$

$$= \int_{-1.5in}^{1.5in} \left[2000 \tfrac{psi}{in}\, y^2 + 500\,psi \cdot y\right] dy \cdot 1in = \left[2000 \tfrac{psi}{in} \frac{y^3}{3} + 500psi \cdot \frac{y^2}{2}\right]_{-1.5in}^{1.5in} \cdot 1in$$

$$= \left\{2000 \tfrac{lb}{in^3} \cdot \tfrac{1}{3}\left[(1.5in)^3 - (-1.5in)^3\right] + 500 \tfrac{lb}{in^2} \cdot \tfrac{1}{2}\left[(1.5in)^2 - (-1.5in)^2\right]\right\} \cdot 1in$$

$$\boxed{= 4.5(10^3)\ lb \cdot in}$$

SUMMARY:

THIS PROBLEM FOCUSES ON THE RELATIONSHIP BETWEEN THE STRESS DISTRIBUTION AND THE INTERNAL FORCE AND MOMENT IN A STRUCTURE. THE SMALL SHADED AREA IN FIGURE REPRESENTS THE DIFFERENTIAL FORCE GENERATED BY THE STRESS DISTRIBUTION. ALL THE DIFFERENTIAL FORCES OVER THE AREA ARE SUMMED THROUGH INTEGRATION TO CALCULATE THE NET INTERNAL FORCE ACTING ON THE CROSS-SECTION. TAKING THE MOMENT OF THE DIFFERENTIAL FORCE AND SUMMING THROUGH INTEGRATION YIELDS THE NET MOMENT ACTING ON THE CROSS-SECTION.