Starting with the stress tensor given in Problem 2.16 from Budynas, 2nd ed. The problem states the state of stress is plane stress (no stress in the z direction). The plane stress condition results from a structure being very thin.

$$[\sigma]=\begin{bmatrix} 40 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa = \begin{bmatrix} 40 & 0 \\ 0 & 10 \end{bmatrix} MPa$$

This principal stress cube was then rotated 15° about the z-axis and resulted in the following state of stress.

$$\left[\sigma_{x'y'z'}\right]=\begin{bmatrix} 38 & 7.5 & 0 \\ 7.5 & 12 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa = \begin{bmatrix} 38 & 7.5 \\ 7.5 & 12 \end{bmatrix} MPa$$

The problem also asked to determine the state of stress where the shear was maximum in the x-y plane. This resulted in the following state of stress.

$$\left[\sigma_{\max Shear}\right]=\begin{bmatrix} 25 & 15 & 0 \\ 15 & 25 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa = \begin{bmatrix} 25 & 15 \\ 15 & 25 \end{bmatrix} MPa$$

Given the material is steel (E=75GPa, υ=0.3), determine the initial engineering strain tensor and the engineering strain tensor after transformation.

SOLUTION:

```
>> Sig=[40 10 0]'*1e6
Sig =
   40000000
   10000000
          0

>> Sig_xyz=[38 12 7.5]'*1e6
Sig_xyz =
   38000000
   12000000
    7500000

>> Sig_maxShear=[25 25 15]'*1e6
Sig_maxShear =
   25000000
   25000000
   15000000
```

```
>> Scomp=[1/75e9 .3/75e9 0; .3/75e9 1/75e9 0; 0 0 2*(1+.3)/75e9]
Scomp =  1.0e-010 *
    0.1333   -0.0400         0
   -0.0400    0.1333         0
         0         0    0.3467

e =  1.0e-003 *
    0.4933
   -0.0267
         0
```

$$[\varepsilon]_{eng} = \begin{bmatrix} \mathbf{493.3} & \mathbf{0} \\ \mathbf{0} & \mathbf{-26.7} \end{bmatrix} \mu\varepsilon \quad [\varepsilon] = \begin{bmatrix} \mathbf{493.3} & \mathbf{0} \\ \mathbf{0} & \mathbf{-26.7} \end{bmatrix} \mu\varepsilon$$

```
>> e_xyz=Scomp*Sig_xyz
e_xyz =  1.0e-003 *
    0.4587
    0.0080
    0.2600
```

$$\left[\varepsilon_{x'y'z'}\right]_{eng} = \begin{bmatrix} \mathbf{458.7} & \mathbf{260} \\ \mathbf{260} & \mathbf{8} \end{bmatrix} \mu\varepsilon \quad \left[\varepsilon_{x'y'z'}\right] = \begin{bmatrix} \mathbf{458.7} & \mathbf{130} \\ \mathbf{130} & \mathbf{8} \end{bmatrix} \mu\varepsilon$$

```
>> e_maxShear=Scomp*Sig_maxShear
e_maxShear =  1.0e-003 *
    0.2333
    0.2333
    0.5200
```

$$\left[\varepsilon_{\max Shear}\right]_{eng} = \begin{bmatrix} \mathbf{233.3} & \mathbf{520} \\ \mathbf{520} & \mathbf{233.3} \end{bmatrix} \mu\varepsilon \quad \left[\varepsilon_{\max Shear}\right] = \begin{bmatrix} \mathbf{233.3} & \mathbf{260} \\ \mathbf{260} & \mathbf{233.3} \end{bmatrix} \mu\varepsilon$$