NAME:

PROBLEM 1: Design a fourbar mechanism to give the two positions shown of coupler motion. Both rockers should be designed to have 30° of travel.

Find: w, θ, W_{1x}, W_{1y}, W_{2x}, W_{2y},
U, σ, U_{1x}, U_{1y}, U_{2x}, U_{2y},

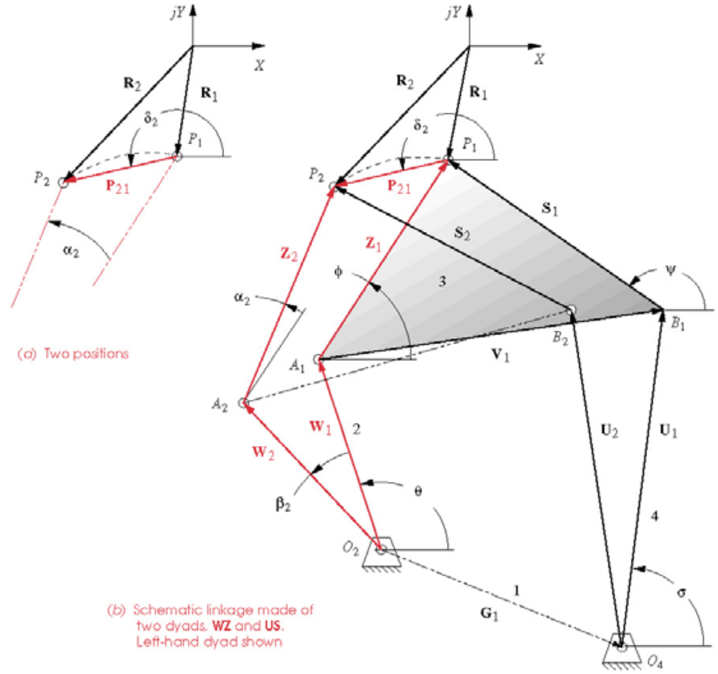
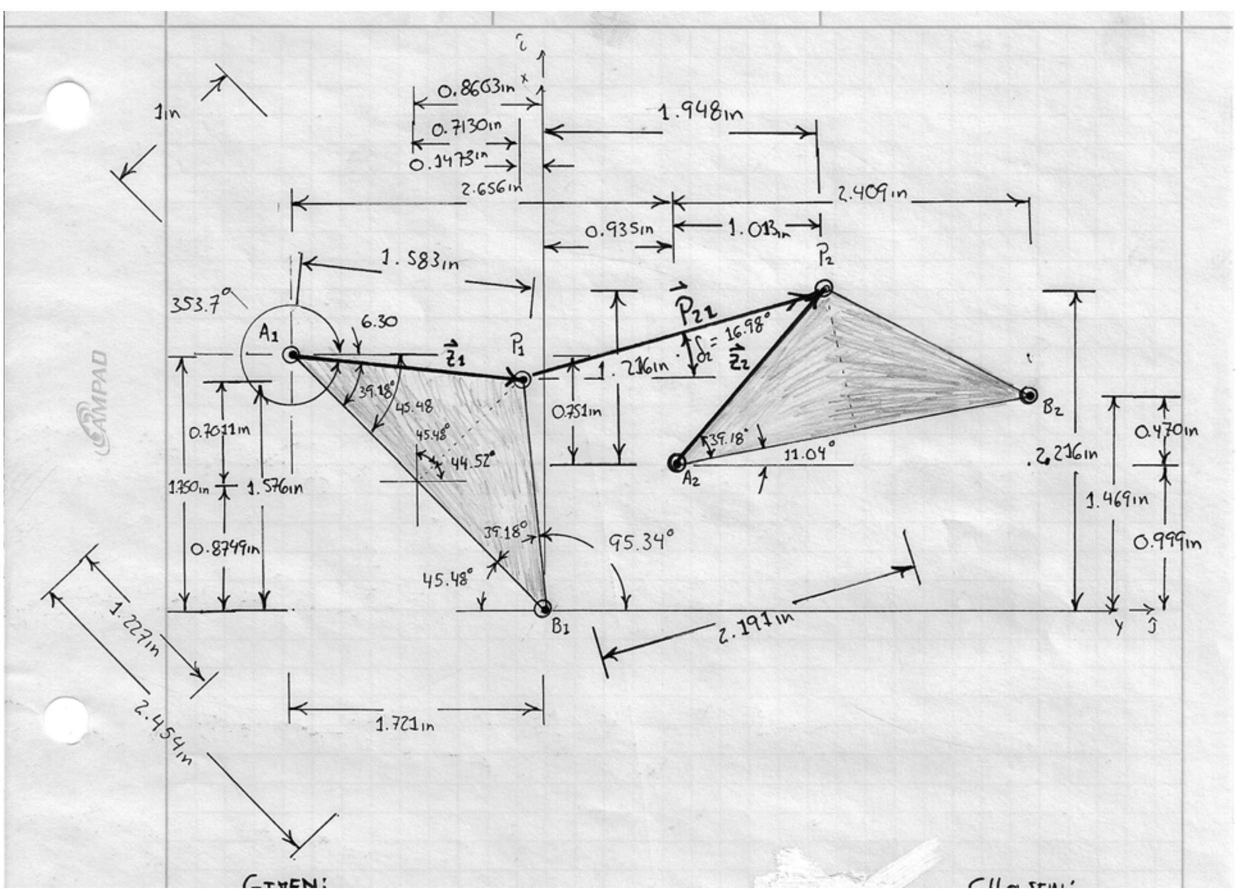


FIGURE 5-1
Two-position analytical synthesis



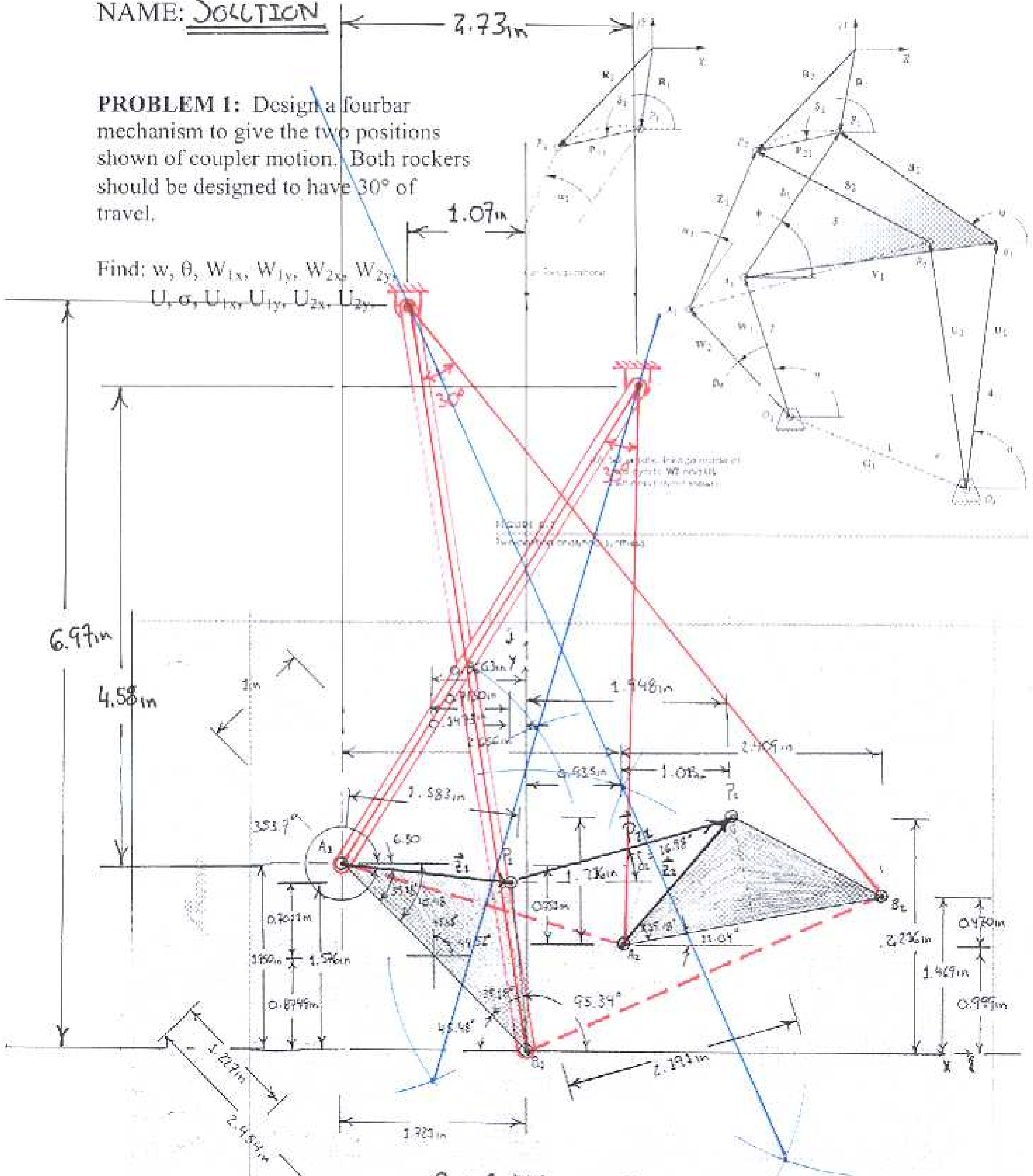
MER312: Advanced Kinematics and Kinetics Exam II
Winter 2008

Union College
Mechanical Engineering

NAME: SOLUTION

PROBLEM 1: Design a fourbar mechanism to give the two positions shown of coupler motion. Both rockers should be designed to have 30° of travel.

Find: w, θ, W_{1x}, W_{1y}, W_{2x}, W_{2y}, U, σ, U_{1x}, U_{1y}, U_{2x}, U_{2y}



$\rho_{12} = 2.191$ in $z = 1.583$ in $s = 1.583$ in

$\delta_2 = 16.98°$ $\phi = 353.7°$ $\psi = 95.34°$

$\alpha_2 = 56.52°$ $\beta_2 = 30°$ $\gamma_2 = 30°$

$= 6.30 + 11.04 + 35.18$

TWO POSITION ANALYTICAL MOTION SYNTHESIS

$$\vec{W}_2 + \vec{Z}_2 = \vec{W}_1 + \vec{Z}_1 + \vec{P}_{21}$$

$$\left|\vec{W}_1\right| = \left|\vec{W}_2\right| = w$$

$$\left|\vec{Z}_1\right| = \left|\vec{Z}_2\right| = z$$

$$\vec{W}_1 = w \cdot \left[\cos(\theta)\hat{i} + \sin(\theta)\hat{j}\right]$$

$$\vec{W}_2 = w \cdot \left[\cos(\theta+\beta_2)\hat{i} + \sin(\theta+\beta_2)\hat{j}\right]$$

$$\vec{Z}_1 = z \cdot \left[\cos(\phi)\hat{i} + \sin(\phi)\hat{j}\right]$$

$$\vec{Z}_2 = z \cdot \left[\cos(\phi+\alpha_2)\hat{i} + \sin(\phi+\alpha_2)\hat{j}\right]$$

$$\vec{P}_{21} = p_{21} \cdot \left[\cos(\delta_2)\hat{i} + \sin(\delta_2)\hat{j}\right]$$

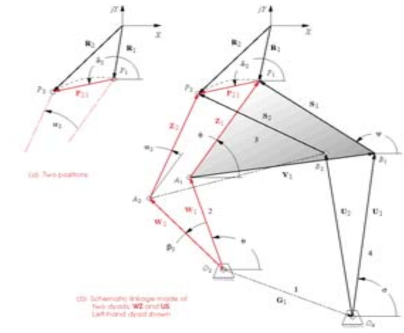


FIGURE 5-1
Two-position analytical synthesis

$$\vec{U}_2 + \vec{S}_2 = \vec{U}_1 + \vec{S}_1 + \vec{P}_{31}$$

$$\left|\vec{U}_1\right| = \left|\vec{U}_2\right| = u$$

$$\left|\vec{S}_1\right| = \left|\vec{S}_2\right| = s$$

$$\vec{U}_1 = u \cdot \left[\cos(\sigma)\hat{i} + \sin(\sigma)\hat{j}\right]$$

$$\vec{U}_2 = u \cdot \left[\cos(\sigma+\gamma_2)\hat{i} + \sin(\sigma+\gamma_2)\hat{j}\right]$$

$$\vec{S}_1 = s \cdot \left[\cos(\psi)\hat{i} + \sin(\psi)\hat{j}\right]$$

$$\vec{S}_2 = s \cdot \left[\cos(\psi+\alpha_2)\hat{i} + \sin(\psi+\alpha_2)\hat{j}\right]$$

$$\vec{P}_{21} = p_{21} \cdot \left[\cos(\delta_2)\hat{i} + \sin(\delta_2)\hat{j}\right]$$

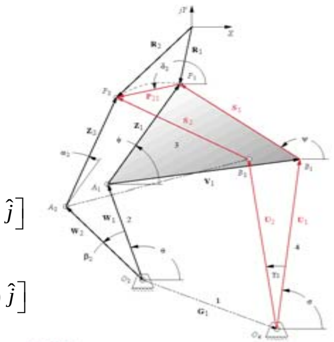


FIGURE 5-2
Right-side dyad shown in two positions

APPROACH B

GIVEN:		CHOSEN:		FIND:	
P12	2.191	**z**	1.583	**w**	5.33
δ2	16.98	**ϕ**	353.7	**θ**	-120.8
α2	56.52	**β2**	30	**W1x**	-2.73
				W1y	-4.58

$$\begin{bmatrix} -0.13397 & -0.5 \\ 0.5 & -0.13397 \end{bmatrix} \begin{Bmatrix} W1x \\ W1y \end{Bmatrix} = \begin{Bmatrix} 2.656058 \\ -0.7504 \end{Bmatrix} \qquad \begin{matrix} -0.5 & 1.866025 \\ -1.86603 & -0.5 \end{matrix}$$

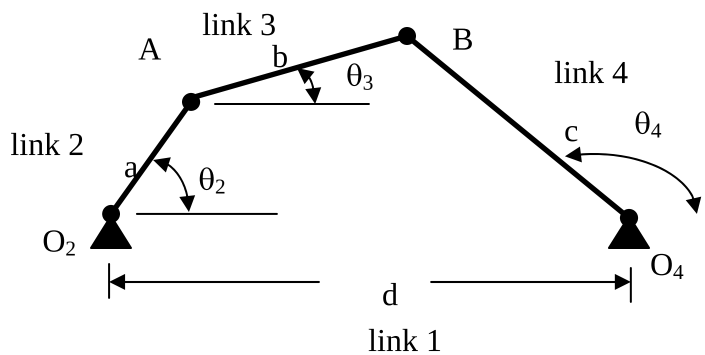
$$\begin{bmatrix} \cos(\beta_2)-1 & -\sin(\beta_2) \\ \sin(\beta_2) & \cos(\beta_2)-1 \end{bmatrix} \begin{Bmatrix} W_{1x} \\ W_{1y} \end{Bmatrix} = \begin{Bmatrix} p_{21}\cdot\cos(\delta_2) - z\cdot\left[\cos(\phi+\alpha_2)-\cos(\phi)\right] \\ p_{21}\cdot\sin(\delta_2) - z\cdot\left[\sin(\phi+\alpha_2)-\sin(\phi)\right] \end{Bmatrix}$$

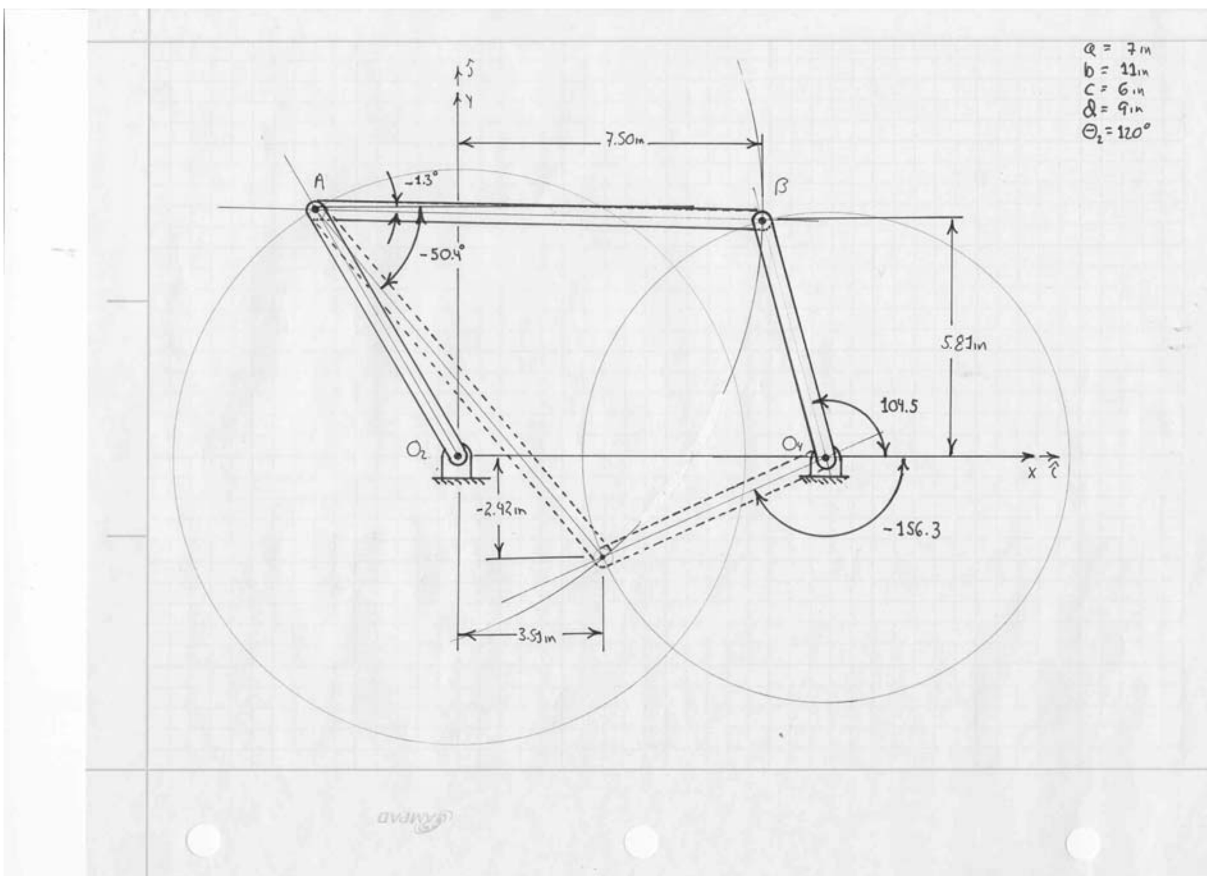
GIVEN:		CHOSEN:		FIND:	
P12	2.191	**s**	1.583	**u**	7.06
δ2	16.98	**ψ**	95.34	**σ**	-81.3
α2	56.52	**γ2**	30	**U1x**	1.07
				U1y	-6.97

$$\begin{bmatrix} -0.13397 & -0.5 \\ 0.5 & -0.13397 \end{bmatrix} \begin{Bmatrix} W1x \\ W1y \end{Bmatrix} = \begin{Bmatrix} 3.34405 \\ 1.469398 \end{Bmatrix} \qquad \begin{matrix} -0.5 & 1.866025 \\ -1.86603 & -0.5 \end{matrix}$$

$$\begin{bmatrix} \cos(\gamma_2)-1 & -\sin(\gamma_2) \\ \sin(\gamma_2) & \cos(\gamma_2)-1 \end{bmatrix} \begin{Bmatrix} U_{1x} \\ U_{1y} \end{Bmatrix} = \begin{Bmatrix} p_{21}\cdot\cos(\delta_2) - s\cdot\left[\cos(\psi+\alpha_2)-\cos(\psi)\right] \\ p_{21}\cdot\sin(\delta_2) - s\cdot\left[\sin(\psi+\alpha_2)-\sin(\psi)\right] \end{Bmatrix}$$

PROBLEM 2: In the four bar linkage shown a=7in, b=11in, c=6in, d=9in, and θ_2=120°. For both the open and crossed configurations determine θ_3, θ_4, B_x, B_y and expressions for the vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4$ in both Cartesian and magnitude-unit vector form.

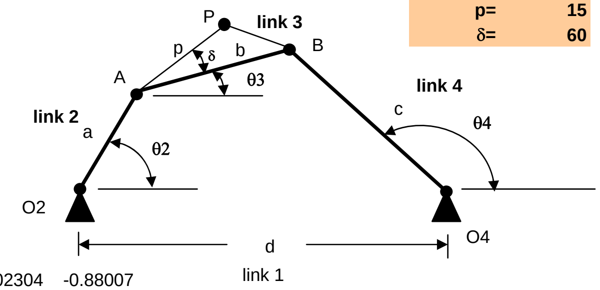




4-Bar Linkage

a=	7	Link 2			
b=	11	Link 3			
c=	6	Link 4			
d=	9	Link 1			
$\theta_2 =$	120	2.094395102			
$\dot{\theta}_2 =$	15	$1/s$			
$\ddot{\theta}_2 =$	-65	$1/s^2$			
By=	5.81	-2.42			
Bx=	7.50	3.51			
$\theta_3 =$	-1.3	-50.4	-0.02304	-0.88007	
$\theta_4 =$	104.5	-156.3	1.823979	-2.72709	
$\dot{\theta}_3 =$	2.6504E+00	9.8626E+00			
$\dot{\theta}_4 =$	1.5539E+01	-3.0259E+00			
$\ddot{\theta}_3 =$	-6.9538E+00	-2.6177E+01			
$\ddot{\theta}_4 =$	-1.2733E+02	9.4202E+01			

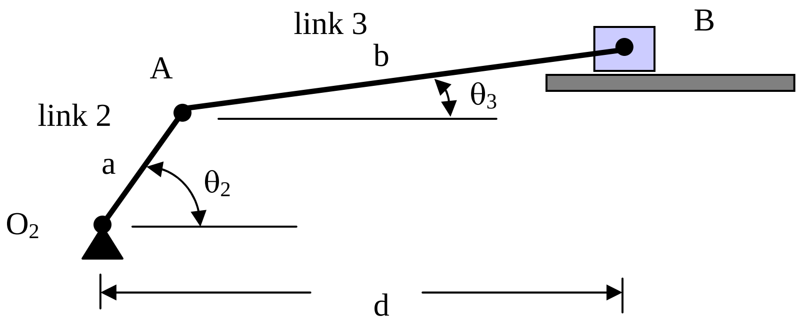
K1=	4.6800E+00
K2=	4.8497E-01
K3=	3.3923E+00
K4=	-1.4036E+01

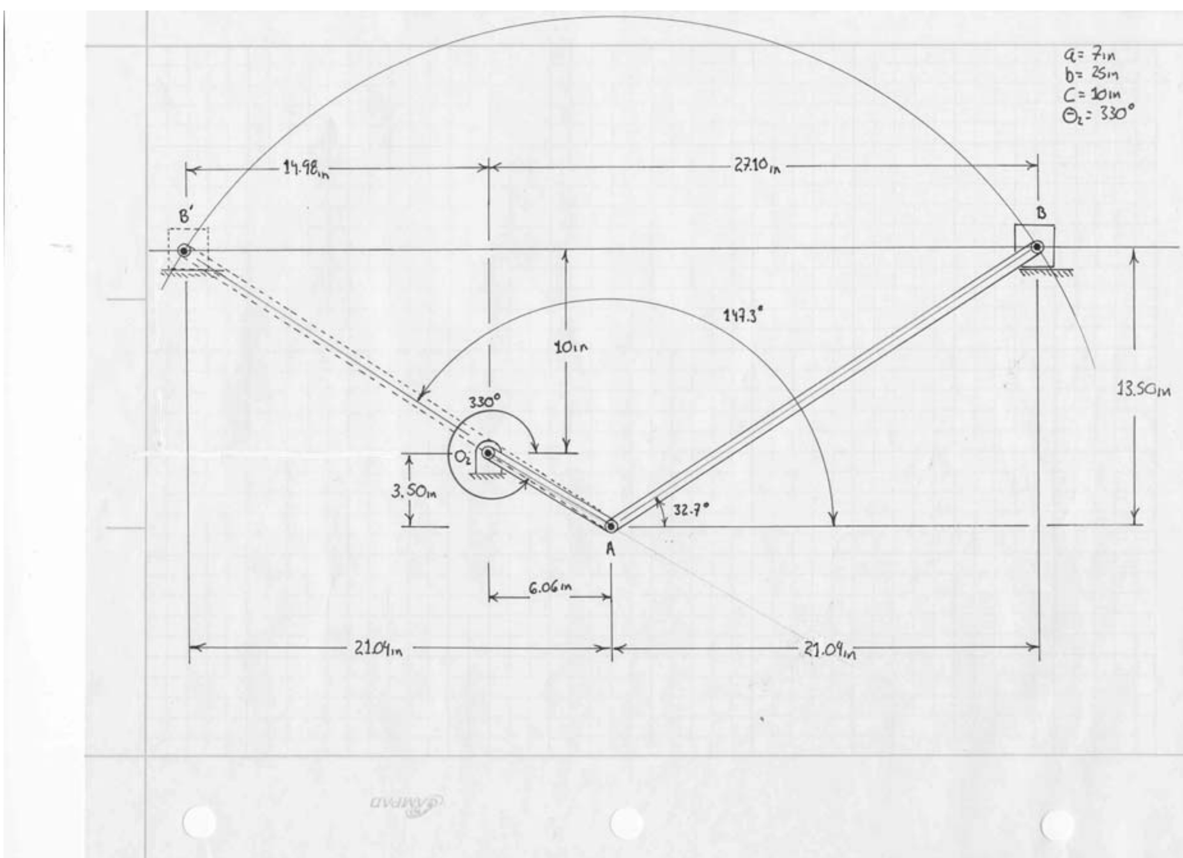


p=	15
δ=	60

	x comp	y comp	mag	angle	i	j
r04=	**9.00**	**0.00**	**9.00**	**0.0**	**1.000**	**0.000**
rA=	**-3.50**	**6.06**	**7.00**	**120.0**	**-0.500**	**0.866**
rBA=	11.00	-0.25	11.00	-1.3	1.000	-0.023
rBO4=	-1.50	5.81	6.00	104.5	-0.250	0.968
rB=	**7.50**	**5.81**	**9.48**	**37.8**	**0.790**	**0.612**
rPA=	7.80	12.81	15.00	58.7	0.520	0.854
rP=	**4.30**	**18.88**	**19.36**	**77.2**	**0.222**	**0.975**
vA=	**-90.93**	**-52.50**	**105.00**	**-150.0**	**-0.866**	**-0.500**
vBA=	0.67	29.15	29.15	88.7	0.023	1.000
vB=	**-90.26**	**-23.35**	**93.23**	**-165.5**	**-0.968**	**-0.250**
vPA=	-33.96	20.67	39.76	148.7	-0.854	0.520
vP=	**-124.89**	**-31.83**	**128.89**	**-165.7**	**-0.969**	**-0.247**
aA=	**1181.54**	**-1136.49**	**1639.41**	**-43.9**	**0.721**	**-0.693**
aBA	-79.01	-74.69	108.73	-136.6	-0.727	-0.687
aB	**1102.53**	**-1211.18**	**1637.84**	**-47.7**	**0.673**	**-0.739**
aPA=	34.33	-144.23	148.26	-76.6	0.232	-0.973
aP=	**1215.88**	**-1280.72**	**1765.96**	**-46.5**	**0.689**	**-0.725**
ALT	**x comp**	**y comp**	**mag**	**angle**	**i**	**j**
rO4=	**9.00**	**0.00**	**9.00**	**0.0**	**1.000**	**0.000**
rA=	**-3.50**	**6.06**	**7.00**	**120.0**	**-0.500**	**0.866**
rBA=	7.01	-8.48	11.00	-50.4	0.637	-0.771
rBO4=	-5.49	-2.42	6.00	-156.3	-0.915	-0.403
rB=	**3.51**	**-2.42**	**4.26**	**-34.6**	**0.824**	**-0.567**
rPA=	14.79	2.50	15.00	9.6	0.986	0.166
rP=	**11.29**	**8.56**	**14.17**	**37.2**	**0.797**	**0.604**
vA=	**-90.93**	**-52.50**	**105.00**	**-150.0**	**-0.866**	**-0.500**
vBA=	83.62	69.12	108.49	39.6	0.771	0.637
vB=	**-7.31**	**16.62**	**18.16**	**113.7**	**-0.403**	**0.915**
vPA=	-24.61	145.88	147.94	99.6	-0.166	0.986
vP=	**-115.54**	**93.38**	**148.56**	**141.1**	**-0.778**	**0.629**
aA=	**1181.54**	**-1136.49**	**1639.41**	**-43.9**	**FALSE**	**-0.693**
aBA	-903.63	641.27	1108.04	144.6	-0.816	0.579
aB	**277.92**	**-495.22**	**567.88**	**-60.7**	**0.489**	**-0.872**
aPA=	-1373.41	-629.90	1510.97	-155.4	-0.909	-0.417
aP=	**-191.87**	**-1766.39**	**1776.78**	**-96.2**	**-0.108**	**-0.994**

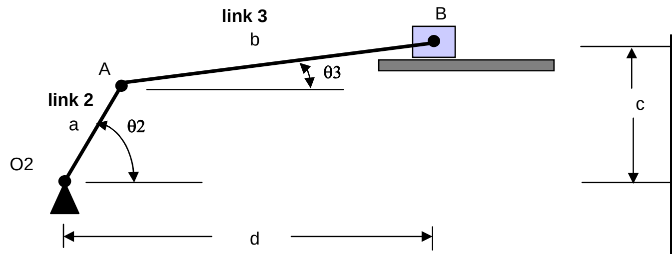
PROBLEM 3: In the four bar slider-crank linkage shown a=7in, b=25in, offset =10in, and θ_2=330°. For both the open and crossed configurations determine θ_3, B_x, B_y and expressions for the vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$ in both Cartesian and magnitude-unit vector form.





Slider Crank

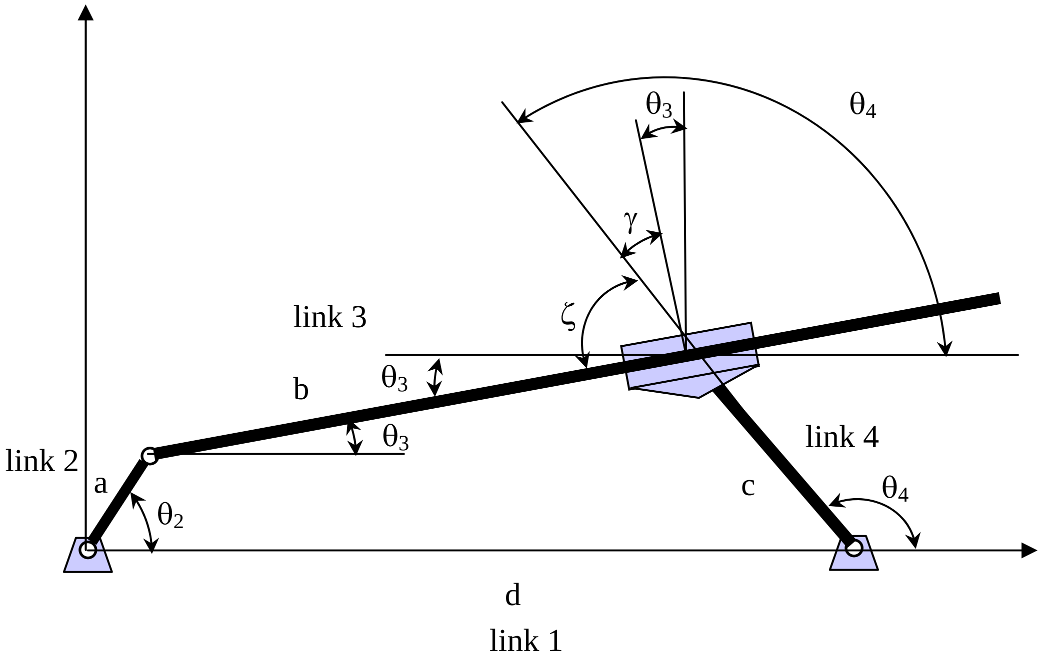
a=	7	Link 2	
b=	25	Link 3	
c=	10	Link 1	
$\theta_2 =$	330	5.759586532	
$\dot{\theta}_2 =$	100	$1/s$	
$\ddot{\theta}_2 =$	18	$1/s^2$	
By=	10.00	10.00	
Bx=	27.10	-14.98	
$\theta_3 =$	32.7	147.3	
$\dot{\theta}_3 =$	-28.81	28.81	
$\ddot{\theta}_3 =$	-1136.01	1136.01	
vB=	738.94	-38.94	
aB=	-77635.22	-43482.34	



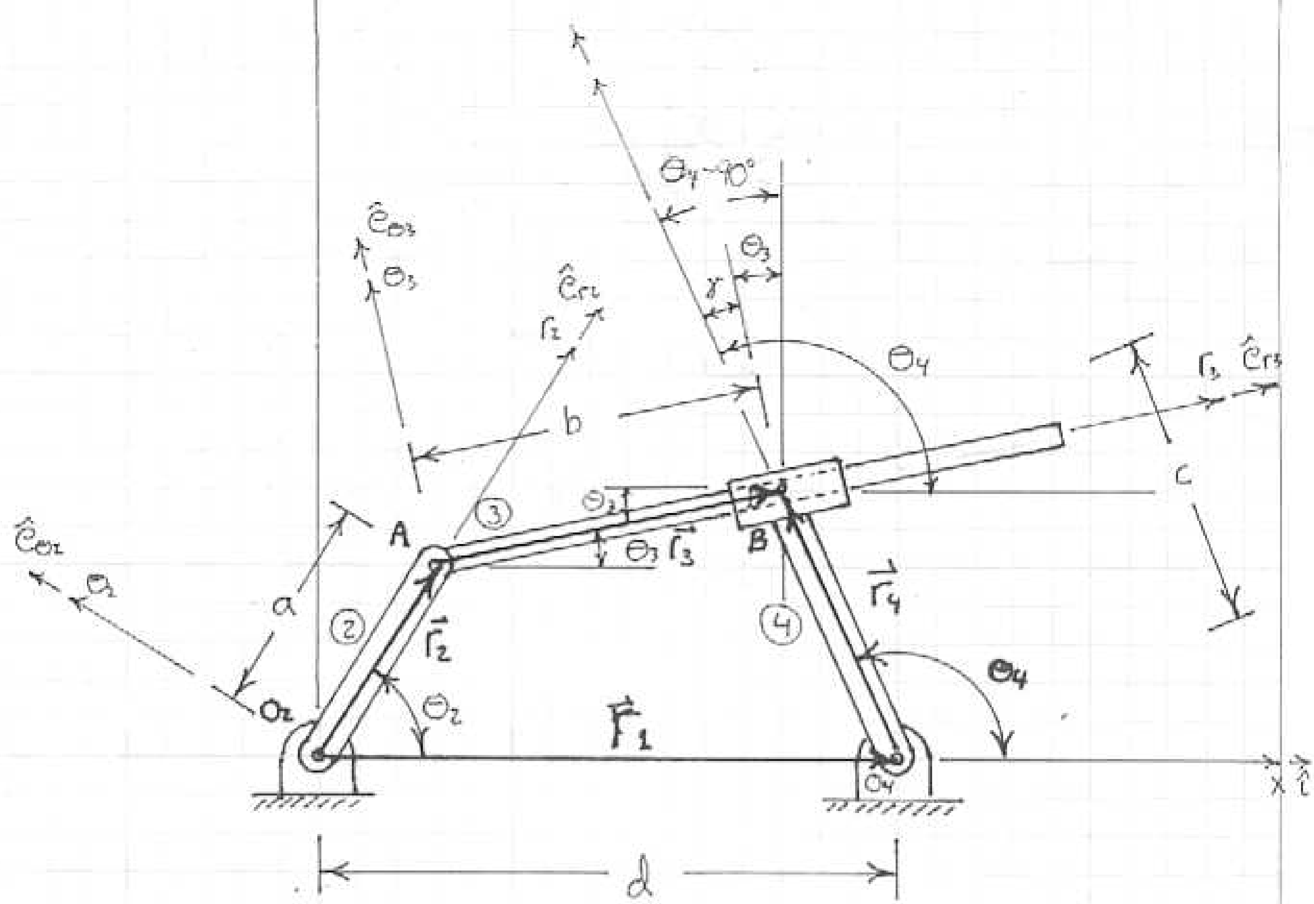
	x comp	y comp	mag	angle	i	j
rB=	27.10	10.00	28.89	20.3	0.938	0.346
rA=	6.06	-3.50	7.00	-30.0	0.866	-0.500
rBA=	21.04	13.50	25.00	32.7	0.842	0.540
vB	738.94	0.00	738.94	0.0	1.000	0.000
vA	350.00	606.22	700.00	60.0	0.500	0.866
vBA	388.94	-606.22	720.26	-57.3	0.540	-0.842
aB	-62687.97	0.00	62687.97	180.0	-1.000	0.000
aA	-60558.78	35109.12	70000.11	149.9	-0.865	0.502
aBA	-2129.19	-35109.12	35173.62	-93.5	-0.061	-0.998

alt	x comp	y comp	mag	angle	i	j
rB=	-14.98	10.00	18.01	146.3	-0.832	0.555
rA=	6.06	-3.50	7.00	-30.0	0.866	-0.500
rBA=	-21.04	13.50	25.00	147.3	-0.842	0.540
vB	-38.94	0.00	38.94	180.0	-1.000	0.000
vA	350.00	606.22	700.00	60.0	0.500	0.866
vBA	-388.94	-606.22	720.26	-122.7	-0.540	-0.842
aB	-58429.59	0.00	58429.59	180.0	-1.000	0.000
aA	-60558.78	35109.12	70000.11	149.9	-0.865	0.502
aBA	2129.19	-35109.12	35173.62	-86.5	0.061	-0.998

PROBLEM 4: Develop general equations that can be used to calculate b, θ_3, & θ_4 in the inverted slider crank when given a, c, d, θ_2, & γ.



INVERTED SLIDER-CRANK LINKAGE



FOR THE INVERTED SLIDER CRANK THE FIXED ANGLE γ DEFINES THE OFFSET BETWEEN THE SLIDER AND LINK 4. THIS RESULTS IN A RELATIONSHIP BETWEEN θ_3 AND θ_4

$$\theta_4 - 90° = \theta_3 + \gamma$$

$$\boxed{\theta_4 = \theta_3 + 90° + \gamma = \theta_3 + \zeta \quad \text{where } \zeta = 90° + \gamma} \qquad (1)$$

NOTE THAT γ IS A CONSTANT. THE LOOP THAT DEFINES THE KINEMATICS OF THIS PROBLEM CAN BE WRITTEN.

$$\vec{r}_2 + \vec{r}_3 = \vec{r}_1 + \vec{r}_4 \qquad (2)$$

$$\vec{r}_2 = r_2 \cdot \hat{e}_{r2} \; (= a \cdot \hat{e}_{r2}) = r_2(\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}) \qquad (3)$$

$$\vec{r}_3 = r_3 \cdot \hat{e}_{r3} \; (= b \cdot \hat{e}_{r3}) = r_3(\cos\theta_3 \hat{i} + \sin\theta_3 \hat{j}) \qquad (4)$$

$$\vec{r}_1 = d \cdot \hat{i} \qquad (5)$$

$$\vec{r}_4 = r_4 \cdot \hat{e}_{r4} \; (= c \cdot \hat{e}_{r4}) = r_4 \cdot (\cos\theta_4 \hat{i} + \sin\theta_4 \hat{j}) \qquad (6)$$

THE LENGTH $r_3 = b$ VARIES AS THE LINKAGE MOVES. THUS $r_3 = b$ IS A VARIABLE THAT MUST BE SOLVED FOR. THIS CREATES AN ADDITIONAL UNKNOWN THAT NEEDS TO BE DETERMINED, $\theta_4, \theta_3, r_3 = b$. HOWEVER, (1) IS A THIRD EQUATION THAT CAN BE USED TO DETERMINE THESE VARIABLES GIVEN θ_2, a, c, d.

STARTING WITH THE LOOP EQUATION (2) AND SUBSTITUTING (3)-(6)

$$r_2\cdot(\cos\theta_2\hat{i}+\sin\theta\hat{j})+r_3\cdot(\cos\theta_3\hat{i}+\sin\theta_3\hat{j})=d\hat{i}+r_4\cdot(\cos\theta_4\hat{i}+\sin\theta_4\hat{j}) \quad (7)$$

DOTTING WITH $\hat{i}$

$$r_2\cdot\cos\theta_2+r_3\cdot\cos\theta_3=d+r_4\cdot\cos\theta_4$$

$$a\cdot\cos\theta_2+r_3\cdot\cos\theta_3=d+c\cdot\cos\theta_4 \quad (8)$$

DOTTING WITH $\hat{j}$

$$r_2\cdot\sin\theta_2+r_3\cdot\sin\theta_3=r_4\cdot\sin\theta_4$$

$$a\cdot\sin\theta_2+r_3\cdot\sin\theta_3=c\cdot\sin\theta_4 \quad (9)$$

EQUATIONS (1), (8), (9) ARE THREE EQUATIONS THAT CONTAIN THREE UNKNOWNS: r_3; θ_3, θ_4. SOLVING (9) FOR r_3

$$\boxed{r_3=\frac{c\cdot\sin\theta_4-a\cdot\sin\theta_2}{\sin\theta_3}} \quad (10)$$

SUBSTITUTING (10) INTO (8)

$$a\cdot\cos\theta_2+\left(\frac{c\cdot\sin\theta_4-a\cdot\sin\theta_2}{\sin\theta_3}\right)\cdot\cos\theta_3=d+c\cdot\cos\theta_4$$

$$a\cdot\cos\theta_2+\frac{c\cdot\sin\theta_4\cos\theta_3-a\cdot\sin\theta_2\cdot\cos\theta_3}{\sin\theta_3}-c\cdot\cos\theta_4-d=0 \quad (11)$$

SUBSTITUTING (1) INTO (11)

$$a\cdot\cos\theta_2+\frac{c\cdot\sin\theta_4\cdot\cos(\theta_4-\gamma)-a\cdot\sin\theta_2\cdot\cos(\theta_4-\gamma)}{\sin(\theta_4-\gamma)}-c\cdot\cos\theta_4-d=0$$

$$a\cdot\cos\theta_2+\frac{c\cdot\sin\theta_4(\cos\theta_4\cos\gamma+\sin\theta_4\cdot\sin\gamma)-a\cdot\sin\theta_2(\cos\theta_4\cos\gamma+\sin\theta_4\cdot\sin\gamma)}{\sin\theta_4\cos\gamma-\cos\theta_4\sin\gamma}$$

$$-c\cdot\cos\theta_4-d=0$$

$$\frac{c\cdot\sin\theta_4\cdot\cos\theta_4\cdot\cos\gamma+c\cdot\sin\theta_4\cdot\sin\theta_4\cdot\sin\gamma-a\cdot\sin\theta_2\cdot\cos\theta_4\cdot\cos\gamma-a\cdot\sin\theta_2\cdot\sin\theta_4\cdot\sin\gamma}{\sin\theta_4\cos\gamma-\cos\theta_4\sin\gamma}$$

$$=c\cos\theta_4+d-a\cdot\cos\theta_2$$

$$c\cdot\cos\gamma\cdot\sin\theta_4\cdot\cos\theta_4+c\cdot\sin\gamma\cdot\sin^2\theta_4-a\cdot\cos\gamma\cdot\sin\theta_2\cos\theta_4-a\cdot\sin\gamma\cdot\sin\theta_2\sin\theta_4$$
$$=(c\cdot\cos\theta_4+d-a\cdot\cos\theta_2)\cdot(\sin\theta_4\cdot\cos\gamma-\cos\theta_4\cdot\sin\gamma)$$

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
AMPAD

$$c\cdot\cos\gamma\cdot\sin\theta_4\cdot\cos\theta_4 + c\cdot\sin\gamma\cdot\sin^2\theta_4 - a\cdot\cos\gamma\cdot\sin\theta_2\cdot\cos\theta_4 - a\cdot\sin\gamma\cdot\sin\theta_2\cdot\sin\theta_4$$
$$= c\cdot\cos\theta_4\cdot\sin\theta_4\cdot\cos\gamma - c\cdot\cos^2\theta_4\cdot\sin\gamma + d\cdot\sin\theta_4\cdot\cos\gamma - d\cdot\cos\theta_4\cdot\sin\gamma$$
$$- a\cdot\cos\theta_2\cdot\sin\theta_4\cdot\cos\gamma + a\cdot\cos\theta_2\,\cos\theta_4\cdot\sin\gamma$$

$$\cancel{c\cdot\cos\gamma\cdot\sin\theta_4\cdot\cos\theta_4} + \cancel{c\cdot\sin\gamma\cdot\sin^2\theta_4} - \cancel{a\cdot\cos\gamma\cdot\sin\theta_2\cdot\cos\theta_4} - \cancel{a\cdot\sin\gamma\cdot\sin\theta_2\cdot\sin\theta_4}$$
$$- \cancel{c\cdot\cos\theta_4\cdot\sin\theta_4\cdot\cos\gamma} + \cancel{c\cdot\cos^2\theta_4\cdot\sin\gamma} - d\cdot\sin\theta_4\cdot\cos\gamma + d\cdot\cos\theta_4\cdot\sin\gamma$$
$$+ \cancel{a\cdot\cos\theta_2\cdot\sin\theta_4\cdot\cos\gamma} - \cancel{a\cdot\cos\theta_2\cdot\cos\theta_4\cdot\sin\gamma} = 0$$

$$c\cdot\sin\gamma\cdot(\sin^2\theta_4+\cos^2\theta_4) - a\cdot(\sin\theta_2\cdot\cos\gamma+\sin\gamma\cdot\cos\theta_2)\cdot\cos\theta_4$$
$$+ a\cdot(\cos\theta_2\cdot\cos\gamma - \sin\theta_2\cdot\sin\gamma)\cdot\sin\theta_4 - d\cdot\cos\gamma\cdot\sin\theta_4 + d\sin\gamma\cdot\cos\theta_4 = 0$$

$$c\cdot\sin\gamma - a\cdot\sin(\theta_2+\gamma)\cdot\cos\theta_4 + a\cdot\cos(\theta_2+\gamma)\cdot\sin\theta_4 - d\cdot\cos\gamma\cdot\sin\theta_4$$
$$+ d\cdot\sin\gamma\cdot\cos\theta_4 = 0$$

$$[a\cdot\cos(\theta_2+\gamma) - d\cdot\cos\gamma]\cdot\sin\theta_4 + [-a\cdot\sin(\theta_2+\gamma) + d\cdot\sin\gamma]\cdot\cos\theta_4$$
$$+ c\cdot\sin\gamma = 0$$

$$\boxed{\begin{aligned} &K_1\cdot\sin\theta_4 + K_2\cdot\cos\theta_4 + K_3 = 0 && (12)\\ &K_1 = a\cdot\cos(\theta_2+\gamma) - d\cdot\cos\gamma && (13)\\ &K_2 = -a\cdot\sin(\theta_2+\gamma) + d\cdot\sin\gamma && (14)\\ &K_3 = c\cdot\sin\gamma && (15)\end{aligned}}$$

USING THE TRIGOMETRIC IDENTITIES

$$\sin 2\cdot\alpha = \frac{2\cdot\tan\alpha}{1+\tan^2\alpha} \quad\Rightarrow\quad \sin\theta = \frac{2\cdot\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \tag{16}$$

$$\cos 2\cdot\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha} \quad\Rightarrow\quad \cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \tag{17}$$

SUBSTITUTING (16) AND (17) INTO (12)

$$K_1\cdot\frac{2\cdot\tan\frac{\theta_4}{2}}{1+\tan^2\frac{\theta_4}{2}} + K_2\cdot\frac{1-\tan^2\frac{\theta_4}{2}}{1+\tan^2\frac{\theta_4}{2}} + K_3 = 0$$

$$2\cdot K_1\cdot\tan\frac{\theta_4}{2} + K_2 - K_2\cdot\tan^2\frac{\theta_4}{2} + K_3 + K_3\cdot\tan^2\frac{\theta_4}{2} = 0$$

$$(K_3-K_2)\cdot\tan^2\frac{\theta_4}{2} + 2\cdot K_1\cdot\tan\frac{\theta_4}{2} + (K_3+K_2) = 0$$

$$\tan^2\frac{\theta_4}{2} + \frac{2\cdot K_1}{K_3-K_2}\cdot\tan\frac{\theta_4}{2} + \frac{K_3+K_2}{K_3-K_2} = 0$$

$$\tan^2\frac{\Theta_4}{2} + \frac{2\cdot K_1}{K_3-K_2}\cdot\tan\frac{\Theta_4}{2} + \left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_1}{K_3-K_2}\right)^2 + \frac{K_3+K_2}{K_3-K_2} = 0$$

$$\left(\tan\frac{\Theta_4}{2} + \frac{K_1}{K_3-K_2}\right)^2 = \left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_3+K_2}{K_3-K_2}\right)$$

$$\tan\frac{\Theta_4}{2} = -\frac{K_1}{K_3-K_2} \pm \sqrt{\left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_3+K_2}{K_3-K_2}\right)}$$

$$\tan\frac{\Theta_4}{2} = -\frac{K_1}{K_3-K_2} \pm \sqrt{\left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_3+K_2}{K_3-K_2}\right)\left(\frac{K_3-K_2}{K_3-K_2}\right)}$$

$$\tan\frac{\Theta_4}{2} = -\frac{K_1}{K_3-K_2} \pm \sqrt{\frac{K_1^2 - (K_3^2-K_2^2)}{(K_3-K_2)^2}}$$

$$\tan\frac{\Theta_4}{2} = \frac{-K_1 \pm \sqrt{K_1^2+K_2^2-K_3^2}}{K_3-K_2}$$

$$\boxed{\Theta_4 = 2\cdot\tan^{-1}\left[\frac{-K_1 \pm \sqrt{K_1^2+K_2^2-K_3^2}}{K_3-K_2}\right]} \qquad (18)$$

THE SOLUTION OF THE INVERTED SLIDER CRANK LINKAGE STARTS WITH THE DEFINATION OF THE LINKAGE PARAMETERS

GIVEN: $a, c, d, \Theta_2,$ & γ

THE PARAMETERS THAT NEED TO BE DETERMINED INCLUDE

FIND: $b, \Theta_3,$ & Θ_4

THESE PARAMETERS ARE FOUND USING (1), (10), (13), (14), (15) AND (18)