4 BAR LINKAGE
ACCELERATION ANALYSIS

THE 4 BAR LINKAGE VELOCITY LOOP WAS FOUND TO BE

$$r_2\cdot\dot{\theta}_2\cdot\hat{e}_{\theta 2} + r_3\cdot\dot{\theta}_3\cdot\hat{e}_{\theta 3} = r_4\cdot\dot{\theta}_4\cdot\hat{e}_{\theta 4}$$

ALL PARAMETERS IN THIS EXPRESSION ARE KNOWN ONCE THE VELOCITY ANALYSIS HAS BEEN COMPLETED. THE ACCELERATION LOOP EQUATION IS FOUND BY TAKING THE DERIVATIVE OF THE VELOCITY LOOP WITH RESPECT TO TIME

$$\cancel{\dot{r}_2}^{0}\cdot\dot{\theta}_2\cdot\hat{e}_{\theta 2} + r_2\cdot\ddot{\theta}_2\cdot\hat{e}_{\theta 2} + r_2\cdot\dot{\theta}_2\cdot\dot{\hat{e}}_{\theta 2} + \cancel{\dot{r}_3}^{0}\cdot\dot{\theta}_3\cdot\hat{e}_{\theta 3} + r_3\cdot\ddot{\theta}_3\cdot\hat{e}_{\theta 3} + r_3\cdot\dot{\theta}_3\cdot\dot{\hat{e}}_{\theta 3}$$
$$= \cancel{\dot{r}_4}^{0}\cdot\dot{\theta}_4\cdot\hat{e}_{\theta 4} + r_4\cdot\ddot{\theta}_4\cdot\hat{e}_{\theta 4} + r_4\cdot\dot{\theta}_4\cdot\dot{\hat{e}}_{\theta 4}$$

$$r_2\cdot\ddot{\theta}_2\cdot\hat{e}_{\theta 2} + r_2\cdot\dot{\theta}_2\cdot(\dot{\theta}_2\hat{k}\times\hat{e}_{\theta 2}) + r_3\cdot\ddot{\theta}_3\cdot\hat{e}_{\theta 3} + r_3\cdot\dot{\theta}_3\cdot(\dot{\theta}_3\hat{k}\times\hat{e}_{\theta 3})$$
$$= r_4\cdot\ddot{\theta}_4\cdot\hat{e}_{\theta 4} + r_4\cdot\dot{\theta}_4\cdot(\dot{\theta}_4\hat{k}\times\hat{e}_{\theta 4})$$

$$r_2\cdot\ddot{\theta}_2\cdot\hat{e}_{\theta 2} - r_2\dot{\theta}_2^2\hat{e}_{r2} + r_3\cdot\ddot{\theta}_3\cdot\hat{e}_{\theta 3} - r_3\cdot\dot{\theta}_3^2\hat{e}_{r3} = r_4\cdot\ddot{\theta}_4\cdot\hat{e}_{\theta 4} - r_4\cdot\dot{\theta}_4^2\cdot\hat{e}_{r4} \qquad (17)$$

IN 4 BAR LINKAGES $\ddot{\theta}_2$ IS TYPICALLY GIVEN. ~~THIS~~ $\ddot{\theta}_2$ REPRESENTS THE ANGULAR ACCELERATION OF THE CRANK LINK. THE VARIABLES $\ddot{\theta}_3$ AND $\ddot{\theta}_4$ REMAIN UNKNOWN AND CAN BE SOLVED ~~A~~ USING THE TWO SCALAR EQUATIONS FOUND WHEN (17) IS DOTTED WITH $\hat{i}$ & $\hat{j}$.

DOTTING WITH $\hat{i}$

$$r_2\cdot\ddot{\theta}_2\cdot(-\sin\theta_2) - r_2\cdot\dot{\theta}_2^2\cdot(\cos\theta_2) + r_3\cdot\ddot{\theta}_3\cdot(-\sin\theta_3) - r_3\cdot\dot{\theta}_3^2\cdot(\cos\theta_3)$$
$$= r_4\cdot\ddot{\theta}_4\cdot(-\sin\theta_4) - r_4\cdot\dot{\theta}_4^2\,(\cos\theta_4)$$

$$-r_2\cdot\ddot{\theta}_2\cdot\sin\theta_2 - r_2\cdot\dot{\theta}_2^2\cdot\cos\theta_2 - r_3\cdot\ddot{\theta}_3\cdot\sin\theta_3 - r_3\cdot\dot{\theta}_3^2\cdot\cos\theta_3$$
$$= -r_4\cdot\ddot{\theta}_4\sin\theta_4 - r_4\cdot\dot{\theta}_4^2\cdot\cos\theta_4$$

$$r_2\cdot\ddot{\theta}_2\cdot\sin\theta_2 + r_2\cdot\dot{\theta}_2^2\cdot\cos\theta_2 + r_3\cdot\ddot{\theta}_3\sin\theta_3 + r_3\cdot\dot{\theta}_3^2\cdot\cos\theta_3$$
$$= r_4\cdot\ddot{\theta}_4\cdot\sin\theta_4 + r_4\cdot\dot{\theta}_4^2\cdot\cos\theta_4 \qquad (18)$$

DOTTING WITH $\hat{j}$

$$r_2\cdot\ddot{\theta}_2\cdot(\cos\theta_2) - r_2\cdot\dot{\theta}_2^2\cdot(\sin\theta_2) + r_3\cdot\ddot{\theta}_3\cdot(\cos\theta_3) - r_3\cdot\dot{\theta}_3^2\cdot(\sin\theta_3)$$
$$= r_4\cdot\ddot{\theta}_4(\cos\theta_4) - r_4\cdot\dot{\theta}_4^2(\sin\theta_4)$$

$$r_2\cdot\ddot{\theta}_2\cos\theta_2 - r_2\cdot\dot{\theta}_2^2\sin\theta_2 + r_3\cdot\ddot{\theta}_3\cdot\cos\theta_3 - r_3\cdot\dot{\theta}_3^2\cdot\sin\theta_3$$
$$= r_4\cdot\ddot{\theta}_4\cdot\cos\theta_4 - r_4\cdot\dot{\theta}_4^2\cdot\sin\theta_4 \qquad (19)$$

(18) IS SOLVED FOR $\ddot{\theta}_4$

$$\boxed{\ddot{\theta}_4 = \frac{r_2\cdot\ddot{\theta}_2\sin\theta_2 + r_2\cdot\dot{\theta}_2^2\cdot\cos\theta_2 + r_3\cdot\ddot{\theta}_3\cdot\sin\theta_3 + r_3\cdot\dot{\theta}_3^2\cdot\cos\theta_3 - r_4\cdot\dot{\theta}_4^2\cdot\cos\theta_4}{r_4\cdot\sin\theta_4}} \qquad (20)$$

(20) CAN NOW BE SUBSTITUTED INTO (19) TO CREATE AN EXPRESSION FOR $\ddot{\theta}_3$

$$r_2\cdot\ddot{\theta}_2\cdot\cos\theta_2 - r_2\cdot\dot{\theta}_2^2\cdot\sin\theta_2 + r_3\cdot\ddot{\theta}_3\cdot\cos\theta_3 - r_3\cdot\dot{\theta}_3^2\cdot\sin\theta_3 + r_4\cdot\dot{\theta}_4^2\cdot\sin\theta_4$$

$$= \frac{\cancel{r_4}\cdot\cos\theta_4}{\cancel{r_4}\cdot\sin\theta_4}\cdot\left[r_2\cdot\ddot{\theta}_2\cdot\sin\theta_2 + r_2\cdot\dot{\theta}_2^2\cdot\cos\theta_2 + r_3\cdot\ddot{\theta}_3\cdot\sin\theta_3 + r_3\cdot\dot{\theta}_3^2\cdot\cos\theta_3 - r_4\cdot\dot{\theta}_4^2\cdot\cos\theta_4\right]$$

$$r_2\cdot\ddot{\theta}_2\cos\theta_2\cdot\sin\theta_4 - r_2\cdot\dot{\theta}_2^2\sin\theta_2\cdot\sin\theta_4 + r_3\cdot\ddot{\theta}_3\cdot\cos\theta_3\cdot\sin\theta_4 - r_3\cdot\dot{\theta}_3^2\cdot\sin\theta_3\cdot\sin\theta_4$$
$$+ r_4\cdot\dot{\theta}_4^2\cdot\sin^2\theta_4 = r_2\cdot\ddot{\theta}_2\cdot\sin\theta_2\cdot\cos\theta_4 + r_2\cdot\dot{\theta}_2^2\cdot\cos\theta_2\cdot\cos\theta_4 + r_3\cdot\ddot{\theta}_3\cdot\sin\theta_3\cdot\cos\theta_4$$
$$+ r_3\cdot\dot{\theta}_3^2\cdot\cos\theta_3\cdot\cos\theta_4 - r_4\cdot\dot{\theta}_4^2\cdot\cos^2\theta_4$$

$$r_2\cdot\ddot{\theta}_2\cos\theta_2\sin\theta_4 - r_2\cdot\dot{\theta}_2^2\cdot\sin\theta_2\cdot\sin\theta_4 + r_3\cdot\ddot{\theta}_3\cdot\cos\theta_3\cdot\sin\theta_4 - r_3\cdot\dot{\theta}_3^2\cdot\sin\theta_3\cdot\sin\theta_4$$
$$+ r_4\cdot\dot{\theta}_4^2\cdot\sin^2\theta_4 - r_2\cdot\ddot{\theta}_2\cdot\sin\theta_2\cdot\cos\theta_4 - r_2\cdot\dot{\theta}_2^2\cdot\cos\theta_2\cdot\cos\theta_4 - r_3\cdot\ddot{\theta}_3\cdot\sin\theta_3\cdot\cos\theta_4$$
$$- r_3\cdot\dot{\theta}_3^2\cdot\cos\theta_3\cdot\cos\theta_4 + r_4\cdot\dot{\theta}_4^2\cdot\cos^2\theta_4 = 0$$

$$r_2\cdot\ddot{\theta}_2\cdot[\cos\theta_2\cdot\sin\theta_4 - \sin\theta_2\cdot\cos\theta_4] - r_2\cdot\dot{\theta}_2^2\cdot[\cos\theta_2\cdot\cos\theta_4 + \sin\theta_2\cdot\sin\theta_4]$$
$$+ r_3\cdot\ddot{\theta}_3[\cos\theta_3\cdot\sin\theta_4 - \sin\theta_3\cdot\cos\theta_4] - r_3\cdot\dot{\theta}_3^2[\cos\theta_3\cdot\cos\theta_4 + \sin\theta_3\cdot\sin\theta_4]$$
$$+ r_4\cdot\dot{\theta}_4^2[\cos^2\theta_4 + \sin^2\theta_4] = 0$$

$$r_2\cdot\ddot{\theta}_2\cdot\sin(\theta_4-\theta_2) - r_2\cdot\dot{\theta}_2^2\cdot\cos(\theta_4-\theta_2) + r_3\cdot\ddot{\theta}_3\sin(\theta_4-\theta_3)$$
$$- r_3\cdot\dot{\theta}_3^2\cdot\cos(\theta_4-\theta_3) + r_4\cdot\dot{\theta}_4^2 = 0$$

$$\boxed{\ddot{\theta}_3 = -\frac{r_2\cdot\ddot{\theta}_2\cdot\sin(\theta_4-\theta_2) - r_2\cdot\dot{\theta}_2^2\cdot\cos(\theta_4-\theta_2) - r_3\cdot\dot{\theta}_3^2\cdot\cos(\theta_4-\theta_3) + r_4\cdot\dot{\theta}_4^2}{r_3\cdot\sin(\theta_4-\theta_3)}} \quad (21)$$

$$= -\frac{r_2\ddot{\theta}_2(\sin\theta_4\cos\theta_2 - \sin\theta_2\cos\theta_4) - r_2\cdot\dot{\theta}_2^2\cdot(\cos\theta_4\cos\theta_2 + \sin\theta_4\sin\theta_2) - r_3\cdot\dot{\theta}_3^2(\cos\theta_4\cos\theta_3 + \sin\theta_4\sin\theta_3) + r_4\cdot\dot{\theta}_4^2}{r_3(\sin\theta_4\cos\theta_3 - \sin\theta_3\cos\theta_4)}$$