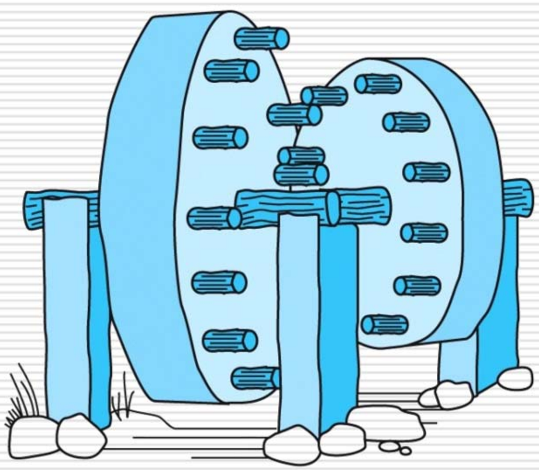
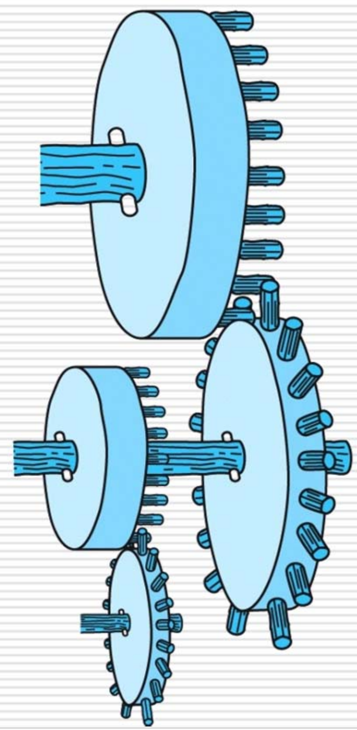
Introduction to Gears

- Basic Nomenclature
- Conjugate Action/Involute

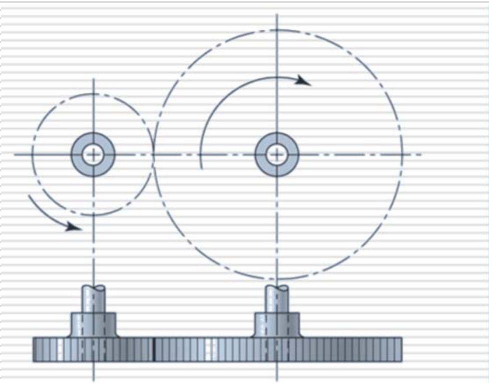


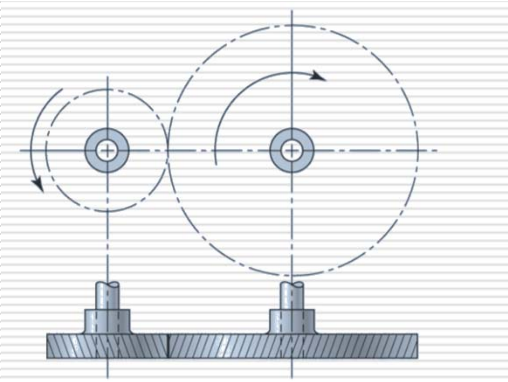
Right-angle gearing



Gears are Classified into Four Basic Types

SPUR

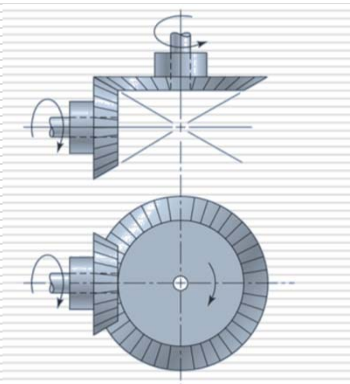


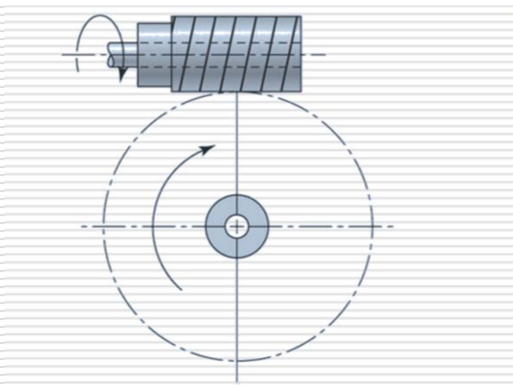


Helical

Bevel

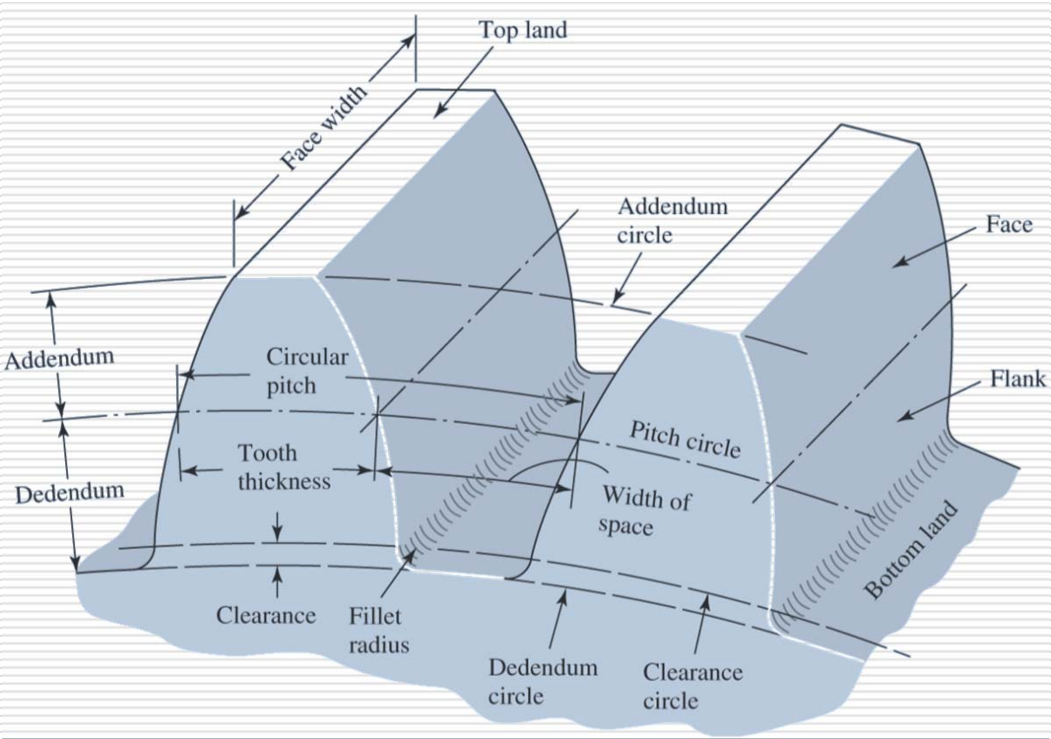
- Straight Tooth
- Spiral
- Hypoid





Worm

Gear Nomenclature



$$P = \frac{N}{d} \equiv \frac{\text{Diametral}}{\text{Pitch}}\left[\frac{\text{teeth}}{\text{in}}\right]$$

$$m = \frac{d}{N} \equiv \text{Module [mm]}$$

$$p = \frac{\pi \cdot d}{N} = \pi \cdot m \equiv \frac{\text{Circular}}{\text{Pitch}}[\text{in or mm}]$$

$$= \frac{\pi \cdot d}{N} = \frac{\pi}{P}$$

$$p \cdot P = \pi$$

$d \equiv$ Pitch Diameter [in or mm]

N $\equiv$ Number of Teath

General Use Tooth Sizes

Diametral Pitch	
Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Modules	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Standard Tooth Systems for Spur Gears

Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

Common Face Width

$$3p < F < 5p$$

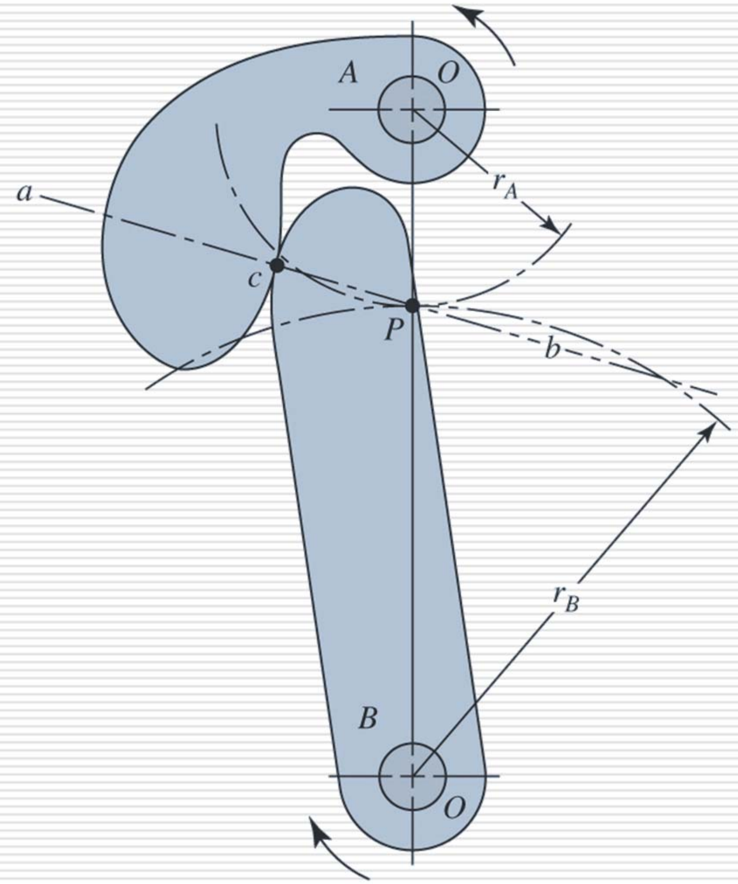
$$p = \frac{\pi}{P}$$

$$\frac{3\pi}{P} < F < \frac{5\pi}{P}$$

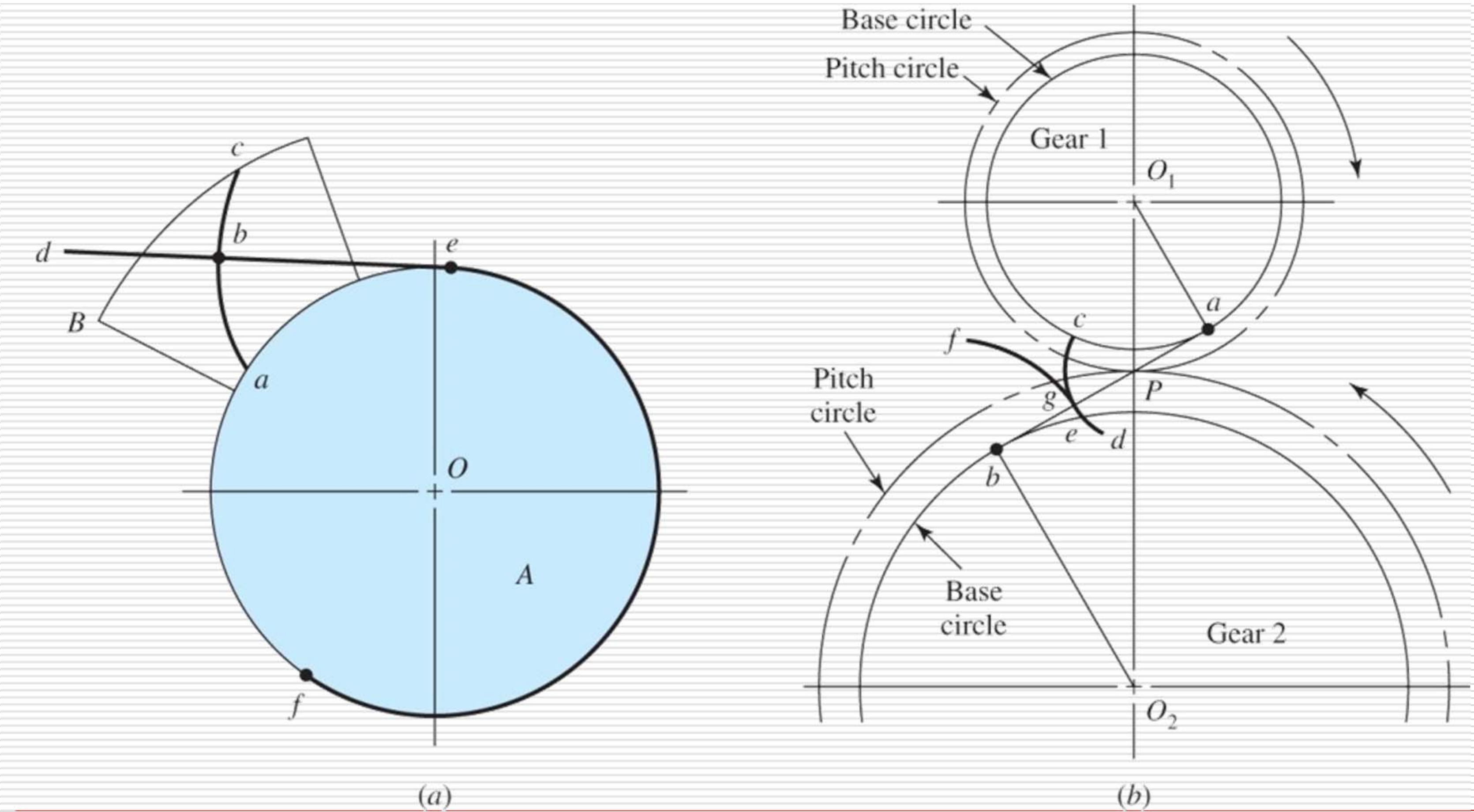
Conjugate Action

- Conjugate Action
  - Surfaces Roll/Slide Against Each Other
  - Produce Constant Angular Velocity

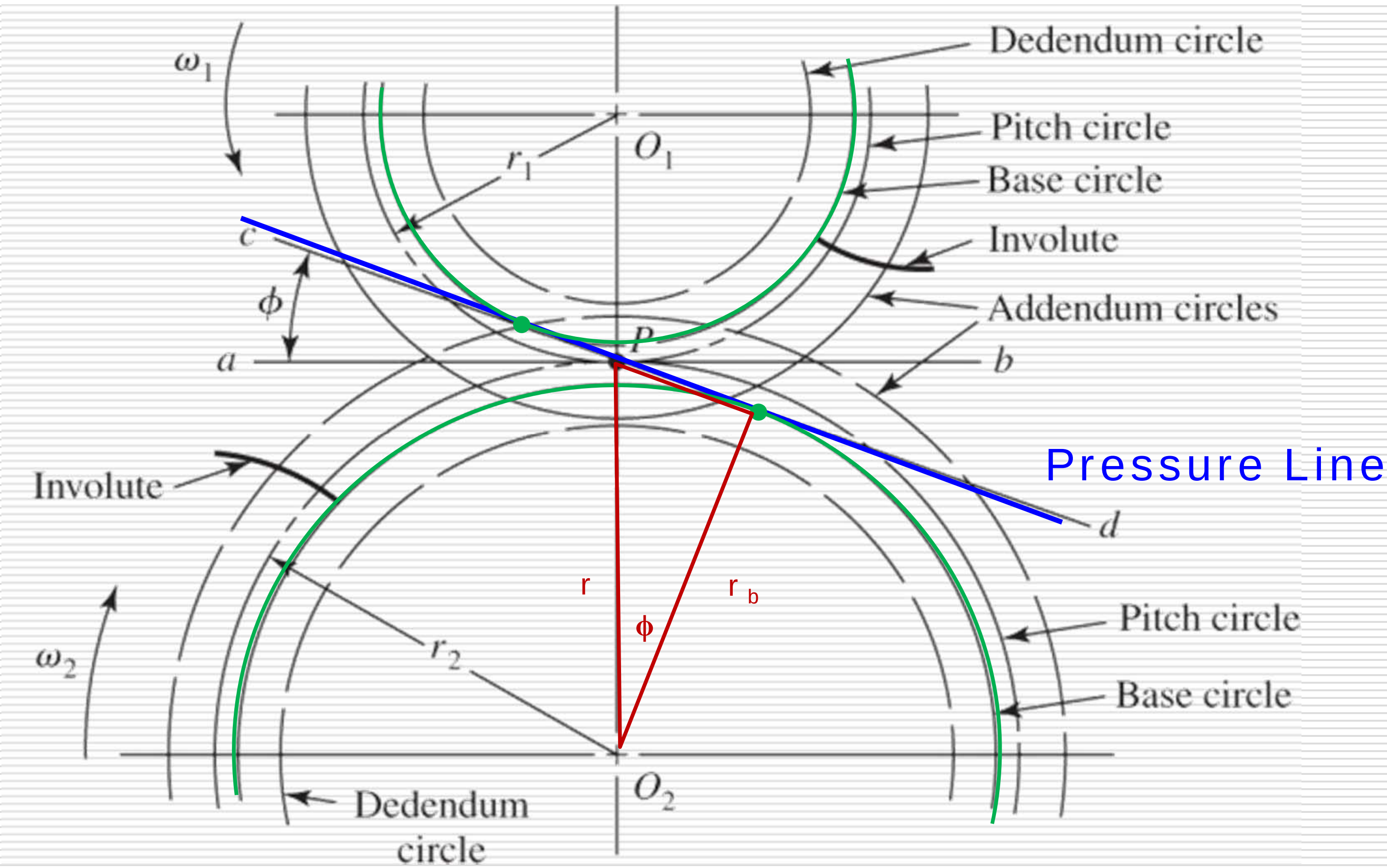
- Instant Center of Velocity Between Bodies Stationary
  - Between Ground ICs



Understanding the Importance of the Involute

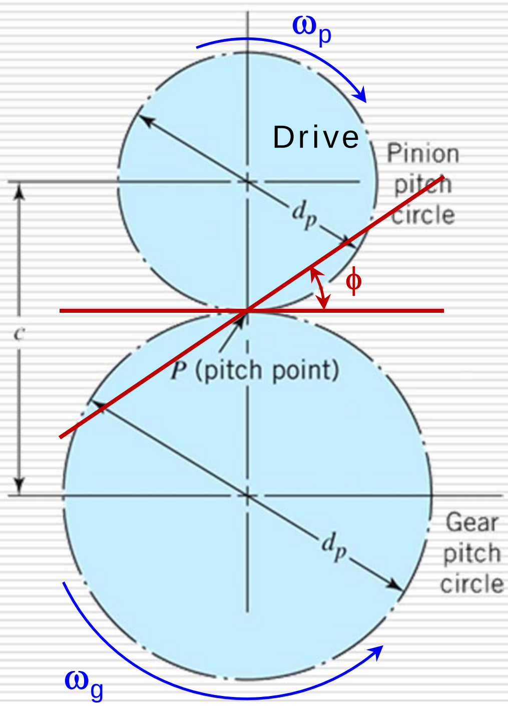


Circles of a Gear Layout

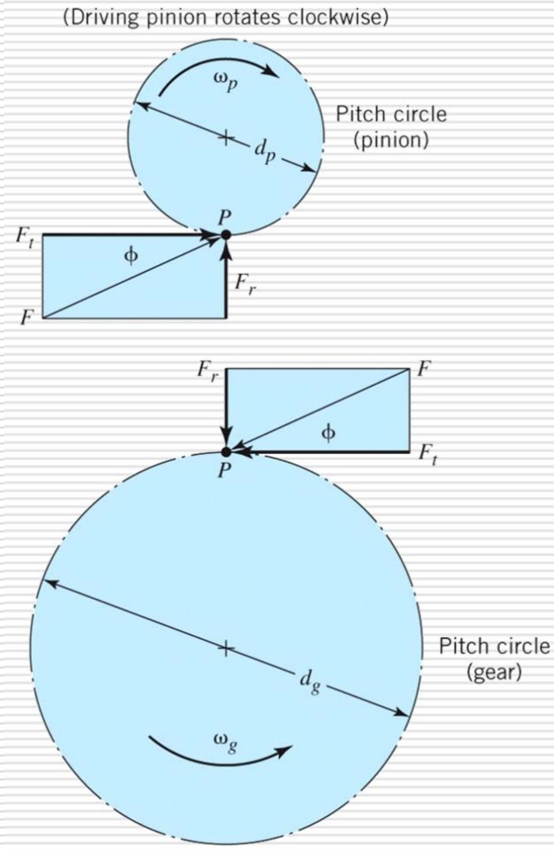


Force Analysis

- F_t x v accounts for power transmission
- F_r does no work



Line of Action of the Force



Torque on Shafts

$$T_p = F_{tp} \cdot \frac{d_p}{2} = F_{tp} \cdot r_p$$

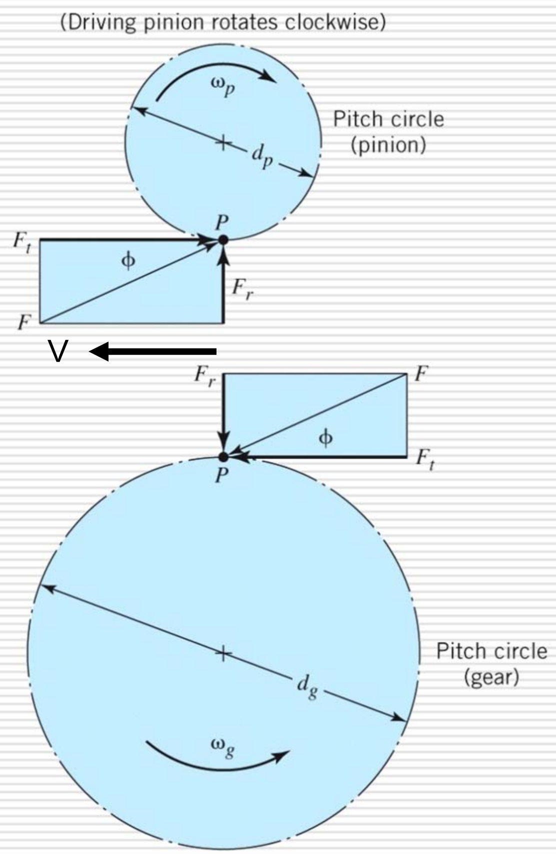
$$T_g = F_{tg} \cdot \frac{d_g}{2} = F_{tg} \cdot r_g$$

Subscripts: p-pinion, g-gear

$T \equiv$ Torque

$F_t \equiv$ Tangential Force at Pitch Point

Force Analysis



Pitch Line Velocity (feet / min)

$$V = r_p \cdot \omega_p = r_g \cdot \omega_g \quad \left[{rad}/{s} \right] or \left[1/s \right]$$

$$= \frac{\pi \cdot d_p \cdot n_p}{12} = \frac{\pi \cdot d_g \cdot n_g}{12} \quad \left[ft/\text{min} \right]$$

Subscripts: p-pinion, g-gear

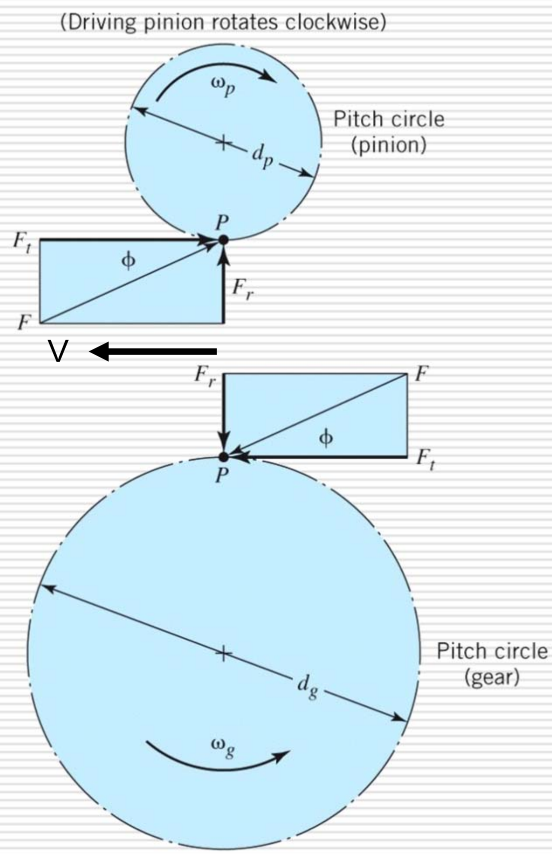
$n \equiv$ speed of the shaft, [rev/min]

$\omega \equiv$ speed of the shaft, [rad/s]

$r \equiv$ pitch radius, [in]

$d \equiv$ pitch diameter, [in]

Power Transmission



Transmitted Power in horsepower [hp], Imperial Units

$$H\left[hp\right] = \frac{F_t[lb]\cdot V[{}^{ft}/_{\min}]}{33,000\left[{}^{ft\cdot lb}/_{hp\cdot \min}\right]} = \frac{T[lb\cdot in]\cdot n[{}^{rev}/_{\min}]}{63025\cdot[\frac{lb\cdot in\cdot rev}{\min\cdot hp}]}$$

Subscripts: p-pinion, g-gear

$H \equiv$ Power

$F_t \equiv$ Tangential Force at pitch Point

$T \equiv$ Torque

$V \equiv$ pitch line velocity

$n \equiv$ angular velocity

Transmitted Power in kilo - Watts [kW], SI Units

$$H\left[kW\right] = F_t \cdot V$$

Subscripts: p-pinion, g-gear

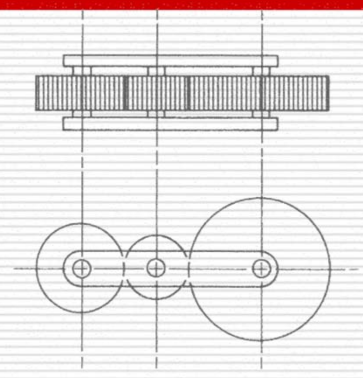
$H \equiv$ Power, [kW]

$F_t \equiv$ Tangential Force at pitch Point, [N]

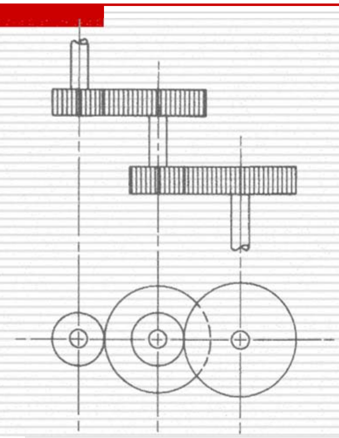
$$V\left[{}^{m}/_{s}\right] \equiv \text{pitch line velocity} = \frac{\pi\cdot d[\text{mm}]\cdot n[{}^{rev}/_{\min}]}{60,000}$$

$d \equiv$ pitch diameter

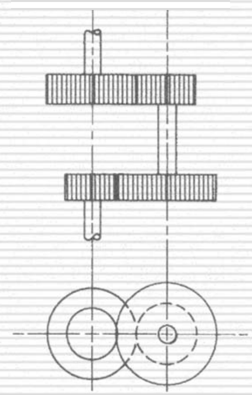
Gear Trains



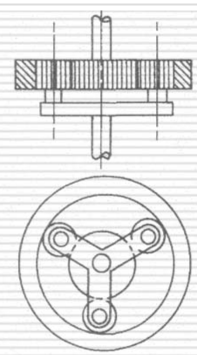
Simple



Compound

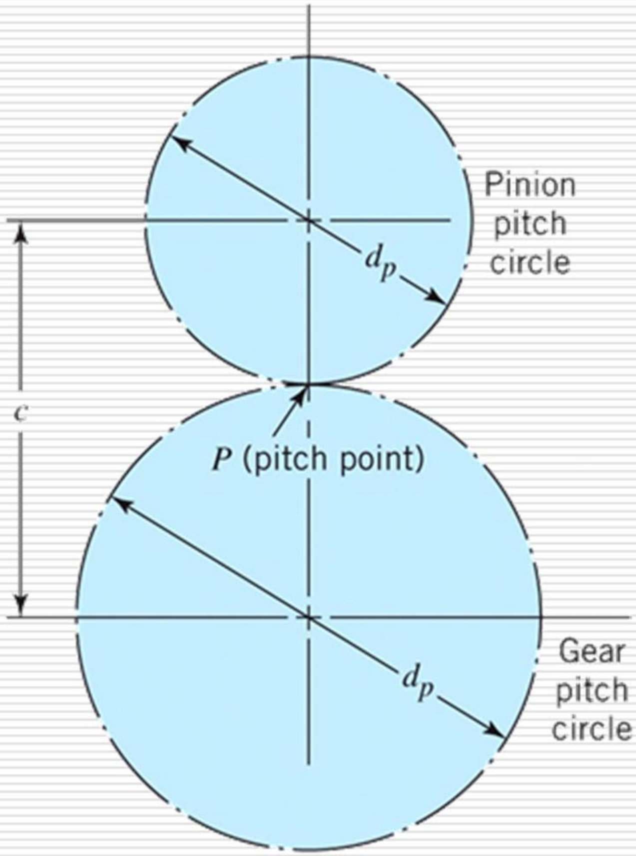


Reverted



Planetary

Gear Train's Gear Ratio



Gear Ratio

$$\left|\frac{\omega_p}{\omega_g}\right| = \left|\frac{n_g}{n_p}\right| = \frac{d_g}{d_p} = \frac{N_g}{N_p}$$

Subscripts: p-pinion, g-gear

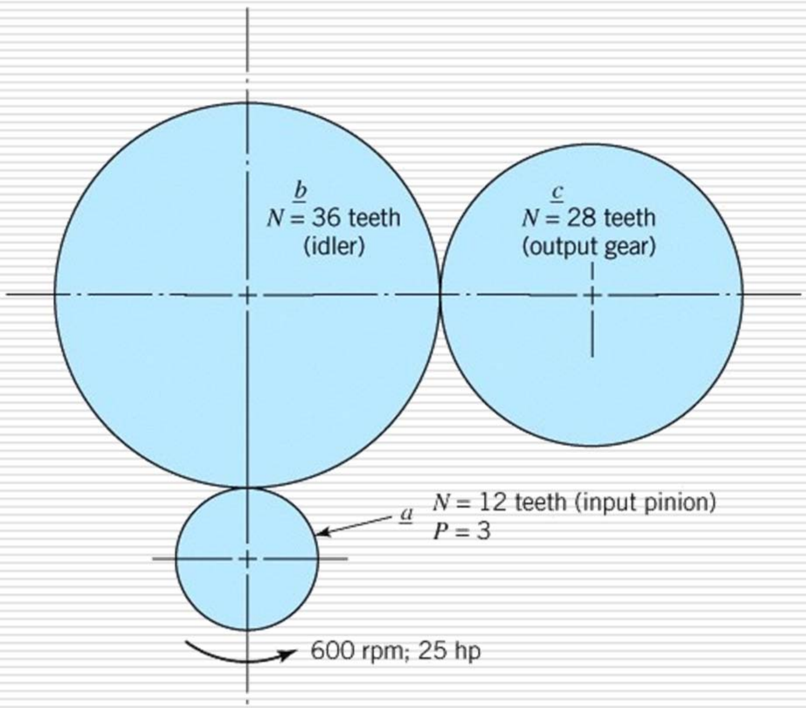
$n \equiv$ speed of the shaft, [rev/min]

$\omega \equiv$ speed of the shaft, [rad/s]

$d \equiv$ pitch diameter, [in]

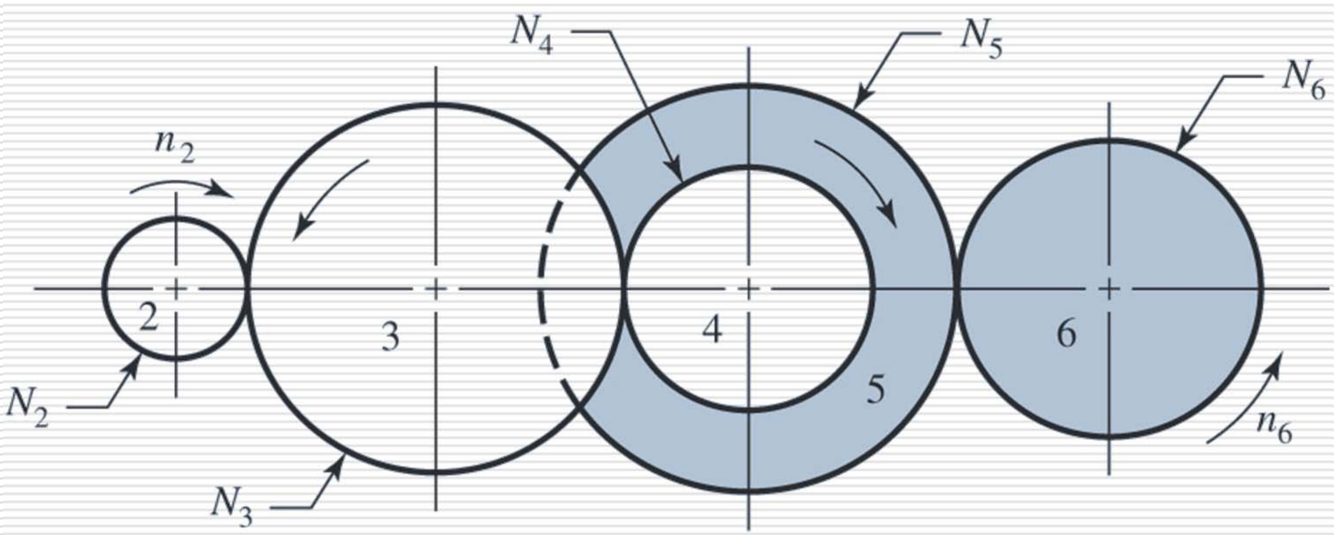
$N \equiv$ teeth

Example



The three gears shown have a pitch of 3, a pressure angle of 20°. Gear *a* is the driving, or input, pinion. It rotates counterclockwise at 600-rpm and transmits 25-hp to the idler gear *b*. Output gear *c* is attached to a shaft that drives a machine. Nothing is attached to the idler shaft, and friction losses in the bearings and gears can be neglected. Determine the resultant load applied by the idler to its shaft.

Gear Train: Train Value



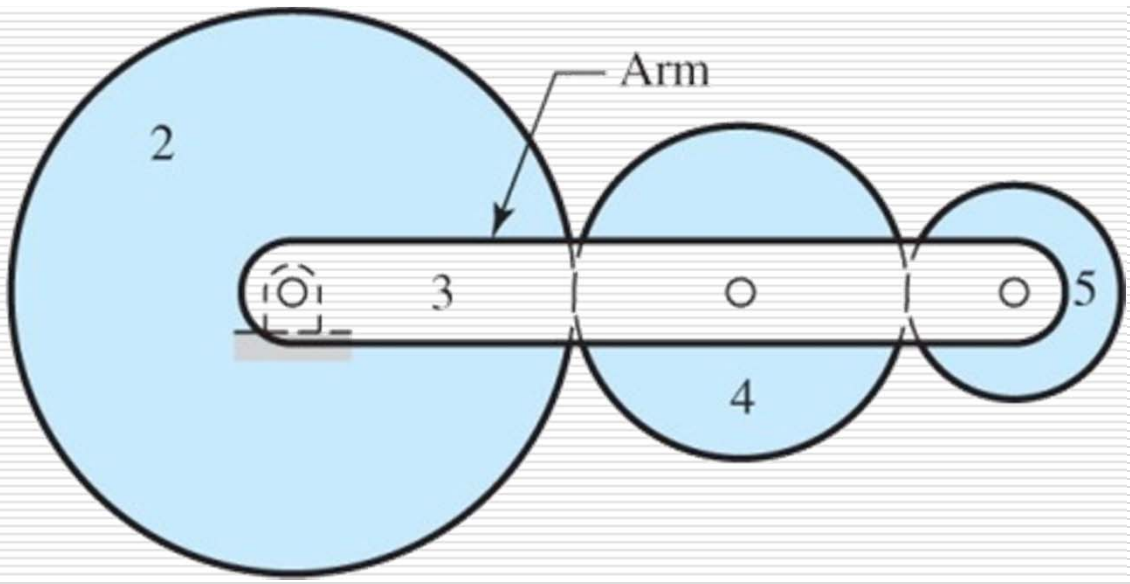
$$e \equiv \text{Train Value} = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}$$

Example

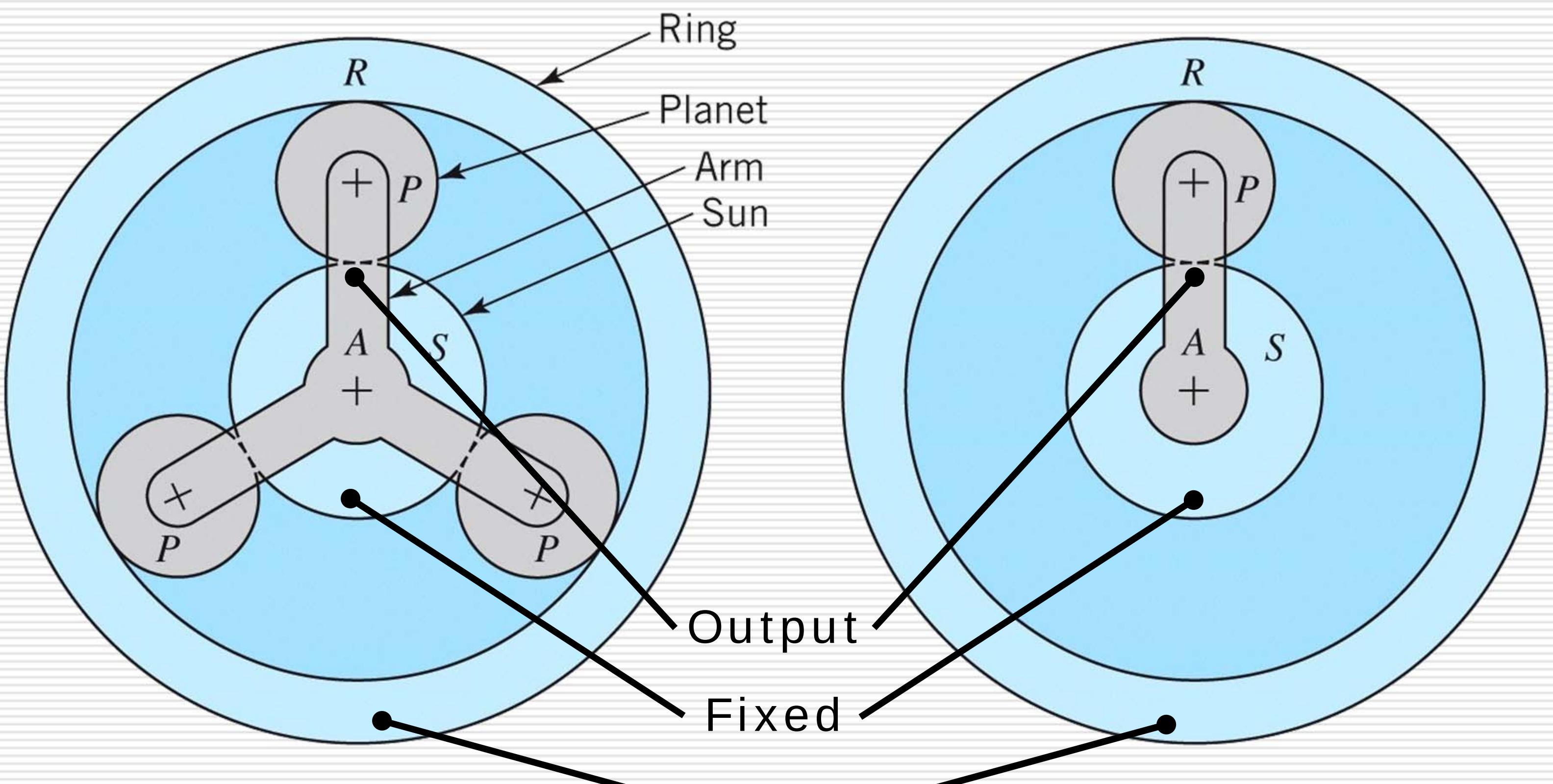
A gearset consists of a 16 tooth pinion driving a 40 tooth gear. The gears are cut using a pressure angle of 20°

a. Compute the addendum and dedendum
b. Compute the circular pitch, the center distance, and the radii of the base circles
c. In mounting these gears, the center distance was incorrectly made ¼ in larger. Compute the new values of the pressure angle and the pitch-circle diameters.

Gear Train on the Arm of a Planetary Gear Train



Planetary or Epicyclic Gear Train



(a)
With three planets (typical)

(b)
With one planet (for analysis only)