EXAMPLE 2 CONSIDER THE ASSEMBLAGE OF THE TRUSS ELEMENTS SHOWN. DETERMINE THE NODAL FORCES, NODAL DISPLACEMENTS,

GIVEN:

1. THREE ELEMENT STRUCTURE
   - TWO OF THE ELEMENTS ARE IN PARALLEL
   - THE THIRD ELEMENT IS IN SERIES WITH THE PARALLEL ELEMENTS
2. THE STIFFNESSES OF THE ELEMENTS ARE: $k_1 = 50\,\text{lb/in}$, $k_2 = 30\,\text{lb/in}$, $k_3 = 70\,\text{lb/in}$
3. THE PARALLEL ELEMENTS ARE ATTACHED TO THE WALL
3. AN EXTERNAL LOAD OF 40 lb IS APPLIED TO THE STRUCTURE

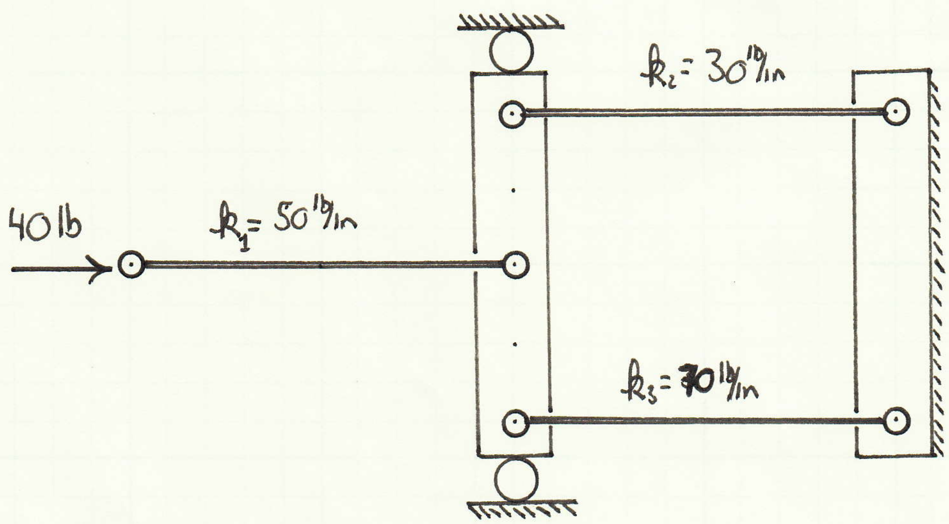
ASSUMPTIONS:

1. ALL ELEMENTS ARE TRUSS ELEMENTS; TWO FORCE MEMBERS
2. ALL SUPPORTING STRUCTURES ARE RIGID
3. ALL FORCES AND DISPLACEMENTS ARE IN THE HORIZONTAL DIRECTION.

FIND:

1. NODAL FORCES
2. NODAL DISPLACEMENTS

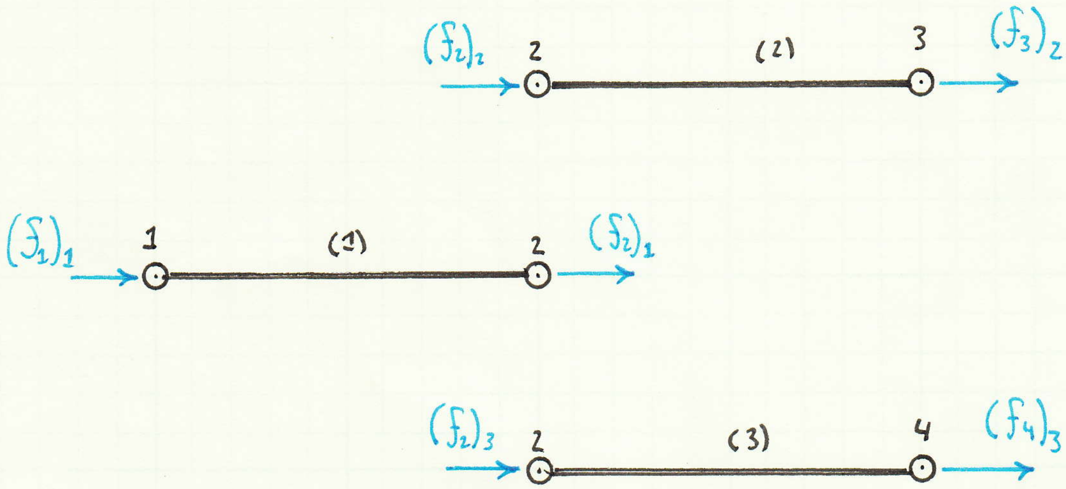
FIGURE:



(a)

SOLUTION:

FIRST THE STRUCTURE IS BROKEN DOWN INTO ELEMENTS.



(b)

CALCULATING THE STIFFNESS MATRICES FOR EACH ELEMENT

ELEMENT 1

$$\begin{Bmatrix} (f_1)_1 \\ (f_2)_1 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} (u_1)_1 \\ (u_2)_1 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \qquad (1)$$

$$\begin{Bmatrix} (f_2)_2 \\ (f_3)_2 \end{Bmatrix} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} (u_2)_2 \\ (u_3)_2 \end{Bmatrix} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} \qquad (2)$$

$$\begin{Bmatrix} (f_2)_3 \\ (f_4)_3 \end{Bmatrix} = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} (u_2)_3 \\ (u_4)_3 \end{Bmatrix} = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} \qquad (3)$$

KNOWING THAT FROM INTERNAL EQUILIBRIUM CONSIDERATIONS

$$F_1 = (f_1)_1 = 40\,\text{lb} \qquad (4)$$

$$F_2 = (f_2)_1 + (f_2)_2 + (f_2)_3 = 0 \qquad (5)$$

$$F_3 = (f_3)_2 \qquad (6)$$

$$F_4 = (f_4)_3 \qquad (7)$$

THE GLOBAL STIFFNESS MATRIX IS CONSTRUCTED BY FIRST EXPANDING (1) - (3)

$$(1) \rightarrow \quad (f_1)_1 = k_1 \cdot u_1 - k_1 \cdot u_2 \tag{8}$$

$$(f_2)_1 = -k_1 \cdot u_1 + k_1 \cdot u_2 \tag{9}$$

$$(2) \rightarrow \quad (f_2)_2 = k_2 \cdot u_2 - k_2 u_3 \tag{10}$$

$$(f_3)_2 = -k_2 \cdot u_2 + k_2 \cdot u_3 \tag{11}$$

$$(3) \rightarrow \quad (f_2)_3 = k_3 \cdot u_2 - k_3 \cdot u_4 \tag{12}$$

$$(f_4)_3 = -k_3 \cdot u_2 + k_3 \cdot u_4 \tag{13}$$

(8) - (13) ARE NOW SUBSTITUTED INTO (4) - (7)

$$F_1 = (f_1)_1 = k_1 \cdot u_1 - k_1 \cdot u_2 = 40\,\text{lb} \tag{14}$$

$$F_2 = (f_2)_1 + (f_2)_2 + (f_2)_3 = 0$$

$$= -k_1 \cdot u_1 + k_1 \cdot u_2 + k_2 \cdot u_2 - k_2 u_3 + k_3 \cdot u_2 - k_3 \cdot u_4 = 0$$

$$= -k_1 \cdot u_1 + (k_1 + k_2 + k_3) \cdot u_2 - k_2 \cdot u_3 - k_3 \cdot u_4 = 0 \tag{15}$$

$$F_3 = (f_3)_2 = -k_2 \cdot u_2 + k_2 \cdot u_3 \tag{16}$$

$$F_4 = (f_4)_3 = -k_3 \cdot u_2 + k_3 u_4 \tag{17}$$

(14) - (17) CAN NOW BE WRITTEN IN MATRIX FORM

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{Bmatrix} 40\,\text{lb} \\ 0 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & (k_1+k_2+k_3) & -k_2 & -k_3 \\ 0 & -k_2 & k_2 & 0 \\ 0 & -k_3 & 0 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \tag{18}$$

FROM THE PROBLEM STATEMENT AND FIGURE IT IS SEEN THAT $u_3 = u_4 = 0$. THESE RESULTS CAN NOW BE ENTERED INTO (18)

$$\begin{Bmatrix} 40\text{lb} \\ 0\text{lb} \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & (k_1+k_2+k_3) & -k_2 & -k_3 \\ 0 & -k_2 & k_2 & 0 \\ 0 & -k_3 & 0 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \end{Bmatrix} \qquad (19)$$

(19) NOW HAS TO BE PARTITIONED. STARTING BY TAKING OUT THE ROWS ASSOCIATED WITH THE KNOWN FORCES

$$\begin{Bmatrix} 40\text{lb} \\ 0\text{lb} \end{Bmatrix} = \left[\begin{array}{cc|cc} k_1 & -k_1 & 0 & 0 \\ -k_1 & (k_1+k_2+k_3) & -k_2 & -k_3 \end{array}\right] \begin{Bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \end{Bmatrix}$$

THIS CAN FURTHER BE REDUCED

$$\begin{Bmatrix} 40\text{lb} + (0)(0) + (0)(0) \\ 0\text{lb} + (k_2)(0) + k_3(0) \end{Bmatrix} = \begin{Bmatrix} 40\text{lb} \\ 0\text{lb} \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1+k_2+k_3) \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \qquad (20)$$

ONCE THE TOP TWO ROWS ARE REMOVED FROM (19) THE REMAINING COMPONENTS OF THE MATRIX HAVE KNOWN DISPLACEMENTS AND UNKNOWN FORCES

$$\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} 0 & -k_2 & k_2 & 0 \\ 0 & -k_3 & 0 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \end{Bmatrix} \qquad (21)$$

TO FIND F_3 & F_4, u_1 & u_2 MUST FIRST BE SOLVED FOR. FROM (20) u_1 & u_2 ARE FOUND BY TAKING THE INVERSE OF THE STIFFNESS MATRIX

$$\begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1+k_2+k_3) \end{bmatrix}^{-1} \begin{Bmatrix} 40\text{lb} \\ 0\text{lb} \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1+k_2+k_3) \end{bmatrix}^{-1} \begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1+k_2+k_3) \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \begin{Bmatrix} 1.2\text{in} \\ 0.4\text{in} \end{Bmatrix} \qquad (22)$$

SUBSTITUTING THE RESULTS IN (22) INTO (21), THE UNKNOWN FORCES CAN NOW BE SOLVED.

$$\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} 0 & -k_2 & k_2 & 0 \\ 0 & -k_3 & 0 & k_3 \end{bmatrix} \begin{Bmatrix} 1.2\text{in} \\ 0.4\text{in} \\ 0 \\ 0 \end{Bmatrix}$$

$$= \begin{bmatrix} 0 & -30\text{ lb/in} & 30\text{ lb/in} & 0 \\ 0 & -70\text{ lb/in} & 0 & 70\text{ lb/in} \end{bmatrix} \begin{Bmatrix} 1.2\text{in} \\ 0.4\text{in} \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -12\text{ lb} \\ -28\text{ lb} \end{Bmatrix} \quad (23)$$

THE MATRIX CALCULATIONS WERE MADE USING THE MINV MATRIX INVERSION FUNCTION AND THE MMULT MATRIX MULTIPLICATION FUNCTION IN EXCEL. THE EXCEL CALCULATIONS ARE FOUND ON THE NEXT PAGE.

SUMMARY:

THIS EXAMPLE ILLUSTRATES THE CONSTRUCTION OF THE GLOBAL STIFFNESS MATRIX FOR A PARALLEL SYSTEM OF ELEMENTS AND HOW THE MATRIX IS PARTITIONED TO SOLVE FOR THE UNKNOWN FORCES AND DISPLACEMENTS.

MER311: Advanced Mechanics, Lecture 03, Example 02

$$\begin{Bmatrix} 40 \\ 0 \end{Bmatrix} = \begin{bmatrix} 50 & -50 \\ -50 & 150 \end{bmatrix} \begin{Bmatrix} u1 \\ u2 \end{Bmatrix}$$

$$\begin{Bmatrix} u1 \\ u2 \end{Bmatrix} = \begin{bmatrix} 0.03 & 0.01 \\ 0.01 & 0.01 \end{bmatrix} \begin{Bmatrix} 40 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 1.2 \\ 0.4 \end{Bmatrix}$$

$$\begin{Bmatrix} F3 \\ F4 \end{Bmatrix} = \begin{bmatrix} 0 & -30 & 30 & 0 \\ 0 & -70 & 0 & 70 \end{bmatrix} \begin{Bmatrix} 1.2 \\ 0.4 \\ 0 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -12 \\ -28 \end{Bmatrix}$$