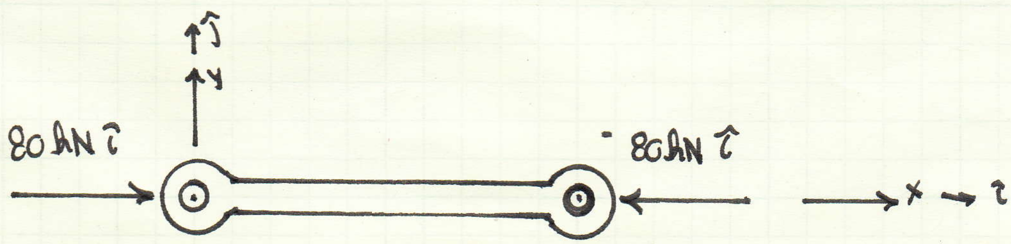
PROBLEM STATEMENT: AN INDUSTRIAL MACHINE REQUIRES A SOLID, ROUND PISTON CONNECTING ROD 200 mm LONG (BETWEEN PINNED ENDS) THAT IS SUBJECTED TO A MAXIMUM COMPRESSIVE FORCE OF 80 kN. USING A SAFETY FACTOR OF 2.5, WHAT DIAMETER IS REQUIRED IF ALUMINUM IS USED, HAVING PROPERTIES OF $S_y = 496\ MPa$, $E = 71\ GPa$.

GIVEN:
1. 200 mm LONG ROD WITH PINNED ENDS
2. SOLID CROSS-SECTION, CIRCULAR
3. MAX COMPRESSIVE FORCE 80 kN
4. FACTOR OF SAFETY 2.5
5. ALUMINUM

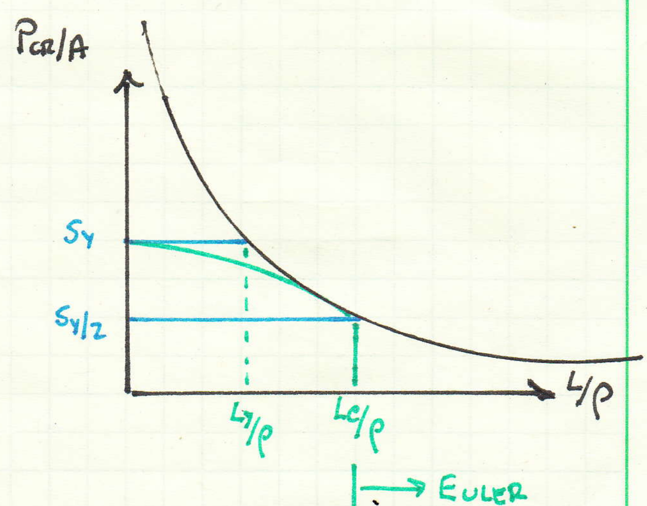
ASSUMPTIONS:
1. LOAD THROUGH CENTER OF CROSS-SECTION
2. LINEAR ELASTIC MATERIAL RESPONSE
3. SMALL DEFLECTIONS

FIND:
1. REQUIRED DIAMETER

FIGURE:



SOLUTION:



P_{CR} IS GOING TO BE DEFINED AS THE MAX LOAD TIMES THE FoS

$$P_{CR} = 80\ kN \cdot 2.5$$

$$= \underline{\underline{200\ kN}} \quad (1)$$

NOW, DETERMINE IF THE BEAM IS IN THE EULER RANGE

FOR THE PINNED - PINNED CASE, $K = 1$

$$\frac{P_{CR}}{A} = \frac{\pi^2 \cdot E}{K^2 \cdot (L/\rho)^2} = \frac{\pi^2 \cdot E}{(L/\rho)^2} \quad (1)$$

$P_{CR} = 200\,kN$

$A = \pi \cdot d^2/4$

$$\rho = \sqrt{\frac{I}{A}} = \sqrt{\frac{\pi \cdot d^4/64}{\pi d^2/4}} = \sqrt{\frac{d^2}{16}} = \frac{d}{4} \quad (2)$$

$$\frac{P_{CR} \cdot 4}{\pi \cdot d^2} = \frac{\pi^2 E \cdot d^2}{L^2 \cdot 16} \Rightarrow d = \sqrt[4]{\frac{P_{CR} \cdot L^2 \cdot 4 \cdot 16}{\pi^2 \cdot E}}$$

$$= \sqrt[4]{\frac{200\,kN \cdot (0.2m)^2 \cdot 64}{\pi^2 \cdot 71(10^9)\frac{N}{m^2}}}$$

$$= 0.02924\,m$$

$$= \underline{\underline{29.2mm}} \quad (3)$$

THE ~~CRITICAL~~ SLENDERNESS RATIO FOR THIS BEAM CAN NOW BE CALCULATED, USING (2)

$$\frac{L}{\rho} = \frac{0.2m \cdot 4}{0.0292m} = \underline{\underline{27.4}} \quad (4)$$

THE CRITICAL SLENDERNESS RATIO IN (4) NOW NEEDS TO BE COMPARED TO THE CRITICAL SLENDERNESS RATIO FOR EULER BUCKLING CAN NOW BE COMPUTED

$$\left(\frac{L}{\rho}\right)_{CR} = \left(L_e/\rho\right) = \sqrt{\frac{\pi^2 E}{1/2\,(P/A)_{CR}}} = \sqrt{\frac{\pi^2 E}{1/2 \cdot S_y}} =$$

$$= \sqrt{\frac{\pi^2 \cdot 71 \times 10^9\,N/m^2}{\frac{1}{2} \cdot 496 \times 10^6\,N/m^2}} = \underline{\underline{53.1}} \quad (5)$$

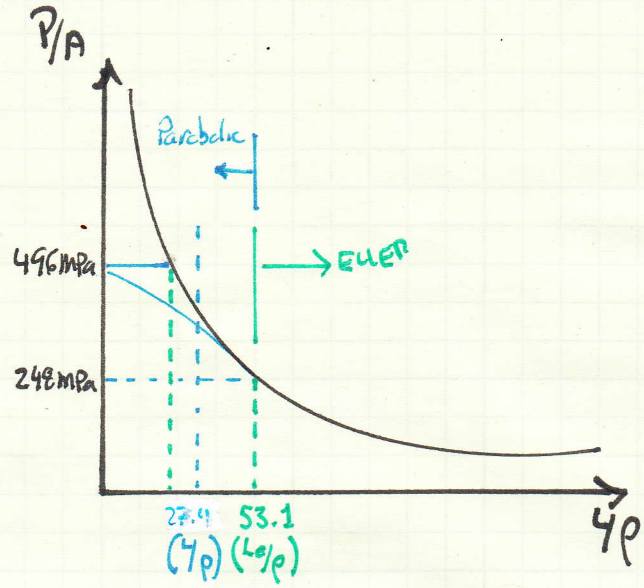
(4) AND (5) ARE PLOTTED ON THE P/A v. L/ρ GRAPH ON THE NEXT PAGE

THE FIGURE SHOWS THAT BECAUSE

$$27.4 < 53.1$$

THE BEAM IS <u>NOT</u> IN THE EULER RANGE.

THE RANGE WHERE THE CURVE IS PARABOLIC NOW NEEDS TO BE EXPLORED.



$$\frac{P_{CR}}{A} = S_y - \frac{S_y^2}{4\pi^2 E}\left(\frac{L}{\rho}\right)^2$$

$$\frac{200\,kN \cdot 4}{\pi \cdot d^2} = 496\times10^6 \frac{N}{m^2} - \frac{(496\times10^6\,N/m^2)^2}{4\cdot\pi\cdot 71\times10^9\,N/m^2}\cdot\frac{(0.2m)^2\cdot16}{d^2}$$

$$\left[\frac{200\,kN\cdot4}{\pi} + \frac{(496\times10^6\,N/m^2)^2\cdot(0.2m)^2\cdot16}{4\cdot\pi\cdot71\times10^9\,N/m^2}\right]\cdot\frac{1}{d^2} = 496\times10^6\frac{N}{m^2}$$

$$d = \underline{\underline{29.5\,mm}} \quad 6 \qquad \Rightarrow \frac{L}{\rho} = 27.1$$

$$\frac{P_{CR}}{A} = \frac{200\,kN\cdot4}{\pi\cdot(.0295m)^2} = 292.6\,MPa$$