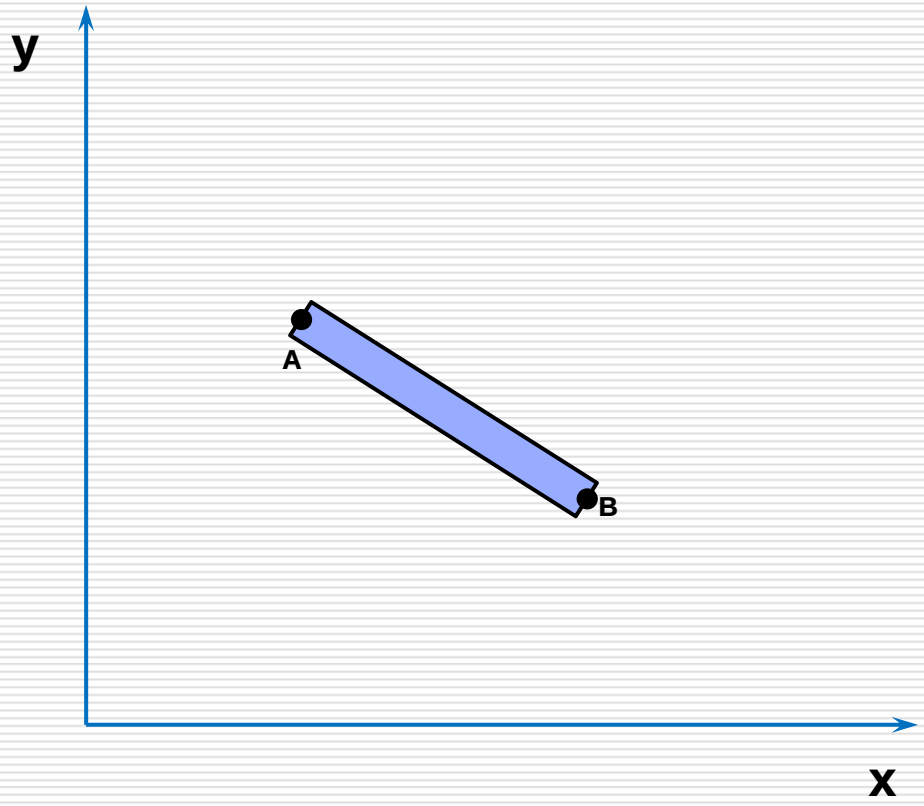
Kinematics of Mechanisms

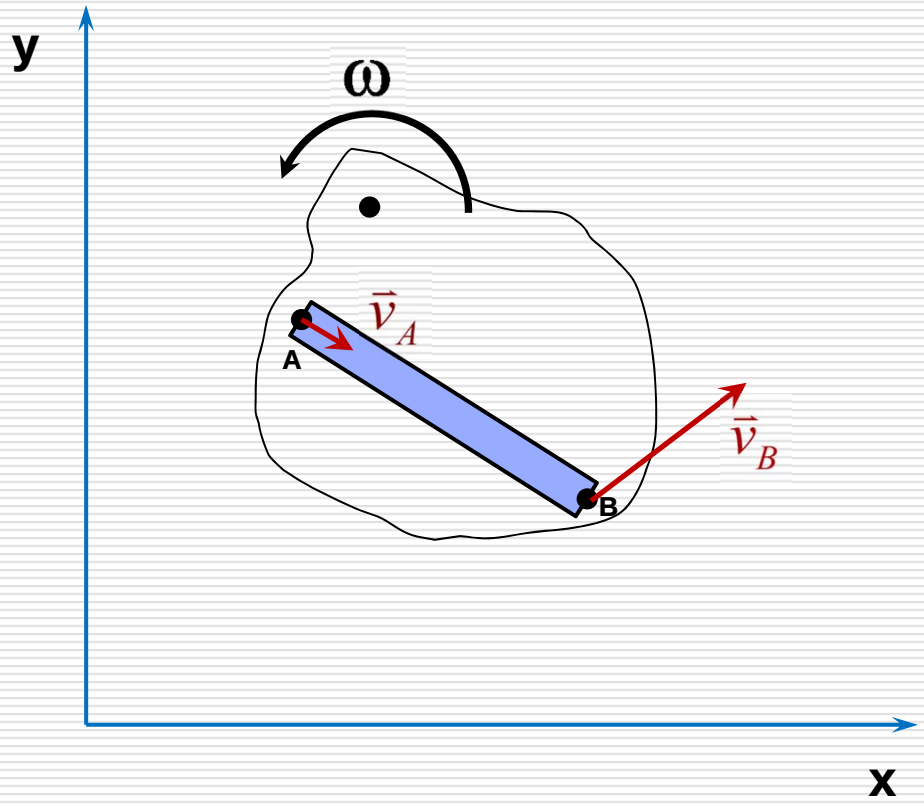
- Instant Centers
- Aronhold-Kennedy Theorem

Planar Body Motion Assumptions



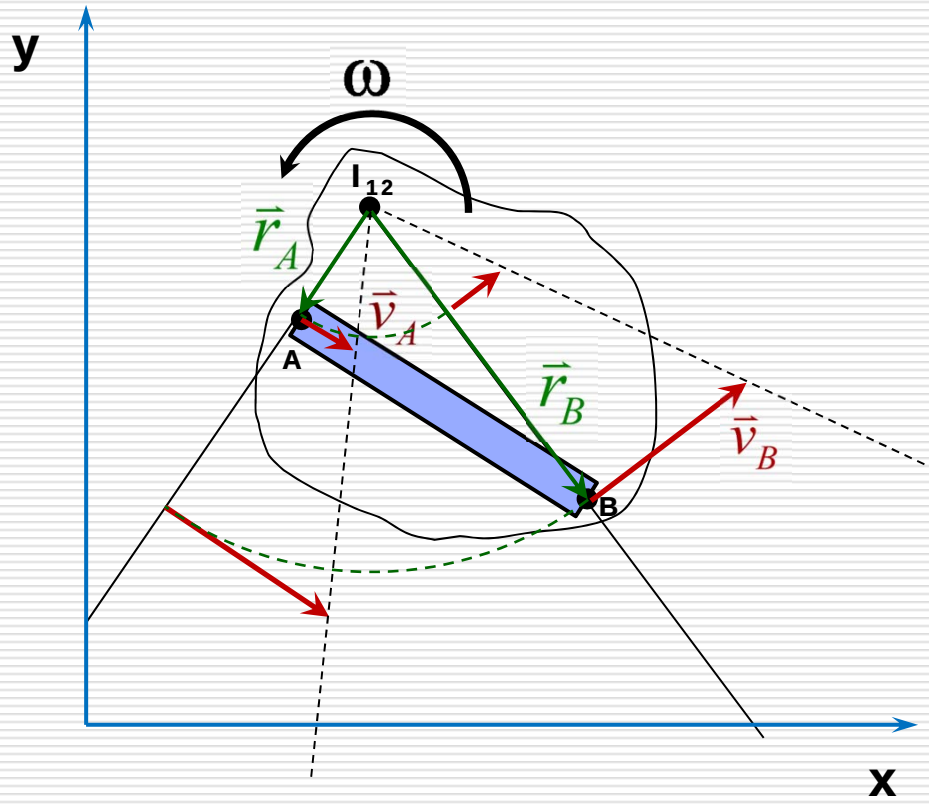
- Rigid Body Motion
  - No axial deformation
  - No twisting
  - No bending
- Planar Motion
  - In a single plane
  - In parallel planes

Velocity at Points on a Rigid Body



- Rigid Body Motion
  - No axial deformation
  - No twisting
  - No bending
- Planar Motion
  - In a single plane
  - In parallel planes
- Link Expansion
  - Any point in the plane can move with the link
- The Link appears to Rotate about a Point

Location of the Instant Centers of Velocity



In General

$$\vec{v} = \vec{\omega} \times \vec{r}$$

For Planar Problems

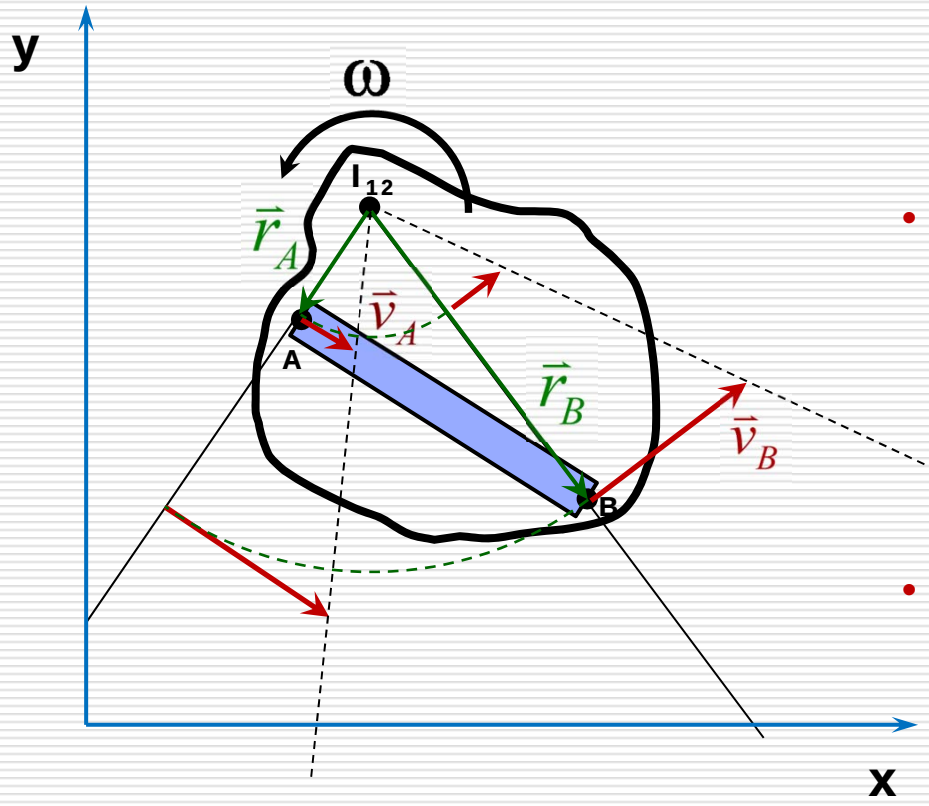
$$\vec{v} = \omega \cdot \hat{k} \times \vec{r}$$

If ω and r are perpendicular

$$v = \omega \cdot r$$

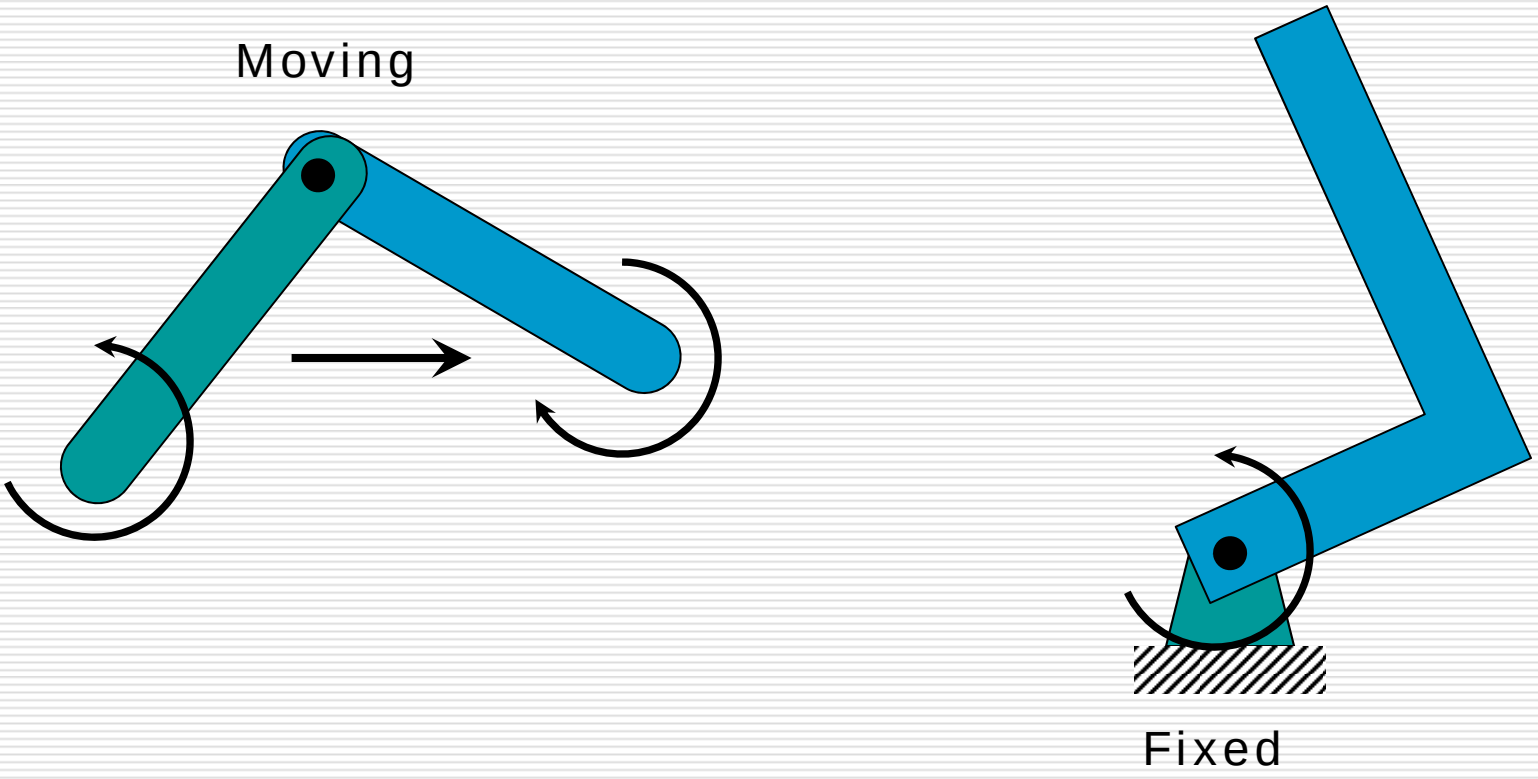
I_{12} is the instant center between the body AB and the ground.

Instant Centers of Velocity Definition:

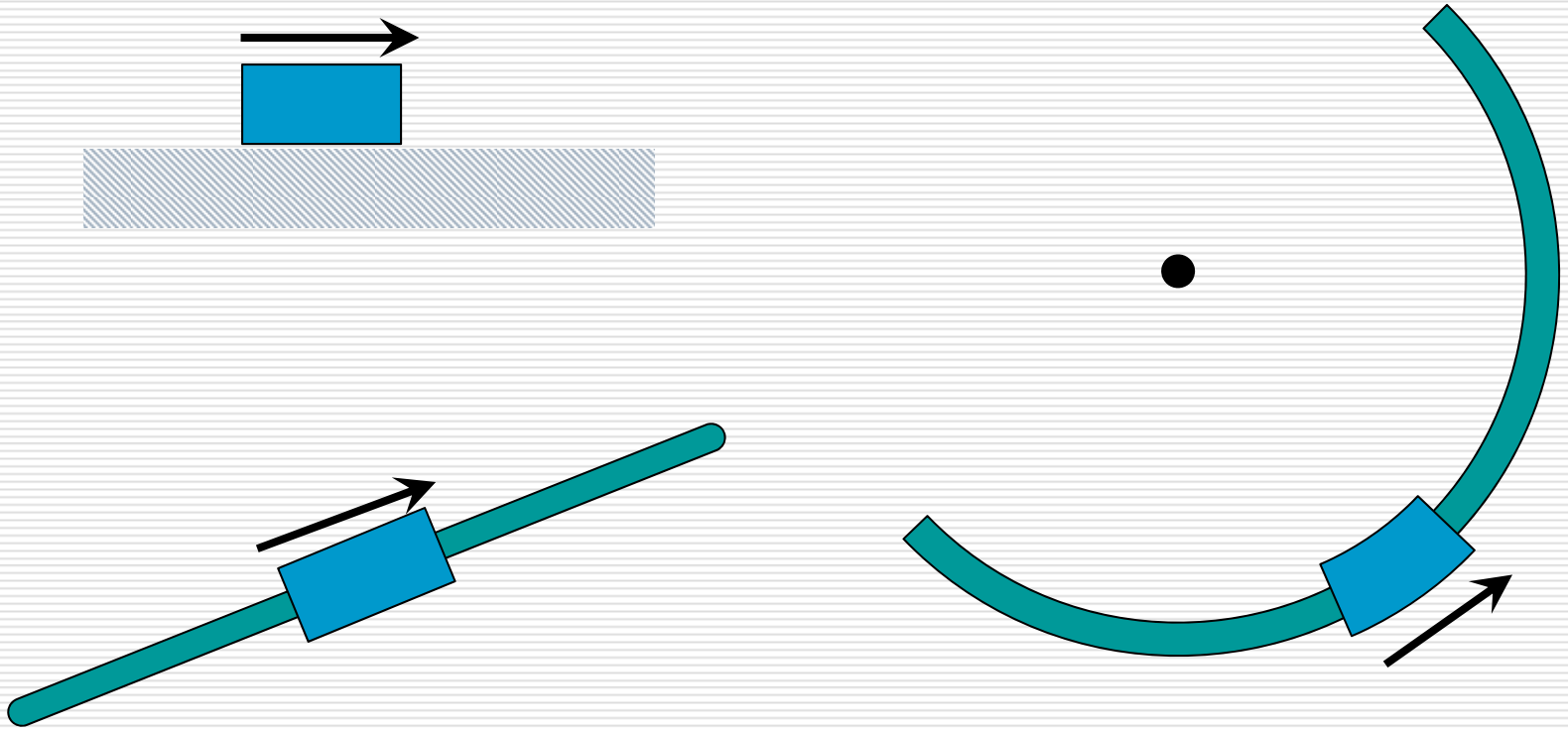


- Instant Centers are sometimes referred to as
  - Poles
  - Centro
  - Instantaneous Centers
- Definition
  - A point in one body about which some other body is rotating either permanently or at the instant
  - A point common to two bodies having the same linear velocity in both magnitude and direction
- Functional Types
  - Fixed Centers
  - Moving Centers

Instant Center
Revolute Joint / Pin Connections

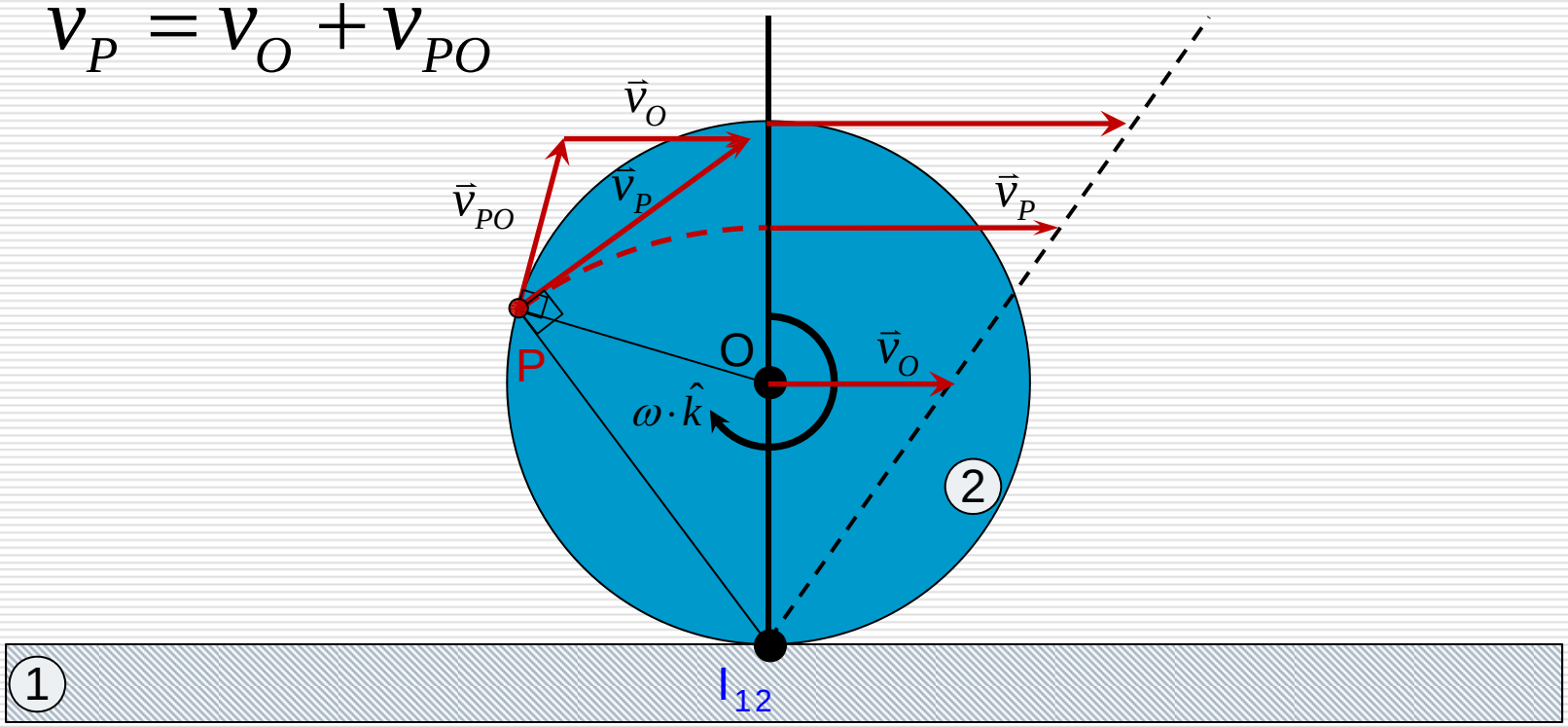


Instant Center
Prismatic Joint / Sliding Body



Instant Center
Rolling-NO Slip Contact

$$\vec{v}_P = \vec{v}_O + \vec{v}_{PO}$$

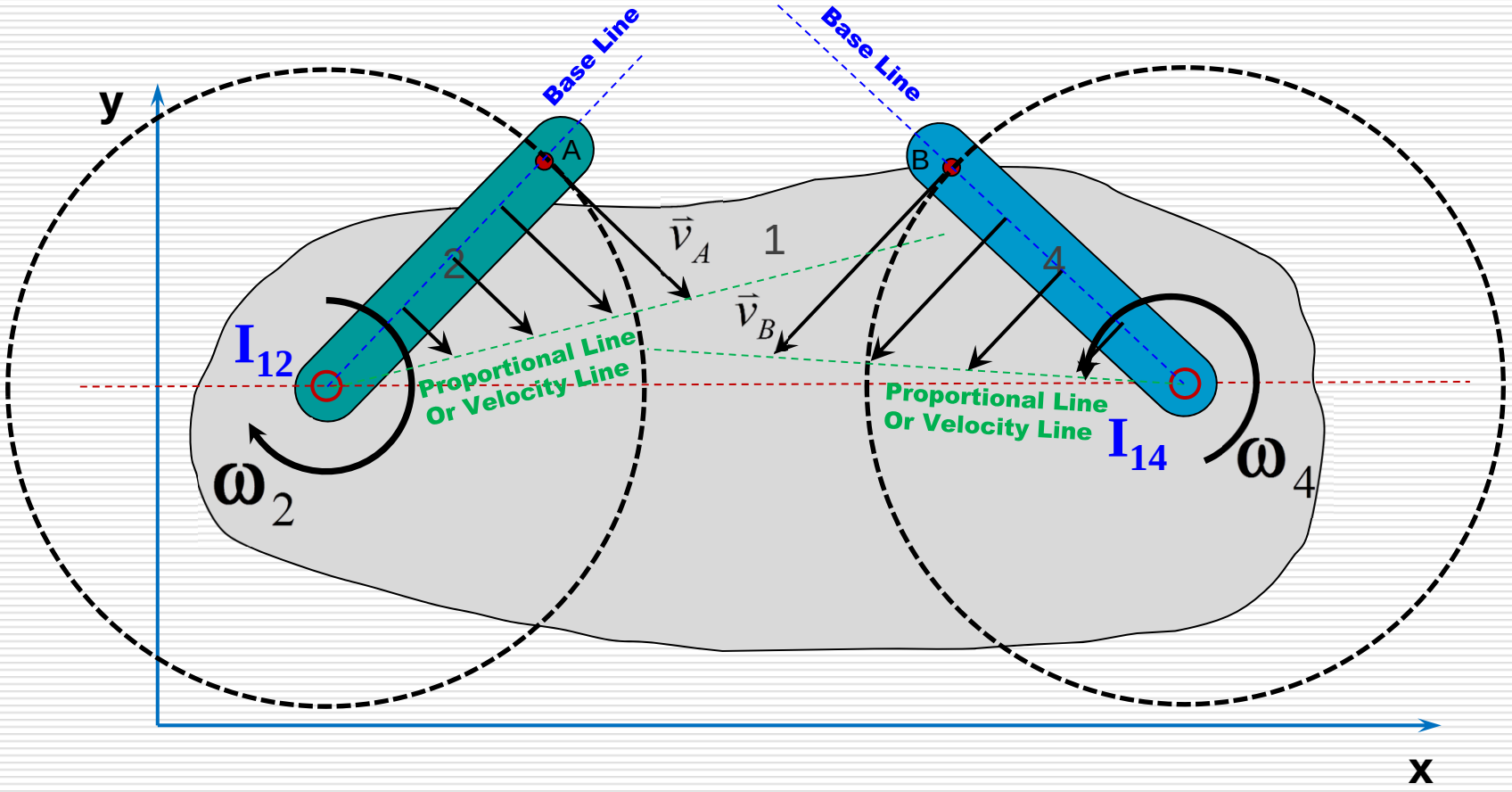


Aronhold-Kennedy Theorem

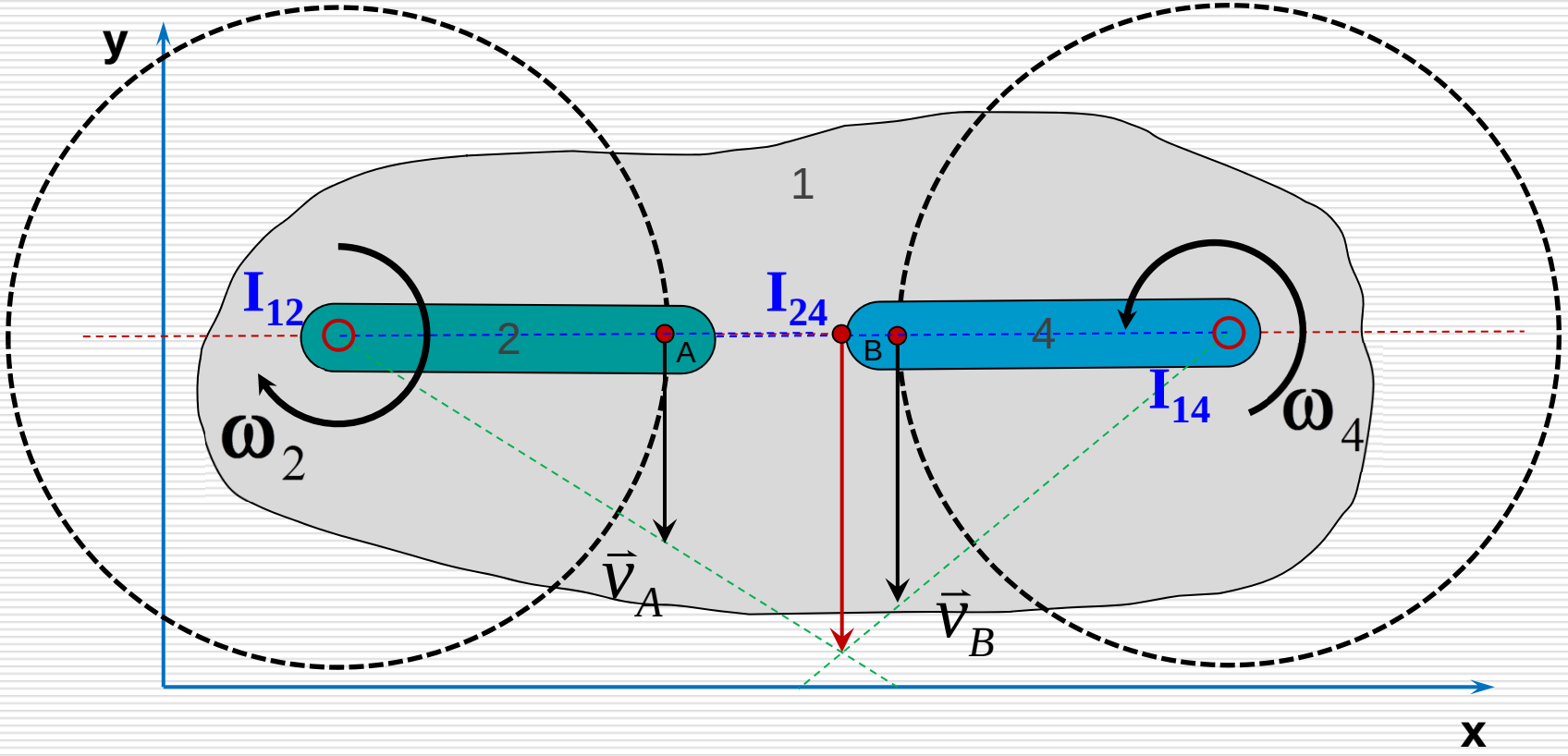
- The three instant centers shared by three rigid bodies in relative motion to one another all lie on the same straight line
- They do not have to be connected

- Independently Discovered
  - S.H. Aronhold (1872) - German Speaking Countries
  - A.B.W. Kennedy (1886) - English Speaking Countries

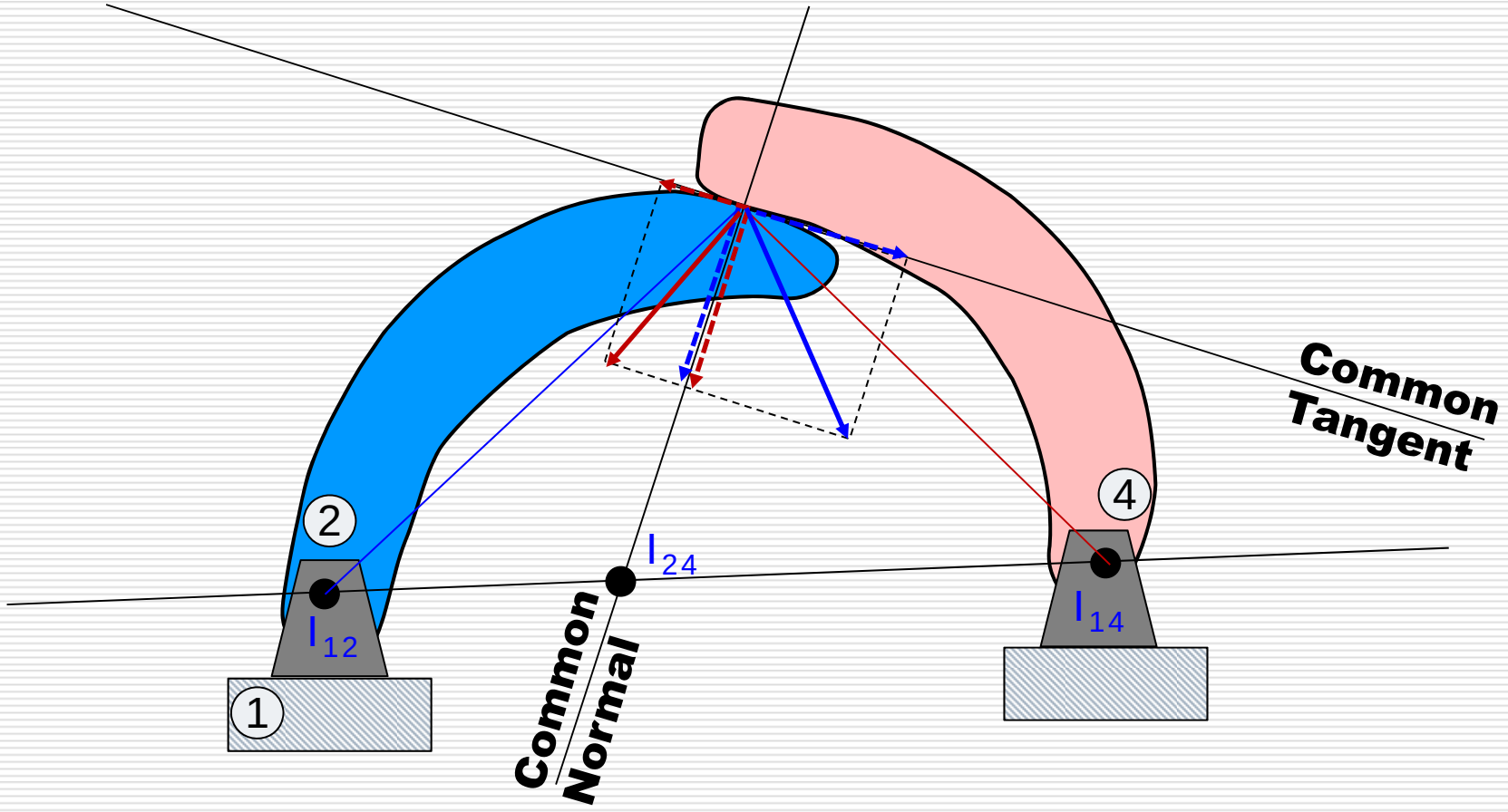
Aronhold-Kennedy Theorem



Aronhold-Kennedy Theorem



Instant Center
Rolling-Slip Contact



The Number of Instant Centers Can Be Calculated

Because any two links in a mechanism have motion relative to one another, they have a common Instant Center.

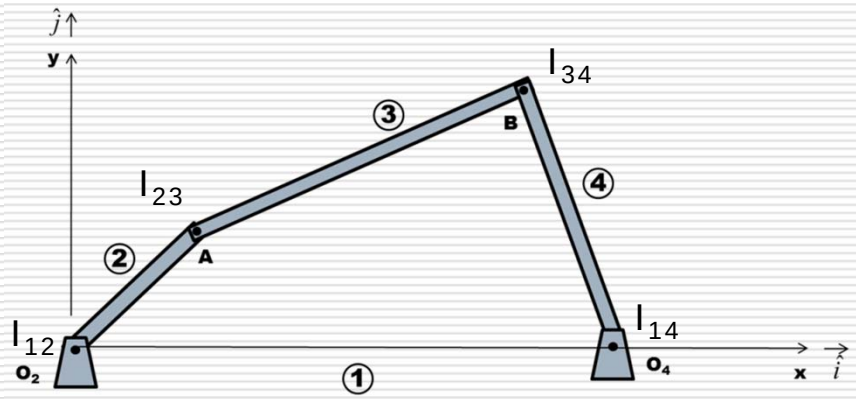
$$N = \frac{n(n-1)}{2}$$

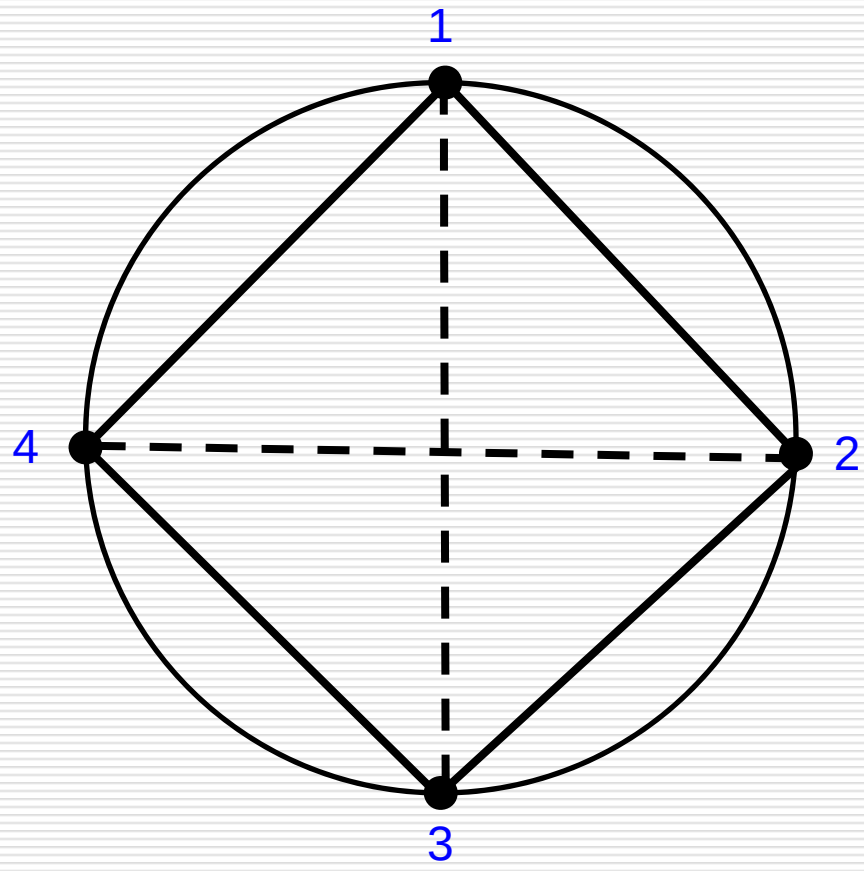
The number of links in a mechanism taken two at a time.

Primary Instant Centers Can Be Found By Inspection

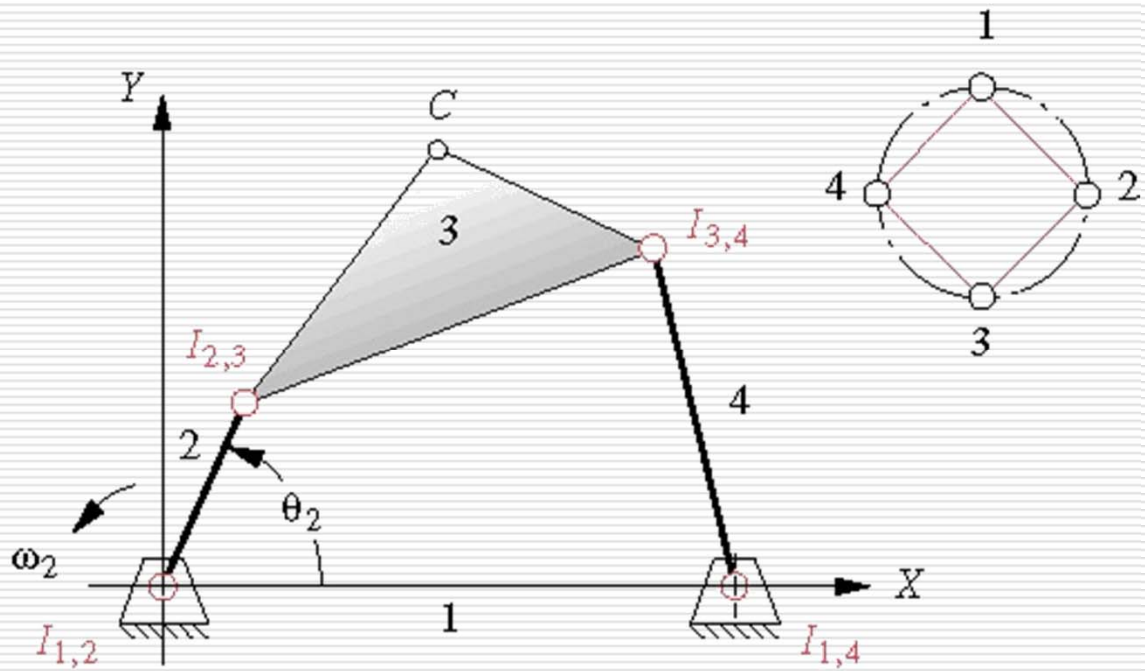
1. Instant Centers for Pin Connected Links
2. Instant Centers for Sliding Bodies
3. Instant Centers for Rolling Bodies
4. Instant Centers for Direct-Contact Mechanisms
   a. Rolling Contact
   b. Sliding Contact

Other ICs Are Found Using Circle Diagram Method Along with A-K

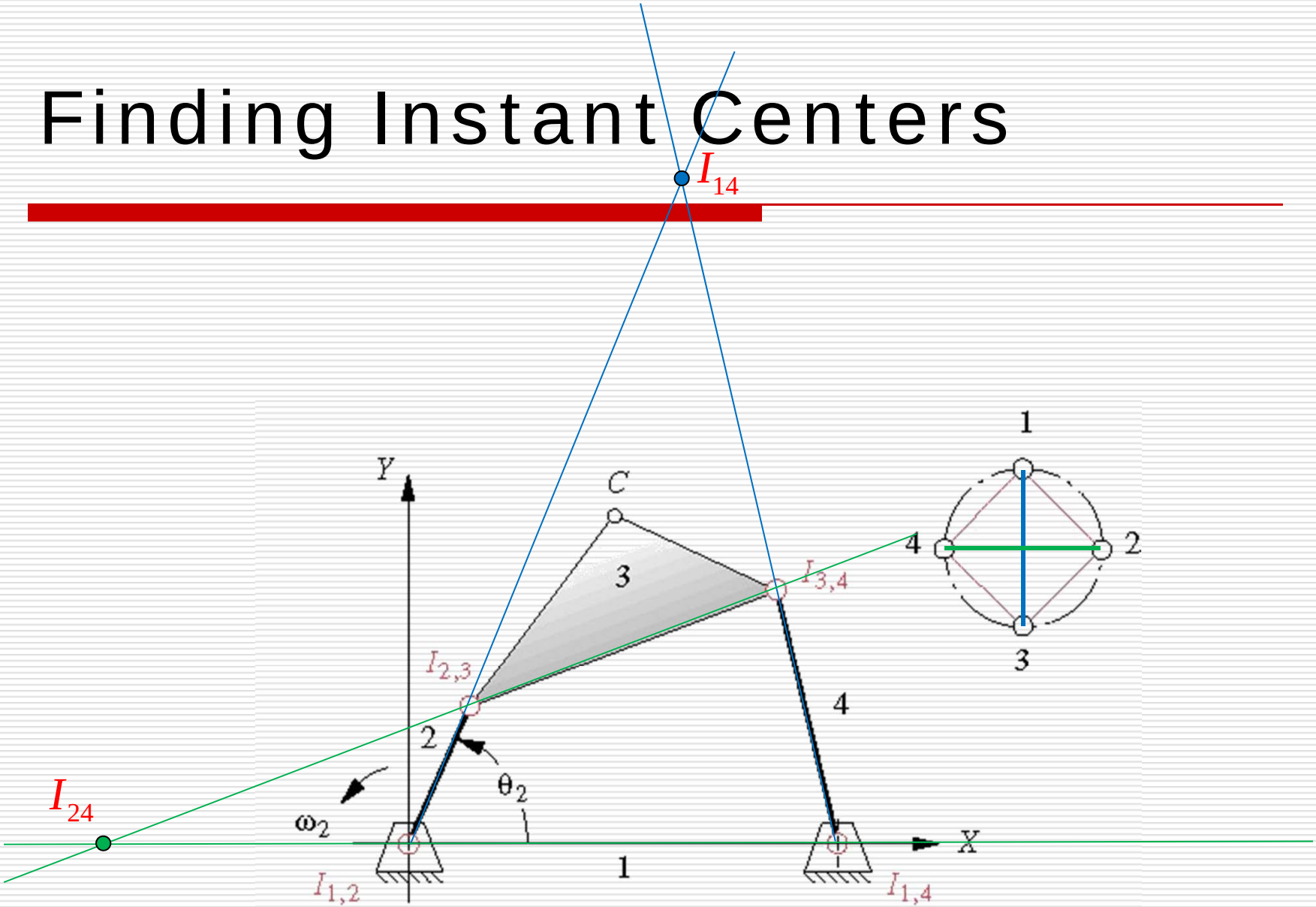




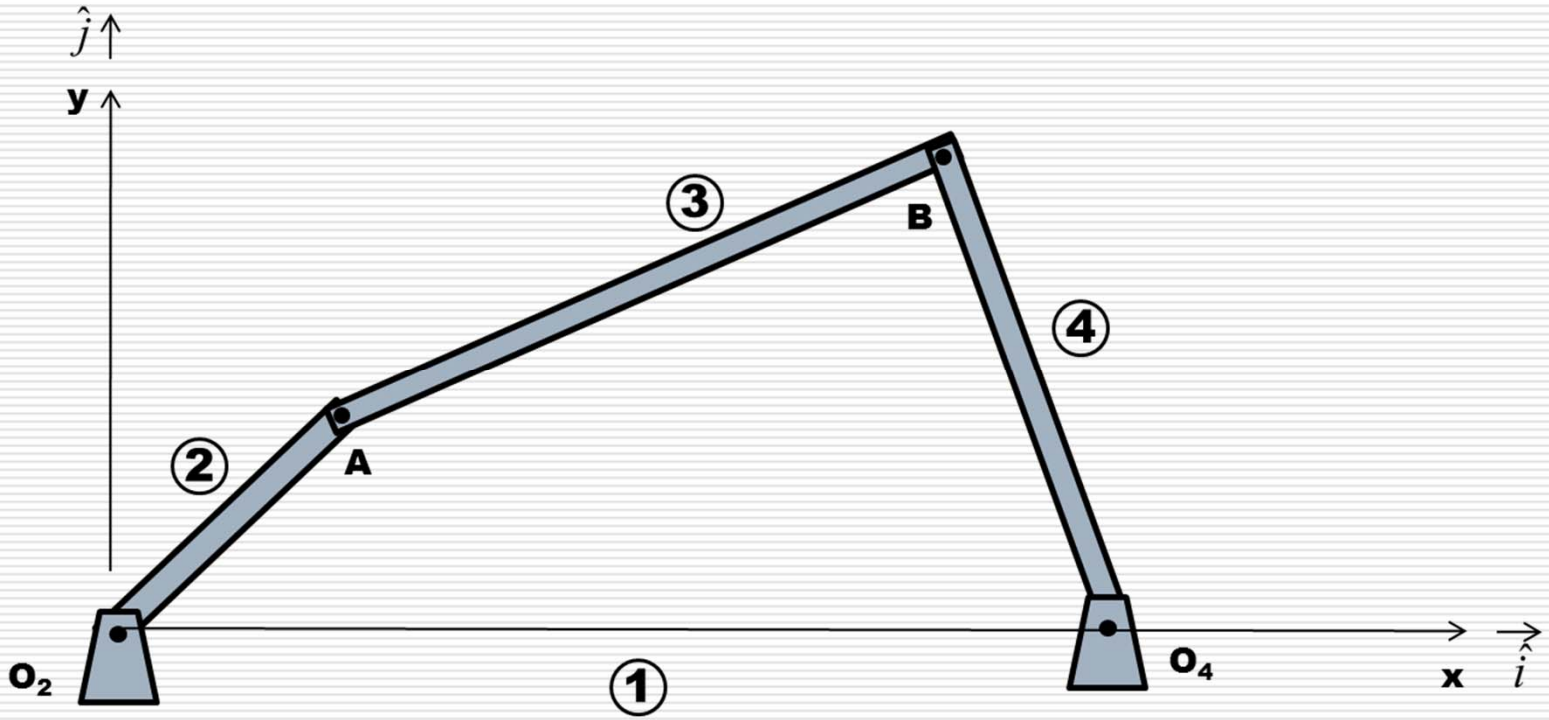
Finding Instant Centers



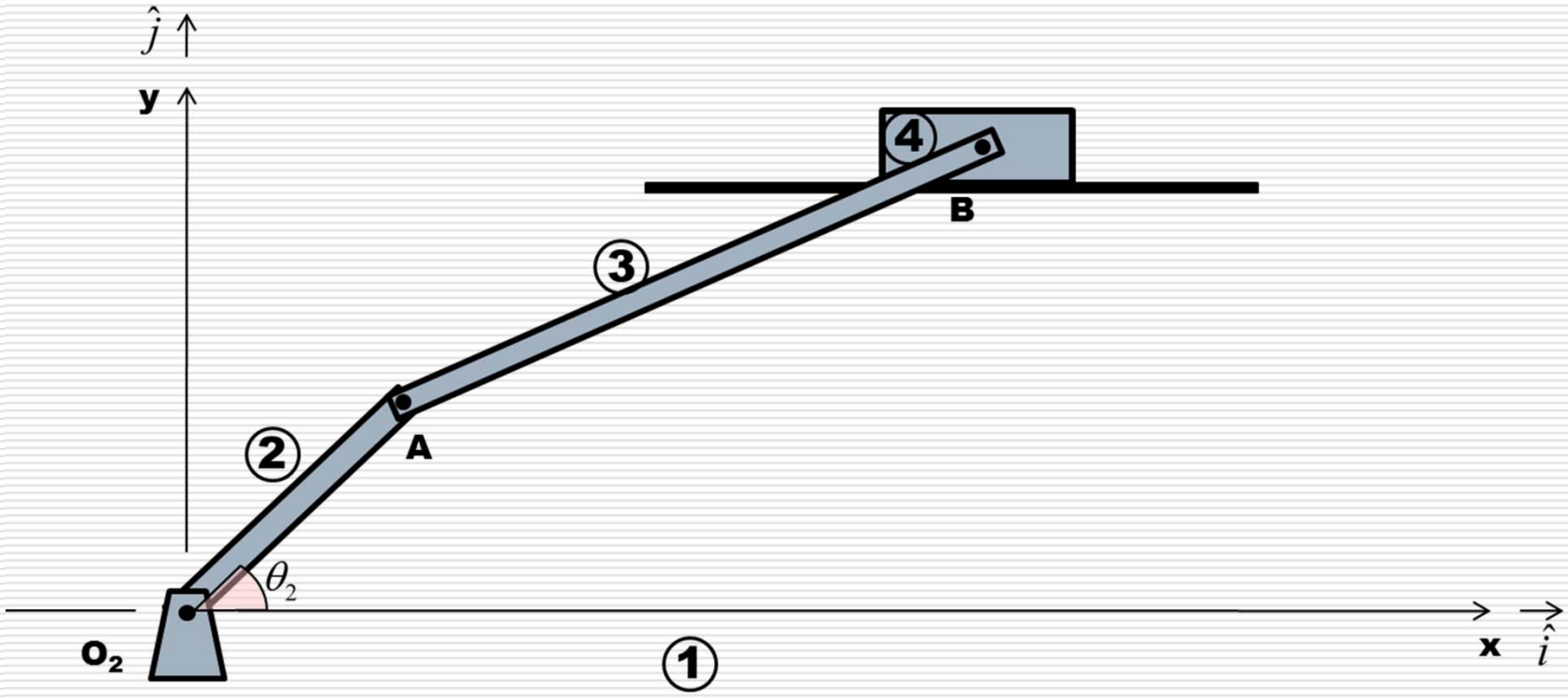
Finding Instant Centers



TYPE I (RRRR): 4 - BAR



TYPE II: (RRRP): SLIDER CRANK



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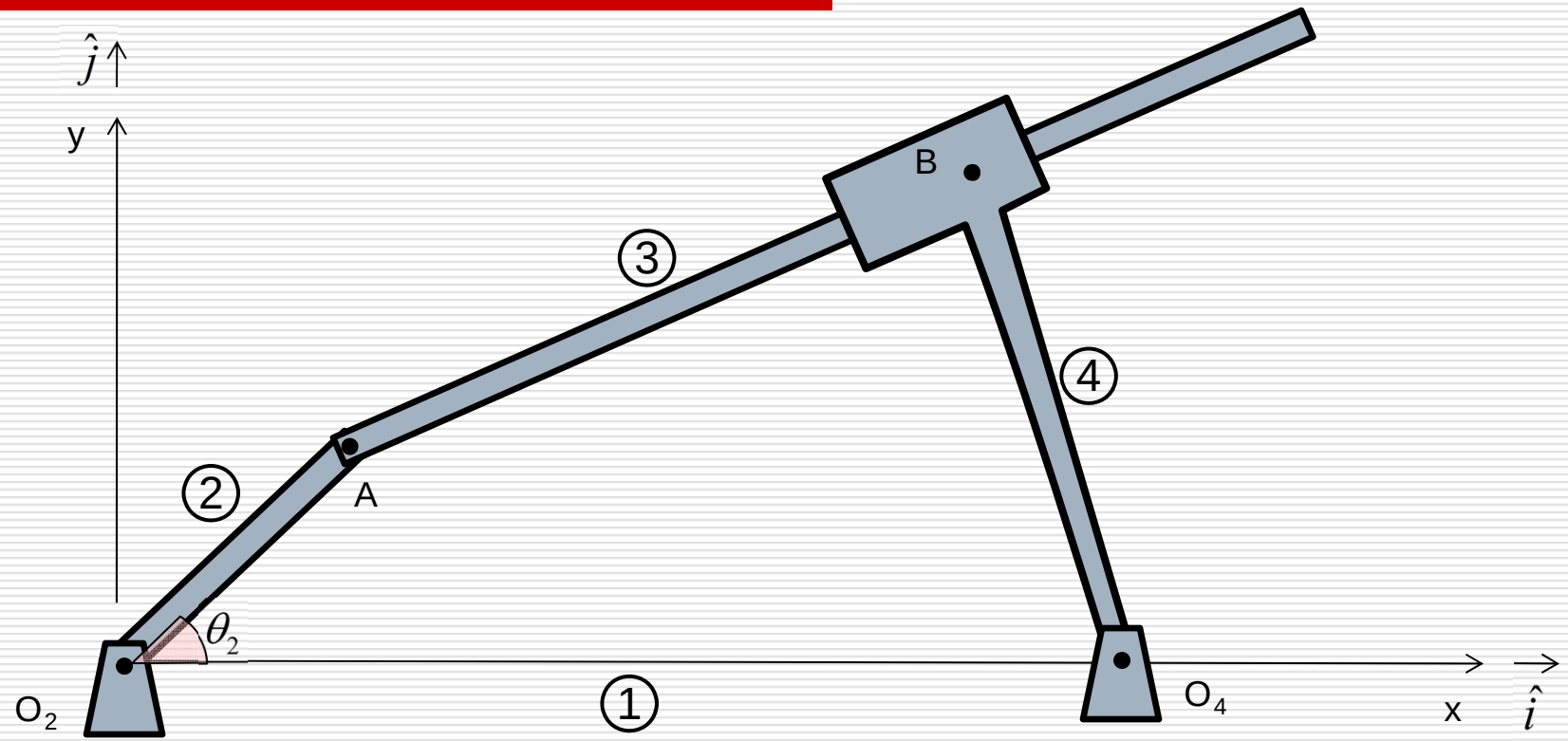
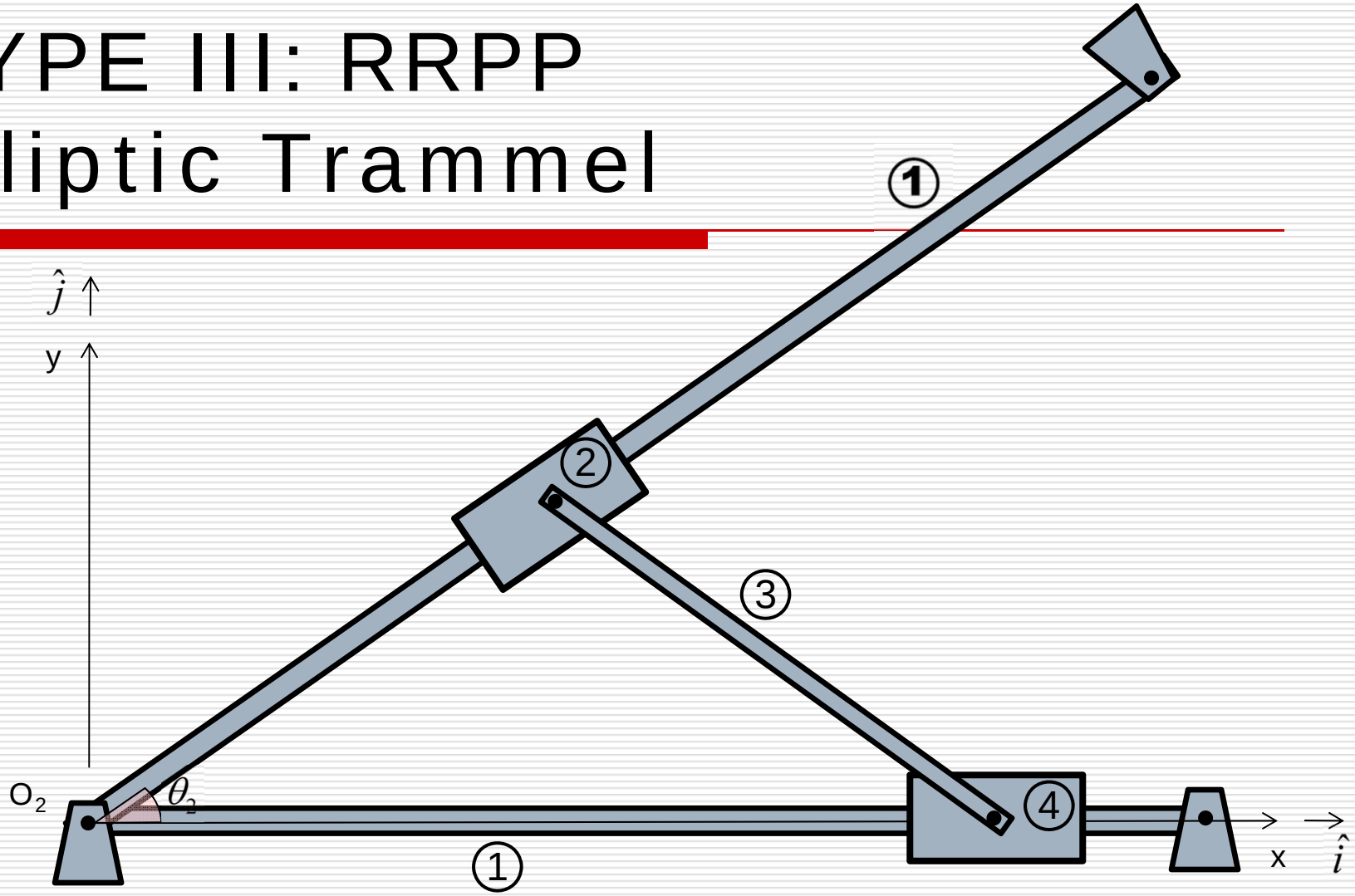


TYPE III: RRPP
Elliptic Trammel



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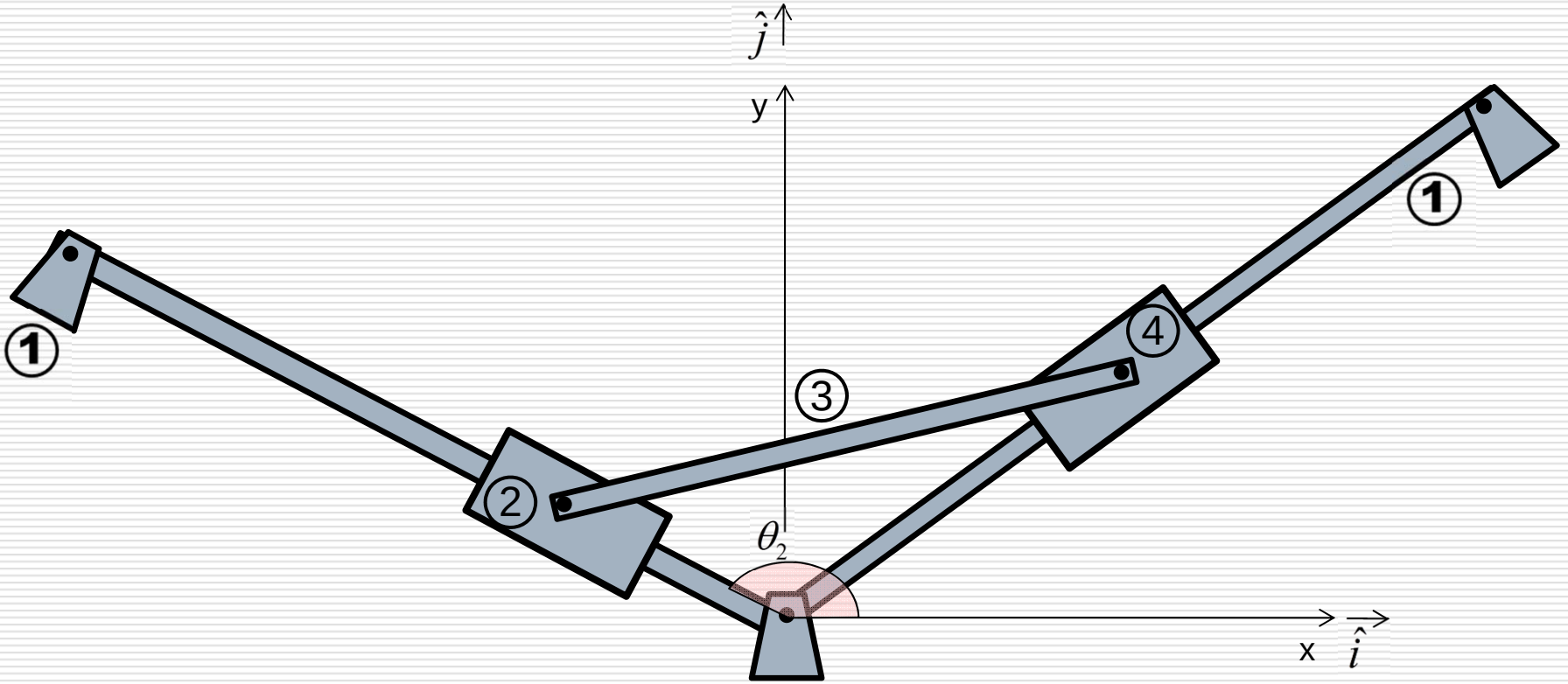

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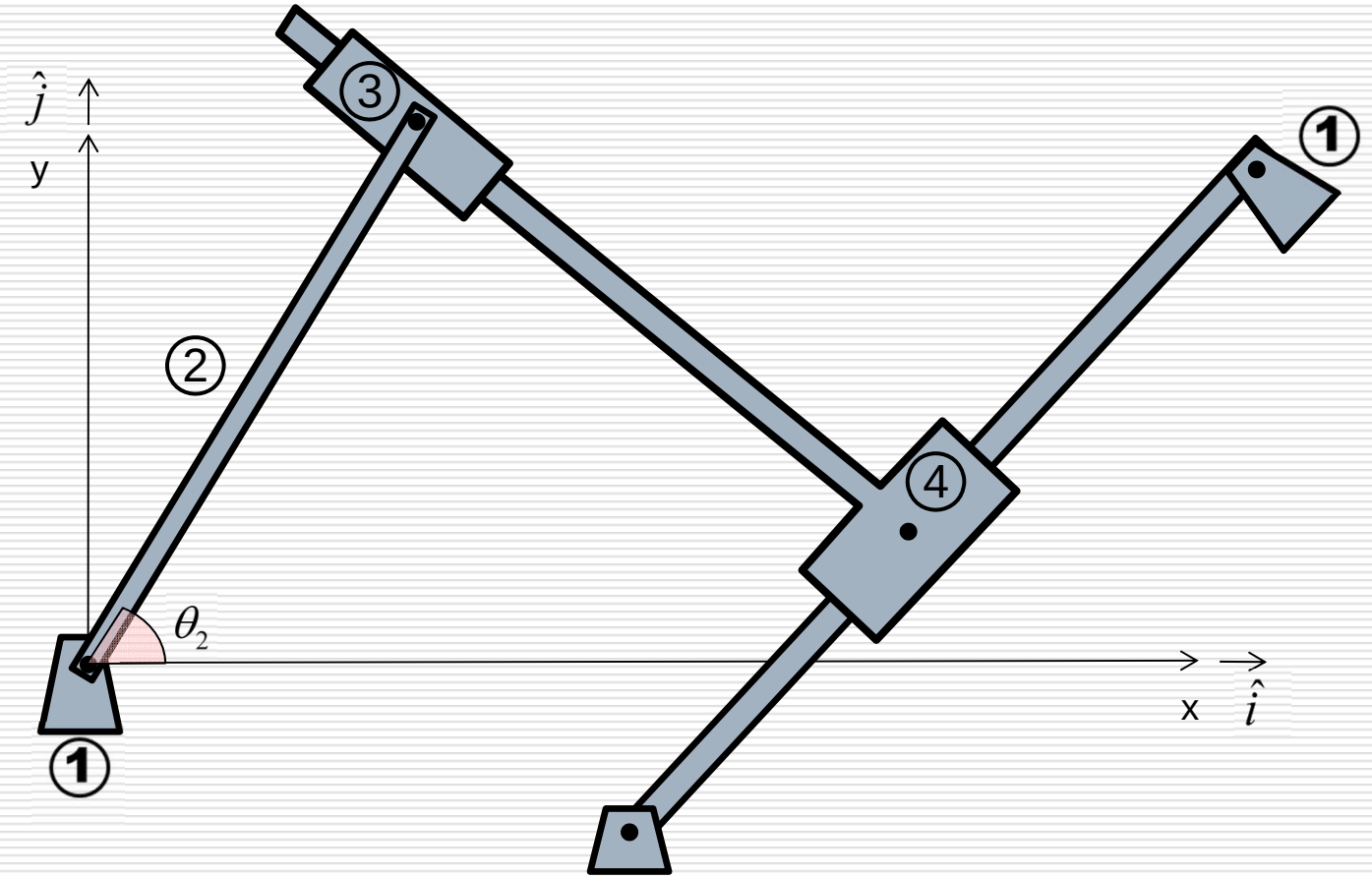
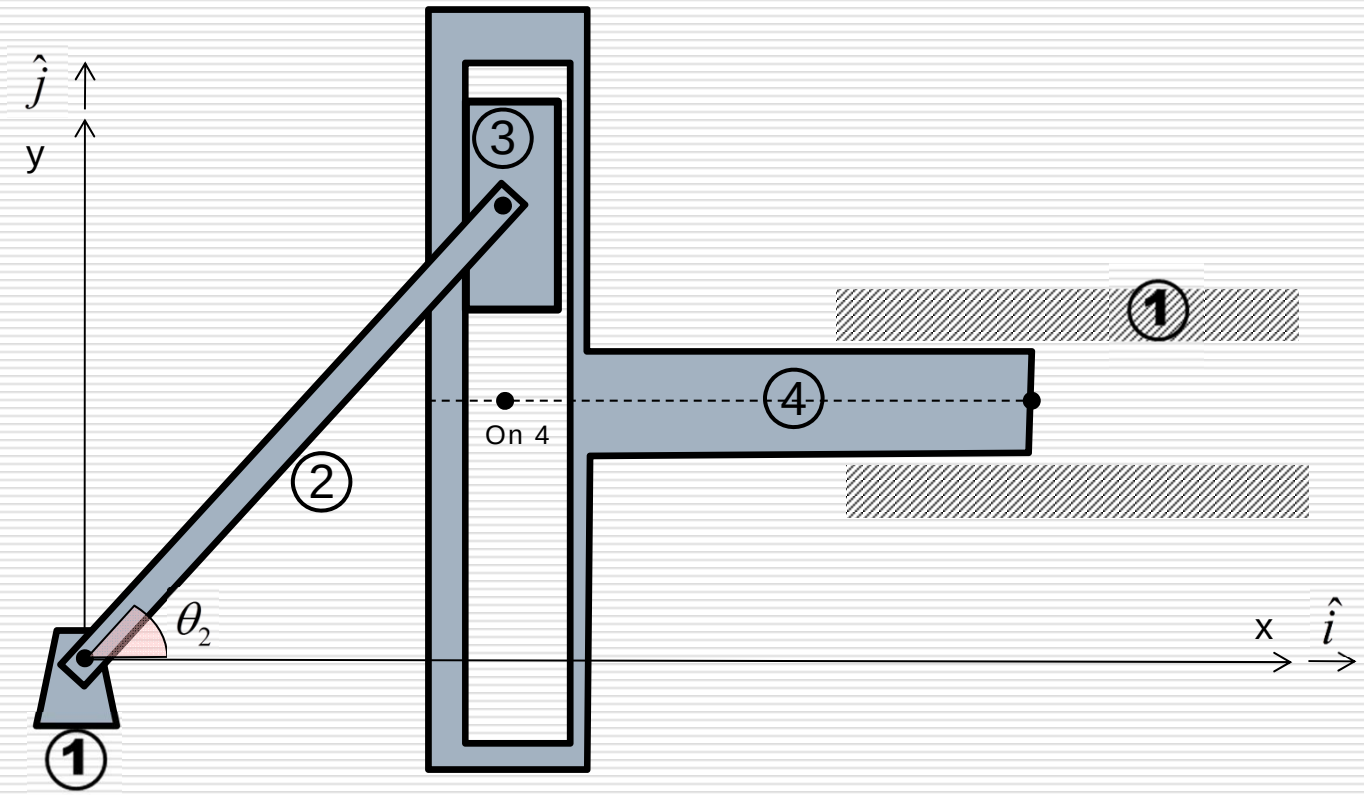
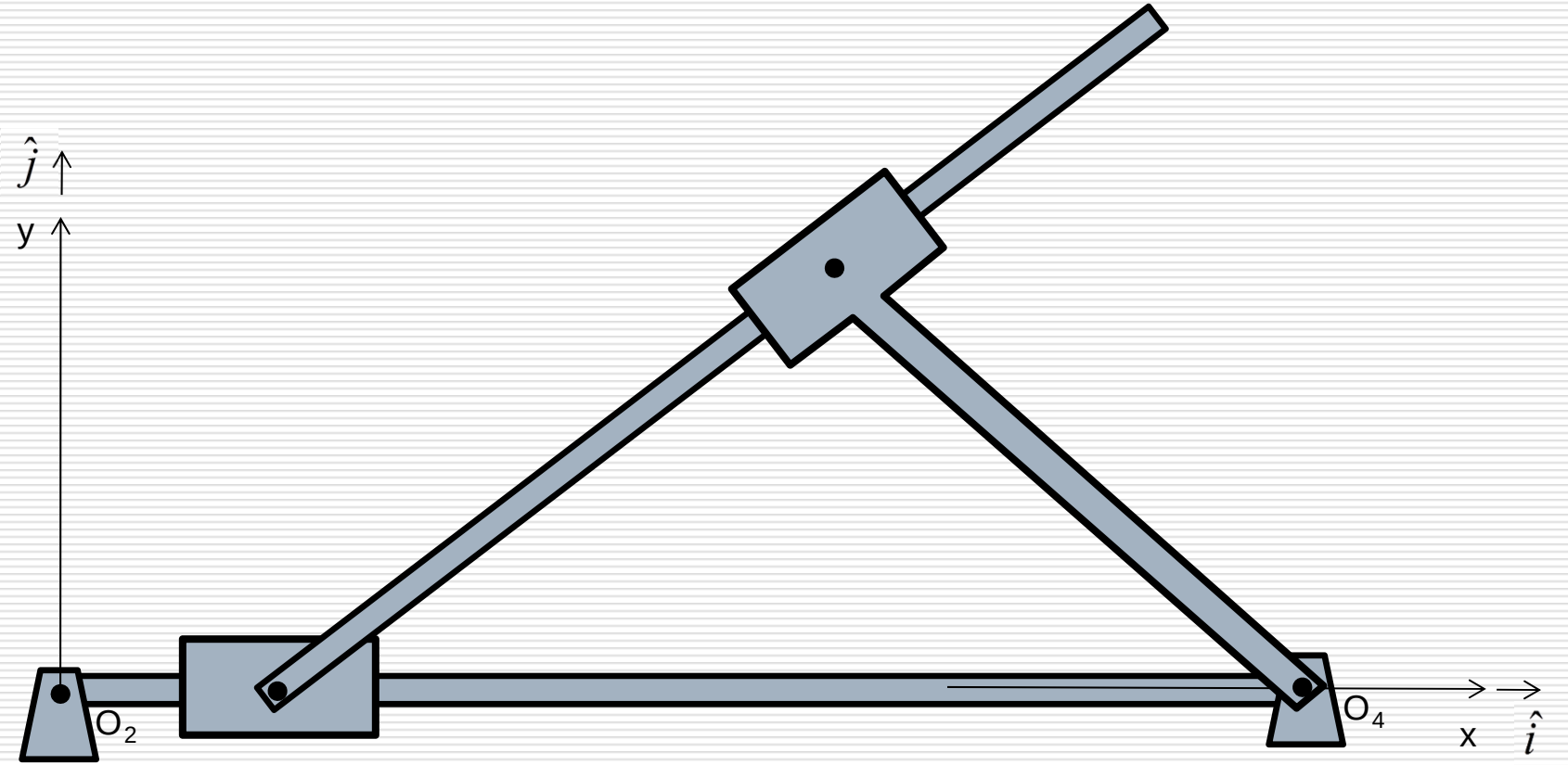


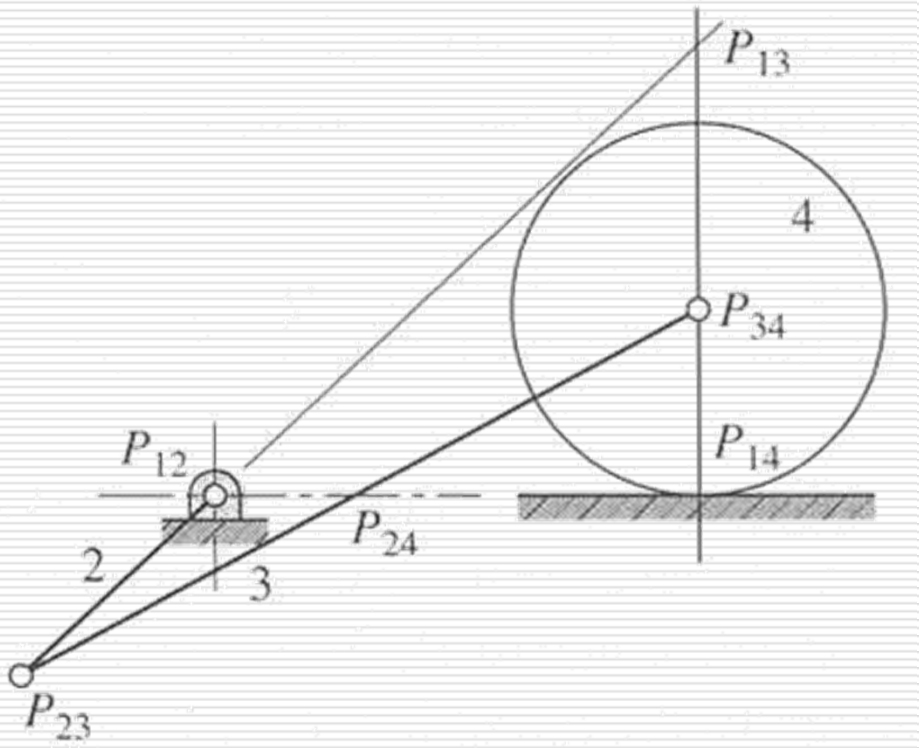
Type III: RRPP
Elliptic Trammel: Scotch Yoke



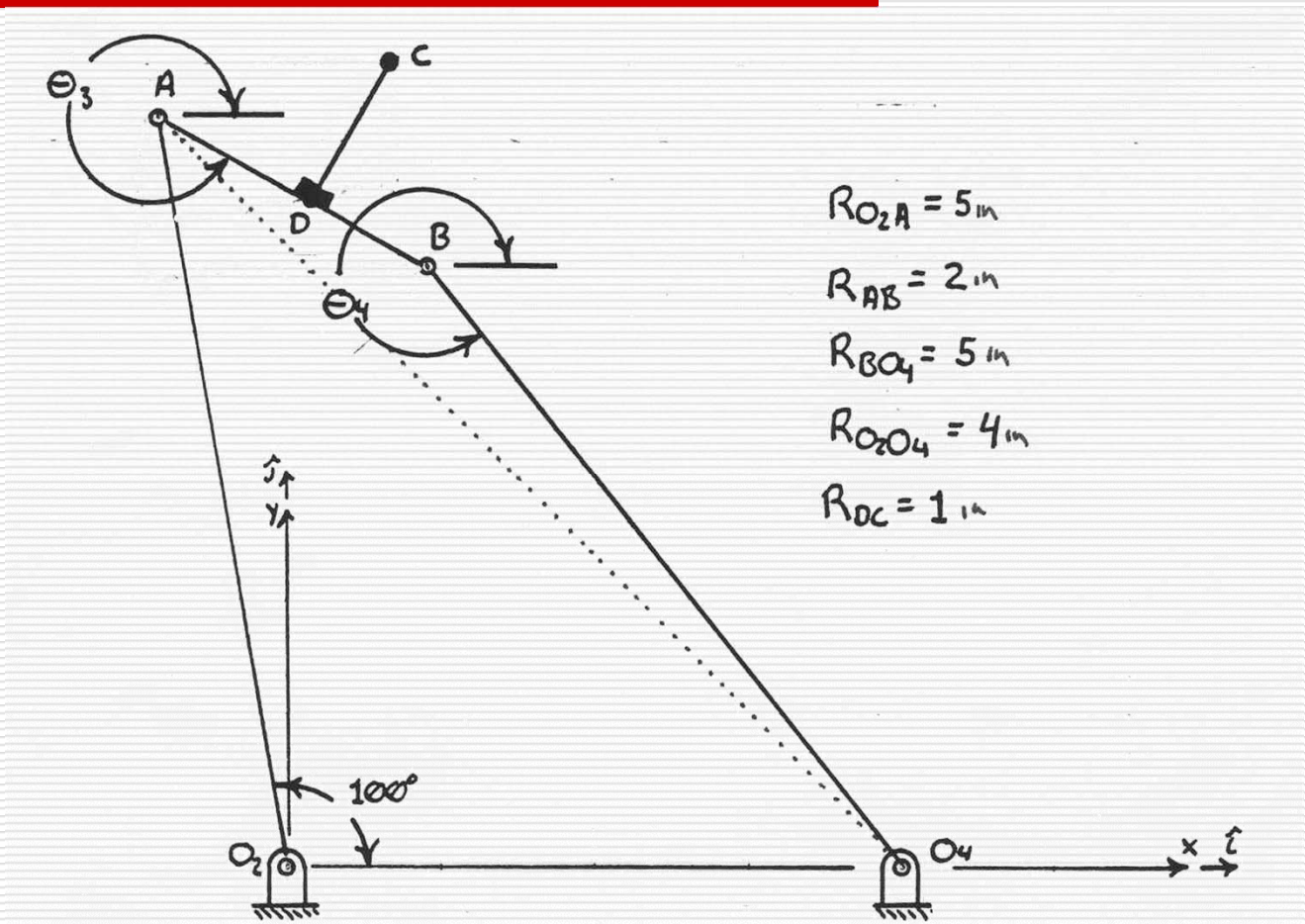
Type IV: RPRP
Rapson Slide Linkage



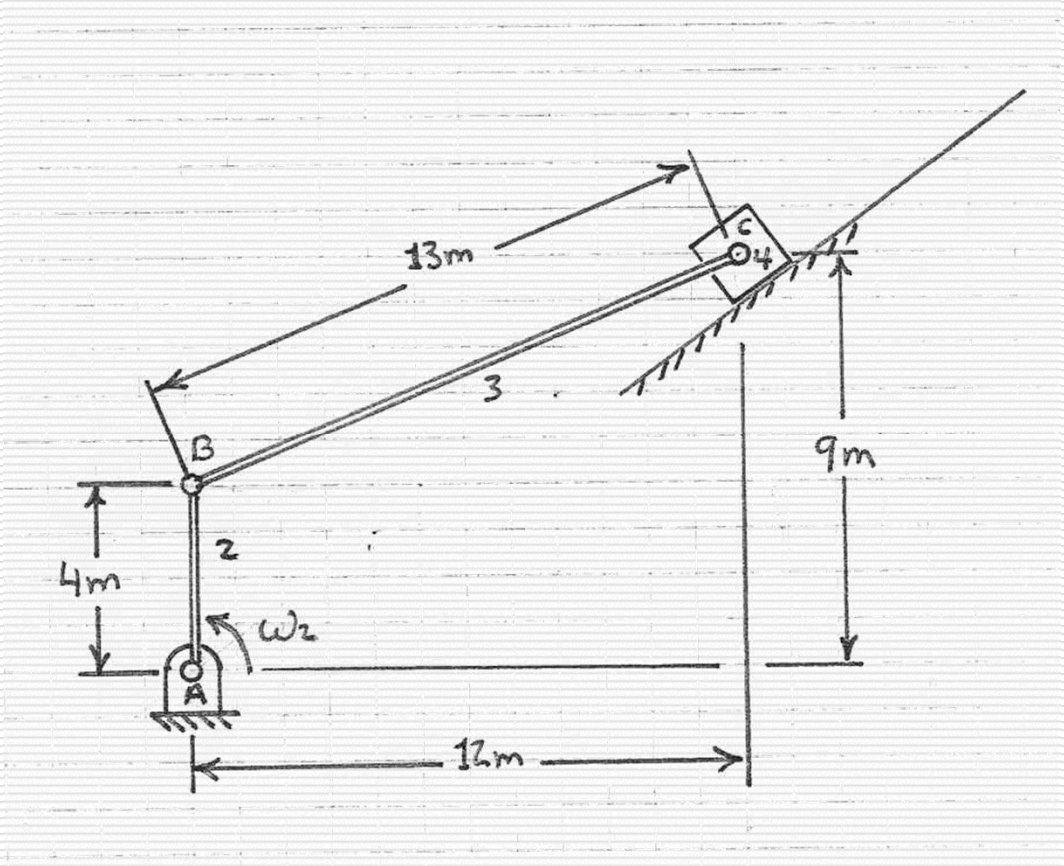
Rolling Instant Centers



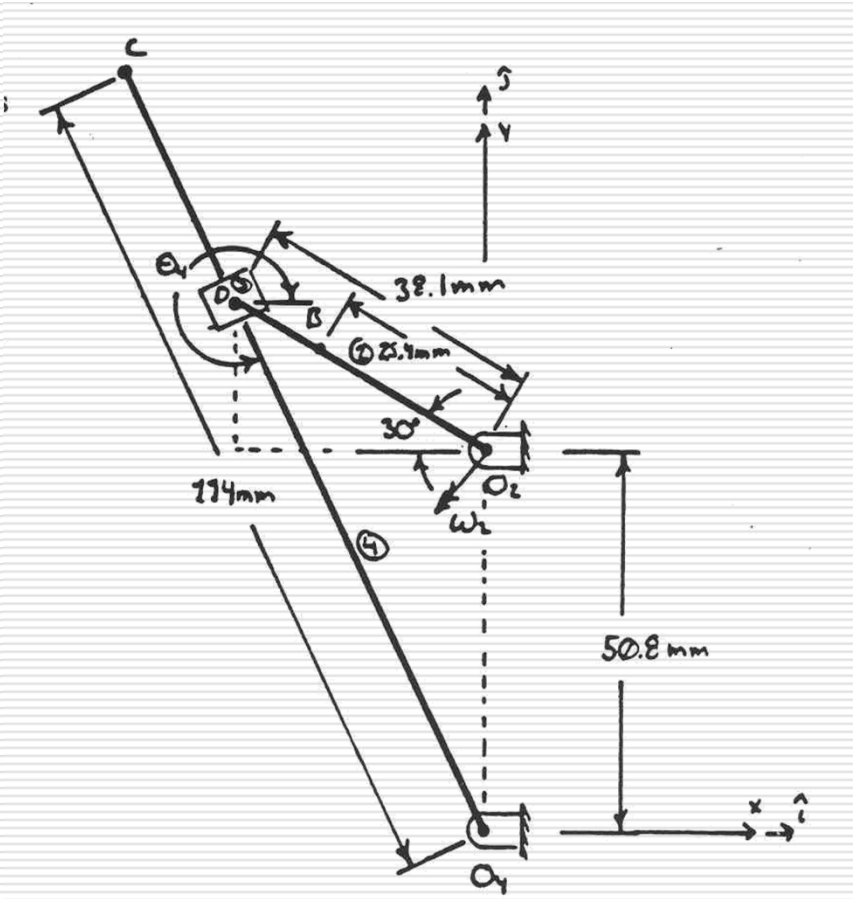
4 Bar Linkage Instant Centers



Slider Crank Instant Centers



Another Slider Crank



Instant Centers