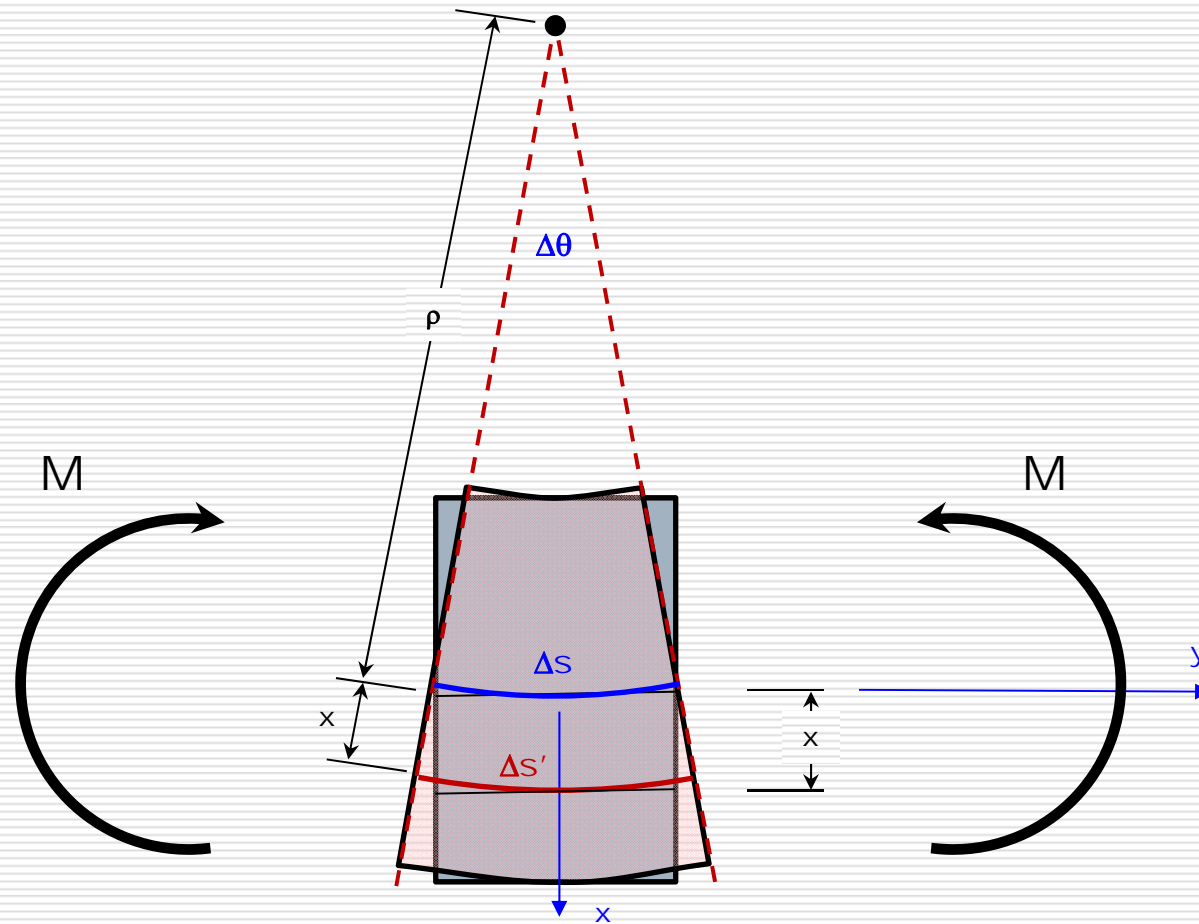


# Deflection of Beams Through Direct Integration

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- Moment-Curvature Relationship
- Moment-Deflection Relationship
- $q$ - $V$ - $M$ - $\theta$ - $u$

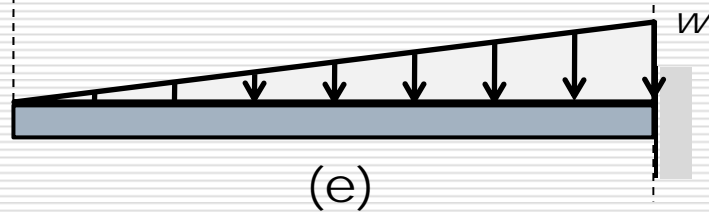
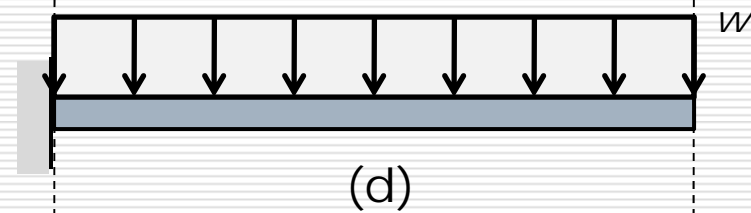
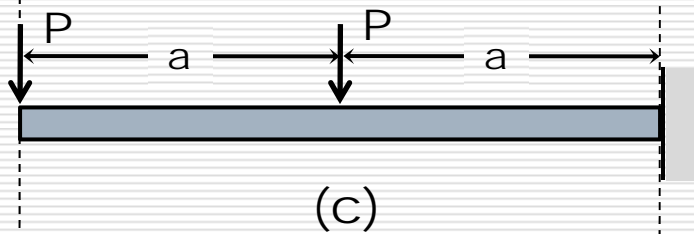
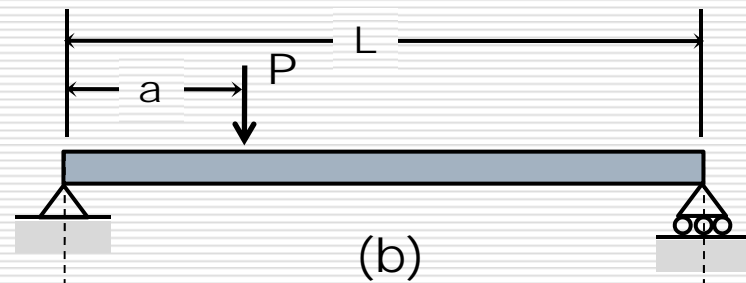
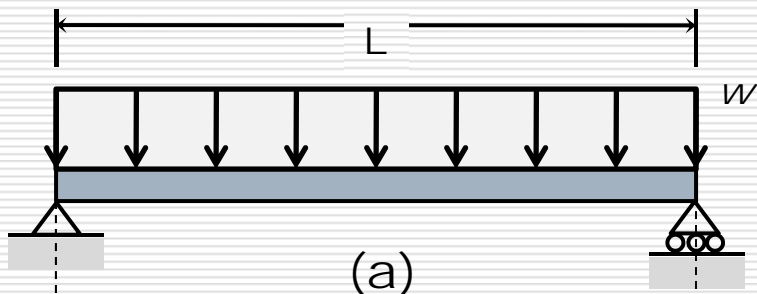
# Defining the Moment Deflection Relationship



# Linear-Elastic Response

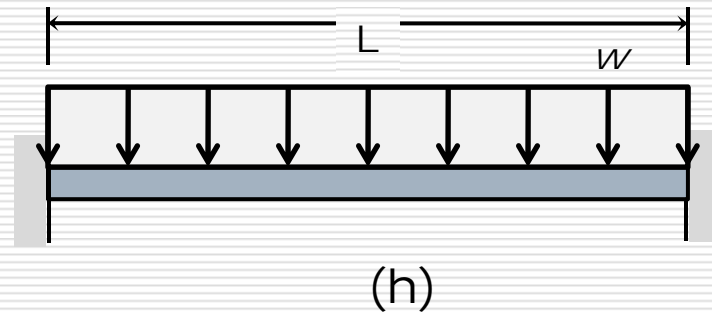
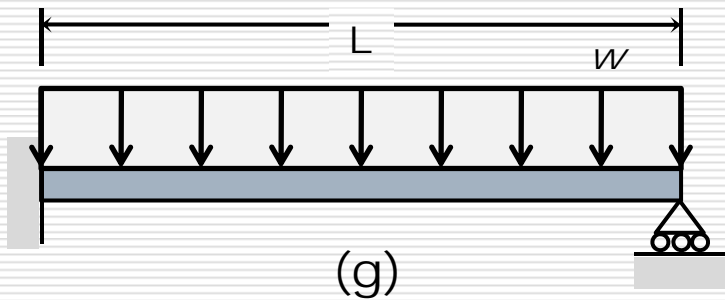
$$\begin{array}{l}
 \begin{array}{c} q \\ \downarrow x \end{array} \begin{array}{c} \rightarrow y \end{array} \quad -q = dV/dy = -\frac{d^2}{dy^2} \left( E \cdot I \cdot \frac{d^2 u}{dy^2} \right) = -(E \cdot I \cdot u'')'' \Rightarrow \frac{\text{Constant}}{E \cdot I} \Rightarrow q(y) = -E \cdot I \cdot d^4 u / dy^4 \\
 \\
 \begin{array}{c} V \\ \uparrow \end{array} \begin{array}{c} \rightarrow y \end{array} \quad V = dM/dy = -\frac{d}{dy} \left( E \cdot I \cdot \frac{d^2 u}{dy^2} \right) = -(E \cdot I \cdot u'')' \Rightarrow \frac{\text{Constant}}{E \cdot I} \Rightarrow V(y) = -E \cdot I \cdot d^3 u / dy^3 \\
 \\
 \begin{array}{c} M \\ \uparrow \end{array} \begin{array}{c} \rightarrow y \end{array} \quad M = -E \cdot I \cdot d^2 u / dy^2 = -E \cdot I \cdot u'' \Rightarrow \frac{\text{Constant}}{E \cdot I} \Rightarrow M(y) = -E \cdot I \cdot d^2 u / dy^2 \\
 \\
 \begin{array}{c} \theta \\ \downarrow \end{array} \begin{array}{c} \rightarrow y \end{array} \quad \theta \equiv du/dy = u' \equiv \text{Slope of the Elastic Curve} \\
 \\
 \begin{array}{c} u \\ \downarrow \end{array} \begin{array}{c} \rightarrow y \end{array} \quad u \equiv \text{Deflection of the Elastic Curve}
 \end{array}$$

# Beams



# Beams – Statically Indeterminate

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# Example

$$0 < y < 2a$$

$$V = -\frac{P}{2}$$

$$M = -\frac{P \cdot y}{2}$$

$$2a < y < 3a$$

$$V = P$$

$$M = -(3 \cdot a - y) \cdot P$$

