MER 311: Chapter 5
Torsion in Thin-Walled Hollow Shafts

- Thin-Walled Hollow Shafts
- Multiple Cell Sections in Torsion

Review of Torsion
From Strength of Materials

- **Torque on a Cross-Section**

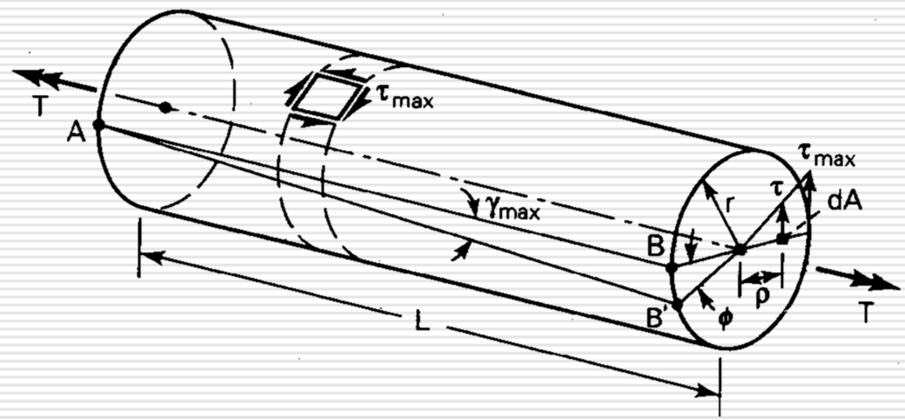
$$\tau = \frac{T \cdot \rho}{J}$$

- **Angle of Twist**

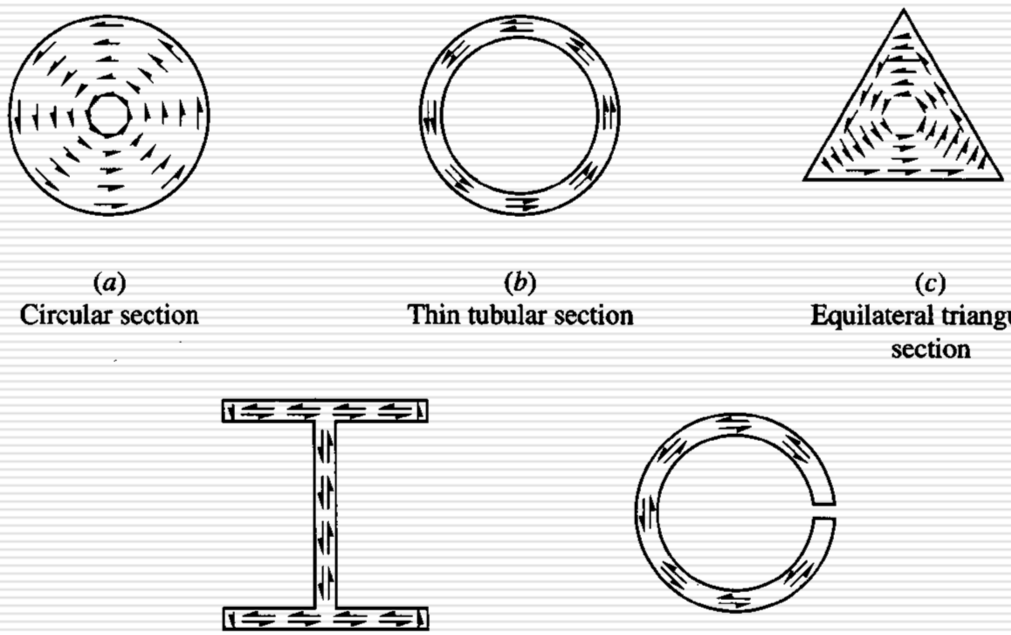
$$\phi = \frac{T \cdot L}{J \cdot G}$$

- **Shear Strain**

$$\gamma = \frac{\phi \cdot r}{L}$$



Shear Flow Due to Torsion



(a)
Circular section

(b)
Thin tubular section

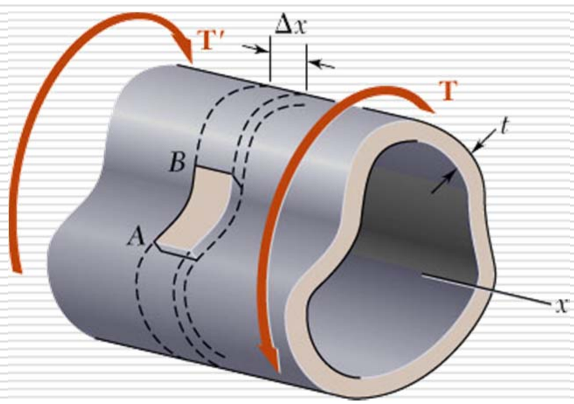
(c)
Equilateral triangular section

(d)
I-section

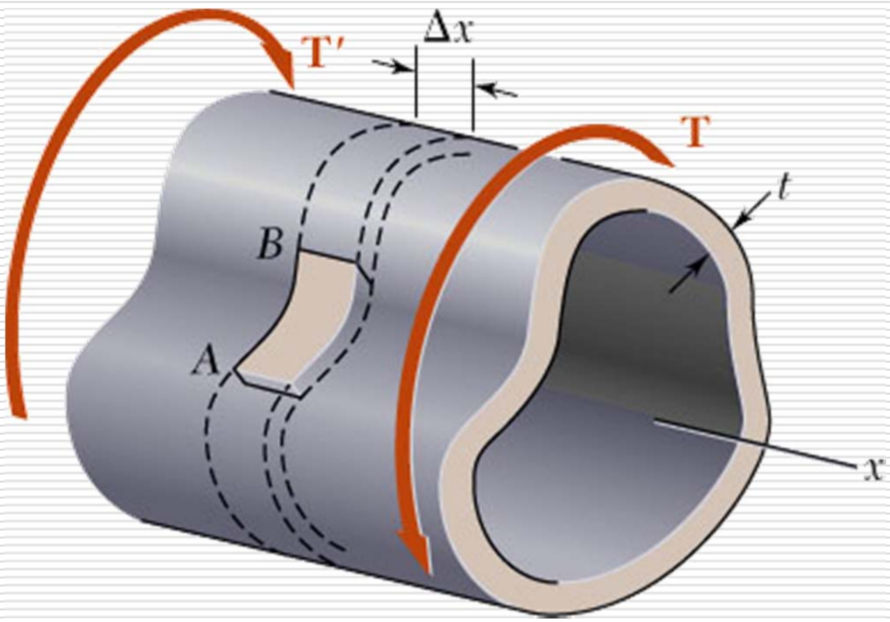
(e)
Thin tubular section with cutout

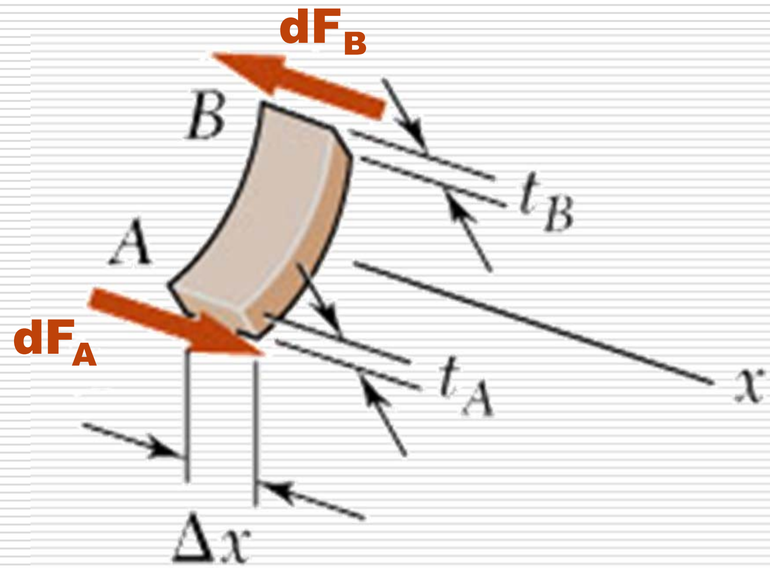
Thin Walled Tubes
Closed Cross Sections

- The member is cylindrical
  - The cross section does not vary along the length of the member
- The cross section is closed
- The wall thickness is small compared with the cross-sectional dimensions of the member
- The member is subjected to end torques only
- The ends are not restrained from warping



Thin Walled Tubes
Closed Cross Sections

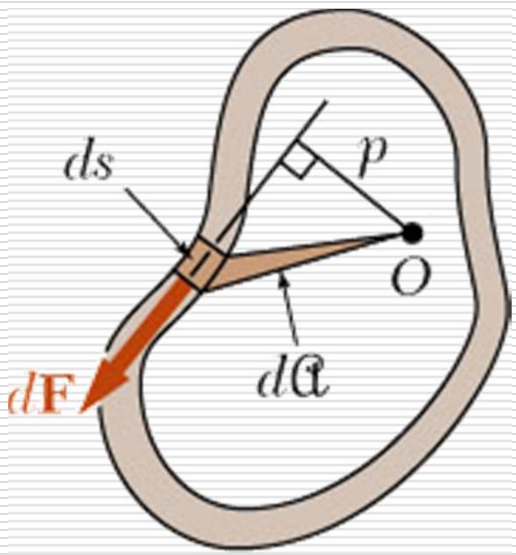


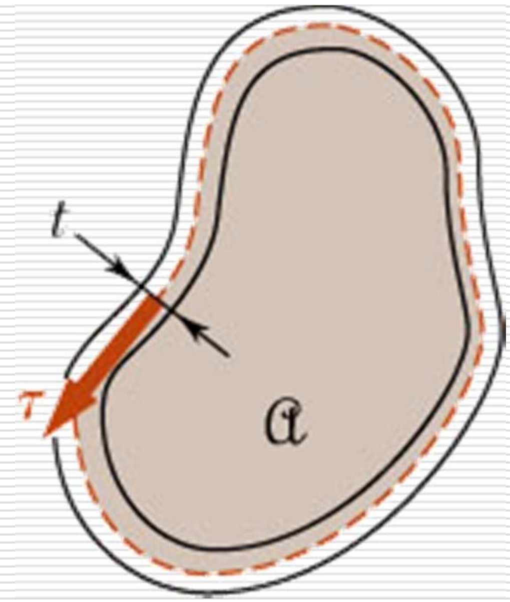


$$\sum F_x = 0 = \tau_A \left(t_A \Delta x\right) - \tau_B \left(t_B \Delta x\right)$$

$$\tau_A t_A = \tau_B t_B = \tau t = q = \text{shear flow}$$

Average Shear Stress





$$dM_0 = p \cdot dF = p \cdot \tau \cdot \left(t \cdot ds\right) = q \cdot \left(p \cdot ds\right) = 2q \cdot d\bar{A}$$

$$T = \oint dM_0 = \oint 2q \cdot d\bar{A} = 2 \cdot q \cdot \bar{A}$$

$$\tau = \frac{T}{2 \cdot t \cdot \bar{A}}$$

Angle of Twist

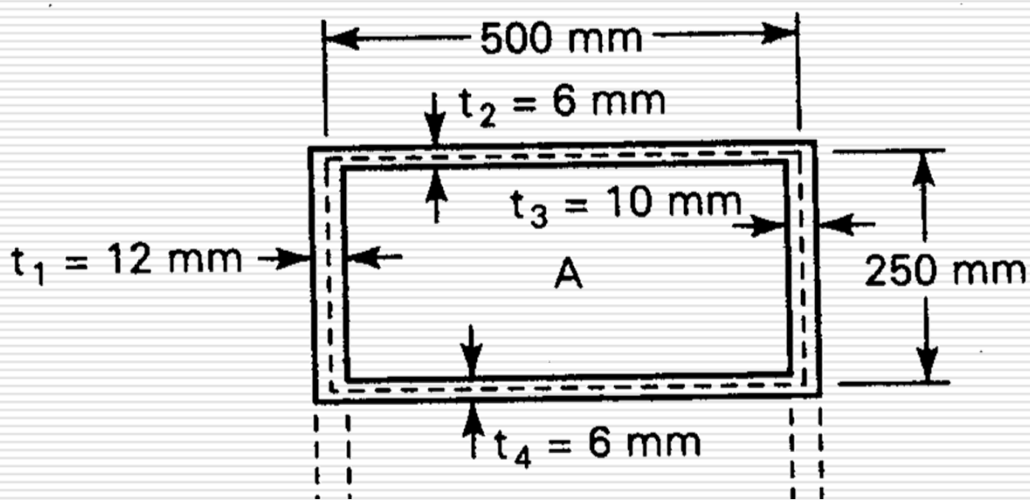
- Expanding using Strength of Materials Principles

$$\phi = \frac{q \cdot L}{2 \cdot \bar{A} \cdot G} \cdot \oint \frac{ds}{t}$$

$$= \frac{T \cdot L}{4 \cdot \bar{A}^2 \cdot G} \cdot \oint \frac{ds}{t} = \frac{T \cdot L}{4 \cdot \bar{A}^2 \cdot G} \cdot \sum \frac{\Delta s_i}{t_i}$$

Example

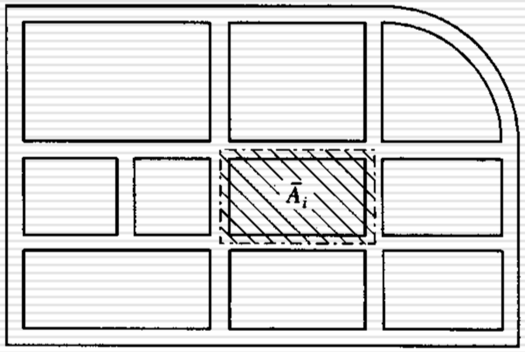
A hollow aluminum tube of rectangular cross section is subjected to a torque of 56.5 kN-m along its longitudinal axis. Determine the shearing stresses and the angle of twist. Assume G=28 GPa.

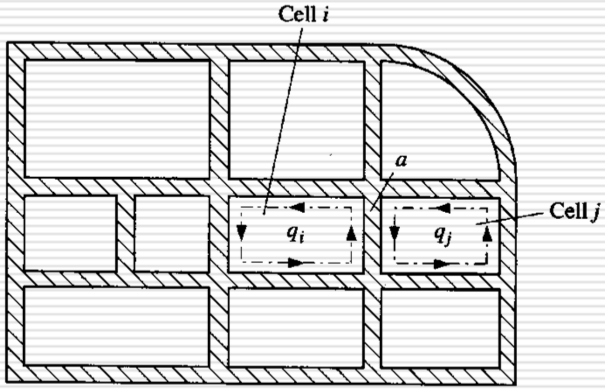


Multiple Cell Sections in Torsion

- Angle of twist Φ of each cell is the same.
- $$T = \sum_{i=1}^{n} T_i = 2 \cdot \sum_{i=1}^{n} q_i \cdot \bar{A}_i$$
- $$\phi_i = \frac{(1+\nu) \cdot L}{E \cdot \bar{A}_i} \cdot \left(\oint \frac{q}{t} \cdot ds \right)_i$$

$$= \frac{L}{2 \cdot G \cdot \bar{A}_i} \cdot \left(\oint \frac{q}{t} \cdot ds \right)_i$$





Example

A multiply connected hollow steel tube resists a torque of 12 kN-m . The wall thicknesses are t_1= t_2= t_3=6mm and t_4= t_5=3mm. Determine the maximum shearing stresses and the angle of twist per unit length. Let G=80GPa. Dimensions are given in millimeters.