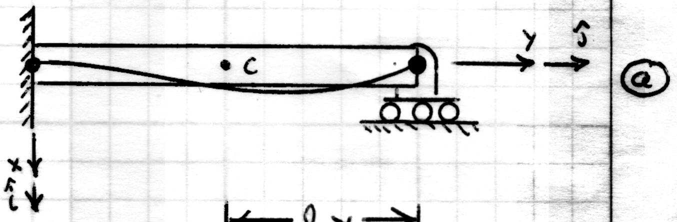
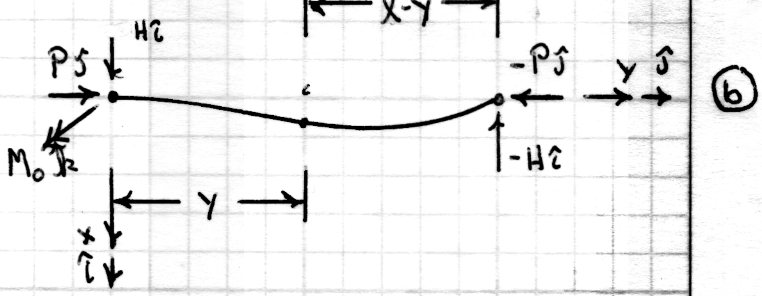
DETERMINE THE CRITICAL LOAD FOR THIS BEAM SUBJECTED TO BUCKLING LOADS.



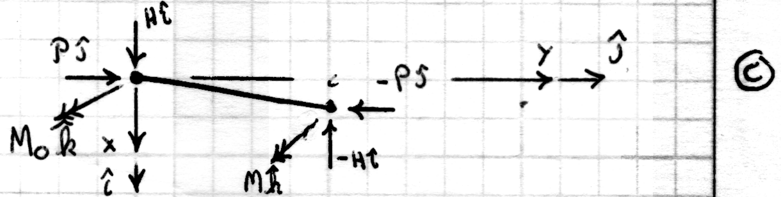
CONSIDER THE FREE BODY DIAGRAM IN FIGURE ⓑ. CONSIDER THE EQUILIBRIUM OF THE BEAM. DETERMINING THE MOMENT AT POINT C.



$$\Sigma M_z/_{@C} = 0 = M + P\cdot u + H(\ell - y)$$

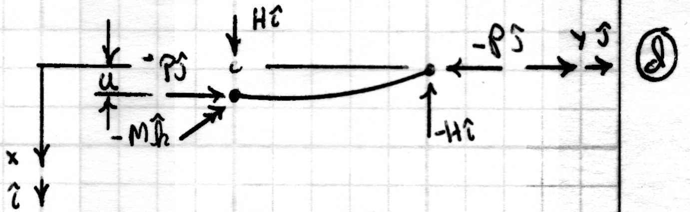
$$M = P\cdot u + H\cdot \ell - H\cdot y \qquad ①$$

FROM STRENGTH OF MATERIALS THE RELATIONSHIP BETWEEN THE MOMENT AND THE DEFLECTION IS GIVEN BY



$$\frac{\partial^2 u}{\partial y^2} = -\frac{P\cdot u - H\cdot \ell + H\cdot y}{EI}$$

$$\frac{\partial^2 u}{\partial y^2} + \frac{P}{EI}u = \frac{H}{EI}(y-\ell) \qquad ②$$



EQUATION ② REPRESENTS AN ORDINARY DIFFERENTIAL EQUATION. THE SOLUTION WILL HAVE BOTH A HOMOGENEOUS AND PARTICULAR PART

$$u = u_H + u_P \qquad ③$$

STARTING WITH THE HOMOGENEOUS PORTION OF THE SOLUTION

$$\frac{\partial^2 u_H}{\partial y^2} + \frac{P}{EI}u_H = 0 \qquad ④$$

ASSUMING THE SOLUTION TO BE OF THE FORM

$$u_H = A_n \cdot e^{S_n \cdot y}$$

$$u_H' = \partial u_H/\partial y = A_n \cdot S_n \cdot e^{S_n \cdot y} \qquad ⑤$$

$$u_H'' = \partial^2 u_H/\partial y^2 = A_n \cdot S_n^2 \cdot e^{S_n \cdot y}$$

SUBSTITUTING THE RESULTS FROM ⑤ INTO ④

$$A_n \cdot S_n^2 \cdot e^{S_n \cdot y} + \frac{P}{EI}\cdot A_n \cdot e^{S_n \cdot y} = 0$$

$$A_n \cdot e^{S_n \cdot y}\left(S_n^2 + \frac{P}{EI}\right) = 0$$

SINCE $A_n = 0$ WOULD LEAD TO A TRIVIAL SOLUTION AND $e^{S_n \cdot y}$ CAN NEVER EQUAL ZERO, THE ONLY WAY THIS EQUATION CAN BE SATISFIED IS FOR

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
AMPAD

$$S_n^2 + \frac{P}{EI} = 0 \Rightarrow S_n = \pm\sqrt{-\frac{P}{EI}} \Rightarrow S_n = \pm i\sqrt{\frac{P}{EI}} \qquad (6)$$

SUBSTITUTE (6) INTO (5)

$$U_H = A_0 e^{+i\sqrt{\frac{P}{EI}}\cdot y} + A_1 e^{-i\sqrt{\frac{P}{EI}}\cdot y}$$

$$= A_0\left(\cos\sqrt{\tfrac{P}{EI}}\cdot y + i\cdot\sin\sqrt{\tfrac{P}{EI}}\cdot y\right) + A_1\left(\cos\sqrt{\tfrac{P}{EI}}\cdot y - i\cdot\sin\sqrt{\tfrac{P}{EI}}\cdot y\right)$$

$$= \underbrace{(A_0 + A_1)}_{C_0}\cdot\cos\sqrt{\tfrac{P}{EI}}\cdot y + \underbrace{(A_0 - A_1)\cdot i}_{C_1}\cdot\sin\sqrt{\tfrac{P}{EI}}\cdot y$$

$$U_H = C_0\cdot\cos\sqrt{\tfrac{P}{EI}}\cdot y + C_1\cdot\sin\sqrt{\tfrac{P}{EI}}\cdot y \qquad (7)$$

NOW CONSIDERING THE PARTICULAR SOLUTION. THE PARTICULAR SOLUTION WILL HAVE THE SAME FORM AS THE RIGHT HAND SIDE OF (2)

$$U_P = C_2\cdot y + C_3$$

$$U_P' = \frac{du_P}{dy} = C_2 \qquad (8)$$

$$U_P'' = \frac{d^2u_P}{dy^2} = 0$$

SUBSTITUTING THE RESULTS FROM (8) INTO (2)

$$0 + \frac{P}{EI}\cdot(C_2\cdot y + C_3) = \frac{H}{EI}(y - l) \Rightarrow C_2\cdot y + C_3 = \frac{H}{P}(y - l)$$

$$\Rightarrow U_P = \frac{H}{P}(y - l) \qquad (9)$$

(9) AND (7) ARE NOW SUBSTITUTED INTO (3)

$$U = C_0\cdot\cos\sqrt{\tfrac{P}{EI}}\cdot y + C_1\cdot\sin\sqrt{\tfrac{P}{EI}}\cdot y + \frac{H}{P}(y - l) \qquad (10)$$

THE BOUNDARY CONDITIONS CAN NOW BE INVOKED TO DETERMINE THE CONSTANTS
FIRST AT $y=0$, $U(0)=0$

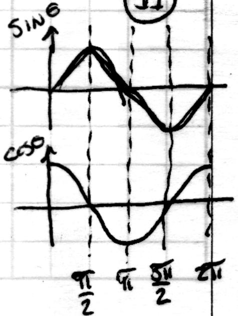
$$U(0) = 0 = C_0\cdot\underbrace{\cos\sqrt{\tfrac{P}{EI}}\cdot 0}_{1} + C_1\cdot\underbrace{\sin\sqrt{\tfrac{P}{EI}}\,0}_{0} + \frac{H}{P}(0 - l)$$

$$0 = C_0 - \frac{Hl}{P} \Rightarrow C_0 = \frac{Hl}{P}$$

$$\Rightarrow U = \frac{Hl}{P}\cdot\cos\sqrt{\tfrac{P}{EI}}\cdot y + C_1\cdot\sin\sqrt{\tfrac{P}{EI}}\cdot y + \frac{H}{P}(y - l) \qquad (11)$$

THE SECOND BOUNDARY CONDITION IS THAT AT $y=0$, $du/dy = 0$

$$\frac{du}{dy} = -\frac{Hl}{P}\sqrt{\frac{P}{EI}}\sin\sqrt{\frac{P}{EI}}\cdot y + C_1\sqrt{\frac{P}{EI}}\cos\sqrt{\frac{P}{EI}}\cdot y + \frac{H}{P}$$



INVOKING THE BOUNDARY CONDITION

$$\frac{du(0)}{dy} = 0 = -\frac{H\cdot l}{P}\sqrt{\frac{P}{EI}}\cdot \cancelto{0}{\sin\sqrt{\frac{P}{EI}}\cdot 0} + C_1\sqrt{\frac{P}{EI}}\cdot \cancelto{1}{\cos\sqrt{\frac{P}{EI}}\cdot 0} + \frac{H}{P}$$

$$0 = C_1\sqrt{\frac{P}{EI}} + \frac{H}{P} \Rightarrow C_1 = -\frac{H}{P}\sqrt{\frac{EI}{P}}$$

$$\Rightarrow u = \frac{H\cdot l}{P}\cdot \cos\sqrt{\frac{P}{EI}}\cdot y - \frac{H}{P}\sqrt{\frac{EI}{P}}\cdot \sin\sqrt{\frac{P}{EI}}\cdot y + \frac{H}{P}(y-l)$$

$$= \frac{H}{P}\left(l\cdot\cos\sqrt{\frac{P}{EI}}\cdot y - \sqrt{\frac{EI}{P}}\cdot \sin\sqrt{\frac{P}{EI}}\cdot y + y - l\right)$$

THE FINAL BOUNDARY CONDITION IS THAT AT $y=L$, $u(L)=0$

$$u(L)=0 = \frac{H}{P}\left(l\cdot\cos\sqrt{\frac{P}{EI}}\cdot L - \sqrt{\frac{EI}{P}}\cdot \sin\sqrt{\frac{P}{EI}}\cdot L + L - L\right)$$

$$0 = l\cdot\cos\sqrt{\frac{P}{EI}}\cdot L - \sqrt{\frac{EI}{P}}\cdot \sin\sqrt{\frac{P}{EI}}\cdot L$$

$$l\cdot\cos\sqrt{\frac{P}{EI}}\cdot L = \sqrt{\frac{EI}{P}}\cdot \sin\sqrt{\frac{P}{EI}}\cdot L$$

$$\frac{\sin\sqrt{\frac{P}{EI}}\cdot L}{\cos\sqrt{\frac{P}{EI}}\cdot L} = \frac{l}{\sqrt{\frac{EI}{P}}} = \sqrt{\frac{P}{EI}}\cdot l \Rightarrow \tan\sqrt{\frac{P}{EI}}\cdot L = \sqrt{\frac{P}{EI}}\cdot l \qquad (12)$$

OTHER THAN $\sqrt{\frac{P}{EI}}\cdot l = 0$, THE SMALLEST VALUE OF $\sqrt{\frac{P}{EI}}\cdot l$ THAT SATISFIES (12) IS

$$\sqrt{\frac{P}{EI}}\cdot L = 4.493 \Rightarrow P_{CR} = \frac{(4.493)^2}{L^2}\cdot EI = \frac{20.19}{L^2}\cdot EI = \frac{2.046\cdot\pi^2 EI}{L^2}$$

$$P_{CR} = \frac{\pi^2\cdot E\cdot I}{.4888\, L^2} = \boxed{\frac{\pi^2\cdot E\cdot I}{(.6992)^2 L^2}}$$