PROBLEM 3 A SIMPLE BEAM AB WITH AN OVERHANG BC IS LOADED BY TWO FORCES P AND A COUPLE Pa THROUGH THE ARRANGEMENT SHOWN IN THE FIGURE. DRAW THE SHEAR AND BENDING MOMENT DIAGRAM FOR BEAM ABC.

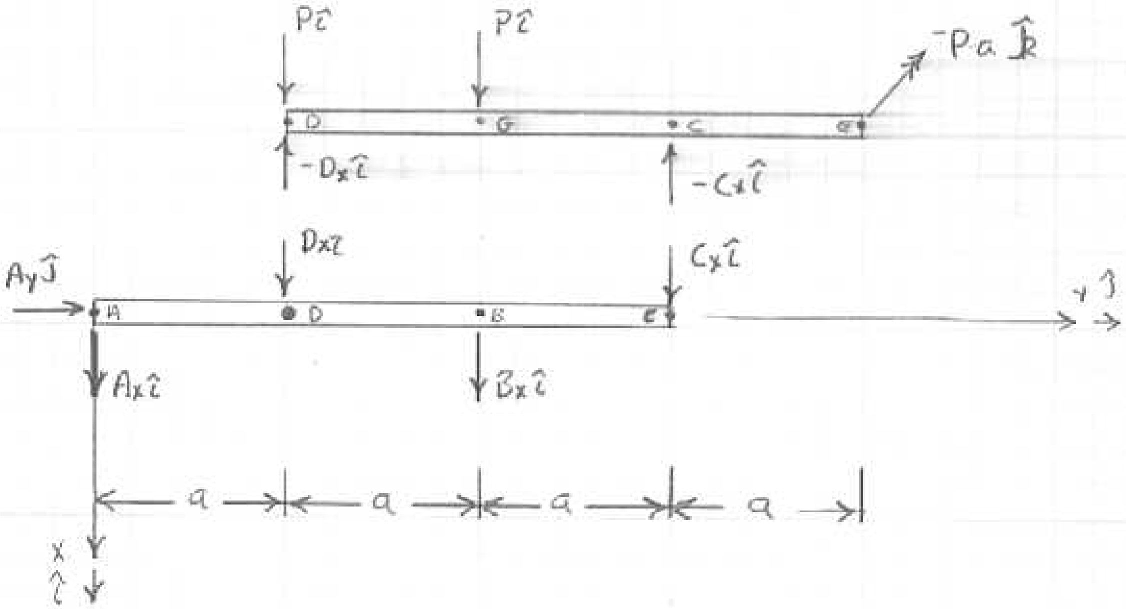
GIVEN:

CONSTRAINTS
1) TWO BEAMS CONNECTED BY PIN JOINTS
2) BEAM LOADED WITH TWO POINT LOADS AND A COUPLE

ASSUMPTIONS
1) MATERIAL RESPONDS IN A LINEAR ELASTIC MANNER
2) DEFLECTIONS ARE SMALL

FIND:
1) SHEAR AND BENDING MOMENT DIAGRAM FOR ABC

FREE BODY DIAGRAM:



ⓐ

MECHANICS:

THE SOLUTION STARTS BY CONSIDERING THE EQUILIBRIUM OF DGCE

$$\sum F_x = \emptyset = 2P - D_x - C_x \Rightarrow \underline{D_x + C_x = 2 \cdot P} \quad (1)$$

$$\sum M_{z/AT\,D} = \emptyset = -Pa - 2aC_x - Pa \Rightarrow \underline{C_x = P} \quad (2)$$

$$(2) \rightarrow (1) \Rightarrow \underline{D_x = P} \quad (3)$$

THESE RESULTS ARE NOW USED TO DETERMINE THE REACTIONS IN ADBC.

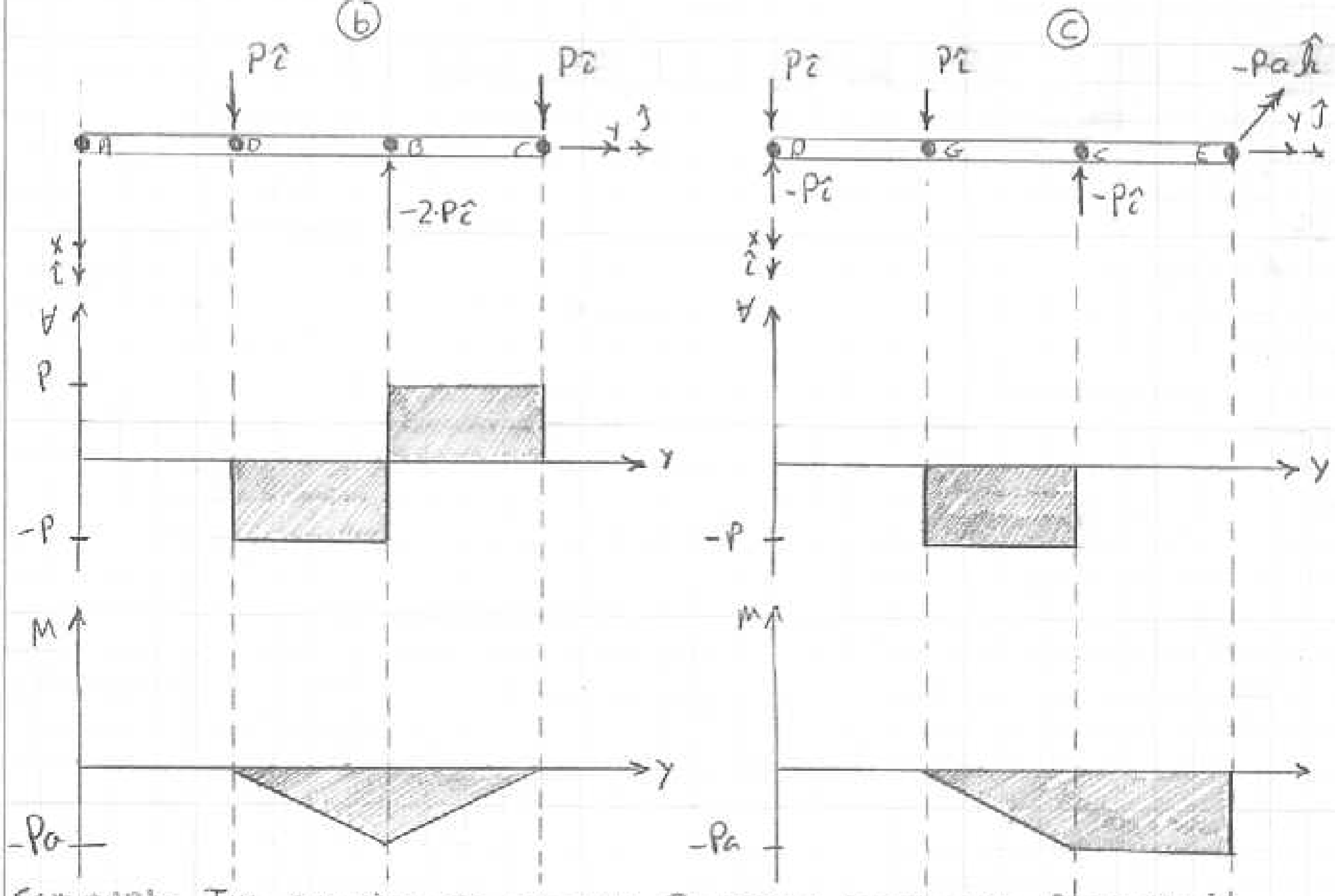
$$\sum F_y = \underline{\emptyset = A_y}$$

$$\sum F_x = \emptyset = A_x + P + B_x + P = \emptyset \Rightarrow \underline{A_x + B_x = -2P} \quad (4)$$

$$\sum M_{z/AT\,A} = \emptyset = -Pa - 2 \cdot a \cdot B_x - 3 \cdot a \cdot P \Rightarrow \underline{B_x = -2P} \quad (5)$$

$$(5) \rightarrow (4) \Rightarrow \underline{A_x = \emptyset} \quad (6)$$

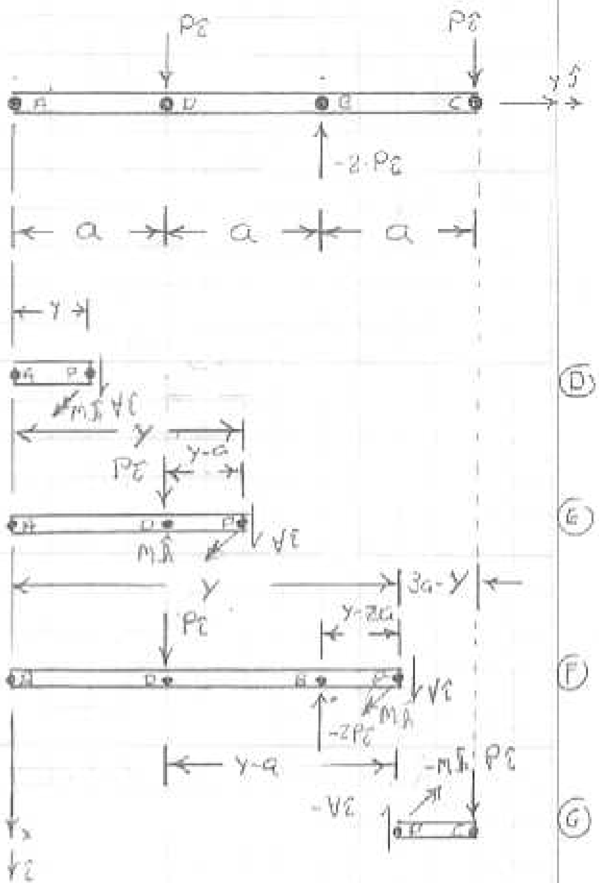
NOW THE SHEAR AND BENDING MOMENT DIAGRAMS CAN BE DRAWN



SUMMARY: THE SOLUTION IS QUICKLY REALIZED ONCE THE PROBLEM IS DECOMPOSED INTO TWO BEAMS. INTERNAL REACTIONS ARE EQUAL BUT OPPOSITE.

PROB 3 SUPPLEMENTAL

FOR BEAM "ADBC"



0 < y < a   USING (D)

$\Sigma F_x = 0 = V \quad \Rightarrow \quad \boxed{V = 0}$

$\Sigma M_{z/p} = 0 = M \quad \Rightarrow \quad \boxed{M = 0}$

a < y < 2a   USING (E)

$\Sigma F_x = 0: V + P \quad \Rightarrow \quad \boxed{V = -P}$

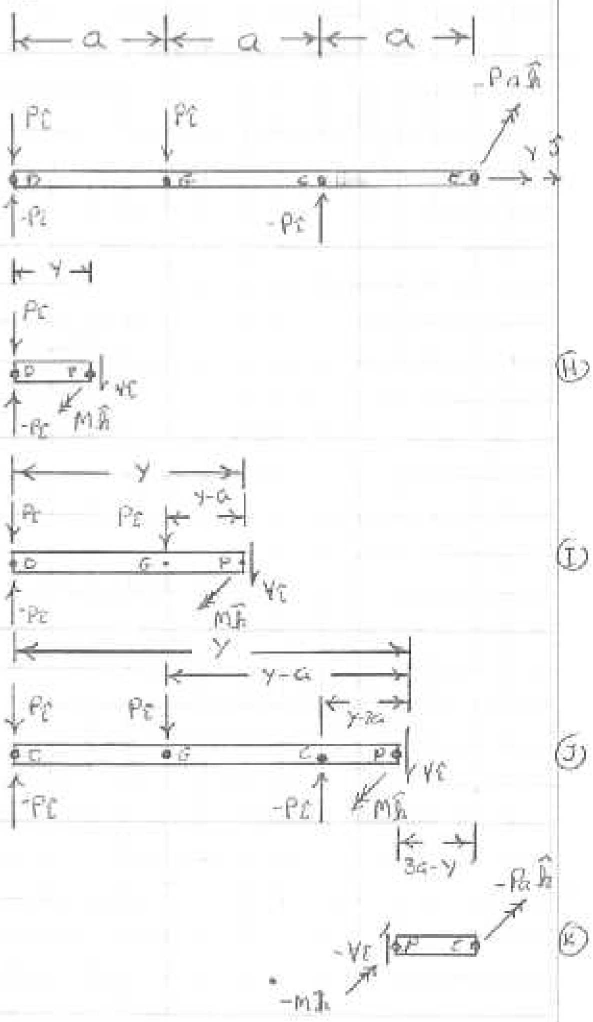
$\Sigma M_{z/p} = 0 = M + (y-a)\cdot P \quad \Rightarrow \quad \boxed{M = -P\cdot y + a\cdot P}$

2a < y < 3a   USING (G)

$\Sigma F_y = 0 = -V + P \quad \Rightarrow \quad \boxed{V = P}$

$\Sigma M_{z/ep} = 0 = -M - (3a-y)\cdot P \quad \Rightarrow \quad \boxed{M = P_y - 3a\cdot P}$

FOR BEAM "DGCE"



0 < y < a   USING (H)

$\Sigma F_y = 0 = P - P + V \quad \Rightarrow \quad \boxed{V = 0}$

$\Sigma M_{z/ep} = 0 = M - P_y + P_y \quad \Rightarrow \quad \boxed{M = 0}$

a < y < 2a   USING (I)

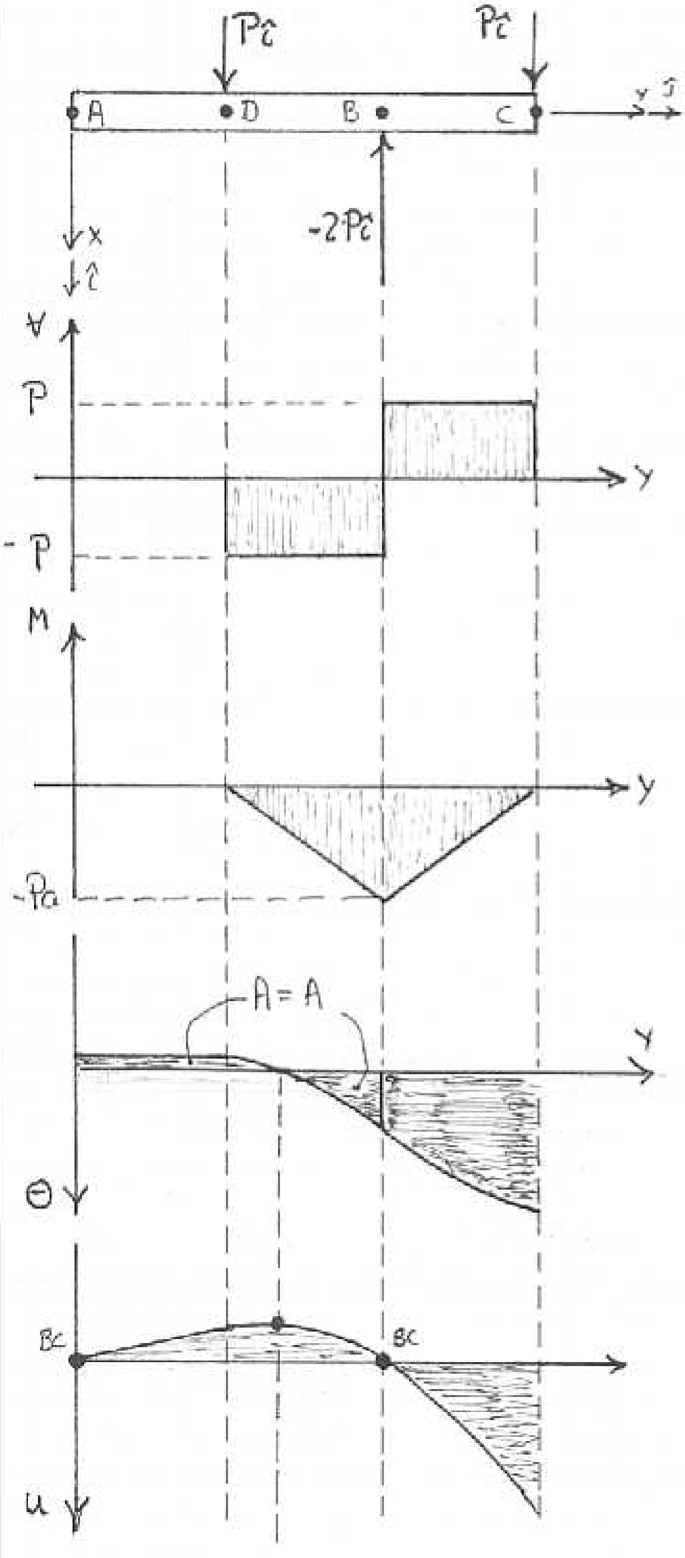
$\Sigma F_x = 0 = P - P + P + V \quad \Rightarrow \quad \boxed{V = -P}$

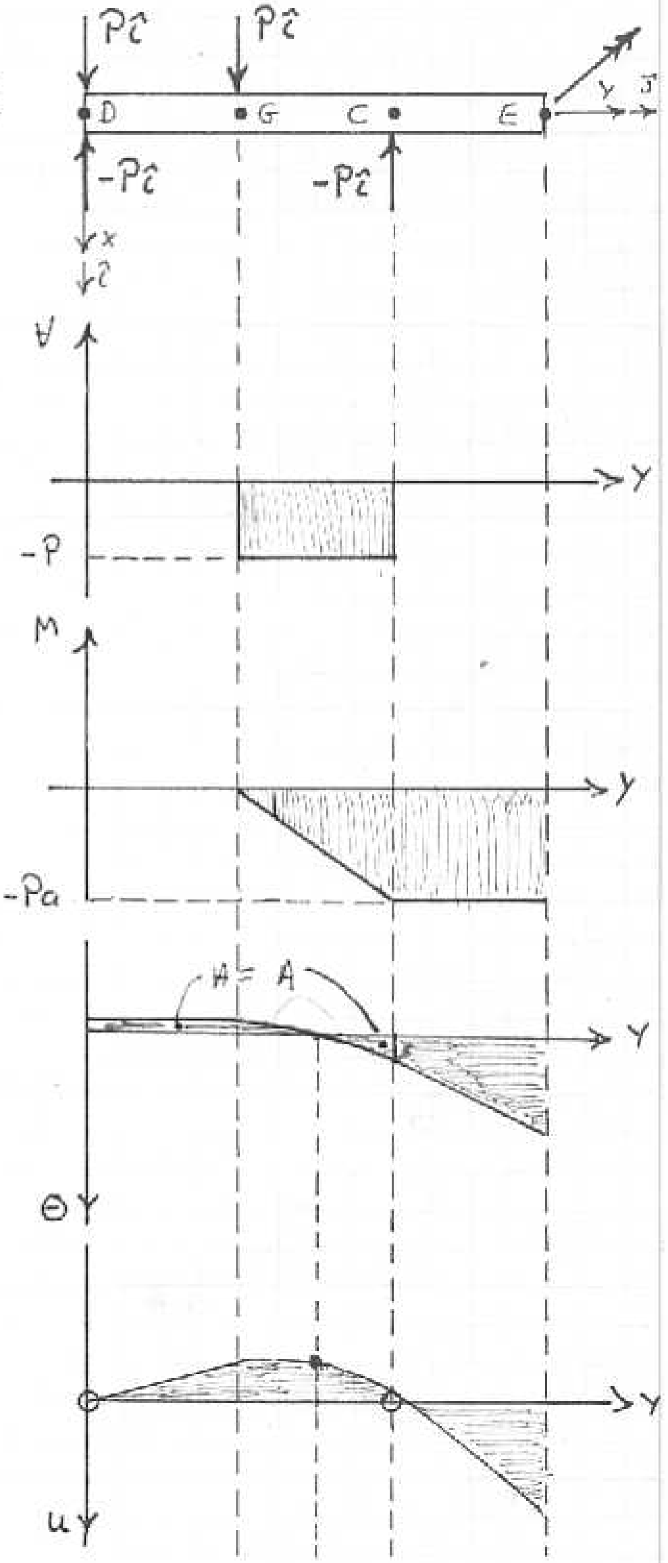
$\Sigma M_{z/ep} = 0 = -P_y + P_y + P(y-a) + M \quad \Rightarrow \quad \boxed{M = -P_y + Pa}$

2a < y < 3a   USING (K)

$\Sigma F_x = 0 = V \quad \Rightarrow \quad \boxed{V = 0}$

$\Sigma M_{z/ep} = 0 = -M - Pa \quad \Rightarrow \quad \boxed{M = -Pa}$





BECAUSE THE BOUNDARY CONDITION AT D & C ON DGCE ARE DEPENDENT ON THE POSITION OF D & C ON ADBC, THE Θ AND u CURVES FOR DGCE ARE RELATIVE TO THE POSITIONS OF D & C ON ADBC AND NOT ABSOLUTE

FOR BEAM ADBC

$$q = P\langle y-a\rangle_{-1} - 2\cdot P\langle y-2a\rangle_{-1} + P\langle y-3a\rangle_{-1} \quad (1)$$

$$V = -\int q\, dy$$

$$= -P\langle y-a\rangle^0 + 2\cdot P\langle y-2a\rangle^0 - P\langle y-3a\rangle^0 \quad (2)$$

$$M = \int V \cdot dy$$

$$= -P\langle y-a\rangle^1 + 2\cdot P\langle y-2a\rangle^1 - P\langle y-3a\rangle^1 \quad (3)$$

$$\theta = \frac{-1}{EI}\int M \cdot dy$$

$$= \frac{P}{2EI}\langle y-a\rangle^2 - \frac{P}{EI}\langle y-2a\rangle^2 + \frac{P}{2EI}\langle y-3a\rangle^2 + C_1 \quad (4)$$

$$u = \int \theta \cdot dy$$

$$= \frac{P}{6EI}\langle y-a\rangle^3 - \frac{P}{3EI}\langle y-2a\rangle^3 + \frac{P}{6EI}\langle y-3a\rangle^3 + C_1 \cdot y + C_2 \quad (5)$$

THE CONSTANTS OF INTEGRATION ARE DETERMINED USING THE BOUNDARY CONDITIONS $u(0)=0$ & $u(2a)=0$. FROM THE FIRST OF THESE BOUNDARY CONDITIONS

$$u(0) = 0 = C_2$$

FROM THE SECOND BOUNDARY CONDITION

$$u(2a) = 0 = \frac{P}{6EI}(2a-a)^3 + C_1 \cdot (2a) = \frac{P \cdot a^3}{6EI} + C_1 \cdot 2a$$

$$\Rightarrow C_1 = -\frac{P \cdot a^3}{6 \cdot E \cdot I} \cdot \frac{1}{2a} = -\frac{P \cdot a^2}{12 \cdot E \cdot I}$$

EQUATIONS (4) & (5) CAN NOW BE REWRITTEN

$$\theta = \frac{P}{2E \cdot I}\langle y-a\rangle^2 - \frac{P}{EI}\langle y-2a\rangle^2 + \frac{P}{2E \cdot I}\langle y-3a\rangle^2 - \frac{P \cdot a^2}{12EI} \quad (6)$$

$$u = \frac{P}{6EI}\langle y-a\rangle^3 - \frac{P}{3EI}\langle y-2a\rangle^3 + \frac{P}{6EI}\langle y-3a\rangle^3 - \frac{P \cdot a^2}{12EI} \cdot y \quad (7)$$

FOR BEAM DGCE IT NEEDS TO BE UNDERSTOOD THAT THE BOUNDARY CONDITIONS AT D AND C ARE RELATIVE TO BEAM ADBC. THIS MEANS THAT THE RIGID BODY DEFORMATIONS THAT RESULT FROM ~~DGCE~~ ADBC DEFORMING UNDER LOAD HAVE TO BE ADDED TO THE FOLLOWING RESULTS.

$$q = \underbrace{P\langle y-0\rangle_{-1} - P\langle y-0\rangle_{-1}}_{\text{TERMS CANCEL}} + P\langle y-a\rangle_{-1} - P\langle y-2a\rangle_{-1} - P\cdot a\langle y-3a\rangle_{-2} \quad (8)$$

$$V = -\int q\cdot dy$$

$$= \underbrace{-P\langle y-0\rangle^0 + P\langle y-0\rangle^0}_{\text{TERMS CANCEL}} - P\langle y-a\rangle^0 + P\langle y-2a\rangle^0 + P\cdot a\langle y-3a\rangle_{-1} \quad (9)$$

$$M = \int V\, dy$$

$$= \underbrace{-P\langle y-0\rangle^1 + P\langle y-0\rangle^1}_{\text{TERMS CANCEL}} - P\langle y-a\rangle^1 + P\langle y-2a\rangle^1 - Pa\langle y-3a\rangle^0 \quad (10)$$

$$\theta = \frac{-1}{EI}\int M\cdot dy$$

$$= \underbrace{\frac{P}{2\cdot EI}\langle y-0\rangle^2 - \frac{P}{2EI}\langle y-0\rangle^2}_{\text{TERMS CANCEL}} + \frac{P}{2EI}\langle y-a\rangle^2 - \frac{P}{2EI}\langle y-2a\rangle^2 + \frac{P\cdot a}{EI}\langle y-3a\rangle^1 + C_3 \quad (11)$$

$$u = \int \theta\, dy$$

$$= \underbrace{\frac{P}{6EI}\langle y-0\rangle^3 - \frac{P}{6EI}\langle y-0\rangle^3}_{\text{TERMS CANCEL}} + \frac{P}{6EI}\langle y-a\rangle^3 - \frac{P}{6EI}\langle y-2a\rangle^3 + \frac{P\cdot a}{2EI}\langle y-3a\rangle^2 + C_3 y + C_4 \quad (12)$$

THE CONSTANTS C_3 AND C_4 ARE DETERMINED USING THE BOUNDARY CONDITIONS $u(0)=0$ AND $u(2a)=0$. THESE BOUNDARY CONDITIONS ARE RELATIVE TO ADBC

$$u(0) = 0 = C_4$$

$$u(2a) = 0 = \frac{P}{6EI}(2a-a)^3 + C_3\cdot 2a = \frac{P\cdot a^3}{6\cdot EI} + 2\cdot a\cdot C_3$$

$$\Rightarrow C_3 = \frac{-Pa^2}{12EI}$$

REWRITING (8)-(12)

$$q = P\langle y-a\rangle_{-1} - P\langle y-2a\rangle_{-1} - Pa\langle y-3a\rangle_{-2}$$

$$V = -P\langle y-a\rangle^0 + P\langle y-2a\rangle^0 + Pa\langle y-3a\rangle_{-1}$$

$$M = -P\langle y-a\rangle^1 + P\langle y-2a\rangle^1 - Pa\langle y-3a\rangle^0$$

$$\theta = \frac{P}{2EI}\langle y-a\rangle^2 - \frac{P}{2EI}\langle y-2a\rangle^2 + \frac{Pa}{EI}\langle y-3a\rangle^1 - \frac{Pa^2}{12EI}$$

$$u = \frac{P}{6EI}\langle y-a\rangle^3 - \frac{P}{6EI}\langle y-2a\rangle^3 + \frac{P\cdot a}{2EI}\langle y-3a\rangle^2 - \frac{Pa^2}{12EI}\cdot y$$