PROBLEM 2.31 ‖ THE ENGINEERING STRAIN MATRIX AT A PARTICULAR POINT IN A STRUCTURE IS

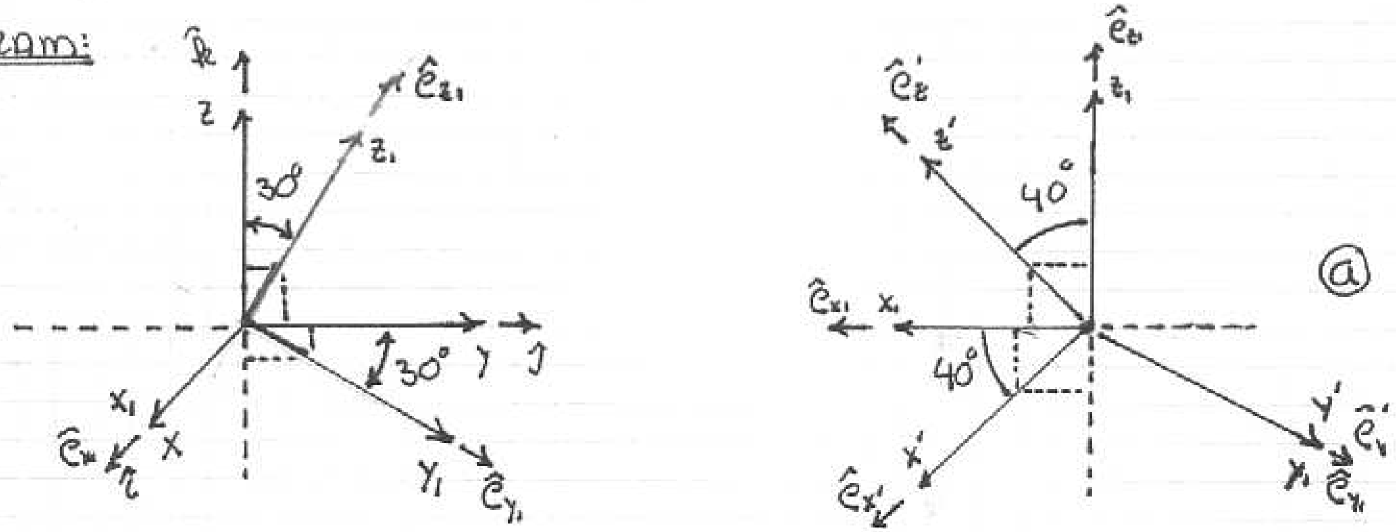
$$[\varepsilon]_{ENG} = \begin{bmatrix} \epsilon_x & \gamma_{xy} & \gamma_{xz} \\ \gamma_{xy} & \epsilon_y & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & \epsilon_z \end{bmatrix} = \begin{bmatrix} -4 & 0 & 2 \\ 0 & 3 & -2 \\ 2 & -2 & -1 \end{bmatrix} \times 10^{-4}$$

DETERMINE THE ENGINEERING STRAIN MATRIX RELATIVE TO A COORDINATE SYSTEM DEFINE BY FIRST ROTATING THE XYZ COORDINATE SYSTEM -30° ABOUT THE X-AXIS, THEN ROTATING 40° ABOUT THE NEW Y-AXIS.

GIVEN:
CONSTRAINTS
1. STRAIN TENSOR GIVEN
2. -30° ROTATION ABOUT THE X-AXIS
3. 40° ABOUT THE NEW Y-AXIS.
ASSUMPTIONS
1. STRAIN IS DEFINED AS A POINT IN THE STRUCTURE

FIND:
1. THE NEW ENGINEERING STATE OF STRAIN

DIAGRAM:



SOLUTION:

THE SOLUTION STARTS BY CONSTRUCTING THE TRANSFORMATION MATRIX FOR GOING FORM THE XYZ COORDINATE SYSTEM TO THE $X_1 Y_1 Z_1$ COORDINATE SYSTEM

$$\begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & \cos 120 \\ 0 & \cos 60 & \cos 30 \end{bmatrix} \cdot \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & -0.50 \\ 0 & 0.50 & 0.866 \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \qquad (1)$$

$$\begin{Bmatrix} X' \\ Y' \\ Z' \end{Bmatrix} = \begin{bmatrix} \cos 40 & 0 & \cos 130 \\ 0 & 1 & 0 \\ \cos 50 & 0 & \cos 40 \end{bmatrix} \cdot \begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} = \begin{bmatrix} .7660 & 0 & -.6428 \\ 0 & 1 & 0 \\ .6428 & 0 & .7660 \end{bmatrix} \begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} \qquad (2)$$

$$\begin{Bmatrix} X' \\ Y' \\ Z' \end{Bmatrix} = \begin{bmatrix} .7660 & 0 & -.6428 \\ 0 & 1 & 0 \\ .6428 & 0 & .7660 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & -0.5 \\ 0 & 0.5 & 0.866 \end{bmatrix} \cdot \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}$$

$$\begin{Bmatrix} X' \\ Y' \\ Z' \end{Bmatrix} = \underbrace{\begin{bmatrix} .7660 & -.3214 & -.5567 \\ 0 & .8660 & -.500 \\ .6428 & .3830 & .6634 \end{bmatrix}}_{[T]} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \qquad (3)$$

THE STRAINS GIVEN ARE ENGINEERING STRAINS. THE STRAIN TENSOR FOR THIS STATE OF STRAIN IS

$$[\varepsilon] = \begin{bmatrix} -4 & 0 & 1 \\ 0 & 3 & -1 \\ 1 & -1 & -1 \end{bmatrix} \times 10^{-4}$$

THE TRANSFORMATION CAN NOW BE CALCULATED BY

$$[\epsilon_{x'y'z'}] = [T]\cdot[\epsilon_{xyz}][T]^T$$

$$= \begin{bmatrix} -0.3558 & -0.1175 & -0.1393 \\ -0.1175 & 0.2866 & 0.0622 \\ -0.1393 & 0.0622 & -0.1308 \end{bmatrix} \times 10^{-3}$$

SUMMARY:

THE CREATION OF THE TRANSFORMATION MATRIX FOR STRAIN IS IDENTICAL TO THE TRANSFORMATION MATRIX FOR STRESS.

$$[\epsilon_{x'y'z'}]_{ENG} = \begin{bmatrix} -0.3558 & -0.2350 & -0.2786 \\ -0.2350 & 0.2866 & 0.1245 \\ -0.2786 & 0.1245 & -0.1308 \end{bmatrix} \times 10^{-3}$$