A bar of steel has S_u 700 MPa S_y 500MPa and a fully co rected endurance im t of S_e 200 MPa For each case below find the factor of safety wh ch guard against stat c and fatigue failures

τ_m 40 MPa

τ 140 MPa τ_a 70 MPa

τ_{xym} 100 MPa σ_{xa} 80 MPa

σ_{xm} 60 MPa σ_{xa} 80 MPa
τ_{xym} 70 MPa τ_{xya} 35 MPa

Solution:

(a) $\tau_m = 140\ MPa$

Using maximum shear theory

$$= 0.5\ S_y = 0.5(500\ MPa) = 250\ MPa$$

$$n = \frac{S_{sy}}{\tau_{max}} = \frac{250\ MPa}{140\ MPa} = 1.78 \quad \text{(static)}$$

(b) $\tau_m = 140\ MPa$, $\tau_a = 70\ MPa$

$$\tau_{max} = \tau_m + \tau_a = 140\ MPa + 70\ MPa = 210\ MPa$$

$$n = \frac{S_{sy}}{\tau_{max}} = \frac{250\ MPa}{210\ MPa} = 1.19 \quad \text{(static)}$$

Now we need to calculate the endurance limit in shear to determine the fatigue factor of safety.

$$0.5\ S_e = 0.5(200\ MPa) = 100\ MPa$$

$$\frac{S_{se}}{\tau_a} = \frac{100\ MPa}{70\ MPa} = \underline{\underline{1.43}} \quad \text{(fatigue)}$$

Note that for shear loading only the alternating component is used to calculate the factor of safety. As previously discussed, the shear endurance limit is not effected by the mean stress level as long as the material does not yield.

(c) $\tau_{xy\,m} = 100\text{ MPa}$, $\sigma_{xa} = 80\text{ MPa}$

The maximum von Mises stress occurs when the alternating component is summed with the mean component

$$\sigma'_{max} = \sqrt{\sigma_{xa}^2 + 3\tau_{xym}^2} = \sqrt{(80\text{MPa})^2 + 3(100\text{MPa})^2} = 191\text{ MPa}$$

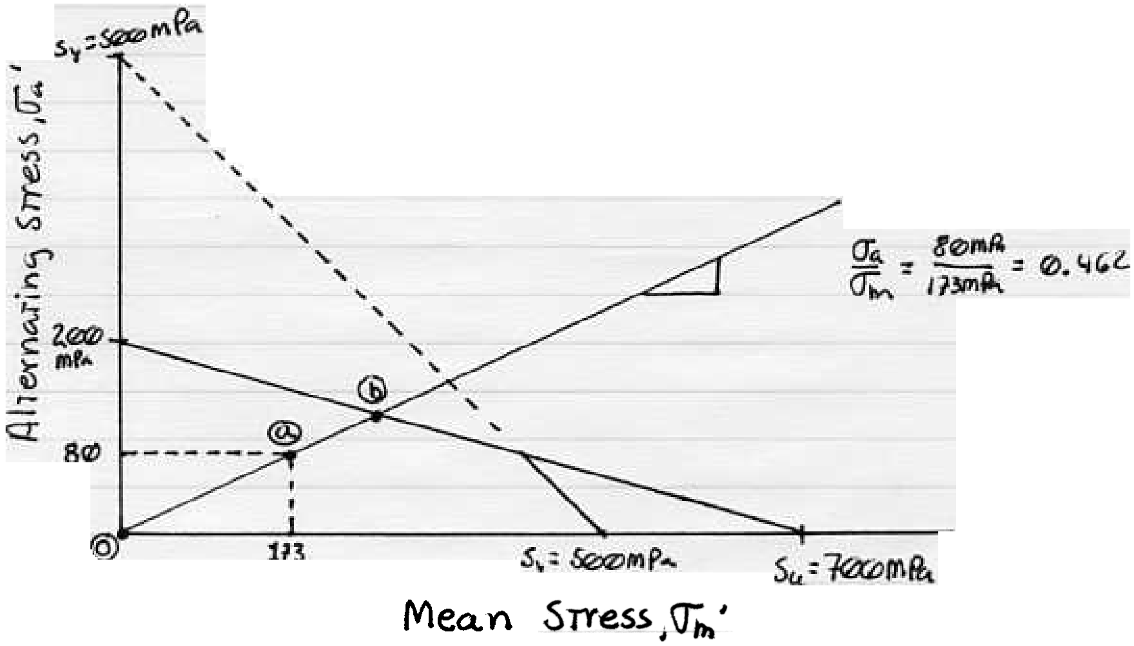
Therefore the static factor of safety is

$$n = \frac{S_y}{\sigma'_{max}} = \frac{500\text{ MPa}}{191\text{ MPa}} = \underline{\underline{2.62}} \qquad \text{(STATIC)}$$

Now lets consider a fatigue failure

$$\sigma'_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} = \sqrt{3(100\text{MPa})^2} = 173\text{ MPa}$$

$$\sigma'_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2} = \sigma_{xa} = 80\text{ MPa}$$



Point ⓐ on the diagram is defined from the von Mises calculations. Point ⓑ is the intersection of the two lines.

Let's determine the equation of the Goodman ine

$\sigma_a \quad \sigma_m + b$ standard equation of a line

b intercept 200 MPa

m slope $\frac{Rise}{Run}$ $\frac{200 \text{ MPa}}{700 \text{ MPa}}$ 0.86

$\sigma_a \quad 0.286\,\sigma_m + 200 \text{ MPa}$ (1)

Now ets consider the equation of he e
that defines the ratio between σ_a and σ

$\sigma_a \quad m\,\sigma_m \quad b$

$b \quad 0$

$m \quad \frac{\sigma_a}{\sigma} \quad 0\,462$

$\sigma_a \quad 0\,462\,\sigma$ (2)

The in rcept is defined where σ_a and σ_m
are equal herefore we can wr e

$\sigma_a^{\prime(2b)} \quad \sigma_a^{(2b)}$

$0\,286\,\sigma^{\prime(b)} + 200 \text{ MPa} \quad 0\,462\,\sigma^{(b)}$

$\sigma^{\prime(b)}$ $\underline{267.4 \text{ MPa}}$

$\sigma_a^{\prime(b)}$ $\underline{23.5 \text{ MPa}}$

Now ets determine o a and o b since

o-b
o

fatigue

$$b = \sqrt{(267.4\ MPa)^2 + (123.5\ MPa)^2} = \underline{294.5\ MPa}$$

$$0-a = \sqrt{(173\ MPa)^2 + (80\ MPa)^2} = \underline{190.6\ MPa}$$

$$n = \frac{294.5\ MPa}{190.6\ MPa} = 1.55$$

(d) $\sigma_{xm} = 60 MPa$, $\sigma_{xa} = 80 MPa$, $\tau_{xym} = 70 MPa$, $\tau_{xya} = 35 MPa$

We will again consider static yielding first

$$\sigma_{x\,max} = \sigma_{xm} + \sigma_{xa} = 60\ MPa + 80\ MPa = 140 MPa$$

$$\tau_{xymax} = \tau_{xym} + \tau_{xya} = 70\ MPa + 35\ MPa = \underline{105 MPa}$$

Now we can compute the maximum von Mises stress

$$\sigma'_{max} = \sqrt{(140\ MPa)^2 + 3(105\ MPa)^2} = \underline{229\ MPa}$$

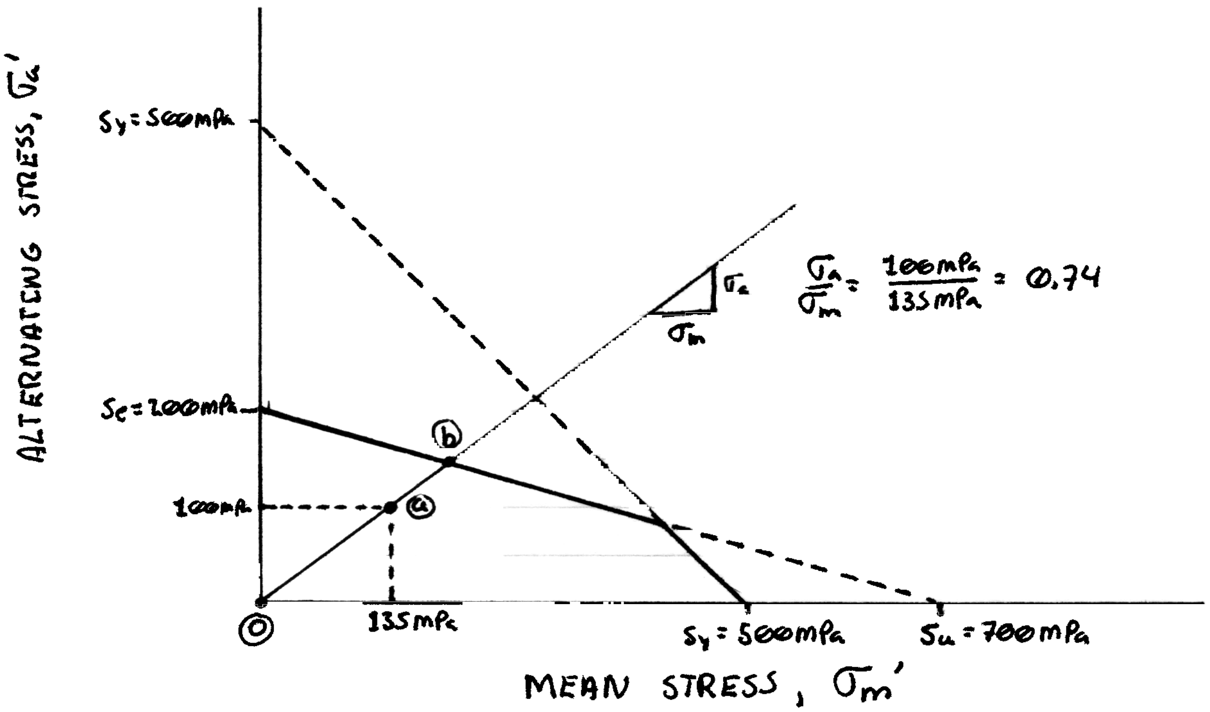
$$n = \frac{S_y}{\sigma'_{max}} = \frac{500\ MPa}{229\ MPa} = 2.18$$

Now let's calculate the fatigue factor of safety

$$= \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} = \sqrt{(60 MPa)^2 + 3(70 MPa)^2} = \underline{135 MPa}$$

$$\sigma_a' = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2} = \sqrt{(80 MPa)^2 + 3(35 MPa)^2} = 100 MPa$$

Once again we draw the σ_m-σ_a diagram



The equation for the Goodman Line is, as before,

$$\sigma_a' = m\sigma_m' + b = -0.286\,\sigma_m' + 200\,MPa \qquad (1)$$

The equation for the line o-a-b is

$$\sigma_a' = m\sigma_m' + b$$

$$b = 0$$
$$m = 0.74$$

$$\sigma_a' = 0.74\,\sigma_m' \qquad (3)$$

(b) can now be found from (1) and (3) intersection

$$\sigma_a'^{(2b)} = \sigma_a'^{(3b)}$$

$$-0.286\,\sigma_m'^{(b)} + 200\,MPa = 0.74\,\sigma_m'^{(b)}$$

$\sigma_m'^{(b)} = \underline{194.8 \text{ MPa}}$

$\sigma_a'^{(b)} = \underline{144.1 \text{ MPa}}$

Knowing

$$n = \frac{0-b}{0-a}$$

$$0-b = \sqrt{(144.1 \text{ MPa})^2 + (194.8 \text{ MPa})^2} = 242.3 \text{ MPa}$$

$$0-a = \sqrt{(100 \text{ MPa})^2 + (135 \text{ MPa})^2} = 168 \text{ MPa}$$

$$n = \frac{242.3 \text{ MPa}}{168 \text{ MPa}} = \underline{\underline{1.44}}$$