

As a student at Union College, I am part of a community that values intellectual effort, curiosity and discovery. I understand that in order to truly claim my educational and academic achievements, I am obligated to act with academic integrity. Therefore, I affirm that I carried out the work on this exam with full academic honesty, and I rely on my fellow students to do the same.

For this Exam, I understand that:

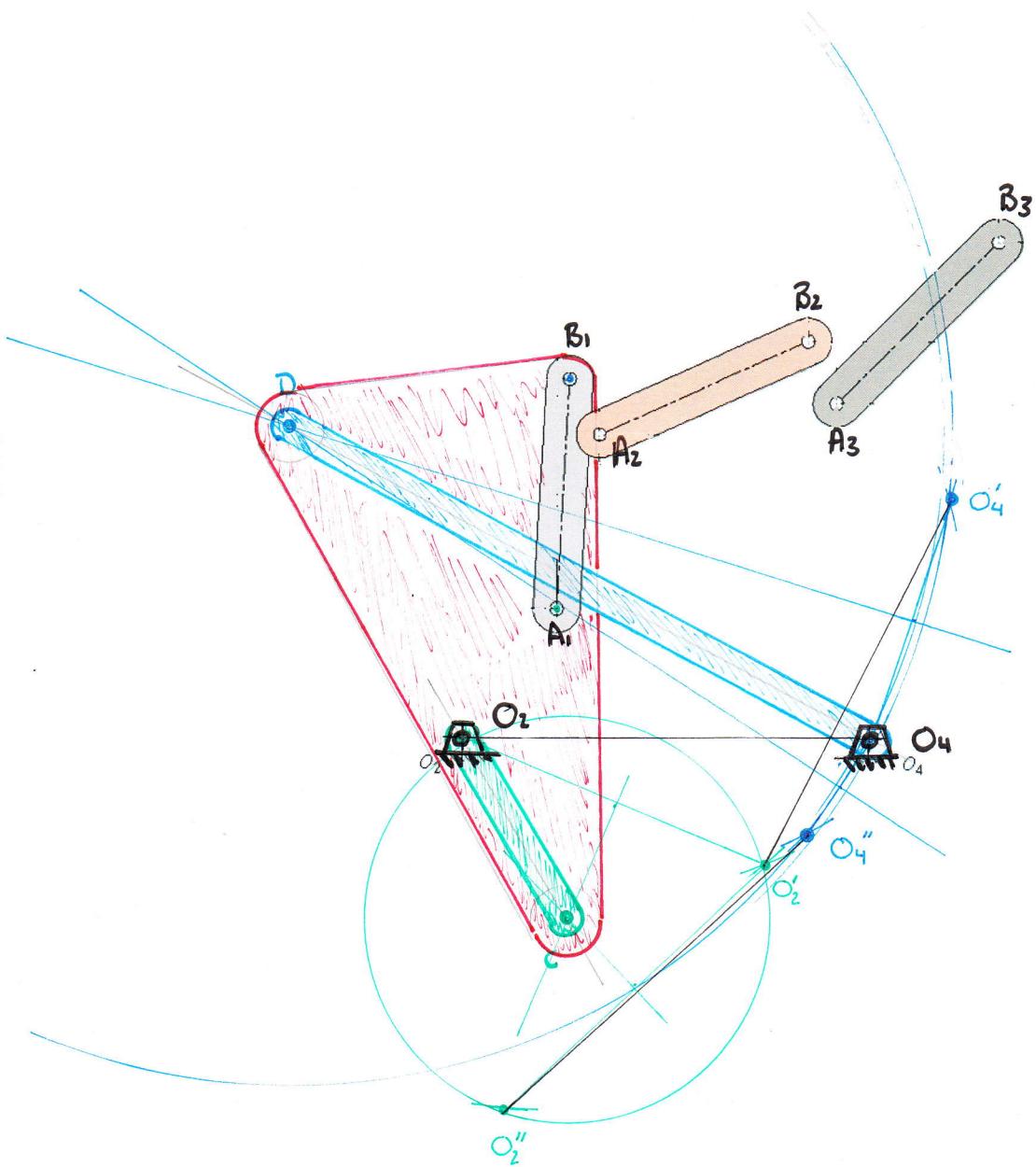
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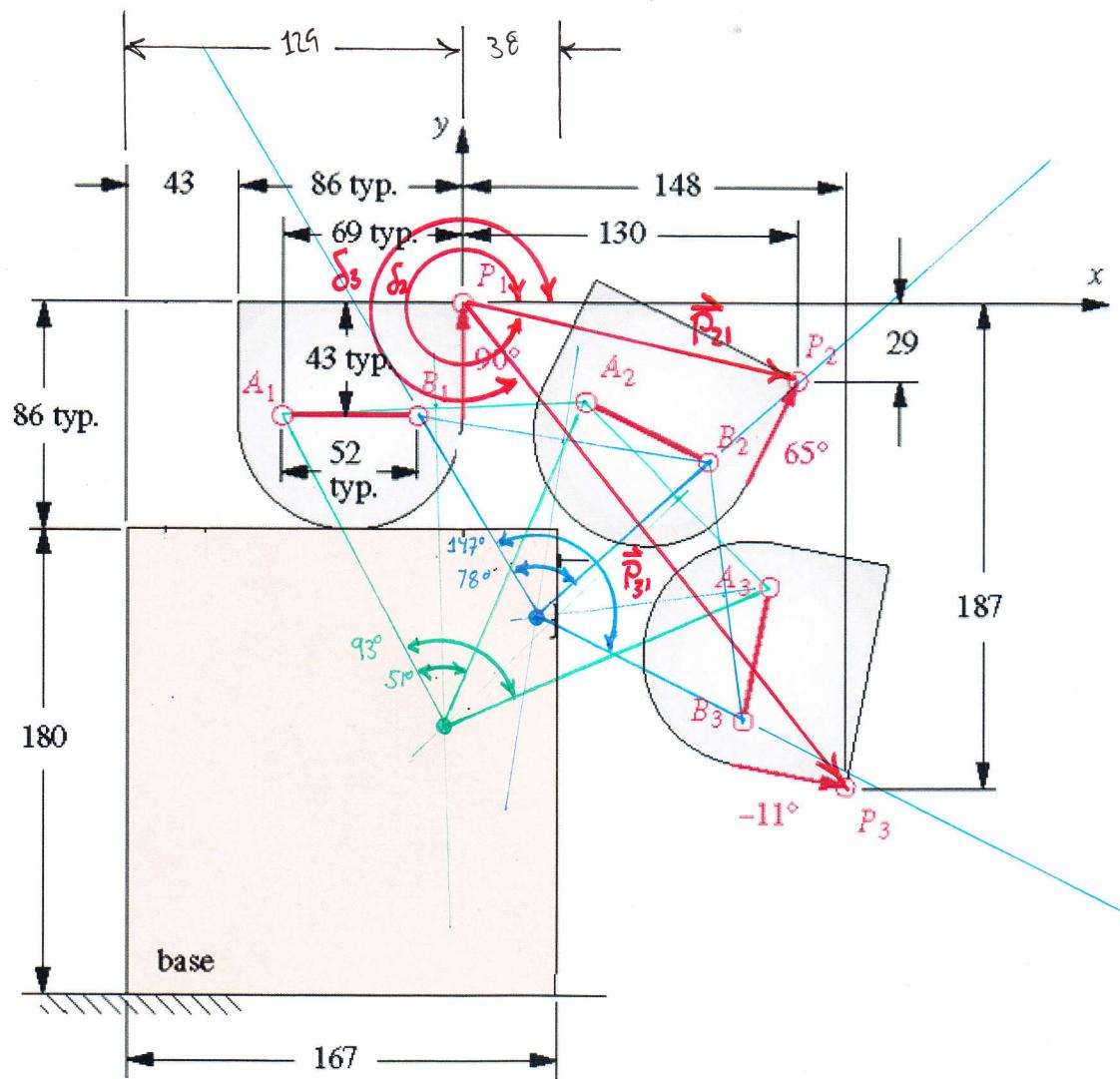
Print Name: SOLUTION

Exam Date: 15 MARCH 2016

PROBLEM 1: A mechanism needs to travel starting at the left position, moving through the middle positon, and ending at the right position as shown below below. Design the mechanism such that it attaches to the ground pivots shown.



PROBLEM 2: Using your three position algorithm, synthesize a mechanism that will take the bucket through the three positions shown. Make sure to use two pivots A and B. The ground pivots must be on the base. All dimensions are in millimeters.



$$(P_{1x}, P_{1y}) = (0 \text{ mm}, 0 \text{ mm})$$

$$(P_{2x}, P_{2y}) = (130 \text{ mm}, -29 \text{ mm})$$

$$(P_{3x}, P_{3y}) = (148 \text{ mm}, -187 \text{ mm})$$

GIVENS (CALCULATED)

$$P_{21} = \sqrt{(29 \text{ mm})^2 + (130 \text{ mm})^2} = 133.20 \text{ mm}$$

$$P_{31} = \sqrt{(148 \text{ mm})^2 + (187 \text{ mm})^2} = 238.98 \text{ mm}$$

FREE CHOICES (MEASURED)

$$\beta_2 = 360^\circ - 51^\circ = 309^\circ$$

$$\beta_3 = 360^\circ - 93^\circ = 267^\circ$$

$$\gamma_2 = 360^\circ - 78^\circ = 282^\circ$$

$$\gamma_3 = 360^\circ - 147^\circ = 213^\circ$$

$$\delta_2 = 360^\circ - \tan^{-1} \frac{29}{130} = 347.43^\circ$$

$$\delta_3 = 360^\circ - \tan^{-1} \frac{187}{148} = 308.36^\circ$$

$$\alpha_2 = 360^\circ - (90^\circ - 65^\circ) = 335^\circ$$

$$\alpha_3 = 360^\circ - (90^\circ + 11^\circ) = 259^\circ$$

First Dyad

FIRST DYAD

GIVEN:	CHOSEN:	FIND:		x-coord	y-coord.
P12	133.20 β_2	309.00 w	136.303	O2	-6.952 -166.197
P13	238.48 β_3	267.00 θ	118.160	A1	-71.279 -46.028
δ_2	347.43	z	84.848	A2	45.955 -40.581
δ_3	308.36	ϕ	32.852	A3	116.419 -108.247
α_2	335.00	W1x	-64.327	P1	0.000 0.000
α_3	259.00	W1y	120.168	P2	130.007 -28.989
		Z1x	71.279	P3	148.001 -186.999
		Z1y	46.028		

$$\begin{bmatrix} -0.3707 & 0.7771 & -0.0937 & 0.4226 \\ -0.7771 & -0.3707 & -0.4226 & -0.0937 \\ -1.0523 & 0.9986 & -1.1908 & 0.9816 \\ -0.9986 & -1.0523 & -0.9816 & -1.1908 \end{bmatrix} \begin{bmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{bmatrix} = \begin{bmatrix} 130.0073 \\ -28.9886 \\ 148.0008 \\ -186.9986 \end{bmatrix}$$

Second Dyad

SECOND DYAD

GIVEN:	CHOSEN:	FIND:		x-coord	y-coord.
P12	133.20 γ_2	282.00 u	91.807	O4	28.859 -120.715
P13	238.48 γ_3	213.00 σ	120.114	B1	-17.203 -41.300
δ_2	347.43	s	44.739	B2	96.962 -59.149
δ_3	308.36	ψ	67.387	B3	110.742 -162.232
α_2	335.00	U1x	-46.062	P1	0.000 0.000
α_3	259.00	U1y	79.416	P2	130.007 -28.989
		S1x	17.203	P3	148.001 -186.999
		S1y	41.300		

$$\begin{bmatrix} -0.7921 & 0.9781 & -0.0937 & 0.4226 \\ -0.9781 & -0.7921 & -0.4226 & -0.0937 \\ -1.8387 & 0.5446 & -1.1908 & 0.9816 \\ -0.5446 & -1.8387 & -0.9816 & -1.1908 \end{bmatrix} \begin{bmatrix} U1x \\ U1y \\ S1x \\ S1y \end{bmatrix} = \begin{bmatrix} 130.0073 \\ -28.9886 \\ 148.0008 \\ -186.9986 \end{bmatrix}$$

THESE RESULTS SHOW THE FIXED ROT-FACTS O₂ AND O₄ ARE ON THE BASE

PROBLEM 3: Design a cam to move a follower from 0 to 4 inches in 60° , dwell for 90° , fall 4 inches in 60° , and dwell for the remainder of the cycle. The total cycle time must take 4 seconds. Use CYCLOIDAL FUNCTIONS to accomplish the task.

Rise

$$s = y = h \cdot \left[\frac{\theta}{\beta} - \frac{1}{2\pi} \sin \left(2\pi \cdot \frac{\theta}{\beta} \right) \right]$$

$$y' = \frac{h}{\beta} \cdot \left[1 - \cos \left(2\pi \cdot \frac{\theta}{\beta} \right) \right]$$

$$y'' = \frac{2\pi \cdot h}{\beta^2} \cdot \sin \left(2\pi \cdot \frac{\theta}{\beta} \right)$$

$$y''' = \frac{4 \cdot \pi^2 \cdot h}{\beta^3} \cdot \cos \left(2\pi \cdot \frac{\theta}{\beta} \right)$$

Fall

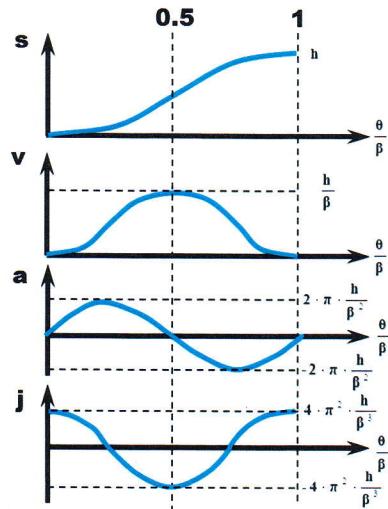
$$s = y = h - h \cdot \left[\frac{\theta}{\beta} - \frac{1}{2\pi} \sin \left(2\pi \cdot \frac{\theta}{\beta} \right) \right]$$

$$y' = -\frac{h}{\beta} \cdot \left[1 - \cos \left(2\pi \cdot \frac{\theta}{\beta} \right) \right]$$

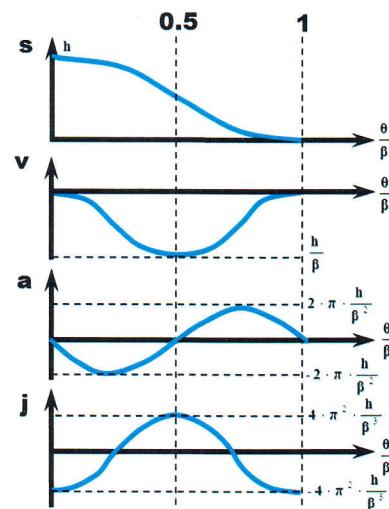
$$y'' = -\frac{2\pi \cdot h}{\beta^2} \cdot \sin \left(2\pi \cdot \frac{\theta}{\beta} \right)$$

$$y''' = -\frac{4 \cdot \pi^2 \cdot h}{\beta^3} \cdot \cos \left(2\pi \cdot \frac{\theta}{\beta} \right)$$

RISE

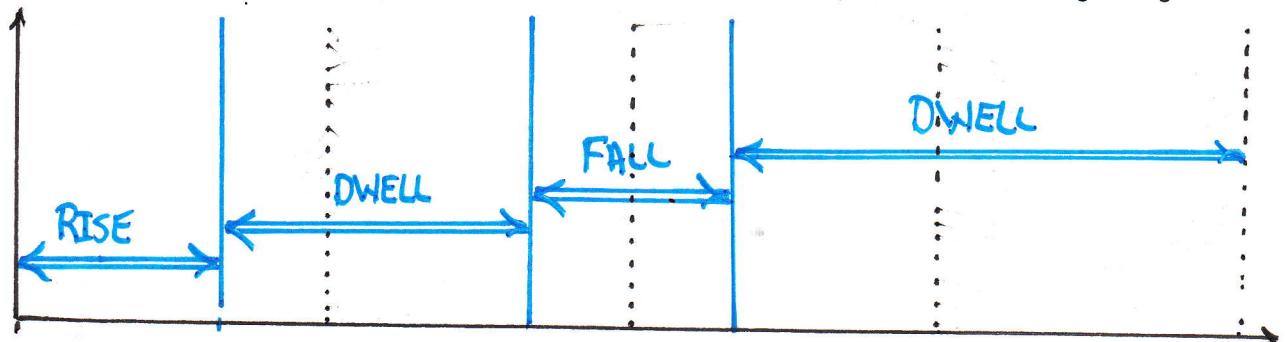


FALL



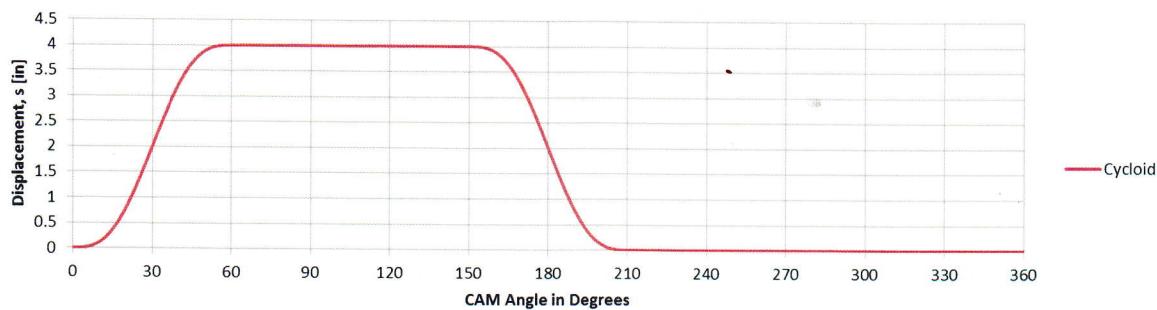
3a: Design this cam using the CYCLOIDAL function for all the rises and falls.

3b: Use your program to plot the s , s' , s'' , and s''' diagrams and attach them after the last page of this problem.

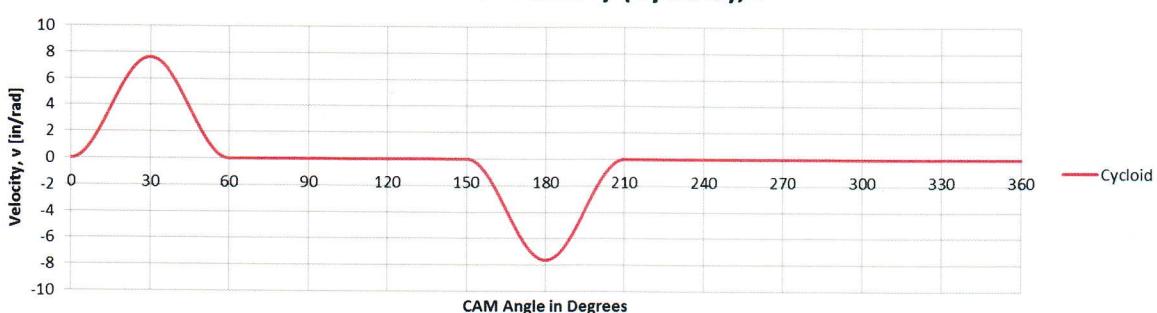


$\Theta [^\circ]$	0	60	90	150	180	210	270	360
$\Theta [^{\text{RAD}}]$	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	2π
$t [s]$	0	0.667s	1s	1.667s	2s	2.333s	3s	4s
$S [in]$	0	4in	0	4in	0	0	0	0
$S' [in/\text{RAD}]$	0	0	0	0	0	0	0	0
$S'' [in/\text{RAD}^2]$	0	0	0	0	0	0	0	0
$S''' [in/\text{RAD}^3]$	-	-	-	-	-	-	-	-

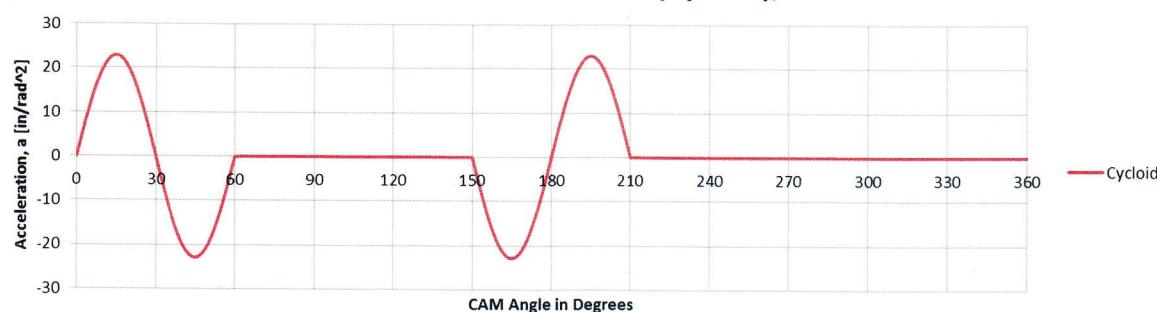
Follower Displacement (Cycloid), s



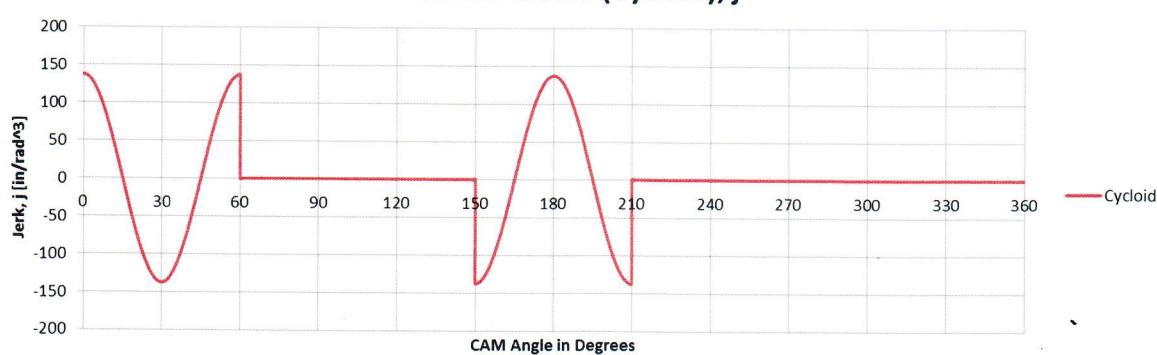
Follower Velocity (Cycloid), v



Follower Acceleration (Cycloid), a



Follower Jerk (Cycloid), j



PROBLEM 4: Design a cam to move a follower from 0 to 6 inches in 45° , dwell for 90° , fall 3 inches in 45° , dwell for 60° , fall 3 more inches in 60° , and dwell the remainder of the cycle. The total cycle time must take 5 seconds. Use POLYNOMIAL FUNCTIONS to accomplish the task.

$$s = C_0 + C_1 \cdot \left(\frac{\theta}{\beta}\right) + C_2 \cdot \left(\frac{\theta}{\beta}\right)^2 + C_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + C_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + C_5 \cdot \left(\frac{\theta}{\beta}\right)^5 + C_6 \cdot \left(\frac{\theta}{\beta}\right)^6 + C_7 \cdot \left(\frac{\theta}{\beta}\right)^7$$

$$= C_0 + \frac{C_1}{\beta} \cdot \theta + \frac{C_2}{\beta^2} \cdot \theta^2 + \frac{C_3}{\beta^3} \cdot \theta^3 + \frac{C_4}{\beta^4} \cdot \theta^4 + \frac{C_5}{\beta^5} \cdot \theta^5 + \frac{C_6}{\beta^6} \cdot \theta^6 + \frac{C_7}{\beta^7} \cdot \theta^7$$

$$s' = \frac{C_1}{\beta} + 2 \cdot \frac{C_2}{\beta^2} \cdot \theta + 3 \cdot \frac{C_3}{\beta^3} \cdot \theta^2 + 4 \cdot \frac{C_4}{\beta^4} \cdot \theta^3 + 5 \cdot \frac{C_5}{\beta^5} \cdot \theta^4 + 6 \cdot \frac{C_6}{\beta^6} \cdot \theta^5 + 7 \cdot \frac{C_7}{\beta^7} \cdot \theta^6$$

$$= \frac{1}{\beta} \left[C_1 + 2 \cdot C_2 \cdot \left(\frac{\theta}{\beta}\right) + 3 \cdot C_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot C_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot C_5 \cdot \left(\frac{\theta}{\beta}\right)^4 + 6 \cdot C_6 \cdot \left(\frac{\theta}{\beta}\right)^5 + 7 \cdot C_7 \cdot \left(\frac{\theta}{\beta}\right)^6 \right]$$

$$s'' = 2 \cdot \frac{C_2}{\beta^2} + 6 \cdot \frac{C_3}{\beta^3} \cdot \theta + 12 \cdot \frac{C_4}{\beta^4} \cdot \theta^2 + 20 \cdot \frac{C_5}{\beta^5} \cdot \theta^3 + 30 \cdot \frac{C_6}{\beta^6} \cdot \theta^4 + 42 \cdot \frac{C_7}{\beta^7} \cdot \theta^5$$

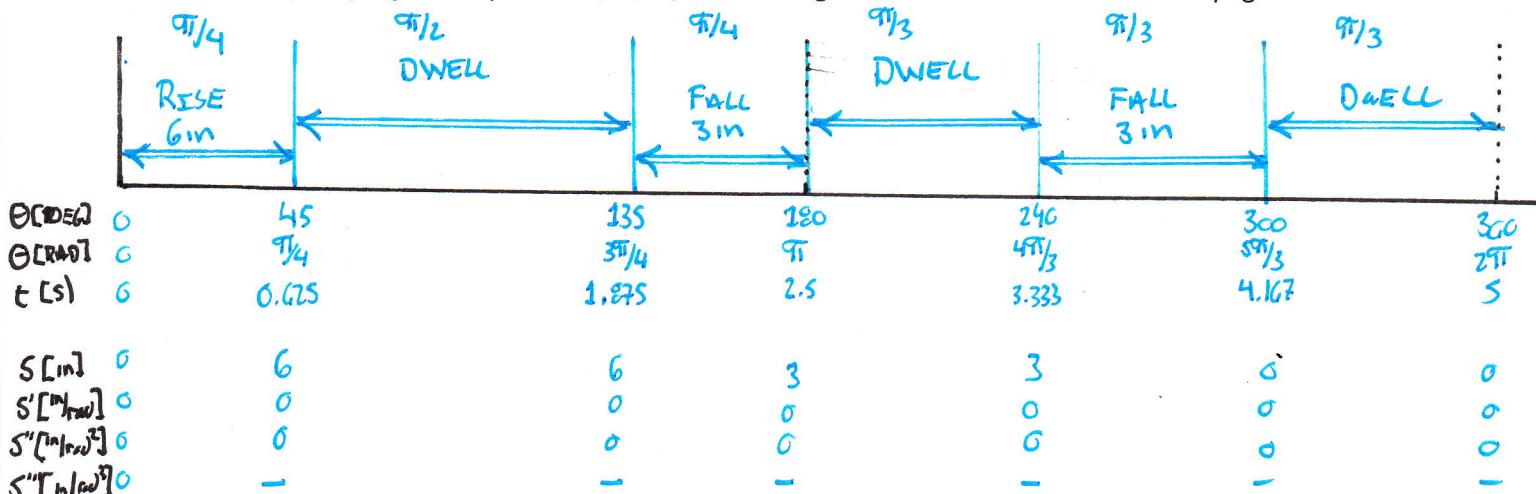
$$= \frac{1}{\beta^2} \left[2 \cdot C_2 + 6 \cdot C_3 \cdot \left(\frac{\theta}{\beta}\right) + 12 \cdot C_4 \cdot \left(\frac{\theta}{\beta}\right)^2 + 20 \cdot C_5 \cdot \left(\frac{\theta}{\beta}\right)^3 + 30 \cdot C_6 \cdot \left(\frac{\theta}{\beta}\right)^4 + 42 \cdot C_7 \cdot \left(\frac{\theta}{\beta}\right)^5 \right]$$

$$s''' = 6 \cdot \frac{C_3}{\beta^3} + 24 \cdot \frac{C_4}{\beta^4} \cdot \theta + 60 \cdot \frac{C_5}{\beta^5} \cdot \theta^2 + 120 \cdot \frac{C_6}{\beta^6} \cdot \theta^3 + 210 \cdot \frac{C_7}{\beta^7} \cdot \theta^4$$

$$= \frac{1}{\beta^3} \left[6 \cdot C_3 + 24 \cdot C_4 \cdot \left(\frac{\theta}{\beta}\right) + 60 \cdot C_5 \cdot \left(\frac{\theta}{\beta}\right)^2 + 120 \cdot C_6 \cdot \left(\frac{\theta}{\beta}\right)^3 + 210 \cdot C_7 \cdot \left(\frac{\theta}{\beta}\right)^4 \right]$$

4a: Design this cam using Polynomial function for all the rises and falls.

4b: Use your program to plot the s , s' , s'' , and s''' diagrams and attach them after this page.



REGION 1: $0 \leq \theta \leq 45^\circ, 0 \leq \theta_1 \leq \beta_1 = 45^\circ = \frac{\pi}{4}$

$C_0 = C_1 = C_2 = 0$ From THE BC $S(0) = 0, S'(0) = 0, S''(0) = 0$

$$S(45^\circ) = 6 \text{ in} = C_3 + C_4 + C_5$$

$$S'(45^\circ) = 0 = 3 \cdot C_3 + 4 \cdot C_4 + 5 \cdot C_5$$

$$S''(45^\circ) = 0 = 6 \cdot C_3 + 12 \cdot C_4 + 20 \cdot C_5$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{bmatrix} \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} 6 \\ -96 \\ 36 \end{Bmatrix}$$

Region 2: $45^\circ \leq \theta \leq 135^\circ, 0 \leq \theta_2 \leq \beta_2 = 90^\circ = \frac{\pi}{2}$

$$S = 6 \text{ in}, \quad S' = 0 \text{ in/rad}, \quad S'' = 0 \text{ in/rad}^2, \quad S''' = 0 \text{ in/rad}^3$$

Region 3: $135^\circ \leq \theta \leq 180^\circ, 0 \leq \theta_3 \leq \beta_3 = 45^\circ = \frac{\pi}{4}$

$$S(0) = 6 \text{ in} = C_0$$

$$S'(0) = 0 = C_1$$

$$S''(0) = 0 = C_2$$

$$S(45^\circ) = 3 \text{ in} = 6 \text{ in} + C_3 + C_4 + C_5 \Rightarrow -3 \text{ in} = C_3 + C_4 + C_5$$

$$S'(45^\circ) = 0 \text{ in/rad} = 3 \cdot C_3 + 4 \cdot C_4 + 5 \cdot C_5$$

$$S''(45^\circ) = 0 \text{ in/rad}^2 = 6 \cdot C_3 + 12 \cdot C_4 + 20 \cdot C_5$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{bmatrix} \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} -3 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} -30 \\ 45 \\ -12 \end{Bmatrix}$$

Region 4: $180^\circ \leq \theta \leq 240^\circ, 0 \leq \theta_4 \leq \beta_4 = 60^\circ = \frac{\pi}{3}$

$$S = 3 \text{ in}, \quad S' = 0 \text{ in/rad}, \quad S'' = 0 \text{ in/rad}^2, \quad S''' = 0 \text{ in/rad}^3$$

Region 5: $240^\circ \leq \theta \leq 300^\circ, 0 \leq \theta_5 \leq \beta_5 = 60^\circ = \frac{\pi}{3}$

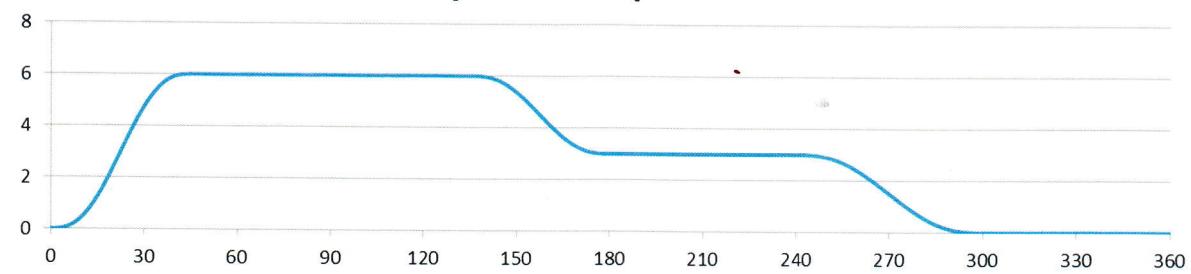
$$S(0) = 3 \text{ in} = C_0, \quad S'(0) = 0 = C_1, \quad S''(0) = 0 = C_2$$

$$C_3 = -30, \quad C_4 = 45, \quad C_5 = -18$$

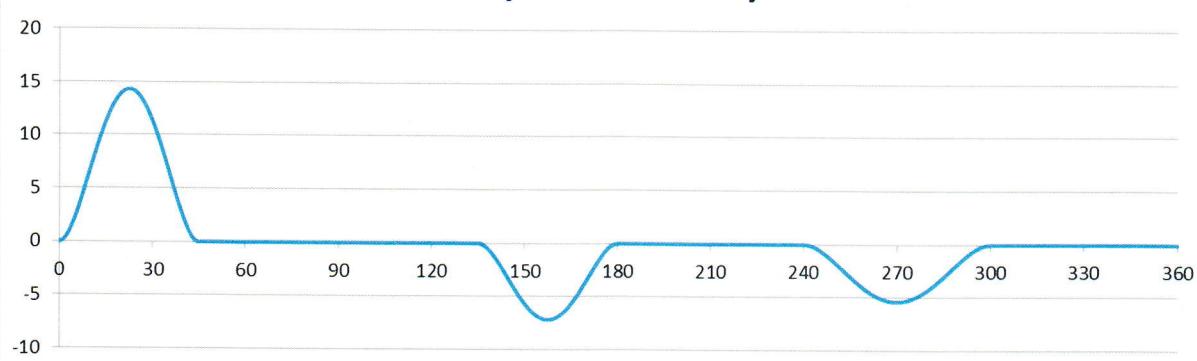
Region 6: $240^\circ \leq \theta \leq 360^\circ, 0 \leq \theta_6 \leq \beta_6 = 60^\circ = \frac{\pi}{3}$

$$S = 0 \text{ in}, \quad S' = 0 \text{ in/rad}, \quad S'' = 0 \text{ in/rad}^2, \quad S''' = 0 \text{ in/rad}^3$$

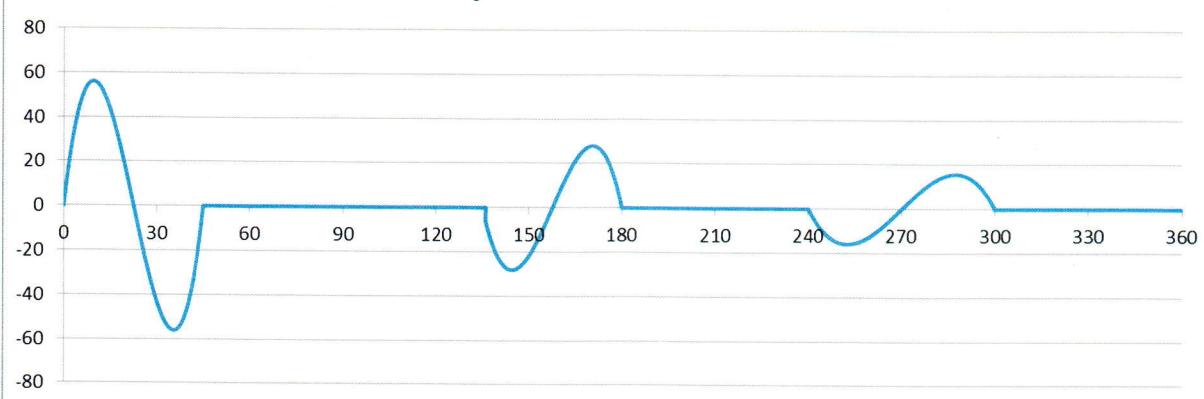
Polynomial Displacement



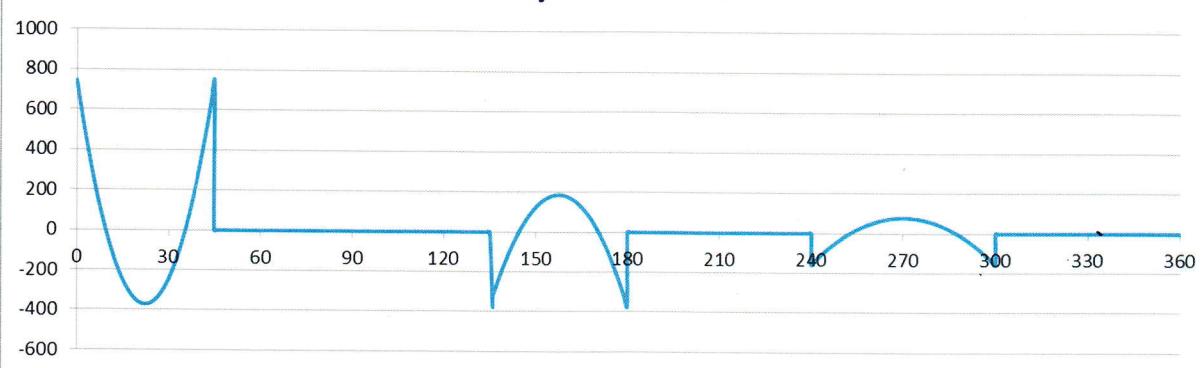
Polynomial Velocity



Polynomial Acceleration



Polynomial Jerk



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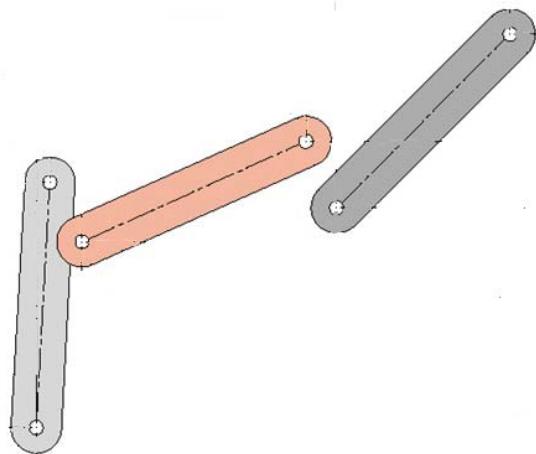
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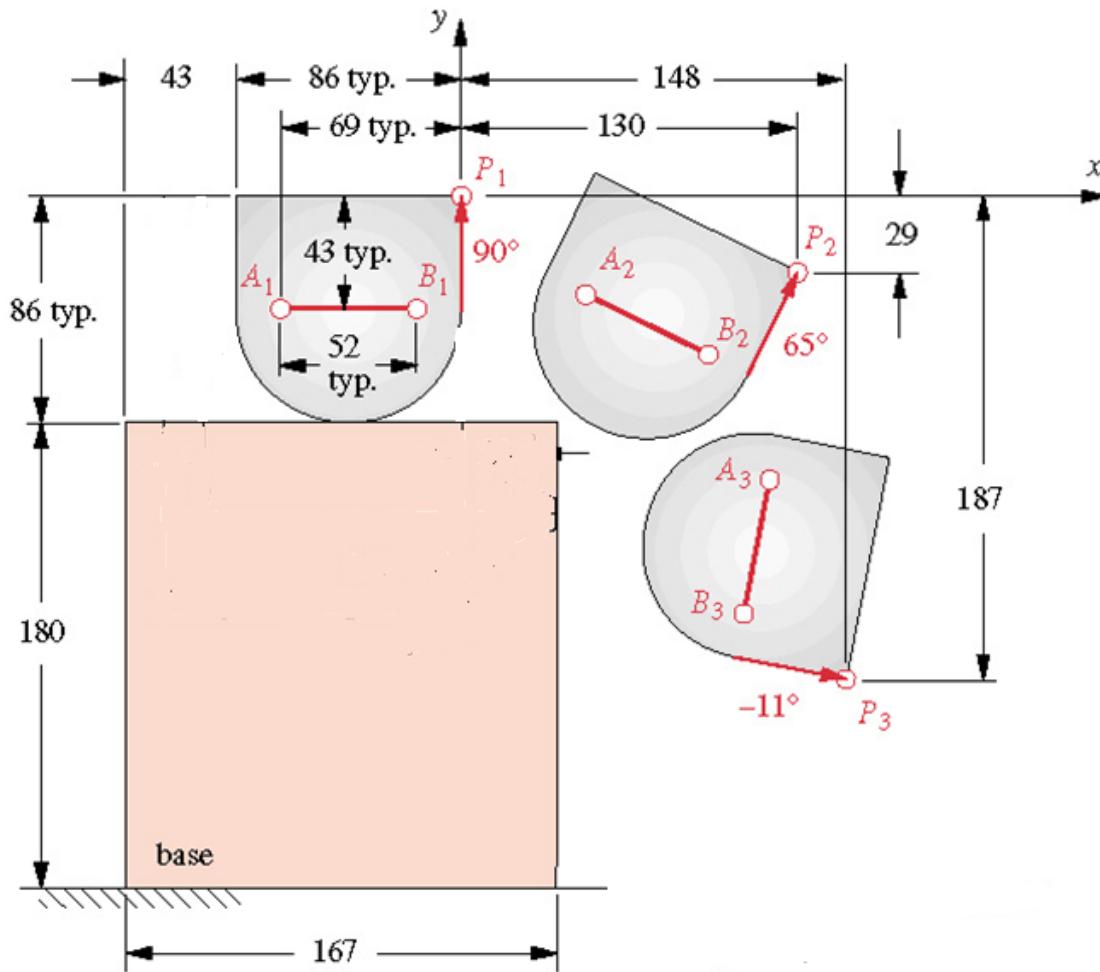
Print Name: _____

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PROBLEM 3: Design a cam to move a follower from 0 to 4 inches in 60° , dwell for 90° , fall 4 inches in 60° , and dwell for the remainder of the cycle. The total cycle time must take 4 seconds. Use CYCLOIDAL FUNCTIONS to accomplish the task.

Rise

$$s = y = h \cdot \left[\frac{\theta}{\beta} - \frac{1}{2\pi} \sin \left(2\pi \cdot \frac{\theta}{\beta} \right) \right]$$

$$y' = \frac{h}{\beta} \cdot \left[1 - \cos \left(2\pi \cdot \frac{\theta}{\beta} \right) \right]$$

$$y'' = \frac{2\pi \cdot h}{\beta^2} \cdot \sin \left(2\pi \cdot \frac{\theta}{\beta} \right)$$

$$y''' = \frac{4 \cdot \pi^2 \cdot h}{\beta^3} \cdot \cos \left(2\pi \cdot \frac{\theta}{\beta} \right)$$

Fall

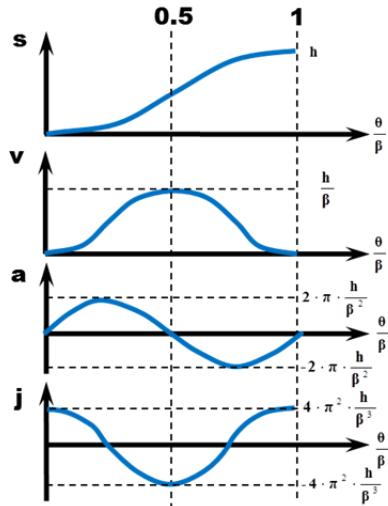
$$s = y = h - h \cdot \left[\frac{\theta}{\beta} - \frac{1}{2\pi} \sin \left(2\pi \cdot \frac{\theta}{\beta} \right) \right]$$

$$y' = -\frac{h}{\beta} \cdot \left[1 - \cos \left(2\pi \cdot \frac{\theta}{\beta} \right) \right]$$

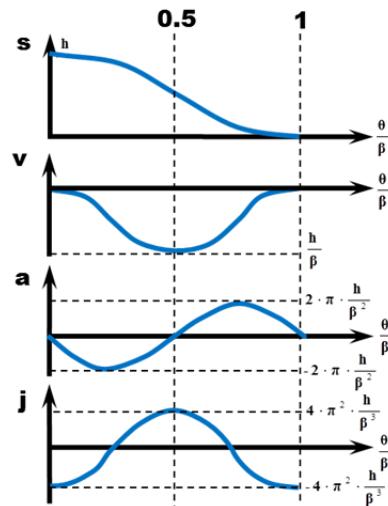
$$y'' = -\frac{2\pi \cdot h}{\beta^2} \cdot \sin \left(2\pi \cdot \frac{\theta}{\beta} \right)$$

$$y''' = -\frac{4 \cdot \pi^2 \cdot h}{\beta^3} \cdot \cos \left(2\pi \cdot \frac{\theta}{\beta} \right)$$

RISE



FALL



3a: Design this cam using the CYCLOIDAL function for all the rises and falls.

3b: Use your program to plot the s , s' , s'' , and s''' diagrams and attach them after the last page of this problem.

PROBLEM 4: Design a cam to move a follower from 0 to 6 inches in 45° , dwell for 90° , fall 3 inches in 45° , dwell for 60° , fall 3 more inches in 60° , and dwell the remainder of the cycle. The total cycle time must take 5 seconds. Use POLYNOMIAL FUNCTIONS to accomplish the task.

$$s = C_0 + C_1 \cdot \left(\frac{\theta}{\beta}\right) + C_2 \cdot \left(\frac{\theta}{\beta}\right)^2 + C_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + C_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + C_5 \cdot \left(\frac{\theta}{\beta}\right)^5 + C_6 \cdot \left(\frac{\theta}{\beta}\right)^6 + C_7 \cdot \left(\frac{\theta}{\beta}\right)^7$$

$$= C_0 + \frac{C_1}{\beta} \cdot \theta + \frac{C_2}{\beta^2} \cdot \theta^2 + \frac{C_3}{\beta^3} \cdot \theta^3 + \frac{C_4}{\beta^4} \cdot \theta^4 + \frac{C_5}{\beta^5} \cdot \theta^5 + \frac{C_6}{\beta^6} \cdot \theta^6 + \frac{C_7}{\beta^7} \cdot \theta^7$$

$$s' = \frac{C_1}{\beta} + 2 \cdot \frac{C_2}{\beta^2} \cdot \theta + 3 \cdot \frac{C_3}{\beta^3} \cdot \theta^2 + 4 \cdot \frac{C_4}{\beta^4} \cdot \theta^3 + 5 \cdot \frac{C_5}{\beta^5} \cdot \theta^4 + 6 \cdot \frac{C_6}{\beta^6} \cdot \theta^5 + 7 \cdot \frac{C_7}{\beta^7} \cdot \theta^6$$

$$= \frac{1}{\beta} \left[C_1 + 2 \cdot C_2 \cdot \left(\frac{\theta}{\beta}\right) + 3 \cdot C_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot C_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot C_5 \cdot \left(\frac{\theta}{\beta}\right)^4 + 6 \cdot C_6 \cdot \left(\frac{\theta}{\beta}\right)^5 + 7 \cdot C_7 \cdot \left(\frac{\theta}{\beta}\right)^6 \right]$$

$$s'' = 2 \cdot \frac{C_2}{\beta^2} + 6 \cdot \frac{C_3}{\beta^3} \cdot \theta + 12 \cdot \frac{C_4}{\beta^4} \cdot \theta^2 + 20 \cdot \frac{C_5}{\beta^5} \cdot \theta^3 + 30 \cdot \frac{C_6}{\beta^6} \cdot \theta^4 + 42 \cdot \frac{C_7}{\beta^7} \cdot \theta^5$$

$$= \frac{1}{\beta^2} \left[2 \cdot C_2 + 6 \cdot C_3 \cdot \left(\frac{\theta}{\beta}\right) + 12 \cdot C_4 \cdot \left(\frac{\theta}{\beta}\right)^2 + 20 \cdot C_5 \cdot \left(\frac{\theta}{\beta}\right)^3 + 30 \cdot C_6 \cdot \left(\frac{\theta}{\beta}\right)^4 + 42 \cdot C_7 \cdot \left(\frac{\theta}{\beta}\right)^5 \right]$$

$$s''' = 6 \cdot \frac{C_3}{\beta^3} + 24 \cdot \frac{C_4}{\beta^4} \cdot \theta + 60 \cdot \frac{C_5}{\beta^5} \cdot \theta^2 + 120 \cdot \frac{C_6}{\beta^6} \cdot \theta^3 + 210 \cdot \frac{C_7}{\beta^7} \cdot \theta^4$$

$$= \frac{1}{\beta^3} \left[6 \cdot C_3 + 24 \cdot C_4 \cdot \left(\frac{\theta}{\beta}\right) + 60 \cdot C_5 \cdot \left(\frac{\theta}{\beta}\right)^2 + 120 \cdot C_6 \cdot \left(\frac{\theta}{\beta}\right)^3 + 210 \cdot C_7 \cdot \left(\frac{\theta}{\beta}\right)^4 \right]$$

4a: Design this cam using Polynomial function for all the rises and falls.

4b: Use your program to plot the s , s' , s'' , and s''' diagrams and attach them after this page.

