PROBLEM 5.22 | THE CROSS SECTION OF A THIN-WALLED BEAM WITH A WALL THICKNESS OF 2mm IS SYMMETRIC WITH RESPECT TO THE Z-AXIS. THE SECTION IS IN BENDING ABOUT THE Z-AXIS AND IS SUPPORTING A VERTICAL TRANSVERSE SHEAR FORCE OF V_y = 4 kN. ALL DIMENSIONS ARE IN MILLIMETERS AND WHERE APPROPRIATE ARE FROM WALL CENTERS. FOR WALL AB DETERMINE HOW THE TRANSVERSE SHEAR STRESS VARIES AND THE VALUES OF THE NET SHEAR FORCE.

GIVEN:

1) THIN WALLED CROSS-SECTION SHOWN
2) WALL THICKNESS 2mm
3) VERTICAL TRANSVERSE SHEAR FORCE OF V_y = 4 kN

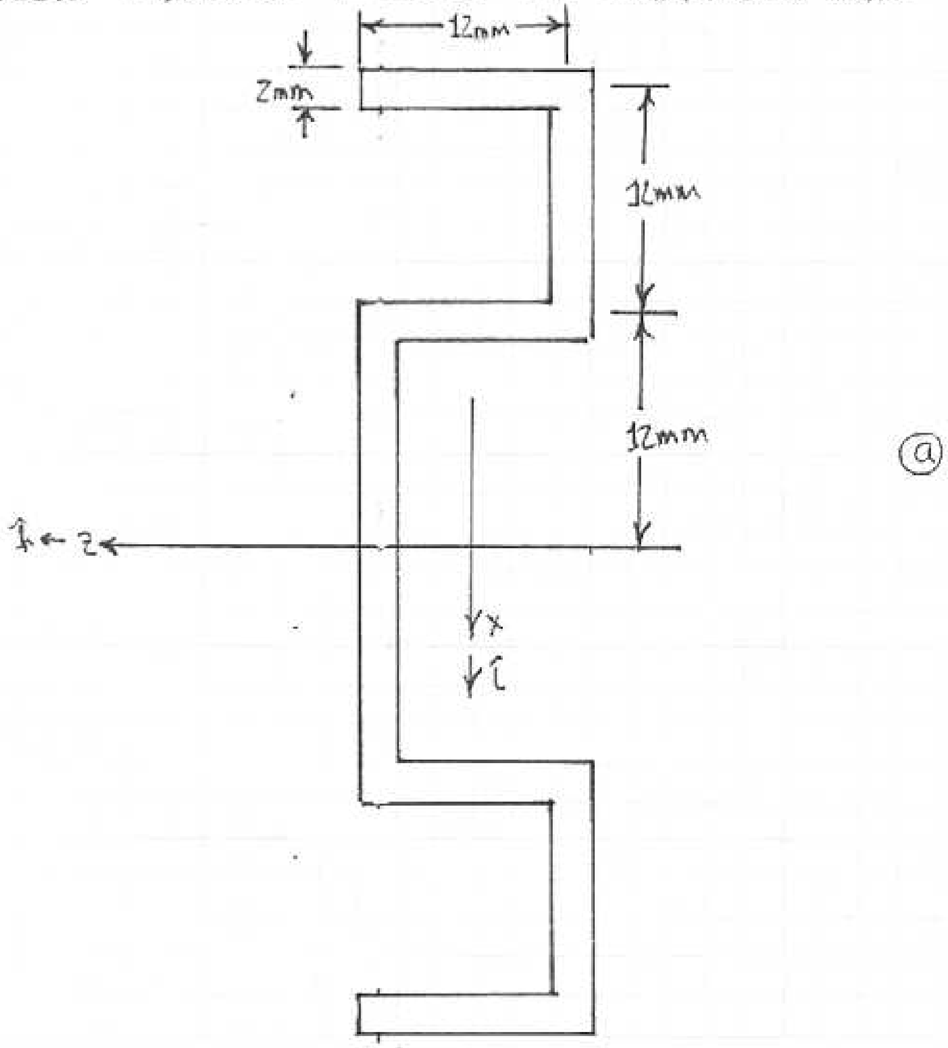
ASSUMPTIONS:

1) SMALL DEFORMATIONS
2) LINEAR-ELASTIC RESPONSE

FIND:

1) DIAGRAM THE SHEAR FLOW THROUGH THE ENTIRE CROSS-SECTION.
2) IF THE CROSS-SECTION IS MADE FROM 2mm THICK BOARDS, SHOW THE NAIL LOCATION THAT WILL PRODUCE THE MAXIMUM NAIL SPACING.

FIGURE:



SOLUTION:

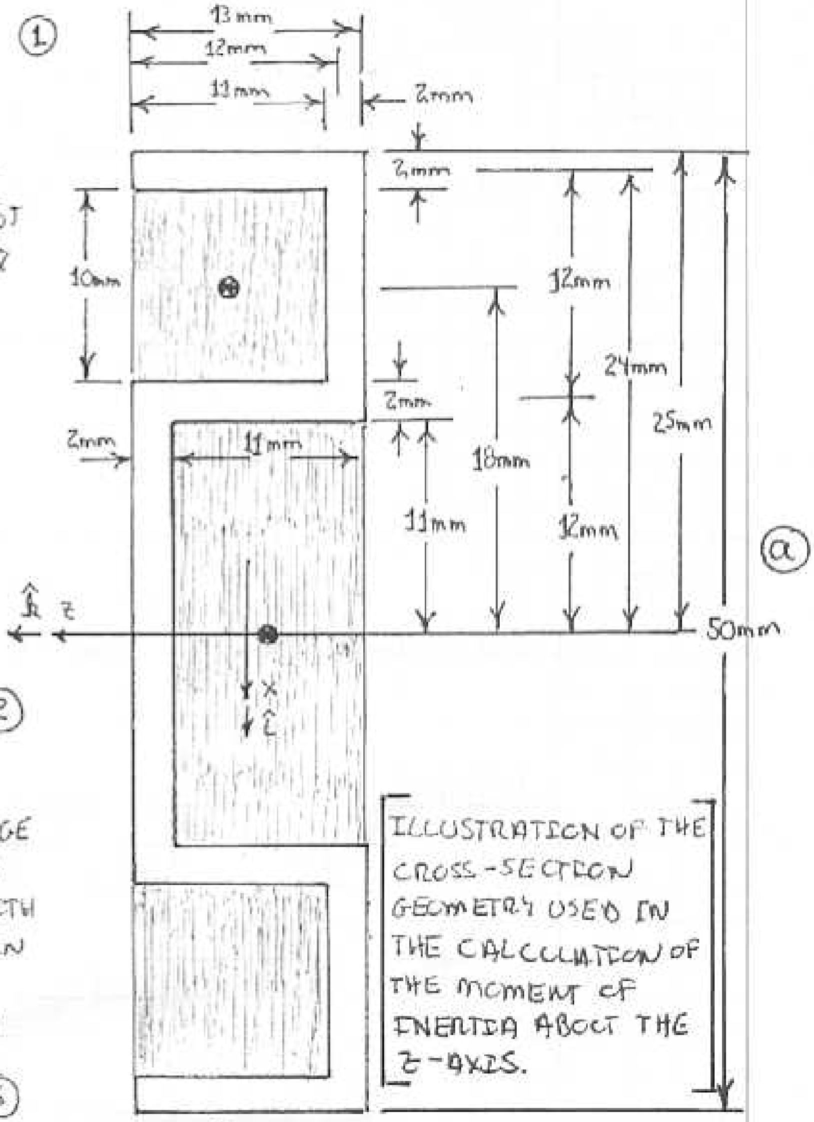
THE SHEAR STRESS IN EACH SECTION OF THE CROSS-SECTION IS CALCULATED FROM THE SHEAR FORCE USING

$$\tau = \frac{VQ}{IT} \quad (1)$$

FOR ALL SECTIONS OF THE CROSS-SECTION THE MOMENT OF INERTIA ABOUT THE Z-AXIS MUST BE CALCULATED. THE GEOMETRY ILLUSTRATED IN (a) IS USED IN THIS CALCULATION

$$I = \frac{1}{12}(0.013m)(0.050m)^3 - 2\cdot\left[\frac{1}{12}(0.011m)(0.010m)^3 + (0.011m)(0.010m)(0.018m)^2\right] - \frac{1}{12}(0.011m)(0.022m)^3$$

$$= \underline{52.54 \times 10^{-9} m^4} \quad (2)$$



THE SHEAR STRESS FOR THE UPPER FLANGE CAN NOW BE CALCULATED USING (1), (2) AND THE GEOMETRY IN (b). STARTING WITH THE CALCULATION OF Q FOR THIS SECTION

$$Q = \bar{X}\cdot A = (0.024m)(0.002m)\cdot S$$

$$\simeq 48.00\times10^{-6} m^2\cdot S \quad (3)$$

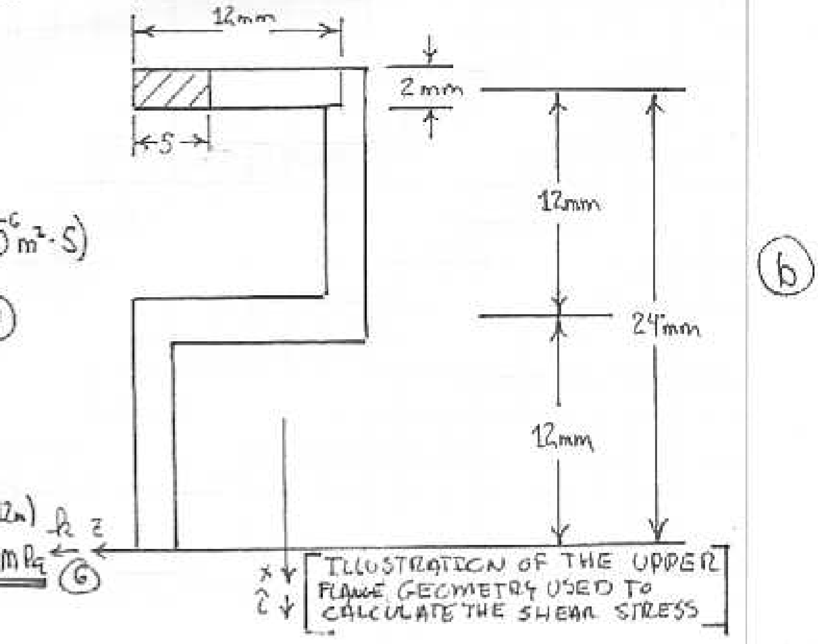
$$\tau_{yz} = \frac{(4\times10^3 N)(48.0\times10^{-6} m^2\cdot S)}{(52.54\times10^{-9} m^4)(0.002m)}$$

$$= \left(38.07\times10^{12}\frac{N}{m^5}\right)(48.0\times10^{-6} m^2\cdot S)$$

$$= \underline{1.8272\times10^{9}\frac{N}{m^3}\cdot S} \quad (4)$$

$$\tau_{yz}(0) = \underline{0} \quad (5)$$

$$\tau_{yz}(0.012m) = 1.8272\times10^{9}\frac{N}{m^3}\cdot(0.012m)$$
$$= 21.93\times10^{6}\frac{N}{m^2} = \underline{21.93 MPa} \quad (6)$$



THE NET SHEAR FORCE IN THE UPPER FLANGE ~~$\forall_{UF}$~~ IS CALCULATED BY INTEGRATING THE SHEAR STRESS (4) ~~τ~~ OVER THE UPPER FLANGE AREA.

$$\forall_{UF} = \int \tau_{yz} \cdot t \cdot ds = \int_0^{0.012m} \left(1.8272 \times 10^9 \tfrac{N}{m^3} \cdot s\right)(0.002m)\, ds$$
$$= \int_0^{0.012} \left(3.642 \times 10^6 \tfrac{N}{m^2} \cdot s\right) ds = \left[\left(3.642 \times 10^6 \tfrac{N}{m^2}\right) \frac{s^2}{2}\right]_0^{0.012m}$$
$$= \underline{262.3\,N} \qquad (7)$$

THE SHEAR STRESS IN THE UPPER WEB IS CALCULATED USING (1), (2), AND THE GEOMETRY ILLUSTRATED IN (C). STARTING WITH THE CALCULATION OF Q FOR THIS SECTION.

$$Q = \Sigma \bar{y} \cdot A$$
$$= (0.024m)(0.002m)(0.012m) + (0.024m - s/2)(0.002m) \cdot s$$
$$= 576.0\times10^{-9} m^3 + 48.0\times10^{-6} m^2 \cdot s - 1.0\times10^{-3} \cdot s^2 \qquad (8)$$

$$\tau_{xy} = \frac{(4\times10^3 N)\left(\begin{matrix} 576.0\times10^{-9} m^3 + 48.0\times10^{-6} m^2 \cdot s \\ - 1.0\times10^{-3} \cdot s^2 \end{matrix}\right)}{(52.54\times10^{-9} m^4)(0.002m)}$$

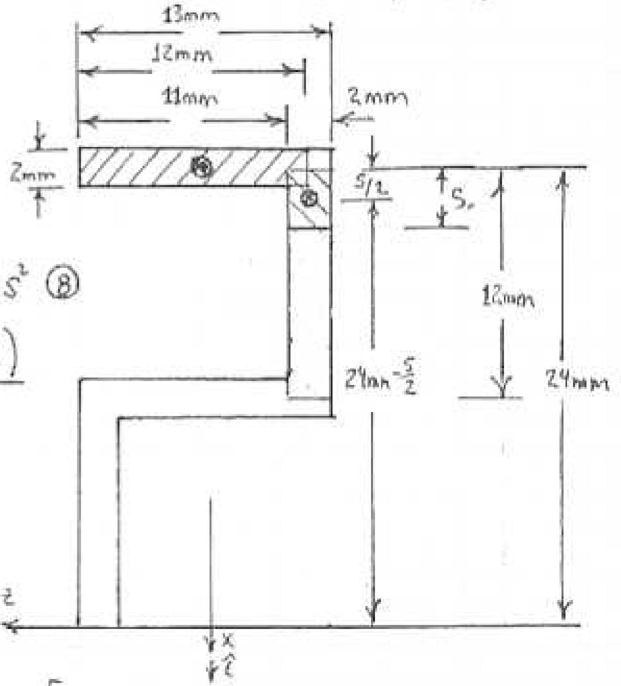
$$= \left(38.07\times10^{12} \tfrac{N}{m^5}\right)\left(576.0\times10^{-9} m^3 + 48.0\times10^{-6} m^2 \cdot s - 1.0\times10^{-3} m \cdot s^2\right)$$

$$= 21.93\times10^6 \tfrac{N}{m^2} + 1.8274\times10^9 \tfrac{N}{m^3} \cdot s - 38.07\times10^9 \tfrac{N}{m^4} \cdot s^2 \qquad (9)$$

$$\tau_{xy}(0) = \underline{21.93\times10^6 \tfrac{N}{m^2}} \qquad (10)$$

$$\tau_{xy}(12mm) = 21.93\times10^6 \tfrac{N}{m^2} + \left(1.8274\times10^9 \tfrac{N}{m^3}\right)(0.012m) - \left(38.07\times10^9 \tfrac{N}{m^4}\right)(0.012m)^2$$

$$= 38.37\times10^6 \tfrac{N}{m^2} = \underline{38.37\,MPa} \qquad (11)$$



(C)

[ILLUSTRATION OF THE UPPER FLANGE GEOMETRY USED IN THE CALCULATION OF THE SHEAR STRESS]

THE NET SHEAR FORCE IN THE UPPER WEB $\forall_{UW}$ ~~$\forall$~~ IS CALCULATED BY INTEGRATING THE SHEAR STRESS OVER THE UPPER WEB AREA

$$\forall_{UW} = \int \tau_{xy} \cdot t \cdot ds = \int_0^{12mm} \left(21.93\times10^6 \tfrac{N}{m^2} + 1.8274\times10^9 \tfrac{N}{m^3} \cdot s - 38.07\times10^9 \tfrac{N}{m^4} \cdot s^2\right)(0.002m)\, ds$$
$$= \int_0^{0.012m} \left(43.86\times10^3 \tfrac{N}{m} + 3.655\times10^6 \tfrac{N}{m^2} \cdot s - 76.14\times10^6 \tfrac{N}{m^3} \cdot s^2\right) \cdot ds$$
$$= \left[43.86\times10^3 \tfrac{N}{m} \cdot s + 3.655\times10^6 \tfrac{N}{m^2} \cdot \frac{s^2}{2} - 76.14\times10^6 \tfrac{N}{m^3} \cdot \frac{s^3}{3}\right]_0^{0.012m}$$
$$= \underline{745.6\,N} \qquad (12)$$

THE SHEAR STRESS IN THE INTERMEDIATE FLANGE IS CALCULATED USING ①, ② AND THE GEOMETRY ILLUSTRATED IN ⓓ. THE CALCULATION STARTS WITH THE DETERMINATION OF AN EXPRESSION FOR Q.

$$Q = \Sigma \bar{x} A$$

$$= (0.024m)(0.012m)(0.002m)$$

$$+ (0.018m)(0.012m)(0.002m)$$

$$+ (0.012m)(s)(0.002m)$$

$$= 1.0080 \times 10^{-6} m^3 + 24.0 \times 10^{-6} m^2 \cdot s$$

$$\tau_{yz} = \frac{(4\times10^3 N)(1.0080\times10^{-6}m^3 + 24.0\times10^{-6}m^2\cdot s)}{(52.54\times10^{-9}m^4)(0.002m)}$$

$$= (38.07\times10^{12}\tfrac{N}{m^5}) \cdot (1.0080\times10^{-6}m^3 + 24.0\times10^{-6}m^2\cdot s)$$

$$= 38.37\times10^6 \tfrac{N}{m^2} + 913.7\times10^6 \tfrac{N}{m^3}\cdot s \quad (13)$$

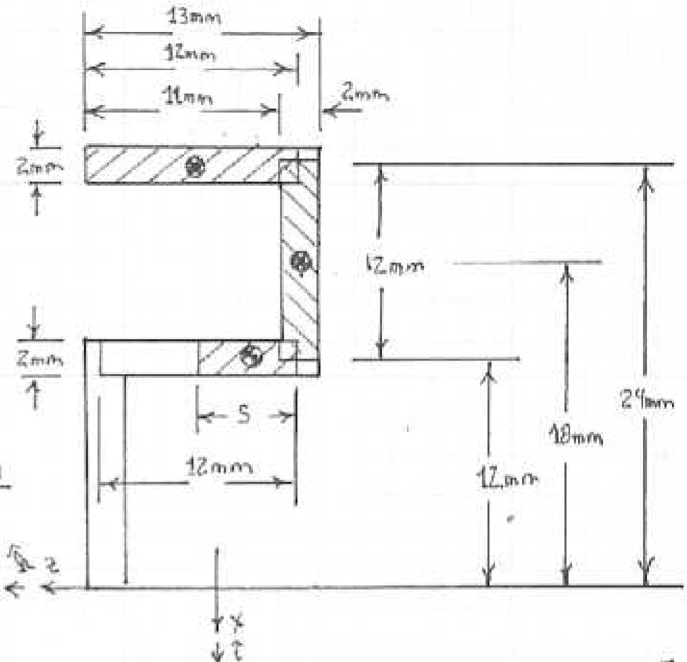


ILLUSTRATION OF THE INTERMEDIATE FLANGE GEOMETRY USED IN THE CALCULATION OF THE SHEAR STRESS

$$\tau_{yz}(0) = \underline{\underline{38.37\,MPa}} \quad (14)$$

$$\tau_{yz}(12mm) = \underline{\underline{49.34\,MPa}} \quad (15)$$

THE NET SHEAR FORCE IN THE INTERMEDIATE FLANGE IS CALCULATED BY INTEGRATING THE SHEAR STRESS IN THE INTERMEDIATE FLANGE OVER THE INTERMEDIATE FLANGE AREA

$$V_{IF} = \int \tau_{yz} t \cdot ds = \int_0^{0.012m} (38.37\times10^6 \tfrac{N}{m^2} + 913.7\times10^6 \tfrac{N}{m^3}\cdot s)(0.002m)\,ds$$

$$= \int_0^{0.012m} (76.74\times10^3 \tfrac{N}{m} + 1.8274\times10^6 \cdot s)\,ds =$$

$$= \left[76.74\times10^3 \tfrac{N}{m}\cdot s + 1.8274\times10^6 \cdot \tfrac{s^2}{2} \right]_0^{0.012m}$$

$$= \underline{\underline{1052.4\,N}} \quad (16)$$

THE SHEAR STRESS IN THE CENTER WEB IS CALCULATED USING ①, ②, AND THE GEOMETRY ILLUSTRATED IN ©. THE CALCULATION STARTS WITH THE DETERMINATION OF THE EXPRESSION FOR Q IN THIS SECTION.

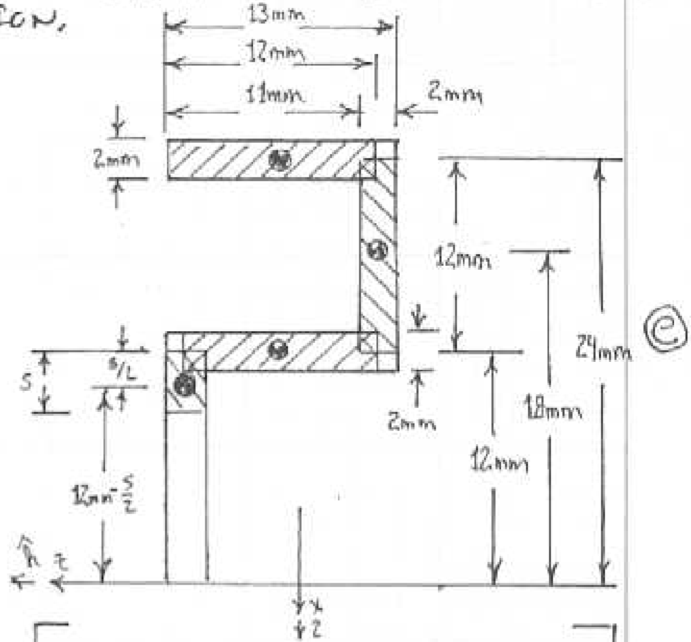


ILLUSTRATION OF THE CENTER WEB GEOMETRY USED IN THE CALCULATION OF THE SHEAR STRESS

$$Q = \Sigma \bar{x} A$$

$$= (0.024m)(0.012m)(0.002m) + (0.018m)(0.012m)(0.002m) + (0.012m)(0.012m)(0.002m) + \left(0.012m - \frac{s}{2}\right)(0.002m)(s)$$

$$= 1.2960\times10^{-6}m^3 + 24.0\times10^{-6}m^2\cdot s - 1.000\times10^{-3}\cdot m\cdot s^2 \quad (17)$$

$$\tau_{xy} = \frac{(4\times10^3N)(1.2960\times10^{-6}m^3 + 24\times10^{-6}m^2\cdot s - 1.00\times10^{-3}m\cdot s^2)}{(52.54\times10^{-9}m^4)(0.002m)}$$

$$= \left(38.07\times10^{12}\frac{N}{m^5}\right)(1.2960\times10^{-6}m^3 + 24\times10^{-6}m^2\cdot s - 1.000\times10^{-3}m\cdot s^2)$$

$$= 48.34\times10^6\frac{N}{m^2} + 913.7\times10^6\frac{N}{m^3}\cdot s - 38.07\times10^9\frac{N}{m^4}\cdot s^2 \quad (18)$$

$$\tau_{xy}(0) = 48.34\times10^6\frac{N}{m^2} = \underline{\underline{48.34\ MPa}} \quad (19)$$

$$\tau_{xy}(0.012m) = 53.82\times10^6\frac{N}{m^2} = \underline{\underline{53.82\ MPa}} \quad (20)$$

THE NET SHEAR FORCE IN THE CENTER WEB IS CALCULATED BY INTEGRATING THE SHEAR STRESS IN THE CENTER WEB OVER THE CENTER WEB AREA.

$$V_{cw} = \int \tau_{xy}\cdot t\cdot ds$$

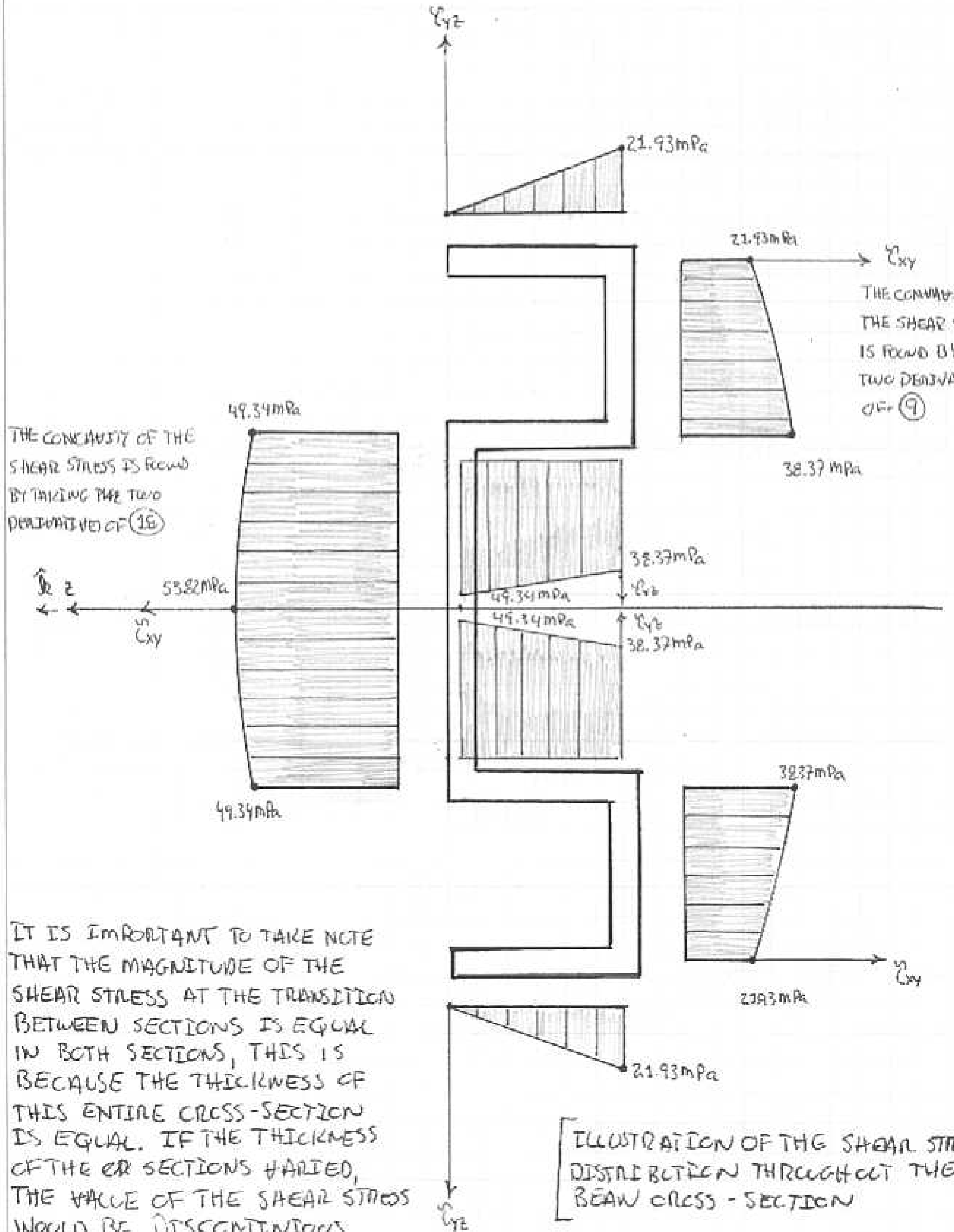
$$= \int_0^{0.012m}\left(48.34\times10^6\frac{N}{m^2} + 913.7\times10^6\frac{N}{m^3}\cdot s - 38.07\times10^9\frac{N}{m^4}\cdot s^2\right)\cdot(0.002m)\,ds$$

$$= \int_0^{0.012m}\left(96.68\times10^3\frac{N}{m} + 1.8274\times10^6\frac{N}{m^2}\cdot s - 76.14\times10^6\frac{N}{m^3}\cdot s^2\right)ds$$

$$= \left[96.68\times10^3\frac{N}{m}\cdot s + 1.8274\times10^6\frac{N}{m^2}\cdot\frac{s^2}{2} - 76.14\times10^6\frac{N}{m^3}\cdot\frac{s^3}{3}\right]_0^{0.012m}$$

$$= \underline{\underline{1247.9N}} \quad (21)$$

THE SHEAR STRESS DISTRIBUTION THROUGHOUT THE CROSS-SECTION IS SUMMARIZED IN (f)



(f)

IT IS IMPORTANT TO TAKE NOTE THAT THE MAGNITUDE OF THE SHEAR STRESS AT THE TRANSITION BETWEEN SECTIONS IS EQUAL IN BOTH SECTIONS, THIS IS BECAUSE THE THICKNESS OF THIS ENTIRE CROSS-SECTION IS EQUAL. IF THE THICKNESS OF THE ~~ER~~ SECTIONS VARIED, THE VALUE OF THE SHEAR STRESS WOULD BE DISCONTINUOUS AT THE INTERFACES.

ILLUSTRATION OF THE SHEAR STRESS DISTRIBUTION THROUGHOUT THE BEAM CROSS-SECTION

THE NET SHEAR FORCES IN EACH SECTION OF THE CROSS-SECTION ARE SUMMERIZED IN (9)

CONSIDERING THE EQUILIBRIUM OF THE CROSS SECTION ILLUSTRATED IN (9), ~~FROM~~ BECAUSE OF THE ANTI-SYMMETRY ABOUT THE Z-AXIS THE HORIZONTAL FORCES ARE SEEN TO SUM TO ZERO.

THE HORIZONTAL FORCES SHOW ON THE WEB SECTIONS ARE NOW SUMMED

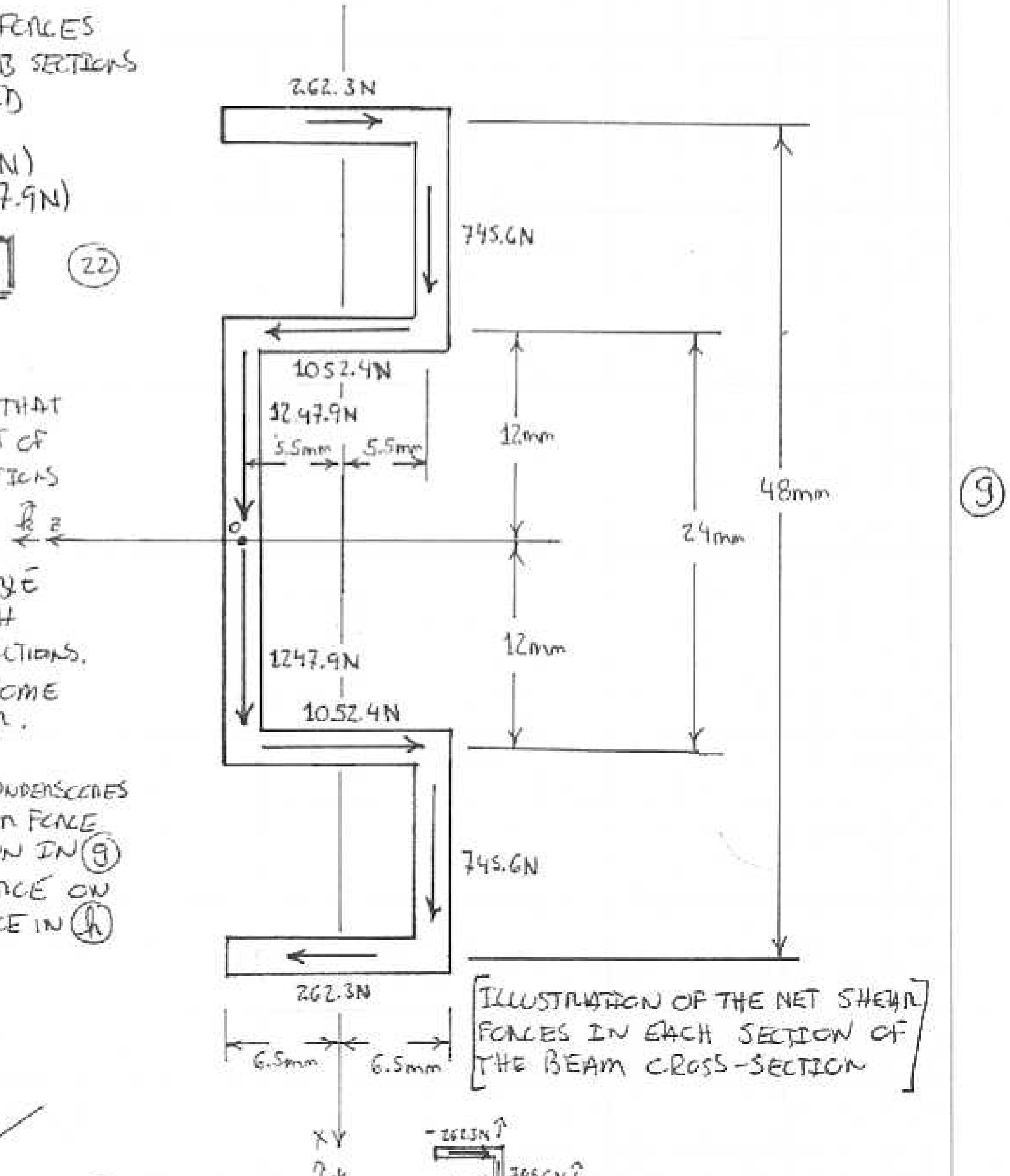
$$V = 2(745.6\text{N}) + 2(1247.9\text{N})$$

$$\approx \boxed{3987\text{N}} \quad (22)$$

$$\approx 4000\text{N}$$

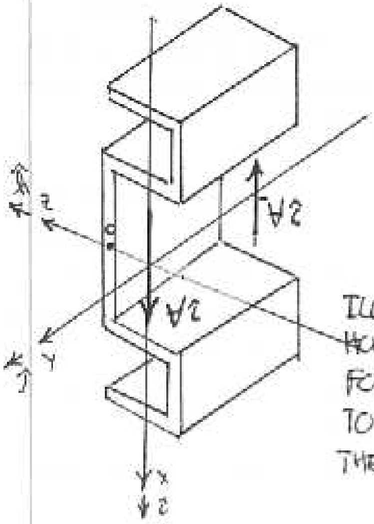
THE SLIGHT ERROR THAT RESULTS IS A RESULT OF THE WAY THE SECTIONS HAVE TO BE DRAWN SO THAT THEY ARE COMPATIBLE WITH THE STRENGTH OF MATERIALS SOLUTIONS. THERE IS ALSO SOME NUMERICAL ERROR.

THE RESULT IN (22) UNDERSCORES THAT THE NET SHEAR FORCE DISTRIBUTION SHOWN IN (9) IS THE SHEAR FORCE ON THE POSITIVE FACE IN (h)



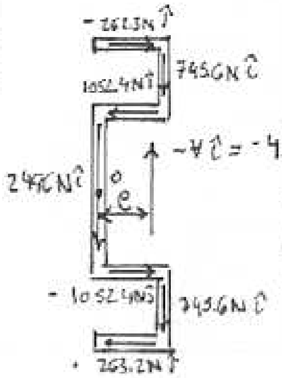
(9)

ILLUSTRATION OF THE NET SHEAR FORCES IN EACH SECTION OF THE BEAM CROSS-SECTION



(h)

ILLUSTRATION OF HOW THE SHEAR FORCES ARE APPLIED TO A SEGMENT OF THE BEAM.



(i)

ILLUSTRATION OF A FRONT VIEW OF THE BEAM SEGMENT SHOWN IN (h)

THE FINAL CHECK TO SEE IF THE BEAM IS IN EQUILIBRIUM IS TO SUM THE MOMENTS ABOUT A POINT ON THE BEAM. THE NET MOMENT THAT RESULTS FROM THE SHEAR FORCES ILLUSTRATED IN (9) ARE

$$M_{@O} = -(262.3N)(0.048m) - 2(745.6N)(0.011m) + (1052.4N)(0.024m)$$

$$\approx -3.736\ N{\cdot}m \quad (23)$$

THE NET MOMENT IN (23) NEEDS TO BE REACTED OUT BY THE SHEAR FORCE ON THE NEGATIVE SIDE OF THE BEAM AS SHOWN IN (2). THIS REPRESENTS THE TOTAL EQUILIBRIUM OF THE BEAM SEGMENT

$$\Sigma M_{y@O} = 0 = -(262.3N)(0.048m) - 2(745.6N)(0.011m) + (1052.4N)(0.024m) + e\cdot(4000N)$$

$$0 = -3.736\ N{\cdot}m + e\cdot 4000N$$

$$e = 934\times10^{-6}m = 0.934\ mm \approx 1mm \quad (24)$$

THIS RESULT INDICATES THAT IF THE LOAD ON THE BEAM IS PLACED AT THE CENTROID OF THE CROSS-SECTION THE BEAM WILL NOT BE IN EQUILIBRIUM AND WILL TWIST. TO AVOID THIS THE LOAD NEEDS TO BE APPLIED APPROXIMATELY 1mm AWAY FROM O.

THE SECOND PART OF THE PROBLEM SUGGESTS THAT THIS BEAM IS TO BE MADE OUT OF ~~SA~~ A BUILD-UP OF STRAIGHT BOARDS. THE BEST CONFIGURATION OF BOARDS ~~IS~~ THAT WILL MINIMIZE NAIL SPACING IS DESIRED. THE ONLY TWO POTENTIAL BUILD-UP CONFIGURATIONS ARE SHOWN IN (j)

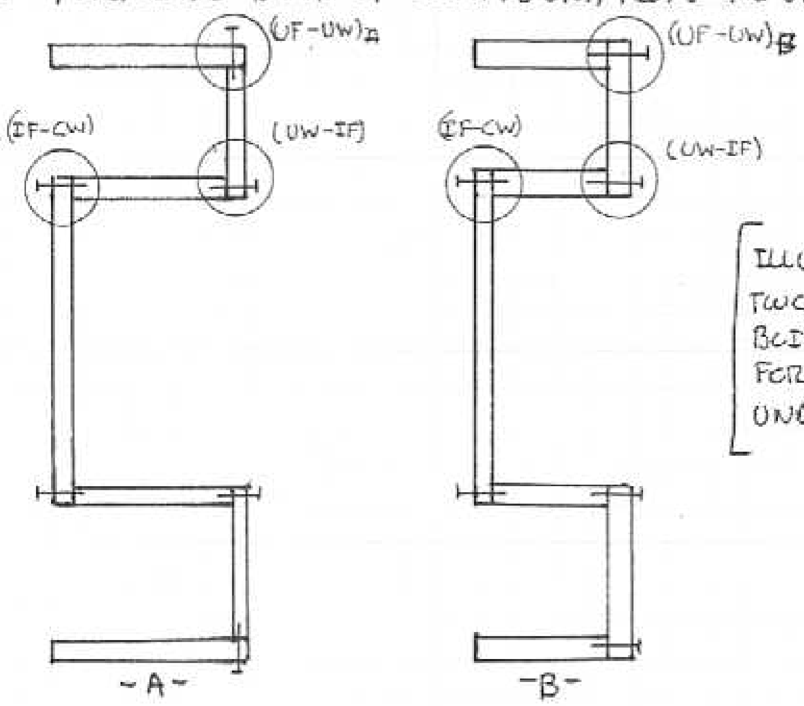


ILLUSTRATION OF THE TWO POTENTIAL BUILD-UP CONFIGURATIONS FOR THE CROSS SECTION UNDER CONSIDERATION (j)

IN (J) THE JOINTS UPPER FLANGE – UPPER WEB (UF-UW), UPPER WEB – INTERMEDIATE FLANGE (UW-IF), AND INTERMEDIATE FLANGE – CENTER WEB (IF-CW) ALL HAVE TO BE EVALUATED TO DETERMINE NAIL SPACING ~~...~~ ALONG THE LENGTH OF THE BEAM (Y-DIRECTION). BECAUSE OF ACCESS CONSIDERATIONS ONLY ONE CONFIGURATION OF IF-CW AND UW-IF NEED TO BE CONSIDERED. THE TWO CONFIGURATIONS OF UF-UW ARE DESIGNATED A AND B.

FIGURE (K) ILLUSTRATES THE GEOMETRY NEEDED TO CALCULATE THE SHEAR FLOW FOR THESE TWO CONFIGURATIONS.

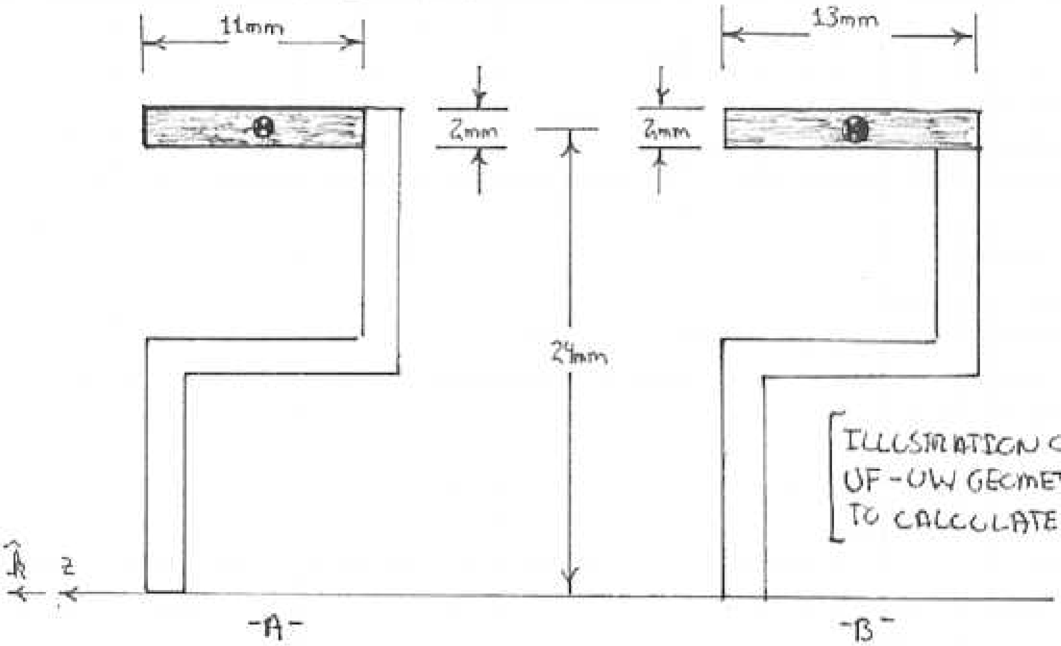


ILLUSTRATION OF THE TWO UF-UW GEOMETRIES USED TO CALCULATE SHEAR FLOW

(K)

$(UF\text{-}UW)_A$

CALCULATING THE SHEAR FLOW USING THE GEOMETRY IN (K)-A

$$q = \frac{V \cdot Q}{I} = \frac{(4\times10^3 N)(0.024m)(0.011m)(0.002m)}{52.54\times10^{-9}\,m^4}$$

$$= \underline{\underline{40.20\times10^3 \frac{N}{m}}} \qquad (25)$$

$(UF\text{-}UW)_B$

CALCULATING THE SHEAR FLOW USING THE GEOMETRY IN (K)-B

$$q = \frac{V \cdot Q}{I} = \frac{(4\times10^3 N)(0.024m)(0.013m)(0.002m)}{52.54\times10^{-9}\,m^4}$$

$$= 47.51\times10^3 \frac{N}{m} \qquad (26)$$

UW-IF

CALCULATING THE SHEAR FLOW USING THE GEOMETRY IN (l)

$$q = \frac{V \cdot Q}{I}$$

$$= \frac{(4\times10^3 N)\left[(0.024m)(0.013m)(0.002m) + (0.017m)(0.012m)(0.002m)\right]}{52.53\times10^{-9} m^4}$$

$$= 78.58\times10^3 \frac{N}{m} \quad (27)$$

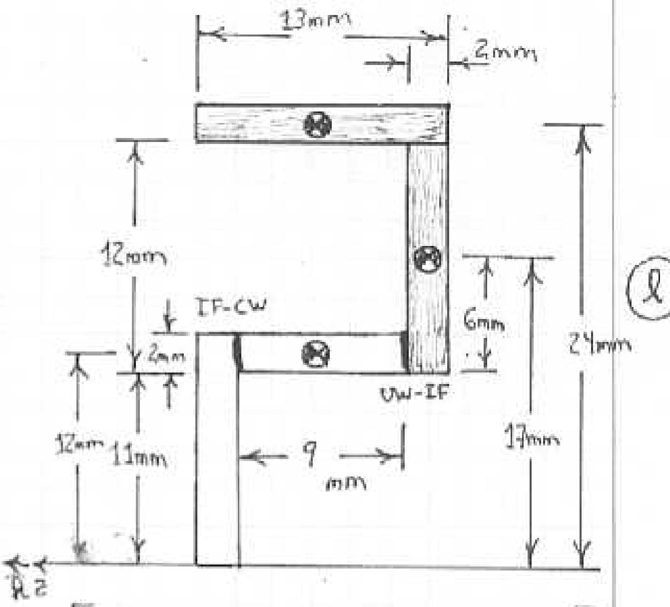


ILLUSTRATION OF THE GEOMETRY USED IN THE CALCULATION OF THE SHEAR FLOW FOR THE IF-CW AND UW-IF JOINTS

IF-CW

CALCULATING THE SHEAR FLOW USING THE GEOMETRY IN (l)

$$q = \frac{V Q}{I}$$

$$= \frac{(4\times10^3 N)\left[(0.024m)(0.013m)(0.002m) + (0.017m)(0.012m)(0.002m) + (0.012m)(0.007m)(0.002m)\right]}{52.53\times10^{-9} m^4}$$

$$= 95.03\times10^3 \frac{N}{m} \quad (28)$$

THE NAIL SPACING IS CALCULATED KNOWING THE SHEAR LOAD CAPACITY OF THE NAILS BEING USED, S_N

$$SP = \frac{S_N}{q} = \frac{[N]}{[N/m]} = [m]$$

THE MAXIMUM SPACING THAT ALLOWS FOR THE SAFEST DESIGN OCCURS WHERE q IS HIGHEST. THUS IF-CW WILL DETERMINE THE NAIL SPACING USED. THE MORE CONSERVATIVE UF-UW JOINT DESIGN WOULD BE "A" BECAUSE THE SHEAR FLOW IS THE LOWEST AT THE JOINT FOR THESE TWO CONFIGURATIONS.