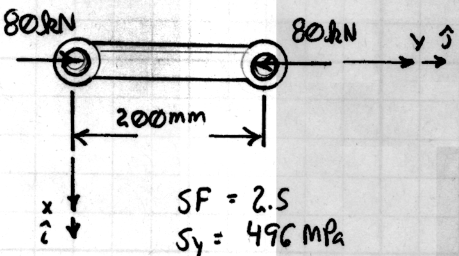
EXAMPLE

AN INDUSTRIAL MACHINE REQUIRES A SOLID, ROUND PISTON CONNECTING ROD 200 mm LONG (BETWEEN PINNED ENDS) THAT IS SUBJECTED TO A MAXIMUM COMPRESSIVE FORCE OF 80,000 N. USING A SAFETY FACTOR OF 2.5, WHAT DIAMETER IS REQUIRED IF ALUMINUM IS USED, HAVING PROPERTIES OF $S_y = 496$ MPa, $E = 71$ GPa.

SOLUTION:

THE SOLUTION STARTS BY DETERMINING THE CRITICAL SLENDERNESS RATIO.
- FIRST ASSUME THE BEAM IS IN THE EULER RANGE



SF = 2.5
S_y = 496 MPa
E = 71 GPa

$$\left(\frac{P}{A}\right)_{cr} = \frac{\pi^2 E}{(L_e/\rho)^2}$$

$$\rho = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi \cdot D^4}{64}}{\pi \cdot D^2/4}} = \sqrt{\frac{D^2}{16}} = \frac{D}{4}$$

$$\left(\frac{P}{A}\right)_{cr} = \frac{\pi^2 \cdot E}{(L_e \cdot \frac{4}{D})^2}$$

$$\frac{P \cdot 4}{\pi \cdot D^2} = \frac{\pi^2 \cdot E \cdot D^2}{L^2 \cdot 16} \Rightarrow D = \sqrt[4]{\frac{64 \cdot P \cdot L_e^2}{\pi^3 \cdot E}}$$

$$= \left[\frac{64 \cdot 80 \times 10^3 \frac{N}{m^2} \cdot (200 \times 10^{-3} m)^2}{2.5 \cdot \pi^3 \cdot 71 \times 10^9 \frac{N}{m^2}}\right]^{1/4}$$

$$= \underline{\underline{0.022 m}}$$

THE ASPECT RATIO FOR THIS BEAM IS

$$\frac{L}{\rho} = \frac{200 \times 10^{-3} m}{0.022 m/4} = \underline{\underline{36.4}}$$

THE CRITICAL ASPECT RATIO IS GIVEN BY

$$\left(\frac{L}{\rho}\right)_{cr} = \sqrt{\frac{\pi^2 \cdot E}{\frac{1}{2}(P/A)_{cr}}} = \sqrt{\frac{\pi^2 \cdot 71 \times 10^9 \ N/m^2}{\underbrace{\frac{1}{2} \cdot 496 (10^6) \ N/m^2}_{S_y/2}}} = \underline{\underline{53.1}}$$

A COMPARISON OF THE BEAMS ASPECT RATIO WITH ~~THAT~~ THE CRITICAL ASPECT RATIO SHOWS THAT THIS BEAM IS TOO SHORT TO USE EULER'S EQUATION. Thus

$$\frac{P}{A} = S_y - \frac{S_y^2}{4\pi^2 \cdot E} \cdot \left(\frac{L}{\rho}\right)^2, \quad A = \frac{\pi \cdot D^2}{4}, \quad \rho = \frac{D}{4}$$

$$\frac{80 \times 10^3 N \cdot 4}{2.5 \cdot \pi \cdot D^2} = 496 \times 10^6 \frac{N}{m^2} - \frac{(496 \times 10^6 \frac{N}{m^2})^2}{4 \cdot \pi^2 \cdot 71 \times 10^9 \frac{N}{m^2}} \cdot \frac{(0.2m)^2 \cdot (16)}{D^2}$$

$$D = \boxed{0.025 m} \Rightarrow \frac{L}{\rho} = \frac{(0.2m) \cdot (4)}{0.025 m} = 32$$