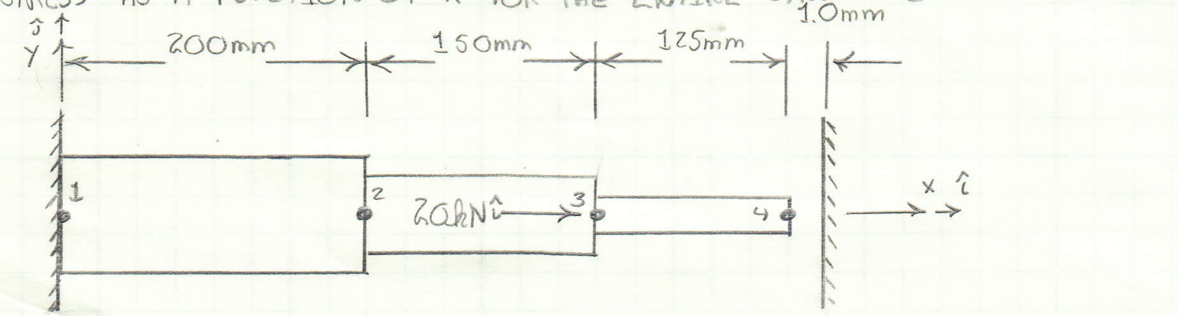
FOR THE STRUCTURE SHOWN IN THE FIGURE CONSIDER THE WALLS TO BE RIGID. USING THE FINITE ELEMENT METHOD DETERMINE THE DEFLECTIONS OF ALL NODES; THE WALL REACTIONS; AND THE DEFLECTION EQUATIONS, STRESSES, AND NODAL FORCES OF EACH ELEMENT. PLOT THE DEFLECTION AND STRESS AS A FUNCTION OF X FOR THE ENTIRE STRUCTURE.



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THE APPLIED FORCE IS F = 20 kN. THE AREAS FOR THE ELEMENTS ARE $A_1 = 100mm^2$, $A_2 = 75mm^2$, AND $A_3 = 50mm^2$. THE ELEMENTS HAVE THE SAME MODULUS OF ELASTICITY, E = 50 GPa.

<u>GIVEN:</u>

1. 1.mm GAP BETWEEN WALL AND STRUCTURE
2. CROSS SECTIONAL AREAS OF STEP SECTIONS $100mm^2$, $75mm^2$, AND $50mm^2$.
3. MODULUS OF THE MATERIAL USED 50 GPa.

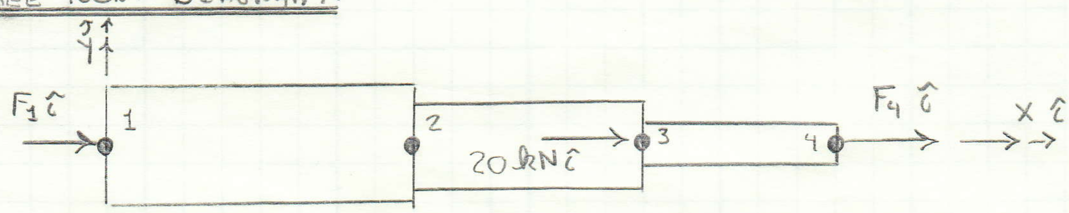
<u>ASSUMPTIONS</u>

1. WALLS ARE RIGID
2. BUCKLING DOES NOT OCCUR
3. DEFLECTIONS ARE SMALL

<u>FIND:</u>

1. DEFLECTIONS AT NODES
2. EXTERNAL FORCES AT NODES

<u>FREE BODY DIAGRAM:</u>



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<u>FINITE ELEMENT SOLUTION:</u>

IT WAS ASSUMED THAT UNDER THE LOADING SHOWN THAT THE STRUCTURE WILL HIT THE WALL. BUT HERE THIS WILL BE SHOWN.

CONSIDER THE STRUCTURE WITHOUT THE WALL PRESENT FOR THE FIRST PART OF THIS SOLUTION.

CALCULATING THE STIFFNESS OF EACH ELEMENT

$$k_1 = \frac{A_1 \cdot E_1}{L_1} = \frac{(100\times10^{-6}\,m^2)\cdot(50\times10^{9}\,\frac{N}{m^2})}{200\times10^{-3}\,m} = 25\times10^{6}\,\frac{N}{m} \quad (1)$$

$$k_2 = \frac{A_2 \cdot E_2}{L_2} = \frac{(75\times10^{-6}\,m^2)\cdot(50\times10^{9}\,\frac{N}{m^2})}{150\times10^{-3}\,m} = 25\times10^{6}\,\frac{N}{m} \quad (2)$$

$$k_3 = \frac{A_2 \cdot E_2}{L_2} = \frac{(50\times10^{-6}\,m^2)\cdot(50\times10^{9}\,\frac{N}{m^2})}{125\times10^{-3}\,m} = 20\times10^{6}\,\frac{N}{m} \quad (3)$$

FOR ELEMENT 1

$$\begin{Bmatrix} f_{x1} \\ f_{x2} \end{Bmatrix}_1 = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \Rightarrow \begin{matrix} (f_{x1})_1 = k_1 \cdot u_1 - k_1 \cdot u_2 & (4) \\ (f_{x2})_1 = -k_1 \cdot u_1 + k_1 \cdot u_2 & (5) \end{matrix}$$

FOR ELEMENT 2

$$\begin{Bmatrix} f_{x2} \\ f_{x3} \end{Bmatrix}_2 = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} \Rightarrow \begin{matrix} (f_{x2})_2 = k_2 \cdot u_2 - k_2 \cdot u_3 & (6) \\ (f_{x3})_2 = -k_2 u_2 + k_2 \cdot u_3 & (7) \end{matrix}$$

FOR ELEMENT 3

$$\begin{Bmatrix} f_{x3} \\ f_{x4} \end{Bmatrix}_3 = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} \Rightarrow \begin{matrix} (f_{x3})_3 = k_3 \cdot u_3 - k_3 u_4 & (8) \\ (f_{v4})_3 = -k_3 u_3 + k_3 u_4 & (9) \end{matrix}$$

WRITING (4) - (9) IN TERMS OF THE GLOBAL FORCES AND DISPLACEMENTS

$$F_1 = (f_{x1})_1 = k_1 \cdot u_1 - k_1 \cdot u_2 \quad (10)$$

$$F_2 = (f_{x2})_1 + (f_{x2})_2 = -k_1 \cdot u_1 + (k_1 + k_2) u_2 - k_2 \cdot u_3 \quad (11)$$

$$F_3 = (f_{x3})_2 + (f_{x3})_3 = -k_2 \cdot u_2 + (k_2 + k_3) \cdot u_3 - k_3 \cdot u_4 \quad (12)$$

$$F_4 = (f_{x4})_3 = -k_3 \cdot u_3 + k_3 \cdot u_4 \quad (13)$$

IN MATRIX FORM

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & (k_1+k_2) & -k_2 & 0 \\ 0 & -k_2 & (k_2+k_3) & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \quad (14)$$

SUBSTITUTING THE VALUES IN 1 - 3 INTO 14

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \left(10^6 \frac{N}{m}\right) \begin{bmatrix} 25 & -25 & 0 & 0 \\ -25 & 50 & -25 & 0 \\ 0 & -25 & 45 & -20 \\ 0 & 0 & -20 & 20 \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \qquad (15)$$

15 IS VALID FOR BOTH THE PROBLEM WITH THE WALL PRESENT AND WITHOUT THE WALL PRESENT. STARTING BY DETERMINING THE DEFLECTION OF THE SHAFT WITHOUT THE WALL PRESENT. THE FORCE AND DISPLACEMENT VECTORS FOR THIS CASE CAN NOW BE WRITTEN

$$\begin{Bmatrix} F_1 \\ 0 \\ 20kN \\ 0 \end{Bmatrix} \quad (16) \; ; \qquad \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \quad (17)$$

REWRITING 15 USING 16 AND 17

$$\begin{Bmatrix} F_1 \\ 0 \\ 20kN \\ 0 \end{Bmatrix} = \left(10^6 \frac{N}{m}\right) \cdot \begin{bmatrix} 25 & -25 & 0 & 0 \\ -25 & 50 & -25 & 0 \\ 0 & -25 & 45 & -20 \\ 0 & 0 & -20 & 20 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \qquad (18)$$

NOW THE PORTION OF 18 ASSOCIATED WITH KNOWN DISPLACEMENTS IS PARTITIONED OUT

$$F_1 = 25(10^6)\frac{N}{m}(0) - 25(10^6)\frac{N}{m} \cdot u_2 + 0 \cdot u_3 + 0 \cdot u_4$$
$$= -25(10^6)\frac{N}{m} \cdot u_2 \qquad (19)$$

THE OTHER PORTION OF THE PARTION OF 18 IS

$$\begin{Bmatrix} 0 \\ 20kN \\ 0 \end{Bmatrix} = \left(10^6 \frac{N}{m}\right) \cdot \begin{bmatrix} -25 & 50 & -25 & 0 \\ 0 & -25 & 45 & -20 \\ 0 & 0 & -20 & 20 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

TAKING THE KNOWNS ON THE RIGHT-HAND SIDE OF THE ABOVE EQUATION TO THE LEFT HAND SIDE.

$$\begin{Bmatrix} 0 + 25(10^6)\frac{N}{m} \cdot 0 \\ 20kN - 0 \\ 0 - 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 20kN \\ 0 \end{Bmatrix} = \left(10^6 \frac{N}{m}\right) \begin{bmatrix} 50 & -25 & 0 \\ -25 & 45 & -20 \\ 0 & -20 & 20 \end{bmatrix} \cdot \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \left(10^6 \frac{N}{m}\right)^{-1} \cdot \begin{bmatrix} 50 & -25 & 0 \\ -25 & 45 & -20 \\ 0 & -20 & 20 \end{bmatrix}^{-1} \cdot \begin{Bmatrix} 0 \\ 20kN \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \left(10^{-6}\,\tfrac{m}{N}\right)\begin{bmatrix} 0.04 & 0.04 & 0.04 \\ 0.04 & 0.08 & 0.08 \\ 0.04 & 0.08 & 0.13 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ 20\,kN \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 800 \\ 1600 \\ 1600 \end{Bmatrix}(10^{-6}\,m) = \underline{\underline{\begin{Bmatrix} 0.8\,mm \\ 1.6\,mm \\ 1.6\,mm \end{Bmatrix}}} \quad (20)$$

CLEARLY, u_4 (= 1.6mm) IS GREATER THAN 1mm, THUS THE BAR WILL NOT DEFORM FREELY IN THE PRESENTS OF THE WALL. SUBSTITUTING THE RESULTS IN (20) INTO (19) TO DETERMINE THE REACTION AT THE WALL.

$$F_1 = -25(10^6)\tfrac{N}{m} \cdot 800(10^{-6})m = 20(10^3)\,N = \underline{\underline{20\,kN}} \quad (21)$$

NOW CONSIDERING THE BAR WITH A 1mm GAP BETWEEN THE BAR AND WALL. FOR THIS CASE (18) IS WRITTEN

$$\begin{Bmatrix} F_1 \\ 0 \\ 20\,kN \\ F_4 \end{Bmatrix} = \left(10^6\,\tfrac{N}{m}\right) \cdot \begin{bmatrix} 25 & -25 & 0 & 0 \\ -25 & 50 & -25 & 0 \\ 0 & -25 & 45 & -20 \\ 0 & 0 & -20 & 20 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ 1(10^{-3})m \end{Bmatrix} \quad (22)$$

PARTITIONING OUT THE EQUATIONS ASSOCIATED WITH KNOWN DISPLACEMENTS

$$\begin{Bmatrix} F_1 \\ F_4 \end{Bmatrix} = \left(10^6\,\tfrac{N}{m}\right) \cdot \begin{bmatrix} 25 & -25 & 0 & 0 \\ 0 & 0 & -20 & 20 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ 1(10^{-3})m \end{Bmatrix} \quad (23)$$

THE REMAINING PORTION OF THE PARTITION OF (22) IS WRITTEN

$$\begin{Bmatrix} 0 \\ 20\,kN \end{Bmatrix} = \left(10^6\,\tfrac{N}{m}\right) \cdot \begin{bmatrix} -25 & 50 & -25 & 0 \\ 0 & -25 & 45 & -20 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ 1(10^{-3})m \end{Bmatrix}$$

TAKING THE KNOWNS TO THE LEFT HAND SIDE OF THE ABOVE EQUATION

$$\begin{Bmatrix} 0 + 25(10^6)\tfrac{N}{m} \cdot 0 - 0 \cdot 1(10^{-3})m \\ 20\,kN + 0 \cdot 0 + 20(10^6)\tfrac{N}{m} \cdot 1(10^{-3})m \end{Bmatrix} = \left(10^6\,\tfrac{N}{m}\right) \cdot \begin{bmatrix} 50 & -25 \\ -25 & 45 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ 40000\,N \end{Bmatrix} = \left(10^6\,\tfrac{N}{m}\right) \cdot \begin{bmatrix} 50 & -25 \\ -25 & 45 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \left(10^6\,\tfrac{N}{m}\right)^{-1} \cdot \begin{bmatrix} 50 & -25 \\ -25 & 45 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 40000\,N \end{Bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = 10^{-6}\,\frac{m}{N}\begin{bmatrix} 0.02769 & 0.01538 \\ 0.01538 & 0.03077 \end{bmatrix}\begin{Bmatrix} 0 \\ 20020\,N \end{Bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 615.4 \\ 1231 \end{Bmatrix}(10^{-6})\,m = \underline{\underline{\begin{Bmatrix} 0.6154\,mm \\ 1.231\,mm \end{Bmatrix}}} \qquad (24)$$

SUBSTITUTING THE RESULTS IN (24) INTO (23)

$$\begin{Bmatrix} F_1 \\ F_4 \end{Bmatrix} = (10^6\,\tfrac{N}{m})\cdot\begin{bmatrix} 25 & -25 & 0 & 0 \\ 0 & 0 & -20 & 20 \end{bmatrix}\cdot\begin{Bmatrix} 0 \\ 0.6154(10^{-3}) \\ 1.231(10^{-3}) \\ 1.0(10^{-3}) \end{Bmatrix} m$$

$$= \begin{Bmatrix} -15380\,N \\ -4615\,N \end{Bmatrix} = \underline{\underline{\begin{Bmatrix} -15.38\,kN \\ -4.615\,kN \end{Bmatrix}}} \qquad (25)$$

FROM THE PREVIOUS DISCUSSION OF F.E. THE DISPLACEMENT ALONG THE LENGTH OF AN ELEMENT IS GIVEN BY

$$u(x) = \frac{1}{L_E}\left[u_i(x_j - x) + u_j(x - x_i)\right]$$

<u>ELEMENT 1</u>

$$u_{(1)}(x) = \frac{1}{200(10^{-3})m}\left[0m\cdot(0.6154(10^{-3})m - x) + 0.6154(10^{-3})m\cdot(x - 0)\right]$$

$$\underline{\underline{u_{(1)}(x) = 3.077(10^{-3})\cdot x}} \qquad (26)$$

<u>ELEMENT 2</u>

$$u_{(2)}(x) = \frac{1}{150(10^{-3})m}\left[0.6154(10^{-3})m\,(350(10^{-3})m - x) + 1.231(10^{-3})m\,(x - 200(10^{-3})m)\right]$$

$$= 4.103(10^{-3})\left[350(10^{-3})m - x\right] + 8.207(10^{-3})m\left[x - 200(10^{-3})m\right]$$

$$= 1.436(10^{-3})m - 4.103(10^{-3})\cdot x + 8.207(10^{-3})m\cdot x - 1.641(10^{-3})m$$

$$\underline{\underline{u_{(2)}(x) = -205.4(10^{-6})m + 4.104(10^{-3})\,x}} \qquad (27)$$

<u>ELEMENT 3</u>

$$u_{(3)}(x) = \frac{1}{125(10^{-3})m}\left[1.231(10^{-3})m\,(475(10^{-3})m - x) + 1.0(10^{-3})m\,(x - 350(10^{-3})m)\right]$$

$$= 9.848(10^{-3})\cdot(475(10^{-3})m - x) + 8(10^{-3})\cdot(x - 350(10^{-3})m)$$

$$= 4.678(10^{-3})m - 9.848(10^{-3})\cdot x + 8(10^{-3})\cdot x - 2.800(10^{-3})m$$

$$u_{(3)}^{(x)} = 1.878(10^{-3})m - 1.848(10^{-3})\cdot x \quad (28)$$

CALCULATING THE STRESS IN EACH ELEMENT

ELEMENT 1

$$(\sigma_x)_1 = \left(\frac{E}{L}\right)_{(1)} \{-1 \quad 1\} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{50(10^9)\frac{N}{m^2}}{200(10^{-3})m} \{-1 \quad 1\} \begin{Bmatrix} 0 \\ 0.6154(10^{-3})m \end{Bmatrix}$$

$$= 153.8(10^6)\frac{N}{m^2} = 153.8 \text{ MPa} \quad (29)$$

$$\begin{Bmatrix} f_{x1} \\ f_{x2} \end{Bmatrix}_1 = 25(10^6)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.6154(10^{-3})m \end{Bmatrix} = \begin{Bmatrix} -15.38(10^3)N \\ 15.38(10^3)N \end{Bmatrix} \quad (30)$$

ELEMENT 2

$$(\sigma_x)_2 = \left(\frac{E}{L}\right)_{(2)} \{-1 \quad 1\} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \frac{50(10^9)\frac{N}{m^2}}{150(10^{-3})m} \{-1 \quad 1\} \begin{Bmatrix} .6154(10^{-3})m \\ 1.2308(10^{-3})m \end{Bmatrix}$$

$$= 205.1(10^6)\frac{N}{m^2} = 205.1 \text{ MPa} \quad (31)$$

$$\begin{Bmatrix} f_2 \\ f_3 \end{Bmatrix}_2 = 25(10^6)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.6154(10^{-3})m \\ 1.2308(10^{-3})m \end{Bmatrix} = \begin{Bmatrix} -15.38(10^3)N \\ 15.38(10^3)N \end{Bmatrix}$$

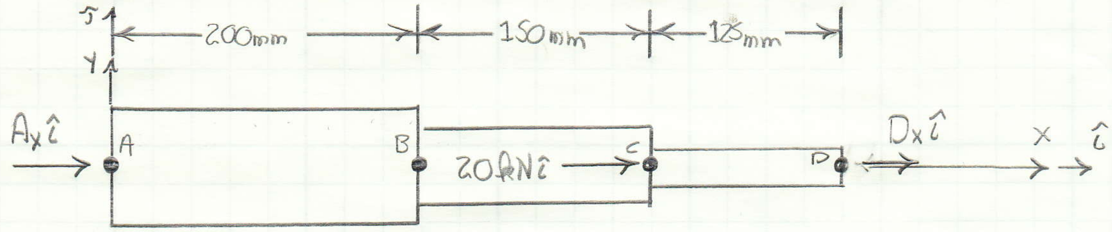
ELEMENT 3

$$(\sigma_x)_3 = \left(\frac{E}{L}\right)_3 \{-1 \quad 1\} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \frac{50(10^9)\frac{N}{m^2}}{125(10^{-3})m} \{-1 \quad 1\} \cdot \begin{Bmatrix} 1.2308(10^{-3})m \\ 1.0(10^{-3})m \end{Bmatrix}$$

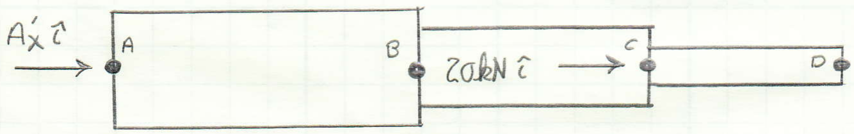
$$= -92.3(10^6)\frac{N}{m^2} = -92.3 \text{ MPa} \quad (32)$$

$$\begin{Bmatrix} f_3 \\ f_4 \end{Bmatrix}_3 = 20(10^6)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 1.2308(10^{-3})m \\ 1.0(10^{-3})m \end{Bmatrix} = \begin{Bmatrix} 4.62(10^3)N \\ -4.62(10^3)N \end{Bmatrix}$$

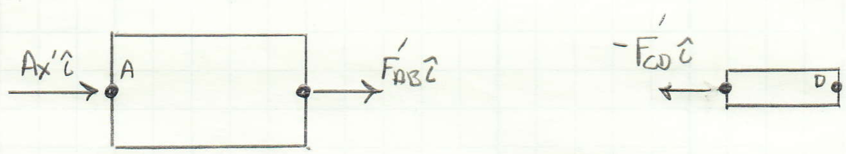
NOW CONSIDER THE ANALYTICAL SOLUTION TO THIS SAME PROBLEM. BELOW IS A ILLUSTRATION OF THE SUPERPOSITION THAT NEEDS TO BE PERFORMED. IT IS IMPORTANT TO NOTE THAT THE SOLUTION STARTS BY ASSUMING THAT THE BAR WILL EXPAND TO THE WALL AND THAT THE WALL WILL CONSTRAIN THE BARS FREE EXPANSION.



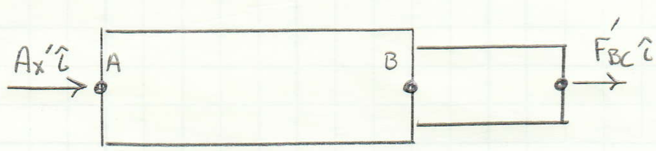
(c)



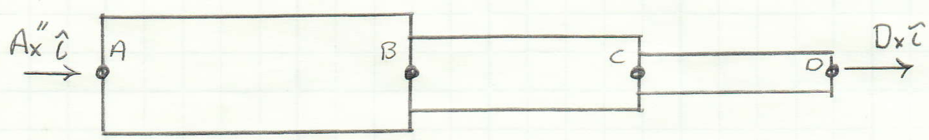
(d)



(e)

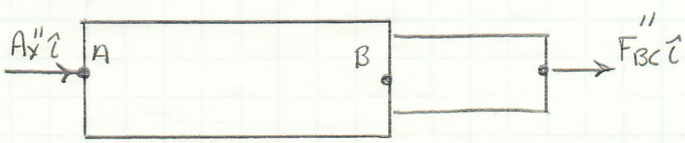


(f)



(g)

(h)



(i)

THE SOLUTION TO THIS PROBLEM REQUIRES THE USE OF THE PRINCIPLE OF SUPERPOSITION. FIGURE (c) IS A FREE BODY DIAGRAM OF THE STRUCTURE AS IT TOUCHES THE WALL AT "D". USING SUPERPOSITION THIS BEAM IS BROKEN DOWN INTO A BEAM WITH A REDUNDANT CONSTRAINT REMOVED, FIGURE (d), AND A SECOND BEAM WITH ALL EXTERNAL LOADS REMOVED AND THE REDUNDANT CONSTRAINT REINTRODUCED AS A PSEUDO-LOAD, FIGURE (g).

STARTING WITH (d)

$$\Sigma F_x = 0 = A_x' + 20kN \Rightarrow \underline{\underline{A_x' = -20kN}} \qquad (33)$$

THE DEFORMATION OF THIS STRUCTURE IS NOW CONSIDERED

$$\delta_{AD}' = \delta_{AB}' + \delta_{BC}' + \delta_{CD}'$$
$$= \frac{F_{AB}' \cdot l_{AB}}{A_{AB} \cdot E} + \frac{F_{BC}' \cdot l_{BC}}{A_{BC} \cdot E} + \frac{F_{CD}' \cdot l_{CD}}{A_{CD} \cdot E} \qquad (34)$$

GIVEN THE RESULT IN (33), THE INTERNAL LOADS F_{AB}', F_{BC}', AND F_{CD}' ARE FOUND BY APPLYING EQUILIBRIUM TO THE BEAM SEGMENTS SHOWN IN FIGURES (e) AND (f).

$$(e): \Sigma F_x = 0 = A_x' + F_{AB}' \Rightarrow F_{AB}' = -A_x' = \underline{\underline{20kN}} \qquad (35)$$

$$\Sigma F_x = 0 = -F_{CD} \Rightarrow \underline{\underline{F_{CD} = 0}} \qquad (36)$$

$$(f): \Sigma F_x = 0 = A_x' + F_{BC}' = F_{BC}' = -A_x' = \underline{\underline{20kN}} \qquad (37)$$

SUBSTITUTING (35) - (37) INTO (34)

$$\delta_{AD}' = \frac{20(10^3)N \cdot 200(10^{-3})m}{100(10^{-6})m^2 \cdot 50(10^9)N/m^2} + \frac{20(10^3)N \cdot 150(10^{-3})m}{75(10^{-6})m^2 \cdot 50(10^9)N/m^2} = \underline{\underline{1.60(10^{-3})m}} \qquad (38)$$

NOW REMOVING ALL EXTERNAL LOADS AND INTRODUCING THE CONSTRAINT AT "D" AS A PSEUDO-LOAD.

$$\delta_{AD}'' = \delta_{AB}'' + \delta_{BC}'' + \delta_{CD}''$$
$$= \frac{F_{AB}'' \cdot l_{AB}}{A_{AB} \cdot E} + \frac{F_{BC}'' \cdot l_{BC}}{A_{BC} \cdot E} + \frac{F_{CD}'' \cdot l_{CD}}{A_{CD} \cdot E} \qquad (39)$$

THE VALUES OF THE INTERNAL LOADS ARE CALCULATED BY CONSIDERING EQUILIBRIUM OF THE BEAM SEGMENTS IN (h) AND (i). STARTING WITH THE OVERALL EQUILIBRIUM OF (g)

$$\Sigma F = 0 = A_x'' + D_x \Rightarrow A_x'' = -D_x \qquad (40)$$

FROM (h)

$$\Sigma F_x = 0 = A_x'' + F_{AB}'' \Rightarrow F_{AB}'' = -A_x'' = \underline{\underline{D_x}} \tag{41}$$

$$\Sigma F_x = 0 = -F_{CD}'' + D_x \Rightarrow F_{CD}'' = \underline{\underline{D_x}} \tag{42}$$

FROM (i)

$$\Sigma F_x = 0 = A_x'' + F_{BC}'' \Rightarrow F_{BC}'' = -A_x'' = \underline{\underline{D_x}} \tag{43}$$

SUBSTITUTING (41)-(43) INTO (39)

$$\delta_{AD}'' = \frac{D_x \cdot 200(10^{-3})m}{100(10^{-6})m^2 \cdot 50(10^9)\frac{N}{m^2}} + \frac{D_x \cdot 150(10^{-3})m}{75(10^{-6})m^2 \cdot 50(10^9)\frac{N}{m^2}} + \frac{D_x \cdot 125(10^{-3})m}{50(10^{-6})m^2 \cdot 50(10^9)\frac{N}{m^2}}$$

$$= 130.0(10^{-9})\frac{m}{N} \cdot D_x \tag{44}$$

THE KINEMATIC CONSTRAINT FOR THIS PROBLEM IS

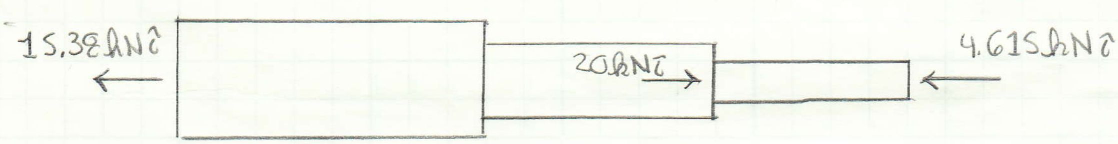
$$\delta_{AD} = \delta_{AD}' + \delta_{AD}'' = 1.0(10^{-3})m \tag{45}$$

SUBSTITUTING (38) AND (44) INTO (45)

$$1.0(10^{-3})m = 1.60(10^{-3})m + 130.0(10^{-9})\frac{m}{N} \cdot D_x$$

$$D_x = \frac{1.0(10^{-3})m - 1.60(10^{-3})m}{130.0(10^{-9})\frac{m}{N}} = \underline{\underline{-4.615(10^3)N}} \tag{46}$$

REDRAWING (c), (d), AND (g) USING THE CALCULATED VALUES



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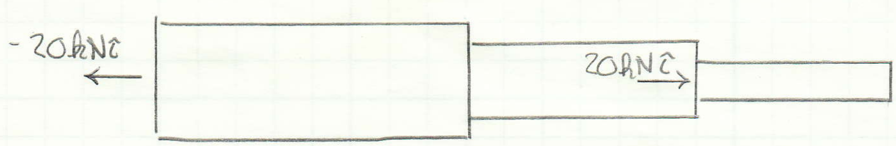

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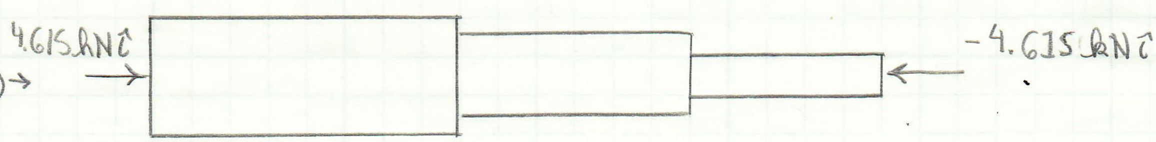
(g)



THESE RESULTS AGREE WITH (25)