NAME: SOLUTION

PROBLEM 1: At a point in a loaded member, the stresses relative to a x-y-z coordinate system are given by:

$$[\sigma]_{xyz} = \begin{bmatrix} 60 & 20 & 10 \\ 20 & -40 & -5 \\ 10 & -5 & 30 \end{bmatrix} MPa$$

1a. Determine the principal stresses for this state of stress and their directions. (If you use MATLAB or Excel to perform calculations, be sure to print out the Command Window or spreadsheet that contains the commands you used to perform the calculations.)

$\sigma_1 = -44.48 \text{ mPa}$ $\quad r_1 = -.1957\hat{i} + .9764\hat{j} + .0918\hat{k}$

$\sigma_2 = 28.42 \text{ MPa}$ $\quad r_2 = .2204\hat{i} + .1350\hat{j} - .9660\hat{k}$

$\sigma_3 = 66.06 \text{ MPa}$ $\quad r_3 = -.9556\hat{i} - .1688\hat{j} - .2416\hat{k}$

```
>> S

S =
```

$$\begin{bmatrix} 60 & 20 & 10 \\ 20 & -40 & -5 \\ 10 & -5 & 30 \end{bmatrix} = [\sigma]$$

```
>> [DS,PS]=eig(S)

DS =
```

$$\begin{array}{ccc} r_1 & r_2 & r_3 \\ -0.1957 & 0.2204 & -0.9556 \\ 0.9764 & 0.1350 & -0.1688 \\ 0.0918 & -0.9660 & -0.2416 \end{array}$$

```
PS =
```

$$\begin{array}{ccc} \sigma_1 & \sigma_2 & \sigma_3 \\ 44.4788 & 0 & 0 \\ 0 & 28.4176 & 0 \\ 0 & 0 & 56.0612 \end{array}$$

```
>>
```

1b. What angles (in degrees) do each of the principal stresses make with the x, y, and z axes?

σ_1: θ_x= 101.3° θ_y= 12.5° θ_z= 84.7°

σ_2: θ_x= 77.3° θ_y= 82.2° θ_z= 165.0°

σ_3: θ_x= 162.9° θ_y= 99.7 θ_z= 104.0°

$\theta_{x1} = \cos^{-1}(-.1957) = 101.3°$
$\theta_{y1} = \cos^{-1}(.9764) = 12.5°$
$\theta_{z1} = \cos^{-1}(.0918) = 84.7°$

$\theta_{x2} = \cos^{-1}(.2204) = 77.3°$
$\theta_{y2} = \cos^{-1}(.1350) = 82.2°$
$\theta_{z2} = \cos^{-1}(-.9660) = 165.0°$

$\theta_{x3} = \cos^{-1}(-.9556) = 162.9°$
$\theta_{y3} = \cos^{-1}(-.1688) = 99.7$
$\theta_{z3} = \cos^{-1}(-.2416) = 104.0°$

1c. What is the absolute maximum shear stress?

$$\tau_{max} = \frac{\sigma_3 - \sigma_1}{2} = \frac{(66.06\ MPa) - (-44.48\ MPa)}{2}$$

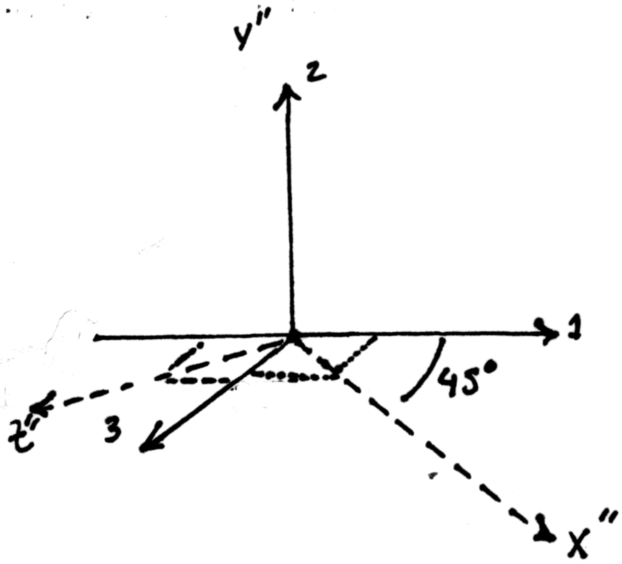
$$= \boxed{55.27\ MPa}$$

1d. What angles does the absolute shear stress make with the x, y, and z coordinates

τ_{max}: $\theta_x =$ 144.5° $\theta_y =$ 55.2° $\theta_z =$ 96.1°

$$\begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} = \begin{bmatrix} \cos 45 & 0 & \cos 45 \\ 0 & 1 & 0 \\ -\cos 45 & 0 & \cos 45 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$

$$= \begin{bmatrix} .7071 & 0 & .7071 \\ 0 & 1 & 0 \\ -.7071 & 0 & .7071 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$



$$\begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} = \begin{bmatrix} .7071 & 0 & .7071 \\ 0 & 1 & 0 \\ -.7071 & 0 & .7071 \end{bmatrix} \begin{bmatrix} -.1957 & .9764 & .0918 \\ .2204 & .1350 & -.9660 \\ -.9556 & -.1688 & -.2416 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$\begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} \doteq \begin{bmatrix} -.8141 & .5710 & -.1059 \\ .2204 & .1350 & -.9660 \\ .5373 & -.8097 & -.235 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

X'' represents THE ORIENTATION OF THE ABSOLUTE MAXIMUM SHEAR STRESS

$\Theta_{xx''} = \cos^{-1}(-.8141) = 144.5°$

$\Theta_{xy''} = \cos^{-1}(.5710) = 55.2°$

$\Theta_{xz''} = \cos^{-1}(-.1059) = 96.1°$

```
  Using Toolbox Path Cache.  Type "help toolbox_path_cache" for more info.

  To get started, select "MATLAB Help" from the Help menu.

>> S=[60,20,10;20,-40,-5;10,-5,30]

S =

    60    20    10
    20   -40    -5
    10    -5    30
```

$= [\sigma]$

```
>> T1=[ 7071,0,.7071;0,1,0;-.7071,0,.7071]

T1 =

    0.7071         0    0.7071
         0    1.0000         0
   -0.7071         0    0.7071
```

TRANSFORMATION FROM THE PRINCIPAL AXES TO THE ORIENTATION OF THE MAXIMUM SHEAR STRESS

```
>> T2=[-.1957, 9764, 0918;.2204, 1350,-.9660;-.9556,-.1688,-.2416]

T2 =

   -0.1957    0.9764    0.0918
    0.2204    0.1350   -0.9660
   -0.9556   -0.1688   -0.2416
```

TRANSFORMATION TO THE PRINCIPAL AXES

```
>> T=T1*T2

T =

   -0.8141    0.5711   -0.1059
    0.2204    0.1350   -0.9660
   -0.5373   -0.8098   -0.2357
```

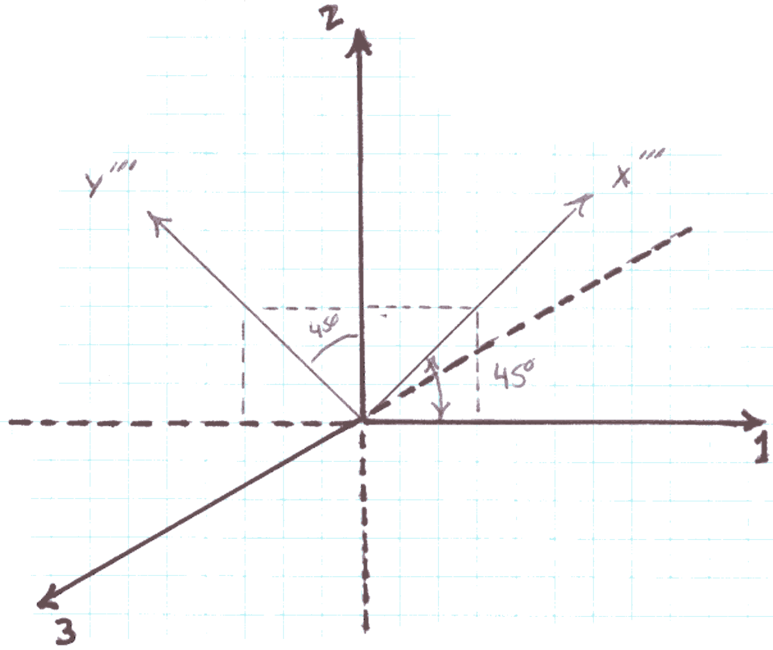
TRANSFORMATION MATRIX FROM THE INITIAL STATE OF STRESS TO THE MAXIMUM SHEAR STRESS ORIENTATION

```
>> T*S*T'

ans =

   10.7905   -0.0000   55.2719
   -0.0000   28.4164   -0.0022
   55.2719   -0.0022   10.7901
```

REALLY 0

STRESS TENSOR FOR THE STATE OF MAXIMUM SHEAR FOR THIS POINT IN THE STRUCTURE

$$\begin{Bmatrix} X''' \\ Y''' \\ Z''' \end{Bmatrix} = \begin{bmatrix} \cos 45^\circ & \cos 45^\circ & 0 \\ -\cos 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$



$$\begin{Bmatrix} X''' \\ Y''' \\ Z''' \end{Bmatrix} = \begin{bmatrix} .7071 & .7071 & 0 \\ -.7071 & .7071 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$

$$\begin{Bmatrix} X''' \\ Y''' \\ Z''' \end{Bmatrix} = \begin{bmatrix} .7071 & .7071 & 0 \\ -.7071 & .7071 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -.1957 & 9764 & .0918 \\ .2204 & 1350 & -.9660 \\ -.9556 & 1688 & -.2416 \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}$$

$$\begin{Bmatrix} X''' \\ Y''' \\ Z''' \end{Bmatrix} = \begin{bmatrix} 0.0175 & 0.7859 & -0.6181 \\ 0.2942 & -0.5950 & -0.7480 \\ -0.9556 & -.1688 & -0.2416 \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}$$

X''' REPRESENTS THE ORIENTATION TO THE MAXIMUM SHEAR STRESS IN THE 1-2 PLANE

$\Theta_{xx'''} = 89^\circ$

$\Theta_{xy'''} = 38^\circ$

128°

```
>> S

S =

    60    20    10
    20   -40    -5
    10    -5    30

   T1

T1 =

    0.7071    0.7071         0
   -0.7071    0.7071         0
         0         0    1.0000
```

TRANSFORMATION FROM THE PRINCIPAL PLANE TO THE MAXIMUM SHEAR STRESS IN THE 1-2 PLANE

```
   T2

T2

   -0.1957    0.9764    0.0918
    0.2204    0.1350   -0.9660
   -0.9556   -0.1688   -0.2416
```

TRANSFORMATION FROM THE ORIGINAL STATE OF STRESS TO THE PRINCIPAL STATE OF STRESS

```
   T=T1*T2

T =

    0.0175    0.7859   -0.6181
    0.2942   -0.5950   -0.7480
   -0.9556   -0.1688   -0.2416
```

TRANSFORMATION FROM THE ORIGINAL STATE OF STRESS TO THE MAXIMUM SHEAR STRESS IN THE 1-2 PLANE

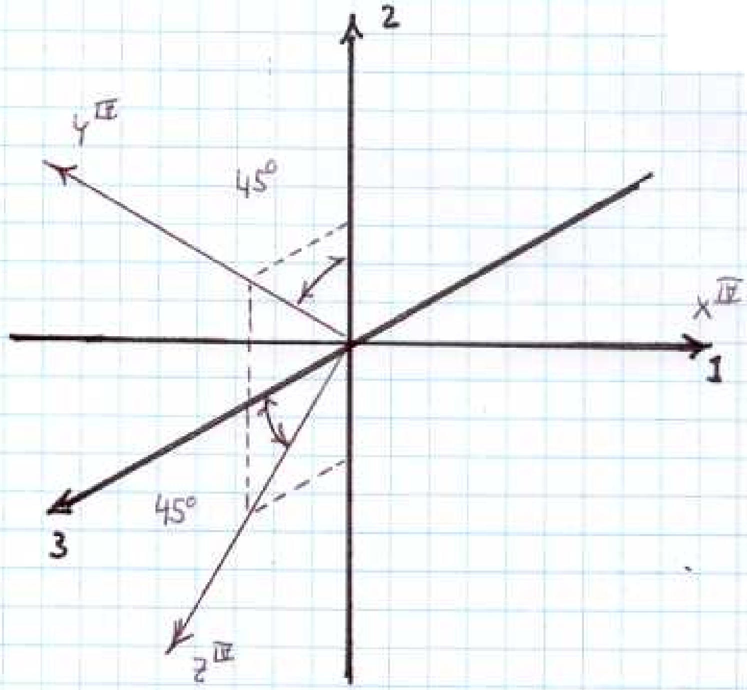
```
   T*S*T'

ans =

   -8.0313   36.4487   -0.0010
   36.4487   -8.0344   -0.0013
   -0.0010   -0.0013   66.0635
```

STATE OF STRESS WHEN THE SHEAR STRESS IS MAXIMIZED IN THE 1-2 PLANE

```
   acos(T)*180/pi

ans

   88.9993    38.1987   128.1809
   72.8890   126.5093   138.4149
  162.8624    99.7181   103.9810

>>
```

ANGLES MADE BETWEEN THE X-Y-Z COORDINATE SYSTEM AND THE X'''-Y'''-Z''' COORDINATE SYSTEM.

$$\begin{Bmatrix} X^{IV} \\ Y^{IV} \\ Z^{IV} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45 & \cos 45 \\ 0 & -\cos 45 & \cos 45 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$



$$\begin{Bmatrix} X^{IV} \\ Y^{IV} \\ Z^{IV} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .7071 & .7071 \\ 0 & -.7071 & .7071 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$

$$\begin{Bmatrix} X^{IV} \\ Y^{IV} \\ Z^{IV} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .7071 & .7071 \\ 0 & -.7071 & .7071 \end{bmatrix} \cdot \begin{bmatrix} -.1957 & .9764 & .0918 \\ .2204 & .1350 & -.9660 \\ -.9556 & -.1688 & -.2416 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$\begin{Bmatrix} X^{IV} \\ Y^{IV} \\ Z^{IV} \end{Bmatrix} = \begin{bmatrix} -.1957 & .9764 & .0918 \\ -.5199 & -.0239 & -.8539 \\ -.8315 & -.2148 & .5122 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

Y^{IV} REPRESENTS THE ORIENTATION TO THE MAXIMUM SHEAR STRESS IN THE 2-3 PLANE

$\theta_{xY^{IV}} = 121°$

$\theta_{yY^{IV}} = 91°$

$\theta_{zY^{IV}} = 149°$

```
>> S

S

    60    20    10
    20   -40    -5
    10    -5    30
```

INITIAL STATE OF STRESS

```
   T2

   -0.1957    0.9764    0.0918
    0.2204    0.1350   -0.9660
   -0.9556   -0.1688   -0.2416
```

TRANSFORMATION FROM THE INITIAL STATE OF STRESS TO THE PRINCIPAL STATE OF STRESS

```
>> T3

T3 =

     0000         0         0
        0    0.7071    0.7071
        0   -0.7071    0.7071
```

TRANSFORMATION FROM THE PRINCIPAL STATE OF STRESS TO THE STATE OF STRESS THAT MAXIMIZES SHEAR IN THE 2-3 PLANE

```
>> T=T3*T2

T =

   -0.1957    0.9764    0.0918
   -0.5199   -0.0239   -0.8539
   -0.8315   -0.2148    0.5122
```

TRANSFORMATION FROM THE INITIAL STATE OF STRESS TO THE STATE OF STRESS THAT MAXIMIZES SHEAR IN THE 2-3 PLANE

```
>> T*S*T'

ans

  -44.4825    0.0012   -0.0010
    0.0012   47.2375   18.8232
   -0.0010   18.8232   47.2406
```

STATE OF STRESS THAT MAXIMIZES SHEAR IN THE 2-3 PLANE

```
>> acos(T)*180/pi

ans =

  101.2856   12.4724   84.7328
  121.3229   91.3695  148.6378
  146.2582  102.4048   59.1880
```

ANGLES MADE BETWEEN THE INITIAL COORDINATE SYSTEM AND THE COORDINATE SYSTEM THAT MAXIMIZES STRESS IN THE 2-3 PLANE

PROBLEM 2: A uniform bar of rectangular cross section 2hxb and specific weight γ hangs in the vertical plan. Its weight results in displacements

$$u = -\frac{\nu \cdot \gamma}{E} \cdot x \cdot z$$

$$v = -\frac{\nu \cdot \gamma}{E} \cdot y \cdot z$$

$$w = \frac{\nu \cdot \gamma}{E} \cdot \left[\left(z^2 - a^2 \right) + \nu \left(x^2 + y^2 \right) \right]$$

2a. Determine the normal strains ε_x, ε_y, and ε_z.

$$\epsilon_x = \frac{\partial u}{\partial x} = \boxed{-\frac{\nu \cdot \gamma}{E} \cdot z}$$

$$= \frac{\partial v}{\partial y} = \boxed{-\frac{\nu \cdot \gamma}{E} \cdot z}$$

$$= \frac{\partial w}{\partial z} = \boxed{2 \cdot \frac{\nu \cdot \gamma}{E} \cdot z}$$

2b. Determine the shearing strains γ_{xy}, γ_{xz}, and γ_{yz}.

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 + 0 = \boxed{0}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = -\frac{\nu \cdot \gamma}{E} \cdot y + \frac{\nu \gamma}{E} \cdot 2 \cdot y \cdot \gamma = 2\frac{\nu \gamma^2}{E} \cdot y - \frac{\nu \gamma}{E} y$$

$$= \frac{\nu \gamma}{E} \cdot y \cdot (2 \cdot \nu - 1) = \boxed{-\frac{\nu \cdot \gamma \cdot (1 - 2\nu)}{E} \cdot y}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = -\frac{\nu \gamma}{E} \cdot x + 2 \cdot x \cdot \nu \cdot \frac{\nu \cdot \gamma}{E}$$

$$= \frac{\nu \cdot \gamma}{E} \cdot x \, (2\nu - 1) = \boxed{-\frac{\nu \cdot \gamma \cdot (1 - 2\nu)}{E} \cdot x}$$

2c. Determine the rigid body rotations Θ_{xy}, Θ_{xz}, and Θ_{yz}.

$$\Theta_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0-0) = \boxed{0}$$

$$\Theta_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = \frac{1}{2}\left[\frac{\nu^2\gamma}{E}\cdot 2\cdot y - \left(-\frac{\nu\cdot\gamma}{E}y\right)\right]$$

$$= \frac{1}{2}\frac{\nu\cdot\gamma}{E}\cdot y(2\cdot\nu+1) = \boxed{\frac{\gamma\cdot\nu(\nu+1/2)}{E}\cdot y = \frac{\gamma\cdot(\nu^2+\nu/2)}{E}\cdot y}$$

$$\Theta_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = \frac{1}{2}\left[\left(-\frac{\nu\cdot\gamma}{E}\cdot x\right) - \left(\frac{\nu^2\cdot\gamma}{E}\cdot 2\cdot x\right)\right]$$

$$= -\frac{1}{2}\frac{\nu\cdot\gamma\cdot x}{E}(1+2\nu) = \boxed{\frac{-\gamma\cdot\nu\cdot(\nu+1/2)}{E}\cdot x = \frac{-\gamma\cdot(\nu^2+\nu/2)}{E}\cdot x}$$

2d. Given that the material under consideration is isotropic with modulus E and Poisson's ration ν, determine the normal stresses σ_x, σ_y, an σ_z.

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)} \cdot \left[(1-\nu)\cdot\epsilon_x + \nu\cdot(\epsilon_y + \epsilon_z)\right]$$

$$= \frac{E}{(1+\nu)\cdot(1-2\nu)} \cdot \left[(1-\nu)\cdot\left(\frac{-\nu\cdot\gamma}{E}\cdot z\right) + \nu\cdot\left(-\frac{\nu\cdot\gamma}{E}\cdot z + 2\cdot\frac{\nu\cdot\gamma}{E}\cdot z\right)\right]$$

$$= \frac{\nu\cdot\gamma\cdot z}{(1+\nu)\cdot(1-2\nu)} \cdot \left[-(1-\nu) + \nu(-1+2)\right] = \frac{\nu\cdot\gamma\cdot z}{(1+\nu)(1-2\nu)} \cdot \left[-1+2\nu\right]$$

$$= -\frac{\nu\cdot\gamma\cdot z\,\cancel{(1-2\nu)}}{(1+\nu)\,\cancel{(1-2\nu)}} = \boxed{-\frac{\nu\cdot\gamma\cdot z}{(1+\nu)}}$$

$$\sigma_y = \frac{E}{(1+\nu)\cdot(1-2\nu)} \cdot \left[(1-\nu)\cdot\epsilon_y + \nu\cdot(\epsilon_x + \epsilon_z)\right]$$

$$= \frac{E}{(1+\nu)\cdot(1-2\nu)} \cdot \left[(1-\nu)\cdot\left(-\frac{\nu\cdot\gamma}{E}\cdot z\right) + \nu\cdot\left(-\frac{\nu\cdot\gamma}{E}\cdot z + 2\cdot\frac{\nu\cdot\gamma}{E}\cdot z\right)\right]$$

$$= \frac{\gamma\cdot\nu\cdot z}{(1+\nu)\cdot(1-2\nu)} \cdot \left[(1-\nu) + \nu(-1+2)\right] = \frac{\gamma\cdot\nu\cdot z}{(1+\nu)\cdot(1-2\nu)} \cdot (-1+\nu+\nu)$$

$$= -\frac{\nu\cdot\gamma\cdot z\,\cancel{(1-2\nu)}}{(1+\nu)\cdot\cancel{(1-2\nu)}} = \boxed{-\frac{\nu\cdot\gamma\cdot z}{(1+\nu)}}$$

$$\sigma_z = \frac{E}{(1+\nu)\cdot(1-2\nu)} \cdot \left[(1-\nu)\cdot\epsilon_z + \nu(\epsilon_x + \epsilon_y)\right]$$

$$= \frac{E}{(1+\nu)\cdot(1-2\nu)} \cdot \left[(1-\nu)\cdot\left(2\cdot\frac{\nu\cdot\gamma}{E}\cdot z\right) + \nu\left(-\frac{\nu\cdot\gamma}{E}\cdot z - \frac{\nu\cdot\gamma}{E}\cdot z\right)\right]$$

$$= \frac{\nu\cdot\gamma\cdot z}{(1+\nu)\cdot(1-2\nu)} \cdot \left[(1-\nu)\cdot 2 - 2\nu\right] = \frac{\nu\cdot\gamma\cdot z}{(1+\nu)\cdot(1-2\nu)} \cdot \left[2-2\nu-2\nu\right]$$

$$= \frac{\nu\cdot\gamma\cdot z}{(1+\nu)\cdot\cancel{(1-2\nu)}} \cdot 2\cdot\cancel{(1-2\nu)} = \boxed{2\cdot\frac{\gamma\cdot\nu}{(1+\nu)}\cdot z}$$

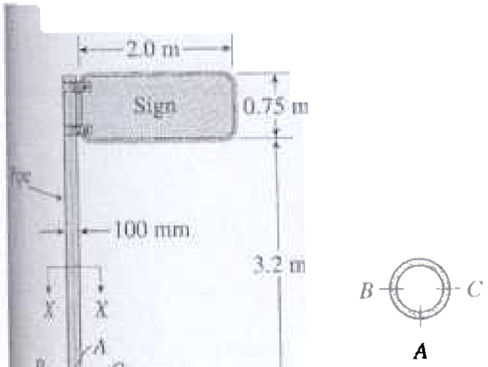
2e. Determine the shear stresses τ_{xy}, τ_{xz}, and τ_{yz}.

$$\tau_{xy} = \frac{E}{2\cdot(1+\nu)}\cdot 0 = \boxed{0}$$

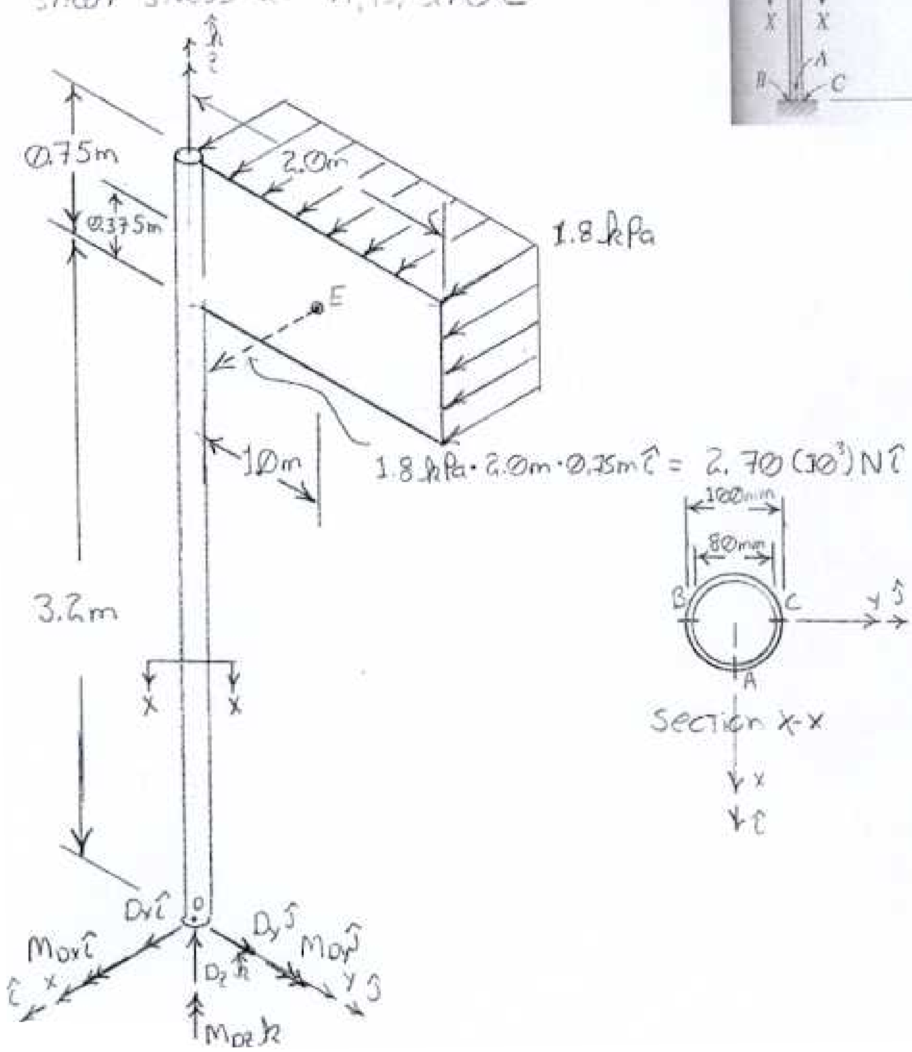
$$\tau_{yz} = \frac{\cancel{E}}{2\cdot(1+\nu)}\cdot\frac{\nu\cdot\gamma}{\cancel{E}}\cdot y\cdot(2\cdot\nu-1) = \boxed{-\frac{\nu\cdot\gamma\cdot(1-2\nu)}{2\,(1+\nu)}\cdot y}$$

$$\tau_{xz} = \frac{\cancel{E}}{2\cdot(1+\nu)}\cdot\frac{\nu\cdot\gamma}{\cancel{E}}\cdot x\cdot(2\nu-1) = \boxed{-\frac{\nu\cdot\gamma\cdot(1-2\nu)}{2\cdot(1+\nu)}\cdot x}$$

PROBLEM 3: A sign is supported by a pipe having outer diameter 100mm and inner diameter 80mm. The dimensions of the sign are 2m x 0.75m, and its lower edge is 3.2m above the base. The wind pressure against the sign is 1.8 kPa. Determine the absolute maximum shear stress in the pole.



Section *X*-*X*



$$\Sigma F_x = 0 = D_x + 2.70(10^3)N \Rightarrow D_x = \underline{\underline{-2.70(10^3)N}}$$

$$\Sigma F_y = \underline{\underline{0 = D_y}}$$

$$\Sigma F_z = \underline{\underline{0 = D_z}}$$

$$\vec{r}_{DE} = 1.05m\,\hat{j} + 3.575m\,\hat{k}$$

$$\vec{F}_{DE} = 2.70(10^3)N\,\hat{i}$$

$$\Sigma \vec{M}_{@D} = \vec{0} = \vec{M}_D + \vec{r}_{DE} \times \vec{F}_E$$

$$= \vec{M}_D + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1.05m & 3.575m \\ 2.70(10^3)N & 0 & 0 \end{vmatrix}$$

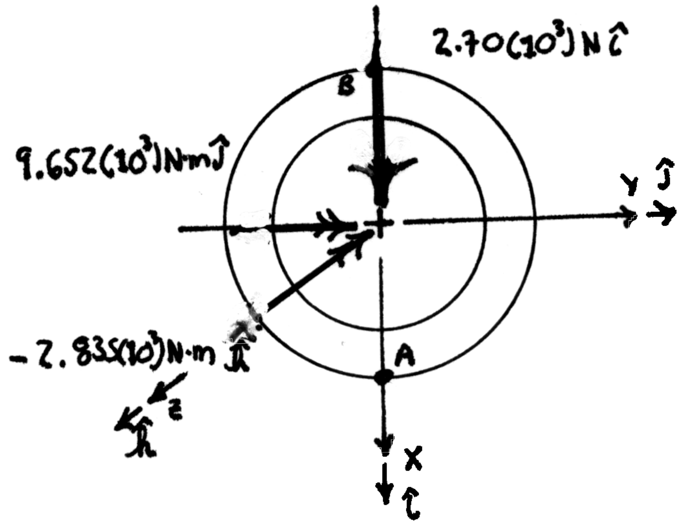
$$= -[-(3.575m)\cdot(2.70\times10^3 N)]\,\hat{j} + [-(1.05m)\cdot(2.70\times10^3 N)]\,\hat{k} + \vec{M}_D$$

$$= 9.652\times10^3 N\cdot m\,\hat{j} - 2.835\times10^3 N\cdot m\,\hat{k} + \vec{M}_D$$

$$\Rightarrow M_{Dx} = \underline{\underline{0}}$$

$$M_{Dy} = \underline{\underline{-9.652\times10^3 N\cdot m}}$$

$$M_{Dz} = \underline{\underline{2.835\times10^3 N\cdot m}}$$



LOADING IN THE PIPE AT THE BASE

THE MAXIMUM TENSILE NORMAL WILL OCCUR AT "B" AND THE MAXIMUM COMPRESSION WILL BE AT "A".

AT "B"

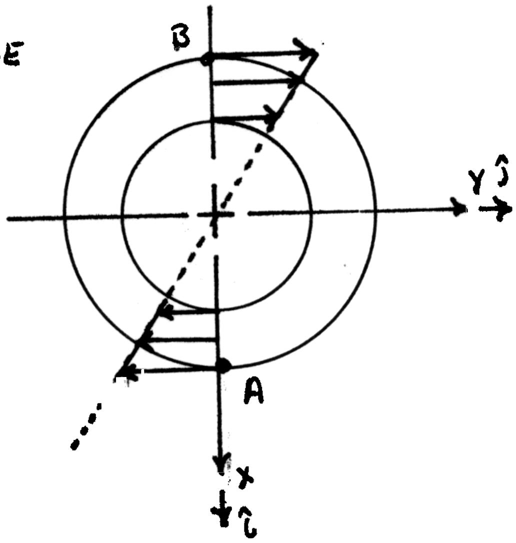
$$\sigma_z^{(B)} = -\frac{M_y \cdot x}{I_{yy}} = \frac{-(9.652\times10^3 N\cdot m)\cdot(-.05m)}{\frac{\pi}{64}[(0.1m)^4 - (.08m)^4]}$$

$$= 166.5(10^6)\frac{N}{m^2} = \boxed{166.5\,MPa}$$

$$\sigma_z^{(A)} = -\frac{M_y \cdot x}{I_{yy}} = -\frac{(9.652\times10^3 N\cdot m)\cdot(.05m)}{\frac{\pi}{64}[(0.1m)^4 - (0.8m)^4]}$$

$$-166.5(10^6)\frac{N}{m^2} = \boxed{-166.5\,MPa}$$

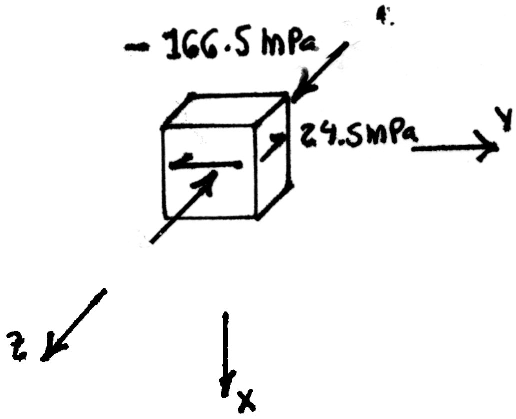
AT "A" ∴ THE ONLY SHEAR IS DUE TO THE TORQUE

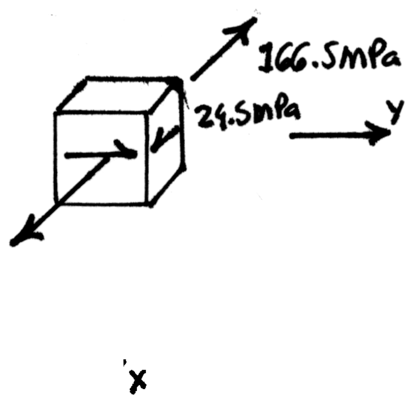


$= \boxed{-24.46\ MPa}$

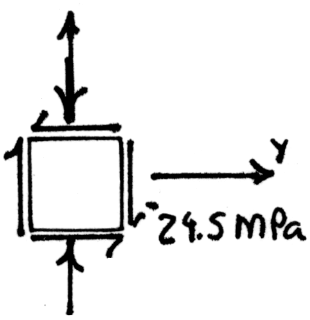
$$\tau_{zy}^{(T)} = -\frac{T\cdot r}{J} = -\frac{(-2.835\times 10^{3} N\cdot m)\cdot(0.05m)}{\frac{\pi}{32}\left[(0.1m)^4-(0.08m)^4\right]}$$

$= \boxed{24.46\ MPa}$

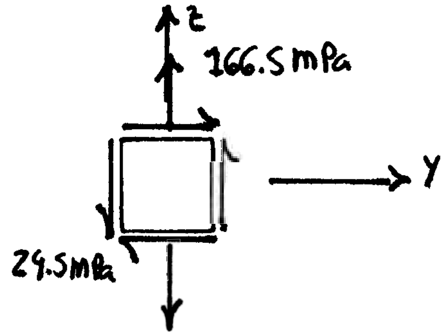




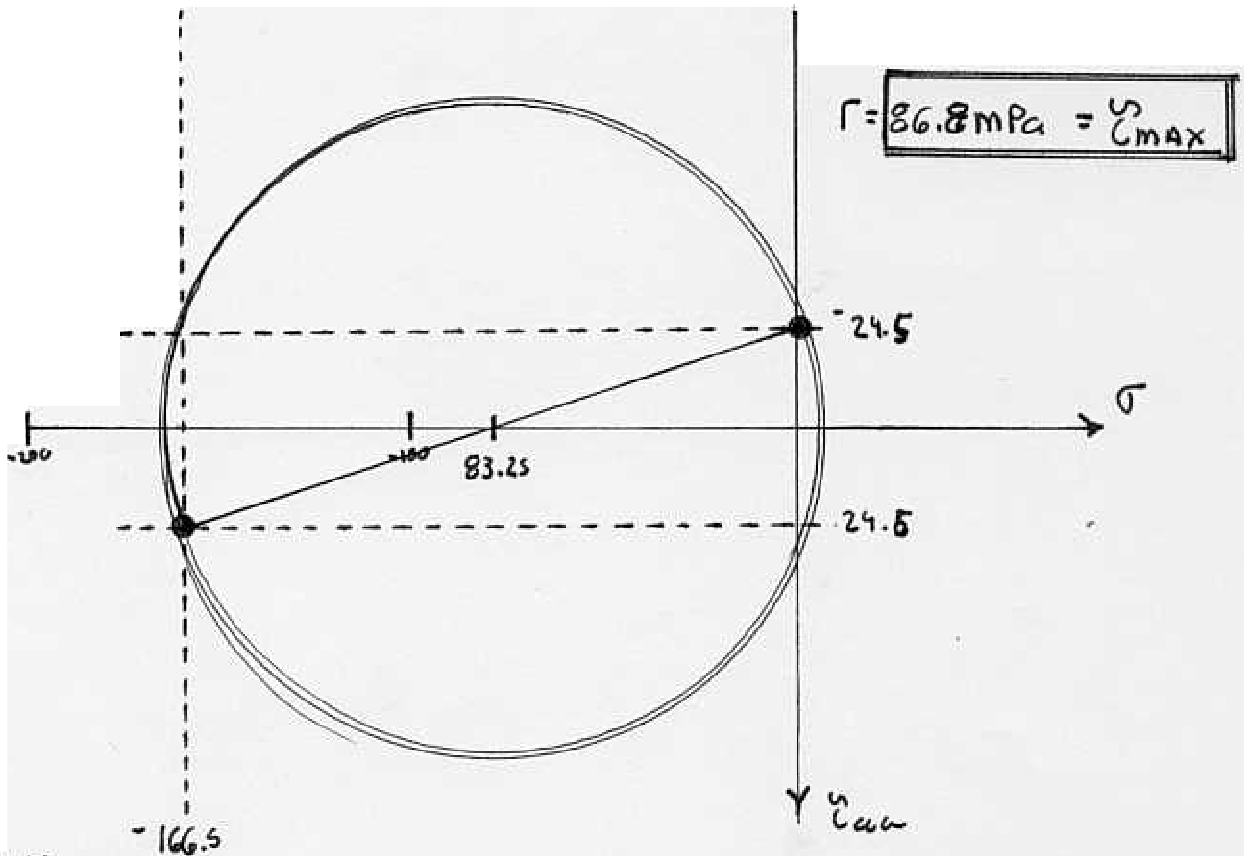
POINT A



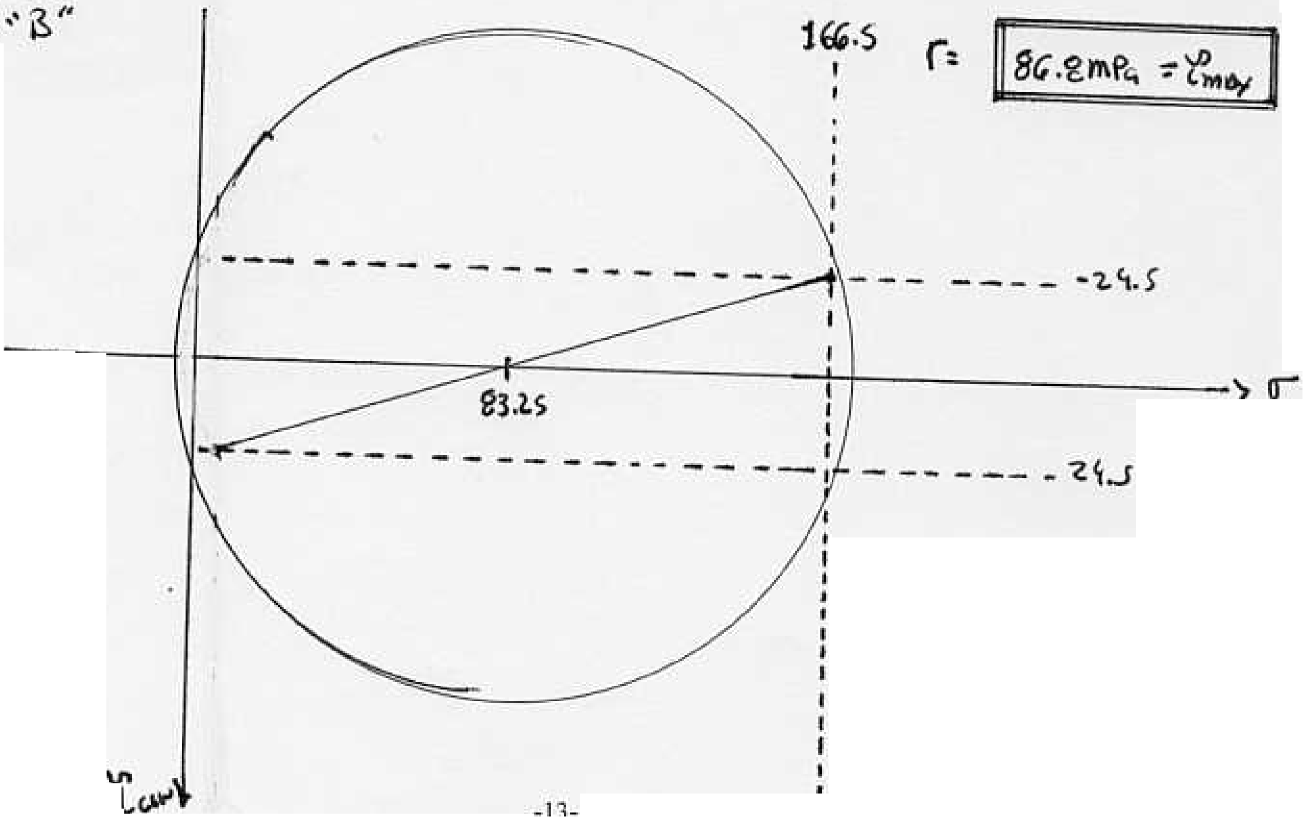
POINT B



MOHR'S CIRCLE FOR POINT "A"



MOHR'S CIRCLE FOR POINT "B"



THE STATES OF STRESS AT POINTS "C" AND "D" RESULT FROM A COMBINATION OF THE SHEARING FORCE AND THE TORQUE APPLIED TO THE CROSS-SECTION. THE BENDING MOMENT DOES NOT ENTER THE PROBLEM BECAUSE "C" AND "D" ARE ON THE NEUTRAL AXIS.

$$= \frac{V \cdot Q}{I \cdot t} + \frac{T \cdot r}{J}$$

FOR THIS

$$.424(.05m)\cdot(.5)\cdot\pi\cdot(.05m)^2 - .424\cdot(.04m)\cdot(.5)\cdot\pi\cdot(.04m)^2$$

$$)\,m^3$$

$$I = \frac{\pi}{64}\left[(.1m)^4 - (.08m)^4\right] = 2.898(10^{-6})m^4$$

$$J = 2 * I = 5.796(10^{-6})m^4$$

THUS AT "C"

$$\frac{2.70(10^3)N \cdot 40.63(10^{-6})m^3}{2.898(10^{-6})m^4 \cdot 2 \cdot (.01m)} + \frac{(-2.835\times10^3)N\cdot m \cdot .05m}{5.796(10^{-6})m^4}$$

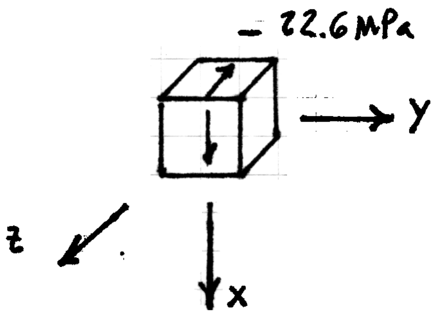
$$= 1.893(10^6)\frac{N}{m^2} - 24.45(10^6)\frac{N}{m^2} = -22.56(10^6)\frac{N}{m^2}$$

$$= -22.56\ MPa$$

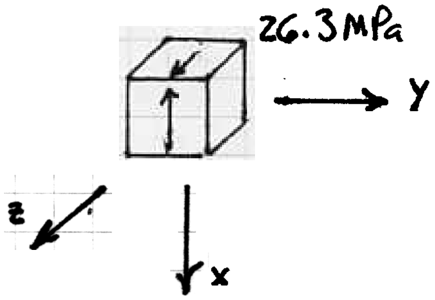
AT "D"

$$\tau_{zx} = 1.893(10^6)\frac{N}{m^2} + 24.45(10^6)\frac{N}{m^2} = 26.34(10^6)\frac{N}{m^2}$$

$$= 26.34\ MPa$$



POINT C



POINT D