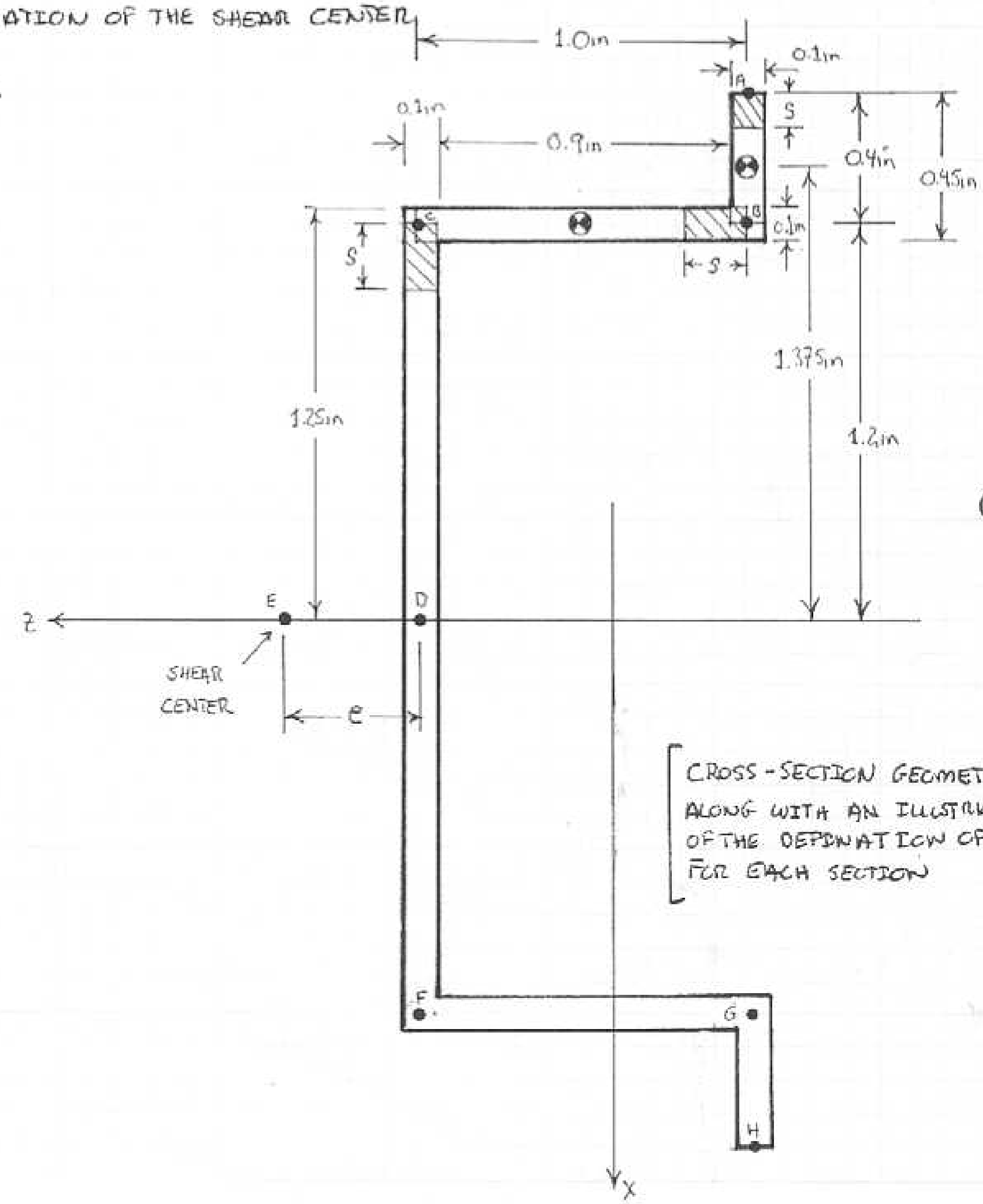
PROBLEM 5-29 | DETERMINE THE LOCATION OF THE SHEAR CENTER e OF THE SECTION SHOWN. ALL DIMENSIONS ARE IN INCHES AND, WHERE APPROPRIATE, ARE FROM THE CENTERS OF THE 0.1in THICK WALLS.

GIVEN:
1. CROSS ~~CHE~~ SECTION SHOWN
2. A LOAD THAT IS CREATING A SHEARING FORCE V

ASSUMPTIONS:
1. LINEAR ELASTIC MATERIAL RESPONSE
2. SMALL DEFORMATIONS

FIND:
1. LOCATION OF THE SHEAR CENTER

FIGURE:



CROSS-SECTION GEOMETRY ALONG WITH AN ILLUSTRATION OF THE DEFINATION OF s FOR EACH SECTION

SOLUTION:

THE SHEAR STRESS NEEDS TO BE CALCULATED IN EACH SECTION AND THEN INTEGRATED IN ORDER TO DETERMINE THE SHEAR FORCE IN EACH SECTION. THE SHEAR FORCE IS GIVEN BY

$$\tau = \frac{\forall Q}{I \cdot t} \qquad (1)$$

THE MOMENT OF INERTIA I IS FOR THE ENTIRE SECTION AND IS GIVEN BY

$$I = \frac{1}{12}(0.1_{in})(2.125_{in})^3 + 2\left[\frac{1}{12}(0.9_{in})(0.1_{in})^3 + (0.9)(0.1_{in})(1.2_{in})^2\right] + 2\left[\frac{1}{12}(0.1_{in})(0.45_{in})^3 + (0.1_{in})(0.45_{in})(1.375_{in})^2\right]$$

$$= \underline{\underline{0.5612\ in^4}} \qquad (2)$$

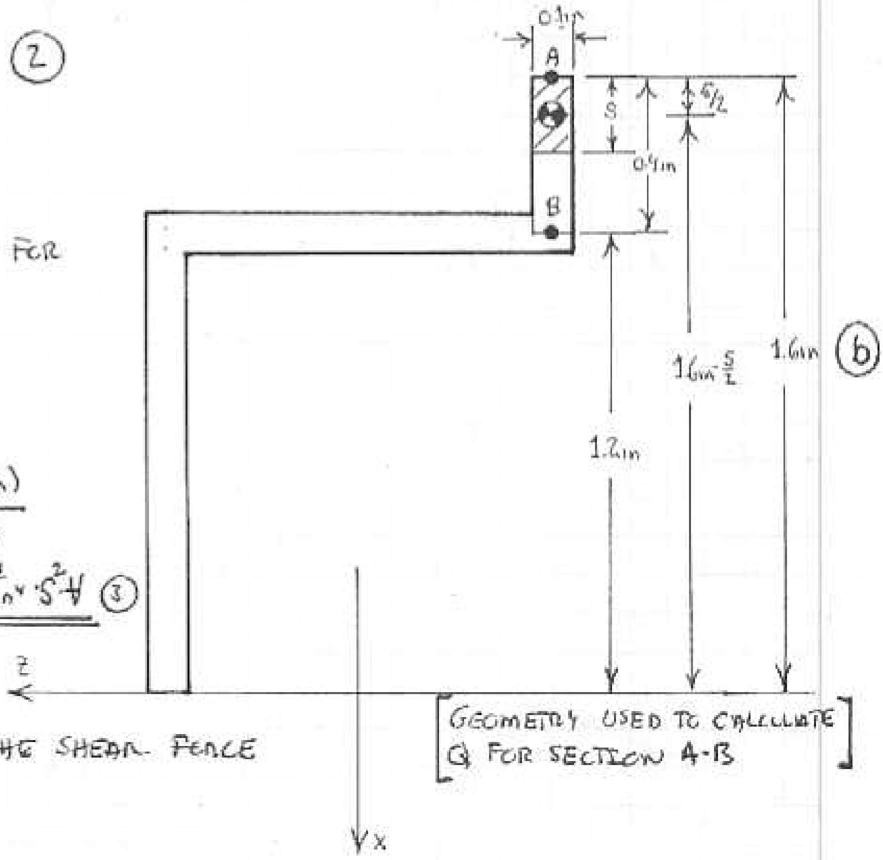
SECTION A-B

CALCULATING AN EXPRESSION FOR THE SHEAR STRESS FROM A TO B

$$\tau_{xy} = \frac{\forall \cdot Q}{I t} = \frac{\forall \cdot \bar{x} A}{I t}$$

$$= \frac{\forall \cdot (1.6_{in} - \frac{s}{2})(s)(0.1_{in})}{(0.5612\ in^4)(0.1_{in})}$$

$$= \underline{\underline{2.851\ \tfrac{1}{in^3} \cdot s \cdot \forall - 0.8909\ \tfrac{1}{in^4} \cdot s^2 \forall}} \qquad (3)$$



GEOMETRY USED TO CALCULATE Q FOR SECTION A-B

THE SHEAR STRESS IS NOW INTEGRATED OVER THE AREA OF SECTION A-B TO DETERMINE THE SHEAR FORCE

$$\forall_{AB} = \int_0^{0.4in} \tau_{xy} \cdot t \cdot ds$$

$$= \int_0^{0.4in} \left(2.581\ \tfrac{1}{in^3} \cdot s\forall - 0.8909\ \tfrac{1}{in^4} \cdot s^2 \cdot \forall\right)(0.1_{in})\, ds$$

$$= \forall \int_0^{0.4in} \left(0.2581\ \tfrac{1}{in^2} \cdot s - 0.08909\ \tfrac{1}{in^3} \cdot s^2\right) ds = \forall \cdot \left[0.2581\ \tfrac{1}{in^2} \cdot \frac{s^2}{2} - 0.08909\ \tfrac{1}{in^3} \cdot \frac{s^3}{3}\right]_0^{0.4in}$$

$$= \underline{\underline{0.018747 \cdot \forall}} \qquad (4)$$

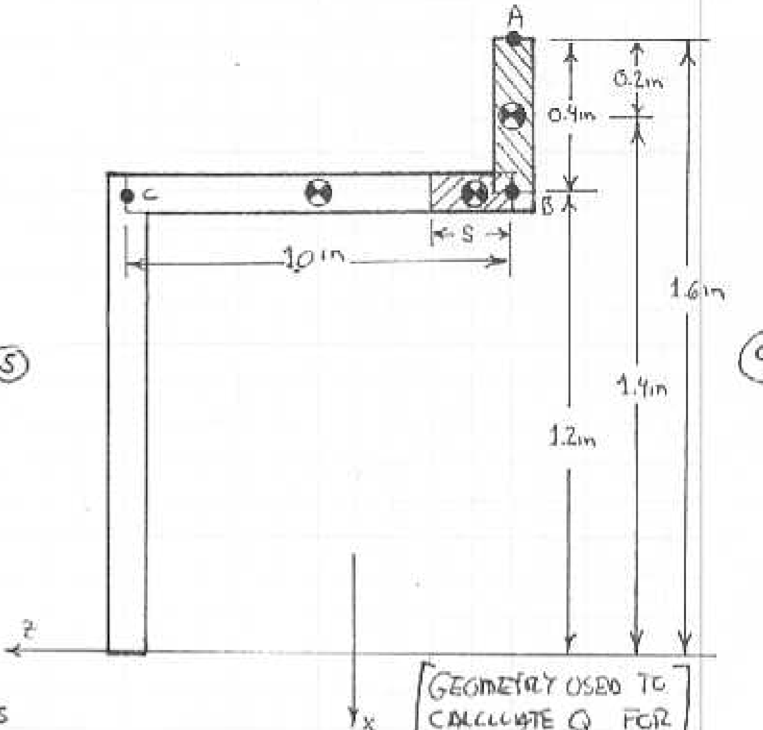
SECTION B-C

CALCULATING AN EXPRESSION FOR THE SHEAR STRESS FROM B TO C

$$\tau_{yz} = \frac{VQ}{It} = \frac{V\cdot(\Sigma \bar{x}A)}{It}$$

$$= \frac{V\cdot[(1.4\,in)(0.1\,in)(0.4\,in) + (1.2\,in)(0.1\,in)\,s]}{(0.5612\,in^4)(0.1\,in)}$$

$$= V\cdot\left[0.9979\,\tfrac{1}{in^2} + 2.138\,\tfrac{1}{in^3}\cdot s\right] \quad (5)$$



[GEOMETRY USED TO CALCULATE Q FOR SECTION B-C]

THE SHEAR STRESS IS NOW INTEGRATED OVER THE AREA OF SECTION B-C TO DETERMINE THE SHEAR FORCE IN THIS SECTION

$$V_{BC} = \int \tau_{yz}\,dA = \int \tau_{yz}\cdot t\cdot ds$$

$$= \int_0^{1.0} V\left[0.9979\,\tfrac{1}{in^2} + 2.138\,\tfrac{1}{in^3}\cdot s\right]\cdot(0.1\,in)\,ds$$

$$= \int_0^{1.0} V\cdot\left[0.09979\,\tfrac{1}{in} + 0.2138\,\tfrac{1}{in^2}\cdot s\right]ds = V\cdot\left[0.09979\,\tfrac{1}{in}\cdot s + 0.2138\,\tfrac{1}{in^2}\cdot\tfrac{s^2}{2}\right]_0^{1.0\,in}$$

$$= 0.2067\cdot V \quad (6)$$

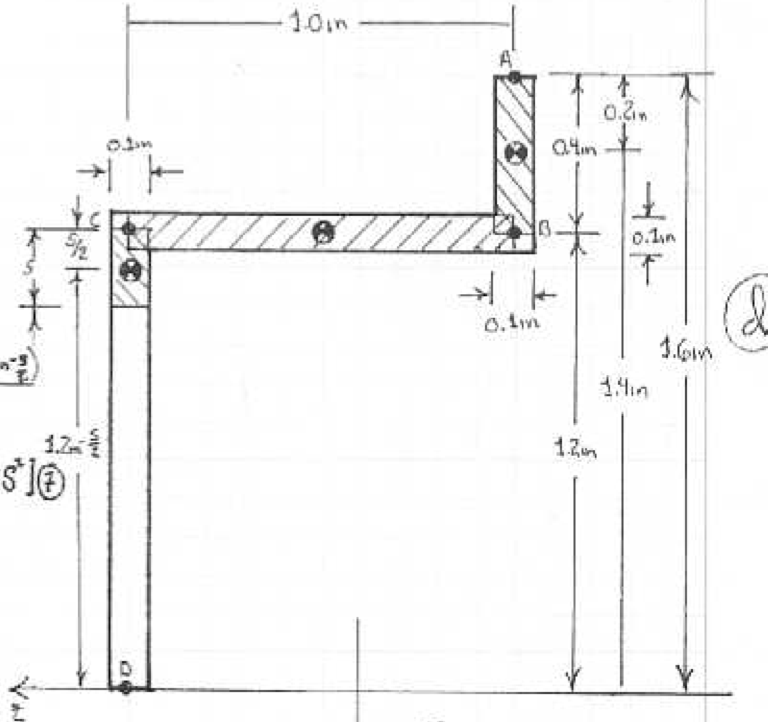
SECTION C-D

CALCULATING AN EXPRESSION FOR THE SHEAR STRESS FROM C-D

$$\tau_{xy} = \frac{V\cdot Q}{I\cdot t} = \frac{V\cdot(\Sigma\bar{x}A)}{I\cdot t}$$

$$= \frac{V\cdot\left[(0.1\,in)(0.4\,in)(1.4\,in) + (0.1\,in)(1.0\,in)(1.2\,in) + (0.1\,in)\cdot s\cdot(1.2\,in - \tfrac{s}{2})\right]}{0.5612\,in^4\cdot 0.1\,in}$$

$$= V\cdot\left[3.136\,\tfrac{1}{in^2} + 2.138\,\tfrac{1}{in^3}\cdot s - 0.8909\,\tfrac{1}{in^4}\,s^2\right] \quad (7)$$



[GEOMETRY USED TO CALCULATE Q FOR SECTION C-D]

THE SHEAR STRESS IS NOW INTEGRATED OVER THE AREA OF SECTION C-D TO DETERMINE THE SHEAR FORCE IN THIS SECTION.

$$V_{CD} = \int \tau_{xy}\cdot dA = \int \tau_{xy}\cdot t\cdot ds$$

$$= \int_0^{1.2\,in} V\cdot\left[3.136\,\tfrac{1}{in^2} + 2.138\,\tfrac{1}{in^3}\cdot s - 0.8909\,\tfrac{1}{in^4}\cdot s^2\right]\cdot(0.1\,in)\cdot ds$$

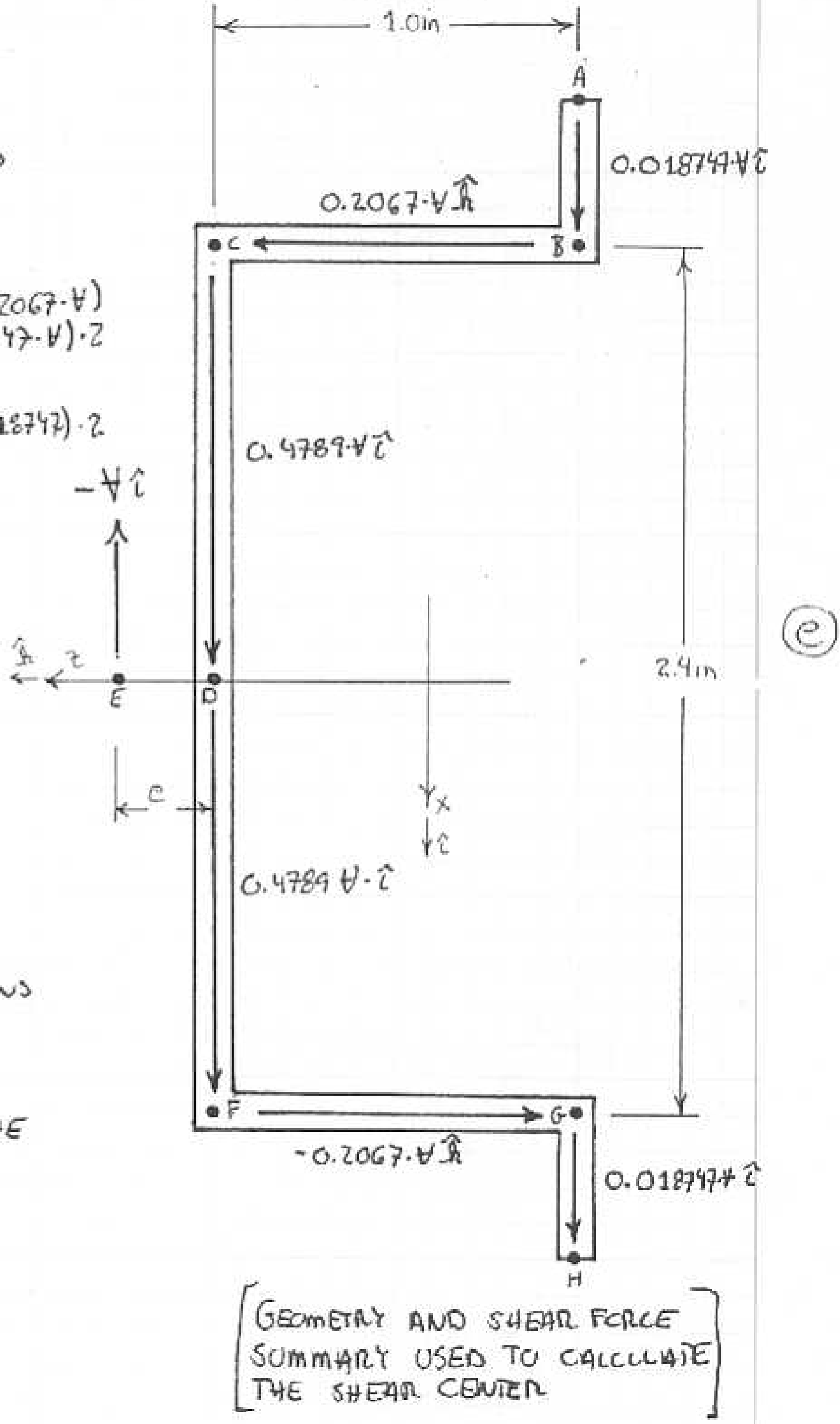
$$= V\cdot\int_0^{1.2\,in}\left[0.3136\,\tfrac{1}{in} + 0.2138\,\tfrac{1}{in^2}\cdot s - 0.08909\,\tfrac{1}{in^3}\cdot s^2\right]\cdot ds$$

$$= V\cdot\left[0.3136\,\tfrac{1}{in}\cdot s + 0.2138\,\tfrac{1}{in^2}\cdot\tfrac{s^2}{2} - 0.08909\,\tfrac{1}{in^3}\cdot\tfrac{s^3}{3}\right]_0^{1.2\,in} = 0.4789\cdot V \quad (8)$$

THE SHEAR CENTER LOCATION WITH RESPECT TO POINT CAN NOW BE CALCULATED USING THE SHEAR FORCES CALCULATED AND GEOMETRY SUMMERIZED IN (e)

$$\sum M_{y@D} = 0 = -e \cdot V + (2.4in)(0.2067 \cdot V) - (1.0in)(0.018747 \cdot V) \cdot 2$$

$$e = (2.4in)(0.2067) - (1.0in)(0.018747) \cdot 2 = \boxed{0.4589 in}$$



Summary:

IN THE SOLUTION TO THE PROBLEM SYMMETRY IS USED TO EVALUATE THE SHEAR STRESS AND SHEAR FORCE IN SECTIONS D-F, F-G, AND G-H. THE LOCATION OF THE SHEAR CENTER DID NOT REQUIRE THE CALCULATION OF THE SHEAR FORCE IN SECTIONS C-D AND D-F. THESE FORCES ARE USED TO VERIFY THAT THE NET FORCE IN THIS CROSS-SECTION IS V ACTING IN THE POSITIVE X-DIRECTION.