NAME: SOLUTION

PROBLEM 1: A beam of length 4a is shown on the next page. Young's modulus for the beam is E and the moment of inertia for the beam is I. A vertical load of P is applied at point B (y=a) and a force of –P at point D (y=3a). A couple of magnitude 2Pa is applied at point C (y=2a). Points A and E are supported by pin joints.

1a. Determine the reactions at A and E, and complete the free body diagram on the next page.

1b. Draw the shear, bending moment, curvature, and displacement diagrams for this beam.

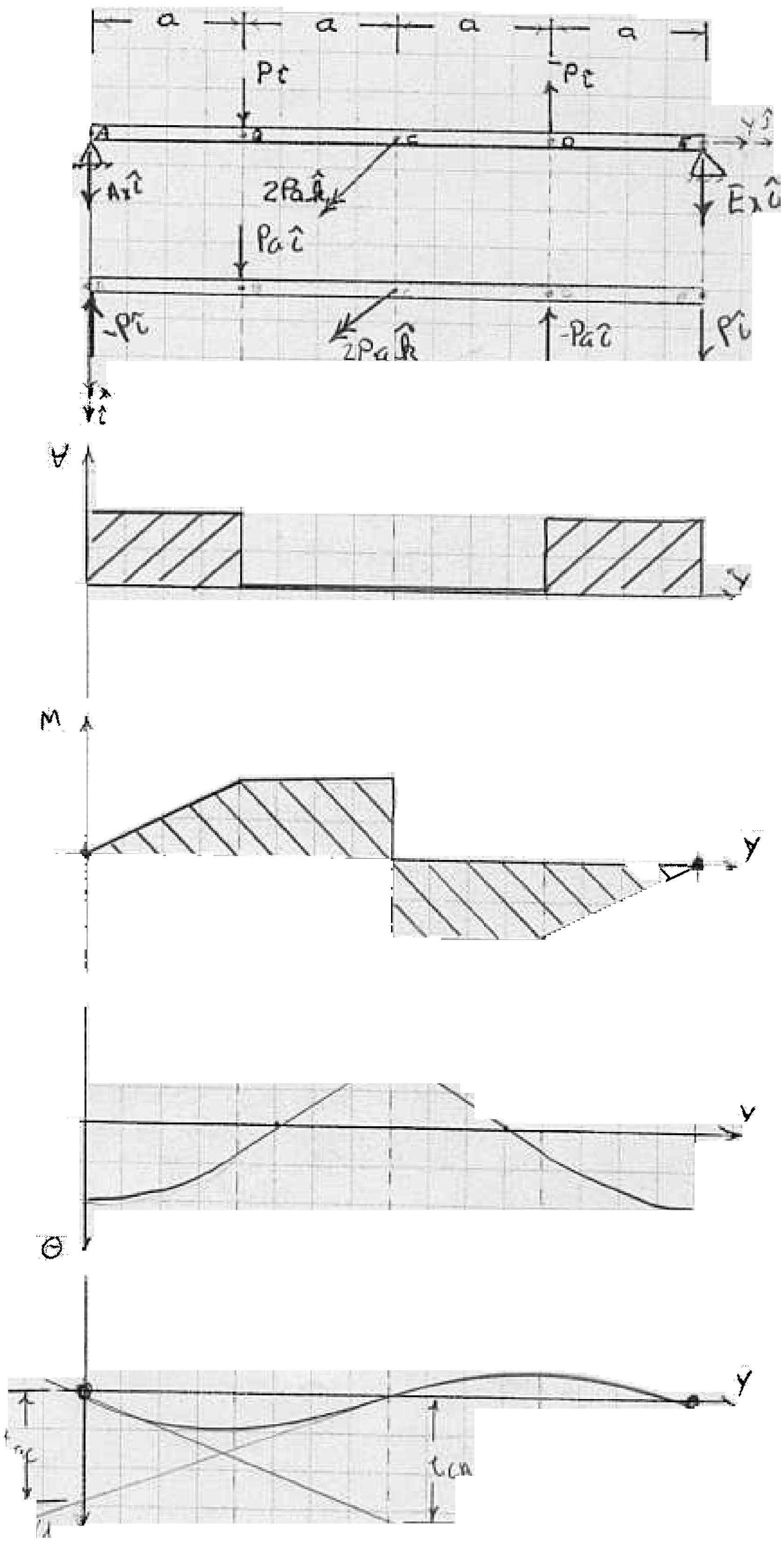
$$\sum F_x = 0 = A_x + E_x + P - P = 0 \Rightarrow \underline{\underline{A_x + E_x = 0}} \quad (1)$$

$$\sum M_z|_{@A} = 0 = -P \cdot a + 2 \cdot P \cdot a + 3 \cdot P \cdot a - 4 \cdot a \cdot E_x$$

$$E_x = \frac{4 \cdot P \cdot a}{4a} = \boxed{P} \quad (2)$$

SUBSTITUTING (2) INTO (1)

$$\boxed{A_x = -P}$$


a
a
a
a
Pî
-Pî
Axî
2Pak̂
Exî
Paî
-Pî
2Pak̂
-Paî
Pî
V
M
Y
Y
θ
Y
t_CA

1c. Determine the deflection of the beam at point B.

$$q(y) = -P\langle y-0\rangle_{-1} + P\langle y-a\rangle_{-1} + 2\cdot P\cdot a\langle y-2a\rangle_{-2} - P\langle y-3a\rangle_{-} + P\langle y-4a\rangle_{-}$$

$$V(y) = P\langle y-0\rangle^{0} - P\langle y-a\rangle^{0} - 2\cdot P\cdot a\langle y-2a\rangle_{-1} + P\langle y-3a\rangle^{0} - P\langle y-4a\rangle^{0}$$

$$M(y) = P\langle y-0\rangle^{1} - P\langle y-a\rangle^{1} - 2P\cdot a\langle y-2a\rangle^{0} + P\langle y-3a\rangle^{1} - P\langle y-4a\rangle^{1}$$

$$\Theta(y) = \frac{1}{EI}\left[-\frac{P}{2}\langle y-0\rangle^{2} + \frac{P}{2}\langle y-a\rangle^{2} + 2\cdot P\cdot a\langle y-2a\rangle^{1} - \frac{P}{2}\langle y-3a\rangle^{2} + \frac{P}{2}\langle y-4a\rangle^{2} + C_1\right]$$

$$u(y) = \frac{1}{EI}\left[-\frac{P}{6}\langle y-0\rangle^{3} + \frac{P}{6}\langle y-a\rangle^{3} + P\cdot a\langle y-2a\rangle^{2} - \frac{P}{6}\langle y-3a\rangle^{3} + \frac{P}{2}\langle y-4a\rangle^{3} + C_1\cdot y + C_2\right]$$

FROM THE BOUNDARY CONDITION AT $y=0$, $u(0)=0$

$$u(0) = \boxed{0 = C_2} \qquad (1)$$

FROM THE BOUNDARY CONDITION AT $y=4a$, $u(4a)=0$

$$u(4a) = 0 = \frac{1}{EI}\left[-\frac{P}{6}(4a)^3 + \frac{P}{6}(3a)^3 + Pa(2a)^2 - \frac{P}{6}(a)^3 + \frac{P}{2}(0)^3 + C_1\cdot 4a\right]$$

$$0 = -\frac{64\cdot a^3\cdot P}{6} + \frac{27\cdot P\cdot a^3}{6} + \frac{6\cdot 4\cdot P\cdot a^3}{6} - \frac{P\cdot a^3}{6} + 0 + C_1\cdot 4a$$

$$\frac{14}{6}P\cdot a^3 + C_1\cdot 4a \Rightarrow C_1 = \frac{\frac{14}{6}P\cdot a^3}{4a} = \frac{14}{24}Pa^2 = \frac{7}{12}Pa^2$$

.5833

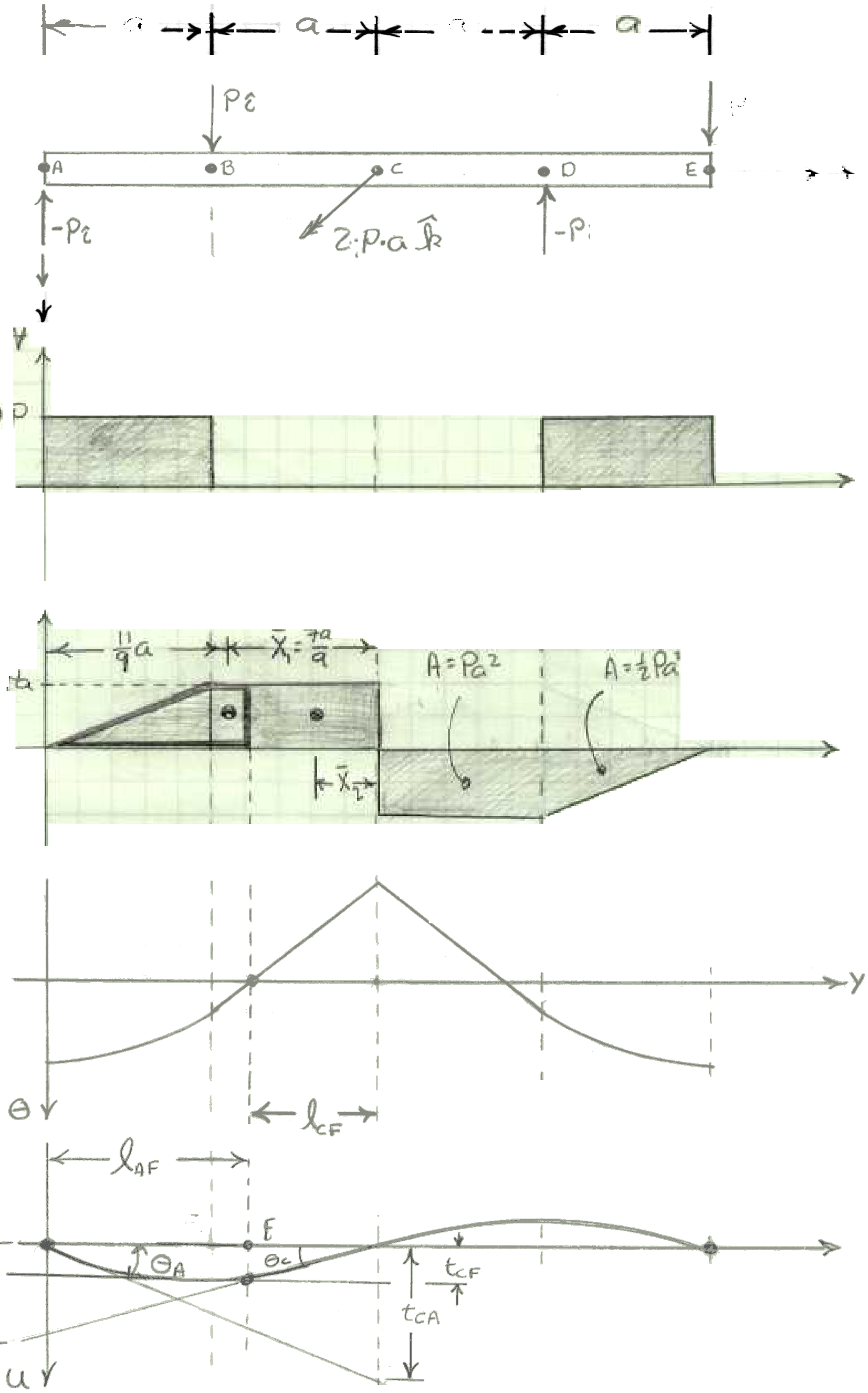
$$u(y) = \frac{1}{EI}\left[-\frac{P}{6}\langle y-0\rangle^{3} + \frac{P}{6}\langle y-a\rangle^{3} + P\cdot a\langle y-2a\rangle^{2} - \frac{P}{6}\langle y-3a\rangle^{3} + \frac{P}{2}\langle y-4a\rangle^{3} + \frac{7}{12}\cdot P a^2\cdot y\right]$$

$$u(a) = \frac{1}{EI}\left[-\frac{P}{6}(a^3) + \frac{7}{12}P\cdot a^3\right] = \frac{1}{EI}\left[-\frac{2P\cdot a^3}{12} + \frac{7}{12}Pa^3\right]$$

$$= \boxed{\frac{5}{12}\frac{P\cdot a^3}{EI}}$$

.4167

AXI DEFLEC BEAM Prob



X_1

$Pa \quad \frac{4a}{3} \quad Pa$

$Pa^2 \quad Pa$

$\frac{Pa \quad 3Pa}{6}$

Pa

Pa

Pa

$\frac{7}{3}a \quad \frac{}{9}a$ ①

$t \quad \frac{\left(\frac{1}{2}Pa^2 + Pa^2\right)}{E} \quad a$

$\frac{Pa}{E} \quad \frac{}{9}a$

$\frac{P}{EI}$ ②

$\frac{t_{CA}}{2a}$

$\frac{Pa}{EI}$

$7 \frac{Pa}{EI}$ ③

$\Delta\theta_{AF} \quad \theta_A$

$\frac{Pa + (a\, l_{CF})Pa}{EI}$

$\frac{Pa}{} \quad \frac{Pa}{} \quad Pa \cdot l_C \qquad \frac{Pa^2 - P \cdot a \cdot l_{CF}}{EI} \qquad 7\frac{Pa}{EI}$

$a\, l_{CF} \quad \frac{7}{12} \quad 12a \quad a \quad a\, l \rightarrow a \quad l_C$

$\underline{l_C \quad 1.\, a}$ (4) $\quad 9a \quad 67a \qquad 1.08$

$$t_{AC} = \frac{3}{2}\frac{Pa^2}{EI} \cdot \frac{11}{9}\cdot\frac{}{3} a = \underline{\underline{\frac{11}{6}\frac{Pa^3}{EI}}} \quad (5)$$

$$\Theta_C = \frac{t_{AC}}{2a} = \frac{\frac{11}{6}\frac{Pa^3}{EI}}{2a} = \underline{\underline{\frac{11}{12}\frac{Pa^2}{EI}}} \quad (6)$$

$$\Delta\Theta_{CF} = \Theta_C = \frac{l_{CF} \cdot \cancel{P}a}{\cancel{EI}} = \frac{11}{12}\frac{\cancel{P}a^2}{\cancel{EI}} \Rightarrow \underline{\underline{l_{CF} = \frac{11}{12}a}} \quad (7)$$

$$\underline{\underline{\bar{X}_2 = \frac{11}{24} \cdot a}} \quad (8)$$

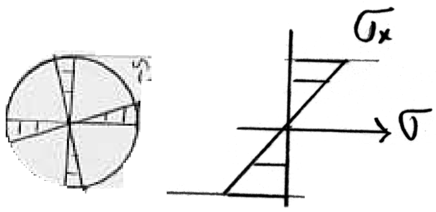
$$t_{CF} = \frac{1}{EI} \cdot \frac{11}{12}a \cdot P \cdot a \cdot \frac{11}{24}a = \underline{\underline{\frac{121}{288}\frac{Pa^3}{EI}}} = \underline{\underline{.4201\frac{P \cdot a^3}{E \cdot I}}} \quad (8)$$

PROBLEM 2: A machined steel shaft has a diameter of 1.0 inches. This shaft is subjected to a purely reversing moment of 1000 lb-in and a constant torque of 1200 lb-in. the ultimate strength of the steel is 100,000lb/in^2 and the yield strength is 80,000lb/in^2.

2a. Construct the modified Goodman diagram for the material in the as manufactured shaft.

$$\frac{Mc}{I} \quad \frac{1000\ \text{lb-in} \cdot .5\,\text{in}}{\frac{\pi (1\,\text{in})^4}{64}} = \underline{\underline{10\ 186\,(10^3)\ \frac{\text{lb}}{\text{in}^2}}} \quad (1)$$

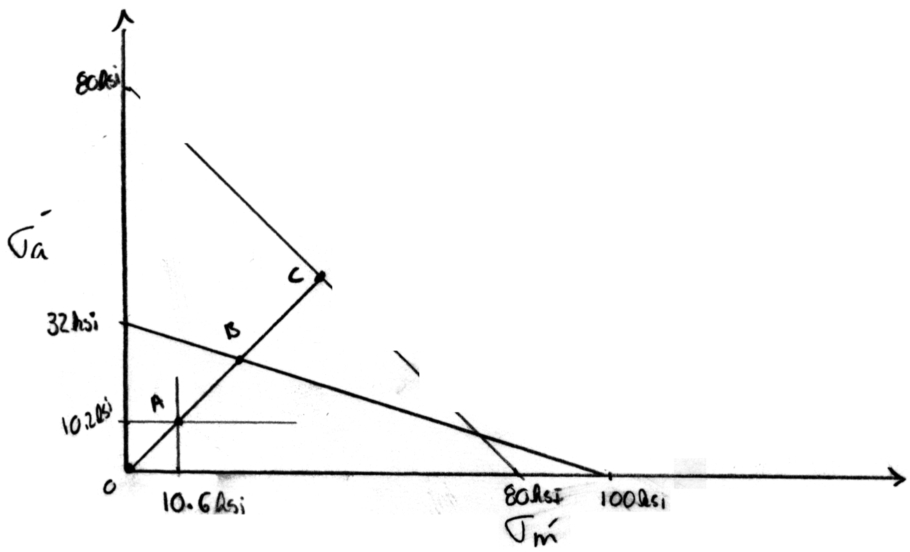
$$\frac{Tr}{J} = \frac{1200\ \text{in-lb} \cdot .5\,\text{in}}{\frac{\pi (1\,\text{in})^4}{32}} = \underline{6.112\,(10^3)\ \frac{\text{lb}}{\text{in}^2}} \quad (2)$$



$$\sigma_m' = \sqrt{3 \cdot (6.112 \times 10^3\ \text{lb/in}^2)} \quad \underline{10.586 \times 10^3\ \text{lb/in}^2} \quad (3)$$

$$\sigma_a' = \sqrt{(10.186 \times 10^3\ \tfrac{\text{lb}}{\text{in}^2})^2} = \underline{10.186 \times 10^3\ \text{lb/in}^2} \quad (4)$$

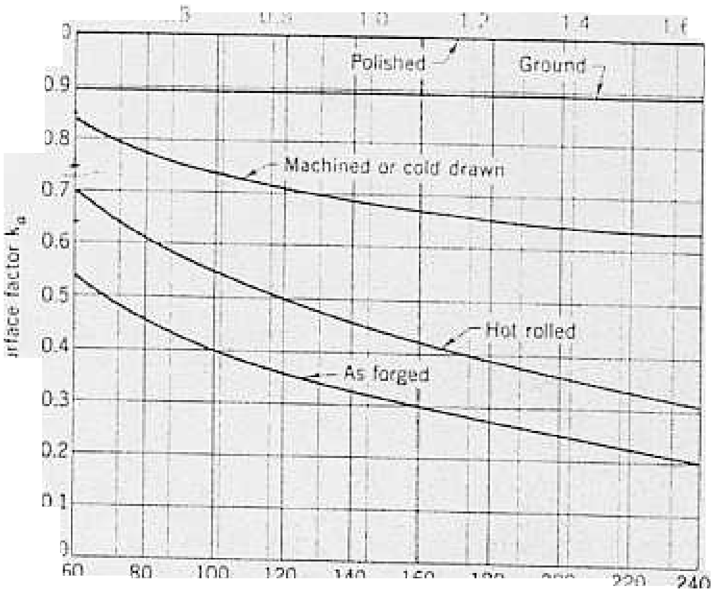
$$S_e = (.75)(.85)(.5)(100{,}000\ \text{lb/in}^2) = \underline{\underline{31.88\,(10^3)\ \text{lb/in}^2}}$$



2b. Determine the static and fatigue factor of safeties for this loading condition.

$$SF_f = \frac{OB}{OA} = \frac{33}{15} =$$

$$SF_s = \frac{OC}{OA} = \frac{57}{15} =$$



DISTORTIONAL ENERGY METHOD

$$\sigma_m' = \sqrt{\sigma_{1,m}^2 - \sigma_{1,m}\cdot\sigma_{2,m} + \sigma_{2,m}^2}$$

$$\sigma_a' = \sqrt{\sigma_{1,a}^2 - \sigma_{1,a}\cdot\sigma_{2,a} + \sigma_{2,a}^2}$$

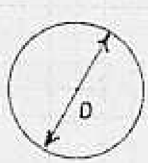
σ_x, τ_{xy} and $\sigma_y = \sigma_z = \tau_{yz} = \tau_{xz} = 0$

$$\sigma_m' = \sqrt{\sigma_{x,m}^2 + 3\cdot\tau_{xy,m}^2}$$

$$\sigma_a' = \sqrt{\sigma_{x,a}^2 + 3\cdot\tau_{xy,a}^2}$$

$I = \frac{\pi D^4}{64}$

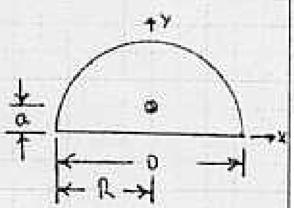
$J = \frac{\pi D^4}{32}$



$a = 0.4244\ R$

$I_x = 0.1098\ R^4$

$I_y = \frac{\pi}{8}\cdot R^4$



$S_e = k_a, k_b, k_c, k_d\cdot k_e\cdot k_f\cdot S_e'$

Size effect

$$k_b = \begin{cases} 1 & d \le 0.3\,in\ (7.6\,mm) \\ 0.85 & 0.3\,in < d \le 2.0\,in \\ 0.75 & d > 2.0\,in\ (50\,mm) \end{cases}$$

$k_c = 1$: Reliability

$k_d = 1$: Temperature effect

$k_e = 1$: stress con. factor

$k_f = 1$