MER311: Advanced Strength of Materials

LECTURE OUTLINE

- Strain Transformatins
- Mohr's Circle for Strain

Strain Tensor

$$[\varepsilon] = \begin{bmatrix} \varepsilon_x & \dfrac{\gamma_{xy}}{2} & \dfrac{\gamma_{xz}}{2} \\ \dfrac{\gamma_{yx}}{2} & \varepsilon_y & \dfrac{\gamma_{yz}}{2} \\ \dfrac{\gamma_{zx}}{2} & \dfrac{\gamma_{zy}}{2} & \varepsilon_z \end{bmatrix}$$

$$= \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \varepsilon_{ij}$$

$$\Rightarrow$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \dfrac{\gamma_{xz}}{2} \\ \dfrac{\gamma_{yz}}{2} \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xz} \\ \varepsilon_{yz} \\ \varepsilon_{xy} \end{Bmatrix}$$

Strain Transformations

$$T = \begin{bmatrix} n_{x',x} & n_{x',y} & n_{x',z} \\ n_{y',x} & n_{y',y} & n_{y',z} \\ n_{z',x} & n_{z',y} & n_{z',z} \end{bmatrix}$$

$$[\varepsilon]_{x'y'z'} = [T] \cdot [\varepsilon]_{xyz} \cdot [T]^T$$

Two Dimensional/Plane Strain Transformations

- General Transformation Equations

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cdot \cos 2\theta + \frac{\gamma_{xy}}{2} \cdot \sin 2\theta$$

$$\frac{\gamma_{x_1 y_2}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2} \cdot \sin 2\theta + \frac{\gamma_{xy}}{2} \cdot \sin 2\theta \cos 2\theta$$

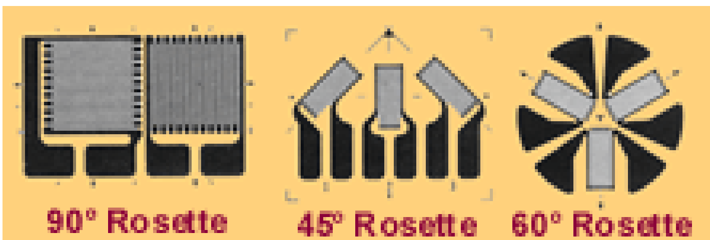
- Principal Strains

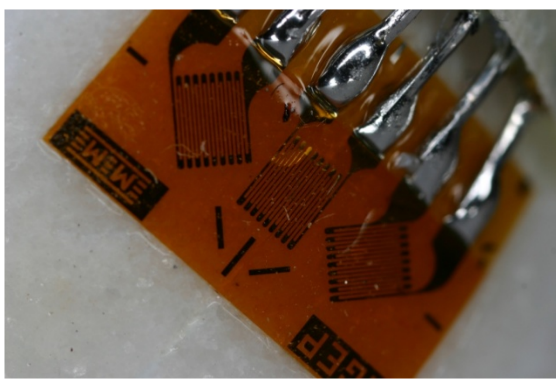
$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

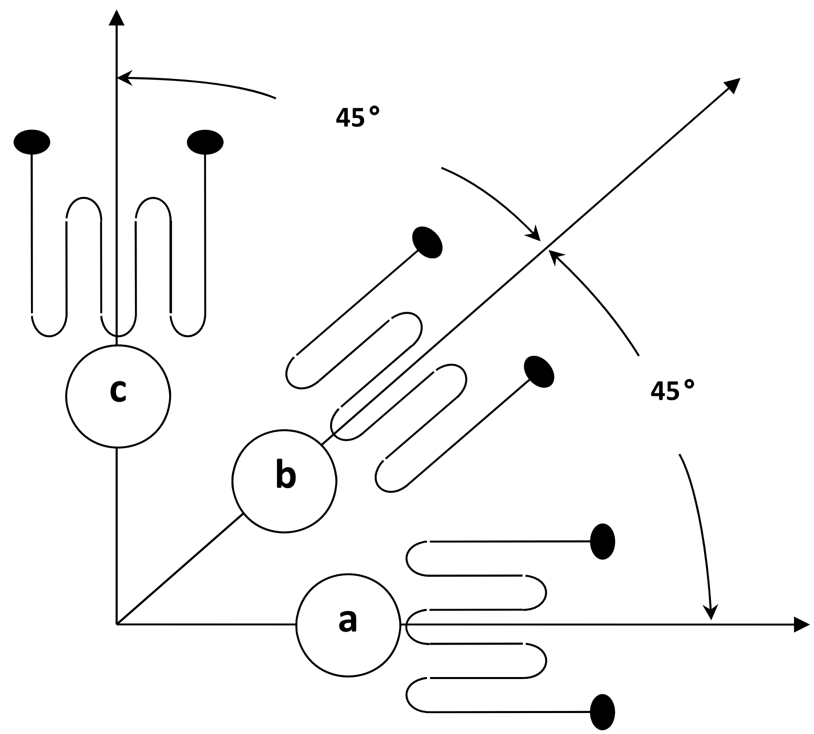
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\frac{\gamma_{\max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

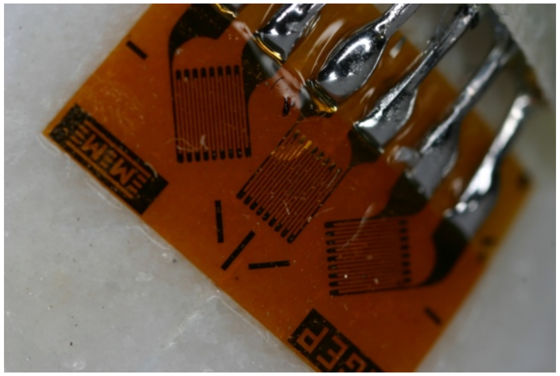
Typical Strain Gage Rosettes

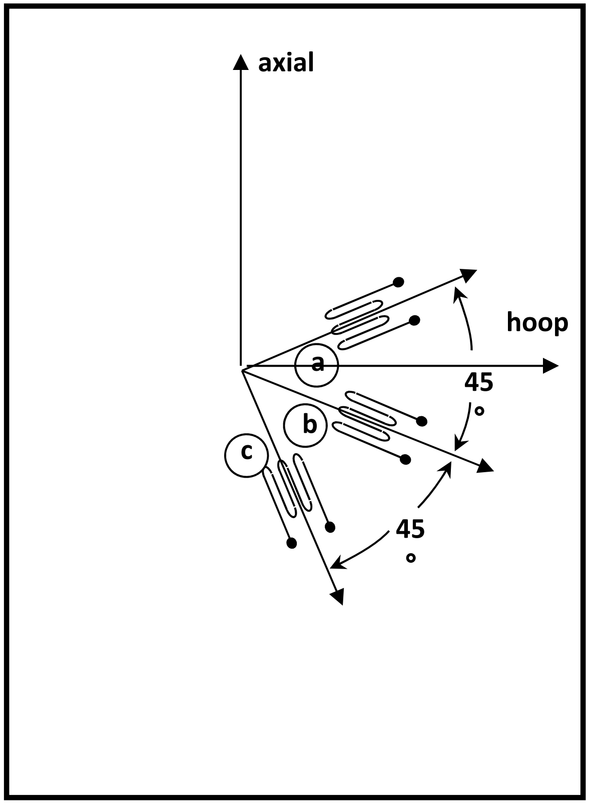






Typical Strain Gage Rosettes





$|\varepsilon_{a^*}| = \varepsilon_a = 1237(10^{-6})$ in/in $= 1237\mu\varepsilon$ (12)

$|\varepsilon_{b^*}| = \varepsilon_b = 1270(10^{-6})$ in/in $= 1270\mu\varepsilon$ (13)

$|\varepsilon_{c^*}| = \varepsilon_c = 402(10^{-6})$ in/in $= 402\mu\varepsilon$ (14)

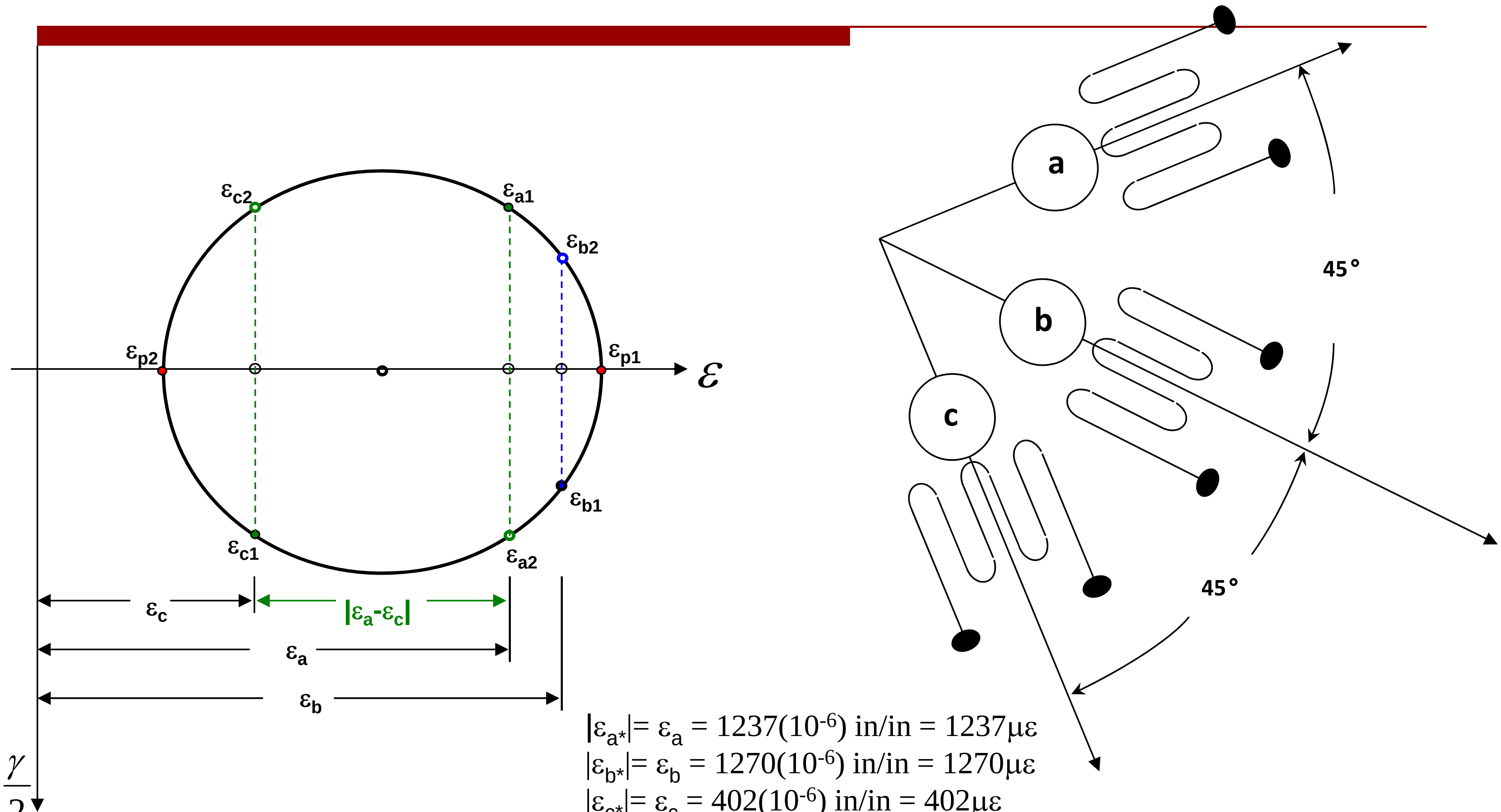
Transverse Sensitivity

$$\varepsilon_a = \frac{\hat{\varepsilon}_a \cdot (1 - \nu_0 \cdot K_a) - K_a \cdot \hat{\varepsilon}_c \cdot (1 - \nu_0 \cdot K_c)}{1 - K_a \cdot K_c} \quad (1)$$

$$\varepsilon_b = \frac{\hat{\varepsilon}_b \cdot (1 - \nu_0 \cdot K_b)}{1 - K_b} - \frac{K_b \cdot [\hat{\varepsilon}_a \cdot (1 - \nu_0 \cdot K_a) \cdot (1 - K_c) + \hat{\varepsilon}_c \cdot (1 - \nu_0 \cdot K_c) \cdot (1 - K_a)]}{(1 - K_a \cdot K_c) \cdot (1 - K_b)} \quad (2)$$

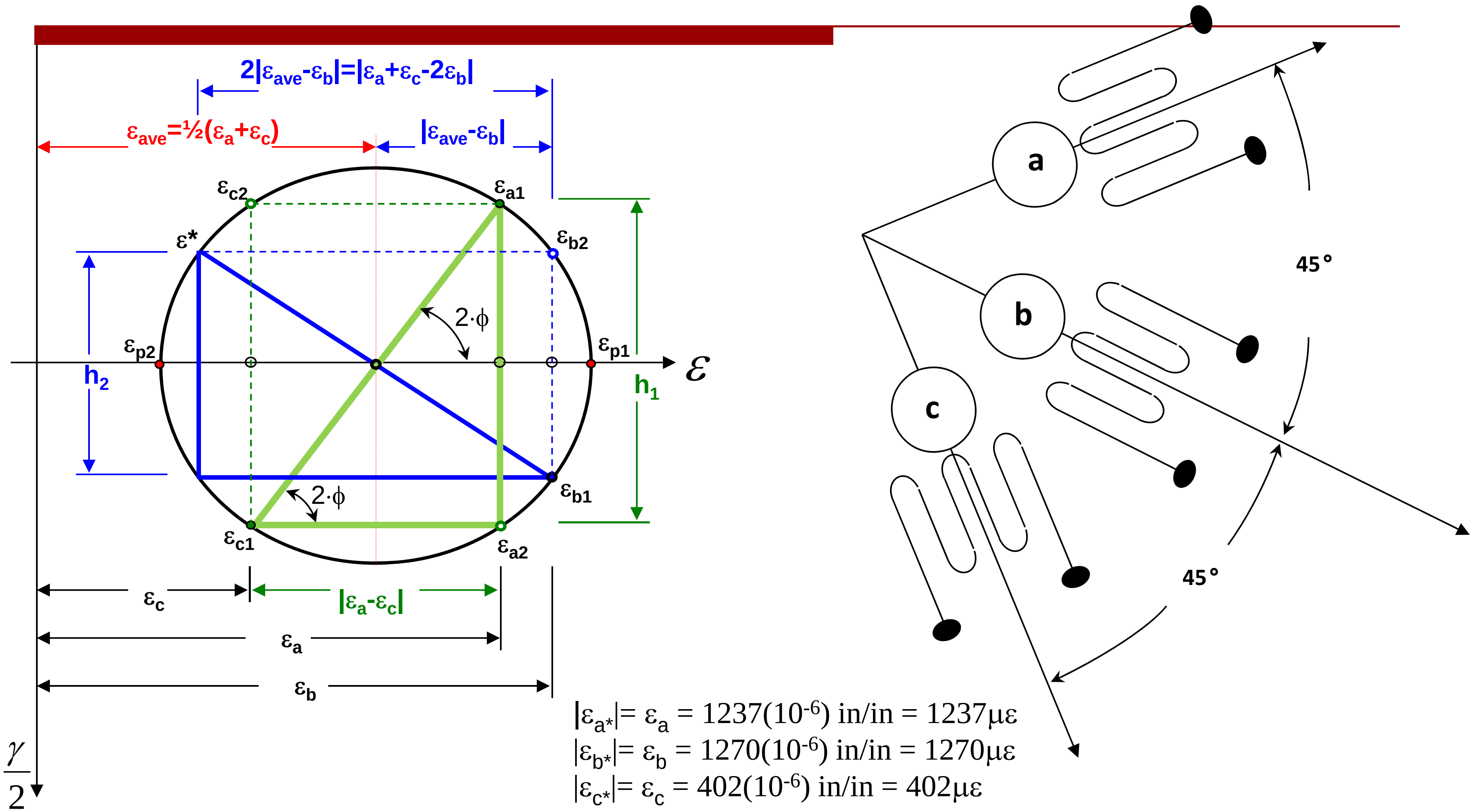
$$\varepsilon_c = \frac{\hat{\varepsilon}_c \cdot (1 - \nu_0 \cdot K_c) - K_c \cdot \hat{\varepsilon}_a \cdot (1 - \nu_0 \cdot K_a)}{1 - K_a \cdot K_c} \quad (3)$$

Mohr's Circle for Strain



$|\varepsilon_{a*}| = \varepsilon_a = 1237(10^{-6})$ in/in = 1237με
$|\varepsilon_{b*}| = \varepsilon_b = 1270(10^{-6})$ in/in = 1270με
$|\varepsilon_{c*}| = \varepsilon_c = 402(10^{-6})$ in/in = 402με

Mohr's Circle for Strain



$|\varepsilon_{a*}| = \varepsilon_a = 1237(10^{-6})$ in/in $= 1237\mu\varepsilon$

$|\varepsilon_{b*}| = \varepsilon_b = 1270(10^{-6})$ in/in $= 1270\mu\varepsilon$

$|\varepsilon_{c*}| = \varepsilon_c = 402(10^{-6})$ in/in $= 402\mu\varepsilon$