PROBLEM 8-9 | DESIGN A SINGLE-DWELL CAM TO MOVE A FOLLOWER 0 TO 2in IN 60°, FALL 2in IN 90°, AND DWELL FOR THE REMAINDER. THE TOTAL CYCLE MUST TAKE 2SEC. CHOOSE SUITABLE FUNCTIONS FOR RISE AND FALL TO MINIMIZE ACCELERATION. PLOT THE S-V-A-J DIAGRAMS.

GIVEN:

1. TOTAL CYCLE TIME 2 SECONDS.
2. RISE FROM 0 TO 2in IN 60° ($\pi/3$).
3. FALL FROM 2in TO 0 in 90° ($\pi/3$).
4. DWELL AT 0in FROM 150° TO 360°

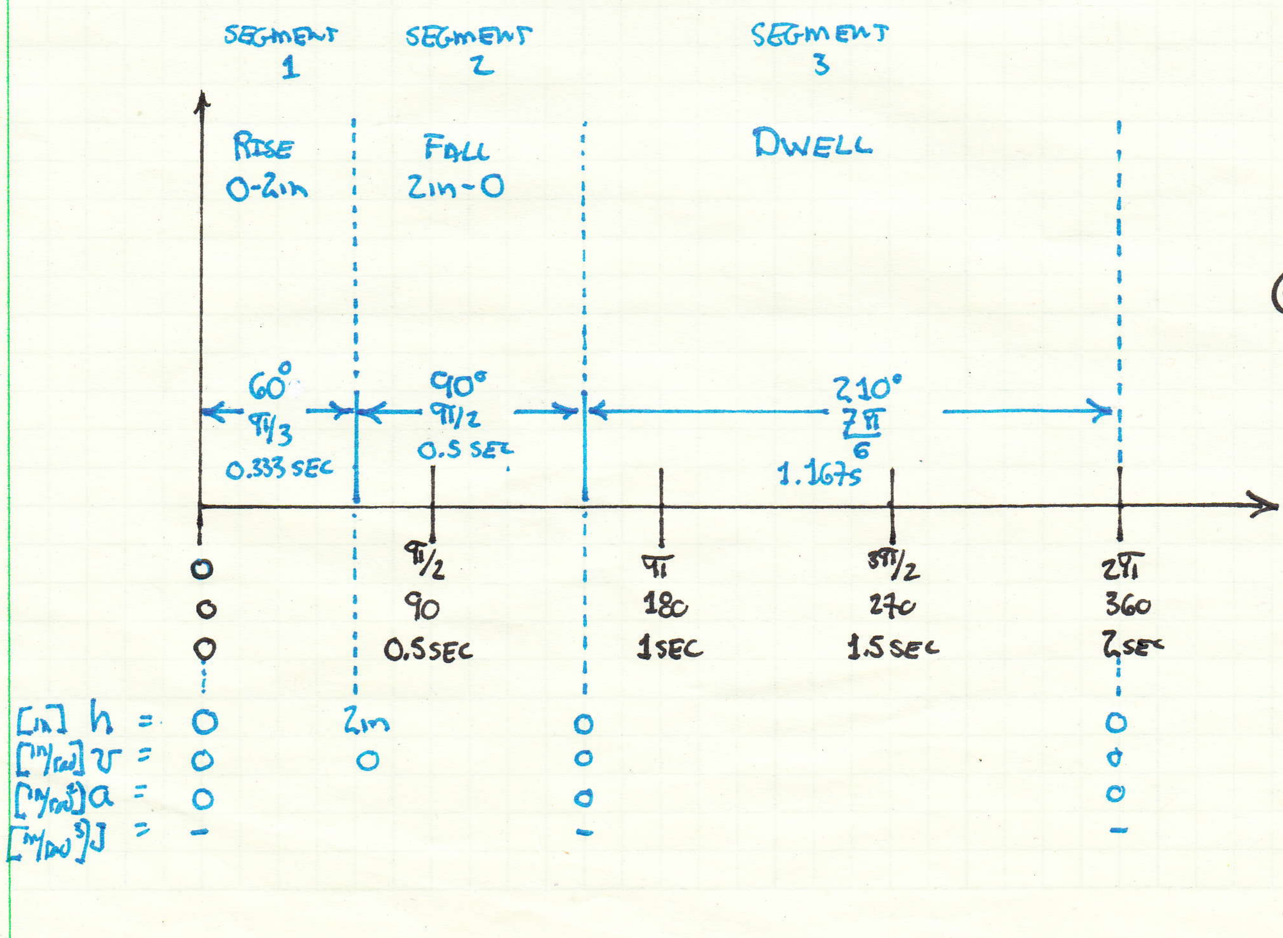
ASSUMPTIONS:

1. THE CAM DISPLACEMENT FUNCTION MUST BE CONTINUOUS THROUGH THE FIRST TWO DERIVATIVES FROM 0 TO 360° (0 TO 2π).
2. THE CAM ROTATES AT A CONSTANT ANGULAR VELOCITY.
3. THE FOLLOWER OFFSET AND DIAMETER ARE NOT CONSIDERED.

FIND:

1. DESIGN A SINGLE DWELL CAM THAT MINIMIZES ACCELERATION.

FIGURE:



SOLUTION:

FOR A CONSTANT ANGULAR VELOCITY CAM, GIVEN THAT A CYCLE IS 2π RADIANS AND THE CYCLE TIME IS 2s., THE ANGULAR VELOCITY IS GIVEN BY

$$\omega = \frac{2\pi \text{ rad}}{2 \text{ sec}} = \pi \text{ rad/s} \qquad (1)$$

FROM FIGURE @, IT IS SEEN THAT THREE SEGMENTS EXIST. IF THE SEGMENT LENGTHS WERE THE SAME A SIMPLE HARMONIC FUNCTION WOULD GIVE US THE LOWEST ACCELERATION; HOWEVER THE SEGMENTS HERE ARE NOT THE SAME LENGTH.

CONSIDER POLYNOMIAL FUNCTIONS FOR EACH OF THE THREE SEGMENTS

SEGMENT 1 $0 \le \theta \le 60°$, $0 \le \theta_1 \le 60°$, $0 \le \theta_1 \le \pi/3$, $\beta_1 = \pi/3$

SINCE THIS SEGMENT IS A SIMPLE RISE TO A FALL, A 3-4-5 POLYNOMIAL APPEARS APPROPRIATE. THE BOUNDARY CONDITIONS FOR THIS POLYNOMIAL ARE

$\theta_1 = 0$ $\qquad\qquad$ $\theta_1 = \beta_1$

$\theta_1/\beta_1 = 0$ $\qquad\qquad$ $\theta_1/\beta_1 = 1$

$S_1 = 0 \text{ in}$ (2) $\qquad$ $S_1 = 2 \text{ in}$ (5)

$S_1' = 0 \text{ in/rad}$ (3) $\qquad$ $S_1' = 0 \text{ in/rad}$ (6)

$S_1'' = 0 \text{ in/rad}^2$ (4) $\qquad$ $S_1'' = 0 \text{ in/rad}^2$ (7)

THE APPROPRIATE POLYNOMIAL FOR SIX BOUNDARY CONDITIONS IS

$$S = \cancelto{0}{C_0} + \cancelto{0}{C_1}\left(\frac{\theta_1}{\beta_1}\right) + \cancelto{0}{C_2}\left(\frac{\theta_1}{\beta_1}\right)^2 + C_3\left(\frac{\theta_1}{\beta_1}\right)^3 + C_4\left(\frac{\theta_1}{\beta_1}\right)^4 + C_5\left(\frac{\theta_1}{\beta_1}\right)^5 \qquad (8)$$

$$S' = \frac{\cancelto{0}{C_1}}{\beta_1} + \frac{2\cdot \cancelto{0}{C_2}}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right) + \frac{3\cdot C_3}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^2 + \frac{4 C_4}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^3 + \frac{5\cdot C_5}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^4 \qquad (9)$$

$$S'' = \frac{2\cdot \cancelto{0}{C_2}}{\beta_1^2} + \frac{6\cdot C_3}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right) + \frac{12\cdot C_4}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^2 + \frac{20\cdot C_5}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^3 \qquad (10)$$

$$S''' = \frac{6\cdot C_3}{\beta_1^3} + \frac{24\cdot C_4}{\beta_1^3}\left(\frac{\theta_1}{\beta_1}\right) + \frac{60\cdot C_5}{\beta_1^3}\left(\frac{\theta_1}{\beta_1}\right)^2 \qquad (11)$$

THE FIRST 3 BOUNDARY CONDITIONS (2)-(4) RESULT IN

$$C_0 = C_1 = C_2 = 0 \qquad (12)$$

THE REMAINING BOUNDARY CONDITIONS ARE USED TO CALCULATE C_3, C_4 & C_5.

$$⑤ \rightarrow ⑧ \Rightarrow \quad S\left(\frac{\theta_1}{\beta_1}=1\right) = \underline{C_3 + C_4 + C_5 = 2in} \qquad (13)$$

$$⑥ \rightarrow ⑨ \Rightarrow \quad S'\left(\frac{\theta_1}{\beta_1}=1\right) = \frac{3 \cdot C_3}{\beta_1} + \frac{4 \cdot C_4}{\beta_1} + \frac{5 \cdot C_5}{\beta_1} = 0$$

$$\underline{3 \cdot C_3 + 4 \cdot C_4 + 5 \cdot C_5 = 0} \qquad (14)$$

$$⑦ \rightarrow ⑩ \Rightarrow \quad S''\left(\frac{\theta_1}{\beta_1}=1\right) = \frac{6 \cdot C_3}{\beta_1^2} + \frac{12 \cdot C_4}{\beta_1^2} + \frac{20 \cdot C_5}{\beta_1^2} = 0$$

$$\underline{6 \cdot C_3 + 12 \cdot C_4 + 20 \cdot C_5 = 0} \qquad (15)$$

EQUATIONS (13)–(15) ARE A SYSTEM OF EQUATIONS WITH UNKNOWNS C_3, C_4, & C_5. IN MATRIX FORM THESE EQUATIONS CAN BE WRITTEN

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{bmatrix} \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} 2in \\ 0in \\ 0in \end{Bmatrix} \qquad (16)$$

$$\begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{bmatrix}^{-1} \begin{Bmatrix} 2in \\ 0in \\ 0in \end{Bmatrix}$$

$$\begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} 20in \\ -30in \\ 12in \end{Bmatrix} \qquad (17)$$

⑧ → ⑪ CAN NOW BE WRITTEN

$$⑧ \rightarrow \quad S = 20in\left(\frac{\theta_1}{\beta_1}\right)^3 - 30in\left(\frac{\theta_1}{\beta_1}\right)^4 + 12in\left(\frac{\theta_1}{\beta_1}\right)^5 \qquad (18)$$

$$⑨ \rightarrow \quad S' = \frac{60in}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^2 - \frac{120in}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^3 + \frac{60in}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^4 \qquad (19)$$

$$⑩ \rightarrow \quad S'' = \frac{120in}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right) - \frac{360in}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^2 + \frac{240in}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^3 \qquad (20)$$

$$⑪ \rightarrow \quad S''' = \frac{120in}{\beta_1^3} - \frac{720in}{\beta_1^3}\left(\frac{\theta_1}{\beta_1}\right) + \frac{720in}{\beta_1^3}\left(\frac{\theta_1}{\beta_1}\right)^2 \qquad (21)$$

NOW CONSIDERING THE FALL SEGMENT

<u>SEGMENT 2:</u> $60° \leq \theta \leq 150°$, $0 \leq \theta_2 \leq 90°$, $0 \leq \theta_2 \leq \pi/2$

THIS SEGMENT IS A SIMPLE FALL FROM 2in TO 0in. AGAIN, A POLYNOMIAL FUNCTION WILL BE CONSIDERED TO MEET THE FOLLOWING BOUNDARY CONDITIONS.

$\theta_2 = 0$ $\qquad\qquad$ $\theta_2 = \beta_2$

$\theta_2/\beta_2 = 0$ $\qquad\qquad$ $\theta_2/\beta_2 = 1$

$S_2 = 2in$ (22) $\qquad\qquad$ $S_2 = 0in$ (25)

$S_2' = 0\ in/rad$ (23) $\qquad\qquad$ $S_2' = 0\ in/rad$ (26)

$S_2'' = 0\ in/rad^2$ (24) $\qquad\qquad$ $S_2'' = 0\ in/rad^2$ (27)

GIVEN SIX BOUNDARY CONDITIONS. A POLYNOMIAL OF DEGREE 5 IS APPROPRIATE FOR THIS SEGMENT

$$S_2 = \cancelto{2in}{C_0} + \cancelto{0}{C_1}\left(\frac{\theta_2}{\beta_2}\right) + \cancelto{0}{C_2}\left(\frac{\theta_2}{\beta_2}\right)^2 + C_3\left(\frac{\theta_2}{\beta_2}\right)^3 + C_4\left(\frac{\theta_2}{\beta_2}\right)^4 + C_5\left(\frac{\theta_2}{\beta_2}\right)^5 \quad (28)$$

$$S_2' = \frac{\cancelto{0}{C_1}}{\beta_2} + \frac{2\cdot \cancelto{0}{C_2}}{\beta_2}\left(\frac{\theta_2}{\beta_2}\right) + \frac{3\cdot C_3}{\beta_2}\left(\frac{\theta_2}{\beta_2}\right)^2 + \frac{4\cdot C_4}{\beta_2}\left(\frac{\theta_2}{\beta_2}\right)^3 + \frac{5\cdot C_5}{\beta_2}\left(\frac{\theta_2}{\beta_2}\right)^4 \quad (29)$$

$$S_2'' = \frac{2\cdot \cancelto{0}{C_2}}{\beta_2^2} + \frac{6\cdot C_3}{\beta_2^2}\left(\frac{\theta_2}{\beta_2}\right) + \frac{12\cdot C_4}{\beta_2^2}\left(\frac{\theta_2}{\beta_2}\right)^2 + \frac{20\cdot C_5}{\beta_2^2}\left(\frac{\theta_2}{\beta_2}\right)^3 \quad (30)$$

$$S_2''' = \frac{6\cdot C_3}{\beta_2^3} + \frac{24\cdot C_4}{\beta_2^3}\left(\frac{\theta_2}{\beta_2}\right) + \frac{60\cdot C_5}{\beta_2^3}\left(\frac{\theta_2}{\beta_2}\right)^2 \quad (31)$$

APPLYING THE FIRST 3 BOUNDARY CONDITIONS, (22)-(24), TO THE CORRESPONDING POLYNOMIAL (28)-(30) TO DETERMINE VALUES FOR C_0, C_1, & C_2. AT $\frac{\theta_2}{\beta_2} = 0$

$$(22) \rightarrow (28) \Rightarrow S\left(\frac{\theta_2}{\beta_2} = 0\right) = \underline{2in = C_0} \quad (32)$$

$$(23) \rightarrow (29) \Rightarrow S'\left(\frac{\theta_2}{\beta_2} = 0\right) = 0\ \tfrac{in}{rad} = \frac{C_1}{\beta_2} \Rightarrow \underline{C_1 = 0} \quad (33)$$

$$(24) \rightarrow (30) \Rightarrow S''\left(\frac{\theta_2}{\beta_2} = 0\right) = 0\ in/rad^2 = \frac{2\cdot C_2}{\beta_2^2} \Rightarrow \underline{C_2 = 0} \quad (34)$$

THE SECOND SET OF 3 BOUNDARY CONDITIONS, (25)-(27), TO DETERMINE VALUES FOR $C_3 - C_5$

(25) → (28) ⇒ $$S_2\left(\frac{\theta_2}{\beta_2}=1\right) = 2\,in + C_3 + C_4 + C_5 = 0\,in$$

$$\underline{\underline{C_3 + C_4 + C_5 = -2\,in}} \quad (35)$$

(26) → (29) ⇒ $$S_2'\left(\frac{\theta_2}{\beta_2}=1\right) = \frac{3\cdot C_3}{\beta_2} + \frac{4\cdot C_4}{\beta_2} + \frac{5\cdot C_5}{\beta_2} = 0\,in/rad$$

$$\underline{\underline{3\cdot C_3 + 4\,C_4 + 5\,C_5 = 0\,in}} \quad (36)$$

(27) → (30) ⇒ $$S''\left(\frac{\theta_2}{\beta_2}=1\right) = \frac{6\cdot C_3}{\beta_2^2} + \frac{12\cdot C_4}{\beta_2^2} + \frac{20\cdot C_5}{\beta_2^2} = 0\,in/rad^2$$

$$\underline{\underline{6\cdot C_3 + 12\cdot C_4 + 20\cdot C_5 = 0\,in}} \quad (37)$$

EQUATIONS (35) - (37) ARE A SYSTEM OF EQUATIONS WITH UNKNOWNS C_3, C_4, & C_5. IN MATRIX FORM THEY CAN BE WRITTEN

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{bmatrix} \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} -2 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{bmatrix}^{-1} \begin{Bmatrix} -2\,in \\ 0\,in \\ 0\,in \end{Bmatrix}$$

$$\begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} -20\,in \\ 30\,in \\ -12\,in \end{Bmatrix}$$

(28)-(31) CAN NOW BE WRITTEN

$$S_2 = 2in - 20in\left(\frac{\theta_2}{\beta_2}\right)^3 + 30in\left(\frac{\theta_2}{\beta_2}\right)^4 - 12in\left(\frac{\theta_2}{\beta_2}\right)^5 \qquad (38)$$

$$S_2' = -\frac{60in}{\beta_2}\left(\frac{\theta_2}{\beta_2}\right)^2 + \frac{120in}{\beta_2}\left(\frac{\theta_2}{\beta_2}\right)^3 - \frac{60in}{\beta_2}\cdot\left(\frac{\theta_2}{\beta_2}\right)^4 \qquad (39)$$

$$S_2'' = -\frac{120in}{\beta_2^2}\left(\frac{\theta_2}{\beta_2}\right) + \frac{360in}{\beta_2^2}\left(\frac{\theta_2}{\beta_2}\right)^2 - \frac{240in}{\beta_2^2}\left(\frac{\theta_2}{\beta_2}\right)^3 \qquad (40)$$

$$S_2''' = -\frac{120in}{\beta_2^3} + \frac{720in}{\beta_2^3}\left(\frac{\theta_2}{\beta_2}\right) - \frac{360in}{\beta_2^3}\left(\frac{\theta_2}{\beta_2}\right)^2 \qquad (41)$$

THE FINAL SEGMENT, SEGMENT 3, IS A DWELL. THEREFORE

$$S_3 = 0 \qquad (42)$$

$$S_3' = 0 \qquad (43)$$

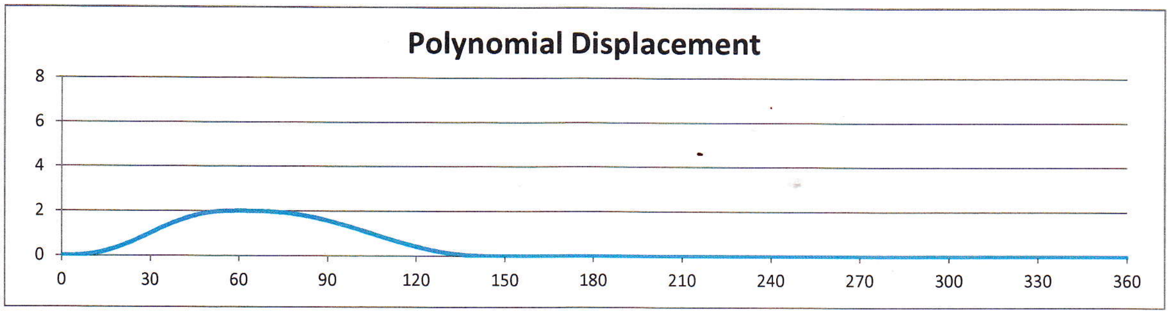
$$S_3'' = 0 \qquad (44)$$

$$S_3''' = 0 \qquad (45)$$

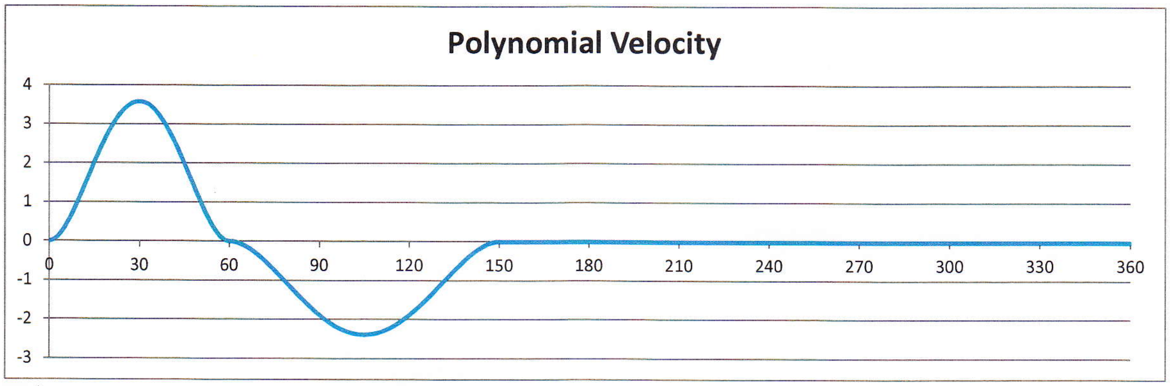
THE S-V-A-J DIAGRAMS FOR THE COMBINATION OF SEGMENTS 1, 2, & 3 ARE SHOWN ON THE NEXT PAGE. THE EQUATIONS USED FOR CONSTRUCTING THESE DIAGRAMS ARE SUMMARIZED BELOW

			SEG 1	SEG 2	SEG 3
DISPLACEMENT	S [IN]	:	(18)	(38)	(42)
VELOCITY	S' [in/rad]	:	(19)	(39)	(43)
ACCELERATION	S'' [in/rad^2]	:	(20)	(40)	(44)
JERK	S''' [in/rad^3]	:	(21)	(41)	(45)

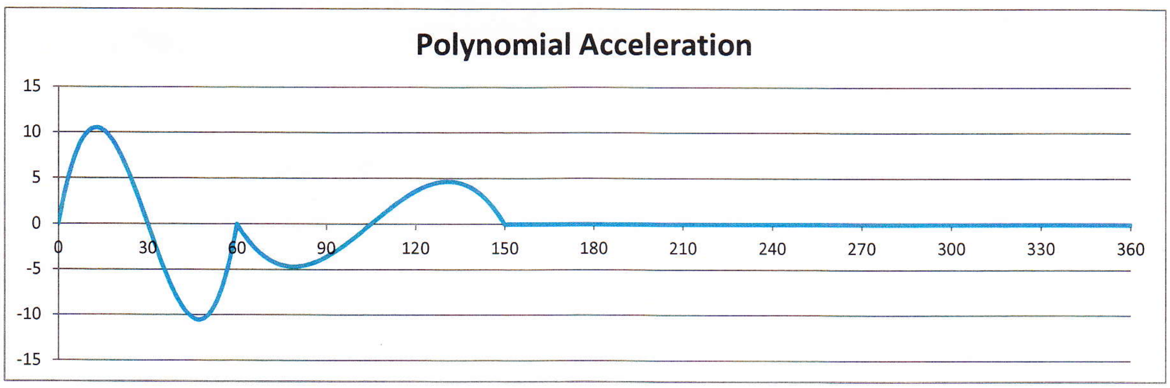
~~ALL~~ THE REQUIREMENT THAT THE DISPLACEMENT FUNCTION IS PIECEWISE CONTINUOUS THROUGH TWO DERIVATIVES IS SATISFIED AND IS ILLUSTRATED IN FIGURES (b) - (e)


Polynomial Displacement
8
6
4
2
0
0 30 60 90 120 150 180 210 240 270 300 330 360

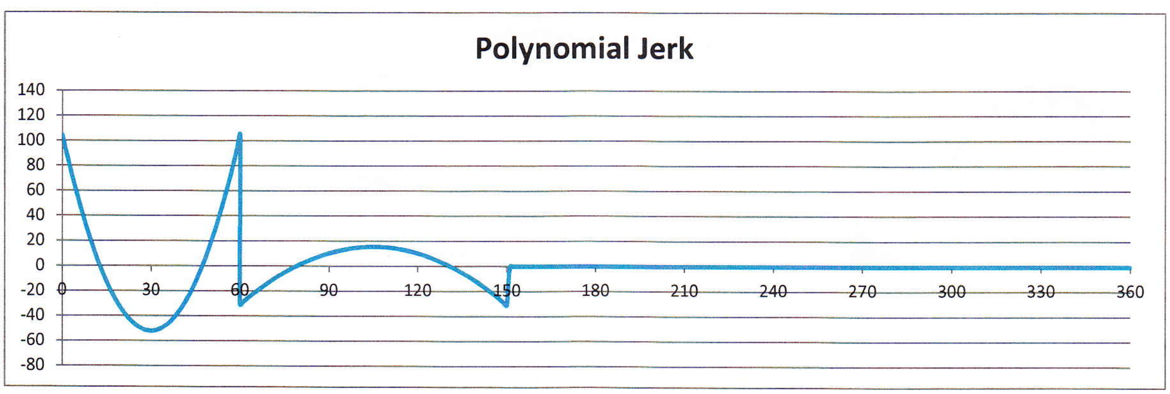
(b)


Polynomial Velocity
4
3
2
1
0
-1
-2
-3
0 30 60 90 120 150 180 210 240 270 300 330 360

(c)

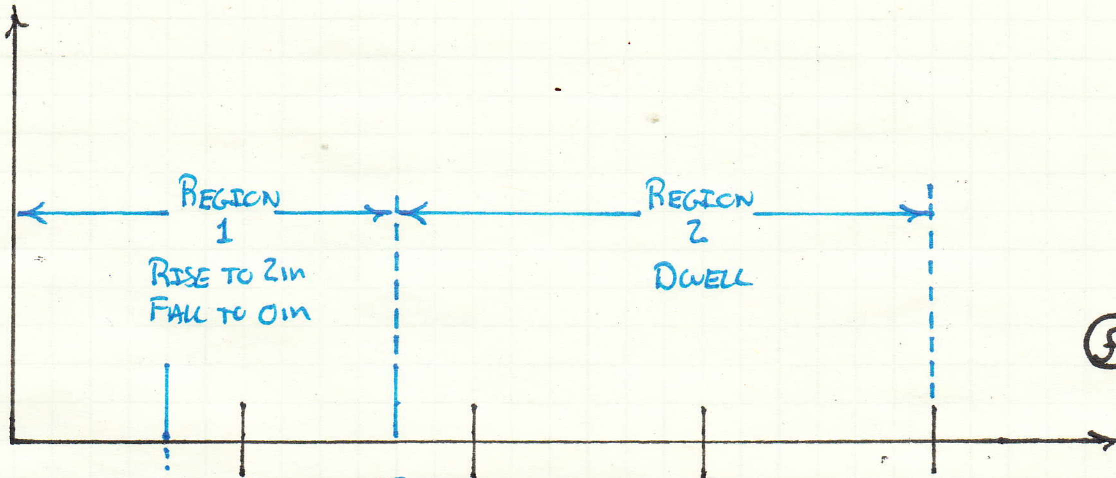

Polynomial Acceleration
15
10
5
0
-5
-10
-15
0 30 60 90 120 150 180 210 240 270 300 330 360

(d)


Polynomial Jerk
140
120
100
80
60
40
20
0
-20
-40
-60
-80
0 30 60 90 120 150 180 210 240 270 300 330 360

(e)

NOW A <u>SECOND FORMULATION</u> OF THE CAM PROFILE WILL BE CONSIDERED. IN THIS FORMULATION ONLY TWO REGIONS ARE CONSIDERED.



θ [rad]	0	$\pi/3$	$\pi/2$	$5\pi/6$	π	$3\pi/2$	2π
θ [°]	0	60	90	150°	180	270	360
t [s]	0	0.333	0.5	0.833	1.0	1.5	2.0

S [in] =	0 (50)	2in (53)	0 (55)	0
S' [in/rad] =	0 (51)	0 (54)	0 (56)	0
S'' [in/rad²] =	0 (52)	–	0 (57)	0
S''' [in/rad³] =	–	–	–	–

<u>SEGMENT 1</u>: $0 \le \theta \le 150°$, $0 \le \theta_1 \le 150°$, $0 \le \theta_1 \le 5\pi/6$, $\beta_1 = 5\pi/6$

SINCE THERE ARE SIX BOUNDARY CONDITIONS AN TWO INTERMEDIATE CONDITIONS, A 7TH DEGREE POLYNOMIAL APPEARS TO BE APPROPRIATE FOR THIS REGION.

$$S_1 = \overset{\overset{(61)}{0}}{C_0} + \overset{\overset{(62)}{0}}{C_1}\left(\frac{\theta_1}{\beta_1}\right) + \overset{\overset{(63)}{0}}{C_2}\left(\frac{\theta_1}{\beta_1}\right)^2 + C_3\left(\frac{\theta_1}{\beta_1}\right)^3 + C_4\left(\frac{\theta_1}{\beta_1}\right)^4 + C_5\left(\frac{\theta_1}{\beta_1}\right)^5 + C_6\left(\frac{\theta_1}{\beta_1}\right)^6 + C_7\left(\frac{\theta_1}{\beta_1}\right)^7 \quad (58)$$

$$S_1' = \frac{\overset{\overset{(62)}{0}}{C_1}}{\beta_1} + \frac{2\cdot \overset{\overset{(63)}{0}}{C_2}}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right) + \frac{3\cdot C_3}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^2 + \frac{4\cdot C_4}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^3 + \frac{5\cdot C_5}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^4 + \frac{6\cdot C_6}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^5 + \frac{7\cdot C_7}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^6 \quad (59)$$

$$S_1'' = \frac{2\cdot \overset{\overset{(63)}{0}}{C_2}}{\beta_1^2} + \frac{6\cdot C_3}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right) + \frac{12\cdot C_4}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^2 + \frac{20\cdot C_5}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^3 + \frac{30\cdot C_6}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^4 + \frac{42\cdot C_7}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^5 \quad (60)$$

$$S_1''' = \frac{6\cdot C_3}{\beta_1^3} + \frac{24\cdot C_4}{\beta_1^3}\left(\frac{\theta_1}{\beta_1}\right) + \frac{60\cdot C_5}{\beta_1^3}\left(\frac{\theta_1}{\beta_1}\right)^2 + \frac{120\cdot C_6}{\beta_1^3}\left(\frac{\theta_1}{\beta_1}\right)^3 + \frac{210\, C_7}{\beta_1^3}\left(\frac{\theta_1}{\beta_1}\right)^4 \quad (61)$$

THE SOLUTION TO THE EIGHT CONSTANTS STARTS WITH APPLYING THE INITIAL CONDITIONS (50) - (52) INTO (58) - (60).

(50) → (58) ⇒ $s_1\left(\frac{\theta_1}{\beta_1}=0\right) = \underline{0 = C_0}$ (62)

(51) → (59) ⇒ $s_1'\left(\frac{\theta_1}{\beta_1}=0\right) = 0 = \frac{C_1}{\beta_1} \Rightarrow \underline{C_1 = 0}$ (63)

(52) → (60) ⇒ $s_1''\left(\frac{\theta_1}{\beta_1}=0\right) = 0 = \frac{2\cdot C_2}{\beta_1} \Rightarrow \underline{C_2 = 0}$ (64)

SUBSTITUTING THE TWO INTERMEDIATE CONDITIONS (53) & (54) INTO (58) & (59) GIVEN

$$\frac{\theta_1}{\beta_1} = \frac{\pi/3}{5\pi/6} = \frac{2}{5}$$

(53) → (58) ⇒ $s_1\left(\frac{\theta_1}{\beta_1}=\frac{2}{5}\right) = \underline{2m = C_3\left(\frac{2}{5}\right)^3 + C_4\left(\frac{2}{5}\right)^4 + C_5\left(\frac{2}{5}\right)^5 + C_6\left(\frac{2}{5}\right)^6 + C_7\left(\frac{2}{5}\right)^7}$ (65)

(54) → (59) ⇒ $s_1'\left(\frac{\theta_1}{\beta_1}=\frac{2}{5}\right) = 0 = \frac{3\cdot C_3}{\beta_1}\left(\frac{2}{5}\right)^2 + \frac{4\cdot C_4}{\beta_1}\left(\frac{2}{5}\right)^3 + \frac{5\cdot C_5}{\beta_1}\left(\frac{2}{5}\right)^4 + \frac{6\cdot C_6}{\beta_1}\cdot\left(\frac{2}{5}\right)^5 + \frac{7\cdot C_7}{\beta_1}\left(\frac{2}{5}\right)^6$

$$\underline{0 = 3\cdot C_3 + 4\cdot C_4\left(\frac{2}{5}\right) + 5\cdot C_5\left(\frac{2}{5}\right)^2 + 6\cdot C_6\cdot\left(\frac{2}{5}\right)^3 + 7\cdot C_7\cdot\left(\frac{2}{5}\right)^4} \quad (66)$$

SUBSTITUTING THE REMAINING END CONDITIONS (55) - (57) INTO (58) - (60)

(55) → (58) ⇒ $s_1\left(\frac{\theta_1}{\beta_1}=1\right) = \underline{0 = C_3 + C_4 + C_5 + C_6 + C_7}$ (67)

(56) → (59) ⇒ $s_1'\left(\frac{\theta_1}{\beta_1}=1\right) = 0 = \frac{3\cdot C_3}{\beta_1} + \frac{4\cdot C_4}{\beta_1} + \frac{5\cdot C_5}{\beta_1} + \frac{6\cdot C_6}{\beta_1} + \frac{7\cdot C_7}{\beta_1}$

$$\underline{0 = 3\cdot C_3 + 4 C_4 + 5\cdot C_5 + 6\cdot C_6 + 7\cdot C_7} \quad (68)$$

(57) → (60) ⇒ $s_1''\left(\frac{\theta_1}{\beta_1}=1\right) = 0 = \frac{6\cdot C_3}{\beta_1^2} + \frac{12\cdot C_4}{\beta_1^2} + \frac{20\cdot C_5}{\beta_1^2} + \frac{30\cdot C_6}{\beta_1^2} + \frac{42\cdot C_7}{\beta_1^2}$

$$\underline{0 = 6\cdot C_3 + 12\cdot C_4 + 20\cdot C_5 + 30\cdot C_6 + 42\cdot C_7} \quad (69)$$

EQUATIONS (65) - (69) ARE FIVE EQUATIONS THAT CONTAIN FIVE UNKNOWN CONSTANTS (C_3, C_4, C_5, C_6, C_7). THESE FIVE EQUATIONS ARE WRITTEN IN MATRIX FORM AND SOLVED SIMULTANEOUSLY.

$$\begin{bmatrix} \left(\frac{2}{3}\right)^3 & \left(\frac{2}{3}\right)^4 & \left(\frac{2}{3}\right)^5 & \left(\frac{2}{3}\right)^6 & \left(\frac{2}{3}\right)^7 \\ 3 & 4\left(\frac{2}{3}\right) & 5\left(\frac{2}{3}\right)^2 & 6\cdot\left(\frac{2}{3}\right)^3 & 7\cdot\left(\frac{2}{3}\right)^4 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 6 & 12 & 20 & 30 & 42 \end{bmatrix} \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{Bmatrix} = \begin{Bmatrix} 2\text{in} \\ 0\text{in} \\ 0\text{in} \\ 0\text{in} \\ 0\text{in} \end{Bmatrix}$$

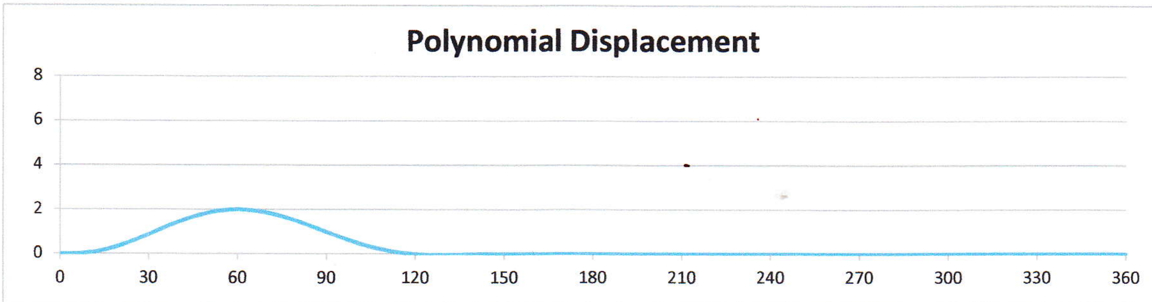
$$\begin{Bmatrix} C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{Bmatrix} = \begin{Bmatrix} 289.35\text{in} \\ -1229.75\text{in} \\ 1953.12\text{in} \\ -1374.42\text{in} \\ 361.69\text{in} \end{Bmatrix} \qquad (70)$$

SUBSTITUTING (62), (63), (64) AND (70) INTO (58) - (61) WILL RESULTS IN EQUATIONS USED TO PLOT THE S, S', S'', & S''' DIAGRAMS SHOWN ON THE NEXT PAGE, (g) - (j).

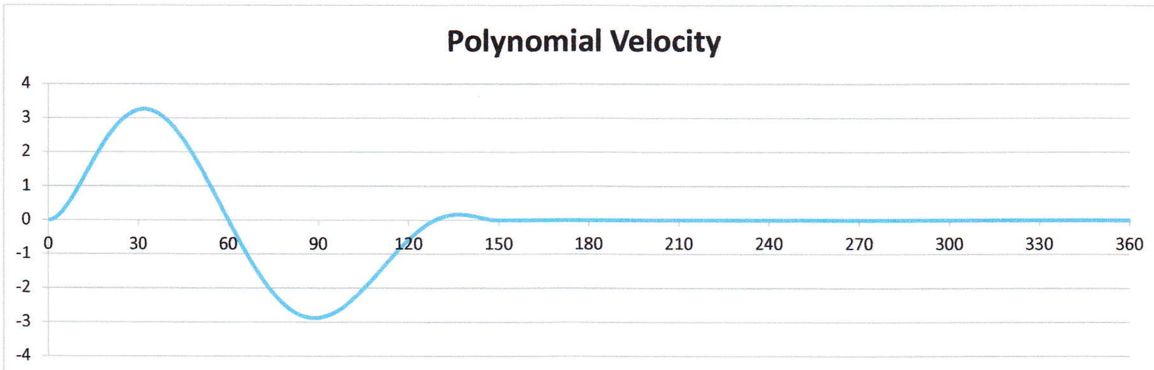
THIS SECOND METHOD RESULTS IN SLIGHTLY LOWER VALUES FOR PEEK VELOCITY AND ACCELERATION. THE SECOND APPROACH ALSO RESULTS IN SMOOTHER CURVES IN ~~REGI~~ SEGMENT 1.

Homework Solution
MER 312: Adv Dynamics & Kinematics

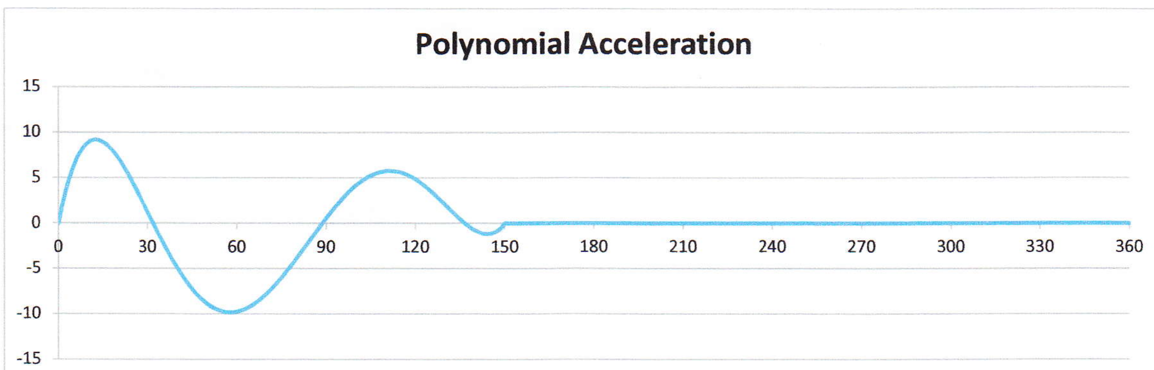
Prob 8-9 Pg 11 of 15
Norton 5th (RBM)



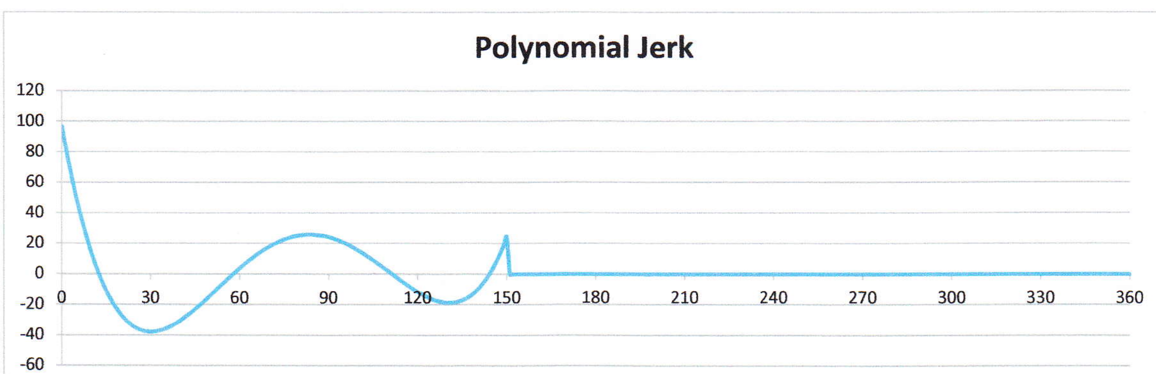
(g)



(h)



(i)



(j)

AS A THIRD FORMULATION, CYCLOID FUNCTIONS WILL BE USED IN SEGMENTS 1 & 2 SHOWN IN FIGURE ⓐ

SEGMENT 1: $0 \le \theta \le 60°$, $0 \le \theta_1 \le 60°$, $0 \le \theta_1 \le \pi/3$, $\beta_1 = \pi/3$, $h = 2\,in$

THE BOUNDARY CONDITIONS FOR THIS SEGMENT ARE

$\theta_1 = 0$ $\qquad\qquad$ $\theta_1 = \beta_1$

$\theta_1/\beta_1 = 0$ $\qquad\qquad$ $\theta_1/\beta_1 = 1$

$s_1 = 0\,in$ (71) $\qquad$ $s_1 = 2\,in$ (74)

$s_1' = 0\,in/rad$ (72) $\qquad$ $s_1' = 0\,in/rad$ (75)

$s_1'' = 0\,in/rad^2$ (73) $\qquad$ $s'' = 0\,in/rad^2$ (76)

THE CYCLOID FUNCTIONS FOR THIS REGION ARE

$$s = h\left[\frac{\theta_1}{\beta_1} - \frac{1}{2\pi}\cdot\sin\left(2\pi\cdot\frac{\theta_1}{\beta_1}\right)\right] \quad (77)$$

$$v = \frac{h}{\beta_1}\left[1 - \cos\left(2\pi\frac{\theta_1}{\beta_1}\right)\right] \quad (78)$$

$$a = 2\pi\frac{h}{\beta_1^2}\cdot\sin\left(2\pi\cdot\frac{\theta_1}{\beta_1}\right) \quad (79)$$

$$j = 4\pi^2\cdot\frac{h}{\beta_1^3}\cdot\cos\left(2\pi\frac{\theta}{\beta}\right) \quad (80)$$

THE FUNCTIONS (77)-(79) SATISFY ~~THE~~ ALL THE BOUNDARY CONDITIONS FOR THIS REGION (71)-(76)

SEGMENT 2: $60° \leq \theta \leq 150°$, $0 \leq \theta_2 \leq 90°$, $0 \leq \theta_2 \leq \pi/2$, $\beta_2 = \pi/2$,

THE BOUNDARY CONDITIONS FOR THIS SEGMENT ARE

$\theta_2 = 0$ $\qquad$ $\theta_2 = \beta_2$

$\theta_2/\beta_2 = 0$ $\qquad$ $\theta_2/\beta_2 = 1$

$S_2 = 2\,in$ (81) $\qquad$ $S_2 = 0\,in$ (84)

$S_2' = 0\,in/rad$ (82) $\qquad$ $S_2' = 0\,in/rad$ (85)

$S_2'' = 0\,in/rad^2$ (83) $\qquad$ $S_2'' = 0\,in/rad^2$ (86)

THE EQUATIONS FOR THE FALL SEGMENT ARE

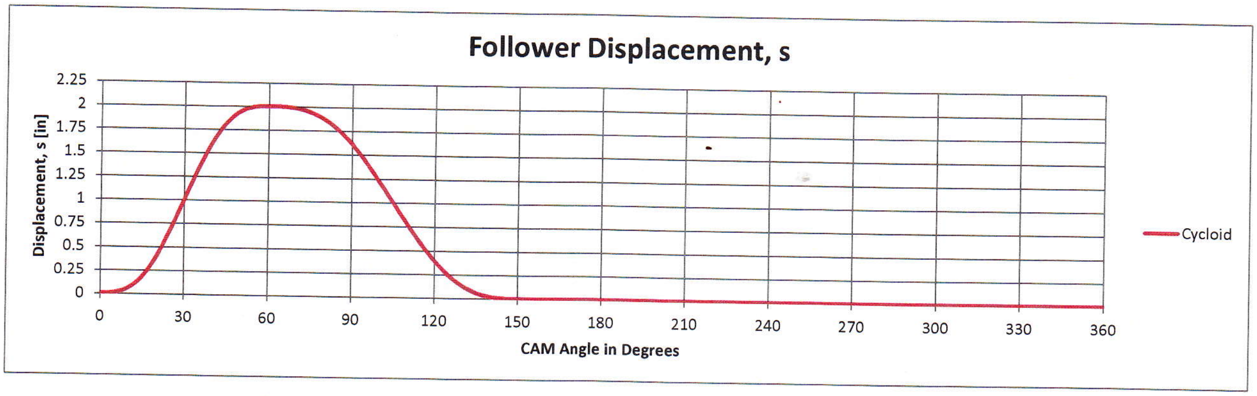
$$S_2 = h - \frac{h}{\beta_2}\left[\theta_2 - \frac{\beta_2}{2\pi}\cdot\sin\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right)\right] \quad (87)$$

$$S_2' = \frac{h}{\beta_2}\cdot\left[\cos\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right) - 1\right] \quad (88)$$

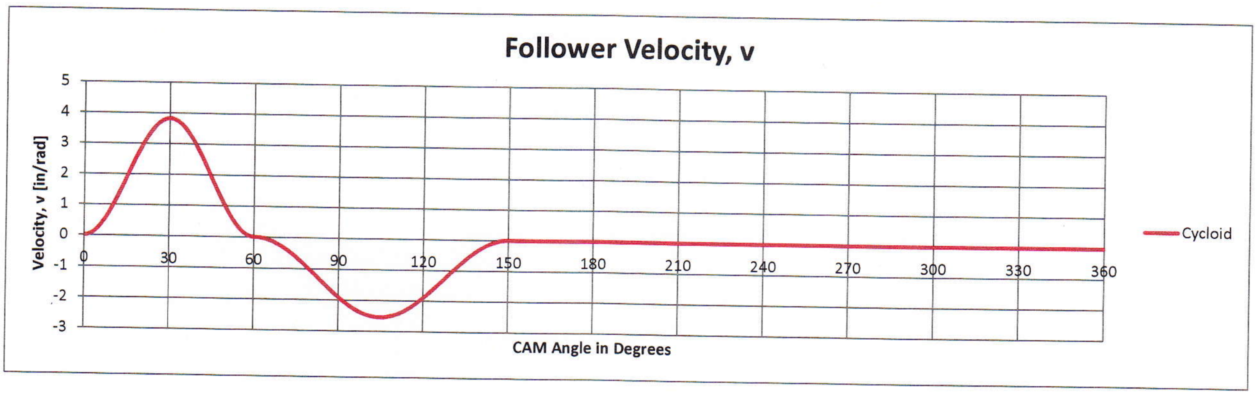
$$S_2'' = -\frac{2\cdot\pi\cdot h}{\beta_2^2}\cdot\sin\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right) \quad (89)$$

$$S_2''' = -\frac{4\cdot\pi^2\cdot h}{\beta_2^3}\cdot\cos\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right) \quad (90)$$

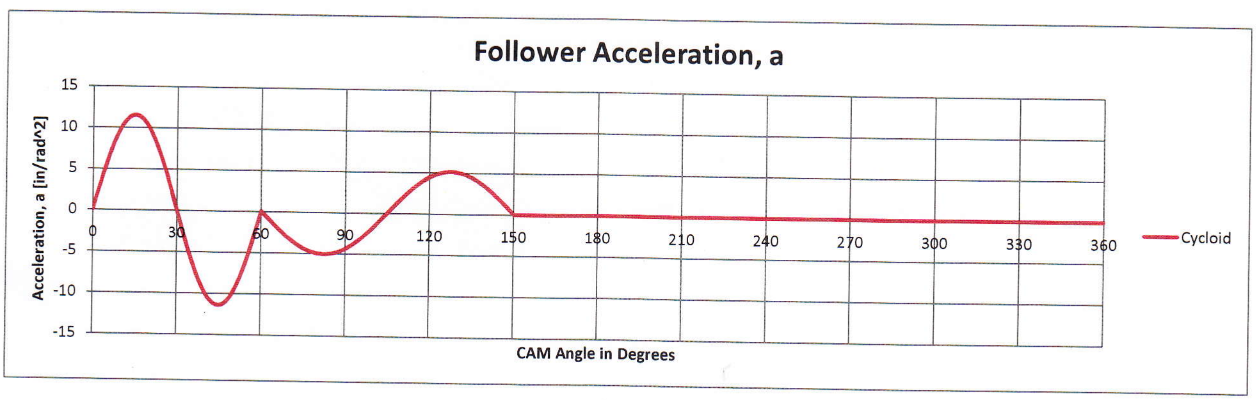
FUNCTIONS (87) - (90) SATISFY ALL THE BOUNDARY CONDITIONS SPECIFIED FOR THIS REGION. THE FUNCTIONS OVER THE ENTIRE RANGE OF THE CAM ARE PLOTTED IN FIGURES (A) - (A).


Follower Displacement, s
Displacement, s [in]
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
0
30
60
90
120
150
180
210
240
270
300
330
360
CAM Angle in Degrees
Cycloid

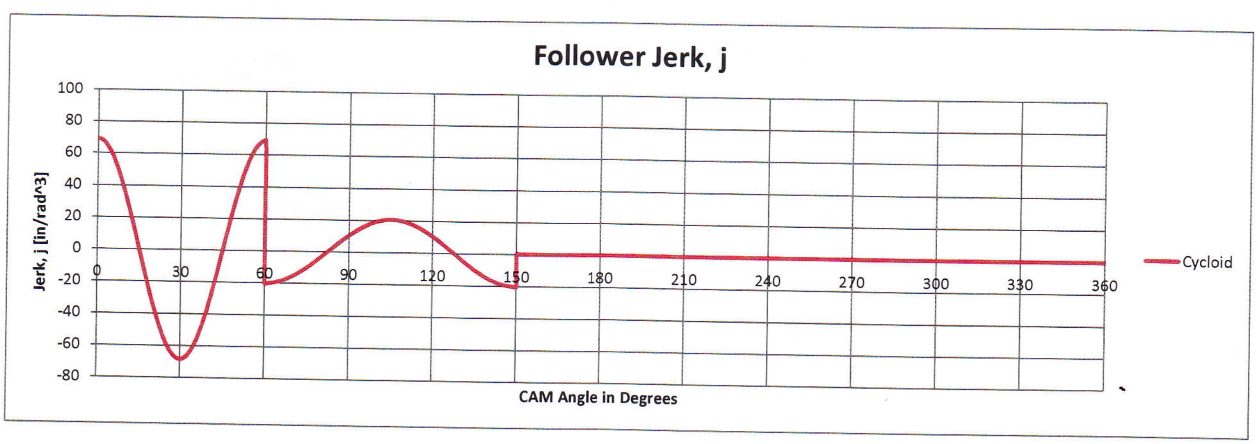
(b)


Follower Velocity, v
Velocity, v [in/rad]
5
4
3
2
1
0
-1
-2
-3
0
30
60
90
120
150
180
210
240
270
300
330
360
CAM Angle in Degrees
Cycloid

(c)


Follower Acceleration, a
Acceleration, a [in/rad^2]
15
10
5
0
-5
-10
-15
0
30
60
90
120
150
180
210
240
270
300
330
360
CAM Angle in Degrees
Cycloid

(m)


Follower Jerk, j
Jerk, j [in/rad^3]
100
80
60
40
20
0
-20
-40
-60
-80
0
30
60
90
120
150
180
210
240
270
300
330
360
CAM Angle in Degrees
Cycloid

(n)

SUMMARY:

THE SOLUTION PRESENTED CONSIDERS THREE APPROACHES TO DETERMINING A CAM PROFILE THAT WILL SATISFY THE RISE-FALL REQUIREMENTS. THE FIRST TWO APPROACHES USED POLYNOMIAL FUNCTIONS AND THE THIRD APPROACH USED CYCLOID FUNCTIONS. ALL THREE APPROACHES SATIFIFIED THE REQUIREMENT THAT THE DISPLACEMENT FUNCTION HAD TO BE CONTINUOUS THROUGH TWO DERIVATIVES. THE DIFFERENCE IN THE FUNCTIONS WAS THE MAGNITUDE OF THE VELOCITIES AND ACCELERATIONS.

THE FUNCTION WITH THE LOWEST VELOCITIES AND ACCELERATIONS WAS THE SECOND APPROACH IN WHICH A POLYNOMIAL OF DEGREE SEVEN WAS USED TO MODEL THE ENTIRE RISE AND FALL OF THE FOLLOWER. THE S-S'-S"-S''' DIAGRAMS FOR THESE FUNCTIONS ARE FOUND IN (g)-(j).

THE FUNCTION WITH THE SECOND LOWEST VELOCITIES & ACCELERATIONS WAS THE FIRST APPROACH IN WHICH A POLYNOMIAL OF DEGREE FIVE WAS USED TO MODEL THE RISE TO 2in AND THE FALL BACK TO 0in SEPERATELY. THE S-S'-S"-S''' DIAGRAMS FOR THESE FUNCTIONS ARE FOUND IN FIGURES (b)-(e).

THE CYCLOIDAL FUNCTIONS PRODUCED THE HIGHEST VELOCITIES AND ACCELERATIONS. ALTHOUGH THE DIFFERENCE BETWEEN THE THREE DEVELOPMENTS WAS ONLY SLIGHT. THE CYCLOIDAL FUNCTIONS ARE PLOTTED IN FIGURES (k)+(h).