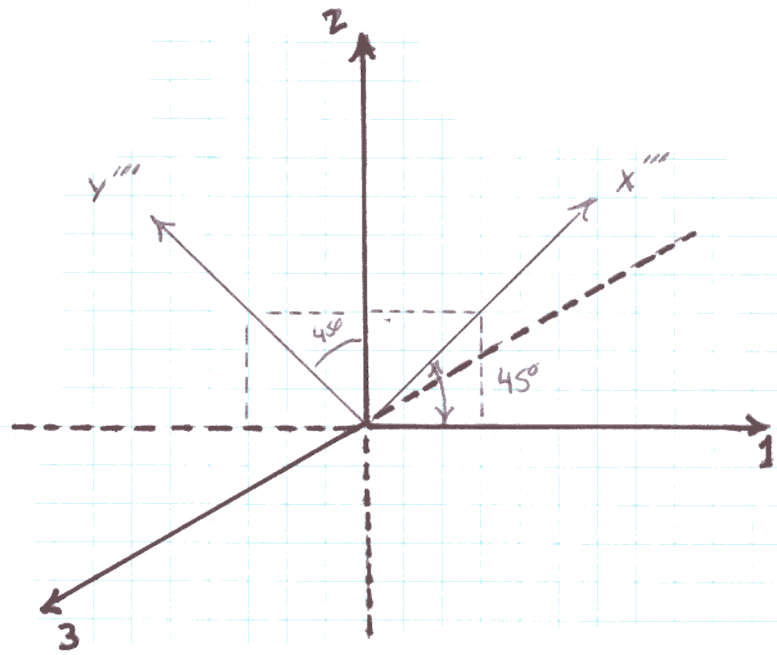
$$\begin{Bmatrix} X''' \\ Y''' \\ Z''' \end{Bmatrix} = \begin{bmatrix} \cos 45^\circ & \cos 45^\circ & 0 \\ -\cos 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$



$$\begin{Bmatrix} X''' \\ Y''' \\ Z''' \end{Bmatrix} = \begin{bmatrix} .7071 & .7071 & 0 \\ -.7071 & .7071 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$

$$\begin{Bmatrix} X''' \\ Y''' \\ Z''' \end{Bmatrix} = \begin{bmatrix} .7071 & .7071 & 0 \\ -.7071 & .7071 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -.1957 & 9764 & .0918 \\ .2204 & 1350 & -.9660 \\ -.9556 & 168^\circ & -.2416 \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}$$

$$\begin{Bmatrix} X''' \\ Y''' \\ Z''' \end{Bmatrix} = \begin{bmatrix} 0.0175 & 0.7859 & -0.6181 \\ 0.2942 & -0.5950 & -0.7480 \\ -0.9556 & -.1688 & -0.2416 \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}$$

X''' REPRESENTS THE ORIENTATION TO THE MAXIMUM SHEAR STRESS IN THE 1-2 PLANE

$\Theta_{XX'''} = 89^\circ$

$\Theta_{XY'''} = 38^\circ$

128°

```
>> S

S =

    60    20    10
    20   -40    -5
    10    -5    30

   T1

T1 =

    0.7071    0.7071         0
   -0.7071    0.7071         0
         0         0    1.0000

   T2

T2

   -0.1957    0.9764    0.0918
    0.2204    0.1350   -0.9660
   -0.9556   -0.1688   -0.2416

   T=T1*T2

T =

    0.0175    0.7859   -0.6181
    0.2942   -0.5950   -0.7480
   -0.9556   -0.1688   -0.2416

   T*S*T'

ans =

   -8.0313   36.4487   -0.0010
   36.4487   -8.0344   -0.0013
   -0.0010   -0.0013   66.0635

   acos(T)*180/pi

ans

   88.9993   38.1987  128.1809
   72.8890  126.5093  138.4149
  162.8624   99.7181  103.9810

>>
```

TRANSFORMATION FROM THE PRINCIPAL PLANE TO THE MAXIMUM SHEAR STRESS IN THE 1-2 PLANE

TRANSFORMATION FROM THE ORIGINAL STATE OF STRESS TO THE PRINCIPAL STATE OF STRESS

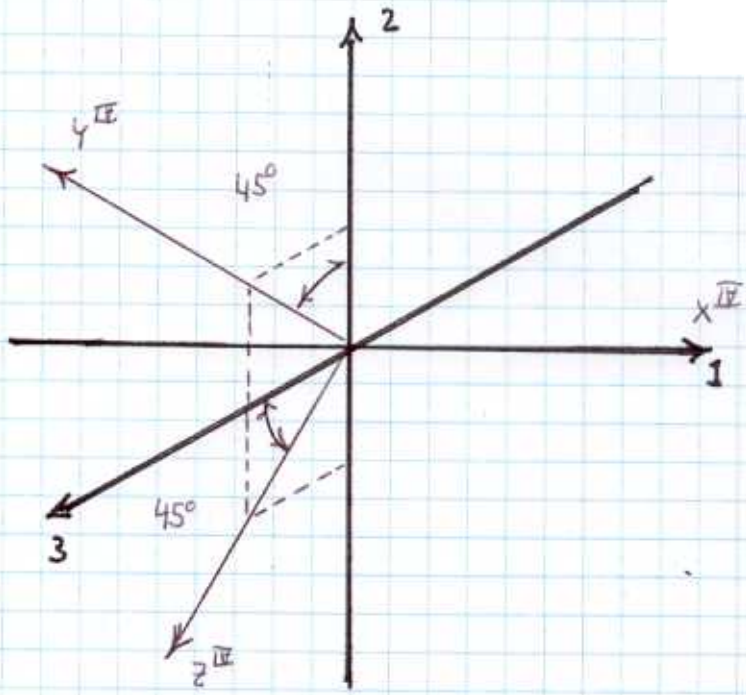
TRANSFORMATION FROM THE ORIGINAL STATE OF STRESS TO THE MAXIMUM SHEAR STRESS IN THE 1-2 PLANE

STATE OF STRESS WHEN THE SHEAR STRESS IS MAXIMIZED IN THE 1-2 PLANE

ANGLES MADE BETWEEN THE X-Y-Z COORDINATE SYSTEM AND THE X'''-Y'''-Z''' COORDINATE SYSTEM.

$$\begin{Bmatrix} X^{III} \\ Y^{III} \\ Z^{III} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45 & \cos 45 \\ 0 & -\cos 45 & \cos 45 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$

$$\begin{Bmatrix} X^{III} \\ Y^{III} \\ Z^{III} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .7071 & .7071 \\ 0 & -.7071 & .7071 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$



$$\begin{Bmatrix} X^{III} \\ Y^{III} \\ Z^{III} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .7071 & .7071 \\ 0 & -.7071 & .7071 \end{bmatrix} \cdot \begin{bmatrix} -.1957 & .9764 & .0918 \\ .2204 & .1850 & -.9660 \\ -.9556 & -.1688 & -.2416 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$\begin{Bmatrix} X^{III} \\ Y^{III} \\ Z^{III} \end{Bmatrix} = \begin{bmatrix} -.1957 & .9764 & .0918 \\ -.5199 & -.0239 & -.8539 \\ -.8315 & -.2148 & .5122 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

Y^{III} REPRESENTS THE ORIENTATION TO THE MAXIMUM SHEAR STRESS IN THE 2-3 PLANE

$\Theta_{xY^{III}} = 121°$

$\Theta_{yY^{III}} = 91°$

$\Theta_{zY^{III}} = 149°$

```
>> S

S

    60    20    10
    20   -40    -5
    10    -5    30
```

INITIAL STATE OF STRESS

```
   T2

   -0.1957    0.9764    0.0918
    0.2204    0.1350   -0.9660
   -0.9556   -0.1688   -0.2416
```

TRANSFORMATION FROM THE INITIAL STATE OF STRESS TO THE PRINCIPAL STATE OF STRESS

```
>> T3

T3 =

       0000         0         0
          0    0.7071    0.7071
          0   -0.7071    0.7071
```

TRANSFORMATION FROM THE PRINCIPAL STATE OF STRESS TO THE STATE OF STRESS THAT MAXIMIZES SHEAR IN THE 2-3 PLANE

```
>> T=T3*T2

T =

   -0.1957    0.9764    0.0918
   -0.5199   -0.0239   -0.8539
   -0.8315   -0.2148    0.5122
```

TRANSFORMATION FROM THE INITIAL STATE OF STRESS TO THE STATE OF STRESS THAT MAXIMIZES SHEAR IN THE 2-3 PLANE

```
>> T*S*T'

ans

  -44.4825    0.0012   -0.0010
    0.0012   47.2375   18.8232
   -0.0010   18.8232   47.2406
```

STATE OF STRESS THAT MAXIMIZES SHEAR IN THE 2-3 PLANE

```
>> acos(T)*180/pi

ans =

  101.2856   12.4724   84.7328
  121.3229   91.3695  148.6378
  146.2582  102.4048   59.1880
```

ANGLES MADE BETWEEN THE INITIAL COORDINATE SYSTEM AND THE COORDINATE SYSTEM THAT MAXIMIZES STRESS IN THE 2-3 PLANE

THE STATES OF STRESS AT POINTS "C" AND "D" RESULT FROM A COMBINATION OF THE SHEARING FORCE AND THE TORQUE APPLIED TO THE CROSS-SECTION. THE BENDING MOMENT DOES NOT ENTER THE PROBLEM BECAUSE "C" AND "D" ARE ON THE NEUTRAL AXIS.

$$= \frac{V \cdot Q}{I \cdot t} + \frac{T \cdot r}{J}$$

FOR THIS

$$.424(.05m)\cdot(.5)\cdot\pi\cdot(.05m)^2 - .424\cdot(.04m)(.5)\cdot\pi\cdot(.04m)^2$$

$$)\,m^3$$

$$I = \frac{\pi}{64}\left[(.1m)^4 - (.08m)^4\right] = 2.898(10^{-6})\,m^4$$

$$J = 2 * I = 5.796(10^{-6})\,m^4$$

THUS AT "C"

$$\frac{2.70(10^3)N \cdot 40.63(10^{-6})m^3}{2.898(10^{-6})m^4 \cdot 2 \cdot (.01m)} + \frac{(-2.835\times10^3)N\cdot m \cdot .05m}{5.796(10^{-6})m^4}$$

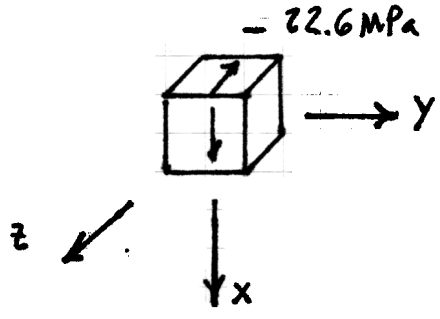
$$= 1.893(10^6)\frac{N}{m^2} - 24.45(10^6)\frac{N}{m^2} = -22.56(10^6)\frac{N}{m^2}$$

$$= -22.56\ mPa$$

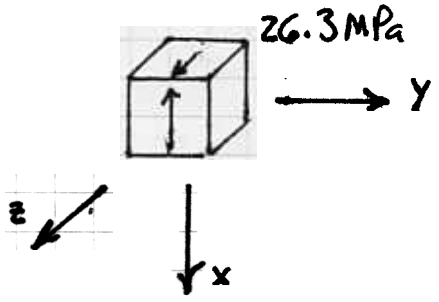
AT "D"

$$\tau_{zx} = 1.893(10^6)\frac{N}{m^2} + 24.45(10^6)\frac{N}{m^2} = 26.34(10^6)\frac{N}{m^2}$$

$$= 26.34\ mPa$$



POINT C



POINT D