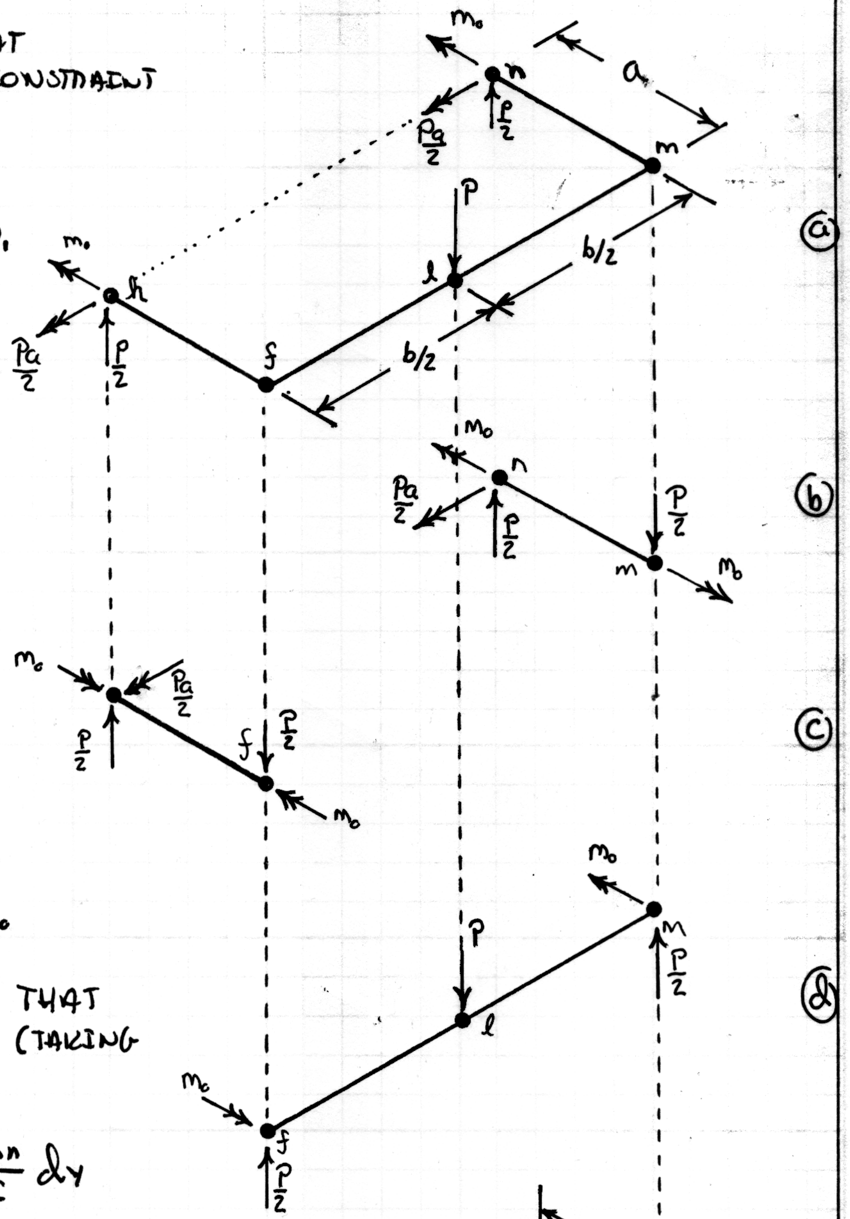
M_o IS A REDUNDANT LOAD THAT RESULTS FROM A REDUNDANT CONSTRAINT AT THE WALL.

TO USE CASTIGLIANO'S METHOD ALL LOADS NEED TO BE KNOWN, THUS M_o NEEDS TO BE SOLVED FOR FIRST.

SYMMETRY IS USED TO SIMPLIFY THIS PROBLEM.

ONLY BENDING AND TORSION ARE CONSIDERED.



FROM (e)

$$M_{mn} = \frac{Py}{2}$$

$$T_{mn} = M_o$$

FROM (f)

$$M_{lm} = -M_o + \frac{Px}{2} = \frac{Px}{2} - M_o$$

THE TOTAL STRAIN ENERGY THAT RESULTS FROM THESE LOADS (TAKING SYMMETRY INTO ACCOUNT)

$$U = 2\cdot\int_0^{b/2} \frac{M_{lm}^2}{2\cdot EI}\,dx + 2\int_0^{a} \frac{M_{mn}^2}{2EI}\,dy$$

$$+\ 2\cdot\frac{T_{mn}^2\cdot a}{2GK}$$

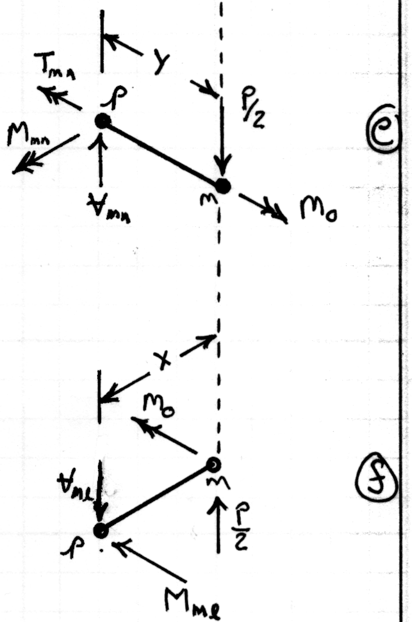
$$= \int_0^{b/2} \frac{\left(\frac{Px}{2}-M_o\right)^2}{EI}\,dx + \int_0^{a} \frac{P^2 y^2}{4\cdot EI}\cdot dy + \frac{M_o^2\cdot a}{GK}$$

$$= \frac{1}{EI}\int_0^{b/2}\left(\frac{Px}{2}-M_o\right)^2 dx + \frac{P^2}{4EI}\int_0^{a} y^2\cdot dy + \frac{M_o^2\cdot a}{GK}$$

$$= \frac{1}{EI}\int_0^{b/2}\left(\frac{P^2x^2}{4} - M_o\cdot P\cdot x + M_o^2\right) dx + \frac{P^2}{4EI}\int_0^{a} y^2\cdot dy + \frac{M_o^2\cdot a}{GK}$$

$$= \frac{1}{EI}\left[\frac{P^2x^3}{12} - \frac{M_o\cdot P\cdot x^2}{2} + M_o^2\cdot x\right]_0^{b/2} + \frac{P^2}{4EI}\,\frac{y^3}{3}\Big|_0^{a} + \frac{M_o^2 a}{GK}$$

$$= \frac{1}{EI}\left[\frac{P^2\cdot b^3}{96} - \frac{M_o\cdot P\cdot b^2}{8} + \frac{M_o^2\cdot b}{2}\right] + \frac{P^2\cdot a^3}{12EI} + \frac{M_o^2 a}{GK}$$



$$U = \frac{P^2 \cdot b^3}{96EI} - \frac{M_o \cdot P \cdot b^2}{8EI} + \frac{M_o^2 \cdot b}{2EI} + \frac{P^2 \cdot a^3}{12EI} + \frac{M_o^2 \cdot a}{GK}$$

$$= \left(\frac{b}{2EI} + \frac{a}{GK}\right) \cdot M_o^2 - \frac{Pb^2}{8EI} \cdot M_o + \frac{a^2}{12EI} \cdot P^2$$

THE ROTATIONAL DEFLECTION DUE TO M_o CAN NOW BE FOUND AND SET TO 0

$$\delta_{M_o} = 0 = \frac{\partial U}{\partial M_o} = 2 \cdot \left(\frac{b}{2EI} + \frac{a}{GK}\right) \cdot M_o - \frac{P \cdot b^2}{8EI}$$

$$M_o = \frac{\frac{P \cdot b^2}{8EI}}{2\left(\frac{b}{2EI} + \frac{a}{GK}\right)} = \frac{P \cdot b^2}{16EI}\left(\frac{1}{\frac{bGK + 2aEI}{2EIGK}}\right) = \frac{P \cdot b^2}{\cancel{16}_{8}\cancel{EI}} \cdot \frac{\cancel{2EI}GK}{(bGK + 2aEI)}$$

$$\delta_{M_o} = \frac{b^2 \cdot G \cdot K}{8(b \cdot G \cdot K + 2a \cdot EI)} \cdot P = A \cdot P \qquad \text{WHERE } A = \frac{b^2 \cdot G \cdot K}{8(b G \cdot K + 2 a \cdot EI)}$$

SUBSTITUTING THIS RESULT FOR M_o INTO THE TOTAL ENERGY EXPRESSION

$$U = \left(\frac{b}{2EI} + \frac{a}{GK}\right) \cdot A^2 \cdot P^2 - \frac{P^2 \cdot b^2 \cdot A}{8EI} + \frac{a^2}{12EI} \cdot P^2$$

$$= \left(\frac{b \cdot A^2}{2EI} + \frac{a \cdot A^2}{GK} - \frac{b^2 \cdot A}{8EI} + \frac{a^2}{12EI}\right) P^2$$

THE DEFLECTION THAT RESULTS FROM P, AT THE LOCATION OF P, AND IN THE DIRECTION OF P CAN NOW BE FOUND BY TAKING THE DERIVATIVE OF THE STRAIN ENERGY WITH RESPECT TO P

$$\frac{\partial U}{\partial P} = \delta_P = 2\left(\frac{b \cdot A^2}{2EI} + \frac{a \cdot A^2}{GK} - \frac{b^2 A}{8EI} + \frac{a^2}{12EI}\right) P$$

$$= \left(\frac{b \cdot A^2}{EI} - \frac{b^2 \cdot A}{4EI} + \frac{a^2}{6EI} + 2 \cdot \frac{a \cdot A^2}{GK}\right) P$$