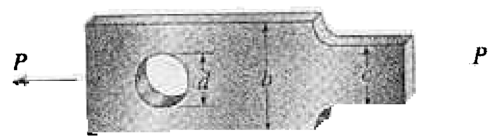
SOLUTION

Union College
Mechanical Engineering

1 A flat bar with a hole has widths b=2.4in and c=1.6in. The fillets have radii equal to 0.2in. what is the diameter d_{max} of the largest hole that can be drilled through the bar without reducing the load-carrying capacity?



1.6in Section

$$\sigma_{AVE} = \frac{P}{A} = \frac{P}{1.6in \cdot t}$$

$$\sigma_{max} = K \cdot \sigma_{AVE} = \frac{K \cdot P}{1.6in \cdot t}$$

FROM THE ATTACHED FIGURES

$$\left.\begin{aligned} \frac{b}{c} &= \frac{2.4in}{1.6in} = 1.5 \\ \frac{r}{c} &= \frac{0.2in}{1.6in} = 0.125 \end{aligned}\right\} K = 2.125 \Rightarrow \sigma_{max} = \frac{2.125\,P}{1.6in \cdot t} = \frac{1.328}{in} \cdot \frac{P}{t} \quad (1)$$

2.4in SECTION

AVERAGE STRESS WITH THE HOLE

$$\sigma_{AVE} = \frac{P}{t \cdot (2.4in - d)}$$

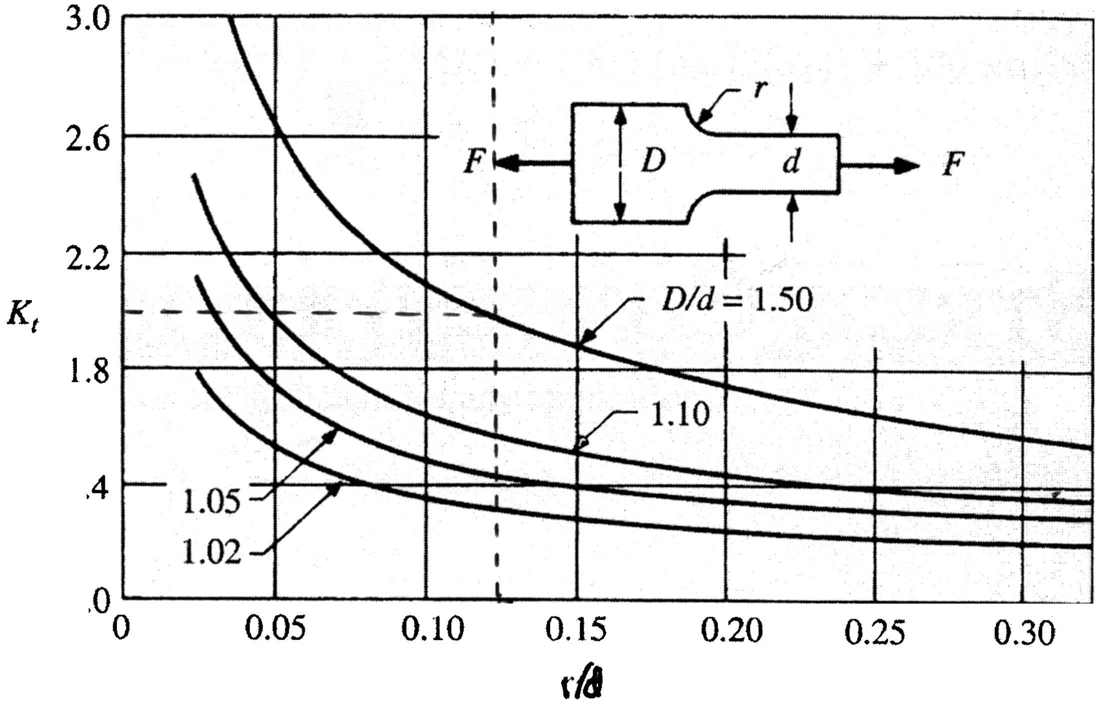
$$\sigma_{max} = \frac{K(d) \cdot P}{t \cdot (2.4in - d)} \quad (2)$$

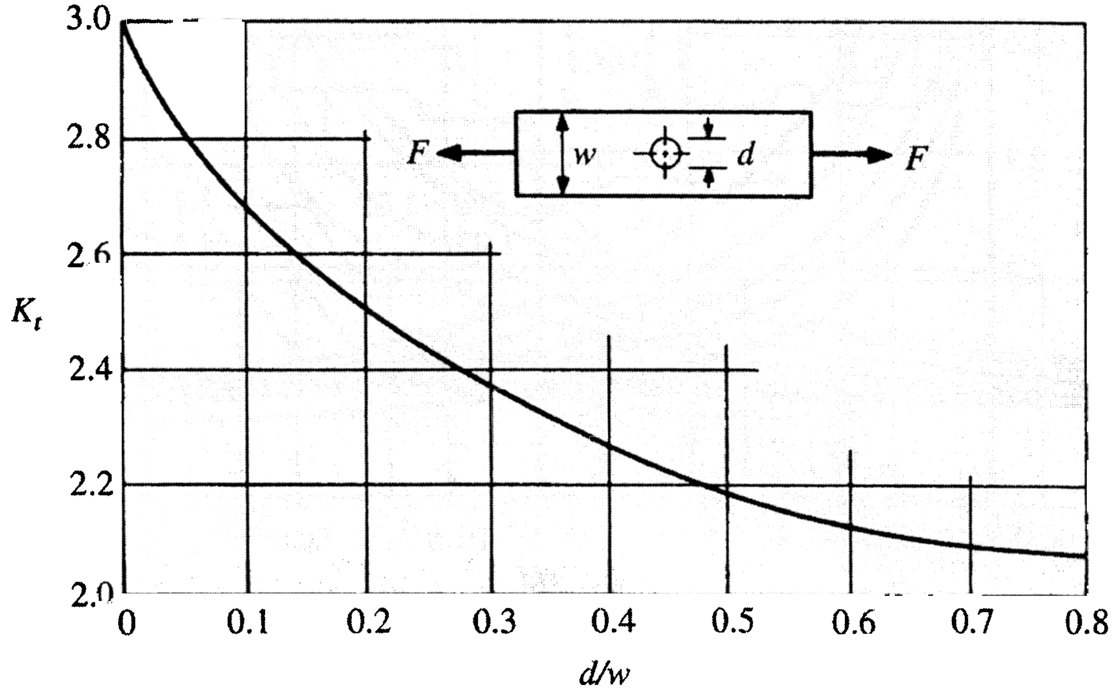
WITHOUT THE HOLE THE STRESS IN THE 1.6in MEMBER REPRESENTS THE MAXIMUM LOADING CONDITION. THUS THE LOAD IN THE 2.4in WIDE SECTION NEEDS TO STAY LESS THAN THIS TO MEET THE STATED CONDITION.

$$\frac{1.328}{in} \cdot \frac{P}{t} > \frac{K(d) \cdot P}{t \cdot (2.4in - d)} \Rightarrow 1.328 > \frac{K(d)}{}$$

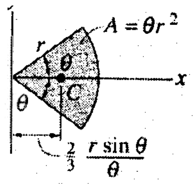
NEED TO TRY A FEW VALUES

d [in]	d/b	K	$\frac{K(d)}{2.4in - d}$
1in	.4167	2.25	1.607
0.5in	.2083	2.5	1.316
0.52in	.2167	2.49	1.325 $\approx$ 1.328 $\Rightarrow \boxed{d_{max} = 0.52in}$


3.0
2.6
2.2
K_t
1.8
.4
.0
0
0.05
0.10
0.15
0.20
0.25
0.30
r/d
F
D
d
r
F
D/d = 1.50
1.10
1.05
1.02


3.0
2.8
2.6
K_t
2.4
2.2
2.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
d/w
F
w
d
F

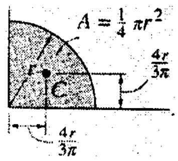
Centroid Location	Area Moment of Inertia



Circular sector area

$I_x = \frac{1}{4} r^4 (\theta - \frac{1}{2}\sin 2\theta)$

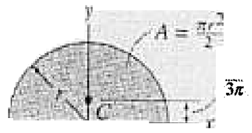
$I_x = \frac{1}{4} r^4 (\theta + \frac{1}{2}\sin 2\theta)$



Quarter circle area

$I_x = \frac{1}{16}\pi r^4$

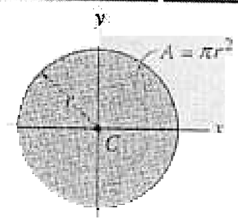
$I_y = \frac{1}{16}\pi r^4$



Semicircular area

$I_x = \frac{1}{8}\pi r^4$

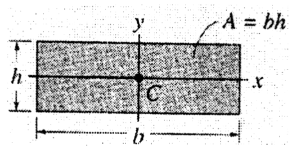
$I_y = \frac{1}{8}\pi r^4$



Circular area

$I_x = \frac{1}{4}\pi r^4$

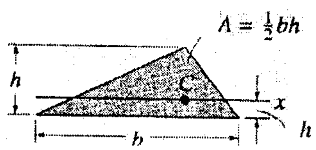
$I_y = \frac{1}{4}\pi r^4$



Rectangular area

$I_x = \frac{1}{12}bh^3$

$I_y = \frac{1}{12}hb^3$



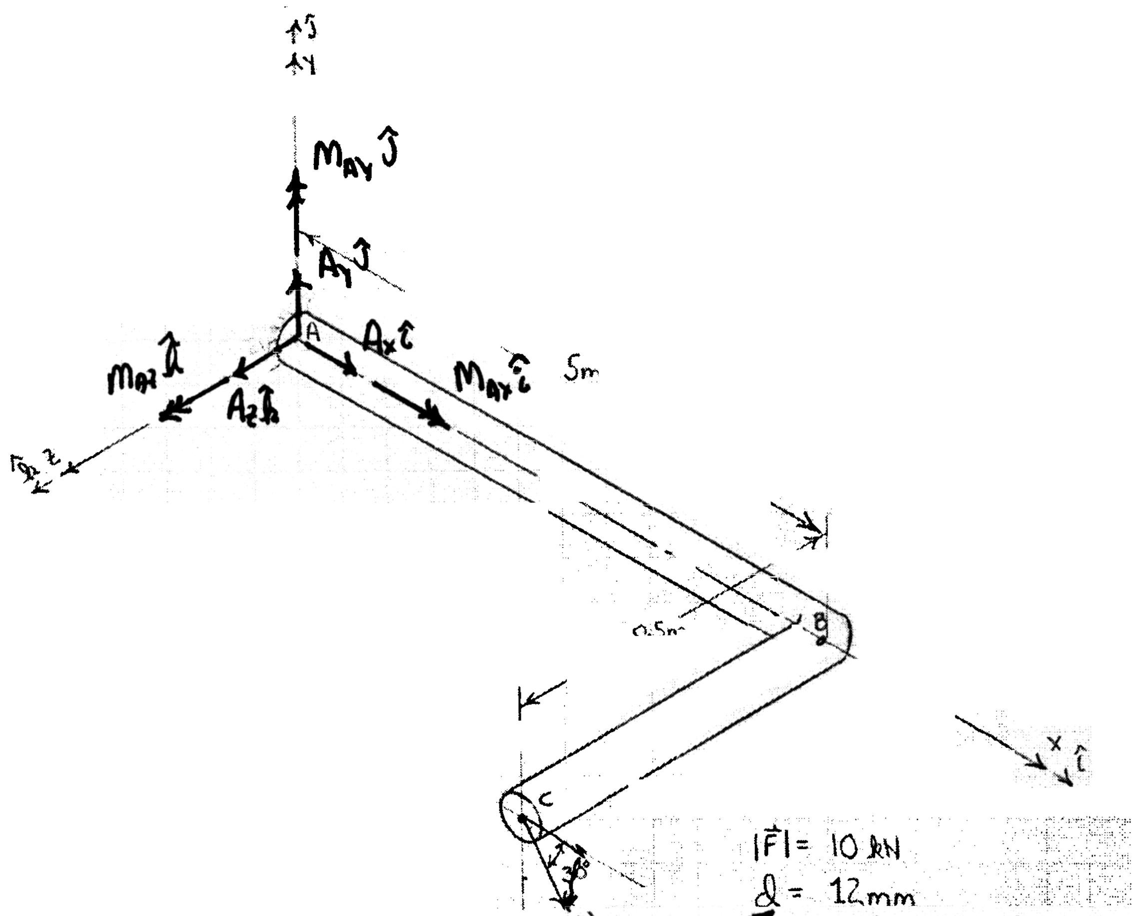
Triangular area

$I_x = \frac{1}{36}bh^3$

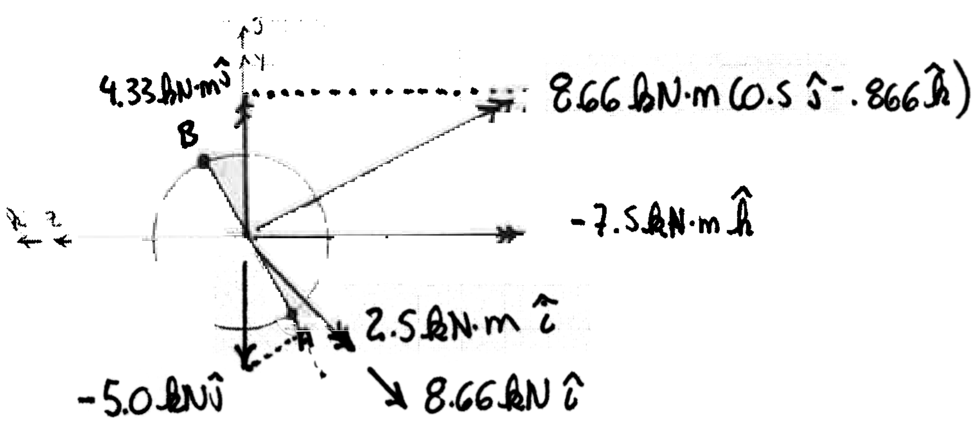
2. The L-bracket shown is 12mm in diameter and is built into the wall at A. at point C a 10kN load is applied as shown.

2a. Using the diagram below, draw the free-body diagram and determine the reactions at A. Using the diagram provided, illustrate the resultant bending moment, torque, normal force, and shearing force at the wall.

$$I = \pi \cdot d^4 / 64 \qquad J = \pi \cdot d^4 / 32$$



$\vec{F} = 10\text{kN}(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}) = 8.66\text{kN}\hat{i} - 5.0\text{kN}\hat{j})$



$$\Sigma F_x = 0 = A_x + 8.66\,kN \Rightarrow \underline{\underline{A_x = -8.66\,kN}}$$

$$\Sigma F_y = 0 = A_y - 5.0\,kN \Rightarrow \underline{\underline{A_y = 5.0\,kN}}$$

$$\Sigma F_z = \underline{\underline{0 = A_z}}$$

$$\Sigma \vec{M}_A = 0 = \vec{r}_{AC} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.5m & 0 & 0.5m \\ 8.66kN & -5kN & 0 \end{vmatrix} + \vec{M}_A$$

$$= [-(0.5m)(-5kN)]\hat{i} - [(0.5m)(8.66kN)]\hat{j} + [(1.5m)(-5kN)]\hat{k} + M_{Ax}\hat{i} + M_{Ay}\hat{j} + M_{Az}\hat{k}$$

$$\hat{i}: \quad 0 = 2.5\,kN\cdot m + M_{Ax} \Rightarrow \underline{\underline{M_{Ax} = -2.5\,kN\cdot m}}$$

$$\hat{j}: \quad 0 = 4.33\,kN\cdot m + M_{Ay} \Rightarrow \underline{\underline{M_{Ay} = -4.33\,kN\cdot m}}$$

$$\hat{k}: \quad 0 = -7.5\,kN\cdot m + M_{Az} \Rightarrow \underline{\underline{M_{Az} = 7.5\,kN\cdot m}}$$

2b. At the point of the maximum bending stress in the beam at the wall, determine the complete state of stress, designate the location on the previous illustration, and draw the resultant stress cube.

MAXIMUM TENSION AT POINT B $76.57(10^6)$ $51.05(10^9)$

$$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{8.66(10^3)N}{\pi[12(10^{-3})m/2]^2} + \frac{8.66(10^3)N\cdot m\cdot[12(10^{-3})m/2]}{\frac{\pi[12(10^{-3})m]^4}{64}}$$

$$51.07(10^9)\frac{N}{m^2} = \boxed{51.07\,GPa}$$

SHEAR STRESS DUE TO TORQUE

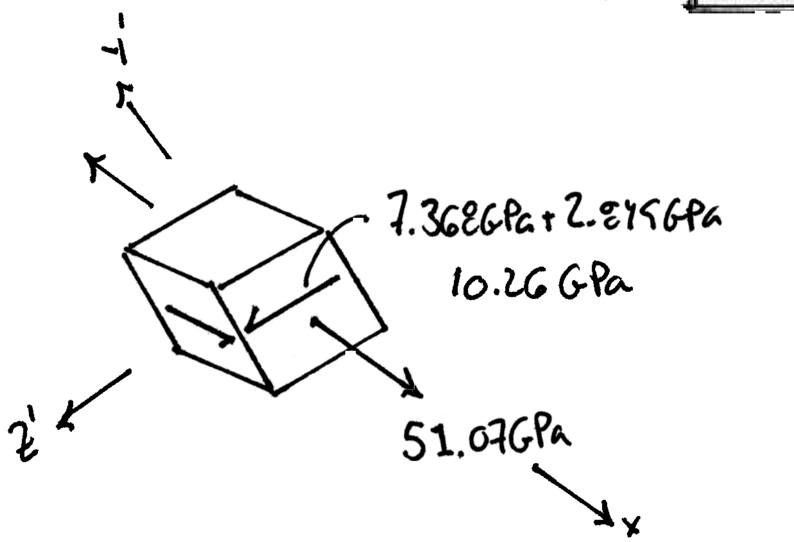
$$\tau_T = \frac{T\cdot r}{J} = \frac{2.5kN\cdot m\cdot 6(10^{-3})m}{\frac{\pi[12(10^{-3})m]^4}{32}} = 7.368(10^9)\frac{N}{m^2} = \boxed{7.368\,GPa}$$

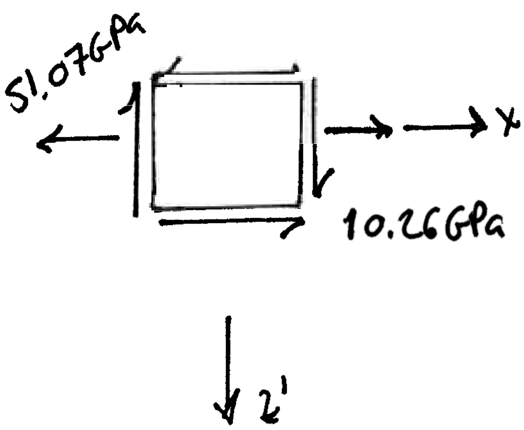
COMPONENT OF THE SHEAR DUE TO THE SHEAR FORCE PERPENDICULAR TO THE MOMENT

$$5.8kN\,\hat{j}\cdot(.5\hat{j} + .866\hat{k}) = 2.9kN$$

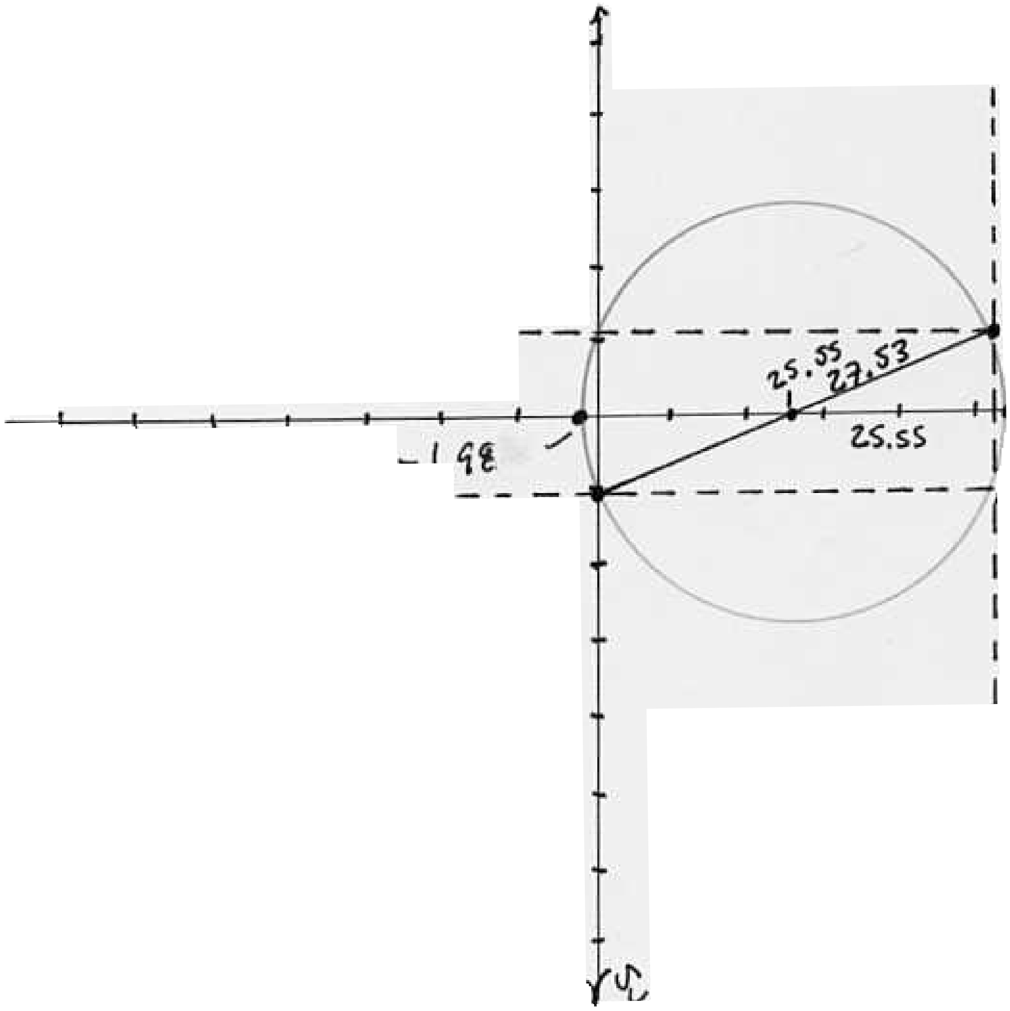
$$\tau_V = \frac{VQ}{It} = \frac{2.9kN\cdot 0.4244\cdot\left(\frac{12(10^{-3})m}{2}\right)\cdot\left(\frac{\pi\cdot 12(10^{-3})m}{8}\right)}{\frac{\pi\cdot[12(10^{-3})m]^4}{64}\cdot 12(10^{-3})m}$$

$$= 2.849(10^9)\frac{N}{m^2} = \boxed{2.849\,GPa}$$





2c. Using the Von Misses and maximum shear stress criterion, determine the factor of safety for this structure if the yield strength is 100GPa.



Max Shear stress

$$N = \frac{50GPa}{27.53GPa} = \underline{1.82}$$

Von Misses

$$N = \frac{100GPa}{\sqrt{\frac{(53.08-0)^2+(0+1.98)^2+(53.08+1.98)^2}{2}}} = \frac{100GPa}{54.09GPa} = \underline{\underline{1.85}}$$