MER311: Advanced Strength of Materials

Energy Methods

- Examples

Summary of Energy Equations

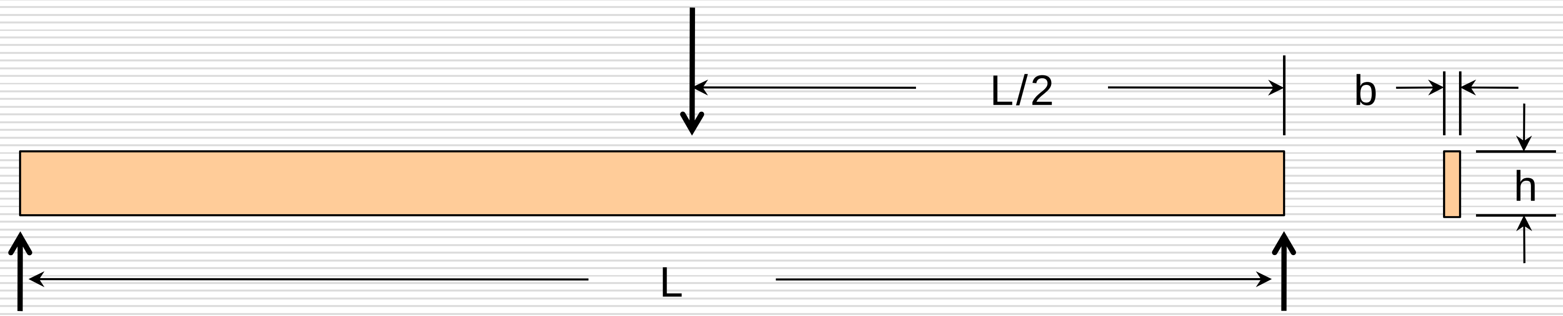
Load Type	Factors Involved	General Equations	For Const Factors
Axial	P,E,A	$U=\int_0^L \frac{P^2}{2\cdot E\cdot A}\cdot dx$	$U=\frac{P^2\cdot L}{2\cdot A\cdot E}$
Bending	M,E,I	$U=\int_0^L \frac{M^2}{2\cdot E\cdot A}\cdot dx$	$U=\frac{M^2\cdot L}{2\cdot A\cdot E}$
Torsion	T,G,k	$U=\int_0^L \frac{T^2}{2\cdot G\cdot k}\cdot dx$	$U=\frac{T^2\cdot L}{2\cdot G\cdot k}$
Tran. Shear	V,G,A	$U=\int_0^L \frac{3\cdot V^2}{5\cdot G\cdot A}\cdot dx$	$U=\frac{3\cdot V^2\cdot L}{5\cdot A\cdot G}$

Summary of Deflection Equations

Load Type	Factors Involved	General Equations	For Const Factors
Axial	P,E,A	$\Delta = \int_0^L \frac{P \cdot \left(\partial P / \partial Q\right)}{E \cdot A} \cdot dx$	$\Delta = \frac{P \cdot L}{A \cdot E}$
Bending	M,E,I	$\Delta = \int_0^L \frac{M \cdot \left(\partial M / \partial Q\right)}{E \cdot A} \cdot dx$	$\Delta = \frac{M \cdot L}{A \cdot E}$
Torsion	T,G,k	$\Delta = \int_0^L \frac{T \cdot \left(\partial T / \partial Q\right)}{G \cdot k} \cdot dx$	$\Delta = \frac{T \cdot L}{G \cdot k}$
Tran. Shear	V,G,A	$\Delta = \int_0^L \frac{6 \cdot V \cdot \left(\partial V / \partial Q\right)}{5 \cdot G \cdot A} \cdot dx$	$\Delta = \frac{6 \cdot V \cdot L}{5 \cdot A \cdot G}$

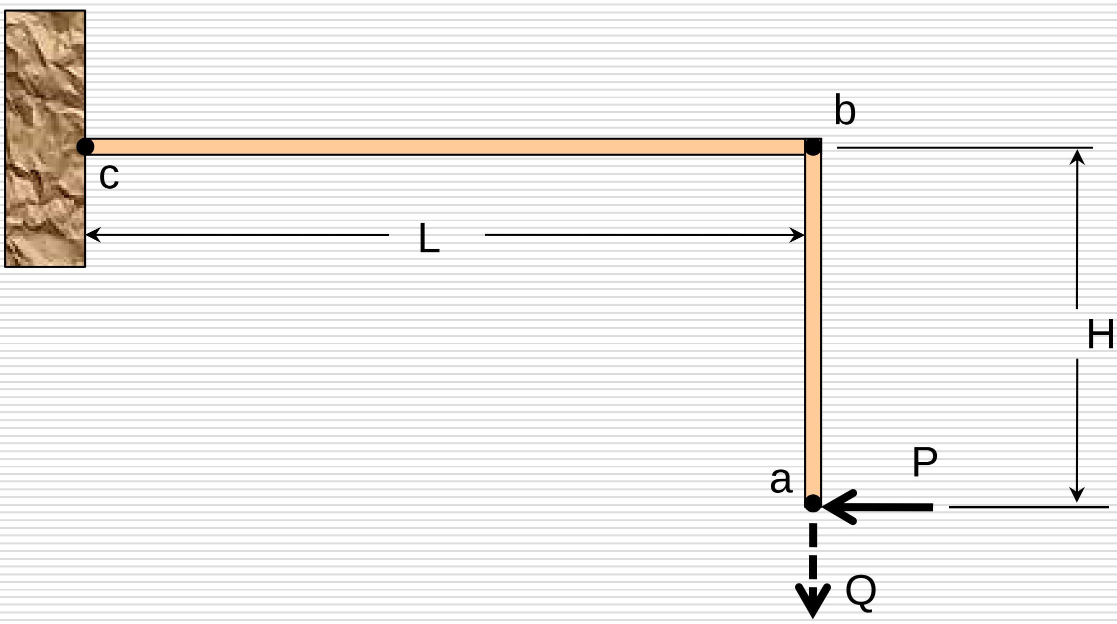
Example 1

Determine the deflection at the center of the beam illustrated due to bending.



Example 2

Find the vertical deflection at the free end of the beam that results from the load P being applied to the beam as shown.

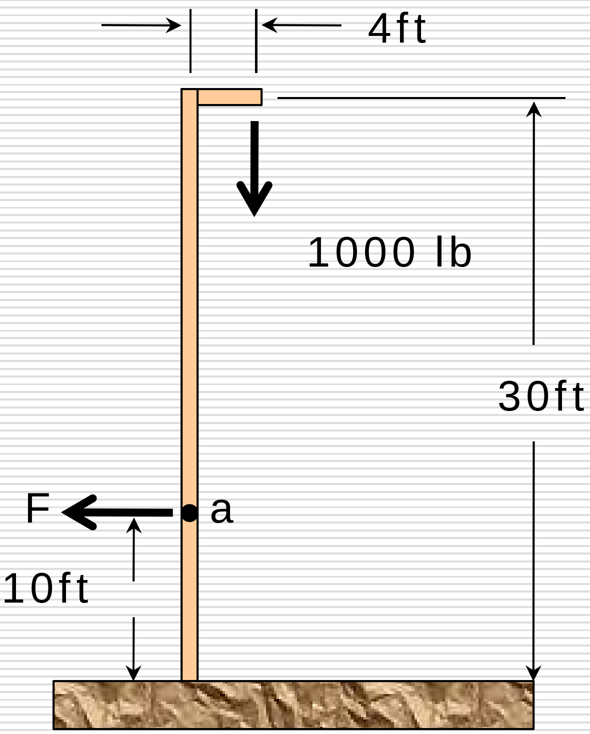


Example 3

A pole supports an eccentric load. The pole is “fixed” at the bottom end, and supported horizontally by a guy wire at point a. The tension F in the guy wire is adjusted to make the pole deflection equal to zero where the wire is attached.

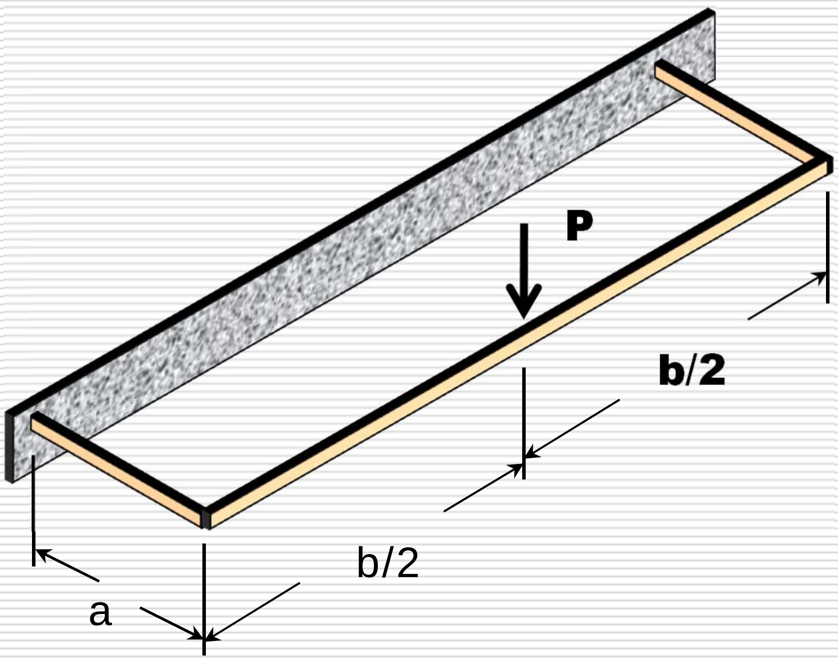
(a) What is the guy wire force?

(b) How do properties E and I of the pole affect this force?



Example 4

A three-sided bracket with a vertical center load is shown. The bracket has the same cross section at all points.



(a) Determine the deflection at the load P.

(b) Determine the torque at the two points of attachment of the bracket, what twisting moment – in addition to the bending moment – would be applied to each weld

Example 5

For the wire form shown the sectional rigidity is EI. Using Castigliano's method determine the change in the gap where the loads are applied. Consider the effect of bending only.