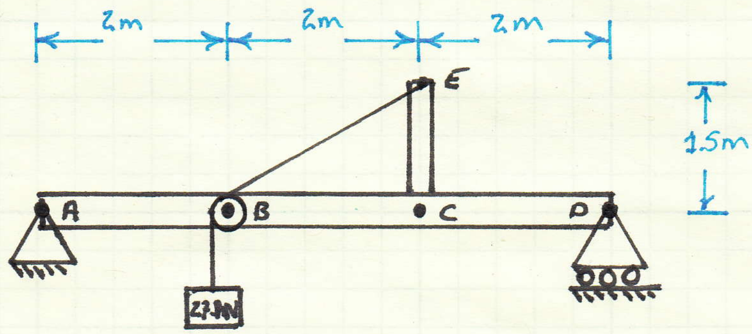
PROBLEM 8 CONSTRUCT THE SHEAR FORCE, BENDING MOMENT, CURVATURE, AND DEFLECTION DIAGRAMS FOR THIS BEAM.

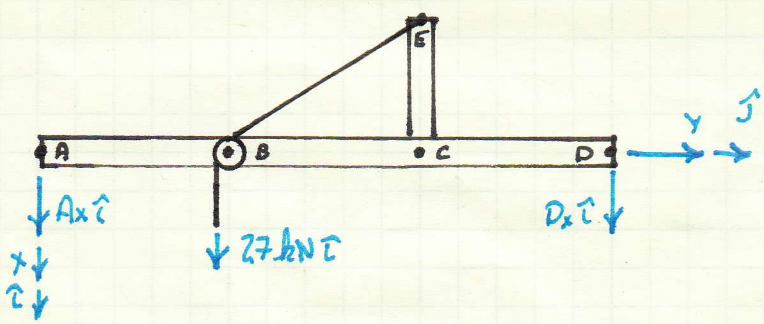
GIVEN:
1. A SIMPLY SUPPORTED BEAM OF LENGTH 6 m.
2. 1.5 m VERTICAL ARM LOCATED 4 m FROM THE LEFT END OF THE BEAM
3. A PULLEY LOCATED ON THE HORIZONTAL BEAM 2 m FROM THE LEFT END.
4. A CABLE ATTACHED AT THE TOP OF THE VERTICAL ARM WRAPPING AROUND THE PULLEY.
5. 27 kN LOAD ATTACHED TO THE CABLE UNDER THE BEAM

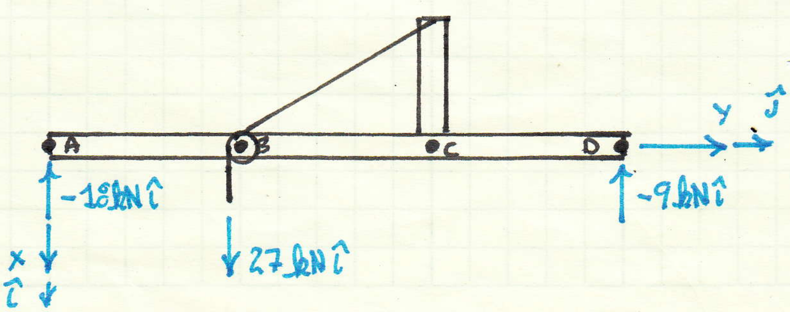
ASSUMPTIONS:
1. THE BEAM IS ORIGINALLY STRAIGHT, THE HORIZONTAL DEFLECTIONS ARE SMALL
2. LINEAR-ELASTIC MATERIAL RESPONSE
3. VERTICAL ARM IS RIGID
4. THE CABLE IS RIGID
5. THE ROLLER IS FRICTIONLESS AND THE RADIUS CAN BE IGNORED.

FIND:
1. SHEAR DIAGRAM
2. BENDING MOMENT DIAGRAM
3. CURVATURE (ELASTIC CURVE SLOPE) DIAGRAM
4. DISPLACEMENT DIAGRAM

FIGURE:







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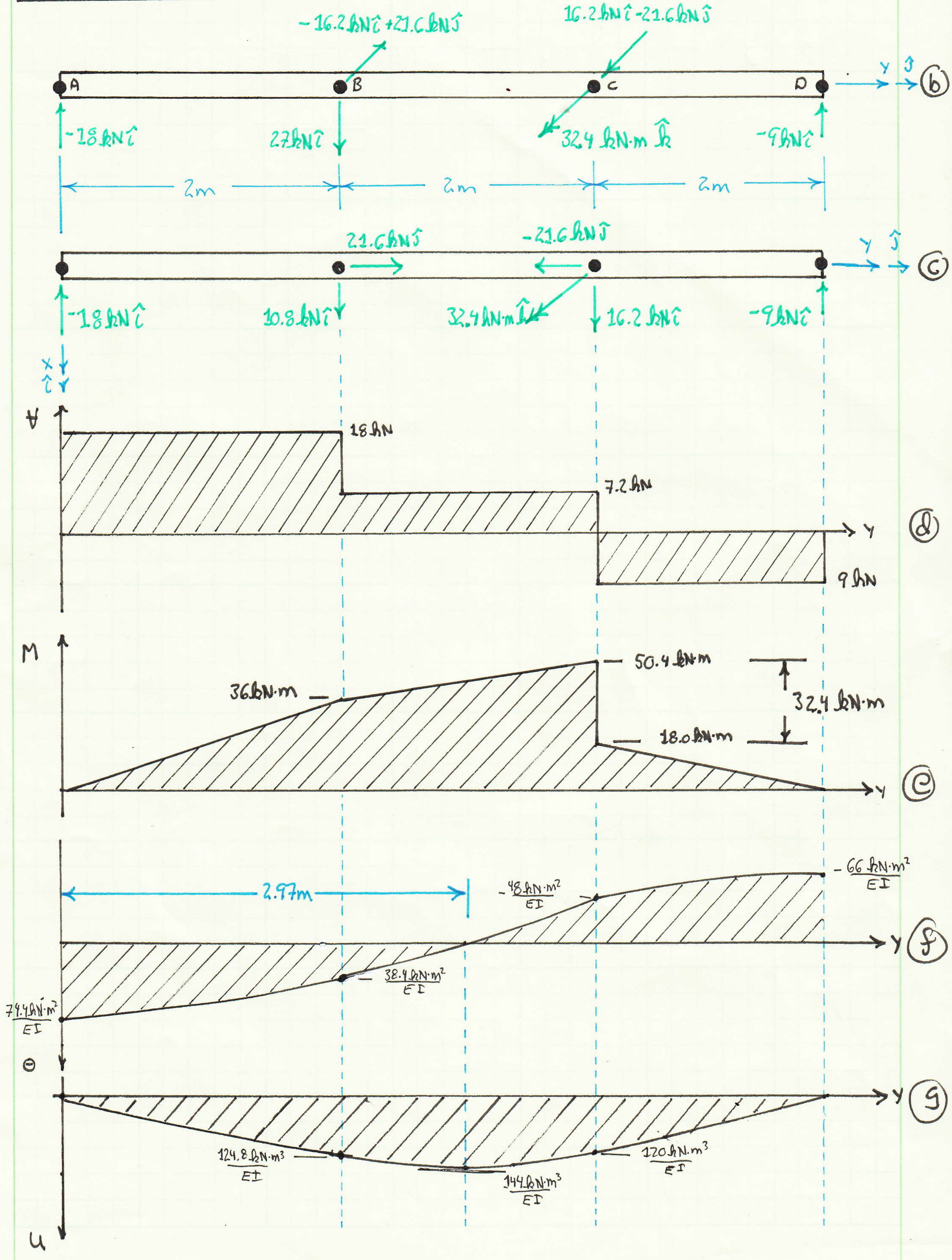
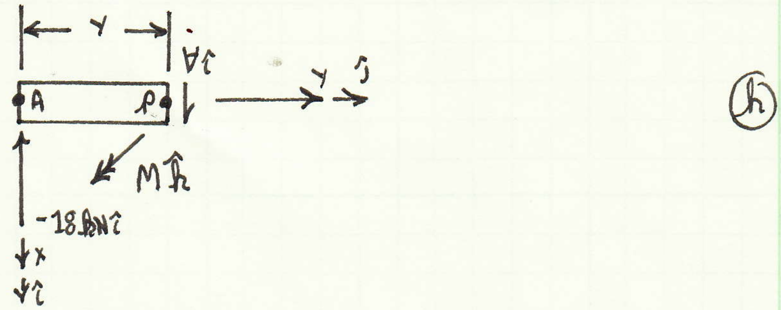
Solution Diagrams:



THE SHEAR AND BENDING MOMENT IN THE BEAM AS A FUNCTION OF Y CAN BE DETERMINED BY CONSIDERING THE INTERNAL EQUILIBRIUM OF ~~THE~~ EACH OF THE LOAD SEGMENTS.

$0 < y < 2m$



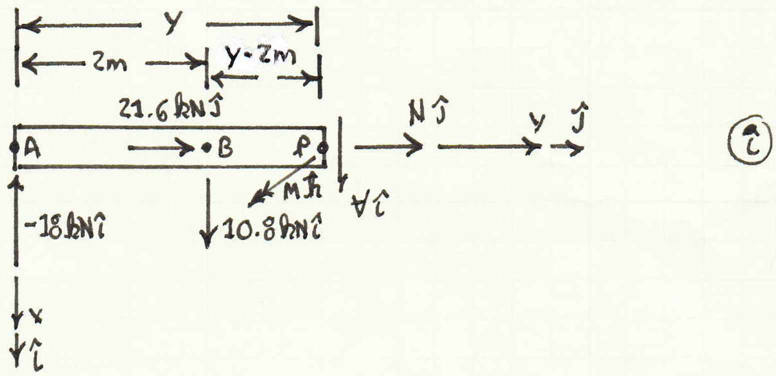
$\Sigma F_x = 0 = -18\,kN + V$

$\Rightarrow \underline{V = 18\,kN}$ (1)

$\Sigma M_{z@P} = 0 = M - y \cdot 18\,kN$

$\Rightarrow \underline{M = 18\,kN \cdot y}$ (2)

$2m < y < 4m$



$\Sigma F_x = 0 = -18\,kN + 10.8\,kN + V$

$\underline{V = 7.2\,kN}$ (3)

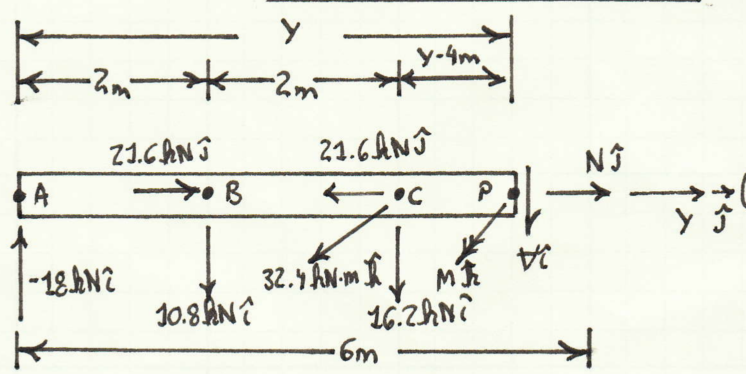
$\Sigma F_y = 0 = 21.6\,kN + N$

$\underline{N = -21.6\,kN}$ (4)

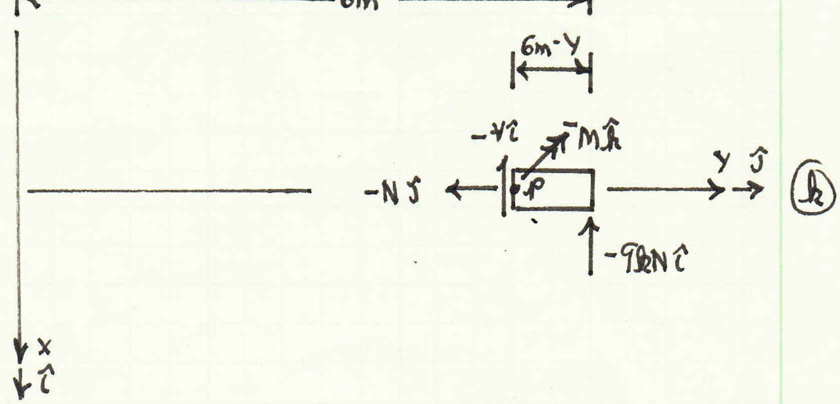
$\Sigma M_{z@P} = 0 = M + (10.8\,kN)\cdot(y - 2m) - 18\,kN \cdot y = M - 10.8\,kN \cdot (2m) + 10.8\,kN \cdot y - 18\,kN \cdot y$

$= M - 21.6\,N{:}m - 7.2\,k_y \cdot y \Rightarrow \underline{M = 7.2\,kN \cdot y + 21.6\,kN \cdot m}$ (5)

$4m < y < 6m$



FOR THIS SECTION EITHER THE SEGMENT OF BEAM IN (j) OR (k) CAN BE CONSIDERED, THEY ARE BOTH EQUIVALENT AS DRAWN. (k) IS LESS complex, SO IT WILL BE CONSIDERED.



$\Sigma F_x = 0 = -V - 9\,kN$

$\Rightarrow \underline{V = -9\,kN}$ (6)

$\Sigma F_y = 0 = -N \Rightarrow \underline{N = 0}$ (7)

$$\Sigma M_{z@p} = 0 = -M + (9\,kN)\cdot(6m - y)$$

$$\Rightarrow M = 9kN\cdot 6m - 9kN\cdot y = \underline{\underline{-9kN\cdot y + 54\,kN\cdot m = M}} \quad (8)$$

FOR THE THREE REGIONS CONSIDERED ABOVE, LET'S CONSIDER THE VALUES OF ~~M~~ THE NORMAL LOAD (N), SHEAR FORCE (V), AND BENDING MOMENT.

<u>$0 < y < 2m$</u>

$(1) \rightarrow V(0) = 18kN$

$(1) \rightarrow V(2m) = 18kN$ (9)

$(2) \rightarrow M(0) = 18kN\,(0m) = 0\,kN\cdot m$

$(2) \rightarrow M(2m) = 18kN\cdot(2m) = 36\,kN\cdot m$

<u>$2m < y < 4m$</u>

$(3) \rightarrow V(2m) = 7.2\,kN$

$(3) \rightarrow V(4m) = 7.2\,kN$ (10)

$(5) \rightarrow M(2m) = 7.2kN\cdot(2m) + 21.6\,kN\cdot m = 36\,kN\cdot m$

$(5) \rightarrow M(4m) = 7.2kN\cdot(4m) + 21.6\,kN\cdot m = 50.4\,kN\cdot m$

<u>$4m < y < 6m$</u>

$(6) \rightarrow V(4m) = -9\,kN$

$(6) \rightarrow V(6m) = -9\,kN$ (11)

$(8) \rightarrow M(4m) = -9kN\cdot(4m) + 54\,kN\cdot m = 18.0\,kN\cdot m$

$(8) \rightarrow M(6m) = -9kN\cdot(6m) + 54\,kN\cdot m = 0\,kN\cdot m$

EXPRESSIONS FOR THE SLOPE OF THE ELASTIC CURVE AND THE DISPLACEMENT OF THE ELASTIC CURVE IN EACH REGION CAN NOW BE DETERMINED USING DIRECT INTEGRATION

$0 < y < 2m$

STARTING WITH (2)

$$\theta(y) = \int \frac{-M}{EI} \cdot dy = \frac{1}{EI} \int -18kN \cdot y \cdot dy = -\frac{18kN \cdot y^2}{2EI} + C_1$$

$$= -\frac{9kN \cdot y^2}{EI} + C_1 \qquad (12)$$

$$u(y) = \int \theta(y) \cdot dy = \int \left[-\frac{9kN}{EI} \cdot y^2 + C_1 \right] dy$$

$$= -\frac{9kN \cdot y^3}{3 \cdot EI} + C_1 \cdot y + C_2 = -\frac{3\ kN \cdot y^3}{EI} + C_1 \cdot y + C_2 \qquad (13)$$

THE ONLY BOUNDARY CONDITION IN THIS REGION IS AT $y=0$, $u(0)=0$. SUBSTITUTING THIS BOUNDARY CONDITION INTO (13)

$$u(0) = 0 = -\frac{3\ kN \cdot (0)^3}{EI} + C_1 \cdot (0) + C_2 \Rightarrow C_2 = 0$$

(13) CAN NOW BE REWRITTEN

$$u(y) = -\frac{3\ kN \cdot y^3}{EI} + C_1 \cdot y \qquad (14)$$

THE VALUES OF (12) AND (14) AT $y=2m$ NEED TO BE CALCULATED BECAUSE THE FORM THE INITIAL CONDITIONS FOR THE NEXT REGION. THIS TYPE OF BOUNDARY CONDITION IS OFTEN REFERRED TO AS A CONTINUITY CONDITION. FOR CLARITY u IN THIS REGION WILL BE REFERD TO AS u_{AB} AND θ AS θ_{AB}.

$$u_{AB}(2m) = -\frac{3\ kN \cdot (2m)^3}{EI} + 2m \cdot C_1 = -\frac{24\ kN \cdot m^3}{EI} + 2m \cdot C_1 \qquad (15)$$

$$\theta_{AB}(2m) = -\frac{9kN}{EI} \cdot (2m)^2 + C_1 = -\frac{36\ kN \cdot m^2}{EI} + C_1 \qquad (16)$$

NOW THE NEXT REGION OF THE BEAM CAN BE CONSIDERED.

$2m < Y < 4m$

THE SLOPE OF THE ELASTIC CURVE IN THIS REGION REQUIRES THE INTEGRATION OF (5)

$$\Theta_{BC}(Y) = \int \frac{-M}{EI}\,dY = \int\left[-\frac{7.2\,kN\cdot Y}{EI} - \frac{21.6\,kN\cdot m}{EI}\right]dY$$

$$= -\frac{7.2\,kN\cdot Y^2}{2EI} - \frac{21.6\,kN\cdot m\cdot Y}{EI} + C_3$$

$$= -\frac{3.6\,kN\cdot Y^2}{EI} - \frac{21.6\,kN\cdot m\cdot Y}{EI} + C_3 \qquad (17)$$

$$U_{BC}(Y) = \int \Theta_{BC}\,dY = \int\left[-\frac{36\,kN\cdot Y^2}{EI} - \frac{21.6\,kN\cdot m}{EI}\cdot Y + C_3\right]dY$$

$$= -\frac{3.6\,kN\cdot Y^3}{3\cdot EI} - \frac{21.6\,kN\cdot m\cdot Y^2}{2EI} + C_3\cdot Y + C_4$$

$$= -\frac{1.2\,kN\cdot Y^3}{EI} - \frac{10.8\,kN\cdot m\cdot Y^2}{EI} + C_3\cdot Y + C_4 \qquad (18)$$

THE ONLY BOUNDARY CONDITIONS IN THIS REGION ARE THE CONTINUITY CONDITIONS AT 2m FROM THE END OF REGION AB IN (15) AND (16).

STARTING WITH (17) AT 2m AND COMPARING IT TO (16)

$$\Theta_{BC}(2m) = \Theta_{AB}(2m)$$

$$-\frac{36\,kN\cdot(2m)^2}{EI} - \frac{21.6\,kN\cdot m\cdot(2m)}{EI} + C_3 = -\frac{36\,kN\cdot m^2}{EI} + C_1$$

$$-\frac{14.4\,kN\cdot m^2}{EI} - \frac{43.2\,kN\cdot m^2}{EI} + C_3 = -\frac{36\,kN\cdot m^2}{EI} + C_1$$

$$C_3 = \frac{21.6\,kN\cdot m^2}{EI} + C_1 \qquad (19)$$

(17) CAN NOW BE REWRITTEN

$$\underline{\underline{\Theta_{BC}(Y) = -\frac{3.6\,kN\cdot Y^2}{EI} - \frac{21.6\,kN\cdot m}{EI}\cdot Y + \frac{21.6\,kN\cdot m^2}{EI} + C_1}} \qquad (20)$$

(18) CAN ALSO BE REWRITTEN

$$u_{BC}(y) = -\frac{1.2\,kN\cdot y^3}{EI} - \frac{10.8\,kN\cdot m\cdot y^2}{EI} + \left[\frac{21.6\,kN\cdot m^2}{EI} + C_1\right]\cdot y + C_4$$

$$= -\frac{1.2\,kN\cdot y^3}{EI} - \frac{10.8\,kN\cdot m\cdot y^2}{EI} + \frac{21.6\,kN\cdot m^2}{EI}\cdot y + C_1\cdot y + C_4 \quad (21)$$

NOW THE VALUE OF (21) AT 2 m CAN BE COMPARED TO THE CONTINUITY CONDITION IN (15)

$$u_{BC}(2m) = u_{AB}(2m)$$

$$-\frac{1.2\,kN}{EI}\cdot(2m)^3 - \frac{10.8\,kN\cdot m}{EI}\cdot(2m)^2 + \frac{21.6\,kN\cdot m^2}{EI}(2m) + C_1(2m) + C_4$$

$$= -\frac{24\,kN\cdot m^3}{EI} + 2m\cdot C_1$$

$$-\frac{9.6\,kN\cdot m^3}{EI} - \frac{43.2\,kN\cdot m^3}{EI} + \frac{43.2\,kN\cdot m^2}{EI} + C_1(2m) + C_4 = -\frac{24\,kN\cdot m^3}{EI} + 2m\cdot C_1$$

$$-\frac{9.6\,kN\cdot m^3}{EI} + \cancel{C_1(2m)} + C_4 = \cancel{2m\cdot C_1} - \frac{24\,kN\cdot m^3}{EI}$$

$$C_4 = -\frac{14.4\,kN\cdot m^3}{EI} \quad (22)$$

(21) CAN NOW BE REWRITTEN

$$u_{BC}(y) = -\frac{1.2\,kN}{EI}\cdot y^3 - \frac{10.8\,kN\cdot m}{EI}\cdot y^2 + \frac{21.6\,kN\cdot m^2}{EI}\cdot y + C_1\cdot y - \frac{14.4\,kN\,m^3}{EI} \quad (23)$$

AS WAS THE CASE IN THE PREVIOUS REGION, THE VALUE OF (20) AND (23) NEED TO BE EVALUATED AT THE OTHER END OF THE REGION, $y = 4m$, TO ESTABLISH THE CONTINUITY CONDITIONS FOR THE REGION BETWEEN C AND D.

$$(20) \rightarrow \Theta_{BC}(4m) = -\frac{3.6\,kN}{EI}\cdot(4m)^2 - \frac{21.6\,kN\cdot m}{EI}\cdot(4m) + \frac{21.6\,kN\cdot m^2}{EI} + C_1$$

$$= -\frac{57.6\,kN\cdot m^2}{EI} - \frac{86.4\,kN\cdot m^2}{EI} + \frac{21.6\,kN\cdot m^2}{EI} + C_1$$

$$= -\frac{122.4\,kN\cdot m^2}{EI} + C_1 \quad (24)$$

$$\text{(21)} \rightarrow u_{BC}(4m) = -\frac{1.2\,kN}{EI}\cdot(4m)^3 - \frac{10.8\,kN\cdot m}{EI}\cdot(4m)^2 + \frac{21.6\,kN\cdot m^2}{EI}\cdot(4m) + C_1\cdot(4m) - \frac{14.4\,kN\cdot m^3}{EI}$$

$$= -\frac{76.8\,kN\cdot m^3}{EI} - \frac{172.8\,kN\cdot m^3}{EI} + \frac{86.4\,kN\cdot m^3}{EI} + 4m\cdot C_1 - \frac{14.4\,kN\,m^3}{EI}$$

$$= -\frac{177.6\,kN\cdot m^3}{EI} + 4m\cdot C_1 \qquad (25)$$

(24) AND (25) ARE CONTINUITY CONDITIONS THAT WILL BE USED AS INITIAL CONDITIONS IN THE REGION FROM C TO D.

$4m < y < 6m$

THE MOMENT IN THIS REGION IS FOUND IN (8)

$$\theta(y) = \int -\frac{M}{EI}\,dy = \int\left[\frac{9\,kN}{EI}\cdot y - \frac{54\,kN\cdot m}{EI}\right]dy$$

$$= \frac{9\,kN}{2EI}y^2 - \frac{54\,kN\cdot m}{EI}\cdot y + C_5$$

$$= \frac{4.5\,kN}{EI}\cdot y^2 - \frac{54\,kN\cdot m}{EI}\cdot y + C_5 \qquad (26)$$

$$u(y) = \int \theta(y)\,dy = \int\left[\frac{4.5\,kN}{EI}\cdot y^2 - \frac{54\,kN\cdot m}{EI}\cdot y + C_5\right]dy$$

$$= \frac{4.5\,kN}{3EI}\cdot y^3 - \frac{54\,kN\cdot m}{2EI}\cdot y^2 + C_5\cdot y + C_6$$

$$= \frac{1.5\,kN}{EI}\cdot y^3 - \frac{27\,kN\cdot m}{EI}\cdot y^2 + C_5\cdot y + C_6 \qquad (27)$$

THE NEXT STEP IN THE SOLUTION WILL BE TO SUBSTITUTE IN THE CONTINUITY CONDITIONS FOUND IN (24) AND (25). THIS WILL ~~EA~~ LEAVE (26) AND (27) STILL WITH THE CONSTANT C_1. THIS FINAL CONSTANT WILL BE SOLVED FOR BY SUBSTITUTING THE END CONDITION AT $u_{CD}(6ft) = 0$.

STARTING WITH THE CONTINUITY CONDITION THAT $\theta_{BC}(4m)$, (24), HAS TO BE EQUAL TO $\theta_{CD}(4m)$. (θ_{CD} IS (26))

$$\theta_{CD}(4m) = \theta_{BC}(4m)$$

$$\frac{4.5\,kN}{EI}\cdot(4m)^2 - \frac{54\,kN\cdot m}{EI}\cdot(4m) + C_5 = -\frac{122.4\,kN\cdot m^2}{EI} + C_1$$

$$\frac{72\,kN\cdot m^2}{EI} - \frac{216\,kN\cdot m^2}{EI} + C_5 = -\frac{122.4\,kN\cdot m^2}{EI} + C_1$$

$$\Rightarrow C_5 = -\frac{122.4\,kN\cdot m^2}{EI} + \frac{144.0\,kN\cdot m^2}{EI} + C_1$$

$$C_5 = \frac{21.6\,kN\cdot m^2}{EI} + C_1 \qquad (28)$$

THIS RESULT CAN NOW BE SUBSTITUTED INTO (26)

$$\underline{\underline{\theta_{CD}(y) = \frac{4.5\,kN}{EI}\cdot y^2 - \frac{54\,kN\cdot m}{EI}\cdot y + \frac{21.6\,kN\cdot m^2}{EI} + C_1}} \qquad (29)$$

THE CONTINUITY CONDITION THAT $u_{BC}(4m)$, (25), HAS TO BE EQUAL TO $u_{CD}(4m)$ ($u_{CD}(y)$ IS 27) CAN NOW BE CONSIDERED

$$u_{CD}(4m) = u_{BC}(4m)$$

$$\frac{1.5\,kN}{EI}\cdot(4m)^3 - \frac{27\,kN\cdot m}{EI}\cdot(4m)^2 + C_5\cdot(4m) + C_6 = -\frac{177.6\,kN\cdot m^3}{EI} + 4m\cdot C_1$$

SUBSTITUTING THE RESULT IN (28)

$$\frac{1.5\,kN}{EI}\cdot(4m)^3 - \frac{27\,kN\cdot m}{EI}\cdot(4m)^2 + (4m)\left[\frac{21.6\,kN\cdot m^2}{EI} + C_1\right] + C_6 = -\frac{177.6\,kN\cdot m^3}{EI} + 4m\cdot C_1$$

$$\frac{1.5\,kN}{EI}\cdot(4m)^3 - \frac{27\,kN\cdot M}{EI}\cdot(4m)^2 + \frac{21.6\,kN\cdot m^2}{EI}\cdot(4m) + \cancel{4m\cdot C_1} + C_6 = -\frac{177.6\,kN\cdot m^3}{EI} + \cancel{4m\,C_1}$$

$$C_6 = -\frac{177.6\,kN\cdot m^3}{EI} - \frac{1.5\,kN}{EI}\cdot(4m)^3 + \frac{27\,kN\cdot m}{EI}\cdot(4m)^2 - \frac{21.6\,kN\cdot m^2}{EI}\cdot(4m)$$

$$= \frac{72.0\,kN\cdot m^3}{EI} \qquad (30)$$

(30) AND (28) ARE SUBSTITUTED INTO (27)

$$u(y) = \frac{1.5\,kN}{EI}\cdot y^3 - \frac{27\,kN\cdot m}{EI}\cdot y^2 + \left[\frac{21.6\,kN\cdot m^2}{EI} + C_1\right]\cdot y + \frac{72.0\,kN\cdot m^3}{EI}$$

$$u(y) = \frac{1.5\,kN}{EI}\cdot y^3 - \frac{27\,kN\cdot m}{EI}\cdot y^2 + \frac{21.6\,kN\cdot m^2}{EI}\cdot y + C_1\cdot y + \frac{72.0\,kN\cdot m^3}{EI} \quad (31)$$

THE CONSTANT C_1 CAN NOW BE DETERMINED BY APPLYING THE BOUNDARY CONDITION THAT AT THE END OF THE BEAM ($y=6m$) THE DEFLECTION IS ZERO, $u_{CD}(6m)=0$.

$$u_{CD}(6m) = \frac{1.5\,kN}{EI}\cdot(6m)^3 - \frac{27\,kN\cdot m}{EI}(6m)^2 + \frac{21.6\,kN\cdot m^2}{EI}(6m) + C_1\cdot(6m) + \frac{72\,kN\,m^3}{EI} = 0$$

$$= \frac{324.0\,kN\cdot m^3}{EI} - \frac{972.0\,kN\cdot m^3}{EI} + \frac{129.6\,kN\cdot m^3}{EI} + \frac{72\,kN\cdot m^3}{EI} + 6m\cdot C_1 = 0$$

$$\Rightarrow C_1 = \frac{446.4\,kN\cdot m^3}{6m\cdot EI} = \frac{74.4\,kN\cdot m^2}{EI} \quad (32)$$

C_1 WAS THE LAST CONSTANT. THIS CONSTANT NOW NEEDS TO BE SUBSTITUTED INTO THE EXPRESSIONS FOR THE SLOPE, θ, AND DEFLECTION, u, OF THE ELASTIC CURVE IN EACH REGION OF THE BEAM

<u>$0<y<2m$</u>

$$u_{AB}(y) = -\frac{3\,kN}{EI}\cdot y^3 + \left[\frac{74.4\,kN\cdot m^2}{EI}\right]y = -\frac{3\,kN}{EI}\cdot y^3 + \frac{74.4\,kN\cdot m^2}{EI}\cdot y \quad (33)$$

$$\theta_{AB}(y) = -\frac{9\,kN}{EI}\cdot y^2 + \frac{74.4\,kN\cdot m^2}{EI} \quad (34)$$

<u>$2m<y<4m$</u>

$$u_{BC}(y) = -\frac{1.2\,kN}{EI}\cdot y^3 - \frac{10.8\,kN\cdot m}{EI}\cdot y^2 + \frac{21.6\,kN\cdot m^2}{EI}\cdot y + \frac{74.4\,kN\cdot m^2}{EI}\cdot y - \frac{14.4\,kN\cdot m^3}{EI}$$

$$= -\frac{1.2\,kN}{EI}\cdot y^3 - \frac{10.8\,kN\cdot m}{EI}\cdot y^2 + \frac{96\,kN\cdot m^2}{EI}\cdot y - \frac{14.4\,kN\cdot m^3}{EI} \quad (35)$$

$$\theta_{BC}(y) = -\frac{3.6\,kN}{EI}\cdot y^2 - \frac{21.6\,kN\cdot m}{EI}\cdot y + \frac{21.6\,kN\cdot m^2}{EI} + \frac{74.4\,kN\cdot m^2}{EI}$$

$$= -\frac{3.6\,kN}{EI}\cdot y^2 - \frac{21.6\,kN\cdot m}{EI}\cdot y + \frac{96\,kN\cdot m^2}{EI} \quad (36)$$

4m<y<6m

$$u_{CD}(y) = \frac{1.5\,kN}{EI}\cdot y^3 - \frac{27\,kN\cdot m}{EI}\cdot y^2 + \frac{21.6\,kN\cdot m^2}{EI}\cdot y + \frac{74.4\,kN\cdot m^2}{EI}\,y + \frac{72.0\,kN\cdot m^3}{EI}$$

$$= \frac{1.5\,kN}{EI}\cdot y^3 - \frac{27\,kN\cdot m}{EI}\cdot y^2 + \frac{96\,kN\cdot m^2}{EI}\cdot y + \frac{72\,kN\cdot m^3}{EI} \quad (37)$$

$$\Theta_{CD}(y) = \frac{4.5\,kN}{EI}\cdot y^2 - \frac{54\,kN\cdot m}{EI}\cdot y + \frac{21.6\,kN\cdot m^2}{EI} + \frac{74.4\,kN\cdot m^2}{EI}$$

$$= \frac{4.5\,kN}{EI}\cdot y^2 - \frac{54\,kN\cdot m}{EI}\cdot y + \frac{96\,kN\cdot m^2}{EI} \quad (38)$$

THE VALUES OF THE SLOPE AND DISPLACEMENT OF THE ELASTIC CURVE AT THE ENDS OF EACH REGION NEED TO BE CHECKED USING (33) THROUGH (38)

0<y<2m

$$(33) \rightarrow u_{AB}(0) = -\frac{3\,kN}{EI}(0)^3 + \frac{74.4\,kN\cdot m^2}{EI}\cdot(0) = 0 \quad (39)$$

$$u_{AB}(2m) = -\frac{3\,kN}{EI}(2m)^3 + \frac{74.4\,kN\cdot m^2}{EI}(2m) = \frac{124.8\,kN\cdot m^3}{EI} \quad (40)$$

$$(34) \rightarrow \Theta_{AB}(0) = -\frac{9\,kN}{EI}\cdot(0)^2 + \frac{74.4\,kN\cdot m^2}{EI} = \frac{74.4\,kN\cdot m^2}{EI} \quad (41)$$

$$\Theta_{BC}(2m) = -\frac{9\,kN}{EI}(2m)^2 + \frac{74.4\,kN\cdot m^2}{EI} = \frac{38.4\,kN\cdot m^2}{EI} \quad (42)$$

2m<y<4m

$$(35) \rightarrow u_{BC}(2m) = -\frac{1.2\,kN}{EI}\cdot(2m)^3 - \frac{10.8\,kN\cdot m}{EI}\cdot(2m)^2 + \frac{96\,kN\cdot m^2}{EI}\cdot(2m) - \frac{14.4\,kN\cdot m^3}{EI}$$

$$= \underline{\underline{\frac{124.8\,kN\cdot m^3}{EI}}} \quad (43)$$

(SAME AS (40), $u_{BC}(2m) = u_{AB}(2m)$ ✓)

$$u_{BC}(4m) = -\frac{1.2\,kN}{EI}(4m)^3 - \frac{10.8\,kN\cdot m}{EI}\cdot(4m)^2 + \frac{96\,kN\cdot m^2}{EI}(4m) - \frac{14.4\,kN\cdot m^3}{EI}$$

$$= \underline{\underline{\frac{120\,kN\cdot m^3}{EI}}} \quad (44)$$

(36) $\rightarrow \Theta_{BC}(2m) = -\frac{3.6\,kN}{EI}\cdot(2m)^2 - \frac{21.6\,kN\cdot m}{EI}\cdot(2m) + \frac{96\,kN\cdot m^2}{EI}$

$= \underline{\frac{38.4\,kN\cdot m^2}{EI}}$ (45) (SAME AS (42), $\Theta_{BC}(2m) = \Theta_{AB}(2m)$ ✓)

$\Theta_{BC}(4m) = -\frac{3.6\,kN}{EI}(4m)^2 - \frac{21.6\,kN\cdot m}{EI}(4m) + \frac{96\,kN\cdot m^2}{EI}$

$= \underline{-\frac{48\,kN\cdot m^2}{EI}}$ (46)

<u>4m < y < 6m</u>

(37) $\rightarrow u_{CD}(4m) = \frac{1.5\,kN}{EI}\cdot(4m)^3 - \frac{27\,kN\cdot m}{EI}\cdot(4m)^2 + \frac{96\,kN\cdot m^2}{EI}(4m) + \frac{72\,kN\cdot m^3}{EI}$

$= \underline{\frac{120\,kN\cdot m^3}{EI}}$ (47) (SAME AS (44), $u_{CD}(4m) = u_{BC}(4m)$ ✓)

$u_{CD}(6m) = \frac{1.5\,kN}{EI}\cdot(6m)^3 - \frac{27\,kN\cdot m}{EI}\cdot(6m)^2 + \frac{96\,kN\cdot m^2}{EI}\cdot(6m) + \frac{72\,kN\cdot m^3}{EI}$

$= \underline{0}$ (48) (THIS AGREES WITH THE BOUNDARY CONDITION)

(38) $\Rightarrow \Theta_{CD}(4m) = \frac{4.5\,kN}{EI}\cdot(4m)^2 - \frac{54\,kN\cdot m}{EI}\cdot(4m) + \frac{96\,kN\cdot m^2}{EI}$

$= \underline{-\frac{48\,kN\cdot m^2}{EI}}$ (49) (SAME AS (46), $\Theta_{CD}(4m) = \Theta_{BC}(4m)$ ✓)

$\Theta_{CD}(6m) = \frac{4.5\,kN}{EI}(6m)^2 - \frac{54\,kN\cdot m}{EI}(6m) + \frac{96\,kN\cdot m^2}{EI}$

$= \underline{-\frac{66\,kN\cdot m^2}{EI}}$ (50)

FROM (45) AND (46), IT CAN BE CONCLUDED THE θ GOES TO ZERO IN THE REGION $2m < y < 4m$, BETWEEN B AND C. BECAUSE THE INTERCEPT OF THE SLOPE OF THE ELASTIC CURVE GOING TO ZERO INDICATES THE LOCATION OF THE MAX/MIN OF THE ELASTIC CURVE, WHERE $\theta_{BC} = 0$ IS IMPORTANT AND NEEDS TO BE LOCATED. SETTING (36) EQUAL TO ZERO

$$(36) \rightarrow \theta_{BC}(y) = 0 = -\frac{3.6\,kN}{EI}\cdot y^2 - \frac{21.6\,kN\cdot m}{EI}\cdot y + 96\,\frac{kN\cdot m^2}{EI}$$

$$\Rightarrow 0 = -3.6\cdot y^2 - 21.6m\cdot y + 96m^2$$

$$0 = y^2 + \frac{21.6m}{3.6}\cdot y - \frac{96m^2}{3.6} = y^2 + 6m\cdot y - 26.27m^2$$

$$\underbrace{y^2 + 6m\cdot y + \left(\frac{6m}{2}\right)^2}_{(y+3m)^2} - \left(\frac{6m}{2}\right)^2 - 26.27m^2 = 0$$

$$(y+3m)^2 = \left(\frac{6m}{2}\right)^2 + 26.27m^2 = 35.67m^2$$

$$y = -3m \pm \sqrt{35.67m^2} = -3m \pm 5.97m$$

$$= \underbrace{-8.97m}_{\text{OUTSIDE THE DOMAIN OF THE REGION}},\ \underline{\underline{2.97m}} \quad (51)$$

LOCATION OF θ INTERCEPT AND MAXIMUM DEFLECTION OF THE ELASTIC CURVE.

THE MAXIMUM DEFLECTION OF THE ELASTIC CURVE CAN NOW BE CALCULATED BY SUBSTITUTING (51) INTO (35)

$$(35) \rightarrow u_{BC}(y) = -\frac{1.2\,kN}{EI}\cdot(2.97m)^3 - \frac{10.8\,kN\cdot m}{EI}\cdot(2.97m)^2 + \frac{96\,kN\cdot m^2}{EI}\cdot(2.97m) - \frac{14.4\,kN\cdot m^3}{EI}$$

$$= \underline{\underline{\frac{144.0\,kN\cdot m^3}{EI}}} \quad (52)$$

DIRECT INTEGRATION SUMMARY

AFTER DETERMINING EXPRESSIONS FOR THE MOMENT IN EACH REGION OF THE BEAM, TWO INTEGRATIONS WERE PERFORMED TO DEVELOP EXPRESSIONS FOR THE SLOPE OF THE ELASTIC CURVE IN EACH SECTION. THESE INTEGRATIONS RESULT IN CONSTANTS OF INTEGRATION THAT ARE SOLVED FOR WITH BOUNDARY AND CONTINUITY CONDITIONS.