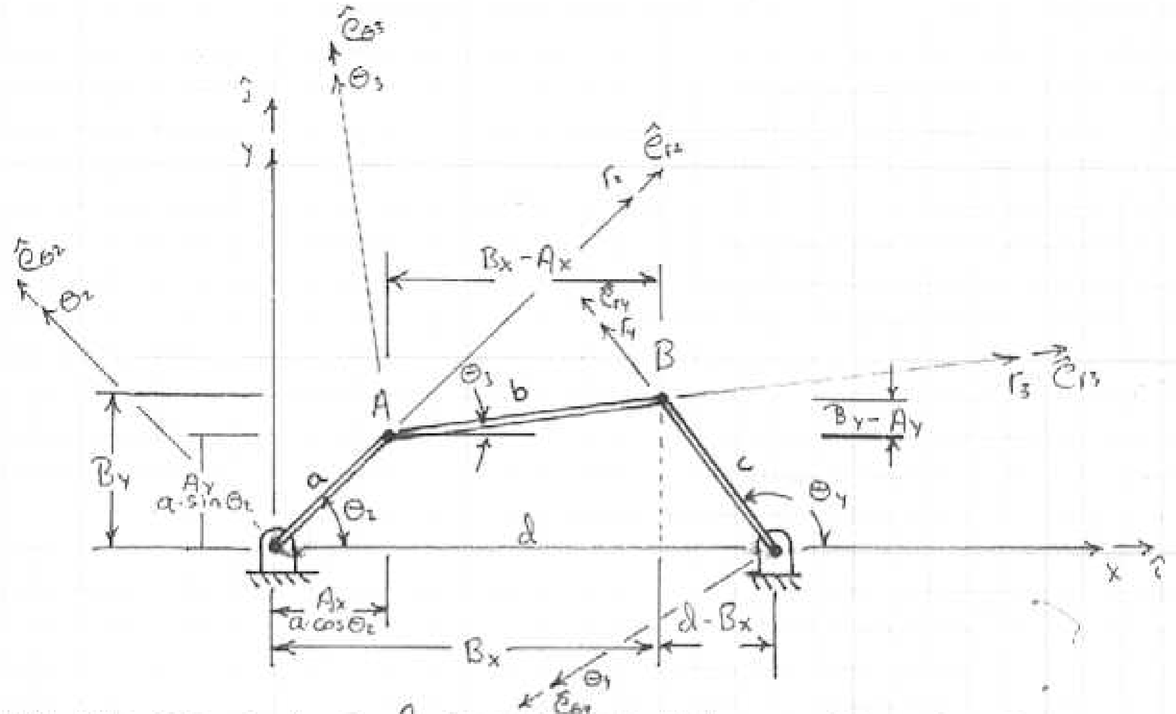

THE VALUES OF a, b, c, d, AND θ_2 ARE KNOWN, THEREFORE

$$A_x = a \cdot \cos\theta_2 \qquad (1)$$

$$A_y = a \cdot \sin\theta_2 \qquad (2)$$

THE LOCATION OF POINT B MUST NOW BE FOUND. THE LENGTHS OF LINKS (b) AND (c) CAN BE WRITTEN

$$b^2 = (B_x - A_x)^2 + (B_y - A_y)^2 \qquad (3)$$

$$c^2 = (d - B_x)^2 + B_y^2 \qquad (4)$$

SUBTRACTING THESE TWO EQUATIONS

$$b^2 - c^2 = (B_x - A_x)^2 - (d - B_x)^2 + (B_y - A_y)^2 - B_y^2$$

$$b^2 - c^2 = B_x^2 - 2 \cdot B_x \cdot A_x + A_x^2 - (d^2 - 2 \cdot d \cdot B_x + B_x^2) + B_y^2 - 2 \cdot B_y \cdot A_y + A_y^2 - B_y^2$$

$$b^2 - c^2 = \cancel{B_x^2} - 2 \cdot B_x \cdot A_x + \underline{A_x^2} - d^2 + 2 \cdot d \cdot B_x - \cancel{B_x^2} + \cancel{B_y^2} - 2 \cdot B_y \cdot A_y + \underline{A_y^2} - \cancel{B_y^2}$$

$$b^2 - c^2 = \underline{-2 \cdot B_x \cdot A_x} - d^2 + \underline{2 \cdot d \cdot B_x} - 2 \cdot B_y \cdot A_y + \underbrace{A_x^2 + A_y^2}_{a^2}$$

$$b^2 - c^2 = a^2 - d^2 + 2 \cdot B_x \cdot (d - A_x) - 2 \cdot B_y \cdot A_y$$

$$2 \cdot B_x \cdot (d - A_x) = b^2 - c^2 - a^2 + d^2 + 2 \cdot B_y \cdot A_y$$

$$B_x = \frac{b^2 - c^2 - a^2 + d^2 + 2 \cdot B_y \cdot A_y}{2 \cdot (d - A_x)} = \underbrace{\frac{b^2 - c^2 - a^2 + d^2}{2 \cdot (d - A_x)}}_{K_1} + \underbrace{\frac{A_y}{d - A_x}}_{K_2} \cdot B_y$$

LETTING

$$K_1 = \frac{b^2 - c^2 - a^2 + d^2}{2\cdot(d - A_x)} = S \qquad \cancel{4}\ \cancel{3}\ (5)$$

$$K_2 = \frac{A_y}{d - A_x} \qquad \cancel{4}\ (6)$$

THUS THE EXPRESSION FOR B_x CAN BE REWRITTEN

$$B_x = K_1\cdot + K_2 \cdot B_y \qquad \cancel{5}\ (7)$$

SUBSTITUTING (7) INTO (4)

$$C^2 = \left[d - (K_1 + K_2 \cdot B_y)\right]^2 + B_y^2$$

$$= \left[d - K_1 - K_2 \cdot B_y\right]^2 + B_y^2$$

$$= \left[(d - K_1) - K_2 \cdot B_y\right]^2 + B_y^2$$

$$C^2 = (d - K_1)^2 - 2\cdot(d - K_1)\cdot K_2 \cdot B_y + K_2^2 \cdot B_y^2 + B_y^2$$

$$B_y^2 \cdot (K_2^2 + 1) - 2\cdot(d - K_1)\cdot K_2 \cdot B_y + (d - K_1)^2 - C^2 = 0$$

$$B_y^2 - \frac{2\cdot(d - K_1)\cdot K_2}{(K_2^2 + 1)} \cdot B_y + \frac{(d - K_1)^2 - C^2}{(K_2^2 + 1)} = 0$$

LETTING

$$K_3 = \frac{2\cdot(d - K_1)\cdot K_2}{K_2^2 + 1} \qquad (8)$$

$$K_4 = \frac{(d - K_1)^2 - C^2}{K_2^2 + 1} \qquad (9)$$

THE POLYNOMIAL EXPRESSION IN B_y CAN NOW BE WRITTEN

$$B_y^2 - K_3 \cdot B_y + K_4 = 0$$

$$B_y^2 - K_3 \cdot B_y + \left(\frac{-K_3}{2}\right)^2 - \left(\frac{-K_3}{2}\right)^2 + K_4$$

$$\left(B_y - \frac{K_3}{2}\right)^2 = \left(\frac{-K_3}{2}\right)^2 - K_4$$

$$B_y = \frac{K_3}{2} \pm \sqrt{\left(\frac{-K_3}{2}\right)^2 - K_4} \qquad (10)$$

STARTING WITH THE LOOP EQUATION

$$\vec{r}_2 + \vec{r}_3 = \vec{r}_1 + \vec{r}_4 \Rightarrow r_2 \cdot \hat{e}_{r2} + r_3 \cdot \hat{e}_{r3} = r_1 \cdot \hat{e}_{r1} + r_4 \cdot \hat{e}_{r4} \qquad (1)$$

WHERE

$$\vec{r}_2 = r_2 \cdot \hat{e}_{r2} = r_2 \cdot (\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}) \qquad (2)$$

$$\vec{r}_3 = r_3 \cdot \hat{e}_{r3} = r_3 \cdot (\cos\theta_3 \hat{i} + \sin\theta_3 \hat{j}) \qquad (3)$$

$$\vec{r}_4 = r_4 \cdot \hat{e}_{r4} = r_4 \cdot (\cos\theta_4 \hat{i} + \sin\theta_4 \hat{j}) \qquad (4)$$

$$\vec{r}_1 = r_1 \cdot \hat{i} = d \cdot \hat{i} \qquad (5)$$

THE UNIT VECTORS FOR THIS SYSTEM ARE DEFINED AS

$$\hat{e}_{r2} = \cos\theta_2 \hat{i} + \sin\theta_2 \hat{j} \qquad (6)$$
$$\hat{e}_{\theta 2} = -\sin\theta_2 \hat{i} + \cos\theta_2 \hat{j} \qquad (7)$$

$$\hat{e}_{r3} = \cos\theta_3 \hat{i} + \sin\theta_3 \hat{j} \qquad (8)$$
$$\hat{e}_{\theta 3} = -\sin\theta_3 \hat{i} + \cos\theta_3 \hat{j} \qquad (9)$$

$$\hat{e}_{r4} = \cos\theta_4 \hat{i} + \sin\theta_4 \hat{j} \qquad (10)$$
$$\hat{e}_{\theta 4} = -\sin\theta_4 \hat{i} + \cos\theta_4 \hat{j} \qquad (11)$$

THE KNOWN INPUTS ARE θ_2, $r_2 = a$, $r_3 = b$, $r_4 = c$, $r_1 = d$. THE UNKNOWNS ARE θ_3 AND θ_4.

THE VELOCITY ANALYSIS STARTS BY KNOWING $\dot{\theta}_2 = \omega_2$. TAKING THE DERIVATIVE OF (1)

$$\cancel{\dot{r}_2}^{\,0} \hat{e}_{r2} + r_2 \cdot \underbrace{\dot{\hat{e}}_{r2}}_{\dot{\theta}_2 \hat{k} \times \hat{e}_{r2}} + \cancel{\dot{r}_3}^{\,0} \cdot \hat{e}_{r3} + r_3 \cdot \underbrace{\dot{\hat{e}}_{r3}}_{\dot{\theta}_3 \hat{k} \times \hat{e}_{r3}} = \cancel{\dot{r}_1}^{\,0} \cdot \hat{e}_{r1} + r_1 \cdot \cancel{\dot{\hat{e}}_{r1}}^{\,0} + \cancel{\dot{r}_4}^{\,0} \cdot \hat{e}_{r4} + r_4 \cdot \underbrace{\dot{\hat{e}}_{r4}}_{\dot{\theta}_4 \hat{k} \times \hat{e}_{r4}}$$

$$r_2 \cdot \dot{\theta}_2 \cdot \hat{e}_{\theta 2} + r_3 \cdot \dot{\theta}_3 \cdot \hat{e}_{\theta 3} = r_4 \cdot \dot{\theta}_4 \cdot \hat{e}_{\theta 4}$$

$$r_2 \cdot \dot{\theta}_2 \cdot (-\sin\theta_2 \hat{i} + \cos\theta_2 \hat{j}) + r_3 \cdot \dot{\theta}_3 \cdot (-\sin\theta_3 \hat{i} + \cos\theta_3 \hat{j}) = r_4 \cdot \dot{\theta}_4 \cdot (-\sin\theta_4 \hat{i} + \cos\theta_4 \hat{j}) \qquad (12)$$

DOTTING (12) WITH $\hat{i}$

$$-r_2 \cdot \dot{\theta}_2 \cdot \sin\theta_2 - r_3 \cdot \dot{\theta}_3 \cdot \sin\theta_3 = -r_4 \cdot \dot{\theta}_4 \cdot \sin\theta_4$$

$$r_2 \cdot \dot{\theta}_2 \cdot \sin\theta_2 + r_3 \cdot \dot{\theta}_3 \cdot \sin\theta_3 = r_4 \cdot \dot{\theta}_4 \cdot \sin\theta_4 \qquad (13)$$

DOTTING (12) WITH $\hat{j}$

$$r_2 \cdot \dot{\theta}_2 \cdot \cos\theta_2 + r_3 \cdot \dot{\theta}_3 \cdot \cos\theta_3 = r_4 \cdot \dot{\theta}_4 \cdot \cos\theta_4 \qquad (14)$$

THE UNKNOWNS IN (13) AND (14) ARE $\dot{\Theta}_3$ AND $\dot{\Theta}_4$, TWO EQUATIONS WITH TWO UNKNOWNS. SOLVING (13) FOR $\dot{\Theta}_4$.

$$\boxed{\dot{\Theta}_4 = \frac{r_2 \cdot \dot{\Theta}_2 \cdot \sin\Theta_2 + r_3 \cdot \dot{\Theta}_3 \cdot \sin\Theta_3}{r_4 \cdot \sin\Theta_4}} \qquad (15)$$

SUBSTITUTING THIS RESULT INTO (14)

$$r_2 \cdot \dot{\Theta}_2 \cdot \cos\Theta_2 + r_3 \cdot \dot{\Theta}_3 \cdot \cos\Theta_3 = r_4 \cdot \cos\Theta_4 \cdot \left[\frac{r_2 \cdot \dot{\Theta}_2 \cdot \sin\Theta_2 + r_3 \cdot \dot{\Theta}_3 \cdot \sin\Theta_3}{r_4 \cdot \sin\Theta_4}\right]$$

$$r_2 \cdot \dot{\Theta}_2 \cdot \cos\Theta_2 + r_3 \cdot \dot{\Theta}_3 \cdot \cos\Theta_3 = \frac{r_2 \cdot \dot{\Theta}_2 \cdot \sin\Theta_2 + r_3 \cdot \dot{\Theta}_3 \cdot \sin\Theta_3}{\tan\Theta_4}$$

$$= \frac{\cos\Theta_4}{\sin\Theta_4}\left[r_2 \cdot \dot{\Theta}_2 \cdot \sin\Theta_2 + r_3 \cdot \dot{\Theta}_3 \cdot \sin\Theta_3\right]$$

$$r_2 \cdot \dot{\Theta}_2 \cdot \cos\Theta_2 \cdot \sin\Theta_4 + r_3 \cdot \dot{\Theta}_3 \cdot \cos\Theta_3 \sin\Theta_4$$
$$= r_2 \cdot \dot{\Theta}_2 \cdot \sin\Theta_2 \cos\Theta_4 + r_3 \cdot \dot{\Theta}_3 \cdot \sin\Theta_3 \cos\Theta_4$$

$$r_3 \cdot \dot{\Theta}_3 (\cos\Theta_3 \cdot \sin\Theta_4 - \sin\Theta_3 \cos\Theta_4)$$
$$= r_2 \cdot \dot{\Theta}_2 \cdot (\sin\Theta_2 \cdot \cos\Theta_4 - \cos\Theta_2 \cdot \sin\Theta_4)$$

$$r_3 \cdot \dot{\Theta}_3 \cdot (\sin\Theta_4 \cdot \cos\Theta_3 - \cos\Theta_4 \cdot \sin\Theta_3) = -r_2 \cdot \dot{\Theta}_2 \cdot (\sin\Theta_4 \cos\Theta_2 - \cos\Theta_4 \cdot \sin\Theta_2)$$

$$r_3 \cdot \dot{\Theta}_3 \sin(\Theta_4 - \Theta_3) = -r_2 \cdot \dot{\Theta}_2 \cdot \sin(\Theta_4 - \Theta_2)$$

$$\boxed{\dot{\Theta}_3 = -\frac{r_2}{r_3} \cdot \dot{\Theta}_2 \cdot \frac{\sin(\Theta_4 - \Theta_2)}{\sin(\Theta_4 - \Theta_3)}} \qquad (16)$$

$$= -\frac{r_2}{r_3} \cdot \dot{\Theta}_2 \cdot \frac{\sin\Theta_4 \cos\Theta_2 - \sin\Theta_2 \cos\Theta_4}{\sin\Theta_4 \cos\Theta_3 - \sin\Theta_3 \cos\Theta_4}$$