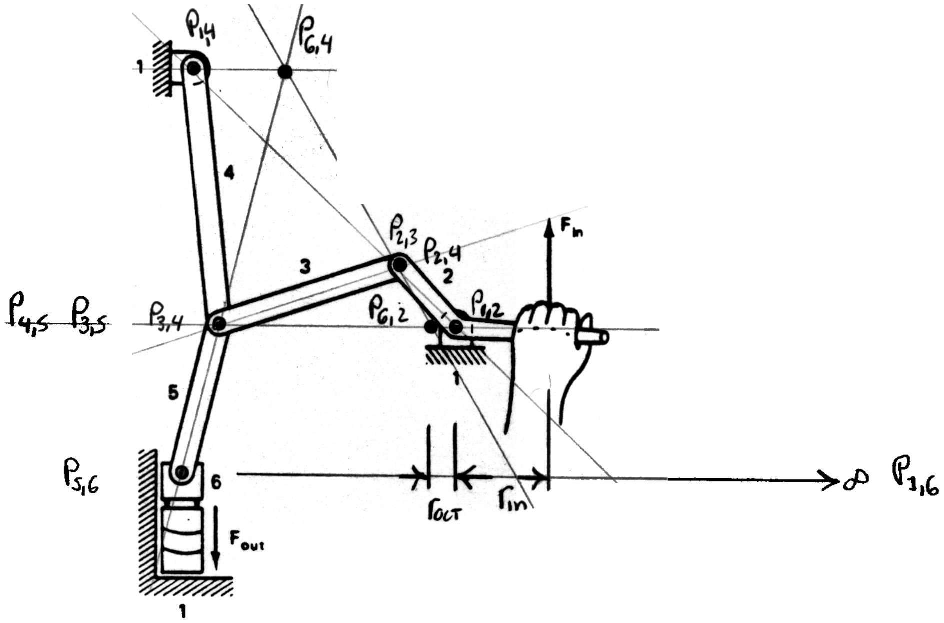
NAME: Solution

PROBLEM 1: For the mechanism shown, determine the mechanical advantage.



Knowing that

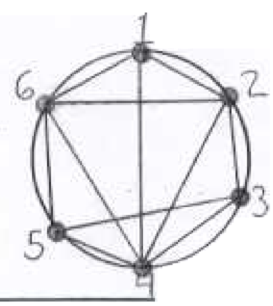
$$P_{in} = P_{out} \Rightarrow T_{in} \cdot \omega_{in} = F_{in} \cdot V_{in} = F_{out}\, V_{out} = T_{out}\, \omega_{out}$$

Because of the constraints on link 6, it can be stated that $V_6 = V_{P_{6,2}}$

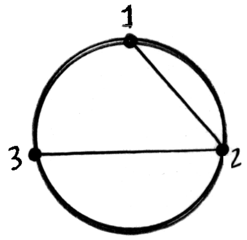
$$V_{in} = r_{in} \cdot \omega_2 \qquad F_{in} \cdot V_{in} = F_{out} \cdot V_{out}$$

$$V_{out} = r_{out} \cdot \omega_2 \quad \Rightarrow \quad F_{in} \cdot r_{in} \cdot \cancel{\omega_2} = F_{out} \cdot r_{out} \cdot \cancel{\omega_2}$$

$$= \frac{F_{out}}{F_{in}} = \frac{r_{in}}{r_{out}} = \frac{15}{4} = \boxed{3.75}$$



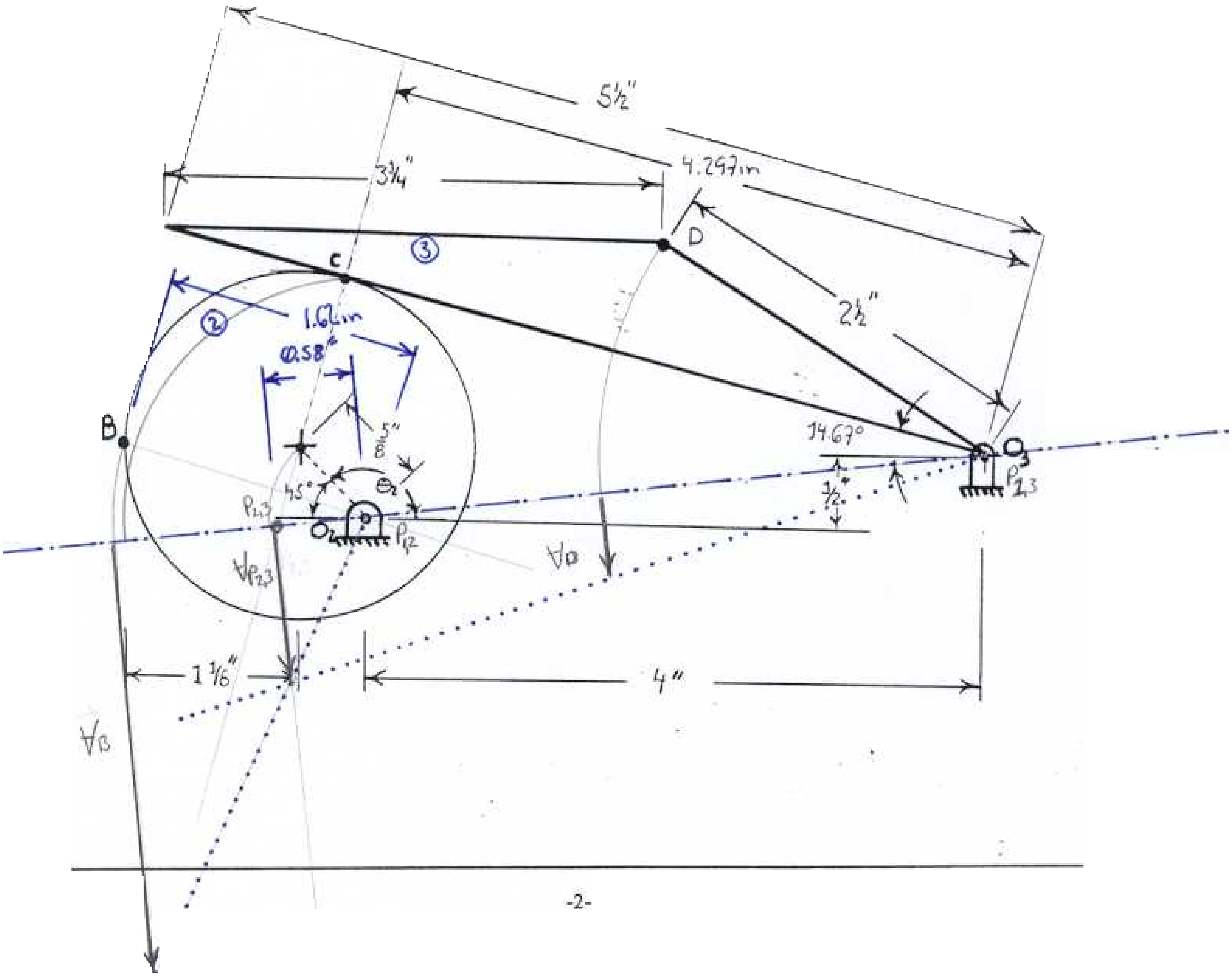
PROBLEM 2: Given that link 2 of the mechanism below rotates at a constant angular velocity of 10.47 1/s, determine the angular velocity of link 3, and the velocity of points D and B using graphical means. The loop closure equation for this mechanism is



$$\cdot\,\hat{e}_{O_2A} + R_{AC}\cdot\hat{e}_{AC} + R_{CO_3}\cdot\hat{e}_{CO_3} = R_{O_2O_3}\cdot\hat{e}_{O_2O_3}$$

where

$$R_{O_2A}\cdot\hat{e}_{O_2A} = 0.625\text{in}\cdot(-0.7071\cdot\hat{i} + 0.7071\cdot\hat{j})$$
$$\hat{e}_{AC} = 1.125\text{in}\cdot(0.2406\cdot\hat{i} + 0.9706\cdot\hat{j})$$
$$\cdot\,\hat{e}_{CO_3} = 4.297\text{in}\cdot(0.9706\cdot\hat{i} - 0.2406\cdot\hat{j})$$
$$\cdot\,\hat{e}_{O_2O_3} = 4.031\text{in}\cdot(0.9923\cdot\hat{i} + 0.1240\cdot\hat{j})$$



$$V_B = \omega_2 R_{O_2B} = (10.47\,{}^1/_s)\cdot(1.62\,in) = \boxed{16.96\ {}^{in}/_s}$$

$$V_{P_{2,3}} = \omega_2 R_{O_2P_{23}} = (10.47\,{}^1/_s)\cdot(0.58\,in) = \boxed{6.073\ {}^{in}/_s}$$

$$= \frac{V_{P_{2,3}}}{R_{O_4P_{23}}} = \frac{6.073\ {}^{in}/_s}{4.58\,in} = \boxed{1.326\ {}^1/_s}$$

$$= \omega_3 R_{O_4D} = (1.326\,{}^1/_s)(2\tfrac{1}{2}\,in) = \boxed{3.315\ {}^{in}/_s}$$

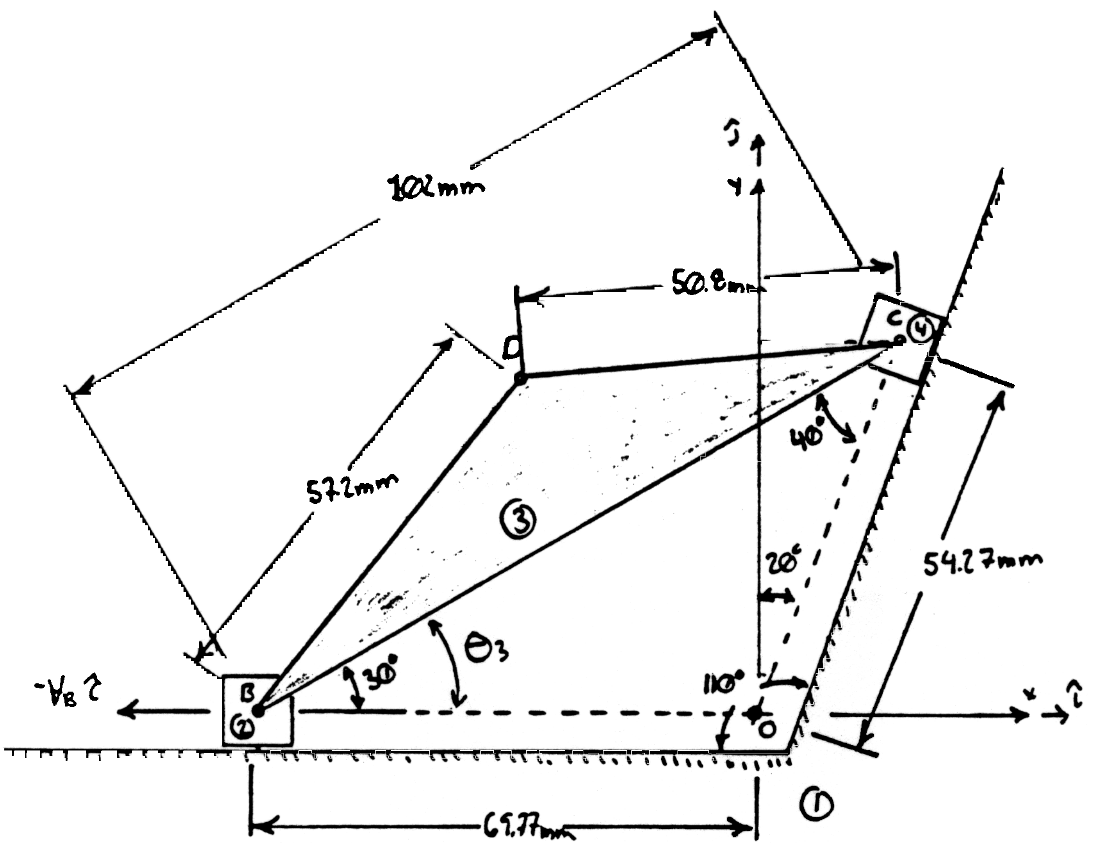
PROBLEM 3: For the mechanism shown below, the loop closure equation can be written,

$$\vec{R}_{BO} + \vec{R}_{OC} = \vec{R}_{BC}$$

where

$$\vec{R}_{BO} = R_{BO} \cdot \hat{e}_{BO} = 69.77\text{mm} \cdot \hat{i} = R_{BO} e^{j \cdot \theta_2} = 69.77 e^{j \cdot 0}$$
$$\vec{R}_{OC} = R_{OC} \cdot \hat{e}_{OC} = 54.27\text{mm} \cdot (0.3420 \cdot \hat{i} + 0.9397 \cdot \hat{j}) = R_{OC} \cdot e^{j \cdot \theta_4} = 54.27\text{mm} \cdot e^{j \cdot 70°}$$
$$\vec{R}_{BC} = R_{BC} \cdot \hat{e}_{BC} = 102.0\text{mm} \cdot (0.8660 \cdot \hat{i} + 0.5 \cdot \hat{j}) = R_{BC} \cdot e^{j \cdot \theta_3} = 102.0\text{mm} \cdot e^{j \cdot 30°}$$

Differentiate the loop closure equation and determine the angular velocity of link 3 and the velocity of point C.



(using vectors)

$$R_{BO}\hat{e}_{BO} + R_{OC}\hat{e}_{OC} = R_{BC}\hat{e}_{BC}$$

$$\dot{R}_{BO}\hat{e}_{BO} + R_{BO}\cancel{\dot{\hat{e}}_{BO}}^{0} + \dot{R}_{OC}\hat{e}_{OC} + R_{OC}\cancel{\dot{\hat{e}}_{OC}}^{0} = \cancel{\dot{R}_{BC}}^{0}\hat{e}_{BC} + R_{BC}\dot{\hat{e}}_{BC}$$

$$\underset{\text{known}}{\dot{R}_{BO}}\hat{e}_{BO} + \underset{\text{unknown}}{\dot{R}_{OC}}\hat{e}_{OC} = R_{BC}\dot{\hat{e}}_{BC} = \underset{\text{known}}{R_{BC}}\,\underset{\text{unknown}}{\dot{\theta}_3}(\hat{k}\times\hat{e}_{BC})$$

Solving for $\dot{R}_{OC}$ by dotting both sides by $\hat{e}_{BC}$

$$\dot{R}_{BO}\hat{e}_{BC}\cdot\hat{e}_{BO} + \dot{R}_{OC}\hat{e}_{BC}\cdot\hat{e}_{OC} = R_{BC}\dot{\theta}_3\underbrace{\hat{e}_{BC}\cdot(\hat{k}\times\hat{e}_{BC})}_{0}$$

$$\Rightarrow \dot{R}_{OC} = -\dot{R}_{BO}\frac{\hat{e}_{BC}\cdot\hat{e}_{BO}}{\hat{e}_{BC}\cdot\hat{e}_{OC}} = +6.10\,\tfrac{m}{s}\left[\frac{(.8660\hat{i}+0.5\hat{j})\cdot(\hat{i})}{(.8660\hat{i}+0.5\hat{j})\cdot(.3420\hat{i}+.9397\hat{j})}\right] = \boxed{6.896\,\tfrac{m}{s}}$$

Solving for $\dot{\theta}_3$ by dotting both sides by $(\hat{k}\times\hat{e}_{OC})$

$$\dot{R}_{BO}\hat{e}_{BO}\cdot(\hat{k}\times\hat{e}_{OC}) + \dot{R}_{OC}\cancel{\hat{e}_{OC}\cdot(\hat{k}\times\hat{e}_{OC})}^{0} = R_{BC}\dot{\theta}_3(\hat{k}\times\hat{e}_{BC})\cdot(\hat{k}\times\hat{e}_{OC})$$

$$\Rightarrow \dot{\theta}_3 = -\frac{\dot{R}_{BO}}{R_{BC}}\,\frac{\hat{e}_{BO}\cdot(\hat{k}\times\hat{e}_{OC})}{(\hat{k}\times\hat{e}_{BC})\cdot(\hat{k}\times\hat{e}_{OC})}$$

$$\hat{k}\times\hat{e}_{OC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ .3420 & .9397 & 0\end{vmatrix} = .9397\hat{i} - .3420\hat{j}$$

$$\hat{k}\times\hat{e}_{BC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ .8660 & .5 & 0\end{vmatrix} = .5\hat{i} - .8660\hat{j}$$

$$\dot{\theta}_3 = \frac{6.10\,\frac{m}{s}}{102\,m}\cdot\frac{\hat{i}\cdot(.9397\hat{i}-.3420\hat{j})}{(.5\hat{i}-.8660\hat{j})\cdot(.9397\hat{i}-.3420\hat{j})} = \boxed{73.36\,{}^{1}/_{s}}$$

(Using Complex Approach)

$$R_{BO} e^{j\theta_2} + R_{OC} e^{j\theta_4} = R_{BC} e^{j\theta_3}$$

$$\dot{R}_{BO} e^{j\theta_2} + R_{BO}\, j\cancel{\dot{\theta}_2}^{\,0} e^{j\theta_2} + \dot{R}_{OC} e^{j\theta_4} + R_{OC}\, j\cancel{\dot{\theta}_4}^{\,0} e^{j\theta_4} = \cancel{\dot{R}_{BC}}^{\,0} e^{j\theta_3} + R_{BC}\, j\dot{\theta}_3 e^{j\theta_3}$$

$$\underset{\text{known}}{\dot{R}_{BO}} e^{j\theta_2} + \underset{\text{unknown}}{\dot{R}_{OC}} e^{j\theta_4} = \underset{\text{known}}{R_{BC}}\, j\, \underset{\text{unknown}}{\dot{\theta}_3}\, e^{j\theta_3}$$

expanding using Euler's equation

$$\dot{R}_{BO}(\cos\theta_2 + j\sin\theta_2) + \dot{R}_{OC}(\cos\theta_4 + j\sin\theta_4) = R_{BC}\, j\dot{\theta}_3(\cos\theta_3 + j\sin\theta_3)$$

$$\dot{R}_{BO}\cos\theta_2 + \dot{R}_{OC}\cos\theta_4 = -R_{BC}\dot{\theta}_3\sin\theta_3$$

$$\dot{R}_{BO}\sin\theta_2 + \dot{R}_{OC}\sin\theta_4 = R_{BC}\dot{\theta}_3\cos\theta_3$$

Solving for $\dot{R}_{OC}$ by multiplying the real equation by $\cos\theta_3$ and the imaginary portion of the equation by $\sin\theta_3$

$$\dot{R}_{BO}\cos\theta_2\cos\theta_3 + \dot{R}_{OC}\cos\theta_4\cos\theta_3 = -R_{BC}\dot{\theta}_3\sin\theta_3\cos\theta_3$$

$$\dot{R}_{BO}\sin\theta_2\sin\theta_3 + \dot{R}_{OC}\sin\theta_4\sin\theta_3 = R_{BC}\dot{\theta}_3\cos\theta_3\sin\theta_3$$

$$\dot{R}_{BO}(\cos\theta_2\cos\theta_3 + \sin\theta_2\sin\theta_3) + \dot{R}_{OC}(\cos\theta_4\cos\theta_3 + \sin\theta_4\sin\theta_3) = 0$$

$$\dot{R}_{BO}\cos(\theta_3-\theta_2) + \dot{R}_{OC}\cos(\theta_4-\theta_3) = 0$$

$$\Rightarrow \dot{R}_{OC} = -\dot{R}_{BO}\frac{\cos(\theta_3-\theta_2)}{\cos(\theta_4-\theta_3)} = +(6.10\tfrac{m}{s})\cdot\frac{\cos(30^\circ-0^\circ)}{\cos(70^\circ-30^\circ)} = \boxed{6.896\tfrac{m}{s}}$$

Solving for $\dot{\theta}_3$ by multiplying the real equation by $\sin\theta_4$ and the imaginary portion of the equation by $\cos\theta_4$

$$\dot{R}_{BO}\cos\theta_2\sin\theta_4 + \dot{R}_{OC}\cos\theta_4\sin\theta_4 = -R_{BC}\dot{\theta}_3\sin\theta_3\sin\theta_4$$

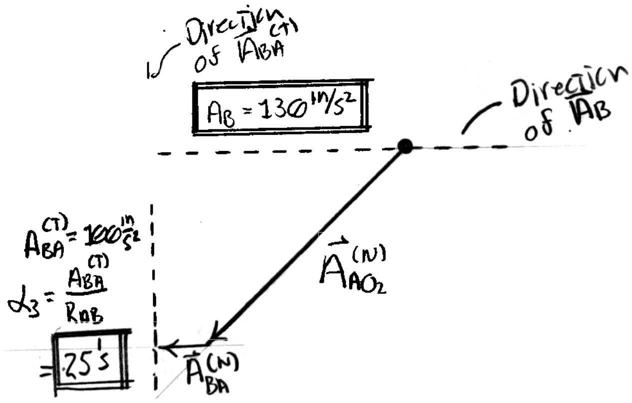
$$[\dot{R}_{BO}\sin\theta_2\cos\theta_4 + \dot{R}_{OC}\sin\theta_4\cos\theta_4 = R_{BC}\dot{\theta}_3\cos\theta_3\cos\theta_4]$$

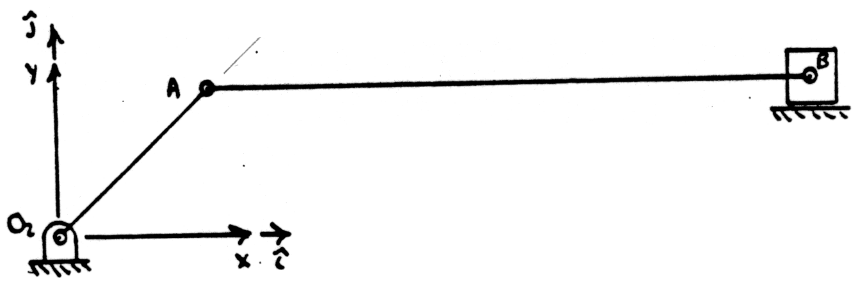
$$\dot{R}_{BO}(\cos\theta_2\sin\theta_4 - \sin\theta_2\cos\theta_4) = -R_{BC}\dot{\theta}_3(\sin\theta_3\sin\theta_4 + \cos\theta_3\cos\theta_4)$$

$$\dot{R}_{BO}\sin(\theta_4-\theta_2) = -R_{BC}\dot{\theta}_3\cos(\theta_4-\theta_3)$$

$$\Rightarrow \dot{\theta}_3 = \frac{-\dot{R}_{BO}}{R_{BC}}\,\frac{\sin(\theta_4-\theta_2)}{\cos(\theta_4-\theta_3)} = \frac{+6.10}{.102m}\,\frac{\sin(70^\circ-0^\circ)}{\cos(70^\circ-30^\circ)} = \boxed{73.36\ {}^{1}/_{s}}$$

PROBLEM 4: Determine the acceleration of point B and the angular acceleration of link 3 for the mechanism shown knowing that the length of link 2 is 1.4 in., link 3 is 4 in., the offset is 1 in.; the angle link 2 makes with the horizontal is 45°; the angular velocity of link 2 is 10 1/s; the angular velocity of link 3 is -2.475 1/s; and the angular acceleration of link 2 is 0.





$$\vec{A}_A = \vec{A}_{O_2} + \vec{A}_{AO_2}^{(T)} + \vec{A}_{AO_2}^{(N)}$$

$$\vec{A}_{AO_2}^{(N)} = \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{r}_{O_2A})$$

$$\vec{A}_{AO_2}^{(T)} = \vec{\alpha}_2 \times \vec{r}_{O_2A}$$

$$A_{AO_2}^{(N)} = \omega_2^2\, r_{O_2A} = (10\tfrac{1}{s})^2 \cdot (1.4in) = 140\ \tfrac{in}{s^2}$$

$$A_{AO_2}^{(T)} = \alpha_2\, r_{O_2A} = (\varnothing)\,(1.4in) = \varnothing$$

$$\vec{A}_B = \vec{A}_A + \vec{A}_{BA}^{(N)} + \vec{A}_{BA}^{(T)}$$

$$\vec{A}_{BA}^{(N)} = \vec{\omega}_3 \times (\vec{\omega}_3 \times \vec{R}_{AB})$$

$$A_{BA}^{(T)} = \vec{\alpha}_3 \times \vec{r}_{AB}$$

$$A_{BA}^{(N)} = \omega_3^2 \cdot r_{AB} = (-2.475\tfrac{1}{s})^2 (4in) = 24.50\ in/s^2$$