COLUMNS: BUCKLING

Buckling

- Introduction
  - Buckling is a mode of failure generally resulting from structural instability due to *compressive* action on the structural member or element involved.
  - Examples
    - Overloaded metal building columns.
    - Compressive members in bridges.
    - Roof trusses.
    - Hull of submarine.

Buckling

- Introduction
  - Examples (cont'd)
    - Metal skin on aircraft fuselages or wings with excessive torsional and/or compressive loading.
    - Any thin-walled torque tube.
    - The thin web of an I-beam with excessive shear load
    - A thin flange of an I-beam subjected to excessive compressive bending effects.

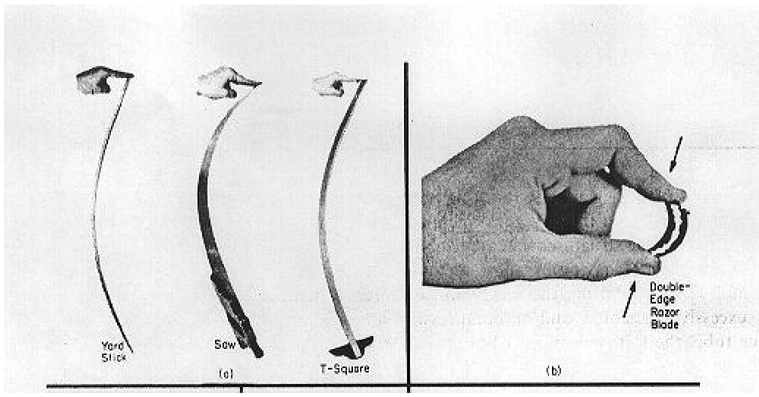
Buckling

- Introduction
  - In view of the above-mentioned examples, it is clear that buckling is a result of compressive action.
  - Overall torsion or shear, as was discussed earlier, may cause a localized compressive action that could lead to buckling.
  - Examples of buckling for commonly seen and used tools (components) are provided in the next few viewgraphs.

Buckling

- Introduction

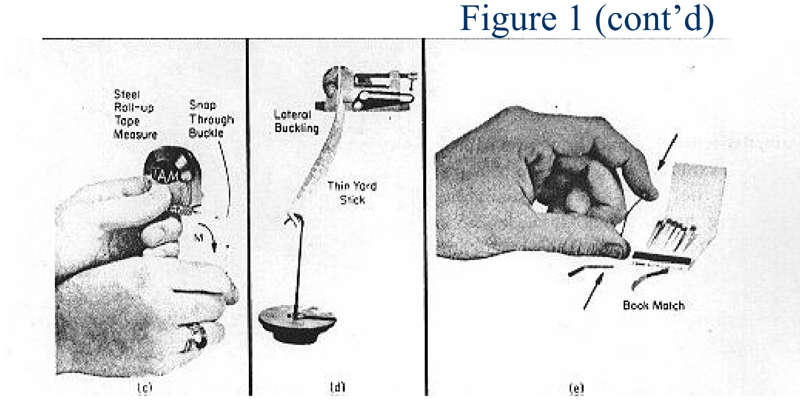
Figure 1



Buckling

- Introduction

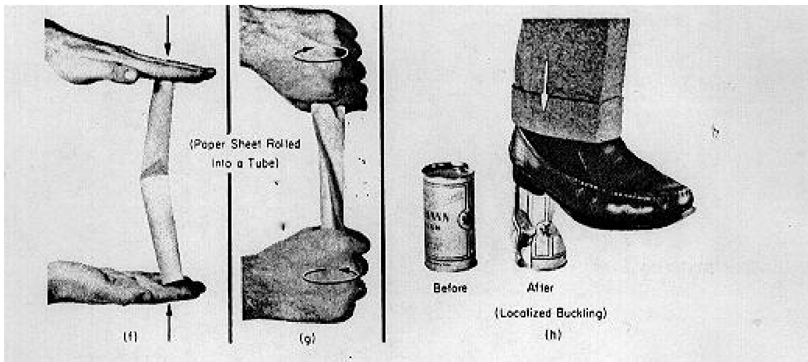
Figure 1 (cont'd)



Buckling

- Introduction

Figure 1 (cont'd)



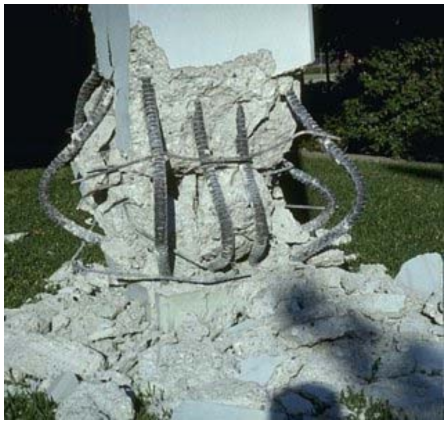
Buckling

- Introduction
  - In Fig. 1, (a) to (d) are examples of temporary or elastic buckling.
  - While (e) to (h) of the same figure are examples of plastic buckling
  - The distinctive feature of buckling is the *catastrophic* and often spectacular nature of failure.

Buckling

- Introduction

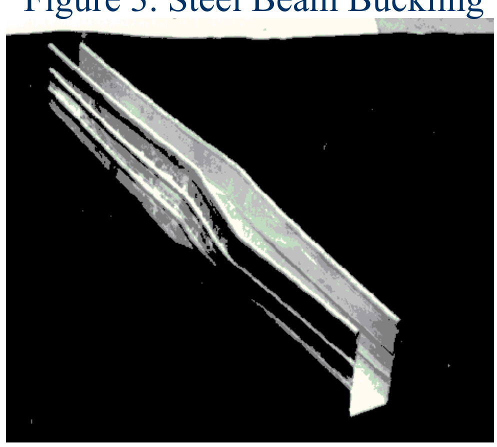
Figure 2. Reinforced Concrete



Buckling

- Introduction

Figure 3. Steel Beam Buckling



Buckling

- Introduction
  - The collapse of a column supporting stands in a stadium or the roof of a building usually draws large headlines and cries of engineering negligence.
  - On a lesser scale, the reader can witness and get a better understanding of buckling by trying to understand a few of the tests shown in Fig. 1.

Buckling

- The Nature of Buckling
  - In the previous chapters, we related load to stress and load to deformation.
  - For these non-buckling cases of axial, torsional, bending, and combined loading, the stress or deformation was the significant quantity in failure.
  - Buckling of a member is uniquely different in that the quantity significant in failure is

Buckling

- The Nature of Buckling
  the buckling load itself.
  - The failure (buckling) load bears no unique relationship to the stress and deformation at failure.
  - Our usual approach of deriving a load-stress and load-deformation relations cannot be used here, instead, the approach to find an expression for the buckling load P_{cr}.

Buckling

- The Nature of Buckling
  - Buckling is unique from our other structural-element considerations in that it results from a state of unstable equilibrium.
  - For example, buckling of a long column is not caused by failure of the material of which the column is composed, but by determination of what was a stable state of equilibrium to an unstable one.

Buckling

- The Nature of Buckling
  - Mechanism of Buckling
    - Let's consider Fig. 4, 5, and 6, and study them very carefully.
    - In Fig. 4, some axial load *P* is applied to the column.
    - The column is then given a small deflection by applying the small lateral force *F.*
    - If the load *P* is sufficiently small, when the force *F* is removed, the column will go back to its original straight condition.

Buckling

- The Nature of Buckling
  - Mechanism of Buckling

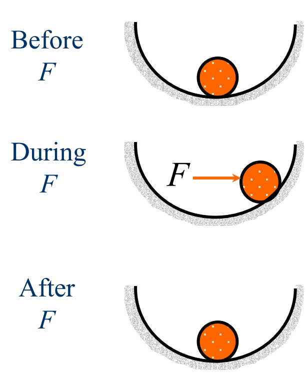
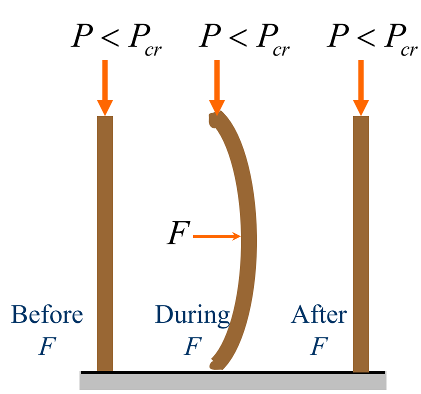


Figure 4



Stable Equilibrium

Buckling

- The Nature of Buckling
  - Mechanism of Buckling
    - The column will go back to its original straight condition just as the ball returns to the bottom of the curved container.
    - In Fig. 4 of the ball and the curved container, gravity tends to restore the ball to its original position, while for the column the elasticity of the column itself acts as restoring force.
    - This action constitutes *stable equilibrium*.

Buckling

- The Nature of Buckling
  - Mechanism of Buckling
    - The same procedure can be repeated for increased value of the load P until some critical value P_{cr} is reached, as shown in Fig. 5.
    - When the column carries this load, and a lateral force F is applied and removed, the column will remain in the slightly deflected position. The elastic restoring force of the column is not sufficient to return the column to its original

Buckling

- The Nature of Buckling
  - Mechanism of Buckling

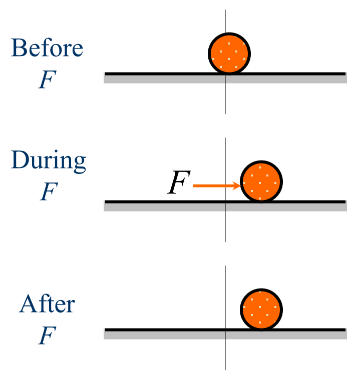
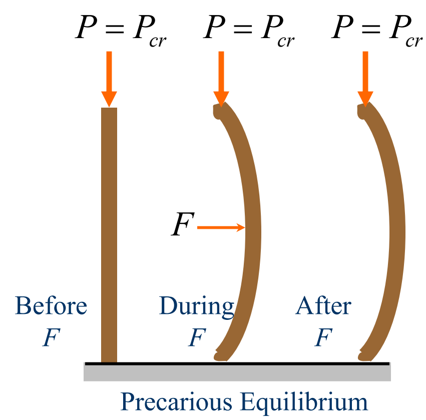


Figure 5



Buckling

- The Nature of Buckling
  - Mechanism of Buckling

    straight position but is sufficient to prevent excessive deflection of the column.

    - In Fig. 5 of the ball and the flat surface, the amount of deflection will depend on the magnitude of the lateral force *F*.
    - Hence, the column can be in equilibrium in an infinite number of slightly bent positions.
    - This action constitutes <u>*neutral or precarious equilibrium*</u>.

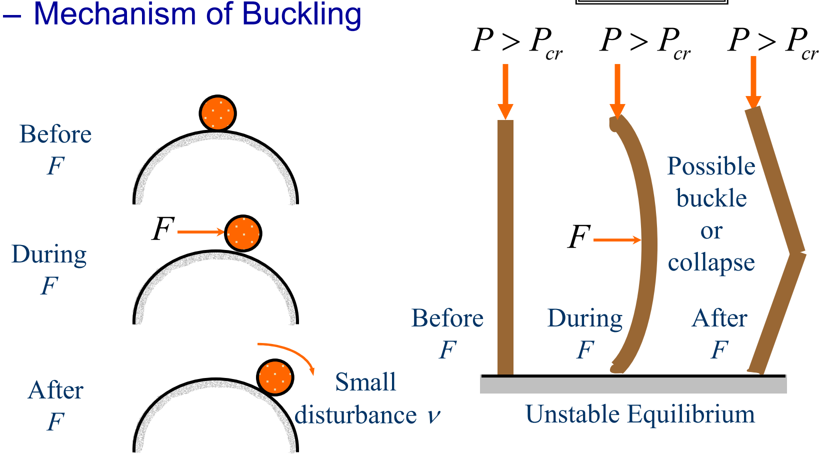
Buckling

- The Nature of Buckling
  - Mechanism of Buckling
    - If the column is subjected to an axial compressive load P that exceeds P_{cr}, as shown in Fig. 6, and a lateral force F is applied and removed, the column will bend considerably.
    - That is, the elastic restoring force of the column is not sufficient to prevent a small disturbance from growing into an excessively large deflection.

Buckling

- The Nature of Buckling
  - Mechanism of Buckling

Figure 6



Buckling

- The Nature of Buckling
  - Mechanism of Buckling
    - Depending on the magnitude of P, the column either will remain in the bent position or will completely collapse and fracture, just as the ball will roll off the curved surface in Fig. 6.
    - This type of behavior indicates that for axial loads greater than P_{cr}, the straight position of a column is one of *unstable equilibrium* in that a small disturbance will tend to grow into an excessive deformation.

Buckling

- The Nature of Buckling

  Definition

  "Buckling can be defined as the sudden large deformation of structure due to a slight increase of an existing load under which the structure had exhibited little, if any, deformation before the load was increased."

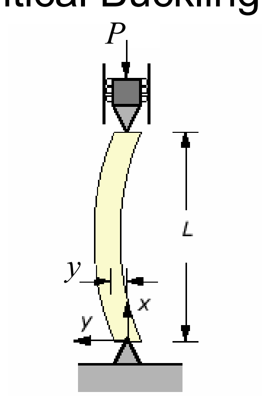
Buckling of Long Straight Columns

- Critical Buckling Load
  - The purpose of this analysis is to determine the minimum axial compressive load for which a column will experience lateral deflection.
  - Governing Differential Equation:
    - Consider a buckled simply-supported column of length L under an external axial compression force P, as shown in the left schematic of Fig. 7. The transverse displacement of the buckled column is represented by δ.

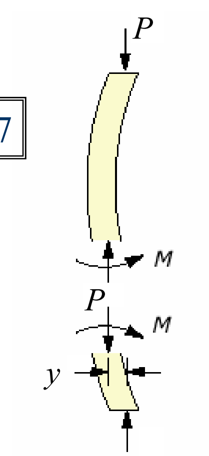
Buckling of Long Straight Columns

- Critical Buckling Load

Figure 7



(a) Simply supported column subjected to axial load F



(b) Free body diagram

Buckling of Long Straight Columns

- Critical Buckling Load
  - Governing Differential Equation:
    - The right schematic of Fig. 7 shows the forces and moments acting on a cross-section in the buckled column. Moment equilibrium on the lower free body yields a solution for the internal bending moment *M*,

$$Py + M = 0 \qquad (1)$$

Buckling of Long Straight Columns

- Critical Buckling Load
  - Governing Differential Equation (cont'd):
    - Recall the relationship between the moment *M* and the transverse displacement *y* for the elastic curve,

$$EI\frac{dy^2}{dx^2} = M \qquad (2)$$

    - Eliminating *M* from Eqs. 1 and 2 results in the governing equation for the buckled slender column,

Buckling of Long Straight Columns

- Critical Buckling Load
  - Governing Differential Equation (cont'd):

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0 \qquad (3)$$

  - Buckling Solution:
    - The governing equation is a second order homogeneous ordinary differential equation with constant coefficients and can be solved by the method of characteristic equations. The solution is found to be,

Buckling of Long Straight Columns

- Critical Buckling Load
  - Buckling Solution (cont'd):

$$y(x) = A\sin px + B\cos px \qquad (4)$$

    - Where $p^2 = P/EI$. The coefficients A and B can be determined by the two boundary conditions, $y(0) = 0$ and $y(L) = 0$, which yields,

$$B = 0$$

$$A\sin pL = 0 \qquad (5)$$

Buckling of Long Straight Columns

- Critical Buckling Load
  - Buckling Solution (cont'd):
    - The coefficient *B* is always zero, and for most values of $m \times L$ the coefficient *A* is required to be zero. However, for special cases of $m \times L$, *A* can be nonzero and the column can be buckled. The restriction on $m \times L$ is also a restriction on the values for the loading *F*; these special values are mathematically called eigenvalues. All other values of *F* lead to trivial solutions (i.e. zero deformation).

Buckling of Long Straight Columns

- Critical Buckling Load
  - Buckling Solution (cont'd):

$$\sin pL = 0$$

$$\Rightarrow pL = 0, \pi, 2\pi, 3\pi, \cdots, n\pi$$

or

$$p = 0, \frac{\pi}{L}, \frac{2\pi}{L}, \frac{3\pi}{L}, \cdots, \frac{n\pi}{L} \qquad (6)$$

- Since $p^2 = P/EI$, therefore,

$$P = 0, \frac{\pi^2 EI}{L^2}, \frac{(2)^2 \pi^2 EI}{L^2}, \frac{(3)^2 \pi^2 EI}{L^2}, \cdots, \frac{n^2 \pi^2 EI}{L^2} \qquad (7)$$

Buckling of Long Straight Columns

- Critical Buckling Load
  - Buckling Solution (cont'd):
    - Or

$$P = EI\left(\frac{n\pi}{L}\right)^2 \quad \text{for} \;\; n = 0,1,2,3\cdots \qquad (8)$$

    - The lowest load that causes buckling is called critical load (n = 1).

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad (9)$$

Buckling of Long Straight Columns

- Critical Buckling Load, P_{cr}

  The critical buckling load (Euler Buckling) for a long column is given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad (9)$$

  where

  E = modulus of elasticity of the material
  I = moment of inertia of the cross section
  L = length of column

Buckling of Long Straight Columns

- Critical Buckling Load
  - Equation 9 is usually called **Euler's formula**. Although Leonard Euler did publish the governing *equation* in 1744, J. L. Lagrange is considered the first to show that a non-trivial solution exists only when n is an *integer*. Thomas Young then suggested the critical load (n = 1) and pointed out the solution was valid when the column is *slender* in his 1807 book. The "slender" column idea was not quantitatively developed until A. Considère performed a series of 32 tests in 1889.

Buckling of Long Straight Columns

- Critical Buckling Load

  Shape function:

  - Substituting the expression of P in Eq. 9, into Eq. 4, and noting that B = 0, the shape function for the buckled shape $y(x)$ is mathematically called an eigenfunction, and is given by,

$$y(x) = A\sin\left(\frac{n\pi x}{L}\right) \qquad (10)$$

Buckling of Long Straight Columns

- Critical Buckling Stress
  - The critical buckling normal stress σ_n is found as follows:

When the moment of inertia I in Eq. 9 is replaced by Ar^2, the result is

$$\frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2} = \sigma_{cr} \qquad (11)$$

where

A = cross-sectional area of column

r = radius of gyration = $\sqrt{\dfrac{I}{A}}$

Buckling of Long Straight Columns

- Critical Buckling Stress

The critical buckling normal stress is given by

$$\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2} \qquad (12)$$

Where

r = radius of gyration = $\sqrt{\dfrac{I}{A}}$

(L/r) = slenderness ratio of column

Buckling of Long Straight Columns

- Critical Buckling Load and Stress
  - The Euler buckling load and stress as given by Eq. 9 or Eq. 12 agrees well with experiment if the slenderness ratio is large (L/r > 140 for steel columns).
  - Short compression members (L/r < 140 for steel columns) can be treated as compression blocks where yielding occurs before buckling.

Buckling of Long Straight Columns

- Critical Buckling Load and Stress
  - Many columns lie between these extremes in which neither solution is applicable.
  - These intermediate-length columns are analyzed by using empirical formulas to be described later.
  - *When calculating the critical buckling for columns, I (or r) should be obtained about the weak axis.*

Buckling of Long Straight Columns

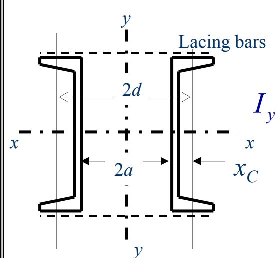
- Review of Parallel-Axis Theorem for Radius of Gyration
  - In dealing with columns that consist of several rolled standard sections, it is sometimes necessary to compute the radius of gyration for the entire section for the purpose of analyzing the buckling load.
  - It was shown that the parallel-axis theorem is a useful tool to calculate the second

Buckling of Long Straight Columns

- Review of Parallel-Axis Theorem for Radius of Gyration
  - Moment of area (moment of inertia) about other axes not passing through the centroid of the overall section.
  - In a similar fashion, the parallel-axis theorem can be used to find radii of gyration of a section about different axis not passing through the centroid.

Buckling of Long Straight Columns

- Review of Parallel-Axis Theorem for Radius of Gyration
  - Consider the two channels, which are laced a distance of 2*a* back to back.



$$I_x = 2I_{x_C} \quad \Rightarrow r_x = \sqrt{\frac{I_x}{A_{\text{overall}}}} = \sqrt{\frac{2I_{x_C}}{2A_{\text{sec}}}} = \sqrt{\frac{I_{x_C}}{A_{\text{sec}}}} = r_{x_C} \quad (13)$$

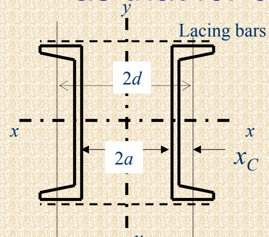
$$I_y = 2\left(I_{y_C} + A_{\text{sec}}d^2\right) = 2\left(A_{\text{sec}}r_{y_C}^2 + A_{\text{sec}}d^2\right) = 2A_{\text{sec}}\left(r_{y_C}^2 + d^2\right)$$

$$\Rightarrow r_y = \sqrt{\frac{I_y}{A_{\text{overall}}}} = \sqrt{\frac{2A_{\text{sec}}\left(r_{y_C}^2 + d^2\right)}{2A_{\text{sec}}}} = \sqrt{r_{y_C}^2 + d^2} \quad (14)$$

Buckling of Long Straight Columns

- Parallel-Axis Theorem for Radius of Gyration

  Eqs. 13 and 14 indicate that the radius of gyration for the two channels is the same as that for one channel, and



$$r_y = \sqrt{r_{y_C}^2 + \left(a + x_C\right)^2} \quad (15)$$

where $a + x_C = d$

Buckling of Long Straight Columns

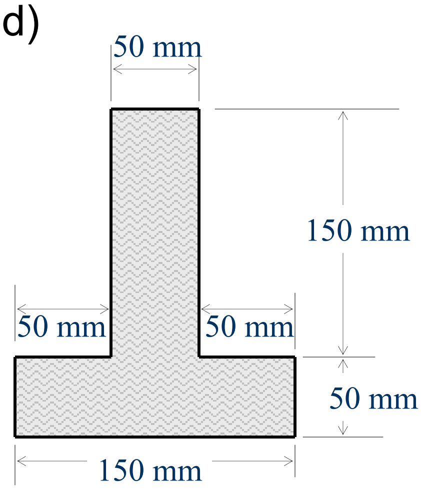
- Example 1

  A 3-m column with the cross section shown in Fig. 8 is constructed from two pieces of timber. The timbers are nailed together so that they act as a unit. Determine (a) the slenderness ratio, (b) the Euler buckling load (E = 13 GPa for timber), and (c) the axial stress in the column when Euler load is applied.

Buckling of Long Straight Columns

- Example 1 (cont'd)

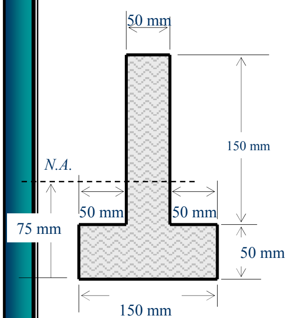
Figure 8



Buckling of Long Straight Columns

- Example 1 (cont'd)

  Properties of the cross section:



$$A = 2(150)(50) = 15{,}000\ \text{mm}^2$$

$$y_C = \frac{25(50\times150)+(50+75)(50\times150)}{15{,}000} = 75.0\ \text{mm from bottom}$$

$$I_x = \frac{1}{3}(50)(125)^3 + \frac{1}{3}(150)(75)^3 - \frac{1}{3}(100)(25)^3 = 53.13\times10^6\ \text{mm}^4$$

$$I_x = \frac{1}{12}(150)(50)^3 + \frac{1}{12}(50)(150)^3 = 15.625\times10^6\ \text{mm}^4$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{15.625\times10^6}{15{,}000}} = 32.3$$

Buckling of Long Straight Columns

- Example 1 (cont'd)

  (a) Slenderness Ratio:

$$\text{Slenderness ratio} = \frac{L}{r} = \frac{3000}{32.27} = \boxed{93}$$

  (b) Euler Buckling Load:

$$P_{cr} = \frac{\pi^2 E I_y}{L^2} = \frac{\pi^2\left(13\times10^9\right)\left(15.625\times10^{-6}\right)}{(3)^2} = \boxed{222.75\ \text{kN}}$$

  (c) Axial Stress:

$$\sigma = \frac{P_{cr}}{A} = \frac{222.75}{15\times10^{-3}} = \boxed{14.85\ \text{MPa (C)}}$$

Buckling of Long Straight Columns

- Example 2

A WT6 × 36 structural steel section is used for an 18-ft column. Determine

(a) The slenderness ratio.

(b) The Euler buckling load. Use E = 29×10^3 ksi.

(c) The axial stress in the column when Euler load is applied.

Buckling of Long Straight Columns

- Example 2 (cont'd)

For a WT6 × 36 section (see Fig 9, or Appendix B of Textbook:

$$A = 10.6\ \text{in}^2 \qquad r_{\min} = 1.48\ \text{in}$$

$$\text{(a)}\quad \frac{L}{r} = \frac{18\times12}{1.48} = 145.9 \cong \boxed{146\ (\text{slender})}$$

$$\text{(b)}\quad P_{cr} = \frac{\pi^2 EA}{(L/r)^2} = \frac{\pi^2(29{,}000)(10.6)}{145.9} = \boxed{142.4\ \text{kips}}$$

$$\text{(c)}\quad \sigma = \frac{P_{cr}}{A} = \frac{142.4}{10.6} = \boxed{13.43\ \text{ksi (C)}}$$

Figure 9

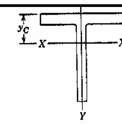


TABLE B-11 Structural Tees (U.S. Customary Units)

Designation*	Area (in.²)	Depth of Tee (in.)	FLANGE Width (in.)	FLANGE Thickness (in.)	Stem Thickness (in.)	AXIS X–X I (in.⁴)	AXIS X–X S (in.³)	AXIS X–X r (in.)	AXIS X–X y_c (in.)	AXIS Y–Y I (in.⁴)	AXIS Y–Y S (in.³)	AXIS Y–Y r (in.)
WT18 × 115	33.8	17.950	16.470	1.260	0.760	934	67.0	5.25	4.01	470	57.1	3.73
× 80	23.5	18.005	12.000	1.020	0.650	740	55.8	5.61	4.74	147	24.6	2.50
WT15 × 66	19.4	15.155	10.545	1.000	0.615	421	37.4	4.66	3.90	98.0	18.6	2.25
× 54	15.9	14.915	10.475	0.760	0.545	349	32.0	4.69	4.01	73.0	13.9	2.15
WT12 × 52	15.3	12.030	12.750	0.750	0.500	189	20.0	3.51	2.59	130	20.3	2.91
× 47	13.8	12.155	9.065	0.875	0.515	186	20.3	3.67	2.99	54.5	12.0	1.98
× 42	12.4	12.050	9.020	0.770	0.470	166	18.3	3.67	2.97	47.2	10.5	1.95
× 31	9.11	11.870	7.040	0.590	0.430	131	15.6	3.79	3.46	17.2	4.90	1.38
WT9 × 38	11.2	9.105	11.035	0.680	0.425	71.8	9.83	2.54	1.80	76.2	13.8	2.61
× 30	8.82	9.120	7.555	0.695	0.415	64.7	9.29	2.71	2.16	25.0	6.63	1.69
× 25	7.33	8.995	7.495	0.570	0.355	53.5	7.79	2.70	2.12	20.0	5.35	1.65
× 20	5.88	8.950	6.015	0.525	0.315	44.8	6.73	2.76	2.29	9.55	3.17	1.27
WT8 × 50	14.7	8.485	10.425	0.985	0.585	76.8	11.4	2.28	1.76	93.1	17.9	2.51
× 25	7.37	8.130	7.070	0.630	0.380	42.3	6.78	2.40	1.89	18.6	5.26	1.59
× 20	5.89	8.005	6.995	0.505	0.305	33.1	5.35	2.37	1.81	14.4	4.12	1.57
× 13	3.84	7.845	5.500	0.345	0.250	23.5	4.09	2.47	2.09	4.80	1.74	1.12
WT7 × 60	17.7	7.240	14.670	0.940	0.590	51.7	8.61	1.71	1.24	247	33.7	3.74
× 41	12.0	7.155	10.130	0.855	0.510	41.2	7.14	1.85	1.39	74.2	14.6	2.48
× 34	9.99	7.020	10.035	0.720	0.415	32.6	5.69	1.81	1.29	60.7	12.1	2.46
× 24	7.07	6.985	8.030	0.595	0.340	24.9	4.48	1.87	1.35	25.7	6.40	1.91
× 15	4.42	6.920	6.730	0.385	0.270	19.0	3.55	2.07	1.58	9.79	2.91	1.49
× 11	3.25	6.870	5.000	0.335	0.230	14.8	2.91	2.14	1.76	3.50	1.40	1.04
WT6 × 60	17.6	6.560	12.320	1.105	0.710	43.4	8.22	1.57	1.28	172	28.0	3.13
× 48	14.1	6.355	12.160	0.900	0.550	32.0	6.12	1.51	1.13	135	22.2	3.09
× 36	10.6	6.125	12.040	0.670	0.430	23.2	4.54	1.48	1.02	97.5	16.2	3.04
× 25	7.34	6.095	8.080	0.640	0.370	18.7	3.79	1.60	1.17	28.2	6.97	1.96
× 15	4.40	6.170	6.520	0.440	0.260	13.5	2.75	1.75	1.27	10.2	3.12	1.52
× 8	2.36	5.995	3.990	0.265	0.220	8.70	2.04	1.92	1.74	1.41	0.706	0.773
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	28.6	6.40	1.32	1.21	118	22.6	2.68

Buckling of Long Straight Columns

- Example 3

  Two C229 × 30 structural steel channels are used for a column that is 12 m long. Determine the total compressive load required to buckle the two members if

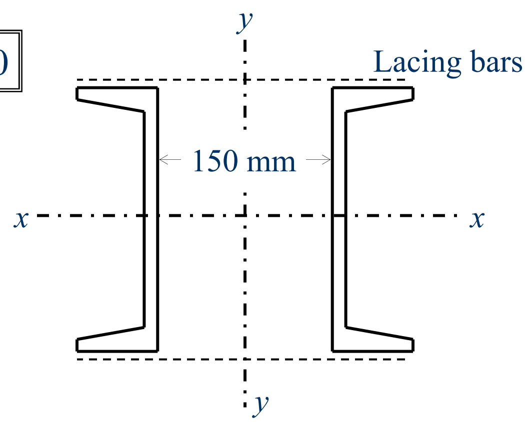
  (a) They act independently of each other. Use E = 200 GPa.

  (b) They are laced 150 mm back to back as shown in Fig. 10.

Buckling of Long Straight Columns

- Example 3 (cont'd)

Figure 10



Buckling of Long Straight Columns

- Example 3 (cont'd)
  - (a) Two channels act independently:
    - If the two channels are not connected and each acts independently, the slenderness ratio is determined by using the minimum radius of gyration r_{min} of the individual section
    - For a C229 × 30 section (see Fig 11, or Appendix B of Textbook):

$$r_{\min} = r_y = 16.3\ \text{mm} \qquad A = 3795\ \text{mm}^2$$

Figure 11

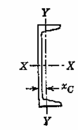


TABLE B-6 Standard Channels (SI Units)

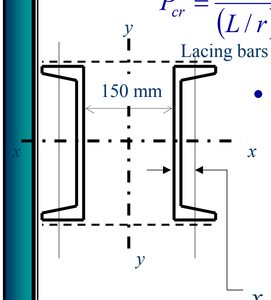
Designation*	Area (mm^2)	Depth (mm)	FLANGE Width (mm)	FLANGE Thickness (mm)	Web Thickness (mm)	AXIS X–X I (10^6 mm^4)	AXIS X–X S (10^3 mm^3)	AXIS X–X r (mm)	AXIS Y–Y I (10^6 mm^4)	AXIS Y–Y S (10^3 mm^3)	AXIS Y–Y r (mm)	x_C (mm)
C457 × 86	11030	457.2	106.7	15.9	17.8	281	1230	160	7.41	87.2	25.9	21.9
× 77	9870	457.2	104.1	15.9	15.2	261	1140	163	6.83	83.1	26.4	21.8
× 68	8710	457.2	101.6	15.9	12.7	241	1055	167	6.29	79.0	26.9	22.0
× 64	8130	457.2	100.3	15.9	11.4	231	1010	169	5.99	76.9	27.2	22.3
C381 × 74	9485	381.0	94.4	16.5	18.2	168	882	133	4.58	61.9	22.0	20.3
× 60	7615	381.0	89.4	16.5	13.2	145	762	138	3.84	55.2	22.5	19.7
× 50	6425	381.0	86.4	16.5	10.2	131	688	143	3.38	51.0	23.0	20.0
C305 × 45	5690	304.8	80.5	12.7	13.0	67.4	442	109	2.14	33.8	19.4	17.1
× 37	4740	304.8	77.4	12.7	9.8	59.9	395	113	1.86	30.8	19.8	17.1
× 31	3930	304.8	74.7	12.7	7.2	53.7	352	117	1.61	28.3	20.3	17.7
C254 × 45	5690	254.0	77.0	11.1	17.1	42.9	339	86.9	1.64	27.0	17.0	16.5
× 37	4740	254.0	73.3	11.1	13.4	38.0	298	89.4	1.40	24.3	17.2	15.7
× 30	3795	254.0	69.6	11.1	9.6	32.8	259	93.0	1.17	21.6	17.6	15.4
× 23	2895	254.0	66.0	11.1	6.1	28.1	221	98.3	0.949	19.0	18.1	16.1
C229 × 30	3795	228.6	67.3	10.5	11.4	25.3	221	81.8	1.01	19.2	16.3	14.8
× 22	2845	228.6	63.1	10.5	7.2	21.2	185	86.4	0.803	16.6	16.8	14.9
× 20	2540	228.6	61.8	10.5	5.9	19.9	174	88.4	0.733	15.7	17.0	15.3
C203 × 28	3555	203.2	64.2	9.9	12.4	18.3	180	71.6	0.824	16.6	15.2	14.4
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	148	75.9	0.637	14.0	15.6	14.0

Buckling of Long Straight Columns

- Example 3 (cont'd)

$$\frac{L}{r} = \frac{12\times 10^3}{16.3} = 736.2\ \text{(slender)}$$

$$P_{cr} = \frac{\pi^2 EA}{(L/r)^2} = \frac{\pi^2\left(200\times 10^9\right)\left[(2)\left(3795\times 10^{-6}\right)\right]}{(736.2)^2} = \boxed{27.64\times 10^3\ \text{N} = 27.6\ \text{kN}}$$



- (b) For a C229 × 30 section (see Fig 11, or Appendix B of Textbook):

$r_{\min} = 81.8\ \text{mm}$ $\quad$ $x_C = 14.8\ \text{mm}$

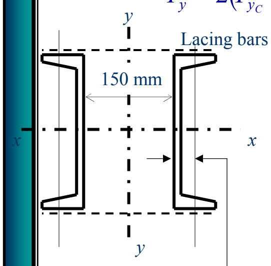
$I_x = 25.3\times 10^6\ \text{mm}^4$ $\quad$ $I_y = 1.01\times 10^6\ \text{mm}$

Buckling of Long Straight Columns

- Example 3 (cont'd)

$$I_x = 2I_{x_C} = 2\left(25.3\times10^6\right) = 50.6\times10^6 \text{ mm}^2 \Rightarrow r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{50.6\times10^6}{2(3795)}} = 81.7$$

$$I_y = 2\left(I_{y_C} + Ad^2\right) = 2\left[1.01\times10^6 + 3795(75+14.8)^2\right] = 63.23\times10^6 \text{ mm}^2$$



$x_C = 14.8$ mm

$$\Rightarrow r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{63.23\times10^6}{2(3795)}} = 91.3 \text{ mm}$$

$$r_x = r_{\min} = 81.7, \text{ therefore, } \frac{L}{r_{\min}} = \frac{12\times10^3}{81.7} = 146.9$$

$$\therefore P_{cr} = \frac{\pi^2 EA}{(L/r_{\min})^2} = \frac{\pi^2\left(200\times10^9\right)\left[2\left(3795\times10^{-6}\right)\right]}{(146.9)^2} \boxed{= 694.3 \text{ kN}}$$

Buckling of Long Straight Columns

- Example 3 (cont'd)
  - An alternate solution for finding r_x and r_y:
    - Using Eqs. 13 and 15,

$$r_x = r_{x_C} = 81.8 \text{ mm}$$

$$r_y = \sqrt{r_{y_C}^2 + (a + x_C)^2} = \sqrt{(16.3)^2 + (75+14.8)^2}$$
$$= 91.3 \text{ mm}$$

    - Therefore, $r_{\min} = r_x = 81.8 \text{ mm}$

The slight difference in the result is due to round-off errors.