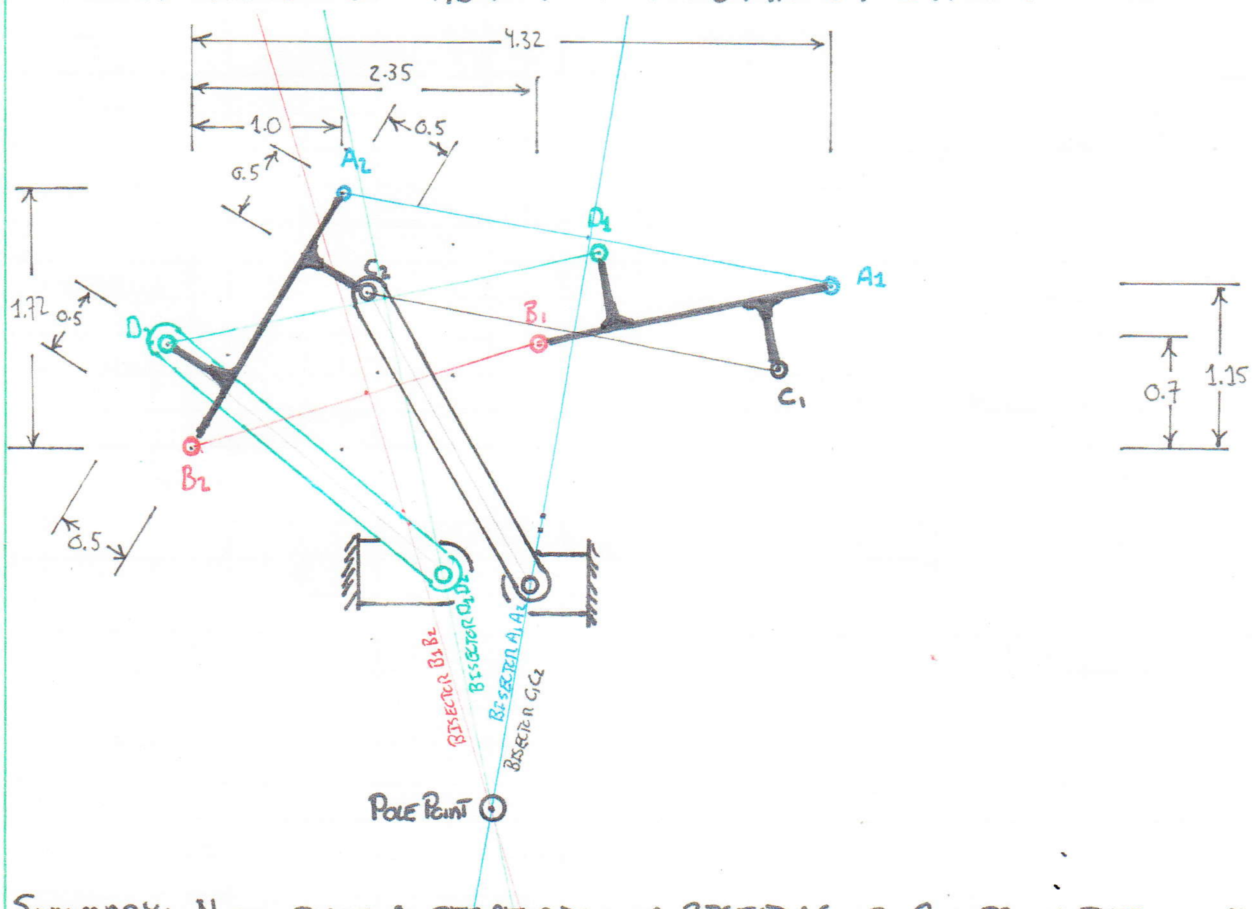
PROBLEM 7.4 | FOR THE TWO COUPLER LINK POSITION INDICATED BY LINES A_1B_1 AND A_2B_2, LOCATE THE POLE POINT. USING POINTS C AND D AS MOVING HINGE PINS, DESIGN A FOUR BAR MECHANISM THAT WILL MOVE LINE AB INTO ITS TWO DESIGNATED POSITIONS.

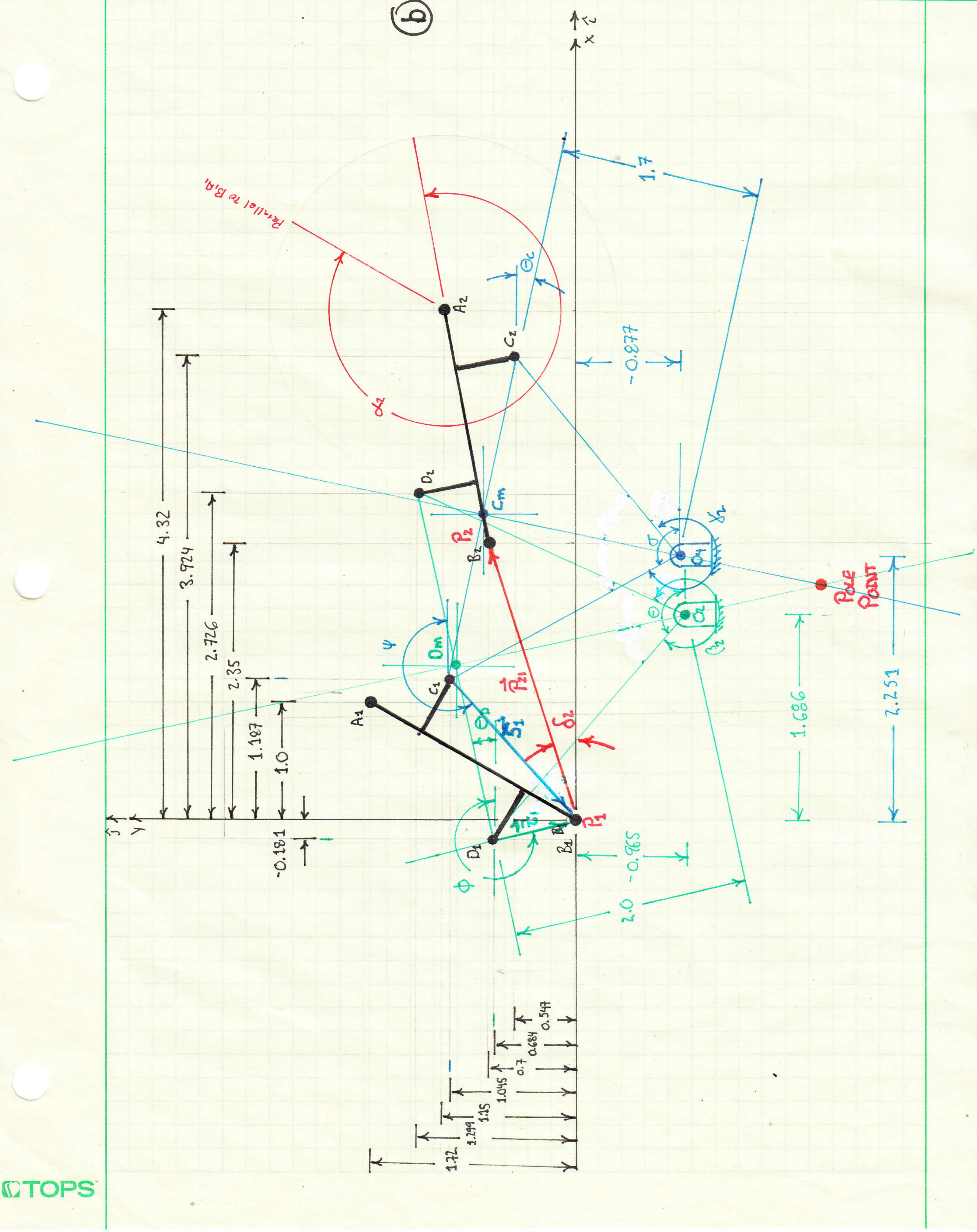
GIVEN:
1) TO POSITIONS OF THE COUPLER LINK AB.
2) ORIENTATION OF POINTS C AND D WITH RESPECT TO A AND B.
3) DESIRE FOR COUPLER MOTION.

ASSUMPTIONS:
1. ALL LINKS ARE RIGID
2. ALL JOINTS ARE FRICTIONLESS
3. INERTIAL AFFECTS ARE IGNORED
4. ALL LINK MOVE IN A SINGLE OR PARALLEL PLANES.
5. POINTS A, B, C, & D ARE ALL ON THE SAME LINK.

FIND:
1. LOCATE POLE POINTS FOR THE COUPLER LINK AB GIVEN POSITIONS A_1B_1 & A_2B_2.
2.) USING POINTS C D AS MOVING HINGE PINS, DESIGN A 4 BAR THAT WILL MOVE AB TO THEIR DESIGNATED POSITIONS.

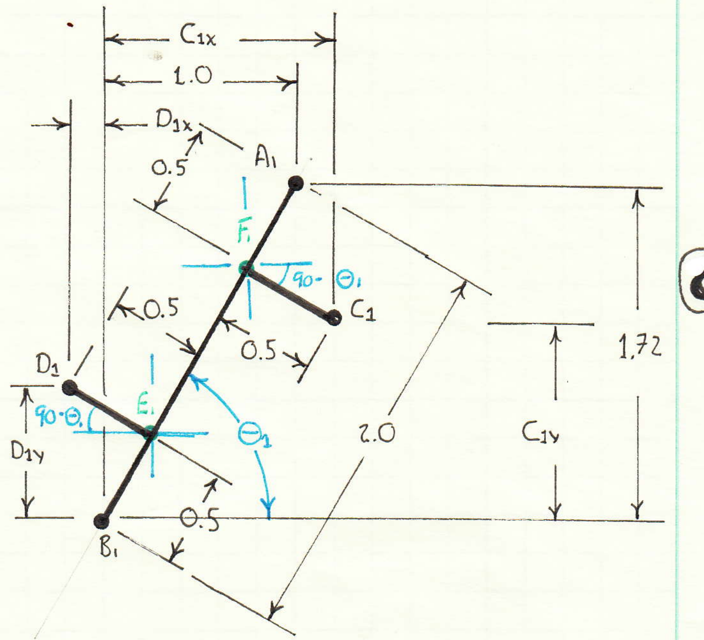


SUMMARY: NOTE THAT A PERPENDICULAR BISECTORS OF POINTS ON THE COUPLER INTERSECT AT A SINGLE POLE POINT. IF THE POLE POINT WERE USED AS THE FIXED PIVOT, THE SYNTHESIS WOULD HAVE RESULTED IN ROCKER OUTPUT.


b
Parallel to B1A1
1.7
A2
C2
D2
Cm
B2
Dm
C1
A1
D1
B1
O2
O4
Pole Point
4.32
3.924
2.726
2.35
1.187
1.0
-0.181
-0.877
-0.965
2.0
1.686
2.251
1.72
1.299
1.15
1.045
0.7
0.684
0.547

STARTING BY DETERMINING THE LOCATION OF D_1 & C_1 WITH RESPECT TO THE POINT B_1 WHERE THE COORDINATE SYSTEM FOR THIS PROBLEM IS CENTERED.

THE ANGLE B_1A_1 MAKES WITH THE HORIZONTAL IS Θ_1



$$\Theta_1 = \tan^{-1}\frac{1.72}{1.0}$$

$$= \underline{59.8^\circ} \qquad (1)$$

THE X-Y COORDINATES OF E_1 & F_1 CAN NOW BE CALCULATED

$$E_{1x} = 0.5 \cdot \cos 59.8^\circ$$

$$= \underline{0.251} \qquad (2)$$

$$E_{1y} = 0.5 \cdot \sin 59.8^\circ$$

$$= \underline{0.432} \qquad (3)$$

$$F_{1x} = 1.5 \cos 59.8^\circ = \underline{0.755} \qquad (4)$$

$$F_{1y} = 1.5 \sin 59.8^\circ = \underline{1.296} \qquad (5)$$

NOW D_1's AND C_1's X-Y COORDINATES CAN BE FOUND

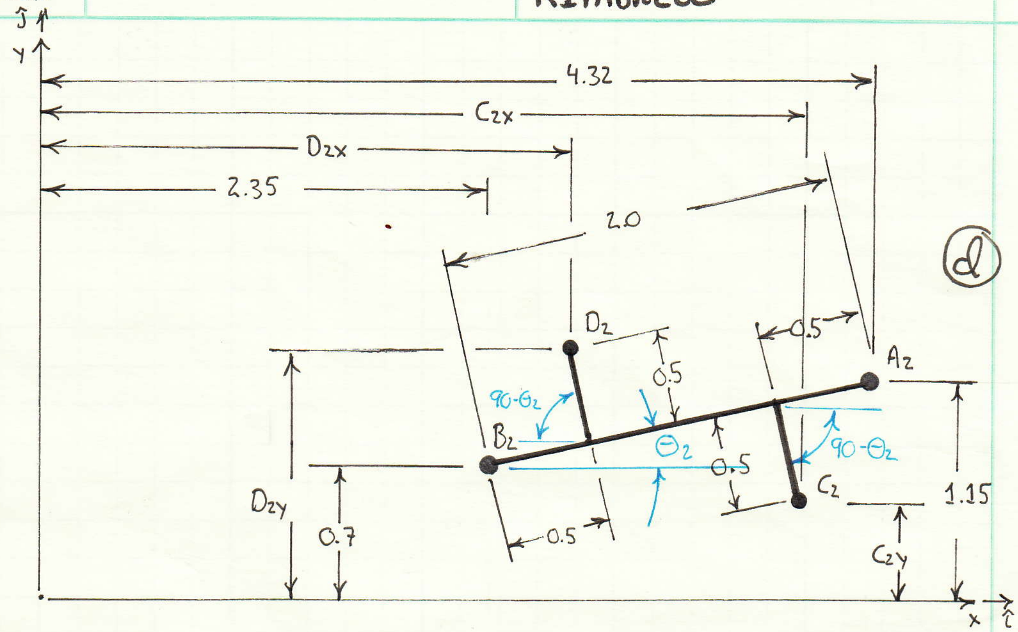
$$D_{1x} = 0.5 \cdot \cos 59.8^\circ - 0.5 \cos(90^\circ - 59.8^\circ) = \underline{-0.181} \qquad (6)$$

$$D_{1y} = 0.5 \cdot \sin 59.8^\circ + 0.5 \cdot \sin(90^\circ - 59.8^\circ) = \underline{0.684} \qquad (7)$$

$$C_{1x} = 1.5 \cdot \cos 59.8^\circ + 0.5 \cdot \cos(90^\circ - 59.8^\circ) = \underline{1.187} \qquad (8)$$

$$C_y = 1.5 \sin 59.8^\circ - 0.5 \sin(90^\circ - 59.8) = \underline{1.045} \qquad (9)$$

THE X-Y COORDINATES OF C_1 & D_1 ~~HA~~ IN 8 & 9 HAVE BEEN ADDED TO (b). NOW THE SECOND POSITION OF THE COUPLER IS CONSIDERED.



THE ANGLE B_2A_2 MAKES WITH THE HORIZONTAL, θ_2, CAN NOW BE CALCULATED

$$\theta_2 = \tan^{-1}\frac{1.15-0.7}{4.32-2.35} = 12.9^\circ \quad (10)$$

THE X-Y COORDINATES OF D_2 & C_2 CAN NOW BE CALCULATED

$$D_{2x} = 2.35 + 0.5\cdot\cos 12.9^\circ - 0.5\cdot\cos(90^\circ-12.9^\circ) = 2.726 \quad (11)$$

$$D_{2y} = 0.7 + 0.5\cdot\sin 12.9^\circ + 0.5\cdot\sin(90^\circ-12.9^\circ) = 1.299 \quad (12)$$

$$C_{2x} = 2.35 + 1.5\cdot\cos 12.9^\circ + 0.5\cdot\cos(90^\circ-12.9^\circ) = 3.924 \quad (13)$$

$$C_{2y} = 0.7 + 1.5\cdot\sin 12.9^\circ - 0.5\cdot\sin(90^\circ-12.9^\circ) = 0.547 \quad (14)$$

NOW THAT THE GEOMETRY OF THE COUPLER LINK IS COMPLETELY DEFINED, THE QUANTITIES ~~ARE~~ CONSIDERED GIVEN IN BOTH APPROACH A & B OF THE ANALYTICAL METHOD CAN NOW BE CALCULATED. THE QUANTITIES CONSIDERED GIVEN ARE ρ_{21}, δ_2, AND α_2. THE GEOMETRY FOR THESE PARAMETERS IS ILLUSTRATED IN b

$$\rho_{21} = \sqrt{(2.35-0)^2+(0.7-0)^2} = \underline{\underline{2.452}} \quad (15)$$

$$\delta_2 = \tan^{-1}\frac{0.7}{2.35} = \underline{\underline{16.6^\circ}} \quad (16)$$

$$\alpha_2 = 360^\circ - (59.8^\circ - 12.9^\circ) = \underline{\underline{313.1^\circ}} \quad (17)$$

TO HELP VERIFY THE ANALYTICAL SOLUTION THE ROTO-POLES O_2 AND O_4 ARE GOING TO BE CHOSEN THROUGH GEOMETRY.

STARTING WITH THE ROTO-POLE FOR D_1D_2, O_2.

THE PERPENDICULAR BISECTOR IS FIRST FOUND BY LOCATING THE MID POINT OF D_1D_2, D_m

$$\theta_D = \tan^{-1}\frac{1.299-0.684}{2.726-(-0.181)} = \underline{\underline{11.95^\circ}} \quad (18)$$

$$l_{D1D2} = \sqrt{(1.299-0.684)^2+(2.726+0.181)^2} = \underline{\underline{2.971}} \quad (19)$$

THE X-Y LOCATION OF D_m CAN NOW BE CALCULATED

$$D_{mx} = -0.181+0.5(2.971)\cos 11.95^\circ = \underline{\underline{1.272}} \quad (20)$$

$$D_{my} = 0.684+0.5(2.971)\sin 11.95^\circ = \underline{\underline{0.992}} \quad (21)$$

IF THE LOCATION OF THE ROTO POLE O_2 IS ~~CHOOSEN~~ CHOSEN TO BE 2.0 BELOW D_m ALONG THE PERPENDICULAR BISECTOR, THE X-Y COORDINATES OF O_2 ARE

$$O_{2x} = 1.272 + 2.0\cdot\sin 11.95^\circ = \underline{\underline{1.686}} \quad (22)$$

$$O_{2y} = 0.992 - 2.0\cdot\cos 11.95^\circ = \underline{\underline{-0.965}} \quad (23)$$

PROCEEDING TO FIND THE ROTO POLE FOR C_1C_2, O_4

THE PERPENDICULAR BISECTOR IS FIRST FOUND BY LOCATING THE MID POINT OF C_1C_2, C_m

$$\theta_C = \tan^{-1}\frac{1.045-0.547}{3.924-1.187} = \underline{\underline{10.31^\circ}} \quad (24)$$

$$l_{C1C2} = \sqrt{(3.924-1.187)^2+(1.045-0.547)^2} = \underline{\underline{2.782}} \quad (25)$$

THE X-Y LOCATION OF C_m CAN NOW BE CALCULATED

$$C_{mx} = 1.187+0.5\cdot(2.782)\cdot\cos(10.31^\circ) = \underline{\underline{2.556}} \quad (26)$$

$$C_{my} = 1.045-0.5\cdot(2.782)\cdot\sin(10.31^\circ) = \underline{\underline{0.796}} \quad (27)$$

IF THE LOCATION OF THE ROTO-POLE IS CHOSEN TO BE 1.7 BELOW C_m ALONG THE PERPENDICULAR BISECTOR, THE X-Y COORDINATES OF O_4 ARE.

$$O_{4x} = 2.556 - 1.7 \cdot \sin 10.31 = \underline{\underline{2.251}} \quad (28)$$

$$O_{4y} = 0.796 - 1.7 \cos 10.31 = \underline{\underline{-0.877}} \quad (29)$$

NOW CONSIDER THE APPROACH A SOLUTION TO THE TWO POSITION SYNTHESIS

THE GIVENS ARE SUMMERIZED (15)-(17) FOR BOTH DYADS.

THE FREE CHOICES FOR THE FIRST DYAD CAN BE MEASURED OR IN THIS CASE CALCULATED. THE FREE CHOICE PARAMETERS ARE Θ, ϕ & β. THESE ARE ILLUSTRATED IN (b).

$$\Theta = 180° - \tan^{-1}\frac{0.965+0.684}{1.686+0.181} = 180° - 41.45° = \underline{\underline{138.55°}} \quad (30)$$

$$\phi = 270° + \tan^{-1}\frac{0.181}{0.684} = 270° + 14.82° = \underline{\underline{284.82}} \quad (31)$$

$$\beta_2 = \tan^{-1}\frac{0.965+0.684}{1.686+0.181} + 180° + \tan^{-1}\frac{0.965+1.299}{2.726-1.686}$$

$$= 41.45° + 180° + 65.33° = \underline{\underline{286.78°}} \quad (32)$$

THE FREE CHOICES FOR THE SECOND DYAD CAN BE MEASURED OR IN THIS CASE CALCULATED. THE FREE CHOICE PARAMETERS ARE σ, ψ & γ_2. THESE ARE ILLUSTRATED IN (b)

$$\sigma = 180° - \tan^{-1}\frac{0.877+1.045}{2.251-1.187} = 180° - 61.03° = \underline{\underline{118.97°}} \quad (33)$$

$$\psi = 180° + \tan^{-1}\frac{1.045}{1.187} = \underline{\underline{221.36}} \quad (34)$$

$$\gamma_2 = 180° + \tan^{-1}\frac{0.877+1.045}{2.251-1.187} + \tan^{-1}\frac{0.877+0.547}{3.924-2.251}$$

$$= 180° + 61.03 + 40.40 = \underline{\underline{281.43}} \quad (35)$$

THE FREE CHOICES IN (30)-(35) ALONG WITH THE GIVEN PARAMETERS IN (15)-(17) ARE INPUT INTO THE ALGORITHM, ~~AND~~ THE RESULTS ARE SHOWN ON THE NEXT PAGE.

APPROACH A | **FIRST DYAD**

GIVEN:		**CHOSEN:**		**FIND:**				**x-coord**	**y-coord**
P12	2.452	θ	138.55	**w**	2.493		**O2**	1.687	-0.963
δ2	16.597	ϕ	284.82	**z**	0.711		**A1**	-0.182	0.687
α2	313.07	β2	286.78		**x-coord**	**y-coord**	**A2**	2.728	1.303
				W1	-1.869	1.651	**P1**	0.000	0.000
				W2	1.041	2.266	**P2**	2.350	0.700
				Z1	0.182	-0.687			
				Z2	-0.378	-0.602			

$$\begin{bmatrix} 1.166925003 & -0.787329649 \\ 0.246760639 & 0.119705 \end{bmatrix} \begin{Bmatrix} w \\ z \end{Bmatrix} = \begin{Bmatrix} 2.349844 \\ 0.700385 \end{Bmatrix}$$

0.358432	2.357495
-0.73887	3.494115

APPROACH A | **SECOND DYAD**

GIVEN:		**CHOSEN:**		**FIND:**				**x-coord**	**y-coord**
P12	2.452	σ	118.97	**u**	2.197		**O4**	2.251	-0.877
δ2	16.597	ψ	221.36	**s**	1.581		**B1**	1.187	1.045
α2	313.07	γ2	281.43		**x-coord**	**y-coord**	**B2**	3.924	0.547
				U1	-1.064	1.922	**P1**	0.000	0.000
				U2	1.673	1.424	**P2**	2.350	0.700
				S1	-1.187	-1.045			
				S2	-1.574	0.153			

$$\begin{bmatrix} 1.24589 & -0.24471 \\ -0.22675 & 0.75785 \end{bmatrix} \begin{Bmatrix} w \\ z \end{Bmatrix} = \begin{Bmatrix} 2.349844 \\ 0.700385 \end{Bmatrix}$$

0.852753	0.27535
0.255149	1.401909

THE ~~A~~ COMPUTED PARAMETERS MATCH THE GEOMETRIC SOLUTION AND THE GRAPHICAL SOLUTION.

NOW CONSIDER THE APPROACH B SOLUTION TO THE TWO POSITION SYNTHESIS

THE PROBLEM GIVENS ARE THE SAME AS APPROACH A AND ARE SUMMARIZED IN 15 – 17

THE FREE CHOICES FOR THE FIRST DYAD CAN BE MEASURED OR IN THIS CASE CALCULATED. THE FREE CHOICE PARAMETERS ~~OF~~ FOR THIS APPROACH ~~ARE~~ FOR THE FIRST DYAD ARE Z, ϕ, & β_2. THESE ARE ILLUSTRATED IN b.

$$Z = \sqrt{(0.181)^2 + (0.684)^2} = \underline{0.7075} \qquad (36)$$

$$\phi = (\text{WAS CALCULATED IN } (31)) = \underline{284.82}$$

$$\beta_2 = (\text{WAS CALCULATED IN } (32)) = \underline{286.78}$$

THE FREE CHOICES FOR THE SECOND DYAD CAN BE MEASURED OR IN THIS CASE CALCULATED. THE FREE CHOICE PARAMETERS FOR THIS APPROACH FOR THE SECOND DYAD ARE S, ψ, & γ_2. THESE ARE ILLUSTRATED IN b.

$$S = \sqrt{(1.187)^2 + (1.045)^2} = \underline{1.582} \qquad (37)$$

$$\psi = (\text{WAS CALCULATED IN } (34)) = \underline{221.36}$$

$$\gamma_2 = (\text{WAS CALCULATED IN } (35)) = \underline{281.43}$$

THE FREE CHOICES 31, 32, 34, 35, 36, 37 ALONG WITH THE GIVEN PARAMETERS ARE INPUT INTO THE ALGORITHM. THE RESULTING SOLUTION IS SHOWN ON THE NEXT PAGE.

APPROACH B	FIRST DYAD

GIVEN:		CHOSEN:		FIND:	
P12	2.452	z	0.707	w	2.491
δ2	16.597	ϕ	284.82	θ	138.572
α2	313.07	β2	286.78	W1x	-1.868
				W1y	1.648

	x-coord	y-coord
W1	-1.868	1.648
W2	1.039	2.264
Z1	0.181	-0.683
Z2	-0.376	-0.599

	x-coord	y-coord
O2	1.687	-0.965
A1	-0.181	0.683
A2	2.726	1.299
P1	0.000	0.000
P2	2.350	0.700

$$\begin{bmatrix} -0.711302388 & 0.95742033 \\ -0.95742033 & -0.711302388 \end{bmatrix} \begin{Bmatrix} W1x \\ W1y \end{Bmatrix} = \begin{Bmatrix} 2.906486 \\ 0.615753 \end{Bmatrix}$$

inverse

-0.5	-0.67301
0.673005	-0.5

APPROACH B	SECOND DYAD

GIVEN:		CHOSEN:		FIND:	
P12	2.452	s	1.582	u	2.197
δ2	16.597	ψ	221.36	σ	118.962
α2	313.07	γ2	281.43	U1x	-1.064
				U1y	1.922

	x-coord	y-coord
U1	-1.0638	1.9221
U2	1.6732	1.4236
S1	-1.1874	-1.0454
S2	-1.5745	0.1536

	x-coord	y-coord
O4	2.251	-0.877
B1	1.187	1.045
B2	3.924	0.547
P1	0.000	0.000
P2	2.350	0.700

$$\begin{bmatrix} -0.80183 & 0.980168 \\ -0.98017 & -0.80183 \end{bmatrix} \begin{Bmatrix} U1x \\ U1y \end{Bmatrix} = \begin{Bmatrix} 2.736968 \\ -0.49853 \end{Bmatrix}$$

inverse

-0.5	-0.61121
0.611207	-0.5

SUMMARY:

THE SOLUTIONS ~~POSTE~~ IN THIS PROBLEM INCLUDES THE GRAPHICAL SYNTHESIS, A GEOMETRIC SYNTHESIS, 2 POSITION-APPROACH SYNTHESIS AND 2 POSITION-APPROACH B SYNTHESIS. ALL METHODS MATCH. THE 2 POSITION APPROACH A & B USED CALCULATED VALUES FOR FOR THE FREE CHOICES. THESE VALUES WERE CALCULATED USING GEOMETRY.