DETERMINE THE CRITICAL LOAD FOR THIS BEAM TO BUCKLE.

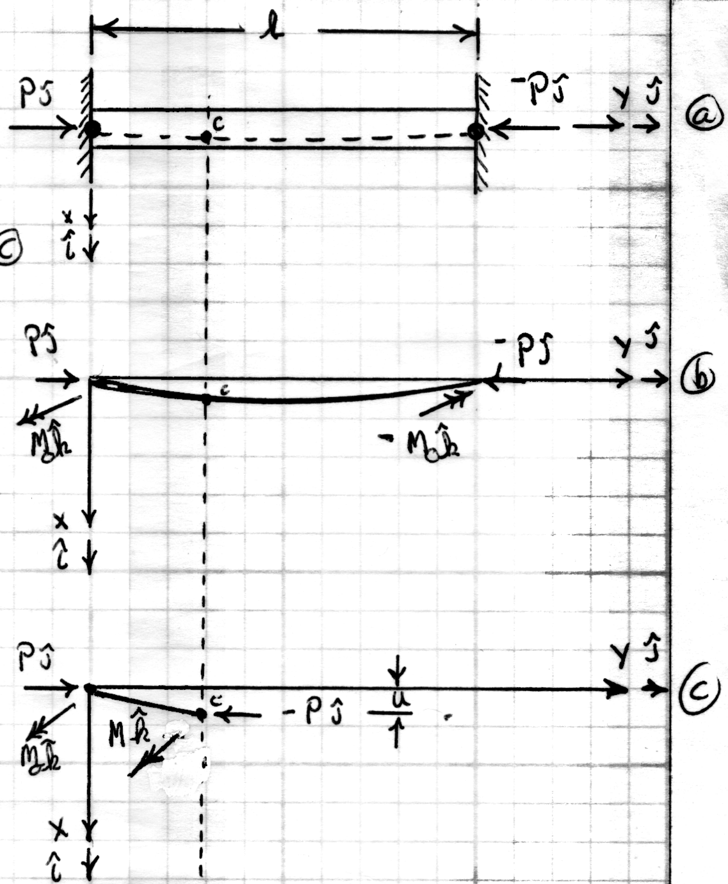

FIGURE ⓐ ILLUSTRATES THE CONSTRAINTS ON A BEAM WITH FIXED CONDITIONS AT BOTH ENDS. FIGURE ⓑ IS THE FREE BODY DIAGRAM FOR THIS BEAM. FIGURE ⓒ ILLUSTRATES THE FREE BODY DIAGRAM THAT IS USED TO CONSIDER INTERNAL EQUILIBRIUM. THE INTERNAL MOMENT "M" IS FOUND BY CONSIDERING THE EQUILIBRIUM OF THE MOMENTS ABOUT POINT "C".

$$\Sigma M_{z/e_c} = 0 = M + M_o - P \cdot u$$

$$M = Pu - M_o \qquad (1)$$

FROM STRENGTH OF MATERIALS, THE RELATIONSHIP BETWEEN THE BENDING MOMENT AND THE DISPLACEMENT ON THE ELASTIC CURVE IS GIVEN BY

$$\frac{\partial^2 u}{\partial y^2} = \frac{-M}{EI} = \frac{-P}{EI} \cdot u + \frac{M_o}{E \cdot I}$$

$$\frac{\partial^2 u}{\partial y^2} + \frac{P}{EI} u = \frac{M_o}{EI} \qquad (2)$$

EQUATION (2) REPRESENTS A LINEAR DIFFERENTIAL EQUATION. BECAUSE THE RIGHT HAND SIDE OF THIS EQUATION IS NOT ZERO, THE SOLUTION WILL HAVE BOTH A HOMOGENIOUS (u_H) AND PARTICULAR (u_p) PORTIONS OF THE SOLUTION.

$$u = u_H + u_p \qquad (3)$$

STARTING WITH THE HOMOGENIOUS PORTION OF THE SOLUTION.

$$\frac{\partial^2 u_H}{\partial y^2} + \frac{P}{EI} u_H = 0 \qquad (4)$$

THE SOLUTION TO (4) IS OF THE FORM

$$u_H = A_n \cdot e^{S_n \cdot y}$$

$$u'_H = \partial u_H / \partial y = A_n \cdot S_n \cdot e^{S_n \cdot y} \qquad (5)$$

$$u''_H = \partial^2 u_H / \partial y^2 = A_n \cdot S_n^2 \cdot e^{S_n y}$$

AMPAD 22-141 50 SHEETS 22-142 100 SHEETS 22-144 200 SHEETS

SUBSTITUTING (5) INTO (4)

$$A_N \cdot S_N^2 \cdot e^{S_n \cdot y} + \frac{P}{EI} \cdot A_N \cdot e^{S_n \cdot y} = 0$$

$$A_N \cdot e^{S_n \cdot y} \left(S_n^2 + \frac{P}{EI} \right) = 0$$

TO FIND SOLUTION TO THE ABOVE EQUATION, $A_N = 0$ RESULTS IN A TRIVIAL SOLUTION, $e^{S_n \cdot y}$ CAN NOT EQUAL ZERO; THEREFORE THE ONLY NON-TRIVIAL SOLUTION IS

$$S_n^2 + \frac{P}{EI} = 0 \Rightarrow S_n = \pm \sqrt{-\frac{P}{EI}} \qquad (6)$$

SUBSTITUTING (6) INTO (5)

$$u_H = A_0 e^{i\sqrt{\frac{P}{EI}} y} + A_1 e^{-i\sqrt{\frac{P}{EI}} y} \qquad (7)$$

SINCE $e^{i\theta} = \cos\theta + i \cdot \sin\theta$ AND $e^{i\theta} = \cos\theta - i \cdot \sin\theta$, (7) CAN BE REWRITEN

$$u_H = A_0 \left(\cos\sqrt{\frac{P}{EI}} \cdot y + i \cdot \sin\sqrt{\frac{P}{EI}} \cdot y \right) + A_1 \left(\cos\sqrt{\frac{P}{EI}} \cdot y - i \cdot \sin\sqrt{\frac{P}{EI}} \cdot y \right)$$

$$= \underbrace{(A_0 + A_1)}_{C_0} \cos\sqrt{\frac{P}{EI}} \cdot y + \underbrace{(A_0 - A_1)}_{C_1} i \sin\sqrt{\frac{P}{EI}} \cdot y$$

$$= C_0 \cdot \cos\sqrt{\frac{P}{EI}} \cdot y + C_1 \cdot \sin\sqrt{\frac{P}{EI}} \cdot y \qquad (8)$$

NOW THE PARTICULAR SOLUTION TO (2) CAN BE CONSIDERED

$$\frac{\partial^2 u_p}{\partial y^2} + \frac{P}{EI} u_p = \frac{M_0}{EI} \qquad (9)$$

THE FORM OF THE PARTICULAR SOLUTION IS THE SAME AS THE FORM OF THE RIGHT HAND SIDE OF (9)

$$u_p = C_2$$

$$u_p' = \partial u_p / \partial y = 0 \qquad (10)$$

$$u_p'' = \partial^2 u_p / \partial y^2 = 0$$

SUBSTITUTING THE RESULTS IN (10) INTO (9)

$$0 + \frac{P}{EI} \cdot C_2 = \frac{M_0}{EI} \Rightarrow C_2 = \frac{M_0}{P} = u_p \qquad (11)$$

NOW (8) AND (11) CAN BE SUBSTITUTED INTO (3)

$$u = C_0 \cdot \cos\sqrt{\frac{P}{EI}} \cdot y + C_1 \cdot \sin\sqrt{\frac{P}{EI}} \cdot y + \frac{M_0}{P} \qquad (12)$$

AMPAD 22-141 50 SHEETS 22-142 100 SHEETS 22-144 200 SHEETS

THE CONSTANTS IN (12) ARE DETERMINE BY CONSIDERING THE BOUNDARY CONDITIONS THAT CONSTRAIN THE BEAM. STARTING AT Y=0

$$u(0)=0 = C_0 \cdot \cancelto{1}{\cos\sqrt{\frac{P}{EI}}\cdot 0} + C_1 \cdot \cancelto{0}{\sin\sqrt{\frac{P}{EI}}\cdot 0} + \frac{M_0}{P}$$

$$0 = C_0 + \frac{M_0}{P} \Rightarrow C_0 = -\frac{M_0}{P}$$

SUBSTITUTING THIS RESULT INTO (12)

$$u = -\frac{M_0}{P}\cdot\cos\sqrt{\frac{P}{EI}}\cdot y + C_1\cdot\sin\sqrt{\frac{P}{EI}}\cdot y + \frac{M_0}{P} \qquad (13)$$

THE SECOND BOUNDARY CONDITION AT Y=0

$$\frac{\partial u(0)}{\partial y} = 0 = \frac{M_0}{P}\cdot\sqrt{\frac{P}{EI}}\cdot\cancelto{0}{\sin\sqrt{\frac{P}{EI}}\cdot 0} + C_1\cdot\sqrt{\frac{P}{EI}}\,\cancelto{1}{\cos\sqrt{\frac{P}{EI}}\cdot 0} \Rightarrow C_1 = 0$$

SUBSTITUTING THIS RESULT INTO (13)

$$u = -\frac{M_0}{P}\cdot\cos\sqrt{\frac{P}{EI}}\cdot y + \frac{M_0}{P} \qquad (14)$$

THE DETERMINATION OF THE CRITICAL LOAD IS DETERMINED BY CONSIDERING THE SYMMETRY CONDITION AT THE CENTER OF THE BEAM

$$\frac{\partial u(L/2)}{\partial y} = 0 = \frac{M_0}{P}\cdot\sqrt{\frac{P}{EI}}\sin\sqrt{\frac{P}{EI}}\cdot\frac{L}{2}$$

THIS EQUATION IS SATISFIED WHEN THE SINE TERM EQUALS ZERO

$$\sin\sqrt{\frac{P}{EI}}\cdot\frac{L}{2} = 0 \Rightarrow \sqrt{\frac{P}{EI}}\cdot\frac{L}{2} = n\cdot\pi,\quad n=0,1,2,\ldots$$

THE LOWEST, NON-TRIVIAL SOLUTION IS FOUND WHEN N=1

$$\sqrt{\frac{P}{EI}}\cdot\frac{L}{2} = \pi \Rightarrow P_{CR} = \frac{4\pi^2}{L^2}\cdot EI$$