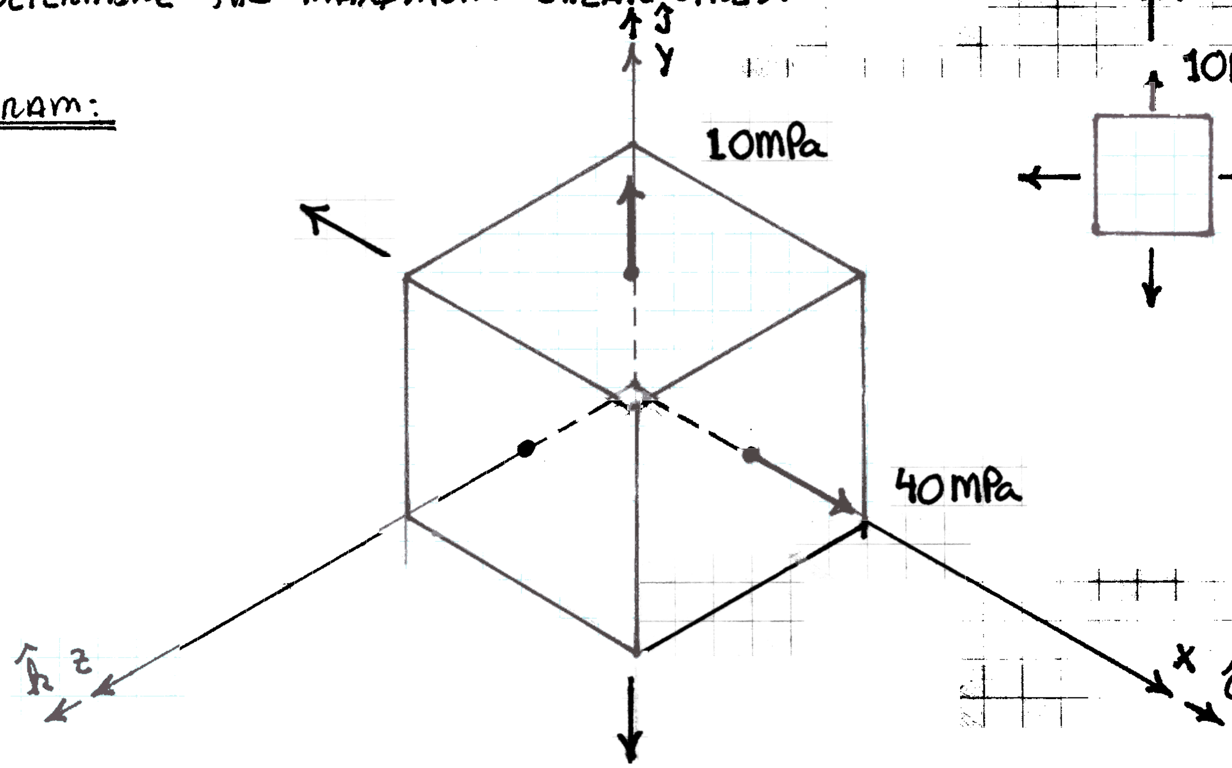
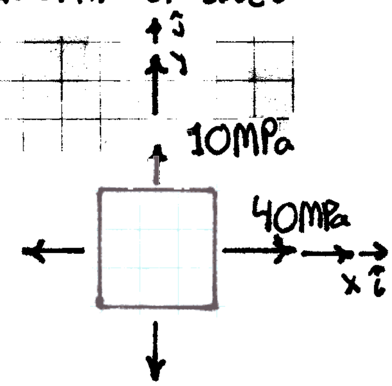
PROBLEM 2.16 | FOR THE PLANE STRESS STATE GIVEN
(a) DRAW THE CORRESPONDING STRESS ELEMENT PROPERLY ORIENTED RELATIVE TO THE XY AXES
(b) DETERMINE THE COMPLETE STRESS ELEMENT ASSOCIATED WITH AN AXIS SYSTEM ROTATED θ (DEFINED POSITIVE COUNTERCLOCKWISE) USING THE TRANSFORMATION EQUATIONS ALONE.
(c) DETERMINE THE PRINCIPAL STRESSES AND THE CORRESPONDING STRESS ELEMENT CONTAINING THE STRESSES PROPERLY ORIENTED RELATIVE TO THE XY AXES USING EQUATIONS ONLY.
(d) REPEAT PARTS (b) AND (c) USING MOHR'S CIRCLE.
(e) DETERMINE THE MAXIMUM AND MINIMUM SHEAR STRESS AND SHOW THE COMPLETE STRESS ELEMENT CONTAINING THESE STRESSES. SHOW THE ELEMENT PROPERLY ORIENTED WITH RESPECT TO THE XYZ COORDINATE SYSTEM.

GIVEN:
CONSTRAINTS
1. $\sigma_x = 40\,MPa$, $\sigma_y = 10\,MPa$, $\tau_{xy} = 0$, $\theta = 15°$
ASSUMPTION
1. PLANE STRESS: $\sigma_z = \tau_{xz} = \tau_{yz} = 0$

FIND:
1. DRAW THE ELEMENT RELATIVE TO THE XY PLANE
2. USING TRANSFORMATION EQUATIONS DETERMINE $[\sigma]$ FOR $\theta = 15°$
3. USING TRANSFORMATION EQUATIONS DETERMINE THE PRINCIPAL STRESSES
4. REPEAT PREVIOUS TWO PARTS USING MOHR'S CIRCLE
5. DETERMINE THE MAXIMUM SHEAR STRESS

DIAGRAM:





SOLUTION:

USING TRANSFORMATION EQUATIONS THE STRESSES ON THE ELEMENT ROTATED 15° ABOUT THE z AXIS ARE

$$\sigma_{x'} = \sigma_x \cdot \cos^2\theta + \sigma_y \cdot \sin^2\theta + 2 \cdot \tau_{xy} \cdot \cos\theta \sin\theta$$

$$= (40\text{MPa}) \cdot \cos^2(15°) + (10\text{MPa}) \sin^2(15°) + 2(0) \cdot \cos(15°) \sin(15°)$$

$$= \boxed{38.0\text{MPa}}$$

$$\sigma_{y'} = \sigma_x \cdot \sin^2\theta + \sigma_y \cdot \cos^2\theta - 2 \cdot \tau_{xy} \cdot \sin\theta \cdot \cos\theta$$

$$= (40\text{MPa}) \cdot \sin^2(15°) + (10\text{MPa}) \cdot \cos^2(15°) - 2 \cdot (0) \cdot \sin(15) \cos(15)$$

$$= \boxed{12.0\text{MPa}}$$

$$\tau_{x'y'} = -(\sigma_x - \sigma_y) \cdot \sin\theta \cdot \cos\theta + \tau_{xy} \cdot (\cos^2\theta - \sin^2\theta)$$

$$= -(40\text{MPa} - 15\text{MPa}) \cdot \sin(15°) \cdot \cos(15°) + (0) \cdot (\cos^2(15°) - \sin^2(15°))$$

$$= \boxed{7.5\text{MPa}}$$

THE PRINCIPAL STRESSES, CALCULATED USING EQUATIONS, ARE

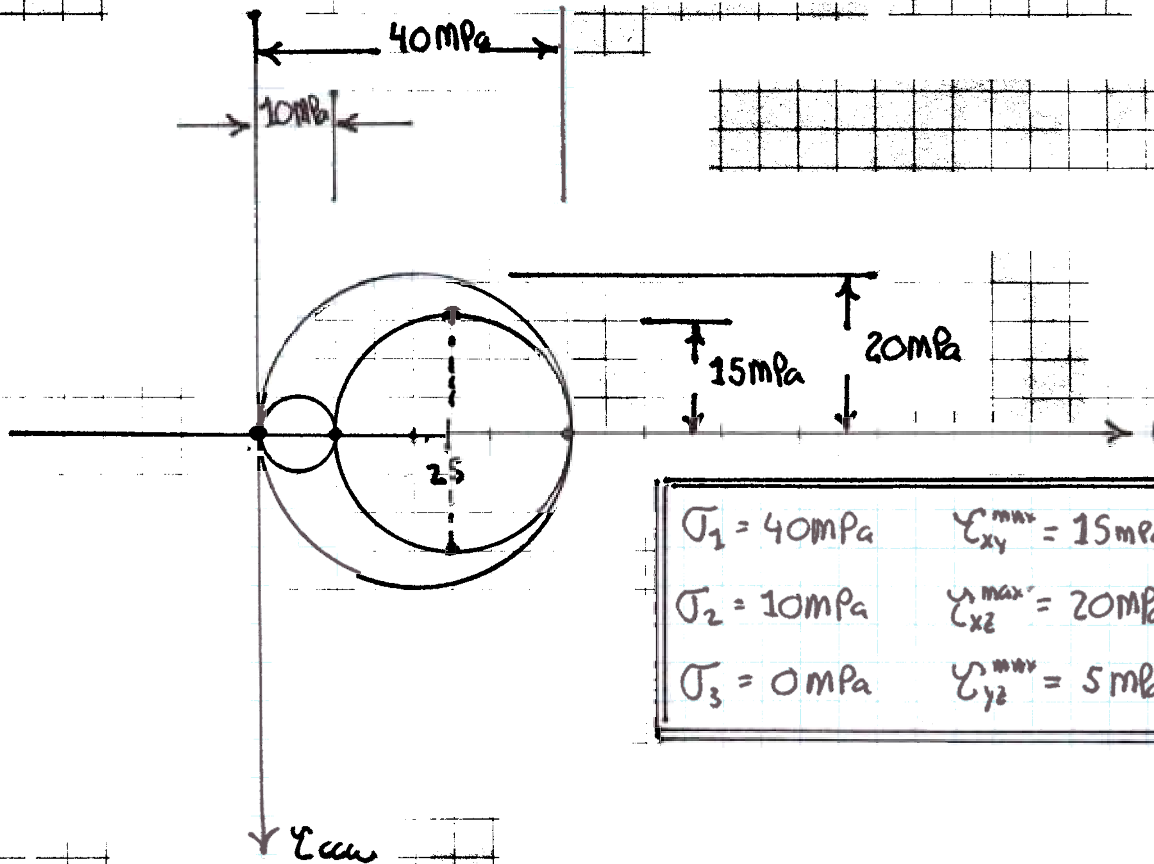
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{(40\text{MPa} + 10\text{MPa})}{2} \pm \sqrt{\left(\frac{40\text{MPa} - 10\text{MPa}}{2}\right)^2 + (0)^2} =$$

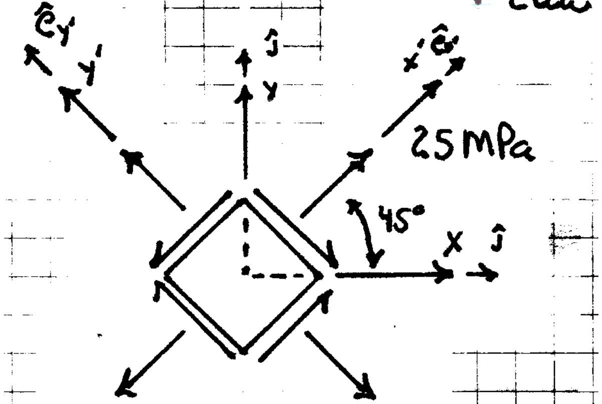
$$= 25\text{MPa} \pm 15\text{MPa}$$

$$\boxed{\begin{aligned} \sigma_1 &= 40\text{MPa} \\ \sigma_2 &= 15\text{MPa} \end{aligned}}$$

$$\tau_{max} = \pm\sqrt{\left(\frac{40\text{MPa} - 10\text{MPa}}{2}\right)^2 + (0)^2} = \boxed{\pm 15\text{MPa}}$$



$\sigma_1 = 40\text{MPa}$	$\tau_{xy}^{max} = 15\text{MPa}$
$\sigma_2 = 10\text{MPa}$	$\tau_{xz}^{max} = 20\text{MPa}$
$\sigma_3 = 0\text{MPa}$	$\tau_{yz}^{max} = 5\text{MPa}$



<u>SUMMARY:</u>

THE ACTUAL MAXIMUM SHEAR STRESS IS NOT IN THE X-Y PLANE. THE MAXIMUM SHEAR STRESS IS IN THE XZ PLANE AND IS 25% HIGHER THAN THE MAXIMUM SHEAR STRESS IN THE X-Y PLANE. NOTE, THE TRANSFORMATION EQUATIONS DID NOT GIVE ANY HINT TO THE STRESSES IN OTHER PLANES. MOHR'S CIRCLE GAVE A CLEAR VISUAL CLUE TO THE POTENTIAL OF OUT-OF-PLANE STRESSES THAT WERE MORE IMPORTANT TO FAILURE THAT THE IN-PLANE STRESSES.