

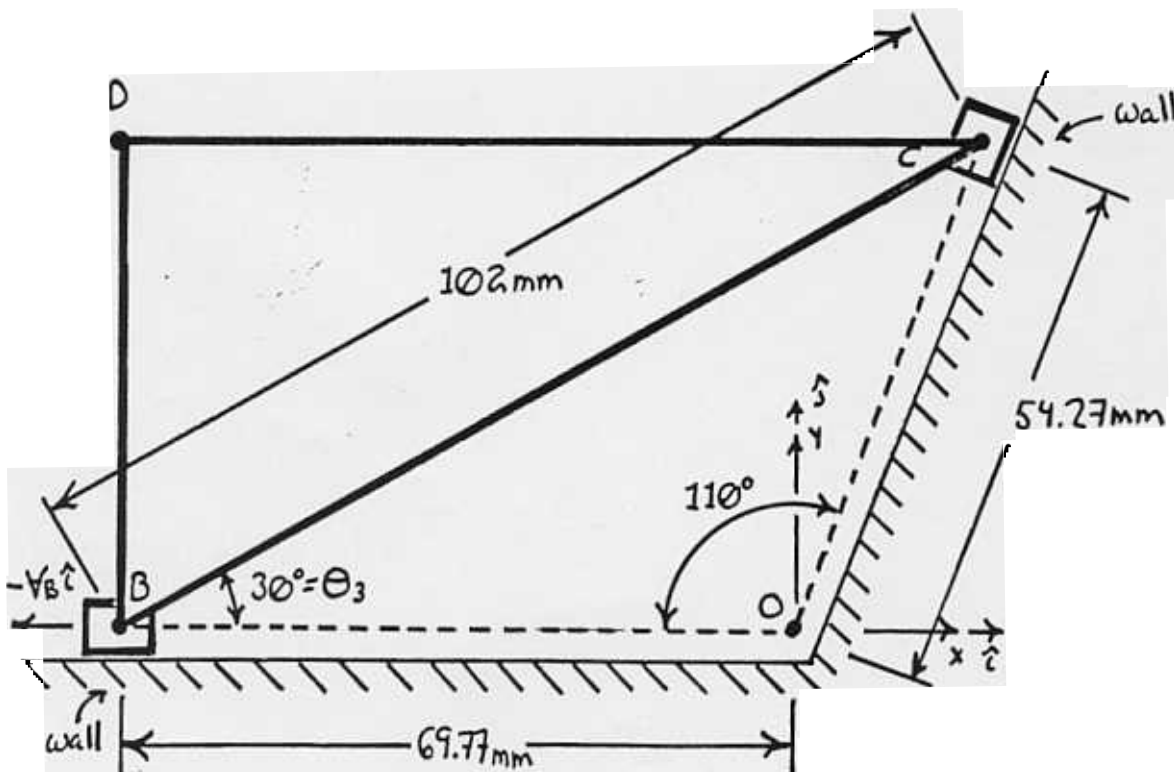
NAME: Solution

PROBLEM #1: Consider the mechanism shown in the figure below. The triangular wedge, coupler CDB, is attached to two sliders at B and C. The joints at B and C are full joints. Both sliders move frictionlessly along the walls shown and are constrained to move along the wall. Point B is being forced to move at a *constant* velocity of 6.10 m/s to the left. For the position shown the loop closure equation is as follows.

$$\vec{R}_{BO} + \vec{R}_{OC} = \vec{R}_{BC}$$

where

$$\begin{aligned} R_{BO} e^{j\theta_1} &= 69.77 \text{ mm} e^{j0^\circ} = 69.77 \text{ mm} \\ R_{BO} \hat{e}_{BO} &= 69.77 \text{ mm} \hat{i} \\ \vec{R}_{OC} &= R_{OC} e^{j\theta_2} = 54.77 \text{ mm} e^{j70^\circ} = 18.56 \text{ mm} + j51.0 \text{ mm} \\ R_{OC} \hat{e}_{OC} &= 54.27 \text{ mm} (0.3420 \hat{i} + 0.9397 \hat{j}) \\ R_{BC} e^{j\theta_3} &= 102 \text{ mm} e^{j30^\circ} = 88.33 \text{ mm} + j51.0 \text{ mm} \\ R_{BC} \hat{e}_{BC} &= 102 \text{ mm} (0.866 \hat{i} + 0.5 \hat{j}) \end{aligned}$$



- 1a) From a velocity analysis it was found that $\dot{\theta}_3 = -73.4 \frac{1}{s}$ and $\dot{R}_{oc} = -5.396 \frac{m}{s}$. Differentiate the loop closure equation twice and determine \ddot{R}_{oc} using one of the analytical approaches.

Complex Method

$$R_{BO} e^{j\theta_1} + R_{oc} e^{j\theta_2} = R_{BC} e^{j\theta_3}$$

$$\dot{R}_{BO} e^{j\theta_1} + R_{BO} j \dot{\theta}_1 e^{j\theta_1} + \dot{R}_{oc} e^{j\theta_2} + R_{oc} j \dot{\theta}_2 e^{j\theta_2} = \dot{R}_{BC} e^{j\theta_3} + R_{BC} j \dot{\theta}_3 e^{j\theta_3}$$

$$\dot{R}_{BO} e^{j\theta_1} + \dot{R}_{oc} e^{j\theta_2} = R_{BC} j \dot{\theta}_3 e^{j\theta_3}$$

$$\ddot{R}_{BO} e^{j\theta_1} + \dot{R}_{BO} j \dot{\theta}_1 e^{j\theta_1} + \ddot{R}_{oc} e^{j\theta_2} + \dot{R}_{oc} j \dot{\theta}_2 e^{j\theta_2} = \ddot{R}_{BC} e^{j\theta_3} + R_{BC} j \ddot{\theta}_3 e^{j\theta_3} + R_{BC} j \dot{\theta}_3^2 e^{j\theta_3}$$

$$\ddot{R}_{oc} e^{j\theta_2} = R_{BC} j \ddot{\theta}_3 e^{j\theta_3} - R_{BC} \dot{\theta}_3^2 e^{j\theta_3}$$

$$\ddot{R}_{oc} (\cos\theta_2 + j \sin\theta_2) = R_{BC} j \ddot{\theta}_3 (\cos\theta_3 + j \sin\theta_3) - R_{BC} \dot{\theta}_3^2 (\cos\theta_3 + j \sin\theta_3)$$

$$\ddot{R}_{oc} \cos\theta_2 = -R_{BC} \ddot{\theta}_3 \sin\theta_3 - R_{BC} \dot{\theta}_3^2 \cos\theta_3$$

$$\ddot{R}_{oc} \sin\theta_2 = R_{BC} \ddot{\theta}_3 \cos\theta_3 - R_{BC} \dot{\theta}_3^2 \sin\theta_3$$

$$\ddot{R}_{oc} \cos\theta_2 \cos\theta_3 = -R_{BC} \ddot{\theta}_3 \sin\theta_3 \cos\theta_3 - R_{BC} \dot{\theta}_3^2 \cos\theta_3 \cos\theta_3$$

$$\ddot{R}_{oc} \sin\theta_2 \sin\theta_3 = R_{BC} \ddot{\theta}_3 \cos\theta_3 \sin\theta_3 - R_{BC} \dot{\theta}_3^2 \sin\theta_3 \sin\theta_3$$

$$\ddot{R}_{oc} \cos(\theta_2 - \theta_3) = -R_{BC} \ddot{\theta}_3$$

$$\ddot{R}_{oc} = \frac{-R_{BC} \ddot{\theta}_3}{\cos(\theta_2 - \theta_3)} = \frac{(102 \text{ mm})(-73.4 \frac{1}{s})^2}{\cos(70^\circ - 30^\circ)} = -77.4 (10^3) \frac{\text{mm}}{\text{s}^2} = -77.4 \frac{\text{m}}{\text{s}^2}$$

Vector Method

$$R_{BO} \hat{e}_{BO} + R_{OC} \hat{e}_{OC} = R_{BC} \hat{e}_{BC}$$

$$\dot{R}_{BO} \hat{e}_{BO} + R_{BO} \dot{\hat{e}}_{BO} + \dot{R}_{OC} \hat{e}_{OC} + R_{OC} \dot{\hat{e}}_{OC} = \dot{R}_{BC} \hat{e}_{BC} + R_{BC} \dot{\hat{e}}_{BC}$$

$$\dot{R}_{BO} \hat{e}_{BO} + \dot{R}_{OC} \hat{e}_{OC} = R_{BC} \dot{\theta}_3 (\hat{j} \times \hat{e}_{BC})$$

$$\ddot{R}_{BO} \hat{e}_{BO} + \dot{R}_{BO} \dot{\hat{e}}_{BO} + \ddot{R}_{OC} \hat{e}_{OC} + \dot{R}_{OC} \dot{\hat{e}}_{OC} = \dot{R}_{BC} \dot{\theta}_3 (\hat{j} \times \hat{e}_{BC}) + R_{BC} \ddot{\theta}_3 (\hat{j} \times \hat{e}_{BC}) + R_{BC} \dot{\theta}_3^2 (\hat{j} \times (\hat{j} \times \hat{e}_{BC}))$$

$$\ddot{R}_{OC} \hat{e}_{OC} = R_{BC} \ddot{\theta}_3 (\hat{j} \times \hat{e}_{BC}) + R_{BC} \dot{\theta}_3^2 (\hat{j} \times (\hat{j} \times \hat{e}_{BC}))$$

Dotting with \hat{e}_{BC}

$$\ddot{R}_{OC} \hat{e}_{BC} \cdot \hat{e}_{OC} = R_{BC} \ddot{\theta}_3 \hat{e}_{BC} \cdot (\hat{j} \times \hat{e}_{BC}) + R_{BC} \dot{\theta}_3^2 \hat{e}_{BC} \cdot (\hat{j} \times (\hat{j} \times \hat{e}_{BC}))$$

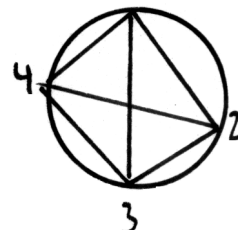
$$\ddot{R}_{OC} = R_{BC} \dot{\theta}_3^2 \frac{\hat{e}_{BC} \cdot (\hat{j} \times (\hat{j} \times \hat{e}_{BC}))}{\hat{e}_{BC} \cdot \hat{e}_{OC}}$$

$$\hat{e}_{BC} \cdot \hat{e}_{OC} = (-.866\hat{i} + .5\hat{j}) \cdot (.3420\hat{i} + .9397\hat{j}) = 0.7660$$

$$\ddot{R}_{OC} = 102\text{mm} (-73.4\frac{1}{s})^2 \frac{-1}{.7660} = 717.4(10^3) \frac{\text{mm}}{s^2} = 717.4 \frac{\text{m}}{s^2}$$

PROBLEM #2: The link shown scale The graph of the links

0
6



2a) Knowing that θ_1 determine the location of all instant centers for this mechanism and the ω_3 and ω_4 graphically

$$v_A = (10 \text{ in}) (-15 \text{ rad/s}) = -150 \text{ in/s}$$

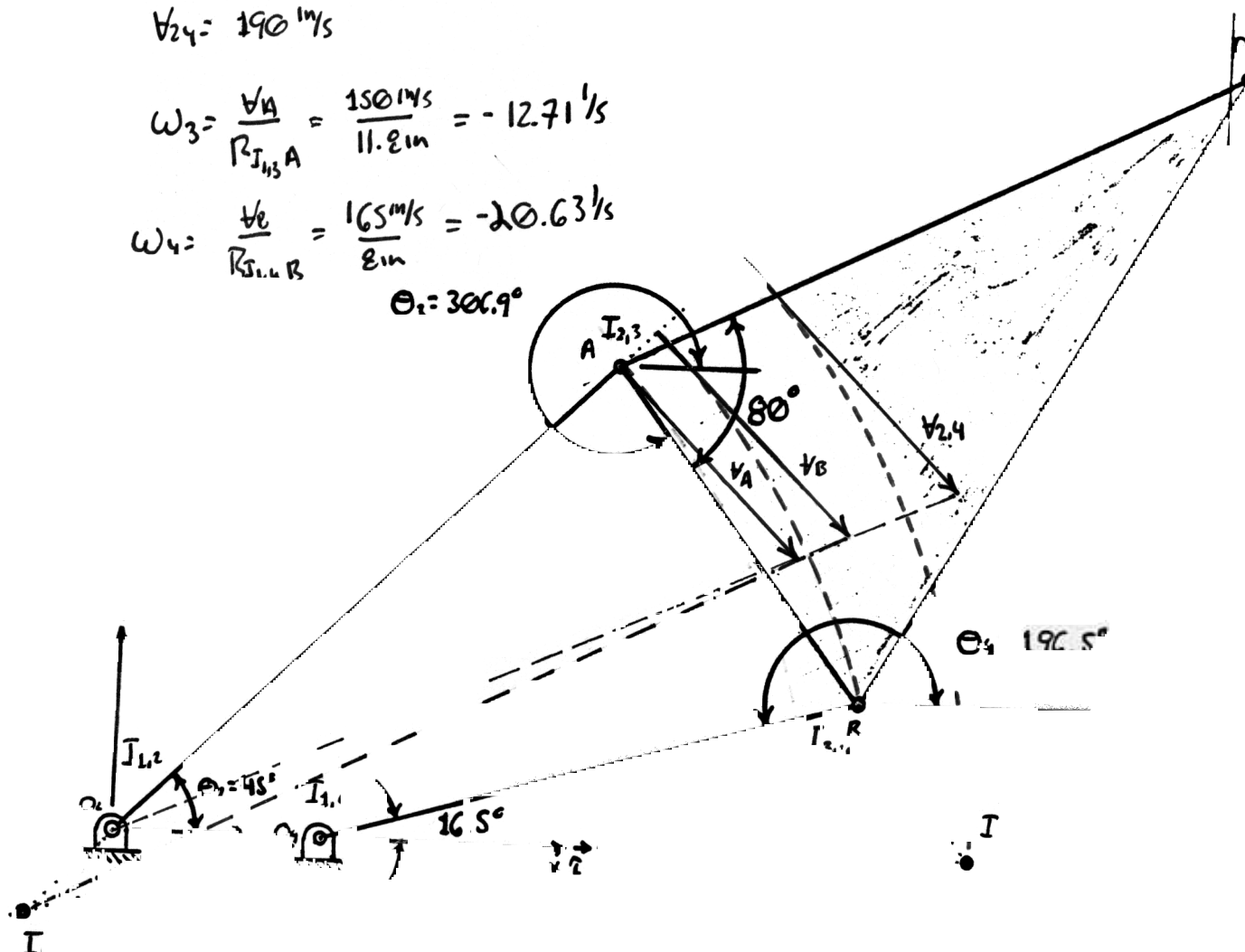
$$v_B = 165 \text{ in/s}$$

$$v_{2,4} = 190 \text{ in/s}$$

$$\omega_3 = \frac{v_A}{R_{I_{1,3}A}} = \frac{150 \text{ in/s}}{11.8 \text{ in}} = -12.71 \text{ rad/s}$$

$$\omega_4 = \frac{v_B}{R_{I_{1,4}B}} = \frac{165 \text{ in/s}}{8 \text{ in}} = -20.63 \text{ rad/s}$$

$$\theta_1 = 306.9^\circ$$



2b) Knowing that $\ddot{\theta}_2 = -15 \frac{1}{s^2}$ determine $\ddot{\theta}_3$ and $\ddot{\theta}_4$ graphically

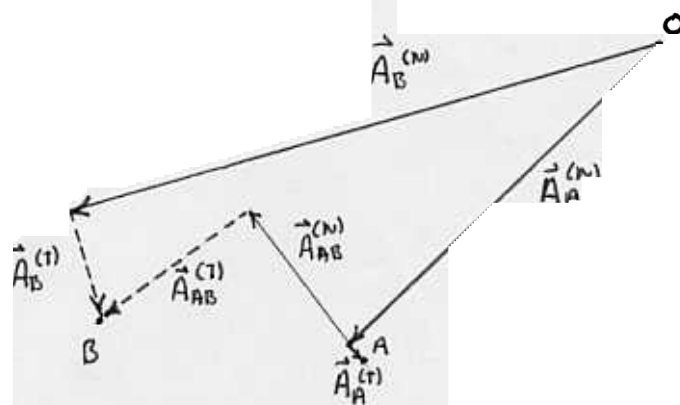
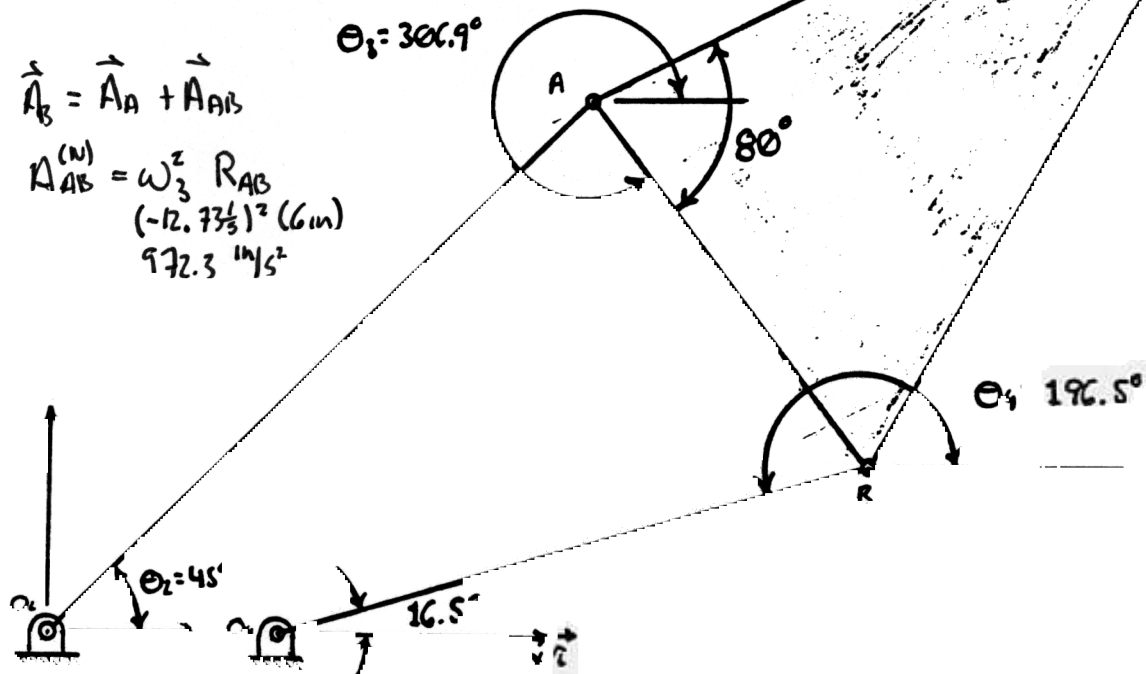
$$A_{O_2A}^{(N)} = A_A^{(N)} = \omega_2^2 R_{O_2A} = (-15 \frac{1}{s^2})(10 \text{ in}) = 2250 \frac{\text{in}}{s^2}$$

$$A_{O_2A}^{(T)} = A_A^{(T)} = \alpha_2 R_{O_2A} = (-10 \frac{1}{s})(10 \text{ in}) = -100 \frac{\text{in}}{s^2}$$

$$A_{O_4B}^{(N)} = A_B^{(N)} = \omega_4^2 R_{O_4B} = (-19.8 \frac{1}{s^2})(8 \text{ in}) = 3136 \frac{\text{in}}{s^2}$$

$$\vec{A}_B = \vec{A}_A + \vec{A}_{AB}$$

$$A_{AB}^{(N)} = \omega_3^2 R_{AB} = (-12.73 \frac{1}{s^2})(6 \text{ in}) = 972.3 \frac{\text{in}}{s^2}$$



$$\ddot{\theta}_4 = \frac{A_B^{(T)}}{R_{O_4B}} = \frac{575 \frac{\text{in}}{s^2}}{8 \text{ in}} = 71.9 \frac{1}{s^2}$$

$$\ddot{\theta}_3 = \frac{A_{AB}^{(T)}}{R_{AB}} = \frac{925 \frac{\text{in}}{s^2}}{6 \text{ in}} = 154 \frac{1}{s^2}$$