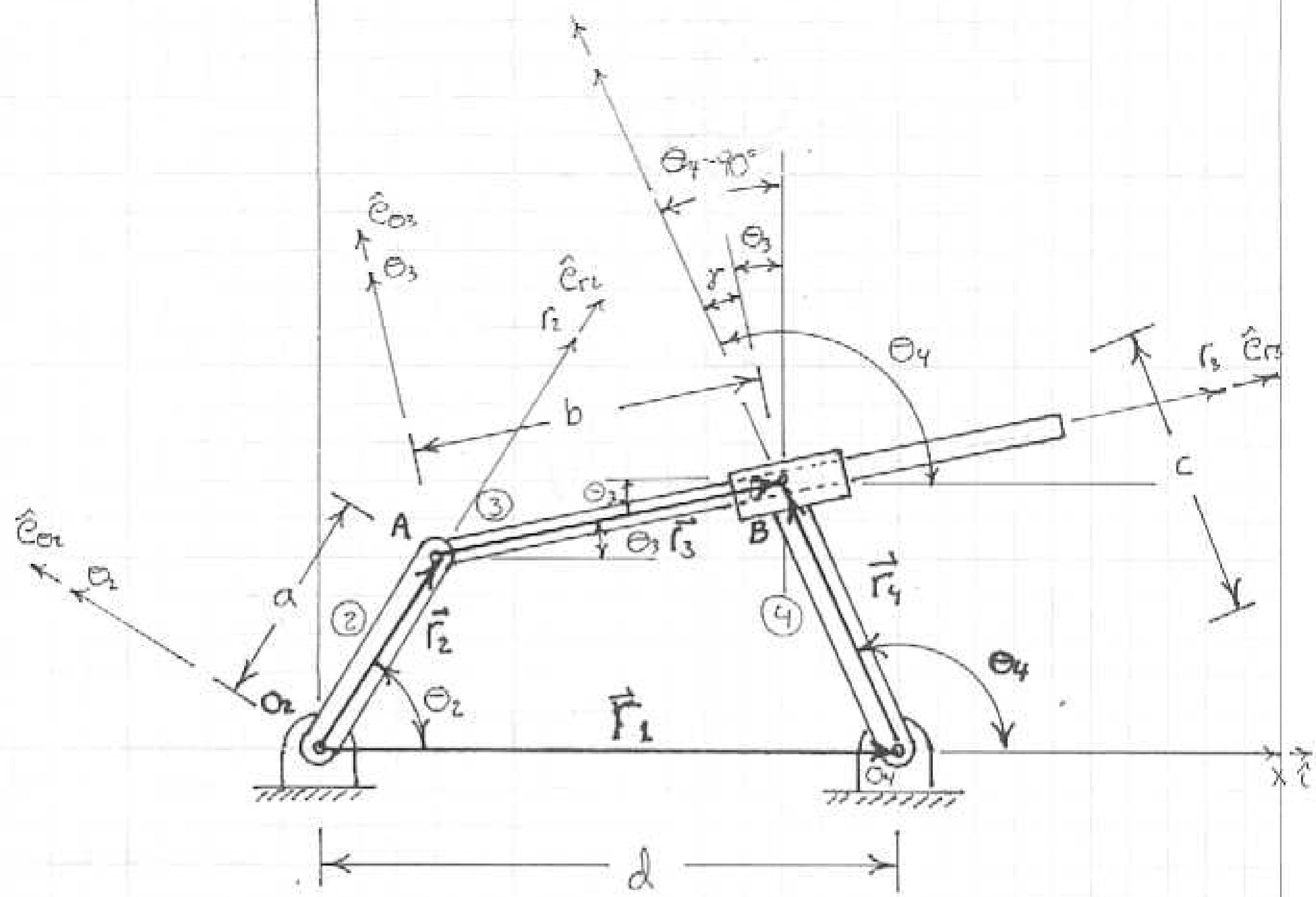
INVERTED SLIDER-CRANK LINKAGE



FOR THE INVERTED SLIDER CRANK THE FIXED ANGLE γ DEFINES THE OFFSET BETWEEN THE SLIDER AND LINK 4. THIS RESULTS IN A RELATIONSHIP BETWEEN θ_3 AND θ_4

$$\theta_4 - 90° = \theta_3 + \gamma$$

$$\boxed{\theta_4 = \theta_3 + 90° + \gamma = \theta_3 + \zeta \quad \text{WHERE } \zeta = 90° + \gamma} \qquad (1)$$

NOTE THAT γ IS A CONSTANT. THE LOOP THAT DEFINES THE KINEMATICS OF THIS PROBLEM CAN BE WRITTEN.

$$\vec{r}_2 + \vec{r}_3 = \vec{r}_1 + \vec{r}_4 \qquad (2)$$

$$\vec{r}_2 = r_2 \cdot \hat{e}_{r2} \; (= a \cdot \hat{e}_{r2}) = r_2 (\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}) \qquad (3)$$

$$\vec{r}_3 = r_3 \cdot \hat{e}_{r3} \; (= b \cdot \hat{e}_{r3}) = r_3 (\cos\theta_3 \hat{i} + \sin\theta_3 \hat{j}) \qquad (4)$$

$$\vec{r}_1 = d \cdot \hat{i} \qquad (5)$$

$$\vec{r}_4 = r_4 \cdot \hat{e}_{r4} \; (= c \cdot \hat{e}_{r4}) = r_4 \cdot (\cos\theta_4 \hat{i} + \sin\theta_4 \hat{j}) \qquad (6)$$

THE LENGTH $r_3 = b$ VARIES AS THE LINKAGE MOVES. THUS $r_3 = b$ IS A VARIABLE THAT MUST BE SOLVED FOR. THIS CREATES AN ADDITIONAL UNKNOWN THAT NEEDS TO BE DETERMINED, $\theta_4, \theta_3, r_3 = b$. HOWEVER, (1) IS A THIRD EQUATION THAT CAN BE USED TO DETERMINE THESE VARIABLES GIVEN θ_2, a, c, d.

STARTING WITH THE LOOP EQUATION (2) AND SUBSTITUTING (3)-(6)

$$r_2\cdot(\cos\theta_2\,\hat{i}+\sin\theta\,\hat{j})+r_3\cdot(\cos\theta_3\,\hat{i}+\sin\theta_3\,\hat{j})=d\,\hat{i}+r_4\cdot(\cos\theta_4\,\hat{i}+\sin\theta_4\,\hat{j}) \quad (7)$$

DOTTING WITH $\hat{i}$

$$r_2\cdot\cos\theta_2+r_3\cdot\cos\theta_3=d+r_4\cdot\cos\theta_4$$

$$a\cdot\cos\theta_2+r_3\cdot\cos\theta_3=d+c\cdot\cos\theta_4 \quad (8)$$

DOTTING WITH $\hat{j}$

$$r_2\cdot\sin\theta_2+r_3\cdot\sin\theta_3=r_4\cdot\sin\theta_4$$

$$a\cdot\sin\theta_2+r_3\cdot\sin\theta_3=c\cdot\sin\theta_4 \quad (9)$$

EQUATIONS (1), (8), (9) ARE THREE EQUATIONS THAT CONTAIN THREE UNKNOWNS: r_3; θ_3, θ_4. SOLVING (9) FOR r_3

$$\boxed{r_3=\frac{c\cdot\sin\theta_4-a\cdot\sin\theta_2}{\sin\theta_3}} \quad (10)$$

SUBSTITUTING (10) INTO (8)

$$a\cdot\cos\theta_2+\left(\frac{c\cdot\sin\theta_4-a\cdot\sin\theta_2}{\sin\theta_3}\right)\cdot\cos\theta_3=d+c\cdot\cos\theta_4$$

$$a\cdot\cos\theta_2+\frac{c\cdot\sin\theta_4\cos\theta_3-a\cdot\sin\theta_2\cdot\cos\theta_3}{\sin\theta_3}-c\cdot\cos\theta_4-d=0 \quad (11)$$

SUBSTITUTING (1) INTO (11)

$$a\cdot\cos\theta_2+\frac{c\cdot\sin\theta_4\cdot\cos(\theta_4-\zeta)-a\cdot\sin\theta_2\cdot\cos(\theta_4-\zeta)}{\sin(\theta_4-\zeta)}-c\cdot\cos\theta_4-d=0$$

$$a\cdot\cos\theta_2+\frac{c\cdot\sin\theta_4(\cos\theta_4\cos\zeta+\sin\theta_4\cdot\sin\zeta)-a\cdot\sin\theta_2(\cos\theta_4\cos\zeta+\sin\theta_4\cdot\sin\zeta)}{\sin\theta_4\cos\zeta-\cos\theta_4\sin\zeta}$$

$$-c\cdot\cos\theta_4-d=0$$

$$\frac{c\cdot\sin\theta_4\cdot\cos\theta_4\cdot\cos\zeta+c\cdot\sin\theta_4\cdot\sin\theta_4\cdot\sin\zeta-a\cdot\sin\theta_2\cdot\cos\theta_4\cdot\cos\zeta-a\cdot\sin\theta_2\cdot\sin\theta_4\cdot\sin\zeta}{\sin\theta_4\cos\zeta-\cos\theta_4\sin\zeta}$$

$$=c\cos\theta_4+d-a\cdot\cos\theta_2$$

$$c\cdot\cos\zeta\cdot\sin\theta_4\cdot\cos\theta_4+c\cdot\sin\zeta\cdot\sin^2\theta_4-a\cdot\cos\zeta\cdot\sin\theta_2\cos\theta_4-a\cdot\sin\zeta\cdot\sin\theta_2\sin\theta_4$$
$$=(c\cdot\cos\theta_4+d-a\cdot\cos\theta_2)\cdot(\sin\theta_4\cdot\cos\zeta-\cos\theta_4\cdot\sin\zeta)$$

$$c \cdot \cos\rho \cdot \sin\theta_4 \cdot \cos\theta_4 + c \cdot \sin\rho \cdot \sin^2\theta_4 - a \cdot \cos\rho \cdot \sin\theta_2 \cdot \cos\theta_4 - a \cdot \sin\rho \cdot \sin\theta_2 \cdot \sin\theta_4$$
$$= c \cdot \cos\theta_4 \cdot \sin\theta_4 \cdot \cos\rho - c \cdot \cos^2\theta_4 \cdot \sin\rho + d \cdot \sin\theta_4 \cdot \cos\rho - d \cdot \cos\theta_4 \cdot \sin\rho$$
$$- a \cdot \cos\theta_2 \cdot \sin\theta_4 \cdot \cos\rho + a \cdot \cos\theta_2 \cos\theta_4 \cdot \sin\rho$$

$$c \cdot \cos\rho \cdot \sin\theta_4 \cdot \cos\theta_4 + c \cdot \sin\rho \cdot \sin^2\theta_4 - a \cdot \cos\rho \cdot \sin\theta_2 \cdot \cos\theta_4 - a \cdot \sin\rho \cdot \sin\theta_2 \cdot \sin\theta_4$$
$$- c \cdot \cos\theta_4 \cdot \sin\theta_4 \cdot \cos\rho + c \cdot \cos^2\theta_4 \cdot \sin\rho - d \cdot \sin\theta_4 \cdot \cos\rho + d \cdot \cos\theta_4 \cdot \sin\rho$$
$$+ a \cdot \cos\theta_2 \cdot \sin\theta_4 \cdot \cos\rho - a \cdot \cos\theta_2 \cdot \cos\theta_4 \cdot \sin\rho = 0$$

$$c \cdot \sin\rho \cdot (\sin^2\theta_4 + \cos^2\theta_4) - a \cdot (\sin\theta_2 \cdot \cos\rho + \sin\rho \cdot \cos\theta_2) \cdot \cos\theta_4$$
$$+ a \cdot (\cos\theta_2 \cdot \cos\rho - \sin\theta_2 \cdot \sin\rho) \cdot \sin\theta_4 - d \cdot \cos\rho \cdot \sin\theta_4 + d \sin\rho \cdot \cos\theta_4 = 0$$

$$c \cdot \sin\rho - a \cdot \sin(\theta_2 + \rho) \cdot \cos\theta_4 + a \cdot \cos(\theta_2 + \rho) \cdot \sin\theta_4 - d \cdot \cos\rho \cdot \sin\theta_4$$
$$+ d \cdot \sin\rho \cdot \cos\theta_4 = 0$$

$$[a \cdot \cos(\theta_2 + \rho) - d \cdot \cos\rho] \cdot \sin\theta_4 + [-a \cdot \sin(\theta_2 + \rho) + d \cdot \sin\rho] \cdot \cos\theta_4$$
$$+ c \cdot \sin\rho = 0$$

$$K_1 \cdot \sin\theta_4 + K_2 \cdot \cos\theta_4 + K_3 = 0 \quad (12)$$
$$K_1 = a \cdot \cos(\theta_2 + \rho) - d \cdot \cos\rho \quad (13)$$
$$K_2 = -a \cdot \sin(\theta_2 + \rho) + d \cdot \sin\rho \quad (14)$$
$$K_3 = c \cdot \sin\rho \quad (15)$$

USING THE TRIGONOMETRIC IDENTITIES

$$\sin 2 \cdot \alpha = \frac{2 \cdot \tan\alpha}{1 + \tan^2\alpha} \Rightarrow \sin\theta = \frac{2 \cdot \tan\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}} \quad (16)$$

$$\cos 2 \cdot \alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha} \Rightarrow \cos\theta = \frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}} \quad (17)$$

SUBSTITUTING (16) AND (17) INTO (12)

$$K_1 \cdot \frac{2 \cdot \tan\frac{\theta_4}{2}}{1 + \tan^2\frac{\theta_4}{2}} + K_2 \cdot \frac{1 - \tan^2\frac{\theta_4}{2}}{1 + \tan^2\frac{\theta_4}{2}} + K_3 = 0$$

$$2 \cdot K_1 \cdot \tan\frac{\theta_4}{2} + K_2 - K_2 \cdot \tan^2\frac{\theta_4}{2} + K_3 + K_3 \cdot \tan^2\frac{\theta_4}{2} = 0$$

$$(K_3 - K_2) \cdot \tan^2\frac{\theta_4}{2} + 2 \cdot K_1 \cdot \tan\frac{\theta_4}{2} + (K_3 + K_2) = 0$$

$$\tan^2\frac{\theta_4}{2} + \frac{2 \cdot K_1}{K_3 - K_2} \cdot \tan\frac{\theta_4}{2} + \frac{K_3 + K_2}{K_3 - K_2} = 0$$

$$\tan^2\frac{\theta_4}{2} + \frac{2\cdot K_1}{K_3-K_2}\cdot\tan\frac{\theta_4}{2} + \left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_1}{K_3-K_2}\right)^2 + \frac{K_3+K_2}{K_3-K_2} = 0$$

$$\left(\tan\frac{\theta_4}{2} + \frac{K_1}{K_3-K_2}\right)^2 = \left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_3+K_2}{K_3-K_2}\right)$$

$$\tan\frac{\theta_4}{2} = -\frac{K_1}{K_3-K_2} \pm \sqrt{\left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_3+K_2}{K_3-K_2}\right)}$$

$$\tan\frac{\theta_4}{2} = -\frac{K_1}{K_3-K_2} \pm \sqrt{\left(\frac{K_1}{K_3-K_2}\right)^2 - \left(\frac{K_3+K_2}{K_3-K_2}\right)\left(\frac{K_3-K_2}{K_3-K_2}\right)}$$

$$\tan\frac{\theta_4}{2} = -\frac{K_1}{K_3-K_2} \pm \sqrt{\frac{K_2^2 - (K_3^2 - K_2^2)}{(K_3-K_2)^2}}$$

$$\tan\frac{\theta_4}{2} = \frac{-K_1 \pm \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_3-K_2}$$

$$\boxed{\theta_4 = 2\cdot\tan^{-1}\left[\frac{-K_1 \pm \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_3-K_2}\right]} \qquad (18)$$

THE SOLUTION OF THE INVERTED SLIDER CRANK LINKAGE STARTS WITH THE DEFINATION OF THE LINKAGE PARAMETERS

GIVEN: $a, c, d, \theta_2,$ & γ

THE PARAMETERS THAT NEED TO BE DETERMINED INCLUDE

FIND: $b, \theta_3,$ & θ_4

THESE PARAMETERS ARE FOUND USING (1), (10), (13), (14), (15) AND (18)