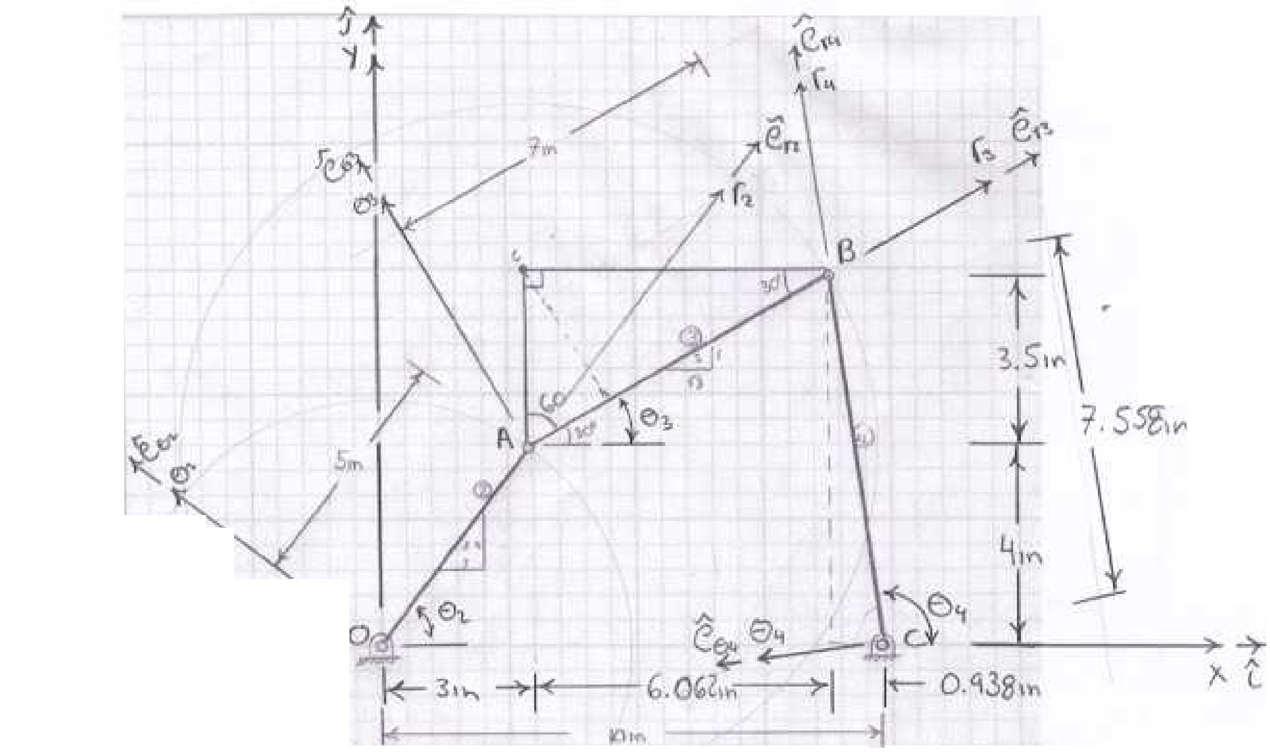
NAME: SOLUTION

PROBLEM 1: Given the linkage below and that link 2 is rotating at a constant angular velocity of 10 1/s answer the following questions.

1a. Write the loop closure equation and determine the values of all the position vectors, there normal unit vector ($\bar{e}_r$), and perpendicular unit vector ($\bar{e}_\theta$).



$$\vec{r}_{OA} + \vec{r}_{AB} = \vec{r}_{OC} + \vec{r}_{CB}$$

$$r_{OA}\hat{e}_{r2} + r_{AB}\hat{e}_{r3} = r_{OC}\hat{i} + r_{CB}\hat{e}_{r4} \quad (1)$$

$$\theta_4 = 180 - \tan^{-1}(7.5in/0.938in) = 97.13^\circ$$

$$\hat{e}_{r2} = \underline{\underline{0.6\hat{i} + 0.8\hat{j}}}$$

$$\hat{e}_{\theta 2} = \underline{\underline{-0.8\hat{i} + 0.6\hat{j}}}$$

$$\hat{e}_{r4} = \cos\theta_4\hat{i} + \sin\theta_4\hat{j} = \underline{\underline{-0.1241\hat{i} + 0.9923\hat{j}}}$$

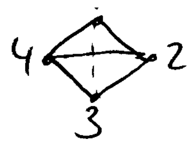
$$e_{\theta 4} \quad -\sin\theta_4\hat{i} + \cos\theta_4\hat{j} = \underline{\underline{-0.9923\hat{i} - 0.1241\hat{j}}}$$

$$\hat{e}_{r3} = \underline{\underline{0.866\hat{i} + 0.5\hat{j}}}$$

$$\hat{e}_{\theta 3} = \underline{\underline{-0.5\hat{i} + 0.866\hat{j}}}$$

$$\vec{r}_{OC} = \vec{r}_{OA} + \vec{r}_{AC} = r_{OA}\hat{e}_{r2} + 3.5in(0.5)\hat{e}_{r3} + 3.5in(.866)\hat{e}_{\theta 3}$$

$$= \underline{\underline{5in\,\hat{e}_{r2} + 1.75in\,\hat{e}_{r3} + 3.031in\,\hat{e}_{\theta 3}}} \quad (2)$$



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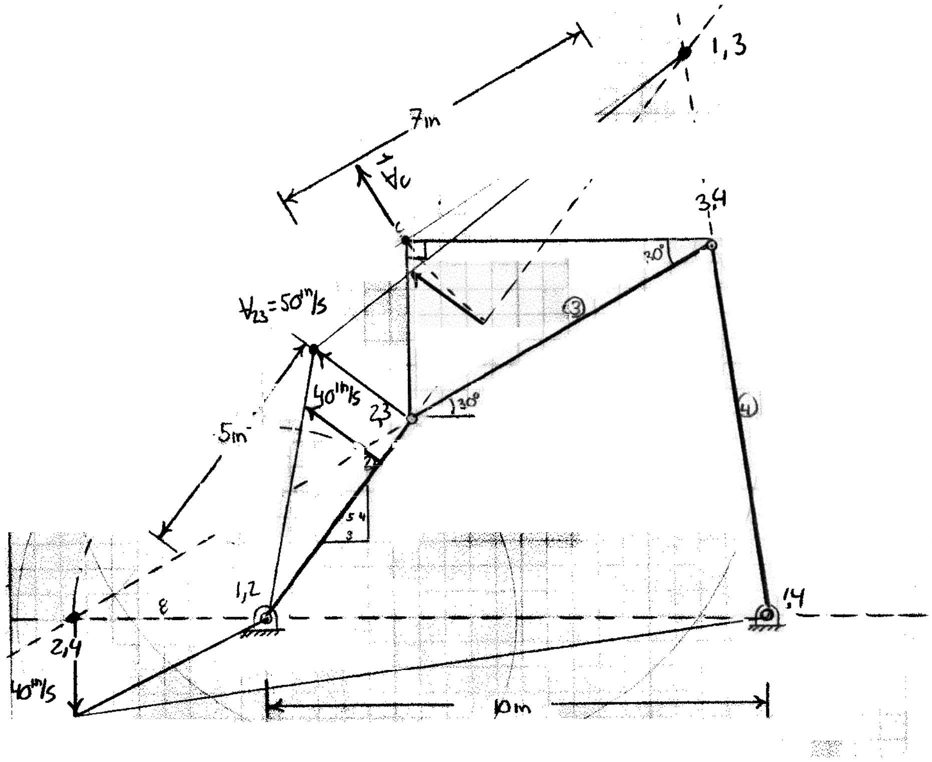


$$\omega_4 = \frac{V_{24}}{r_{14,24}} = \frac{40^{in}/s}{18in} = 2.22\ 1/s$$

$$\omega_3 = \frac{V_{23}}{r_{13,23}} = \frac{50^{in}/s}{9.5in} = 5.26\ 1/s$$

$$V_c = \omega_3\, r_{13,c} = 5.26\ 1/s \cdot 7in = \boxed{36.8^{in}/s}$$

1b. **Determine the velocity of point C using the loop closure method.**

STARTING BY DETERMINING THE UNKNOWN ANGULAR VELOCITIES $\dot{\theta}_3$ AND $\dot{\theta}_4$, FROM (1)

$$\cancel{\dot{r}_{OA}}^{\,0}\hat{e}_{r2} + r_{OA}\underbrace{\dot{\hat{e}}_{r2}}_{\dot{\theta}_2\hat{k}\times\hat{e}_{r2}} + \cancel{\dot{r}_{AB}}^{\,0}\hat{e}_{r3} + r_{AB}\underbrace{\dot{\hat{e}}_{r3}}_{\dot{\theta}_3\hat{k}\times\hat{e}_{r3}} = \cancel{\dot{r}_{OC}}^{\,0}\hat{i} + \cancel{\dot{r}_{CB}}^{\,0}\hat{e}_{r4} + r_{CB}\underbrace{\dot{\hat{e}}_{r4}}_{\dot{\theta}_4\hat{k}\times\hat{e}_{r4}}$$

$$r_{OA}\dot{\theta}_2\hat{e}_{\theta 2} + r_{AB}\underline{\underline{\dot{\theta}_3}}\hat{e}_{\theta 3} = r_{CB}\underline{\underline{\dot{\theta}_4}}\hat{e}_{\theta 4} \qquad (3)$$

UNKNOWN

$$r_{OA}\dot{\theta}_2(-0.8\hat{i} + 0.6\hat{j}) + r_{AB}\dot{\theta}_3(-0.5\hat{i} + 0.866\hat{j}) = r_{CB}\cdot\dot{\theta}_4(-0.9923\hat{i} - 0.1241\hat{j})$$

DOTTING WITH $\hat{i}$

$$-0.8\cdot r_{OA}\cdot\dot{\theta}_2 - 0.5\cdot r_{AB}\cdot\dot{\theta}_3 = -0.9923\, r_{CB}\dot{\theta}_4$$

$$0.8\, r_{OA}\dot{\theta}_2 + 0.5\, r_{AB}\dot{\theta}_3 = 0.9923\, r_{CB}\dot{\theta}_4$$

DOTTING WITH $\hat{j}$

$$0.6\, r_{OA}\dot{\theta}_2 + .866\, r_{AB}\dot{\theta}_3 = -0.1241\, r_{CB}\dot{\theta}_4$$

FROM (5), SOLVING FOR $\dot{\theta}_4$

$$\dot{\theta}_4 = -\frac{0.6\, r_{OA}\dot{\theta}_2 + 0.866\, r_{AB}\dot{\theta}_3}{0.1241\cdot r_{CB}} \qquad (6)$$

SUBSTITUTING THIS INTO (4)

$$0.8\, r_{AO}\dot{\theta}_2 + 0.5\, r_{AB}\dot{\theta}_3 = 0.9923\, \cancel{r_{CB}}\left(-\frac{0.6\, r_{CA}\dot{\theta}_2 + 0.866\, r_{AB}\dot{\theta}_3}{0.1241\,\cancel{r_{CB}}}\right)$$

$$0.8\, r_{AO}\dot{\theta}_2 + \frac{0.9923}{0.1241}\cdot 0.6\, r_{AO}\dot{\theta}_2 = -0.5\, r_{AB}\dot{\theta}_3 - \frac{0.9923}{0.1241}\cdot 0.866\, r_{AB}\dot{\theta}_3$$

$$\left(0.8 + \frac{0.9923}{0.1241}0.6\right) r_{AO}\cdot\dot{\theta}_2 = -\left(0.5 + \frac{0.9923}{0.1241}0.866\right) r_{AB}\dot{\theta}_3$$

$$5.598\, r_{AO}\dot{\theta}_2 = -7.425\, r_{AB}\dot{\theta}_3 \Rightarrow \dot{\theta}_3$$

$$\Rightarrow \dot{\theta}_3 = -\frac{5.598\, r_{AO}}{7.425\, r_{AB}}\dot{\theta}_2 = -\frac{5.598\,(5in)}{7.425\,(7in)}\, 10\tfrac{1}{s} = \underline{\underline{-5.385\,{}^1\!/_s}} \qquad (7)$$

FROM (6)

$$\dot{\theta}_4 = -\frac{0.6\cdot(5in)\cdot(10\,{}^1\!/_s) + 0.866(7in)(-5.385\,{}^1\!/_s)}{0.1241\cdot(7.558in)} = 2.819\,{}^1\!/_s \qquad (8)$$

USING EQUATION (2) THE VELOCITY IS DETERMINED BY TAKING THE DERIVATIVE OF THIS EQUATION

$$\dot{\vec{r}}_{OC} = 5in\,\dot{\theta}_2\,\hat{e}_{\theta 2} + 1.75in\,\dot{\theta}_3\,\hat{e}_{\theta 3} - 3.031in\cdot\dot{\theta}_3\,\hat{e}_{r3} \qquad (9)$$

$$5in\,(10^1/s)(-0.8\hat{i} + 0.6\hat{j}) + 1.75in(-5.385^1/s)(-0.5\hat{i} + 0.866\hat{j})$$
$$- 3.031in\cdot(-5.385^1/s)(0.866\hat{i} + 0.5\hat{j})$$

$$= [(5in)(10^1/s)(-0.8) + (1.75in)(-5.385^1/s)(-0.5) - (3.031in)\cdot(-5.385^1/s)(0.866)]\,\hat{i}$$
$$+ [(5in)(10^1/s)(0.6) + 1.75in(-5.385^1/s)(0.866) - (3.031in)(-5.385^1/s)(0.5)]\,\hat{j}$$

$$21.15^{in}/s\,\hat{i} + 30.0^{in}/s\,\hat{j} \qquad \boxed{36.71^{in}/s(-0.5761\hat{i} + 0.8172\hat{j})}$$

1c. Determine the acceleration of point C using loop closure methods.

BEFORE THE ACCELERATION OF POINT C CAN BE DETERMINED, THE ANGULAR VELOCITIES $\ddot{\theta}_3$ AND $\ddot{\theta}_4$ MUST BE DETERMINED. START BY TAKING THE DERIVATIVE OF (3)

$$r_{OA}\dot{\theta}_2\underbrace{\dot{\hat{e}}_{\theta 2}}_{\substack{\dot{\theta}_2 k\times\hat{e}_{\theta 2}\\ =-\dot{\theta}_2\hat{e}_{r2}}} + r_{AB}\ddot{\theta}_3\hat{e}_{\theta 3} + r_{AB}\dot{\theta}_3\underbrace{\dot{\hat{e}}_{\theta 3}}_{\substack{\dot{\theta}_3 k\times\hat{e}_{\theta 3}\\ =-\dot{\theta}_3\hat{e}_{r3}}} = r_{CB}\ddot{\theta}_4\hat{e}_{\theta 4} + r_{CB}\cdot\dot{\theta}_4\underbrace{\dot{\hat{e}}_{\theta 4}}_{\substack{\dot{\theta}_4 k\times\hat{e}_{\theta 4}\\ =-\dot{\theta}_4\hat{e}_{r4}}}$$

$$-r_{OA}\dot{\theta}_2^2\hat{e}_{r2} + r_{AB}\underline{\ddot{\theta}_3}\hat{e}_{\theta 3} - r_{AB}\cdot\dot{\theta}_3^2\hat{e}_{r3} = r_{CB}\underline{\ddot{\theta}_4}\hat{e}_{\theta 4} - r_{CB}\cdot\dot{\theta}_4^2\hat{e}_{r4}$$

$$-r_{OA}\dot{\theta}_2^2(0.6\hat{i}+0.8\hat{j}) + r_{AB}\ddot{\theta}_3(-0.5\hat{i}+0.866\hat{j}) - r_{AB}\dot{\theta}_3^2(0.866\hat{i}+0.5\hat{j})$$
$$= r_{CB}\ddot{\theta}_4(-0.9923\hat{i}-0.1241\hat{j}) - r_{CB}\dot{\theta}_4^2(-0.1241\hat{i}+0.9923\hat{j})$$

DOTTING WITH $\hat{i}$

$$-0.6\cdot r_{OA}\dot{\theta}_2^2 - 0.5\cdot r_{AB}\cdot\ddot{\theta}_3 - 0.866\cdot r_{AB}\cdot\dot{\theta}_3^2 = -0.9923\cdot r_{CB}\cdot\ddot{\theta}_4 + 0.1241\cdot r_{CB}\dot{\theta}_4^2$$

$$\ddot{\theta}_4 = \frac{0.6\cdot r_{OA}\dot{\theta}_2^2 + 0.5\cdot r_{AB}\ddot{\theta}_3 + 0.866\, r_{AB}\dot{\theta}_3^2 + 0.1241\cdot r_{CB}\cdot\dot{\theta}_4^2}{0.9923\cdot r_{CB}} \quad (10)$$

DOTTING WITH $\hat{j}$

$$0.8\cdot r_{OA}\cdot\dot{\theta}_2^2 + 0.866\cdot r_{AB}\cdot\ddot{\theta}_3 - 0.5\cdot r_{AB}\cdot\dot{\theta}_3^2 = -0.124\ r_{CB}\ \ddot{\theta}_4 - 0.9923\cdot r_{CB}\cdot\dot{\theta}_4^2$$

$$\ddot{\theta}_3 = \frac{0.8 r_{OA}\cdot\dot{\theta}_2^2 + 0.5\cdot r_{AB}\cdot\dot{\theta}_3^2 - 0.1241\cdot r_{CB}\cdot\ddot{\theta}_4 - 0.9923\ r_{CB}\cdot\dot{\theta}_4^2}{0.866\cdot r_{AB}}$$

$$\frac{\left[0.8\cdot r_{OA}\dot{\theta}_2^2 + 0.5\, r_{AB}\dot{\theta}_3^2 - 0.1241\, \cancel{r_{CB}}\left(\frac{0.6\, r_{OA}\cdot\dot{\theta}_2^2 + 0.5\cdot r_{AB}\cdot\ddot{\theta}_3 + 0.866\cdot r_{AB}\dot{\theta}_3^2 + 0.1241\cdot r_{CB}\cdot\dot{\theta}_4^2}{0.9923\cdot\cancel{r_{CB}}}\right) + 0.9923\ r_{CB}\dot{\theta}_4^2\right]}{0.866\cdot r_{AB}}$$

$$= \frac{\left[r_{OA}\dot{\theta}_2^2\left(0.8-\frac{0.1241}{0.9923}\cdot 0.6\right) - \frac{0.1241}{0.9923}\cdot 0.5\, r_{AB}\cdot\ddot{\theta}_3 + r_{AB}\dot{\theta}_3^2\left(0.5-\frac{0.1241}{0.9923}\cdot 0.866\right) + r_{CB}\cdot\dot{\theta}_4^2\left(0.9923-\frac{0.1241}{0.9923}\cdot 0.1241\right)\right]}{0.866\, r_{AB}}$$

$$\ddot{\theta}_3\left(1+\frac{\frac{0.1241}{0.9923}\cdot 0.5\,\cancel{r_{AB}}}{0.866\,\cancel{r_{AB}}}\right) = \frac{r_{OA}\dot{\theta}_2^2\left(0.8-\frac{0.1241}{0.9923}\cdot 0.6\right) + r_{AB}\cdot\dot{\theta}_3^2\left(0.5-\frac{0.1241}{0.9923}\cdot 0.866\right) + r_{CB}\cdot\dot{\theta}_4^2\left(0.9923-\frac{0.1241}{0.9923}0.1241\right)}{0.866\ r_{AB}}$$

$$\ddot{\theta}_3\left(1+\frac{0.1241\cdot 0.5}{0.9923\cdot 0.866}\right)$$

$$\ddot{\Theta}_3 = \frac{r_{OA}\cdot\dot{\Theta}_2^2\left(0.8-\frac{0.1241}{0.9923}\cdot 0.6\right) + r_{AB}\cdot\dot{\Theta}_3^2\left(0.5-\frac{0.1241}{0.9923}\cdot 0.866\right) + r_{CB}\cdot\dot{\Theta}_4^2\left(0.9923-\frac{0.1241}{0.9923}\cdot 0.1241\right)}{0.866\left(1+\frac{0.1241}{0.9923}\cdot\frac{0.5}{0.866}\right) r_{AB}}$$

$$\frac{0.7250\cdot r_{OA}\cdot\dot{\Theta}_2^2 + 0.3917\cdot r_{AB}\cdot\dot{\Theta}_3^2 + 0.9768\cdot r_{CB}\cdot\dot{\Theta}_4^2}{0.9285\, r_{AB}}$$

$$\frac{0.7250\cdot 5_{in}\,(10\,^1/_s)^2 + 0.3917\,(7_{in})(-5.385\,^1/_s)^2 + 0.9768\,(7.558_{in})(2.819\,^1/_s)^2}{0.9285\cdot 7_{in}}$$

$\underline{\underline{77.03\ ^1/_{s^2}}}$ (11)

SUBSTITUTING THIS RESULT INTO (10)

$$\ddot{\Theta}_4 = \frac{0.6\cdot 5_{in}\cdot(10\,^1/_s)^2 + 0.5\cdot(7_{in})(77.03\,^1/_{s^2}) + 0.866\,(7_{in})(-5.385\,^1/_s)^2 + 0.1241(7.558_{in})(2.819\,^1/_s)^2}{0.9923\cdot(7.558_{in})}$$

$\underline{\underline{100.4\ ^1/_{s^2}}}$ (12)

NOW THE ACCELERATION OF C CAN BE CALCULATED BY TAKING THE DERIVATIVE OF (9)

$$\ddot{\vec{r}}_C = -5_{in}\,\dot{\Theta}^2\hat{e}_{r2} - 1.75_{in}\,\dot{\Theta}_3^2\hat{e}_{r3} + .75_{in}\,\ddot{\Theta}_3\hat{e}_{\Theta 3} - 3.031_{in}\,\dot{\Theta}_3^2\hat{e}_{\Theta 3} - 3.031\,\ddot{\Theta}_3\hat{e}_{r3}$$

$$-5_{in}(10\tfrac{1}{s})^2(0.6\hat{\imath} + 0.8\hat{\jmath}) - 1.75_{in}(-5.385\tfrac{1}{s})^2(0.866\,\hat{\imath} + 0.5\hat{\jmath})$$
$$+ 1.75_{in}(77.03\,^1/_{s^2})(-0.5\hat{\imath} + 0.866\hat{\jmath}) - 3.031_{in}(-5.385\,^1/_s)^2(-0.5\hat{\imath} + 0.866\hat{\jmath})$$
$$- 3.031(77.03\,^1/_{s^2})\cdot(0.866\,\hat{\imath} + 0.5\hat{\jmath})$$

$$\Big[(-5_{in})(10\tfrac{1}{s})^2(.6) - 1.75_{in}(-5.385\tfrac{1}{s})^2(0.866) + .75_{in}(77.03\,^1/_{s^2})(-0.5) - 3.031_{in}(-5.385\,^1/_s)^2(-.5)$$
$$- 3.031(77.03\,^1/_{s^2})(0.866)\Big]\hat{\imath} +$$
$$\Big[-5(10\tfrac{1}{s})^2(0.8) - 1.75_{in}(-5.385\tfrac{1}{s})^2(0.5) + 1.75_{in}(77.03\,^1/_{s^2})(0.866) - 3.031_{in}(-5.385\,^1/_s)^2(.866)$$
$$3.031(77.03\,^1/_{s^2})(0.5)\Big]\hat{\jmath}$$

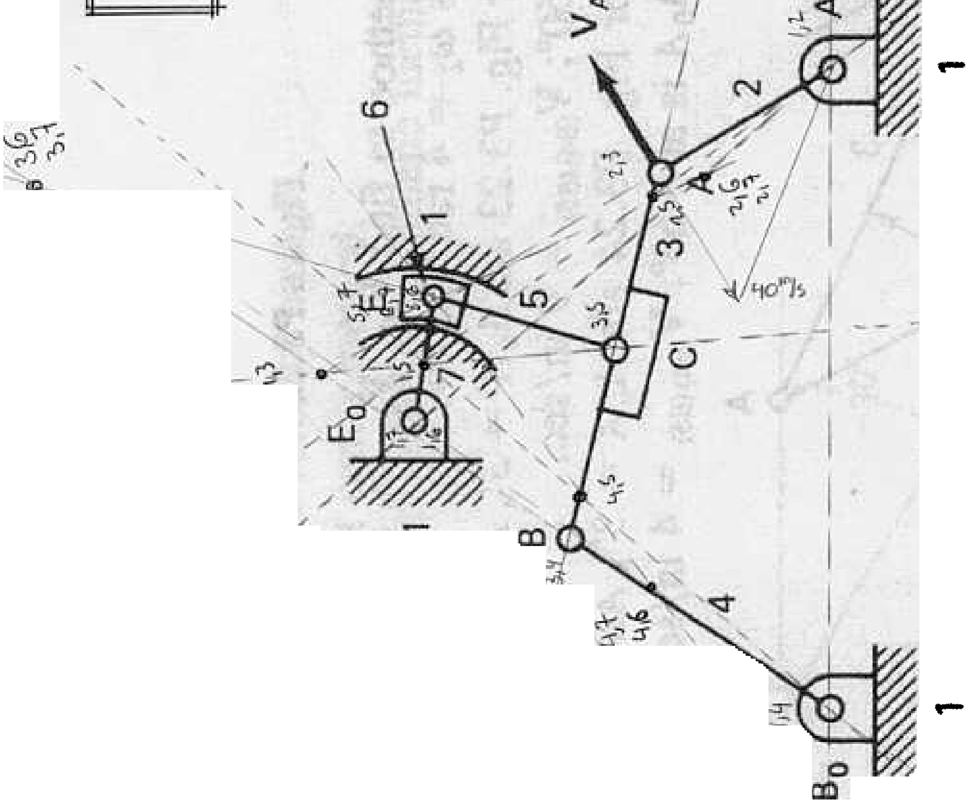
$$657.5\,^{in}/_{s^2}\,\hat{\imath} - 501.5\,^{in}/_{s^2}\,\hat{\jmath} = \boxed{-826.9(0.7951\hat{\imath} + 0.6065\hat{\jmath})}$$

Problem 2. Determine the location of the instant centers for the structure shown. Given link 2 is rotating at 10 1/s, determine the adsolute velocity of link 6 and the angular velocity of links 3, 4, 5, 7.

$V_{Link6} = V_{67} = r_{16,67} \cdot \omega_7$

$= 4.4\,\tfrac{1}{s} \cdot 2.5\,in$

$= \boxed{11\,in/s}$



$V_A = r_{12,23}\,\omega_2$

$= 4\,in \cdot 10\,\tfrac{1}{s}$

$= \boxed{40\,in/s}$

$\omega_3 = \dfrac{V_A}{r_{13,23}} = \dfrac{40\,in/s}{8\,in}$

$= \boxed{5\,\tfrac{1}{s}}$

$V_{24} = r_{12,24} \cdot \omega_2$

$= 13\,in \cdot 10\,\tfrac{1}{s}$

$= 130\,in/s$

$\omega_4 = \dfrac{V_{24}}{r_{14,24}} = \dfrac{130\,in/s}{26.5\,in}$

$= \boxed{4.9\,\tfrac{1}{s}}$

$V_{25} = r_{12,25} \cdot \omega_2$

$= 4.5\,in \cdot 10\,\tfrac{1}{s}$

$= 45\,in/s$

$\omega_5 = \dfrac{V_{25}}{r_{15,25}} = \dfrac{45\,in/s}{6\,in}$

$= \boxed{7.5\,\tfrac{1}{s}}$

$r_{12,27} \cdot \omega_2$

$= 3.5\,in \cdot 10\,\tfrac{1}{s}$

$35\,in/s$

$\omega_7 = \dfrac{V_{27}}{r_{17,27}} = \dfrac{35\,in/s}{8\,in}$

$= \boxed{4.4\,\tfrac{1}{s}}$

Problem 3. Determine the Mechanical advantage for the mechanism shown. List as many ways as possible to increase the mechanical advantage for this mechanism.

$$MA = \frac{r_{in}}{r_{out}} \cdot \frac{d_{out}}{d_{in}}$$

$$\frac{2.2}{2.8} = \boxed{0.79}$$