PROBLEM 8.5-16 A sign is supported by a pipe having outside diameter 100mm and inside diameter 80mm. The dimensions of the sign are 2.0m x 0.75m, and its lower edge is 3.2m above the support. The wind pressure against the sign is 1.8 kPa. Determine the maximum in-plane shear stress due to the wind pressure on the sign at points A, B, and C, located at the base of the pipe.

GIVEN:

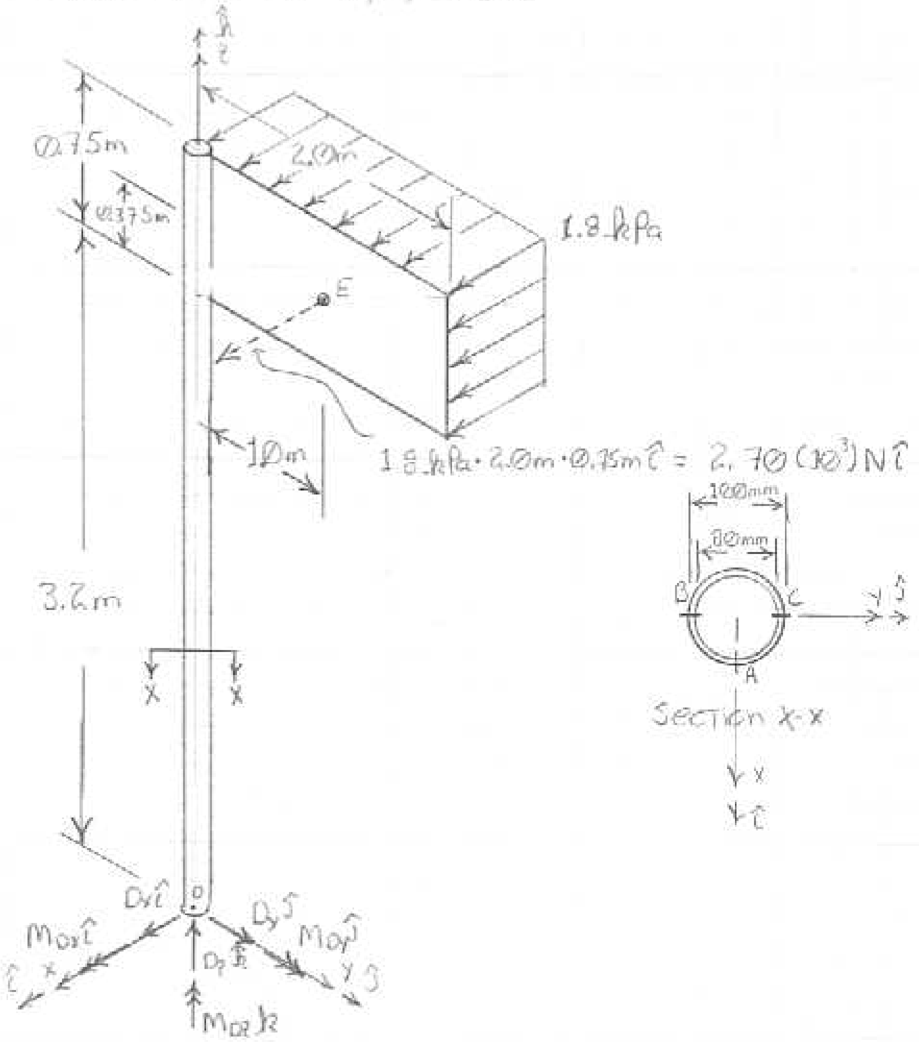
1) Constraints
- sign mounted on a hollow pipe
- wind pressure on sign of 1.8 kPa

2) Assumptions
- The wind pressure against the pipe is negligible
- Ground is restricting the pipe from rotating and translating
- material is linear elastic
- All deformations are small
- weight of the sign is neglected

FIND:

1) maximum in-plane shear stress at A, B, and C

FREE BODY DIAGRAM:



STATICS:

First we need to solve for the reactions at D

$\Sigma F_x = \emptyset = D_x + 2.70(10^3)\,N \Rightarrow \underline{D_x = -2.70(10^3)\,N}$ (1)

$\Sigma F_y = \emptyset = D_y$ (2)

$\Sigma F_z = \emptyset = D_z$ (3)

$\Sigma \vec{M}/_{atD} = \vec{\emptyset} = \vec{r}_{DE} \times \vec{F}_E + \vec{M}_D$

$\vec{r}_{DE} = 1.05\,m\,\hat{j} + 3.575\,m\,\hat{k}$

$\vec{F}_{DE} = 2.70(10^3)\,N\,\hat{i}$

$\vec{M}_D = M_{Dx}\hat{i} + M_{Dy}\hat{j} + M_{Dz}\hat{k}$

$$\Sigma \vec{M}/_{atD} = \vec{\emptyset} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \emptyset & 1.05m & 3.575m \\ 2.70(10^3)N & \emptyset & \emptyset \end{vmatrix} + M_{Dx}\hat{i} + M_{Dy}\hat{j} + M_{Dz}\hat{k}$$

$$= -[-(2.70\times10^3 N)\cdot(3.575m)]\,\hat{j} + [-(2.70\times10^3 N)\cdot(1.05m)]\,\hat{k} + M_{Dx}\hat{i} + M_{Dy}\hat{j} + M_{Dz}\hat{k}$$

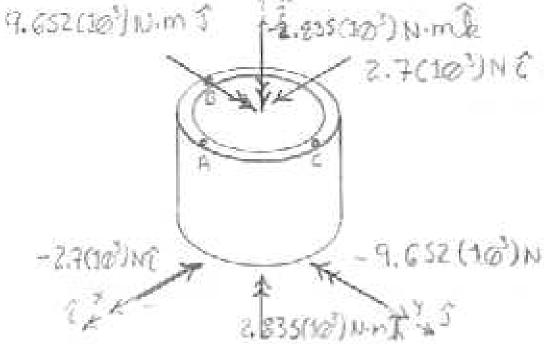
$$= 9.652\times10^3\,N\cdot m\,\hat{j} - 2.835\times10^3\,N\cdot m\,\hat{k} + M_{Dx}\hat{i} + M_{Dy}\hat{j} + M_{Dz}\hat{k}$$

$\Sigma \vec{M}/_{atD}\cdot\hat{i} = \Sigma M_x/_{atD} = \underline{\emptyset = M_{Dx}}$ (4)

$\Sigma \vec{M}/_{atD}\cdot\hat{j} = \Sigma M_y/_{atD} = \emptyset = M_{Dy} + 9.652\times10^3\,N\cdot m \Rightarrow \underline{M_{Dy} = -9.652\times10^3\,N\cdot m}$ (5)

$\Sigma \vec{M}/_{atD}\cdot\hat{k} = \Sigma M_z/_{atD} = \emptyset = M_{Dz} - 2.70\times10^3\,N\cdot m \Rightarrow \underline{M_{Dz} = 2.835\times10^3\,N\cdot m}$ (6)

Now we can consider the internal loads in the pole an infinitesimal distance from D in order to determine the state of stress at A, B, & C.



MECHANICS:

The internal moment in the y direction gives rise to normal stress in the z direction

$$\sigma_z = \frac{-M_y \cdot x}{I_{yy}} = \frac{-9.652(10^3)\,N\cdot m \cdot x}{\frac{\pi}{64}\left[(0.1m)^4-(0.08m)^4\right]} = \underline{\underline{-3.330(10^9)\tfrac{N}{m^3}\cdot x}} \quad (7)$$

From (7) it is seen that σ_z at B and C equal zero and σ_z at A is compressive

$$\underline{\sigma_{z,C} = \sigma_{z,B} = 0} \quad (8)$$

$$\sigma_{zA} = -3.330(10^9)\tfrac{N}{m^3}\cdot(0.05m) = -166.5(10^6)\tfrac{N}{m^2}$$

$$= \underline{-166.5\ MPa} \quad (9)$$

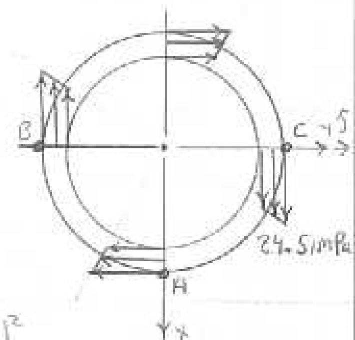
The torque in the z direction gives rise to shear stress

$$\tau = \frac{T\cdot r}{J} = \frac{2.835(10^3)\,N\cdot m \cdot r}{\frac{\pi}{32}\left[(0.1m)^4-(0.08m)^4\right]} = \underline{\underline{489.1(10^6)\tfrac{N}{m^3}\cdot r}} \quad (10)$$

At the outer surface of the pole where A, B, and C are located the shear stress is given by

$$\tau = 489.1(10^6)\tfrac{N}{m^3}\cdot 0.05m = 24.46(10^6)\tfrac{N}{m^2} = \underline{24.46\ MPa} \quad (11)$$

The force directed along the x-axis also gives rise to a shearing stress. This stress will equal zero at A and will be maximum at B and C.



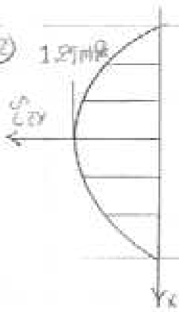
$$\tau_{zx} = \frac{VQ}{It}$$

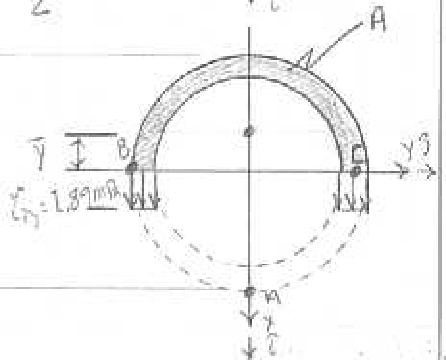
$$Q = \Sigma \bar{y}A = \frac{4\cdot(0.05m)}{3\pi}\cdot\frac{\pi(0.05)^2}{2} - \frac{4(0.04m)}{3\pi}\cdot\frac{\pi(0.04)^2}{2}$$

$$= \underline{40.67(10^{-6})\,m^3} \quad (12)$$

$$\tau_{zx} = \frac{2.7(10^3)N\cdot 40.67(10^{-6})m^3}{\frac{\pi}{64}\left[(0.1m)^4-(0.08m)^4\right]\cdot 0.02m}$$

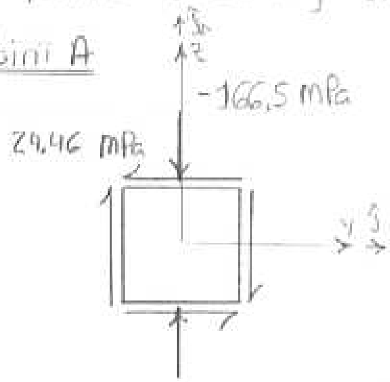
$$= \underline{1.894(10^3)\tfrac{N}{m^2}} \quad (13)$$

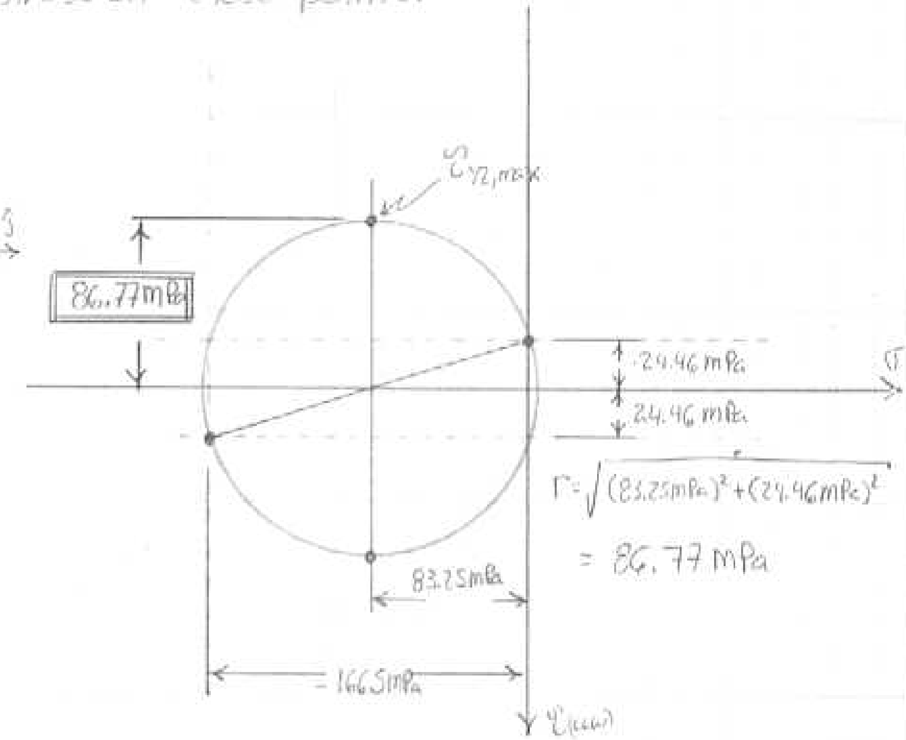




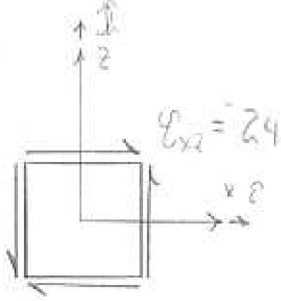
Now the stress elements at A, B, and C can be considered along with mohr's circle. This will allow us to determine the maximum in plane shearing stress at these points.

Point A



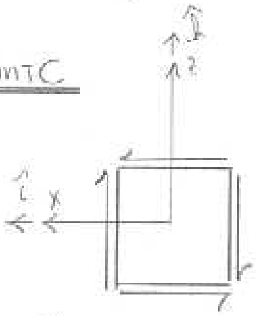


Point B



$\tau_{xz} = 24.5\text{ MPa} - 1.89\text{ MPa} = \boxed{22.57\text{ MPa}}$

Point C



$\tau_{xz} = 24.5\text{ MPa} + 1.85\text{ MPa} = \boxed{26.35\text{ MPa}}$

SUMMARY:

This problem takes us through statics, determination of stress in a cross-section, to the determination of maximum shear stresses. At this point the maximum shear stress criterion can be applied to determine the margin of safety in the structure.