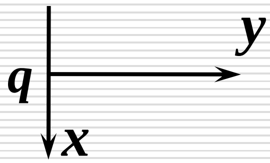
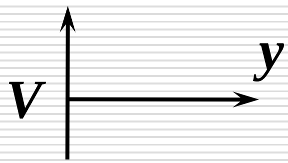
Deflection of Beams Using Singularity Functions

- Introduction to Singularity Functions
  - Macaulay Functions
- Applying Singularity Functions to Beams

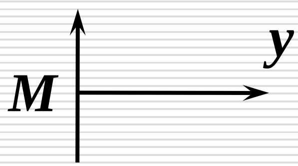
Linear-Elastic Response



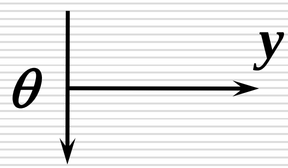
$$-q = {}^{dV}\!/_{dy} = -\frac{d^2}{dy^2}\left(E\cdot I\cdot\frac{d^2u}{dy^2}\right) = -\left(E\cdot I\cdot u''\right)'' \quad\Rightarrow\quad \frac{Constant}{E\cdot I} \Rightarrow V(y) = -E\cdot I\cdot {}^{d^4u}\!/_{dy^4}$$



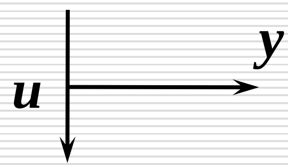
$$V = {}^{dM}\!/_{dy} = -\frac{d}{dy}\left(E\cdot I\cdot\frac{d^2u}{dy^2}\right) = -\left(E\cdot I\cdot u''\right)' \quad\Rightarrow\quad \frac{Constant}{E\cdot I} \Rightarrow V(y) = -E\cdot I\cdot {}^{d^3u}\!/_{dy^3}$$



$$M = -E\cdot I\cdot {}^{d^2u}\!/_{dy^2} = -E\cdot I\cdot u'' \quad\Rightarrow\quad \frac{Constant}{E\cdot I} \Rightarrow M(y) = -E\cdot I\cdot {}^{d^2u}\!/_{dy^2}$$

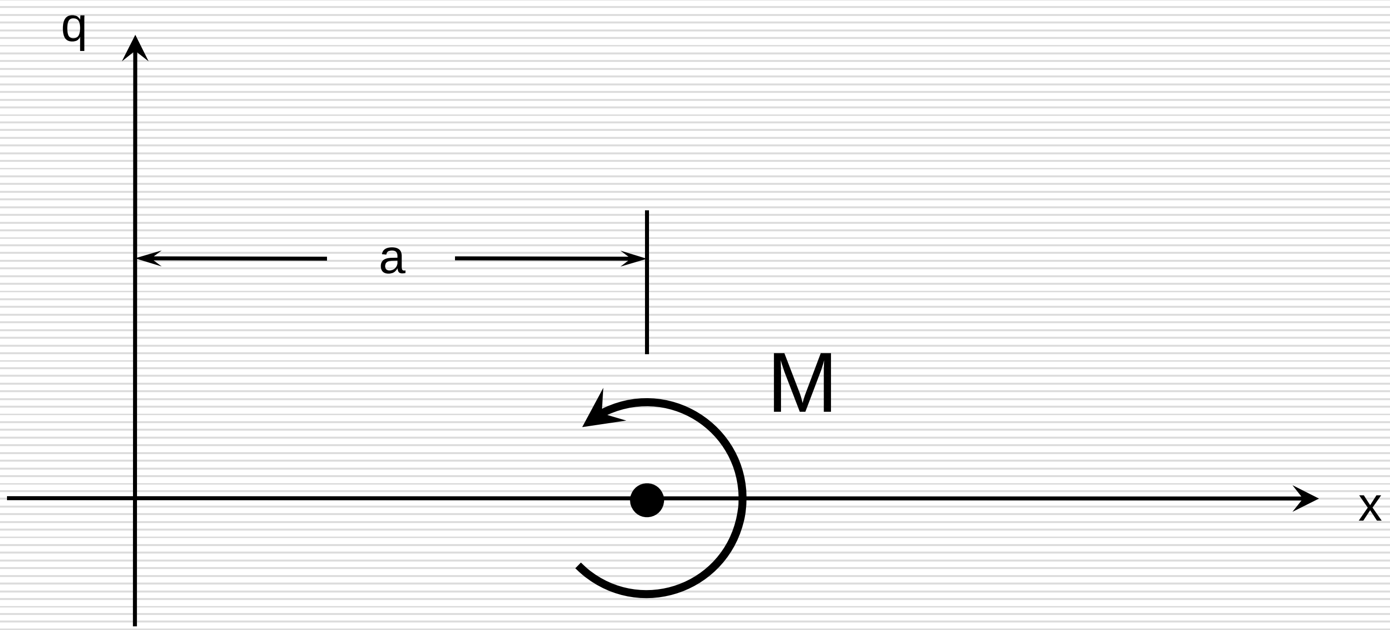


$$\theta \equiv {}^{du}\!/_{dy} = u' \equiv Slope \quad of \quad the \quad Elastic \quad Curve$$



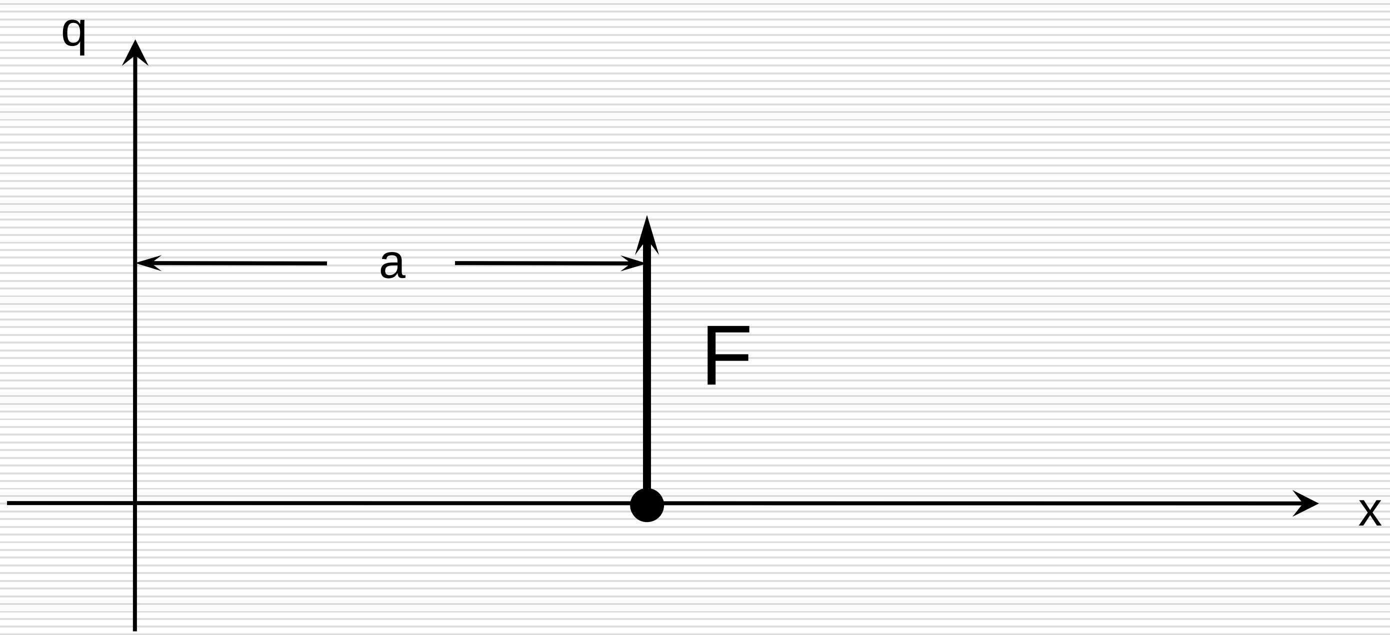
$$u \equiv Deflection \quad of \quad the \quad Elastic \quad Curve$$

Concentrated Moment (Unit Doublet)



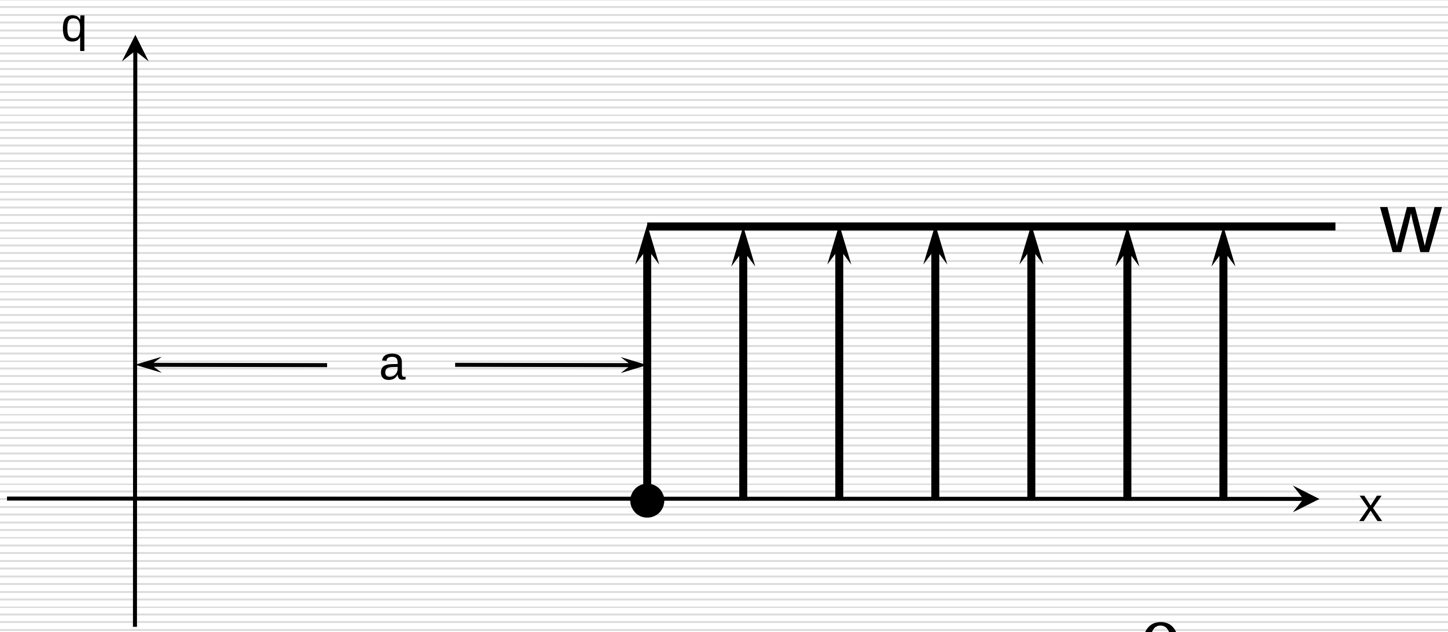
$$q(x) = M\langle x-a\rangle_{-2}$$

Concentrated Force (unit Impulse, Dirac delta)



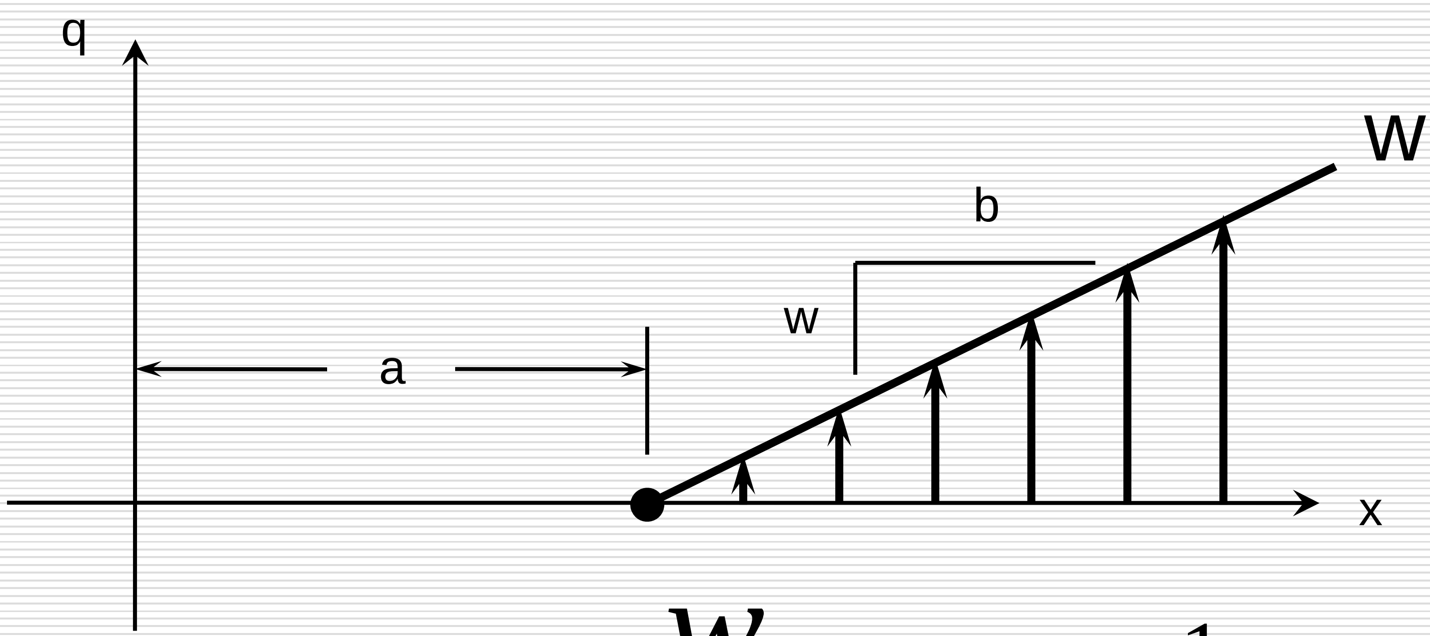
$$q(x) = F\langle x-a \rangle_{-1}$$

Distributed Force (Unit Step)



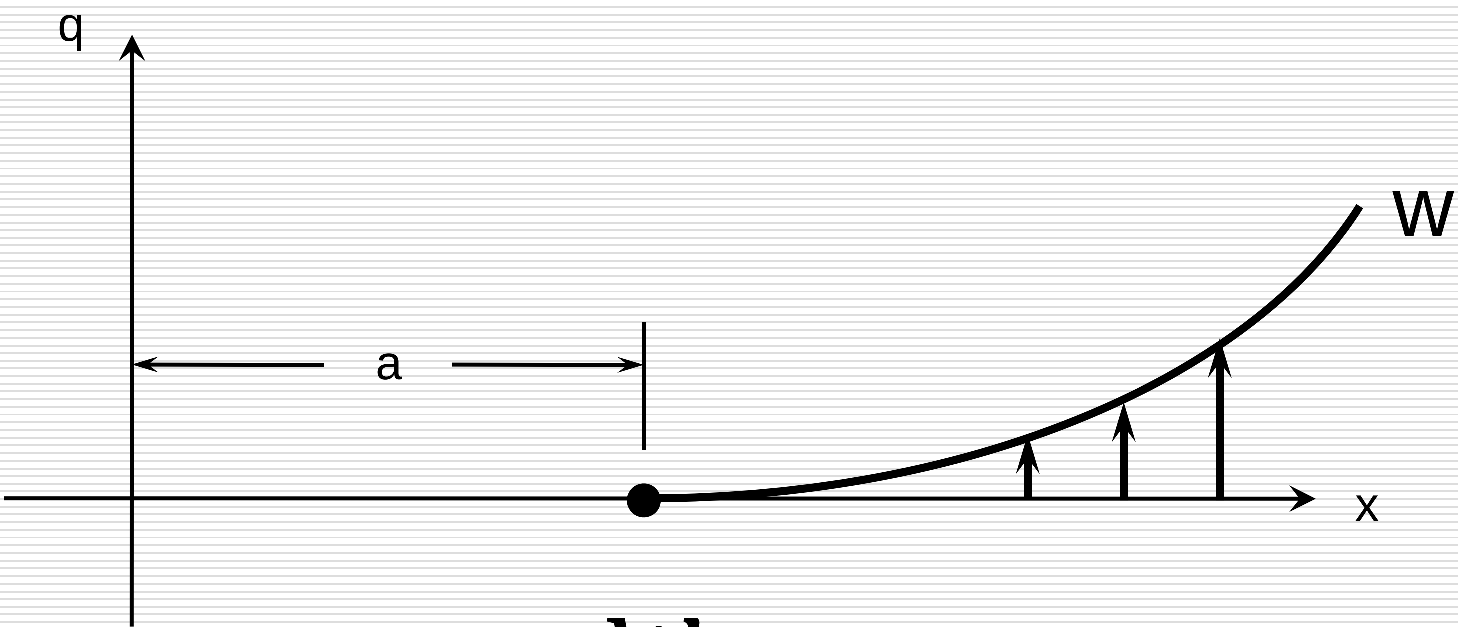
$$q(x) = w\langle x - a \rangle^0$$

Ramp Function



$$q(x) = \frac{w}{b}\langle x - a \rangle^{1}$$

Polynomial Function



$$q(x) = \frac{w}{b^2} \langle x - a \rangle^2$$

Notation Significance

$$q(x) = \langle x - a \rangle^n, \quad n \geq 0$$

If the quantity in the "<>" is negative
- q(x) is zero

If the quantity in the "<>" is positive
- $q(x) = (x-a)^n$

Summary

$$q(x) = \langle x - a \rangle^n = \begin{cases} (x-a) & x > a \\ 0 & x \le a \end{cases}$$

$$q(x) = \langle x - a \rangle_{-2} = \begin{cases} \pm\infty & x = a \\ 0 & x \ne a \end{cases}$$

$$q(x) = \langle x - a \rangle_{-1} = \begin{cases} \infty & x = a \\ 0 & x \ne 0 \end{cases}$$

Integration
n Greater or equal to 0

$$\int_{-\infty}^{x} \left\langle x^{*} - a \right\rangle^{n} \cdot dx^{*} = \frac{\left\langle x - a \right\rangle^{n+1}}{n+1}, \quad n \geq 0$$

Integration
n Less Than 0

$$\int_{-\infty}^{x} \left\langle x^{*} - a \right\rangle^{n} \cdot dx^{*} = \left\langle x - a \right\rangle^{n+1}, \quad n < 0$$

Example

$$0 < y < 2a$$

$$V = -\frac{P}{2}$$

$$M = -\frac{P \cdot y}{2}$$

$$2a < y < 3a$$

$$V = P$$

$$M = -(3 \cdot a - y) \cdot P$$