PROB 3.66 | THE WATT SIX BAR LINKAGE SHOWN BELOW IS DESIGNED FOR USE AS A PARALLEL MOTION SUPER MARKET BASED HAND OPERATED CAN CRUSHER TO SPUR RECYCLING OF MATERIALS. SUCH MACHINES REQUIRE ~~LARGE~~ AS LARGE A MECHANICAL ADVANTAGE AS POSSIBLE TO AMPLIFY THE CRUSH FORCE DEVELOPED FROM A LIMITED HUMAN INPUT. DETERMINE THE MECHANICAL ADVANTAGE OF THIS LINKAGE IN THE POSITION SHOWN WHEN THE CRUSH PLATE FIRST CONTACTS THE CAN.

GIVEN:

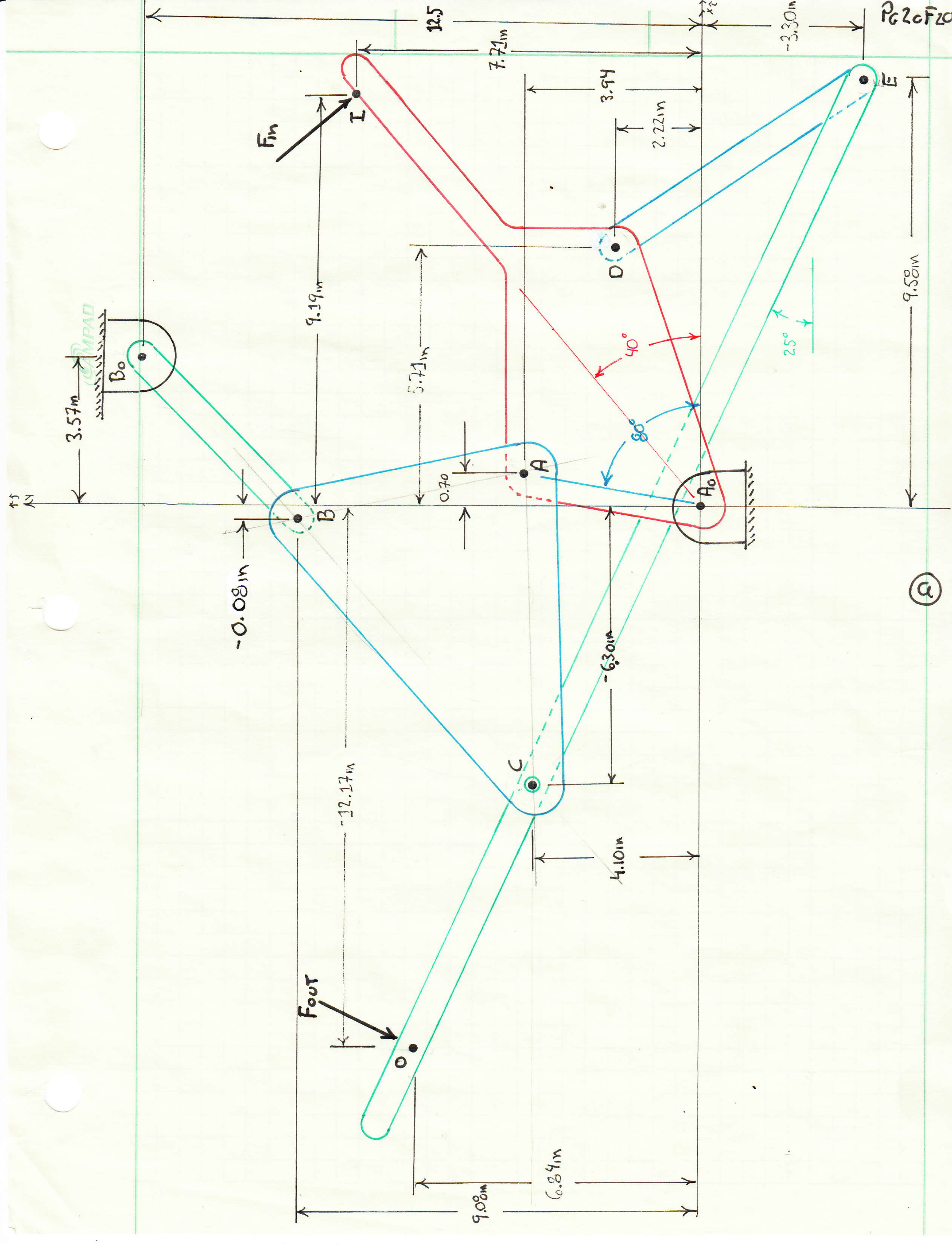
1. THE HANDLE ROTATES AT 1 $^r/s$ CW
2. $A_oA = 4"$, $A_oD = 6.13"$, $A_oF_{in} = 12in$
3. A_oA IS AT $+80°$
4. $AB = 5.2"$, $AC = 7in$
5. $BC = 7.97in$, $BB_o = 5"$
6. $CE = 17.52"$, $DE = 6.74"$, $EF_{OUT} = 24"$
7. CE IS AT $-25°$
8. THE HANDLE IS AT $+40°$

ASSUMPTIONS:

1. ALL LINKS ARE RIGID
2. ALL JOINTS ARE FRICTIONLESS
3. INERTIAL LOADING IS IGNORED
4. ALL LINKS ARE IN A SINGLE PLANE OR PARALLEL PLANES

FIND:

1. THE MECHANICAL ADVANTAGE OF THE SYSTEM IN THE CONFIGURATION SHOWN.


12.5
7.71in
-3.30in
3.94
2.22in
9.58in
Fin
I
E
D
40°
60°
25°
9.19in
5.71in
3.57in
Bo
A
Ao
B
0.70
-0.08in
-6.30in
-12.17in
C
4.10in
Fout
O
6.84in
9.08in
a

SOLUTION:

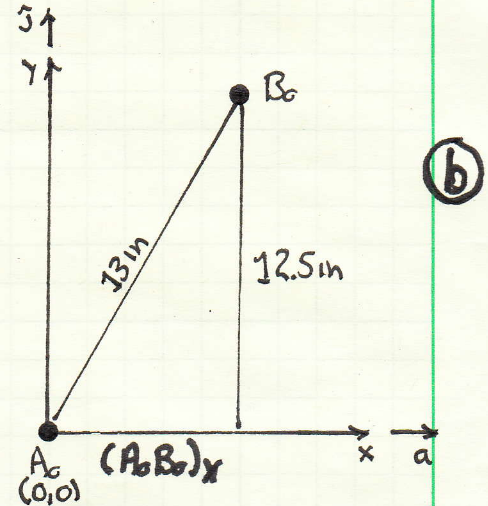
BEFORE THE EQUILIBRIUM ANALYSIS IS PERFORMED, THE COORDINATES OF EACH OF THE JOINTS IN THE MECHANISM MUST BE DETERMINED. FIXING THE COORDINATE SYSTEM TO POINT A_0 IN (a).

THE INFORMATION PROVIDED IN THE PROBLEM STATEMENT STATES $A_0B_0 = 13in$ AND $(A_0B_0)_y = 12.5in$. USING THE GEOMETRY ILLUSTRATED IN (b)

$$(A_0B_0)_x^2 = (A_0B_0)^2 - (A_0B_0)_y^2$$

$$(A_0B_0)_x = \sqrt{(13in)^2 + (12.5in)^2}$$

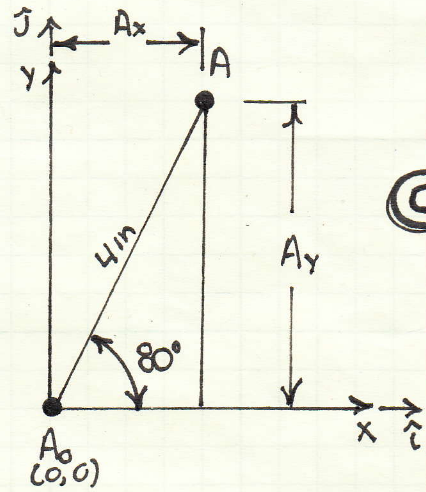
$$= \underline{\underline{3.57in}} \quad (1)$$



THE PROBLEM STATEMENT ALSO GIVES $(A_0A) = 4in$ AND $\Theta_{A_0A} = 80°$. THIS GEOMETRY IS ILLUSTRATED IN (c) AND ENABLES THE CALCULATION OF THE A COORDINATES

$$A_x = 4in \cdot \cos 80° = \underline{0.70in} \quad (2)$$

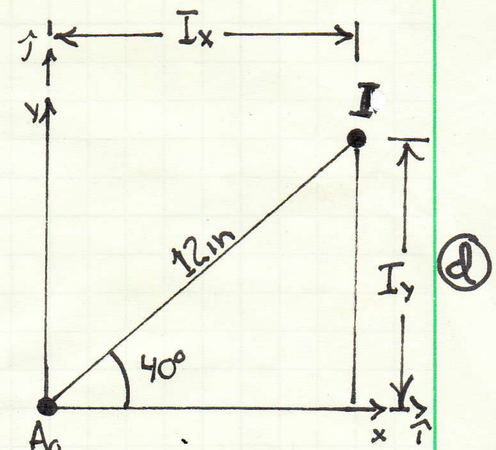
$$A_y = 4in \cdot \sin 80° = \underline{\underline{3.94in}} \quad (3)$$



THE PROBLEM STATEMENT GIVES $(A_0I) = 12in$ AND $\Theta_{A_0I} = 40°$. THIS GEOMETRY IS ILLUSTRATED IN (d) AND ENABLES THE CALCULATION OF THE I COORDINATES

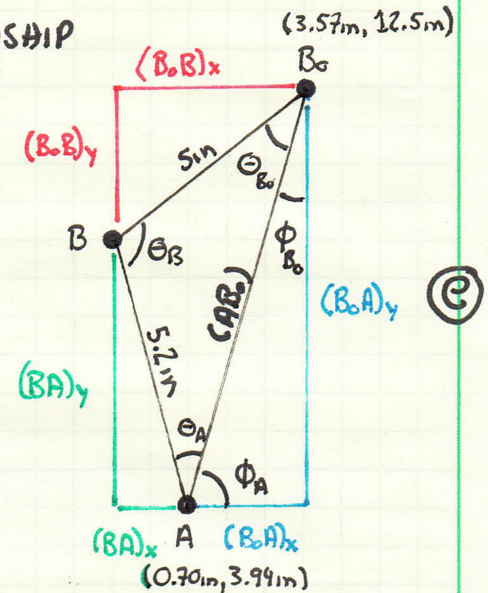
$$I_x = 12in \cdot \cos 40° = \underline{9.19in} \quad (4)$$

$$I_y = 12in \cdot \sin 40° = \underline{\underline{7.71in}} \quad (5)$$



TOPS

IN (a), THE LOCATIONS OF A & B_0 HAVE BEEN CALCULATED. THE PROBLEM STATEMENT GIVES $(B_0B) = 5in$ AND $(AB) = 5.2in$. THE GEOMETRIC RELATIONSHIP BETWEEN $B - B_0 - A$ IS ILLUSTRATED IN (c). THE COORDINATES OF B CAN BE FOUND.



FROM THE COORDINATES OF A & B_0, THE LENGTH (AB_0) IS CALCULATED

$$(AB_0)^2 = (B_0A)_x^2 + (B_0A)_y^2$$

$$= (3.57in - 0.70in)^2 + (12.5in - 3.94in)^2$$

$$= 81.51 in^2$$

$$(AB_0) = \sqrt{81.51in^2} = \underline{\underline{9.03in}} \quad (6)$$

$$(AB_0)_x = (3.57in - 0.70in) = \underline{\underline{2.87in}} \quad (7)$$

$$(AB_0)_y = (12.5in - 3.94in) = \underline{\underline{8.56in}} \quad (8)$$

$$\phi_{B_0} = \tan^{-1}\frac{(B_0A)_x}{(B_0A)_y} = \tan^{-1}\frac{2.87in}{8.56in} = \underline{\underline{18.5^\circ}} \quad (9)$$

$$\phi_A = \tan^{-1}\frac{(B_0A)_y}{(B_0A)_x} = \tan^{-1}\frac{8.56in}{2.87in} = \underline{\underline{71.5^\circ}} \quad (10)$$

USING THE LAW OF COSINES $(c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\Theta_c)$

$$\Theta_A = \cos^{-1}\left[\frac{(5.2in)^2 + (9.03in)^2 - (5in)^2}{2\cdot(5.2in)\cdot(9.03in)}\right] = \underline{\underline{27.1^\circ}} \quad (11)$$

$$\Theta_{B_0} = \cos^{-1}\left[\frac{(5.0in)^2 + (9.03in)^2 - (5.2in)^2}{2\cdot(5.0in)\cdot(9.03in)}\right] = \underline{\underline{28.3^\circ}} \quad (12)$$

$$\Theta_B = \cos^{-1}\left[\frac{(5.0in)^2 + (5.2in)^2 - (9.03in)^2}{2\cdot(5.0in)\cdot(5.2in)}\right] = \underline{\underline{124.6^\circ}} \quad (13)$$

FROM THE GEOMETRY ILLUSTRATED IN (e), THE COORDINATE LOCATION OF B CAN NOW BE DETERMINED USING EITHER B_o OR A AS A REFERENCE. FIRST B_o WILL BE USED AND THEN THE RESULT WILL BE CHECKED WITH THE SOLUTION THAT USES A.

THE COMPLEMENT ANGLE TO θ_{B_o} IN (12) AND ϕ_{B_o} IN (9) IS USED IN CONJUNCTION WITH THE GIVEN LENGTH OF (B_oB) AND THE LOCATION OF B_o TO CALCULATE THE COORDINATE LOCATION OF B.

$$B_x = B_{ox} - (B_oB)\cdot\cos(90° - \theta_{B_o} - \phi_{B_o})$$

$$= 3.57in - (5in)\cdot\cos(90° - 28.3° - 18.5°)$$

$$= 3.57in - 5in\cdot\cos(43.2°) = \underline{\underline{-0.08\ in}} \quad (14)$$

$$B_y = B_{oy} - (B_oB)\cdot\sin(90° - \theta_{B_o} - \phi_{B_o})$$

$$= 12.5in - 5in\cdot\sin(90° - 28.3° - 18.5°)$$

$$= 12.5in - 5in\cdot\sin(43.2°) = \underline{\underline{9.08in}} \quad (15)$$

THE SUPPLEMENTARY ANGLE TO θ_A IN (11) AND ϕ_A IN (10) IS USED IN CONJUNCTION WITH THE GIVEN LENGTH OF (BA) AND THE LOCATION OF A TO CALCULATE THE COORDINATE LOCATION OF B IN ORDER TO CHECK (14) AND (15).

$$B_x = A_x - (BA)\cdot\cos(180 - \theta_A - \phi_A)$$

$$= (0.70in) - (5.2in)\cdot\cos(180 - 27.1° - 71.5°)$$

$$= (0.70in) - (5.2in)\cdot\cos(81.4°) = -0.08in \text{ SAME AS (14)} \checkmark$$

$$B_y = A_y + (BA)\cdot\sin(180 - \theta_A - \phi_A)$$

$$= 3.94in + (5.2in)\cdot\sin(180° - 27.1° - 71.5°)$$

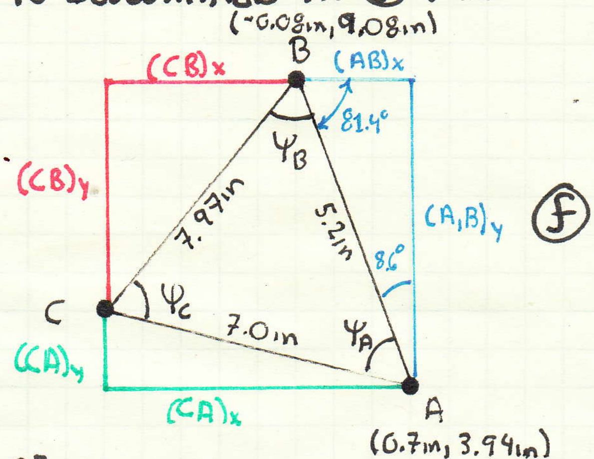
$$= 3.94in + (5.2in)\cdot\sin(81.4°) = \underline{\underline{9.08in}} \text{ SAME AS (15)} \checkmark$$

NOW THAT THE LOCATIONS OF A AND B ARE KNOWN ALONG WITH THE GIVEN LENGTHS OF (BC) AND (AC), THE COORDINATE LOCATION OF POINT C CAN NOW BE DETERMINED. AS WAS PREVIOUSLY DONE, THE CALCULATION WILL BE MADE USING BOTH A AND B IN ORDER TO CONFIRM THE COORDINATE LOCATION WITH INDEPENDENT CALCULATIONS

TOPS

THE GEOMETRY ASSOCIATED WITH A-B-C IS ILLUSTRATED IN (f) (THE ILLUSTRATION IS NOT to scale)

BEFORE THE COORDINATES OF C CAN BE CALCULATED FROM THE LOCATIONS OF A AND B, THE INTERIOR ANGLES ψ_A, ψ_B, & ψ_C NEED TO BE DETERMINED USING THE LAW OF COSINES ($c^2 = a^2 + b^2 - 2\cdot a\cdot b\cdot \cos\theta_c$)



$$\psi_A = \cos^{-1}\left[\frac{(CA)^2 + (AB)^2 - (CB)^2}{2\cdot(CA)\cdot(AB)}\right]$$

$$= \cos^{-1}\left[\frac{(7.0in)^2 + (5.2in)^2 - (7.97in)^2}{2\cdot(7.0in)^2\cdot(5.2in)^2}\right] = \underline{\underline{80.1^\circ}} \quad (16)$$

$$\psi_B = \cos^{-1}\left[\frac{(CB)^2 + (AB)^2 - (CA)^2}{2\cdot(CB)\cdot(AB)}\right]$$

$$= \cos^{-1}\left[\frac{(7.97in)^2 + (5.2in)^2 - (7.0in)^2}{2\cdot(7.97in)(5.2in)}\right] = \underline{\underline{59.9^\circ}} \quad (17)$$

$$\psi_C = \cos^{-1}\left[\frac{(CB)^2 + (CA)^2 - (AB)^2}{2\cdot(CB)\cdot(CA)}\right]$$

$$= \cos^{-1}\left[\frac{(7.97in)^2 + (7.0in)^2 - (5.2in)^2}{2\cdot(7.97in)\cdot(7.0in)}\right] = \underline{\underline{40^\circ}} \quad (18)$$

THE COORDINATES OF C ARE FIRST CALCULATED USING THE LOCATION OF POINT B, THE GIVEN LENGTH OF CB, AND THE SUPPLEMENTARY ANGLE TO ψ_B IN (17) AND THE 81.4° ANGLE THAT AB MAKES WITH THE HORIZONTAL.

$$C_x = B_x - (CB)_x = B_x - (CB)\cdot\cos(180 - \psi_B - 81.4^\circ)$$

$$= -0.08in - 7.97in\cdot\cos(180^\circ - 59.9^\circ - 81.4^\circ)$$

$$= -0.08in - 7.97in\cdot\cos(38.7^\circ) = \underline{\underline{-6.30in}} \quad (19)$$

$$C_y = B_y - (CB)_y = B_y - (CB)\cdot\sin(180 - \psi_B - 81.4^\circ)$$

$$= 9.08in - 7.97in\cdot\cos(38.7^\circ) = \underline{\underline{4.10in}} \quad (20)$$

TOPS

THE COORDINATES OF C ARE NOW CALCULATED WITH RESPECT TO A IN ORDER TO CHECK 19 & 20. THIS CALCULATION WILL USE THE GEOMETRY SHOWN IN E, THE COORDINATES OF A, THE GIVEN LENGTH OF CA, ψ_A IN 16, AND THE COMPLEMENTARY ANGLE TO ψ_A AND THE ANGLE AB MAKES WITH THE VERTICLE.

$$C_x = A_x - (CA)_x = A_x - (CA)\cdot\cos(90 - \psi_A - 8.6°)$$

$$= 0.70in - 7.0in\cdot\cos(90 - 80.1° - 8.6°)$$

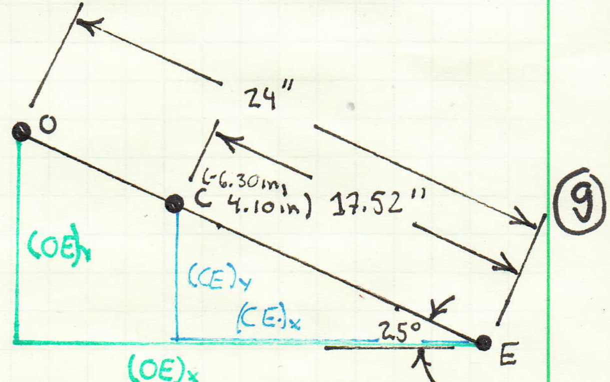
$$= 0.70in - 7.0in\cdot\cos(1.3°) = -6.30in, \text{ SAME AS } 19 \checkmark$$

$$C_y = A_y + (CA)_y = A_y + (CA)\sin(90 - \psi_A - 8.6°)$$

$$= 3.94in + 7.0in\cdot\sin(90° - 80.1° - 8.6°)$$

$$= 3.94in + 7.0in\sin(1.3°) = 4.10in, \text{ SAME AS } 20 \checkmark$$

FROM POINT C, THE COORDINATES OF E ARE FOUND FROM THE GIVEN LENGTH CE = 17.52", AND THAT THE ANGLE CE MAKES WITH THE HORIZONTAL IS 25°. USING THE GEOMETRY ILLUSTRATED IN 9.



$$E_x = C_x + (CE)_x$$

$$= C_x + (CE)\cos 25°$$

$$= -6.30in + 17.52in\cdot\cos 25°$$

$$= \underline{\underline{9.58in}} \quad 21$$

$$E_y = C_y - (CE)_y = C_y - (CE)\cdot\sin 25°$$

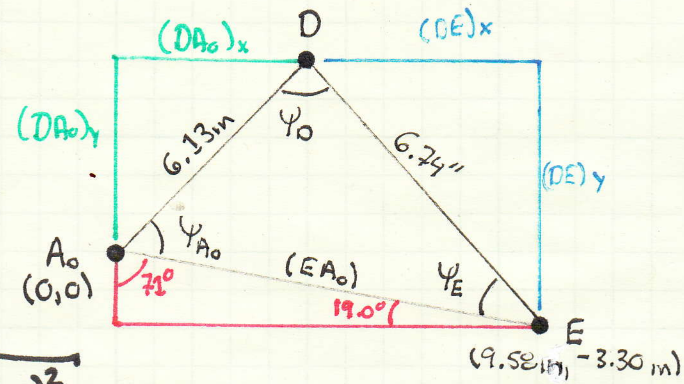
$$= 4.10in - 17.52in\cdot\sin 25° = \underline{\underline{-3.30in}} \quad 22$$

FROM POINT E, THE COORDINATES OF O ARE FOUND FROM THE GIVEN LENGTH (EO) = 24in, THE GIVEN ANGLE EO, AND THE GEOMETRY ILLUSTRATED IN 9

$$O_x = E_x - (OE)_x = E_x - (OE)\cdot\cos 25° = 9.58in - 24in\cdot\cos 25° = \underline{\underline{-12.17in}} \quad 23$$

$$O_y = E_y + (OE)_y = E_y + (OE)\cdot\sin 25° = -3.30in + 24in\cdot\sin 25° = \underline{\underline{6.84in}}$$

D IS THE FINAL POINT ON THE MECHANISM THAT NEEDS TO BE LOCATED. THE PROCESS OF CALCULATING THE COORDINATES OF D, STARTS WITH DETERMINING THE DISTANCE BETWEEN A_0 & E USING THE COORDINATES OF A_0 & E



n

$$(EA_0) = \sqrt{(E_x - A_{0x})^2 + (E_y - A_{0y})^2}$$

$$= \sqrt{(9.58\,in - 0\,in)^2 + (-3.30\,in - 0\,in)^2} = 10.13\,in \quad (24)$$

NOW THE INTERIOR ANGLES ψ_{A_0}, ψ_D, & ψ_E ARE FOUND USING THE LAW OF COSINES ($c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\theta_c$)

$$\psi_{A_0} = \cos^{-1}\left[\frac{(DA_0)^2 + (EA_0)^2 - (DE)^2}{2 \cdot (DA_0) \cdot (EA_0)}\right]$$

$$= \cos^{-1}\left[\frac{(6.13\,in)^2 + (10.13\,in)^2 - (6.74\,in)^2}{2 \cdot (6.13\,in) \cdot (10.13\,in)}\right] = \underline{\underline{40.3^\circ}} \quad (25)$$

$$\psi_D = \cos^{-1}\left[\frac{(DA_0)^2 + (DE)^2 - (EA_0)^2}{2 \cdot (DA_0) \cdot (EA_0)}\right]$$

$$= \cos^{-1}\left[\frac{(6.13\,in)^2 + (6.74\,in)^2 - (10.13\,in)^2}{2 \cdot (6.13\,in) \cdot (6.74\,in)}\right] = \underline{\underline{103.7^\circ}} \quad (26)$$

$$\psi_E = \cos^{-1}\left[\frac{(EA_0)^2 + (DE)^2 - (DA_0)^2}{2 \cdot (EA_0) \cdot (DE)}\right]$$

$$= \cos^{-1}\left[\frac{(10.13)^2 + (6.74\,in)^2 - (6.13\,in)^2}{2 \cdot (10.13\,in) \cdot (6.74\,in)}\right] = \underline{\underline{36.0^\circ}} \quad (27)$$

THE ANGLE BETWEEN EA_0 AND THE HORIZONTAL IS

$$\tan^{-1}\frac{3.30\,in}{9.58\,in} = 19.0^\circ \quad (28)$$

AND THE ANGLE BETWEEN EA_0 AND THE VERTICAL IS

$$\tan^{-1}\frac{9.58\,in}{3.30\,in} = 71.0^\circ \quad (29)$$

USING THE GEOMETRY SHOWN IN (h), THE COORDINATE LOCATION OF D CAN BE FOUND. FIRST THE COORDINATES OF D ARE FOUND USING THE GIVEN LENGTH DA_o, THE ANGLE ψ_{A_o} IN (25), AND THE SUPPLEMENTAL ANGLE TO ψ_{A_o} AND THE ANGLE EA_o MAKES WITH THE VERTICAL IN (29)

$$D_x = A_{ox} + (DA_o)_x = A_{ox} + (DA_o)\cdot\sin(180 - \psi_{A_o} - 71_o)$$

$$= 0 + 6.13in\cdot\sin(180° - 40.3° - 71.0°)$$

$$= 6.13in\cdot\sin(68.7°) = \underline{\underline{5.71in}} \quad (30)$$

$$D_y = A_{oy} + (DA_o)_y = A_{oy} + (DA_o)\cos(180 - \psi_{A_o} - 71°)$$

$$= 0.0in + 6.13in\cdot\cos(180° - 40.3° - 71.0°)$$

$$= 6.13in\cdot\cos(68.7°) = \underline{\underline{2.22in}} \quad (31)$$

(30) AND (31) CAN NOW BE CHECKED BY FINDING THE COORDINATES OF D USING THE COORDINATES OF POINT E, THE GIVEN LENGTH DE, THE CALCULATED ANGLE ψ_E IN (27) AND THE COMPLEMENTARY ANGLE TO ψ_E AND THE ANGLE A_oE MAKES WITH THE HORIZONTAL IN (28)

$$D_x = E_x - (DE)_x = E_x - (DE)\cdot\sin(90 - \psi_E - 19.0°)$$

$$= 9.58in - 6.74in\cdot\sin(90 - 36.0° - 19.0°)$$

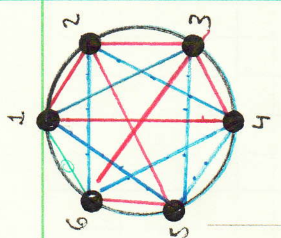
$$= 9.58in - 6.74in\cdot\sin(35°) = \underline{\underline{5.71in}}$$ THIS IS THE SAME AS (30) ✓

$$D_y = E_y + (DE)_y = E_y + (DE)\cos(90 - \psi_E - 19.0°)$$

$$= -3.30in + 6.74in\cdot\cos(90 - 36.0° - 19.0°)$$

$$= -3.30in + 6.74in\cdot\cos(35°) = 2.22in$$ THIS IS THE SAME AS (31)

FIRST INSTANT CENTERS WILL BE USED TO ~~LOCATE~~ CALCULATE THE MECHANICAL ADVANTAGE. (i) SHOWS THE LOCATION OF ALL THE INSTANT CENTERS AND (j) SHOWS THE INSTANT CENTERS NEEDED TO CALCULATE THE MECHANICAL ADVANTAGE.


1
2
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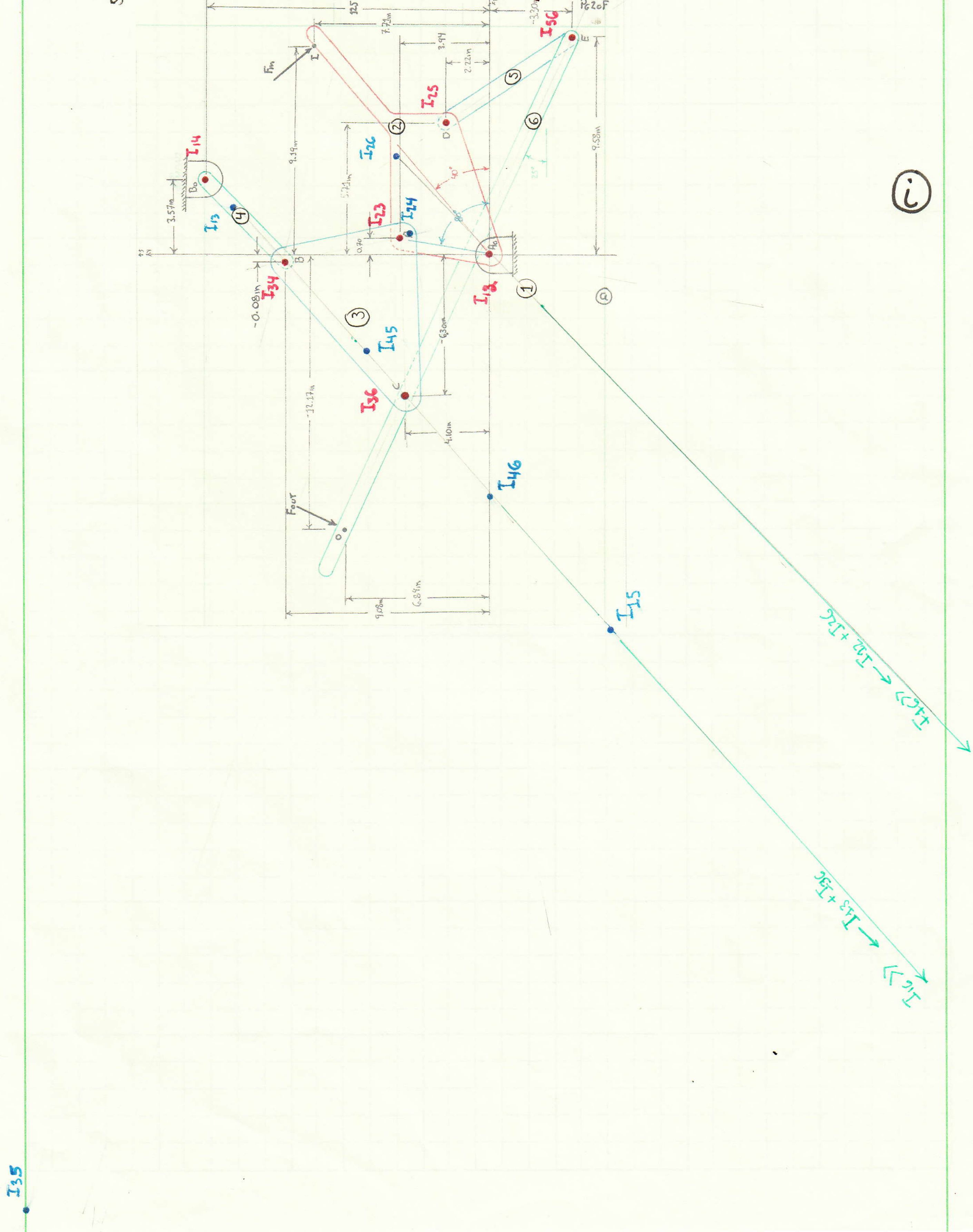
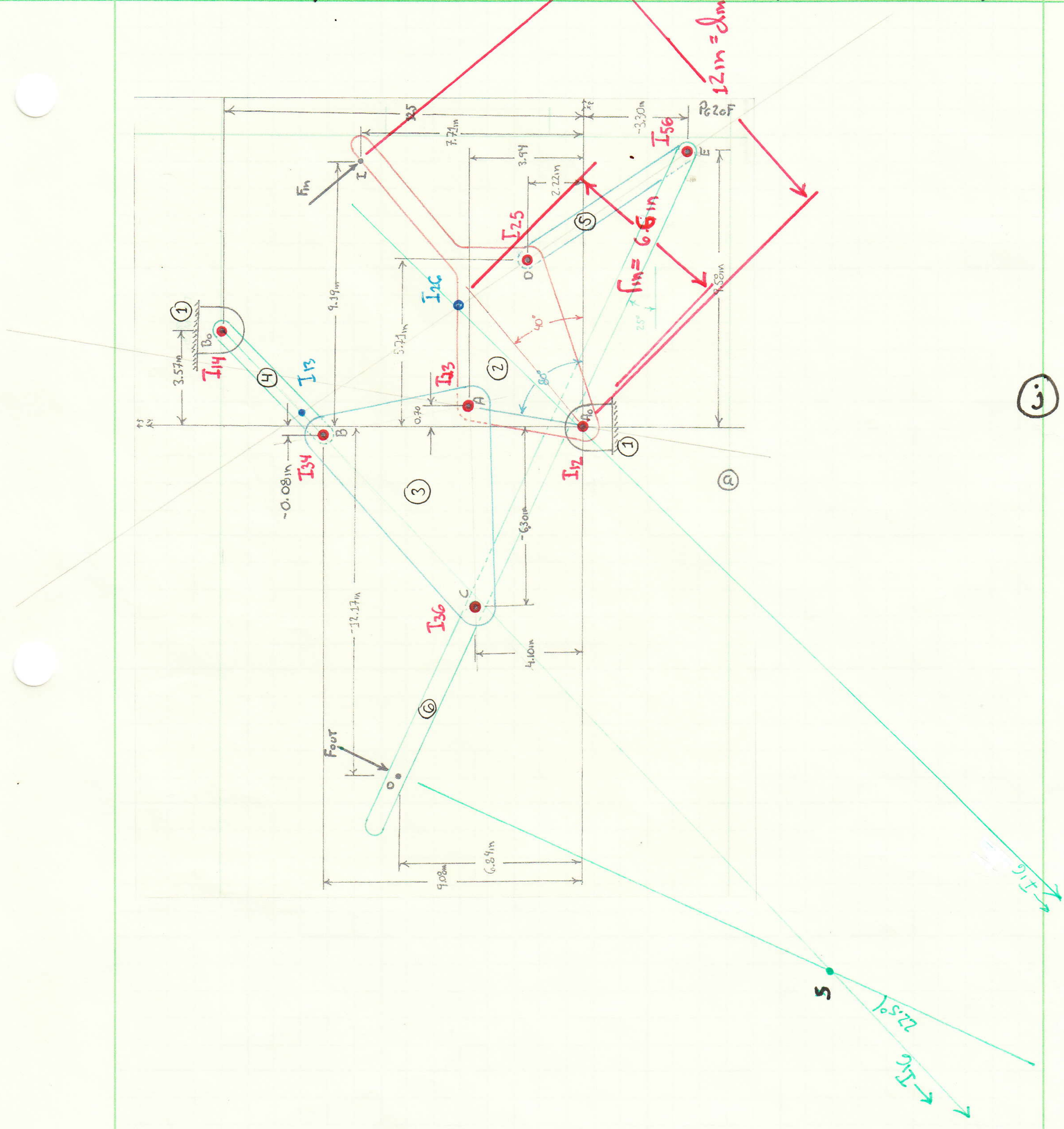

I_{14}
I_{13}
I_{34}
I_{23}
I_{24}
I_{26}
I_{25}
I_{56}
I_{12}
I_{45}
I_{36}
I_{46}
I_{15}
I_{35}
F_{in}
F_{OUT}
(i)


12 in = 1 in
1 in = 6.6 in
I_{14}
I_{13}
I_{12}
I_{23}
I_{25}
I_{56}
I_{34}
I_{36}
I_{2C}
I_{16}
F_{in}
Foot
B_o
A_o
A
B
C
D
E
I
Pg 2 of F
3.57 in
9.19 in
5.71 in
7.71 in
3.94
2.22 in
-3.30 in
0.70
-0.08 in
-6.30 in
-12.17 in
4.10 in
6.84 in
9.08 in
40°
80°
25°
22.5°
5
(1)
(2)
(3)
(4)
(5)
(6)
(a)
(j)

FOR ROTARY INPUT AND OUTPUT THE MECHANICAL ADVANTAGE IS GIVEN BY

$$MA = \frac{d_{in}}{d_{out}} \cdot \frac{r_{out}}{r_{in}} \quad (32)$$

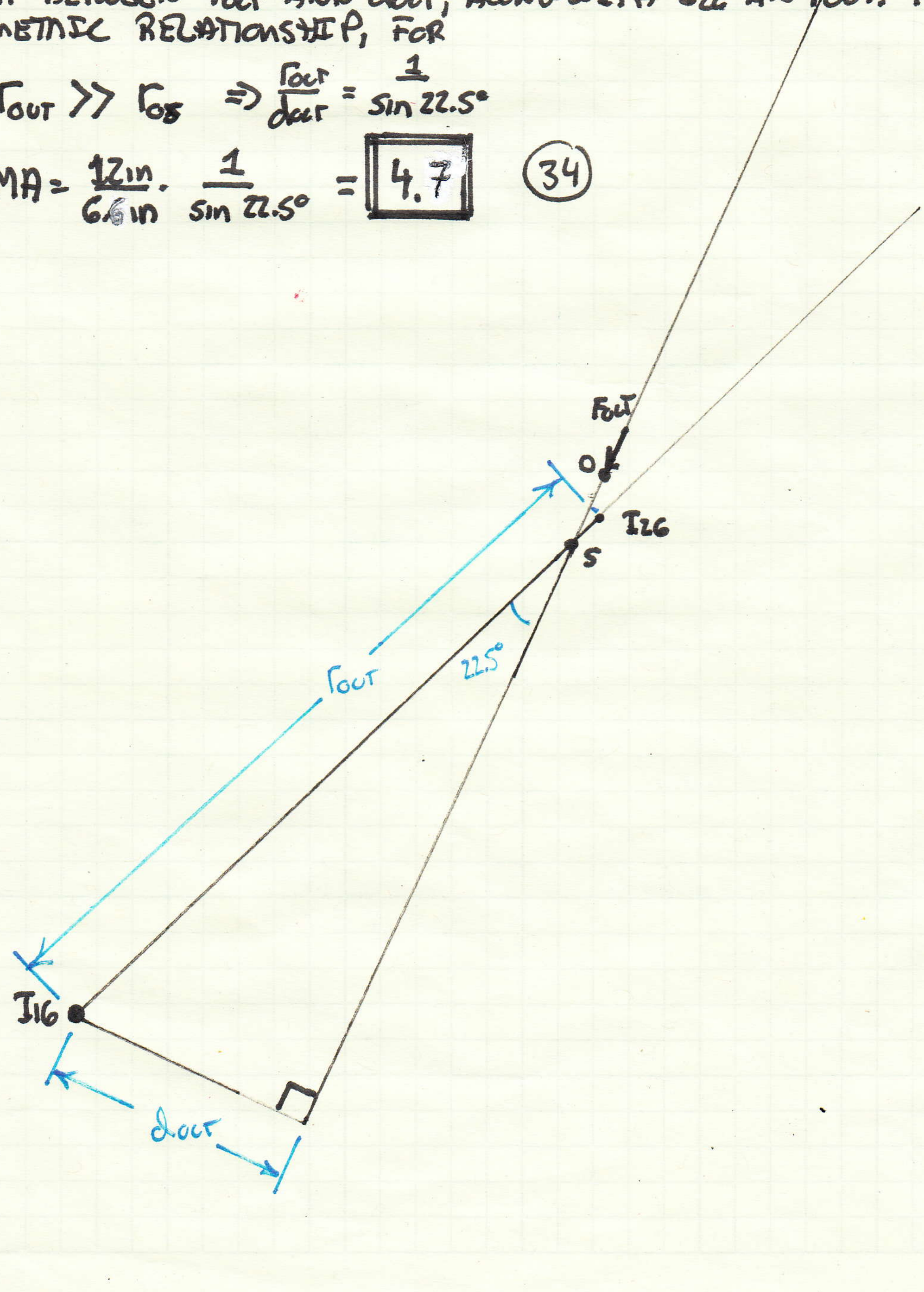
FROM (J) $d_{in} = 12in$ & $r_{in} = 6.4in$

$$MA = \frac{12in}{6.4in} \cdot \frac{r_{out}}{d_{out}} \quad (33)$$

THE LOCATION OF r_{out} & d_{out} are COMPLICATED BY THE DISTANCE OF I_{16} FROM ~~THE~~ ANY POINT ON THE STRUCTURE IS LARGE, THUS THE RATIO OF r_{out}/d_{out} NEEDS TO BE APPROXIMATED. (K) ILLUSTRATES THE RELATIONSHIP BETWEEN r_{out} AND d_{out}, ALONG WITH I_{26} AND r_{out}. FROM THIS GEOMETRIC RELATIONSHIP, FOR

$$r_{out} \gg r_{OS} \Rightarrow \frac{r_{out}}{d_{out}} = \frac{1}{\sin 22.5°}$$

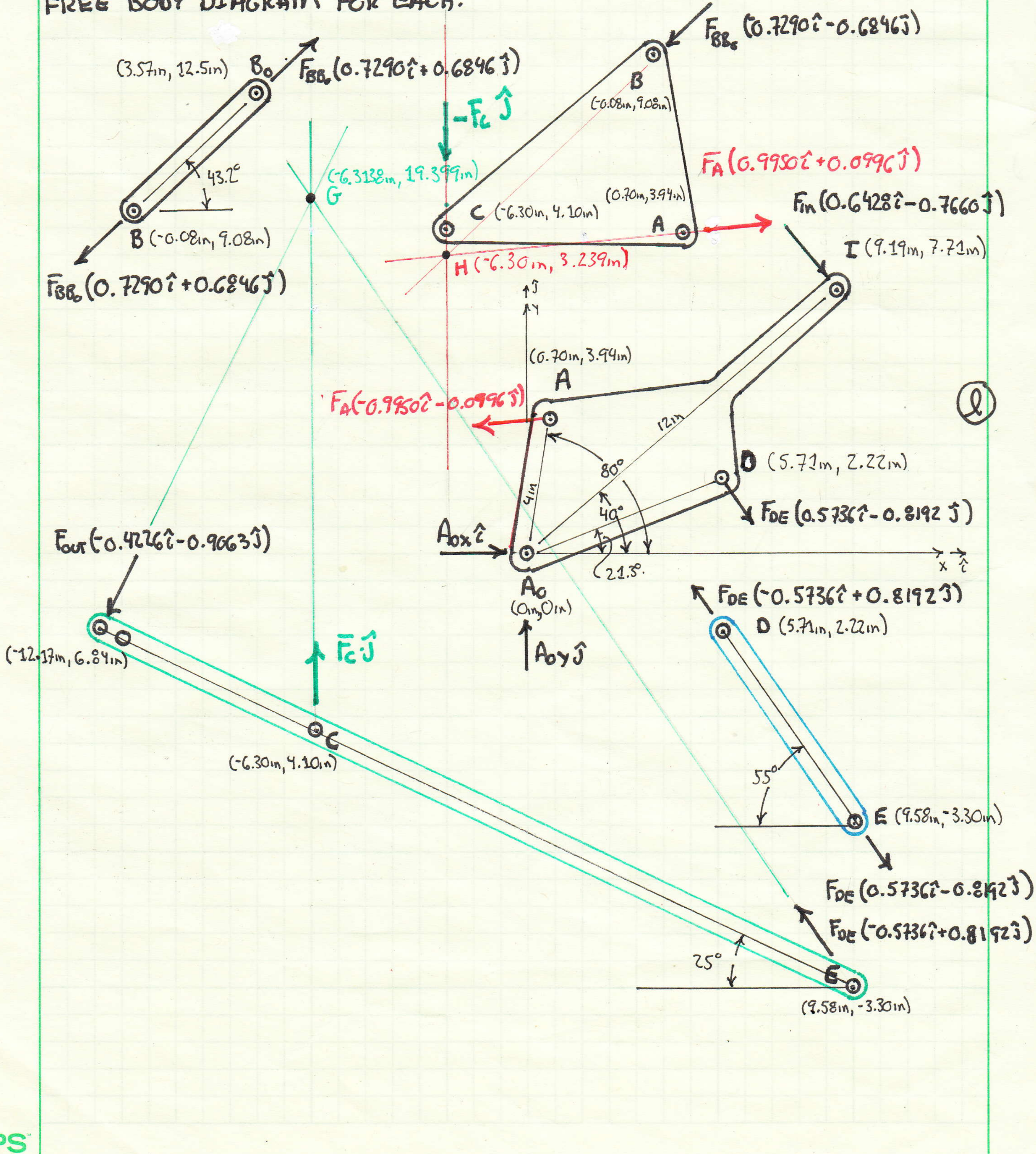
$$MA = \frac{12in}{6.6in} \cdot \frac{1}{\sin 22.5°} = \boxed{4.7} \quad (34)$$



(K)

THE LOCATION OF ALL THE JOINTS HAVE BEEN FOUND FOR THE MECHANISM IN THE CONFIGURATION SPECIFIED. THIS IS NECESSARY IN THE CALCULATION OF MECHANICAL ADVANTAGE USING INTERNAL EQUILIBRIUM CONCEPTS FROM STATICS.

THIS ALTERNATE CALCULATION OF MECHANICAL ADVANTAGE STARTS BY DECOMPOSING THE MECHANISM INTO INDIVIDUAL LINKS AND DRAWING FREE BODY DIAGRAM FOR EACH.



BOTH ABC AND OCE ARE THREE FORCE MEMBERS. THIS WILL HELP IN REDUCING THE NUMBER OF UNKNOWNS WHEN EQUILIBRIUM IS CONSIDERED BECAUSE IN THREE FORCE MEMBERS ALL THREE FORCES ACT THROUGH A COMMON POINT OF CONCURRANCE. FOR OCE THIS IS G AND FOR ABC IT IS POINT H. POINTS G & H NOW NEED TO BE LOCATED.

FIRST LOCATING G.

THE LINE OF ACTION FOR THE FORCE AT O IS THE UNIT VECTOR

$$\hat{e}_O = -0.4226\hat{i} - 0.9063\hat{j}$$

THE EQUATION FOR THIS LINE CAN BE WRITTEN

$$Y_O = m \cdot X_O + b$$

$$m = \frac{-0.9063}{-0.4226} = 2.1446$$

$$Y_O = 2.1446 \cdot X_O + b$$

TO FIND b THE LOCATION OF POINT O IS SUBSTITUTED INTO THE ABOVE EQUATION.

$$6.84\text{in} = 2.1446 \cdot (-12.17) + b$$

$$\Rightarrow b = 6.84\text{in} + (2.1446)\cdot(12.17) = 32.940\text{in}$$

$$\underline{\underline{Y_O = 2.1446 \cdot X_O + 32.940\text{in}}} \qquad (35)$$

THE LINE OF ACTION FOR THE FORCE AT E IS THE UNIT VECTOR

$$\hat{e}_{DE} = -0.5736\hat{i} + 0.8192\hat{j}$$

THE EQUATION FOR THIS LINE CAN BE WRITTEN

$$Y_{DE} = m \cdot X_{DE} + b$$

$$m = \frac{0.8192}{-0.5736} = -1.4282$$

$$Y_{DE} = -1.4282 \cdot X_{DE} + b$$

TO FIND b THE LOCATION OF POINT E IS SUBSTITUTED INTO THE ABOVE EQUATION.

TOPS

$$-3.30\text{in} = -1.4282\cdot(9.58\text{in}) + b$$

$$\Rightarrow b = -3.30\text{in} + 1.4282\cdot(9.58\text{in}) = 10.382\text{in}$$

$$\underline{\underline{Y_{DE} = -1.4282\cdot X_{DE} + 10.382\text{in}}} \quad (36)$$

THE LOCATION OF G IS WHERE $Y_{DE} = Y_o = Y_G$ AND $X_{DE} = X_o = X_G$ USING (35) AND (36)

$$2.1446\cdot X_G + 32.940\text{in} = -1.4282\cdot X_G + 10.382\text{in}$$

$$(2.1446 + 1.4282)\,X_G = 10.382\text{in} - 32.940\text{in}$$

$$X_G = \frac{10.382\text{in} - 32.940\text{in}}{2.1446 + 1.4282} = \underline{\underline{-6.3138\text{in}}}$$

Y_G CAN BE CALCULATED USING (35) OR (36), BOTH ARE USED HERE TO SHOW THE CALCULATION IS THE SAME.

$$(35) \rightarrow Y_G = 2.1446\cdot(-6.3138\text{in}) + 32.940\text{in}$$
$$= 19.399\text{in} \checkmark$$

$$(36) \rightarrow Y_G = -1.4282\cdot(-6.3138\text{in}) + 10.382\text{in}$$
$$= \underline{\underline{19.399\text{in}}} \checkmark$$

$$\Rightarrow \quad \underline{\underline{G = (-6.3138\text{in},\ 19.399\text{in})}} \quad (37)$$

KNOWING G'S LOCATION ALLOWS THE LINE OF ACTION OF THE FORCE AT C, F_C TO BE CALCULATED USING THE POSITION VECTOR $\vec{r}_{CG}$

$$\vec{r}_{CG} = (-6.3138\text{in} - (-6.30\text{in}))\,\hat{\imath} + (19.399\text{in} - 4.10\text{in})\,\hat{\jmath}$$

$$= -0.0138\text{in}\,\hat{\imath} + 15.299\text{in}\,\hat{\jmath}$$

$$r_{CG} = \sqrt{(-0.0138\text{in})^2 + (15.299\text{in})^2} = 15.299\text{in}$$

$$\hat{e}_{CG} = -\frac{0.0138}{15.299\text{in}}\,\hat{\imath} + \frac{15.299\text{in}}{15.299\text{in}}\,\hat{\jmath} = 1.0\hat{\jmath} \quad (38)$$

$$\Rightarrow \quad \underline{\underline{\vec{F}_C = F_C\,\hat{\jmath}}} \quad (39)$$

TOPS

LOCATING H

THE LINE OF ACTION OF THE FORCE AT C ON ABC IS

$$\hat{e}_{HE} = -\hat{j}$$

THE EQUATION FOR THIS LINE IS WRITTEN

$$\underline{\underline{X_{HE} = -6.30\text{in}}} \quad (36)$$

THE LINE OF ACTION FOR THE FORCE AT B ON ABC, F_{BB_0} IS

$$\hat{e}_{BB_0} = -0.7290\,\hat{i} - 0.6846\,\hat{j}$$

THE EQUATION FOR THIS LINE CAN BE WRITTEN

$$Y_{BB_0} = m \cdot X_{BB_0} + b$$

$$m = \frac{-0.6846}{-0.7290} = 0.9391$$

$$\Rightarrow Y_{BB_0} = 0.9391 \cdot X_{BB_0} + b$$

TO FIND b THE LOCATION OF POINT B IS SUBSTITUTED INTO THE ABOVE EQUATION

$$9.08\text{in} = 0.9391 \cdot (-0.08\text{ in}) + b$$

$$b = 9.08\text{in} + 0.9391 \cdot (0.08\text{in}) = 9.155\text{in}$$

$$\Rightarrow \underline{\underline{Y_{BB_0} = 0.9391 \cdot X_{BB_0} + 9.155\text{in}}} \quad (37)$$

THE LOCATION OF H, (X_H, Y_H) IS LOCATED WHERE (36) AND (37) INTERSECT

$$\Rightarrow X_H = X_{BB_0} = X_{HE}$$

$$\Rightarrow Y_H = Y_{BB_0} = 0.9391 \cdot (-6.30\text{in}) + 9.155\text{in}$$

$$= \underline{\underline{3.239\text{in}}}$$

$$\Rightarrow \underline{\underline{H = (-6.30\text{in}, 3.239\text{in})}} \quad (38)$$

TOPS

KNOWING H ENABLES THE CALCULATION OF THE LINE OF ACTION OF THE FORCE AT A TO BE MADE, STARTING WITH THE POSITION VECTOR r_{AH}

$$\vec{r}_{AH} = [0.70\text{in} - (-6.30\text{in})]\hat{\imath} + [3.94\text{in} - 3.239\text{in}]\hat{\jmath}$$

$$= 7.00\text{in}\,\hat{\imath} + 0.701\text{in}\,\hat{\jmath}$$

$$r_{AH} = \sqrt{(7.00\text{in})^2 + (0.701\text{in})^2} = 7.035\text{in}$$

$$\hat{e}_{AH} = \frac{7.00\text{in}}{7.035\text{in}}\hat{\imath} + \frac{0.701\text{in}}{7.035\text{in}}\hat{\jmath} =$$

$$= \underline{\underline{0.9950\hat{\imath} + 0.0996\hat{\jmath}}} \qquad (39)$$

(39) IS THE LINE OF ACTION OF THE FORCE AT A.

$$\vec{F}_A = F_A(0.9950\hat{\imath} + 0.0996\hat{\jmath}) \qquad (40)$$

THE EQUILIBRIUM OF EACH OF THE BODIES NOW CAN BE EVALUATED

<u>OCE</u>

$$\Sigma F_x = 0 = -0.4226 \cdot F_{OUT} - 0.5736 \cdot F_{DE} \qquad (41)$$

$$\Sigma F_y = 0 = -0.9063 \cdot F_{OUT} + F_C + 0.8192 \cdot F_{DE} \qquad (42)$$

<u>ABC</u>

$$\Sigma F_x = 0 = -0.7290 \cdot F_{BB_0} + 0.9950 \cdot F_A \qquad (43)$$

$$\Sigma F_y = 0 = -F_C - 0.6846 \cdot F_{BB_0} + 0.0996 \cdot F_A \qquad (44)$$

<u>A_0ADI</u>

$$\Sigma F_x = 0 = A_{ox} - 0.9950 \cdot F_A + 0.5736 \cdot F_{DE} + \underline{0.6428 \cdot F_{in}} \qquad (45)$$

$$\Sigma F_y = 0 = A_{oy} - 0.0996 \cdot F_A - 0.8192 \cdot F_{DE} \underline{- 0.7660 \cdot F_{in}} \qquad (46)$$

TOPS

OCE AND ABC ARE THREE FORCE MEMBERS. IN THREE FORCE MEMBERS THE SUM OF THE MOMENTS IN THE Z-DIRECTION IS IDENTICALLY SATISFIED BECAUSE ALL THREE FORCES ACT THROUGH A COMMON POINT OF CONCURRENCE. THIS MEANS THAT SUMMING THE MOMENTS IN THE Z-DIRECTION WILL NOT YIELD AN EQUATION THAT IS INDEPENDENT OF THE SUM OF THE FORCES IN THE X AND Y.

A_0ADI IS NOT A THREE FORCE MEMBER AND THEREFORE TO COMPLETE THE ASSESSMENT OF THE EQUILIBRIUM OF THIS MEMBER THE SUM OF THE MOMENTS MUST BE CONSIDERED.

$$\Sigma M_{z@A_0} = 0 = \vec{r}_{AA_0} \times \vec{F}_A + \vec{r}_{IA_0} \times \vec{F}_{in} + \vec{r}_{DA_0} \times \vec{F}_{DE}$$

THE POSITION VECTORS HAVE BEEN PREVIOUSLY DETERMINED: $\vec{r}_{AA_0}$ IN (2) & (3), $\vec{r}_{IA_0}$ IN (4) & (5), AND $\vec{r}_{DA_0}$ IN (30) & (31)

$$0 = (0.70\text{in}\,\hat{i} + 3.94\text{in}\,\hat{j}) \times (-0.9950 \cdot F_A - 0.0996 \cdot F_A \hat{j})$$
$$+ (9.19\text{in}\,\hat{i} + 7.71\text{in}\,\hat{j}) \times (0.6428 \cdot F_{in}\,\hat{i} - 0.7660 \cdot F_{in}\,\hat{j})$$
$$+ (5.71\text{in}\,\hat{i} + 2.22\text{in}\,\hat{j}) \times (0.5736 \cdot F_{DE}\,\hat{i} - 0.8192 \cdot F_{DE}\,\hat{j})$$

$$0 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.70\text{in} & 3.94\text{in} & 0 \\ -.9950 \cdot F_A & -0.0996 F_A & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 9.19\text{in} & 7.71\text{in} & 0 \\ 0.6428 \cdot F_{in} & -0.7660 \cdot F_{in} & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5.71\text{in} & 2.22\text{in} & 0 \\ 0.5736 \cdot F_{DE} & -0.8192 \cdot F_{DE} & 0 \end{vmatrix}$$

$$0 = (0.70\text{in})(-0.0996 \cdot F_A) - (3.94\text{in})(-0.9950 \cdot F_A)$$
$$+ (9.19\text{in})(-0.7660 \cdot F_{in}) - (7.71\text{in})(0.6428 \cdot F_{in})$$
$$+ (5.71\text{in})(-0.8192 \cdot F_{DE}) - (2.22\text{in})(0.5736 \cdot F_{DE})$$

$$0 = 3.8506\,\text{in} \cdot F_A - 11.996\,\text{in}\,F_{in} - 5.951\,\text{in} \cdot F_{DE}$$

$$0 = 3.8506 \cdot F_A - 11.996 \cdot F_{in} - 5.951 \cdot F_{DE} \qquad (47)$$

(41) THROUGH (47) REPRESENT SEVEN INDEPENDENT EQUATIONS THAT CONTAIN SEVEN UNKNOWNS (F_{OUT}, F_{DE}, F_{BB_0}, F_A, A_{0x}, A_{0y}, F_C) AND ONE KNOWN (F_{in}). A SYSTEM OF EQUATIONS IN MATRIX FORM CAN NOW BE SET UP FOR SOLUTION.

$$
\begin{array}{c}
 \\ (41)\rightarrow \\ (42)\rightarrow \\ (43)\rightarrow \\ (44)\rightarrow \\ (45)\rightarrow \\ (46)\rightarrow \\ (47)\rightarrow
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} F_{OUT} & F_C & F_{DE} & F_{BB_0} & F_A & A_{0x} & A_{0y} \end{array} \\
\begin{bmatrix}
-0.4226 & 0 & -0.5736 & 0 & 0 & 0 & 0 \\
-0.9063 & 1 & 0.8192 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.7290 & 0.9950 & 0 & 0 \\
0 & -1 & 0 & -0.6846 & 0.0996 & 0 & 0 \\
0 & 0 & 0.5736 & 0 & -0.9950 & 1 & 0 \\
0 & 0 & -0.8192 & 0 & -0.0996 & 0 & 1 \\
0 & 0 & -5.951 & 0 & 3.8506 & 0 & 0
\end{bmatrix}
\end{array}
\begin{Bmatrix} F_{OUT} \\ F_C \\ F_{DE} \\ F_{BB_0} \\ F_A \\ A_{0x} \\ A_{0y} \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.6428 \cdot F_{in} \\ 0.7660 \cdot F_{in} \\ 11.996 \cdot F_{in} \end{Bmatrix}
$$

IF F_{in} IS SET TO 1 lb, USING EXCEL

$$
\begin{Bmatrix} F_{OUT} \\ F_C \\ F_{DE} \\ F_{BB_0} \\ F_A \\ A_{0x} \\ A_{0y} \end{Bmatrix}
=
\begin{Bmatrix} -4.7 \\ -7.0 \\ 3.4 \\ 11.5 \\ 8.4 \\ 7.0 \\ 4.4 \end{Bmatrix} [\text{lb}]
$$

$$\Rightarrow MA = \frac{F_{OUT}}{F_{in}} = \frac{4.7}{1} = \boxed{4.7}$$

SUMMARY:

THIS SOLUTION FIRST DETERMINED THE MECHANICAL ADVANTAGE USING INSTANT CENTERS AND THEN THE MECHANICAL ADVANTAGE WAS DETERMINED BY APPLYING STATIC EQUILIBRIUM TO THIS MECHANISM. THE LATER APPROACH LED TO A SYSTEM OF EQUATIONS THAT WERE SOLVED USING MATRIX ALGEBRA. BOTH APPROACHES YIELDED THE SAME SOLUTION WITH IN REASON. THE GRAPHICAL IS VERY SUSEPTABLE TO ERROR DUE TO THE ACCURACY OF THE MEASUREMENT TOOLS USED.

THE APPLICATION OF INSTANT CENTERS TO THE DETERMINATION OF THE MECHANICAL ADVANTAGE OF THIS MECHANISM WAS COMPLICATED BY I_{16} BEING LOCATED FAR AWAY FROM THE MECHANISM. THIS MADE THE DIRECT MEASUREMENT OF r_{out} AND d_{out} NOT POSSIBLE; THEREFORE, AN APPROXIMATION OF THE RATIO OF r_{out}/d_{out} WAS MADE USING THE CONSTRUCTION IN (k). IT IS ONLY BECAUSE I_{16} IS SO FAR FROM O AND I_{26} THAT THE ERROR IN THE CALCULATION CAN BE IGNORED.