**PROBLEM 3-84** THE CANTILEVERED BAR IN THE FIGURE IS MADE FROM A DUCTILE MATERIAL AND IS STATICALLY LOADED WITH $F_y = 250$ lb AND $F_x = F_z = 0$ lb. ANALYZE THE STRESS ~~DISTRIBUTION~~ SITUATION IN THE SMALL DIAMETER AT THE SHOULDER AT A BY OBTAINING THE FOLLOWING INFORMATION.

(a) DETERMINE THE PRECISE LOCATION OF THE CRITICAL STRESS ELEMENT AT THE CROSS-SECTION AT A.

(b) SKETCH THE CRITICAL STRESS ELEMENT AND DETERMINE MAGNITUDES AND DIRECTIONS OF ALL STRESSES ACTING ON IT. (TRANSVERSE SHEAR MAY BE NEGLECTED IF YOU CAN JUSTIFY THIS DECISION.)

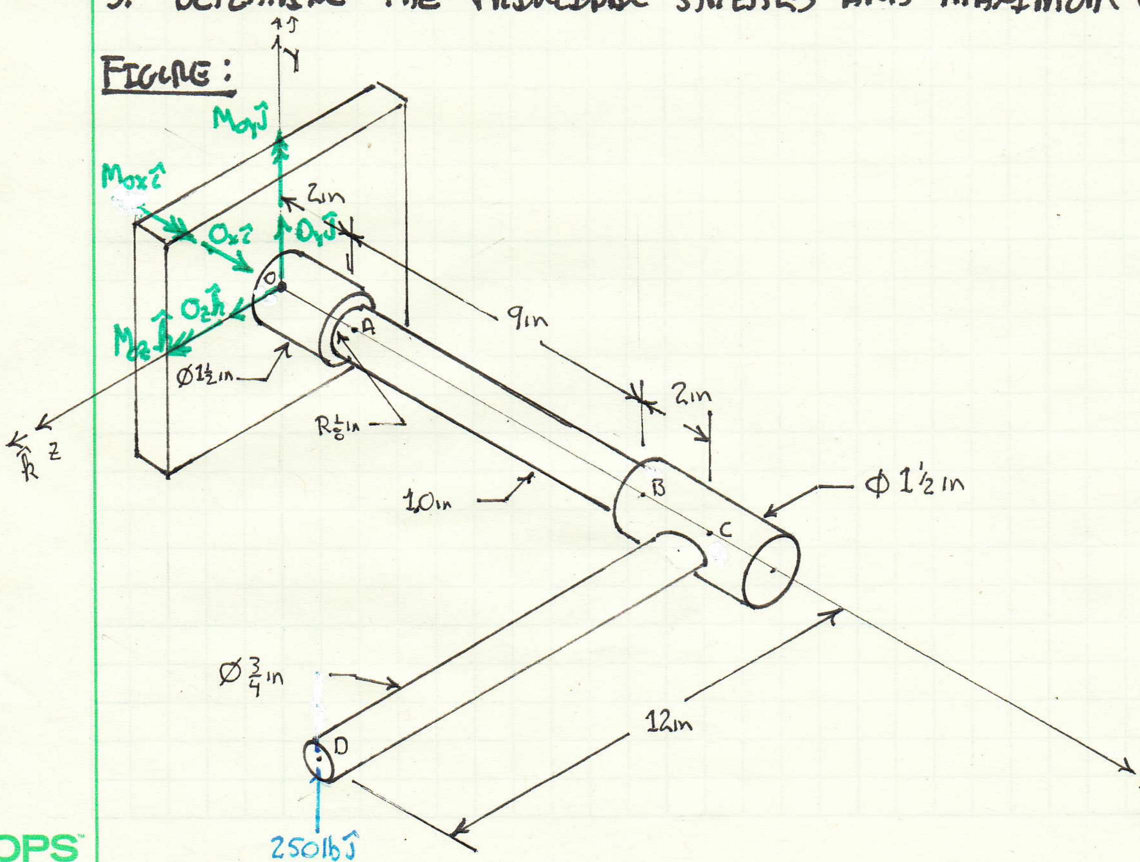
(c) FOR THE CRITICAL STRESS ELEMENT, DETERMINE THE PRINCIPAL STRESSES AND THE MAXIMUM SHEAR STRESS.

**GIVEN:**
1. THE CANTILEVERED BAR SHOWN
2. A 250 lb FORCE IN THE Y-DIRECTION
3. BAR CONSTRUCTED FROM A DUCTILE MATERIAL

**ASSUMPTIONS:**
1. THE DEFORMATIONS THAT RESULT FOR THE LOADING OF THE BAR ARE SMALL
2. THE WEIGHT OF THE BAR CAN BE NEGLECTED.

**FIND:**
1. DETERMINE THE CRITICAL STRESS LOCATION IN CROSS-SECTION A
2. DETERMINE THE MAGNITUDES AND DIRECTIONS OF THE CRITICAL STRESS
3. DETERMINE THE PRINCIPAL STRESSES AND MAXIMUM SHEAR STRESS.

**FIGURE:**




(a)

SOLUTION:

CONSIDERING EQUILIBRIUM IN ORDER TO DETERMINE THE REACTIONS AT THE WALL. USING @ AS THE FREE BODY DIAGRAM

$$\Sigma F_x = 0 = O_x \quad (1)$$

$$\Sigma F_y = 0 = O_y + 250\,\text{lb} \Rightarrow O_y = -250\,\text{lb} \quad (2)$$

$$\Sigma F_z = 0 = O_z \quad (3)$$

SUMMING MOMENTS ABOUT O

$$\Sigma \vec{M}_{@O} = \vec{0} = M_{Ox}\hat{\imath} + M_{Oy}\hat{\jmath} + M_{Oz}\hat{k} + \vec{r}_{OD} \times \vec{F}_D \quad (4)$$

WHERE

$$\vec{r}_{OD} = 11\,\text{in}\,\hat{\imath} + 12\,\text{in}\,\hat{k} \quad (5)$$

$$\vec{F}_D = 250\,\text{lb}\,\hat{\jmath} \quad (6)$$

SUBSTITUTING (5) & (6) INTO (4)

$$\vec{0} = M_{Ox}\hat{\imath} + M_{Oy}\hat{\jmath} + M_{Oz}\hat{k} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 13\,\text{in} & 0\,\text{in} & 12\,\text{in} \\ 0 & 250\,\text{lb} & 0 \end{vmatrix}$$

$$= M_{Ox}\hat{\imath} + M_{Oy}\hat{\jmath} + M_{Oz}\hat{k}$$
$$+ [(0) - (12\,\text{in})\cdot(250\,\text{lb})]\hat{\imath} - [(11\,\text{in})(0) - (12\,\text{in})(0)]\hat{\jmath} + [(13\,\text{in})(250\,\text{lb}) - 0]\hat{k}$$

$$= [M_{Ox} - 3000\,\text{in}\cdot\text{lb}]\hat{\imath} + [M_{Oy}]\hat{\jmath} + [M_{Oz} + 3250\,\text{in}\cdot\text{lb}]\hat{k}$$

DOTTING WITH $\hat{\imath}$

$$0 = M_{Ox} - 3000\,\text{in-lb} \Rightarrow M_{Ox} = 3000\,\text{in-lb} \quad (7)$$

DOTTING WITH $\hat{\jmath}$

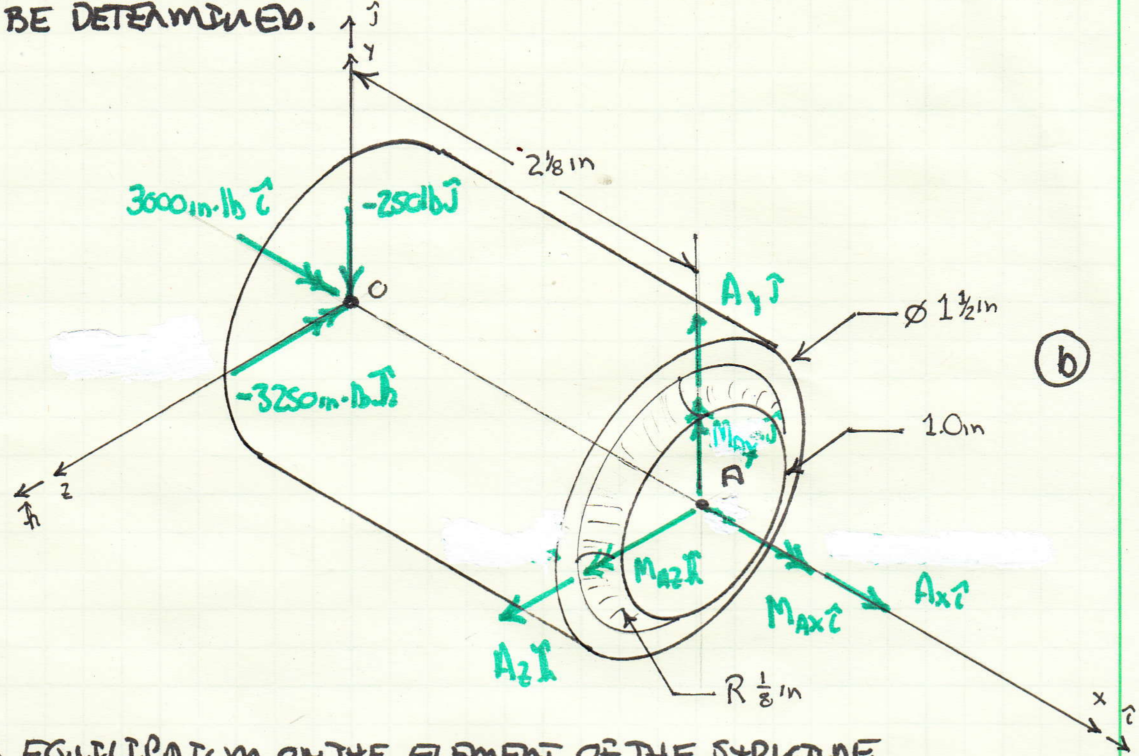
$$0 = M_{Oy} \quad (8)$$

DOTTING WITH $\hat{k}$

$$0 = M_{Oz} + 3250\,\text{in}\cdot\text{lb} \Rightarrow M_{Oz} = -3250\,\text{in}\cdot\text{lb} \quad (9)$$

TOPS

NOW THE INTERNAL FORCES AND MOMENTS AT THE BOTTOM OF THE FILLET AT A CAN BE DETERMINED.




(b)

CONSIDERING EQUILIBRIUM ON THE ELEMENT OF THE STRUCTURE SHOWN IN (b)

$$\Sigma F_x = 0 = A_x \qquad (10)$$

$$\Sigma F_y = 0 = -250\,\text{lb} + A_y \Rightarrow A_y = 250\,\text{lb} \qquad (11)$$

$$\Sigma F_z = 0 = A_z \qquad (12)$$

TAKING THE SUM OF THE MOMENTS ABOUT A

$$\Sigma \vec{M}_{@A} = \vec{0} = M_{Ax}\hat{i} + M_{Ay}\hat{j} + M_{Az}\hat{k} + 3000\,\text{in}\cdot\text{lb}\,\hat{i} - 3250\,\text{in}\cdot\text{lb}\,\hat{k} + (2\tfrac{1}{8}\,\text{in})(250\,\text{lb})\,\hat{k}$$

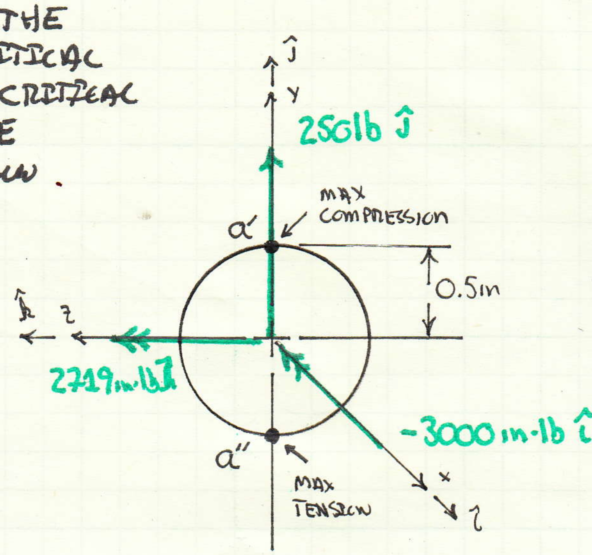
DOTTING WITH THE UNIT VECTORS

DOTTING WITH $\hat{i}$: $M_{Ax} + 3000\,\text{in}\cdot\text{lb} = 0 \Rightarrow M_{Ax} = -3000\,\text{in}\cdot\text{lb}$ (13)

DOTTING WITH $\hat{j}$: $M_{Ay} = 0$ (14)

DOTTING WITH $\hat{k}$: $M_{Az} - 3250\,\text{in}\cdot\text{lb} + (2\tfrac{1}{8}\,\text{in})(250\,\text{lb}) = 0 \Rightarrow M_{Az} = 2719\,\text{in}\cdot\text{lb}$ (15)

Ⓒ ILLUSTRATES THE INTERNAL LOADS AT THE BOTTOM OF THE FILLET AT A. THE CRITICAL TENSILE STRESS ARE AT a" AND THE CRITICAL COMPRESSIVE STRESSES ARE AT a'. THE STRESS CUBES AT THESE LOCATIONS NOW NEED TO BE DETERMINED.




(c)

__LOCATION a":__

$$\sigma_x = K_b \frac{M_z \cdot y}{I_{zz}}$$

BECAUSE OF THE FILLET, A STRESS CONCENTRATION IS PRESENT. USING FIGURE A-15-9 IN THE TEXTBOOK

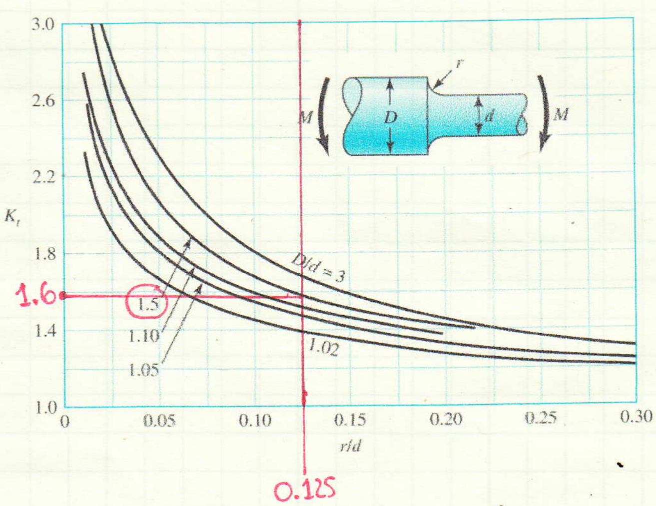
$$\frac{r}{d} = \frac{1/8\,\text{in}}{1\,\text{in}} = 0.125;\quad \frac{D}{d} = \frac{1\tfrac{1}{2}\,\text{in}}{1\,\text{in}} = 1.5 \Rightarrow K_b = 1.6$$

$$\sigma_x = 1.6 \cdot \frac{(2719\,\text{in·lb})(0.5\,\text{in})}{\pi(1\,\text{in})^4/64} = 44.31(10^3)\,\frac{\text{lb}}{\text{in}^2} = \underline{44.31\,\text{ksi}}$$ (16)

ON THE SURFACE THAT IS NORMAL TO A ALONG THE X-AXIS THE 250lb SHEAR FORCE IN THE Y-DIRECTION DOES NOT CONTRIBUTE TO THE STATE OF STRESS AT a' OR a". IT HAS TO BE ZERO AT THESE LOCATIONS BECAUSE THE OUTER SURFACE OF THE ROD IS FREE OF TRACTION / SHEAR LOADING.

**Figure A-15-9**

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

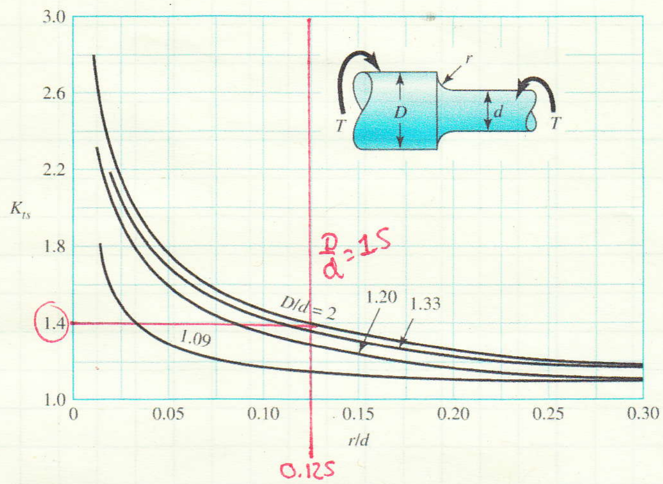



(d)

THE STRESS CONCENTRATION AT THE BOTTOM OF THE FILLET ALSO HAS TO BE TAKEN INTO ACCOUNT WHEN CALCULATING THE TORSIONAL STRESS. FROM SHIGLEY 10TH, FIGURE A-15-8:

**Figure A-15-8**

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



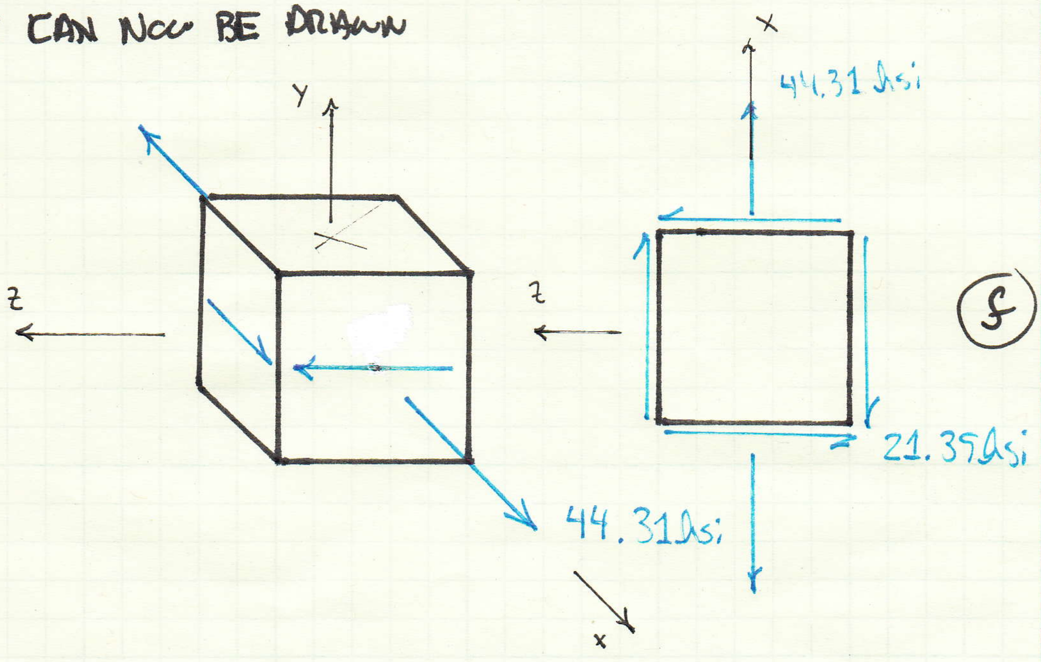

(c)

$$\frac{r}{d} = \frac{1/8\,in}{1\,in} = 0.125; \quad \frac{D}{d} = \frac{1\tfrac{1}{2}\,in}{1\,in} = 1.5 \Rightarrow K_s = 1.4$$

THE SHEAR STRESS DUE TO TORSION CAN NOW BE CALCULATED

$$\tau_{xr} = K_s \cdot \frac{T \cdot r}{J} = 1.4\,\frac{(-3000\,in \cdot lb)(0.5\,in)}{\pi(1\,in)^4/32} = 21.39(10^3)\,\frac{lb}{in^2} = 21.39\,ksi \quad (17)$$

THE STRESS CUBE CAN NOW BE DRAWN

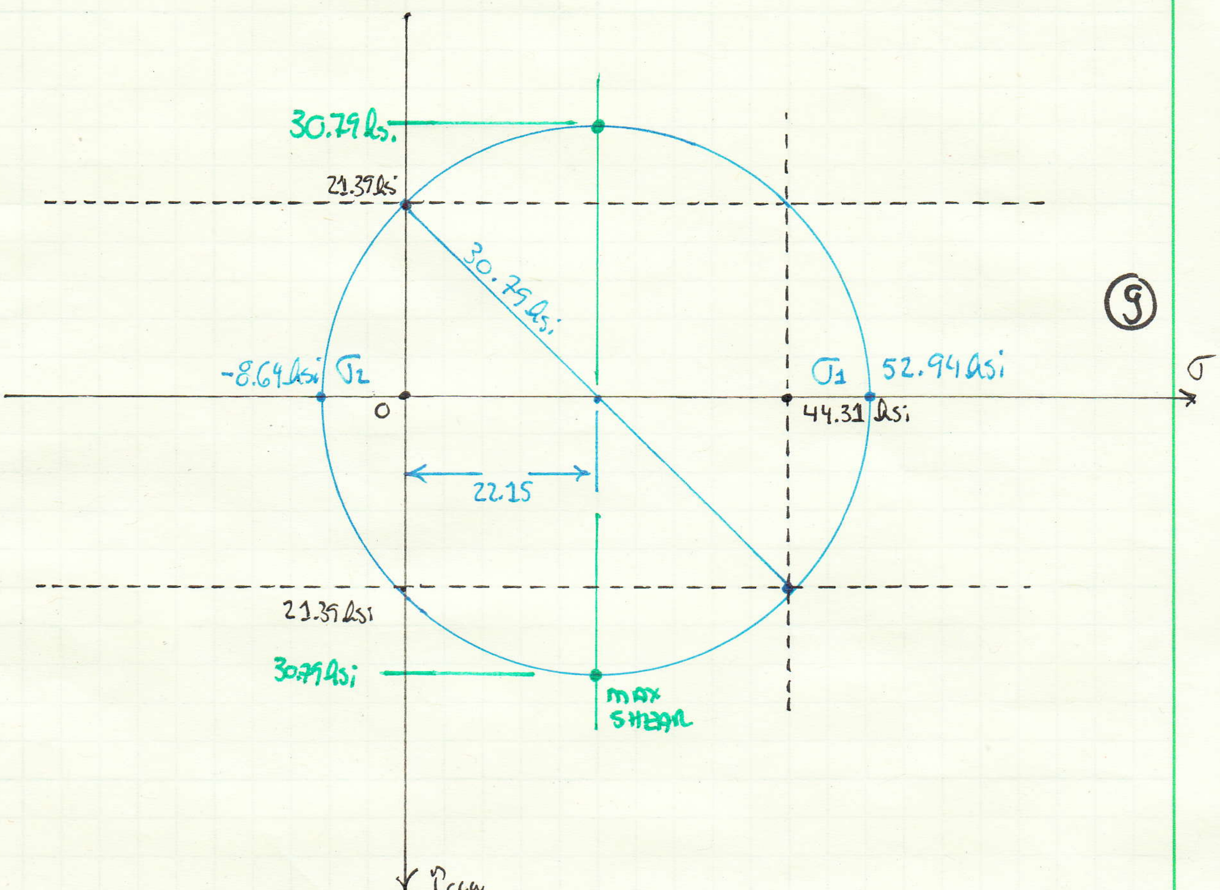



(f)

THE PRINCIPAL STRESSES AND ~~[illegible]~~ MAXIMUM SHEAR STRESS CAN BE DETERMINED USING MOHR'S CIRCLE

THE RADIUS OF MOHR'S CIRCLE FOR THIS STATE OF STRESS

$$r = \sqrt{(21.39\,ksi)^2 + (22.15\,ksi)^2} = 30.79\,ksi$$




(9)

FROM THE MOHR'S CIRCLE REPRESENTATION OF THIS STATE OF STRESS SHOWN IN (9), THE PRINCIPAL STRESS ARE

$$\sigma_1 = \underline{\underline{52.94\,ksi}} \quad (17)$$

$$\sigma_2 = \underline{\underline{-8.64\,ksi}} \quad (18)$$

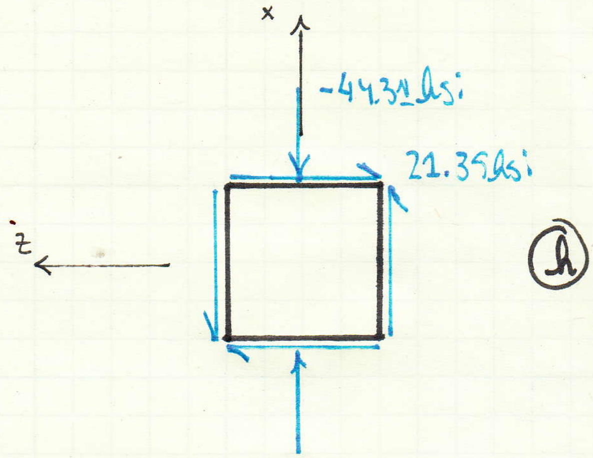
AND THE MAXIMUM SHEAR STRESS IS

$$\tau_{max} = \underline{\underline{30.79\,ksi}} \quad (19)$$

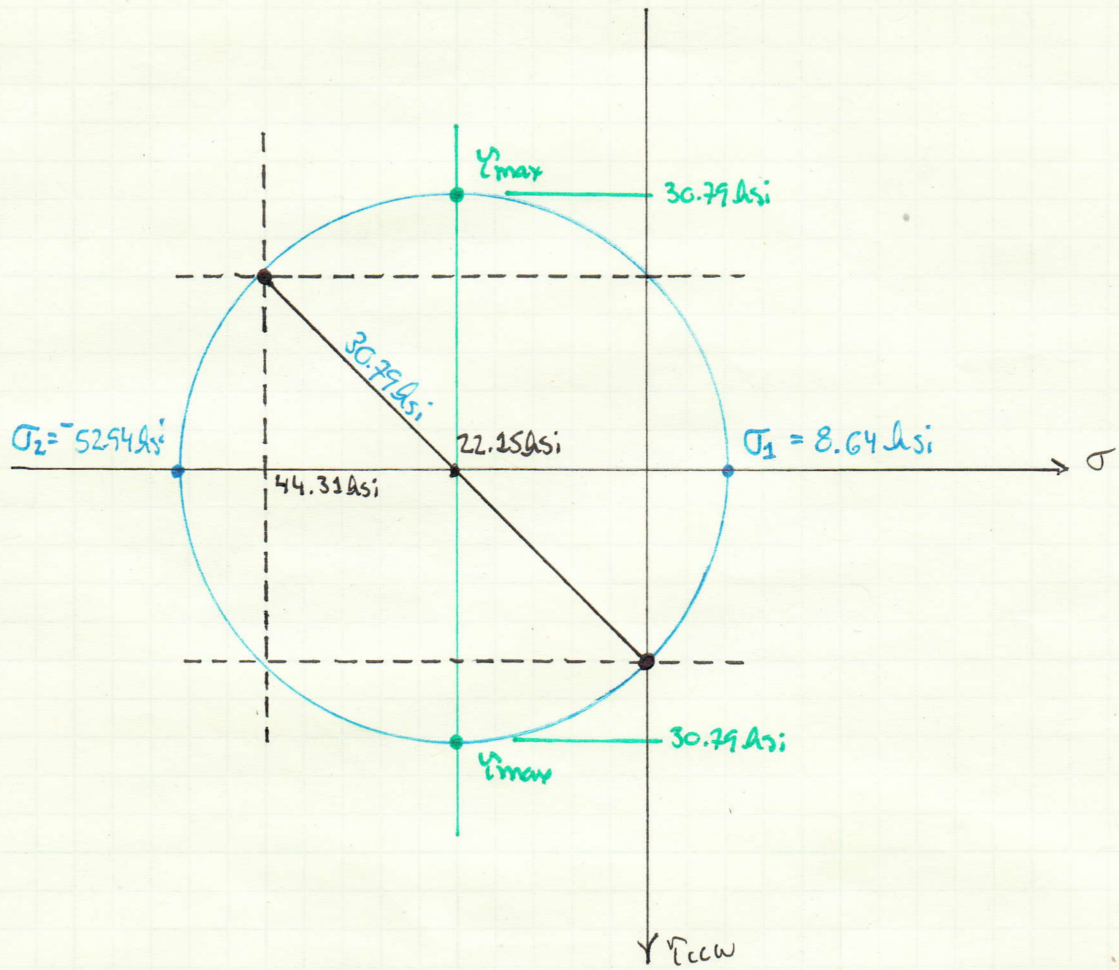
## LOCATION a'

$$\sigma_x = -44.31 \text{ ksi}$$

$$\tau_{xz} = -21.39 \text{ ksi}$$




THE STRESS CUBE FOR THIS LOCATION IS SHOWN IN (h). NOW MOHR'S CIRCLE CAN BE DRAWN.




## SUMMARY:

THIS PROBLEM COULD HAVE INITIALLY EXPEDITED BY NOT DETERMINING THE REACTIONS AT O AND ONLY HAVE DETERMINED THE REACTIONS AT A. THE SYMMETRY IN THE STATES OF STRESS AT a' AND a'' IS A DIRECT RESULT OF V & M BEING PERPENDICULAR TO EACH OTHER.