PROBLEM 7 | CONSTRUCT THE SHEAR FORCE, BENDING MOMENT, CURVATURE AND DEFLECTION DIAGRAMS FOR THIS BEAM. USING THE SINGULARITY FUNCTIONS DISCUSSED IN CLASS, WRITE EXPRESSIONS FOR THE SHEAR FORCE, BENDING MOMENT, CURVATURE, AND DEFLECTION OF THE BEAM.

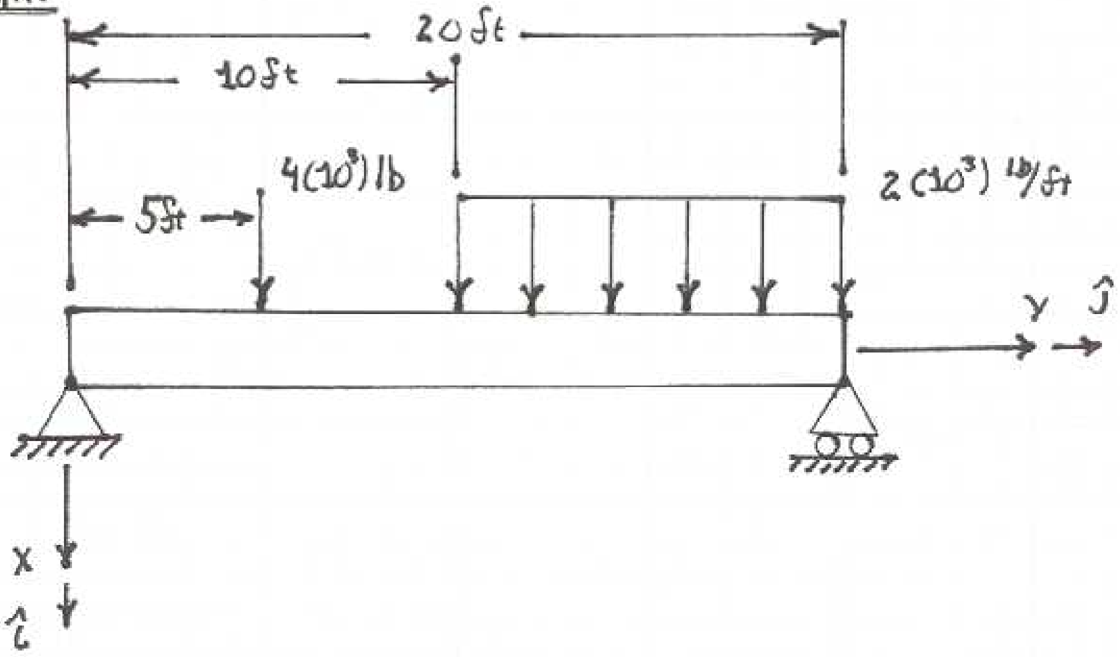
GIVEN:

CONSTRAINT:
1. 20 ft BEAM THAT IS PINNED ON ON END AND SUPPORTED BY ROLLERS ON THE OTHER
2. $2(10^3)$ lb/ft DISTRIBUTED LOAD OVER HALF THE BEAM
3. $4(10^3)$ lb LOAD APPLIED AT A QUATER SPAN ON THE SIDE WITHOUT THE DISTRIBUTED LOAD.

ASSUMPTIONS:
1. THE BEAM RESPONDS IN A LINEAR-ELASTIC MANNER
2. THE DEFORMATION IN THE BEAM IS CONSIDERED SMALL
3. STRAINS IN THE BEAM ARE SMALL
4. PINS IN JOINTS ARE FRICTIONLESS
5. ROLLER JOINT IS FRICTIONLESS

FIND:
1. DRAW SHEAR, BENDING MOMENT, CURVATURE, AND DEFLECTION DIAGRAMS.
2. WRITE EXPRESSIONS FOR THE SHEAR, BENDING MOMENT, CURVATURE AND DEFLECTION DIAGRAMS.

DIAGRAM:



SOLUTION:

STARTING WITH THE CONSTRUCTION OF THE DIAGRAMS.

$$\Sigma F_x = 0 = A_x + B_x + 4(10^3)\,lb + 20(10^3)\,lb$$

$$0 = A_x + B_x + 24(10^3)\,lb \quad (1)$$

$$\Sigma M_{z/@A} = 0 = -(5ft)\cdot(4\times10^3 lb) - 15ft\cdot 20\times10^3 lb + 20ft\cdot B_x$$

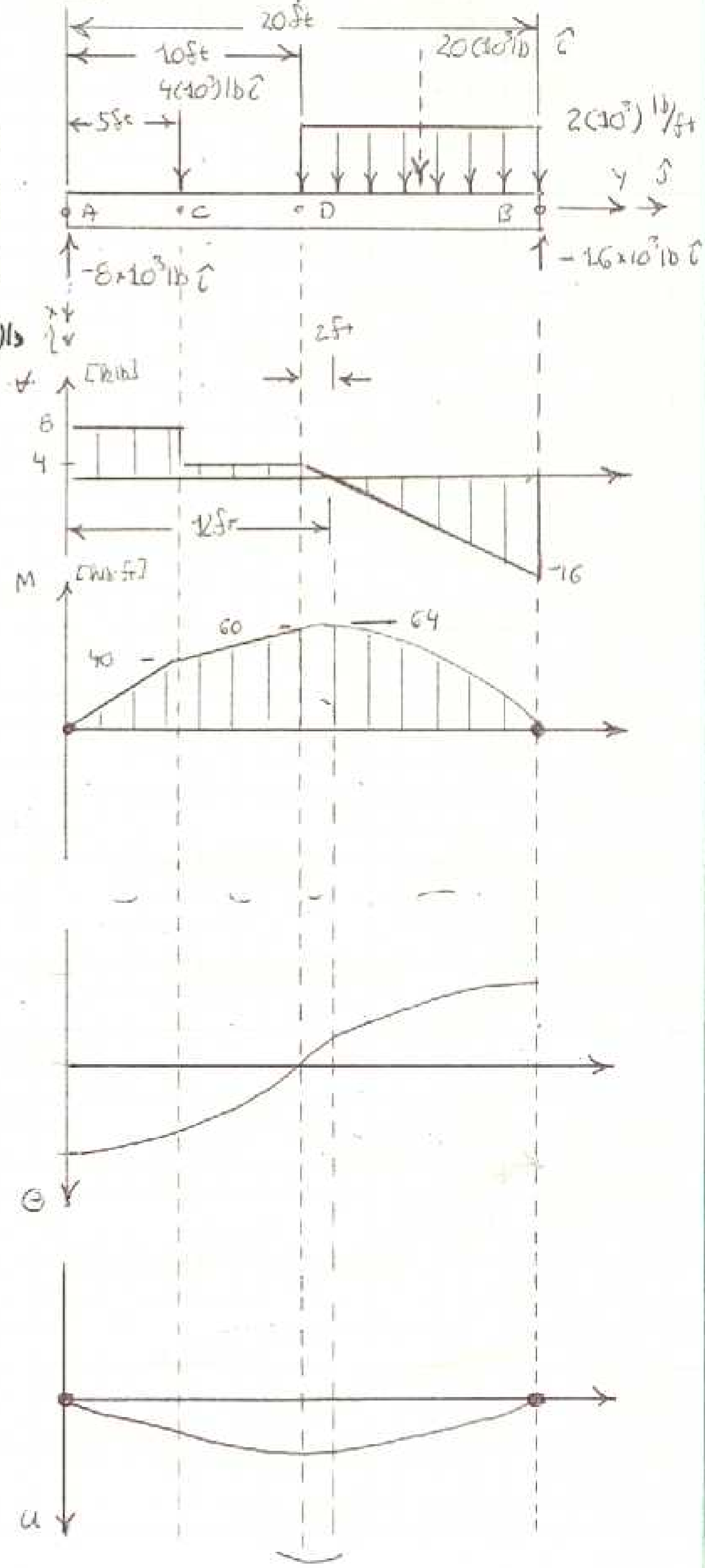
$$B_x = \frac{(5ft)\cdot(4\times10^3 lb) + (15ft)\cdot(20\times10^3)\,lb}{20ft}$$

$$\underline{B_x = -16(10^3)\,lb}$$

FROM (1)

$$A_x = -(-16\times10^3 lb) - 24\times10^3 lb$$

$$= \underline{-8\times10^3 lb}$$



STARTING WITH THE CLOSED FORM SOLUTION

$$q(y) = -8(10^3)\,\text{lb}\,\langle y-0\rangle_{-1} + 4(10^3)\,\text{lb}\,\langle y-5\text{ft}\rangle_{-1} + 2(10^3)\,\tfrac{\text{lb}}{\text{ft}}\cdot\langle y-10\text{ft}\rangle^{0} - 16(10^3)\,\text{lb}\,\langle y-20\text{ft}\rangle_{-1}$$

$$V(y) = \int -q(y)\,dy = 8(10^3)\,\text{lb}\,\langle y-0\rangle^{0} - 4(10^3)\,\text{lb}\,\langle y-5\text{ft}\rangle^{0} - 2(10^3)\,\tfrac{\text{lb}}{\text{ft}}\,\langle y-10\text{ft}\rangle^{1} + 16(10^3)\,\text{lb}\,\langle y-20\text{ft}\rangle^{0}$$

$$M(y) = \int V(y)\,dy = 8(10^3)\,\text{lb}\,\langle y-0\rangle^{1} - 4(10^3)\,\text{lb}\,\langle y-5\text{ft}\rangle^{1} - 10(10^3)\,\tfrac{\text{lb}}{\text{ft}}\,\langle y-10\text{ft}\rangle^{2} + 16(10^3)\,\text{lb}\,\langle y-20\text{ft}\rangle^{1}$$

$$\theta(y) = -\frac{1}{EI}\int M(y)\,dy = \frac{1}{EI}\Big[-4(10^3)\,\text{lb}\,\langle y-0\rangle^{2} + 2(10^3)\,\text{lb}\,\langle y-5\text{ft}\rangle^{2} + \tfrac{10}{3}(10^3)\,\tfrac{\text{lb}}{\text{ft}}\,\langle y-10\text{ft}\rangle^{3} - 8(10^3)\,\text{lb}\,\langle y-20\text{ft}\rangle^{2} + C_1\Big]$$

$$u(y) = \int \theta\,dy = \frac{1}{EI}\Big[-\tfrac{4}{3}(10^3)\,\text{lb}\,\langle y-0\rangle^{3} + \tfrac{2}{3}(10^3)\,\text{lb}\,\langle y-5\text{ft}\rangle^{3} + \tfrac{10}{12}(10^3)\,\tfrac{\text{lb}}{\text{ft}}\,\langle y-10\text{ft}\rangle^{4} - \tfrac{8}{3}(10^3)\,\text{lb}\,\langle y-20\text{ft}\rangle^{3} + C_1\cdot y + C_2\Big]$$

FROM THE BOUNDARY CONDITION $u(0)=0$

$$u(0) = 0 = \frac{1}{EI}\Big[-\tfrac{4}{3}(10^3)\,\text{lb}\cdot(0) + C_1(0) + C_2\Big] \Rightarrow C_2 = 0$$

$$u(y) = \frac{1}{EI}\Big[-\tfrac{4}{3}(10^3)\,\text{lb}\,\langle y-0\rangle^{3} + \tfrac{2}{3}(10^3)\,\text{lb}\,\langle y-5\text{ft}\rangle^{3} + \tfrac{10}{12}(10^3)\,\tfrac{\text{lb}}{\text{ft}}\,\langle y-10\text{ft}\rangle^{4} - \tfrac{8}{3}(10^3)\,\text{lb}\,\langle y-20\text{ft}\rangle^{3} + C_1\cdot y\Big]$$

NOW CONSIDERING THE SECOND BOUNDARY CONDITION

$$u(20\text{ft}) = 0 = \frac{1}{EI}\Big[-\tfrac{4}{3}(10^3)\,\text{lb}\cdot(20\text{ft})^{3} + \tfrac{2}{3}(10^3)\,\text{lb}\,(20\text{ft}-5\text{ft})^{3} + \tfrac{10}{12}(10^3)\,\tfrac{\text{lb}}{\text{ft}}\cdot(20\text{ft}-10\text{ft})^{4} - \tfrac{8}{3}(10^3)\,\text{lb}\,(20\text{ft}-20\text{ft})^{3} + C_1\cdot 20\text{ft}\Big]$$

$$C_1 = \frac{\frac{4}{3}(10^3)\,lb\cdot(20ft)^3 - \frac{2}{3}(10^3)\,lb\cdot(20ft-5ft)^3 - \frac{10}{12}(10^3)\frac{lb}{ft}\cdot(20ft-10ft)^4 + \frac{8}{3}(10^3)\,lb\cdot(20ft-20ft)^3}{20ft}$$

$$= 4.167(10^3)\,lb\cdot ft^3$$

NOW THE COMPLETE EXPRESSION FOR THE CURVATURE AND DEFLECTION CAN BE WRITTEN

$$\theta(y) = \frac{1}{EI}\Big[-4(10^3)\,lb\cdot\langle y-0\rangle^2 + 2(10^3)\,lb\cdot\langle y-5ft\rangle^2 + \frac{10}{3}(10^3)\frac{lb}{ft}\langle y-10ft\rangle^3$$
$$- 8(10^3)\,lb\,\langle y-20ft\rangle^2 + 4.167(10^3)\,lb\cdot ft^3\cdot y\Big]$$

$$u(y) = \frac{1}{EI}\Big[-\frac{4}{3}(10^3)\,lb\langle y-0\rangle^3 + \frac{2}{3}(10^3)\,lb\langle y-5ft\rangle^3 + \frac{10}{12}(10^3)\frac{lb}{ft}\langle y-10ft\rangle^4$$
$$-\frac{8}{3}(10^3)\,lb\langle y-20ft\rangle^3 + 4.167(10^3)\,lb\cdot ft^3\cdot y$$