NAME: SOLUTION

PROBLEM 1: Given the state of stress

$$[\sigma]=\begin{bmatrix}40 & 40 & 30\\ 40 & 20 & 0\\ 30 & 0 & 20\end{bmatrix} MPa$$

a. Determine the stress invariants.

Using MatLab

```
STRE=[40 40 30; 40 20 0; 30 0 20]

STRE =

   40   40   30
   40   20    0
   30    0   20


poly(STRE)
```

ans = 1.0e+004 *(σ^3 -0.0080 σ^2 -0.0500 σ^3 + 3.4000)

$\boxed{\begin{array}{l}\mathbf{I_1=80MPa}\\ \mathbf{I_2=-500MPa^2}\\ \mathbf{I_3=-34{,}000MPa^3}\end{array}}$

b. Determine the principal stresses.

Using MatLab

```
>> [V P]=eig(STRE)

V =

  -0.6340    0.0000    0.7733
   0.6187   -0.6000    0.5072
   0.4640    0.8000    0.3804

P =

  -20.9902         0         0
         0   20.0000         0
         0         0   80.9902
```

$\boxed{\begin{array}{l}\mathbf{\sigma_1=-21\ MPa}\\ \mathbf{\sigma_2=20\ MPa}\\ \mathbf{\sigma_3=81\ MPa}\end{array}}$

c. **Determine the direction cosines to each of the principal stresses and calculate $\theta_{x'x}$, $\theta_{x'y}$, $\theta_{x'z}$, $\theta_{y'x}$, $\theta_{y'y}$, $\theta_{y'z}$, $\theta_{z'x}$, $\theta_{z'y}$, and $\theta_{z'z}$.**

Using MatLab

```
>> acos(V)*180/pi

ans =

  129.3450   90.0000   39.3450
   51.7807  126.8699   59.5231
   62.3541   36.8699   67.6420
```

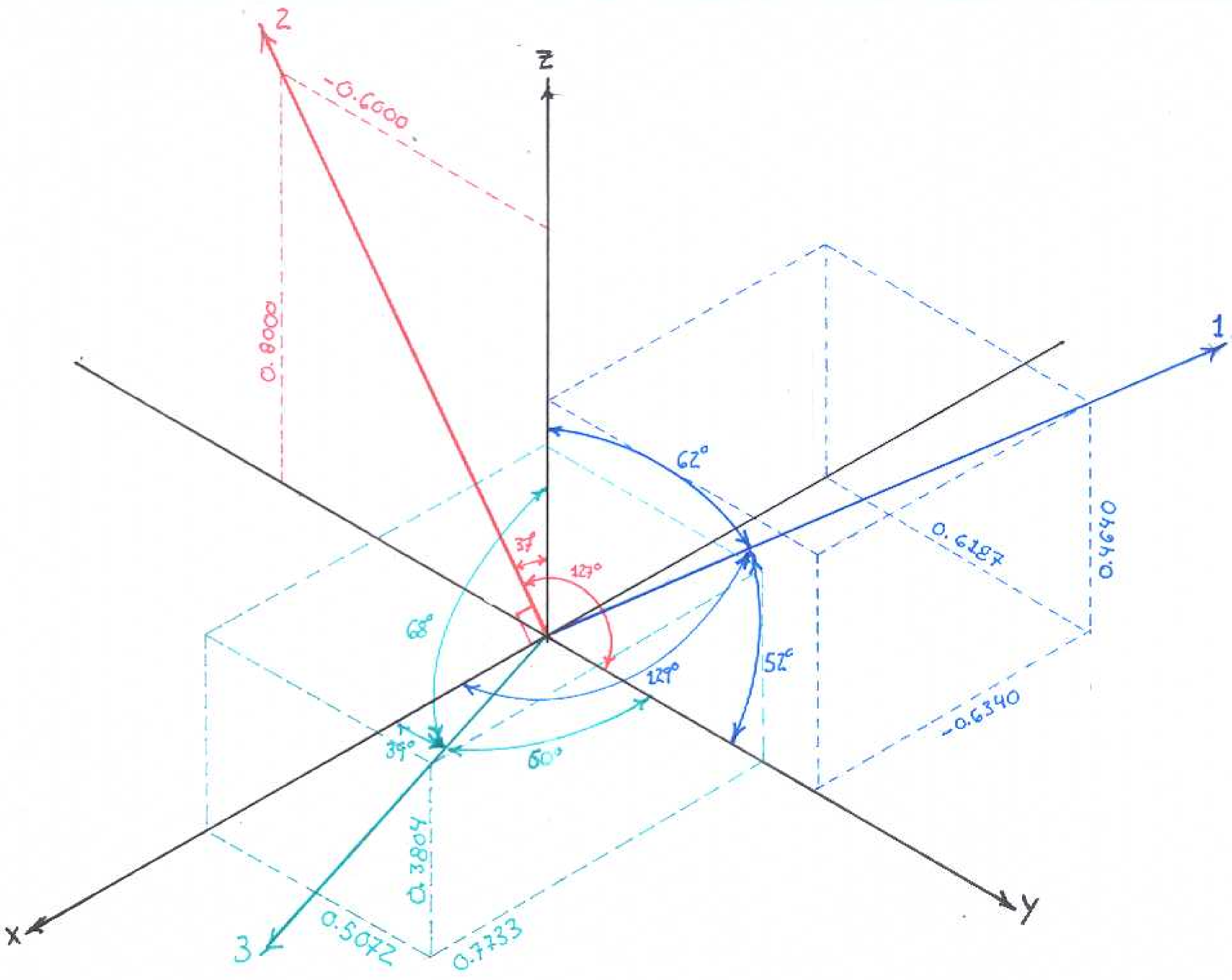
$\theta_{x'x}$=129.3	**$\theta_{x'y}$=51.8**	**$\theta_{x'z}$=62.4**
$\theta_{y'x}$=90.0	**$\theta_{y'y}$=126.9**	**$\theta_{y'z}$=36.9**
$\theta_{z'x}$=39.3	**$\theta_{z'y}$=59.5**	**$\theta_{z'z}$=67.6**

d. **Determine the transformation matrix from the original state of stress to the principal state of stress and prove that it is the transformation matrix by using it to transform the original state of stress.**

```
>> T=V'

T =
```

-0.6340	**0.6187**	**0.4640**
0.0000	**-0.6000**	**0.8000**
0.7733	**0.5072**	**0.3804**

```
>> T*STRE*T'

ans =

  -20.9902   -0.0000   -0.0000
   -0.0000   20.0000    0.0000
    0.0000    0.0000   80.9902

>> P

P =

  -20.9902         0         0
         0   20.0000         0
         0         0   80.9902
```



2
Z
-0.6000
0.8000
1
62°
37°
127°
0.6187
0.4640
68°
127°
52°
-0.6340
39°
60°
0.3804
X
3
0.5072
0.7733
Y

e. Determine the state of stress defined by rotating x,y plane in the original state of stress through an angle of 30° clockwise about the z axis.

Using MatLab

```
>> T2=[0.866 -.5 0; .5 0.866 0; 0 0 1]

T2 =
    0.8660   -0.5000         0
    0.5000    0.8660         0
         0         0    1.0000

>> T2*STRE*T2'

ans =
    0.3582   28.6582   25.9800
   28.6582   59.6391   15.0000
   25.9800   15.0000   20.0000
```

$\sigma_{x'}$= 0.4 MPa	**$\tau_{x'y'}$= 28.7 MPa**	**$\tau_{x'z'}$= 26.0 MPa**
$\tau_{x'y'}$= 28.7 MPa	**$\sigma_{y'}$= 60.0 MPa**	**$\tau_{y'z'}$= 15.0 MPa**
$\tau_{x'z'}$=26.0 MPa	**$\tau_{y'z'}$= 15.0 MPa**	**$\sigma_{z'}$= 20.0 MPa**

f. Determine the maximum shear stress for this state of stress.

Using MatLab

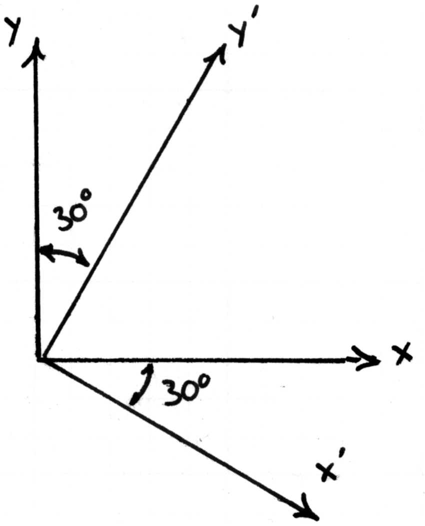
```
>> (P(1,1)-P(3,3))/2

ans =  -50.9902
```

τ_{max}=51 MPa

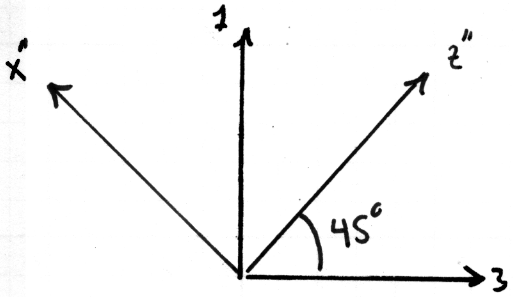
TRANSFORMATION MATRIX FOR A 30° CLOCKWISE ROTATION ABOUT THE Z-AXIS.

$$[T] = \begin{bmatrix} \cos\theta_{x'x} & \cos\theta_{x'y} & \cos\theta_{x'z} \\ \cos\theta_{y'x} & \cos\theta_{y'y} & \cos\theta_{y'z} \\ \cos\theta_{z'x} & \cos\theta_{z'y} & \cos\theta_{z'z} \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



TRANSFORMING THE PRINCIPAL STATE OF STRESS 45° ABOUT THE 2-AXIS



$$[T] = \begin{bmatrix} \cos\theta_{x''1} & \cos\theta_{x''2} & \cos\theta_{x''3} \\ \cos\theta_{y''1} & \cos\theta_{y''2} & \cos\theta_{y''3} \\ \cos\theta_{z''1} & \cos\theta_{z''2} & \cos\theta_{z''3} \end{bmatrix} = \begin{bmatrix} 0.7071 & 0 & -0.7071 \\ 0 & 1 & 0 \\ 0.7071 & 0 & 0.7071 \end{bmatrix}$$

g. Determine the transformation matrix that needs to be used to transform the original state of stress to a state of stress that contains the maximum shear stress on two of the faces and a principal state of stress on the third.

To get to the maximum shear stress state, the original stress state is transformed to the principal state of stress and then the principal state of stress is rotated 45° about the 2-axis. The transformation is [T3]

```
>> T3=[.7071 0 -.7071; 0 1 0; .7071 0 .7071]

T3 =
   0.7071        0   -0.7071
        0   1.0000         0
   0.7071        0    0.7071
```

The transformation matrix from the original state of stress to the state of stress where one surface is in the principal state and on the other two surfaces the maximum shear stress exits can not be calculated.

```
>> T4=T3*T
```

T4 =

-0.9951	**0.0788**	**0.0591**
0.0000	**-0.6000**	**0.8000**
0.0985	**0.7961**	**0.5971**

This result is proven by first calculating the desired state of stress through a transformation to the principal state of stress and then a second transformation about the 2-axis to the state of maximum shear.

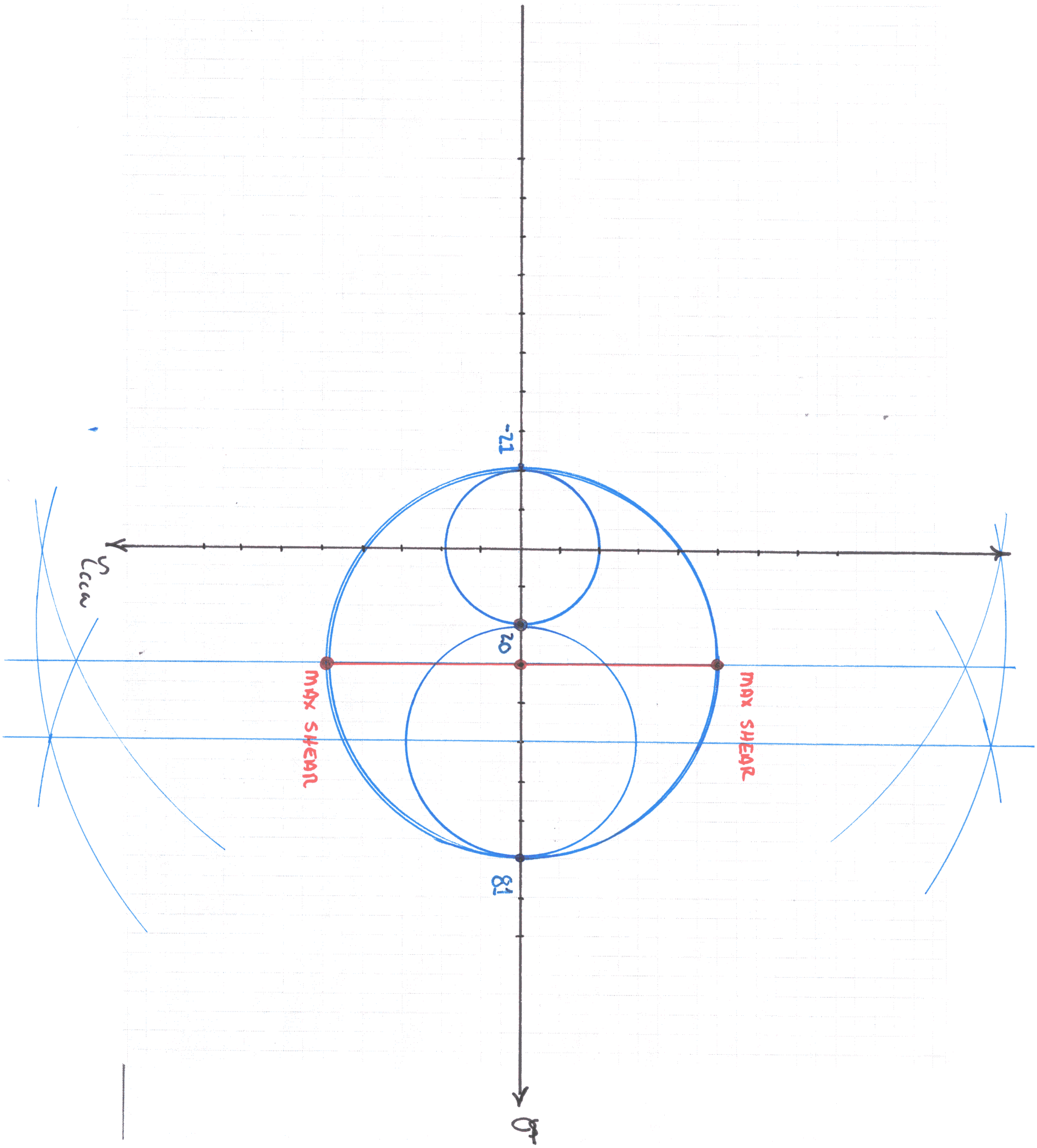
```
>> T3*T*STRE*T'*T3'

ans =
   29.9994   -0.0000  -50.9892
   -0.0000   20.0000    0.0000
  -50.9892    0.0000   29.9994
```

This is compared to the single calculation that was calculated above.

```
>> T4*STRE*T4'

ans =
   29.9994   -0.0000  -50.9892
   -0.0000   20.0000    0.0000
  -50.9892    0.0000   29.9994
```

The two solutions match; therefore, [T4] is the desired transformation matrix

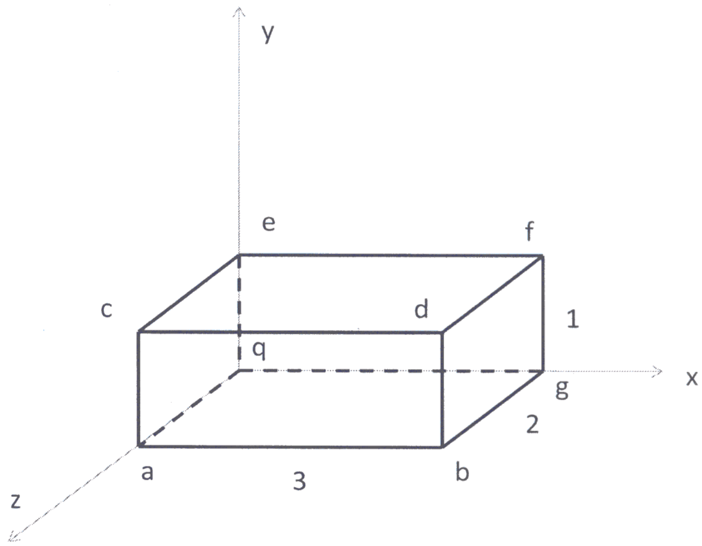
h. Draw the Mohr’s circle that defines the bounds for this state of stress.

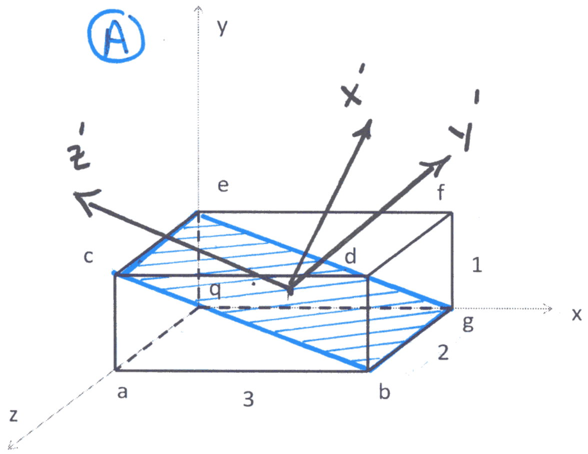

-21
20
81
MAX SHEAR
MAX SHEAR
τ_{ccw}
σ

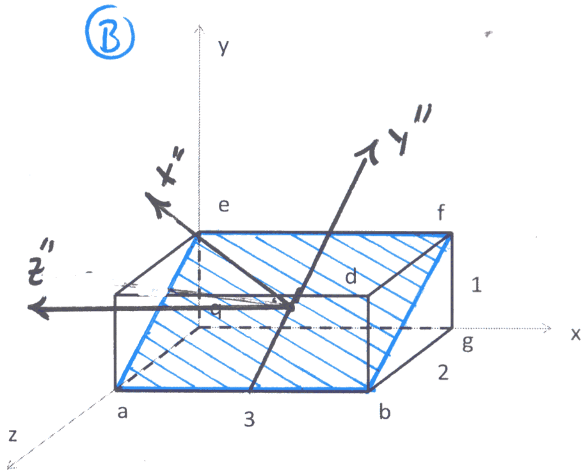
PROBLEM 3: Determine the transformation matrix for rotating a state of stress on the cube shown to the surface parallel to the following surfaces:

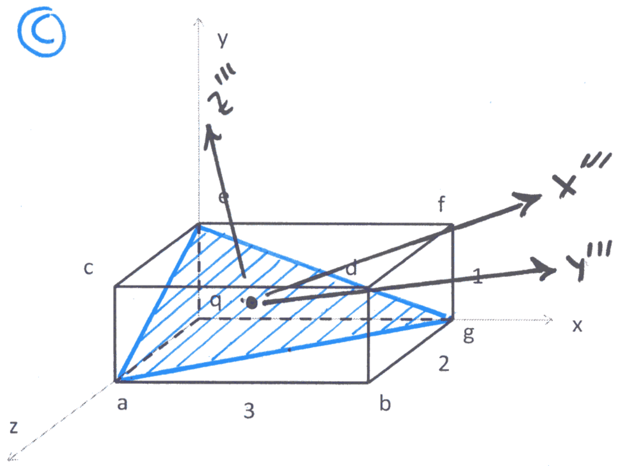
a. CEBG
b. ABEF
c. AEG

Be sure to describe your justification for each of the coordinates used. Draw the transformed coordinate system with respect to the cube.









The transformation matrix for CEBG

```
>> RAbg=[0 0 -2]

RAbg =
     0     0    -2
>> RAbc=[-3 1 0]
RAbc =
    -3     1     0

>> N=cross(RAbg,RAbc)
N =
     2     6     0

>> Uxp=N/norm(N)
Uxp =
    0.3162    0.9487         0

>> Uzp=RAbc/norm(RAbc)
Uzp =
   -0.9487    0.3162         0

>> Uyp=cross(Uzp,Uxp)
Uyp =
         0         0   -1.0000

>> Tp=[Uxp; Uyp; Uzp]
```

Tp =

0.3162	**0.9487**	**0**
0	**0**	**-1.0000**
-0.9487	**0.3162**	**0**

The transformation matrix for ABEF

```
>> RBbf=[0 1 -2]
RBbf =
    0     1    -2

>> RBba=[-3 0 0]
RBba =
   -3     0     0

>> Npp=cross(RBbf,RBba)
Npp =
    0     6     3

>> Uxpp=Npp/norm(Npp)
Uxpp =
         0    0.8944    0.4472

>> Uypp=RBbf/norm(RBbf)
Uypp =
         0    0.4472   -0.8944

>> Uzpp=RBba/norm(RBba)
Uzpp =
   -1     0     0

>> Tpp=[Uxpp; Uypp; Uzpp]
Tpp =
         0    0.8944    0.4472
         0    0.4472   -0.8944
   -1.0000         0         0
```

The transformation matrix for AEG

```
>> RCag=[3 0 -2]
RCag =
     3     0    -2

>> RCae=[0 1 -2]
RCae =
     0     1    -2

>> Nppp=cross(RCag,RCae)
Nppp =
     2     6     3

>> Uxppp=Nppp/norm(Nppp)
Uxppp =
    0.2857    0.8571    0.4286

>> Uyppp=RCag/norm(RCag)
Uyppp =
    0.8321         0   -0.5547

>> Uzppp=cross(Uxppp,Uyppp)
Uzppp =
   -0.4755    0.5151   -0.7132

>> Tppp=[Uxppp; Uyppp; Uzppp]
```

Tppp =

0.2857	**0.8571**	**0.4286**
0.8321	**0**	**-0.5547**
-0.4755	**0.5151**	**-0.7132**

PROBLEM 3: For the state of stress in Problem 1, determine the state of strain given E=70GPa and ν=0.3.

```
>> C=[(1/70e9) (-.3/70e9) (-.3/70e9) 0 0 0;
(-.3/70e9) (1/70e9) (-.3/70e9) 0 0 0;
(-.3/70e9) (-.3/70e9) (1/70e9) 0 0 0;
0 0 0 (2*(1+.3)/70e9) 0 0;
0 0 0 0 (2*(1+.3)/70e9) 0;
0 0 0 0 0 (2*(1+.3)/70e9)]

C =
  1.0e-010 *
    0.1429   -0.0429   -0.0429        0        0        0
   -0.0429    0.1429   -0.0429        0        0        0
   -0.0429   -0.0429    0.1429        0        0        0
         0         0         0   0.3714        0        0
         0         0         0        0   0.3714        0
         0         0         0        0        0   0.3714

>> S=inv(C)
S =
  1.0e+010 *
    9.4231    4.0385    4.0385        0        0        0
    4.0385    9.4231    4.0385        0        0        0
    4.0385    4.0385    9.4231        0        0        0
         0         0         0   2.6923        0        0
         0         0         0        0   2.6923        0
         0         0         0        0        0   2.6923

>> VSTRE=[40e6 20e6 20e6 0 30e6 40e6]'
VSTRE =
    40000000
    20000000
    20000000
           0
    30000000
    40000000

>> VSTRA=C*VSTRE
VSTRA =
    0.0004000
    0.0000286
    0.0000286
         0
    0.001143
    0.0014857
```

ε_x= 400 με
ε_y= 29 με
ε_z=29 με
γ_{zy}=0
γ_{zx}=1143 με
γ_{xy}=1486 με