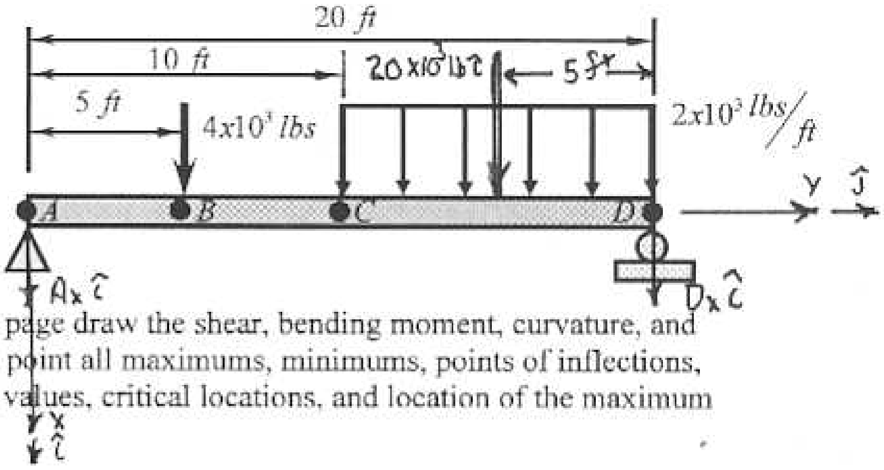
NAME: SOLUTION

PROBLEM 1: Answer the following questions for the beam shown below.



1a. Using the diagram on the next page draw the shear, bending moment, curvature, and deflection diagrams. Make sure to point all maximums, minimums, points of inflections, boundary conditions, critical point values, critical locations, and location of the maximum deflection.

$$\sum F_x = 0 = A_x + D_x + 4\times10^3 lb + 20\times10^3 lb = A_x + D_x + 24\times10^3 lb \quad (1)$$

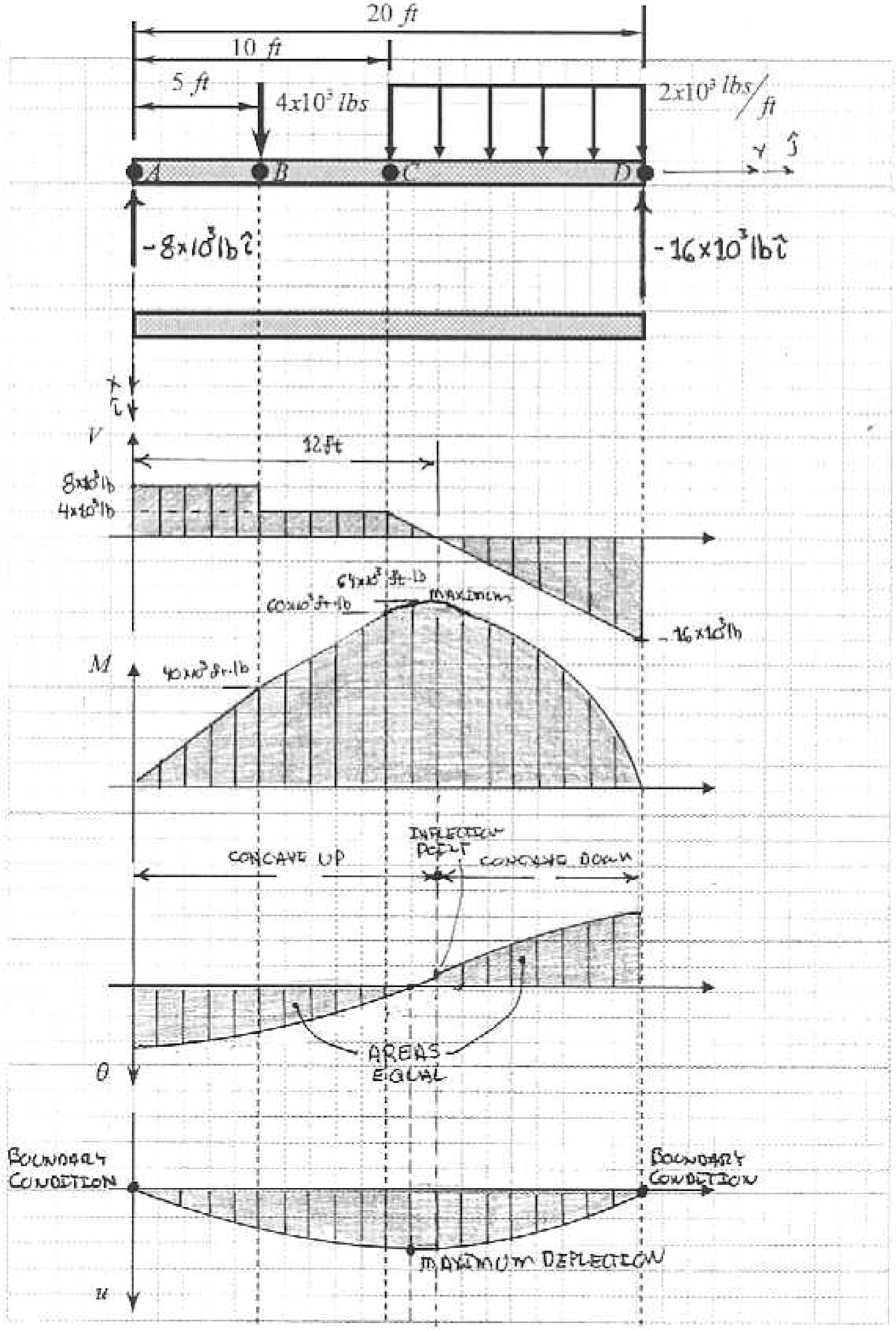
$$\sum M_{z@A} = 0 = -(5ft)\cdot(4\times10^3 lb) - (15ft)(20\times10^3 lb) + 20ft\cdot D_x$$

$$D_x = \frac{(5ft)(4\times10^3 lb) + (15ft)(20\times10^3 lb)}{20ft} = \underline{\underline{-16\times10^3 lb}}$$

SUBSTITUTING THIS RESULT INTO (1)

$$A_x = -(24\times10^3 lb) - (-16\times10^3 lb) = \underline{\underline{-8\times10^3 lb}}$$

1b.

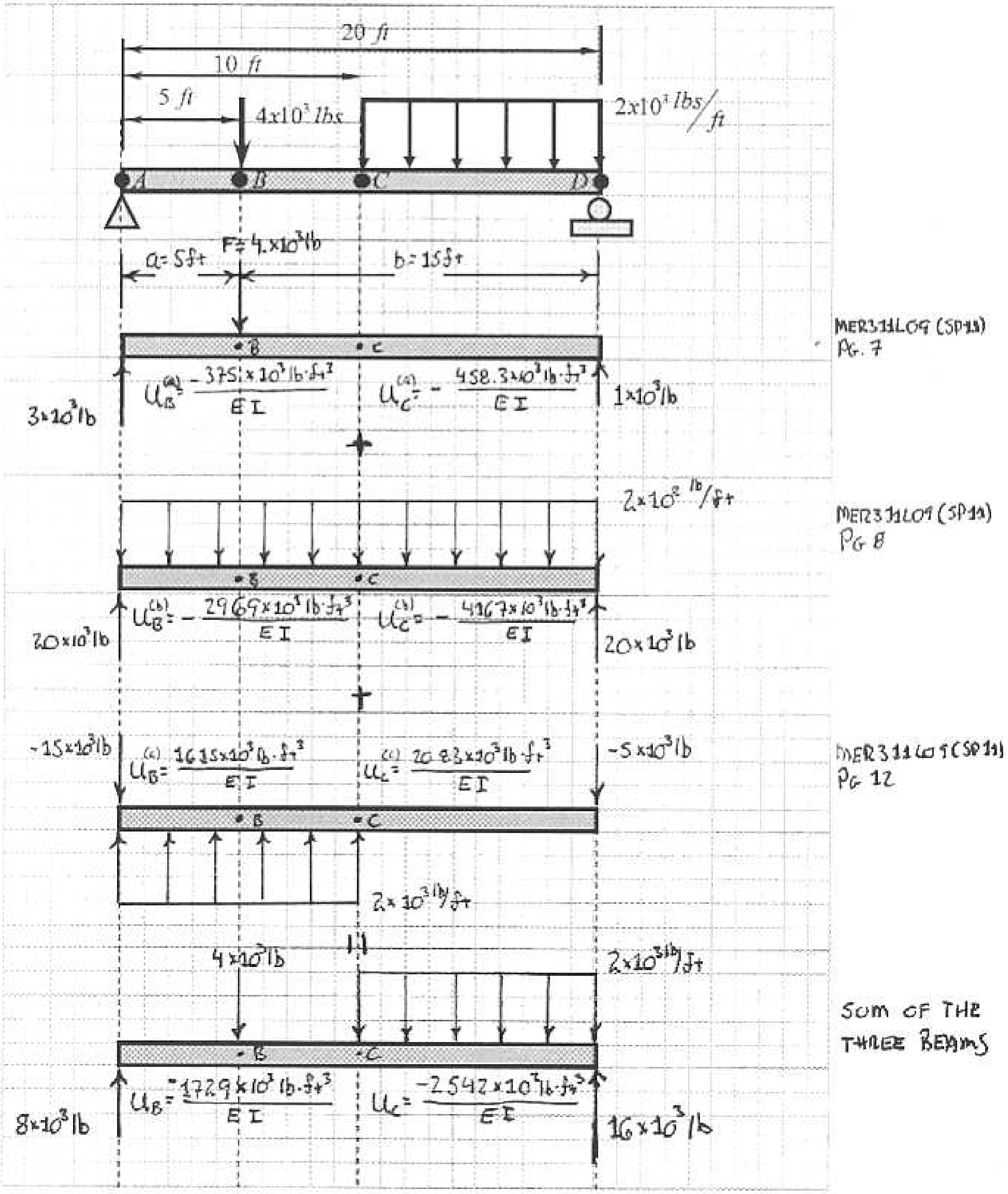

20 ft
10 ft
5 ft
4x10³ lbs
2x10³ lbs/ft
A
B
C
D
-8x10³ lb î
-16x10³ lb î
V
12 ft
8x10³ lb
4x10³ lb
-16x10³ lb
64x10³ ft-lb
60x10³ ft-lb
MAXIMUM
M
40x10³ ft-lb
INFLECTION POINT
CONCAVE UP
CONCAVE DOWN
AREAS EQUAL
θ
BOUNDARY CONDITION
BOUNDARY CONDITION
MAXIMUM DEFLECTION
u

Using the beam bending tables provided determine the reactions at A and D, and the deflections at B and C. Identify and illustrate the beams used in the solution on the figure provided below.

(a)

(b)

(c)



Beam (a) - MER311LO9 (SP11), Pg. 7

$$R_A = \frac{4\times10^3\,lb \cdot 15\,ft}{20\,ft} = \underline{\underline{3\times10^3\,lb}} \qquad R_D = \frac{4\times10^3\,lb\cdot 5\,ft}{20\,ft} = \underline{\underline{1\times10^3\,lb}}$$

$$u_B = \frac{4\times10^3\,lb\cdot15\,ft\cdot5\,ft}{6\cdot EI\cdot 20\,ft}\left[(5\,ft)^2 + (15\,ft)^2 - (20\,ft)^2\right] = \underline{\underline{-\frac{375\times10^3\,lb\cdot ft^3}{EI}}}$$

$$u = \frac{4\times10^3\,lb\cdot 5\,ft\,(20\,ft - 10\,ft)}{6\cdot EI\ 20\,ft}\left[(10\,ft)^2 + (5\,ft)^2 - 2\cdot20\,ft\cdot10\,ft\right] = \underline{\underline{-\frac{458.3\times10^3\,lb\cdot ft^3}{EI}}}$$

Beam (b) - MER311LO9 (SP11) Pg 8

$$R_A = R_B = \frac{2\times10^3\,lbs/ft\cdot20\,ft}{2}\ \underline{\underline{20\times10^3\,lb}}$$

$$u_B = \frac{2\times10^3\,lbs/ft\cdot5\,ft}{24\,E\cdot I}\cdot\left[2\cdot20\,ft\cdot(5\,ft)^2 - (5\,ft)^3 - (20\,ft)^3\right] = \underline{\underline{-\frac{2969\times10^3\,lb\cdot ft^3}{EI}}}$$

$$u_C = \frac{2\times10^3\,lbs/ft\cdot10\,ft}{24\,EI}\left[2\cdot20\,ft\cdot(10\,ft)^2 - (10\,ft)^3 - (20\,ft)^3\right] = \underline{\underline{-\frac{4167\times10^3\,lb\cdot ft^3}{EI}}}$$

Beam (c) - MER311LO9 (SP11) Pg 12

$$R_A = -\frac{2\times10^3\,lb/ft\cdot10\,ft}{2\cdot20\,ft}(2\cdot20\,ft - 10\,ft) = \underline{\underline{-15.0\times10^3\,lb}}$$

$$R_B = -\frac{2\times10^3\,lb/ft\cdot(10\,ft)^2}{2\cdot20\,ft} = \underline{\underline{-5.0\times10^3\,lb}}$$

$$u_B = -\frac{2\times10^3\,lb/ft\cdot5\,ft}{24\,EI\cdot20\,ft}\left[2\times10\,ft\cdot(5\,ft)^2(2\cdot20\,ft - 10\,ft) - 20\,ft\cdot(5\,ft)^3 - (10\,ft)^2(2\cdot20\,ft - 10\,ft)^2\right]$$

$$= \underline{\underline{\frac{1615\times10^3\,lb\cdot ft^3}{EI}}}$$

$$u_C = -\frac{2\times10^3\,lb/ft\cdot(10\,ft)}{24\cdot EI\cdot20\,ft}\left[2\cdot10\,ft\cdot(10\,ft)^2\cdot(2\cdot20\,ft - 10\,ft) - 20\,ft\cdot(10\,ft)^3 - (10\,ft)^2\cdot(2\cdot20\,ft - 10\,ft)^2\right]$$

$$= \underline{\underline{\frac{2083\times10^3\,lb\cdot ft^3}{EI}}}$$

1c. Write a general expression for the load, shear, bending moment, curvature, and deflection of the beam. Make sure to calculate all constants. Using these equations determine the deflections at B and C.

$$q(y) = -8\times10^3\,lb\,\langle y-0\rangle_{-1} + 4\times10^3\,lb\,\langle y-5ft\rangle_{-1} + 2\times10^3\,\tfrac{lb}{ft}\,\langle y-10ft\rangle^0 - 16\times10^3\,lb\,\langle y-20ft\rangle_{-1}$$

$$V(y) = 8\times10^3\,lb\,\langle y-0\rangle^0 - 4\times10^3\,lb\,\langle y-5ft\rangle^0 - 2\times10^3\,\tfrac{lb}{ft}\,\langle y-10ft\rangle^1 + 16\times10^3\,lb\,\langle y-20ft\rangle^0$$

$$M(y) = 8\times10^3\,lb\,\langle y-0\rangle^1 - 4\times10^3\,lb\,\langle y-5ft\rangle^1 - 2\times10^3\,\tfrac{lb}{ft}\cdot\tfrac{1}{2}\langle y-10ft\rangle^2 + 16\times10^3\,lb\,\langle y-20ft\rangle^1$$

$$= 8\times10^3\,lb\,\langle y-0\rangle^1 - 4\times10^3\,lb\,\langle y-5ft\rangle^1 - 1\times10^3\,\tfrac{lb}{ft}\,\langle y-10ft\rangle^2 + 16\times10^3\,lb\,\langle y-20ft\rangle^1$$

$$\Theta = \frac{-4\times10^3\,lb\,\langle y-0\rangle^2}{EI} + \frac{2\times10^3\,lb\,\langle y-5ft\rangle^2}{EI} + \frac{\frac{1}{3}\times10^3\,\frac{lb}{ft}\,\langle y-10ft\rangle^3}{EI} - \frac{8\times10^3\,lb\,\langle y-20ft\rangle^2}{EI} + C_1$$

$$u = \frac{-\frac{4}{3}\times10^3\,lb\,\langle y-0\rangle^3}{EI} + \frac{\frac{2}{3}\times10^3\,lb\,\langle y-5ft\rangle^3}{EI} + \frac{\frac{1}{12}\times10^3\,\frac{lb}{ft}\,\langle y-10ft\rangle^4}{EI} - \frac{\frac{8}{3}\times10^3\,lb\,\langle y-20ft\rangle^3}{EI} + C_1\cdot y + C_2$$

THE CONSTANTS C_1 & C_2 ARE DETERMINED FROM THE BOUNDARY CONDITIONS
$u(0)=0$ AND $u(20ft)=0$

$$u(0) = 0 = C_2$$

$$u(20ft) = 0 = -\tfrac{4}{3}\times10^3\,lb\cdot\frac{(20ft)^3}{EI} + \tfrac{2}{3}\times10^3\,lb\cdot\frac{(15ft)^3}{EI} + \tfrac{1}{12}\times10^3\,\tfrac{lb}{ft}\,\frac{(10ft)^4}{EI} + C_1\cdot 20ft$$

$$\Rightarrow C_1 = \frac{379.2\times10^3\,lb\cdot ft^2}{EI}$$

$$\Theta(y) = \frac{-4\times10^3\,lb}{EI}\langle x-0\rangle^2 + \frac{2\times10^3\,lb}{EI}\langle x-5ft\rangle^2 + \frac{\frac{1}{3}\times10^3\,\frac{lb}{ft}}{EI}\langle x-10ft\rangle^3 - \frac{8\times10^3\,lb}{EI}\langle x-20ft\rangle^2 + \frac{379.2\times10^3\,lb\cdot ft^2}{EI}$$

$$u(y) = \frac{-\frac{4}{3}\times10^3\,lb}{EI}\langle x-0\rangle^3 + \frac{\frac{2}{3}\times10^3\,lb}{EI}\langle x-5ft\rangle^3 + \frac{\frac{1}{12}\times10^3\,\frac{lb}{ft}}{EI}\langle x-10ft\rangle^4 - \frac{\frac{8}{3}\times10^3\,lb}{EI}\langle x-20ft\rangle^3 + \frac{379.2\times10^3\,lb\cdot ft^2}{EI}\cdot y$$

$$u_B = u(5ft) = -\frac{\frac{4}{3}\times10^3\,lb}{EI}\cdot(5ft)^3 + \frac{379.2\times10^3\,lb\cdot ft^2}{EI}\cdot(5ft) = \boxed{\frac{1729\times10^3\,lb\cdot ft^3}{EI}}$$

$$u_C = u(10ft) = -\frac{\frac{4}{3}\times10^3\,lb}{EI}\cdot(10ft)^3 + \frac{\frac{2}{3}\times10^3\,lb}{EI}(5ft)^3 + \frac{379.2\times10^3\,lb\cdot ft^2}{EI}\cdot(10ft)$$
$$= \boxed{\frac{2542\times10^3\,lb\cdot ft^3}{EI}}$$

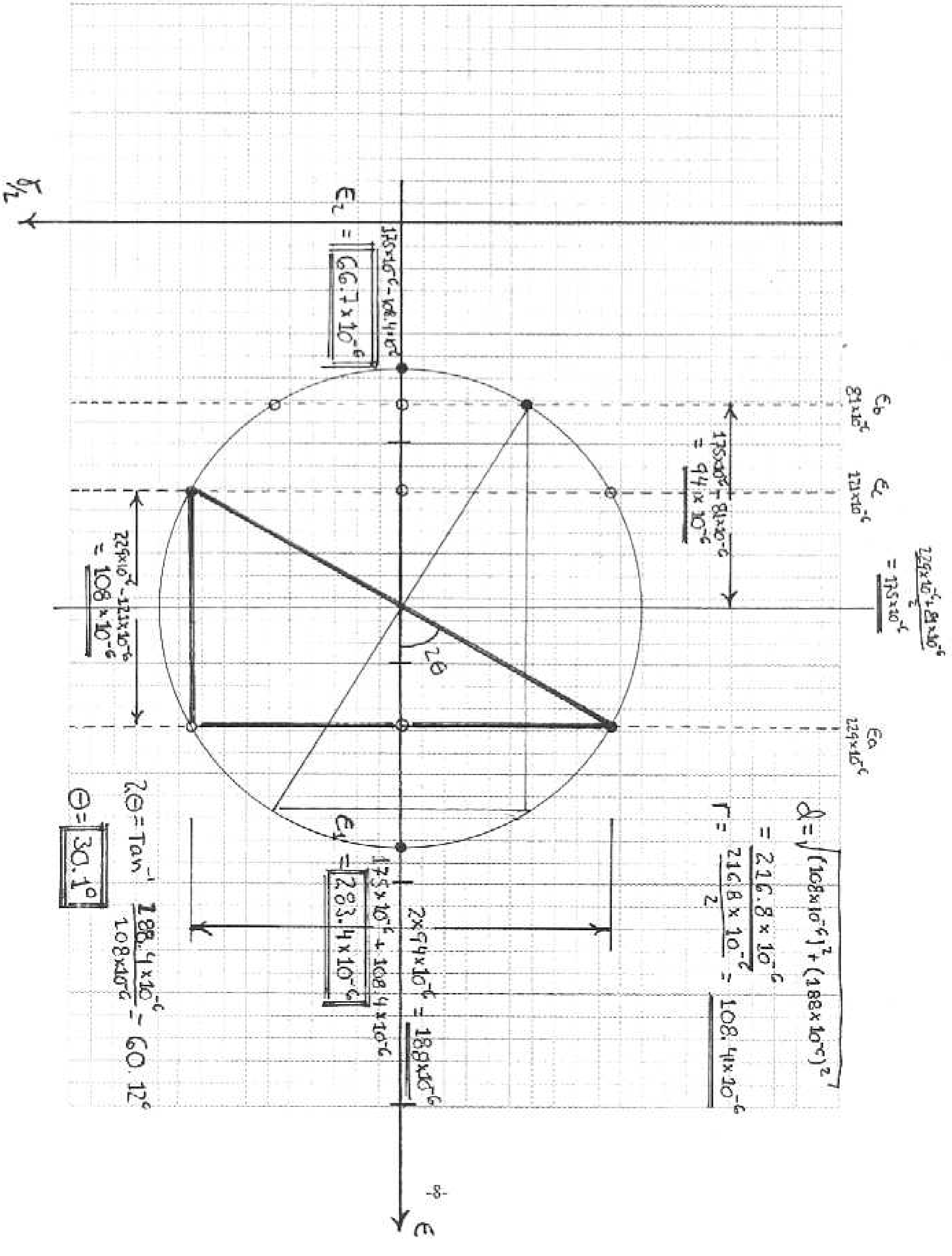
PROBLEM 2: A strain gage rosette is used on a pressure vessel that is 20 inches in diameter and has a wall thickness of 0.25 inches. The pressure vessel is made of steel ($E=10\times10^6$ psi, $\nu=0.3$). The three gage's on the rosette are positioned 45° apart and the gage was attached prior to any pressure being placed in the vessel. After pressurization the gages are read in a counter-clockwise manner (a-b-c), corrected for transverse sensitivity, and read as follows:

$\varepsilon_a=229\times10^{-6}$
$\varepsilon_b=81\times10^{-6}$
$\varepsilon_c=121\times10^{-6}$

Using the grid paper on the next page construct Mohr's circle for strain, determine the principal strains, find the angle that the gages are offset, and determine the pressure in vessel by using both the axial and circumferential stresses.

$$\sigma_1 = \frac{10\times10^6 \frac{lb}{in^2}}{1-(0.3)^2}\left[283.4\times10^{-6} + 0.3\cdot(66.7\times10^{-6})\right] = 3339\,\frac{lb}{in^2} = \frac{P\cdot 10_{in}}{0.25_{in}} \Rightarrow \boxed{P = 83.35\,\frac{lb}{in^2}}$$

$$\sigma_2 = \frac{10\times10^6 \frac{lb}{in^2}}{1-(0.3)^2}\left[(66.7\times10^{-6}) + 0.3(283.4\times10^{-6})\right] = 1667\,\frac{lb}{in^2} = \frac{P\cdot 10_{in}}{2(0.25_{in})} \Rightarrow \boxed{P = 83.36\,\frac{lb}{in^2}}$$

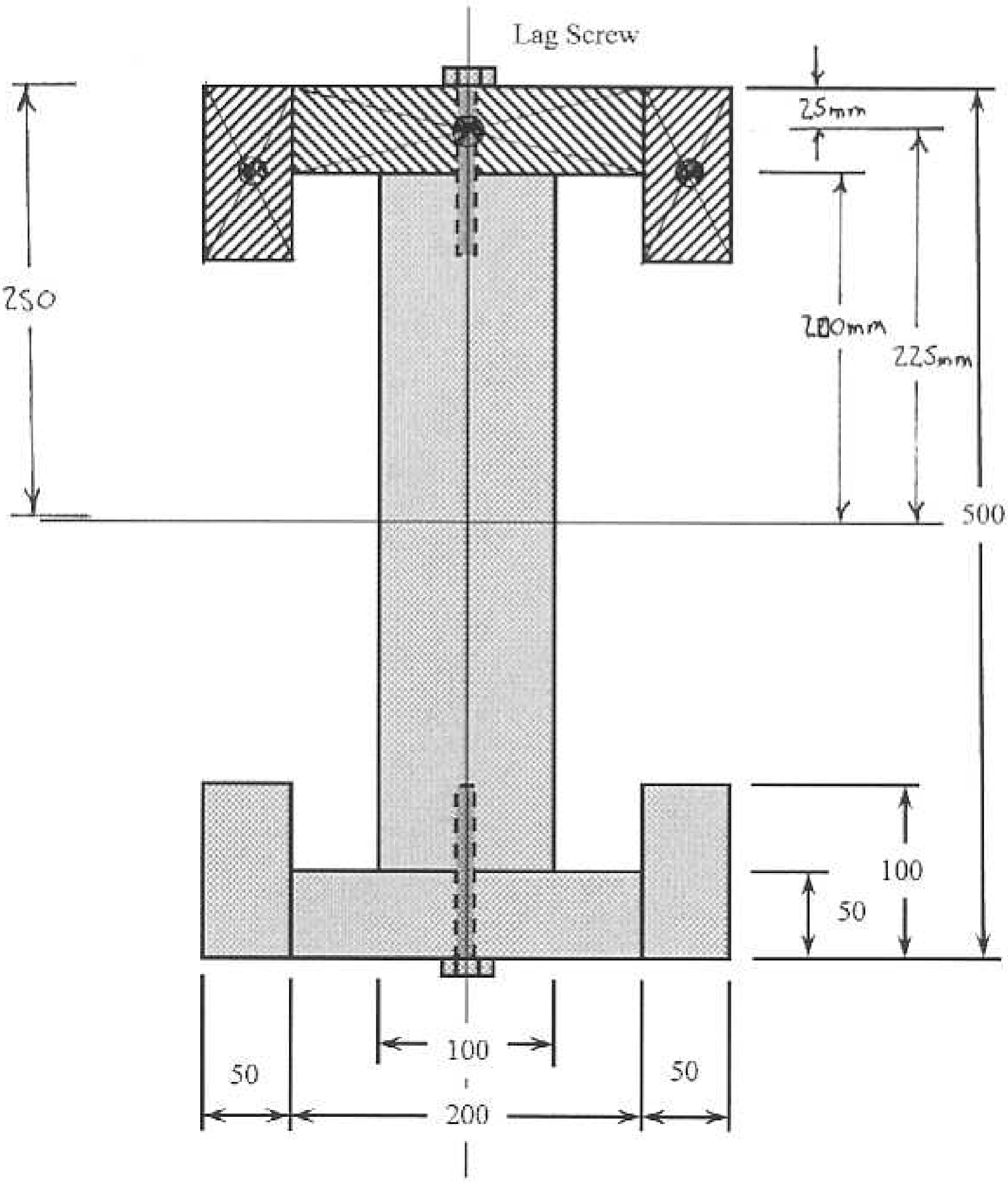

γ/2
ε
ε2 = 175×10^-6 − 108.4×10^-6 = 66.7×10^-6
εb 81×10^-6
εc 121×10^-6
εa 229×10^-6
(229×10^-6 + 81×10^-6)/2 = 175×10^-6
175×10^-6 − 81×10^-6 = 94×10^-6
229×10^-6 − 121×10^-6 = 108×10^-6
2θ
ε1 = 175×10^-6 + 108.4×10^-6 = 283.4×10^-6
2×94×10^-6 = 188×10^-6
d = √((108×10^-6)^2 + (188×10^-6)^2) = 216.8×10^-6
r = 216.8×10^-6 / 2 = 108.4×10^-6
2θ = Tan^-1 (188.4×10^-6 / 108×10^-6) = 60.12°
θ = 30.1°

PROBLEM 3: A simple beam carries a vertical shear load of 4.5kN. The beam's cross section is made from several full-sized wooden pieces as shown in the figure (all dimensions are in mm). Specify the spacing of the 10mm lag screws shown which are necessary to fasten this beam together. Assume that one 10mm lag screw is good for 2kN when transmitting lateral load parallel to the grain of the wood. For the entire cross section, I is equal to $2.36 \times 10^{-3} m^4$.

$$q = \frac{VQ}{I} = \frac{(4.5\times10^3 \text{ N})[(0.225\text{m})(.200\text{m})(0.050\text{m}) + 2\cdot(.200\text{ m})(0.050\text{m})(0.100\text{m})]}{2.36\times10^{-3}\text{ m}^4}$$

$$= 8.104\times10^3 \frac{\text{N}}{\text{m}}$$

$$\text{SPACING} = \frac{S}{q} = \frac{2\times10^3}{8.104\times10^3 \frac{\text{N}}{\text{m}}} = 0.2468\text{ m} \approx \boxed{250\text{mm}}$$


Lag Screw
25mm
250
200mm
225mm
500
100
50
100
50
200
50

NAME: ______________________________

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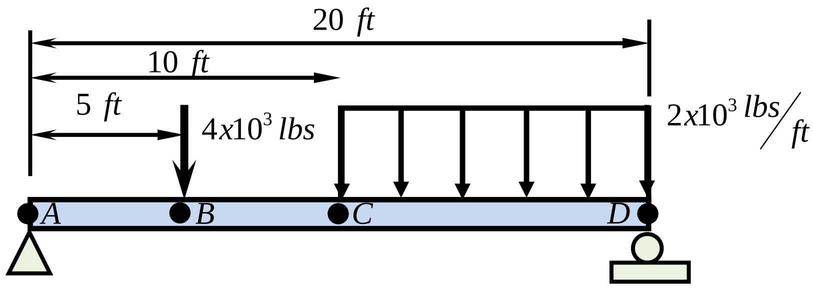

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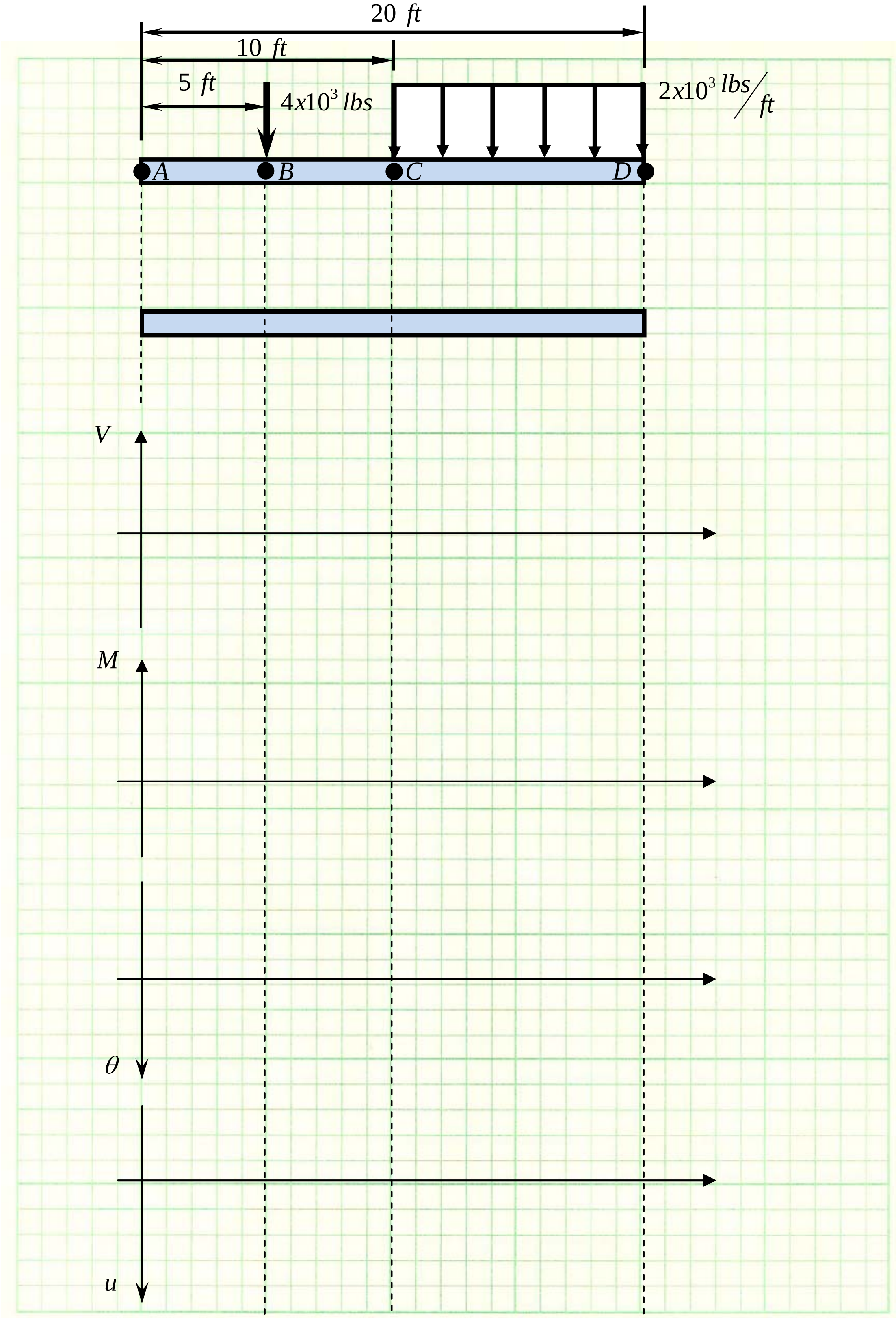

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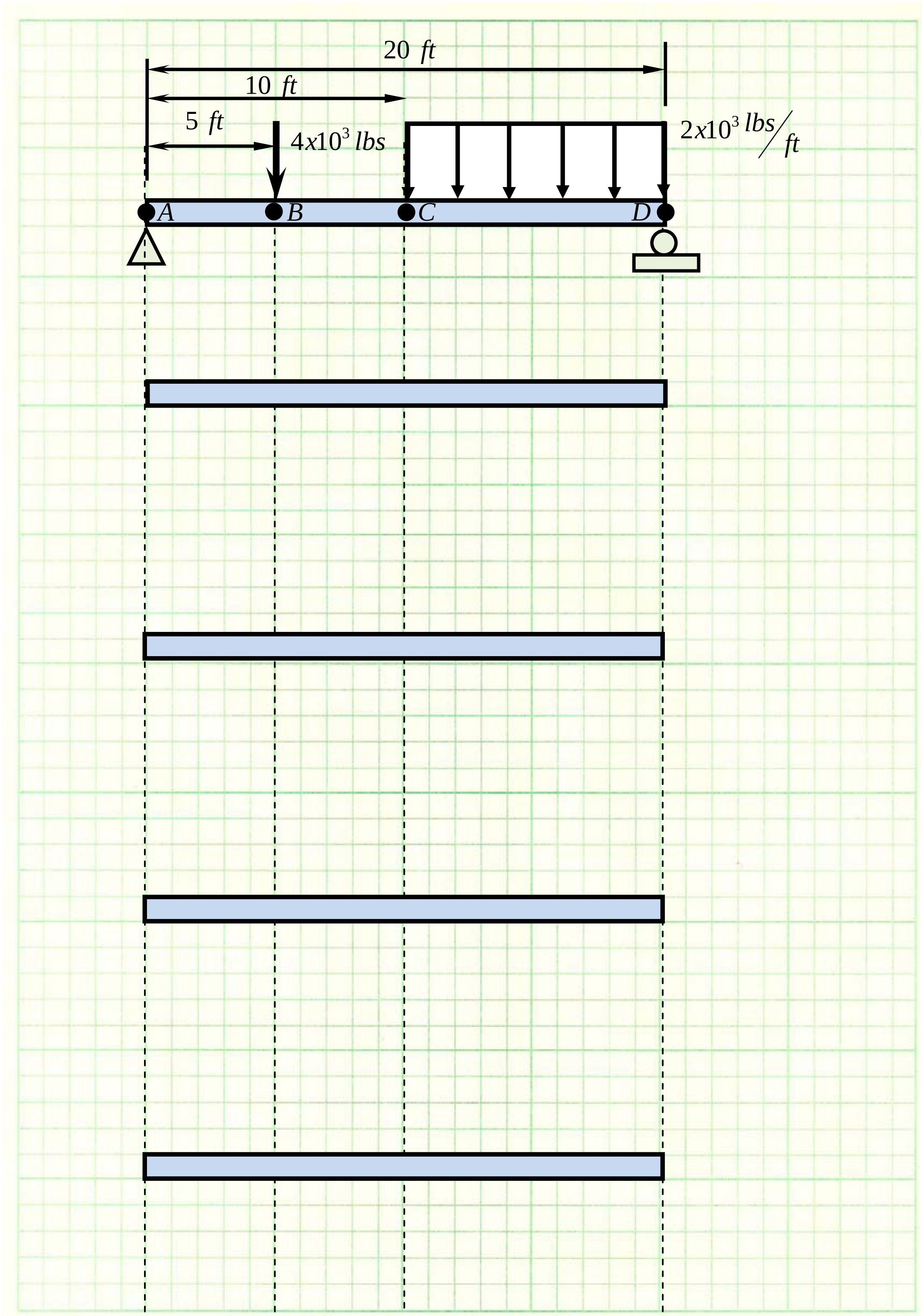

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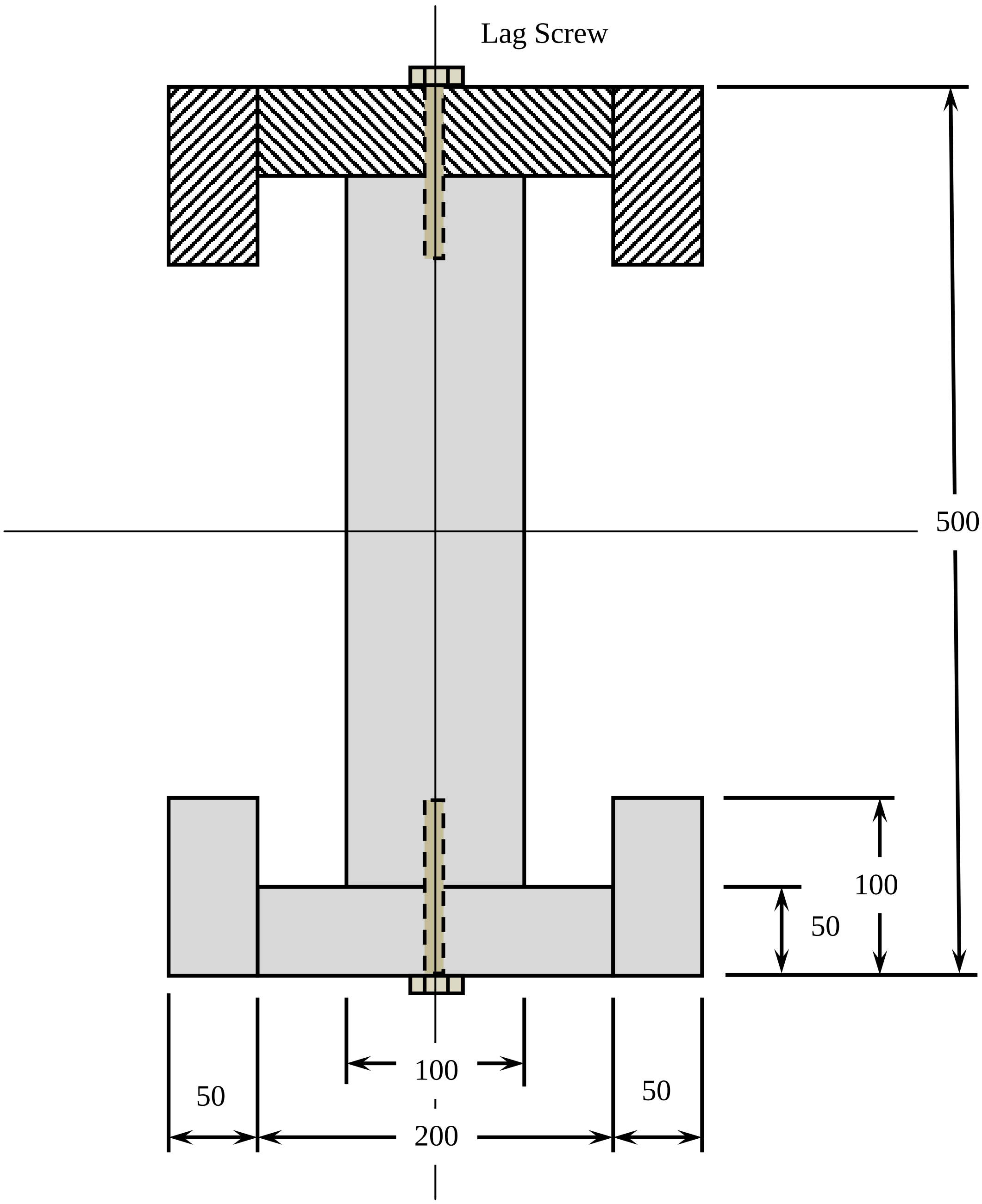
Lag Screw
500
100
50
50
100
50
200