PROBLEM 5.1 DESIGN A FOURBAR MECHANISM TO GIVE THE TWO POSITIONS SHOWN USING THE ANALYTICAL APPROACH.

GIVEN:

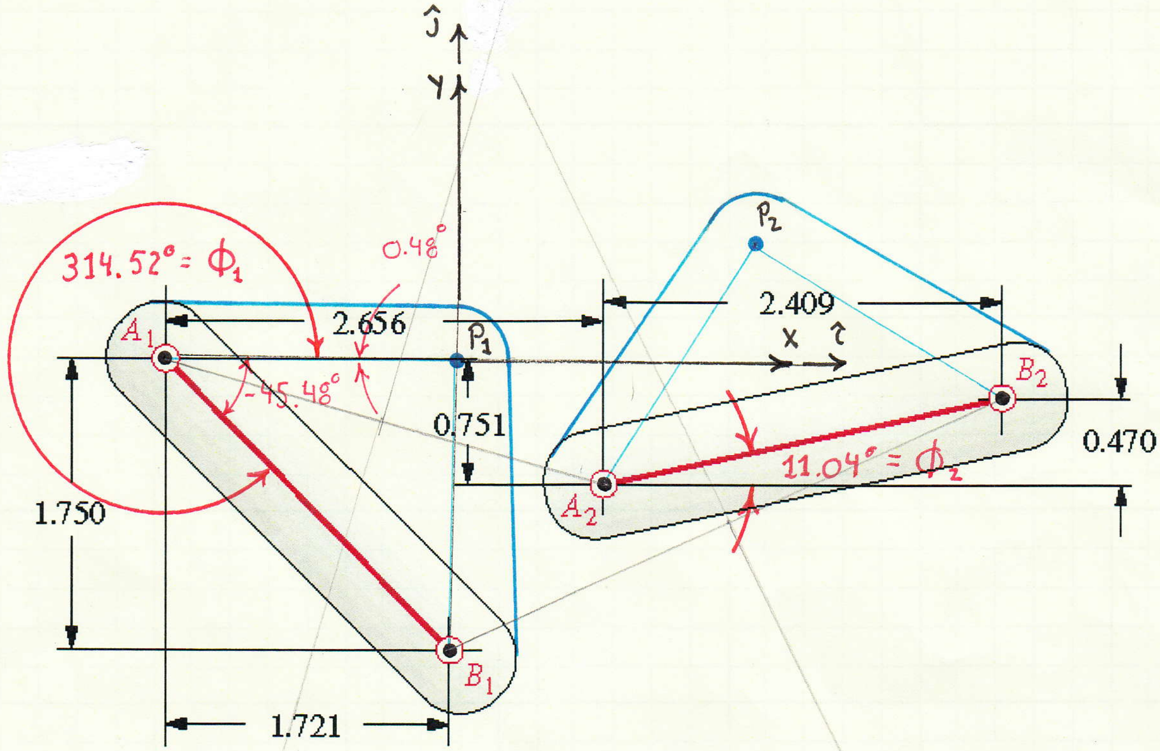
1. POSITIONS OF THE MECHANISM'S COUPLER LINK SHOWN BELOW.
2. LINK LENGTH 2.45

ASSUMPTION:

1. ALL LINKS ARE RIGID
2. ALL JOINTS ARE IDEAL AND FRICTIONLESS
3. COUPLER LINK IS A TERNARY LINK CONFIGURED AS SHOWN.

FIND:

1. SYNTHESIZE A MECHANISM.



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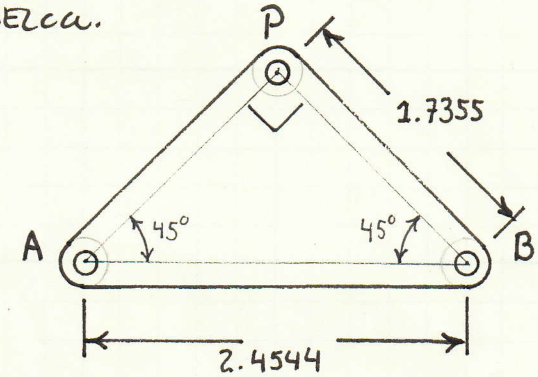
SOLUTION:

THE FIRST STEP IN THE SOLUTION IS TO DETERMINE THE ANGLES A_1B_1 AND A_2B_2 MAKE WITH THE POSITIVE (TO THE RIGHT) HORIZONTAL AXIS.

$$\phi_1 = \tan^{-1}\left(\frac{-1.750}{1.721}\right) = 314.52^\circ \qquad (1)$$

$$\phi_2 = \tan^{-1}\left(\frac{0.470}{2.409}\right) = 11.04^\circ \qquad (2)$$

THE ANALYTICAL SOLUTION REQUIRES THE COUPLER LINK TO BE A TERNARY LINK. THE LOCATION OF THE THIRD NODE ON THIS LINK IS A FREE CHOICE. HERE IT IS CHOSEN TO BE LOCATED AT THE INTERSECTION OF A LINE DRAW FROM POINT A AT A 45° (CCW) ANGLE FROM LINE AB AND A LINE DRAWN FROM POINT B AT A 45° (CW) ANGLE FROM LINE AB. THE DIMENSIONS OF THIS LINK ARE SHOWN BELOW.



(b)

THE CALCULATION OF THE TWO DYADS REQUIRES THE LOCATION OF THE ORIGIN OF THE INERTIAL COORDINATE SYSTEM, THIS IS A FREE CHOICE. FOR THIS SOLUTION THE LOCATION OF THE INERTIAL COORDINATE SYSTEM IS

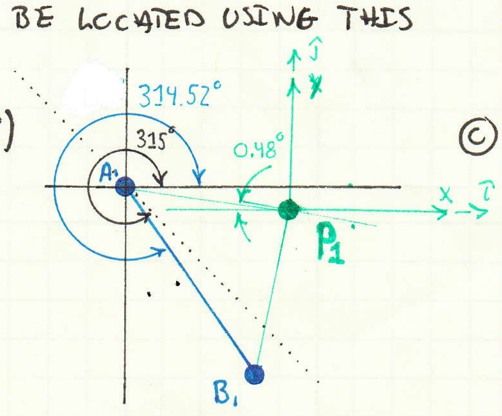
$$(P_{1x}, P_{1y}) = \underline{\underline{(0, 0)}} \qquad (3)$$

WHERE THE X-AXIS EXTENDS FROM THIS POINT HORIZONTALLY TO THE RIGHT AND THE Y-AXIS EXTENDS FROM THIS POINT VERTICALLY UPWARD.

NOW POINTS A_1, B_1, A_2, B_2, AND P_2 NEEE TO BE LOCATED USING THIS COORDINATE SYSTEM.

$$(A_{1x}, A_{1y}) = (P_{1x} - 1.7355\cdot\cos 0.48^\circ,\ P_{1y} + 1.7355\sin 0.48^\circ)$$

$$= \underline{\underline{(-1.7354, 0.0145)}} \qquad (4)$$



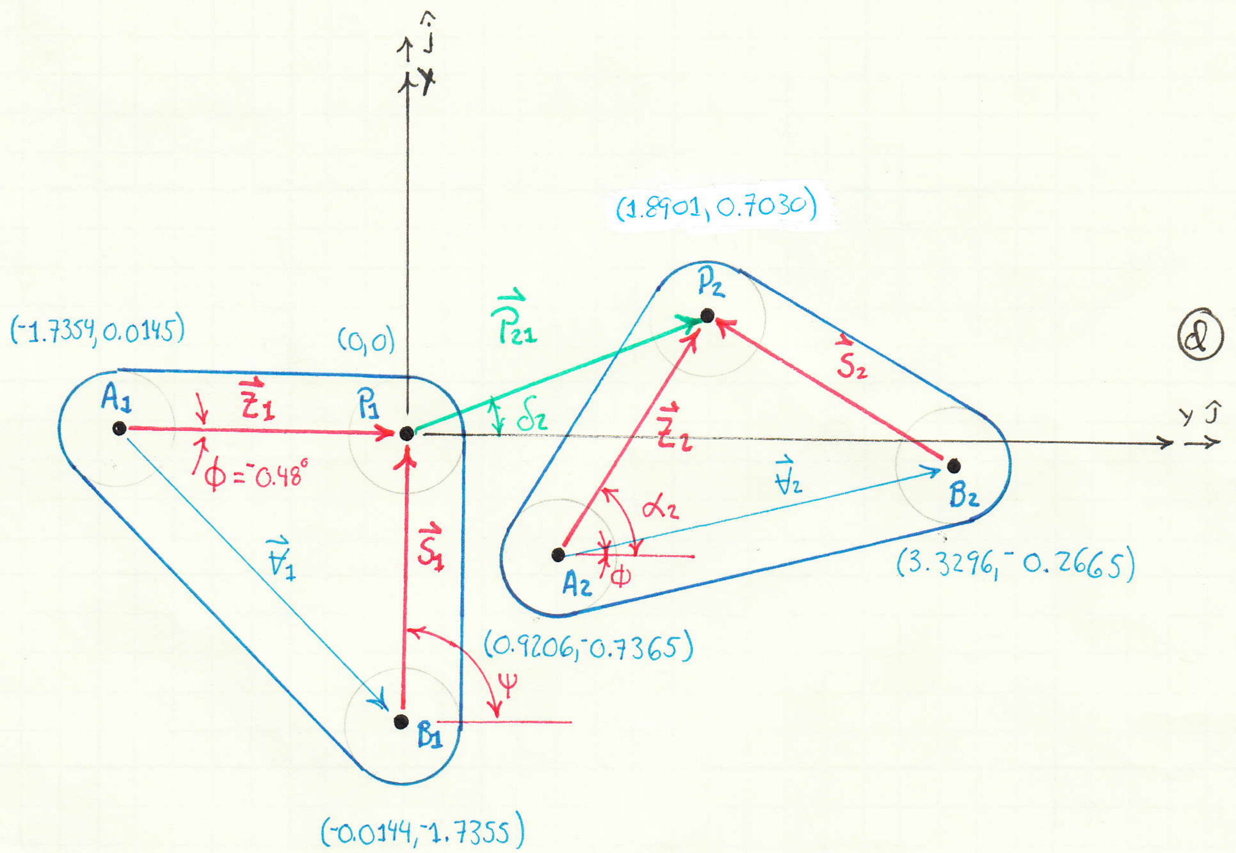
(c)

$$(B_{1x}, B_{1y}) = (A_{1x} + 1.721,\ A_{1y} - 1.750)$$
$$= (-1.7354 + 1.721,\ 0.0145 - 1.750) = \underline{\underline{(-0.0144, -1.7355)}} \quad (5)$$

$$(A_{2x}, A_{2y}) = (A_{1x} + 2.656,\ A_{2y} - 0.751)$$
$$= (-1.7354 + 2.656,\ 0.0145 - 0.751) = \underline{\underline{(0.9206, -0.7365)}} \quad (6)$$

$$(B_{2x}, B_{2y}) = (A_{1x} + 2.656 + 2.409,\ A_{1y} - 0.751 + 0.470)$$
$$= (-1.7354 + 2.656 + 2.409,\ 0.0145 - 0.751 + 0.470) = \underline{\underline{(3.3296, -0.2665)}} \quad (7)$$

$$(P_{2x}, P_{2y}) = (A_{2x} + 1.7355 \cdot \cos(11.04° + 45°),\ A_{2y} + 1.7355 \cdot \sin(11.04° + 45°))$$
$$= (0.9206 + 1.7355 \cdot \cos(56.04°),\ -0.7365 + 1.7355 \cdot \sin(56.04°))$$
$$= \underline{\underline{(1.8901, 0.7030)}} \quad (8)$$



GIVEN THE INFORMATION GIVEN AND THE CHOICES TAKEN, ~~A TWO~~ THE ANALYTICAL APPROACH ~~TO BE~~ MOST APPROPRIATE FOR THE SOLUTION IS THE TWO-POSITION, APPROACH B ANALYSIS. FOR THIS ANALYSIS THE GIVENS ARE

$$p_{21} = |\vec{P}_{21}| = \sqrt{(P_{2x} - P_{1x})^2 + (P_{2y} - P_{1y})^2}$$
$$= \sqrt{(1.8901 - 0)^2 + (0.7030 - 0)^2} = \underline{\underline{2.0166}} \quad (9)$$

$$\delta_2 = \tan^{-1}\left(\frac{P_{2y} - P_{1y}}{P_{2x} - P_{1x}}\right) = \tan^{-1}\left(\frac{0.7030 - 0}{1.8901 - 0}\right) = \underline{\underline{20.40^\circ}} \quad (10)$$

$$\alpha_2 = 0.48^\circ + \tan^{-1}\left(\frac{P_{2y} - A_{2y}}{P_{2x} - A_{2x}}\right) = 0.48^\circ + \tan\left(\frac{0.7030 - (-0.7365)}{1.8901 - 0.9206}\right)$$

$$= 0.48^\circ + 56.05 = \underline{\underline{56.53^\circ}} \quad (11)$$

(9) - (11) ARE THE SAME FOR BOTH DYADS OF THE LINKAGE BEING SYNTHESIZED. FOR THE FIRST DYAD, THE CHOSEN VALUES ARE

$$z = \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} = \sqrt{(0 - (-1.7354))^2 + (0 - 0.0145)^2}$$

$$= \underline{\underline{1.7355}} \quad (12)$$

$$\phi = \tan^{-1}\left(\frac{P_{1y} - A_{1y}}{P_{1x} - A_{1x}}\right) = \tan^{-1}\left(\frac{0 - 0.0145}{0 - (-1.7354)}\right) = \underline{\underline{-0.48^\circ}} \quad (13)$$

THE LAST VALUE TO CHOOSE REPRESENTS THE ANGLE THE DYAD WILL SWEEP OUT

$$\beta_2 = 40^\circ \quad (14)$$

FOR THE SECOND DYAD, THE CHOOSE VALUES ARE

$$s = \sqrt{(P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2} = \sqrt{(0 - (-0.0144))^2 + (0 - (-1.7355))^2}$$

$$= \underline{\underline{1.7355}} \quad (15)$$

$$\psi = \tan^{-1}\left(\frac{P_{1y} - B_{1y}}{P_{1x} - B_{1x}}\right) = \tan^{-1}\left(\frac{0 - (-1.7355)}{0 - (-0.0144)}\right) = \underline{\underline{89.52^\circ}} \quad (16)$$

THE LAST VALUE TO CHOOSE REPRESENTS THE ANGLE THE DYAD WILL SWEEP OUT

$$\underline{\underline{\gamma_2 = 80^\circ}} \quad (17)$$

NOW (9) - (17) CAN BE USED TO ANALYTICALLY SYNTHESIZE A FOUR BAR LINKAGE WITH THE DESIRED COUPLER MOTION.

APPROACH B | **FIRST DYAD**

GIVEN:		CHOSEN:		FIND:	
P12	2.0166	**z**	1.7355	**w**	4.04
δ2	20.4	**ϕ**	-0.48	**θ**	-125.8
α2	56.53	**β2**	40	**W1x**	-2.36
				W1y	-3.27

	x-coord	y-coord
W1	-2.3602	-3.2735
W2	0.2961	-4.0247
Z1	1.7354	-0.0145
Z2	0.9692	1.4396

	x-coord	y-coord
O2	0.62	3.29
A1	-1.74	0.01
A2	0.92	-0.74
P1	0.00	0.00
P2	1.89	0.70

$$\begin{bmatrix} -0.23396 & -0.64279 \\ 0.642788 & -0.23396 \end{bmatrix} \begin{Bmatrix} \mathbf{W1x} \\ \mathbf{W1y} \end{Bmatrix} = \begin{Bmatrix} 2.656339 \\ -0.75125 \end{Bmatrix}$$

inverse

-0.5	1.373739
-1.37374	-0.5

$$\begin{bmatrix} \cos(\beta_2)-1 & -\sin(\beta_2) \\ \sin(\beta_2) & \cos(\beta_2)-1 \end{bmatrix} \begin{Bmatrix} W_{1x} \\ W_{1y} \end{Bmatrix} = \begin{Bmatrix} p_{21}\cdot\cos(\delta_2)-z\cdot\left[\cos(\phi+\alpha_2)-\cos(\phi)\right] \\ p_{21}\cdot\sin(\delta_2)-z\cdot\left[\sin(\phi+\alpha_2)-\sin(\phi)\right] \end{Bmatrix}$$

$$\begin{bmatrix} \cos(\beta_2)-1 & -\sin(\beta_2) \\ \sin(\beta_2) & \cos(\beta_2)-1 \end{bmatrix} \begin{Bmatrix} W_{1x} \\ W_{1y} \end{Bmatrix} = \begin{Bmatrix} p_{21}\cdot\cos(\delta_2)-z\cdot\left[\cos\phi\cdot\cos\alpha_2-\sin\phi\cdot\sin\alpha_2-\cos(\phi)\right] \\ p_{21}\cdot\sin(\delta_2)-z\cdot\left[\sin\phi\cdot\cos\alpha_2+\cos\phi\cdot\sin\alpha_2-\sin(\phi)\right] \end{Bmatrix}$$

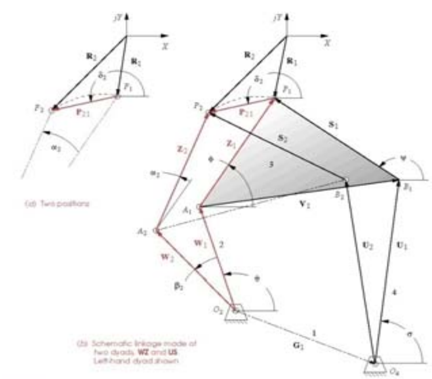


FIGURE 6-1
Two-position analytical synthesis

APPROACH B | **SECOND DYAD**

GIVEN:		CHOSEN:		FIND:	
P12	2.0166	**s**	1.7355	**u**	2.84
δ2	20.4	**ψ**	89.52	**σ**	-106.3
α2	56.53	**γ2**	80	**U1x**	-0.80
				U1y	-2.73

	x-coord	y-coord
U1	-0.7967	-2.7274
U2	2.5476	-1.2582
S1	0.0145	1.7354
S2	-1.4396	0.9692

	x-coord	y-coord
O4	0.78	0.99
B1	-0.01	-1.74
B2	3.33	-0.27
P1	0.00	0.00
P2	1.89	0.70

$$\begin{bmatrix} -0.82635 & -0.98481 \\ 0.984808 & -0.82635 \end{bmatrix} \begin{Bmatrix} \mathbf{W1x} \\ \mathbf{W1y} \end{Bmatrix} = \begin{Bmatrix} 3.344303 \\ 1.469146 \end{Bmatrix}$$

inverse

-0.5	0.595877
-0.59588	-0.5

$$\begin{bmatrix} \cos(\gamma_2)-1 & -\sin(\gamma_2) \\ \sin(\gamma_2) & \cos(\gamma_2)-1 \end{bmatrix} \begin{Bmatrix} U_{1x} \\ U_{1y} \end{Bmatrix} = \begin{Bmatrix} p_{21}\cdot\cos(\delta_2)-s\cdot\left[\cos(\psi+\alpha_2)-\cos(\psi)\right] \\ p_{21}\cdot\sin(\delta_2)-s\cdot\left[\sin(\psi+\alpha_2)-\sin(\psi)\right] \end{Bmatrix}$$

$$\begin{bmatrix} \cos(\gamma_2)-1 & -\sin(\gamma_2) \\ \sin(\gamma_2) & \cos(\gamma_2)-1 \end{bmatrix} \begin{Bmatrix} U_{1x} \\ U_{1y} \end{Bmatrix} = \begin{Bmatrix} p_{21}\cdot\cos(\delta_2)-s\cdot\left[\cos\psi\cdot\cos\alpha_2-\sin\psi\cdot\sin\alpha_2-\cos\psi\right] \\ p_{21}\cdot\sin(\delta_2)-s\cdot\left[\sin\psi\cdot\cos\alpha_2+\cos\psi\cdot\sin\alpha_2-\sin\psi\right] \end{Bmatrix}$$

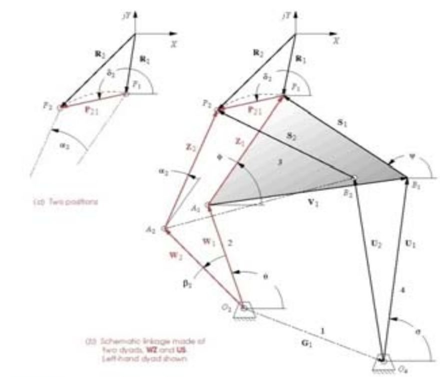
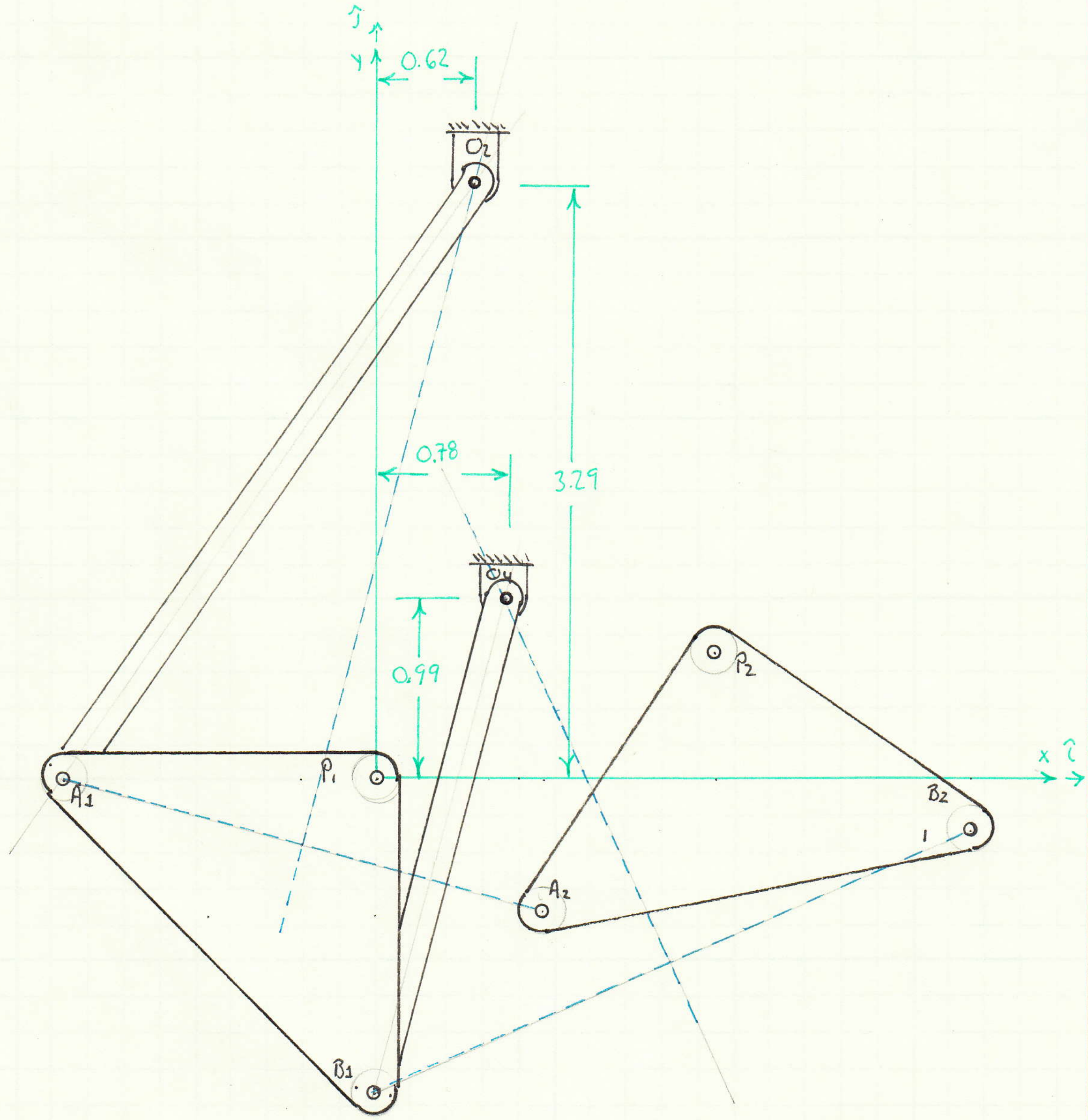


FIGURE 6-1
Two-position analytical synthesis

SOLUTION DIAGRAM

THE FIGURE BELOW ILLUSTRATES THE RESULTS OF THE ANALYTICAL SOLUTION AND THE CONSTRUCTION LINES FOR THE GRAPHICAL SOLUTION ARE SHOWN IN BLUE TO ILLUSTRATE HOW THE TWO SOLUTION APPROACHES ARE RELATED.



SUMMARY:

THE INFORMATION THAT IS PROVIDED IN THE PROBLEM STATEMENT ~~MAKES~~ IS MOST APPROPRIATE FOR THE 2-POSITION-APPROACH-B ANALYTICAL SOLUTION. THE LOCATION OF POINT P ON THE TERNARY LINK AND THE ANGLE SWEPT OUT BY O_2A & O_4B ARE FREE CHOICES FOR THE DESIGNER.