

PROBLEM 3-20 SUP THE STATE OF STRESS AT A POINT IS

$$\sigma_x = -6 \text{ ksi}$$

$$\tau_{xy} = 9 \text{ ksi}$$

$$\sigma_y = 18 \text{ ksi}$$

$$\tau_{yz} = 6 \text{ ksi}$$

$$\sigma_z = -12 \text{ ksi}$$

$$\tau_{zx} = -15 \text{ ksi}$$

GIVEN THAT THE MATERIAL IS BRASS ($E = 15.4 \text{ Msi}$, $\nu = 0.324$), DETERMINE THE STATE OF STRAIN IN THE MATERIAL

GIVEN:

1. THE STATE OF STRESS $[\sigma] = \begin{bmatrix} -6 & 9 & -15 \\ 9 & 18 & 6 \\ -15 & 6 & -12 \end{bmatrix} \text{ ksi}$
2. THE MATERIAL IS BRASS (FROM THE TEXT)
 - $E = 15.4 \text{ Msi}$
 - $\nu = 0.324$

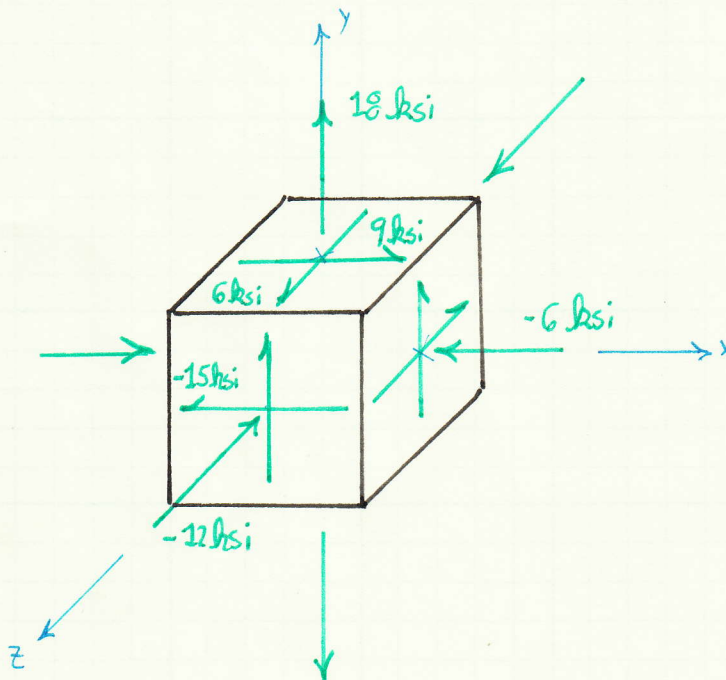
ASSUMPTIONS

1. THE MATERIAL IS RESPONDING IN A LINEAR-ELASTIC MANNER
2. SMALL DEFORMATIONS

FIND:

1. THE COMPLETE STATE OF STRAIN IN THE MATERIAL

FIGURE:



SOLUTION:

KNOWING THAT THE RELATIONSHIP BETWEEN STRESS AND STRAIN IN A LINEAR - ISOTROPIC - HOMOGENEOUS MATERIAL IS

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} \quad (1)$$

FOR BRASS, $E = 15.4(10^6)$ PSI AND $\nu = 0.324$

$$\frac{1}{E} = \frac{1}{15.4(10^6)} \frac{\text{in}^2}{\text{lb}} = 64.94(10^{-9}) \frac{\text{in}^2}{\text{lb}} \quad (2)$$

$$\frac{\nu}{E} = \frac{0.324}{15.4(10^6)} \frac{\text{in}^2}{\text{lb}} = 21.04(10^{-9}) \frac{\text{in}^2}{\text{lb}} \quad (3)$$

$$\frac{2 \cdot (1+\nu)}{E} = \frac{2 \cdot (1+0.324)}{15.4(10^6)} \frac{\text{in}^2}{\text{lb}} = 172.0(10^{-9}) \frac{\text{in}^2}{\text{lb}} \quad (4)$$

THEREFORE

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} 64.94 & -21.04 & -21.04 & 0 & 0 & 0 \\ -21.04 & 64.94 & -21.04 & 0 & 0 & 0 \\ -21.04 & -21.04 & 64.94 & 0 & 0 & 0 \\ 0 & 0 & 0 & 172.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 172.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 172.0 \end{bmatrix} \begin{Bmatrix} -6 \text{ ksi} \\ 18 \text{ ksi} \\ -12 \text{ ksi} \\ 6 \text{ ksi} \\ -15 \text{ ksi} \\ 9 \text{ ksi} \end{Bmatrix} \times (10^{-9}) \frac{\text{in}^2}{\text{lb}} = \begin{Bmatrix} -500 \\ 1500 \\ -100 \\ 100 \\ -260 \\ -1500 \end{Bmatrix} \times (10^{-6})$$

Summary:

THE CALCULATION MADE ABOVE IS IN TERMS OF ENGINEERING STRAIN. THE CALCULATIONS WERE PERFORMED USING MATLAB AND ARE SHOWN ON THE NEXT PAGE.

4/21/16 9:20 AM MATLAB Command Window

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```
>> S=[-6.0e3 18.0e3 -12.0e3 6.0e3 -15.0e3 9.0e3]'
```

S =

```
-6000
18000
-12000
6000
-15000
9000
```

```
>> C=[64.94 -21.04 -21.04 0 0 0;
-21.04 64.94 -21.04 0 0 0;
-21.04 -21.04 64.94 0 0 0;
0 0 0 172.0 0 0;
0 0 0 0 172.0 0;
0 0 0 0 0 172.0]*1e-9
```

C =

1.0e-06 *

```
0.0649 -0.0210 -0.0210 0 0 0
-0.0210 0.0649 -0.0210 0 0 0
-0.0210 -0.0210 0.0649 0 0 0
0 0 0 0.1720 0 0
0 0 0 0 0.1720 0
0 0 0 0 0 0.1720
```

```
>> e=C*S
```

e =

```
-0.0005 =  $\epsilon_x$ 
0.0015 =  $\epsilon_y$ 
-0.0010 =  $\epsilon_z$ 
0.0010 =  $\gamma_{yz}$ 
-0.0026 =  $\gamma_{xz}$ 
0.0015 =  $\gamma_{xy}$ 
```

```
>>
```