PROBLEM 1.21 | A THIN, UNIFORMLY THICK PLATE OF THICKNESS "t" IS HANGING UNDER ITS WEIGHT AS SHOWN. THE CORRESPONDING DISPLACEMENT FIELD IN THE XY PLANE CAN BE APPROXIMATED BY:

$$u(x,y) = \frac{\rho \cdot g}{2 \cdot E}(2 \cdot b \cdot x - x^2 - \nu \cdot y^2) \qquad (1)$$

$$v(x,y) = -\nu \cdot \frac{\rho\, g}{E} \cdot y \cdot (b - x) \qquad (2)$$

WHERE ρ AND E ARE THE MASS DENSITY AND YOUNG'S MODULUS OF THE PLATE MATERIAL, RESPECTIVELY, AND g IS THE GRAVITY CONSTANT.

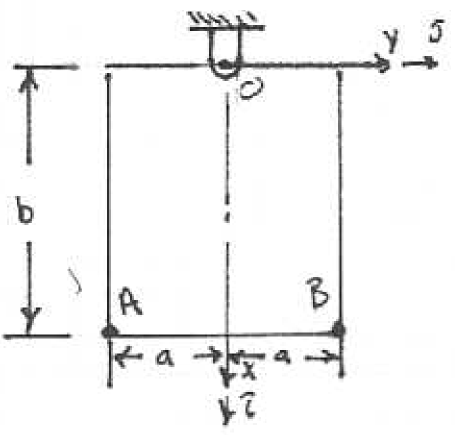
(a) DETERMINE THE CORRESPONDING PLANE STRESS FIELD $\sigma_x(x,y)$, $\sigma_y(x,y)$, AND $\tau_{xy}(x,y)$ AND COMMENT ON THE VALIDITY OF THE RESULTS.
(b) QUALITATIVELY, DRAW THE DEFORMED AND UNDEFORMED SHAPE OF THE EDGES ON THE SAME DRAWING. FOR THE SAKE OF CLARITY, EXAGGERATE THE DEFLECTIONS OF THE EDGES
(c) DETERMINE THE ROTATION OF THE PLATE AT POINTS "A" AND "B". DO THE ROTATIONS AGREE WITH THE SKETCH OF PART (b)?

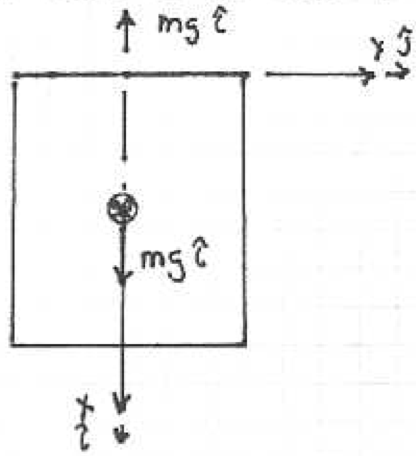
GIVEN:
CONSTRAINTS
1. PLATE OF DIMENSIONS $2a \times b$ THAT IS "t" THICK
2. PLATE HAS DENSITY "ρ", MODULUS OF ELASTICITY "E", AND POISSON'S RATIO "ν"
3. PLATE HANGES FROM THE PIN SUPPORT AT O.
4. THE DISPLACEMENTS OF THE PLATE ARE APPROXIMATED BY u, v in (1) & (2)

ASSUMPTIONS
1. PLATE IS SUBJECTED TO SMALL DISPLACEMENTS AND STRAINS
2. THE PLATE MATERIAL IS HOMOGENIOUS AND ISOTROPIC
3. GRAVITY ACTS IN THE VERTICAL DIRECTION
4. THE PIN AT "O" is FRICTIONLESS

FIND:
1. DETERMINE THE PLANE STRESS FIELD $\sigma_x(x,y)$, $\sigma_y(x,y)$, $\tau_{xy}(x,y)$
2. DRAW THE UNDEFORMED AND DEFORMED EDGES OF THE PLATE.
3. DETERMINE THE ROTATIONS OF THE PLATE AT POINTS "A" AND "B".

DIAGRAM





SOLUTION:

STARTING BY DETERMINING THE STRAINS FROM THE DISPLACEMENT FIELD

$$\epsilon_x = \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left[\frac{\rho \cdot g}{2 \cdot E}(2 \cdot b \cdot x - x^2 - \nu \cdot y^2)\right]$$

$$= \frac{\rho \cdot g}{2E}\left[2 \cdot b - 2 \cdot x\right] = \underline{\underline{\frac{\rho \cdot g}{E}\left[b - x\right]}} \qquad (1)$$

$$\epsilon_y = \frac{\partial v}{\partial y} = \frac{\partial}{\partial y}\left[-\nu \cdot \frac{\rho \cdot g}{E} \cdot y \cdot (b - x)\right]$$

$$= -\nu \frac{\rho \cdot g}{E}\left[(b - x)\right] = \underline{\underline{-\nu \frac{\rho \cdot g}{E} \cdot (b - x)}} \qquad (2)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial}{\partial y}\left[\frac{\rho \cdot g}{2 \cdot E} \cdot (2 \cdot b \cdot x - x^2 - \nu y^2)\right]$$

$$+ \frac{\partial}{\partial x}\left[-\nu \cdot \frac{\rho \cdot g}{E} \cdot y \cdot (b - x)\right]$$

$$= \frac{\rho \cdot g}{2 \cdot E}(-2 \cdot \nu \cdot y) - \nu \cdot \frac{\rho \cdot g}{E} \cdot y(-1)$$

$$= \frac{\rho \cdot g}{E}\left[-\nu \cdot y + \nu \cdot y\right] = \underline{\underline{0}} \qquad (3)$$

USING THE ~~ST~~ PLANE STRES STRESS-STRAIN RELATIONS, EXPRESSIONS FOR THE STATE OF STRESS CAN BE DEVELOPED

$$\sigma_x = \frac{E}{1 - \nu^2}\left[\epsilon_x + \nu \epsilon_y\right] = \frac{E}{1 - \nu^2}\left[\frac{\rho \cdot g}{E}(b - x) - \nu^2 \cdot \frac{\rho \cdot g}{E}(b - x)\right]$$

$$= \frac{E}{1 - \nu^2} \cdot \frac{\rho \cdot g}{E} \cdot (b - x) \cdot (1 - \nu^2) = \underline{\underline{\rho \cdot g \cdot (b - x)}} \qquad (4)$$

$$\sigma_y = \frac{E}{1 - \nu^2}\left[\epsilon_y + \nu \epsilon_x\right] = \frac{E}{1 - \nu^2}\left[-\nu \frac{\rho \cdot g}{E}(b - x) + \nu \frac{\rho \cdot g}{E}(b - x)\right] = \underline{\underline{0}} \qquad (5)$$

$$\tau_{xy} = \frac{E}{2(1 + \nu)} \cdot \gamma_{xy} = \frac{E}{2(1 + \nu)} \cdot (0) = \underline{\underline{0}}$$

IT APPEARS TO MAKE SENSE THAT THE ONLY STRESS IS IN THE ~~[illegible]~~ DIRECTION OF GRAVITY; HOWEVER, THE SOLUTION DOES NOT APPEAR TO ACCOUNT FOR THE REACTION AT "O" AND THE FREE SURFACE ALONG THE TOP OF THE PLATE.

AN EXPRESSION FOR THE ROTATIONS ~~ARE~~ CAN BE DEVELOPED FROM THE DISPLACEMENTS

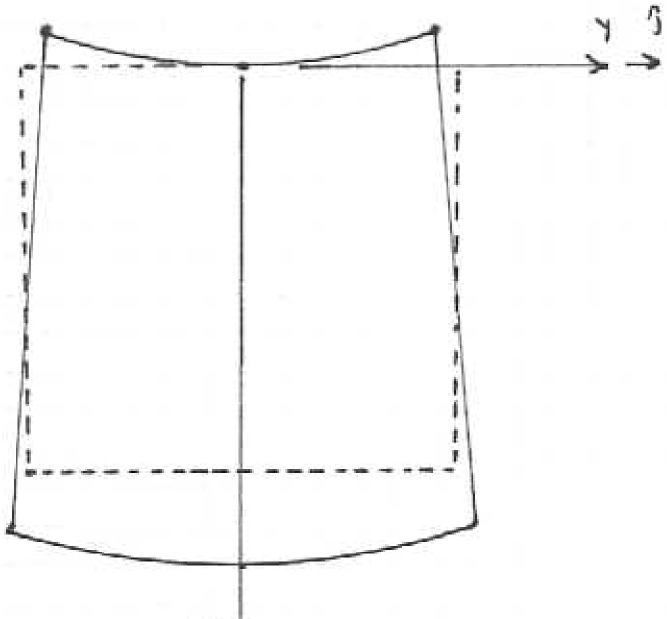
$$\Theta_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left\{\frac{\partial}{\partial x}\left[-\nu \cdot \frac{\rho \cdot g}{E} \cdot y \cdot (b-x)\right]\right.$$

$$\left. - \frac{\partial}{\partial y}\left[\frac{\rho \cdot g}{2 \cdot E} \cdot (2 \cdot b \cdot x - x^2 - \nu \cdot y^2)\right]\right\}$$

$$= \frac{1}{2}\left\{\nu \cdot \frac{\rho \cdot g}{E} \cdot y - \frac{\rho \cdot g}{2 \cdot E}(-2 \nu \cdot y)\right\} = \frac{1}{2}\left\{\nu \frac{\rho \cdot g}{E} \cdot y + \nu \cdot \frac{\rho \cdot g}{E} \cdot y\right\}$$

$$= \underline{\frac{\nu \cdot \rho \cdot g}{E} \cdot y} \qquad (6)$$

$$\Theta_{xy}^{(A)}(A) = -\frac{\nu \cdot \rho \cdot g}{E} \cdot a \qquad \Theta_{xy}^{(B)}(a) = \frac{\nu \cdot \rho \cdot g}{E} a$$



SUMMARY:

THE PLACE WHERE THIS SOLUTION APPEARS TO FALL APPART IS ON THE SURFACE WHERE IT IS PIN CONNECTED. SINCE THIS IS A PRESSURE FREE SURFACE, THE STRESS HERE SHOULD BE ZERO. THE ERROR IS A RESULT OF THE PIN CONNECTION. THE PIN CONNECTION IS A STRESS CONCENTRATION. THIS SOLUTION WOULD BE MORE APPROPRIATE FOR A PLATE FIXED TO THE WALL.