PROBLEM 3.35 | THE CLOSED TUBE SHOWN HAS AN OUTSIDE DIAMETER OF 40mm AND A WALL THICKNESS OF 2mm. THE TUBE IS SUBJECTED TO THE LOADS SHOWN AND AN INTERNAL PRESSURE OF 3MPa.

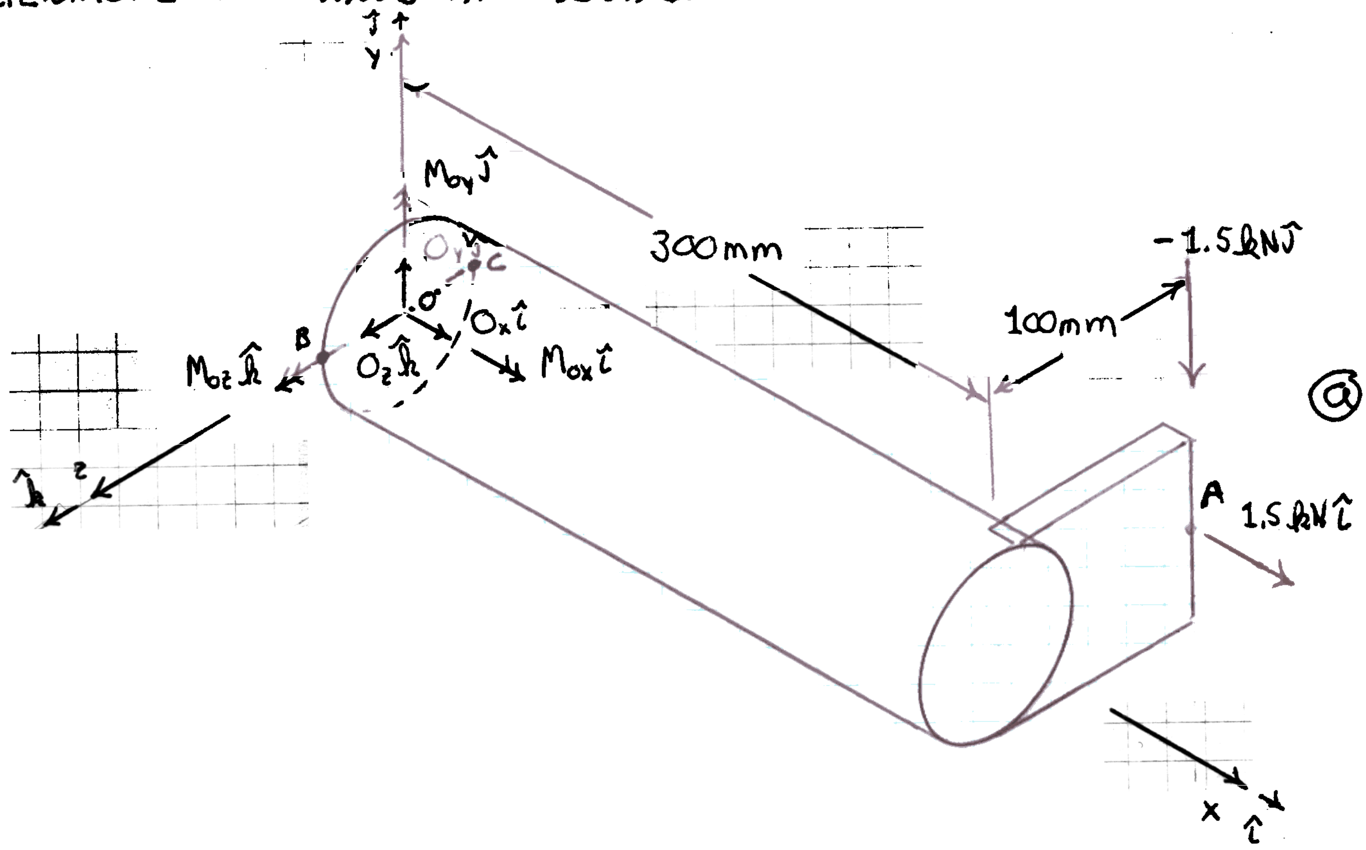
(a) ASSUMING AN IDEAL MODEL, DETERMINE THE STATE OF STRESS AT (0, 0, ± 20mm) AND WHERE THE TENSILE BENDING STRESS IS MAXIMUM.
(b) DETERMINE THE VALUE AND LOCATION OF THE MAXIMUM SHEAR STRESS.
(c) DETERMINE THE VALUE AND LOCATION OF THE MAXIMUM TENSILE STRESS

GIVEN:

CONSTRAINTS
1. 40mm TUBE WITH 2mm WALL THICKNESS
2. TUBE HAS 3MPa INTERNAL PRESSURE
3. 1.5 KN LOAD IN THE HORIZONTAL DIRECTION APPLIED AT THE END OF THE TUBE 100mm FROM THE CENTERLINE
4. 1.5KN LOAD IN THE AXIAL DIRECTION APPLIED AT THE END OF THE TUBE 100mm FROM THE CENTER LINE

ASSUMPTIONS:
1. LINEAR-ELASTIC MATERIAL RESPONSE
2. SMALL DEFORMATIONS AND STRAINS
3. THE WALL IS RIGID

FIND:
1. DETERMINE THE STATE OF STRESS AT (0, 0, ± 20mm)
2. DETERMINE WHERE THE TENSILE BENDING STRESS IS MAXIMUM
3. DETERMINE THE VALUE AND LOCATION OF THE MAXIMUM SHEAR STRESS
4. DETERMINE THE VALUE AND LOCATION OF THE MAXIMUM TENSILE STRESS



(a)

SOLUTION:

BEFORE THE STRESS AT THE WALL CAN BE DETERMINED, THE FORCES AND MOMENTS THAT RESULT FROM THE CONSTRAINT OF THE WALL MUST BE CALCULATED

$$\Sigma F_x = 0 = O_x + 1.5\,kN \Rightarrow \underline{O_x = -1.5\,kN}$$

$$\Sigma F_y = 0 = O_y - 1.5\,kN \Rightarrow \underline{O_y = 1.5\,kN}$$

$$\Sigma F_z = 0 = \underline{O_z}$$

$$\Sigma \vec{M}_{@O} = \vec{0} = \vec{M}_O + \vec{r}_{OA} \times \vec{F}_A$$

$$= \vec{M}_O + (.3m\,\hat{i} - .1m\,\hat{k}) \times (1.5\,kN\,\hat{i} - 1.5\,kN\,\hat{j})$$

$$= \vec{M}_O + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ .3m & 0 & -.1m \\ 1.5\,kN & -1.5\,kN & 0 \end{vmatrix}$$

$$= \vec{M}_O + [-(-1.5\,kN)\cdot(-.1m)]\,\hat{i} - [-(1.5\,kN)(-.1m)]\,\hat{j} + [(.3m)\cdot(-1.5\,kN)]\,\hat{k}$$

$$= \vec{M}_O - 0.15\,kN\cdot m\,\hat{i} - 0.15\,kN\cdot m\,\hat{j} - 0.45\,kN\cdot m\,\hat{k}$$

$$0 = M_{Ox} - 0.15\,kN\cdot m \Rightarrow \underline{M_{Ox} = 0.15\,kN\cdot m}$$

$$0 = M_{Oy} - 0.15\,kN\cdot m \Rightarrow \underline{M_{Oy} = 0.15\,kN\cdot m}$$

$$0 = M_{Oz} - 0.45\,kN\cdot m \Rightarrow \underline{M_{Oz} = 0.45\,kN\cdot m}$$

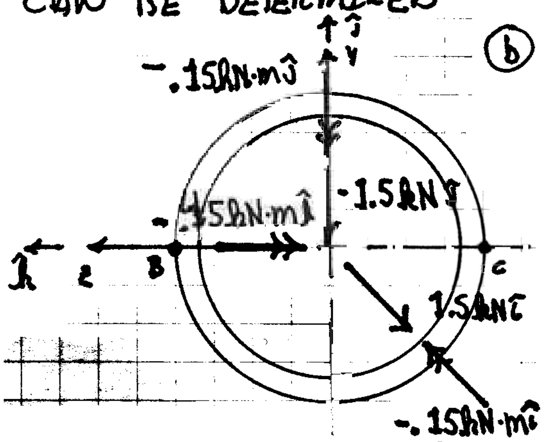
NOW THE STATE OF STRESS AT POINTS "B" AND "C" CAN BE DETERMINED. THE NORMAL STRESS ON "B" AND "C". THE AREA AND MOMENT OF INERTIA FOR THIS CROSS SECTION ARE

$$A = \pi(r_o^2 - r_i^2) = \pi[(0.02m)^2 - (0.018m)^2]$$

$$= \underline{0.2388(10^{-3})\,m^2}$$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}[(.04m)^4 - (.036m)^4]$$

$$= \underline{0.04322(10^{-6})\,m^4}$$



LOAD IN THE TUBE

THE STATE OF STRESS IN THE X-DIRECTION IS A COMBINATIONS OF THE STRESS CAUSED BY F_x, M_y, M_z, AND THE HOOP STRESS

$$= \frac{F_x}{A} + \frac{M_y \cdot z}{I_{yy}} - \frac{M_z \cdot y}{I_{zz}} + \frac{P \cdot r}{2 \cdot t} \quad (1)$$

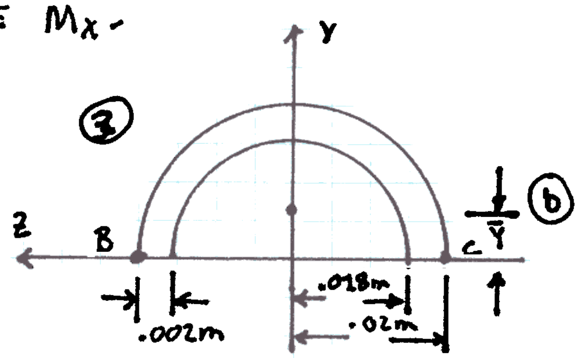
FOR POINT "B"

$$\frac{1.5(10^3)N}{.2388(10^{-3})m^2} + \frac{(-.15 \cdot 10^3 N \cdot m) \cdot (.02m)}{.04322(10^{-6})m^4} - \frac{(-.15 \cdot 10^3 N \cdot m) \cdot (0)}{.04322(10^{-6})m^4} + \frac{3(10^6)\frac{N}{m^2} \cdot (.019m)}{2 \cdot (.002m)}$$

$$-48.88(10^6)\frac{N}{m^2} = \boxed{-48.88 \text{ MPa}} \quad (2)$$

THE SHEAR STRESS IS A COMBINATION OF THE SHEAR THAT RESULTS FROM F_y AND THE SHEAR THAT RESULTS FROM THE TORQUE M_x.

$$= \frac{F_x \cdot Q}{I \cdot C} - \frac{M_x \cdot r}{J} \quad (3)$$



FOR THIS CROSS-SECTION AT POINTS 'B' AND 'C'

$$Q = \bar{Y} \cdot A = \Sigma \bar{Y}_i A_i$$

$$= 0.4244 \cdot (.02m) \cdot \tfrac{1}{2} \cdot \pi (.02m)^2 - 0.4244(.018m) \cdot \tfrac{1}{2} \cdot \pi \cdot (.018m)^2$$

$$= \underline{\underline{1.445(10^{-6})m^3}}$$

$$J = 2 \cdot I = 2 \cdot 0.04322(10^{-6})m^4 = \underline{\underline{0.08644(10^{-6})m^4}}$$

THE SHEAR STRESS AT POINT "B" CAN NOW BE CALCULATED

$$\tau_{xy}^{(B)} = \frac{-1.5(10^3)N \cdot 1.445(10^{-6})m^3}{0.04322(10^{-6})m^4 \cdot 2 \cdot (.002m)} - \frac{(-.15 \times 10^3 N \cdot m) \cdot 0.02m}{0.08644(10^{-6})m^4}$$

$$= -12.54(10^6)\frac{N}{m^2} + 34.71(10^6)\frac{N}{m^2} = \boxed{22.17(10^6)\frac{N}{m^2}} \quad (4)$$

THE NORMAL AND SHEAR STRESSES AT POINT "C" CAN ALSO BE CALCULATED USING (1) AND (3)

$$\sigma_x^{(C)} = \frac{1.5 \times 10^3 N}{.2388 \times 10^{-3} m^2} + \frac{(-.15 \times 10^3 N \cdot m) \cdot (-.02m)}{.04322 \times 10^{-6} m^4} - \frac{(-.15 \times 10^3 N \cdot m)(0)}{.04322 \times 10^{-6} m^4} + \frac{(3 \times 10^6 \frac{N}{m^2}) \cdot (.019m)}{2 \cdot (.002m)}$$

$$= 89.94(10^6)\frac{N}{m^2} = \boxed{89.94 \text{ MPa}}$$

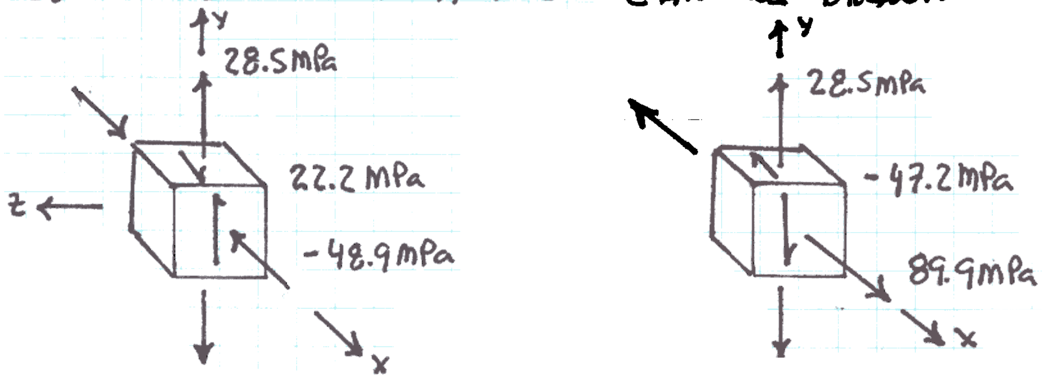
$$\tau_{xy}^{(C)} = \frac{-1.5\times10^{3}\,N \cdot 1.445\times10^{-6}\,m^{3}}{.04322\times10^{-6}\,m^{4}\cdot 2\cdot .002\,m} - \frac{(-.15\times10^{3}\,N\cdot m)\cdot(-0.02\,m)}{0.08644\times10^{-6}\,m^{4}}$$

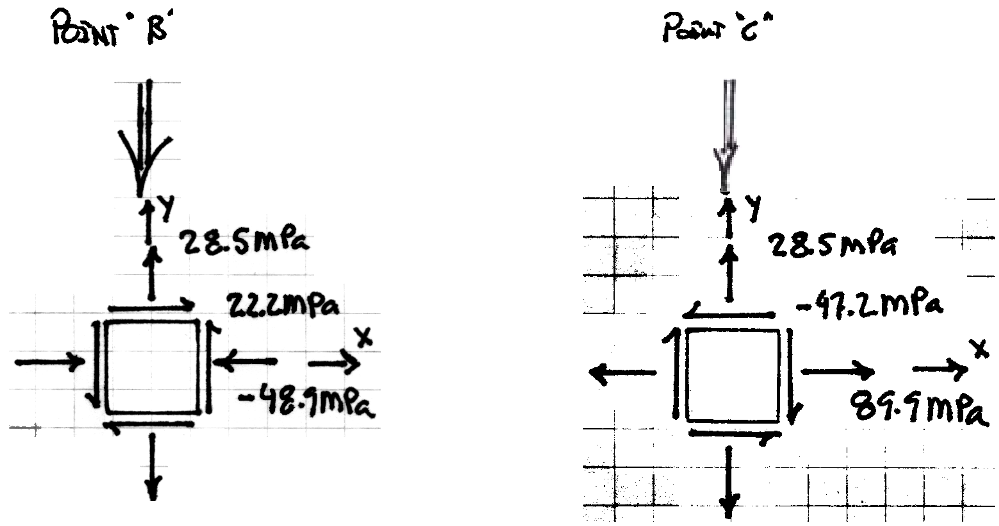
$$= -47.25(10^{6})\frac{N}{m^2} = \boxed{-47.25\,MPa}$$

At both points B and C the hoop stress, σ_y, can be calculated

$$\frac{P\cdot r}{t} = \frac{3\times10^{6}\frac{N}{m^2}\cdot(.019m)}{(.002m)} = 28.5\times10^{6}\frac{N}{m^2} = \boxed{28.5\,MPa}$$

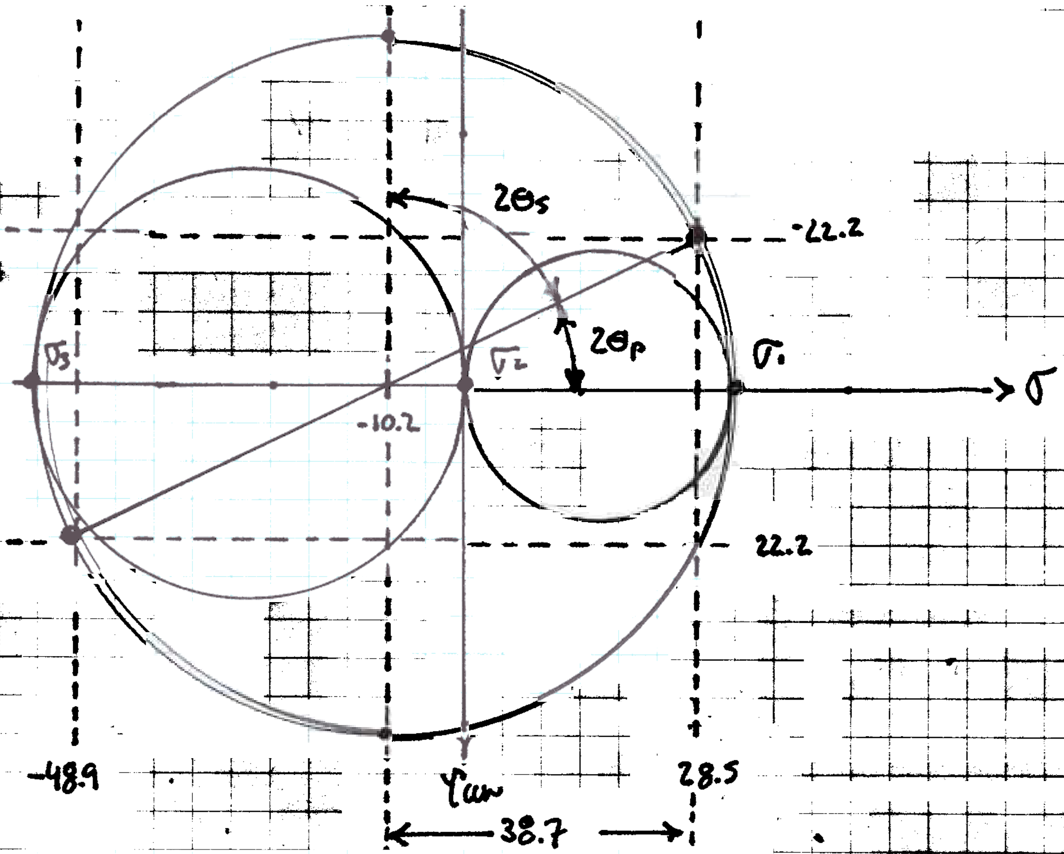
Now the stress cubes at "B" and "C" can be drawn





Mohr's circle will be used to determine the maximum normal (principal) and shearing stresses at these two points along with the orientation ~~the~~ ~~the~~ with the coordinate axes

Point B



$$r = \sqrt{(22.2\,\text{MPa})^2 + (48.9\,\text{MPa} - 10.2\,\text{MPa})^2} = \underline{44.62\ \text{MPa} = \tau_{max}}$$

$$\sigma_1 = -10.2 + 44.62\,\text{MPa} = \underline{34.42\,\text{MPa}}$$

$$\sigma_2 = 0$$

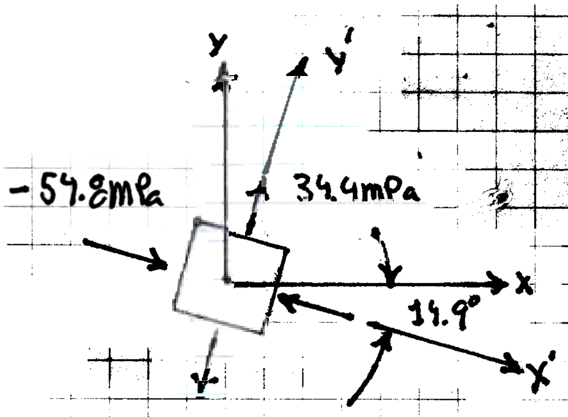
$$\sigma_3 = \underline{-54.82\ \text{MPa}}$$

$$2\theta_p = \tan^{-1}\left(\frac{22.2}{38.7}\right) = 29.84$$

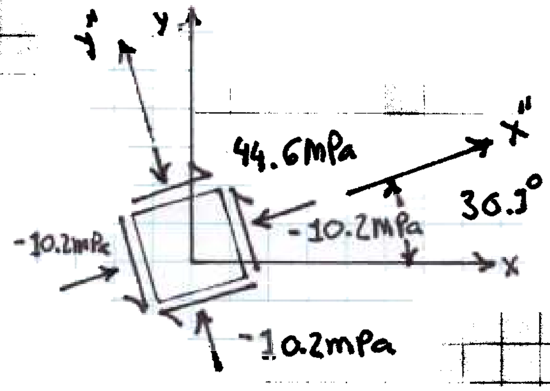
$$\underline{\theta_p = 14.92}$$

$$2\theta_s = \tan^{-1}\left(\frac{38.7}{22.2}\right) = 60.16$$

$$\underline{\theta_s = 30.08}$$

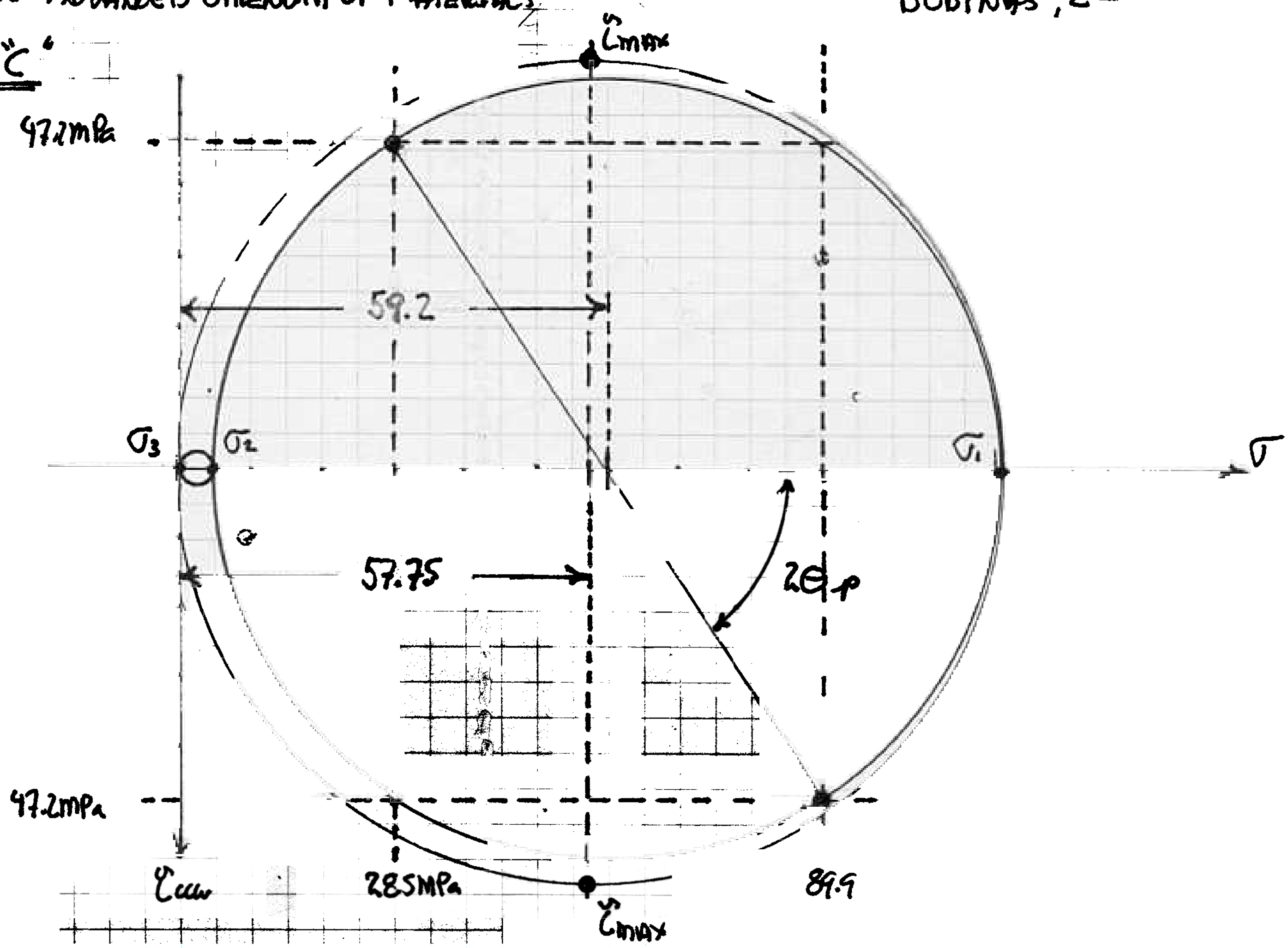


Principal Stress



Maximum Shear Stress

POINT "C"



$= \sqrt{(47.2\,MPa)^2 + (89.9\,MPa - 59.2\,MPa)^2} = 56.31\,MPa = \tau_{xy}^{max}$

$59.2\,MPa + 56.31\,MPa = \underline{115.51\,MPa}$

$2\theta_p = \tan^{-1}\left(\frac{47.2}{89.9 - 59.2}\right)$

$= 59.2\,MPa - 56.31\,MPa = \underline{2.89\,MPa}$

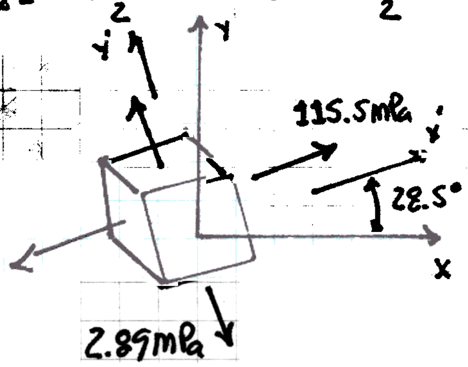
$= 56.96$

$= \underline{0}$

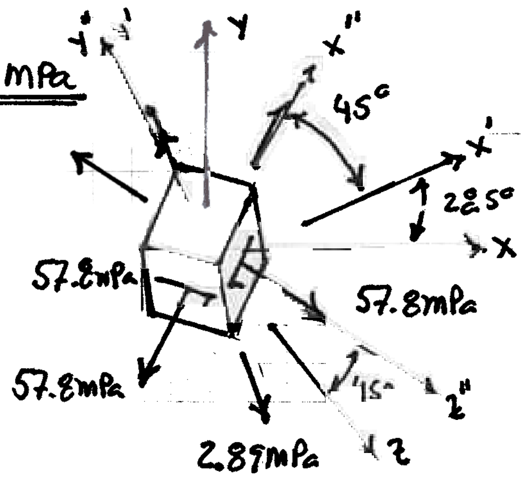
$\theta_p = \underline{28.48°}$

THE ABSOLUTE MAXIMUM SHEAR STRESS IS NOT FOUND IN THE X-Y PLANE, IT IS IN THE X-Z PLANE

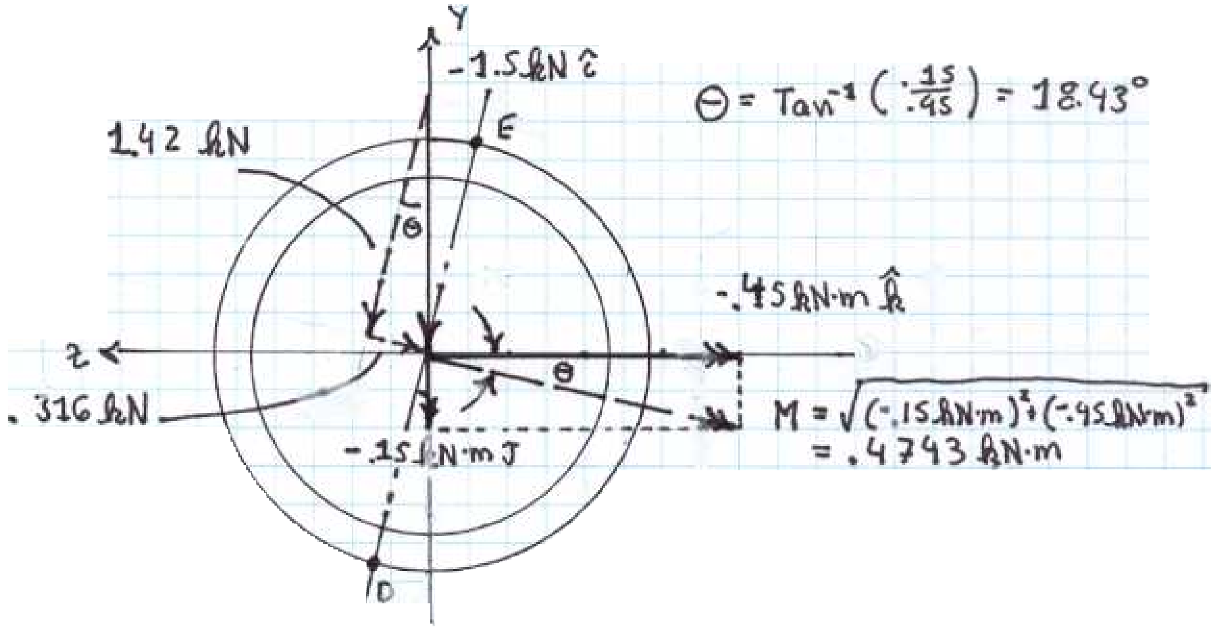
$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{115.5\,MPa - 0\,MPa}{2} = \underline{57.8\,MPa}$



PRINCIPAL STRESS



NOW WE NEED TO FIND WHERE THE NORMAL STRESS IS MAXIMUM. THIS IS FOUND BY COMBINING THE MOMENTS AND CONSIDERING THE STRESSES AT THE EXTREME POINTS PARALLEL TO THE RESULTANT



$$\Theta = \tan^{-1}\left(\frac{.15}{.45}\right) = 18.43^\circ$$

$$M = \sqrt{(-.15\,kN\cdot m)^2 + (-.45\,kN\cdot m)^2} = .4743\,kN\cdot m$$

THE MAXIMUM NORMAL STRESS WILL OCCUR A "E" BECAUSE THIS IS WHERE THE BENDING MOMENT, NORMAL FORCE, AND AXIAL PRESSURE ALL CAUSE POSITIVE STRESS. USING ①

$$\sigma_x^{(E)} = \frac{1.5\times10^3\,N}{.2388\times10^{-3}\,m^2} + \frac{.4734\times10^3\,N\cdot m \cdot .02m}{.04322\times10^{-6}\,m^4} + \frac{3\times10^6\,\frac{N}{m^2}\cdot(.019m)}{2\cdot .002m}$$

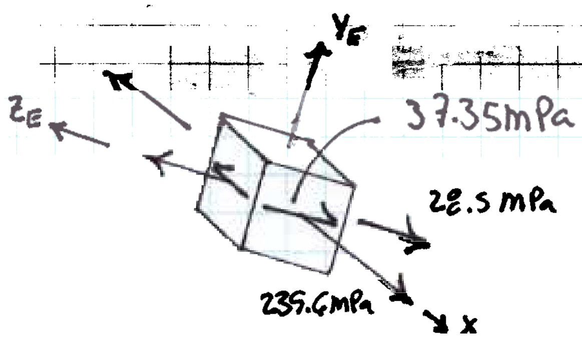
$$= \underline{\underline{239.6\ mPa}}$$

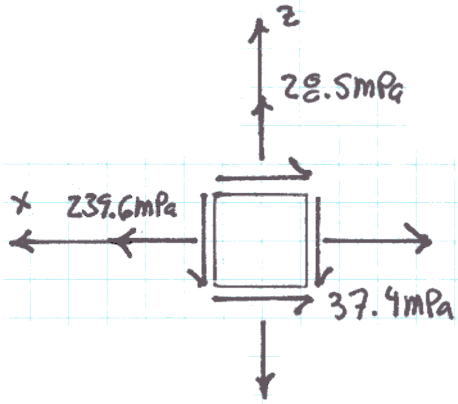
$$\tau^{(E)} = \frac{.316\times10^3\,N\cdot 1.445\times10^{-6}\,m^3}{.04322\times10^{-6}\,m^4\cdot 2\cdot .002m} + \frac{.15\times10^3\,N\cdot m\cdot 0.02m}{.08644\times10^{-6}\,m^4}$$

$$\underline{\underline{37.35\ mPa}}$$

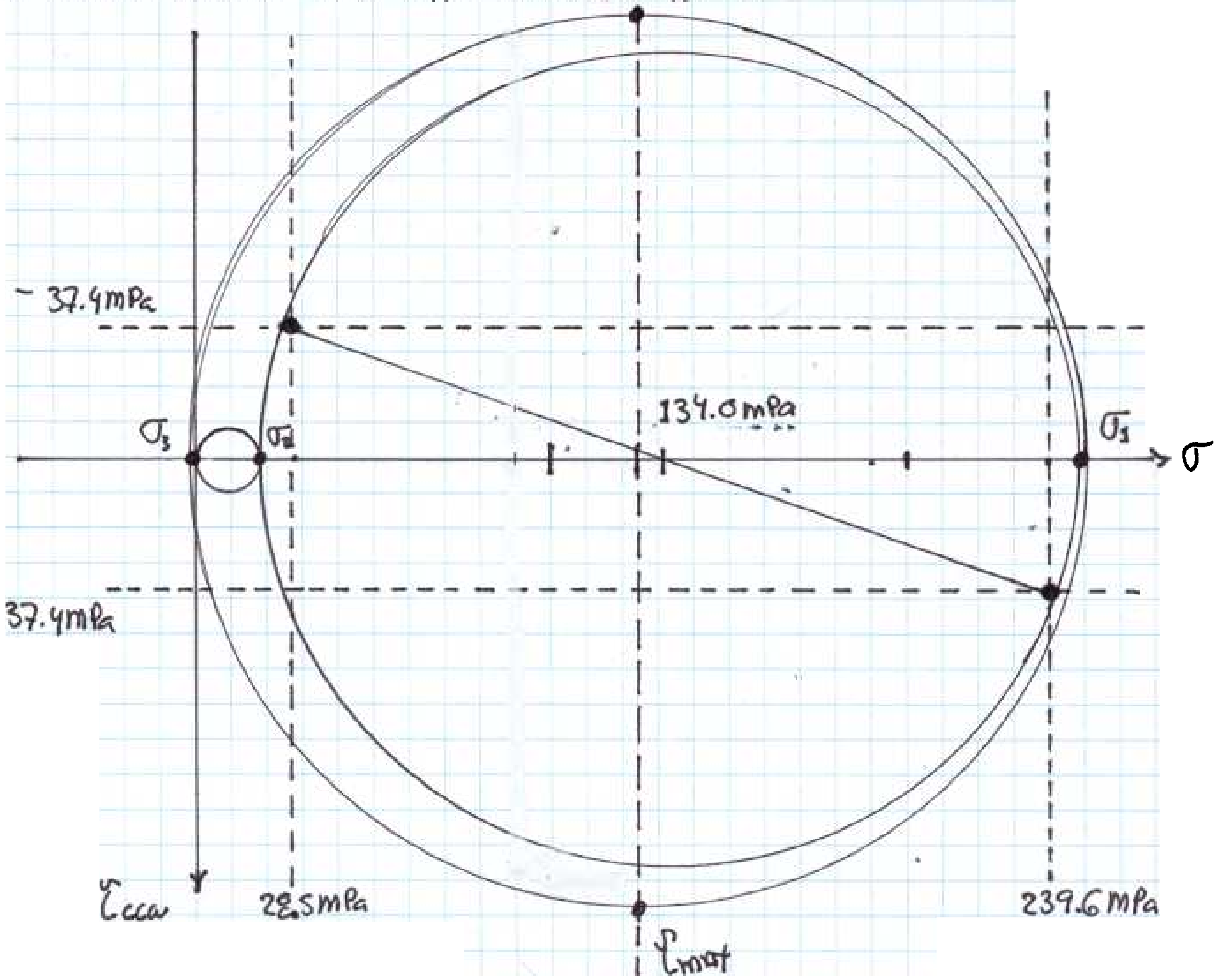
$$\sigma_H^{(E)} = \frac{3\times10^6\,\frac{N}{m^2}\cdot .019m}{.002m} = \underline{\underline{28.5\ mPa}} = \sigma_{zE}$$

NOW THE APPROPRIATE STRESS CUBE CAN BE CREATED





DRAWING MOHR'S CIRCLE FOR THIS ELEMENT AT "E"



$$r = \sqrt{(37.4\,MPa)^2 + (239.6\,MPa - 134.0\,MPa)^2} = 112.0\,MPa$$

$$\sigma_1 = 134.0\,MPa + 112.0\,MPa = \boxed{246\,MPa}$$

$$\sigma_2 = 134.0\,MPa - 112.0\,MPa = 22\,MPa$$

$$0$$

$$\tau_{max} = \frac{\sigma_3}{2} = \frac{246\,MPa}{2} = \boxed{123\,MPa}$$

SUMMARY:

WHEN EVALUATING STRESS IN A STRUCTURE IT IS IMPORTANT TO ALWAYS CONSIDER THE COMPLETE 3-D STATE OF STRESS. IF σ_1 AND σ_2 ARE BOTH POSITIVE, THE MAXIMUM SHEAR STRESS WILL NOT OCCUR IN THE SAME PLANE AS σ_1 & σ_2