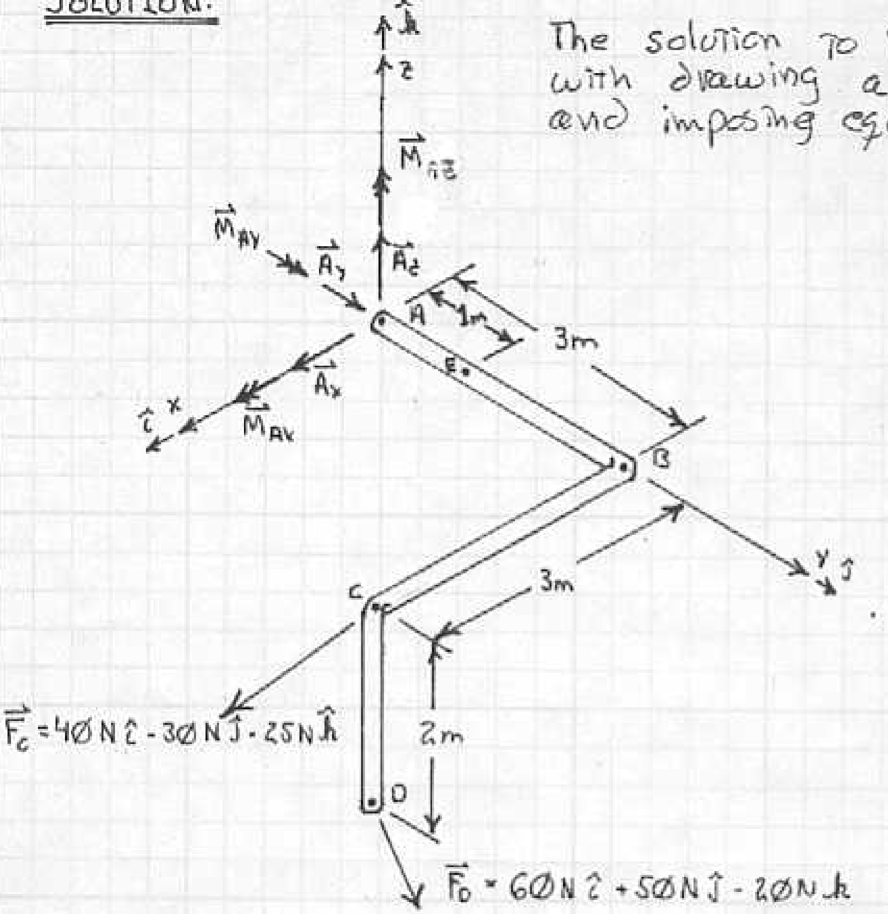
SOLUTION:

The solution to this problem starts with drawing a freebody diagram and imposing equilibrium



$$\Sigma\vec{F} = \emptyset = \vec{A} + \vec{F}_C + \vec{F}_D$$

$$\Sigma\vec{F}\cdot\hat{\imath} = \Sigma F_x = \emptyset = A_x + 40N + 60N \implies A_x = \boxed{-100N} \quad (1)$$

$$\Sigma\vec{F}\cdot\hat{\jmath} = \Sigma F_y = \emptyset = A_y - 30N + 50N \implies A_y = \boxed{-20N} \quad (2)$$

$$\Sigma\vec{F}\cdot\hat{k} = \Sigma F_z = \emptyset = A_z - 25N - 20N \implies A_z = \boxed{45N} \quad (3)$$

$$\Sigma\vec{M}_{/@A} = \vec{M}_A + \vec{r}_{AC}\times\vec{F}_C + \vec{r}_{AD}\times\vec{F}_D$$

$$\vec{r}_{AC} = 3m\cdot\hat{\imath} + 3m\cdot\hat{\jmath}$$

$$\vec{r}_{AD} = 3m\cdot\hat{\imath} + 3m\cdot\hat{\jmath} - 2m\cdot\hat{k}$$

$$\Sigma \vec{M}_{@A} = \vec{0} = \vec{M}_A + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3m & 3m & 0m \\ 40N & -30N & -25N \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3m & 3m & -2m \\ 60N & 50N & -20N \end{vmatrix}$$

$$= \vec{M}_A + [(3m)(-25N)]\cdot\hat{i} - [(3m)(-25N)]\hat{j} + [(3m)(-30N) - (3m)(40N)]\hat{k}$$

$$+ [(3m)(-20N) - (-2m)(50N)]\cdot\hat{i} - [(3m)(-20N) - (-2m)(60N)]\hat{j}$$

$$+ [(3m)(50N) - (3m)(60N)]\cdot\hat{k}$$

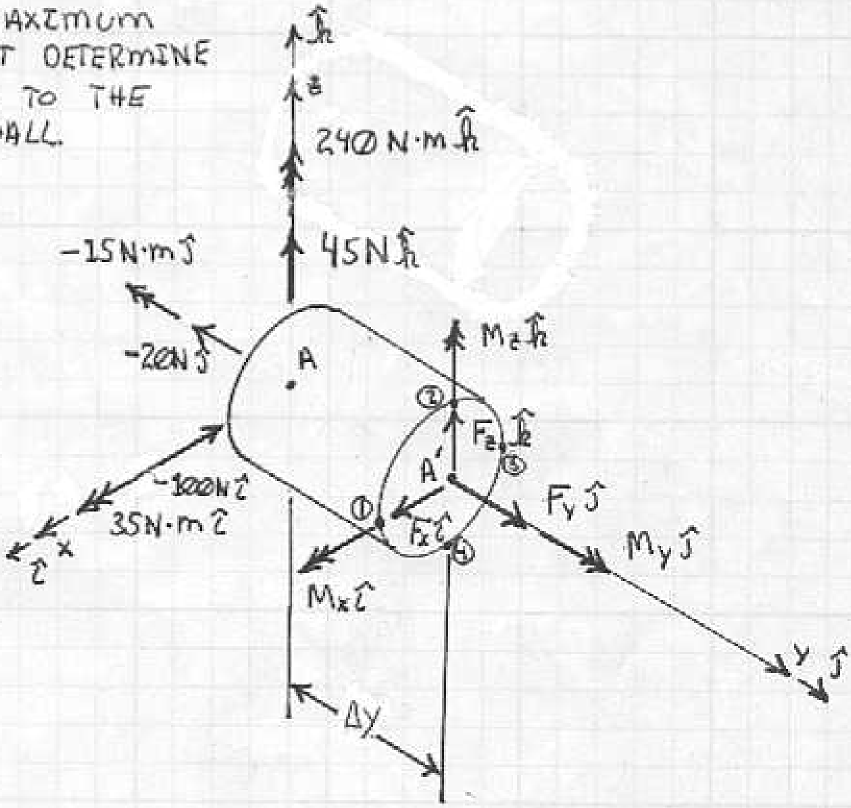
$$= \vec{M}_A - 35\ N\cdot m\ \hat{i} + 15\ N\cdot m\ \hat{j} - 240\ N\cdot m\ \hat{k}$$

$$\Rightarrow \boxed{\vec{M}_A = 35\ N\cdot m\ \hat{i} - 15\ N\cdot m\ \hat{j} + 240\ N\cdot m\ \hat{k}} \qquad (4)$$

NOW THAT THE REACTIONS AT THE WALL HAVE BEEN DETERMINED THE MAXIMUM STRESS IN THE STRUCTURE NEEDS TO BE DETERMINED. THE MAXIMUM STRESS WILL OCCUR AT THE WALL.

TO DETERMINE THE MAXIMUM STRESS WE MUST FIRST DETERMINE THE LOADS INTERNAL TO THE STRUCTURE AT THE WALL.

Let's consider THE FBD AS $\Delta Y \to 0$



$$\Sigma\vec{F} = \vec{\emptyset} = \vec{A} + \vec{F}_{A'}$$

$$\Sigma F_x = \emptyset = -100N + F_x \Rightarrow \underline{\underline{F_x = 100N}}$$

$$\Sigma F_y = \emptyset = -20N + F_y \Rightarrow \underline{\underline{F_y = 20N}}$$

$$\Sigma F_z = \emptyset = 45N + F_z \Rightarrow \underline{\underline{F_z = -45N}}$$

$$\Sigma \vec{M}/_{@A'} = \vec{\emptyset} = \vec{M}_A + \vec{M}_{A'} + \vec{r}_{A'A} \times \vec{F}_A$$

$$= \vec{M}_A + \vec{M}_{A'} + \underbrace{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \emptyset & -\Delta Y & \emptyset \\ -100N & -20N & 45N \end{vmatrix}}_{\substack{\Delta y \to \emptyset \\ \emptyset}}$$

$$\therefore \underline{\underline{\vec{M}_{A'} = -\vec{M}_A = -35\,N\cdot m\,\hat{i} + 15N\cdot m\,\hat{j} - 240\,N\cdot m\,\hat{j}}}$$

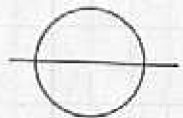
THE INTERNAL STRESS IN THIS STRUCTURE CAN NOW BE COMPUTED. THE STRESS IN THE VARIOUS REGIONS OF THIS STRUCTURE WILL BE DIFFERENT. LET'S CONSIDER THE STRESS AT POINTS ①, ②, ③, & ④. ASSUME THE CROSS-SECTION IS CIRCULAR AND THE RADIUS IS .1m (d=.2m)

①

$$\sigma_{y1} = \frac{20N}{\pi\cdot(.1m)^2} + \frac{(-240N\cdot m)\cdot(.1m)}{\pi(.1m)^4/4} - \frac{(-35N\cdot m)\cdot(\emptyset)}{\pi(.1m)^4/4}$$

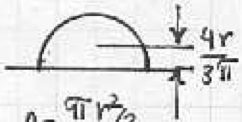
$$= 636.6\,N/m^2 - 305{,}577\tfrac{N}{m^2} = -304{,}900\,\tfrac{N}{m^2}$$

$$= -304.9(10^3)\,Pa = \underline{-304.9\ kPa}$$



$A = \pi r^2$
$I = J/_2 = \frac{\pi r^4}{4}$

$$\tau_{yz1} = \frac{(45N)\cdot(\frac{4}{3\pi})(.1m)(\frac{\pi}{2})(.1m)^2}{\frac{\pi}{4}\cdot(.1m)^4\cdot(.2m)} - \frac{(15\ N\cdot m)(.1m)}{\pi\cdot(.1m)^4/2}$$



$A = \pi r^2/_2$
$I = \emptyset.11\emptyset r^4$

$$= -1.909(10^3)\,Pa - 9.55(10^3)\,Pa$$

$$= \underline{-11.46\ kPa}$$

(2)

$$\sigma_{y2} = \frac{20\,N}{\pi\cdot(.1m)^2} + \frac{(-240\,N\cdot m)\cdot(0)}{\pi(.1m)^4/4} - \frac{(-35\,N\cdot m)\cdot(.1m)}{\pi\cdot(.1m)^4/4}$$

$$= 0.637\,kPa + 44.56\,kPa = \underline{\underline{45.20\ kPa}}$$

$$\tau_{xy2} = \frac{(100\,N)\cdot(\frac{4}{3\pi})\cdot(.1m)\cdot(\frac{\pi}{2})\cdot(.1)^2}{\frac{\pi}{4}\cdot(.1m)^4\cdot(.2m)} + \frac{(15\,N\cdot m)\cdot(.1m)}{\pi\cdot(.1m)^4/2}$$

$$= 4.244\ kPa + 9.549\,kPa = \underline{\underline{13.79\ kPa}}$$

(3)

$$\sigma_{y3} = \frac{20\,N}{\pi\cdot(.1m)^2} + \frac{(-240\,N\cdot m)\cdot(-.1m)}{\pi\ (.1m)^4/4} - \frac{(-35\,N\cdot m)\,(0)}{\pi\cdot(.1m)^4/4}$$

$$= 0.637\,kPa + 305.57\,kPa = \underline{\underline{306.2\ kPa}}$$

$$\tau_{yz3} = \frac{(-45\,N)\cdot(\frac{4}{3\pi})\cdot(.1m)\cdot(\frac{\pi}{2})\cdot(.1\,m)^2}{\frac{\pi}{4}\cdot(.1m)^4\cdot(.2m)} + \frac{(15\,N\cdot m)\,(.1m)}{\pi\cdot(.1m)^4/2}$$

$$= -1.910\ kPa + 9.55\ kPa = \underline{\underline{-7.640\ kPa}}$$

(4)

$$\sigma_{y4} = \frac{20\,N}{\pi\cdot(.1m)^2} + \frac{(-240\,N\cdot m)\cdot(0)}{\pi(.1m)^4/4} - \frac{(-35\,N\cdot m)\cdot(-.1m)}{\pi\cdot(.1m)^4/4}$$

$$= 0.637\,kPa - 44.56\,kPa = \underline{\underline{-43.92\,kPa}}$$

$$\tau_{xy4} = \frac{(100\,N)\cdot(\frac{4}{3\pi})\,(.1m)\cdot(\frac{\pi}{2})\cdot(.1)^2}{\frac{\pi}{4}\cdot(.1m)^4\cdot(.2m)} - \frac{(15\,N\cdot m)\cdot(.1m)}{\pi\cdot(.1m)^4/2}$$

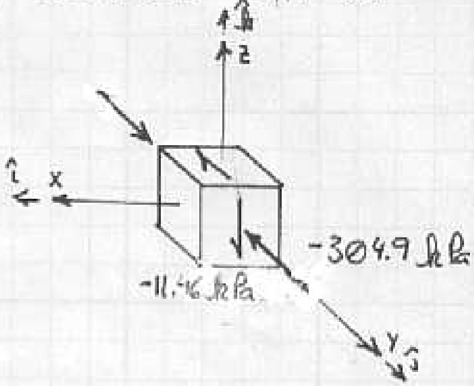
$$= 4.244\ kPa - 9.55\,kPa = \underline{\underline{-5.306\ kPa}}$$

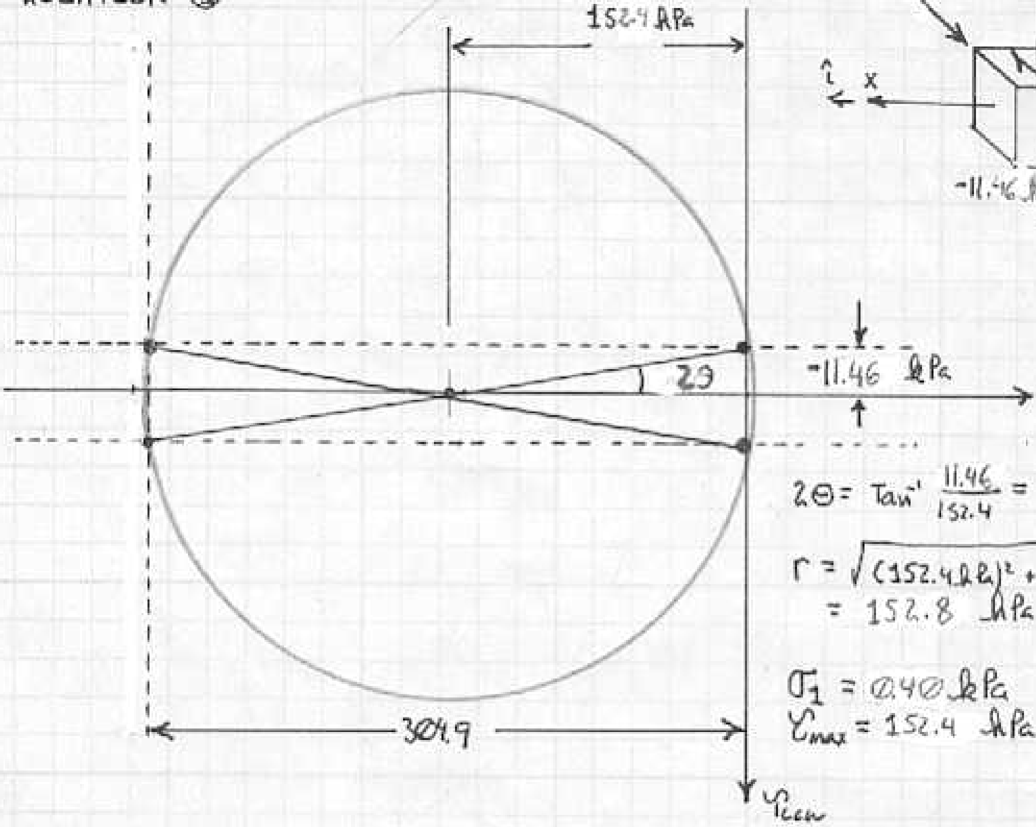
Which Point has the highest stress-state?

Is there another point, on the structure where the stress is maximum?

THE MAXIMUM STRESS AT EACH OF THESE LOCATIONS CAN BE DETERMINED USING MOHR'S CIRCLE

LOCATION ①



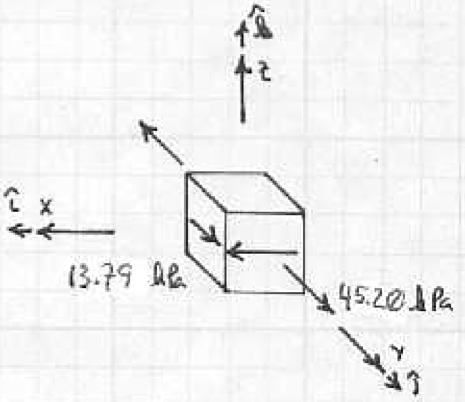


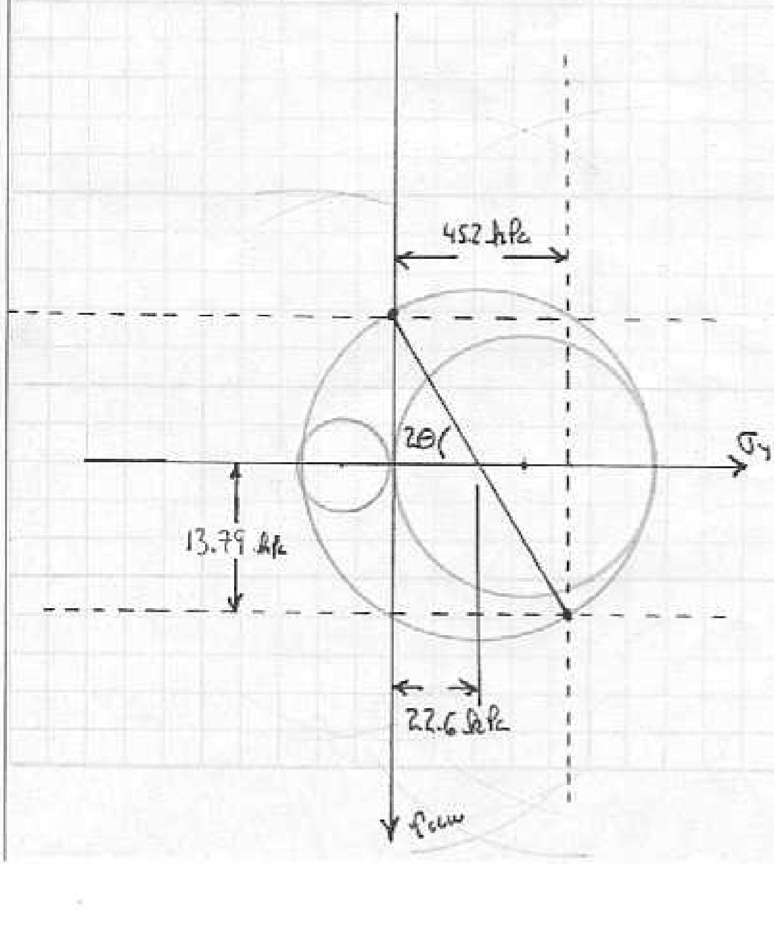
$2\Theta = \tan^{-1}\frac{11.46}{152.4} = 4.30^\circ$

$r = \sqrt{(152.4\,kPa)^2 + (11.46\,kPa)^2}$
$= 152.8\ kPa$

$\sigma_1 = 0.40\ kPa \qquad \sigma_2 = -305.2\ kPa$
$\tau_{max} = 152.4\ kPa$

LOCATION ②





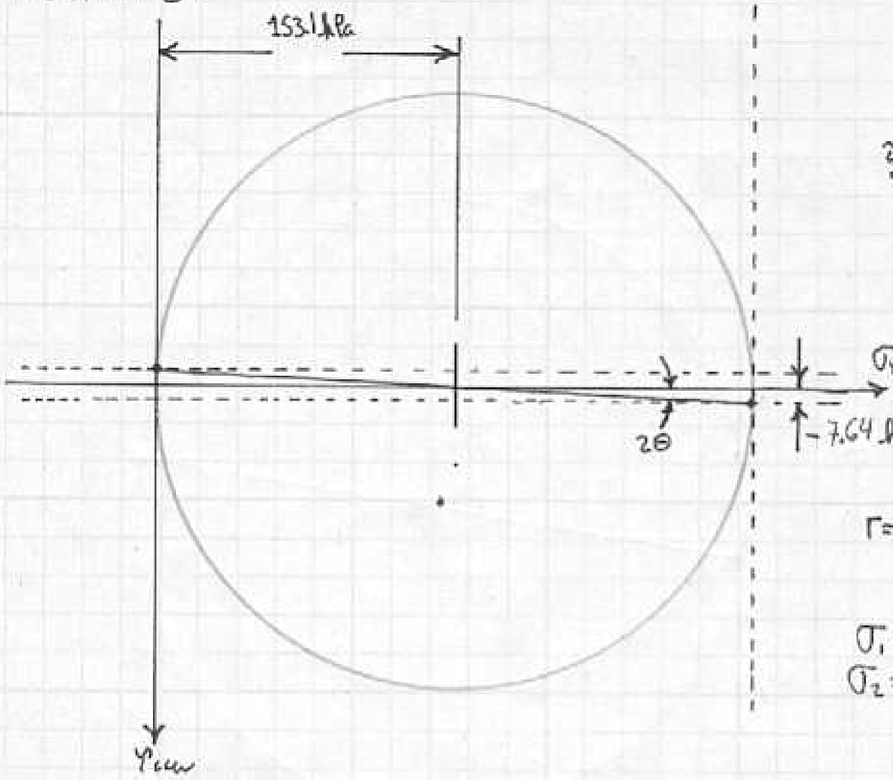
$2\Theta = \tan^{-1}\frac{13.79}{22.6} = 31.39$
$\Theta = 15.70$

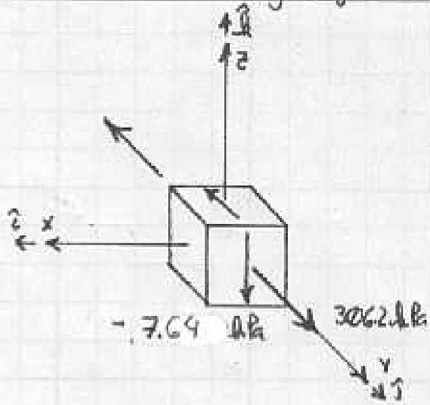
$r = \tau_{max} = \sqrt{(39.85\,kPa)^2 + (13.79\,kPa)^2}$
$= 42.17\ kPa$

$\sigma_1 = 64.77\ kPa$

$\sigma_2 = -19.57\ kPa$

Location (3)





$$2\theta = \tan^{-1}\frac{7.64}{153.1} = 2.86^\circ$$

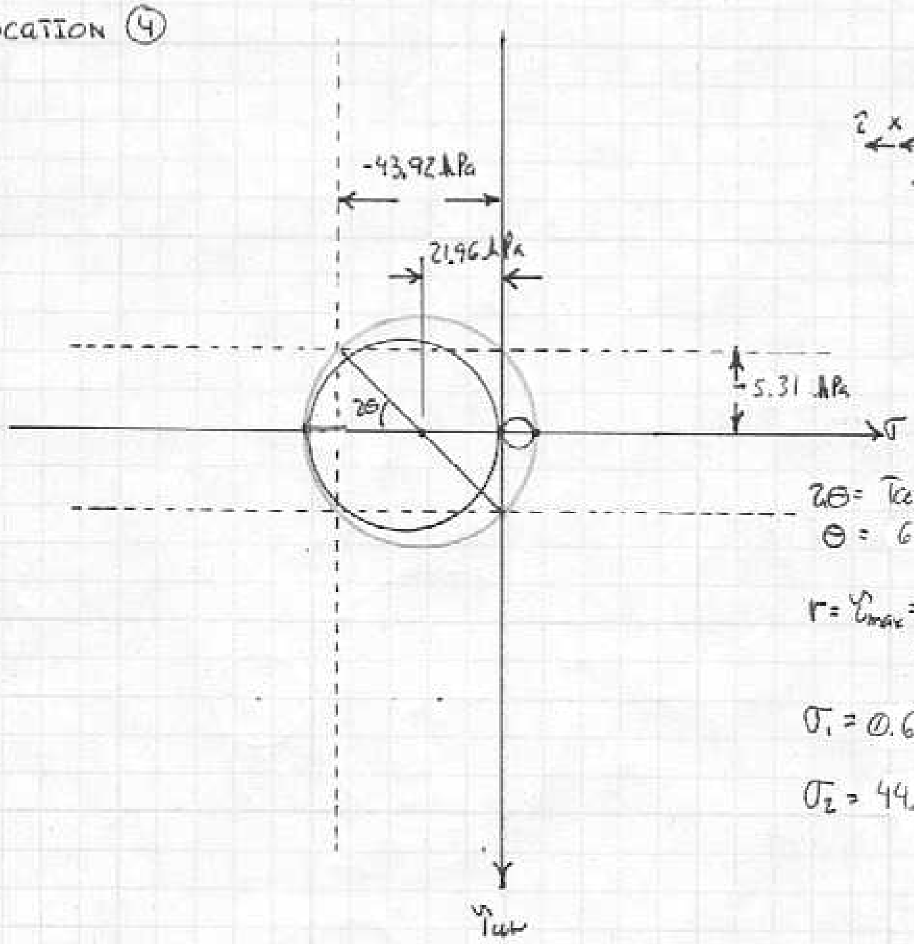
$$\theta = 1.43^\circ$$

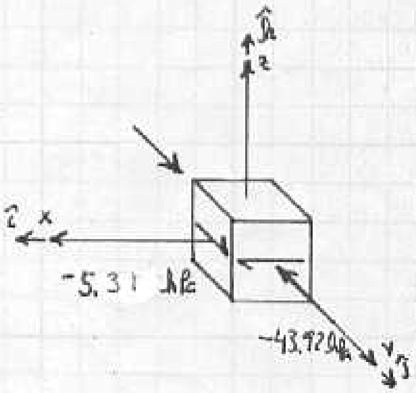
$$r = \tau_{max} = \sqrt{(153.1\ kPa)^2 + (7.64\ kPa)^2}$$
$$= 153.3\ kPa$$

$\sigma_1 = 306.4$ kPa

$\sigma_2 = -0.2$ kPa

Location (4)





$$2\theta = \tan^{-1}\frac{5.31}{21.96} = 13.59$$

$$\theta = 6.80$$

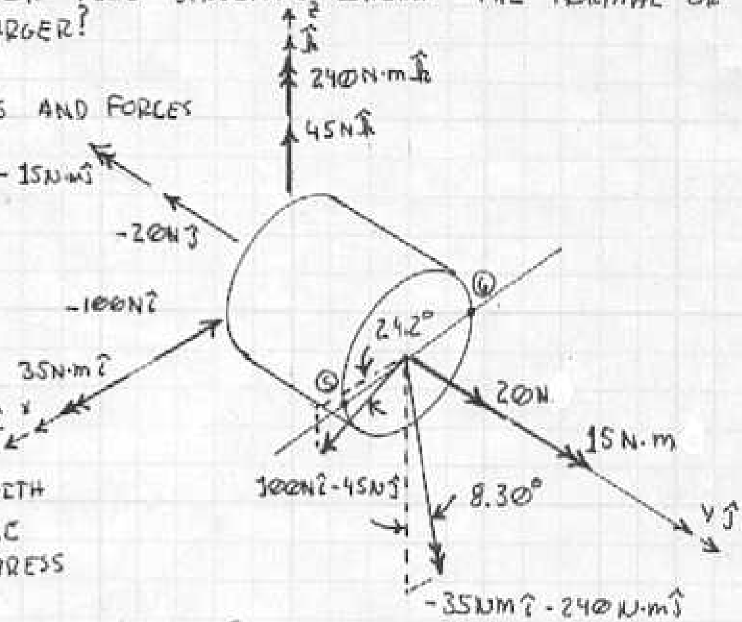
$$r = \tau_{max} = \sqrt{(21.96\ kPa)^2 + (5.31\ kPa)^2}$$
$$= 22.59\ kPa$$

$\sigma_1 = 0.63$ kPa

$\sigma_2 = 44.55$ kPa

WE SEE THAT THE MAXIMUM TENSILE STRESS IS AT LOCATION (3), THE MAXIMUM COMPRESSIVE STRESS is AT LOCATION (1), AND THE maximum SHEAR STRESS IS AT LOCATION (2).

ARE THERE OTHER POINTS ON THIS STRUCTURE WHERE THE NORMAL OR SHEARING STRESSES ARE LARGER?



COMBINING THE MOMENTS AND FORCES IN THE X-Z PLANE WE SEE THAT AT POINTS (5) AND (6) THE NORMAL STRESS DUE TO BENDING WILL BE MAXIMUM. AT THESE POINTS THE INTERNAL LOAD IN THE Y-DIRECTION CONTRIBUTES TO THE NORMAL STRESS, AND THE TORQUE IN THE Y-DIRECTION ALONG WITH THE FORCES IN THE X-Z PLANE CONTRIBUTE TO THE SHEAR STRESS

THE STATE OF STRESS AT POINTS (5) and (6) ARE GOING TO BE MUCH GREATER THAN AT POINTS (7) AND (8).

THE NORMAL STRESS CAN BE COMPUTED USING

$$\sigma_y = \frac{P_y}{A} + \frac{M_{z'} \cdot X'}{I_{z'z'}}$$

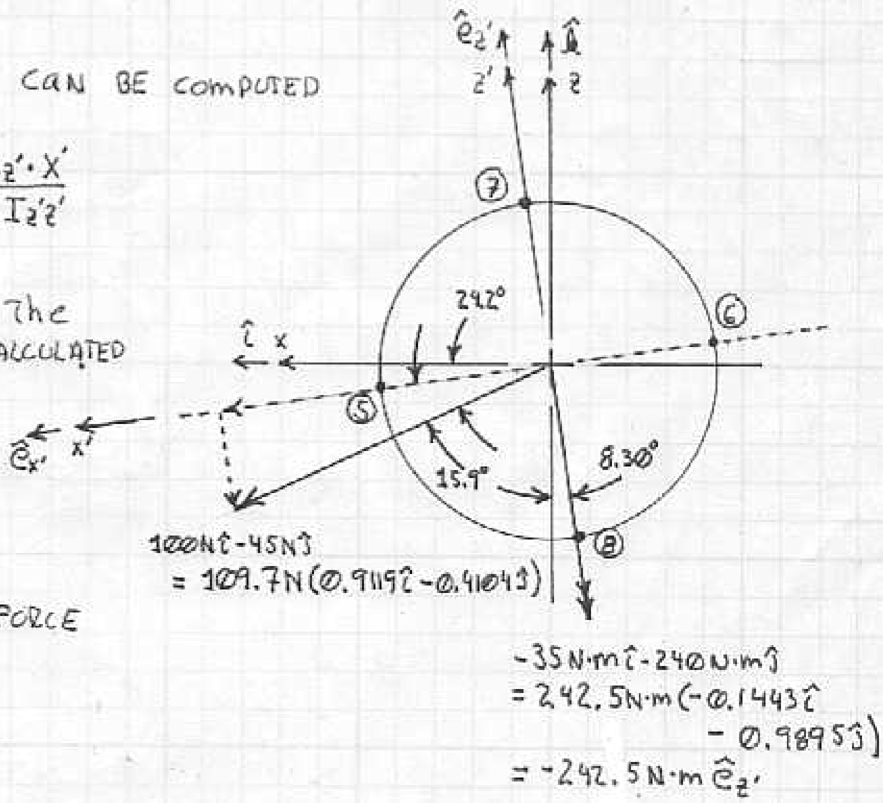
The components of the SHEAR STRESS ARE CALCULATED

1) DUE TO TORQUE

$$\tau = \frac{T \cdot r}{J}$$

2) DUE TO SHEAR FORCE

$$\tau = \frac{V \cdot Q}{I \cdot t}$$



AT POINTS ⑤ AND ⑥, IT IS NECESSARY TO DECOMPOSE THE SHEAR FORCE INTO COMPONENTS ALONG THE X′ AN Z′ COORDINATES.

$$\vec{V} = 100N\,\hat{i} - 45N\,\hat{j} = 109.7N(0.9119\,\hat{i} - 0.4104\,\hat{j})$$

$$V_{x'} = \vec{V}\cdot\hat{e}_{x'} = (100N\,\hat{i} - 45N\,\hat{k})\cdot(0.9895\,\hat{i} - 0.1443\,\hat{j})$$
$$= 105.4\,N$$

$$V_{z'} = \vec{V}\cdot\hat{e}_{y'} = (100N\,\hat{i} - 45N\,\hat{k})\cdot(0.1443\,\hat{i} + 0.9895\,\hat{j})$$
$$= -30.09\,N$$

$$\vec{V} = 105.4N\,\hat{e}_{x'} - 30.09N\,\hat{e}_{z'} = 109.7N(0.9616\,\hat{e}_{x'} - 0.2745\,\hat{e}_{z'})$$

THE COMPONENT OF THE SHEAR FORCE IN THE X′ DIRECTION WILL NOT GIVE RISE TO A SHEAR STRESS BECAUSE THERE IS NO SHEAR STRESS ON A FREE SURFACE.

THE COMPONENT OF THE SHEAR FORCE IN THE Z′ DIRECTION DOES GIVE RISE TO SHEAR STRESSES

$$\tau_{yz'} = \frac{V_{z'}\,Q}{I\,t} = \frac{(-30.09N)\cdot\frac{4\cdot(.1m)}{3\pi}\cdot\frac{\pi\,(.1m)^2}{2}}{\frac{\pi\cdot(.1m)^4}{4}\cdot(.2m)} = -1.277\,kPa$$

THE COMPONENT OF THE SHEAR STRESS DUE TO THE TORQUE IS GIVEN BY

$$\tau_{yz'} = \frac{T\cdot r}{J} = \frac{(15\,N\cdot m)\cdot(.1m)}{\frac{\pi\cdot(.1)^4}{4}} = 19.10\,kPa$$

THE SHEAR STRESS LIKE THE NORMAL STRESS AT LOCATIONS ⑤ & ⑥ WILL NOT BE THE SAME. LET'S CALCULATE THE COMPLETE STATE OF STRESS AT THESE POINTS.

LOCATION ⑤

$$\sigma_y = \frac{P}{A} + \frac{M_{z'}\cdot x'}{I_{z'z'}} = \frac{20N}{\pi\cdot(.1m)^2} + \frac{(-242.5\,N\cdot m)\cdot(.1m)}{\frac{\pi\cdot(.1m)^4}{4}}$$

$$= \underline{-308.1\,kPa}$$

$$\tau_{yz'} = -1.277\,kPa - 19.10\,kPa = \underline{-20.34\,kPa}$$

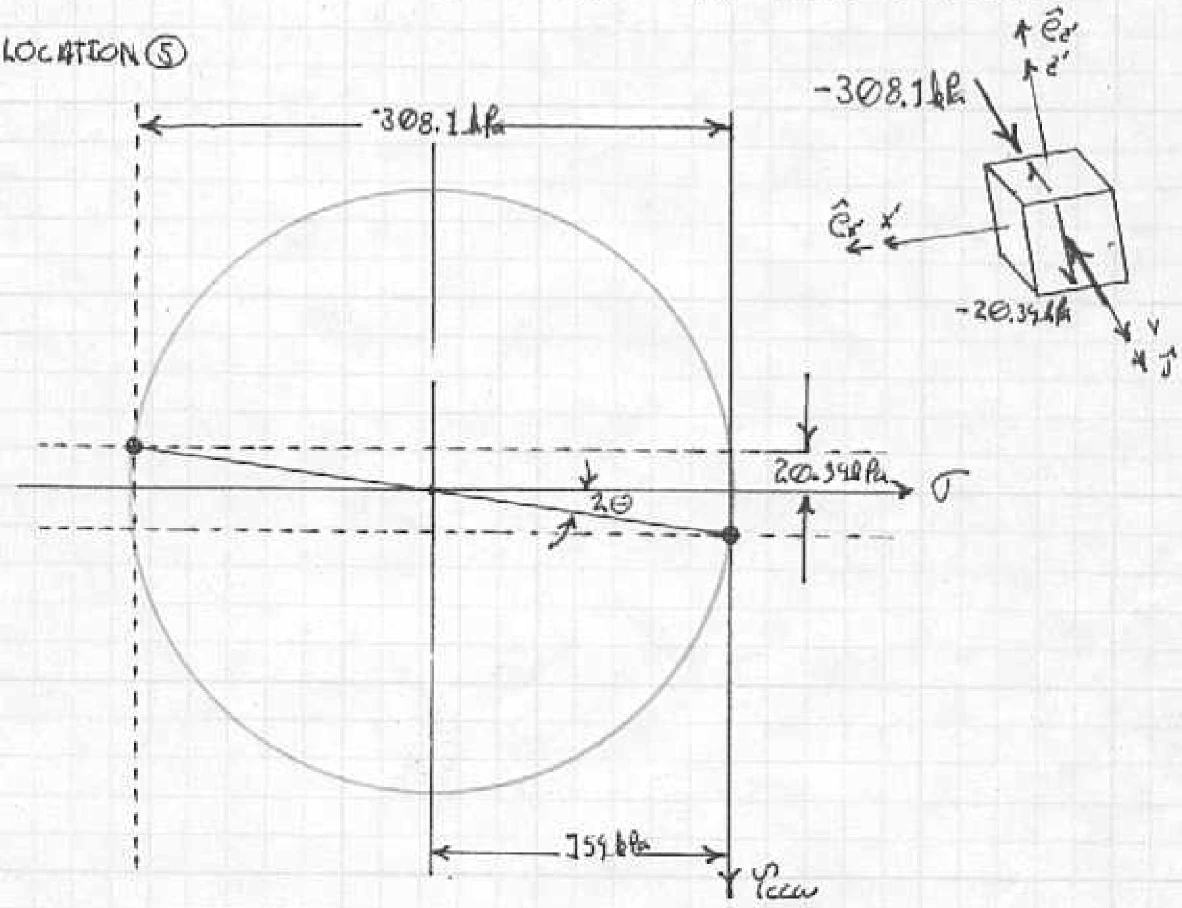
LOCATION ⑥

$$\sigma_y = \frac{20N}{\pi\cdot(.1m)^2} + \frac{(-242.5\,N\cdot m)\cdot(.1m)}{\frac{\pi\cdot(.1m)^4}{4}} = \underline{\underline{309.4\,kPa}}$$

$$\tau_{yz'} = -1.277\,kPa + 19.10\,kPa = \underline{\underline{17.82\,kPa}}$$

NOW LET'S FIND THE MAXIMUM STRESS AT THESE LOCATIONS.

LOCATION ⑤



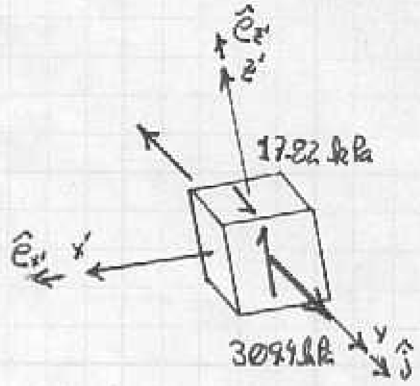
$$2\theta = \tan^{-1}\frac{20.34}{154} = 7.52° \Rightarrow \underline{\underline{\theta = 3.76°}}$$

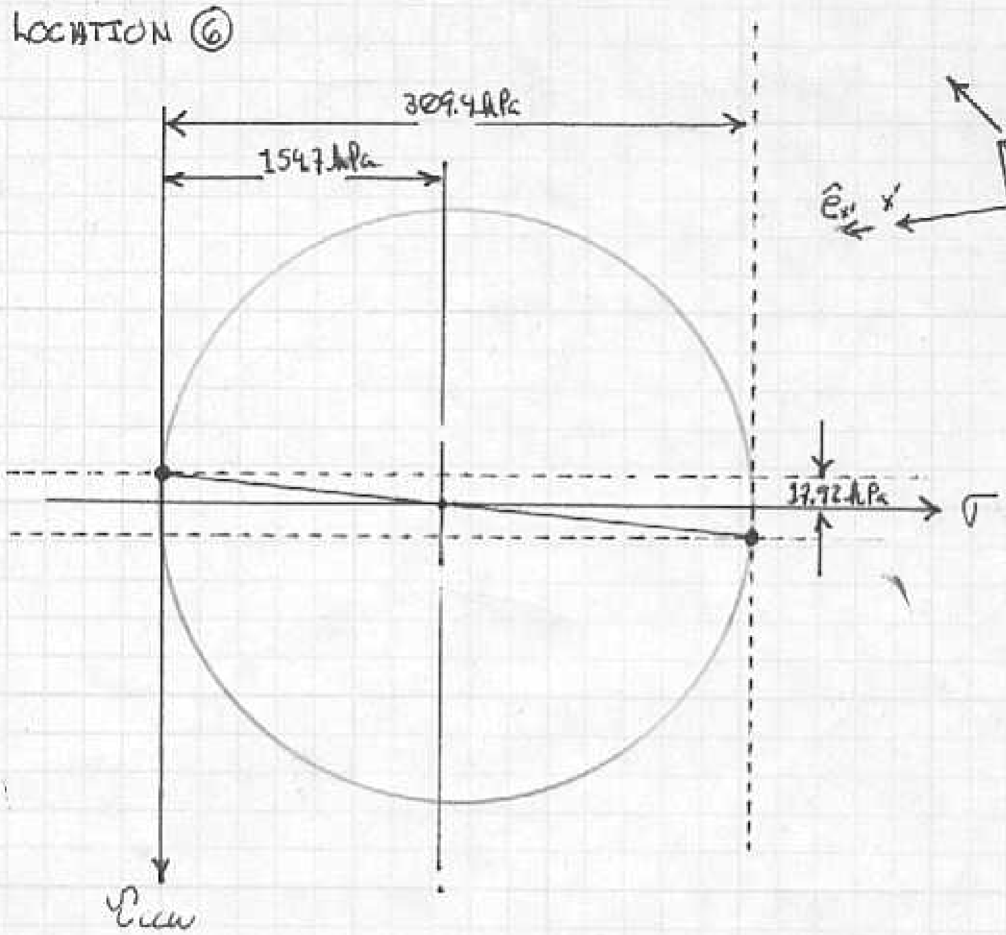
$$r = \tau_{max} = \sqrt{(20.34\,kPa)^2 + (154\,kPa)^2} = \underline{\underline{155.3\,kPa}}$$

$$\sigma_1 = \underline{-309.3\,kPa}$$

$$\sigma_2 = \underline{1.3\,kPa}$$

LOCATION 6





$$2\theta = \tan^{-1}\frac{17.92}{154.7} = 6.61^\circ \implies \underline{\underline{\theta = 3.30^\circ}}$$

$$r = \sqrt{(154.7\,kPa)^2 + (17.92\,kPa)^2} = \underline{\underline{155.7\,kPa}} = \tau_{max}$$

$$\sigma_1 = \underline{\underline{310.4\,kPa}}$$

$$\sigma_2 = \underline{\underline{-1\,kPa}}$$