MER311: Advanced Strength of Materials

Failure Criteria

Design Factors or Factors of Safety

- **Factor of Safety**

$$n = \frac{\text{Allowable Stress (strength)}}{\text{Calculated Stress}} = \frac{S}{\sigma}$$

- **Margin of Safety**

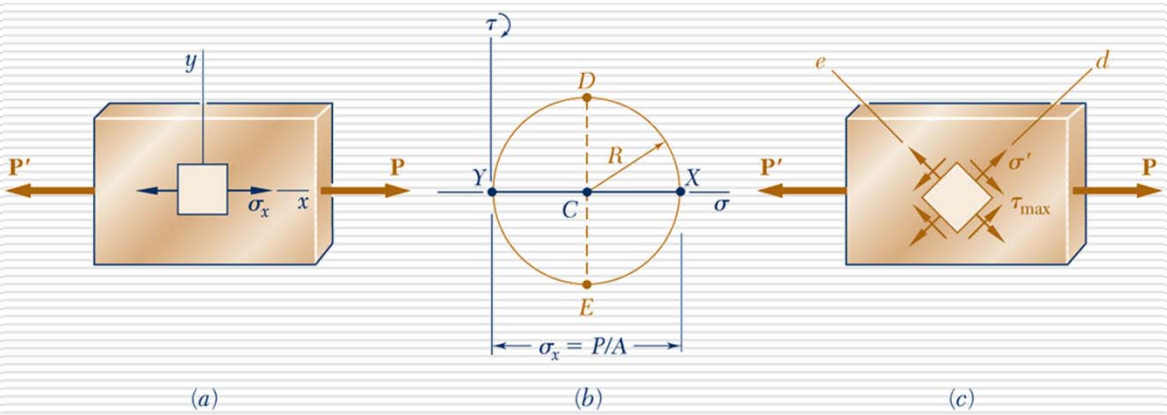
$$m = n - 1$$

Material Properties

Material	Specific Weight, lb/in^3	Ultimate Strength			Yield Strength[3]		Modulus of Elasticity, 10^6 psi	Modulus of Rigidity, 10^6 psi
		Tension, ksi	Compression,[2] ksi	Shear, ksi	Tension, ksi	Shear, ksi		
Steel								
Structural (ASTM-A36)	0.284	58			36	21	29	11.2
High-strength-low-alloy								
ASTM-A709 Grade 50	0.284	65			50		29	11.2
ASTM-A913 Grade 65	0.284	80			65		29	11.2
ASTM-A992 Grade 50	0.284	65			50		29	11.2
Quenched & tempered								
ASTM-A709 Grade 100	0.284	110			100		29	11.2
Stainless, AISI 302								
Cold-rolled	0.286	125			75		28	10.8
Annealed	0.286	95			38	22	28	10.8
Reinforcing Steel								
Medium strength	0.283	70			40		29	11
High strength	0.283	90			60		29	11
Cast Iron								
Gray Cast Iron								
4.5% C, ASTM A-48	0.260	25	95	35			10	4.1
Malleable Cast Iron								
2% C, 1% Si, ASTM A-47	0.264	50	90	48	33		24	9.3
Aluminum								
Alloy 1100-H14 (99% Al)	0.098	16		10	14	8	10.1	3.7
Alloy 2014-T6	0.101	66		40	58	33	10.9	3.9
Alloy 2024-T4	0.101	68		41	47		10.6	
Alloy 5456-H116	0.095	46		27	33	19	10.4	
Alloy 6061-T6	0.098	38		24	35	20	10.1	3.7
Alloy 7075-T6	0.101	83		48	73		10.4	4
Copper								
Oxygen-free copper (99.9% Cu)								
Annealed	0.322	32		22	10		17	6.4
Hard-drawn	0.322	57		29	53		17	6.4
Yellow-Brass								

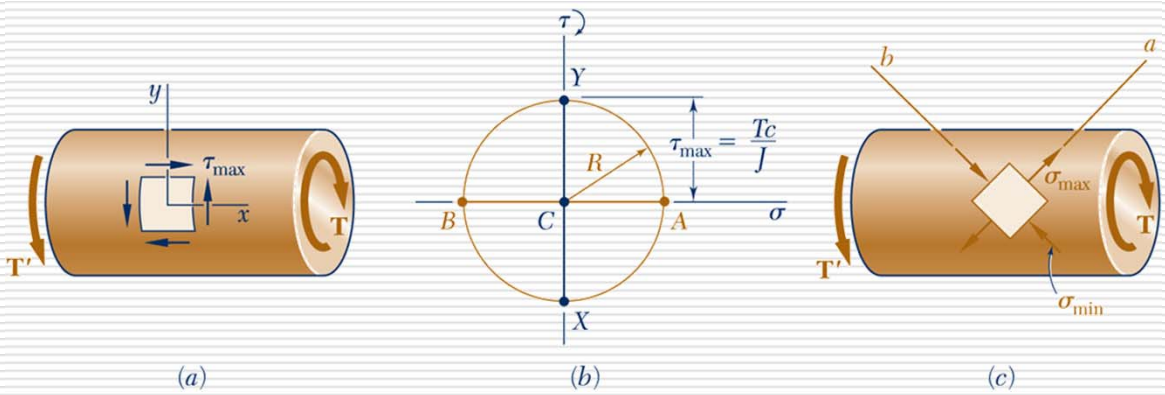
Material	Density kg/m^3	Ultimate Strength			Yield Strength[3]		Modulus of Elasticity, GPa	Modulus of Rigidity, GPa
		Tension, MPa	Compression,[2] MPa	Shear, MPa	Tension, MPa	Shear, MPa		
Steel								
Structural (ASTM-A36)	7860	400			250	145	200	77.2
High-strength-low-alloy								
ASTM-A709 Grade 345	7860	450			345		200	77.2
ASTM-A913 Grade 450	7860	550			450		200	77.2
ASTM-A992 Grade 345	7860	450			345		200	77.2
Quenched & tempered								
ASTM-A709 Grade 690	7860	760			690		200	77.2
Stainless, AISI 302								
Cold-rolled	7920	860			520		190	75
Annealed	7920	655			260	150	190	75
Reinforcing Steel								
Medium strength	7860	480			275		200	77
High strength	7860	620			415		200	77
Cast Iron								
Gray Cast Iron								
4.5% C, ASTM A-48	7200	170	655	240			69	28
Malleable Cast Iron								
2% C, 1% Si, ASTM A-47	7300	345	620	330	230		165	65
Aluminum								
Alloy 1100-H14 (99% Al)	2710	110		70	95	55	70	26
Alloy 2014-T6	2800	455		275	400	230	75	27
Alloy-2024-T4	2800	470		280	325		73	
Alloy-5456-H116	2630	315		185	230	130	72	
Alloy 6061-T6	2710	260		165	240	140	70	26
Alloy 7075-T6	2800	570		330	500		72	28
Copper								
Oxygen-free copper (99.9% Cu)								
Annealed	8910	220		150	70		120	44
Hard-drawn	8910	390		200	265		120	44
Yellow-Brass								

Example



Let P=5000lb and the cross-sectional area = 0.2 in^2. What is the Factor of Safety and Margin of Safety?

Example

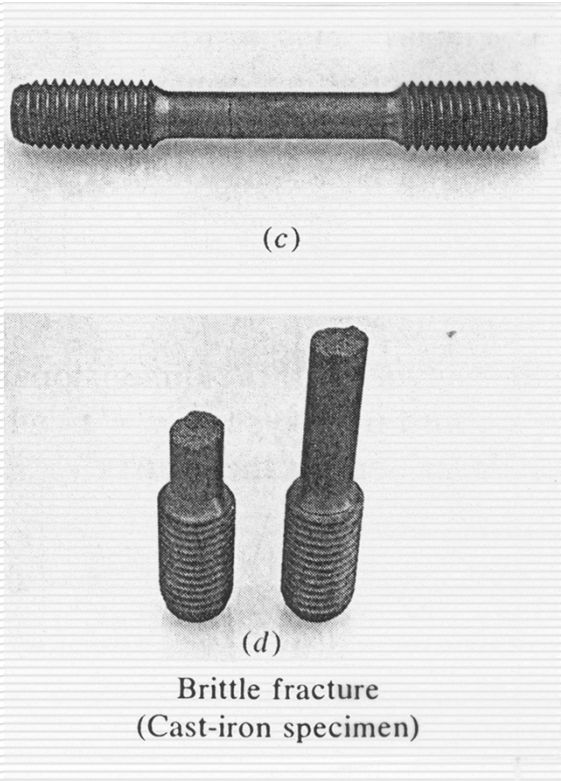


Let T=200 ft-lb and r= 0.2 in ($J=2.51 \times 10^{-3}$ in^4)
What is the Factor of Safety and Margin of Safety?

Unacceptable Deformation Under Load w/ or w/o Fracture

- **Excessive Elastic Deflection**
  - Stretching, Twisting, or Bending
  - Buckling
  - Vibration
- **Yielding**
  - Plastic Deformation @ Room Temp
  - Creep at Elevated Temperatures
- **Fracture**
  - Sudden Fracture of Brittle Materials
  - Fatigue (Progressive Fracture)
  - Stress Rupture at Elevated Temperatures

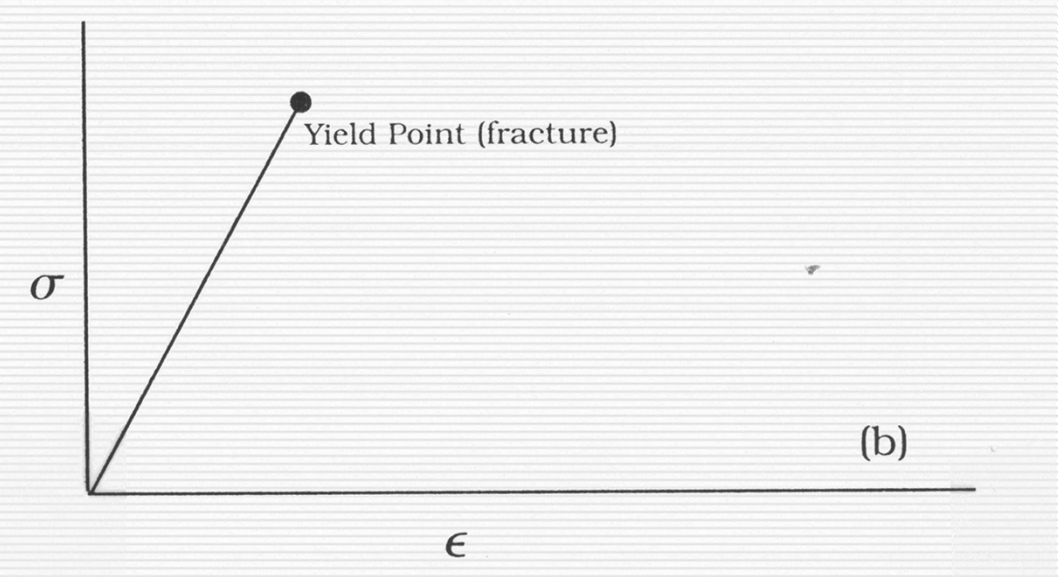
Typical σ-ε Curves - Brittle



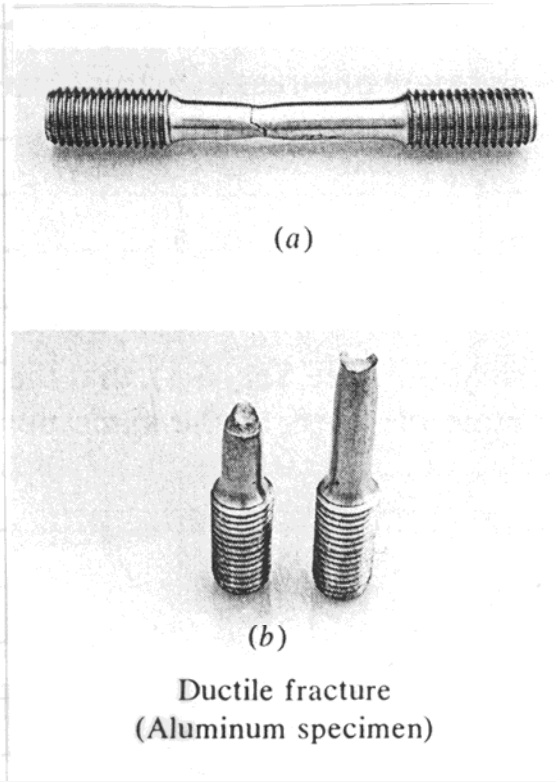
(c)

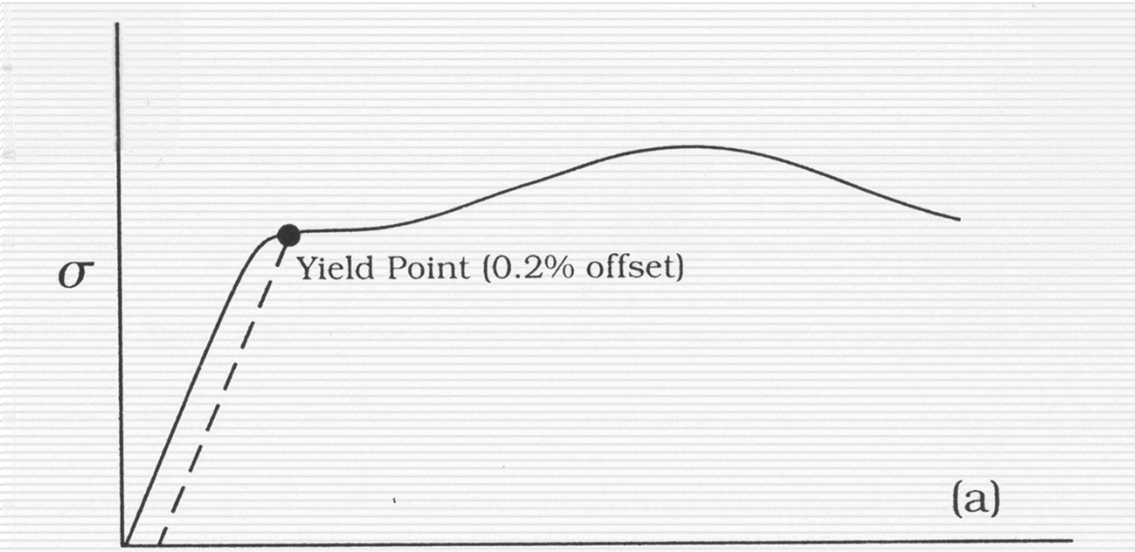
(d)

Brittle fracture
(Cast-iron specimen)

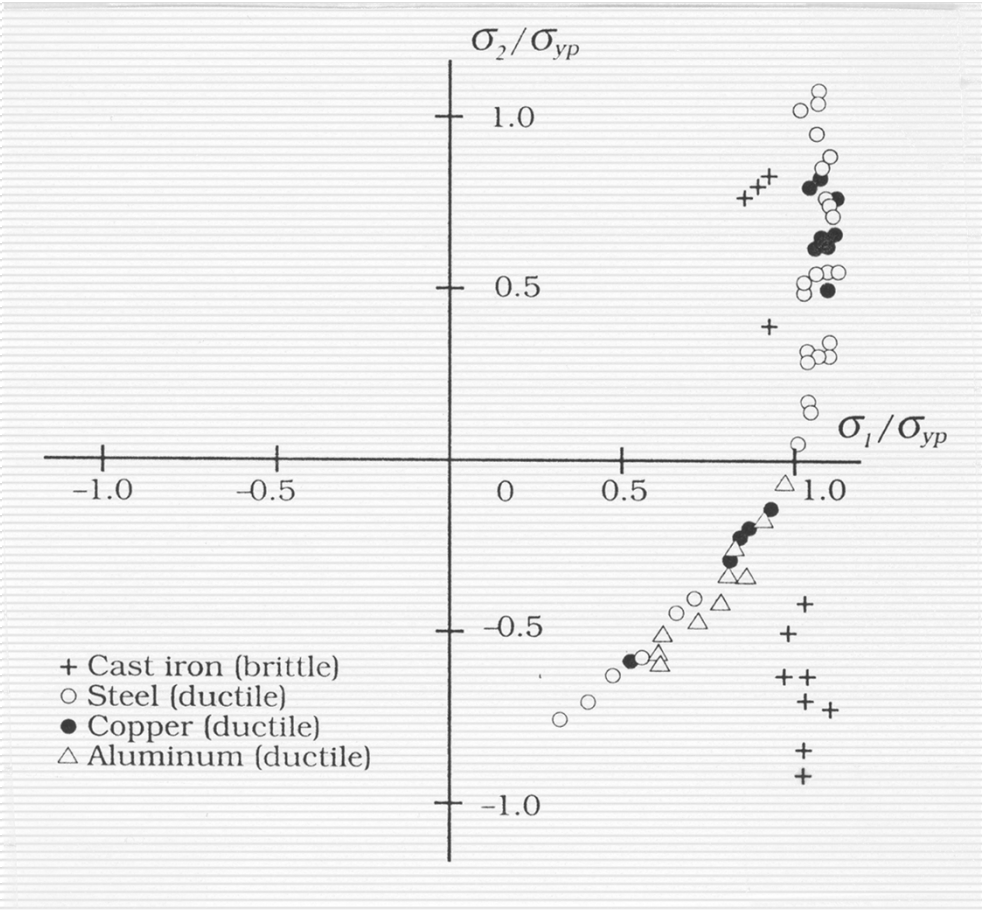


Typical σ-ε Curves - Ductile

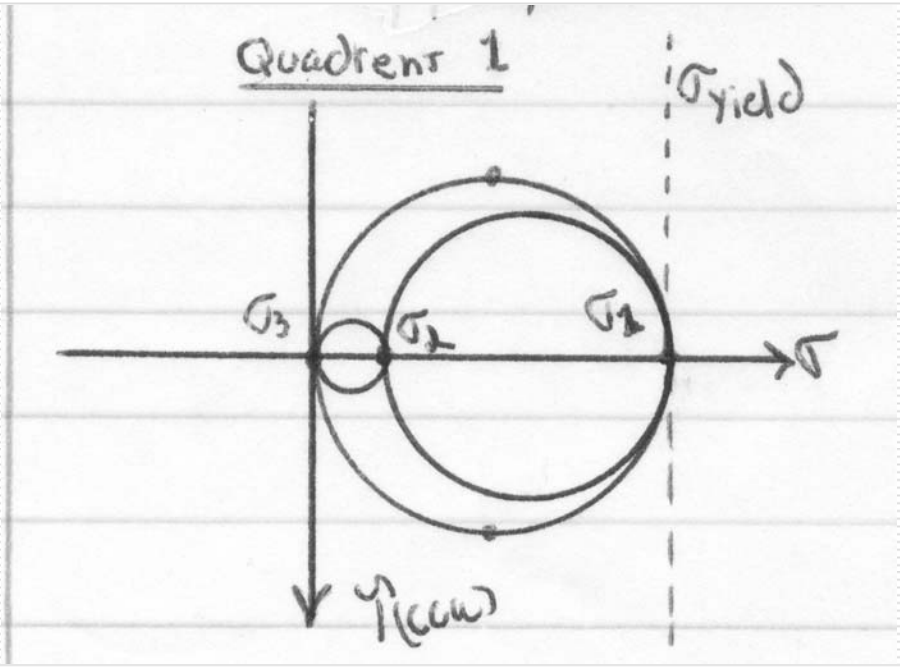


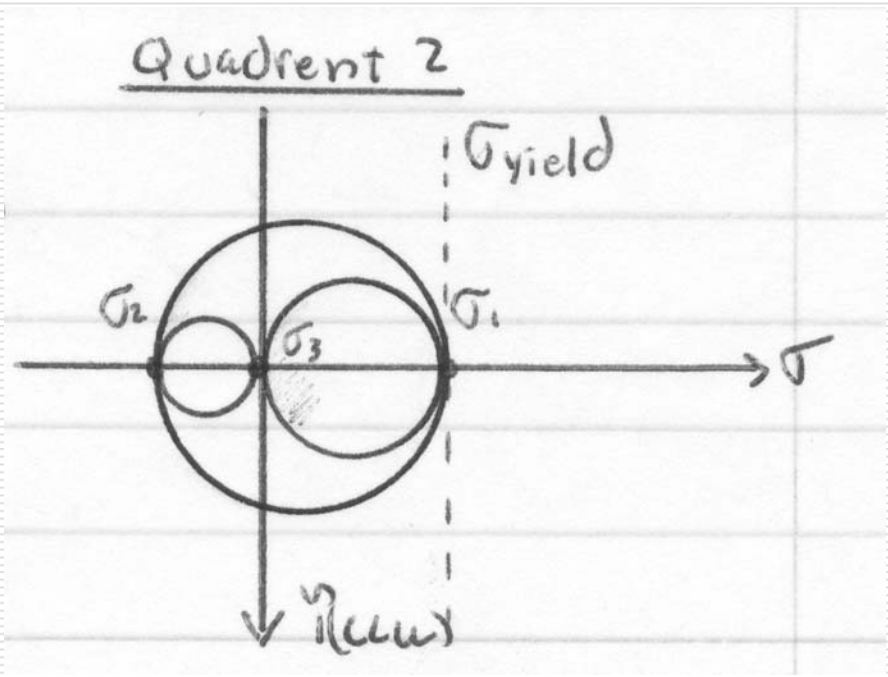


Multi-Axial Loading

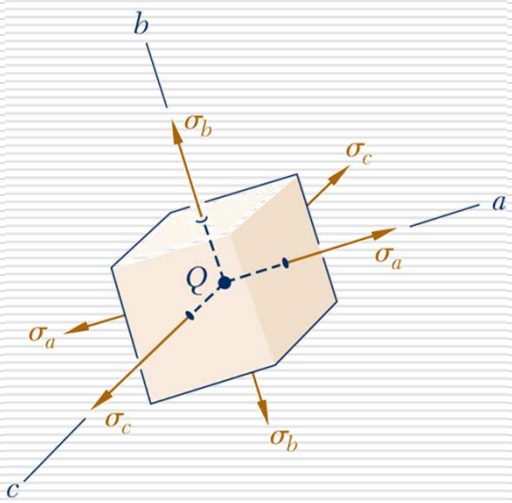


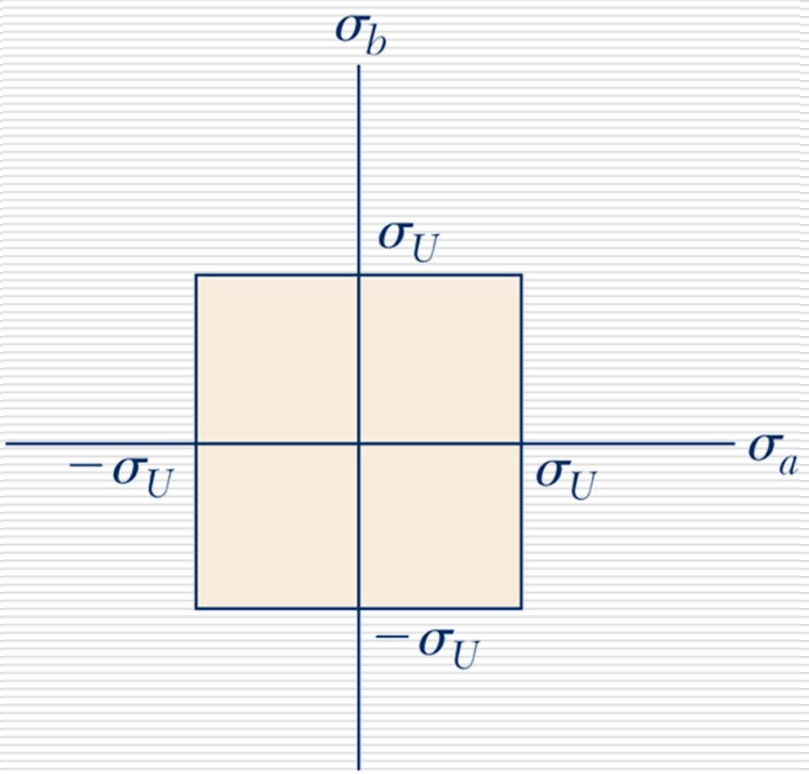
Multi-Axial Load



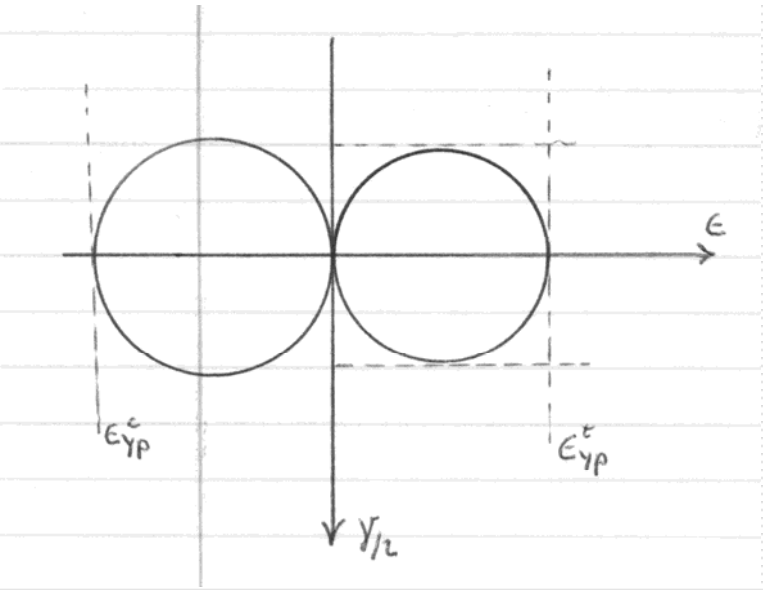


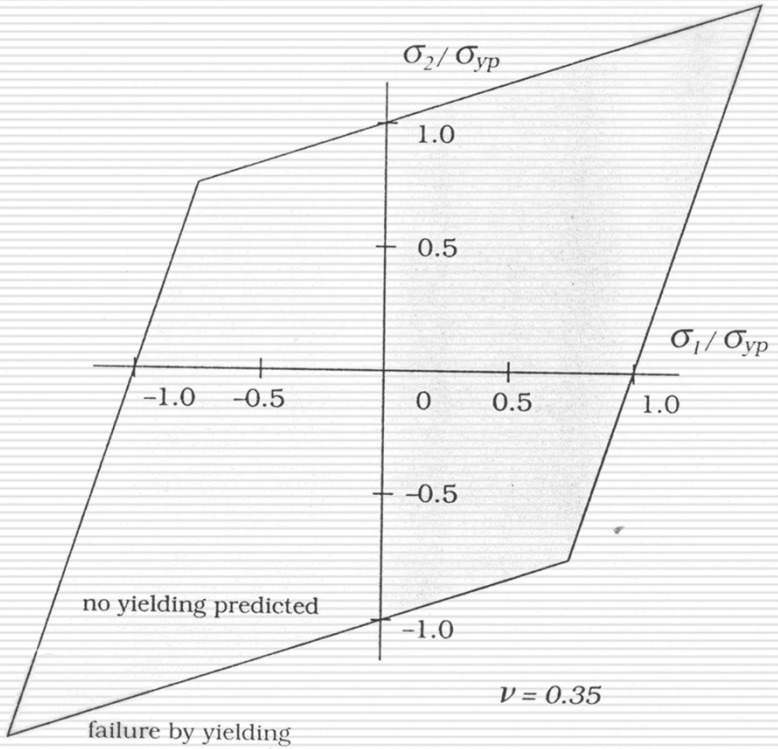
Maximum Normal Stress Theory
Rankine Criteria



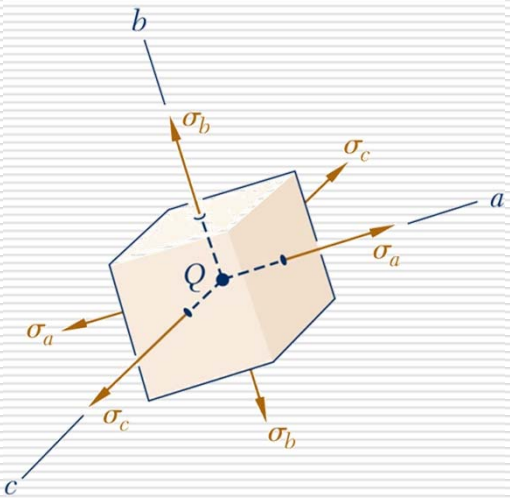


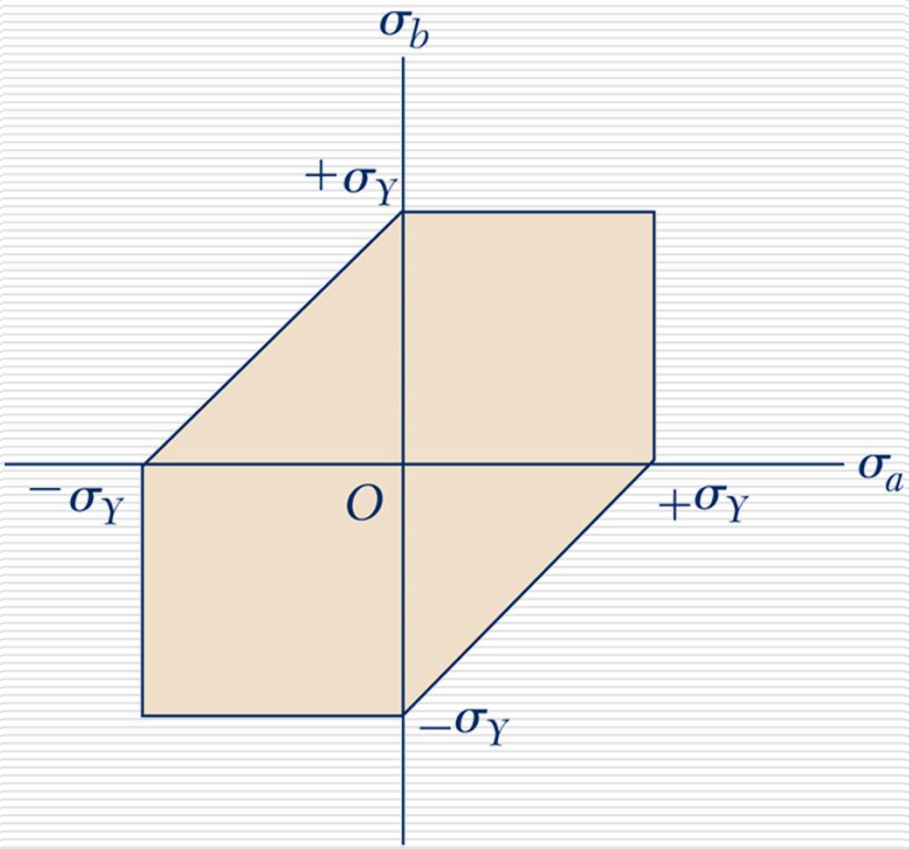
Maximum Strain Theory
St. Venant Criateria



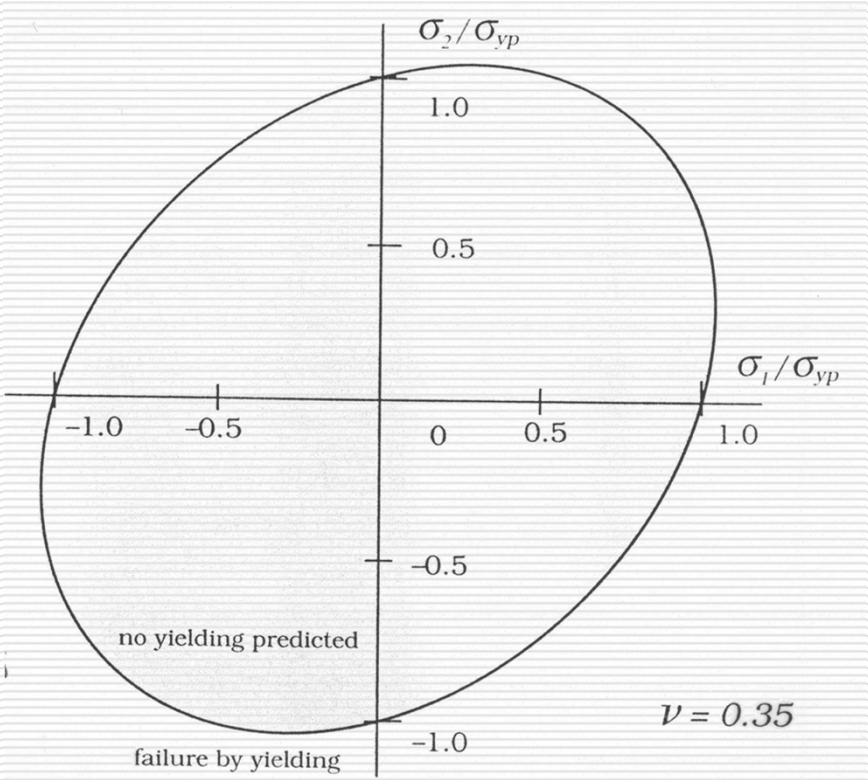


Maximum shear Theory
Tresca Criteria

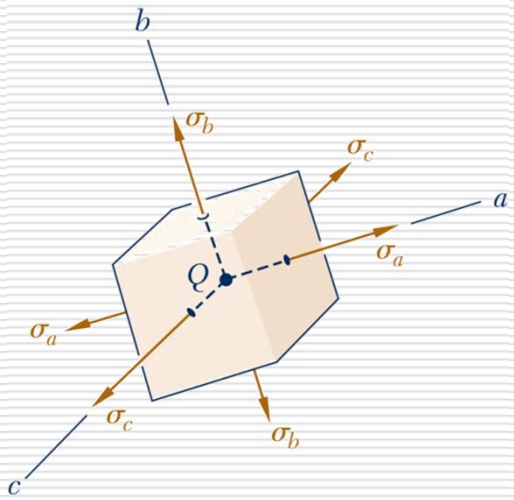


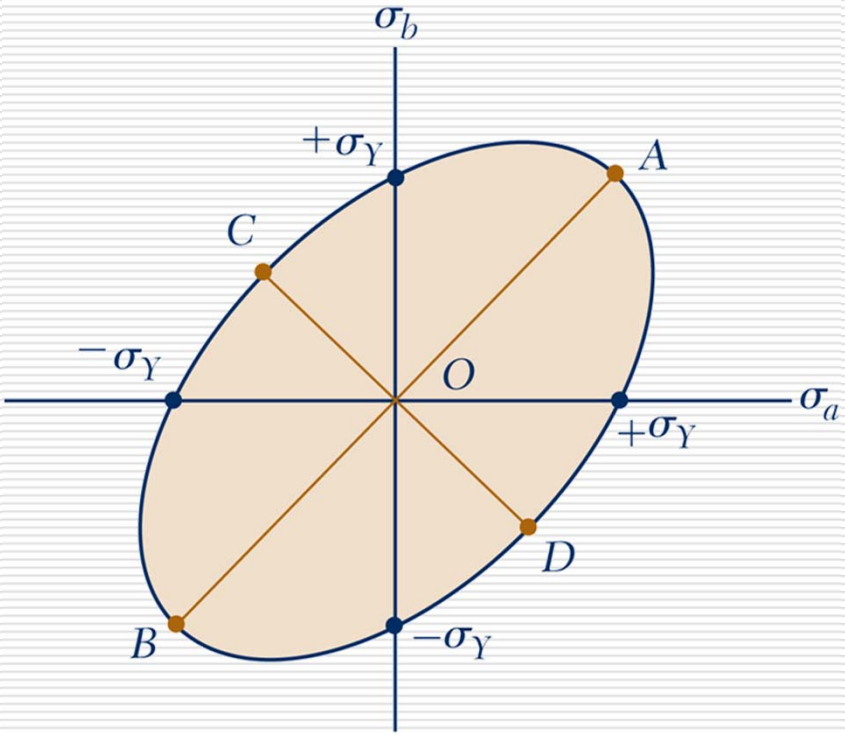


Maximum strain Energy Theory
Haig Criterion



Maximum energy of Distortion Theory – von Mises Criteria





$$\sigma_{vonMises} = \sqrt{0.5\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]} = s_y$$

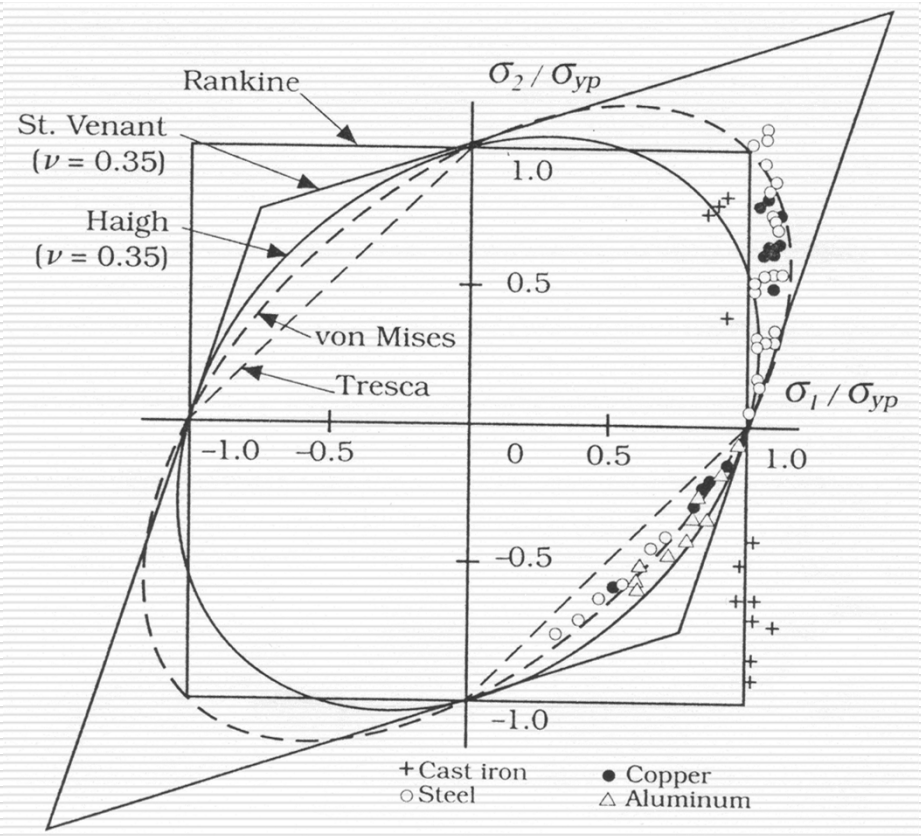
Maximum energy of Distortion Theory – von Mises Criteria

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

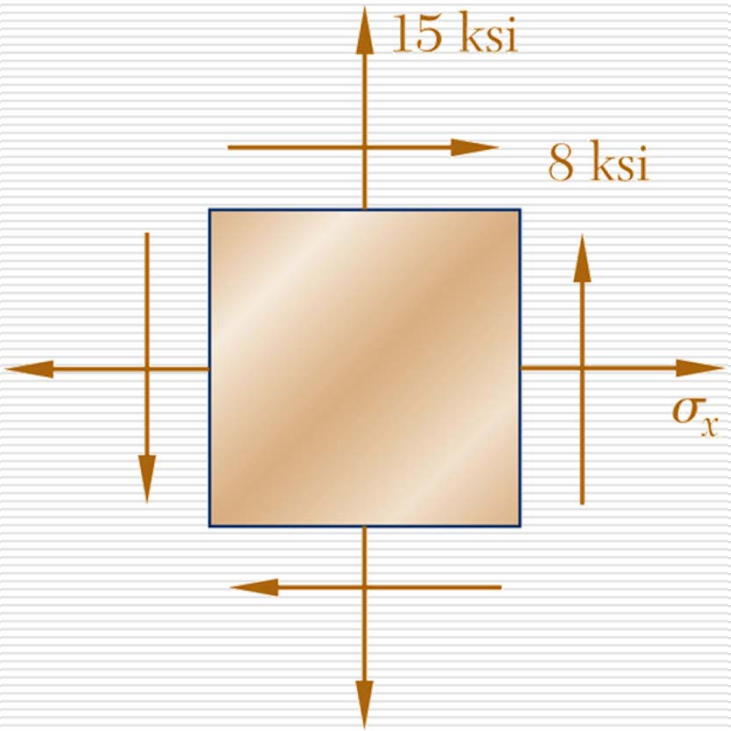
$$= \begin{bmatrix} \sigma_{ave} & 0 & 0 \\ 0 & \sigma_{ave} & 0 \\ 0 & 0 & \sigma_{ave} \end{bmatrix} + \begin{bmatrix} \sigma_x - \sigma_{ave} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_{ave} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_{ave} \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_{ave} & 0 & 0 \\ 0 & \sigma_{ave} & 0 \\ 0 & 0 & \sigma_{ave} \end{bmatrix} + \begin{bmatrix} \sigma_1 - \sigma_{ave} & 0 & 0 \\ 0 & \sigma_2 - \sigma_{ave} & 0 \\ 0 & 0 & \sigma_3 - \sigma_{ave} \end{bmatrix}$$

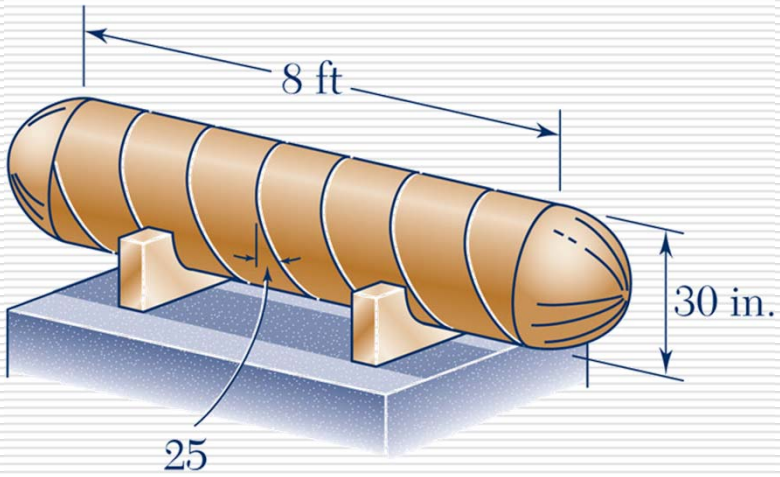
Comparison with Data

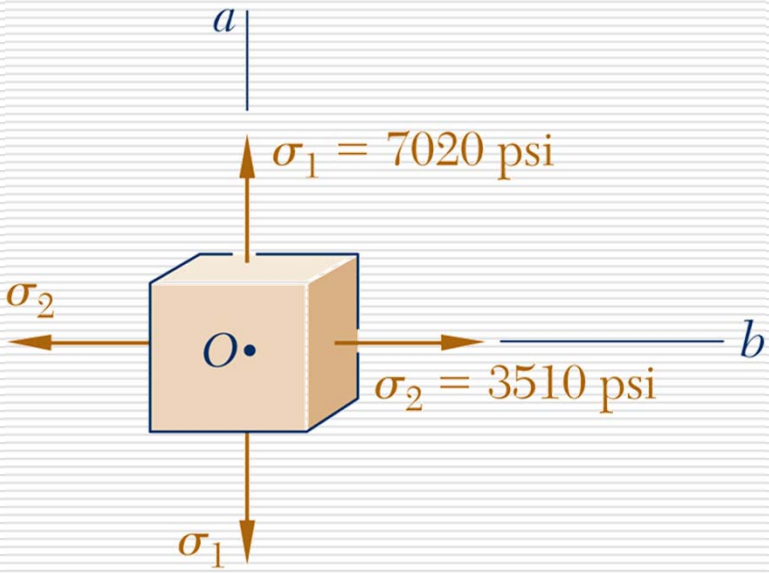


Example



Example


8 ft
30 in.
25


a
$\sigma_1 = 7020$ psi
σ_2
O
b
$\sigma_2 = 3510$ psi
σ_1