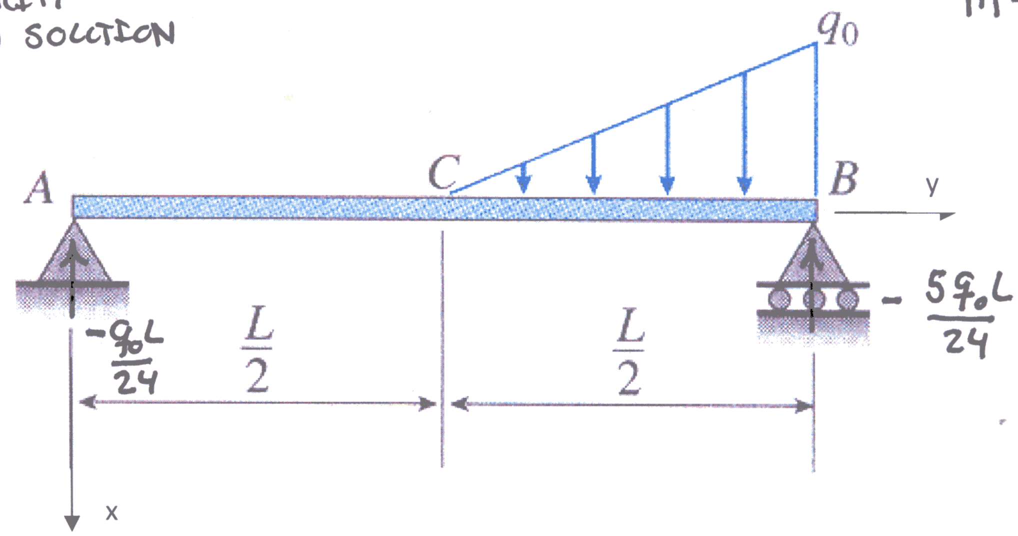
NAME: SOLUTION

PROBLEM 1: Beam AB is supporting a triangularly distributed load of maximum intensity q_o acting on the right-hand half of the beam. For this loading condition A_x=-q_oL/24 and B_x=-5q_oL/24.

SINGULARITY FUNCTION SOLUTION

$$m = \frac{q_0}{L/2} = 2\cdot\frac{q_0}{L}$$



1a. Derive equations that express how V, M, θ, and u change as a function of the distance along the length of the beam.

$$q(y) = -\frac{q_0 L}{24}\langle y-0\rangle_{-1} + \frac{2\cdot q_0}{L}\langle y-L/2\rangle^1 - \frac{5\cdot q_0 L}{24}\langle y-L\rangle_{-1} \quad (1)$$

$$V(y) = -\int q(y)\cdot dy = \frac{q_0 L}{24}\langle y-0\rangle^0 - \frac{q_0}{L}\langle y-\tfrac{L}{2}\rangle^2 + \frac{5\cdot q_0 L}{24}\langle y-L\rangle^0 \quad (2)$$

$$M(y) = \int V(y)\cdot dy = \frac{q_0 L}{24}\langle y-0\rangle^1 - \frac{q_0}{3L}\langle y-\tfrac{L}{2}\rangle^3 + \frac{5\cdot q_0 L}{24}\langle y-L\rangle^1 \quad (3)$$

$$\theta(y) = -\frac{1}{EI}\int M(y)\cdot dy = -\frac{q_0 L}{48EI}\langle y-0\rangle^2 + \frac{q_0}{12L\cdot EI}\langle y-L/2\rangle^4 - \frac{3}{48}\cdot\frac{q_0 L}{EI}\langle y-L\rangle^2 + C_1 \quad (4)$$

$$u(y) = \int \theta(y)\cdot dy = -\frac{q_0 L}{144EI}\langle y-0\rangle^3 + \frac{q_0}{60EIL}\langle y-L/2\rangle^5 - \frac{5}{144}\frac{q_0 L}{EI}\langle y-L\rangle^3 + C_1\cdot y + C_2 \quad (5)$$

BOUNDARY CONDITIONS ARE USED TO FIND THE CONSTANTS C_1 & C_2. THE FIRST BOUNDARY CONDITION

$$u(0) = 0 = C_2$$

(5) CAN NOW BE REWRITTEN

$$u(y) = -\frac{q_0 L}{144EI}\langle y-0\rangle^3 + \frac{q_0}{60EIL}\langle y-L/2\rangle^5 - \frac{5}{144}\cdot\frac{q_0 L}{EI}\langle y-L\rangle^3 + C_1\cdot y \quad (6)$$

THE SECOND BOUNDARY CONDITION

$$u(L) = 0 = -\frac{q_0 L^4}{144EI} + \frac{q_0 L^4}{1920\cdot EI} + C_1\cdot L \Rightarrow C_1 = \frac{q_0 L^3}{EI}\left[\frac{2^3\cdot 5}{144\cdot 2^3\cdot 5} - \frac{3}{1920\cdot 3}\right]$$

$$= \frac{q_0 L^3}{EI}\frac{37}{5760} \quad (7)$$

THE EQUATIONS ① - ⑤ CAN NOW BE WRITTEN

$$q(y) = -\frac{q_0 L}{24}\langle y-0\rangle_{-1} + \frac{2 \cdot q_0}{L}\langle y-L/2\rangle^1 - \frac{5}{24}\cdot q_0 L\langle y-L\rangle_{-1} \quad ①$$

$$V(y) = -\int q(y)\cdot dy = \frac{q_0 L}{24}\langle y-0\rangle^0 - \frac{q_0}{L}\langle y-L/2\rangle^2 + \frac{5}{24}\cdot q_0 L\langle y-L\rangle^0 \quad ②$$

$$M(y) = \int V(y)\cdot dy = \frac{q_0 L}{24}\langle y-0\rangle^1 - \frac{q_0}{3L}\langle y-L/2\rangle^3 + \frac{5}{24}\cdot q_0 L\langle y-L\rangle^1 \quad ③$$

$$\Theta(y) = -\frac{1}{EI}\int M(y)\cdot dy = -\frac{q_0 L}{48EI}\langle y-0\rangle^2 + \frac{q_0}{12LEI}\langle y-L/2\rangle^4 - \frac{5}{48}\frac{q_0 L}{EI}\langle y-L\rangle^2 + \frac{q_0 L^3}{EI}\frac{37}{5760} \quad ④$$

$$u(y) = \int \Theta(y)\, dy = -\frac{q_0 L}{144EI}\langle y-0\rangle^3 + \frac{q_0}{60LEI}\langle y-L/2\rangle^5 - \frac{5}{144}\frac{q_0 L}{EI}\langle y-L\rangle^3 + \frac{37}{5760}\cdot\frac{q_0 L^3}{EI}\cdot y \quad ⑤$$

0.00694 0.01667 0.03472 0.06607

1b. Using the graph paper on the next page draw the V, M, θ, and u diagrams for this beam. For the following values calculate the magnitude, illustrate the location, and show the distance from the origin on the coordinate system.

1. Maximum and minimum values of the shear force.
2. Maximum and minimum values of the bending moment.
3. Maximum and minimum values of the curvature.
4. Maximum value of the deflection.
5. All intercepts and points of inflection on all of the diagrams.

1. SHEAR FORCE DIAGRAM POINTS OF INTEREST USING (2)

- $V(0) = \frac{q_0 L}{24}(0)^0 = \underline{\underline{\frac{q_0 L}{24}}}$ (maximum)
- $V(L) = \frac{q_0 L}{24}(L)^0 - \frac{q_0}{L}\left(\frac{L}{2}\right)^2 + \frac{5}{24}q_0 L(0)^0 = \underline{\underline{0}}$ THIS IS BECAUSE THE END LOAD IS INCLUDED. IF THE END LOAD IS NOT INCLUDED (LAST TERM) $\underline{\underline{V = -\frac{5}{24}q_0 L}}$
- $V(y) = 0 = \frac{q_0 L}{24}(y)^0 - \frac{q_0}{L}(y - L/2)^2 = \frac{q_0 L}{24} - \frac{q_0}{L}\left(y^2 - yL + \frac{L^2}{4}\right) = q_0\left[\frac{L}{24} - \frac{y^2}{L} + y - \frac{L}{4}\right]$

$$0 = \frac{1}{24}\left(L - \frac{y^2}{L}\cdot 24 + 24y - 6L\right) = -\frac{y^2}{L}\cdot 24 + 24y - 5L$$

$$0 = 24\left(\frac{y}{L}\right)^2 - 24\left(\frac{y}{L}\right) + 5 \Rightarrow 0 = \left(\frac{y}{L}\right)^2 - \left(\frac{y}{L}\right) + \frac{5}{24} = \left(\frac{y}{L}\right)^2 - \left(\frac{y}{L}\right) + \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 + \frac{5}{24}$$

$$= \left(\frac{y}{L} - \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{5}{24} = \left(\frac{y}{L} - \frac{1}{2}\right)^2 - \frac{1}{24}$$

$$\frac{y}{L} - \frac{1}{2} = \pm\sqrt{1/24} \Rightarrow \frac{y}{L} = \frac{1}{2} \pm \sqrt{1/24} = 0.5 \pm 0.2041 = 0.7041,\ 0.2959$$

$\Rightarrow \underline{\underline{y = 0.7041 \cdot L}}$, $\cancel{0.2959L}$ THIS SOLUTION IS NOT IN THE RANGE BEING CONSIDERED.

2. BENDING MOMENT DIAGRAM POINTS OF INTEREST USING (3)

- THE BOUNDARY CONDITIONS FOR THE PROBLEM DICTATE THAT $M(0) = M(L) = 0$
- THE MAXIMUM BENDING MOMENT IS CALCULATED WHERE THE SHEAR FORCE GOES TO ZERO

$$M(0.7041 \cdot L) = \frac{q_0 L}{24}(0.7041 \cdot L) - \frac{q_0}{3L}(0.7041L - 0.5L)^3 = \frac{q_0 L}{24}(0.7041 \cdot L) - \frac{q_0}{3L}(0.2041 \cdot L)^3$$

$$= \underline{\underline{0.02650 \cdot q_0 L}} \text{ (maximum)}$$

3. CURVATURE DIAGRAM POINTS OF INTEREST USING (4)

- BECAUSE THE MOMENT DIAGRAM SHOWS THE MOMENT TO BE ZERO AT $y=0$ AND $y=L$, THESE ARE THE LOCATIONS OF MAX AND MINS.

$$\Theta(0) = -\frac{q_0 L}{48EI}(0)^2 + \frac{37}{5760}\frac{q_0 L^3}{EI} = \underline{\underline{\frac{37}{5760}\frac{q_0 L^3}{EI}}} \quad (\text{maximum})$$

$$\Theta(L) = -\frac{q_0 L}{48EI}(L)^2 + \frac{q_0}{12LEI}\left(\frac{L}{2}\right)^4 - \frac{5}{48}\frac{q_0 L}{EI}(0)^2 + \frac{37}{5760}\frac{q_0 L^3}{EI}$$

$$= -\frac{q_0 L^3}{48EI} + \frac{q_0 L^3}{192EI} + \frac{37}{5760}\frac{q_0 L^3}{EI} = -\frac{120}{120}\cdot\frac{q_0 L^3}{48EI} + \frac{30}{30}\cdot\frac{q_0 L^3}{192EI} + \frac{37}{5760}\frac{q_0 L^3}{EI}$$

$$= \underline{\underline{-\frac{53}{5760}\frac{q_0 L^3}{EI}}} \quad (\text{minimum})$$

- THE LOCATION WHERE $\Theta(y)=0$ NEEDS TO BE LOCATED. BOTH ~~[illegible]~~ REGIONS $0<y<L/2$ AND $L/2<y<L$ NEED TO BE CHECKED

$\underline{0<y<L/2}$

$$0 = -\frac{q_0 L}{48EI}y^2 + \frac{q_0 L^3}{EI}\frac{37}{5760} \Rightarrow \frac{y^2}{48} = \frac{37}{5760}L^2 \Rightarrow y^2 = \frac{37\cdot 48}{5760}\cdot L^2 = \frac{1776}{5760}\cdot L^2$$

$$\Rightarrow y = \pm\sqrt{\frac{1776}{5760}}\cdot L = \pm 0.555\cdot L$$

BOTH SOLUTIONS ARE OUTSIDE THE RANGE BEING CONSIDERED.

$\underline{L/2<y<L}$

$$0 = -\frac{q_0 L}{48EI}\cdot y^2 + \frac{q_0}{12EIL}(y-L/2)^4 + \frac{37}{5760}\frac{q_0 L^3}{EI} = -\frac{q_0 L}{48EI}y^2 + \frac{q_0}{12EIL}(y^2-yL+L^2/4)(y^2-yL+L^2/4) + \frac{37}{5760}\frac{q_0 L^3}{EI}$$

$$0 = -\frac{q_0 L}{48EI}y^2 + \frac{q_0}{12EIL}\left(y^4 - y^3L + \frac{y^2L^2}{4} - y^3L + y^2L^2 - \frac{yL^3}{4} + \frac{y^2L^2}{4} - \frac{yL^3}{4} + \frac{L^4}{16}\right) + \frac{37}{5760}\frac{q_0 L^3}{EI}$$

$$0 = -\frac{q_0 L y^2}{48EI} + \frac{q_0}{12EIL}\left(y^4 - 2y^3L + \frac{3}{2}y^2L^2 - \frac{1}{2}yL^3 + \frac{1}{16}L^4\right) + \frac{37}{5760}\frac{q_0 L^3}{EI}$$

$$0 = -\frac{1}{48}\cdot\frac{120}{120}L\cdot y^2 + \frac{1}{12}\cdot\frac{480}{480}\frac{y^4}{L} - \frac{2}{12}\cdot\frac{480}{480}y^3 + \frac{3}{24}\cdot\frac{240}{240}y^2L - \frac{1}{24}\cdot\frac{240}{240}y\cdot L^2 + \frac{1}{192}\cdot\frac{30}{30}L^3 + \frac{37}{5760}L^3$$

$$0 = 480\left(\frac{y}{L}\right)^4 - 960\left(\frac{y}{L}\right)^3 + 600\left(\frac{y}{L}\right)^2 - 240\left(\frac{y}{L}\right) + 67$$

$$\frac{y}{L} = \underbrace{\cancel{1.2473}}_{\text{OUTSIDE RANGE}}, 0.5554, \underbrace{\cancel{0.0987+i0.4379}}_{\text{OUTSIDE RANGE}}, \underbrace{\cancel{0.0987-i0.4379}}_{\text{OUTSIDE RANGE}}$$

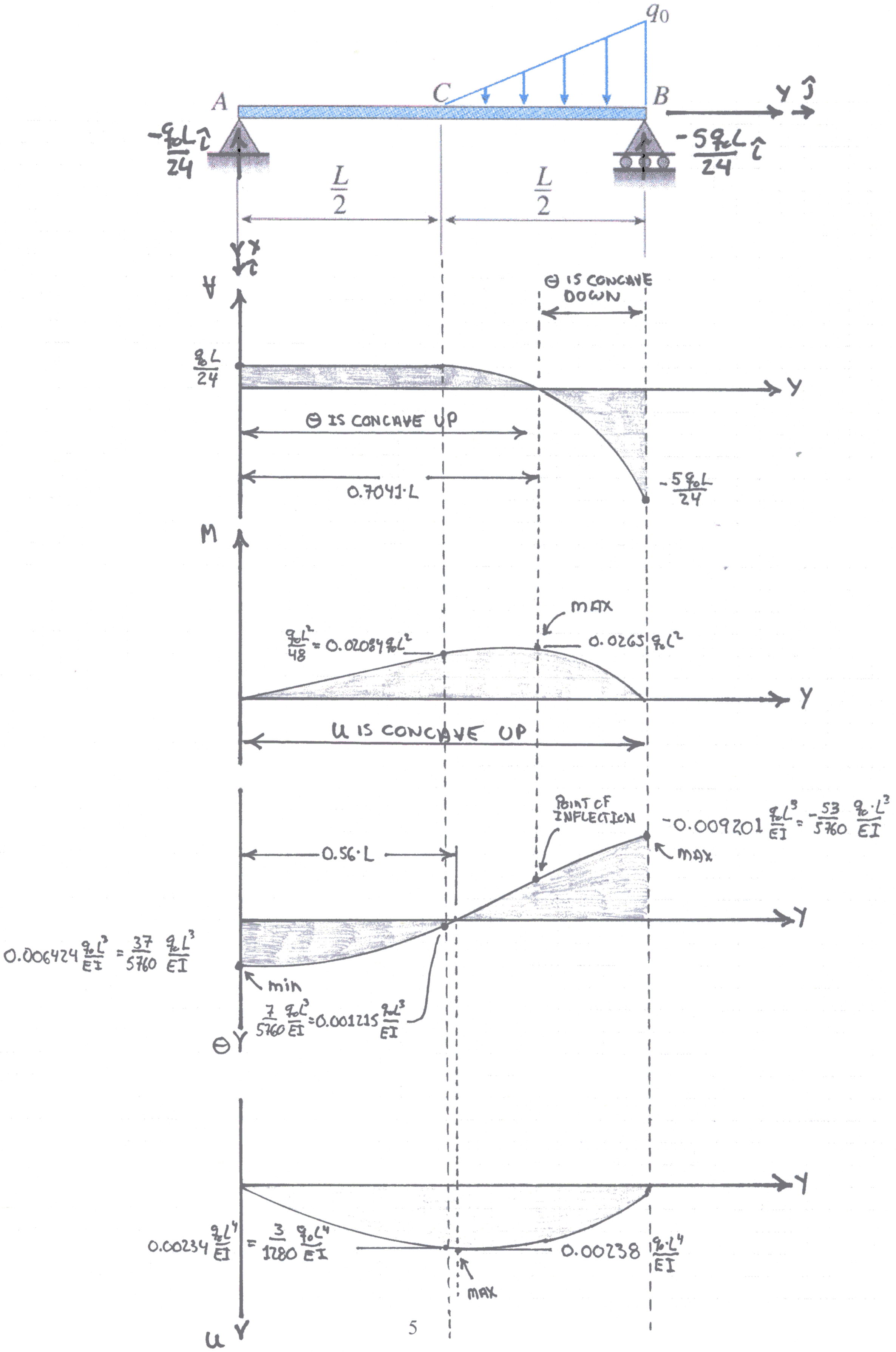
$\Rightarrow \underline{\underline{y = 0.5554\cdot L}}$ (LOCATION WHERE $\Theta(y)=0$)

4. DEFLECTION DIAGRAM POINTS OF INTEREST USING (5)

- THE LOCATION OF THE MAXIMUM DEFLECTION IS LOCATED WHERE THE CURVATURE EQUALS ZERO

$$u(0.5554\cdot L) = -\frac{q_0 L}{144EI}(0.5554\cdot L)^3 + \frac{q_0}{60LEI}(0.5554L - 0.5L)^5 + \frac{37}{5760}\frac{q_0 L^3}{EI}(0.5554L) = \underline{\underline{0.00238\frac{q_0 L^4}{EI}}}$$

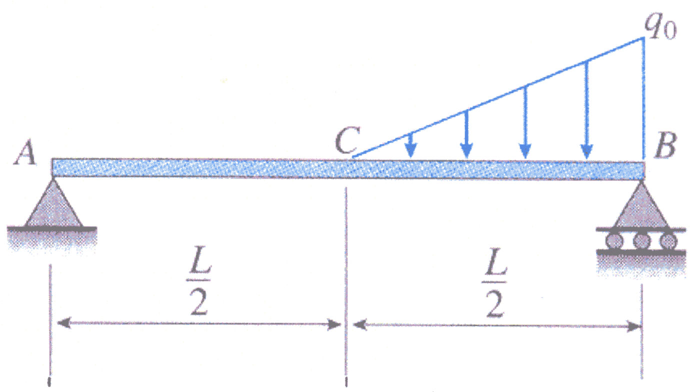
(MAXIMUM)

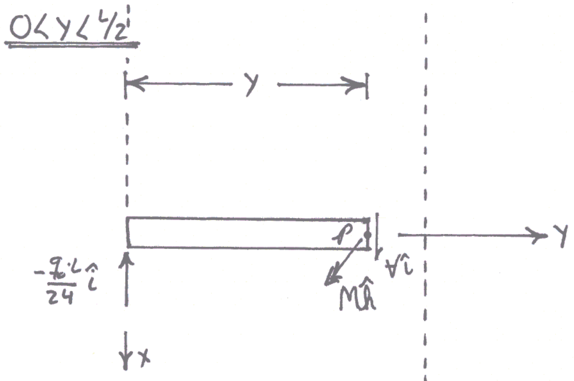

q0
A
C
B
y ĵ
-q0L/24 î
-5q0L/24 î
L/2
L/2
V
q0L/24
θ IS CONCAVE DOWN
θ IS CONCAVE UP
0.7041·L
-5q0L/24
M
MAX
q0L²/48 = 0.02084 q0L²
0.0265 q0L²
u IS CONCAVE UP
POINT OF INFLECTION
-0.009201 q0L³/EI = -53/5760 q0·L³/EI
MAX
0.56·L
0.006424 q0L³/EI = 37/5760 q0L³/EI
min
7/5760 q0L³/EI = 0.001215 q0L³/EI
θ
0.00234 q0L⁴/EI = 3/1280 q0L⁴/EI
0.00238 q0·L⁴/EI
MAX
u

DIRECT INTEGRATION SOLUTION.

$m = \frac{2 \cdot q_0}{L}$

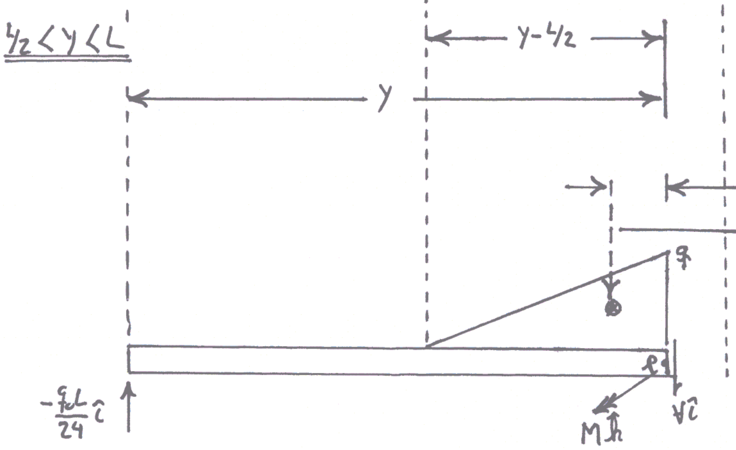
$q = \frac{2 q_0}{L} \cdot y - q_0 = q_0\left(\frac{2y}{L} - 1\right)$





$\Sigma F_x = 0 = -\frac{q_0 L}{24} + V \Rightarrow \underline{V = \frac{q_0 L}{24}}$ (1)

$\Sigma M_{zep} = 0 = M - \frac{q_0 L}{24} \cdot y \Rightarrow \underline{M = \frac{q_0 L}{24} \cdot y}$ (2)



$\frac{1}{3}\left(y - \frac{L}{2}\right) = \frac{y}{3} - \frac{L}{6}$

$\frac{1}{2}\left(y - \frac{L}{2}\right) \cdot q_0 \cdot \left(\frac{2 \cdot y}{L} - 1\right)\hat{i}$

$= \frac{q_0}{2}\left(\frac{2 \cdot y^2}{L} - y - y + \frac{L}{2}\right)\hat{i}$

$= \frac{q_0}{2}\left(\frac{2y^2}{L} - 2y + \frac{L}{2}\right)\hat{i}$

$= \frac{q_0}{L}\left(y^2 - y \cdot L + \frac{L^2}{4}\right)\hat{i}$

$\Sigma F_x = 0 = -\frac{q_0 \cdot L}{24} + \frac{q_0}{L}\left(y^2 - y \cdot L + \frac{L^2}{4}\right) + V$

$V = \frac{q_0 L}{24} - \frac{q_0}{L}\left(y^2 - y \cdot L + \frac{L^2}{4}\right) = \frac{q_0 L}{24} - \frac{q_0 24 L^2}{L\, 24\, L^2}\left(y^2 - y \cdot L + \frac{L^2}{4}\right) = \frac{q_0 L}{24} - \frac{q_0 L}{24} \cdot \frac{24}{L^2}\left(y^2 - y \cdot L + \frac{L^2}{4}\right)$

$= \frac{q_0 L}{24}\left(1 - 24\frac{y^2}{L^2} + 24\frac{y}{L} - 6\right) = \underline{-\frac{q_0 L}{24}\left(24\frac{y^2}{L^2} - 24\frac{y}{L} + 5\right)}$ (3)

$V\left(\frac{L}{2}\right) = -\frac{q_0 L}{24}\left(\frac{L^2/4}{L^2} \cdot 24 - \frac{L/2}{L} \cdot 24 + 5\right) = -\frac{q_0 L}{24}(6 - 12 + 5) = \boxed{\frac{q_0 L}{24}}$ MAX (4)

$V(L) = -\frac{q_0 L}{24}\left(24 \cdot \frac{L^2}{L^2} - 24\frac{L}{L} + 5\right) = \boxed{-\frac{5 q_0 L}{24}}$ MIN (5)

FINDING THE INTERCEPT

$0 = -\frac{q_0 L}{24}\left(24\frac{y^2}{L^2} - 24\frac{y}{L} + 5\right) = \frac{24}{L^2} \cdot y^2 - \frac{24}{L}y + 5 = y^2 - y \cdot L + \frac{5}{24} \cdot L^2 = \underbrace{y^2 - y \cdot L + \left(-\frac{L}{2}\right)^2}_{(y - L/2)^2} - \left(-\frac{L}{2}\right)^2 + \frac{5}{24}L^2$

$(y - L/2)^2 = \left(\frac{1}{4} \cdot \frac{6}{6} - \frac{5}{24}\right)L^2 = \frac{L^2}{24} \Rightarrow y = \frac{L}{2} \pm \sqrt{1/24} \cdot L$

$= (0.5 \pm 0.2041) \cdot L = \boxed{0.7041 \cdot L}$ (6) $\lessgtr$ $0.2959L$

OUTSIDE THE RANGE UNDER CONSIDERATION

$\boxed{\text{ALT SOL 1}}$

$$\Sigma M_{zep} = 0 = M - \frac{q_o L}{24}\cdot y + \frac{1}{3}\left(y-\frac{L}{2}\right)\cdot\frac{q_o}{L}\left(y^2 - y\cdot L + \frac{L^2}{4}\right) = M - \frac{q_o L}{24}\cdot y + \frac{q_o}{3L}\cdot\frac{8}{8}\left(y-\frac{L}{2}\right)\left(y^2-y\cdot L+L^2/4\right)$$

$$= M - \frac{q_o L}{24}\cdot y + \frac{8 q_o}{24L}\left(y^3 - y^2 L + \frac{L^2}{4}\cdot y - \frac{L\cdot y^2}{2} + \frac{L^2 y}{2} - \frac{L^3}{8}\right)$$

$$= M - \frac{q_o L}{24}\cdot y + \frac{8\cdot q_o}{24}\left(\frac{y^3}{L} - \frac{3}{2}\cdot y^2 + \frac{3}{4}y\cdot L - \frac{1}{8}L^2\right)$$

$$\Rightarrow M = \frac{q_o}{24}\left(Ly - 8\frac{y^3}{L} + 12\cdot y^2 - 6\,y\cdot L + L^2\right) = \underline{\underline{\frac{q_o}{24}\left(-8\frac{y^3}{L} + 12\cdot y^2 - 5\cdot L\cdot y + L^2\right)}} \qquad (7)$$

THE MAXIMUM VALUE OF THE MOMENT IS LOCATED WHERE $V=0$, FROM (6)

$$M(0.7041\cdot L) = \frac{q_o}{24}\left[-\frac{8}{L}(0.7041\cdot L)^3 + 12(0.7041\cdot L)^2 - 5\cdot L\cdot(0.7041\cdot L) + L^2\right]$$

$$= \boxed{0.2650\,q_o\cdot L} \qquad (8) \text{ MAX}$$

AN EXPRESSION FOR THE CURVATURE AND DEFLECTION OF THE BEAM IS FOUND BY DIRECTLY INTEGRATING THE MOMENT. THESE INTEGRATIONS ARE DEPENDENT ON THE REGION OF THE BEAM

$\underline{0 < y < L/2}$

STARTING WITH (2)

$$\theta = -\frac{1}{EI}\int \frac{q_o\cdot L}{24} y\cdot dy = -\frac{q_o\cdot L}{24EI}\frac{y^2}{2} + C_1 = \underline{\underline{-\frac{q_o\cdot L\cdot y^2}{48EI} + C_1}} \qquad (9)$$

$$u = \int\left[-\frac{q_o L\cdot y^2}{48EI} + C_1\right]dy = \underline{\underline{-\frac{q_o\cdot L\cdot y^3}{144\cdot EI} + C_1\cdot y + C_2}} \qquad (10)$$

THE CONSTANT C_2 IS EVALUATED USING THE BOUNDARY CONDITION $u(0)=0$

$$u(0) = 0 = -\frac{q_o\cdot L}{144EI}(0)^3 + C_1(0) + C_2 \quad\Rightarrow\quad \underline{\underline{C_2 = 0}} \qquad (11)$$

AT $y = L/2$

$$u(L/2) = -\frac{q_o\cdot L\cdot(L/2)^3}{144\cdot EI} + C_1(L/2) = -\frac{q_o\cdot L^4}{1152\cdot EI} + \frac{C_1\cdot L}{2} \qquad (12)$$

THE BOUNDARY CONDITION AT $L/2$ IS A CONTINUITY CONDITION THAT WILL BE APPLIED AFTER THE DEFORMATION IN THE OTHER HALF OF THE BEAM IS SOLVED FOR.

$\underline{\underline{L/2 < y < L}}$

STARTING WITH (7)

$$\theta = -\frac{1}{EI}\frac{q_o}{24}\int\left(-8\frac{y^3}{L} + 12\cdot y^2 - 5L\cdot y + L^2\right)dy = -\frac{q_o}{24EI}\left(-\frac{8}{L}\frac{y^4}{4} + 12\frac{y^3}{3} - 5\cdot L\frac{y^2}{2} + L^2 y\right) + C_3$$

$$= -\frac{q_o}{24EI}\left(-\frac{2\cdot y^4}{L} + 4\cdot y^3 - \frac{5}{2}\cdot y^2 L + L^2\cdot y\right) + C_3 = \underline{\underline{-\frac{q_o}{24EI}\left(-\frac{2\cdot y^4}{L} + 4\cdot y^3 - \frac{5}{2}L\cdot y^2 + L^2\cdot y + C_4\right)}} \qquad (13)$$

$$-C_3\frac{q_o}{24EI}\cdot\frac{24\cdot EI}{q_o} = C_4\cdot\frac{24\cdot EI}{q_o} \Rightarrow C_3 = -\frac{24EI}{q_o}\cdot C_4$$

ALT SOL 2

$$u = \int \left[\frac{-q_0}{24EI}\left(-\frac{2}{L}y^4 + 4\cdot y^3 - \frac{5L}{2}y^2 + L^2\cdot y\right) + C_3 \right] dy = \frac{-q_0}{24EI}\left(-\frac{2}{L}\cdot\frac{y^5}{5} + 4\frac{y^4}{4} - \frac{5L}{2}\cdot\frac{y^3}{3} + L^2\cdot\frac{y^2}{2}\right) + C_3\cdot y + C_5$$

$$= -\frac{q_0}{24EI}\left(-\frac{2}{5}\cdot\frac{y^5}{L} + y^4 - \frac{5}{6}\cdot L\cdot y^3 + \frac{1}{2}L^2\cdot y^2\right) + C_3\cdot y + C_5$$

$$= -\frac{q_0}{24\cdot EI}\left(-\frac{2}{5}\cdot\frac{6}{6}\cdot\frac{y^5}{L} + \frac{30}{30}\cdot y^4 - \frac{5}{6}\cdot\frac{5}{5}\cdot L\cdot y^3 + \frac{1}{2}\cdot\frac{15}{15}L^2\cdot y^2\right) + C_3\cdot y + C_5$$

$$= -\frac{q_0}{24EI}\left(\frac{-12\cdot y^5/L + 30y^4 - 25L\cdot y^3 + 15L^2y^2}{30}\right) + C_3\cdot y + C_5$$

$$= \frac{-q_0}{720EI}\left(-12\frac{y^5}{L} + 30y^4 - 25L\cdot y^3 + 15L^2\cdot y^2\right) + C_3 y + C_5 \quad (14)$$

APPLYING THE BOUNDARY CONDITION AT $y=L$, $u(L)=0$

$$u(L) = \frac{-q_0}{720EI}\left(-12\frac{L^5}{L} + 30L^4 - 25L^4 + 15L^4\right) + C_3 L + C_5 = \frac{-8\cdot q_0\cdot L^4}{720EI} + C_3 L + C_5$$

$$0 = -\frac{q_0 L^4}{90EI} + C_3 L + C_5 \quad \Rightarrow \quad C_5 = \frac{q_0 L^4}{90EI} - C_3\cdot L \quad (15)$$

BACK SUBSTITUTING (15) INTO (14)

$$u = -\frac{q_0}{720EI}\left(-12\frac{y^5}{L} + 30\cdot y^4 - 25L\cdot y^3 + 15L^2\cdot y^2\right) + C_3 y + \frac{q_0 L^4\cdot 8}{90EI\cdot 8} - C_3\cdot L$$

$$u = -\frac{q_0}{720EI}\left(-12\frac{y^5}{L} + 30\cdot y^4 - 25L\cdot y^3 + 15L^2\cdot y^2 - 8L^4\right) + C_3(y-L)$$

$$u = -\frac{q_0}{720EI}\left(-12\frac{y^5}{L} + 30\cdot y^4 - 25L\cdot y^3 + 15L^2\cdot y^2 - 8L^4\right) - C_3(L-y) \quad (16)$$

$0 < y < L/2$

$$\Theta_1 = -\frac{q_0 \cdot L \cdot y^2}{48EI} + C_1 \qquad (9)$$

$$u_1 = -\frac{q_0 \cdot L \cdot y^3}{144EI} + C_1 \cdot y \qquad (10)$$

$L/2 < y < L$

$$\Theta_2 = \frac{q_0}{24EI}\left(\frac{2}{L}\cdot y^4 - 4\cdot y^3 + \frac{5\cdot L}{2}\cdot y^2 - L^2\cdot y\right) + C_3 \qquad (13)$$

$$u_2 = \frac{q_0}{720EI}\left(\frac{12}{L}\cdot y^5 - 30\cdot y^4 + 25\cdot L\cdot y^3 - 15\cdot L^2\cdot y^2\right) + C_3\cdot y + C_5 \qquad (14)$$

$$= \frac{q_0}{720EI}\left(\frac{12}{L}\cdot y^5 - 30\cdot y^4 + 25\cdot L\cdot y^3 - 15\cdot L^2\cdot y^2 + 8L^4\right) - C_3(L-y) \qquad (16)$$

SINCE THE BEAM IS CONTINIOUS AT $L/2$, $\Theta_1(L/2) = \Theta_2(L/2)$

$$\cancel{-\frac{q_0 L}{48EI}\left(\frac{L}{2}\right)^2 + C_1 = \frac{q_0}{24EI}\left[\frac{2}{L}\left(\frac{L}{2}\right)^5 - 30\left(\frac{L}{2}\right)^4 + 25 L\left(\frac{L}{2}\right)\right.}$$

$$-\frac{q_0\cdot L}{48\cdot EI}\left(\frac{L}{2}\right)^2 + C_1 = \frac{q_0}{24EI}\left[\frac{2}{L}\cdot\left(\frac{L}{2}\right)^4 - 4\left(\frac{L}{2}\right)^3 + \frac{5\cdot L}{2}\cdot\left(\frac{L}{2}\right)^2 - L^2\cdot\left(\frac{L}{2}\right)\right] + C_3$$

$$\cancel{-\frac{q_0 L}{48EI}}$$

$$-\frac{q_0 L^3}{192EI} + C_1 = \frac{q_0 L^3}{24EI}\left[\frac{1}{8} - \frac{1}{2} + \frac{5}{8} - \frac{1}{2}\right] + C_3 = \frac{-q_0 L^3}{96EI} + C_3$$

$$+\frac{q_0 L^3}{96EI} - \frac{q_0 L^3}{192EI} = C_3 - C_1 \Rightarrow \frac{2 q_0 L^3}{192EI} - \frac{q_0 L^3}{192EI} = C_3 - C_1$$

$$\frac{q_0 L^3}{192EI} = C_3 - C_1 \Rightarrow \underline{C_1 = C_3 - \frac{q_0\cdot L^3}{192EI}} \qquad (17)$$

(9) $\rightarrow$ $$\Theta_1 = \frac{-q_0 L\cdot y^2}{48EI} + C_3 - \frac{q_0\cdot L^3}{192EI} = \frac{-4 q_0 L\cdot y^2}{192EI} - \frac{q_0\cdot L^3}{192EI} + C_3$$

$$= -\frac{q_0}{192EI}\left(4L\cdot y^2 + L^3\right) + C_3 \qquad (18)$$

(10) $\rightarrow$ $$u_1 = -\frac{q_0\cdot L\cdot y^3}{144EI} + \left(C_3 - \frac{q_0 L^3}{192EI}\right)\cdot y = -\frac{q_0\cdot L\cdot y^3}{144\cdot E\cdot I} - \frac{q_0 L^3\cdot y}{192EI} + C_3\cdot y$$

$$= \frac{-q_0}{576EI}\left(4\cdot L\cdot y^3 + 3\cdot L^3\cdot y\right) + C_3\cdot y \qquad (19)$$

THE CONTINUITY OF THE BEAM AT $y = L/2$ ALSO REQUIRES THAT $u_1(L/2) = u_2(L/2)$

$$-\frac{q_0}{576EI}\left[4\cdot L\left(\frac{L}{2}\right)^3 + 3\cdot L^3\cdot\left(\frac{L}{2}\right)\right] + C_3\cdot\left(\frac{L}{2}\right)$$

$$= \frac{q_0}{720EI}\left[\frac{12}{L}\underset{32}{\left(\frac{L}{2}\right)^5} - 30\underset{16}{\left(\frac{L}{2}\right)^4} + 25\cdot L\cdot\underset{8}{\left(\frac{L}{2}\right)^3} - 15\cdot L^2\cdot\underset{4}{\left(\frac{L}{2}\right)^2} + 8\cdot L^4\right) - C_3\frac{L}{2}$$

$$\frac{-q_0}{576EI}\left[\frac{1}{2}L^4 + \frac{3}{2}L^4\right] + C_3\left(\frac{L}{2}\right)$$

$$= \frac{q_0}{720EI}\left[\overset{\frac{3}{8}}{\frac{12}{32}}L^4 - \overset{\frac{15}{8}}{\frac{30}{16}}L^4 + \frac{25}{8}L^4 - \overset{\frac{30}{8}}{\frac{15}{4}}L^4 + \overset{\frac{64}{8}}{8}L^4\right] - C_3\frac{L}{2}$$

$$-\frac{2q_0L^4}{576EI} + \frac{C_3L}{2} = \frac{q_0L^4}{720EI}\left[\frac{47}{8}\right] - C_3\frac{L}{2}$$

$$-\frac{20q_0L^4}{5760EI} + \frac{C_3L}{2} = \frac{47q_0L^4}{5760EI} - C_3\frac{L}{2} \Rightarrow C_3 = \frac{67\cdot q_0L^3}{5760EI} \quad (20)$$

SUBSTITUTING (20) INTO (17)

$$C_1 = \frac{67q_0L^3}{5760EI} - \frac{q_0L^3}{192EI}\cdot\frac{30}{30} = \frac{37q_0L^3}{5760EI} \quad (21)$$

A REVIEW OF THE EQUATIONS IN THE TWO REGIONS OF THE BEAM

$0 < y < L/2$

$$V_1 = \frac{q_0L}{24} \quad (22)$$

$$M_1 = \frac{q_0L}{24}\cdot y \quad (23)$$

$$\Theta_1 = \frac{-q_0}{192EI}(4L\cdot y^2 + L^3) + \frac{67}{5760}\frac{q_0L^3}{EI} = \frac{q_0}{5760EI}(37L^3 - 120Ly^2) \quad (24)$$

$$u_1 = -\frac{q_0}{576}(4\cdot L\cdot y^3 + 3\cdot L^3\cdot y) + \frac{67q_0L^3}{5760EI}y = \frac{q_0}{5760EI}(37L^3y - 40L\cdot y^3) \quad (25)$$

$L/2 < y < L$

$$V_2 = -\frac{q_0L}{24}\left(24\cdot\frac{y^2}{L^2} - 24\frac{y}{L} + 5\right) \quad (26)$$

$$M_2 = \frac{q_0}{24}\left(-8\frac{y^3}{L} + 12y^2 - 5L\cdot y + L^2\right) = \frac{q_0L^2}{24}\left(-8\frac{y^3}{L^3} + 12\frac{y^2}{L^2} - 5\frac{y}{L} + 1\right) \quad (27)$$

$$\Theta_2 = \frac{q_0L^3}{24EI}\cdot\left(2\cdot\frac{y^4}{L^4} - 4\frac{y^3}{L^3} + \frac{5}{2}\frac{y^2}{L^2} - \frac{y}{L}\right) + \frac{67q_0L^3}{5760EI} = \frac{q_0L^3}{5760EI}\left(480\frac{y^4}{L^4} - 960\frac{y^3}{L^3} + 600\frac{y^2}{L^2} - 240\frac{y}{L} + 67\right) \quad (28)$$

$$u_2 = \frac{q_0L^4}{720EI}\left(12\frac{y^5}{L^5} - 30\frac{y^4}{L^4} + 25\frac{y^3}{L^3} - 15\frac{y^2}{L^2} + 8\right) - (L-y)\frac{67\cdot q_0\cdot L^3}{5760EI}$$

$$= \frac{q_0L^4}{5760EI}\left(96\cdot\frac{y^5}{L^5} - 240\frac{y^4}{L^4} + 200\frac{y^3}{L^3} - 120\frac{y^2}{L^2} + 67\frac{y}{L} - 3\right) \quad (29)$$

FOR THE REGION $0 < y < L/2$, THE VALUES FOR (22) - (25) AT $y=0$ AND $y = L/2$

$$V_1(0) = \frac{q_o L}{24} = 0.0417 \cdot q_o L \qquad V_1(\tfrac{L}{2}) = \frac{q_o L}{24} = 0.0417 \cdot q_o L$$

$$M_1(0) = 0 \qquad M_1(\tfrac{L}{2}) = \frac{q_o L}{24} \cdot \frac{L}{2} = \frac{q_o L^2}{48} = 0.02083 q_o L^2$$

$$\Theta_1(0) = \frac{37}{5760} \frac{q_o L^3}{EI} = 0.003424 \frac{q_o L^3}{EI}$$

$$\Theta_1(\tfrac{L}{2}) = \frac{q_o}{5760 EI}(37L^3 - 120 \cdot L \cdot (\tfrac{L}{2})^2) = \frac{7 q_o L^3}{5760 EI} = 0.001215 \frac{q_o L^3}{EI}$$

$$u_1(0) = 0$$

$$u_1(\tfrac{L}{2}) = \frac{q_o}{5760 EI}(37 L^3 (\tfrac{L}{2}) - 40 L \cdot (\tfrac{L}{2})^3) = \frac{3}{1280} \cdot \frac{q_o L^4}{EI} = 0.002344 \frac{q_o L^4}{EI}$$

FOR THE REGION $L/2 < y < L$, THE VALUES OF (26) - (29) AT $y = L/2$ AND $y = L$

$$V_2(\tfrac{L}{2}) = -\frac{q_o L}{24}\left(\frac{24}{L^2} \cdot \frac{L^2}{4} - \frac{24}{L} \cdot \frac{L}{2} + 5\right) = \frac{q_o L}{24} = 0.0417 \cdot q_o L$$

$$V_2(L) = -\frac{q_o L}{24}\left(24\frac{L^2}{L^2} - 24\frac{L}{L} + 5\right) = -\frac{5 q_o L}{24} = -0.2083 q_o L$$

$$M_2(\tfrac{L}{2}) = \frac{q_o L^2}{24}\left(-\frac{8}{L^3} \cdot \frac{L^3}{8} + \frac{12}{L^2} \cdot \frac{L^2}{4} - \frac{5}{L} \cdot \frac{L}{2} + 1\right) = \frac{q_o L^2}{48} = 0.02083 \cdot q_o L^2$$

$$M_2(L) = \frac{q_o L^2}{24}\left(-8\frac{L^3}{L^3} + 12\frac{L^2}{L^2} - 5\frac{L}{L} + 1\right) = 0$$

$$\Theta_2(\tfrac{L}{2}) = \frac{q_o L^3}{5760 EI}\left(\frac{480}{L^4} \cdot \frac{L^4}{16} - \frac{960}{L^3} \cdot \frac{L^3}{8} + \frac{600}{L^2} \cdot \frac{L^2}{4} - \frac{240}{L} \cdot \frac{L}{2} + 67\right) = \frac{7}{5760} \frac{q_o L^3}{EI} = 0.001215 \frac{q_o L^3}{EI}$$

$$\Theta_2(L) = \frac{q_o L^3}{5760 EI}\left(480\frac{L^4}{L^4} - 960\frac{L^3}{L^3} + 600\frac{L^2}{L^2} - 240\frac{L}{L} + 67\right) = \frac{-53 q_o L^3}{5760 EI} = -0.009201 \cdot \frac{q_o L^3}{EI}$$

$$u_2(\tfrac{L}{2}) = \frac{q_o L^4}{5760 EI}\left[\frac{96}{L^5} \cdot \frac{L^5}{32} - \frac{240}{L^4} \cdot \frac{L^4}{16} + \frac{200}{L^3} \cdot \frac{L^3}{8} - \frac{120}{L^2} \cdot \frac{L^2}{4} + \frac{67}{L} \cdot \frac{L}{2} - 3\right] = \frac{3 q_o L^4}{1280 EI} = 0.002344 \frac{q_o L^4}{EI}$$

$$u_2(L) = \frac{q_o L}{5760 EI}\left[96\frac{L^5}{L^5} - 240\frac{L^4}{L^4} + 200\frac{L^3}{L^3} - 120\frac{L^2}{L^2} + 67\frac{L}{L} - 3\right) = 0$$

IN THE REGION $L/2 < y < L$ THE ABOVE RESULTS INDICATE THAT THE SHEAR FORCE CHANGES SIGN. THE LOCATION WHERE $V=0$ NEEDS TO BE LOCATED.

$$V_2(y) = 0 = -\frac{q_o L}{24}\left(24\frac{y^2}{L^2} - 24\frac{y}{L} + 5\right)$$

$$0 = 24\frac{y^2}{L^2} - 24\frac{y}{L} + 5$$

$$0 = y^2 - y \cdot L + \frac{5}{24} \cdot L^2 = y^2 - y \cdot L + (-\tfrac{L}{2})^2 - (-\tfrac{L}{2})^2 + \frac{5}{24} L^2$$

$$0 = (y - \tfrac{L}{2})^2 - \frac{L^2}{24}$$

$$(y - \tfrac{L}{2})^2 = \frac{L^2}{24}$$

$$y - \frac{L}{2} = \pm\sqrt{1/24} \cdot L$$

$$y = \frac{L}{2} \pm \sqrt{1/24} \cdot L$$

OUTSIDE THE RANGE OF (26)

$$y = (0.5 \pm 0.2041) \cdot L = 0.7041 \cdot L, \quad \cancel{0.2959 \cdot L} \qquad (30)$$

ALT SOL 6

THE LOCATION WHERE THE SHEAR STRESS CROSSES THE Y AXIS IS ALSO WHERE THE BENDING MOMENT IS A MAXIMUM. SUBSTITUTING (30) INTO (27) TO DETERMINE THE MAXIMUM BENDING MOMENT.

$$M_2(0.7041 \cdot L) = \frac{q_0 \cdot L^2}{24}\left(-8\frac{(.7041L)^3}{L^3} + 12\frac{(.7041L)^2}{L^2} - 5\frac{.7041 \cdot L}{L} + 1\right)$$

$$= \underline{\underline{0.0265 \cdot q_0 \cdot L^2}} \qquad (31)$$

IN THE REGION $L/2 < Y < L$ THE END POINT RESULTS INDICATE THAT THE CURVATURE OF THE BEAM CHANGES SIGN. THE LOCATION WHERE $\Theta = 0$ NEEDS TO BE FOUND. (TECHNICALLY BOTH REGIONS SHOULD BE CHECKED FOR ZEROS BECAUSE THIS IS A FORTH ORDER POLYNOMIAL). STARTING WITH (28)

$$\Theta_2(y) = 0 = \frac{q_0 L^3}{5760EI}\left(480\frac{y^4}{L^4} - 960\frac{y^3}{L^3} + 600\frac{y^2}{L^2} - 240\frac{y}{L} + 67\right)$$

$$0 = \left(\frac{y}{L}\right)^4 - \frac{960}{480}\left(\frac{y}{L}\right)^3 + \frac{600}{480}\left(\frac{y}{L}\right)^2 - \frac{240}{480}\left(\frac{y}{L}\right) + \frac{67}{480}$$

$$0 = \left(\frac{y}{L}\right)^4 - 2\cdot\left(\frac{y}{L}\right)^3 + 1.25\left(\frac{y}{L}\right)^2 - 0.5\left(\frac{y}{L}\right) + 0.1396$$

$$\frac{y}{L} = \underbrace{\cancel{1.2473}}_{\text{OUTSIDE RANGE}},\ 0.5554,\ \underbrace{\cancel{0.0987 + i\,0.4379}}_{\text{OUTSIDE RANGE}},\ \underbrace{\cancel{0.0987 - i\,0.4379}}_{\text{OUTSIDE RANGE}}$$

$$y = 0.5554 \cdot L \qquad (32)$$

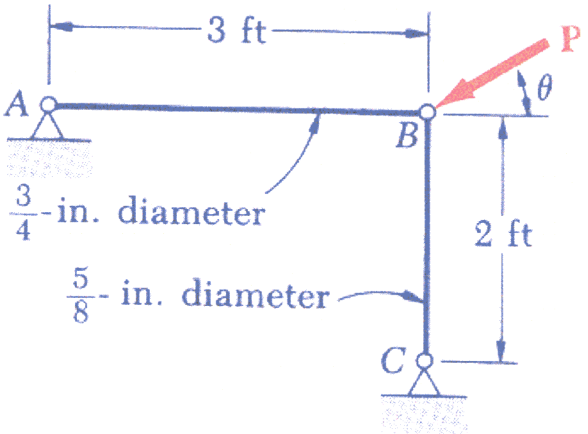
THE LOCATION WHERE THE CURVATURE IS ZERO, $\Theta = 0$, IS THE LOCATION OF THE MAXIMUM BENDING MOMENT. (32) IS SUBSTITUTED INTO (29)

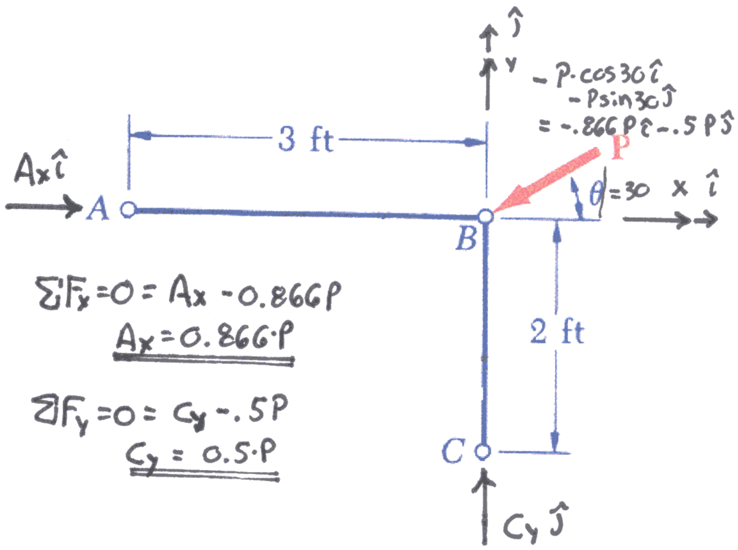
$$u_2 = \frac{q_0 L^4}{5760EI}\left[96(0.5554)^5 - 240(0.5554)^4 + 200(0.5554)^3 - 120(0.5554)^2 + 67(0.5554) - 3\right]$$

$$= \underline{\underline{0.00238\frac{q_0 L^4}{EI}}}$$

PROBLEM 2: For the structure being shown, θ=30° and E=29x10^6 psi.

2a. Using Euler's formula determine the largest load P which may be applied to the structure when only buckling in the plane of the structure is considered.



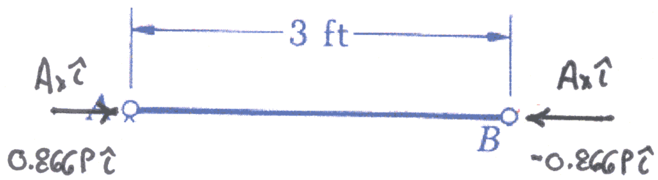


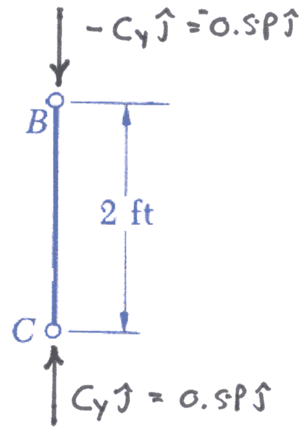
$\Sigma F_x = 0 = A_x - 0.866P$

$\underline{A_x = 0.866 \cdot P}$

$\Sigma F_y = 0 = C_y - .5P$

$\underline{C_y = 0.5 \cdot P}$





STARTING WITH COLUMN AB

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^2 = \frac{\pi}{4}\left(\frac{1}{2}\cdot\frac{3}{4}in\right)^4 = 0.01553\ in^4$$

$$P_{CR} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \cdot 29\times10^6 \frac{lb}{in^2} \cdot 0.01553\ in^4}{(36\ in)^2} = 3430\ lb = F_{AB} = 0.866P$$

$$\Rightarrow \boxed{P = 3961\ lb}$$

FOR COLUMN BC

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^2 = \frac{\pi}{4}\left(\frac{1}{2}\cdot\frac{3}{8}in\right)^4 = 0.00749\ in^4$$

$$P_{CR} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \cdot 29\times10^6 \frac{lb}{in^2} \cdot 0.00749\ in^4}{(24\ in)^2} = 3722\ lb = F_{BC} = 0.5P$$

$$\Rightarrow P = 7444\ lb$$

2b. Knowing that a factor of safety of 2.8 is required, what is the largest load P which can be applied to the structure.

$$\frac{P}{S.F.} = \frac{3961\ lb}{2.8} = \boxed{1415\ lb}$$