PROBLEM 7.50 A MECHANICAL PART IS MADE OF STEEL WITH THE PROPERTIES $S_u = 560\,MPa$, $S_y = 490\,MPa$, AND $S_e = 210\,MPa$. THE PART IS SUBJECTED TO A BENDING STRESS THAT ALTERNATES BETWEEN 100 MPa AND 200 MPa. DETERMINE THE EQUIVALENT REVERSING STRESS AND EVALUATE THE FACTOR OF SAFETY CORRESPONDING TO A LIFE OF 500,000 CYCLES.

GIVEN:

1. STEEL PART: $S_u = 560\,MPa$, $S_y = 490\,MPa$, $S_e = 210\,MPa$
2. ALTERNATING BENDING STRESS 100 MPa – 200 MPa

ASSUMPTIONS:

1. LINEAR ELASTIC MATERIAL RESPONSE
2. NO YIELDING, DUCTILE MATERIAL

FIND:

1. EQUIVALENT REVERSING STRESSES
2. FACTOR OF SAFETY CORRESPONDING TO 500,000 CYCLES.

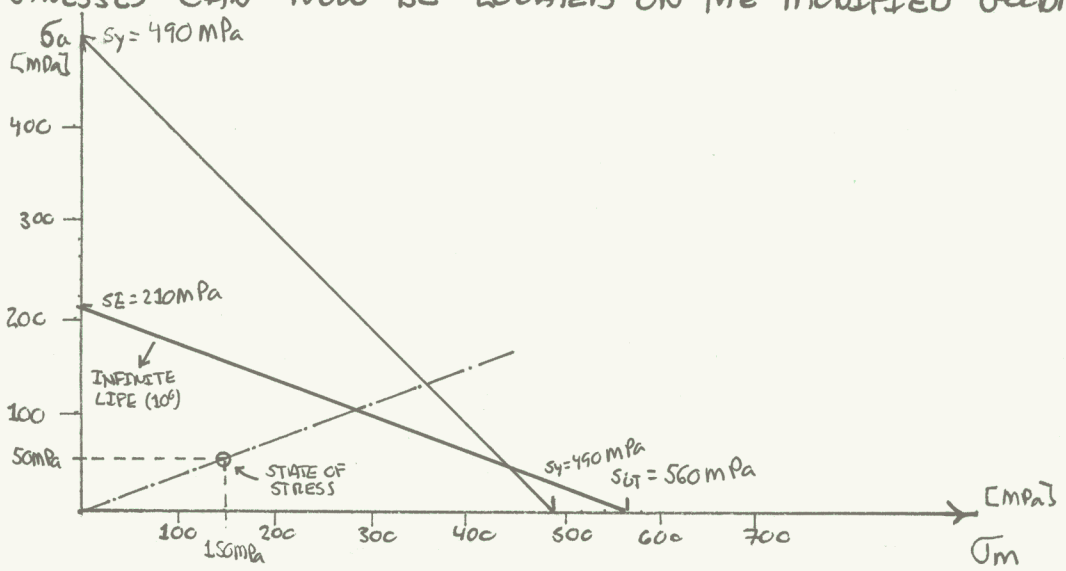
SOLUTION:

FOR THE LOADING PRESCRIBED, THE MEAN AND ALTERNATING STRESSES ARE CALCULATED.

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{(200\,MPa) + (100\,MPa)}{2} = 150\,MPa \qquad (1)$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{(200\,MPa) - (100\,MPa)}{2} = 50\,MPa \qquad (2)$$

THESE STRESSES CAN NOW BE LOCATED ON THE MODIFIED GOODMAN DIAGRAM



(a)

[MODIFIED GOODMAN DIAGRAM ILLUSTRATING THE STATE OF STRESS UNDER CONSIDERATION AND ITS RELATIONSHIP TO FRACTURE AND YIELD FAILURES.]

THE LINE CORRESPONDING TO 10^5 CYCLES NEEDS TO BE ~~DRAWN~~ DRAWN. THIS LINE WILL INTERSECT THE HORIZONTAL σ_m AXIS AT S_{UT}. SINCE THE VERTICAL AXIS CORRESPONDS TO REVERSE LOADING, THE CALCULATIONS FOR FINITE LIFE ON THE S-N DIAGRAM CAN BE USED TO CALCULATE THE FATIGUE STRENGTH THAT CORRESPONDS TO 500,000 CYCLES. THE VALUE OF THE FATIGUE STRENGTH IS THE VERTICAL INTERCEPT THAT ~~will~~ IS NEEDED TO ESTABLISH THE 500,000 CYCLE FINITE LIFE LINE ON THE MODIFIED GOODMAN DIAGRAM.

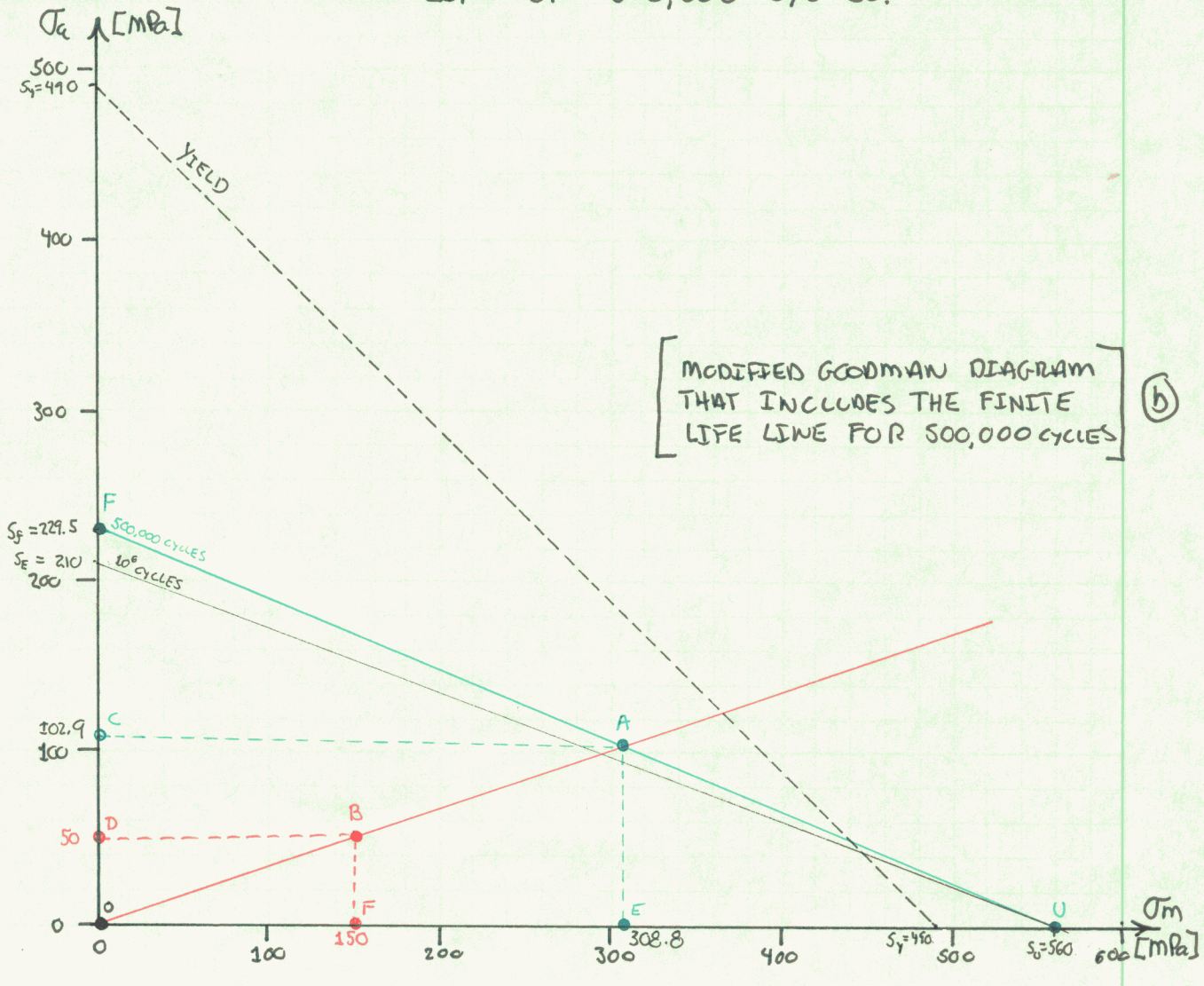
$$m = \frac{1}{3}\log\frac{(0.9)(560\text{MPa})}{210\text{MPa}} = 0.12674 \quad (3)$$

$$b = \log\frac{[(0.9)(560\text{MPa}]^2}{210\text{MPa}} = 3.083\log(\text{MPa}) \quad (4)$$

$$S_f = \frac{10^b}{N^m} = \frac{10^{3.083\log\text{MPa}}}{(500{,}000)^{0.12674}} = \frac{10^{3.083}\,10^{\log(\text{MPa})}}{(500{,}000)^{0.12674}} = \frac{10^{3.083}\,\text{MPa}}{(500{,}000)^{0.12674}} \quad (5)$$

$$= 229.5\text{ MPa}$$

THE MODIFIED GOODMAN CAN NOW BE DRAW WITH A LINE THAT REPRESENTS THE FINITE LIFE OF 500,000 CYCLES.



[MODIFIED GOODMAN DIAGRAM THAT INCLUDES THE FINITE LIFE LINE FOR 500,000 CYCLES] (b)

POINT "B" ON FIGURE (b) REPRESENTS THE STATE OF STRESS UNDER CONSIDERATION. POINT "A" REPRESENTS THE MAXIMUM VALUES OF σ_m AND σ_a THAT MAINTAIN THE ORIGINAL σ_m/σ_a RATIO. THERE ARE SEVERAL WAYS TO DETERMINE THE FACTOR OF SAFETY. USING THE POINTS LABELED ON (b), THE FACTOR OF SAFETY IS CALCULATED AS FOLLOWS.

$$n = \frac{OA}{OB} = \frac{OC}{OD} = \frac{OE}{OF} \qquad (5)$$

THE CALCULATION OF THE VALUES IS (5) REQUIRES THE DETERMINATION OF THE σ_m, σ_A COORDINATES OF POINT "A".

DEVELOPING AN EQUATION FOR THE LINE $\overline{OBA}$

$$\sigma_{a,OBA} = m \cdot \sigma_{m,OBA} + b$$

$$m = \frac{50\,MPa}{150\,MPa} = \frac{1}{3}$$

$$b = 0$$

$$\underline{\sigma_{a,OBA} = \frac{1}{3} \cdot \sigma_{m,OBA}} \qquad (6)$$

DEVELOPING AN EQUATION FOR LINE $\overline{FAU}$

$$\sigma_{a,FAU} = m \cdot \sigma_{m,FAU} + b$$

$$m = \frac{(0 - 229.5\,MPa)}{(560\,MPa - 0)} = -0.4098$$

$$b = 229.5\ MPa$$

$$\underline{\sigma_{a,FAU} = -0.4098 \cdot \sigma_{m,FAU} + 229.5\,MPa} \qquad (7)$$

THE INTERSECTION OF LINES (6) AND (7) OCCUR WHERE $\sigma_{a,OBA} = \sigma_{a,FUA} = \sigma_a$ AND $\sigma_{m,OBA} = \sigma_{m,FAU} = \sigma_m$, OR

$$\frac{1}{3} \cdot \sigma_m = -0.4098 \cdot \sigma_m + 229.5\,MPa$$

$$\sigma_m = \frac{229.5\,MPa}{\frac{1}{3} + 0.4098} = \underline{308.8\,MPa} \qquad (8)$$

$$\sigma_a = \frac{1}{3} \cdot 308.8\,MPa = \underline{102.9\,MPa} \checkmark \qquad (9)$$

$$= -0.4098 \cdot (308.8\,MPa) + 229.5\,MPa = 102.9\,MPa \checkmark$$

NOW THE FACTOR OF SAFETY OF THIS STATE OF STRESS GIVEN THE FINITE LIFE OF 500,000 CYCLES CAN BE COMPUTED USING (5)

$$n = \frac{OA}{OB} = \frac{\sqrt{(308.8\,MPa)^2 + (102.9\,MPa)^2}}{\sqrt{(150\,MPa)^2 + (50\,MPa)^2}} = \frac{325.5\,MPa}{158.11\,MPa} = \underline{2.06} \checkmark$$

$$= \frac{OC}{OD} = \frac{102.9}{50} = 2.06 \checkmark$$

$$= \frac{OE}{OF} = \frac{308.8}{150} = 2.06 \checkmark$$

SUMMARY:

THE LINES THAT EXTEND FROM S_E AND S_J AND EXTEND TO S_{UT} ~~CONSTD~~ ARE LINES THAT REPRESENT THE ONSET OF FRACTURE. BOTH OF THESE LINES INTERSECT THE LINE THAT STARTS AT S_y ON THE VERTICAL AXES AND EXTENDS TO S_y ON THE HORIZONTAL AXIS. IN FATIGUE SITUATIONS YIELDING SHOULD BE AVOIDED.