PROBLEM 12 DETERMINE THE DEFLECTION AND CURVATURE OF POINTS "C" AND "D" ALONG WITH THE LOCATION OF THE MAXIMUM DEFLECTION USING THE MOMENT AREA METHOD.

GIVEN:

CONSTRAINTS

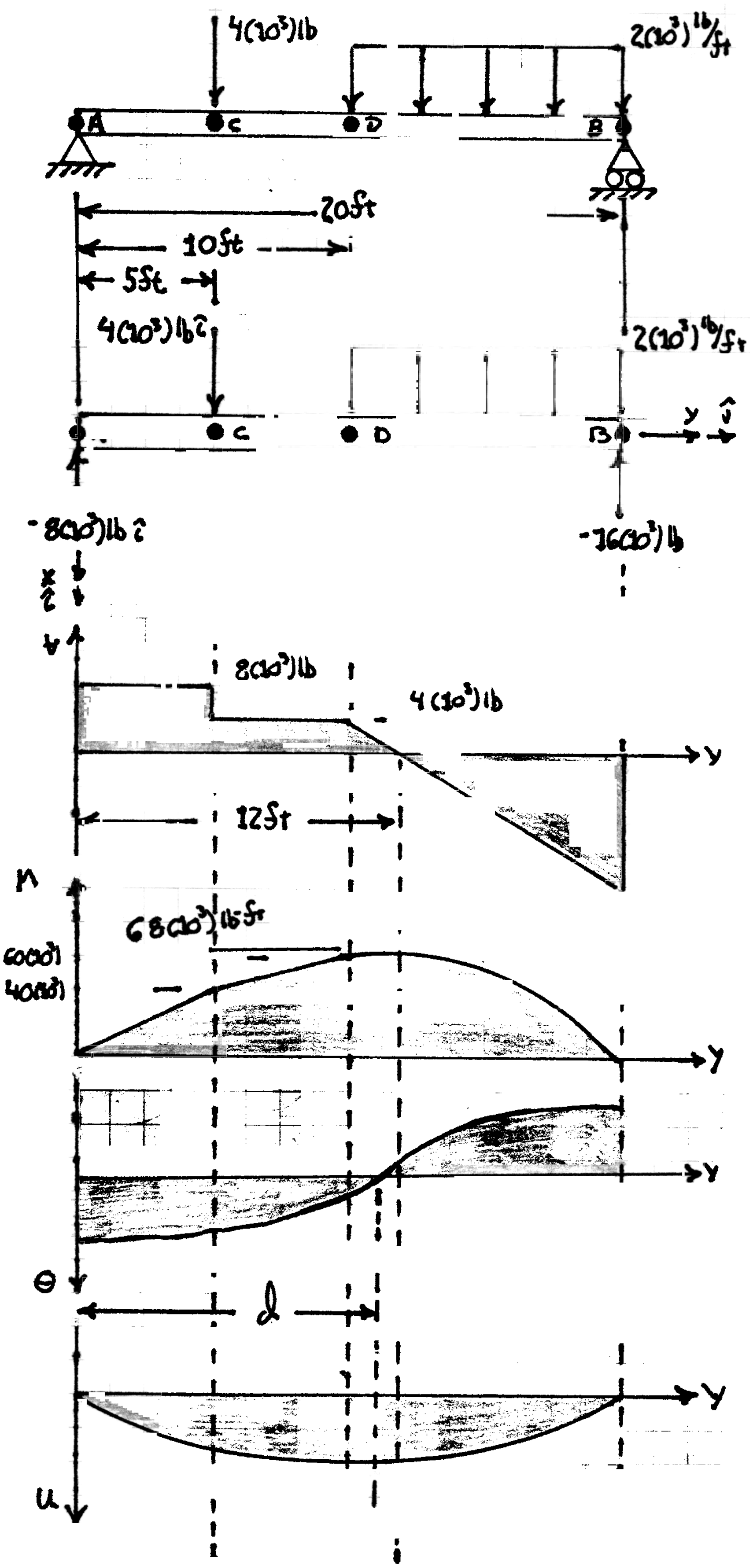
1. 20 ft LONG BEAM WITH SIMPLE SUPPORTS AT BOTH ENDS
2. A DISTRIBUTED LOAD OF $2(10^3)$ lb/ft IS APPLIED ~~OVER~~ FROM THE MID-SPAN TO THE END OF THE BEAM.
3. $4(10^3)$ lb LOAD APPLIED AT QUARTER SPAN

ASSUMPTIONS

1. THE BEAM IS MADE OF LINEAR ELASTIC MATERIAL
2. ALL DEFLECTIONS AND STRAINS IN THE BEAM ARE SMALL

FIND:

1. DETERMINE THE DEFLECTION AT POINTS "C" AND "D"
2. DETERMINE THE CURVATURE AT POINTS "C" AND "D"



SOLUTION:

THE FIGURES TO THE RIGHT SHOW THE ORIGIONAL BEAM, A FREE BODY DIAGRAM OF THE BEAM, THE SHEAR FORCE DIAGRAM, BENDING MOMENT DIAGRAM, AND DEFLECTION DIAGRAM. THE FOUR DIAGRAMS WERE DRAWN BY INSPECTION FROM THE FREE-BODY DIAGRAM AND AN UNDERSTANDING OF THE INTEGRAL RELATIONSHIP BETWEEN THE FIGURES.

THE SOLUTION NEEDS TO START WITH THE LOCATION OF THE MAXIMUM DEFLECTION OF THE ELASTIC CURVE.

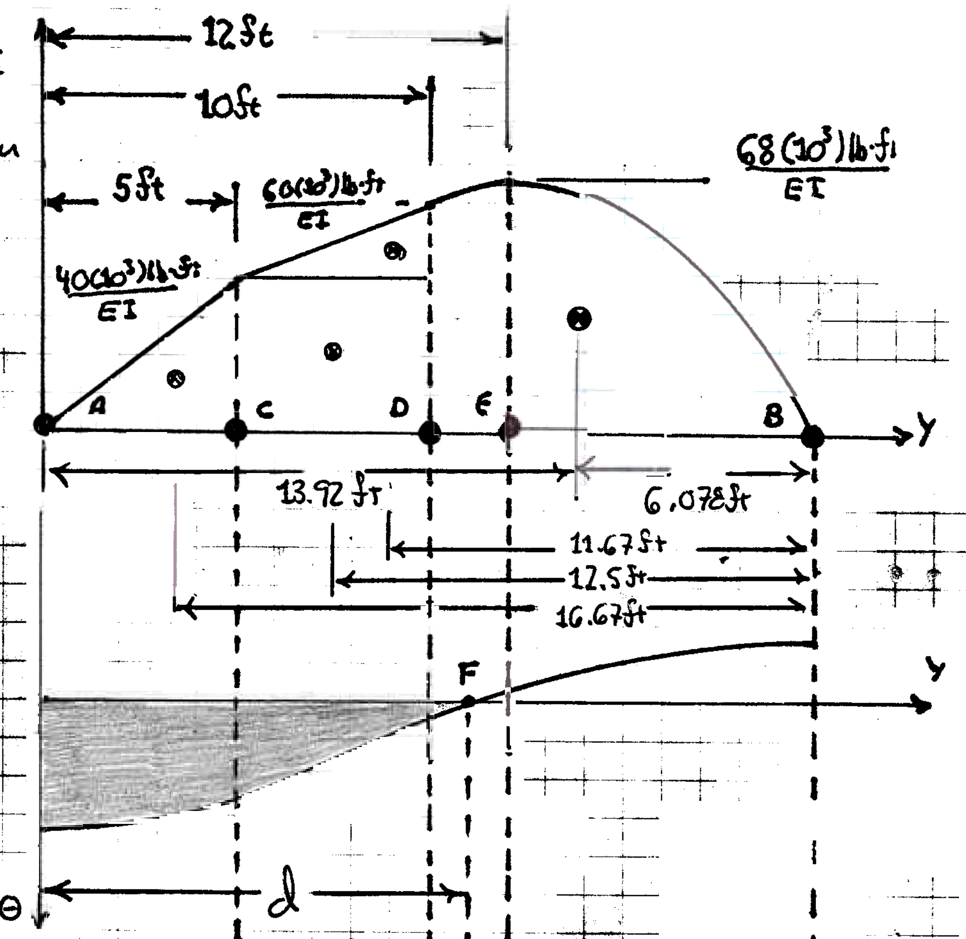
THE SHAPE OF THE MOMENT $\frac{M}{EI}$ DIAGRAM BETWEEN "D" AND "B" IS PARABOLIC. THIS MAKES THE MOMENT AREA CALCULATION MORE DIFFICULT. WE NEED TO FIRST FIND A FUNCTION THAT REPRESENTS THE M/EI DIAGRAM BETWEEN "D" AND "B"

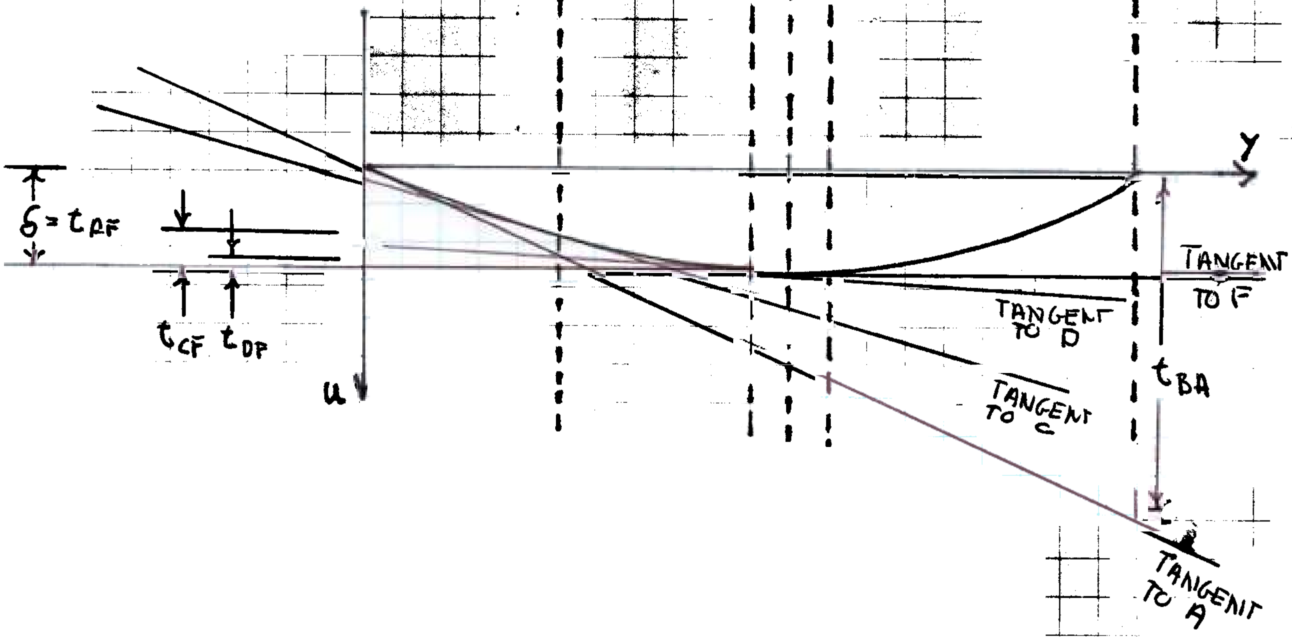
$$q = -8(10^3)\,lb\langle y-0\rangle_{-1} + 4(10^3)\,lb\langle y-5ft\rangle_{-1} + 2(10^3)\,lb/ft\langle y-10ft\rangle^0$$

$$V = -\int q\,dy$$

$$= 8(10^3)\,lb\langle y-0\rangle^0 - 4(10^3)\,lb\langle y-5ft\rangle^0 - 2(10^3)\,lb/ft\langle y-10ft\rangle^1$$

$$M = 8(10^3)\,lb\langle y-0\rangle^1 - 4(10^3)\,lb\langle y-5ft\rangle^1 - 1(10^3)\,lb/ft\langle y-10ft\rangle^2$$





$$\frac{M}{EI} = \frac{1}{EI}\left[8(10^3)\,lb\langle y-0\rangle^1 - 4(10^3)\,lb\langle y-5ft\rangle^1 - 1(10^3)\,lb/ft\,\langle y-10ft\rangle^2\right]$$

IN THE REGION FROM 10 ft TO 20 ft (BETWEEN "D" AND "B")

$$\frac{M^*}{EI} = \frac{1}{EI}\Big[8(10^3)\,lb\cdot y - 4(10^3)\,lb\cdot y + 20(10^3)\,lb\cdot ft$$
$$- 1(10^3)\tfrac{lb}{ft}\cdot\left(y^2 - 20ft\cdot y + 100\,ft^2\right)\Big]$$

$$= \frac{1}{EI}\left[24(10^3)\,lb\cdot y - 80(10^3)\,lb\cdot ft - 1(10^3)\tfrac{lb}{ft}\cdot y^2\right]$$

THE AREA UNDER THE M/EI DIAGRAM BETWEEN "D" AND "B" IS CALCULATED

$$A = \int \frac{M^*}{EI}\,dy = \frac{1}{EI}\int_{10}^{20}\left[24(10^3)\,lb\cdot y - 80(10^3)\,lb\cdot ft - 1(10^3)\tfrac{lb}{ft}\cdot y^2\right]dy$$

$$= \frac{1}{EI}\left[12(10^3)\,lb\cdot y^2 - 80(10^3)\,lb\cdot ft\cdot y - .3333(10^3)\tfrac{lb}{ft}\cdot y^3\right]_{10}^{20}$$

$$= \frac{466.9(10^3)\,lb\cdot ft^2}{EI}$$

THE CENTROID OF THIS CROSS SECTION WITH RESPECT TO THE ORIGIN

$$\bar{y} = \frac{\int y\cdot\frac{M^*}{EI}\cdot dy}{A}$$

$$\int y\cdot\frac{M^*}{EI}dy = \frac{1}{EI}\int_{10}^{20}\left[24(10^3)\,lb\cdot y^2 - 80(10^3)\,lb\cdot ft\cdot y - 1(10^3)\tfrac{lb}{ft}\cdot y^3\right]dy$$

$$= \frac{1}{EI}\left[8(10^3)\,lb\cdot y^3 - 40(10^3)\,lb\cdot ft\cdot y^2 - .25(10^3)\tfrac{lb}{ft}\cdot y^4\right]_{10}^{20}$$

$$= \frac{6.500(10^6)\,lb\cdot ft^3}{EI}$$

$$\bar{y}_J = \frac{6.50(10^6)\,lb\cdot ft^3}{EI} \Big/ \frac{466.9(10^3)\,lb\cdot ft^2}{EI} = 13.92\,ft$$

NOW t_{BA} CAN BE CALCULATED

$$t_{BA} = (16.67ft)\cdot\tfrac{1}{2}\cdot(5ft)\cdot\frac{40(10^3)\,lb\cdot ft}{EI} + (12.5ft)\cdot(5ft)\cdot\left(\frac{40(10^3)\,lb\cdot ft}{EI}\right)$$
$$+(11.67ft)\cdot\tfrac{1}{2}\cdot(5ft)\cdot\left(\frac{20(10^3)\,lb\cdot ft}{EI}\right) + (6.078ft)\cdot\left(\frac{466.9(10^3)\,lb\cdot ft^2}{EI}\right)$$
$$= \frac{1}{EI}\cdot 7.588(10^6)\,lb\cdot ft^3$$

NOW THE ANGLE Θ_A CAN BE CALCULATED USING THEOREM II

$$\Theta_A \approx \tan\Theta_A = \frac{7.588(10^6)\,lb\cdot ft^3}{EI\cdot 20ft} = \frac{379.4(10^3)\,lb\cdot ft^2}{EI}$$

NOW THE ANGLE Θ_A IS CALCULATED WITH THE USE OF THEOREM I

$$\Theta_A = \int_0^d \frac{M}{EI}\,dy = \tfrac{1}{2}\cdot(5ft)\cdot\left(\frac{40(10^3)\,lb\cdot ft}{EI}\right) + (5ft)\cdot\left(\frac{40(10^3)\,lb\cdot ft}{EI}\right) + \tfrac{1}{2}\cdot(5ft)\cdot\left(\frac{20(10^3)\,lb\cdot ft}{EI}\right)$$
$$+\frac{1}{EI}\int_{10ft}^{d}\left[24(10^3)\,lb\cdot y - 80(10^3)\,lb\cdot ft - 1(10^3)\tfrac{lb}{ft}\cdot y^2\right]dy$$
$$= \frac{350.0(10^3)\,lb\cdot ft^2}{EI} + \frac{1}{EI}\left[12(10^3)\,lb\cdot y^2 - 80(10^3)\,lb\cdot ft\cdot y - .3333(10^3)\tfrac{lb}{ft}\,y^3\right]_{10ft}^{d}$$
$$= \frac{350.0(10^3)\,lb\cdot ft^2}{EI} + \frac{1}{EI}\left[12(10^3)\,lb\cdot d^2 - 80(10^3)\,lb\cdot ft\cdot d - .3333(10^3)\tfrac{lb}{ft}\,d^3\right.$$
$$\left. - \frac{66.7(10^3)\,lb\cdot ft^2}{EI}\right]$$
$$= \frac{1}{EI}\left[-0.3333(10^3)\tfrac{lb}{ft}\cdot d^3 + 12(10^3)\,lb\cdot d^2 - 80(10^3)\,lb\cdot ft\cdot d + 283.3(10^3)\,lb\,ft^2\right]$$

NOW THE TWO APPROACHES TO CALCULATING Θ_A CAN BE EQUATED

$$\frac{379.4(10^3)\,lb\cdot ft^2}{EI} = \frac{-0.3333(10^3)\tfrac{lb}{ft}\cdot d^3 + 12(10^3)\,lb\cdot d^2 - 80(10^3)\,lb\cdot ft\cdot d + 283.3(10^3)\,lb\,ft^2}{EI}$$
$$0 = 0.3333(10^3)\tfrac{lb}{ft}\cdot d^3 - 12(10^3)\,lb\cdot d^2 + 80(10^3)\,lb\cdot ft\cdot d + 96.10(10^3)\,lb\cdot ft^2$$
$$0 = d^3 - 36ft\cdot d^2 + 240ft^2 d + 288.3ft^3$$

$$d = -1.04,\ 10.48,\ 26.55$$

THE ONLY ROOT IN THE DOMAIN OF THE PROBLEM IS

$$\underline{d = 10.48}$$

NOW THAT THE LOCATION OF THE MAXIMUM DEFLECTION ON THE ELASTIC CURVE HAS FOUND THE CURVATURES AT POINTS "C" AND "D" CAN BE CALCULATED

$$\Theta_D = \frac{1}{EI}\int_{10}^{10.48}\left[24(10^3)\,lb\cdot y - 80(10^3)\,lb\cdot ft - 1(10^3)\frac{lb}{ft}\cdot y^2\right]dy$$

$$\frac{1}{EI}\left[12(10^3)\,lb\cdot y^2 - 80(10^3)\,lb\cdot ft\cdot y - .3333(10^3)\frac{lb}{ft}\cdot y^3\right]_{10\,ft}^{10.48\,ft}$$

$$= \boxed{\frac{29.23\,(10^3)\,lb\cdot ft^2}{EI}}$$

$$\Theta_C = \frac{1}{2}\cdot(5\,ft)\cdot\left(\frac{20(10^3)\,lb\cdot ft}{EI}\right) + (5\,ft)\cdot\left(\frac{40(10^3)\,lb\cdot ft}{EI}\right) + \frac{29.23\,(10^3)\,lb\cdot ft^2}{EI}$$

$$= \boxed{\frac{279.2\,(10^3)\,lb\cdot ft^2}{EI}}$$

THE DEFLECTIONS AT "C" AND "D" CAN NOW BE CALCULATED

$$\delta_D = t_{AF} - t_{DF}$$

TO CALCULATE THE MOMENT AREAS ASSOCIATED WITH t_{AF} AND t_{DF}, THE CENTROID OF SECTION DF NEEDS TO BE CALCULATED.

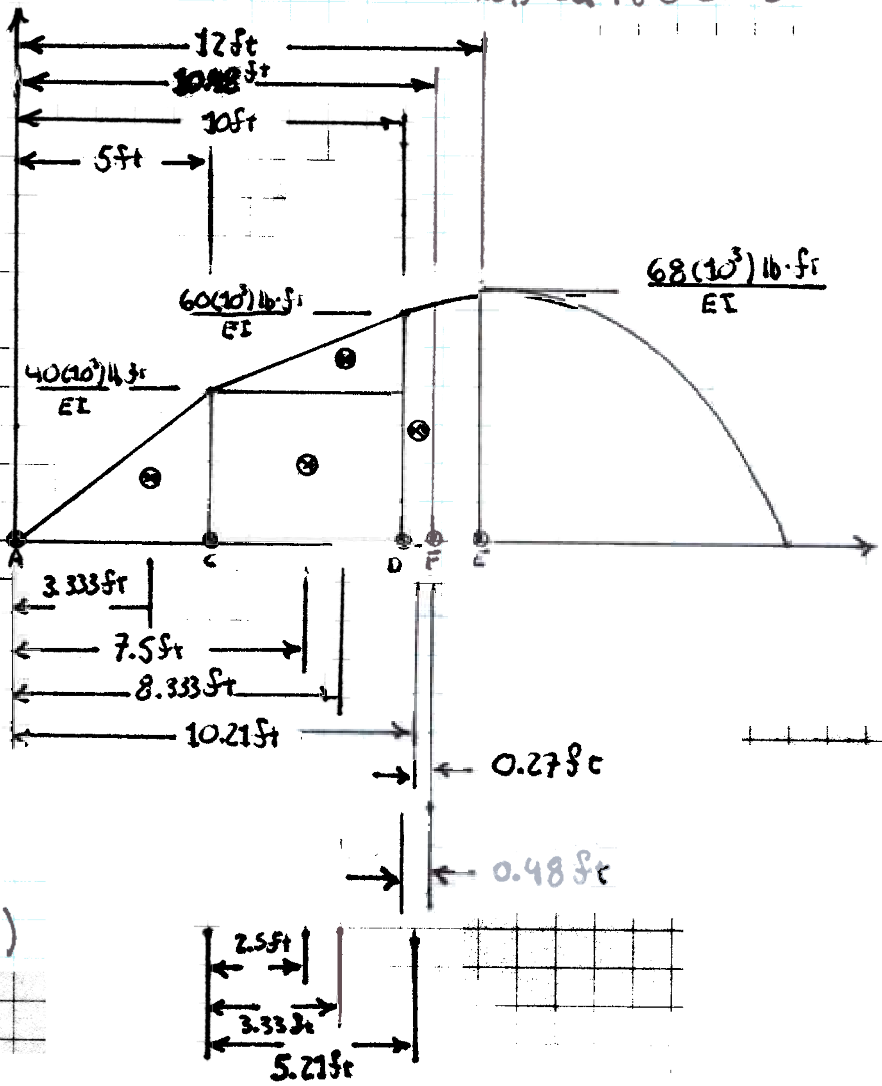
$$\bar{Y}_F = \frac{\int_{10}^{10.48\,ft} y\,\frac{M}{EI}\,dy}{A}$$

$$\int_{10\,ft}^{10.48\,ft}\frac{y\cdot M}{EI}\,dy = \int_{10\,ft}^{10.48\,ft}\frac{1}{EI}\left[24(10^3)\,lb\cdot y^2 - 80(10^3)\,lb\cdot ft\cdot y - 1(10^3)\frac{lb}{ft}\cdot y^3\right]dy$$

$$= \frac{1}{EI}\left[8(10^3)\,lb\cdot y^3 - 40(10^3)\,lb\cdot ft\cdot y^2 - .25(10^3)\frac{lb}{ft}\cdot y^4\right]_{10\,ft}^{10.48\,ft}$$

$$= \frac{1}{EI}\cdot 299.3(10^3)\,lb\cdot ft^3$$

$$\bar{Y}_F = \frac{\frac{299.3(10^3)\,lb\cdot ft^3}{EI}}{\frac{29.23(10^3)\,lb\cdot ft^2}{EI}} = \underline{\underline{10.21\,ft}}$$



STARTING WITH THE CALCULATION OF t_{AF}, THE MAXIMUM DEFLECTION ON THE ELASTIC CURVE.

$$\delta = t_{AF}$$

$$= (3.333\,ft)\cdot\left(\tfrac{1}{2}\right)\cdot(5\,ft)\cdot\frac{40(10^3)\,lb\cdot ft}{EI} + (7.5\,ft)\cdot(5\,ft)\cdot\frac{40(10^3)\,lb\cdot ft}{EI} + (8.333\,ft)\cdot\tfrac{1}{2}\cdot(5\,ft)\cdot\frac{20(10^3)\,lb\cdot ft}{EI} + (10.21\,ft)\cdot\frac{29.23(10^3)\,lb\cdot ft^2}{EI}$$

$$= \frac{2.548(10^6)\,lb\cdot ft^3}{EI}$$

$$t_{DF} = (0.27\,ft)\cdot\left(\frac{29.23(10^3)\,lb\cdot ft^2}{EI}\right) = \frac{7.892(10^3)\,lb\cdot ft^3}{EI}$$

$$\delta_D = \frac{2.548(10^6)\,lb\cdot ft^3}{EI} - \frac{7.89(10^3)\,lb\cdot ft^3}{EI} = \boxed{\frac{2.540(10^6)\,lb\cdot ft^3}{EI}}$$

NOW THE DEFLECTION AT POINT C CAN BE CALCULATED.

$$\delta_C = \delta - t_{CF}$$

$$t_{CF} = (2.5\,ft)\cdot(5\,ft)\cdot\left(\frac{40(10^3)\,lb\cdot ft}{EI}\right) + (3.33\,ft)\cdot\tfrac{1}{2}\cdot(5\,ft)\cdot\left(\frac{20(10^3)\,lb\cdot ft}{EI}\right) + (5.21\,ft)\cdot\left(\frac{29.23(10^3)\,lb\cdot ft^3}{EI}\right)$$

$$= \frac{818.8(10^3)\,lb\cdot ft^3}{EI}$$

$$\delta_C = \frac{2.548(10^6)\,lb\cdot ft^3}{EI} - \frac{818.8(10^3)\,lb\cdot ft^3}{EI} = \boxed{\frac{1.729(10^3)\,lb\cdot ft^3}{EI}}$$

SUMMARY: THIS MAY HAVE NOT BEEN THE BEST WAY TO SOLVE THIS PROBLEM. THE DISTRIBUTED LOAD CAUSES A PARABOLIC DISTRIBUTION IN THE MOMENT THAT FORCES THE AREA AND MOMENT AREA TO BE CALCULATED THROUGH DIRECT INTEGRATION.