A bar of steel has $S_u = 700$ MPa, $S_y = 500$ MPa, and a fully corrected endurance limit of $S_e = 200$ MPa. For each case below find the factor of safety which guard against static and fatigue failures

(a) $\tau_m = 140$ MPa

(b) $\tau_m = 140$ MPa, $\tau_a = 70$ MPa

(c) $\tau_{xym} = 100$ MPa, $\sigma_{xa} = 80$ MPa

(d) $\sigma_{xm} = 60$ MPa, $\sigma_{xa} = 80$ MPa
$\tau_{xym} = 70$ MPa, $\tau_{xya} = 35$ MPa

Solution:

(a) $\tau_m = 140\ MPa$

Using maximum shear theory

$$S_{sy} = 0.5\ S_y = 0.5(500\ MPa) = 250\ MPa$$

$$n = \frac{S_{sy}}{\tau_{max}} = \frac{250\ MPa}{140\ MPa} = \underline{\underline{1.78}} \quad (\text{static})$$

(b) $\tau_m = 140\ MPa$, $\tau_a = 70\ MPa$

$$\tau_{max} = \tau_m + \tau_a = 140\ MPa + 70\ MPa = 210\ MPa$$

$$n = \frac{S_{sy}}{\tau_{max}} = \frac{250\ MPa}{210\ MPa} = \underline{\underline{1.19}} \quad (\text{static})$$

Now we need to calculate the endurance limit in shear to determine the fatigue factor of safety.

$$S_{se} = 0.5\ S_e = 0.5(200\ MPa) = 100\ MPa$$

$$n = \frac{S_{se}}{\tau_a} = \frac{100\ MPa}{70\ MPa} = \underline{\underline{1.43}} \quad (\text{fatigue})$$

Note that for shear loading only the alternating component is used to calculate the factor of safety. As previously discussed, the shear endurance limit is not effected by the mean stress level as long as the material does not yield.

(c) $\tau_{xy\,m} = 100$ MPa , $\sigma_{xa} = 80$ MPa

The maximum von Mises stress occurs when the alternating component is summed with the mean component

$$\sigma'_{max} = \sqrt{\sigma_{xa}^2 + 3\tau_{xym}^2} = \sqrt{(80\,MPa)^2 + 3(100\,MPa)^2} = 191\,MPa$$

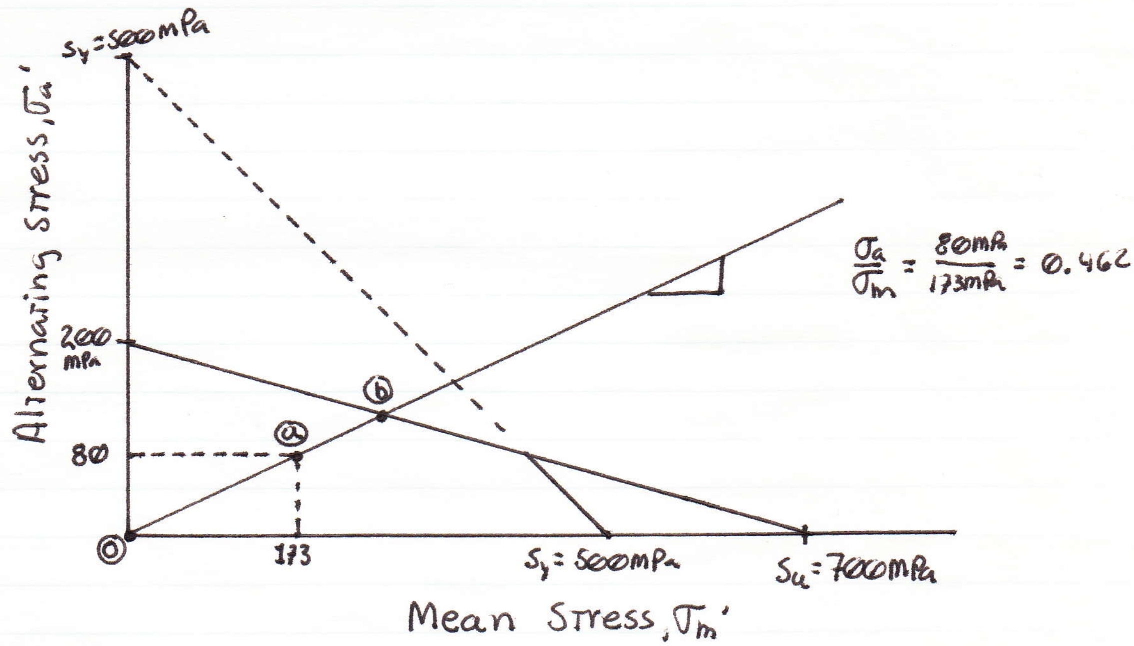
Therefore the static factor of safety is

$$n = \frac{S_y}{\sigma'_{max}} = \frac{500\,MPa}{191\,MPa} = \underline{\underline{2.62}} \qquad \text{(STATIC)}$$

Now lets consider a fatigue failure

$$\sigma'_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} = \sqrt{3(100\,MPa)^2} = 173\,MPa$$

$$\sigma'_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2} = \sigma_{xa} = 80\,MPa$$



Point ⓐ on the diagram is defined from the von Mises calculations. Point ⓑ is the intersection of the two lines.

Let's determine the equation of the Goodman line.

$$\sigma_a' = m\,\sigma_m' + b \qquad \text{(standard equation of a line)}$$

$$b \equiv \text{intercept} = 200 \text{ MPa}$$

$$m \equiv \text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{-200 \text{ MPa}}{700 \text{ MPa}} = -0.286$$

$$\sigma_a' = -0.286\,\sigma_m' + 200 \text{ MPa} \qquad (1)$$

Now lets consider the equation of the line that defines the ratio between σ_a and σ_m

$$\sigma_a' = m\,\sigma_m' + b$$

$$b = 0$$

$$m = \frac{\sigma_a}{\sigma_m} = 0.462$$

$$\sigma_a' = 0.462\,\sigma_m' \qquad (2)$$

The intercept is defined where σ_a' and σ_m' are equal. Therefore we can write

$$\sigma_a'^{(2b)} = \sigma_a'^{(2b)}$$

$$-0.286\,\sigma_m'^{(b)} + 200 \text{ MPa} = 0.462\,\sigma_m'^{(b)}$$

$$\sigma_m'^{(b)} = \underline{267.4 \text{ MPa}}$$

$$\sigma_a'^{(b)} = \underline{123.5 \text{ MPa}}$$

Now lets determine o-a and o-b since

$$n = \frac{o\text{-}b}{o\text{-}a} \qquad \text{(fatigue)}$$

$$o-b = \sqrt{(267.4\,MPa)^2 + (123.5\,MPa)^2} = \underline{294.5\,MPa}$$

$$o-a = \sqrt{(173\,MPa)^2 + (80\,MPa)^2} = \underline{190.6\,MPa}$$

$$n = \frac{294.5\,MPa}{190.6\,MPa} = \underline{\underline{1.55}}$$

(d) $\sigma_{xm} = 60\,MPa$, $\sigma_{xa} = 80\,MPa$, $\tau_{xym} = 70\,MPa$, $\tau_{xya} = 35\,MPa$

We will again consider static yielding first

$$\sigma_{x\,max} = \sigma_{xm} + \sigma_{xa} = 60\,MPa + 80\,MPa = \underline{140\,MPa}$$

$$\tau_{xy\,max} = \tau_{xym} + \tau_{xya} = 70\,MPa + 35\,MPa = \underline{105\,MPa}$$

Now we can compute the maximum von Mises stress

$$\sigma'_{max} = \sqrt{(140\,MPa)^2 + 3(105\,MPa)^2} = \underline{229\,MPa}$$

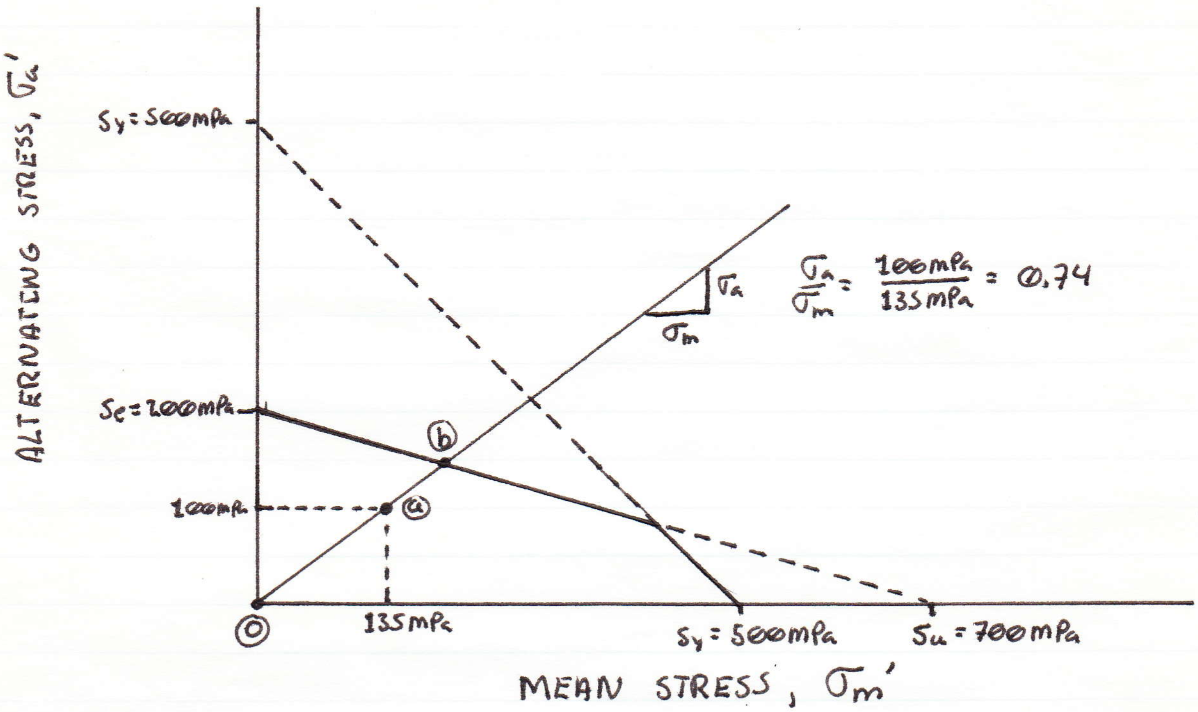
$$n = \frac{S_y}{\sigma'_{max}} = \frac{500\,MPa}{229\,MPa} = \underline{\underline{2.18}}$$

Now let's calculate the fatigue factor of safety

$$\sigma_m' = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} = \sqrt{(60\,MPa)^2 + 3(70\,MPa)^2} = \underline{135\,MPa}$$

$$\sigma_a' = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2} = \sqrt{(80\,MPa)^2 + 3(35\,MPa)^2} = \underline{100\,MPa}$$

Once again we draw the σ_m-σ_a diagram



The equation for the Goodman Line is, as before,

$$\sigma_a' = m\sigma_m' + b = -0.286\,\sigma_m' + 200\,\text{mPa} \qquad (1)$$

The equation for the line 0-a-b is

$$\sigma_a' = m\sigma_m' + b$$

$$b = 0$$
$$m = 0.74$$

$$\sigma_a' = 0.74\,\sigma_m' \qquad (3)$$

(b) can now be found from (1) and (3) intersection

$$\sigma_a'^{(2b)} = \sigma_a'^{(3b)}$$

$$-0.286\,\sigma_m'^{(b)} + 200\,\text{mPa} = 0.74\,\sigma_m'^{(b)}$$

$\sigma_m'^{(b)} = \underline{194.8 \text{ Mpa}}$

$\sigma_a'^{(b)} = \underline{144.1 \text{ MPa}}$

Knowing

$$n = \frac{0-b}{0-a}$$

$$0\text{-}b = \sqrt{(144.1 \text{ MPa})^2 + (194.8 \text{ MPa})^2} = 242.3 \text{ MPa}$$

$$0\text{-}a = \sqrt{(100 \text{ MPa})^2 + (135 \text{ MPa})^2} = 168 \text{ MPa}$$

$$n = \frac{242.3 \text{ MPa}}{168 \text{ MPa}} = \underline{\underline{1.44}}$$