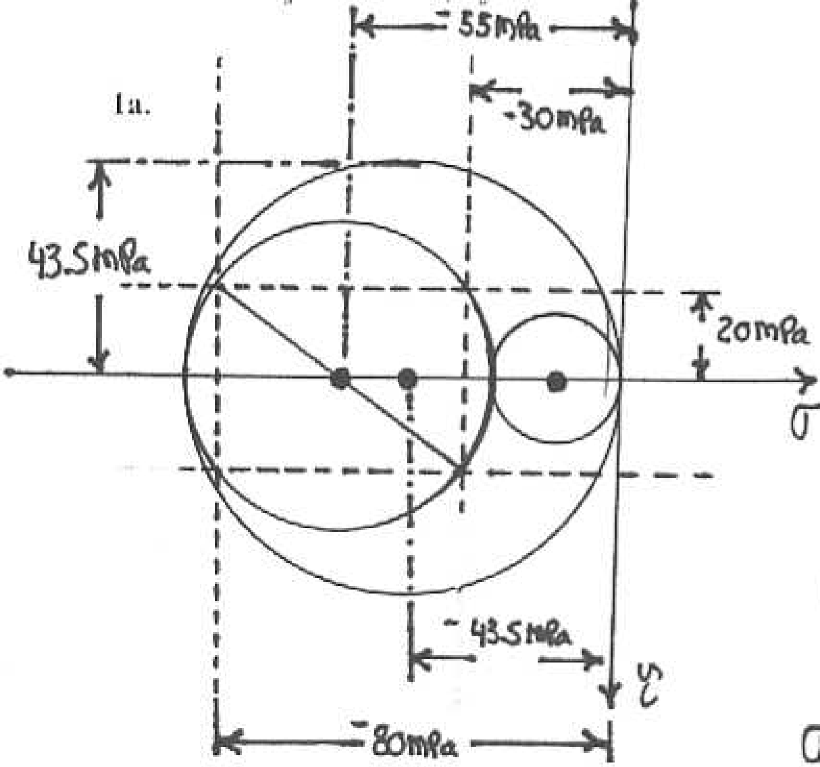
NAME: SOLUTION

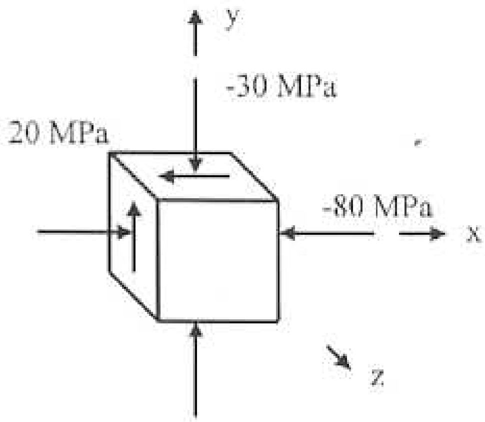
PROBLEM 1: For each of the stress states listed below evaluate the failure of the material for the two materials listed. Determine if failure occurs and if it does not, determine the safety factor. Be sure to state the criteria you are using for failure.

$Steel: S_y = 250MPa$

$Cast\ Iron: S_u^t = 170MPa,\ S_u^c = 655MPa$

1a.





$\tau = \sqrt{(25\,mPa)^2 + (20\,mPa)^2}$

$= 32.02\ mPa$

$\sigma_2 = -22.98\ mPa = \underline{-23.0\,mPa}$

$\sigma_3 = -87.02\ mPa = \underline{-87.0\,mPa}$

$\sigma_1 = \underline{0\,mPa}$

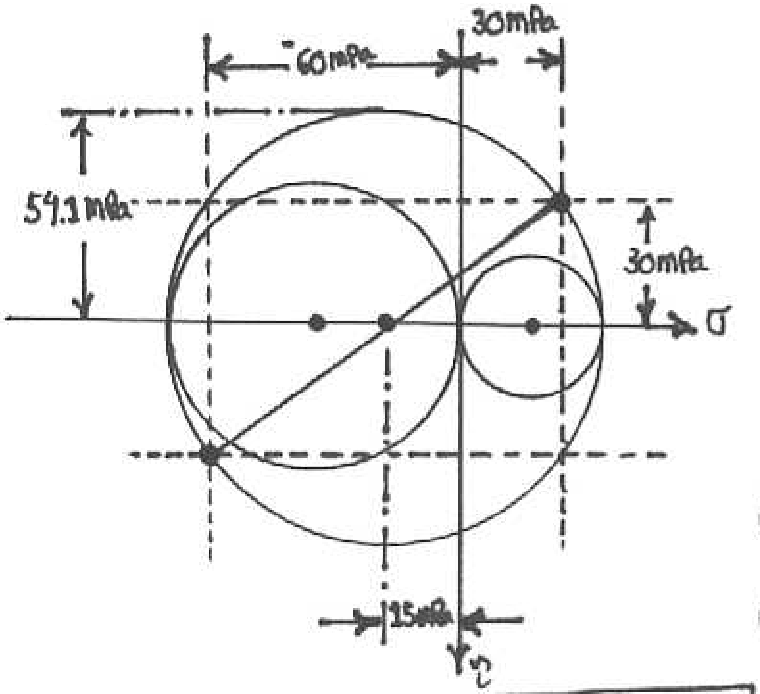
$\sigma_v = \sqrt{\frac{1}{2}\left[(0+23mPa)^2 + (-23mPa + 87mPa)^2 + (0+87mPa)^2\right]}$

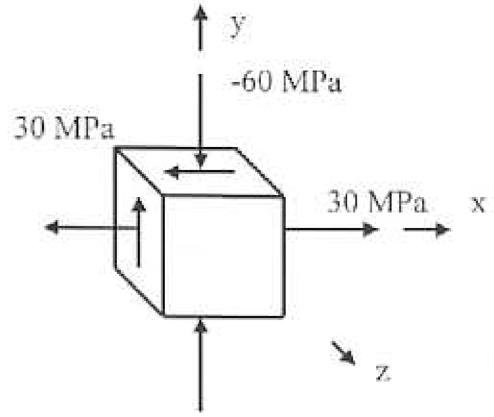
$= 78.08\,mPa$

STEEL: $n_s = \frac{250\,mPa}{78.08\,mPa} = \boxed{3.20}$

CAST IRON: $n_{CI}^c = \frac{655\,mPa}{87\ \ mPa} = \boxed{7.40}$

1b.





$r = \sqrt{(-45\text{mPa})^2 + (30\text{mPa})^2} = 54.08\text{mPa}$

$\sigma_1 = \underline{39.1\text{ mPa}}$

$\sigma_2 = \underline{0\text{ mPa}}$

$\sigma_3 = \underline{-69.1\text{ mPa}}$

$\tau_{max} = \underline{54.1\text{ mPa}}$

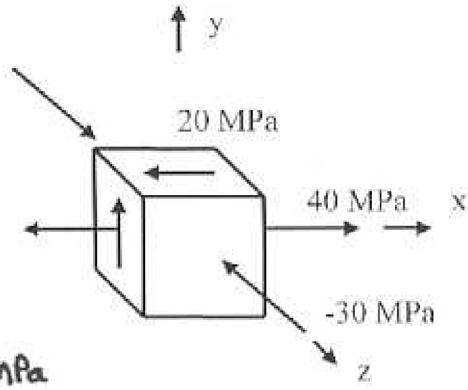
$\sigma_v = \sqrt{\frac{1}{2}\left[(39.1\text{mPa} - 0)^2 + (0 + 69.1\text{mPa})^2 + (39.1\text{mPa} + 69.1\text{mPa})^2\right]}$
$= 94.90\text{ mPa}$

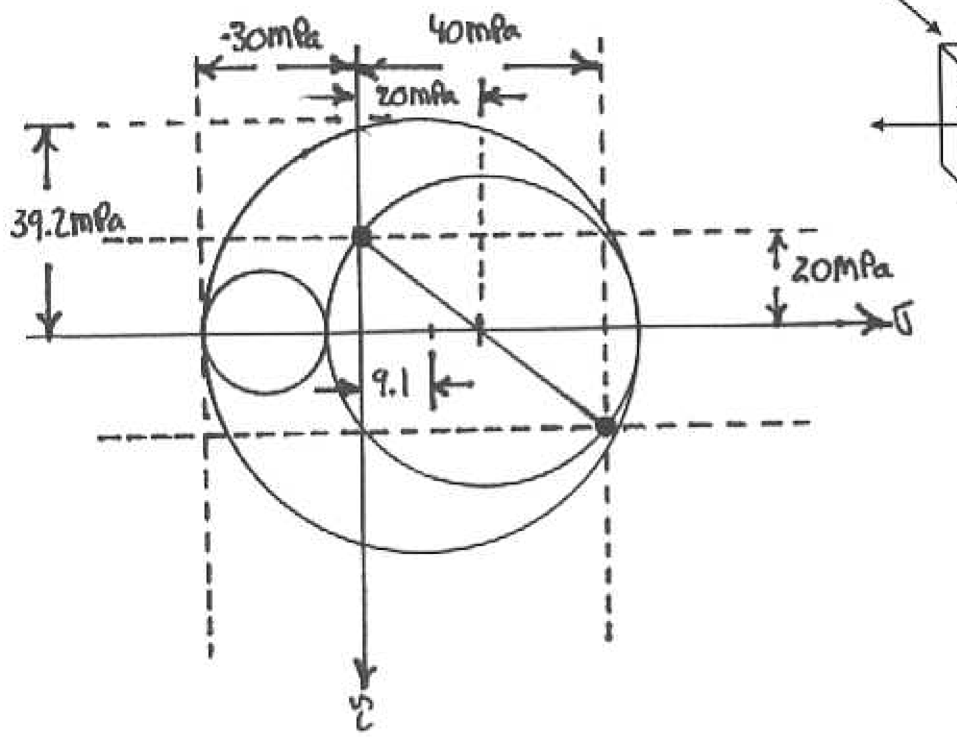
STEEL: $n_s = \frac{250\text{ mPa}}{94.9\text{ mPa}} = \boxed{2.63}$

CAST IRON: $n_{CI}^{T} = \frac{170\text{mPa}}{39.1\text{mPa}} = \boxed{4.35}$

$n_{CI}^{c} = \frac{655\text{mPa}}{69.1\text{mPa}} = 9.48$

1c.





$r = \sqrt{(20\,mPa)^2 + (20\,mPa)^2}$

$= 28.28\,mPa$

$\sigma_1 = \underline{\underline{48.3\,mPa}}$

$\sigma_2 = \underline{\underline{-8.3\,mPa}}$

$\sigma_3 = \underline{\underline{-30\,mPa}}$

$\tau_{max} = \underline{\underline{39.2\,mPa}}$

$\sigma_v = \sqrt{\frac{1}{2}\left[(48.3\,mPa + 8.3\,mPa)^2 + (48.3\,mPa + 30\,mPa)^2 + (-8.3\,mPa + 30\,mPa)^2\right]}$

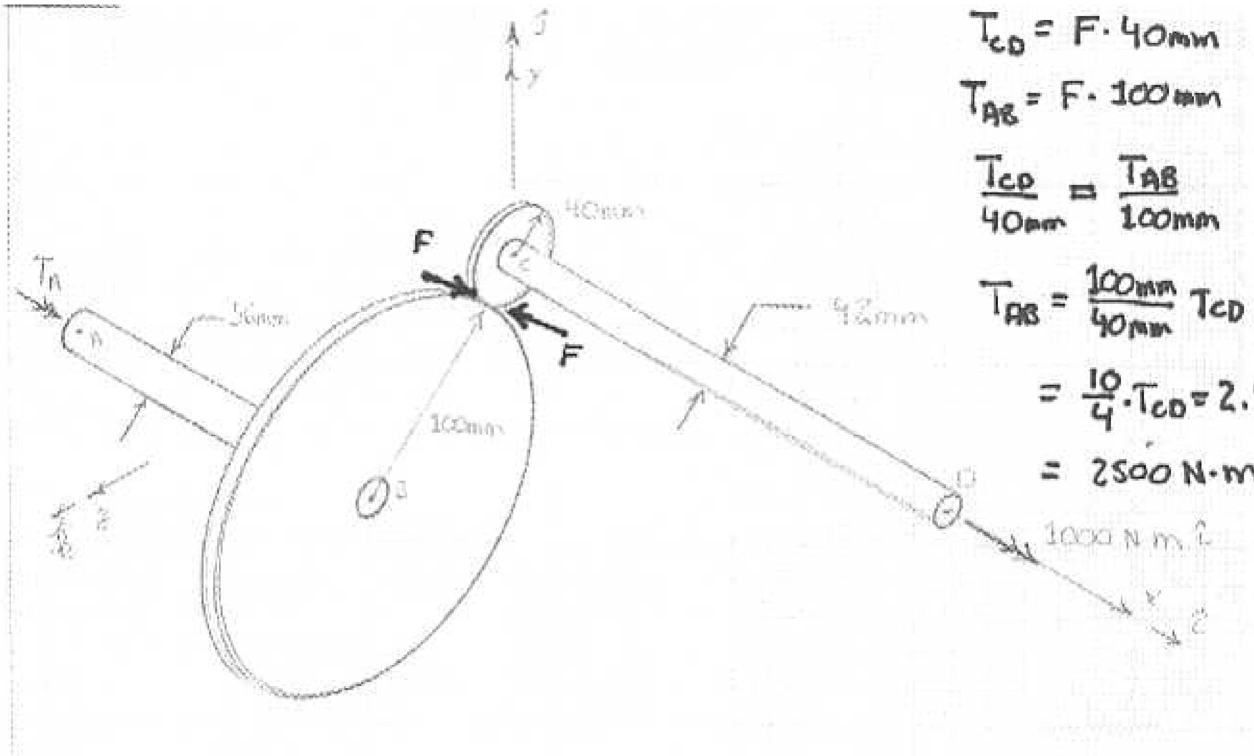
$= \underline{\underline{70.02}}$

STEEL: $n_s = \dfrac{250\,mPa}{70.02} = \boxed{3.57}$

CAST IRON: $n_{CI}^{T} = \dfrac{170\,mPa}{48.3\,mPa} = \boxed{3.52}$

$n_{CI}^{C} = \dfrac{655\,mPa}{30\,mPa} = 21.83$

PROBLEM 2: A torque of magnitude T=1000 N·m is applied at "D". The diameter of shaft "AB" is 56mm and the diameter of shaft "CD" is 42mm. Ignore the stress concentration at the gears. The shaft is made of G43400 Steel (first one in table), with a machine finish, and .9999 reliability. S_y=550MPa.



$$T_{CD} = F \cdot 40\text{mm}$$

$$T_{AB} = F \cdot 100\text{mm}$$

$$\frac{T_{CD}}{40\text{mm}} = \frac{T_{AB}}{100\text{mm}}$$

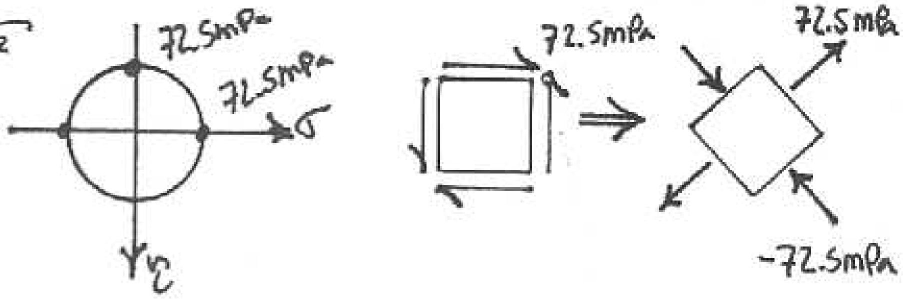
$$T_{AB} = \frac{100\text{mm}}{40\text{mm}} T_{CD} = \frac{10}{4} \cdot T_{CD} = 2.5 T_{CD} = 2500\ \text{N·m}$$

2a. If the load is applied in a fully reversible manner (i.e., 1000N·m to -1000N·m), what is the expected life of the system.

$$\tau_{CD} = \frac{(1000\text{N·m}) \cdot \left(\frac{.042\text{m}}{2}\right)}{\frac{1}{2}\pi\left(\frac{.042\text{m}}{2}\right)^4} = 68.7\ \text{MPa} \longrightarrow -68.7\ \text{MPa}$$

$$\tau_{AB} = \frac{(2500\text{N·m}) \left(\frac{.056\text{m}}{2}\right)}{\frac{1}{2}\pi\left(\frac{.056\text{m}}{2}\right)^4} = \underline{72.5\ \text{MPa} \longrightarrow -72.5\ \text{MPa}} \quad \text{MAX}$$

$$\sigma' = \sqrt{\tfrac{1}{2}\left[(72.5-0)^2+(0+72.5)^2+(72.5+72.5)^2\right]}$$
$$= 125.6\ \text{MPa}$$



$S_E = 489\,\text{MPa} \cdot 0.69 \cdot 0.75 \cdot 0.702 = 177.7\,\text{MPa}$

$m = -\frac{1}{3} \cdot \log\left(\frac{0.9\,(965\,\text{MPa})}{177.7\,\text{MPa}}\right) = -.2297$

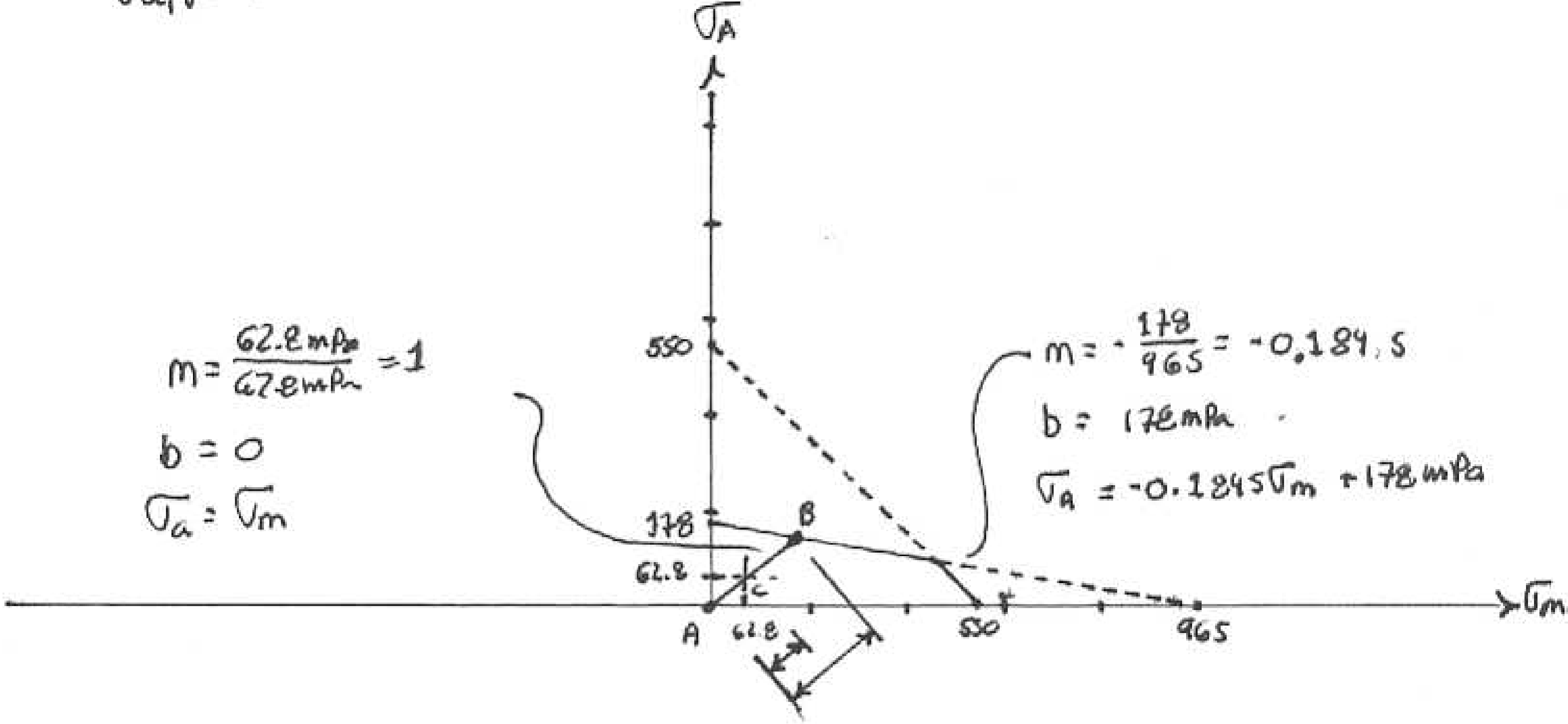
$b = \log\left[\frac{(0.9\,(965\,\text{MPa}))^2}{177.7\,\text{MPa}}\right] = 3.628\ \log(\text{MPa})$

SINCE $S_S < S_e$ LIFE IS INFINITE

2b. If the load is cycled from 1000N·m to 0N·m, what is the factor of safety for the shaft?

$\sigma_{m,\psi} = 62.8\,mPa$

$\sigma_{a,\psi} = 62.8\,mPa$



$m = \frac{62.8\,mPa}{62.8\,mPa} = 1$

$b = 0$

$\sigma_a = \sigma_m$

$m = -\frac{178}{965} = -0.1845$

$b = 178\,mPa$

$\sigma_a = -0.1845\,\sigma_m + 178\,mPa$

Point B

$\sigma_m^{(b)} = -.1845 \cdot \sigma_m^{(b)} + 178\,mPa$

$1.1845\,\sigma_m^{(b)} = 178\,mPa \Rightarrow \sigma_m^{(b)} = 150.3\,mPa$

$\sigma_a^{(b)} = 150.3\,mPa$

$AB = \sqrt{(150.3\,mPa)^2 + (150.3\,mPa)^2} = 212.6\,mPa$

$AC = \sqrt{(62.8\,mPa)^2 + (62.8\,mPa)^2} = 88.8\,mPa$

$n = \frac{AB}{AC} = \frac{212.6\,mPa}{88.8\,mPa} = \boxed{2.39}$

Table 5-1 STANDARD DEVIATIONS OF ENDURANCE LIMIT*

Material† UNS No.	Tensile strength S_{ut}		Endurance limit S_e'		Standard deviation	
	MPa	kpsi	MPa	kpsi	kpsi	%
G43400 steel	965	140	489	71	3.5	4.9
	1310	190	586	85	6.7	7.8
	1580	230	620	90	5.3	5.9
	1790	260	668	97	6.3	6.5
G43500 steel	2070	300	689	100	4.4	4.4
R50001-series titanium alloy	1000	145	579	84	5.4	6.4
A97076 aluminum alloy	524	76	186	27	1.6	6.0
C63000 aluminum bronze	806	117	331	48	4.5	9.4
C17200 beryllium copper	1210	175	248	36	2.7	7.5

* Reported by F. B. Stulen, H. N. Cummings, and W. C. Schulte, Preventing Fatigue Failures, Part 5, *Machine Design*, vol. 33, p. 161, June 22, 1961.
† Alloys are heat-treated, hot-worked, specimens smooth, subjected to long-life rotating-beam tests.