PROBLEM 2.17 FOR THE PLANE STRESS STATE: 40MPa, -10 MPa, 0 MPa, $\Theta = -15°$

(a) DRAW THE CORRESPONDING STRESS ELEMENT PROPERLY ORIENTED RELATIVE TO THE XY AXIS

(b) DETERMINE THE COMPLETE STRESS ELEMENT ASSOCIATED WITH AN AXIS SYSTEM ROTATED Θ (DEFINED POSITIVE COUNTERCLOCKWISE) USING THE TRANSFORMATION EQUATIONS ALONE.

(c) DETERMINE THE PRINCIPAL STRESSES AND THE CORRESPONDING STRESS ELEMENT CONTAINING THE STRESSES PROPERLY ORIENTED RELATIVE TO THE XY AXES USING EQUATIONS ONLY.

(d) REPEAT PARTS (b) AND (c) USING MOHR'S CIRCLE.

(e) DETERMINE THE MAXIMUM AND MINIMUM SHEAR STRESS AND SHOW THE COMPLETE STRESS ELEMENT CONTAINING THESE STRESSES. SHOW THE ELEMENT PROPERLY ORIENTED WITH RESPECT TO THE XYZ COORDINATE SYSTEM

GIVEN:

CONSTRAINTS

1. $\sigma_x = 40\,MPa$, $\sigma_y = -10\,MPa$, $\tau_{xy} = 0\,MPa$, $\Theta = -15°$

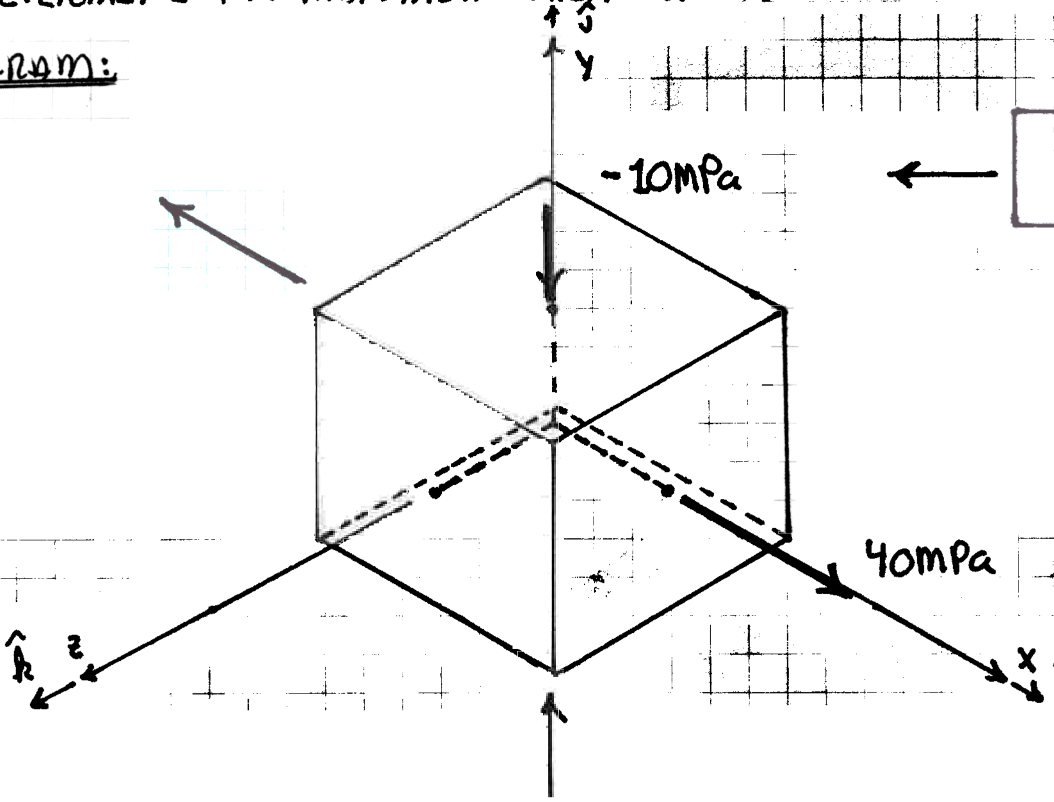
ASSUMPTIONS

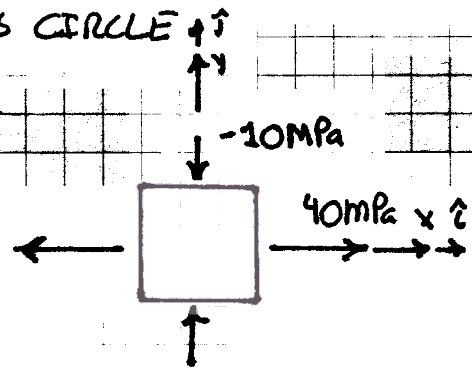
1. PLANE STRESS : $\sigma_z = \tau_{xz} = \tau_{yz} = 0$

FIND:

1. DRAW THE ELEMENT RELATIVE TO THE XY PLANE
2. USING TRANSFORMATION EQUATIONS DETERMINE $[\sigma]$ FOR $\Theta = -15°$
3. USING TRANSFORMATION EQUATIONS DETERMINE THE PRINCIPAL STRESSES
4. REPEAT THE PREVIOUS TWO PARTS USING MOHR'S CIRCLE
5. DETERMINE THE MAXIMUM SHEAR STRESS

DIAGRAM:





SOLUTION:

USING TRANSFORMATION EQUATIONS, THE STRESS ON THE ELEMENT ROTATED -15° ABOUT THE Z-AXES ARE

$$\sigma_{x'} = \sigma_x \cdot \cos^2\theta + \sigma_y \cdot \sin^2\theta + 2 \cdot \tau_{xy} \cdot \cos\theta \sin\theta$$

$$= (40\text{MPa})\cos^2(-15°) + (-10\text{ MPa}) \cdot \sin^2(-15°) + 2 \cdot (0) \cdot \cos(-15°)\sin(-15°)$$

$$= \boxed{36.65\text{ MPa}}$$

$$\sigma_{y'} = \sigma_x \cdot \sin^2\theta + \sigma_y \cdot \cos^2\theta - 2 \cdot \tau_{xy} \cdot \sin\theta \cdot \cos\theta$$

$$= (40\text{MPa}) \cdot \sin^2(-15°) + (-10\text{MPa}) \cdot \cos^2(-15°) - 2 \cdot (0) \cdot \sin(-15°) \cdot \cos(-15°)$$

$$= \boxed{-6.65\text{ MPa}}$$

$$\tau_{x'y'} = -(\sigma_x - \sigma_y) \cdot \sin\theta \cdot \cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -(40\text{MPa} - (-10\text{MPa}))\sin(-15°) \cdot \cos(-15°) + 0(\cos^2(-15°) - \sin^2(-15°))$$

$$= \boxed{12.5\text{ MPa}}$$

THE PRINCIPAL STRESSES, CALCULATED USING EQUATIONS ARE

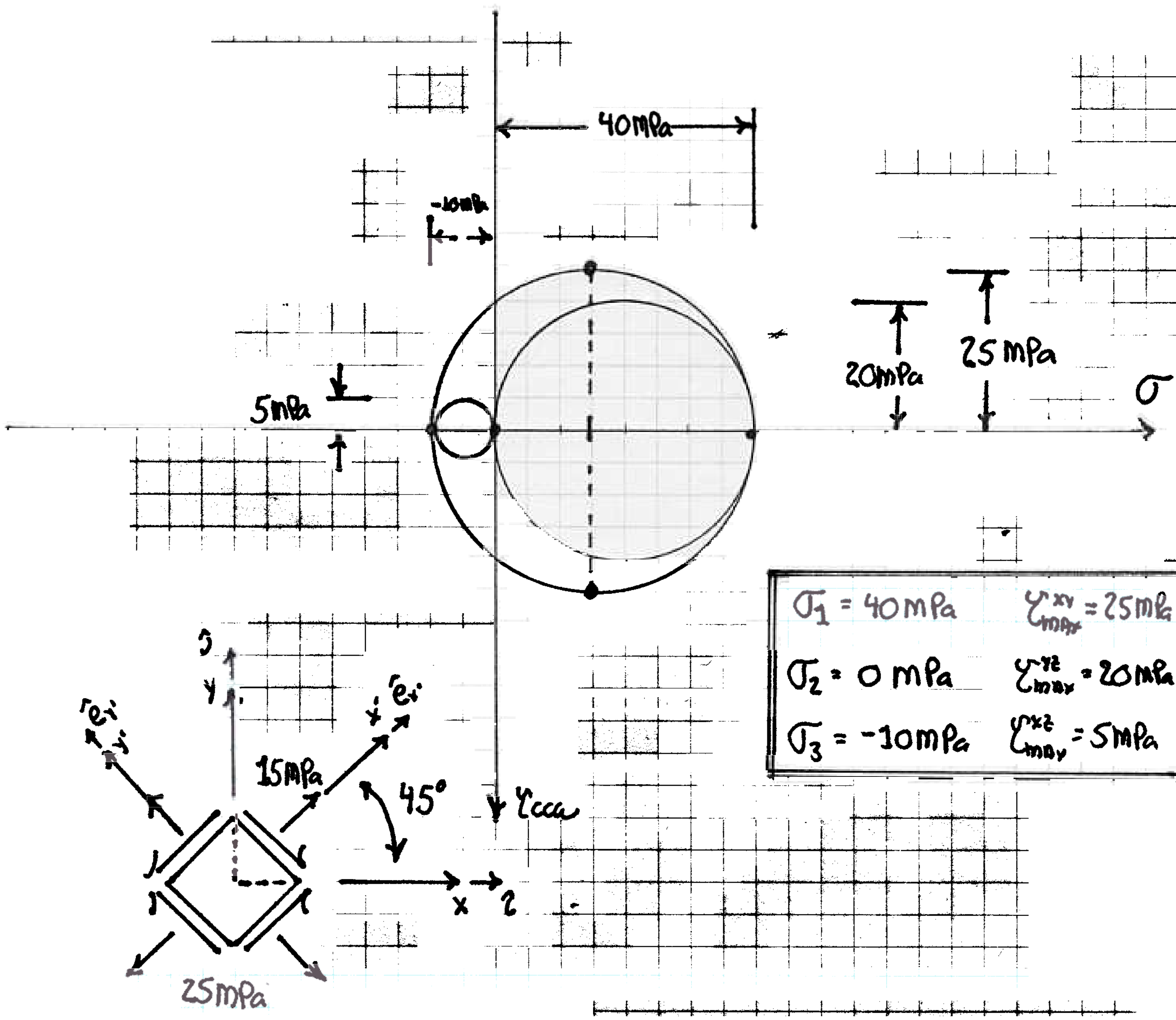
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{(40\text{MPa} + (-10\text{MPa}))}{2} \pm \sqrt{\left(\frac{(40\text{MPa} - (-10\text{MPa}))}{2}\right)^2 + (0)^2}$$

$$= 15\text{MPa} \pm 25\text{MPa}$$

$$\boxed{\begin{aligned} \sigma_1 &= 40\text{ MPa} \\ \sigma_2 &= -10\text{MPa} \end{aligned}}$$

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\sqrt{\left(\frac{40\text{MPa} - (-10\text{MPa})}{2}\right)^2 + (0)^2} = \boxed{\pm 25\text{MPa}}$$



$\sigma_1 = 40\,MPa$	$\tau^{xy}_{max} = 25\,MPa$
$\sigma_2 = 0\,MPa$	$\tau^{yz}_{max} = 20\,MPa$
$\sigma_3 = -10\,MPa$	$\tau^{xz}_{max} = 5\,MPa$

SUMMARY:

IN THIS PROBLEM THE MAXIMUM SHEAR STRESS IS IN PLANE DEFINED BY THE PLANE STRESS PROBLEM. IF THIS PROBLEM IS EXTRAPOLATED TO WHERE $\sigma_x = -\sigma_y$, THE MAXIMUM SHEAR STRESS WOULD BE ORIENTED ON AN ELEMENT WITH ZERO NORMAL STRESS.