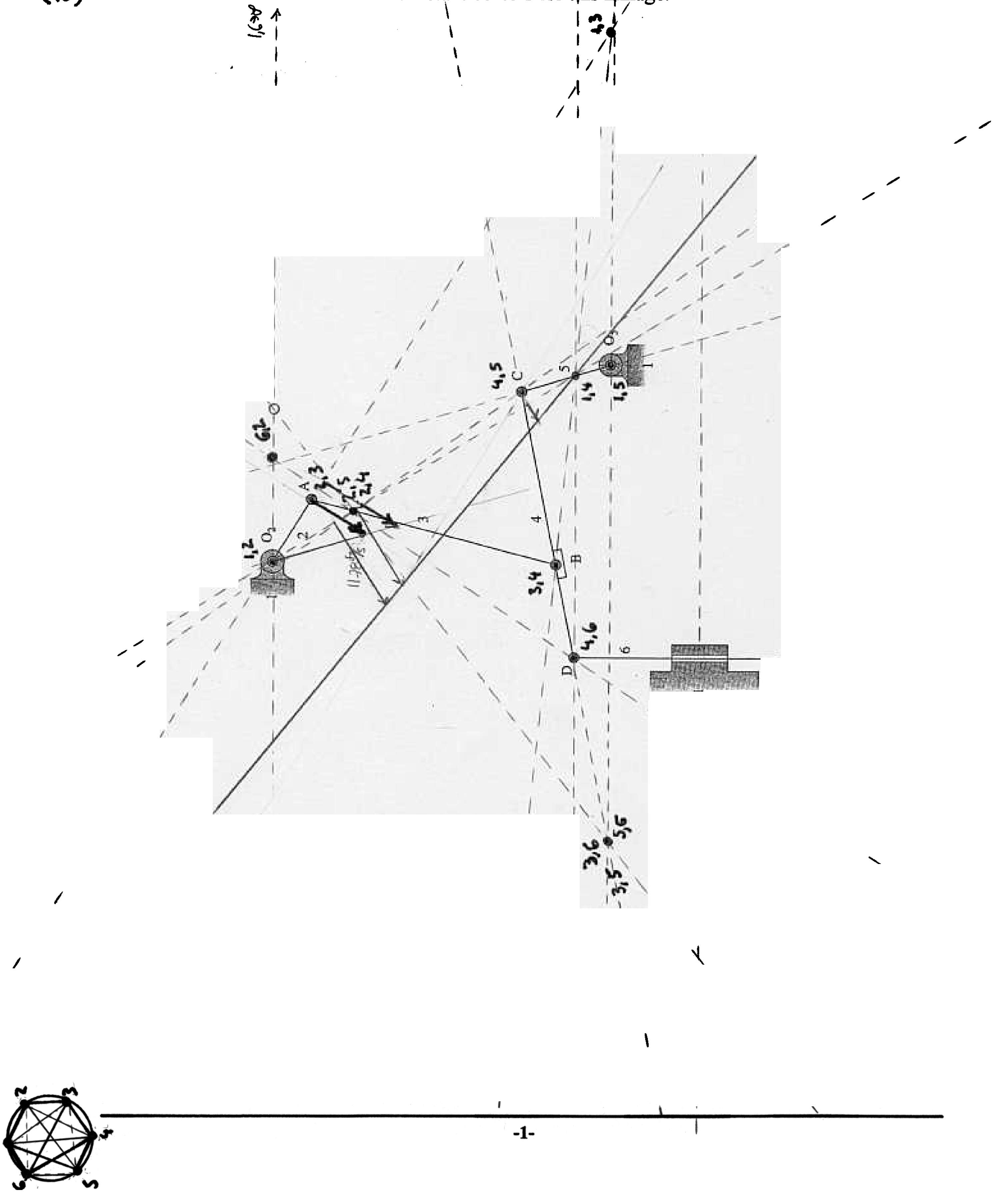
NAME: Solution

(10) **PROBLEM 1: a.** Locate all of the instant centers for this linkage.



(15) **1b.** Link 2 in the mechanism is turning clockwise at the rate of 600 rpm. Determine the linear velocity of points C and D by the use of instant centers. Space scale: 3in. = 1ft. Velocity scale: 1in =500 fpm.

$\omega = 600 \, rev/min \cdot \frac{min}{60 sec} \cdot \frac{2\pi \, rad}{rev} = 62.83 \, 1/s = 3770 \, 1/min$

$O_2A = \frac{9}{16} in. \cdot \frac{1 ft}{3 in} = 0.1875'$

$V_A = \omega \cdot R_{O_2A} = 62.83 \frac{1}{s} \cdot 0.1875' = 11.78 \, ft/s = 706.8 \, ft/min$

$V_{25} = V_{24} = 17 \, ft/s = 1020 \, ft/min$

$V_D = 20 \, ft/s = \boxed{1200 \, ft/min}$

$V_C = 4 \, ft/s = \boxed{240 \, ft/min}$

(15) **1c.** Determine the angular velocity of links 3 and 5 in radians per second, and indicate the direction of each.

$$V_A = V_{23} = 11.78\ ft/s = 706.8\ ft/min$$

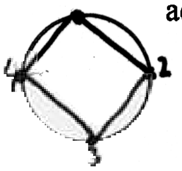
$$\omega_3 = \frac{V_{23}}{R_{I_{13}I_{23}}} = \frac{706.8\ ft/min}{4\frac{5}{16} \cdot \frac{1\ ft}{3\ in}} = \frac{706.8\ ft/min}{1.438\ ft} =$$

$$= \boxed{491.7\ 1/min} = 8.195\ 1/s \quad ccw$$

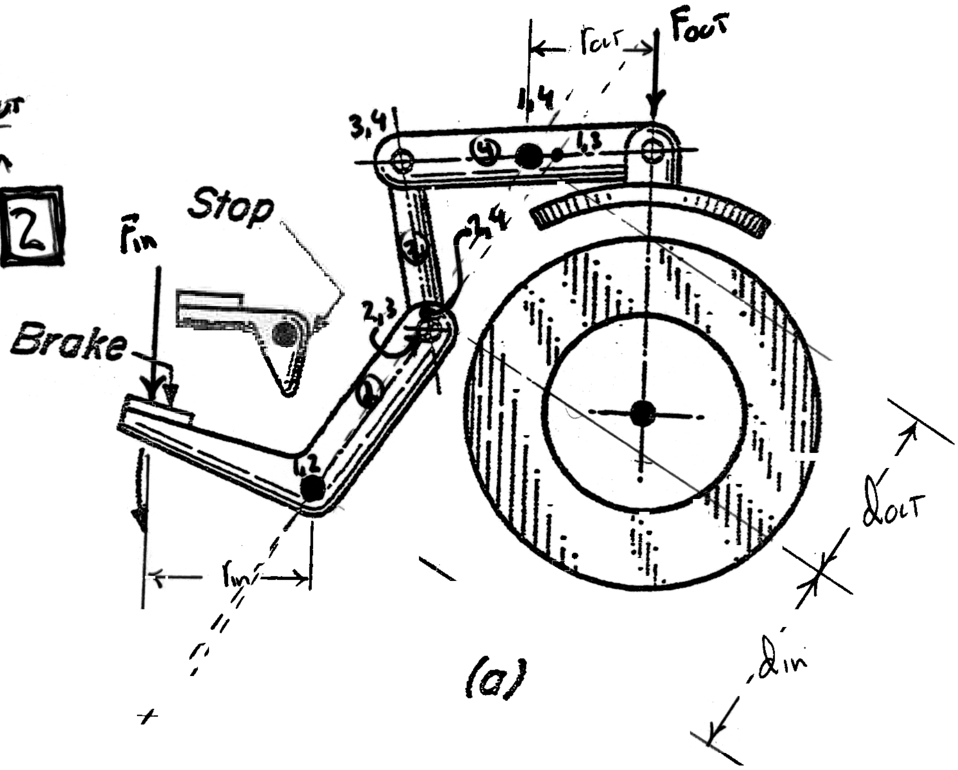
$$\omega_5 = \frac{V_{25}}{R_{I_{15}I_{25}}} = \frac{1020\ ft/min}{2\frac{5}{16} \cdot \frac{1\ ft}{3\ in}} = \frac{1020\ ft/min}{0.7708\ ft}$$

$$= \boxed{1323\ 1/min} = 22.05\ 1/s \quad ccw$$

(20) **PROBLEM 2:** A rear-wheel brake is shown in the (a) released and (b) brake set positions. The black circles are pivot points that are fixed relative to the frame. Determine the mechanical advantage of this system in both the released and break set positions.

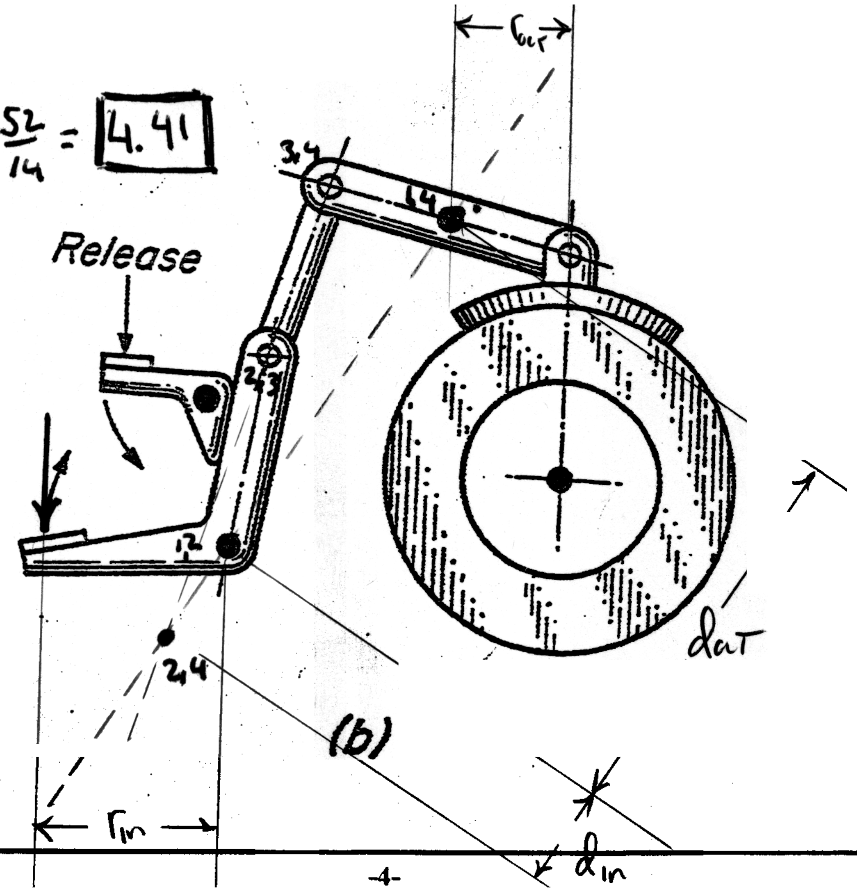


$$MA = \frac{r_{in}}{r_{out}} \frac{d_{out}}{d_{in}} = \frac{22}{12}\,\frac{24}{22} = \boxed{2}$$



(a)

$$MA = \frac{r_{in}}{r_{out}} \cdot \frac{d_{out}}{d_{in}} = \frac{19}{16}\,\frac{52}{14} = \boxed{4.41}$$



(b)

PROBLEM 3: For the mechanism shown on the next page, it is known that link O_2D rotates with a constant angular velocity of $\omega_2 = \dot{\theta}_2 = \mathbf{150\ rev/min}$. The slider at D travels along the link O_4C. The loop closure equation for this mechanism is,

$$\vec{\mathbf{R}}_{O_2D} + \vec{\mathbf{R}}_{DO_4} = \vec{\mathbf{R}}_{O_2O_4}$$

where

$$\vec{\mathbf{R}}_{O_2D} = \mathbf{R}_{O_2D} \cdot \hat{\mathbf{e}}_{O_2D} = \mathbf{38.1mm} \cdot (\mathbf{-0.886\hat{i} + 0.5\hat{j}})$$
$$= \mathbf{R}_{O_2D} \cdot \mathbf{e}^{j \cdot \theta_2} = \mathbf{38.1mm} \cdot \mathbf{e}^{j \cdot 150^\circ}$$
$$\vec{\mathbf{R}}_{DO_4} = \mathbf{R}_{DO_4} \cdot \hat{\mathbf{e}}_{DO_4} = \mathbf{77.3mm} \cdot (\mathbf{0.4271\hat{i} - 0.9042\hat{j}})$$
$$= \mathbf{R}_{DO_4} \cdot \mathbf{e}^{j \cdot \theta_4} = \mathbf{77.3mm} \cdot \mathbf{e}^{j \cdot 295.3^\circ}$$
$$\vec{\mathbf{R}}_{O_2O_4} = \mathbf{R}_{O_2O_4} \cdot \hat{\mathbf{e}}_{O_2O_4} = \mathbf{-50.8mm} \cdot \hat{\mathbf{j}}$$
$$= \mathbf{R}_{O_2O_4} \cdot \mathbf{e}^{j \cdot \theta_1} = \mathbf{50.8mm} \cdot \mathbf{e}^{j \cdot 270^\circ}$$

(10) **3a.** Differentiate the loop closure equation for this mechanism. Identify each of the terms that are equal to zero and explain why. Write the resulting equation in terms of non-zero components only. DO NOT SOLVE FOR THE UNKNOWNS.

$$\dot{R}_{O_2D}\hat{e}_{O_2D} + R_{O_2D}\dot{\hat{e}}_{O_2D} + \dot{R}_{DO_4}\hat{e}_{DO_4} + R_{DO_4}\dot{\hat{e}}_{DO_4} = \dot{R}_{O_2O_4}\hat{e}_{O_2O_4} + R_{O_2O_4}\dot{\hat{e}}_{O_2O_4}$$

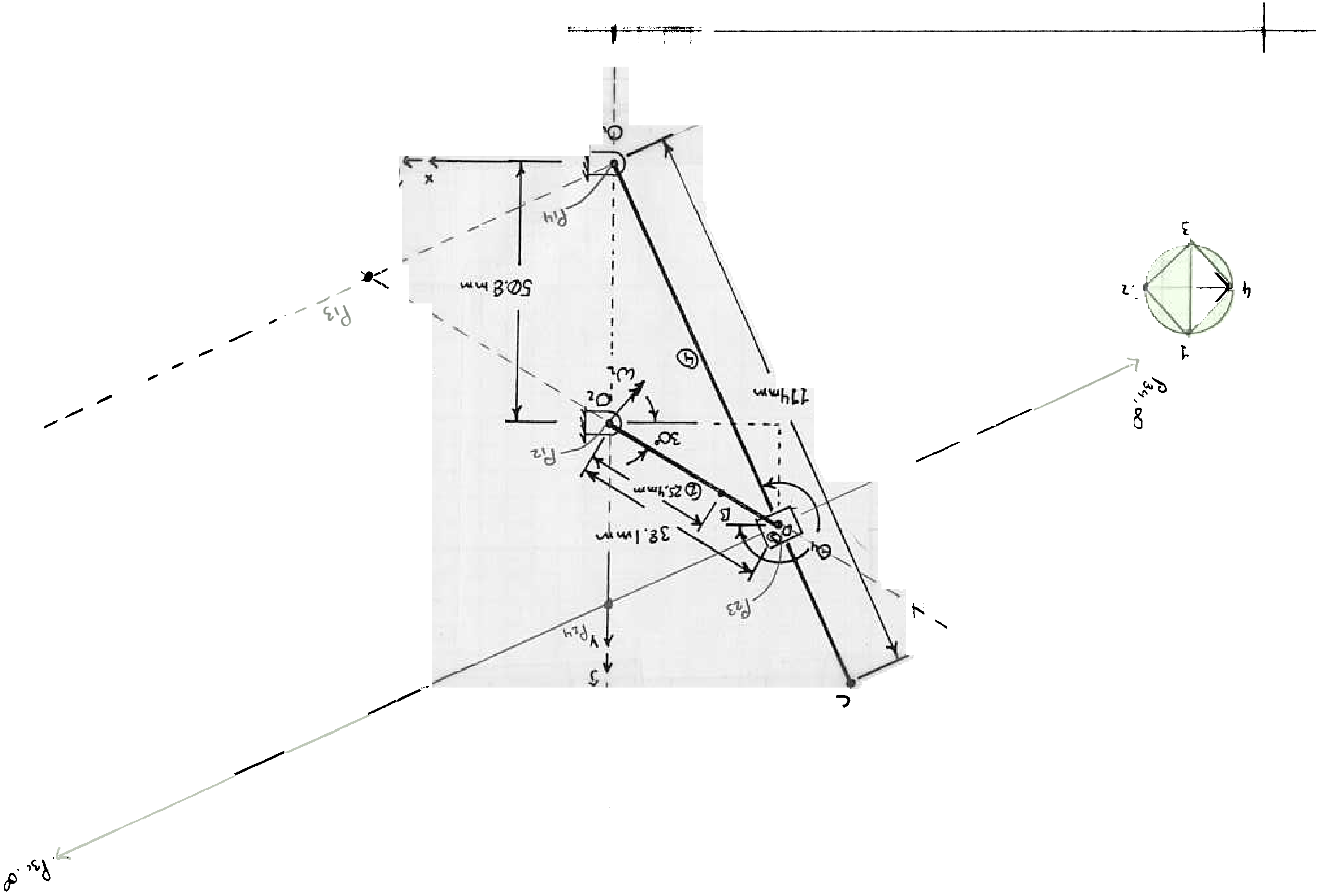
Fixed length 0

$$\cancel{\dot{R}_{O_2D}}\hat{e}_{O_2D} + R_{O_2D}\dot{\theta}_2(\hat{k}\times\hat{e}_{O_2D}) + \dot{R}_{DO_4}\hat{e}_{DO_4} + R_{DO_4}\dot{\theta}_4(\hat{k}\times\hat{e}_{DO_4})$$

Fixed length 0 | Fixed angle 0

$$= \cancel{\dot{R}_{O_2O_4}}\hat{e}_{O_2O_4} + R_{O_2O_4}\cancel{\dot{\theta}_1}(\hat{k}\times\hat{e}_{O_2O_4})$$

$$R_{O_2D}\dot{\theta}_2(\hat{k}\times\hat{e}_{O_2D}) + \dot{R}_{DO_4}\hat{e}_{DO_4} + R_{DO_4}\dot{\theta}_4(\hat{k}\times\hat{e}_{DO_4}) = 0$$


50.8 mm
38.1 mm
25.4 mm
114 mm
30°
O2
ω2
C
x
y
P14
P12
P13
P23
P24
P34,∞
1
2
3
4

(20) **3b.** From part (a) of this problem it can be shown that for this mechanism at the instant shown

$$\dot{\theta}_4 = 60.8\frac{\text{rev}}{\text{min}} = 6.367\frac{\text{rad}}{\text{s}} \qquad \dot{\theta}_2 = 15.7\ 1/s$$

$$\dot{R}_{DO_4} = -340.8\frac{\text{mm}}{\text{s}}$$

DO NOT verify these values by calculating them from the equation that results from the loop closure equation differentiation. Determine $\ddot{\theta}_4$ and $\ddot{R}_{DO_4}$.

$$\cancel{\dot{R}_{O_2D}}^{0}\dot{\theta}_2(\hat{k}\times\hat{e}_{O_2D}) + R_{O_2D}\cancel{\ddot{\theta}_2}^{0}(\hat{k}\times\hat{e}_{O_2D}) + R_{O_2D}\,\dot{\theta}_2(\hat{k}\times\dot{\hat{e}}_{O_2D})$$
$$+ \ddot{R}_{DO_4}\hat{e}_{DO_4} + \dot{R}_{DO_4}\dot{\hat{e}}_{DO_4} + \dot{R}_{DO_4}\dot{\theta}_4(\hat{k}\times\hat{e}_{DO_4}) + R_{DO_4}\cdot\ddot{\theta}_4(\hat{k}\times\hat{e}_{DO_4})$$
$$+ R_{DO_4}\dot{\theta}_4(\hat{k}\times\dot{\hat{e}}_{DO_4}) = 0$$

$$R_{O_2D}\dot{\theta}_2^2\,\hat{k}\times(\hat{k}\times\hat{e}_{O_2D}) + \ddot{R}_{DO_4}\hat{e}_{DO_4} + \dot{R}_{DO_4}\dot{\theta}_4(\hat{k}\times\hat{e}_{DO_4}) + \dot{R}_{DO_4}\dot{\theta}_4(\hat{k}\times\hat{e}_{DO_4})$$
$$+ R_{DO_4}\cdot\ddot{\theta}_4(\hat{k}\times\hat{e}_{DO_4}) + R_{DO_4}\dot{\theta}_4^2\,\hat{k}\times(\hat{k}\times\hat{e}_{DO_4}) = 0$$

$$R_{O_2D}\dot{\theta}_2^2\,\hat{k}\times(\hat{k}\times\hat{e}_{O_2D}) + \underline{\underline{\ddot{R}_{DO_4}}}\hat{e}_{DO_4} + 2\dot{R}_{DO_4}\dot{\theta}_4(\hat{k}\times\hat{e}_{DO_4})$$
$$+ R_{DO_4}\underline{\underline{\ddot{\theta}_4}}(\hat{k}\times\hat{e}_{DO_4}) + R_{DO_4}\dot{\theta}_4^2\,\hat{k}\times(\hat{k}\times\hat{e}_{DO_4}) = 0 \qquad (1)$$

THE UNKNOWNS ARE $\ddot{R}_{DO_4}$ AND $\ddot{\theta}_4$. STARTING BY SOLVING FOR $\ddot{R}_{DO_4}$ BY DOTING THE (1) WITH $\hat{e}_{DO_4}$

$$R_{O_2D}\dot{\theta}_2^2\,\hat{e}_{DO_4}\cdot\hat{k}\times(\hat{k}\times\hat{e}_{O_2D}) + \ddot{R}_{DO_4}\underbrace{\hat{e}_{DO_4}\cdot\hat{e}_{DO_4}}_{1} + 2\dot{R}_{DO_4}\dot{\theta}_4\underbrace{\hat{e}_{DO_4}\cdot(\hat{k}\times\hat{e}_{DO_4})}_{0}$$
$$+ R_{DO_4}\ddot{\theta}_4\underbrace{\hat{e}_{DO_4}\cdot(\hat{k}\times\hat{e}_{DO_4})}_{0} + R_{DO_4}\dot{\theta}_4^2\,\hat{e}_{DO_4}\cdot\hat{k}\times(\hat{k}\times\hat{e}_{DO_4}) = 0$$

$$\ddot{R}_{DO_4} = -\left(R_{O_2D}\dot{\theta}_2^2\,\hat{e}_{DO_4}\cdot\hat{k}\times(\hat{k}\times\hat{e}_{O_2D}) + R_{DO_4}\dot{\theta}_4^2\,\hat{e}_{DO_4}\cdot\hat{k}\times(\hat{k}\times\hat{e}_{DO_4})\right) \qquad (2)$$

$$\hat{k}\times\hat{e}_{O_2D} = \begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ -.886 & .5 & 0\end{matrix} = -.5\,i + -.886\,j = -.5i - .886j$$

$$\hat{k}\times(\hat{k}\times\hat{e}_{O_2D}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ -.5 & -.886 & 0\end{vmatrix} = .886\hat{i} - .5\hat{j}$$

$$\hat{e}_{DO_4}\circ\hat{k}\times(\hat{k}\times\hat{e}_{O_2D}) = (0.4271\hat{i} - .9042\hat{j})\circ(.886\hat{i} - .5\hat{j}) = .8305$$

$$\hat{k}\times\hat{e}_{DO_4} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ .4271 & -.9042 & 0\end{vmatrix} = .9042\hat{i} + .4271\hat{j}$$

$$\hat{k}\times(\hat{k}\times\hat{e}_{DO_4}) = \begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ .9042 & .4271 & 0\end{matrix} = -.4271\hat{i} + .9042\hat{j}$$

$$\hat{e}_{DO_4}\circ\hat{k}\times(\hat{k}\times\hat{e}_{DO_4}) = (.4271\hat{i} - .9042\hat{j})\cdot(-.4271\hat{i} + .9042\hat{j}) = -$$

$$② \rightarrow \ddot{R}_{DO_4} = -(38.1\text{mm})\cdot(15.7\,{}^1/_s)^2(.8305) - (77.3\text{mm})(6.367\,{}^1/_s)^2(-1)$$

$$= \boxed{-4{,}708\ {}^{mm}/_{s^2}}$$

Now solving for $\ddot{\Theta}_4$ By eliminating $\ddot{R}_{DO_4}$ From ① BY DOTTING ① WITH $\hat{k}\times\hat{e}_{DO_4}$

$$R_{O_2D}\dot{\Theta}_2^2(\hat{k}\times\hat{e}_{DO_4})\circ\hat{k}\times(\hat{k}\times\hat{e}_{O_2D}) + \ddot{R}_{DO_4}\underbrace{(\hat{k}\times\hat{e}_{DO_4})\cdot\hat{e}_{DO_4}}_{0} + 2\dot{R}_{DO_4}\dot{\Theta}_4\underbrace{(\hat{k}\times\hat{e}_{DO_4})\cdot(\hat{k}\times\hat{e}_{DO_4})}_{1}$$

$$+ R_{DO_4}\ddot{\Theta}_4\underbrace{(\hat{k}\times\hat{e}_{DO_4})\cdot(\hat{k}\times\hat{e}_{DO_4})}_{1} + R_{DO_4}\dot{\Theta}_4^2\underbrace{(\hat{k}\times\hat{e}_{DO_4})\circ\hat{k}\times(\hat{k}\times\hat{e}_{DO_4})}_{0})$$

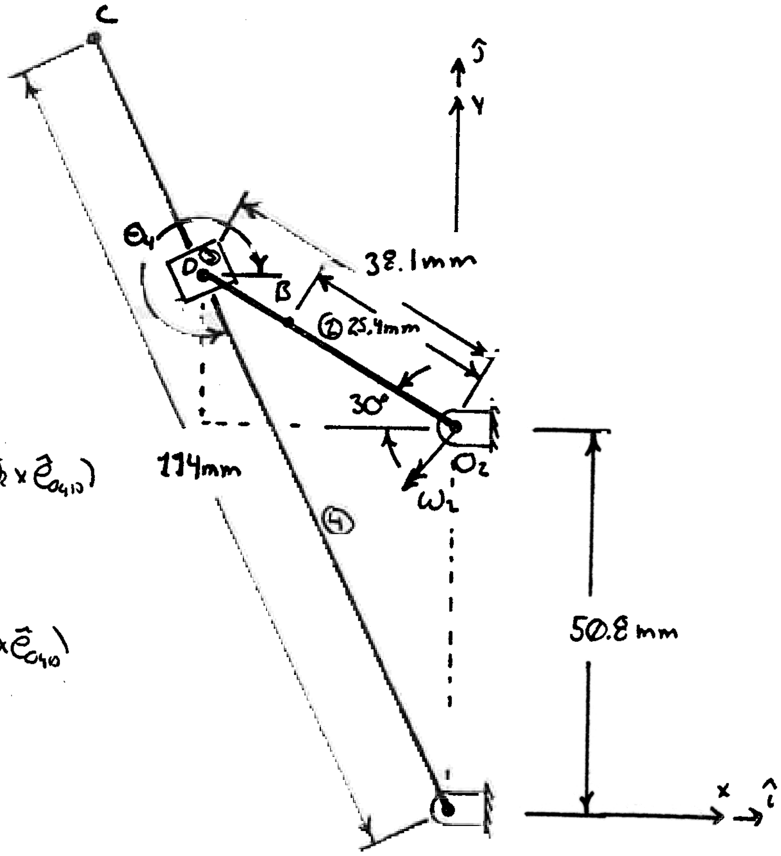
$$\ddot{\Theta}_4 = \frac{-R_{O_2D}\dot{\Theta}_2^2(\hat{k}\times\hat{e}_{DO_4})\circ\hat{k}\times(\hat{k}\times\hat{e}_{O_2D}) - 2\dot{R}_{DO_4}\dot{\Theta}_4}{R_{DO_4}}$$

$$(\hat{k}\times\hat{e}_{DO_4})\circ(\hat{k}\times(\hat{k}\times\hat{e}_{O_2D})) = (.9042\hat{i} + .4271\hat{j})\circ(.886\hat{i} - .5\hat{j}) = 0.5876$$

$$\ddot{\Theta}_4 = \frac{-(38.1\text{mm})(15.7\,{}^1/_s)^2(0.5876) - 2\cdot(-340.8\,{}^{mm}/_s)(6.367\,{}^1/_s)}{77.3}$$

$$= \boxed{-15.2\ {}^1/_s}$$

(10) **3c.** Using acceleration polygons, verify the results in the previous section.



$a_{O_2D} = R_{O_2D}\dot{\theta}^2(\hat{k}\times(\hat{k}\times\hat{e}_{O_2D}))$

$= 38.1mm\cdot(15.7 \,^1/_s)^2 = 9400 \,^{mm}/_{s^2}$

$a_{O_4D} = \ddot{R}_{O_4D}\hat{e}_{O_4D} + 2\dot{R}_{O_4D}\dot{\theta}_4(\hat{k}\times\hat{e}_{O_4D}) + R_{O_4D}\ddot{\theta}_4(\hat{k}\times\hat{e}_{O_4D})$
$+ R_{O_4D}\dot{\theta}_4^2\,\hat{k}\times(\hat{k}\times\hat{e}_{O_4D})$

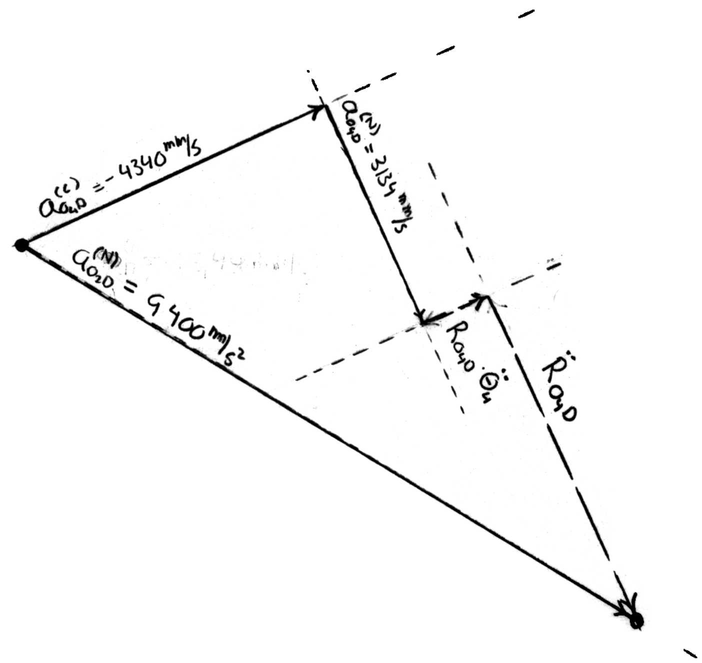
$\doteq \ddot{R}_{O_4D}\hat{e}_{O_4D} + 2\cdot(-340.8\,^{mm}/_s)(6.367\,^1/_s)(\hat{k}\times\hat{e}_{O_4D}) + 77.3mm\cdot\ddot{\theta}_4(\hat{k}\times\hat{e}_{O_4D})$

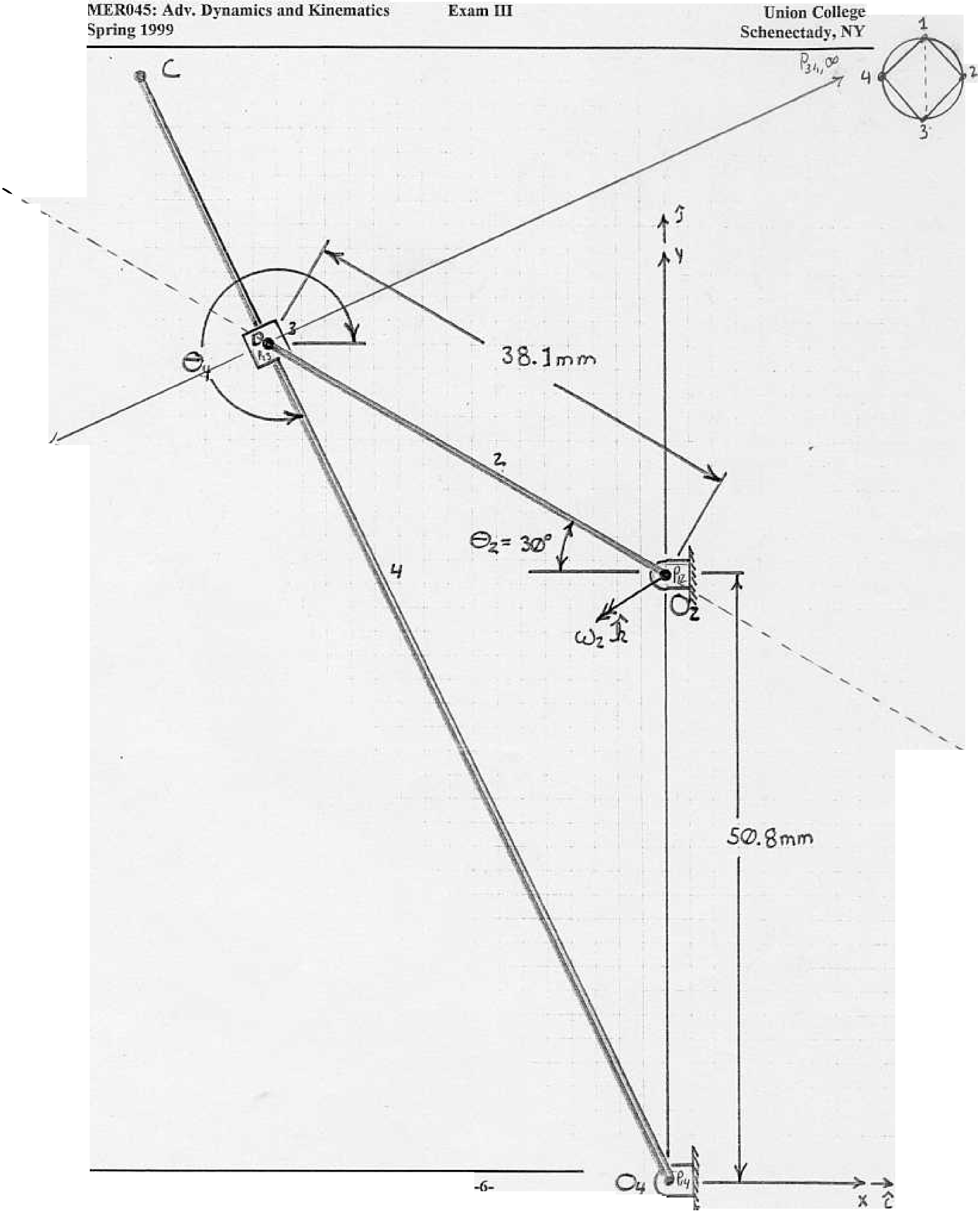
$+ 77.3mm\,(6.367\,^1/_s)^2\,\hat{k}\times(\hat{k}\times\hat{e}_{O_4D})$

$\ddot{R}_{O_4D}\hat{e}_{O_4D} - 4340\,\frac{mm}{s^2}(\hat{k}\times\hat{e}_{O_4D}) + 77.3mm\,\ddot{\theta}_4(\hat{k}\times\hat{e}_{O_4D})$
$+ 3134\,\frac{mm}{s^2}\,\hat{k}\times(\hat{k}\times\hat{e}_{O_4D})$

$\boxed{\ddot{R}_{O_4D} = -4700 \,^{mm}/_{s^2}}$

$\ddot{\theta}_4 = \frac{-900\,^{mm}/_{s^2}}{77.3mm} = \boxed{11.6 \,^1/_{s^2}}$




C
$P_{34,\infty}$
1
2
3
4
3
P_{23}
Θ_4
38.1mm
2
$\Theta_2 = 30°$
4
P_{12}
O_2
$\omega_2 \hat{k}$
$\hat{j}$
y
50.8mm
O_4
P_{14}
x
$\hat{i}$