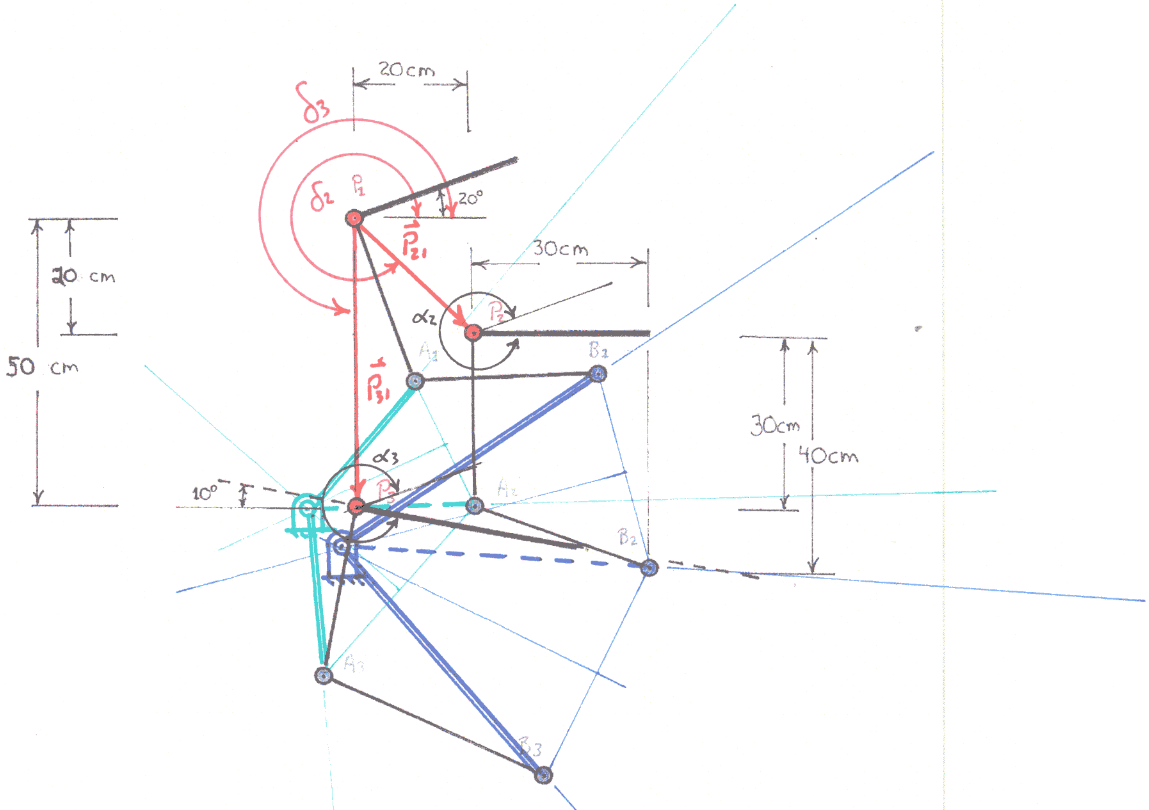
NAME: **SOLUTION**

PROBLEM 1 (50pts): Your first job out of Union is with car maker Bugatti, a company that is known for making the fastest and most expensive road cars in the world. Your asked to redesign the mechanism for the rear spoiler on the Bugatti Veryon.

1a. On the next page is an illustration of the bracket that you must connect your mechanism to in three critical positions. Design a mechanism that will connect to the car below the dotted line which represents the rear contour of the car.



$\rho_{21} = \sqrt{(20cm)^2 + (20cm)^2} = 28.28cm$

$\delta_2 = \tan^{-1} \frac{-20cm}{20cm} = 315°$

$\rho_{31} = 50cm$

$\delta_3 = 270°$

$\alpha_2 = 340°$

$\alpha_3 = 330°$

$\beta_2 = 312°$

$\beta_3 = 224°$

$\gamma_2 = 323°$

$\gamma_3 = 278°$

THREE POSITION ANALYTICAL MOTION SYNTHESIS

$$\vec{W}_2+\vec{Z}_2=\vec{W}_1+\vec{Z}_1+\vec{P}_{21};\quad \vec{W}_3+\vec{Z}_3=\vec{W}_1+\vec{Z}_1+\vec{P}_{31}$$

$$\left|\vec{W}_1\right|=\left|\vec{W}_2\right|=\left|\vec{W}_3\right|=w;\quad \left|\vec{Z}_1\right|=\left|\vec{Z}_2\right|=\left|\vec{Z}_3\right|=z$$

$$\vec{W}_1=w\cdot\left[\cos(\theta)\hat{i}+\sin(\theta)\hat{j}\right]$$

$$\vec{W}_2=w\cdot\left[\cos(\theta+\beta_2)\hat{i}+\sin(\theta+\beta_2)\hat{j}\right]$$

$$\vec{W}_3=w\cdot\left[\cos(\theta+\beta_3)\hat{i}+\sin(\theta+\beta_3)\hat{j}\right]$$

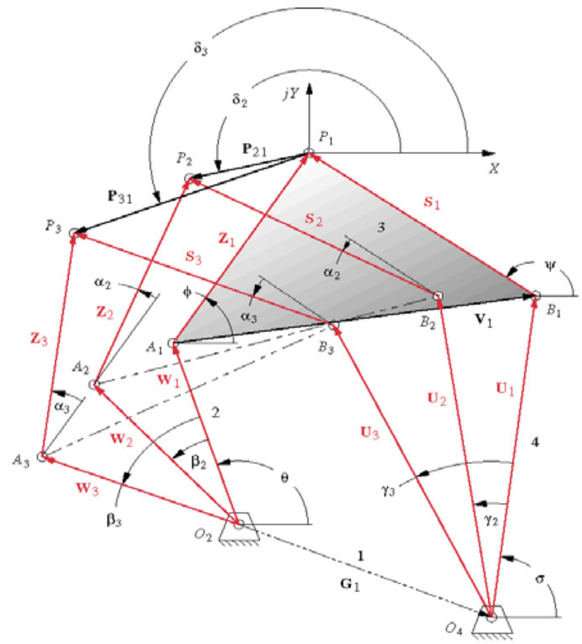
$$\vec{Z}_1=z\cdot\left[\cos(\phi)\hat{i}+\sin(\phi)\hat{j}\right]$$

$$\vec{Z}_2=z\cdot\left[\cos(\phi+\alpha_2)\hat{i}+\sin(\phi+\alpha_2)\hat{j}\right]$$

$$\vec{Z}_3=z\cdot\left[\cos(\phi+\alpha_3)\hat{i}+\sin(\phi+\alpha_3)\hat{j}\right]$$

$$\vec{P}_{21}=p_{21}\cdot\left[\cos(\delta_2)\hat{i}+\sin(\delta_2)\hat{j}\right]$$

$$\vec{P}_{31}=p_{31}\cdot\left[\cos(\delta_3)\hat{i}+\sin(\delta_3)\hat{j}\right]$$



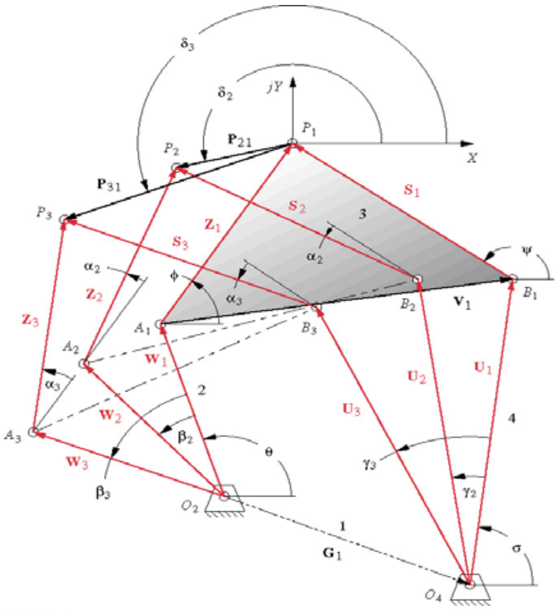
FIRST DYAD

GIVEN:		CHOSEN:		FIND:	
P12	28.28	β2	312.00	w	27.220
P13	50.00	β3	224.00	θ	51.086
δ2	315.00			z	28.561
δ3	270.00			ϕ	98.365
α2	340.00			W1x	17.098
α3	330.00			W1y	21.179
				Z1x	-4.155
				Z1y	28.257

	x-coord	y-coord.
O2	-12.943	-49.436
A1	4.155	-28.257
A2	14.237	-47.971
A3	-10.530	-76.548
P1	0.000	0.000
P2	19.997	-19.997
P3	0.000	-50.000

$$\begin{bmatrix} -0.3309 & 0.7431 & -0.0603 & 0.3420 \\ -0.7431 & -0.3309 & -0.3420 & -0.0603 \\ -1.7193 & 0.6947 & -0.1340 & 0.5000 \\ -0.6947 & -1.7193 & -0.5000 & -0.1340 \end{bmatrix} \begin{Bmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{Bmatrix} = \begin{Bmatrix} 19.9970 \\ -19.9970 \\ 0.0000 \\ -50.0000 \end{Bmatrix}$$

$$\begin{bmatrix} \cos\beta_2-1 & -\sin\beta_2 & \cos\alpha_2-1 & -\sin\alpha_2 \\ \sin\beta_2 & \cos\beta_2-1 & \sin\alpha_2 & \cos\alpha_2-1 \\ \cos\beta_3-1 & -\sin\beta_3 & \cos\alpha_3-1 & -\sin\alpha_3 \\ \sin\beta_3 & \cos\beta_3-1 & \sin\alpha_3 & \cos\alpha_3-1 \end{bmatrix}\cdot\begin{Bmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{Bmatrix}=\begin{Bmatrix} p_{21}\cdot\cos\delta_2 \\ p_{21}\cdot\sin\delta_2 \\ p_{31}\cdot\cos\delta_3 \\ p_{31}\cdot\sin\delta_3 \end{Bmatrix}$$



$$\vec{U}_2+\vec{S}_2=\vec{U}_1+\vec{S}_1+\vec{P}_{21};\quad \vec{U}_3+\vec{S}_3=\vec{U}_1+\vec{S}_1+\vec{P}_{31}$$

$$\left|\vec{U}_1\right|=\left|\vec{U}_2\right|=\left|\vec{U}_3\right|=u;\quad \left|\vec{S}_1\right|=\left|\vec{S}_2\right|=\left|\vec{S}_3\right|=s$$

$$\vec{U}_1=u\cdot\left[\cos(\sigma)\hat{i}+\sin(\sigma)\hat{j}\right]$$

$$\vec{U}_2=u\cdot\left[\cos(\sigma+\gamma_2)\hat{i}+\sin(\sigma+\gamma_2)\hat{j}\right]$$

$$\vec{U}_3=u\cdot\left[\cos(\sigma+\gamma_3)\hat{i}+\sin(\sigma+\gamma_3)\hat{j}\right]$$

$$\vec{S}_1=s\cdot\left[\cos(\psi)\hat{i}+\sin(\psi)\hat{j}\right]$$

$$\vec{S}_2=s\cdot\left[\cos(\psi+\alpha_2)\hat{i}+\sin(\psi+\alpha_2)\hat{j}\right]$$

$$\vec{S}_3=s\cdot\left[\cos(\psi+\alpha_3)\hat{i}+\sin(\psi+\alpha_3)\hat{j}\right]$$

$$\vec{P}_{21}=p_{21}\cdot\left[\cos(\delta_2)\hat{i}+\sin(\delta_2)\hat{j}\right]$$

$$\vec{P}_{31}=p_{31}\cdot\left[\cos(\delta_3)\hat{i}+\sin(\delta_3)\hat{j}\right]$$

SECOND DYAD

GIVEN:		CHOSEN:		FIND:	
P12	28.28	γ2	323.00	u	64.865
P13	50.00	γ3	278.00	σ	18.920
δ2	315.00			s	83.248
δ3	270.00			ψ	147.062
α2	340.00			U1x	61.361
α3	330.00			U1y	21.033
				S1x	-69.867
				S1y	45.265

	x-coord	y-coord.
O4	8.506	-66.298
B1	69.867	-45.265
B2	70.169	-86.428
B3	37.874	-124.134
P1	0.000	0.000
P2	19.997	-19.997
P3	0.000	-50.000

$$\begin{bmatrix} -0.2014 & 0.6018 & -0.0603 & 0.3420 \\ -0.6018 & -0.2014 & -0.3420 & -0.0603 \\ -0.8608 & 0.9903 & -0.1340 & 0.5000 \\ -0.9903 & -0.8608 & -0.5000 & -0.1340 \end{bmatrix} \begin{Bmatrix} U1x \\ U1y \\ S1x \\ S1y \end{Bmatrix} = \begin{Bmatrix} 19.9970 \\ -19.9970 \\ 0.0000 \\ -50.0000 \end{Bmatrix}$$

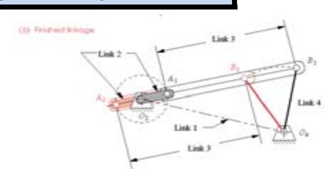
$$\begin{bmatrix} \cos\gamma_2-1 & -\sin\gamma_2 & \cos\alpha_2-1 & -\sin\alpha_2 \\ \sin\gamma_2 & \cos\gamma_2-1 & \sin\alpha_2 & \cos\alpha_2-1 \\ \cos\gamma_3-1 & -\sin\gamma_3 & \cos\alpha_3-1 & -\sin\alpha_3 \\ \sin\gamma_3 & \cos\gamma_3-1 & \sin\alpha_3 & \cos\alpha_3-1 \end{bmatrix}\cdot\begin{Bmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{Bmatrix}=\begin{Bmatrix} p_{21}\cdot\cos\delta_2 \\ p_{21}\cdot\sin\delta_2 \\ p_{31}\cdot\cos\delta_3 \\ p_{31}\cdot\sin\delta_3 \end{Bmatrix}$$

1b. Design a drive dyad that will carry this mechanism through its motion in a non-quick return manner. The motor for the dyad must also be below the dotted lines.

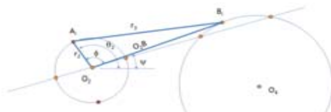
NON-QUICK-RETURN (From Three Position Results)

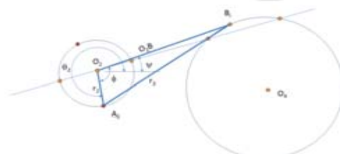
	X-pos	Y-pos	mag	angle	i	j
3P-O2 => O4	**-12.943**	**-49.436**	**51.102**	**-104.7**	**-0.2533**	**-0.9674**
3P-A1	**4.155**	**-28.257**	**28.561**	**-81.6**	**0.1455**	**-0.9894**
3P-A2	**14.237**	**-47.971**	**50.039**	**-73.5**	**0.2845**	**-0.9587**
3P-A3	**-10.530**	**-76.548**	**77.269**	**-97.8**	**-0.1363**	**-0.9907**

Factors		
P	0.5	**% dist up Link 4**
K	2.5	**Length of Link 3+Link 2 wrt B1B2**
Link 1	**50.732**	
Link 2	**12.619**	
Link 3	**50.475**	
Link 4	**13.610**	**Grashof**



ϕ=	59.6
ψ=	65.1
θ_{2i}=	124.8
θ_{2ii}=	365.5





$$r_3^2 = r_2^2 + (O_2B)^2 - 2 \cdot r_2 \cdot (O_2B) \cdot \cos\phi$$

$$\phi = \cos^{-1} \frac{r_2^2 + (O_2B)^2 - r_3^2}{2 \cdot r_2 \cdot (O_2B)}$$

$$\psi = \tan^{-1} \frac{B_{iy} - O_{2y}}{B_{ix} - O_{2x}}$$

$$\theta_{2i} = \phi + \psi$$

$$\theta_{2ii} = 360 + \psi - \phi$$

					Normal (r)		Perpendicular (θ)	
	x comp	**y comp**	**mag**	**angle**	**i**	**j**	**i**	**j**
rO4	**-12.943**	**-49.436**	**51.102**	**-104.7**	**-0.2533**	**-0.9674**	**0.9674**	**-0.2533**
rP3O2-A1	17.098	21.179	27.220	51.1	0.6282	0.7781	-0.7781	0.6282
rP3O2-A2	27.180	1.465	27.220	3.1	0.9985	0.0538	-0.0538	0.9985
rP3O2-A3	2.413	-27.112	27.220	-84.9	0.0887	-0.9961	0.9961	0.0887
rB1	**-4.394**	**-38.846**	**39.094**	**-96.5**	**-0.1124**	**-0.9937**	**0.9937**	**-0.1124**
rO4B1	8.549	10.590	13.610	51.1	0.6282	0.7781	-0.7781	0.6282
rB2	**-11.737**	**-62.992**	**64.076**	**-100.6**	**-0.1832**	**-0.9831**	**0.9831**	**-0.1832**
rO4B2	1.207	-13.556	13.610	-84.9	0.0887	-0.9961	0.9961	0.0887
rBi	**0.647**	**-48.703**	**48.708**	**-89.2**	**0.0133**	**-0.9999**	**0.9999**	**0.0133**
rO4Bi	13.590	0.733	13.610	3.1	0.9985	0.0538	-0.0538	0.9985
rB1B2	-7.342	-24.146	25.238	-106.9	-0.2909	-0.9567	0.9567	-0.2909
rO2	**-22.750**	**-99.211**	**101.786**	**-102.9**	**-0.2235**	**-0.9747**	**0.9747**	**-0.2235**
rB102	-18.356	-60.365	63.094	-106.9	-0.2909	-0.9567	0.9567	-0.2909
rBi02	-23.397	-50.508	55.664	-114.9	-0.4203	-0.9074	0.9074	-0.4203
rB202	-11.014	-36.219	37.856	-106.9	-0.2909	-0.9567	0.9567	-0.2909
rA1	**-19.079**	**-87.138**	**89.202**	**-102.4**	**-0.2139**	**-0.9769**	**0.9769**	**-0.2139**
rO2A1	3.671	12.073	12.619	73.1	0.2909	0.9567	-0.9567	0.2909
rA2	**-26.422**	**-111.284**	**114.377**	**-103.4**	**-0.2310**	**-0.9730**	**0.9730**	**-0.2310**
rO2A2	-3.671	-12.073	12.619	-106.9	-0.2909	-0.9567	0.9567	-0.2909
rAi	-29.950	-88.848	93.760	-108.6	-0.3194	-0.9476	0.9476	-0.3194
rO2Ai	-7.200	10.363	12.619	124.8	-0.5705	0.8213	-0.8213	-0.5705
rAii	-10.190	-98.001	98.530	-95.9	-0.1034	-0.9946	0.9946	-0.1034
rO2Aii	12.561	1.210	12.619	5.5	0.9954	0.0959	-0.0959	0.9954
rB1A1	-14.685	-48.292	50.475	-106.9	-0.2909	-0.9567	0.9567	-0.2909
rBiAi	-30.597	-40.144	50.475	-127.3	-0.6062	-0.7953	0.7953	-0.6062
rBiAii	-10.837	-49.298	50.475	-102.4	-0.2147	-0.9767	0.9767	-0.2147
rB2A2	-14.685	-48.292	50.475	-106.9	-0.2909	-0.9567	0.9567	-0.2909
rO4O2	-9.807	-49.775	50.732	-101.1	-0.1933	-0.9811	0.9811	-0.1933

$\dot{\theta}_2 =$	1.047 1/s
$\ddot{\theta}_2 =$	0.000 1/s^2
ω3-1	-0.262 1/s
ω3-i	0.292 1/s
ω3-ii	0.011 1/s
ω3-2	0.262 1/s
ω4-1	0.000 1/s
ω4-i	-1.213 1/s
ω4-ii	0.959 1/s
ω4-2	0.000 1/s
α3-1	-0.848 1/s^2
α3-i	-0.783 1/s^2
α3-ii	0.027 1/s^2
α3-2	-0.509 1/s^2
α4-1	-3.392 1/s^2
α4-i	1.259 1/s^2
α4-ii	-0.069 1/s^2
α4-2	2.035 1/s^2

Kinematics					Normal (r)		Perpendicular (θ)	
	x comp	**y comp**	**mag**	**angle**	**i**	**j**	**i**	**j**
r1	9.807	49.775	50.732	78.9	0.1933	0.9811	-0.9811	0.1933
r4-1	8.549	10.590	13.610	51.1	0.6282	0.7781	-0.7781	0.6282
r4-i	13.590	0.733	13.610	3.1	0.9985	0.0538	-0.0538	0.9985
r4-2	1.207	-13.556	13.610	-84.9	0.0887	-0.9961	0.9961	0.0887
r2-1	3.671	12.073	12.619	73.1	0.2909	0.9567	-0.9567	0.2909
r2-i	-7.200	10.363	12.619	124.8	-0.5705	0.8213	-0.8213	-0.5705
r2-ii	12.561	1.210	12.619	5.5	0.9954	0.0959	-0.0959	0.9954
r2-2	-3.671	-12.073	12.619	-106.9	-0.2909	-0.9567	0.9567	-0.2909
r3-1	14.685	48.292	50.475	73.1	0.2909	0.9567	-0.9567	0.2909
r3-i	-30.597	-40.144	50.475	-127.3	-0.6062	-0.7953	0.7953	-0.6062
r3-ii	-10.837	-49.298	50.475	-102.4	-0.2147	-0.9767	0.9767	-0.2147
r3-2	14.685	48.292	50.475	73.1	0.2909	0.9567	-0.9567	0.2909
vA-1	-12.640	3.844	13.212	163.1	-0.9567	0.2909	-0.2909	-0.9567
vA-i	-10.850	-7.538	13.212	-145.2	-0.8213	-0.5705	0.5705	-0.8213
vA-ii	-1.266	13.151	13.212	95.5	-0.0959	0.9954	-0.9954	-0.0959
vA-2	12.640	-3.844	13.212	-16.9	0.9567	-0.2909	0.2909	0.9567
vB-1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
vB-i	0.889	-16.485	16.509	-86.9	0.0538	-0.9985	0.9985	0.0538
vB-ii	-0.702	13.027	13.046	93.1	-0.0538	0.9985	-0.9985	-0.0538
vB-2	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
aA-1	-4.024	-13.234	13.833	-106.9	-0.2909	-0.9567	0.9567	-0.2909
aA-i	7.892	-11.360	13.833	-55.2	0.5705	-0.8213	0.8213	0.5705
aA-ii	-13.769	-1.326	13.833	-174.5	-0.9954	-0.0959	0.0959	-0.9954
aA-2	4.024	13.234	13.833	73.1	0.2909	0.9567	-0.9567	0.2909
aB-1	35.915	-28.994	46.158	-38.9	0.7781	-0.6282	0.6282	0.7781
aB-i	-20.920	16.026	26.353	142.5	-0.7938	0.6081	-0.6081	-0.7938
aB-ii	-12.437	-1.612	12.541	-172.6	-0.9917	-0.1285	0.1285	-0.9917
aB-2	27.586	2.455	27.695	5.1	0.9961	0.0887	-0.0887	0.9961

BONUS - Mechanism Kinematics

KINEMATIC ANALYSIS - CRITICAL POSITIONS					Normal (r)		Perpendicular (θ)	
	x-coord	y-coord.	mag	angle	i	j	i	j
O2	-12.943	-49.436	51.102	-104.7	-0.2533	-0.9674	0.9674	-0.2533
A1	4.155	-28.257	28.561	-81.6	0.1455	-0.9894	0.9894	0.1455
A2	14.237	-47.971	50.039	-73.5	0.2845	-0.9587	0.9587	0.2845
A3	-10.530	-76.548	77.269	-97.8	-0.1363	-0.9907	0.9907	-0.1363
P1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
P2	19.997	-19.997	28.280	-45.0	0.7071	-0.7071	0.7071	0.7071
P3	0.000	-50.000	50.000	-90.0	0.0000	-1.0000	1.0000	0.0000

KINEMATIC ANALYSIS - CRITICAL POSITIONS					Normal (r)		Perpendicular (θ)	
	x-coord	y-coord.	mag	angle	i	j	i	j
O4	8.506	-66.298	66.841	-82.7	0.1273	-0.9919	0.9919	0.1273
B1	69.867	-45.265	83.248	-32.9	0.8393	-0.5437	0.5437	0.8393
B2	70.169	-86.428	111.326	-50.9	0.6303	-0.7764	0.7764	0.6303
B3	37.874	-124.134	129.783	-73.0	0.2918	-0.9565	0.9565	0.2918
P1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
P2	19.997	-19.997	28.280	-45.0	0.7071	-0.7071	0.7071	0.7071
P3	0.000	-50.000	50.000	-90.0	0.0000	-1.0000	1.0000	0.0000

SYNTHESIZED LINKAGE KINEMATICS

Input from Drive Link	DLω4-1 => ω2-1	0.0000	1/s
	DLω4-2i => ω2-2i	-1.2130	1/s
	DLω4-2ii => ω2-2ii	0.9586	1/s
	DLω4-3 => ω2-3	0.0000	1/s
	DLα4-1 => α2-1	-3.3915	1/s^2
	DLα4-2i => α2-2i	1.2586	1/s^2
	DLα4-2ii => α2-2ii	-0.0691	1/s^2
	DLα4-3 => α2-3	2.0349	1/s^2
Angular Velocity	ω3-1	0.0000	1/s
	ω3-2i	-0.6209	1/s
	ω3-2ii	0.4907	1/s
	ω3-3	0.0000	1/s
	ω4-1	0.0000	1/s
	ω4-2i	-1.0979	1/s
	ω4-2ii	0.8676	1/s
	ω4-3	0.0000	1/s
Angular Acceleration	α3-1	-1.3142	1/s^2
	α3-2i	-0.1754	1/s^2
	α3-2ii	-0.5471	1/s^2
	α3-3	0.9531	1/s^2
	α4-1	-2.3524	1/s^2
	α4-2i	0.2077	1/s^2
	α4-2i	-0.6442	1/s^2
	α4-3	1.7381	1/s^2

						Normal (r)		Perpendicular (θ)	
		x comp	y comp	mag	angle	i	j	i	j
Displacements	r1	21.449	-16.862	27.284	-38.2	0.7862	-0.6180	0.6180	0.7862
	r4-1	61.361	21.033	64.865	18.9	0.9460	0.3243	-0.3243	0.9460
	r4-2i	61.663	-20.130	64.865	-18.1	0.9506	-0.3103	0.3103	0.9506
	r4-2ii	61.663	-20.130	64.865	-18.1	0.9506	-0.3103	0.3103	0.9506
	r4-3	29.368	-57.836	64.865	-63.1	0.4528	-0.8916	0.8916	0.4528
	r2-1	17.098	21.179	27.220	51.1	0.6282	0.7781	-0.7781	0.6282
	r2-2i	27.180	1.465	27.220	3.1	0.9985	0.0538	-0.0538	0.9985
	r2-2ii	27.180	1.465	27.220	3.1	0.9985	0.0538	-0.0538	0.9985
	r2-3	2.413	-27.112	27.220	-84.9	0.0887	-0.9961	0.9961	0.0887
	r3-1	65.712	-17.008	67.878	-14.5	0.9681	-0.2506	0.2506	0.9681
	r3-2i	55.932	-38.457	67.878	-34.5	0.8240	-0.5666	0.5666	0.8240
	r3-2ii	55.932	-38.457	67.878	-34.5	0.8240	-0.5666	0.5666	0.8240
	r3-3	48.404	-47.586	67.878	-44.5	0.7131	-0.7011	0.7011	0.7131
	rAP-1	-4.155	28.257	28.561	98.4	-0.1455	0.9894	-0.9894	-0.1455
	rAP-2i	5.760	27.974	28.561	78.4	0.2017	0.9795	-0.9795	0.2017
	rAP-2ii	5.760	27.974	28.561	78.4	0.2017	0.9795	-0.9795	0.2017
	rAP-3	10.530	26.548	28.561	68.4	0.3687	0.9295	-0.9295	0.3687
Velocities	vA-1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
	vA-2i	1.778	-32.971	33.019	-86.9	0.0538	-0.9985	0.9985	0.0538
	vA-2ii	-1.405	26.054	26.092	93.1	-0.0538	0.9985	-0.9985	-0.0538
	vA-3	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
	vB-1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
	vB-2i	-22.101	-67.699	71.216	-108.1	-0.3103	-0.9506	0.9506	-0.3103
	vB-2ii	17.465	53.497	56.276	71.9	0.3103	0.9506	-0.9506	0.3103
	vB-3	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
	vP-1	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
	vP-2i	19.147	-36.547	41.259	-62.4	0.4641	-0.8858	0.8858	0.4641
	vP-2ii	-15.130	28.880	32.604	117.6	-0.4641	0.8858	-0.8858	-0.4641
	vP-3	0.000	0.000	0.000	undefined	undefind	undefind	undefind	undefind
Accelerations	aA-1	71.830	-57.988	92.316	-38.9	0.7781	-0.6282	0.6282	0.7781
	aA-2i	-41.839	32.052	52.705	142.5	-0.7938	0.6081	-0.6081	-0.7938
	aA-2ii	-24.873	-3.224	25.082	-172.6	-0.9917	-0.1285	0.1285	-0.9917
	aA-3	55.171	4.910	55.389	5.1	0.9961	0.0887	-0.0887	0.9961
	aB-1	49.478	-144.346	152.590	-71.1	0.3243	-0.9460	0.9460	0.3243
	aB-2i	-70.146	37.071	79.339	152.1	-0.8841	0.4672	-0.4672	-0.8841
	aB-2ii	-59.381	-24.569	64.263	-157.5	-0.9240	-0.3823	0.3823	-0.9240
	aB-3	100.525	51.044	112.742	26.9	0.8916	0.4528	-0.4528	0.8916
	aP-1	108.964	-52.528	120.965	-25.7	0.9008	-0.4342	0.4342	0.9008
	aP-2i	-39.154	20.258	44.084	152.6	-0.8882	0.4595	-0.4595	-0.8882
	aP-2ii	-10.955	-13.110	17.084	-129.9	-0.6412	-0.7674	0.7674	-0.6412
	aP-3	29.868	14.947	33.399	26.6	0.8943	0.4475	-0.4475	0.8943

PROBLEM 2 (50pts): A constant velocity of 2in/sec must be matched for 1sec. Then the follower must return to your choice of start points. The total cycle time is 2.75 sec.

2a. Calculate the angular velocity of the cam.

$$\omega = \frac{1\,rev}{2.75\,s} \cdot \frac{2\pi\,rad}{1\,rev} = 2.285\,\frac{rad}{s}$$

2b. Identify the number of cam segments, the length of the segments in radians and time, the boundary conditions for each segment, and the type of function that will be used in each segment.

Segment	Length of Segment: Time (s)	Length of Segment: Angle in Radians (Degrees)	Boundary Conditions	Type of Function (Order)
1	1	0.7273π=2.285 (130.91°)	$s_1(0)=0,\ \dot{s}_1(0)=2\,in/s,\ \ddot{s}_1(0)=0\,in/s^2$ $\dot{s}_1(1s)=2\,in/s,\ \ddot{s}_1(1s)=0\,in/s^2$	Poly (2nd)
2	1.75	1.2727π=4.0 (229.09°)	$s_2(0)=s_1(1s),\ \dot{s}_2(0)=2\,in/s,\ \ddot{s}_2(0)=0\,in/s^2$ $s_2(1.75s)=0,\ \dot{s}_2(1.75s)=2\,in/s,\ \ddot{s}_2(1.75s)=0\,in/s^2$	Poly (5th)

2c. Determine expressions for the position, velocity, acceleration, and jerk for each segment.

From calculus

$$\dot{s} = v = \frac{ds}{dt} = \frac{ds}{d\theta}\cdot\frac{d\theta}{dt} = \frac{ds}{d\theta}\cdot\omega \Rightarrow s' = \frac{ds}{d\theta} = \frac{v}{\omega}$$
$$\ddot{s} = a = \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt} = \frac{dv}{d\theta}\cdot\omega \Rightarrow s'' = \frac{dv}{d\theta} = \frac{a}{\omega}$$

SEGMENT 1 ($0 \le \theta_1 \le \beta_1$=130.91°=2.285 rad): Constant Velocity

The basic form of the equation for position (s), velocity (s'), acceleration (s''), and jerk (s''') are

$$s_1 = C_0 + C_1\cdot\left(\frac{\theta_1}{\beta_1}\right) = C_0 + C_1\cdot\left(\frac{\theta_1}{2.285\,rad}\right)$$
$$s_1' = \frac{C_1}{\beta_1} = \frac{C_1}{2.285\,rad}$$
$$s_1'' = 0$$
$$s_1''' = 0$$

Converting the time based velocity to rotation based velocity

$$s_1' = \frac{v}{\omega} = \frac{2\,{}^{in}\!/_{s}}{2.285\,{}^{rad}\!/_{s}} = 0.8753\,{}^{in}\!/_{rad}$$

From the first boundary condition, s(0)=0

$$s_1\left(0\right) = 0 = C_0 + C_1 \cdot \left(\frac{0}{2.285\ rad}\right) = C_0 \Rightarrow C_0 = 0$$

From the velocity of the segment being stated to be constant at 2in/s,

$$s_1' = 0.8753\,{}^{in}\!/_{rad} = \frac{C_1}{\beta_1} = \frac{C_1}{2.285\ rad} \Rightarrow C_1 = 2.0in$$

The equations for position (s), velocity (s'), acceleration (s''), and jerk (s''') in this segment can now be written as follows.

$$s_1 = 2.0in \cdot \left(\frac{\theta_1}{2.285\ rad}\right) = 0.8753\,{}^{in}\!/_{rad} \cdot \theta_1 \qquad s_1(0) = 0,\ s_1(2.285rad) = 2in$$

$$s_1' = \frac{2.0in}{2.285\ rad} = 0.8753\,{}^{in}\!/_{rad} \qquad s_1'(0) = s_1'(2.285rad) = s_1' = 0.8753\,{}^{in}\!/_{rad}$$

$$s_1'' = 0 \qquad s_1''(0) = s_1''(2.285rad) = s_1'' = 0$$

$$s_1''' = 0 \qquad s_1'''(0) = s_1'''(2.285rad) = s_1''' = 0$$

<u>SEGMENT 2 ($0 \le \theta_2 \le \beta_2$=229.09°=4.0 rad):</u> 5th degree Polynomial

The basic form of the equation for position (s), velocity (s'), acceleration (s''), and jerk (s''') are

$$s_2 = C_0 + C_2 \cdot \left(\frac{\theta_2}{\beta_2}\right) + C_2 \cdot \left(\frac{\theta_2}{\beta_2}\right)^2 + C_3 \cdot \left(\frac{\theta_2}{\beta_2}\right)^3 + C_4 \cdot \left(\frac{\theta_2}{\beta_2}\right)^4 + C_5 \cdot \left(\frac{\theta_2}{\beta_2}\right)^5$$

$$s_2' = \frac{C_2}{\beta_2} + \frac{2 \cdot C_2}{\beta_2} \cdot \left(\frac{\theta_2}{\beta_2}\right) + \frac{3 \cdot C_3}{\beta_2} \cdot \left(\frac{\theta_2}{\beta_2}\right)^2 + \frac{4 \cdot C_4}{\beta_2} \cdot \left(\frac{\theta_2}{\beta_2}\right)^3 + \frac{5 \cdot C_5}{\beta_2} \cdot \left(\frac{\theta_2}{\beta_2}\right)^4$$

$$s_2'' = \frac{2 \cdot C_2}{\beta_2^2} + \frac{6 \cdot C_3}{\beta_2^2} \cdot \left(\frac{\theta_2}{\beta_2}\right) + \frac{12 \cdot C_4}{\beta_2^2} \cdot \left(\frac{\theta_2}{\beta_2}\right)^2 + \frac{20 \cdot C_5}{\beta_2^2} \cdot \left(\frac{\theta_2}{\beta_2}\right)^3$$

$$s_2''' = \frac{6 \cdot C_3}{\beta_2^3} + \frac{24 \cdot C_4}{\beta_2^3} \cdot \left(\frac{\theta_2}{\beta_2}\right) + \frac{60 \cdot C_5}{\beta_2^3} \cdot \left(\frac{\theta_2}{\beta_2}\right)^2$$

Substituting in for the β for Segment 2

$$s_2 = C_0 + \frac{C_1}{4rad} \cdot \theta_2 + \frac{C_2}{16rad^2} \cdot \theta_2^2 + \frac{C_3}{64rad^3} \cdot \theta_2^3 + \frac{C_4}{256rad^4} \cdot \theta_2^4 + \frac{C_5}{1024rad^5} \cdot \theta_2^5$$

$$s_2' = \frac{C_1}{4rad} + \frac{2 \cdot C_2}{16rad^2} \cdot \theta_2 + \frac{3 \cdot C_3}{64rad^3} \cdot \theta_2^2 + \frac{4 \cdot C_4}{256rad^4} \cdot \theta_2^3 + \frac{5 \cdot C_5}{1024rad^5} \cdot \theta_2^4$$

$$= \frac{C_1}{4rad} + \frac{C_2}{8rad^2} \cdot \theta_2 + \frac{C_3}{21.333rad^3} \cdot \theta_2^2 + \frac{C_4}{64rad^4} \cdot \theta_2^3 + \frac{C_5}{204.8rad^5} \cdot \theta_2^4$$

$$s_2'' = \frac{2 \cdot C_2}{16rad^2} + \frac{6 \cdot C_3}{64rad^3} \cdot \theta_2 + \frac{12 \cdot C_4}{256rad^4} \cdot \theta_2^2 + \frac{20 \cdot C_5}{1024rad^5} \cdot \theta_2^3$$

$$= \frac{C_2}{8rad^2} + \frac{C_3}{10.667rad^3} \cdot \theta_2 + \frac{C_4}{21.333rad^4} \cdot \theta_2^2 + \frac{C_5}{51.2rad^5} \cdot \theta_2^3$$

$$s_2''' = \frac{6 \cdot C_3}{64rad^3} + \frac{24 \cdot C_4}{256rad^4} \cdot \theta_2 + \frac{60 \cdot C_5}{1024rad^5} \cdot \theta_2^2$$

$$= \frac{C_3}{10.667rad^3} + \frac{C_4}{10.667rad^4} \cdot \theta_2 + \frac{C_5}{170.67rad^5} \cdot \theta_2^2$$

Evaluating the five constants (C_0, C_1, C_2, C_3, C_4, C_5) using boundary conditions.

$$s_2(0) = 2in = C_0 + \frac{C_1}{4rad} \cdot 0 + \frac{C_2}{16rad^2} \cdot 0^2 + \frac{C_3}{64rad^3} \cdot 0^3 + \frac{C_4}{256rad^4} \cdot 0^4 + \frac{C_5}{1024rad^5} \cdot 0^5$$

$$\Rightarrow C_0 = 2in$$

$$s_2{}'(0) = \frac{2.0in}{2.285\ rad} = 0.8753\, {}^{in}\!/_{rad} = \frac{C_1}{4rad} + \frac{C_2}{8rad^2}\cdot 0 + \frac{C_3}{21.333rad^3}\cdot 0 + \frac{C_4}{64rad^4}\cdot 0 + \frac{C_5}{204.8rad^5}\cdot 0$$

$$\Rightarrow C_1 = \frac{2.0in}{2.285\ rad}\cdot 4rad = 0.8753\, {}^{in}\!/_{rad}\cdot 4rad = 3.501in$$

$$s_2{}''(0) = 0 = \frac{C_2}{8rad^2} + \frac{C_3}{10.667rad^3}\cdot 0 + \frac{C_4}{21.333rad^4}\cdot 0 + \frac{C_5}{51.2rad^5}\cdot 0$$

$$\Rightarrow C_2 = 0$$

$$s_2(4rad) = 0 = 2in + \frac{3.501in}{4rad}\cdot 4rad + \frac{C_3}{64rad^3}\cdot(4rad)^3 + \frac{C_4}{256rad^4}\cdot(4rad)^4 + \frac{C_5}{1024rad^5}\cdot(4rad)^5$$

$$\Rightarrow -5.501in = C_3 + C_4 + C_5$$

$$s_2{}''(4rad) = 0 = \frac{C_3}{10.667rad^3}\cdot(4rad) + \frac{C_4}{21.333rad^4}\cdot(4rad)^2 + \frac{C_5}{51.2rad^5}\cdot(4rad)^3$$

$$= 0.375\, {}^{1}\!/_{rad^2}\cdot C_3 + 0.75\, {}^{1}\!/_{rad^2}\cdot C_4 + 1.25\, {}^{1}\!/_{rad^2}\cdot C_5$$

$$\Rightarrow 0 = 0.375\cdot C_3 + 0.75\cdot C_4 + 1.25\cdot C_5$$

$$s_2{}'(4rad) = \frac{2.0in}{2.285\ rad} = 0.8753\, {}^{in}\!/_{rad} = \frac{3.501in}{4rad} + \frac{C_3}{21.333rad^3}\cdot(4rad)^2 + \frac{C_4}{64rad^4}\cdot(4rad)^3 + \frac{C_5}{204.8rad^5}\cdot(4rad)^4$$

$$= 0.8753\, {}^{in}\!/_{rad} + 0.750\, {}^{1}\!/_{rad}\cdot C_3 + 1\, {}^{1}\!/_{rad}\cdot C_4 + 1.250\, {}^{1}\!/_{rad}\cdot C_5$$

$$\Rightarrow 0 = 0.750\cdot C_3 + C_4 + 1.250\cdot C_5$$

This leaves three equations with three unknowns that can be placed in matrix form and solved using Excel.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.75 & 1 & 1.25 \\ 0.375 & 0.75 & 1.25 \end{bmatrix} \cdot \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} -5.501in \\ 0 \\ 0 \end{Bmatrix}$$

[M]				inv[M]				V		
1	1	1		10	-16	8		-5.501	C3=	-55.01
0.75	1	1.25		-15	28	-16		0	C4=	82.515
0.375	0.75	1.25		6	-12	8		0	C5=	-33.006

The resulting equations for Segment 2 now become

$$\begin{aligned} s_2 &= 2in + \frac{3.5in}{4rad}\cdot\theta_2 + \frac{(-55.01in)}{64rad^3}\cdot\theta_2^{\,3} + \frac{82.52in}{256rad^4}\cdot\theta_2^{\,4} + \frac{(-33.01in)}{1024rad^5}\cdot\theta_2^{\,5} \\ &= 2in + 0.875\,{}^{in}\!/_{rad}\cdot\theta_2 - 0.8595\,{}^{in}\!/_{rad^3}\cdot\theta_2^3 + 0.3223\,{}^{in}\!/_{rad^4}\cdot\theta_2^4 - 0.03224\,{}^{in}\!/_{rad^5}\cdot\theta_2^5 \end{aligned}$$

$$\begin{aligned} s_2' &= \frac{3.5in}{4rad} + \frac{3\cdot(-55.01in)}{64rad^3}\cdot\theta_2^{\,2} + \frac{4\cdot 82.52in}{256rad^4}\cdot\theta_2^{\,3} + \frac{5\cdot(-33.01in)}{1024rad^5}\cdot\theta_2^{\,4} \\ &= 0.875\,{}^{in}\!/_{rad} - 2.5786\,{}^{in}\!/_{rad^3}\cdot\theta_2^{\,2} + 1.2894\,{}^{in}\!/_{rad^4}\cdot\theta_2^{\,3} - 0.1612\,{}^{in}\!/_{rad^5}\cdot\theta_2^{\,4} \end{aligned}$$

$$\begin{aligned} s_2'' &= \frac{6\cdot(-55.01in)}{64rad^3}\cdot\theta_2 + \frac{12\cdot 82.52in}{256rad^4}\cdot\theta_2^{\,2} + \frac{20\cdot(-33.01in)}{1024rad^5}\cdot\theta_2^{\,3} \\ &= -5.157\,{}^{in}\!/_{rad^3}\cdot\theta_2 + 3.868\,{}^{in}\!/_{rad^4}\cdot\theta_2^{\,2} - 0.6447\,{}^{in}\!/_{rad^5}\cdot\theta_2^{\,3} \end{aligned}$$

$$\begin{aligned} s_2''' &= \frac{6\cdot(-55.01in)}{64rad^3} + \frac{24\cdot 82.52in}{256rad^4}\cdot\theta_2 + \frac{60\cdot(-33.01in)}{1024rad^5}\cdot\theta_2^{\,2} \\ &= -5.157\,{}^{in}\!/_{rad^3} + 7.736\,{}^{in}\!/_{rad^4}\cdot\theta_2 - 1.934\,{}^{in}\!/_{rad^5}\cdot\theta_2^{\,2} \end{aligned}$$

2d. Draw the position, velocity, acceleration, and jerk of the follower for each segment.