HOMEWORK SOLUTION
ESC 30: MECHANICS OF DEFORMABLE SOLIDS
ASSIGNMENT #6

PROBLEM 9.110 Two L-shaped bars are welded at points B and D to the rolled steel beam AE. For the loading shown determine (a) the slope at A, (b) the slope at B, (c) the deflection at B. Use $E = 29(10^6)$ psi.

A) GIVEN:

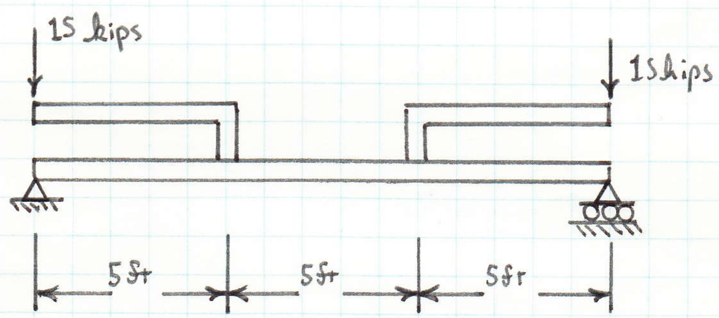
A.1) CONSTRAINTS
(1) L-shaped bars welded to beam
(2) Steel used to make beam, W 10×33
- $E = 29(10^6)\text{ psi} \cdot \left(\frac{12\text{ in}}{\text{ft}}\right)^2 = 4.176(10^9)\frac{\text{lb}}{\text{ft}^2}$

A.2) ASSUMPTIONS
(1) small deflections and rotations.
(2) linear elastic response.

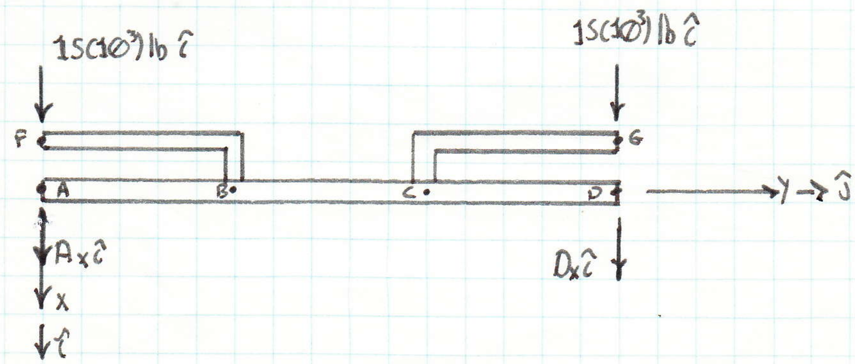
B) FIND:

(1) Slope at A
(2) Slope at B
(3) Deflection at B

C) SKETCH:



D) FREE BODY DIAGRAM:



E) EQUILIBRIUM:

$$\Sigma F_x = 0 = 15(10^3)\,lb + 15(10^3)\,lb + A_x + D_x = 0$$

$$\Rightarrow \quad A_x + D_x = -30(10^3)\,lb \qquad (1)$$

$$\Sigma M_{z/at\,D} = 0 = (15\,ft)(15)(10^3)\,lb + (15\,ft)(A_x)$$

$$\Rightarrow \quad \underline{\underline{A_x = -15(10^3)\,lb}}$$

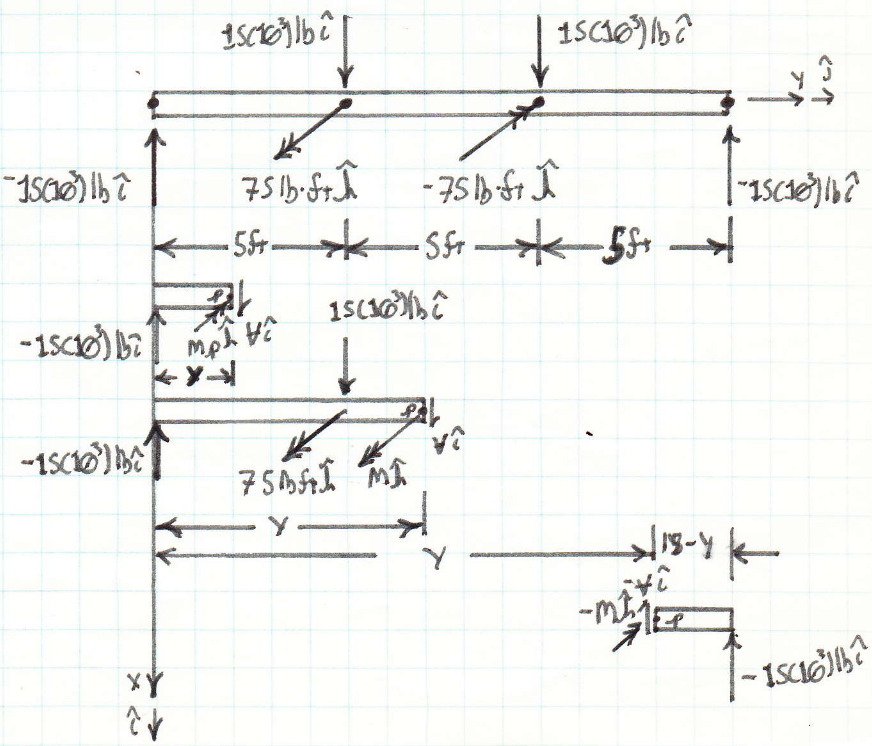
From (1)

$$A_x + D_x = -15(10^3)\,lb + D_x = -30(10^3)\,lb$$

$$\Rightarrow \quad \underline{\underline{D_x = -15(10^3)\,lb}}$$

F) MECHANICS

To solve for the slope and deflections, let's first take the applied loads and react them out at B and C. In doing so we can then draw the shear and bending moment diagrams that will enable us to determine the appropriate slopes and displacements



$0 < y < 5$

$$\Sigma F_y = 0 = V - 15(10^3)\,lb$$

$$\underline{\underline{V = 15(10^3)\,lb}} \qquad (2)$$

$$\Sigma M_{z/at\,p} = 0 = M_p - 15(10^3)\,lb \cdot y$$

$$\underline{\underline{M_p = 15(10^3)\,lb \cdot y}} \qquad (3)$$

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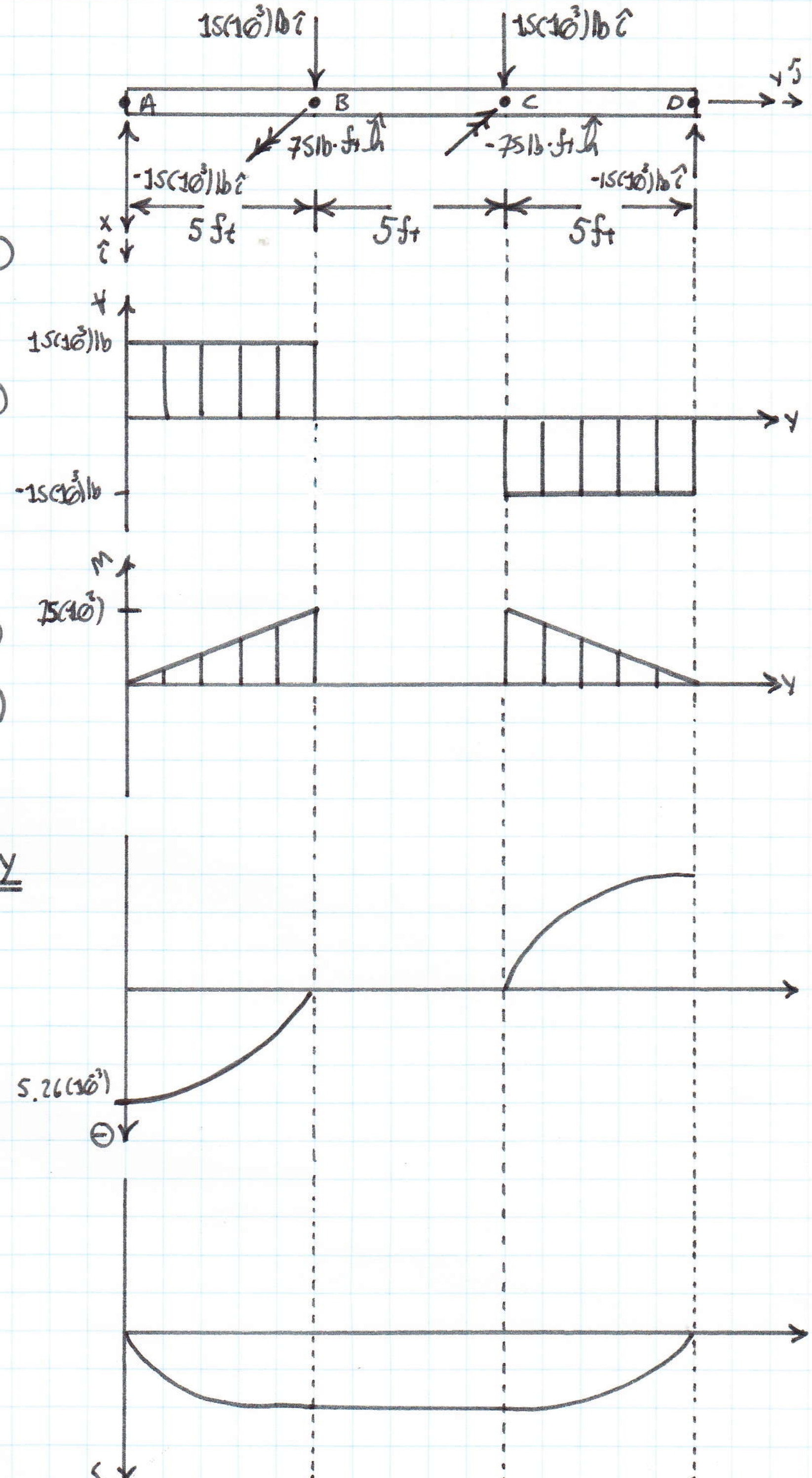

$5 < y < 10$ ft

$\Sigma F_x = 0 = V - 15(10^3)\,lb + 15(10^3)\,lb$

$\Rightarrow \underline{\underline{V = 0}}$ (4)

$\Sigma M_{z/ap} = 0 = 75(10^3)\,ft\,lb + M - 15(10^3)\,lb(y) + 15(10^3)\,lb(y - 5\,ft)$

$\Rightarrow \underline{M_p = 0}$ (5)

$10 < y < 15$ ft

$\Sigma F_x = 0 = -V - 15(10^3)\,lb$

$\underline{\underline{V = -15(10^3)\,lb}}$ (6)

$\Sigma M_{z/ap} = 0 = -M_p + 15(10^3)\,lb\,(15 - y)$

$\Rightarrow M_p = 15(10^3)\,lb\,(15\,ft - y)$

$\underline{\underline{M_p = 225(10^3)\,lb\cdot ft - 15(10^3)\,lb\cdot y}}$

To determine the slope at A

$$\theta_A = -\int_A^B -\frac{M}{EI}\,dy$$

The moment of inertia for a W 10x33 beam is 177 in^4

$$I = 177\,in^4\left(\frac{ft}{12\,in}\right)^4 = 8.54(10^{-3})\,ft^4$$

$$\theta_A = \frac{1}{4.176(10^9)\frac{lb}{ft^2}\cdot 8.54(10^{-3})\,ft^4}\int_0^{5ft} 15(10^3)\,lb\cdot y\,dy$$

$$= \frac{15(10^3)\,lb\cdot\frac{y^2}{2}}{4.176(10^9)\frac{lb}{ft^2}\cdot 8.54(10^{-3})\,ft^4}\Bigg|_0^{5ft}$$

$$\theta_A = \boxed{5.258(10^{-3})\,rad}$$

$$\boxed{\theta_B = 0}$$

The deflection at B can be determined through integration of ③

$$EI \frac{d^2u}{dy^2} = -M(y) = -15(10^3)\,lb \cdot y$$

$$\frac{d^2u}{dy^2} = \frac{-15(10^3)\,lb}{EI} \cdot y \quad \Rightarrow \quad \frac{du}{dy} = \frac{-15(10^3)\,lb}{2EI} y^2 + C_1 \qquad ⑦$$

Knowing that at $y = 5\,ft$, $\frac{du}{dy} = 0$

$$0 = -\frac{15(10^3)\,lb \cdot (5\,ft)^2}{2EI} + C_1 \quad \Rightarrow \quad \underline{\underline{C_1 = \frac{187.5(10^3)\,lb \cdot ft^2}{EI}}}$$

$$\frac{du}{dy} = -\frac{15(10^3)\,lb}{2EI} y^2 + \frac{187.5(10^3)\,lb \cdot ft^2}{EI}$$

$$u = -\frac{15(10^3)\,lb}{6EI} y^3 + \frac{187.5(10^3)\,lb\,ft^2}{EI} y + C_2$$

Knowing that at $y = 0$, $u = 0$

$$0 = C_2$$

$$u = \frac{15(10^3)\,lb}{6EI} y^3 - \frac{187.5(10^3)\,lb \cdot ft^2}{EI} y \quad ; \quad 0 \le y \le 5\,ft$$

$$= \frac{1}{EI}\left[2.5(10^3)\,lb \cdot y^3 - 187.5(10^3)\,lb \cdot ft^2 \cdot y\right] \quad ; \quad 0 \le y \le 5\,ft$$

Therefore $\delta(5\,ft) = 0.0175\,ft \cdot \left(\frac{12\,in}{ft}\right) = \boxed{0.210\,in}$