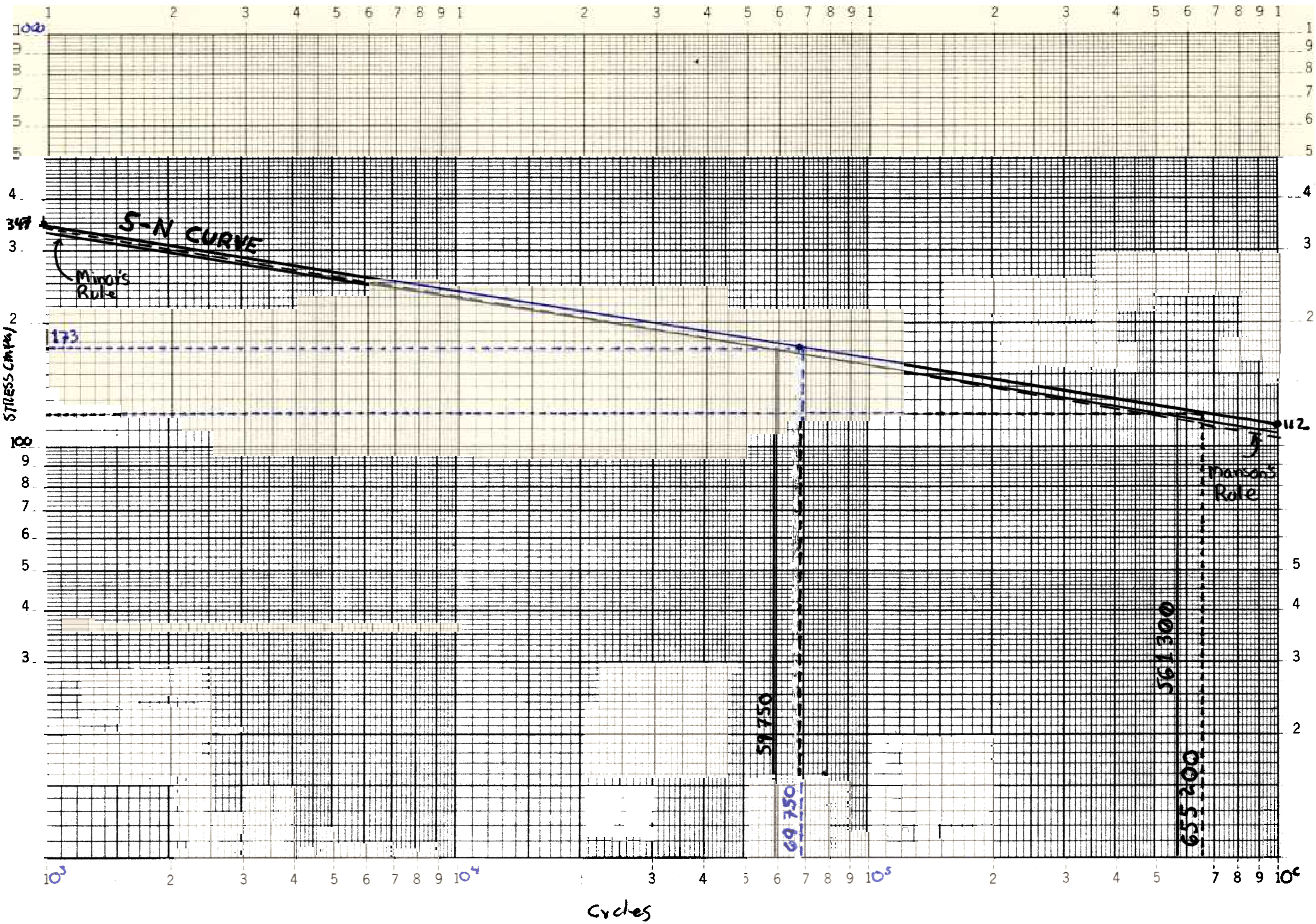
NAME: Solution

Problem 1: The fully corrected endurance limit of a steel member is 112 MPa and the tensile strength is 385 MPa.

(10) **1a.** Using the graph paper supplied on the next page, draw the S-N diagram for this member. Be sure to label all of the important points of interest.

at 10^3 cycles $\sigma_1 = .9 \cdot S_{ut} = .9 \times 385\,\text{MPa} = 346.5\,\text{MPa}$

at 10^6 cycles $\sigma_2 = 112\,\text{MPa} = S_e$



S-N CURVE
347
Miner's Rule
173
112
Manson's Rule
STRESS (MPa)
59 750
69 750
561 300
655 200
10³
10⁴
10⁵
10⁶
Cycles

(10) **1b.** How many cycles will this member survive at an operating load of 173 MPa given R=-1

equation of a line on the S-N curve in log-log space is

$\log \sigma = m \cdot \log N + b$

We know two points on this curve, therefore we can solve for m & b

$\log 346.3 = m \cdot \log 10^3 + b$

$2.539 = m \cdot 3 + b$

$\log 112\,MPa = m \cdot \log 10^6 + b$

$2.049 = m \cdot 6 + b$

$b = 2.049 - 6 \cdot m$

$2.539 = 3 \cdot m + 2.049 - 6m$

$0.4900 = -3 \cdot m \Rightarrow m = -.1633 \Rightarrow b = 3.029 = \log 1069\,MPa$

equation of the S-N curve in log-log space can now be written

$\log \sigma = -0.1633 \cdot \log N + 3.029$

Using this equation we can directly solve for N when $\sigma = 173\,MPa$

$\log 173\,MPa = -0.1633 \cdot \log N + 3.029$

$\boxed{N = 69750 \text{ cycles}}$

1c. If the 173 MPa load was applied for 10,000 cycles, how many cycles could this member withstand at 120 Mpa.

- Solving this first using Minor's Rule. We start by determining the fatigue life at 120 MPa. Since the equation for the S-N curve is

$$\log \sigma_2 = -0.1633 \cdot \log N_2 + 3.029$$

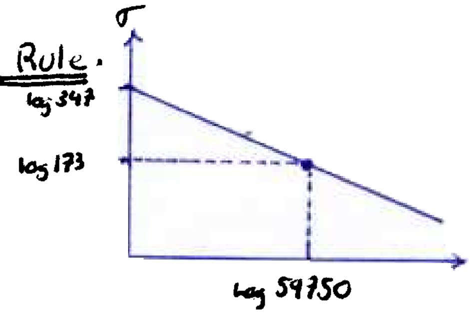
$$\log 120\text{ MPa} = -0.1633 \cdot \log N_2 + 3.029$$

$$N_2 = 655\,200 \text{ cycles}$$

Minors Rule can now be written

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \Rightarrow \frac{10\,000}{69\,750} + \frac{n_2}{655\,200} = 1 \Rightarrow \boxed{n_2 = 561\,300 \text{ cycles}}$$

- Now let's solve the same problem using Manson's Rule. We start by solving for the equation for the Manson curve knowing it starts at 10^3 cycles with a value of 346.5 and a second point on the curve is at 59 750 cycles with a value of 173 MPa.



$$\log \sigma = m \cdot \log N + b$$

$$\log 346.5 = m \cdot \log 10^3 + b$$
$$2.540 = m \cdot 3 + b$$
$$b = -3 \cdot m + 2.540$$

$$\log 173 = m \cdot \log 59\,750 + b$$
$$2.238 = m \cdot 4.776 + b$$

$$2.238 = 4.776 \cdot m - 3 \cdot m + 2.540 \Rightarrow m = -0.1700 \Rightarrow b = 3.050$$

$$\log \sigma = -0.1700 \cdot \log N + 3.050$$

From this equation we can determine the equivalent number of cycles at 120 MPa that causes failure after a 173 MPa load has been cycled for 10^4 cycles

$$\log 120 = -0.1700 \cdot \log N + 3.050$$

$$\boxed{n_2 = 513\,700}$$

1d. After cycling this member at 173 MPa for 10,000 cycles, what is the new endurance limit for this material.

- First let's solve this problem using Minor's Rule. Two points are known on the Minor's rule curve

at 59 750 cycles $\sigma = 173$ MPa

at 561 300 cycles $\sigma = 120$ MPa

$\log 173 = m \cdot \log 59750 + b$ $\log 120 = m \cdot \log 561300 + b$

$2.238 = m \cdot 4.776 + b$ $2.079 = m \cdot 5.749 + b$

$b = 2.238 - 4.776 \cdot m$

$2.079 = 5.749 \cdot m + 2.238 - 4.776 \cdot m$

$\Rightarrow m = -0.1634 \Rightarrow b = 3.018$

$\log \sigma = -0.1634 \cdot \log N + 3.018$

Now the new endurance limit is calculated by finding the value of σ_e when $N = 10^6$ cycles

$\log \sigma_e = -0.1634 \cdot \log 10^6 + 3.018$

$$\boxed{\sigma_e = 109.0 \text{ MPa}}$$

- Now let's solve for the new endurance limit using Manson's rule. From the previous part to this problem, we know the equation for the manson curve

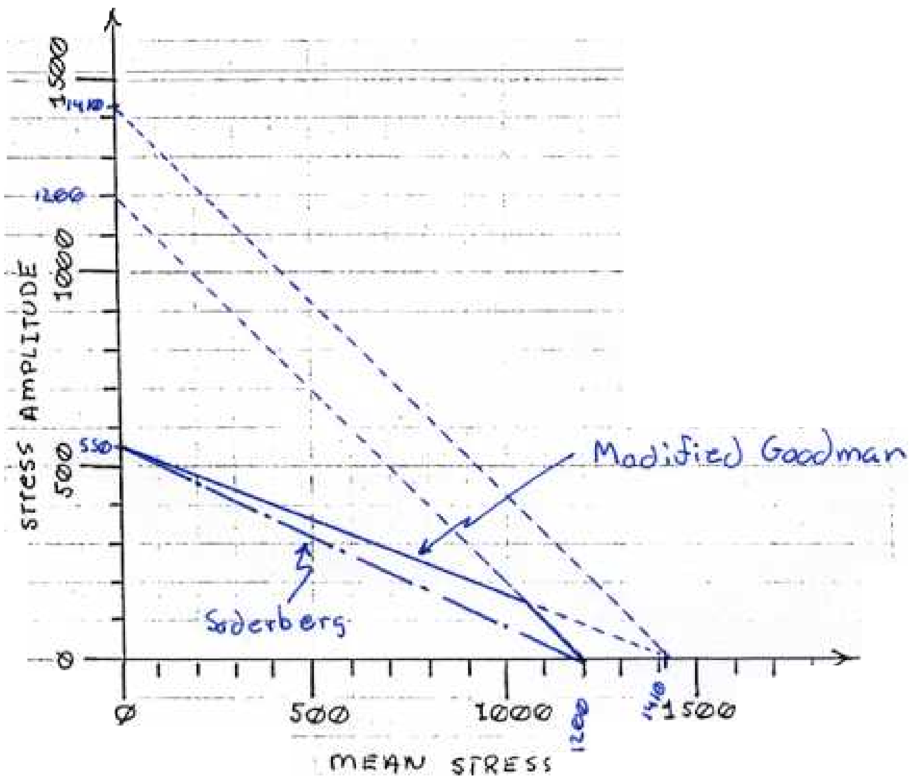
$\log \sigma = -0.1700 \cdot \log N + 3.050$

$\log \sigma_e = -0.1700 \cdot \log 10^6 + 3.050$

$$\boxed{\sigma_e = 107.2 \text{ MPa}}$$

Problem 2: A steel member is manufactured using 4340 Steel with a tensile strength of 1410 MPa, a yield strength of 1200 MPa, and a fully corrected endurance limit of 550 MPa.

(10) **2a.** Using the figure provided, draw the Modified Goodman or Soderberg line on this σ_a versus σ_m diagram.



(6 pts)

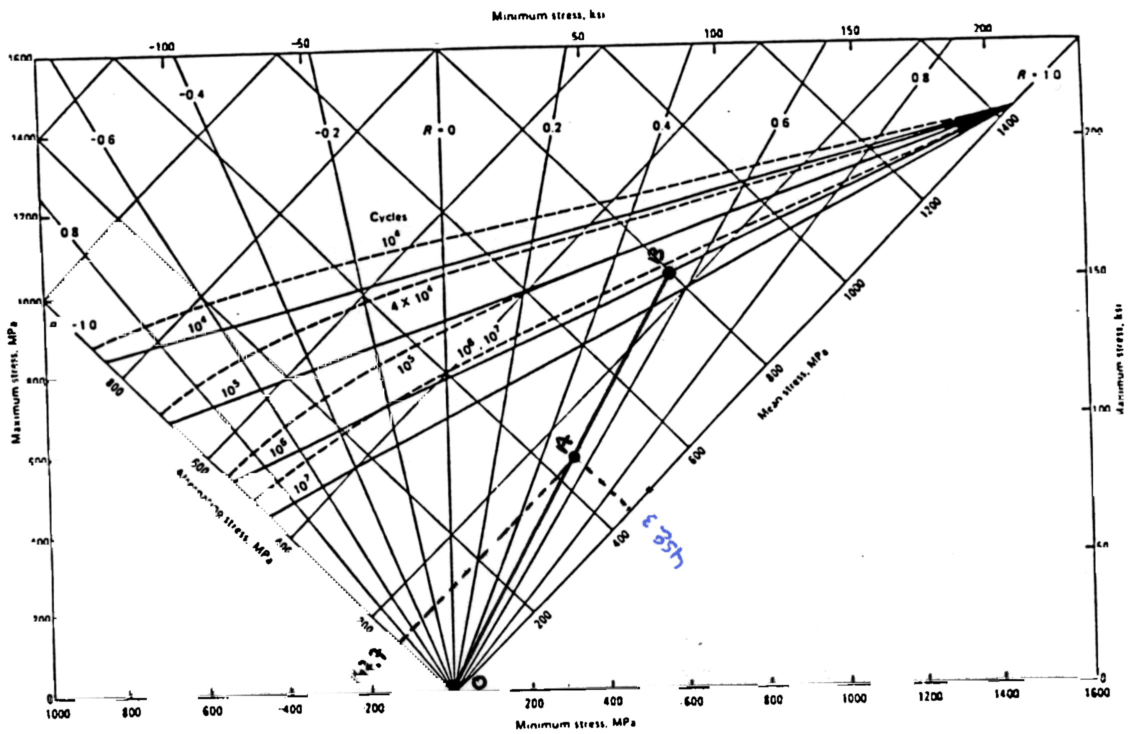
2b. Using the diagram below, determine the factor of safety for a component with the following loading condition

$\sigma_{x,m}$=460 MPa $\sigma_{x,a}$=140 MPa

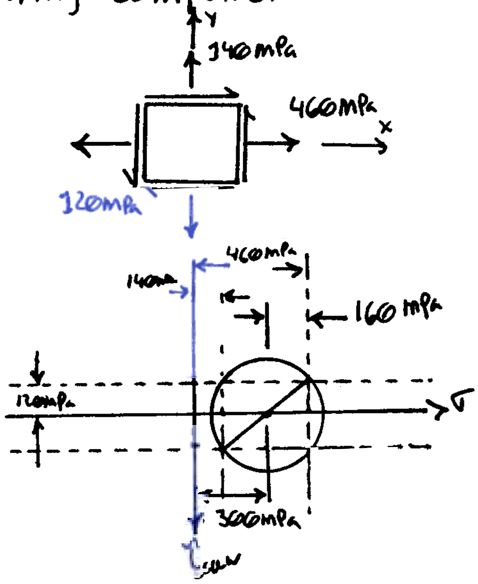
$\sigma_{y,m}$=140 MPa $\sigma_{y,a}$=56 MPa

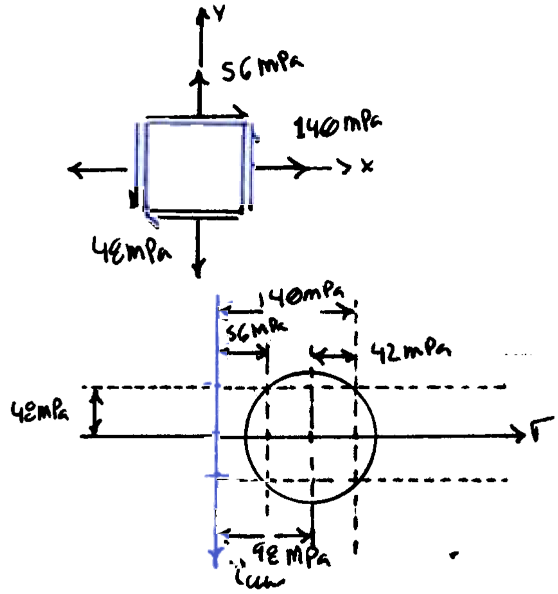
$\tau_{xy,m}$=120 MPa $\tau_{xy,a}$=48 MPa

that is being designed to cycle for 10^5 cycles. (Use the solid lines on the diagram.)



First the principal stresses must be determined for the mean and alternating components





$r = \sqrt{(160\,mPa)^2 + (120\,mPa)^2} = 200\,mPa$

$\sigma_{1,m} = 500\,mPa$

$\sigma_{2,m} = 100\,mPa$

$r = \sqrt{(42\,mPa)^2 + (48\,mPa)^2} = 63.8\,mPa$

$\sigma_{1,a} = 161.8\,mPa$

$\sigma_{2a} = 34.2\,mPa$

$\sigma_m' = \sqrt{(500\,mPa)^2 - (500\,mPa)(100\,mPa) + (100\,mPa)^2} = 458.3\,MPa$

$\sigma_a' = \sqrt{(161.8\,mPa)^2 - (161.8\,mPa)(34.2\,mPa) + (34.2\,mPa)^2} = 147.7\,MPa$

From the figure, the fatigue Factor of safety is defined as

$$\eta = \frac{OB}{OA} = \frac{2.5}{1.4} = \boxed{1.79}$$