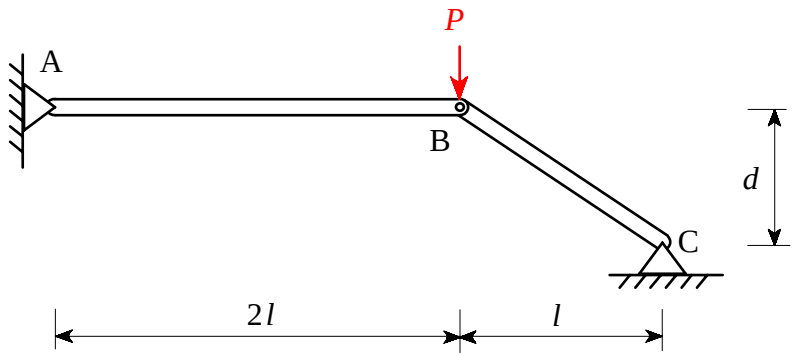
Examples of Buckling

Problem 1

The structure shown below is built of steel (E = 200 GPa) beams of square cross section (b = 30 mm) and is acted upon by force P. The dimensions are l = 2 m and d = 1.3 m.

a) Which bar will buckle first?

b) What is the buckling load P_{cr} of the structure?



solution:

a)

$$\frac{P_{BC}}{P_{AB}} = \frac{\sqrt{\ell^2 + d^2}}{\ell} \approx 1.044 \quad \text{and} \quad \frac{P_{crBC}}{P_{crAB}} = \frac{L_{AB}^2}{L_{BC}^2} \approx 3.67$$

The AB bar will buckle first.

b)

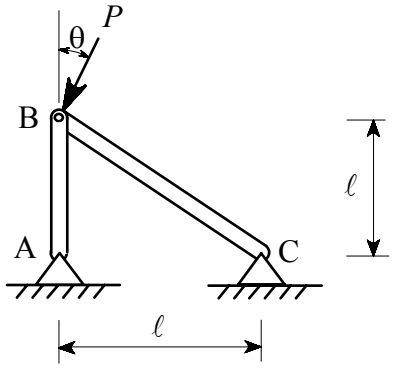
$$P_{AB} = P\frac{\ell}{d}$$

$$P_{cr} = \frac{d}{\ell} P_{crAB} = \frac{\pi^2 d E I}{4\ell^3}, \quad \text{where} \quad I = \frac{b^4}{12}$$

$$P_{cr} \approx 9993 \text{ N}$$

Problem 3

The structure ($\ell = 3$ m) shown below is built of steel ($E = 210$ GPa) beams of square cross section ($b = 23$ mm) and is acted upon by force P. Plot the buckling load of the structure as a function of θ for all possible loading directions. What is the lowest buckling load and at what loading direction does it occur?



solution:

$$P_{crAB} = \frac{\pi^2 I E}{L_{AB}^2} \approx 5{,}370.4\,\text{N} \quad \text{and} \quad P_{crBC} = \frac{\pi^2 I E}{L_{BC}^2} \approx 2{,}685.2\,\text{N}$$

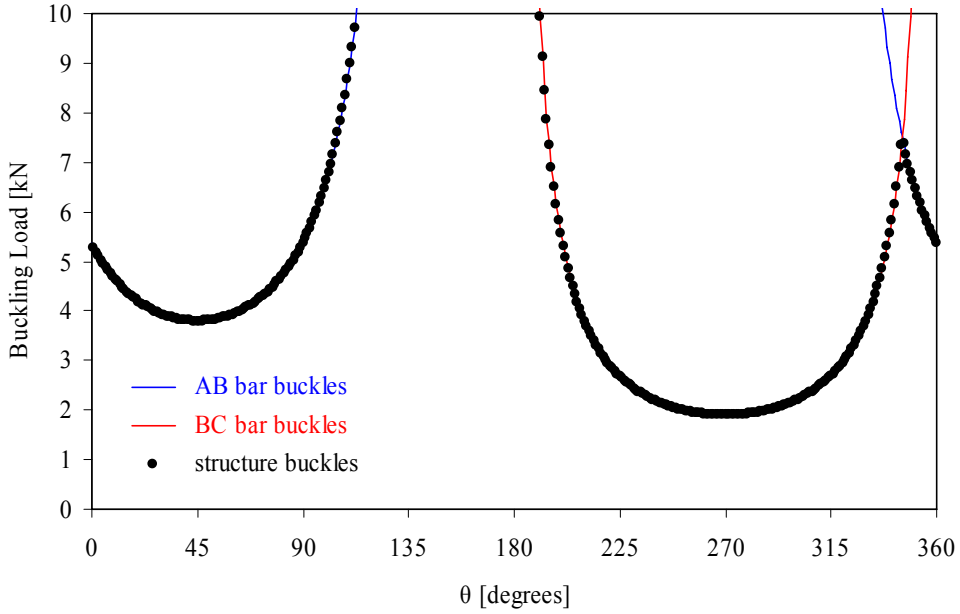
$$P\cos\theta = P_{AB} + \frac{P_{BC}}{\sqrt{2}} \quad \text{and} \quad P\sin\theta = -\frac{P_{BC}}{\sqrt{2}}$$

$$P = -\frac{P_{BC}}{\sqrt{2}\sin\theta} \quad \text{and} \quad P = \frac{P_{AB}}{\cos\theta + \sin\theta}$$

$$P_{cr1} = -\frac{P_{crBC}}{\sqrt{2}\sin\theta} \quad \text{if} \quad \sin\theta < 0 \quad \text{and} \quad P_{cr1} = \infty \quad \text{else}$$

$$P_{cr2} = \frac{P_{crAB}}{\cos\theta + \sin\theta} \quad \text{if} \quad \cos\theta + \sin\theta > 0 \quad \text{and} \quad P_{cr1} = \infty \quad \text{else}$$

$$P_{cr} = \min\left\{P_{cr1},\ P_{cr2}\right\}$$



$$P_{cr\,\min} = \frac{P_{crBC}}{\sqrt{2}} = 1{,}898.7\,\text{N} \quad \text{at} \quad \theta = 270^\circ\ (-90^\circ)$$