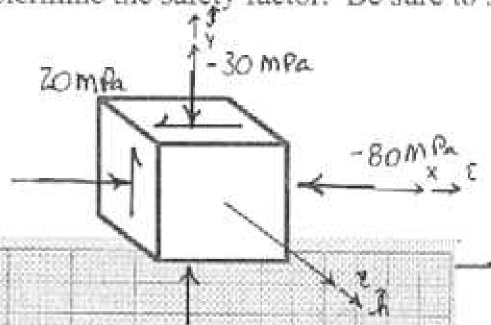
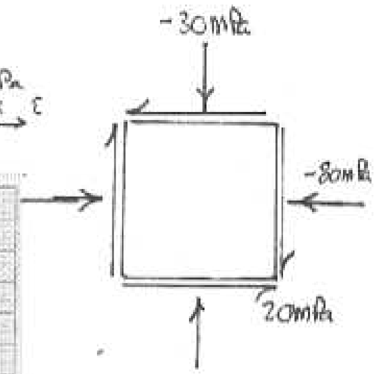
NAME: Solution

PROBLEM 1: For each of the stress states listed below evaluate the failure of the material for the following two materials: Steel S_{ys}=250MPa and Cast Iron S^t_{uci}=170MPa/ S^c_{uci}=655MPa. Determine if failure occurs and if it does not, determine the safety factor. Be sure to state the criteria you are using for failure.

1a (10pts). $\sigma = \begin{bmatrix} -80 & -20 & 0 \\ -20 & -30 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$





$\Gamma_1 = \sqrt{[(-80mPa)-(-55mPa)]^2 + (-20mPa)^2} = 32.02 mPa$

$\Gamma_3 = |-87.02mPa|/2 = 43.51 mPa$

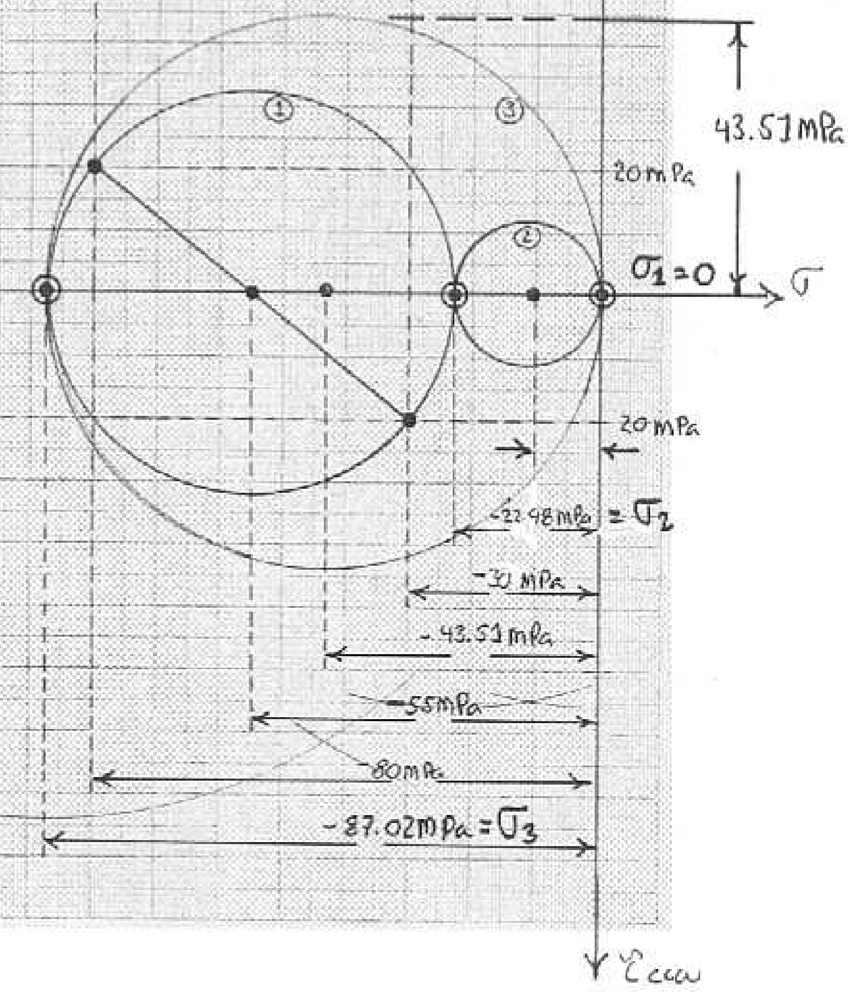
$\sigma_{VM} = \sqrt{\tfrac{1}{2}\{[0-(-22.98mPa)]^2 + [(-22.98mPa)-(-87.02mPa)]^2 + [0-(-87.02mPa)]^2\}}$

$= 78.12 mPa$

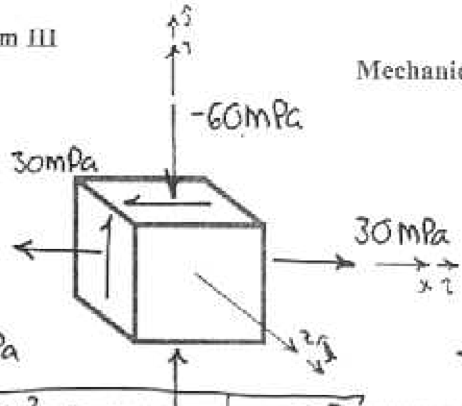
Steel: $n = \frac{250 mPa}{78.12 mPa} = \boxed{3.20}$

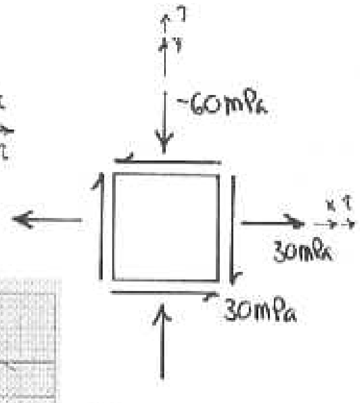
Cast Iron: $n^c = \frac{655 mPa}{87.02 mPa}$

$= \boxed{7.53}$



1b (10pts). $\sigma = \begin{bmatrix} 30 & -30 & 0 \\ -30 & -60 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$





$\tau_1 = \sqrt{[(-60MPa)-(-15MPa)]^2+[-30MPa]^2} = 54.08MPa$

$\sigma_{vm} = \sqrt{\frac{1}{2}\{[39.08MPa-0]^2+[39.08MPa-(-69.08MPa)]^2+[0-(-69.08MPa)]^2\}}$

$\doteq 94.86\ MPa$

STEEL: $n_S = \frac{250MPa}{94.86MPa}$

$= \boxed{2.64}$

CAST IRON

$n_{CI}^{c} = \frac{655MPa}{69.08MPa}$

$= \boxed{9.48}$

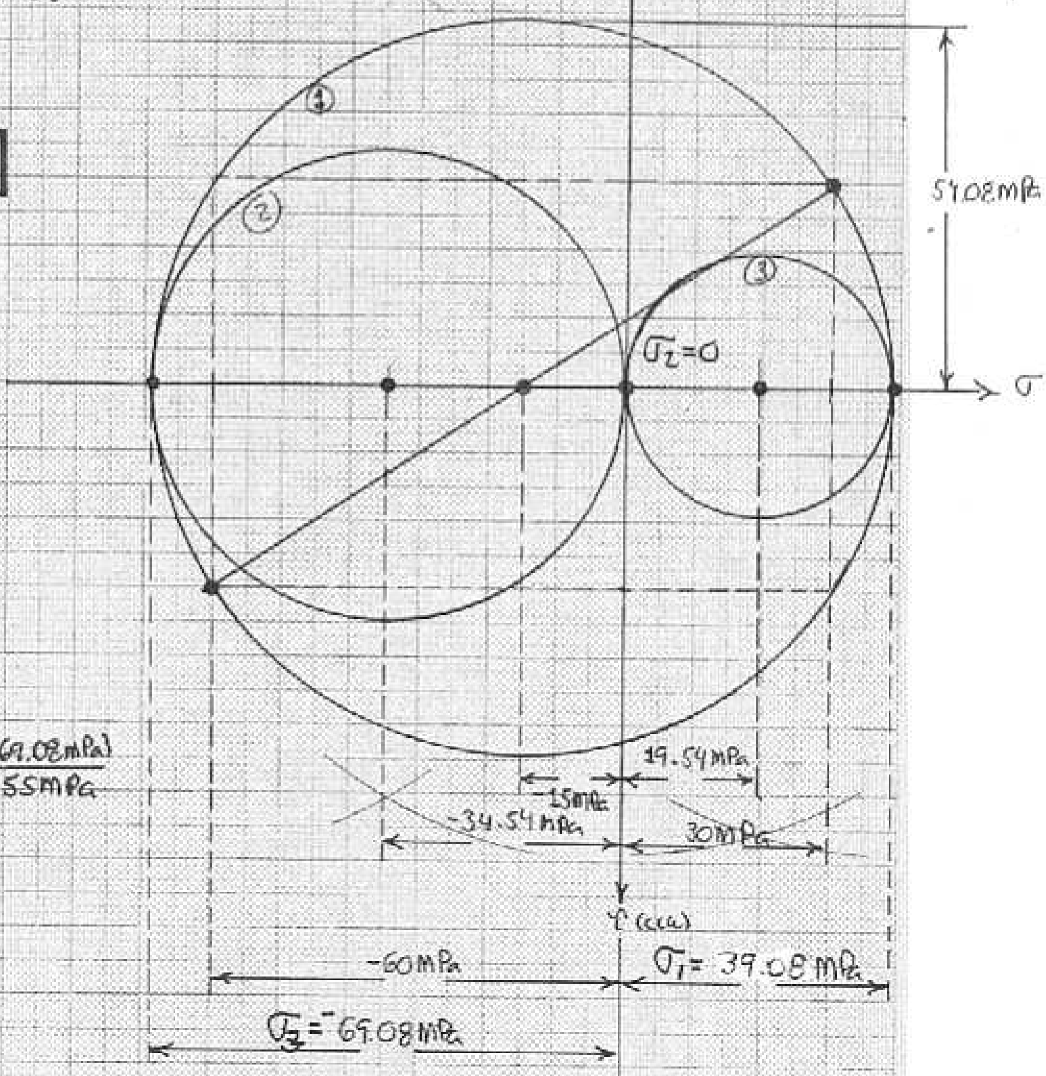
$n_{CI}^{t} = \frac{170\ MPa}{39.08\ MPa}$

$= \boxed{4.35}$

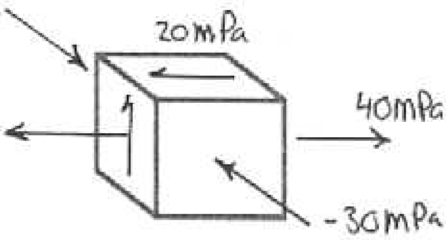
$\frac{1}{n_{CI}^{mc}} = \frac{39.08MPa}{170MPa} - \frac{(-69.08MPa)}{655MPa}$

$\frac{1}{n_{CI}^{m-c}} = 0.3353$

$n_{CI}^{m-c} = \boxed{2.98}$



1c (10pts). $\sigma = \begin{bmatrix} 40 & -20 & 0 \\ -20 & 0 & 0 \\ 0 & 0 & -30 \end{bmatrix} MPa$



$\tau_1 = \sqrt{(20\,MPa)^2 + (20\,MPa)^2} = 28.28\,MPa$

$\tau_3 = 48.28\,MPa - 9.14\,MPa = 39.14\,MPa$

$\sigma_{vm} = \sqrt{\frac{1}{2}\{[48.28\,MPa - (-8.28\,MPa)]^2 + [48.28\,MP - (-30\,MPa)]^2 + [(-8.28\,MP) - (-30\,MPa)]^2\}}$

$= 69.99\,MPa$

STEEL

$n_s = \frac{250\,MPa}{69.99\,MPa} = \boxed{3.57}$

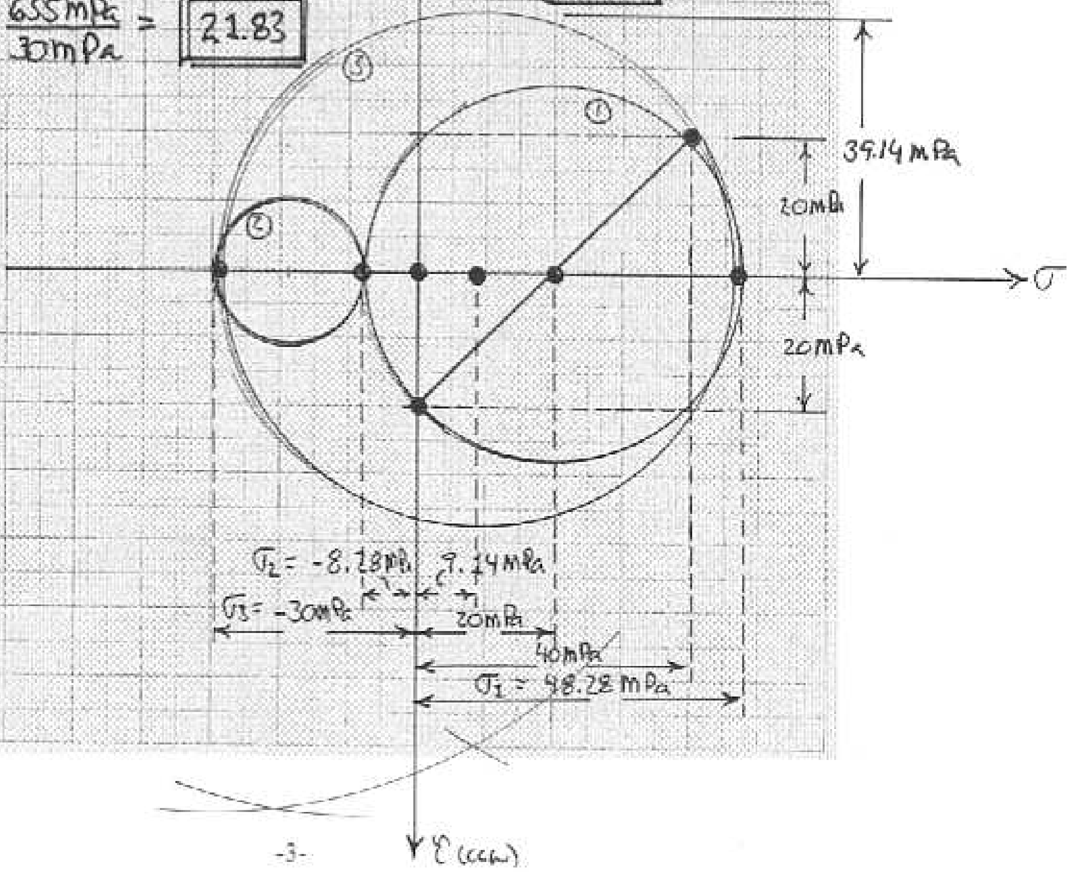
CAST IRON

$n_{CI}^{T} = \frac{170\,MPa}{48.28\,MPa} = \boxed{3.52}$

$\frac{1}{n_{CI}^{mc}} = \frac{48.28\,MPa}{170\,MPa} - \frac{(-30\,MPa)}{655\,MPa} = 0.3298$

$n_{CI}^{mc} = \boxed{3.03}$

$n_{CI}^{C} = \frac{655\,MPa}{30\,MPa} = \boxed{21.83}$



PROBLEM 2: A 34 foot long hollow circular column is constructed out of structural steel with a yield strength of σ_y=30 ksi and a modulus of elasticity of E=30 Msi. The cross-sectional properties of the column are A=8.640 in^2 and I=32.94 in^4.

2a (10pts). Given both ends are pinned, what is the critical buckling load?

$$P_{CR} = \frac{\pi^2 \cdot E \cdot I}{K^2 \cdot L^2} = \frac{\pi^2 \cdot (30\times10^6 \frac{lb}{in^2}) \cdot (32.94\, in^4)}{(1.0)^2 \cdot \left(34\, ft \cdot \frac{12\, in}{1\, ft}\right)^2} = 58.59\times10^3\ lb$$

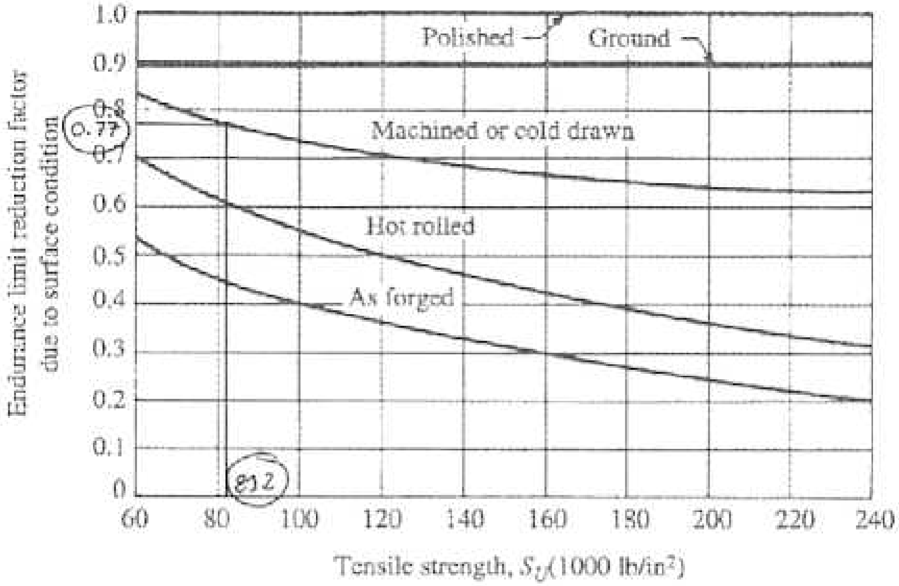
$$= \boxed{58.59\ kips}$$

2b (10pts). Given both ends are fixed, what is the critical buckling load?

$$P_{CR} = \frac{\pi^2 \cdot E \cdot I}{K^2 \cdot L^2} = \frac{\pi^2 \cdot (30 \times 10^6 \frac{lb}{in^2}) \cdot (32.94 in^4)}{(0.5)^2 \cdot (34 ft \cdot \frac{12 in}{ft})^2} = 234.4 \times 10^3 \text{ lb}$$

$$= \boxed{234.4 \text{ kips}}$$

PROBLEM 3: A mechanical part is made of machined steel with the properties S_u=560 MPa (81.2 ksi) and S_y=490 MPa (71.1ksi).



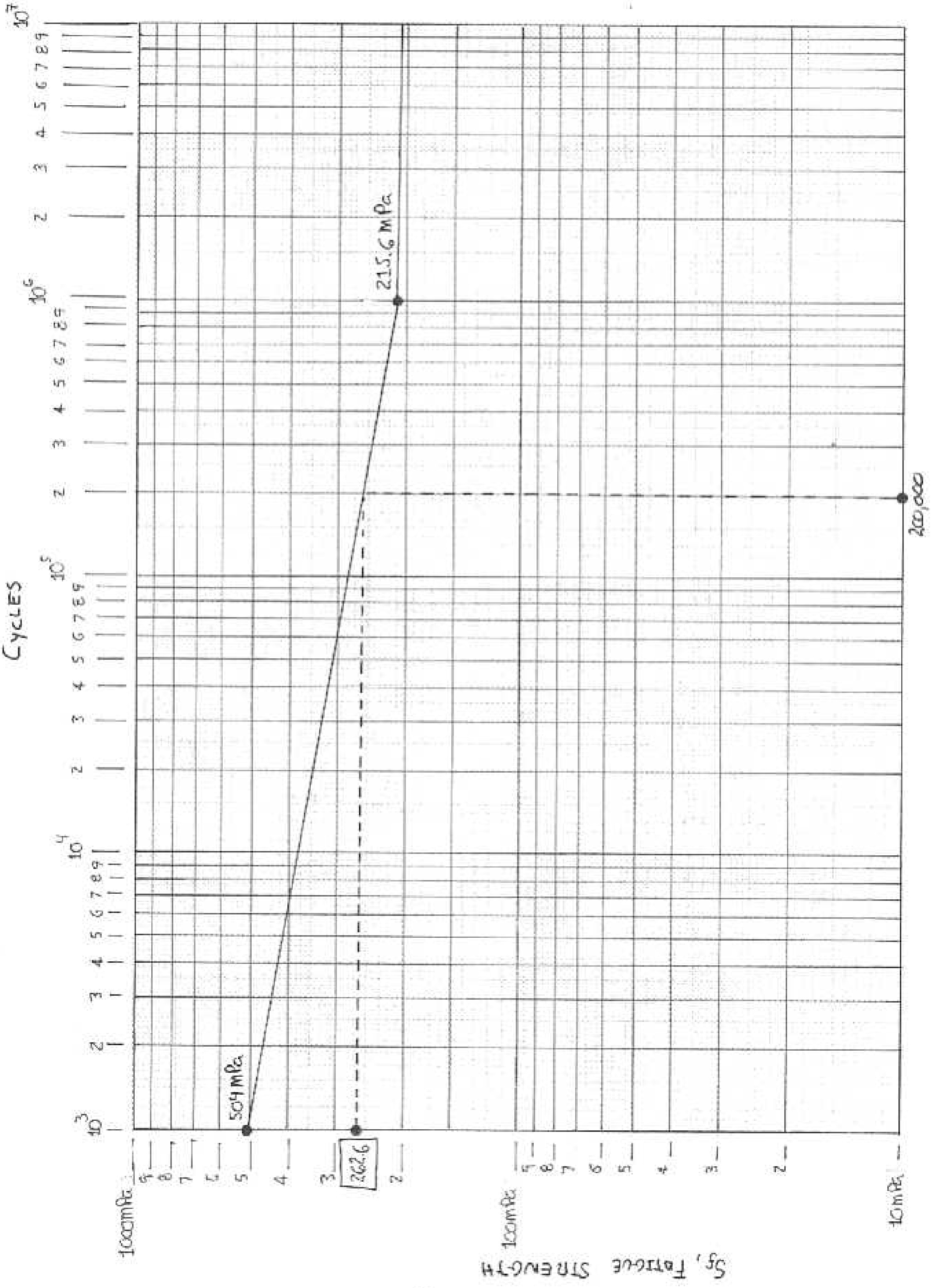
3a (10pts). Draw the S-N diagram for this material on the paper provided on the next page. Make sure to label the axes and all important points with values.

THERE ARE TWO CRITICAL POINTS ON THE S-N DIAGRAM

10^3 CYCLES: $0.9 \cdot S_{UT} = 0.9 \cdot (560 \text{ MPa}) = \underline{504 \text{ MPa}}$

10^6 CYCLES: $S_e = k_a \cdot 0.5 \cdot S_{UT} = (0.77) \cdot \underbrace{(0.5) \cdot (560 \text{ MPa})}_{S_e \text{ ESTIMATE FOR STEEL}}$

$= \underline{215.6 \text{ MPa}}$


Cycles
215.6 MPa
200,000
504 MPa
262.6
1000 MPa
100 MPa
10 MPa
Sf, Fatigue Strength

3b (10 points). What is the equation of the S-N diagrams line? What is the slope of the line? What is the lines intercept?

THE EQUATION OF THE S-N DIAGRAM IS GIVEN BY

$$\log S_f' = -m \cdot \log N + b$$

$$m = \frac{1}{3} \log \frac{0.9 \cdot S_{UT}}{S_e} = \frac{1}{3} \log \frac{0.9 \cdot (560\,MPa)}{215.6\,MPa} = \boxed{0.12293 = m}$$

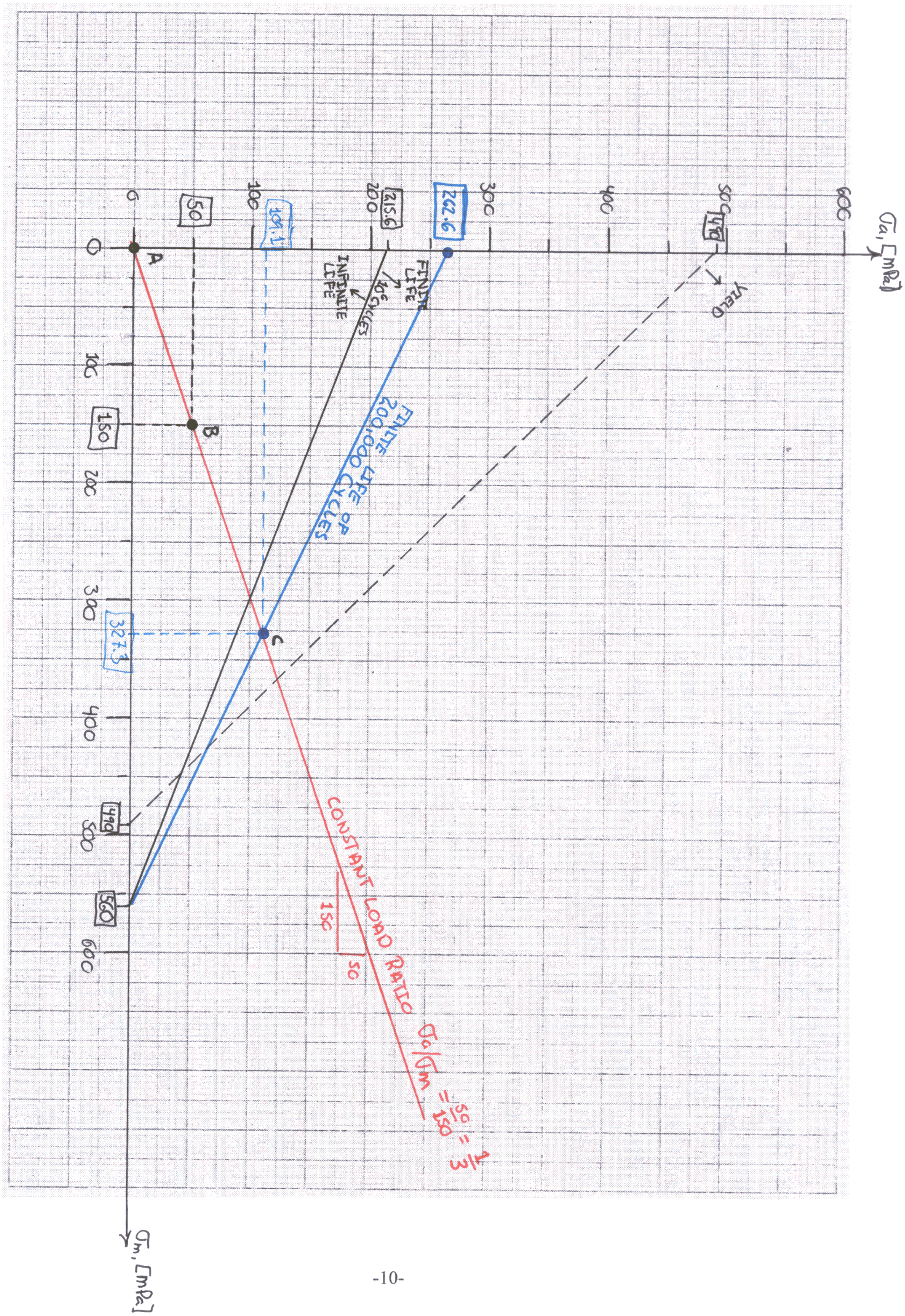
$$b = \log \frac{(0.9 \cdot S_{UT})^2}{S_e} = \log \frac{[0.9 \cdot (560\,MPa)]^2}{215.6\,MPa} = \boxed{3.071 \log(MPa) = b}$$

$$\boxed{\log S_f = -0.12293 \cdot \log N + 3.071 \log(MPa)}$$

3c(10pts). The part is subjected to a bending stress that alternates between 100 MPa and 200 MPa. Using the paper provided on the next page draw the Modified Goodman diagram for this material and illustrate the location of the state of stress under consideration. Make sure to label all important values on the diagram.

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{200\,mPa + 100\,mPa}{2} = 150\,mPa$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{200\,mPa - 100\,mPa}{2} = 50\,mPa$$


σa, [MPa]
σm, [MPa]
A
B
C
50
150
215.6
262.6
109.1
327.3
490
560
FINITE LIFE
INFINITE LIFE
10^6 CYCLES
FINITE LIFE OF 200,000 CYCLES
YIELD
CONSTANT LOAD RATIO
150
50
σa/σm = 50/150 = 1/3

3d (10 pts). Add to the Modified Goodman diagram a line that corresponds to the finite life of 200,000 cycles.

USING THE PARAMETERS PREVIOUSLY CALCULATED IN 3b, THE FATIGUE STRENGTH FOR A SPECIMEN SUBJECTED TO REVERSE LOADING (ZERO MEAN) THAT CORRESPONDS TO 200,000 CYCLES IS.

$$S_f = \frac{10^b}{N^m} = \frac{10^{3.071 \log(MPa)}}{(2\times 10^5)^{0.12293}} = \frac{10^{3.071}\ 10^{\log(MPa)}}{(2\times10^5)^{0.12293}} = \frac{10^{3.071}}{(2\times10^5)^{0.12293}} MPa$$

$$= \underline{\underline{262.6 \ MPa}}$$

THIS VALUE ALONG WITH THE CORRESPONDING 200,000 CYCLES IS PLOTTED ON THE S-N (LOG-LOG) DIAGRAM THAT WAS PART OF 3a ON PAGE 7.

THIS VALUE IS ALSO PLOTTED ON THE VERTICAL AXIS (ORDINATE), WHERE THE HORIZONTAL AXIS (ABSCISSA) IS ZERO. A BLUE LINE IS EXTENDED FROM THIS POINT TO S_{UT} (560 MPa) ON THE HORIZONTAL AXIS (ABSCISSA) TO FORM THE LINE THAT REPRESENTS THE FINITE LIFE OF 200,000 CYCLES.

3e (10 pts). Evaluate the factor of safety corresponding to a finite life of 200,000 cycles.

FROM THE DIAGRAM ON PAGE 10 IT IS CLEAR THAT YIELDING IS NOT A CONSIDERATION FOR THE LOAD RATIO, AT THIS FINITE LIFE.

THE LOCATION OF POINT C MUST BE FOUND. TO DO THIS THE EQUATIONS FOR THE LOAD RATIO AND FINITE LIFE CURVE CORRESPONDING TO 200,000 CYCLES MUST BE FOUND.

LOAD RATIO CURVE

$$\sigma_{a,LR} = \frac{1}{3}\cdot\sigma_{m,LR} \quad (1)$$

FINITE LIFE CURVE

$$\sigma_{a,FL} = \frac{-262.6\text{ MPa}}{560\text{ MPa}}\cdot\sigma_{m,FL} + 262.6\text{ MPa} = -0.4689\cdot\sigma_{m,FL} + 262.6\text{ MPa} \quad (2)$$

POINT C IS LOCATED WHERE $\sigma_{a,LR} = \sigma_{a,FL} = \sigma_a$ AND $\sigma_{m,LR} = \sigma_{m,FL} = \sigma_m$

$$\frac{1}{3}\cdot\sigma_m = -0.4689\cdot\sigma_m + 262.6\text{ MPa}$$

$$\sigma_m = \frac{262.6\text{ MPa}}{\frac{1}{3}+0.4689} = 327.3\text{ MPa}$$

$$(1)\rightarrow \sigma_a = \frac{1}{3}(327.3\text{ MPa}) = 109.1\text{ MPa} \checkmark$$

$$(2)\rightarrow \sigma_a = -0.4689(327.3\text{ MPa}) + 262.6\text{ MPa} = 109.1\text{ MPa} \checkmark$$

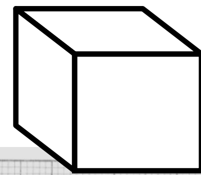
NOW THE FACTOR OF SAFETY CAN BE CALCULATED

$$n = \frac{\sqrt{(109.1\text{ MPa})^2+(327.3\text{ MPa})^2}}{\sqrt{(50\text{ MPa})^2+(150\text{ MPa})^2}} = \frac{345.0\text{ MPa}}{158.11\text{ MPa}} = \boxed{2.182}$$

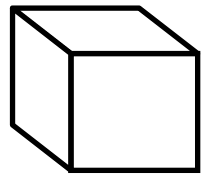
NAME: ____________________

PROBLEM 1: For each of the stress states listed below evaluate the failure of the material for the following two materials: Steel S_{ys}=250MPa and Cast Iron S^{t}_{uci}=170MPa/ S^{c}_{uci}=655MPa. Determine if failure occurs and if it does not, determine the safety factor. Be sure to state the criteria you are using for failure.

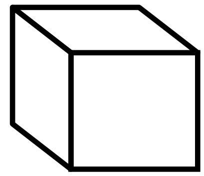
1a (10pts). $\sigma = \begin{bmatrix} -80 & -20 & 0 \\ -20 & -30 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$



1b (10pts). $\sigma = \begin{bmatrix} 30 & -30 & 0 \\ -30 & -60 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$



1c (10pts). $\sigma = \begin{bmatrix} 40 & -20 & 0 \\ -20 & 0 & 0 \\ 0 & 0 & -30 \end{bmatrix} MPa$

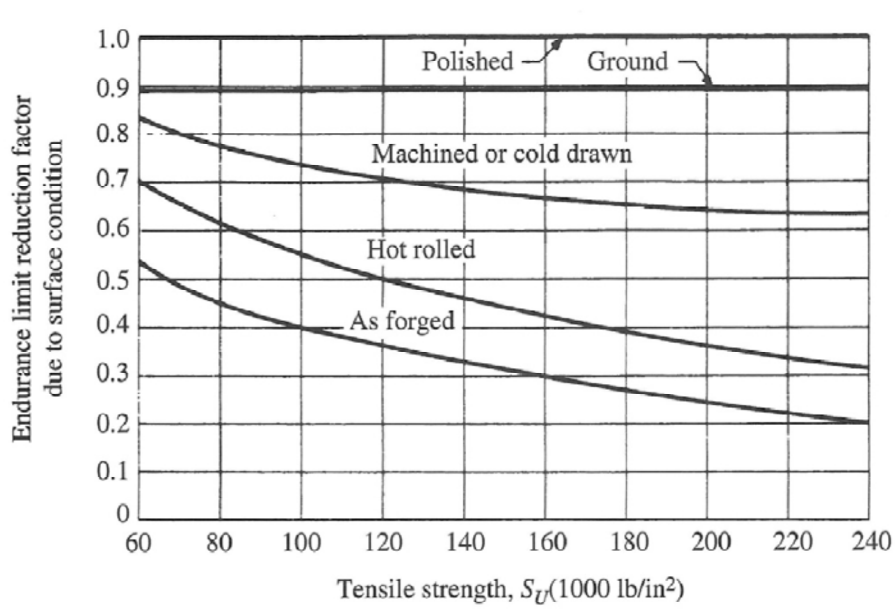


PROBLEM 2: A 34 foot long hollow circular column is constructed out of structural steel with a yield strength of σ_y=30 ksi and a modulus of elasticity of E=30 Msi. The cross-sectional properties of the column are A=8.640 in^2 and I=32.94 in^4.

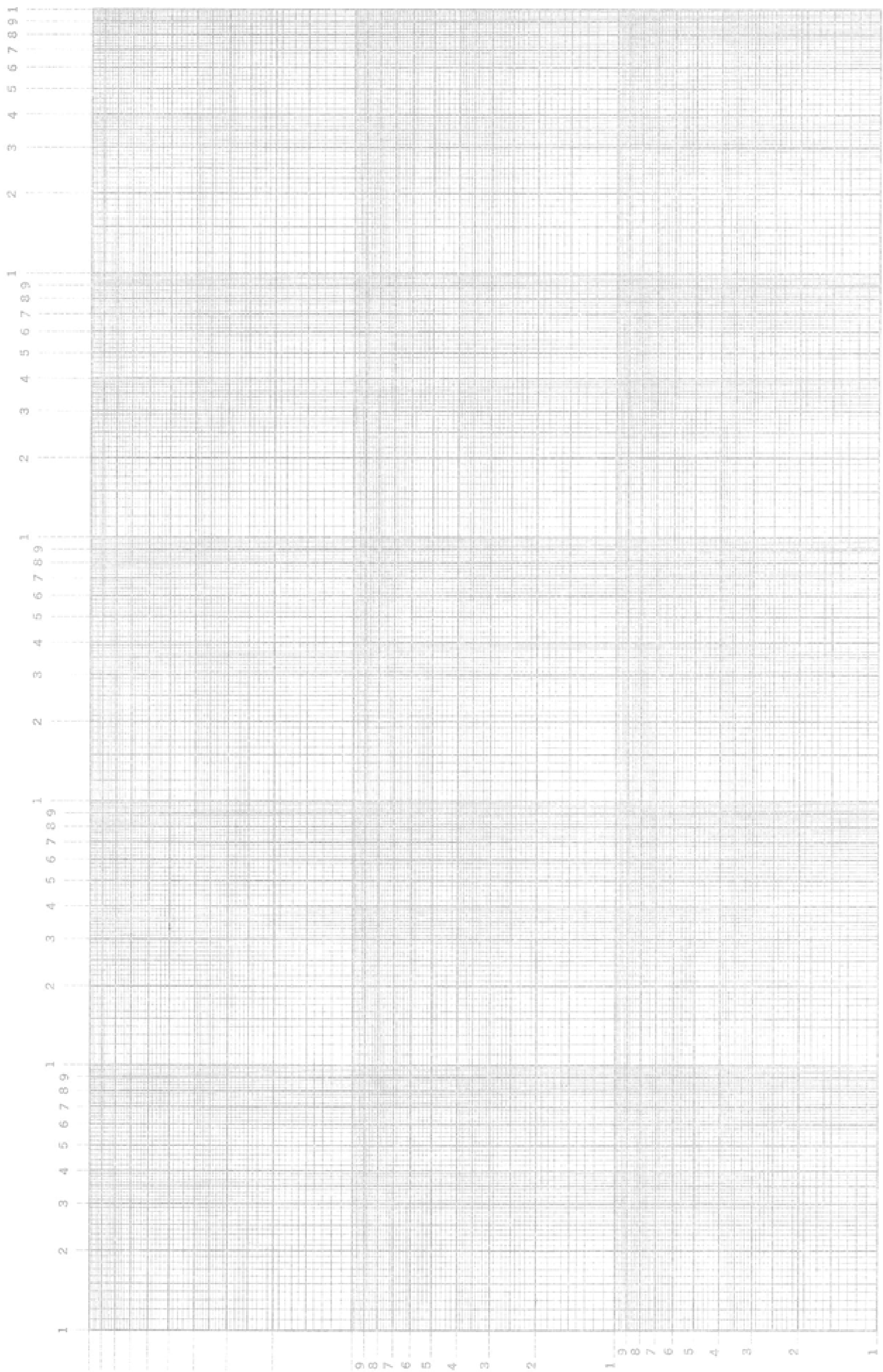
2a (10pts). Given both ends are pinned, what is the critical buckling load?

2b (10pts). Given both ends are fixed, what is the critical buckling load?

PROBLEM 3: A mechanical part is made of machined steel with the properties S_u=560 MPa (81.2 ksi) and S_y=490 MPa (71.1ksi).



3a (10pts). Draw the S-N diagram for this material on the paper provided on the next page. Make sure to label the axes and all important points with values.



3b (10 points). What is the equation of the S-N diagrams line? What is the slope of the line? What is the lines intercept?

3c(10pts). The part is subjected to a bending stress that alternates between 100 MPa and 200 MPa. Using the paper provided on the next page draw the Modified Goodman diagram for this material and illustrate the location of the state of stress under consideration. Make sure to label all important values on the diagram.

3d (10 pts). Add to the Modified Goodman diagram a line that corresponds to the finite life of 200,000 cycles.

3e (10 pts). Evaluate the factor of safety corresponding to a finite life of 200,000 cycles.