PROBLEM 8-8 DESIGN A DOUBLE-DWELL CAM TO MOVE A FOLLOWER FROM 0 TO 1.5" IN 45°, DWELL FOR 150°, FALL 1.5" IN 90°, AND DWELL FOR THE REMAINDER. THE TOTAL CYCLE MUST TAKE 6 S. CHOOSE SUITABLE FUNCTIONS FOR RISE AND FALL TO MINIMIZE VELOCITIES. PLOT THE S-V-A-J DIAGRAMS.

GIVEN:

1. THE SEGMENT REQUIREMENTS FOR THIS PROBLEM ARE AS FOLLOWS

SEGMENT	INTERVAL LENGTH	DESCRIPTION
1	$\beta_1 = 45° = \pi/4$ rad	RISE, $h_1 = 1.5$ in
2	$\beta_2 = 150° = 5\pi/6$ rad	DWELL, $h_2 = 0$
3	$\beta_3 = 90° = \pi/2$ rad	FALL, $h_3 = -1.5$ in
4	$\beta_4 = 75° = 5\pi/12$ rad	DWELL, $h_4 = 0$

2. TOTAL CYCLE TIME 6 SEC.

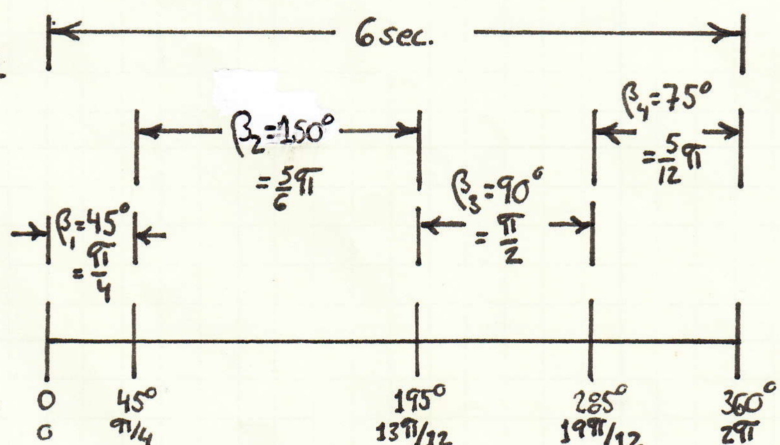
ASSUMPTIONS:

1. THE CAM FUNCTION MUST BE CONTINUOUS THROUGH THE FIRST AND SECOND DERIVATIVES
2. THERE IS THIRD ORDER CONTINUITY AT ALL BOUNDARIES

FIND:

1. DESCRIPTION OF THE FUNCTIONS THAT WILL ACHIEVE THE SEGMENT REQUIREMENTS AND MINIMIZE VELOCITIES
2. DRAW THE POSITION (s) - VELOCITY (v) - ACCELERATION (a) - JERK (j) DIAGRAMS.

FIGURE:



BOUNDARY CONDITIONS	0 (0)	45° ($\pi/4$)	195° ($13\pi/12$)	285° ($19\pi/12$)	360° (2π)
h =	0	1.5 in = h	1.5 in = h	0	0
v =	0	0	0	0	0
a =	0	0	0	0	0
j =	–	–	–	–	–

SOLUTION:

GIVEN THAT THE TOTAL CYCLE TIME IS 6 sec. AND A COMPLETE CYCLE IS ONE REVOLUTION OF THE CAM (OR 2π RADIANS), THE CONSTANT ANGULAR VELOCITY OF THE CAM IS CALCULATED AS

$$\omega = \frac{2\pi \text{ rad}}{6 \text{ sec.}} = \frac{\pi}{3} \frac{\text{rad}}{\text{sec}} \qquad (1)$$

IN THIS PROBLEM THERE ARE FOUR SEGMENTS OF THE CAM THAT MUST BE DESIGNED. EACH SEGMENT WILL HAVE TO BE DESIGNED WITH A UNIQUE FUNCTION THAT SATISFIES THE BOUNDARY CONDITIONS AT THE START AND END OF THE SEGMENT.

SEGMENT 1: $\beta_1 = 45°$, $0 \leq \theta \leq 45°$, $0 \leq \theta_1 \leq 45°$

GIVEN THE REQUIREMENT THAT THE VELOCITY NEEDS TO BE MINIMIZED AND THAT THE FUNCTION DESCRIBING THE PATH OF THE FOLLOWER MUST BE CONTINUOUS THROUGH TWO DERIVATIVES. FOR THE BOUNDARY CONDITIONS SPECIFIED THE FUNCTION MUST ALLOW THE ACCELERATION TO BE ZERO AT THE START AND END OF THE SEGMENT. CONSIDER

$$a_1 = C_1 \cdot \sin\left(\frac{2\pi}{\beta_1} \cdot \theta_1\right) \qquad (2)$$

THE ACCELERATION BOUNDARY CONDITIONS FOR THIS SEGMENT ARE $a_1(0) = 0$ AND $a_1(\beta_1) = 0$. SUBSTITUTING THESE INTO (2)

$$a_1(0) = 0 = C_1 \cdot \sin\left(\frac{2\pi}{\beta_1} \cdot 0\right) = 0 \checkmark$$

$$a_1(\beta_1) = 0 = C_1 \cdot \sin\left(\frac{2\pi}{\beta_1} \cdot \beta_1\right) = C_1 \cdot \sin(2\pi) = 0 \checkmark$$

(2) SATISFIES BOTH BOUNDARY CONDITIONS.

THE NEXT SET OF BOUNDARY CONDITIONS THAT NEED TO BE CONSIDERED ARE WITH RESPECT TO VELOCITY. (2) NEEDS TO BE INTEGRATED IN ORDER TO DEVELOP AN EXPRESSION FOR VELOCITY.

$$a_1 = \frac{dv_1}{d\theta_1} \Rightarrow v_1 = \int a_1 \cdot d\theta_1 = \int C_1 \cdot \sin\left(\frac{2\pi}{\beta_1} \cdot \theta_1\right) d\theta$$

$$v_1 = -C_1 \cdot \frac{\beta_1}{2\pi} \cdot \cos\left(\frac{2\pi}{\beta_1} \cdot \theta_1\right) + C_2 = C_2 - C_1 \frac{\beta_1}{2\pi} \cdot \cos\left(\frac{2\pi}{\beta_1} \cdot \theta_1\right) \qquad (3)$$

THE VELOCITY BOUNDARY CONDITIONS FOR THIS SEGMENT ARE $v_1(0) = 0$ AND $v_1(\beta_1) = v_1(\pi/4) = 0$. SUBSTITUTING THESE BOUNDARY CONDITIONS INTO (3)

$$v_1(0) = 0 = C_2 - C_1 \cdot \frac{\beta_1}{2\pi} \cdot \overbrace{\cos(0)}^{1} \Rightarrow C_2 = C_1 \cdot \frac{\beta_1}{2\pi}$$

$$v_1 = C_1 \cdot \frac{\beta_1}{2\pi} - C_1 \frac{\beta_1}{2\pi} \cdot \cos\left(\frac{2\pi}{\beta_1} \cdot \theta_1\right) = C_1 \cdot \frac{\beta_1}{2\pi}\left[1 - \cos\left(\frac{2\pi}{\beta_1} \cdot \theta_1\right)\right]$$

$$v_1(\beta_1) = v(\pi/4) = 0 = C_1 \cdot \frac{\beta_1}{2\pi}\left[1 - \cos\left(\frac{2\pi}{\beta_1}\cdot \beta_1\right)\right] = C_1 \cdot \frac{\beta_1}{2\pi}\left[1 - \underbrace{\cos(2\pi)}_{1}\right] = 0 \checkmark$$

BOTH VELOCITY BOUNDARY CONDITIONS ARE SATISFIED; HOWEVER, THE SECOND BOUNDARY CONDITION DID NOT ASSIST IN THE DETERMINATION OF C_1. THE FORM OF THE VELOCITY IN THIS SEGMENT IS

$$v_1 = C_1 \cdot \frac{\beta_1}{2\pi}\left[1 - \cos\left(\frac{2\pi}{\beta_1}\cdot \theta_1\right)\right] \quad (4)$$

THE NEXT SET OF BOUNDARY CONDITIONS THAT NEED TO BE CONSIDERED RELATE TO THE DISPLACEMENT OF THE FOLLOWER. THE DISPLACEMENT EQUATION IS DEVELOPED BY INTEGRATING ~~EQ~~ (4) WITH RESPECT TO θ_1

$$v_1 = \frac{dS_1}{d\theta_1} \Rightarrow S_1 = \int v_1 \cdot d\theta_1 = \int C_1 \cdot \frac{\beta_1}{2\pi}\left[1 - \cos\left(\frac{2\pi}{\beta_1}\cdot \theta_1\right)\right] d\theta_1$$

$$S_1 = C_1 \cdot \frac{\beta_1}{2\pi}\left[\theta_1 - \frac{\beta_1}{2\pi}\cdot \sin\left(\frac{2\pi}{\beta_1}\cdot \theta_1\right)\right] + C_3 \quad (5)$$

THE DISPLACEMENT BOUNDARY CONDITIONS IN THIS REGION ARE $S(0) = 0\,in$ AND $S(\beta_1) = S(\pi/4) = 1.5\,in = h$. APPLYING THESE TO (5)

$$S_1(0) = 0\,in = C_1 \cdot \frac{\beta_1}{2\pi}\left[0 - \frac{\beta_1}{2\pi}\cdot \sin\left(\frac{2\pi}{\beta_1}\cdot 0\right)\right] + C_3$$

$$= C_1 \frac{\beta_1}{2\pi}\left[0 - \frac{\beta_1}{2\pi}(0)\right] + C_3 \Rightarrow \underline{\underline{C_3 = 0}}$$

$$S_1(\beta_1) = S_1(\tfrac{\pi}{4}) = 1.5\,in = h = C_1 \cdot \frac{\beta_1}{2\pi}\cdot\left[\beta_1 - \frac{\beta_1}{2\pi}\cdot \sin\left(\frac{2\pi}{\beta_1}\cdot \beta_1\right)\right]$$

$$h = C_1 \cdot \frac{\beta_1}{2\pi}\cdot\left[\beta_1 - \frac{\beta_1}{2\pi}(0)\right] = C_1 \cdot \frac{\beta_1^2}{2\pi}$$

$$\Rightarrow \underline{\underline{C_1 = \frac{h \cdot 2\pi}{\beta_1^2}}} \quad (6)$$

NOW (6) CAN BE BACK SUBSTITUTED INTO (2), (4), & (5)

$$(5) \rightarrow S_1 = \frac{h \cdot \cancel{2\pi}}{\beta_1^{\cancel{2}}}\cdot \frac{\cancel{\beta_1}}{\cancel{2\pi}}\left[\theta_1 - \frac{\beta_1}{2\pi}\cdot \sin\left(\frac{2\pi}{\beta_1}\cdot \theta_1\right)\right]$$

$$= \underline{\underline{h\left[\frac{\theta_1}{\beta_1} - \frac{1}{2\pi}\cdot \sin\left(\frac{2\pi}{\beta_1}\cdot \theta_1\right)\right]}}$$

$$= 1.5\,in\left[\frac{4\cdot\theta_1}{\pi} - \frac{1}{2\pi}\cdot \sin\left(\frac{2\cdot\pi}{\pi/4}\cdot \theta_1\right)\right] = \boxed{1.5\,in\left[\frac{4\cdot\theta_1}{\pi} - \frac{1}{2\pi}\cdot \sin(8\theta_1)\right]} \quad (7)$$

$$(4) \rightarrow v_1 = \frac{h \cdot \cancel{2\pi}}{\beta_1^{\cancel{2}}}\cdot \frac{\cancel{\beta_1}}{\cancel{2\pi}}\left[1 - \cos\left(\frac{2\pi}{\beta_1}\cdot \theta_1\right)\right] = \underline{\underline{\frac{h}{\beta_1}\left[1 - \cos\left(\frac{2\pi}{\beta_1}\cdot \theta_1\right)\right]}}$$

$$= \frac{4\cdot h}{\pi}\left[1 - \cos\left(\frac{2\pi}{\pi/4}\cdot \theta_1\right)\right] = \frac{4\cdot 1.5\,in}{\pi}\left[1 - \cos\left(\frac{2\pi}{\pi/4}\right)\cdot \theta_1\right]$$

$$= \frac{4\cdot h}{\pi}\left[1 - \cos(8\cdot\theta_1)\right] = \boxed{\frac{6\,in}{\pi}\left[1 - \cos(8\cdot\theta_1)\right]} \quad (8)$$

$$⑦ \rightarrow a_1 = \frac{h \cdot 2\pi}{\beta_1^2} \cdot \sin\left(\frac{2\pi}{\beta_1} \cdot \theta_1\right) = \frac{(1.5in) \cdot 2\pi}{\pi^2/16} \cdot \sin\left(\frac{2\pi}{\pi/4} \cdot \theta_1\right)$$

$$= \boxed{\frac{48\,in}{\pi} \cdot \sin(8 \cdot \theta_1)} \qquad ⑨$$

THE EXPRESSION FOR THE JERK IN THIS SECTION OF THE CAM IS DEVELOPED BY TAKING THE DERIVATIVE OF THE ACCELERATION WITH RESPECT TO θ.

$$j_1 = \frac{da_1}{d\theta_1} = \frac{d}{d\theta_1}\left[\frac{2 \cdot \pi \cdot h}{\beta_1^2} \cdot \sin\left(\frac{2\pi}{\beta_1} \cdot \theta_1\right)\right]$$

$$= \frac{4 \cdot \pi^2 \cdot h}{\beta_1^3} \cdot \cos\left(\frac{2\pi}{\beta_1} \cdot \theta_1\right)$$

$$= \frac{4 \cdot \pi^2 \cdot 1.5in}{\pi^3/64} \cdot \cos\left(\frac{2\pi}{\pi/4} \cdot \theta_1\right) = \boxed{\frac{384\,in}{\pi} \cdot \cos(8 \cdot \theta_1)} \qquad ⑩$$

THE BOUNDARY CONDITIONS PREDICTED BY ⑦, ⑧, ⑨, & ⑩ NEED TO BE CHECKED TO INSURE THEY SATISFY THE DESIRED BOUNDARY CONDITIONS

$$⑦ \rightarrow s_1(0) = 1.5in\left[\frac{4}{\pi} \cdot (0) - \frac{1}{2\pi}\sin(8 \cdot 0)\right] = 1.5[0 - 0] = 0 \checkmark$$

$$⑦ \rightarrow s_1(\pi/4) = 1.5in\left[\frac{4}{\pi}\left(\frac{\pi}{4}\right) - \frac{1}{2\pi}\sin\left(8 \cdot \frac{\pi}{4}\right)\right] = 1.5in\left[1 - \frac{1}{2\pi}\sin(2\pi)\right]$$

$$= 1.5in\left[1 - \frac{1}{2\pi}(0)\right] = 1.5in \checkmark$$

$$⑧ \rightarrow v_1(0) = \frac{6}{\pi}\frac{in}{rad}\left[1 - \cos(8 \cdot 0)\right] = \frac{6}{\pi}\frac{in}{rad}\left[1 - \cos(0)\right]$$

$$= \frac{6}{\pi}\frac{in}{rad}[1 - 1] = 0 \checkmark$$

$$⑧ \rightarrow v_1\left(\frac{\pi}{4}\right) = \frac{6}{\pi}\frac{in}{rad}\left[1 - \cos\left(8 \cdot \frac{\pi}{4}\right)\right] = \frac{6}{\pi}\frac{in}{rad}\left[1 - \cos(2\pi)\right]$$

$$= \frac{6}{\pi}\frac{in}{rad}[1 - 1] = 0 \checkmark$$

$$⑨ \rightarrow a_1(0) = \frac{48}{\pi}\frac{in}{rad^2} \cdot \sin(8 \cdot 0) = \frac{48}{\pi}\frac{in}{rad^2} \cdot \sin 0 = 0 \checkmark$$

$$⑨ \rightarrow a_1\left(\frac{\pi}{4}\right) = \frac{48}{\pi}\frac{in}{rad^2} \cdot \sin\left(8 \cdot \frac{\pi}{4}\right) = \frac{48}{\pi}\frac{in}{rad^2} \cdot \sin(2\pi) = 0 \checkmark$$

$$⑩ \rightarrow j_1(0) = \frac{384}{\pi}\frac{in}{rad^3} \cdot \cos(8 \cdot 0) = \frac{384}{\pi}\frac{in}{rad^3} = 122.2\ in/rad^3$$

$$⑩ \rightarrow j_1\left(\frac{\pi}{4}\right) = \frac{384}{\pi}\frac{in}{rad^3} \cdot \cos\left(8 \cdot \frac{\pi}{4}\right) = \frac{384}{\pi}\frac{in}{rad^3} \cdot \cos(2\pi) = \frac{384}{\pi}\ in/rad^3 = 122.2\ in/rad^3$$

SEGMENT 2: $\beta_2 = 150° = \frac{5}{6}\pi$, $45° \le \theta \le 195°$, $0 \le \theta_2 \le 150°$

THIS SEGMENT OF THE CAM IS A DWELL AT 1.5in, THEREFORE

$$S_2 = 1.5\,in$$

$$v_2 = 0\ in/rad$$

$$a_2 = 0\ in/rad^2$$

$$j_2 = 0\ in/rad^3$$

SEGMENT 3: $\beta_3 = 90° = \pi/2$, $195° \le \theta \le 285°$, $0 \le \theta_3 \le 90°$

IN THIS SEGMENT OF THE CAM THE FOLLOWER STARTS AT 1.5in AND ENDS GOING INTO A DWELL AT 0in. THIS IS THE OPPOSITE OF WHAT HAPPENED IN SEGMENT 1. THEREFORE, THE GENERAL EXPRESSION FOR THE DISPLACEMENT IN SEGMENT IS MODIFIED SLIGHTLY TO SATISFY THE BOUNDARY CONDITIONS THAT ARE REQUIRED FOR THIS SEGMENT.

$$S_3 = h\left[1 - \frac{\theta_3}{\beta_3} + \frac{1}{2\pi}\cdot\sin\left(\frac{2\pi}{\beta_3}\cdot\theta_3\right)\right]$$

$$= 1.5in\left[1 - \frac{2\cdot\theta_3}{\pi} + \frac{1}{2\pi}\cdot\sin\left(\frac{2\cdot\pi}{\pi/2}\cdot\theta_3\right)\right]$$

$$= 1.5in\left[1 - \frac{2\cdot\theta_3}{\pi} + \frac{1}{2\pi}\cdot\sin(4\cdot\theta_3)\right] \qquad (11)$$

TAKING THE DERIVATIVE OF S_3 WITH RESPECT TO θ_3 TO DEVELOP AN EXPRESSION FOR v_3

$$v_3 = \frac{dS_3}{d\theta_3} = \frac{d}{d\theta_3}\left\{h\left[1 - \frac{\theta_3}{\beta_3} + \frac{1}{2\pi}\cdot\sin\left(\frac{2\pi}{\beta_3}\cdot\theta_3\right)\right]\right\}$$

$$= h\left[-\frac{1}{\beta_3} + \frac{1}{\cancel{2\pi}}\cdot\frac{\cancel{2\pi}}{\beta_3}\cdot\cos\left(\frac{2\pi}{\beta_3}\cdot\theta_3\right)\right] = \frac{h}{\beta_3}\left[\cos\left(\frac{2\pi}{\beta_3}\cdot\theta_3\right) - 1\right]$$

$$= \frac{1.5}{\pi/2}\frac{in}{rad}\left[\cos\left(\frac{2\pi}{\pi/2}\cdot\theta_3\right) - 1\right] = \frac{3.0}{\pi}\frac{in}{rad}\left[\cos(4\cdot\theta_3) - 1\right] \qquad (12)$$

TAKING THE DERIVATIVE OF v_3 WITH RESPECT TO θ_3 TO DEVELOP AN EXPRESSION FOR a_3

$$a_3 = \frac{dv_3}{d\theta_3} = \frac{d}{d\theta_3}\left\{\frac{h}{\beta_3}\left[\cos\left(\frac{2\pi}{\beta_3}\cdot\theta_3\right) - 1\right]\right\}$$

$$= -\frac{h}{\beta_3}\cdot\frac{2\pi}{\beta_3}\cdot\sin\left(\frac{2\pi}{\beta_3}\cdot\theta_3\right) = -\frac{2\cdot\pi\cdot h}{\beta_3^2}\cdot\sin\left(\frac{2\pi}{\beta_3}\cdot\theta_3\right)$$

$$= -\frac{2\pi\cdot 1.5}{\pi^2/4}\frac{in}{rad^2}\cdot\sin\left(\frac{2\pi}{\pi/2}\cdot\theta_3\right) = -\frac{12}{\pi}\frac{in}{rad^2}\cdot\sin(4\theta_3) \qquad (13)$$

TAKING THE DERIVATIVE OF a_3 WITH RESPECT TO θ_3 TO DEVELOP AN EXPRESSION FOR j_3

$$j_3 = \frac{da_3}{d\theta_3} = \frac{d}{d\theta_3}\left\{-\frac{2\cdot\pi\cdot h}{\beta_3^2}\cdot\sin\left(\frac{2\pi}{\beta_3}\cdot\theta_3\right)\right\}$$

$$= -\frac{4\pi^2 h}{\beta_3^3}\cdot\cos\left(\frac{2\pi}{\beta_3}\cdot\theta_3\right)$$

$$= -\frac{4\pi^2\cdot 1.5}{\pi^3/8}\cdot\cos\left(\frac{2\pi}{\pi/2}\cdot\theta_3\right) = -\frac{48}{\pi}\ \frac{in}{rad^3}\cdot\cos(4\cdot\theta_3) \qquad (14)$$

(11), (12), (13), & (14) NEED TO BE CHECKED AT THE BOUNDARIES OF THIS SEGMENT TO ENSURE THE FUNCTION SATISFIES THE DESIRED BOUNDARY CONDITIONS.

$$(11) \rightarrow S_3(0) = 1.5in\left[1 - \frac{2\cdot 0}{\pi} + \frac{1}{2\pi}\cdot\sin(4\cdot 0)\right] = 1.5in \checkmark$$

$$(11) \rightarrow S_3\left(\frac{\pi}{2}\right) = 1.5in\left[1 - \frac{2\cdot\pi/2}{\pi} + \frac{1}{2\pi}\cdot\sin\left(4\cdot\frac{\pi}{2}\right)\right]$$

$$= 1.5in\left[1 - 1 - \frac{1}{2\pi}\cdot\sin(2\pi)\right] = 0 \checkmark$$

$$(12) \rightarrow v_3(0) = \frac{3.0}{\pi}\frac{in}{rad}\left[\cos(4\cdot 0) - 1\right] = \frac{3.0}{\pi}[1-1] = 0 \checkmark$$

$$(12) \rightarrow v\left(\frac{\pi}{2}\right) = \frac{3.0}{\pi}\frac{in}{rad}\left[\cos\left(4\cdot\frac{\pi}{2}\right) - 1\right] = \frac{3.0}{\pi}[1-1] = 0 \checkmark$$

$$(13) \rightarrow a_3(0) = -\frac{12}{\pi}\frac{in}{rad^2}\cdot\sin(4\cdot 0) = 0 \checkmark$$

$$(13) \rightarrow a_3\left(\frac{\pi}{2}\right) = -\frac{12}{\pi}\frac{in}{rad^2}\cdot\sin\left(4\cdot\frac{\pi}{2}\right) = -\frac{12}{\pi}\frac{in}{rad^2}\cdot\sin(2\pi) = 0 \checkmark$$

$$(14) \rightarrow j_3(0) = -\frac{48}{\pi}\frac{in}{rad^3}\cdot\cos(4\cdot 0) = -\frac{48}{\pi}\frac{in}{rad^3} = -15.28\frac{in}{rad^3}$$

$$(14) \rightarrow j_3\left(\frac{\pi}{2}\right) = -\frac{48}{\pi}\frac{in}{rad^3}\cdot\cos\left(4\cdot\frac{\pi}{2}\right) = -\frac{48}{\pi}\frac{in}{rad^3} = -15.28\frac{in}{rad^3}$$

SEGMENT 4: $\beta_4 = 75° = \frac{5}{12}\pi$, $285° \le \theta \le 360°$, $0 \le \theta_4 \le 75°$

$S_4 = 0\ in$

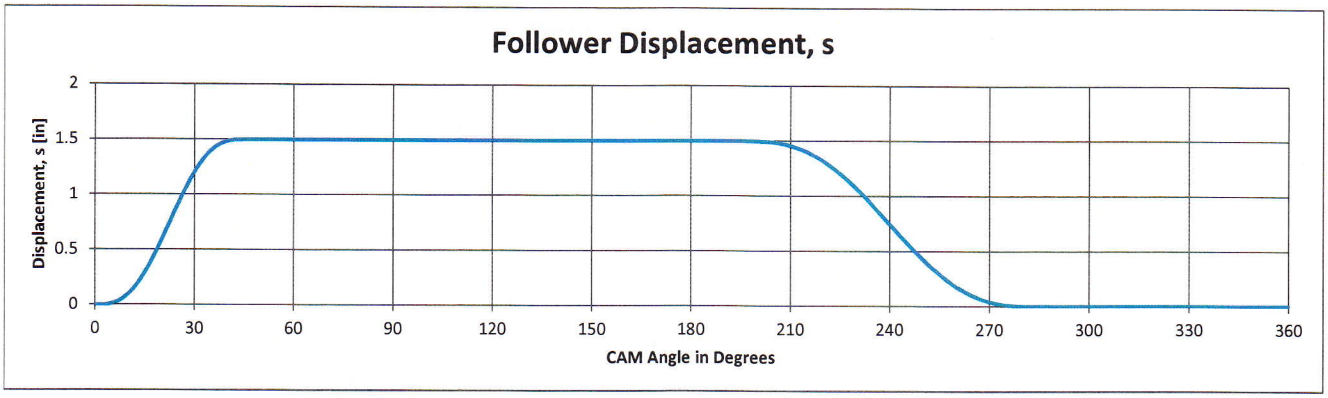
$v_4 = 0\ in/rad$

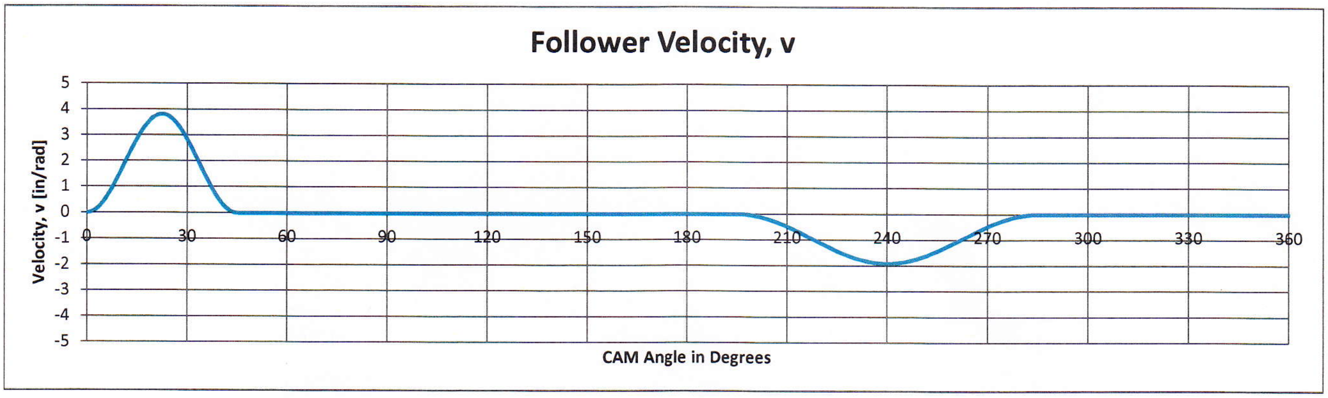
$a_4 = 0\ in/rad^2$

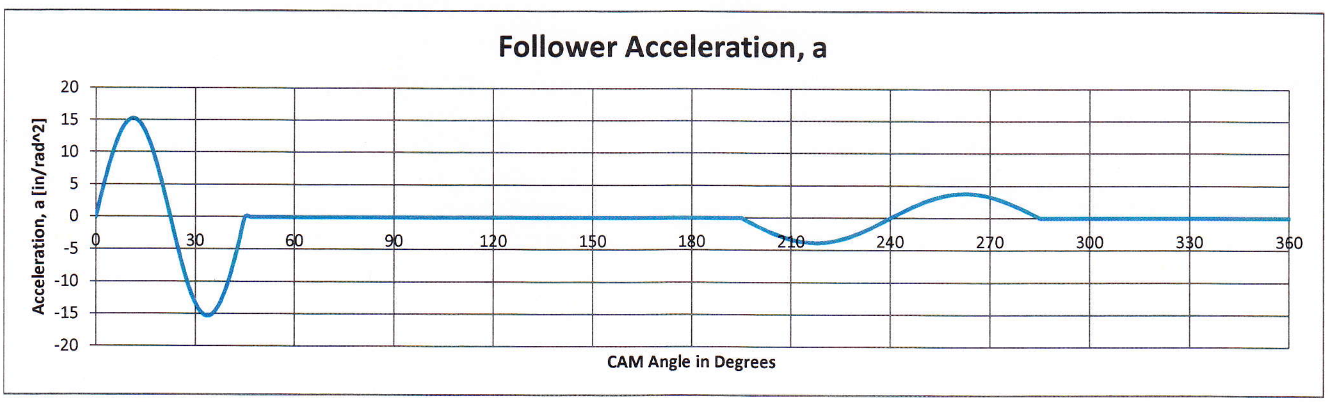
$j_4 = 0\ in/rad^3$

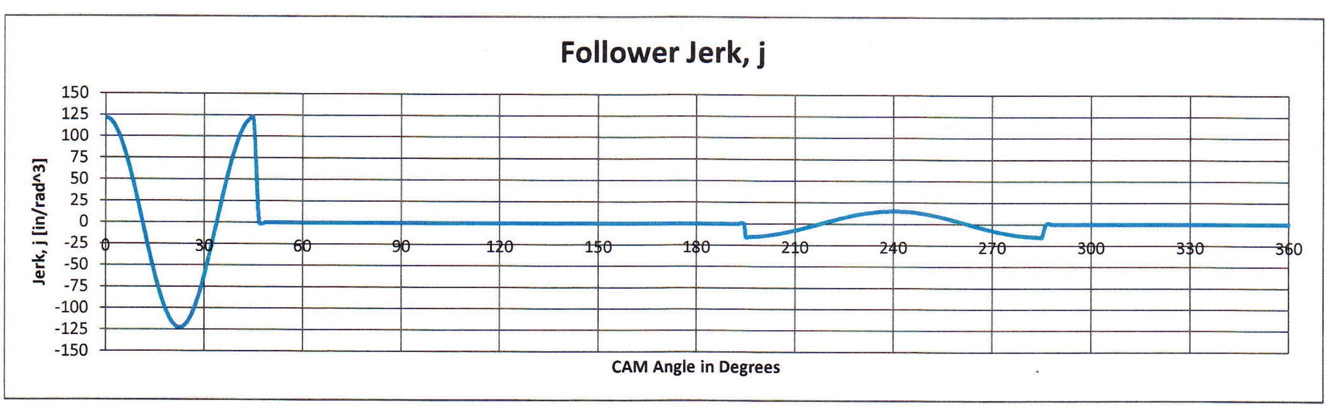
HOMEWORK SOLUTION
MER312: ADV. DYN & KIN

PROB 8-8 PG 7 OF 8
NORTON 5TH









<u>SUMMARY:</u>

THE CYCLOIDAL FUNCTION WAS CHOSEN FOR THIS CAM DESIGN BECAUSE OF THE REQUIREMENT ~~TH~~ THAT THE VELOCITY AND ACCELERATION COMMING OUT OF A DWELL AND GOING INTO A DWELL HAVE TO BE ZERO.

THE IS A SLIGHT MODIFICATION TO THE CYCLOIDAL FUNCTION IN SECTION 1 TO TURN IT INTO A FALL CYCLOIDAL FUNCTION THAT CAN BE USED IN SECTION 3.

THE s, v, a, & j DIAGRAMS WERE PLOTTED IN EXCEL USING THE FUNCTIONS DEVELOPED FOR EACH OF THE SECTIONS.