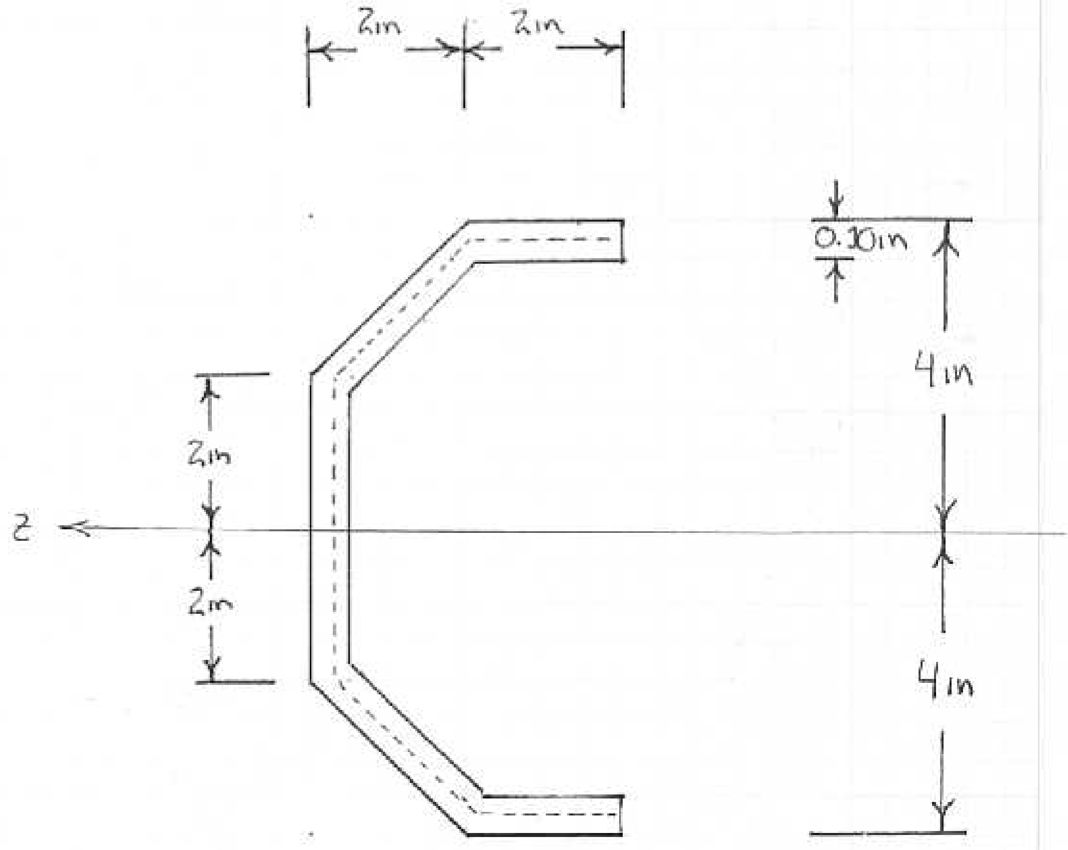
PROBLEM 5.33 | IF BENDING IS ABOUT THE z-AXIS FOR THE CROSS-SECTION SHOWN, DETERMINE THE LOCATION OF THE SHEAR CENTER FROM THE CENTER OF THE VERTICAL WALL. ALL DIMENSIONS ARE IN INCHES AND, WHERE APPROPRIATE, ARE FROM THE WALL CENTERS.

GIVEN:
1. MOMENT ABOUT THE z-AXIS
2. CROSS-SECTION SHOWN

ASSUMPTIONS:
1. SMALL DEFORMATIONS
2. LINEAR-ELASTIC RESPONSE

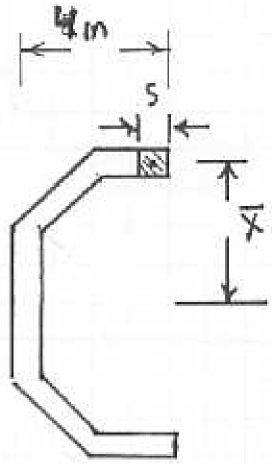
FIND:
1. LOCATION OF THE SHEAR CENTER.

FIGURES:



SOLUTION:

IN THE HORIZONTAL SECTION



$$\tau_1 = \frac{VQ}{It} = \frac{V}{It}\left[(4\text{in})\cdot s\cdot t\right]$$

$$= \frac{4\text{in}\cdot V}{I}\cdot s \qquad (1)$$

THE FORCE IN THE HORIZONTAL SECTION OF THE CROSS-SECTION CAN NOW BE CALCULATED

$$F_1 = \int \tau_1 \cdot t \cdot ds = \int_0^{2\text{in}} \frac{4\text{in}\cdot V}{I}\cdot s\cdot(0.1\text{in})\,ds$$

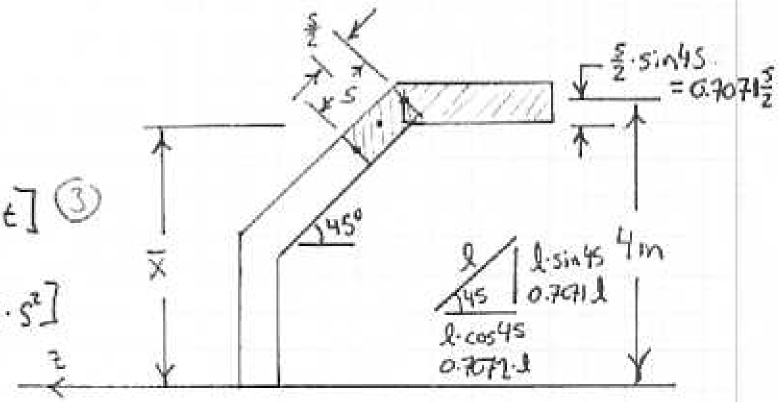
$$= \frac{V}{I}\cdot(0.4\text{in}^2)\cdot\frac{s^2}{2}\Big|_0^{2\text{in}} = \frac{V}{I}\cdot(0.2\text{in}^2)(2\text{in})^2 = \underline{0.8\text{in}^4\cdot\frac{V}{I}} \qquad (2)$$

IN THE DIAGONAL SECTION OF THE CROSS-SECTION

$$\tau_2 = \frac{VQ}{It} =$$

$$= \frac{V}{It}\cdot\left[(4\text{in})\cdot(2\text{in})\cdot t + \left(4\text{in} - \frac{s}{2}\cdot 0.7071\right)\cdot s\cdot t\right] \qquad (3)$$

$$= \frac{V}{I}\left[8\text{in}^2 + 4\text{in}\,s - 0.3536\cdot s^2\right]$$



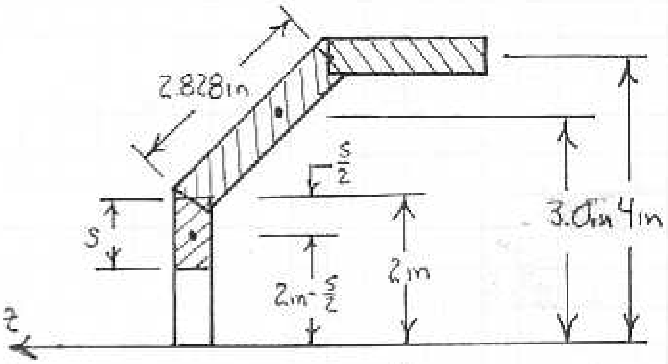
THE FORCE GENERATED IN THE DIAGONAL SECTION AS A RESULT OF THE SHEAR STRESS CAN NOW BE CALCULATED

$$F_2 = \int \tau\cdot t\cdot ds = \frac{V}{I}(0.1\text{in})\int_0^{2.828\text{ in}}\left[8\text{in}^2 + 4\text{in}\cdot s - 0.3536\,s^2\right]ds$$

$$= \frac{V}{I}\cdot(0.1\text{in})\left[8\text{in}^2(2.828\text{ in}) + \frac{4\text{in}}{2}\cdot(2.828\text{ in})^2 - \frac{0.3536}{3}(2.828\text{ in})^3\right]$$

$$= \underline{\underline{3.595\ \text{in}^4\cdot\frac{V}{I}}} \qquad (4)$$

IN THE HORIZONTAL SECTION OF THE CROSS-SECTION



$$\tau_3 = \frac{VQ}{It}$$

$$= \frac{V}{It}\left[(4\text{in})(2\text{in})\cdot t + (3.0\text{ in})(2.828\text{in})\cdot t + \left(2\text{in} - \frac{s}{2}\right)(s)\,t\right] = \frac{V}{I}\left[16.484\text{ in}^2 + 2\text{in}\cdot s - \frac{s^2}{2}\right] \qquad (5)$$

THE FORCE GENERATED IN THE VERTICAL LEG OF THE CROSS-SECTION CAN NOW BE CALCULATED.

$$F_3 = \int \tau \cdot t \cdot ds = \frac{V}{I}(0.1in)\int_0^{2in}\left[16.484in^2 + 2in \cdot s - \frac{s^2}{2}\right] ds$$

$$= (0.1in)\frac{V}{I}\left[16.484in^2 \cdot s + \frac{2in}{2}s^2 - \frac{s^3}{2 \cdot 3}\right]_0^{2in} = (0.1in)\frac{V}{I}\cdot\left[16.484in^2 \cdot 2in + 1in \cdot (2in)^2 - \frac{(2in)^3}{6}\right]$$

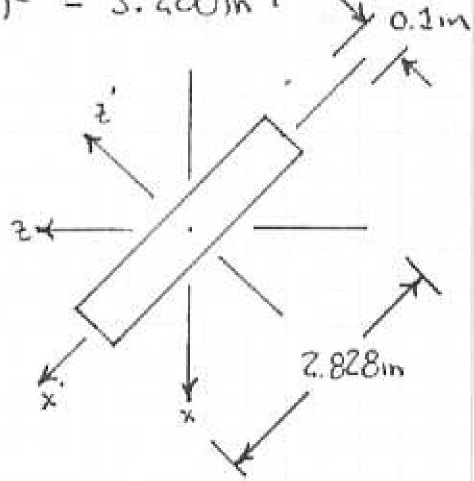
$$= 3.563in^4 \cdot \frac{V}{I} \quad (6)$$

EQUATIONS (2), (4), AND (6) WILL BE USED IN THE CALCULATION OF THE SHEAR CENTER AND EACH CONTAINS THE CROSS-SECTION MOMENT OF INERTIA. THE CROSS-SECTION MOMENT OF INERTIA MUST NOW BE CALCULATED.

$$I = 2 \cdot I_1 + 2 I_2 + I_3 \quad (7)$$

$$I_1 = \frac{1}{12}\cdot(2in)\cdot(0.1in)^3 + (2in)(0.1in)\cdot(4in)^2 = 3.200in^4$$

$$I_3 = \frac{1}{12}\cdot(0.1in)\cdot(4in)^3 = 0.5333in^4$$

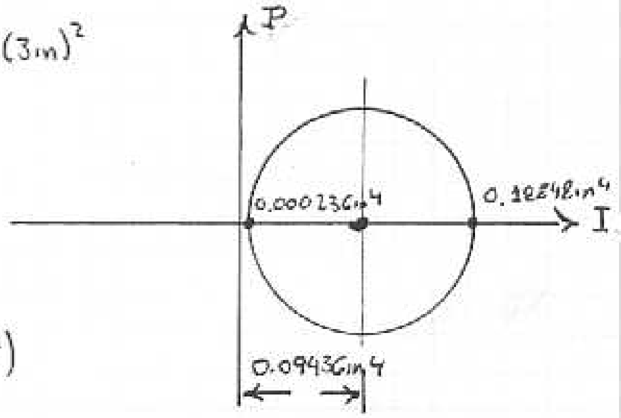


THE CALCULATION OF $I_3 = I_{3zz}$ STARTS WITH THE CALCULATION OF $I_{3z'z'}$ AND $I_{3x'x'}$

$$I_{3z'z'} = \frac{1}{12}\cdot(0.1in)(2.828in)^3 = 0.18848in^4$$

$$I_{3x'x'} = \frac{1}{12}(2.828in)(0.1in)^3 = 0.0002357in^4$$

TRANSFORMING THESE TO THE X-Z COORDINATE SYSTEM USING MOHR'S CIRCLE GIVES



$$I_{zz} = 0.09436in^4 + (0.1in)(2.828in)(3in)^2$$

$$= 2.640in^4 = I_2$$

THE TOTAL MOMENT OF INERTIA FOR THIS CROSS-SECTION CAN NOW BE CALCULATED USING (7)

$$I = 2\cdot(3.200in^4) + 2\cdot(2.640in^4) + 0.5333in^4$$

$$= 12.212in^4 \quad (8)$$

THE FORCES CAN NOW BE REWRITTEN

$$F_1 = \frac{0.8in^4}{12.212in^4}\cdot V = 0.06551 \cdot V$$

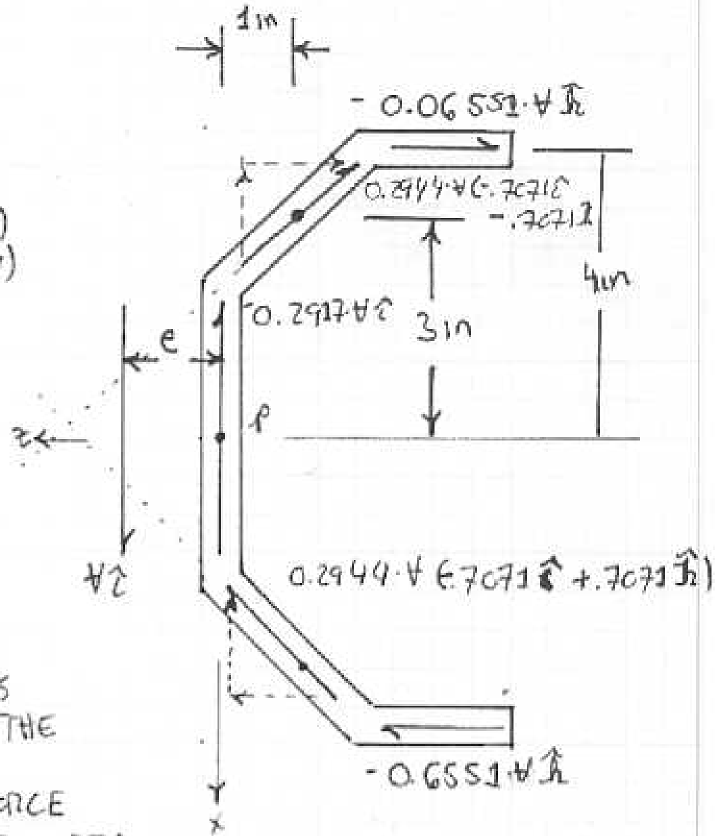
$$F_2 = \frac{3.595in^4}{12.212in^4}\cdot V = 0.2944 \cdot V$$

$$F_3 = \frac{3.563in^4}{12.212in^4}\cdot V = 0.2917 \cdot V$$

THE SHEAR CENTER IS NOW COMPUTED BY SUMMING MOMENTS ABOUT P.

$$\sum M_{y@P} = 0 = 2\cdot(4in)\cdot(0.06551\cdot V) - 2\cdot(3in)(.7071)(0.2944\cdot V) + 2\cdot(1in)(.7071)(0.2944\cdot V) + e\cdot V$$

$$\boxed{e = 1.357in}$$



SUMMARY

THE PROBLEM STARTS WITH THE DETERMINATION OF THE SHEAR STRESS DISTRIBUTION IN THE CROSS-SECTION. THE SHEAR STRESS IN EACH LEG IS THEN INTEGRATED TO DETERMINE THE FORCE IN THAT LEG. THE MOMENT OF INERTIA IS CALCULATED BECAUSE IT APPEARS IN EACH FORCE, BUT NOT IN THE SHEAR LOAD APPLIED TO THE OTHER END OF THE DIFFERENTIAL ELEMENT. FINALLY MOMENTS ARE SUMMED ABOUT P TO DETERMINE e.