As a student at Union College, I am part of a community that values intellectual effort, curiosity and discovery. I understand that in order to truly claim my educational and academic achievements, I am obligated to act with academic integrity. Therefore, I affirm that I carried out the work on this exam with full academic honesty, and I rely on my fellow students to do the same.

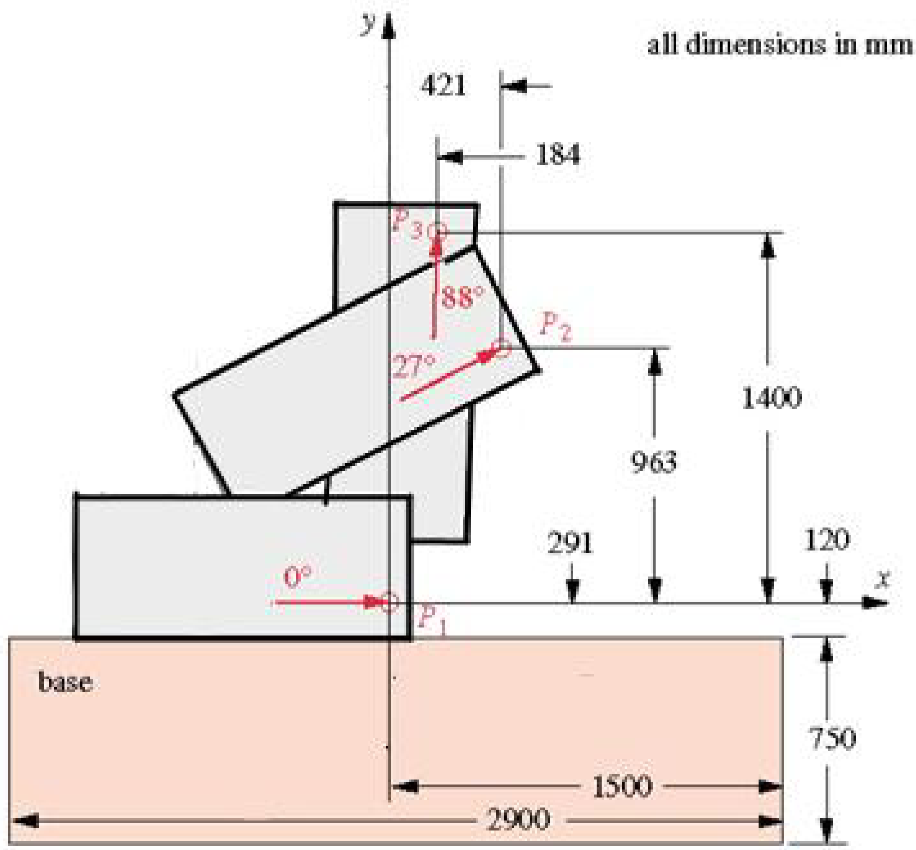
For this exam I understand that:

1. I **must** work alone in writing out the answers to this exam.
2. I **cannot** copy solutions to these problems from any person or resource.
3. I **cannot** use any electronic resources, other than the program I wrote as part of this class, to assist me in the solution to the questions on this exam.
4. I **can** use one formula sheet (both sides) that I prepared as a reference for this exam, I **must** staple this sheet to the back of my exam, and this sheet **cannot** contain any solutions to problems.

Signature: ______________________________

Print Name: Solution

PROBLEM 1 (30 pts): A box sits on top of the base shown in Position 1 and needs to be moved to Position 2, and then to Position 3. (Guess β_2=-50, β_3=-100, γ_2=-50, and γ_3=-80)



1a. Using the program that you developed in class, perform an analytical synthesis to design a linkage that will move the box from positions 1 to 2 to 3, and has ground pivots on the base. Show all work need to calculate the parameters used in your computer model below. Staple a copy of the computer solution directly after the next page of this exam.

From the figure inputs to the numerical analytical synthesis spreadsheet need to be calculated.

First and Second Dyad

$$\left(P_{1x}, P_{1y}\right) = \left(0\,mm, 0\,mm\right)$$

$$\left(P_{2x}, P_{2y}\right) = \left(421\,mm, 963\,mm\right)$$

$$\left(P_{3x}, P_{3y}\right) = \left(184\,mm, 1400\,mm\right)$$

$$p_{21} = \sqrt{\left(421\,mm - 0\,mm\right)^2 + \left(963\,mm - 0\,mm\right)^2} = \boxed{1051\,mm}$$

$$p_{31} = \sqrt{\left(184\,mm - 0\,mm\right)^2 + \left(1400\,mm - 0\,mm\right)^2} = \boxed{1412\,mm}$$

$$\delta_2 = \tan^{-1}\left(\frac{963\,mm}{421\,mm}\right) = \boxed{66.4^\circ}$$

$$\delta_3 = \tan^{-1}\left(\frac{1400\,mm}{184\,mm}\right) = \boxed{82.5^\circ}$$

$$\alpha_2 = \boxed{27^\circ}$$

$$\alpha_3 = \boxed{88^\circ}$$

Free Choices

$$\beta_2 = \boxed{-50^\circ}$$

$$\beta_3 = \boxed{-100^\circ}$$

$$\gamma_2 = \boxed{-50^\circ}$$

$$\gamma_3 = \boxed{-80^\circ}$$

FIRST DYAD

GIVEN:		CHOSEN:		FIND:				x-coord	y-coord.
P12	1051.00	$\beta 2$	-50.00	w	864.570		O2	-306.819	-402.654
P13	1412.00	$\beta 3$	-100.00	θ	155.216		A1	-1091.757	-40.228
$\delta 2$	66.40			z	1092.498		A2	-533.733	431.606
$\delta 3$	82.50			ϕ	2.110		A3	186.405	307.424
$\alpha 2$	27.00			W1x	-784.938		P1	0.000	0.000
$\alpha 3$	88.00			W1y	362.426		P2	420.767	963.097
				Z1x	1091.757		P3	184.303	1399.920
				Z1y	40.228				

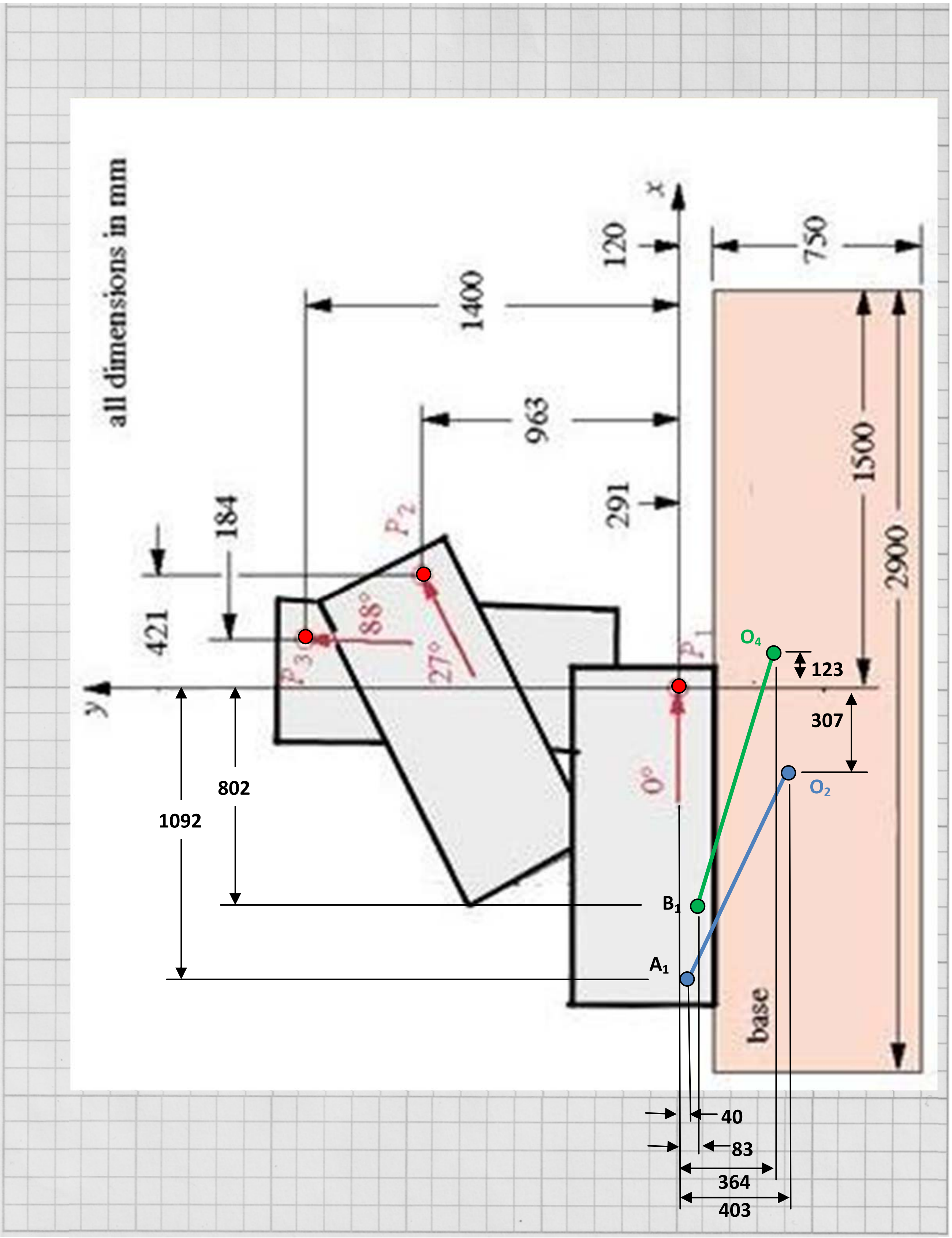
$$\begin{bmatrix} -0.3572 & 0.7660 & -0.1090 & -0.4540 \\ -0.7660 & -0.3572 & 0.4540 & -0.1090 \\ -1.1736 & 0.9848 & -0.9651 & -0.9994 \\ -0.9848 & -1.1736 & 0.9994 & -0.9651 \end{bmatrix} \begin{Bmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{Bmatrix} = \begin{Bmatrix} 420.7668 \\ 963.0972 \\ 184.3030 \\ 1399.9201 \end{Bmatrix}$$

SECOND DYAD

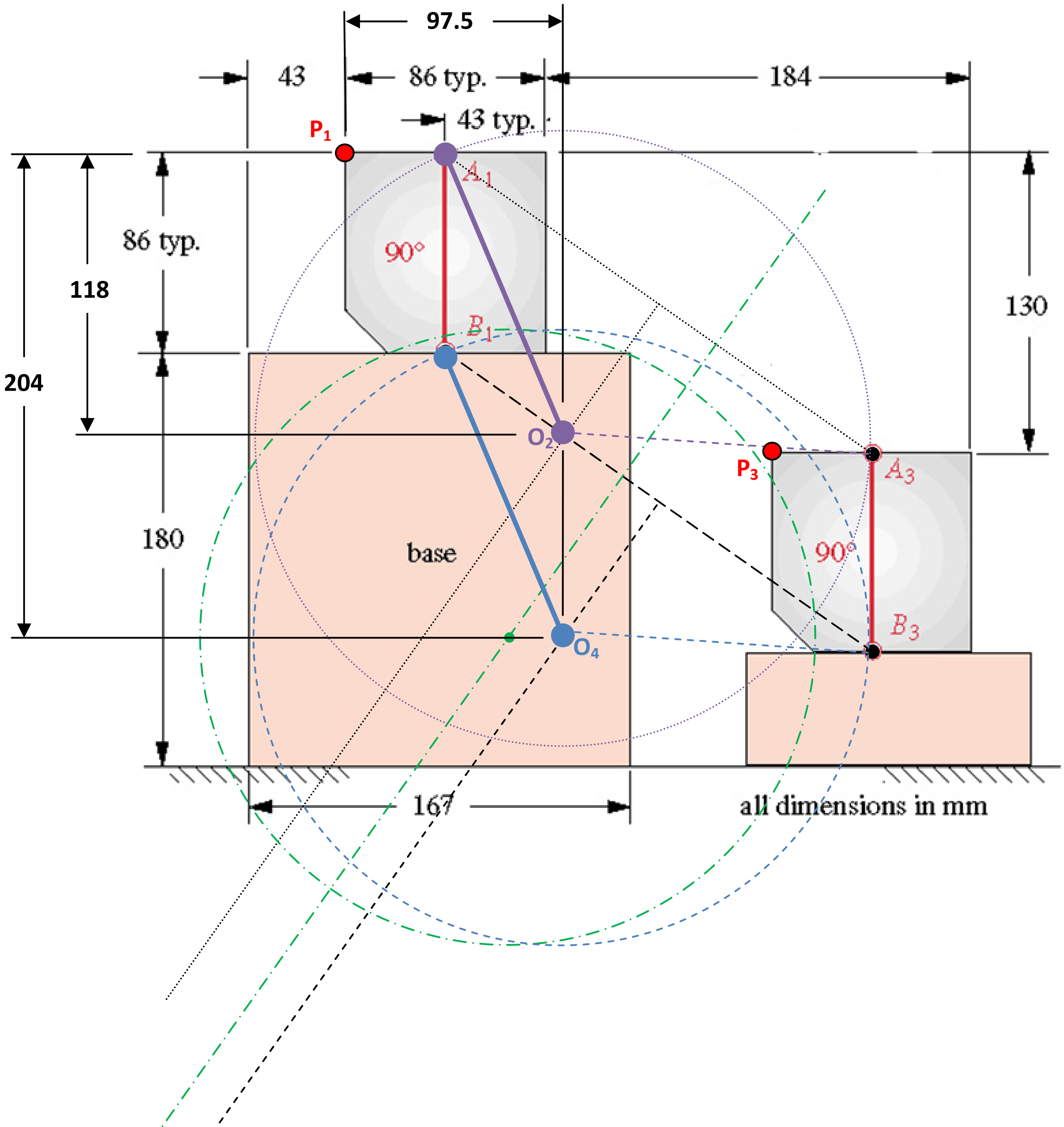
GIVEN:		CHOSEN:		FIND:				x-coord	y-coord.
P12	1051.00	$\gamma 2$	-50.00	u	966.678		O4	122.789	-364.194
P13	1412.00	$\gamma 3$	-80.00	σ	163.084		B1	-802.064	-82.925
$\delta 2$	66.40			s	806.340		B2	-256.231	525.081
$\delta 3$	82.50			ψ	5.903		B3	239.186	595.450
$\alpha 2$	27.00			U1x	-924.853		P1	0.000	0.000
$\alpha 3$	88.00			U1y	281.269		P2	420.767	963.097
				S1x	802.064		P3	184.303	1,399.920
				S1y	82.925				

$$\begin{bmatrix} -0.3572 & 0.7660 & -0.1090 & -0.4540 \\ -0.7660 & -0.3572 & 0.4540 & -0.1090 \\ -0.8264 & 0.9848 & -0.9651 & -0.9994 \\ -0.9848 & -0.8264 & 0.9994 & -0.9651 \end{bmatrix} \begin{Bmatrix} U1x \\ U1y \\ S1x \\ S1y \end{Bmatrix} = \begin{Bmatrix} 420.7668 \\ 963.0972 \\ 184.3030 \\ 1399.9201 \end{Bmatrix}$$

1b. Draw the final synthesized mechanism to scale in Position 1.



PROBLEM 2 (30 pts): A container of liquid sit on top of a base (A_1B_1) and is required to move to the position on the lower step (A_3B_3).



Above is an illustration of the graphical two positions synthesis. This configuration 1) insures that the orientation of the container remains the same throughout the motion from the top of the base to the top of the lower platform and is the minimum configuration for the container clearing the corner of the base. The black lines represent the construction lines and perpendicular bisectors of those lines; it is along the perpendicular bisectors that the rotopoles or fixed pivots have to be located.

2a. Using the program that you developed in class, perform an analytical synthesis to design a linkage that will move the box from the top positions (A_1B_1) to the bottom position (A_3B_3), and has ground pivots on the base. Show all work need to calculate the parameters used in your computer model below. Staple a copy of the computer solution directly after this page.

This particular configuration lends itself to the Analytical Two-Position Synthesis – Approach B. The GIVEN parameters that can be calculated from the above figure are found below.

FIRST DYAD

$$(P_{1x}, P_{1y}) = (0\,mm, 0\,mm)$$

$$(P_{2x}, P_{2y}) = (86\,mm + 184\,mm - 86\,mm, -130\,mm) = (184\,mm, -130\,mm)$$

$$(A_{1x}, A_{1y}) = (43\,mm, 0\,mm)$$

$$(A_{2x}, A_{2y}) = (43\,mm + 184\,mm, -130\,mm) = (227\,mm, -130\,mm)$$

$$p_{21} = \sqrt{(184\,mm - 0\,mm)^2 + (-130\,mm - 0\,mm)^2} = \boxed{225\,mm}$$

$$\delta_2 = \tan^{-1}\left(\frac{-130\,mm}{184\,mm}\right) = -35.2° = \boxed{324.8°}$$

$$\alpha_2 = \boxed{0°}$$ (The orientation of the container does not change)

$$z = \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} = \boxed{43\,mm}$$ (Dependent on the chosen location of P_2)

$$\phi = \tan^{-1}\left(\frac{P_{1y} - A_{1y}}{P_{1x} - A_{1x}}\right) = \tan^{-1}\left(\frac{0\,mm}{43\,mm}\right) = \boxed{180°}$$ (Dependent on chosen location of P_2)

$$\beta_2 = \boxed{-120°}$$ (Measured from the figure, not a unique value.)

APPROACH B			FIRST DYAD							
GIVEN:		CHOSEN:		FIND:					x-coord	y-coord
P12	225.3	z	43	w	130.077			O2	97.561	-118.081
δ2	324.8	ϕ	180	θ	114.800			A1	43.000	0.000
α2	0	β2	-120	W1x	-54.561			A2	227.103	-129.870
				W1y	118.081			P1	0.000	0.000
					x-coord	y-coord		P2	184.103	-129.870
				W1	-54.561	118.081				
				W2	129.542	-11.789				
				Z1	-43.000	0.000				
				Z2	-43.000	0.000				
									inverse	
	-1.5	0.866025404		W1x	=		184.1027		-0.5	-0.28868
	-0.866025404	-1.5		W1y			-129.87		0.288675	-0.5

SECOND DYAD

$$\left(P_{1x}, P_{1y}\right) = \left(0\,mm, 0\,mm\right)$$

$$\left(P_{2x}, P_{2y}\right) = \left(86\,mm + 184\,mm - 86\,mm, -130\,mm\right) = \left(184\,mm, -130\,mm\right)$$

$$\left(B_{1x}, B_{1y}\right) = \left(43\,mm, -86\,mm\right)$$

$$\left(B_{2x}, B_{2y}\right) = \left(43\,mm + 184\,mm, -130\,mm - 86\,mm\right) = \left(227\,mm, -216\,mm\right)$$

$$p_{21} = \sqrt{\left(P_{2x} - P_{1x}\right)^2 + \left(P_{2y} - P_{1y}\right)^2} = \sqrt{\left(184\,mm - 0\,mm\right)^2 + \left(-130\,mm - 0\,mm\right)^2} = \boxed{225\,mm}$$

$$\delta_2 = \tan^{-1}\left(\frac{P_{2y} - P_{1y}}{P_{2x} - P_{1x}}\right) = \tan^{-1}\left(\frac{-130\,mm}{184\,mm}\right) = -35.2° = \boxed{324.8°}$$

$\alpha_2 = \boxed{0°}$ (The orientation of the container does not change)

$s = \sqrt{\left(P_{1x} - B_{1x}\right)^2 + \left(P_{1y} - B_{1y}\right)^2} = \sqrt{\left(-43\,mm\right)^2 + \left(86\,mm\right)^2} = \boxed{96.2\,mm}$ (Dependent on the chosen location of P_2)

$\psi = \tan^{-1}\left(\frac{P_{1y} - B_{1y}}{P_{1x} - B_{1x}}\right) = \tan^{-1}\left(\frac{86\,mm}{-43\,mm}\right) = \boxed{116.6°}$ (Dependent on chosen location of P_2)

$\gamma_2 = \boxed{-120°}$ (Measured from the figure, not a unique value.)

APPROACH B			**SECOND DYAD**								
GIVEN:		**CHOSEN:**		**FIND:**					**x-coord**	**y-coord**	
P12	225.3	**s**	96.2	**u**	130.077			**O4**	97.635	-204.099	
δ2	324.8	ψ	116.6	σ	114.800			**B1**	43.074	-86.018	
α**2**	0	γ2	-120	**U1x**	-54.561			**B2**	227.177	-215.888	
				U1y	118.081			**P1**	0.000	0.000	
					x-coord	**y-coord**		**P2**	184.103	-129.870	
				U1	-54.5611	118.0810					
				U2	129.5417	-11.7892					
				S1	-43.0744	86.0176					
				S2	-43.0744	86.0176					
									inverse		
	-1.5	0.866025		**U1x**		**=**	184.1027		-0.5	-0.28868	
	-0.86603	-1.5		**U1y**			-129.87		0.288675	-0.5	

2b. Does your linkage design require the base to be modified in any way? Explain. Can it be designed so that the base does not have to be modified? How?

The configuration that was synthesized above minimally satisfies the constraint of not having to modify the base. If the fix pivots are positioned any lower on the bisectors of the moving pivots, the corner of the container will run into the base. If this were allowed to happen, the base will have to be modified. If the fixed pivots are positioned higher than the minimal point on the bisectors of the moving pivots, the gap between the container and the corner of the base will increase in distance. The limit to how far above the base the container can be carried is limited by knowing the fixed pivots must be positioned on the bisector, but below the line between the moving pivots.

Because the synthesized mechanism is designed to be a parallelogram, all points on the container will sweep out a circular path of the same diameter. This means that the line between B and the left corner of the container will remain horizontal during the motion of the container and that the location of the center of the corners path is horizontal and to the left of the center of point B's path, at a distance that is equal to the distance from point B to the corner. This geometry enabled the minimal configuration to be identified as shown in the figure.

2c. Will the linkage design enable the container to hold a liquid without spilling it as it travels from the upper base position to the lower platform position? Explain. Can the linkage be designed to hold the container in the same position throughout its motion from the upper to lower base? How?

The linkage designed above will always stay in the same orientation throughout the motion of the container from the top of the base to the lower platform. This is assured, as explained above, by the parallelogram nature of the mechanism design. Because the top of the container will remain horizontal, the mechanism is designed to maximize the ability of the container to hold a liquid without spilling it during the motion of the mechanism.

PROBLEM 3 (40 pts): Design a single-dwell cam to move a follower from 0 to 2” in 60°, fall 2” in 90°, and dwell for the remainder. The total cycle must take 2 seconds.

Using **Cycloid** Functions:

Region 1: $0 \le \theta \le 60°,\quad 0 \le \theta \le \frac{\pi}{3},\quad \beta_1 = 60° = \frac{\pi}{3},\quad 0 \le \frac{\theta_1}{\beta_1} \le 1,\quad 0 \le \theta_1 \le 60°,\quad 0 \le \theta_1 \le \frac{\pi}{3}$

$$s = h\left[\frac{\theta_1}{\beta_1} - \frac{1}{2\cdot\pi}\cdot\sin\left(2\pi\cdot\frac{\theta_1}{\beta_1}\right)\right] = 2in\cdot\left[\frac{\theta_1}{\beta_1} - \frac{1}{2\cdot\pi}\cdot\sin\left(2\pi\cdot\frac{\theta_1}{\beta_1}\right)\right] = \boxed{2in\cdot\left[\frac{3\cdot\theta_1}{\pi} - \frac{1}{2\cdot\pi}\cdot\sin(6\cdot\theta_1)\right]}$$

$$v = \frac{h}{\beta_1}\left[1-\cos\left(2\pi\cdot\frac{\theta_1}{\beta_1}\right)\right] = \frac{6}{\pi}\cdot\frac{in}{rad}\left[1-\cos\left(2\pi\cdot\frac{\theta_1}{\beta_1}\right)\right] = \boxed{\frac{6}{\pi}\cdot\frac{in}{rad}\left[1-\cos(6\cdot\theta_1)\right]}$$

$$a = \frac{2\cdot\pi\cdot h}{\beta_1^2}\cdot\sin\left(2\pi\cdot\frac{\theta_1}{\beta_1}\right) = \frac{36}{\pi}\cdot\frac{in}{rad^2}\cdot\sin\left(2\pi\cdot\frac{\theta_1}{\beta_1}\right) = \boxed{\frac{36}{\pi}\cdot\frac{in}{rad^2}\cdot\sin(6\cdot\theta_1)}$$

$$j = \frac{4\cdot\pi^2\cdot h}{\beta_1^3}\cdot\cos\left(2\pi\cdot\frac{\theta_1}{\beta_1}\right) = \frac{216}{\pi}\cdot\frac{in}{rad^3}\cdot\cos\left(2\pi\cdot\frac{\theta_1}{\beta_1}\right) = \boxed{\frac{216}{\pi}\cdot\frac{in}{rad^3}\cdot\cos(6\cdot\theta_1)}$$

Region 2: $60 \le \theta \le 150°,\quad \frac{\pi}{3} \le \theta \le \frac{5\pi}{6},\quad \beta_2 = 90° = \frac{\pi}{2},\quad 0 \le \frac{\theta_2}{\beta_2} \le 1,\quad 0 \le \theta_2 \le 90°,\quad 0 \le \theta_2 \le \frac{\pi}{2}$

$$s = h - h\left[\frac{\theta_2}{\beta_2} - \frac{1}{2\cdot\pi}\cdot\sin\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right)\right] = 2in - 2in\cdot\left[\frac{\theta_2}{\beta_2} - \frac{1}{2\cdot\pi}\cdot\sin\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right)\right] = \boxed{2in - 2in\cdot\left[\frac{2\cdot\theta_2}{\pi} - \frac{1}{2\cdot\pi}\cdot\sin(4\cdot\theta_2)\right]}$$

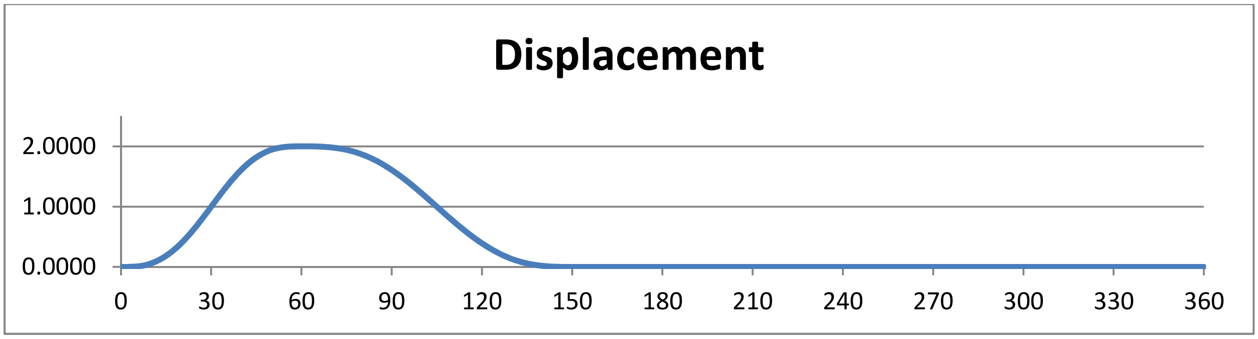
$$v = -\frac{h}{\beta_2}\left[1-\cos\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right)\right] = -\frac{4}{\pi}\cdot\frac{in}{rad}\left[1-\cos\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right)\right] = \boxed{-\frac{4}{\pi}\cdot\frac{in}{rad}\left[1-\cos(4\cdot\theta_2)\right]}$$

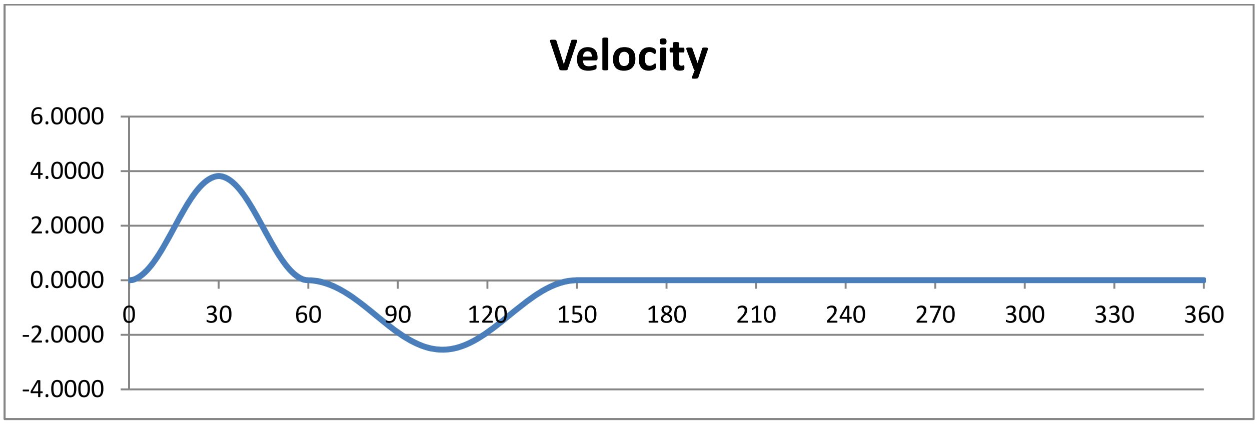
$$a = -\frac{2\cdot\pi\cdot h}{\beta_2^2}\cdot\sin\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right) = -\frac{16}{\pi}\cdot\frac{in}{rad^2}\cdot\sin\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right) = \boxed{-\frac{16}{\pi}\cdot\frac{in}{rad^2}\cdot\sin(4\cdot\theta_2)}$$

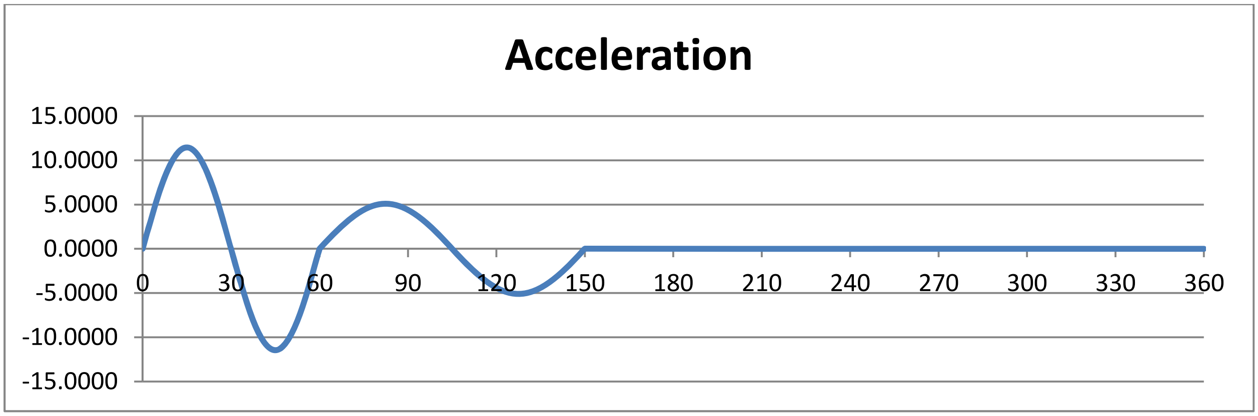
$$j = -\frac{4\cdot\pi^2\cdot h}{\beta_2^3}\cdot\cos\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right) = -\frac{64}{\pi}\cdot\frac{in}{rad^3}\cdot\cos\left(2\pi\cdot\frac{\theta_2}{\beta_2}\right) = \boxed{-\frac{64}{\pi}\cdot\frac{in}{rad^3}\cdot\cos(4\cdot\theta_2)}$$

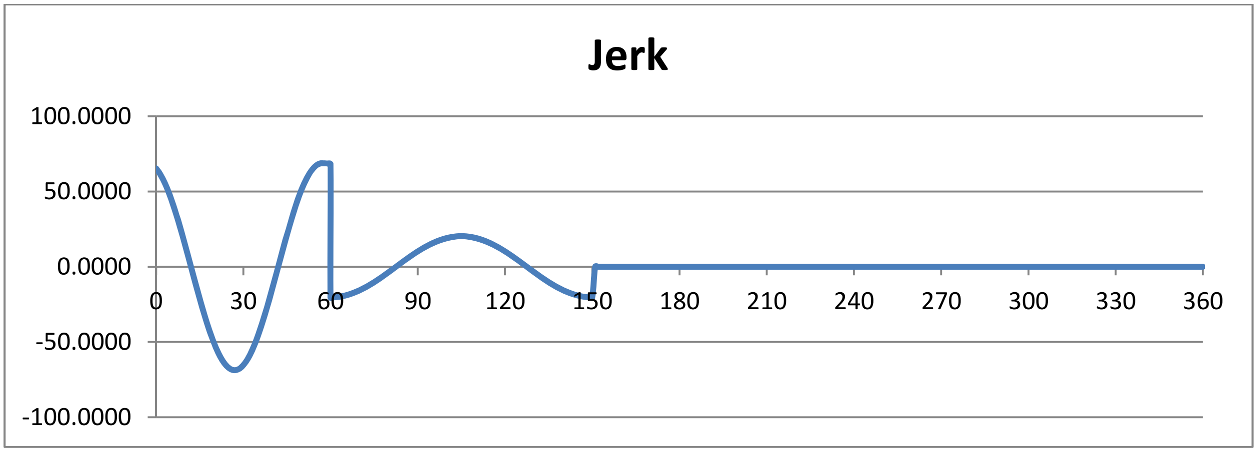
Region 3: $150 \le \theta \le 360°,\quad \frac{5\pi}{6} \le \theta \le 2\pi,\quad \beta_3 = 210° = \frac{7\pi}{6},\quad 0 \le \frac{\theta_3}{\beta_3} \le 1,\quad 0 \le \theta_3 \le 210°,\quad 0 \le \theta_3 \le \frac{7\pi}{6}$

$$s = 0$$
$$v = 0$$
$$a = 0$$
$$j = 0$$


Displacement
2.0000
1.0000
0.0000
0 30 60 90 120 150 180 210 240 270 300 330 360


Velocity
6.0000
4.0000
2.0000
0.0000
-2.0000
-4.0000
0 30 60 90 120 150 180 210 240 270 300 330 360


Acceleration
15.0000
10.0000
5.0000
0.0000
-5.0000
-10.0000
-15.0000
0 30 60 90 120 150 180 210 240 270 300 330 360


Jerk
100.0000
50.0000
0.0000
-50.0000
-100.0000
0 30 60 90 120 150 180 210 240 270 300 330 360

Using **<u>3-4-5 Polynomial</u>** Functions, starting with a function of the form

$$s(\tfrac{\theta}{\beta}) = C_0 + C_1\cdot\left(\tfrac{\theta}{\beta}\right) + C_2\cdot\left(\tfrac{\theta}{\beta}\right)^2 + C_3\cdot\left(\tfrac{\theta}{\beta}\right)^3 + C_4\cdot\left(\tfrac{\theta}{\beta}\right)^4 + C_5\cdot\left(\tfrac{\theta}{\beta}\right)^5$$

Region 1: $0 \le \theta \le 60°,\quad 0 \le \theta \le \frac{\pi}{3},\quad \beta_1 = 60° = \frac{\pi}{3},\quad 0 \le \frac{\theta_1}{\beta_1} \le 1,\quad 0 \le \theta_1 \le 60°,\quad 0 \le \theta_1 \le \frac{\pi}{3}$

$\theta_1 = 0,\quad \frac{\theta_1}{\beta_1} = 0 \quad\Rightarrow\quad s(\frac{\theta_1}{\beta_1}=0) = 0\ in,\ v(\frac{\theta_1}{\beta_1}=0) = 0\,\frac{in}{rad},\ a(\frac{\theta_1}{\beta_1}=0) = 0\,\frac{in}{rad^2}$

(The above BC are used to determine that $\mathbf{C_0 = C_1 = C_2 = 0}$.)

$\theta_1 = 60 = \frac{\pi}{3} \quad\Rightarrow\quad s(\frac{\theta_1}{\beta_1}=1) = 2\ in,\ v(\frac{\theta_1}{\beta_1}=1) = 0\,\frac{in}{rad},\ a(\frac{\theta_1}{\beta_1}=1) = 0\,\frac{in}{rad^2}$

$$s(\tfrac{\theta_1}{\beta_1}=1) = 2\ in = C_3\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 + C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4 + C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^5 \quad\Rightarrow\quad 2\ in = C_3 + C_4 + C_5$$

$$v(\tfrac{\theta_1}{\beta_1}=1) = 0\,\tfrac{in}{rad} = \tfrac{1}{\beta}\left[3\cdot C_3\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2 + 4\cdot C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 + 5\cdot C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4\right] \quad\Rightarrow\quad 0\,\tfrac{in}{rad} = 3\cdot C_3 + 4\cdot C_4 + 5\cdot C_5$$

$$a(\tfrac{\theta_1}{\beta_1}=1) = 0\,\tfrac{in}{rad^2} = \tfrac{1}{\beta^2}\left[6\cdot C_3\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2 + 12\cdot C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 + 20\cdot C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4\right] \quad\Rightarrow\quad 0\,\tfrac{in}{rad^2} = 6\cdot C_3 + 12\cdot C_4 + 20\cdot C_5$$

Solving the three equations simultaneously results in $\mathbf{C_3=20in, C_4=-30in,}$ **and** $\mathbf{C_5=12in}$. For this region,

$$s(\tfrac{\theta_1}{\beta_1}) = C_3\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 + C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4 + C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^5 = \boxed{20in\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 - 30in\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4 + 12in\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^5}$$

$$v(\tfrac{\theta_1}{\beta_1}) = \tfrac{1}{\beta_1}\left[3\cdot C_3\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2 + 4\cdot C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 + 5\cdot C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4\right] = \boxed{\tfrac{3}{\pi}\cdot\tfrac{in}{rad}\left[60\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2 - 120\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 + 60\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4\right]}$$

$$a(\tfrac{\theta_1}{\beta_1}) = \tfrac{1}{\beta_1^2}\left[6\cdot C_3\cdot\left(\tfrac{\theta_1}{\beta_1}\right) + 12\cdot C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2 + 20\cdot C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3\right] = \boxed{\tfrac{9}{\pi^2}\cdot\tfrac{in}{rad}\left[120\cdot\left(\tfrac{\theta_1}{\beta_1}\right) - 360\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2 + 240\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3\right]}$$

$$j(\tfrac{\theta_1}{\beta_1}) = \tfrac{1}{\beta_1^3}\left[6\cdot C_3 + 36\cdot C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right) + 60\cdot C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2\right] = \boxed{\tfrac{27}{\pi^3}\cdot\tfrac{in}{rad}\left[120 - 1080\cdot\left(\tfrac{\theta_1}{\beta_1}\right) + 720\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2\right]}$$

Region 2: $60 \le \theta \le 150°,\quad \frac{\pi}{3} \le \theta \le \frac{5\pi}{6},\quad \beta_2 = 90° = \frac{\pi}{2},\quad 0 \le \frac{\theta_2}{\beta_2} \le 1,\quad 0 \le \theta_2 \le 90°,\quad 0 \le \theta_2 \le \frac{\pi}{2}$

$\theta_2 = 0,\quad \frac{\theta_2}{\beta_2} = 0 \quad\Rightarrow\quad s(\frac{\theta_2}{\beta_2}=0) = 2\ in,\ v(\frac{\theta_2}{\beta_2}=0) = 0\,\frac{in}{rad},\ a(\frac{\theta_2}{\beta_2}=0) = 0\,\frac{in}{rad^2}$

(The above BC are used to determine $\mathbf{C_0 = 2in, C_1 = 0,}$ **and** $\mathbf{C_2 = 0}$.)

$\theta_2 = 90 = \frac{\pi}{2} \quad\Rightarrow\quad s(\frac{\theta_2}{\beta_2}=1) = 2\ in,\ v(\frac{\theta_2}{\beta_2}=1) = 0\,\frac{in}{rad},\ a(\frac{\theta_2}{\beta_2}=1) = 0\,\frac{in}{rad^2}$

$$s(\tfrac{\theta_2}{\beta_2}=1) = 0\ in = 2in + C_3\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3 + C_4\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^4 + C_5\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^5 \quad\Rightarrow\quad -2\ in = C_3 + C_4 + C_5$$

$$v(\tfrac{\theta_2}{\beta_2}=1) = 0\,\tfrac{in}{rad} = \tfrac{1}{\beta}\left[3\cdot C_3\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^2 + 4\cdot C_4\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3 + 5\cdot C_5\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^4\right] \quad\Rightarrow\quad 0\,\tfrac{in}{rad} = 3\cdot C_3 + 4\cdot C_4 + 5\cdot C_5$$

$$a(\tfrac{\theta_2}{\beta_2}=1) = 0\,\tfrac{in}{rad^2} = \tfrac{1}{\beta^2}\left[6\cdot C_3\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^2 + 12\cdot C_4\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3 + 20\cdot C_5\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^4\right] \quad\Rightarrow\quad 0\,\tfrac{in}{rad^2} = 6\cdot C_3 + 12\cdot C_4 + 20\cdot C_5$$

Solving the three equations simultaneously results in $\mathbf{C_3=-20in, C_4=30in,}$ **and** $\mathbf{C_5=-12in}$. For this region,

$$s(\tfrac{\theta_2}{\beta_2}) = 2in + C_3\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3 + C_4\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^4 + C_5\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^5 = \boxed{2in - 20in\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3 + 30in\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^4 - 12in\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^5}$$

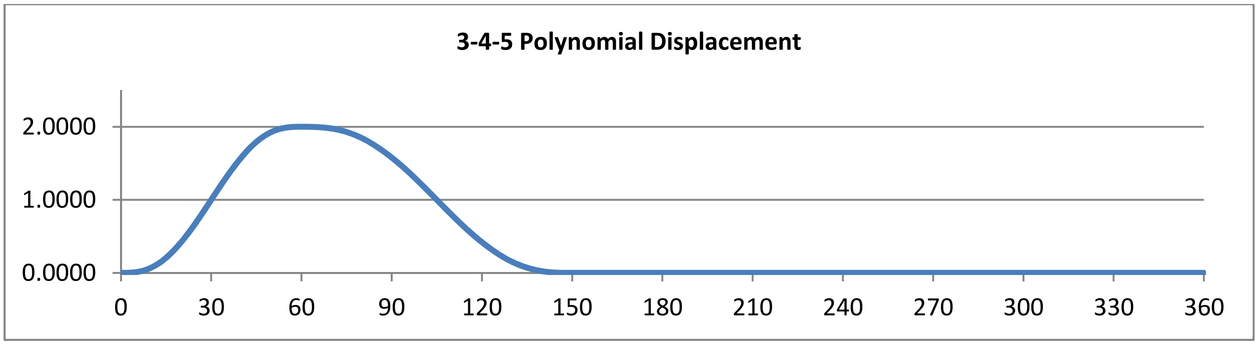
$$v(\tfrac{\theta_2}{\beta_2}) = \tfrac{1}{\beta_2}\left[3\cdot C_3\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^2 + 4\cdot C_4\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3 + 5\cdot C_5\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^4\right] = \boxed{\tfrac{2}{\pi}\cdot\tfrac{in}{rad}\left[-60\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^2 + 120\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3 - 60\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^4\right]}$$

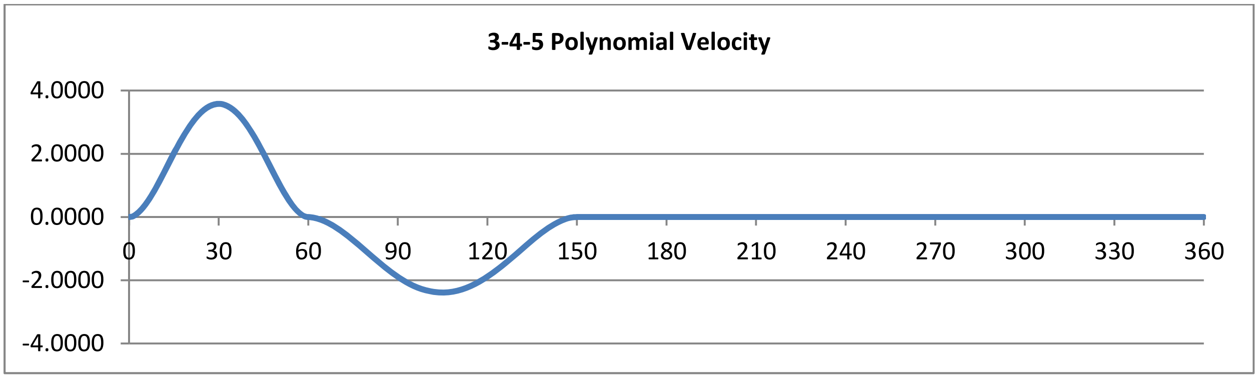
$$a(\tfrac{\theta_2}{\beta_2}) = \tfrac{1}{\beta_2^2}\left[6\cdot C_3\cdot\left(\tfrac{\theta_2}{\beta_2}\right) + 12\cdot C_4\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^2 + 20\cdot C_5\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3\right] = \boxed{\tfrac{4}{\pi^2}\cdot\tfrac{in}{rad}\left[-120\cdot\left(\tfrac{\theta_2}{\beta_2}\right) + 360\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^2 - 240\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3\right]}$$

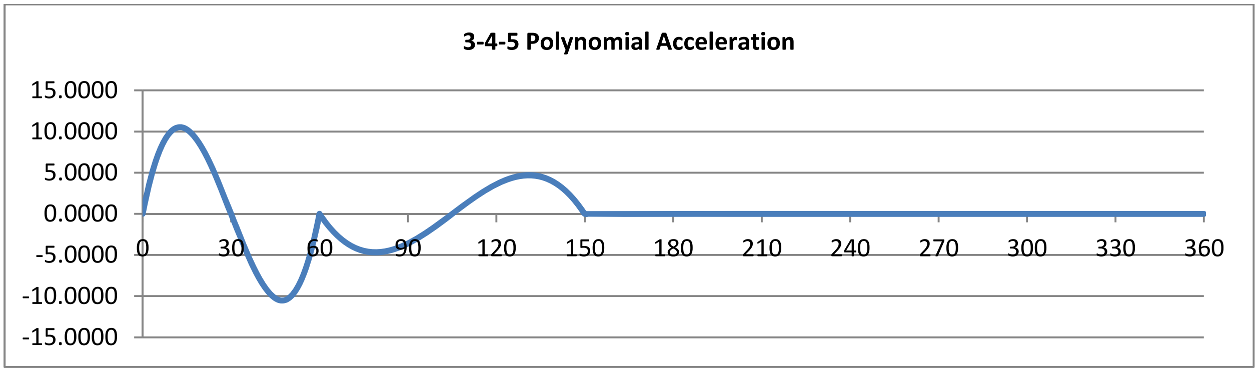
$$j(\tfrac{\theta_2}{\beta_2}) = \tfrac{1}{\beta_2^3}\left[6\cdot C_3 + 36\cdot C_4\cdot\left(\tfrac{\theta_2}{\beta_2}\right) + 80\cdot C_5\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^2\right] = \boxed{\tfrac{8}{\pi^3}\cdot\tfrac{in}{rad}\left[-120 + 1080\cdot\left(\tfrac{\theta_2}{\beta_2}\right) - 720\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^2\right]}$$

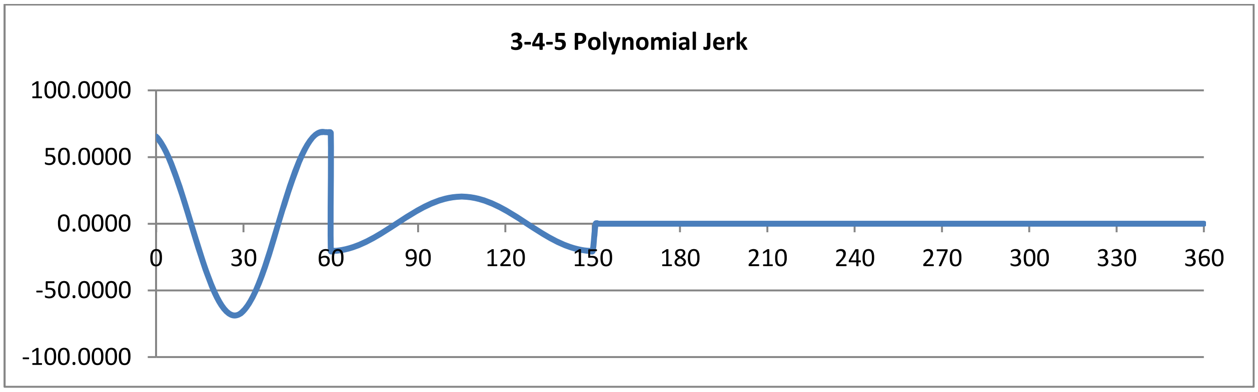
Region 3: $150 \le \theta \le 360°, \quad \frac{5\pi}{6} \le \theta \le 2\pi, \quad \beta_3 = 210° = \frac{7\pi}{6}, \quad 0 \le \frac{\theta_3}{\beta_3} \le 1, \quad 0 \le \theta_3 \le 210°, \quad 0 \le \theta_3 \le \frac{7\pi}{6}$

$$s = 0, \quad v = 0, \quad a = 0, \quad j = 0$$









Using **<u>4-5-6-7 Polynomial</u>** Functions, starting with a function of the form

$$s(\tfrac{\theta}{\beta}) = C_0 + C_1 \cdot \left(\tfrac{\theta}{\beta}\right) + C_2 \cdot \left(\tfrac{\theta}{\beta}\right)^2 + C_3 \cdot \left(\tfrac{\theta}{\beta}\right)^3 + C_4 \cdot \left(\tfrac{\theta}{\beta}\right)^4 + C_5 \cdot \left(\tfrac{\theta}{\beta}\right)^5 + C_6 \cdot \left(\tfrac{\theta}{\beta}\right)^6 + C_7 \cdot \left(\tfrac{\theta}{\beta}\right)^7$$

Region 1: $0 \le \theta \le 60°, \quad 0 \le \theta \le \tfrac{\pi}{3}, \quad \beta_1 = 60° = \tfrac{\pi}{3}, \quad 0 \le \tfrac{\theta_1}{\beta_1} \le 1, \quad 0 \le \theta_1 \le 60°, \quad 0 \le \theta_1 \le \tfrac{\pi}{3}$

$$\theta_1 = 0, \quad \tfrac{\theta_1}{\beta_1} = 0 \quad \Rightarrow \quad s(\tfrac{\theta_1}{\beta_1} = 0) = 0\,in,\ v(\tfrac{\theta_1}{\beta_1} = 0) = 0\,\tfrac{in}{rad},\ a(\tfrac{\theta_1}{\beta_1} = 0) = 0\,\tfrac{in}{rad^2},\ j(\tfrac{\theta_1}{\beta_1} = 0) = 0\,\tfrac{in}{rad^3}$$

(The above BC are used in determining that $\mathbf{C_0 = C_1 = C_2 = C_3 = 0}$.)

$$\theta_1 = 60 = \tfrac{\pi}{3} \quad \Rightarrow \quad s(\tfrac{\theta_1}{\beta_1} = 1) = 2\,in,\ v(\tfrac{\theta_1}{\beta_1} = 1) = 0\,\tfrac{in}{rad},\ a(\tfrac{\theta_1}{\beta_1} = 1) = 0\,\tfrac{in}{rad^2},\ j(\tfrac{\theta_1}{\beta_1} = 1) = 0\,\tfrac{in}{rad^3}$$

$$s(\tfrac{\theta_1}{\beta_1} = 1) = 2\,in = C_4 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4 + C_5 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^5 + C_6 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^6 + C_7 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^7 \quad \Rightarrow \quad 2\,in = C_4 + C_5 + C_6 + C_7$$

$$v(\tfrac{\theta_1}{\beta_1} = 1) = 0\,\tfrac{in}{rad} = \tfrac{1}{\beta}\left[4 \cdot C_4 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^3 + 5 \cdot C_5 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4 + 6 \cdot C_6 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^5 + 7 \cdot C_7 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^6\right] \quad \Rightarrow \quad 0\,in = 4 \cdot C_4 + 5 \cdot C_5 + 6 \cdot C_6 + 7 \cdot C_7$$

$$a(\tfrac{\theta_1}{\beta_1} = 1) = 0\,\tfrac{in}{rad^2} = \tfrac{1}{\beta^2}\left[12 \cdot C_4 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^2 + 20 \cdot C_5 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^3 + 30 \cdot C_6 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4 + 42 \cdot C_7 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^5\right] \quad \Rightarrow \quad 0\,in = 12 \cdot C_4 + 20 \cdot C_5 + 30 \cdot C_6 + 42 \cdot C_7$$

$$j(\tfrac{\theta_1}{\beta_1} = 1) = 0\,\tfrac{in}{rad^2} = \tfrac{1}{\beta^3}\left[24 \cdot C_4 \cdot \left(\tfrac{\theta_1}{\beta_1}\right) + 60 \cdot C_5 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^2 + 120 \cdot C_6 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^3 + 210 \cdot C_7 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4\right] \quad \Rightarrow \quad 0\,in = 24 \cdot C_4 + 60 \cdot C_5 + 120 \cdot C_6 + 210 \cdot C_7$$

Solving the three equations simultaneously results in **C_4=70in, C_5=-168in, C_6=140in, and C_7=-40in**. For this region,

$$s(\tfrac{\theta_1}{\beta_1}) = C_4 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4 + C_5 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^5 + C_6 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^6 + C_7 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^7 = \boxed{70in \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4 - 168in \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^5 + 140in \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^6 - 40in \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^7}$$

$$v(\tfrac{\theta_1}{\beta_1}) = \tfrac{1}{\beta_1}\left[4 \cdot C_4 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^3 + 5 \cdot C_5 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4 + 6 \cdot C_4 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^5 + 7 \cdot C_5 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^6\right] = \boxed{\tfrac{3}{\pi} \cdot \tfrac{in}{rad}\left[280 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^3 - 840 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4 + 840 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^5 - 280 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^6\right]}$$

$$a(\tfrac{\theta_1}{\beta_1}) = \tfrac{1}{\beta_1^2}\left[12 \cdot C_4 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^2 + 20 \cdot C_5 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^3 + 30 \cdot C_4 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4 + 42 \cdot C_5 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^5\right] = \boxed{\tfrac{9}{\pi^2} \cdot \tfrac{in}{rad}\left[840 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^2 - 3360 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^3 + 4200 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4 - 1680 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^5\right]}$$

$$j(\tfrac{\theta_1}{\beta_1}) = \tfrac{1}{\beta_1^3}\left[36 \cdot C_4 \cdot \left(\tfrac{\theta_1}{\beta_1}\right) + 60 \cdot C_5 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^2 + 120 \cdot C_4 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^3 + 210 \cdot C_5 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4\right] = \boxed{\tfrac{27}{\pi^3} \cdot \tfrac{in}{rad}\left[1680 \cdot \left(\tfrac{\theta_1}{\beta_1}\right) - 10080 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^2 + 16800 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^3 - 8400 \cdot \left(\tfrac{\theta_1}{\beta_1}\right)^4\right]}$$

Region 2: $60 \le \theta \le 150°, \quad \frac{\pi}{3} \le \theta \le \frac{5\pi}{6}, \quad \beta_2 = 90° = \frac{\pi}{2}, \quad 0 \le \frac{\theta_2}{\beta_2} \le 1, \quad 0 \le \theta_2 \le 90°, \quad 0 \le \theta_2 \le \frac{\pi}{2}$

$$\theta_2 = 0, \quad \tfrac{\theta_2}{\beta_2} = 0 \quad \Rightarrow \quad s(\tfrac{\theta_2}{\beta_2}=0) = 2\,in,\, v(\tfrac{\theta_2}{\beta_2}=0) = 0\,\tfrac{in}{rad},\, a(\tfrac{\theta_2}{\beta_2}=0) = 0\,\tfrac{in}{rad^2},\, j(\tfrac{\theta_2}{\beta_2}=0) = 0\,\tfrac{in}{rad^3}$$

(The above BC are used in determining that $\mathbf{C_0 = 2in, C_1 = C_2 = C_3 = 0}$.)

$$\theta_2 = 60 = \tfrac{\pi}{3} \quad \Rightarrow \quad s(\tfrac{\theta_2}{\beta_2}=1) = 0\,in,\, v(\tfrac{\theta_2}{\beta_2}=1) = 0\,\tfrac{in}{rad},\, a(\tfrac{\theta_2}{\beta_2}=1) = 0\,\tfrac{in}{rad^2},\, j(\tfrac{\theta_2}{\beta_2}=1) = 0\,\tfrac{in}{rad^3}$$

$$s(\tfrac{\theta_2}{\beta_2}=1) = 0\,in = 2in + C_4\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^4 + C_5\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^5 + C_6\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^6 + C_7\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^7 \quad \Rightarrow \quad -2\,in = C_4 + C_5 + C_6 + C_7$$

$$v(\tfrac{\theta_2}{\beta_2}=1) = 0\,\tfrac{in}{rad} = \tfrac{1}{\beta_2}\left[4\cdot C_4\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3 + 5\cdot C_5\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^4 + 6\cdot C_6\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^5 + 7\cdot C_7\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^6\right] \quad \Rightarrow \quad 0\,in = 4\cdot C_4 + 5\cdot C_5 + 6\cdot C_6 + 7\cdot C_7$$

$$a(\tfrac{\theta_2}{\beta_2}=1) = 0\,\tfrac{in}{rad^2} = \tfrac{1}{\beta_2^2}\left[12\cdot C_4\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^2 + 20\cdot C_5\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3 + 30\cdot C_6\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^4 + 42\cdot C_7\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^5\right] \quad \Rightarrow \quad 0\,in = 12\cdot C_4 + 20\cdot C_5 + 30\cdot C_6 + 42\cdot C_7$$

$$j(\tfrac{\theta_2}{\beta_2}=1) = 0\,\tfrac{in}{rad^2} = \tfrac{1}{\beta_2^3}\left[24\cdot C_4\cdot\left(\tfrac{\theta_2}{\beta_2}\right) + 60\cdot C_5\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^2 + 120\cdot C_6\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^3 + 210\cdot C_7\cdot\left(\tfrac{\theta_2}{\beta_2}\right)^4\right] \quad \Rightarrow \quad 0\,in = 24\cdot C_4 + 60\cdot C_5 + 120\cdot C_6 + 210\cdot C_7$$

Solving the three equations simultaneously results in **C_4=-70in, C_5=168in, C_6=-140in, and C_7=40in**. For this region,

$$s(\tfrac{\theta_1}{\beta_1}) = 2in + C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4 + C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^5 + C_6\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^6 + C_7\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^7 = \boxed{2in - 70in\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4 + 168in\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^5 - 140in\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^6 + 40in\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^7}$$

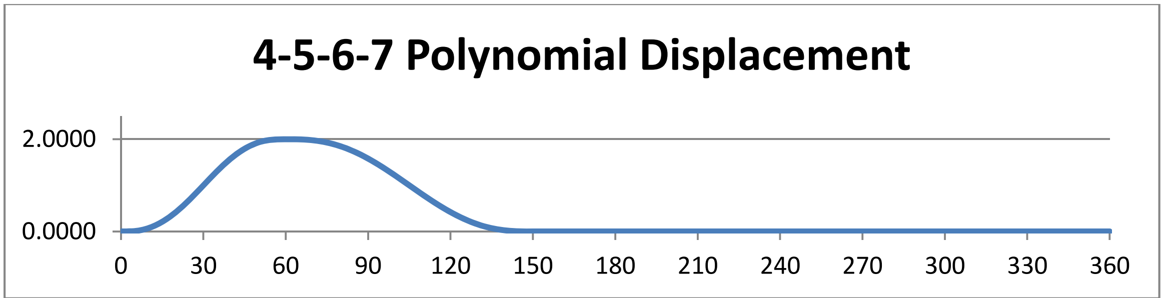
$$v(\tfrac{\theta_1}{\beta_1}) = \tfrac{1}{\beta_1}\left[4\cdot C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 + 5\cdot C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4 + 6\cdot C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^5 + 7\cdot C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^6\right] = \boxed{\tfrac{2}{\pi}\cdot\tfrac{in}{rad}\left[-280\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 + 840\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4 - 840\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^5 + 280\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^6\right]}$$

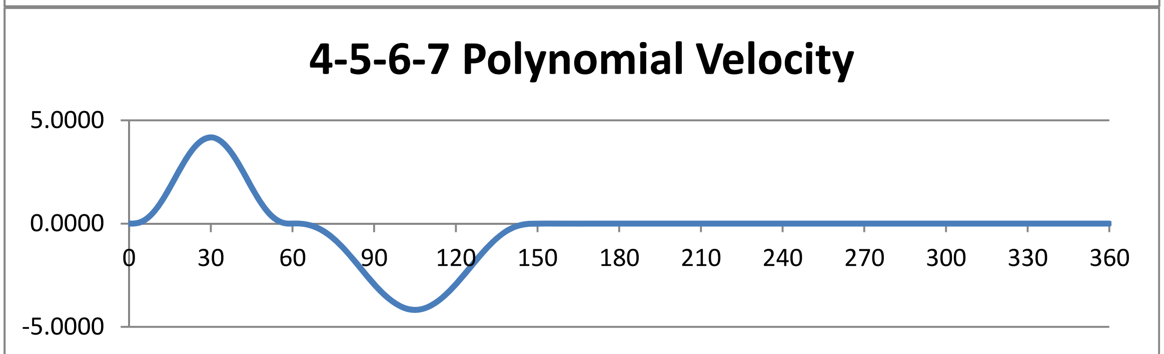
$$a(\tfrac{\theta_1}{\beta_1}) = \tfrac{1}{\beta_1^2}\left[12\cdot C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2 + 20\cdot C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 + 30\cdot C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4 + 42\cdot C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^5\right] = \boxed{\tfrac{4}{\pi^2}\cdot\tfrac{in}{rad}\left[-840\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2 + 3360\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 - 4200\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4 + 1680\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^5\right]}$$

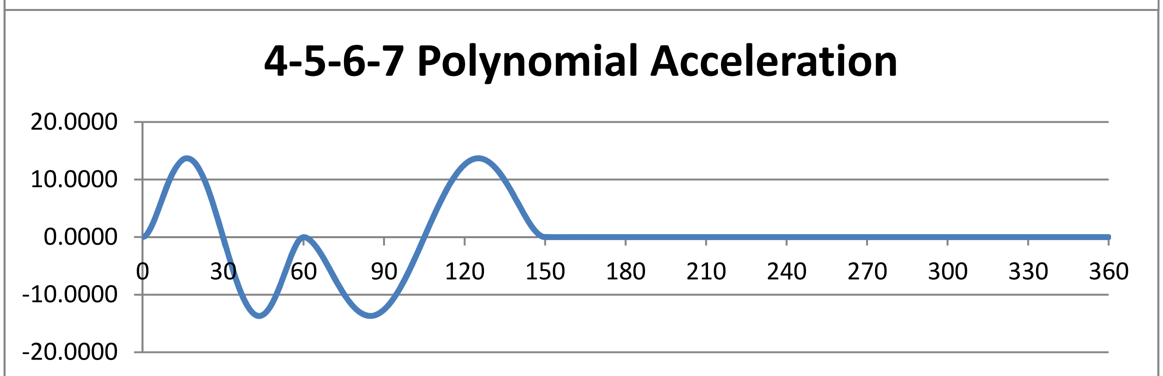
$$j(\tfrac{\theta_1}{\beta_1}) = \tfrac{1}{\beta_1^3}\left[36\cdot C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right) + 60\cdot C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2 + 120\cdot C_4\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 + 210\cdot C_5\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4\right] = \boxed{\tfrac{8}{\pi^3}\cdot\tfrac{in}{rad}\left[-1680\cdot\left(\tfrac{\theta_1}{\beta_1}\right) + 10080\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^2 - 16800\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^3 + 8400\cdot\left(\tfrac{\theta_1}{\beta_1}\right)^4\right]}$$

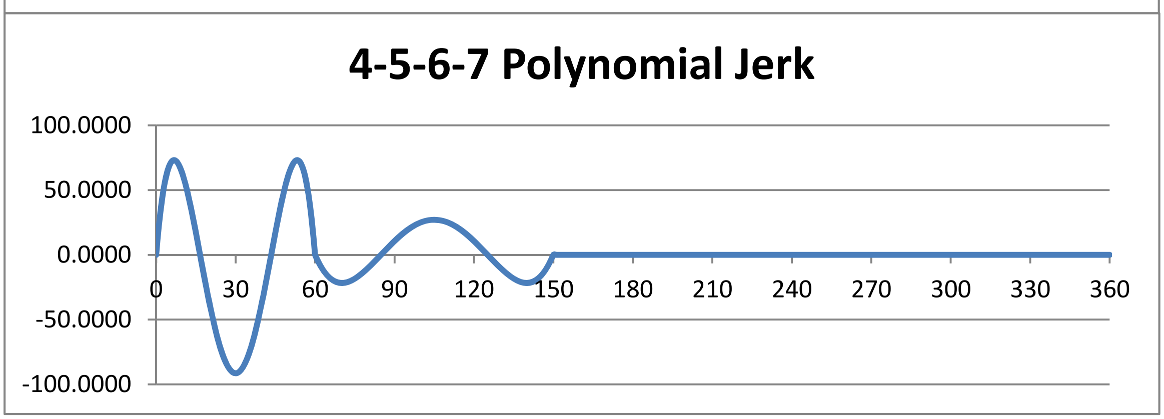
Region 3: $150 \le \theta \le 360°, \quad \frac{5\pi}{6} \le \theta \le 2\pi, \quad \beta_3 = 210° = \frac{7\pi}{6}, \quad 0 \le \frac{\theta_3}{\beta_3} \le 1, \quad 0 \le \theta_3 \le 210°, \quad 0 \le \theta_3 \le \frac{7\pi}{6}$

$$s = 0, \quad v = 0, \quad a = 0, \quad j = 0$$


4-5-6-7 Polynomial Displacement
2.0000
0.0000
0
30
60
90
120
150
180
210
240
270
300
330
360


4-5-6-7 Polynomial Velocity
5.0000
0.0000
-5.0000
0
30
60
90
120
150
180
210
240
270
300
330
360


4-5-6-7 Polynomial Acceleration
20.0000
10.0000
0.0000
-10.0000
-20.0000
0
30
60
90
120
150
180
210
240
270
300
330
360


4-5-6-7 Polynomial Jerk
100.0000
50.0000
0.0000
-50.0000
-100.0000
0
30
60
90
120
150
180
210
240
270
300
330
360

As a student at Union College, I am part of a community that values intellectual effort, curiosity and discovery. I understand that in order to truly claim my educational and academic achievements, I am obligated to act with academic integrity. Therefore, I affirm that I carried out the work on this exam with full academic honesty, and I rely on my fellow students to do the same.

For this exam I understand that:

1. I **must** work alone in writing out the answers to this exam.
2. I **cannot** copy solutions to these problems from any person or resource.
3. I **cannot** use any electronic resources, other than the program I wrote as part of this class, to assist me in the solution to the questions on this exam.
4. I **can** use one formula sheet (both sides) that I prepared as a reference for this exam, I **must** staple this sheet to the back of my exam, and this sheet **cannot** contain any solutions to problems.

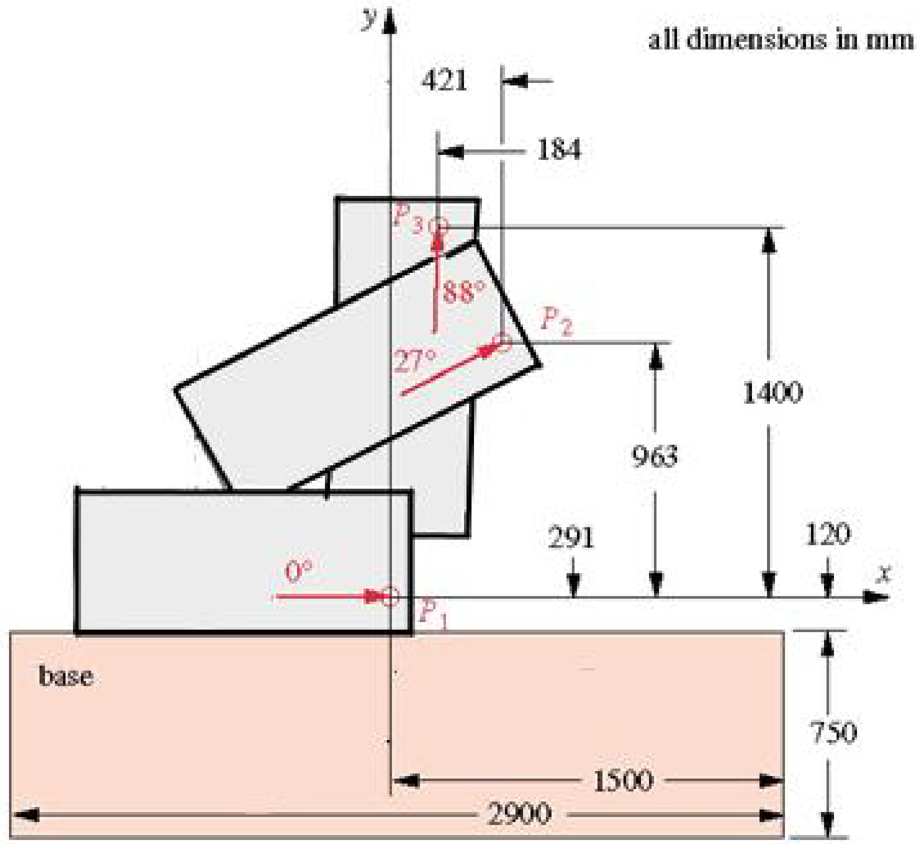
Signature: ______________________________

Print Name: ______________________________

Assignment: ______________________________

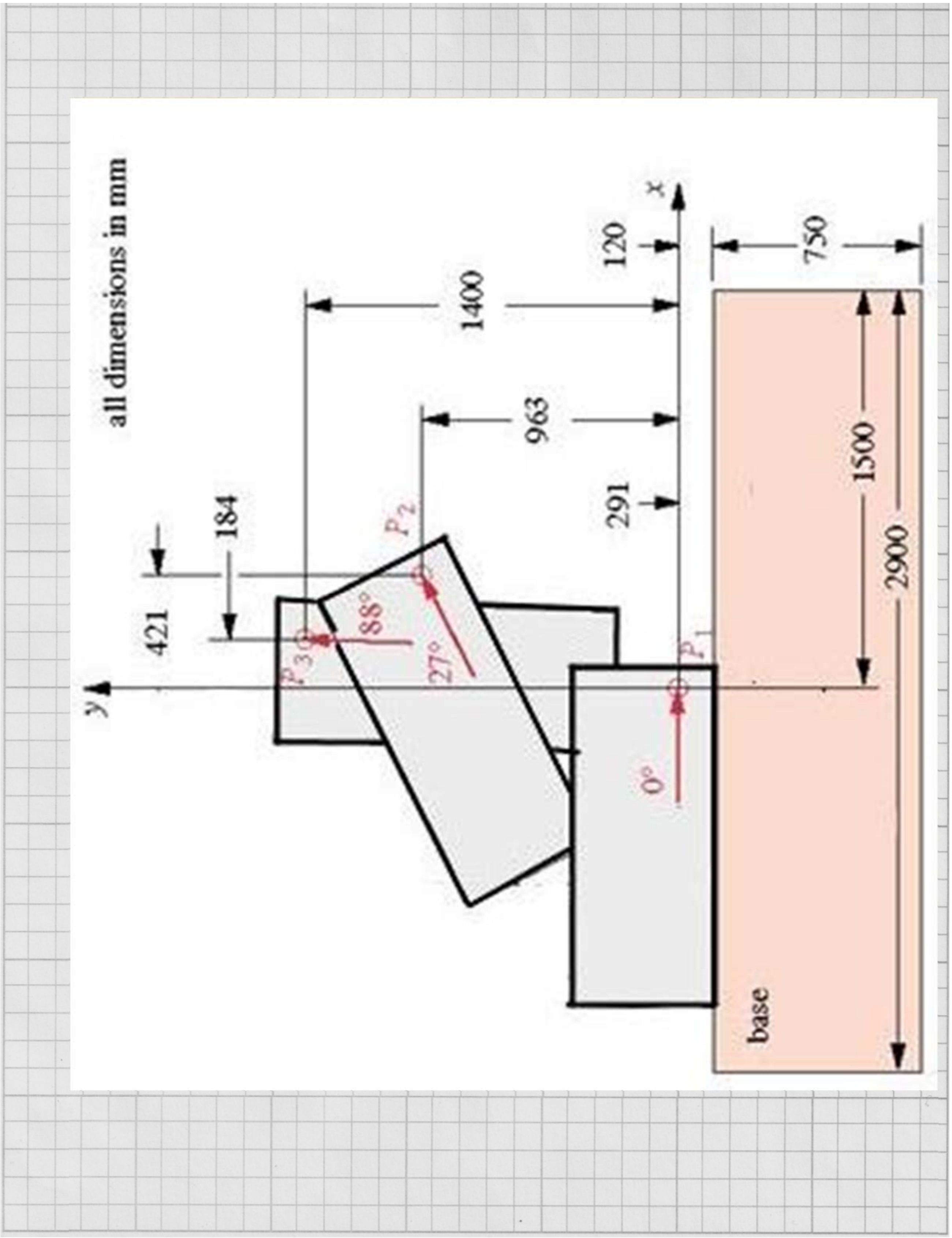
Due Date: ______________________________

PROBLEM 1 (30 pts): A box sits on top of the base shown in Position 1 and needs to be moved to Position 2, and then to Position 3. (Guess β_2=-50, β_3=-100, γ_2=-50, and γ_3=-80)

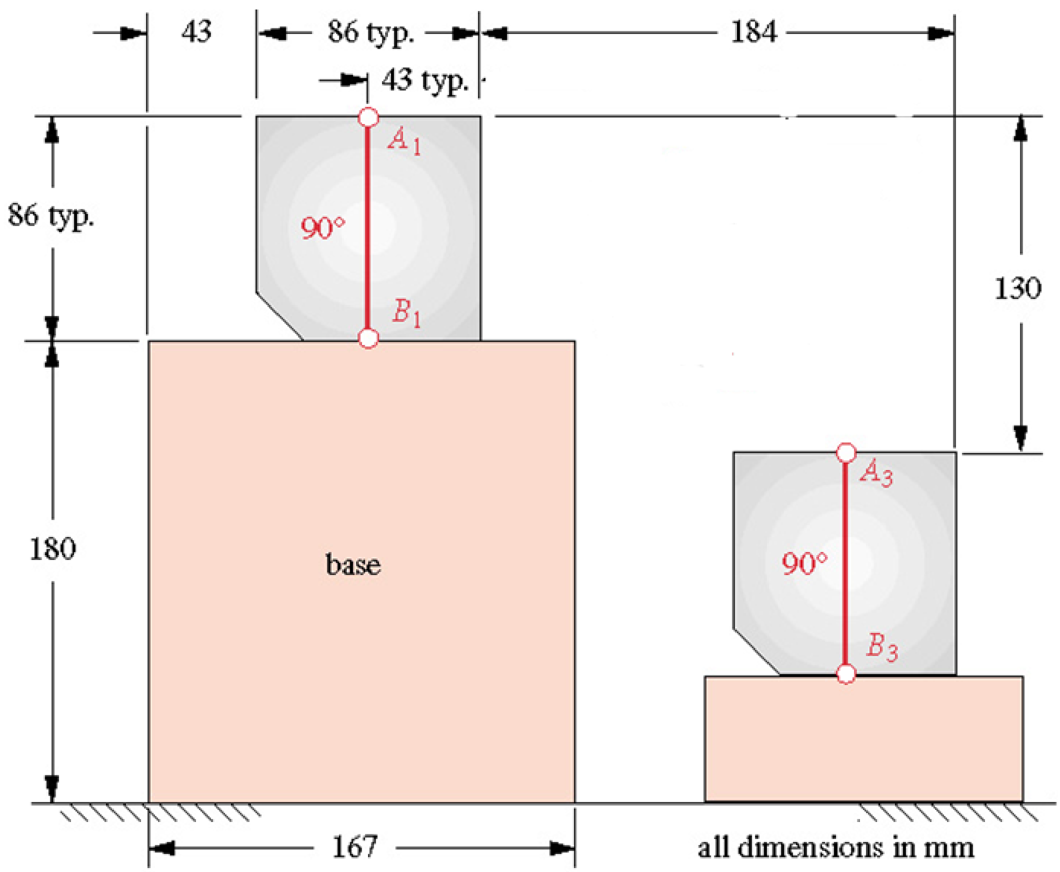


1a. Using the program that you developed in class, perform an analytical synthesis to design a linkage that will move the box from positions 1 to 2 to 3, and has ground pivots on the base. Show all work need to calculate the parameters used in your computer model below. Staple a copy of the computer solution directly after this page.

1b. Draw the final synthesized mechanism to scale in Position 1.



PROBLEM 2 (30 pts): A container of liquid sit on top of a base (A_1B_1) and is required to move to the position on the lower step (A_3B_3).



2a. Using the program that you developed in class, perform an analytical synthesis to design a linkage that will move the box from the top positions (A_1B_1) to the bottom position (A_3B_3), and has ground pivots on the base. Show all work need to calculate the parameters used in your computer model below. Staple a copy of the computer solution directly after this page.

2b. Does your linkage design require the base to be modified in any way? Explain. Can it be designed so that the base does not have to be modified? How?

2c. Will the linkage design enable the container to hold a liquid without spilling it as it travels from the upper base position to the lower platform position? Explain. Can the linkage be designed to hold the container in the same position throughout its motion from the upper to lower base? How?

PROBLEM 3 (40 pts): Design a single-dwell cam to move a follower from 0 to 2” in 60°, fall 2” in 90°, and dwell for the remainder. The total cycle must take 2 seconds.