PROBLEM 2.39 | THE STRAINS SHOWN ARE FOR PLANE STRESS.

$$\epsilon_x = -120\mu,\quad \epsilon_y = -200\mu,\quad \gamma_{xy} = -60\mu,\quad \theta = 25°$$

(a) DETERMINE THE STRAINS ASSOCIATED WITH AN AXIS SYSTEM ROTATED θ (DEFINED POSITIVE COUNTERCLOCKWISE) USING THE TRANSFORMATION EQUATIONS ALONE

(b) DETERMINE THE PRINCIPAL STRAINS AND THE DIRECTION EACH STRAIN MAKES WITH THE X-Y AXES USING EQUATIONS ONLY

(c) REPEAT PARTS (a) AND (b) USING MOHR'S CIRCLE

GIVEN:

CONSTRAINTS

1. $\epsilon_x = -120\mu,\ \epsilon_y = -200\mu,\ \gamma_{xy} = -60\mu,\ \theta = 25°$

ASSUMPTION

1. PLANE STRESS

FIND:

1. $\epsilon_{x'},\ \epsilon_{y'},\ \gamma_{x'y'}$ FOR $\theta = 25°$
2. THE PRINCIPAL STRAINS

SOLUTION

STARTING WITH EQUATIONS ONLY

$$\epsilon_{x'} = \epsilon_x \cdot \cos^2\theta + \epsilon_y \cdot \sin^2\theta + \gamma_{xy}\cos\theta\sin\theta$$

$$= (-120\mu)\cos^2(25°) + (-200\mu)\sin^2(25°) + (-60\mu)\cos(25)\cdot\sin(25)$$

$$= \boxed{-157.3\mu}$$

$$\epsilon_{y'} = \epsilon_x \cdot \sin^2\theta + \epsilon_y \cos^2\theta - \gamma_{xy}\cdot\sin\theta\cdot\cos\theta$$

$$= (-120\mu)\cdot\sin^2(25) + (-200\mu)\cdot\cos^2(25) - (-60\mu)\sin(25)\cdot\cos(25)$$

$$= \boxed{-162.7\mu}$$

$$\gamma_{x'y'} = -(\epsilon_x - \epsilon_y)\cdot\sin\theta\cdot\cos\theta + \frac{\gamma_{xy}}{2}(\cos^2\theta - \sin^2\theta)$$

$$= -\left[(-120\mu) - (-200\mu)\right]\sin 25°\cdot\cos 25° + \frac{(-60\mu)}{2}\left[\cos^2(25) - \sin^2 25\right]$$

$$= \boxed{-49.93\mu}$$

THE PRINCIPAL STRAINS CAN NOW BE CALCULATED

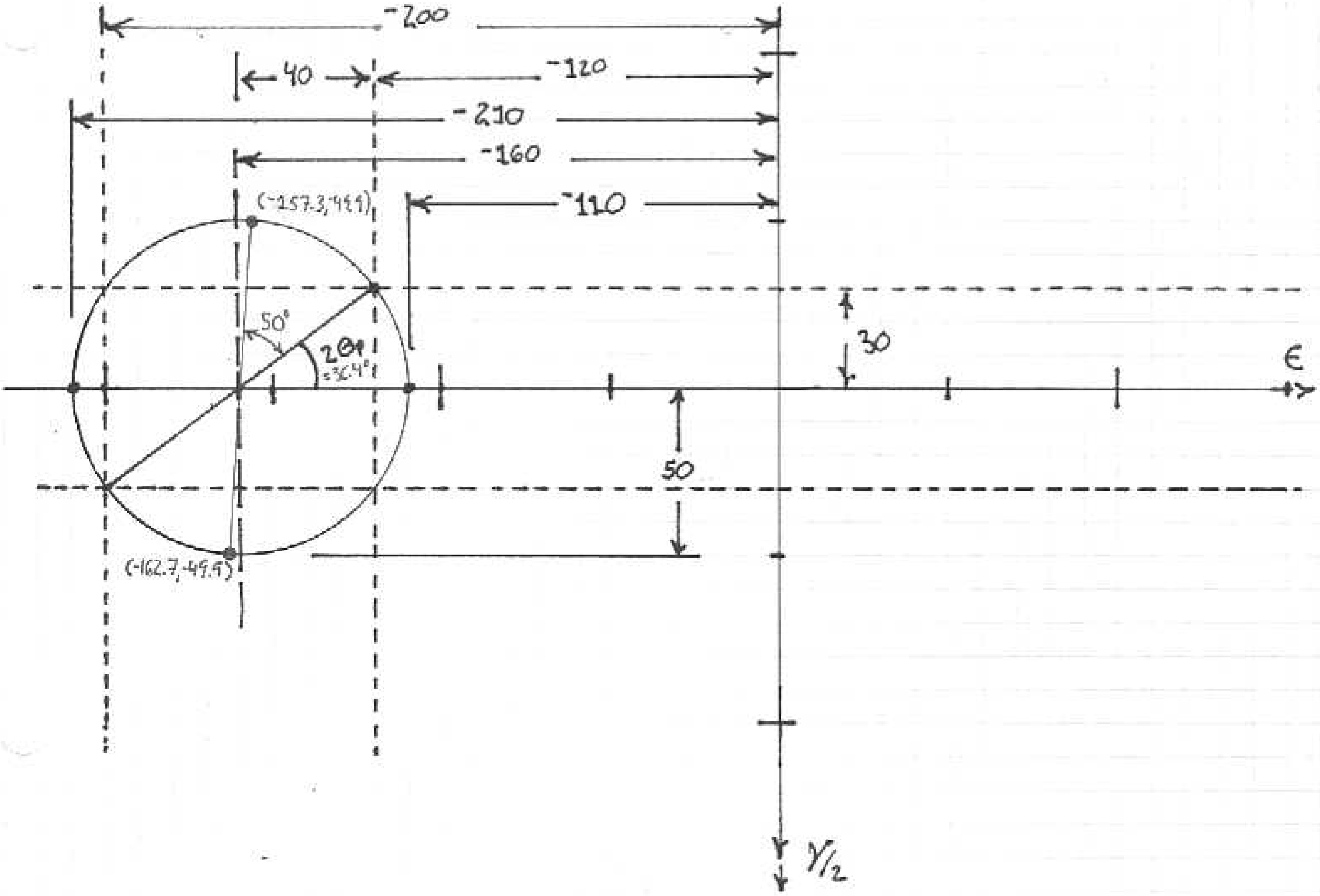
$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{(-120\mu) + (-200\mu)}{2} \pm \sqrt{\left[\frac{(-120\mu) - (-200\mu)}{2}\right]^2 + \left[\frac{-60\mu}{2}\right]^2}$$

$$= -160\mu \pm 50\mu$$

$$\boxed{\begin{aligned} \epsilon_1 &= -110\mu \\ \epsilon_2 &= -210\mu \end{aligned}}$$

$$\frac{\gamma_{max}}{2} = \pm\sqrt{\left[\frac{(-120\mu) - (-200\mu)}{2}\right]^2 + \left[\frac{-60\mu}{2}\right]^2} = \boxed{\pm 50\mu}$$



THE ANGLE TO THE PRINCIPAL PLANE IS CALCULATED

$$\tan 2\theta_p = \frac{30}{40} \Rightarrow 2\theta_p = \tan^{-1}\frac{30}{40} = 36.37$$

$$\boxed{\theta_p = 18.43^\circ}$$

SUMMARY:

WHEN USING MOHR'S CIRCLE FOR STRAIN IT IS IMPORTANT TO KEEP IN MIND THAT THIS IS A TENSOR TRANSFORMATION. BECAUSE OF THIS TENSORIAL, NOT ENGINEERING, STRAIN IS USED. SO $\gamma/2$ IS PLOTTED ALONG THE VERTICAL AXES.

THE ADVANTAGE TO USING MOHR'S CIRCLE IS THAT THERE IS NO NEED FOR TRANSFORMATION EQUATIONS.