PROBLEM 8 CONSTRUCT THE SHEAR FORCE, BENDING MOMENT, CURVATURE, AND DEFLECTION DIAGRAMS FOR THIS BEAM.

GIVEN:

1. A SIMPLY SUPPORTED BEAM OF LENGTH 6 m.
2. 1.5 m VERTICAL ARM LOCATED 4 m FROM THE LEFT END OF THE BEAM
3. A PULLEY LOCATED ON THE HORIZONTAL BEAM 2 m FROM THE LEFT END.
4. A CABLE ATTACHED AT THE TOP OF THE VERTICAL ARM WRAPPING AROUND THE PULLEY.
5. 27 kN LOAD ATTACHED TO THE CABLE UNDER THE BEAM

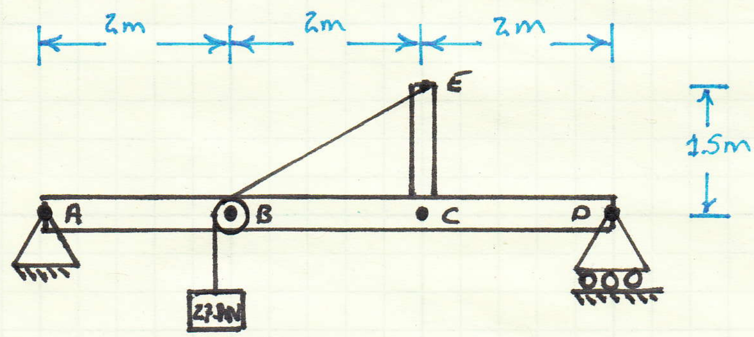
ASSUMPTIONS:

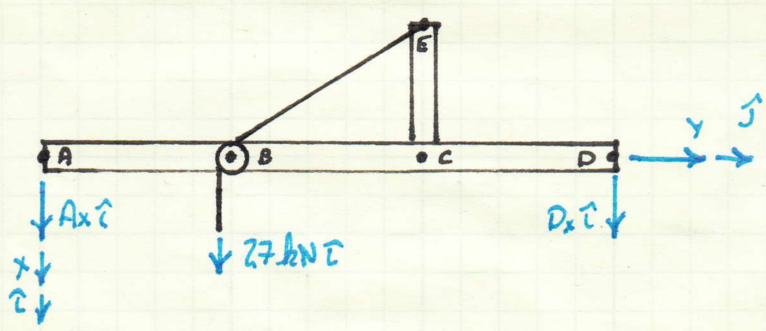
1. THE BEAM IS ORIGINALLY STRAIGHT, THE HORIZONTAL DEFLECTIONS ARE SMALL
2. LINEAR-ELASTIC MATERIAL RESPONSE
3. VERTICAL ARM IS RIGID
4. THE CABLE IS RIGID
5. THE ROLLER IS FRICTIONLESS AND THE RADIUS CAN BE IGNORED.

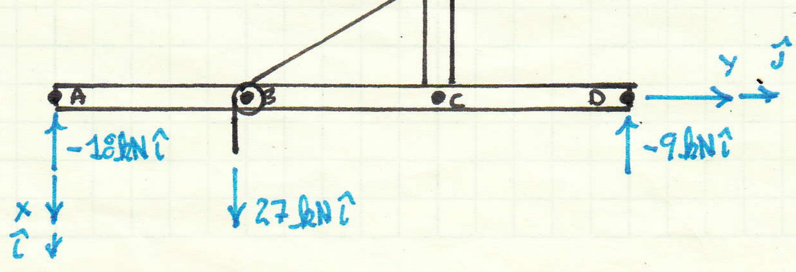
FIND:

1. SHEAR DIAGRAM
2. BENDING MOMENT DIAGRAM
3. CURVATURE (ELASTIC CURVE SLOPE) DIAGRAM
4. DISPLACEMENT DIAGRAM

FIGURE:

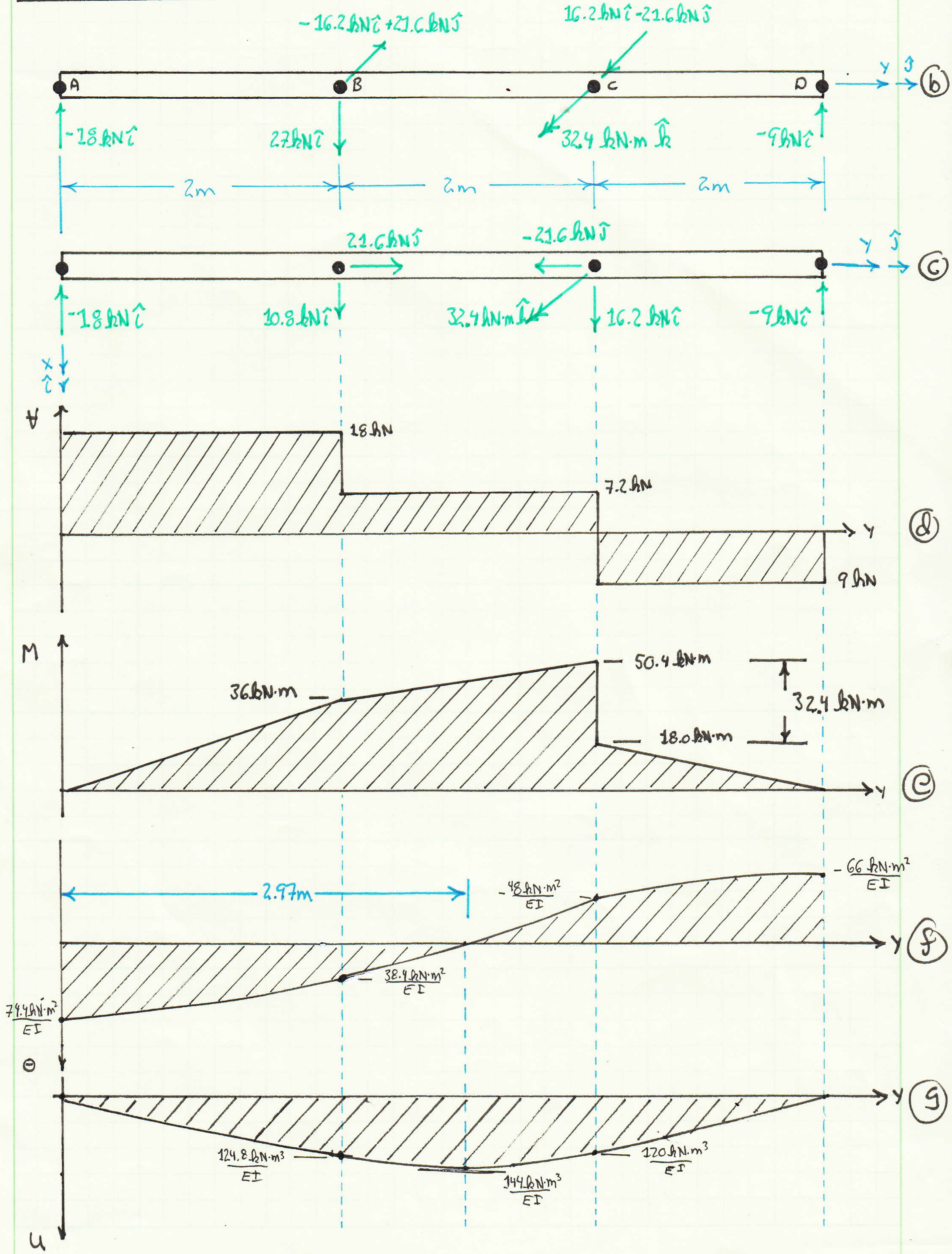






(a)

SOLUTION DIAGRAMS:



SINGULARITY FUNCTION SOLUTION.

STARTING WITH THE BEAM LOADING ILLUSTRATED IN (C)

$$q(y) = -18\,kN\langle y-0\rangle_{-1} + 10.8\,kN\langle y-2m\rangle_{-1} + 32.4\,kN\cdot m\langle y-4m\rangle_{-2} + 16.2\,kN\langle y-4m\rangle_{-1} - 9\,kN\langle y-6m\rangle_{-1} \tag{53}$$

$$V(y) = -\int q(y)\,dy$$

$$= \int\Big[18\,kN\langle y-0\rangle_{-1} - 10.8\,kN\langle y-2m\rangle_{-1} - 32.4\,kN\cdot m\langle y-4m\rangle_{-2} - 16.2\,kN\langle y-4m\rangle_{-1} + 9\,kN\langle y-6m\rangle_{-1}\Big]\,dy$$

$$= 18\,kN\langle y-0\rangle^{0} - 10.8\,kN\langle y-2m\rangle^{0} - 32.4\,kN\cdot m\langle y-4m\rangle_{-1} - 16.2\,kN\langle y-4m\rangle^{0} + 9\,kN\langle y-6m\rangle^{0} \tag{54}$$

$$M(y) = \int V(y)\,dy$$

$$= \int\Big[18\,kN\langle y-0\rangle^{0} - 10.8\,kN\langle y-2m\rangle^{0} - 32.4\,kN\langle y-4m\rangle_{-1} - 16.2\,kN\langle y-4m\rangle^{0} + 9\,kN\langle y-6m\rangle^{0}\Big]\,dy$$

$$= 18\,kN\langle y-0\rangle^{1} - 10.8\,kN\langle y-2m\rangle^{1} - 32.4\,kN\cdot m\langle y-4m\rangle^{0} - 16.2\,kN\langle y-4m\rangle^{1} + 9\,kN\langle y-6m\rangle^{1} \tag{55}$$

$$\Theta = -\frac{1}{EI}\int M(y)\,dy$$

$$= \int\Big[-\frac{18\,kN}{EI}\langle y-0\rangle^{1} + \frac{10.8\,kN}{EI}\langle y-2m\rangle^{1} + \frac{32.4\,kN\cdot m}{EI}\langle y-4m\rangle^{0} + \frac{16.2\,kN}{EI}\langle y-4m\rangle^{1} - \frac{9\,kN}{EI}\langle y-6m\rangle^{1}\Big]\,dy$$

$$= -\frac{18\,kN}{2EI}\langle y-0\rangle^{2} + \frac{10.8\,kN}{2EI}\langle y-2m\rangle^{2} + \frac{32.4\,kN\cdot m}{EI}\langle y-4m\rangle^{1} + \frac{16.2\,kN}{2EI}\langle y-4m\rangle^{2} - \frac{9\,kN}{2EI}\langle y-6m\rangle^{2} + C_4$$

$$\Theta = -\frac{9\,kN}{EI}\langle y-0\rangle^2 + \frac{5.4\,kN}{EI}\langle y-2m\rangle^2 + \frac{32.4\,kN\cdot m}{EI}\langle y-4m\rangle^1 + \frac{8.1\,kN}{EI}\langle y-4m\rangle^2 - \frac{4.5\,kN}{EI}\langle y-6m\rangle^2 + C_1 \quad (56)$$

$$u(y) = \int \Theta(y)\,dy$$

$$= \int\left[-\frac{9\,kN}{EI}\langle y-0\rangle^2 + \frac{5.4\,kN}{EI}\langle y-2m\rangle^2 + \frac{32.4\,kN\cdot m}{EI}\langle y-4m\rangle^1 + \frac{8.1\,kN}{EI}\langle y-4m\rangle^2 - \frac{4.5\,kN}{EI}\langle y-6m\rangle^2 + C_1\right]dy$$

$$= -\frac{9\,kN}{3EI}\langle y-0\rangle^3 + \frac{5.4\,kN}{3EI}\langle y-2m\rangle^3 + \frac{32.4\,kN\cdot m}{2EI}\langle y-4m\rangle^2 + \frac{8.1\,kN}{3EI}\langle y-4m\rangle^3 - \frac{4.5\,kN}{3EI}\langle y-6m\rangle^3 + C_1\cdot y + C_2$$

$$= -\frac{3\,kN}{EI}\langle y-0\rangle^3 + \frac{1.8\,kN}{EI}\langle y-2m\rangle^3 + \frac{16.2\,kN\cdot m}{EI}\langle y-4m\rangle^2 + \frac{2.7\,kN}{EI}\langle y-4m\rangle^3 - \frac{1.5\,kN}{EI}\langle y-6m\rangle^3 + C_1\cdot y + C_2 \quad (57)$$

THE TWO CONSTANTS IN (56) AND (57), C_1 & C_2, ARE DETERMINED USING THE DISPLACEMENT BOUNDARY CONDITIONS.

THE FIRST BOUNDARY CONDITION IS AT $y=0$, $u=0$.

$$u(0) = 0 = -\frac{3\,kN}{EI}(0-0)^3 + C_1(0) + C_2 \Rightarrow \underline{\underline{C_2 = 0}} \quad (58)$$

THE SECOND BOUNDARY CONDITION IS AT $y=6m$, $u(6m)=0$

$$u(6m) = -\frac{3\,kN}{EI}(6m-0)^3 + \frac{1.8\,kN}{EI}(6m-2m)^3 + \frac{16.2\,kN\cdot m}{EI}(6m-4m)^2 + \frac{2.7\,kN}{EI}(6m-4m)^3 - \frac{1.5\,kN}{EI}(6m-6m)^3 + C_1\cdot 6m = 0$$

$$0 = -\frac{446.4\,kN\cdot m^3}{EI} + C_1\cdot 6m \Rightarrow C_1 = \frac{74.4\,kN\cdot m^2}{EI} \quad (59)$$

(58) AND (59) CAN NOW BE SUBSTITUTED INTO (56) AND (57)

$$(56) \rightarrow \Theta = -\frac{9\,kN}{EI}\langle y-0\rangle^2 + \frac{5.4\,kN}{EI}\langle y-2m\rangle^2 + \frac{32.4\,kN\cdot m}{EI}\langle y-4m\rangle^1 + \frac{8.1\,kN}{EI}\langle y-4m\rangle^2 - \frac{4.5\,kN}{EI}\langle y-6m\rangle^2 + \frac{74.4\,kN\cdot m^2}{EI} \quad (60)$$

$$(57) \rightarrow u = -\frac{3\,kN}{EI}\langle y-0\rangle^3 + \frac{1.8\,kN}{EI}\langle y-2m\rangle^3 + \frac{16.2\,kN\cdot m}{EI}\langle y-4m\rangle^2 + \frac{2.7\,kN}{EI}\langle y-4m\rangle^3 - \frac{1.5\,kN}{EI}\langle y-6m\rangle^3 + \frac{74.4\,kN\cdot m^2}{EI}\cdot y \quad (61)$$

(60) AND (61) CAN NOW BE COMPARED WITH THE RESULTS FROM (33)-(38) THAT ARE SUMMARIZED IN FIGURES (f) & (g)

$$\Theta(0) = -\frac{9\,kN}{EI}(0-0)^2 + \frac{74.4\,kN\cdot m^2}{EI} = \frac{74.4\,kN\cdot m^2}{EI} \checkmark \text{ w/ } (41)$$

$$u(0) = -\frac{3\,kN}{EI}(0-0)^3 + \frac{74.4\,kN\cdot m^2}{EI}\cdot(0) = 0 \checkmark \text{ w/ } (33)$$

$$\Theta(2m) = -\frac{9\,kN}{EI}(2m-0)^2 + \frac{5.4\,kN}{EI}(2m-2m)^2 + \frac{74.4\,kN\cdot m^2}{EI} = \frac{38.4\,kN\cdot m^2}{EI} \checkmark \text{ w/ } (42) \,\&\, (45)$$

$$u(2m) = -\frac{3\,kN}{EI}(2m-0)^3 + \frac{1.8\,kN}{EI}(2m-2m)^3 + \frac{74.4\,kN\cdot m^2}{EI}(2m)$$

$$= \frac{124.8\,kN\cdot m^2}{EI} \checkmark \text{ w/ } (40) \,\&\, (43)$$

$$\Theta(2.97m) = -\frac{9\,kN}{EI}(2.97m-0)^2 + \frac{5.4\,kN}{EI}(2.97m-2m)^2 + \frac{74.4\,kN\cdot m^2}{EI}$$

$$= 0 \checkmark \text{ w/ } (51)$$

$$u(2.97m) = -\frac{3\,kN}{EI}(2.97m-0)^3 + \frac{1.8\,kN}{EI}(2.97m-2m)^3 + \frac{74.4\,kN\cdot m^2}{EI}(2.97m)$$

$$= \frac{144.0\,kN\cdot m^3}{EI} \checkmark \text{ w/ } (52)$$

$$\theta(4m) = -\frac{9\,kN}{EI}(4m-0)^2 + \frac{5.4\,kN}{EI}(4m-2m)^2 + \frac{32.4\,kN\cdot m}{EI}(4m-4m) + \frac{8.1\,kN}{EI}(4m-4m)^2 + \frac{74.4\,kN\cdot m^2}{EI}$$

$$= -\frac{48.0\,kN\cdot m^2}{EI} \quad \checkmark \text{ w/ } (46) \ \& \ (49)$$

$$u(4m) = -\frac{3\,kN}{EI}(4m-0)^3 + \frac{1.8\,kN}{EI}(4m-2m)^3 + \frac{16.2\,kN\cdot m}{EI}(4m-4m)^2 + \frac{2.7\,kN}{EI}(4m-4m)^3 + \frac{74.4\,kN\cdot m^2}{EI}(4m)$$

$$= \frac{120\,kN\cdot m^3}{EI} \quad \checkmark \text{ w/ } (44) \ \& \ (47)$$

$$\theta(6m) = -\frac{9\,kN}{EI}(6m-0)^2 + \frac{5.4\,kN}{EI}(6m-2m)^2 + \frac{32.4\,kN\cdot m}{EI}(6m-4m) + \frac{8.1\,kN}{EI}(6m-4m)^2 - \frac{4.5\,kN}{EI}(6m-6m)^2 + \frac{74.4\,kN\cdot m^2}{EI}$$

$$= -\frac{66.0\,kN\cdot m^2}{EI} \quad \checkmark \text{ w/ } (50)$$

$$u(6m) = -\frac{3\,kN}{EI}(6m-0)^3 + \frac{1.8\,kN}{EI}(6m-2m)^3 + \frac{16.2\,kN\cdot m}{EI}(6m-4m)^2 + \frac{2.7\,kN}{EI}(6m-4m)^3 - \frac{1.5\,kN}{EI}(6m-6m)^3 + \frac{74.4\,kN\cdot m^2}{EI}(6m)$$

$$= 0 \quad \checkmark \text{ w/ } (48)$$

SUMMARY FOR SINGULARITY FUNCTIONS

THE EQUATIONS FOR THE SHEAR FORCE, BENDING MOMENT, SLOPE OF THE ELASTIC CURVE, AND DISPLACEMENT OF THE ELASTIC CURVE YIELD THE SAME VALUES AT THE CRITICAL POINTS ON THIS BEAM. THE USE OF SINGULARITY FUNCTIONS REMOVES THE NEED TO CONSIDER CONTINUITY CONDITIONS BETWEEN THE VARIOUS REGIONS OF THE BEAM. IT IS IMPORTANT TO NOTE THAT LIKE ALL SOLUTIONS AT THIS LEVEL OF THEORY, THE LOADING IS EXTREMELY IDEAL AND FURTHER ANALYSIS SHOULD ONLY BE CONSIDERED AWAY FROM THE LOAD APPLICATION POINTS.