

PROBLEM 9.4-10 DERIVE THE EQUATIONS OF THE DEFLECTION CURVE FOR A SIMPLE BEAM AB SUPPORTING A TRIANGULARLY DISTRIBUTED LOAD OF MAXIMUM INTENSITY  $q_0$  ACTING ON THE RIGHT-HAND HALF OF THE BEAM. ALSO, DETERMINE THE ANGLES OF ROTATION  $\theta_A$  AND  $\theta_B$  AT THE ENDS AND THE DEFLECTION  $s_c$  AT THE MIDPOINT.

GIVEN:

CONSTRAINTS

- 1) BEAM WITH PIN SUPPORT AT LEFT HAND SIDE AND ROLLER SUPPORT AT RIGHT-HAND SIDE.
- 2) TRIANGULAR LOAD STARTING AT MID SPAN AND INCREASING LINEARLY TO THE RIGHT HAND SUPPORT

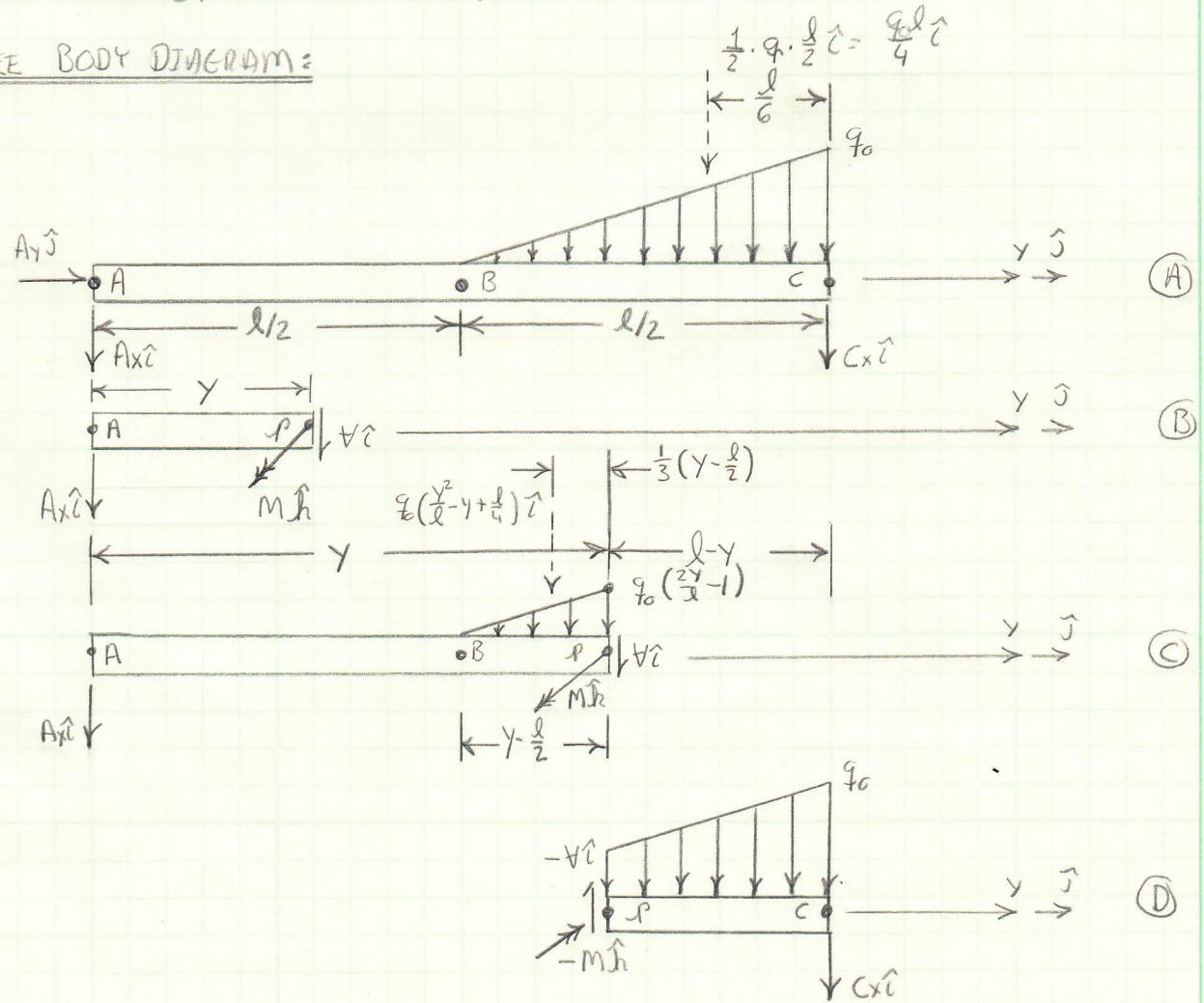
ASSUMPTIONS

- 1) DEFLECTIONS ARE SMALL
- 2) MATERIAL RESPONDS IN A LINEAR ELASTIC MANNER

FIND:

- 1) ANGLES OF ROTATION AT THE END POINTS
- 2) DEFLECTION AT THE MIDPOINT

FREE BODY DIAGRAM:



STATICS:

FIRST LET'S DETERMINE THE REACTIONS AT A S.C. USING (A)

$$\sum F_x = 0 = Ax + C_x + \frac{q_0 l}{4} = 0 \Rightarrow Ax + C_x = -\frac{q_0 l}{4} \quad (1)$$

$$\sum M_{at C} = 0 = Ax \cdot l + \frac{l}{6} \cdot \frac{q_0 l}{4} \Rightarrow Ax = -\frac{q_0 l}{24} \quad (2)$$

$$(2) \rightarrow (1) \Rightarrow C_x = -\frac{q_0 l}{4} \cdot \frac{6}{6} + \frac{q_0 l}{24} = -\frac{5 q_0 l}{24} \quad (3)$$

BEFORE THE SOLUTION PROCEEDS AN EQUATION FOR THE DISTRIBUTED LOAD MUST BE DEVELOPED

$$q = m \cdot y + b$$

$$m = \frac{\text{RISE}}{\text{RUN}} = \frac{q_0 / h_2}{l} = \frac{2 q_0}{l}; \quad b = -q_0 \quad (\text{DUE TO SYMMETRY})$$

$$q = \frac{2 \cdot q_0}{l} \cdot y - q_0 = q_0 \left( \frac{2y}{l} - 1 \right) \quad (4)$$

STARTING WITH THE FREE BODY DIAGRAM (B) THE EQUATIONS FOR THE INTERNAL EQUILIBRIUM FOR THE REGION  $0 < y < l/2$

$$\sum F_x = 0 = -\frac{q_0 l}{24} + V \Rightarrow V = \frac{q_0 l}{24} \quad (5)$$

$$\sum M_{at p} = 0 = -\frac{q_0 l}{24} \cdot y + M \Rightarrow M = \frac{q_0 l}{24} \cdot y \quad (6)$$

NOW LET'S CONSIDER THE REGION  $\frac{l}{2} < y < l$ . THE EQUIVALENT FORCE THAT IS A RESULT OF THE DISTRIBUTED LOAD IS

$$\frac{1}{2} \cdot \left( y - \frac{l}{2} \right) q_0 \left( \frac{2y}{l} - 1 \right) = \frac{q_0}{2} \left( \frac{2y^2}{l} - y - y + \frac{l}{2} \right) = \frac{q_0^2}{2} \left( \frac{y^2}{l} - y + \frac{l}{4} \right) \quad (7)$$

NOW CONSIDERING THE EQUILIBRIUM OF (C)

$$\begin{aligned} \sum F_x = 0 &= -\frac{q_0 l}{24} + \frac{q_0}{2} \left( \frac{y^2}{l} - y + \frac{l}{4} \right) + V \\ \Rightarrow V &= \frac{q_0 l}{24} - \frac{q_0}{2} \left( \frac{y^2}{l} - y + \frac{l}{4} \right) = -\frac{q_0}{2} \left( \frac{y^2}{l} - y + \frac{5l}{24} \right) \end{aligned} \quad (8)$$

$$\sum M_{at p} = M - \frac{q_0 l}{24} \cdot y + \frac{q_0}{2} \left( \frac{y^2}{l} - y + \frac{l}{4} \right) \cdot \frac{1}{3} \left( y - \frac{l}{2} \right)$$

$$\Rightarrow M = \frac{q_0 l}{24} y - \frac{q_0}{2} \left( \frac{y^2}{l} - y + \frac{l}{4} \right) \cdot \frac{1}{3} \cdot \left( y - \frac{l}{2} \right)$$

$$= \frac{q_0 l}{24} y - \frac{q_0}{3} \left( \frac{y^3}{l} - \frac{3}{2} y^2 + \frac{yl}{4} - \frac{y^2}{2} + \frac{yl^2}{2} - \frac{l^3}{8} \right)$$

$$M = \frac{q_0 \cdot l}{24} \cdot y - \frac{q_0}{3} \left( \frac{y^3}{l} - \frac{3}{2} y^2 + \frac{3 \cdot y \cdot l}{4} - \frac{l^2}{8} \right) = \frac{q_0 l \cdot y}{24} - q_0 \left( \frac{y^3}{3l} - \frac{y^2}{2} + \frac{y \cdot l}{4} - \frac{l^2}{24} \right) \quad (9)$$

$$M = -q_0 \left( \frac{y^3}{3l} - \frac{y^2}{2} + \frac{5 \cdot y \cdot l}{24} - \frac{l^2}{24} \right) = \underline{\underline{q_0 \left( \frac{-y^3}{3l} + \frac{y^2}{2} - \frac{5}{24} y \cdot l + \frac{l^2}{24} \right)}} \quad (9)$$

JUST AS A CHECK, THE DERIVATIVE OF (9) DOES EQUAL (8).

### MECHANICS:

NOW THAT EQUATIONS FOR THE BENDING MOMENTS IN BOTH SECTIONS OF THE BEAM HAVE BEEN DEVELOPED, EQUATIONS FOR THE CURVATURE AND DEFLECTION IN BOTH OF THESE SECTIONS CAN NOW BE DETERMINED

$$0 < y < \frac{l}{2}$$

$$\frac{d^2 u}{dy^2} = -\frac{M}{EI} = -\frac{q_0 \cdot l \cdot y}{24 \cdot EI}$$

$$\Theta = \frac{du}{dy} = -\frac{q_0 \cdot l}{24 \cdot EI} \int y \cdot dy = -\frac{q_0 \cdot l}{24 \cdot EI} \frac{y^2}{2} + C_1 = -\frac{q_0 \cdot l}{48 \cdot EI} y^2 + C_1 \quad (10)$$

$$u = \int \left( -\frac{q_0 \cdot l}{48 \cdot EI} y^2 + C_1 \right) dy = -\frac{q_0 \cdot l}{48 \cdot EI} \frac{y^3}{3} + C_1 \cdot y + C_2$$

$$= -\frac{q_0 \cdot l}{144 \cdot EI} y^3 + C_1 y + C_2 \quad (11)$$

THE CONSTANTS  $C_1$  &  $C_2$  IN (10) AND (11) MUST BE DETERMINED THROUGH BOUNDARY CONDITIONS. IN THE REGION UNDER CONSIDERATION THE ONLY BOUNDARY CONDITION AVAILABLE AT THIS TIME IS

$$u(0) = 0$$

SUBSTITUTING THIS INTO (11)

$$u(0) = 0 = -\frac{q_0 \cdot l}{144 \cdot EI} \cdot (0)^3 + C_1(0) + C_2 = \underline{\underline{C_2 = 0}}$$

THUS THE CURVATURE AND DEFLECTION IN THE REGION FROM  $0 < y < \frac{l}{2}$  BECOME

$$\Theta = -\frac{q_0 \cdot l}{48 \cdot EI} y^2 + C_1 \quad (10)$$

$$u = -\frac{q_0 \cdot l}{144 \cdot EI} y^3 + C_1 y \quad (11)$$

THE CONSTANT  $C_3$  WILL BE DETERMINED THROUGH THE CONTINUITY CONDITIONS BETWEEN THE TWO SECTIONS OF THE BEAM. NOW LETS CONSIDER THE OTHER REGION.

$l/2 < y \leq l$

$$\frac{d^2u}{dy^2} = -\frac{M}{EI} = \frac{q_0}{EI} \left( \frac{y^3}{3l} - \frac{y^2}{2} + \frac{5l}{24}y - \frac{l^2}{24} \right)$$

$$\Theta = \frac{du}{dy} = \frac{q_0}{EI} \int \left( \frac{y^3}{3l} - \frac{y^2}{2} + \frac{5l}{24}y - \frac{l^2}{24} \right) dy$$

$$= \frac{q_0}{EI} \left( \frac{y^4}{12l} - \frac{y^3}{6} + \frac{5l}{48}y^2 - \frac{l^2}{24}y \right) + C_3 \quad (12)$$

$$u = \frac{q_0}{EI} \left( \frac{y^5}{60l} - \frac{y^4}{24} + \frac{5l}{144}y^3 - \frac{l^2}{48}y^2 \right) + C_3y + C_4 \quad (13)$$

THE BOUNDARY CONDITION IN THIS SECTION OF THE BEAM THAT IS AVAILABLE AT THIS TIME IS

$$u(l) = 0$$

$$u(l) = 0 = \frac{q_0}{EI} \left( \frac{l^4}{60l} \cdot \frac{12}{12} - \frac{l^4}{24} \cdot \frac{30}{30} + \frac{5l^5}{144} \cdot \frac{5}{5} - \frac{l^4}{48} \cdot \frac{15}{15} \right) + C_3l + C_4$$

$$= -\frac{8}{720} \frac{q_0 l^4}{EI} + C_3l + C_4$$

$$\Rightarrow C_4 = \frac{8}{720} \frac{q_0 l^4}{EI} - C_3l \quad (14)$$

FROM THIS RESULT (13) CAN BE REWRITTEN

$$u = \frac{q_0}{EI} \left( \frac{y^5}{60l} - \frac{y^4}{24} + \frac{5l}{144}y^3 - \frac{l^2}{48}y^2 \right) + C_3y + \frac{8}{720} \frac{q_0 l^4}{EI} - C_3l$$

$$= \frac{q_0}{EI} \left( \frac{y^5}{60l} - \frac{y^4}{24} + \frac{5l}{144}y^3 - \frac{l^2}{48}y^2 \right) - C_3(l-y) + \frac{8}{720} \frac{q_0 l^4}{EI} \quad (15)$$

TO COMPLETE THE SOLUTION TO THIS PROBLEM THE CURVATURE AND DISPLACEMENT AT THE INTERFACE BETWEEN THESE TWO REGIONS MUST BE CONSIDERED. THE VALUE OF THESE QUANTITIES AT THIS POINT FOR THE TWO REGIONS MUST BE THE SAME IN ORDER TO SATISFY CONTINUITY.

STARTING BY CONSIDERING THE VALUE OF THE CURVATURES AT  $\frac{l}{2}$  FROM ⑩ AND ⑫

$$⑩ \rightarrow \Theta\left(\frac{l}{2}\right) = -\frac{q_0 \cdot l}{48 \cdot EI} \left(\frac{l}{2}\right)^2 + C_1 = -\frac{q_0 \cdot l^3}{192 \cdot EI} + C_1$$

$$⑫ \rightarrow \Theta\left(\frac{l}{2}\right) = \frac{q_0}{EI} \left[ \left(\frac{l}{2}\right)^4 - \frac{(l/2)^3}{6} + \frac{5l}{48} \cdot \left(\frac{l}{2}\right)^2 - \frac{l^2}{24} \left(\frac{l}{2}\right) \right] + C_3 \\ = \frac{q_0}{EI} \left( \frac{l^3}{192} - \frac{4 \cdot l^3}{448} + \frac{5l^3}{192} - \frac{4 \cdot l^3}{448} \right) + C_3$$

$$= -\frac{2q_0 l^3}{192 EI} + C_3$$

$$\frac{-q_0 l^3}{192 EI} + C_1 = -\frac{2q_0 l^3}{192 EI} + C_3$$

$$C_1 - C_3 = -\frac{q_0 \cdot l^3}{192 \cdot EI} \Rightarrow C_1 = -\frac{q_0 \cdot l^3}{192 \cdot EI} + C_3$$

⑯

NOW THE DEFLECTIONS AT  $\frac{l}{2}$  FOR BOTH REGIONS CAN BE DETERMINED USING ⑪ AND ⑯

$$⑪ \rightarrow U\left(\frac{l}{2}\right) = -\frac{q_0 \cdot l}{144 \cdot EI} \left(\frac{l}{2}\right)^3 + C_1 \left(\frac{l}{2}\right) \\ = -\frac{q_0 \cdot l^4}{1152 \cdot EI} + C_1 \frac{l}{2}$$

$$⑯ \rightarrow U\left(\frac{l}{2}\right) = \frac{q_0}{EI} \left( \frac{\left(\frac{l}{2}\right)^5}{60l} - \frac{\left(\frac{l}{2}\right)^4}{24} + \frac{5 \cdot l}{144} \left(\frac{l}{2}\right)^3 - \frac{l^2}{48} \left(\frac{l}{2}\right)^2 \right) - C_3 \cdot \left(l - \frac{l}{2}\right) + \frac{8}{720} \frac{q_0 l^4}{EI} \\ = \frac{q_0}{EI} \left( \frac{l^{4.3}}{1920 \cdot 3} - \frac{l^{4.15}}{384 \cdot 15} + \frac{5 \cdot l^{4.5}}{1152 \cdot 5} - \frac{l^{4.30}}{192 \cdot 30} \right) - C_3 \cdot \frac{l}{2} + \frac{8}{720} \frac{q_0 l^4}{EI} \cdot \frac{8}{3} \\ = -\frac{q_0 l^{4.17}}{5760 EI} - C_3 \frac{l}{2} + \frac{64}{5760} \frac{q_0 l^4}{EI} = \frac{4.7}{5760} \frac{q_0 l^4}{EI} - C_3 \frac{l}{2}$$

$$\therefore -\frac{q_0 \cdot l^4}{1152 EI} + C_1 \frac{l}{2} = \frac{4.7}{5760} \frac{q_0 \cdot l^4}{EI} - C_3 \frac{l}{2}$$

$$\frac{l}{2} (C_1 + C_3) = \frac{52}{5760} \frac{q_0 \cdot l^4}{EI}$$

⑰

$$\textcircled{16} \rightarrow \textcircled{17} \Rightarrow \frac{l}{2} \left[ -\frac{q_0 l^3}{192 EI} + C_3 + C_3 \right] = \frac{52}{5760} \frac{q_0 l^4}{EI}$$

$$-\frac{q_0 l^4}{384 EI \cdot 15} + C_3 l = \frac{52}{5760} \frac{q_0 l^4}{EI}$$

$$\Rightarrow C_3 = \frac{67}{5760} \frac{q_0 l^3}{EI}$$

$$\textcircled{16} \rightarrow C_1 = -\frac{q_0 l^3}{192 EI} \cdot \frac{30}{30} + \frac{67}{5760} \frac{q_0 l^3}{EI} = \frac{37}{5760} \frac{q_0 l^3}{EI} \quad \textcircled{19}$$

NOW THE CURVATURE AND DEFLECTION IN THE REGION FROM  $0 < y < \frac{l}{2}$  CAN BE WRITTEN

$$\textcircled{20} \rightarrow \Theta = \frac{\frac{q_0}{120} \frac{q_0 \cdot l}{48 \cdot EI} \cdot y^2 + \frac{37}{5760} \frac{q_0 l^3}{EI}}{\frac{q_0 \cdot l}{5760 EI} (-120y^2 + 37l^2)} \quad \textcircled{20}$$

$$\textcircled{21} \rightarrow u = \frac{\frac{q_0}{40} \frac{q_0 l}{144 EI} \cdot y^3 + \frac{37}{5760} \frac{q_0 l^3}{EI} \cdot y}{\frac{q_0 \cdot l}{5760 \cdot EI} (-40 \cdot y^3 + 37 \cdot l^2 \cdot y)} \quad \textcircled{21}$$

THUS AT THE LEFT MOST SUPPORT THE ROTATION AND DEFLECTION ARE

$$\textcircled{20} \rightarrow \Theta(0) = \frac{\frac{q_0 \cdot l}{5760 EI} (-120(0)^2 + 37l^2)}{\frac{37q_0 l^3}{5760 EI}} \quad \boxed{\frac{37q_0 l^3}{5760 EI}}$$

$$\textcircled{21} \rightarrow u(0) = \frac{\frac{q_0 \cdot l}{5760 EI} (-40(0)^3 + 37 \cdot l^2(0))}{\frac{q_0 \cdot l}{5760 EI} (-40 \cdot (0)^3 + 37 \cdot l^2(0))} = 0 \quad \text{THIS AGREES WITH THE BOUNDARY CONDITION}$$

NOW THE DEFLECTION AND CURVATURE AT  $\frac{l}{2}$  CAN BE CALCULATED

$$\textcircled{20} \rightarrow \Theta\left(\frac{l}{2}\right) = \frac{\frac{q_0 \cdot l}{5760 EI} (-120\left(\frac{l}{2}\right)^2 + 37l^2)}{\frac{7}{5760} \frac{q_0 l^3}{EI}} \quad \textcircled{22}$$

$$\textcircled{21} \rightarrow u\left(\frac{l}{2}\right) = \frac{\frac{q_0 \cdot l}{5760 EI} (-40\left(\frac{l}{2}\right)^3 + 37l^2\left(\frac{l}{2}\right))}{\frac{27}{11520 EI} \frac{q_0 l^4}{EI}} = \boxed{\frac{3}{1280} \frac{q_0 l^4}{EI}} \quad \textcircled{23}$$

NOW THE CURVATURE AND DEFLECTION IN THE REGION  $\frac{l}{2} < y < l$   
CAN BE WRITTEN USING THE RESULTS IN (18) AND (19)

$$(12) \rightarrow \Theta = \frac{q_0}{EI} \left( \frac{y^4}{12l} - \frac{y^3}{6} + \frac{5}{48} l \cdot y^2 - \frac{l^2}{24} y \right) + \frac{67}{5760} \frac{q_0 l^3}{EI}$$

$$= \frac{q_0}{5760 \cdot EI} \left( \frac{480 \cdot y^4}{l} - 960 \cdot y^3 + 600l \cdot y^2 - 240 \cdot l^2 \cdot y + 67l^3 \right) \quad (24)$$

$$(15) \rightarrow u = \frac{q_0}{EI} \left( \frac{y^5}{60l} - \frac{y^4}{24} + \frac{5}{144} l \cdot y^3 - \frac{l^2 \cdot y^2}{48} \right) - \frac{67}{5760} \frac{q_0 l^3}{EI} (l - y) + \frac{8}{720} \frac{q_0 l^4}{EI}$$

$$= \frac{q_0}{EI} \left( \frac{y^5}{60l} - \frac{y^4}{24} + \frac{5 \cdot l \cdot y^3}{144} - \frac{l^2 \cdot y^2}{48} \right) - \frac{67}{5760} \frac{q_0 l^4}{EI} + \frac{67}{5760} \frac{q_0 l^3}{EI} y + \frac{8}{720} \frac{q_0 l^4}{EI}$$

$$= \frac{q_0}{5760 \cdot EI} \left( \frac{96 \cdot y^5}{l} - 240 \cdot y^4 + 2000 \cdot l \cdot y^3 - 120 \cdot l^2 \cdot y^2 + 67l^3 y - 3l^4 \right) \quad (25)$$

THE CURVATURE AND DEFLECTION AT THE FAR RIGHT SUPPORT ARE

$$(24) \rightarrow \Theta(l) = \frac{q_0}{5760 \cdot EI} (480l^3 - 960l^3 + 600l^3 - 240l^3 + 67l^3) = \boxed{-\frac{53}{5760} \frac{q_0 l^3}{EI}}$$

$$(25) \rightarrow u(l) = \frac{q_0}{5760 \cdot EI} (96l^4 - 240l^4 + 200l^4 - 120l^4 + 67l^4 - 3l^4) = \textcircled{1} \quad \begin{matrix} \text{checks with} \\ \text{Boundary Cond.} \end{matrix}$$

NOW CONSIDER THE DEFLECTION AND CURVATURE AT MID SPAN

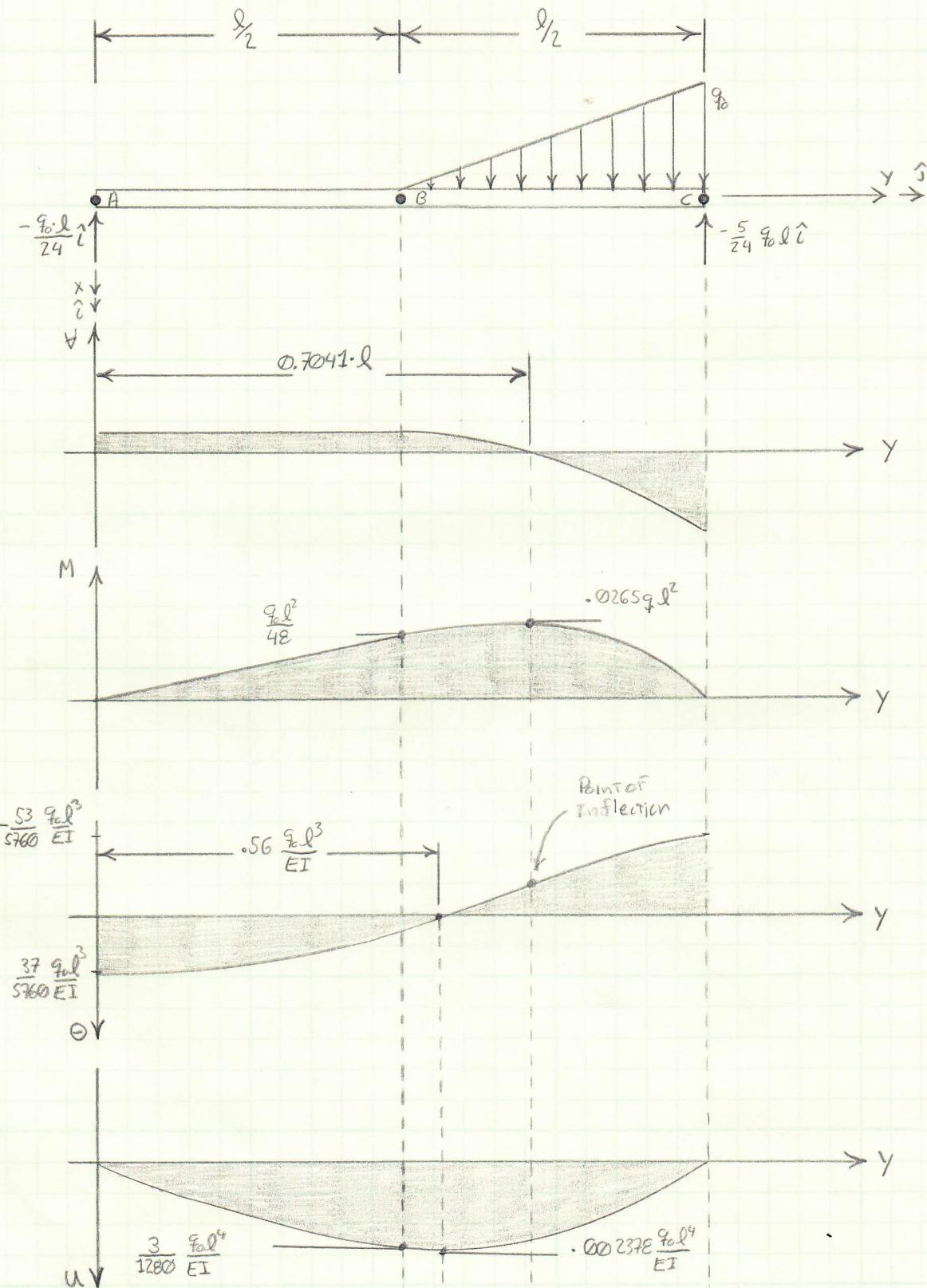
$$(24) \rightarrow \Theta\left(\frac{l}{2}\right) = \frac{q_0}{5760 \cdot EI} \left( \frac{480\left(\frac{l}{2}\right)^4}{l} - 960\left(\frac{l}{2}\right)^3 + 600l\left(\frac{l}{2}\right)^2 - 240l^2\left(\frac{l}{2}\right) + 67l^3 \right)$$

$$= \frac{7}{5760} \frac{q_0 l^3}{EI} \quad \text{THIS VALUE IS THE SAME AS } \textcircled{22} \text{ THE  
CONTINUITY CONDITION IS SATISFIED.}$$

$$(25) \rightarrow u\left(\frac{l}{2}\right) = \frac{q_0}{5760 \cdot EI} \left( \frac{96\left(\frac{l}{2}\right)^5}{l} - 240 \cdot \left(\frac{l}{2}\right)^4 + 200l \cdot \left(\frac{l}{2}\right)^3 - 120l^2 \cdot \left(\frac{l}{2}\right)^2 + 67l^3 \left(\frac{l}{2}\right) - 3l^4 \right)$$

$$= \frac{27}{11,520} \frac{q_0 l^4}{EI} = \boxed{\frac{3}{1280} \frac{q_0 l^4}{EI}} \quad \text{THIS VALUE IS THE  
SAME AS } \textcircled{23}$$

Now LET'S DRAW THE V, M,  $\Theta$ , AND  $u$  DIAGRAMS FOR THIS BEAM.



DETERMINE THE LOCATION WHERE  $\theta = 0$

$$(8) \rightarrow \theta = 0 = -\frac{q_0}{l} \left( \frac{y^2}{l} - y + \frac{5}{24} l \right)$$

$$0 = y^2 - yl + \frac{5}{24} l^2 = y^2 - yl + \left(-\frac{l}{2}\right)^2 - \left(-\frac{l}{2}\right)^2 + \frac{5}{24} l^2$$

$$\theta = \left(y - \frac{l}{2}\right)^2 - \frac{l^2}{24} \Rightarrow y = \frac{l}{2} \pm \sqrt{\frac{l^2}{24}}$$

SINCE WE ARE ONLY CONSIDERING THE DOMAIN BETWEEN  $0 \leq y \leq l$

$$y = \frac{l}{2} + l\sqrt{\frac{1}{24}} = 0.7041 \cdot l$$

THE MAXIMUM VALUE OF THE MOMENT OCCURS AT  $0.7041l$  AND EQUALS

$$(9) \rightarrow M(0.7041l) = q_0 \left( -\frac{(0.7041l)^3}{3l} + \frac{(0.7041l)^2}{2} - \frac{5}{24}(0.7041l) \cdot l + \frac{l^2}{24} \right) \\ = 0.0265 q_0 l^2$$

THE LOCATION WHERE THE CURVATURE IS 0 NEEDS TO BE LOCATED. THIS IS WHERE THE DEFLECTION IS MAXIMUM. TO DO THIS WE NEED TO CHECK FOR INTERSECTS IN BOTH REGIONS OF THE BEAM

$$(10) \rightarrow \frac{q_0 l}{5760EI} (-120y^2 + 37l^2) = 0$$

$$-120y^2 + 37l^2 = 0 \Rightarrow y^2 = \frac{37}{120}l^2 \Rightarrow y = \pm 0.555l$$

SINCE THIS IS OUTSIDE THE DOMAIN FOR EQUATION (10) WE WILL NOW CONSIDER THE OTHER REGION OF THE BEAM

$$(24) \rightarrow \theta = \frac{q_0}{5760EI} \left( \frac{480y^4}{l} - 960y^3 + 600ly^2 - 240l^2 \cdot y + 67l^3 \right)$$

$$= \frac{q_0}{5760EI} \left( 480y^4 - 960l \cdot y^3 + 600l^2y^2 - 240l^3 \cdot y + 67l^4 \right)$$

$$y \approx .56l$$

THE MAXIMUM DEFLECTION AT  $.56l$  CAN NOW BE CALCULATED

$$\begin{aligned}
 (25) \rightarrow U(0.56l) &= \frac{q_0}{5760EI} \left( \frac{96 \cdot y^5}{l} - 240 \cdot y^4 + 200 \cdot l \cdot y^3 - 120 \cdot l^2 \cdot y^2 + 67 \cdot l^3 \cdot y - 3 \cdot l^4 \right) \\
 &= \frac{q_0}{5760lEI} \left( 96y^5 - 240l \cdot y^4 + 200l^2 \cdot y^3 - 120l^3 \cdot y^2 + 67l^4 \cdot y - 3l^5 \right) \\
 &= \frac{q_0}{5760lEI} \left( 96(0.56l)^5 - 240l \cdot (0.56l)^4 + 200l^2 \cdot (0.56l)^3 - 120l^3 \cdot (0.56l)^2 \right. \\
 &\quad \left. + 67l^4 \cdot (0.56l) - 3l^5 \right) \\
 &= 0.002378 \frac{q_0 l^4}{EI}
 \end{aligned}$$

### Summary:

THE SOLUTION PRESENTED GOES BEYOND THE QUESTION ASKED; HOWEVER, I FELT THAT IT WAS NECESSARY TO PRESENT THE COMPLETE SOLUTION. IT IS IMPORTANT TO SEE THE CONNECTION BETWEEN THE  $V$ ,  $M$ ,  $\Theta$ , AND  $U$  DIAGRAMS. IT IS IMPORTANT TO TAKE NOTE OF THE DEFLECTION DIAGRAM AND SEE THAT THE MAXIMUM DEFLECTION OCCURS AT  $0.56l$  AND NOT AT MIDSPEC. DEFLECTION IS OFTEN USED AS A CRITERION FOR FAILURE; HOWEVER, THE MAXIMUM STRESS IN THE BEAM MUST ALSO BE CONSIDERED. THE STRESS AT  $0.7041l$  LOOKS TO BE THE MAXIMUM SINCE THIS IS WHERE THE BENDING MOMENT IS MAXIMUM. ONE SHOULD ALSO CHECK THE LOCATION OF MAXIMUM SHEAR STRESS TO MAKE SURE THIS IS NOT CAUSING AN UNACCEPTABLE PROBLEM.