

**PROBLEM 4-12** | USING SUPERPOSITION, FIND THE DEFLECTION OF THE STEEL SHAFT AT A. FIND THE DEFLECTION AT THE MID SPAN. BY WHAT PERCENTAGE DO THESE TWO VALUES DIFFER?

GIVEN:

1. 39in SPAN BEAM WITH SIMPLE SUPPORTS AT BOTH ENDS
2. BEAM CROSS-SECTIONAL DIAMETER 1.5in
3. 150 lb/ft DISTRIBUTED LOAD APPLIED OVER THE ENTIRE LENGTH OF THE BEAM
4. A 340 lb force APPLIED 15in FROM THE LEFT HAND SUPPORT

ASSUMPTIONS:

1. THE BEAM IS INITIALLY STRAIGHT
2. THE BEAM IS LINEARLY ELASTIC AND ISOTROPIC
3. SMALL DEFLECTION RESULT FROM THE PRESCRIBED LOADS
4. THE DEFORMATION IN THE HORIZONTAL DIRECTION IS NOT RESTRICTED

FIND:

1. THE DEFLECTION AT THE MID SPAN
2. THE DEFLECTION AT POINT A.
3. PERCENT DIFFERENCE BETWEEN THE TWO DEFLECTIONS.

FIGURE / SOLUTION:

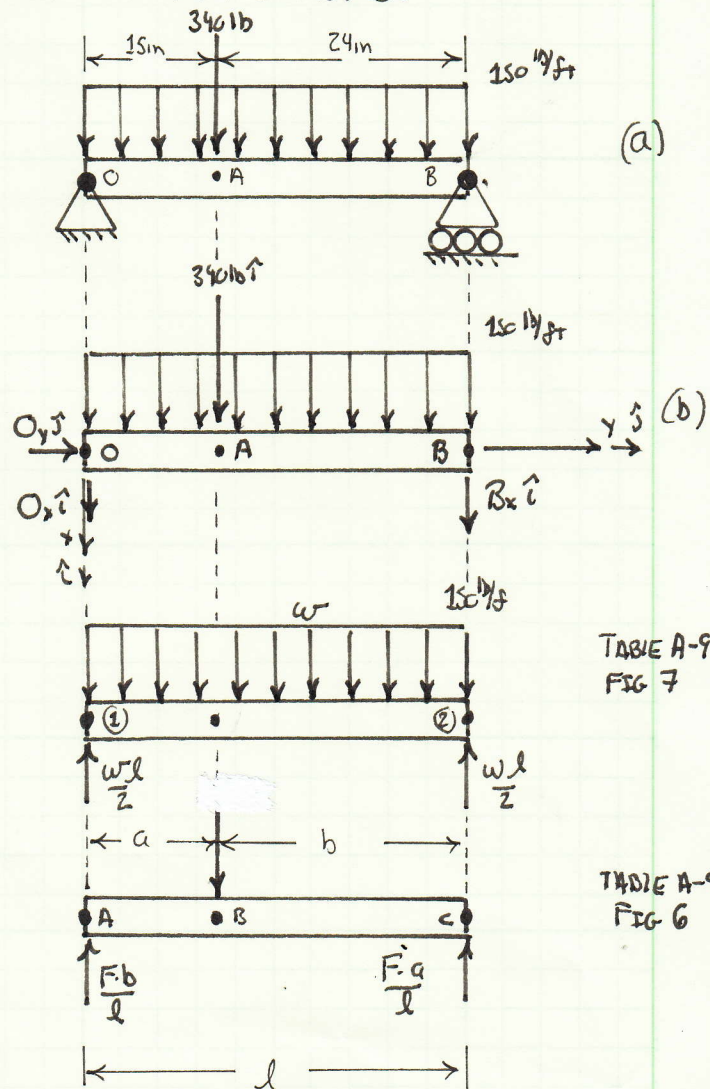
$$I = \frac{\pi}{64} (1.5\text{in})^4 = 0.2485\text{in}^4$$

$$u' = -\frac{w \cdot y}{24 \cdot EI} (2ly^2 - y^3 - l^3)$$

$$u'_{\max} = \frac{5w \cdot l^4}{384EI}$$

$$u''_{AB} = \frac{F \cdot b \cdot y}{6 \cdot EI \cdot l} (y^2 + b^2 - l^2)$$

$$u''_{BC} = \frac{F \cdot a \cdot (l-y)}{6 \cdot EI \cdot l}$$



THE DISPLACEMENTS CAN NOW BE CALCULATED

$$U(15in) = U'(15in) + U''_{AB}(15in)$$

$$= \frac{150 \frac{lb}{ft} \cdot \frac{1}{12} in \cdot 15in}{24 \cdot 30(10^9) \frac{lb}{in^2} \cdot 0.2485in^4} \left[ (39in)^3 + (15in)^3 - 2 \cdot 39in \cdot (15in)^2 \right]$$

$$+ \frac{340lb \cdot 24in \cdot 15in}{6 \cdot 30(10^9) \frac{lb}{in^2} \cdot (0.2485in^4) \cdot 39in} \left[ (39in)^2 - (24in)^2 - (15in)^2 \right]$$

$$= \boxed{0.0978in}$$

$$U(19.5in) = U'(19.5in) + U_{BC}(19.5in)$$

$$= \frac{150 \frac{lb}{ft} \cdot \frac{1}{12} in \cdot 19.5in}{24 \cdot 30(10^9) \frac{lb}{in^2} \cdot 0.2485in^4} \left[ 2 \cdot (39in) \cdot (19.5in)^2 + (19.5in)^3 + (39in)^3 \right]$$

$$+ \frac{340lb \cdot (15in)(19.5in)}{6 \cdot 30(10^9) \frac{lb}{in^2} \cdot 0.2485in^4 \cdot 39in} \left[ (39in)^2 - (15in)^2 - (19.5in)^2 \right]$$

$$= \boxed{0.1027in}$$

$$\% \text{ DIFFERENCE} = \frac{0.1027 - 0.0978}{0.0978} \cdot 100 = \boxed{5.01\%}$$

## SUMMARY

THE TABLES ON PAGE 1 WERE USED TO CALCULATE THE DEFLECTIONS WITHOUT THE NEED TO FIRST CALCULATE THE INTERNAL MOMENT. SUPERPOSITION ALLOWS THE DEFLECTIONS TO SIMPLY BE ADDED AS LONG AS THE STATED ASSUMPTIONS ARE NOT VIOLATED.