



If a, b, c, d are given along with θ_2 , the position of point B remains to be determined.

$$b^2 = (B_x - A_x)^2 + (B_y - A_y)^2$$

From the problem $B_y = c$ (1)

$$b^2 = (B_x - A_x)^2 + (c - A_y)^2$$

$$b^2 = B_x^2 - 2 \cdot B_x \cdot A_x + A_x^2 + (c - A_y)^2$$

$$B_x^2 - 2 \cdot B_x \cdot A_x + A_x^2 + (c - A_y)^2 - b^2 = 0$$

$$B_x^2 - 2 \cdot B_x \cdot A_x + (A_x)^2 - (A_x)^2 + A_x^2 + (c - A_y)^2 - b^2 = 0$$

$$(B_x - A_x)^2 = b^2 - (c - A_y)^2$$

$$B_x = A_x \pm \sqrt{b^2 - (c - A_y)^2}$$

$B_y = c$

FOR THE SLIDER CRANK, THE VECTOR LOOP EQUATION CAN BE WRITTEN

$$\vec{r}_2 + \vec{r}_3 = r_{1x}\hat{i} + r_{1y}\hat{j} \quad (1)$$

WHERE

$$\vec{r}_2 = r_2 \cdot \hat{e}_{r2} \quad (2)$$

$$\vec{r}_3 = r_3 \cdot \hat{e}_{r3} \quad (3)$$

$$\vec{r}_1 = r_{1x}\hat{i} + r_{1y}\hat{j} \quad (4)$$

THE UNIT VECTORS FOR THIS PROBLEM ARE DEFINED AS FOLLOWS

$$\hat{e}_{r2} = \cos \theta_2 \hat{i} + \sin \theta_2 \hat{j} \quad (5)$$

$$\hat{e}_{\theta 2} = -\sin \theta_2 \hat{i} + \cos \theta_2 \hat{j} \quad (6)$$

$$\hat{e}_{r3} = \cos \theta_3 \hat{i} + \sin \theta_3 \hat{j} \quad (7)$$

$$\hat{e}_{\theta 3} = -\sin \theta_3 \hat{i} + \cos \theta_3 \hat{j} \quad (8)$$

SUBSTITUTING (2) → (4) INTO (1)

$$r_2 \cdot \hat{e}_{r2} + r_3 \cdot \hat{e}_{r3} = r_{1x}\hat{i} + r_{1y}\hat{j} \quad (9)$$

TAKING THE DERIVATIVE OF (9) WITH RESPECT TO TIME YIELDS AN EXPRESSION THAT RELATES THE VELOCITY COMPONENTS IN THIS LINKAGE.

$$\underbrace{\dot{r}_2}_{\dot{\theta}_2 \hat{k} \times \hat{e}_{r2}} \cdot \hat{e}_{r2} + r_2 \cdot \underbrace{\dot{\hat{e}}_{r2}}_{\dot{\theta}_2 \hat{k} \times \hat{e}_{r2}} + \underbrace{\dot{r}_3}_{\dot{\theta}_3 \hat{k} \times \hat{e}_{r3}} \cdot \hat{e}_{r3} + r_3 \cdot \underbrace{\dot{\hat{e}}_{r3}}_{\dot{\theta}_3 \hat{k} \times \hat{e}_{r3}} = \dot{r}_{1x}\hat{i} + \dot{r}_{1y}\hat{j}$$

$$r_2 \cdot \dot{\theta}_2 \cdot \hat{e}_{\theta 2} + r_3 \cdot \dot{\theta}_3 \cdot \hat{e}_{\theta 3} = \dot{r}_{1x}\hat{i} \quad (10)$$

(10) IS A SINGLE VECTOR EQUATION WHICH REPRESENTS TWO SCALAR EQUATIONS. FOR THIS TYPE OF PROBLEM $\dot{\theta}_2$ IS GIVEN LEAVING TWO UNKNOWN $\dot{\theta}_3$ AND \dot{r}_{1x} . SOLVING FOR THE UNKNOWN STARTS BY SUBSTITUTING (6) AND (7) INTO (10)

$$r_2 \cdot \dot{\theta}_2 \cdot (-\sin \theta_2 \hat{i} + \cos \theta_2 \hat{j}) + r_3 \cdot \dot{\theta}_3 \cdot (-\sin \theta_3 \hat{i} + \cos \theta_3 \hat{j}) = \dot{r}_{1x}\hat{i} \quad (11)$$

DOTTING (11) WITH \hat{i}

$$-r_2 \cdot \dot{\theta}_2 \cdot \sin \theta_2 - r_3 \cdot \dot{\theta}_3 \cdot \sin \theta_3 = \dot{r}_{1x} \quad (12)$$

DOTTING (12) WITH \hat{j}

$$r_2 \cdot \dot{\theta}_2 \cdot \cos \theta_2 + r_3 \cdot \dot{\theta}_3 \cdot \cos \theta_3 = 0$$

$$\boxed{\dot{\theta}_3 = -\frac{r_2 \cdot \dot{\theta}_2 \cdot \cos \theta_2}{r_3 \cdot \cos \theta_3}} \quad (13)$$

SUBSTITUTING (13) INTO (12)

$$\dot{r}_{1x} = r_3 \sin \theta_3 \left[-\frac{r_2 \dot{\theta}_2 \cos \theta_2}{r_3 \cos \theta_3} \right] - r_2 \dot{\theta}_2 \sin \theta_2$$

$$\dot{r}_{1x} = r_2 \dot{\theta}_2 \left[\tan \theta_3 \cos \theta_2 - \sin \theta_2 \right]$$

(14)