PROBLEM 1.3 FOR THE FRAME SHOWN, DETERMINE ALL SUPPORT AND CONNECTION FORCES ON EACH MEMBER. THE 400 lb-ft COUPLE M IS IN THE PLANE OF THE PAGE AND APPLIED TO MEMBER BF. NEGLECT FRICTION ON ALL MATING PARTS.

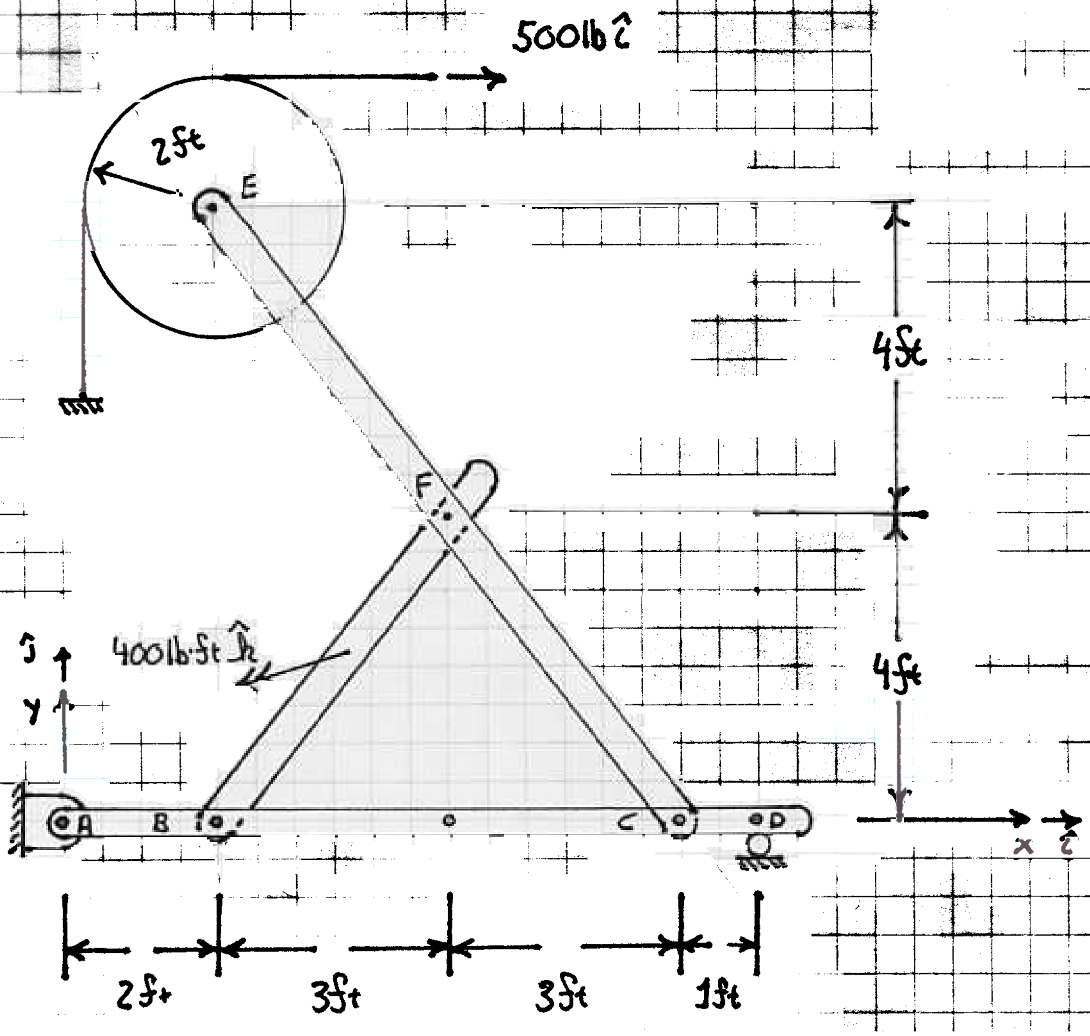
GIVEN:

CONSTRAINTS:
1. 500 lb FORCE APPLIED TO THE CABLE THAT IS WRAPPED AROUND A WHEEL AND THEN ATTACHED TO THE GROUND
2. PIN JOINTS AT E, F, A, B, AND C
3. 400 lb-ft COUPLE IS APPLIED TO MEMBER BF.

ASSUMPTIONS
1. ALL PIN JOINTS ARE FRICTIONLESS
2. ALL MEMBERS ARE RIGID
3. THE WEIGHT OF ALL THE MEMBERS CAN BE NEGLECTED
4. VERTICAL TRANSLATION IS RESTRICTED AT D

FIND:
1. DETERMINE THE SUPPORT REACTIONS
2. DETERMINE THE CONNECTION FORCES

DIAGRAM:



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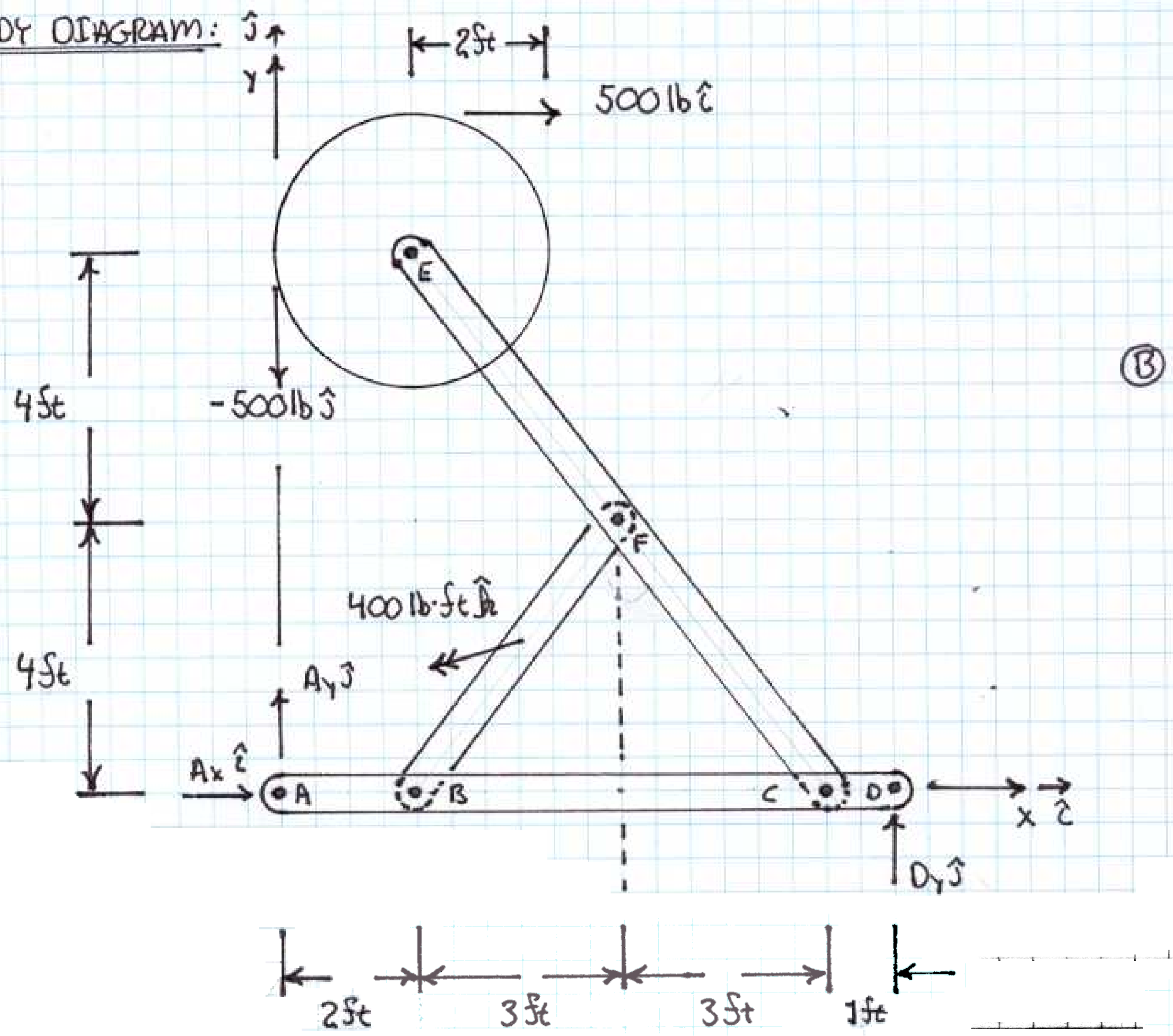
FREEBODY DIAGRAM:



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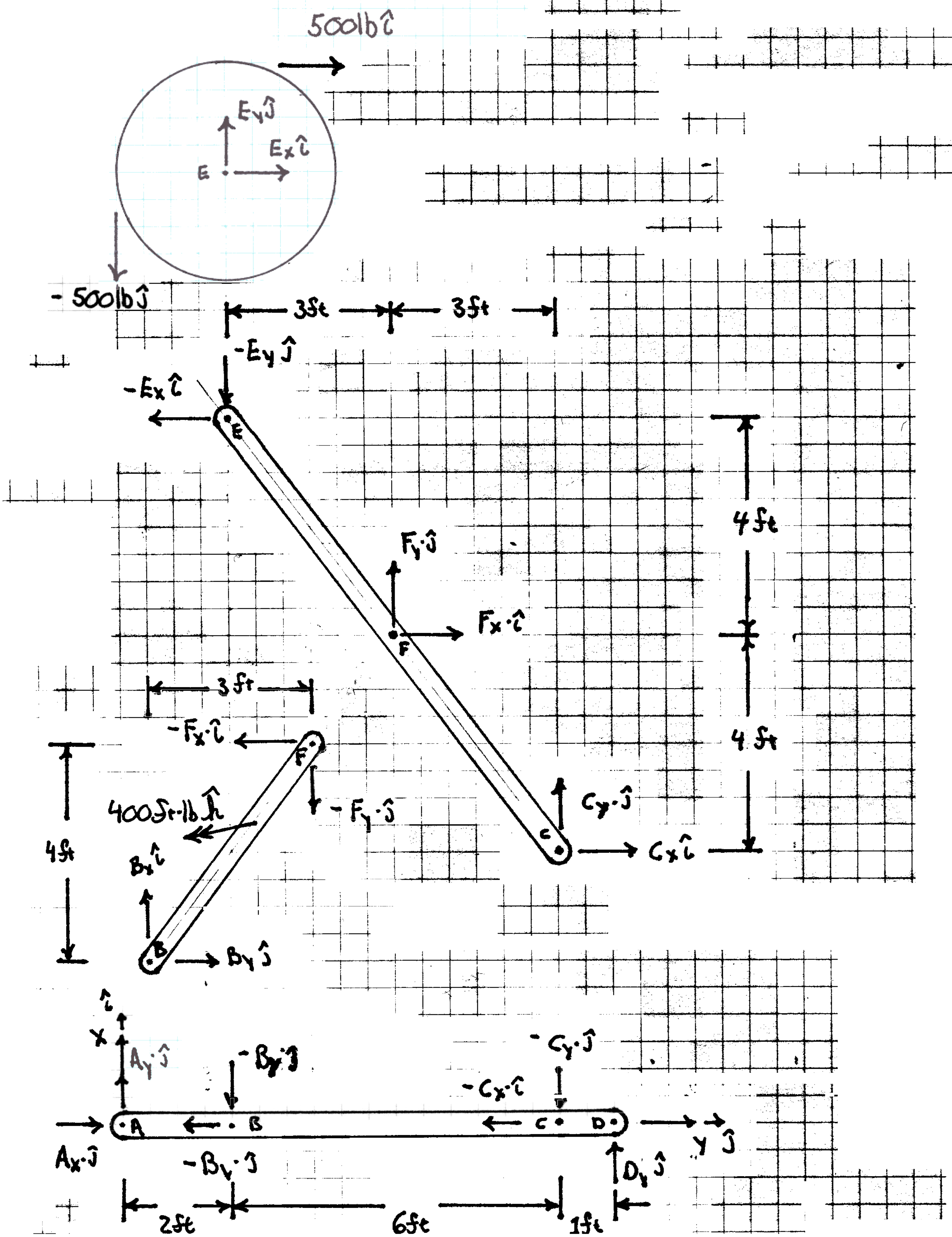

C

D

E

F

SOLUTION:

THE SOLUTION STARTS BY CONSIDERING THE OVERALL EQUILIBRIUM OF THE STRUCTURE. THE FREE BODY DIAGRAM FOR THE OVERALL STRUCTURE IS SHOWN IN B. USING EQUILIBRIUM

$$\Sigma F_x = 0 = A_x + 500\,lb \Rightarrow \underline{A_x = -500\,lb}$$

$$\Sigma F_y = 0 = A_y + D_y - 500\,lb \Rightarrow A_y + D_y = 500\,lb$$

$$\Sigma M_{z/@A} = 0 = 400\,lb\cdot ft - (10ft)\cdot(500lb) + (9ft)\cdot D_y$$

$$\Rightarrow D_y = \frac{-400lb\cdot ft + (10ft)\cdot(500lb)}{9ft} = \underline{511.1\,lb}$$

SUBSTITUTING 3 INTO 2

$$A_y = 500lb - D_y = 500lb - (511.1\,lb) = \underline{-11.1\ lb}$$

NOW THAT THE EXTERNAL REACTIONS ARE KNOWN, THE INTERNAL REACTIONS CAN NOW BE CONSIDERED.

WHEEL E

CONSIDERING THE EQUILIBRIUM OF THE WHEEL, FIGURE C

$$\Sigma F_x = 0 = E_x + 500lb \Rightarrow \underline{E_x = -500\,lb}$$

$$\Sigma F_y = 0 = E_y - 500\,lb \Rightarrow \underline{E_y = 500\,lb}$$

$$\vec{E} = -500lb\,\hat{i} + 500lb\,\hat{j} = 707.1\,lb\,(-.7071\hat{i} + .7071\hat{j})$$

BEAM EFC

CONSIDERING EQUILIBRIUM, FIGURE D

$$\Sigma F_x = 0 = -E_x + F_x + C_x = 500lb + F_x + C_x$$

$$\Sigma F_y = 0 = -E_y + F_y + C_y = -500lb + F_y + C_y$$

$$\Sigma M_{z/@C} = 0 = (8ft)\cdot(E_x) - (4ft)\cdot(F_x) + (6ft)\cdot(E_y) - (3ft)\cdot(F_y)$$

$$= (8ft)\cdot(-500lb) - (4ft)\cdot(F_x) + (6ft)\cdot(500lb) - 3ft\cdot F_y$$

$$= -4000\,ft\cdot lb + 3000\,ft\cdot lb - 4ft\cdot F_x - 3ft\cdot F_y$$

$$= -1000\,ft\cdot lb - 4ft\cdot F_x - 3ft\cdot F_y$$

$$-1000\,ft\cdot lb = 4ft\cdot F_x + 3ft\cdot F_y \quad (10)$$

IN THE THREE EQUATIONS (8) - (10), AND THERE ARE FOUR UNKNOWNS: F_x, F_y, C_x, AND C_y. SINCE NO UNIQUE SOLUTION CAN BE DETERMINED, OTHER ELEMENTS IN THE STRUCTURE NEED TO BE CONSIDERED

BEAM BF

CONSIDERING EQUILIBRIUM USING, FIGURE (E)

$$\Sigma F_x = 0 = B_x - F_x = 0 \Rightarrow B_x = F_x \quad (11)$$

$$\Sigma F_y = 0 = B_y - F_y = 0 \Rightarrow B_y = F_y \quad (12)$$

$$\Sigma M_{z/@B} = 400\,ft\cdot lb + 4ft\cdot F_x - 3ft\cdot F_y$$

$$400\,ft\cdot lb = 3ft\cdot F_y - 4ft\cdot F_x \quad (13)$$

WITH THE ADDITION OF THE BEAM "BF" EQUATIONS THERE ARE NOW SIX EQUATIONS (8)-(13) AND SIX UNKNOWNS. THIS SET OF EQUATIONS AND UNKNOWNS CAN BE SOLVED SIMULTANEOUSLY. A MATRIX WILL BE USED HERE TO SOLVE THESE EQUATIONS. FIRST THE SIX EQUATIONS NEED TO BE REWRITTEN AND THE PLACED IN MATRIX FORM

(8) $\rightarrow$ $-500\,lb = C_x + F_x$

(9) $\rightarrow$ $500\,lb = C_y + F_y$

(10) $\rightarrow$ $-1000\,ft\cdot lb = 4ft\cdot F_x + 3ft\cdot F_y$

(11) $\rightarrow$ $0 = B_x - F_x$

(12) $\rightarrow$ $0 = B_y - F_y$

(13) $\rightarrow$ $400\,ft\cdot lb = -4ft\cdot F_x + 3ft\cdot F_y$

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(8) $\rightarrow$ $-500\,lb = 0\cdot B_x + 0\cdot B_y + C_x + 0\cdot C_y + F_x + 0\cdot F_y$

(9) $\rightarrow$ $500\,lb = 0\cdot B_x + 0\cdot B_y + 0\cdot C_x + C_y + 0\cdot F_x + F_y$

(11) $\rightarrow$ $0 = B_x + 0\cdot B_y + 0\cdot C_x + 0\cdot C_y - F_x + 0\cdot F_y$

(12) $\rightarrow$ $0 = 0\cdot B_x + B_y + 0\cdot C_x + 0\cdot C_y + 0\cdot F_x - F_y$

(10) $\rightarrow$ $-1000\,ft\cdot lb = 0\cdot B_x + 0\cdot B_y + 0\cdot C_x + 0\cdot C_y + 4ft\cdot F_x + 3ft\cdot F_y$

(13) $\rightarrow$ $400\,ft\cdot lb = 0\cdot B_x + 0\cdot B_y + 0\cdot C_x + 0\cdot C_y - 4ft\cdot F_x + 3ft\cdot F_y$

WRITING THESE EQUATIONS IN MATRIX FORM

$$\begin{Bmatrix} -500\,lb \\ 500\,lb \\ 0 \\ 0 \\ -1000\,lb\cdot ft \\ 400\,lb\cdot ft \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 4ft & 3ft \\ 0 & 0 & 0 & 0 & -4ft & 3ft \end{bmatrix} \cdot \begin{Bmatrix} B_x \\ B_y \\ C_x \\ C_y \\ F_x \\ F_y \end{Bmatrix} \quad (14)$$

USING EITHER EXCEL® OR MATLAB®, THE SIX UNKNOWNS CAN NOW BE SOLVED.

$$B_x = -175\ lb \qquad C_x = -325\ lb \qquad F_x = -175\ lb$$
$$B_y = -100\ lb \qquad C_y = 600\ lb \qquad F_y = -100\ lb \quad (15)$$

NOW THE FINAL ELEMENT OF THE STRUCTURE CAN BE CONSIDERED

BEAM ABCD

CONSIDERING EQUILIBRIUM USING FIGURE (F)

$$\Sigma F_x = 0 = A_x - B_x - C_x$$
$$0 = (-500\,lb) - (-175\,lb) - (-325\,lb) = 0 \ \checkmark$$

$$\Sigma F_y = 0 = A_y - B_y - C_y + D$$
$$0 = (-11\,lb) - (-100\,lb) - (600\,lb) + D \Rightarrow D = 511\ lb$$

SUMMARY:

THE ADDITION OF THE COUPLE ON BEAM "BF" TURNED THIS INTO A VERY INTERESTING PROBLEM. BY DOING THIS, "BF" WAS NO LONGER A TWO-FORCE MEMBER. THIS FORCED THE SOLUTION TO CONSIDER TWO ELEMENTS OF THE STRUCTURE SIMULTANIOUSLY TO SOLVE FOR THE INTERNAL UNKNOWNS. IT IS ALSO IMPORTANT TO NOTE THAT BEFORE THE INTERNAL EQUILIBRIUM COULD BE CONSIDERED, THE EXTERNAL REACTIONS HAD TO BE SOLVED FOR FIRST. THIS PROBLEM ALSO EMPHASIZES THE NEED TO ADHERE TO STRICK SIGN CONVENTIONS.