PROBLEM 3-79 A TORQUE $T = 900\,lb\cdot in$ IS APPLIED TO THE SHAFT EFG, WHICH IS RUNNING AT CONSTANT SPEED AND CONTAINS GEAR F. GEAR F TRANSMITS TORQUE TO SHAFT ABCD THROUGH GEAR C, WHICH DRIVES THE CHAIN SPROCKET AT B, TRANSMITTING A FORCE P AS SHOWN. SPROCKET B, GEAR C, AND GEAR F HAVE PITCH DIAMETERS

$a = 6\,in,\ b = 5\,in,\ c = 10\,in$

RESPECTIVELY. THE CONTACT FORCE BETWEEN THE GEARS IS TRANSMITTED THROUGH THE PRESSURE ANGLE $\phi = 20°$. ASSUMING NO FRICTIONAL LOSSES AND CONSIDERING THE BEARINGS AT A, D, E, & G TO BE SIMPLE SUPPORTS, LOCATE THE POINT ON SHAFT ABCD THAT CONTAINS THE MAXIMUM TENSILE AND MAXIMUM TORSIONAL SHEAR STRESSES. COMBINE THESE STRESSES AND DETERMINE THE MAXIMUM PRINCIPAL NORMAL AND SHEAR STRESSES IN THE SHAFT.

OTHER DIMENSIONS OF INTEREST INCLUDE $d = 1.375\,in$, $e = 4\,in$, $f = 10\,in$, $g = 6\,in$

GIVEN:

1. SHAFT EFG IS SUBJECTED TO A 900 lb·in TORQUE
2. THE TORQUE ON SHAFT EFG IS TRANSMITTED TO SHAFT ABCD THROUGH A 10 in (PITCH) DIAMETER PINION ON EFG AND A 5 in (PITCH) DIAMETER ~~ON~~ GEAR ON ABCD
3. THE TORQUE TRANSMITTED TO SHAFT ABCD THROUGH THE GEAR AT C IS TRANSMITTED TO THE 6 in DIAMETER SPROCKET AT C.
4. THE PRESSURE ANGLE FOR THE GEAR SET IS $\phi = 20°$

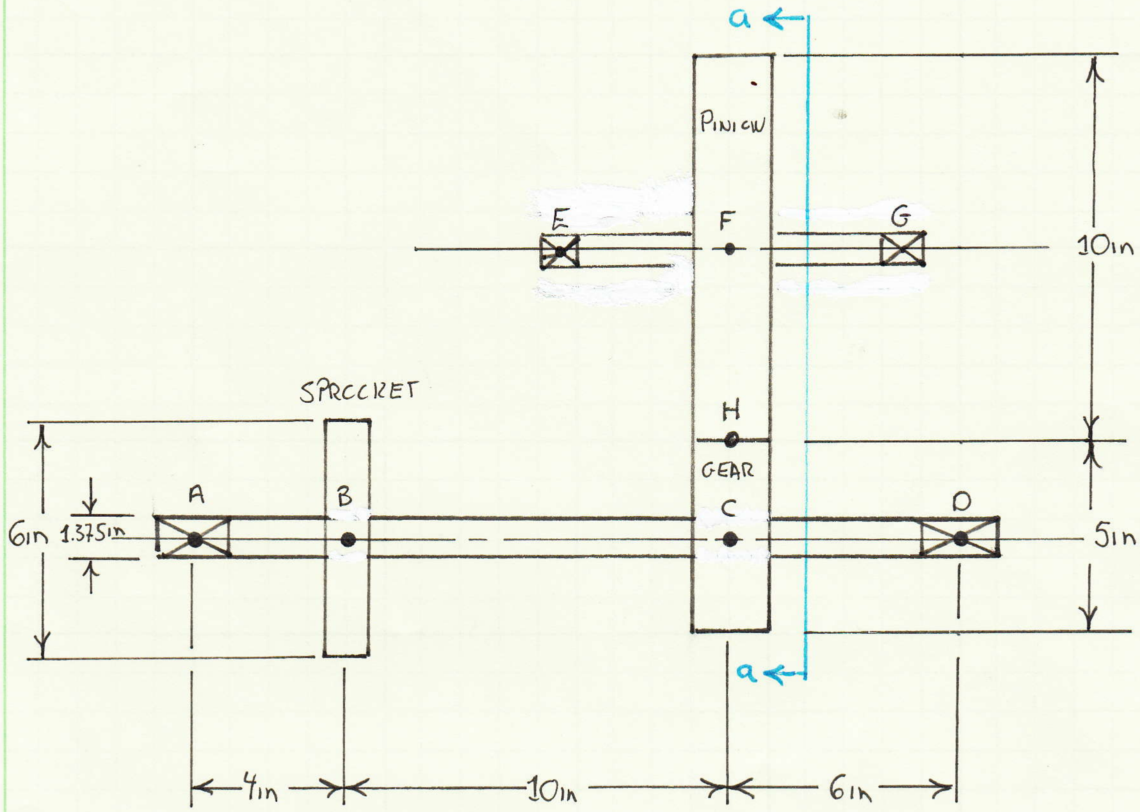
ASSUMPTIONS:

1. THE SHAFTS ARE RIGID
2. THERE IS NO SLIP BETWEEN THE SHAFTS, SPROCKET, AND GEARS.
3. THE SYSTEM IS IN A STATE OF EQUILIBRIUM

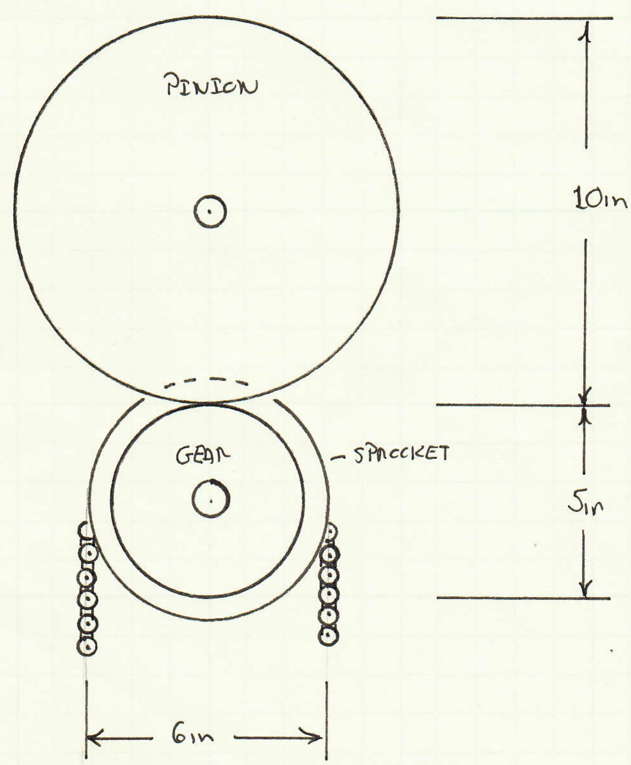
FIND:

1. LOCATE THE POINT ON SHAFT ABCD THAT CONTAINS THE MAXIMUM TENSILE AND MAXIMUM TORSIONAL SHEAR STRESSES.
2. COMBINE THESE STRESSES TO DETERMINE THE MAXIMUM PRINCIPAL NORMAL AND SHEAR STRESSES IN THE SHAFT.

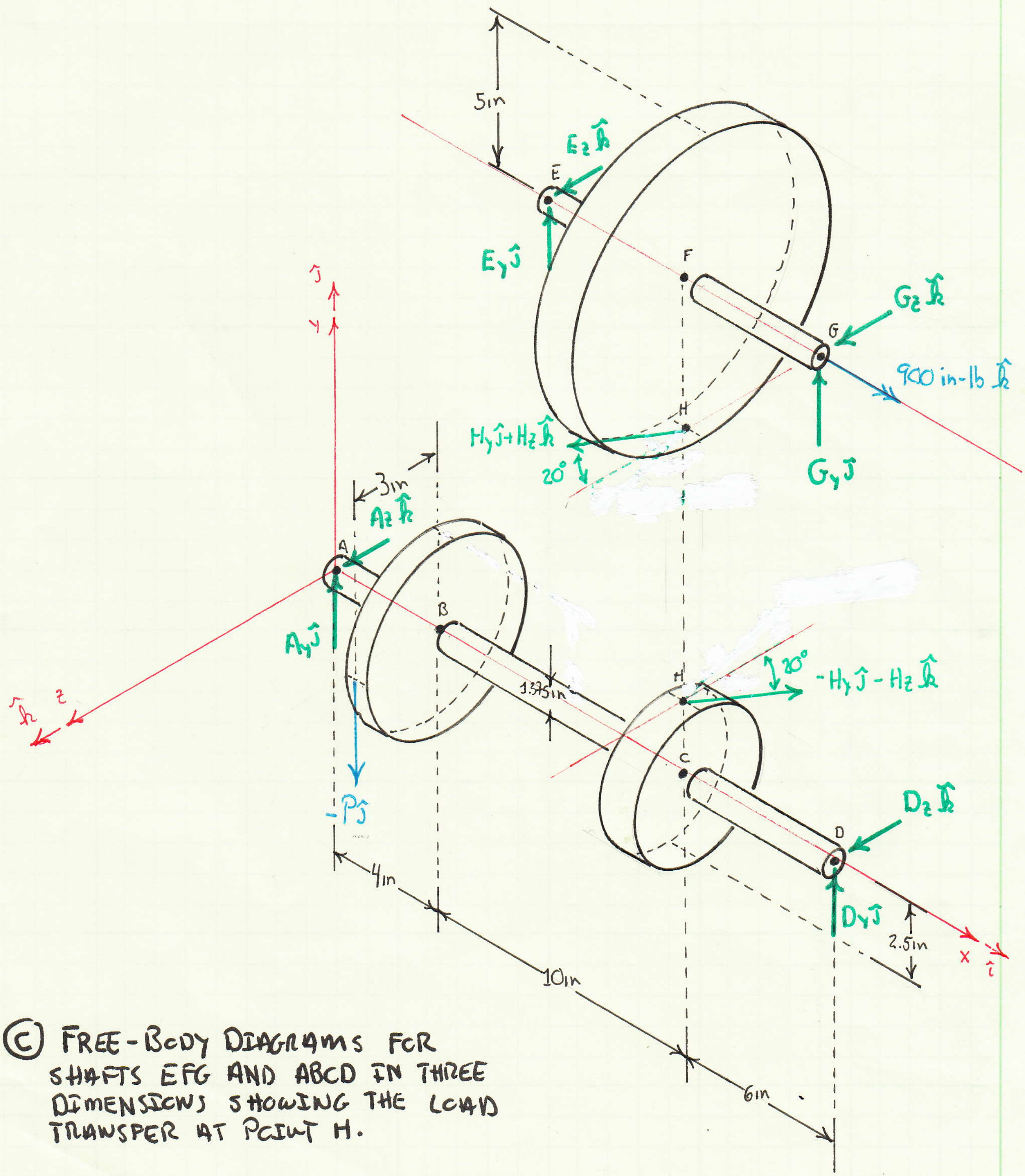
FIGURES



ⓐ GEAR TRAIN DIMENSIONAL LAY-OUT WITHOUT LOADS SHOWN



ⓑ SECTION a-a VIEW OF SYSTEM LAY-OUT



Ⓒ FREE-BODY DIAGRAMS FOR SHAFTS EFG AND ABCD IN THREE DIMENSIONS SHOWING THE LOAD TRANSFER AT POINT H.

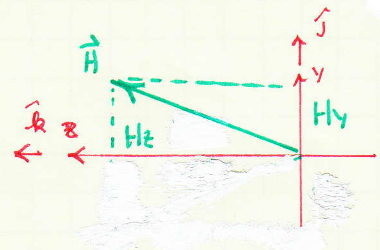
SOLUTION:

NOT ENOUGH INFORMATION IS GIVEN ABOUT THE GEOMETRY OF SHAFT EFG TO DETERMINE THE REACTIONS AT E AND G. THE ONE EQUATION OF EQUILIBRIUM THAT IS SIGNIFICANT TO THIS PROBLEM AND HAS ENOUGH INFORMATION IS

$$\Sigma M_{x@F} = 0 = 900\,\text{in}\cdot\text{lb} - 5\,\text{in}\cdot H_z$$

$$\Rightarrow H_z = \frac{900\,\text{in}\cdot\text{lb}}{5\,\text{in}} = \underline{\underline{180\,\text{lb}}} \quad ①$$

KNOWING THE PRESSURE ANGLE FOR THE GEARS IS 20°, USING THE GEOMETRY SHOWN IN ⓓ



ⓓ ILLUSTRATION OF THE COMPONENTS OF H ON THE GEAR ON SHAFT EFG.

$$\tan 20° = \frac{H_y}{H_z}$$

$$\Rightarrow H_y = H_z \cdot \tan 20°$$

$$= 180\,\text{lb}\cdot\tan 20° = \underline{\underline{65.51\,\text{lb}}} \quad ②$$

AS SHOWN ON THE GEAR AT C IN ⓒ, ① AND ② ARE APPLIED TO THE SHAFT ABCD IN THE OPPOSITE DIRECTIONAL SENSE AT H. NOW P CAN BE CALCULATED FROM THE ~~COUPLE~~ EQUILIBRIUM OF MOMENTS IN THE X-DIRECTION ON SHAFT ABCD

$$\Sigma M_{x@B} = 0 = -(2.5\,\text{in})(180\,\text{lb}) + 3\,\text{in}\cdot P$$

$$\Rightarrow P = \frac{(2.5\,\text{in})(180\,\text{lb})}{3\,\text{in}} = \underline{\underline{150\,\text{lb}}} \quad ③$$

CONTINUING TO APPLY EQUILIBRIUM TO SHAFT ABCD IN ⓒ

$$\Sigma F_y = 0 = A_y - P - H_y + D_y$$

$$\Rightarrow A_y + D_y = P + H_y = 150\,\text{lb} + 65.51\,\text{lb} = 215.5\,\text{lb} \quad ④$$

$$\Sigma F_z = 0 = A_z - H_z + D_z$$

$$\Rightarrow A_z + D_z = H_z = 180\,\text{lb} \quad ④$$

$$\Sigma M_{z@A} = 0 = -4\,\text{in}\cdot P - 14\,\text{in}\cdot H_y + 20\,\text{in}\cdot D_y$$

$$\Rightarrow D_y = \frac{+4\,\text{in}\cdot P + 14\,\text{in}\cdot H_y}{20\,\text{in}} = \frac{(4\,\text{in})(150\,\text{lb}) + (14\,\text{in})(65.51\,\text{lb})}{20\,\text{in}} = 75.86\,\text{lb} \quad ⑤$$

$$= \underline{\underline{75.9}}$$

A_y IS FOUND BY SUBSTITUTING (5) INTO (4)

$$A_y + 75.9\,lb = 215.5\,lb \Rightarrow A_y = 215.5\,lb - 75.9\,lb = \underline{\underline{139.6\,lb}} \quad (6)$$

APPLYING THE EQUILIBRIUM OF THE MOMENTS IN THE Y-DIRECTION

$$\Sigma M_{y@A} = 0 = (14in)\cdot H_z - (20in)\cdot D_z$$

$$\Rightarrow D_z = \frac{14in\cdot H_z}{20in} = \frac{(14in)\cdot(180lb)}{20in} = \underline{\underline{126\,lb}} \quad (7)$$

$$(4) \Rightarrow A_z = 180lb - D_z = 180\,lb - 126\,lb = \underline{\underline{54\,lb}} \quad (8)$$

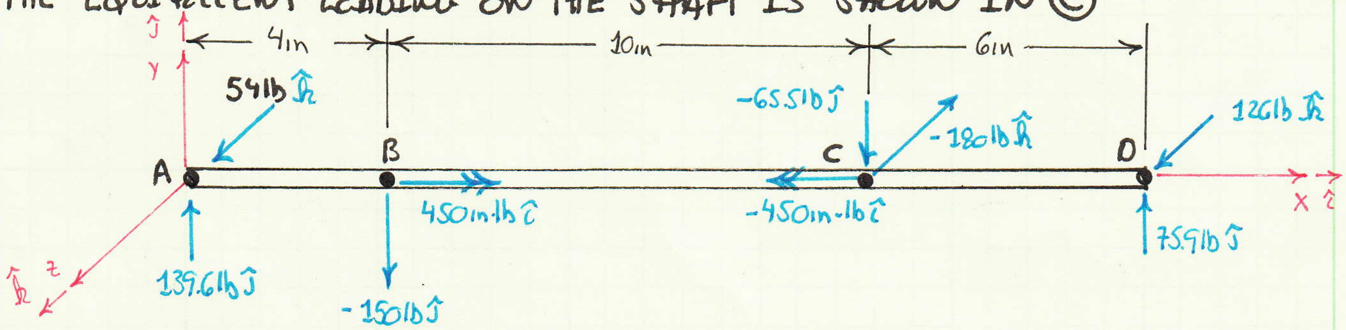
NEXT, THE EQUIVALENT LOADING ON THE SHAFT NEEDS TO BE CALCULATED. THIS REQUIRES THE LOAD P ON THE SPROCKET BE MOVED TO B ON THE SHAFT, THIS WILL REQUIRE THE EQUIVALENT COUPLE/TORQUE T_B TO BE CALCULATED

$$T_B = 3in\cdot P = 3in\cdot 150\,lb \Rightarrow \underline{\underline{450\,in\cdot lb}} \quad (9)$$

IN A SIMILAR MANNER H_y AND H_z ON THE GEAR NEED TO BE MOVED TO C ON THE SHAFT. THE ~~COUPLE GEN~~ EQUIVALENT COUPLE/TORQUE GENERATED IS

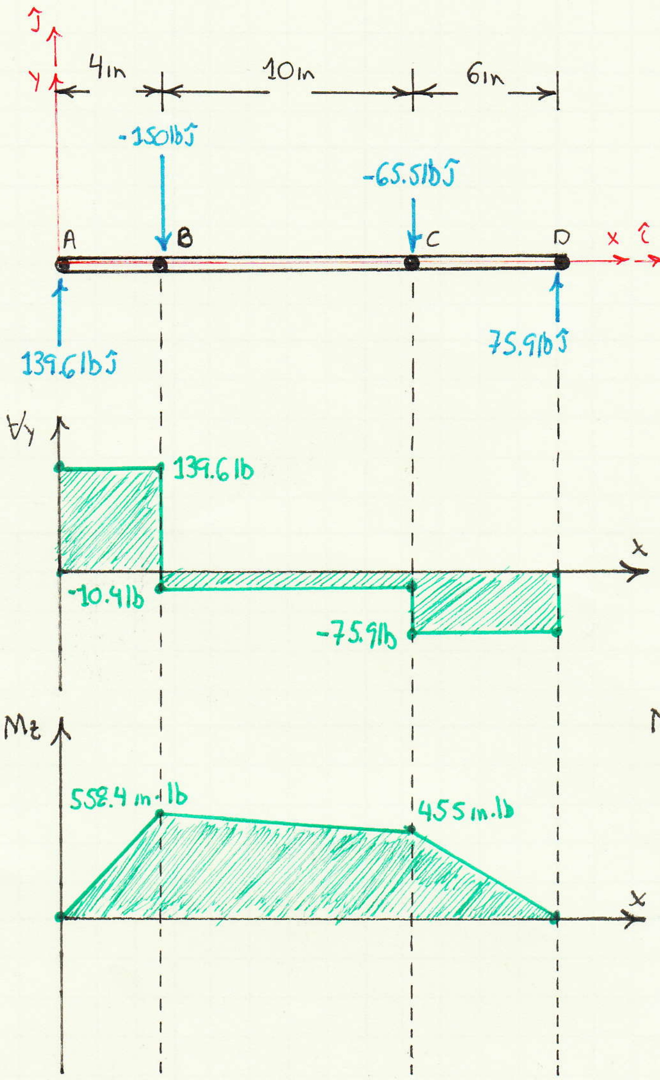
$$T_C = 2.5in\cdot H_z = 2.5in\cdot 180\,lb = \underline{\underline{450\,in\cdot lb.}} \quad (10)$$

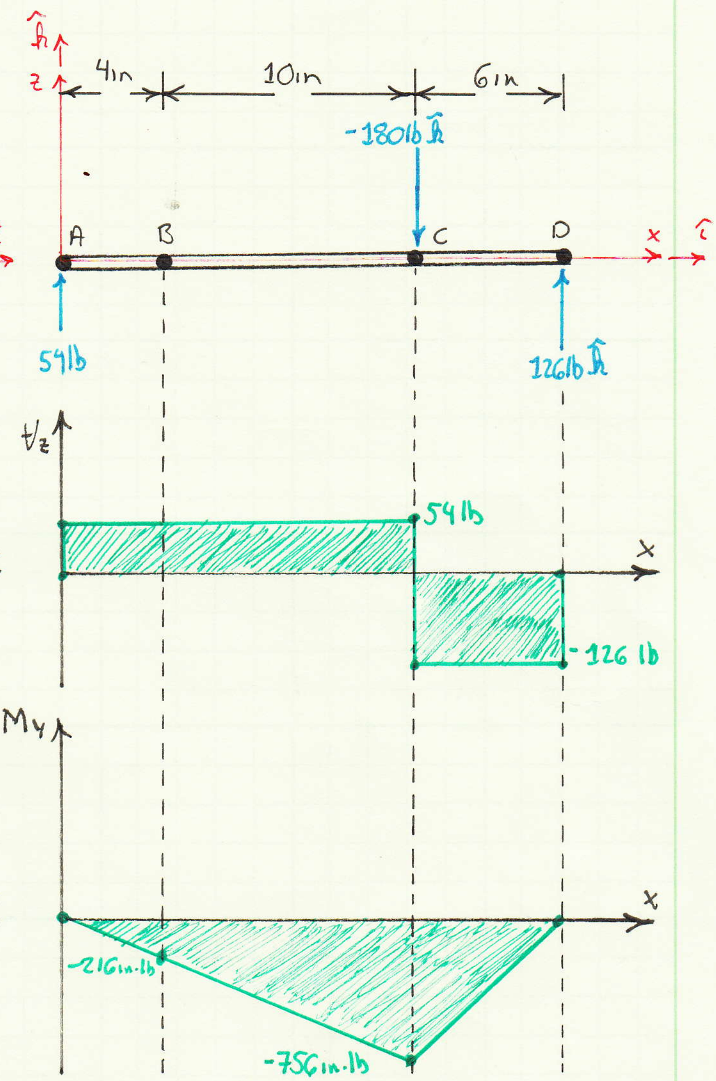
THE EQUIVALENT LOADING ON THE SHAFT IS SHOWN IN (c)



(c) EQUIVALENT LOADING ON SHAFT ABCD IN A FREE BODY DIAGRAM FORMAT

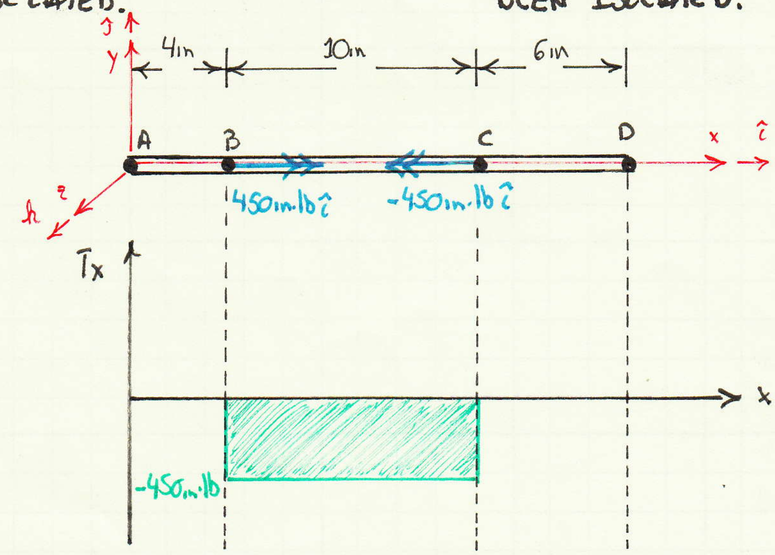
THE SHEAR AND BENDING MOMENT DIAGRAMS IN THE X-Y AND X-Z PLANES NEED TO BE CONSTRUCTED TO ASSIST IN THE IDENTIFICATION OF THE MAXIMUM BENDING MOMENT.





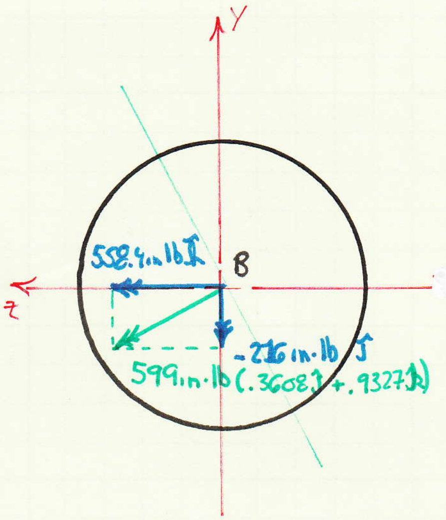
(f) SHEAR AND BENDING MOMENT DIAGRAMS FOR SHAFT ABCD IN THE X-Y PLANE. THE FORCE LOADING IN THE X-Y PLANE HAS BEEN ISOLATED.

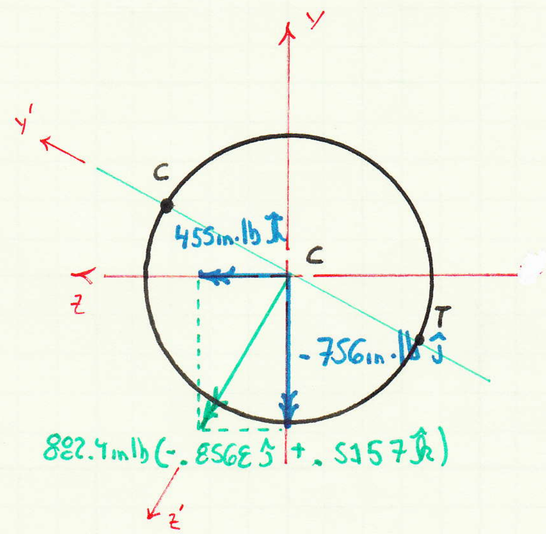
(g) SHEAR AND BENDING MOMENT DIAGRAMS FOR THE SHAFT IN THE ~~xx~~ X-Z PLANE. THE FORCE LOADING IN THE X-Z PLANE HAS BEEN ISOLATED.



(h) THE TORSIONAL DIAGRAM FOR SHAFT ABCD. THE TORQUES ON THE SHAFT ARE ISOLATED

THE LOCATION OF THE MAXIMUM NORMAL STRESS CAN ONLY BE DETERMINED BY COMBINING THE MOMENTS & M_z AND M_y. THE TWO LOCATIONS THAT MUST BE CHECKED ARE B & C.





(i) THE INTERNAL BENDING MOMENTS AT B ARE COMBINED INTO A SINGLE BENDING MOMENT

(j) THE INTERNAL BENDING MOMENTS AT C ARE COMBINED INTO A SINGLE BENDING MOMENT.

USING THE FIGURES (i) AND (j) THE COMBINED BENDING MOMENTS ARE CALCULATED

$$M_B = \sqrt{(558.4\,\text{in·lb})^2 + (-216\,\text{in·lb})^2} = 599\,\text{in·lb} \qquad (11)$$

$$\hat{e}_{M_B} = -\frac{216\,\text{in·lb}}{599\,\text{in·lb}}\hat{j} + \frac{558.4\,\text{in·lb}}{599\,\text{in·lb}}\hat{k} = -0.3608\hat{j} + 0.9327\hat{k} \qquad (12)$$

$$M_C = \sqrt{(-756\,\text{in·lb})^2 + (455\,\text{in·lb})^2} = \boxed{882.4\,\text{in·lb}} \qquad (13)$$

$$\hat{e}_{M_C} = -\frac{756\,\text{in·lb}}{882.4\,\text{in·lb}}\hat{j} + \frac{455\,\text{in·lb}}{882.4\,\text{in·lb}}\hat{k} = -0.8568\hat{j} + 0.5157\hat{k} \qquad (14)$$

THE MAXIMUM BENDING MOMENT IS AT C.

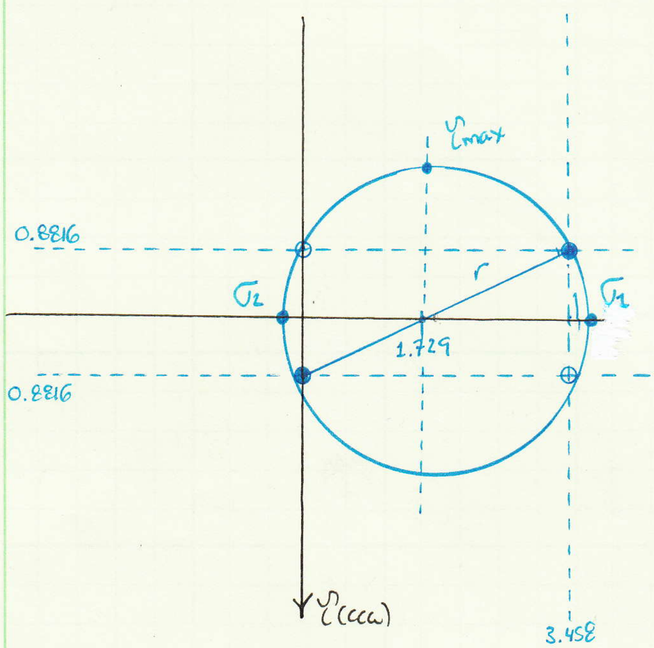
THE MAXIMUM NORMAL STRESS AT C IN (i) IS GIVEN BY

$$\sigma_c = \frac{M \cdot c}{I} = \frac{(882.4\,\text{in}\cdot\text{lb}) \cdot \left(\frac{1.375\,\text{in}}{2}\right)}{\frac{\pi \cdot (1.375\,\text{in})^4}{64}} = 3.458\,\text{ksi} \qquad (15)$$

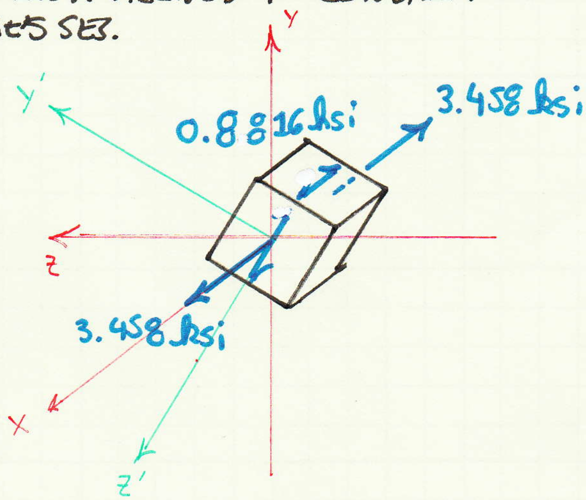
AN THE MAX SHEAR STRESS

$$\tau = \frac{T \cdot r}{J} = \frac{(450\,\text{in}\cdot\text{lb}) \left(\frac{1.375\,\text{in}}{2}\right)}{\frac{\pi \cdot (1.375\,\text{in})^4}{32}} = 0.8816\,\text{ksi} \qquad (16)$$

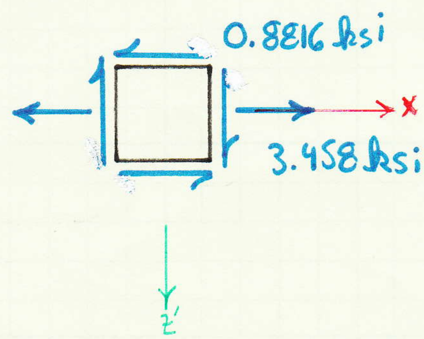
THE STRESSES IN (15) AND (16) ARE ILLUSTRATED IN (k) AND (l). THE STATE OF STRESS IN (l) PROVIDES THE INFORMATION NEEDED TO CONSTRUCT MOHR'S CIRCLE AND THE PRINCIPLE STRESSES.



(m) MOHR'S CIRCLE FOR THE STATE OF STRESS AT C WHERE THE NORMAL STRESS IS MAXIMUM



(k) ILLUSTRATION OF THE STATE OF STRESS AT THE SITE OF MAXIMUM NORMAL STRESS IN THE CROSS-SECTION OF THE SHAFT AT C



(l) A TWO DIMENSIONAL VIEW OF THE STATE OF STRESS IN THE SHAFT AT POINT C WHERE THE NORMAL STRESS IS MAXIMUM. THE VIEW IS LOOKING DOWN THE Y-AXIS.

USING MOHR'S CIRCLE

$$r = \sqrt{(1.729\,\text{ksi})^2 + (0.8816\,\text{ksi})^2} = 1.9408\,\text{ksi} \qquad (17)$$

$$\sigma_1 = 1.729\,\text{ksi} + 1.9408\,\text{ksi} = \boxed{3.670\,\text{ksi}} \qquad (18)$$

$$\sigma_2 = 1.729\,\text{ksi} - 1.9408\,\text{ksi} = \boxed{-0.212\,\text{ksi}} \qquad (19)$$

$$\tau_{max} = \boxed{1.941\,\text{ksi}} \qquad (20)$$

PROBLEM 5-48 | FOR THE PROBLEM PREVIOUSLY SOLVED (3-79), DETERMINE THE MINIMUM FACTOR OF SAFETY FOR YIELDING. USE BOTH THE MAXIMUM-SHEAR-STRESS THEORY AND THE DISTORTION-ENERGY THEORY, AND COMPARE THE RESULTS. THE MATERIAL IS 1018 CD STEEL.

GIVEN:

1. ALL GIVENS FROM 3-79
2. THE MATERIAL USED FOR THE SHAFT IS 1018 CD.

ASSUMPTIONS

1. ALL ASSUMPTIONS FROM 3-79
2. SMALL DEFORMATIONS
3. ISOTROPIC AND LINEAR ELASTIC MATERIAL BEHAVIOR

FIND:

1. DETERMINE THE MINIMUM FACTOR OF SAFETY FOR YIELD USING MAX-SHEAR STRESS
2. DETERMINE THE MINIMUM FACTOR OF SAFETY FOR YIELD USING DISTORTIONAL ENERGY

SOLUTION:

THE MATERIAL PROPERTIES FOR 1018 CD STEEL ARE

$$S^{UT} = 64 \text{ ksi}$$

$$S^{Y} = 54 \text{ ksi}$$

THE MAXIMUM SHEAR STRESS THEORY CAN BE WRITTEN

$$\tau_{max} = \frac{S^Y}{2 \cdot n} \Rightarrow n = \frac{S^Y}{2 \cdot \tau_{max}} = \frac{54 \text{ ksi}}{2 \cdot (1.941 \text{ ksi})} = \boxed{13.9} \quad (21)$$

THE DISTORTIONAL ENERGY THEORY CAN BE WRITTEN

$$n = \frac{S_Y}{\sigma'} = \frac{S_Y}{\left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]^{1/2}}$$

USING THE RESULTS (18) AND (19), $\sigma_3 = 0$ AS SHOWN IN FIGURE (a)

$$n = \frac{54 \text{ ksi}}{\left[\frac{(3.670 \text{ ksi} + 0.212 \text{ ksi})^2 + (-0.212 \text{ ksi} - 0)^2 + (0 - 3.670 \text{ ksi})^2}{2}\right]^{1/2}}$$

$$= \boxed{14.3} \quad (22)$$

PROBLEM 6-48 BUILDING ON THE RESULTS OF PROBLEMS 3-79 & 5-48, DETERMINE THE MINIMUM FACTOR OF SAFETY FOR FATIGUE BASED ON INFINITE LIFE, USING THE MODIFIED GOODMAN CRITERION. THE SHAFT ROTATES AT CONSTANT SPEED, HAS A CONSTANT DIAMETER, AND IS MADE FROM COLD DRAWN AISI 1018 STEEL.

GIVEN:

1. ALL GIVENS IN 3-79 & 5-48
2. INFINITE LIFE, MODIFIED GOODMAN CRITERION
3. SHAFT ROTATES AT CONSTANT SPEED

ASSUMPTIONS:

1. ALL ASSUMPTIONS FROM 3-79 & 5-48
2. NO PLASTIC DEFORMATION
3. NO STRESS CONCENTRATIONS

FIND:

1. DETERMINE THE MINIMUM FACTOR OF SAFETY FOR FATIGUE BASED ON THE MODIFIED GOODMAN DIAGRAM.

SOLUTION:

THE MATERIAL PROPERTIES FOR 1018 CD STEEL ARE

$$S^{UT} = 64 \text{ ksi}$$

$$S^{Y} = 54 \text{ ksi}$$

FIRST THE ENDURANCE LIMIT FOR THE STEEL NEEDS TO BE ESTIMATED. S_e' IS THE ENDURANCE LIMIT FOR THE ROTATING BEAM SPECIMEN AND CAN BE ESTIMATED BY

$$S_e' = 0.5 \cdot S^{UT} = 0.5 \cdot (64 \text{ ksi}) = 32 \text{ ksi} \qquad (23)$$

NOW THE ENDURANCE LIMIT MUST BE MODIFIED TO REFLECT THE DIFFERENCE BETWEEN THE ROTATING BEAM SPECIMEN AND THE ACTUAL SPECIMEN.

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S_e' \qquad (24)$$

$$k_a \equiv \text{SURFACE FACTOR} = a\, S_{UT}^{b} \quad \text{(a \& b ARE GIVEN BY TABLE 6-2)}$$

$$= 2.70 (64 \text{ ksi})^{-0.265} = 0.8969$$

$k_b \equiv$ SIZE FACTOR, FOR A SHAFT BETWEEN $0.11 \le d \le 2\,in$

$$= 0.879 \cdot (1.375\,in)^{-0.107} = 0.850$$

$k_c \equiv$ LOAD FACTOR (BENDING AND TORSION) = 1

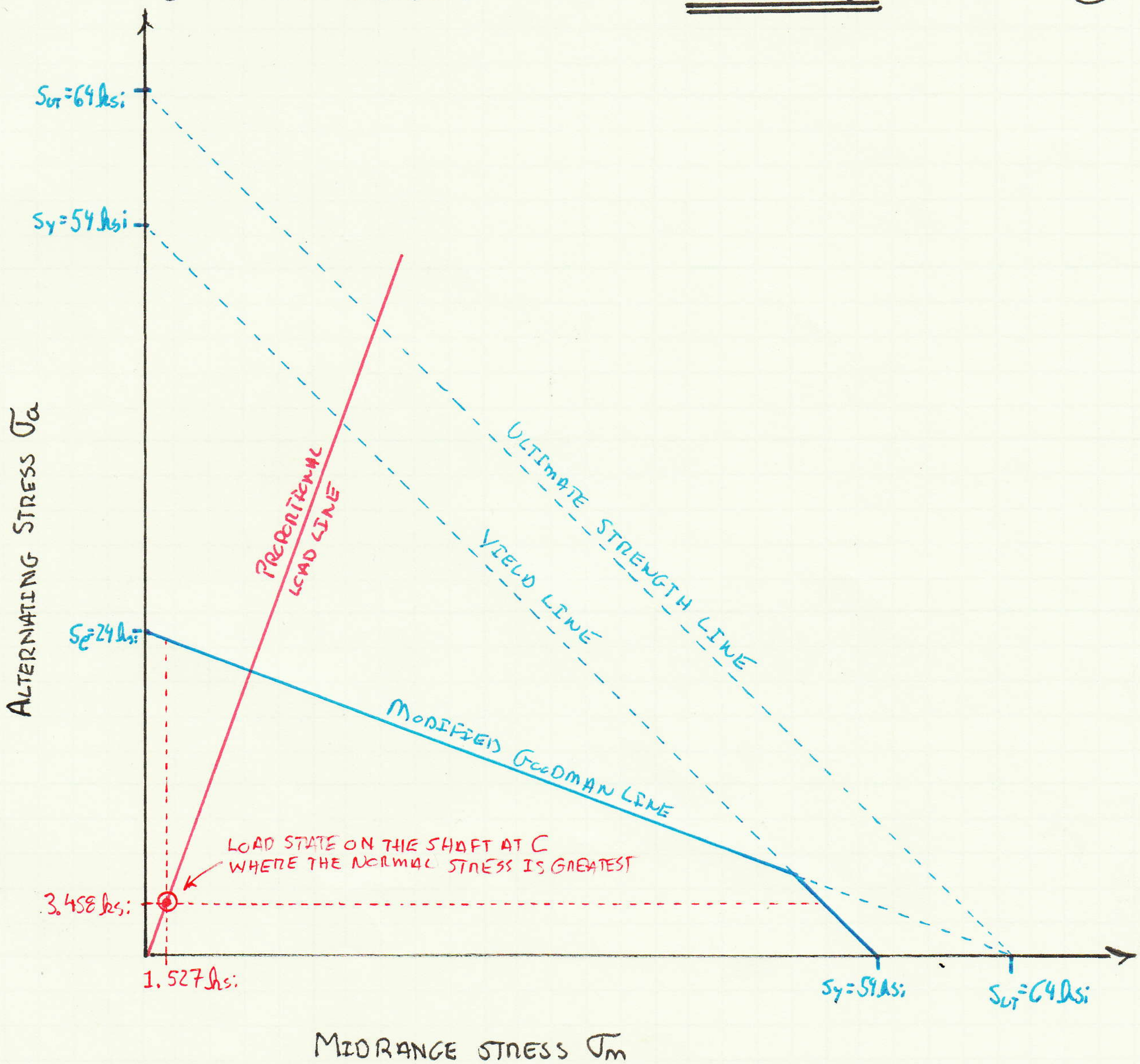
$k_d \equiv$ TEMPERATURE FACTOR = 1

$k_e \equiv$ RELIABILITY FACTOR = 1

$k_f \equiv$ MISCELLANEOUS FACTOR = 1

SUBSTITUTING ALL THE FACTORS INTO (24) ALONG WITH THE RESULT IN (23)

$$S_e = (0.8969)(0.850) \cdot 32\,ksi = \underline{\underline{24.40\,ksi}} \quad (25)$$



(n) ILLUSTRATION OF THE MODIFIED GOODMAN LINE FOR THE SHAFT IN THE EXAMPLE WITH THE LOADING FOR THE POINT OF MAX NORMAL STRESS SHOWN.

FOR THE CYCLING OF THE SHAFT IN THE EXAMPLE

$$\sigma_m = 0 \qquad \sigma_a = 3.458\text{ ksi}$$

$$\tau_m = 0.8816\text{ ksi} \qquad \tau_a = 0.$$

THE MIDRANGE AND ALTERNATING VON MISES STRESSES FOR THE ABOVE LOADING CONDITION ARE

$$\sigma_m' = (\sigma_m^2 + 3\cdot\tau_m^2)^{1/2} = [(0)^2 + 3\cdot(0.8816\text{ ksi})^2]^{1/2} = 1.527\text{ ksi}$$

$$\sigma_a' = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [(3.458\text{ ksi})^2 + 3\cdot(0)]^{1/2} = 3.458\text{ ksi}$$

NOW THE MODIFIED GOODMAN FAILURE CRITERION CAN BE USED TO CALCULATE THE FACTOR OF SAFETY

$$\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{UT}} = \frac{3.458\text{ ksi}}{24.4\text{ ksi}} + \frac{1.527\text{ ksi}}{64\text{ ksi}} = 0.1656$$

$$\Rightarrow n = \boxed{6.04}$$

SUMMARY:

THE ANALYSIS THAT HAS BEEN PERFORMED HERE IS VERY PRELIMINARY DESIGN CALCULATIONS. BECAUSE STRESS CONCENTRATIONS ON THE SHAFT WERE NOT CONSIDERED (BECAUSE OF LACK OF INFORMATION), THE RESULTS ARE NOT CONSERVATIVE. HOWEVER, THE CALCULATIONS ARE TYPICAL OF THE INITIAL CALCULATIONS PERFORMED WHEN DESIGNING SHAFTS.

THE MODIFIED GOODMAN DIAGRAM IN (D) ILLUSTRATES THE LOADING STATE IN THE SHAFT AT POINT C WHERE THE TENSILE STRESS IS MAXIMUM. THE CALCULATED FACTOR OF SAFETY ~~CAN A~~ IS THE SAME AS THE PORTION OF THE PROPORTIONAL LOAD LINE FROM THE ORIGIN TO THE MODIFIED GOODMAN LINE DIVIDED BY THE PORTION OF THE SAME LINE FROM THE ORIGIN TO THE LOAD STATE.