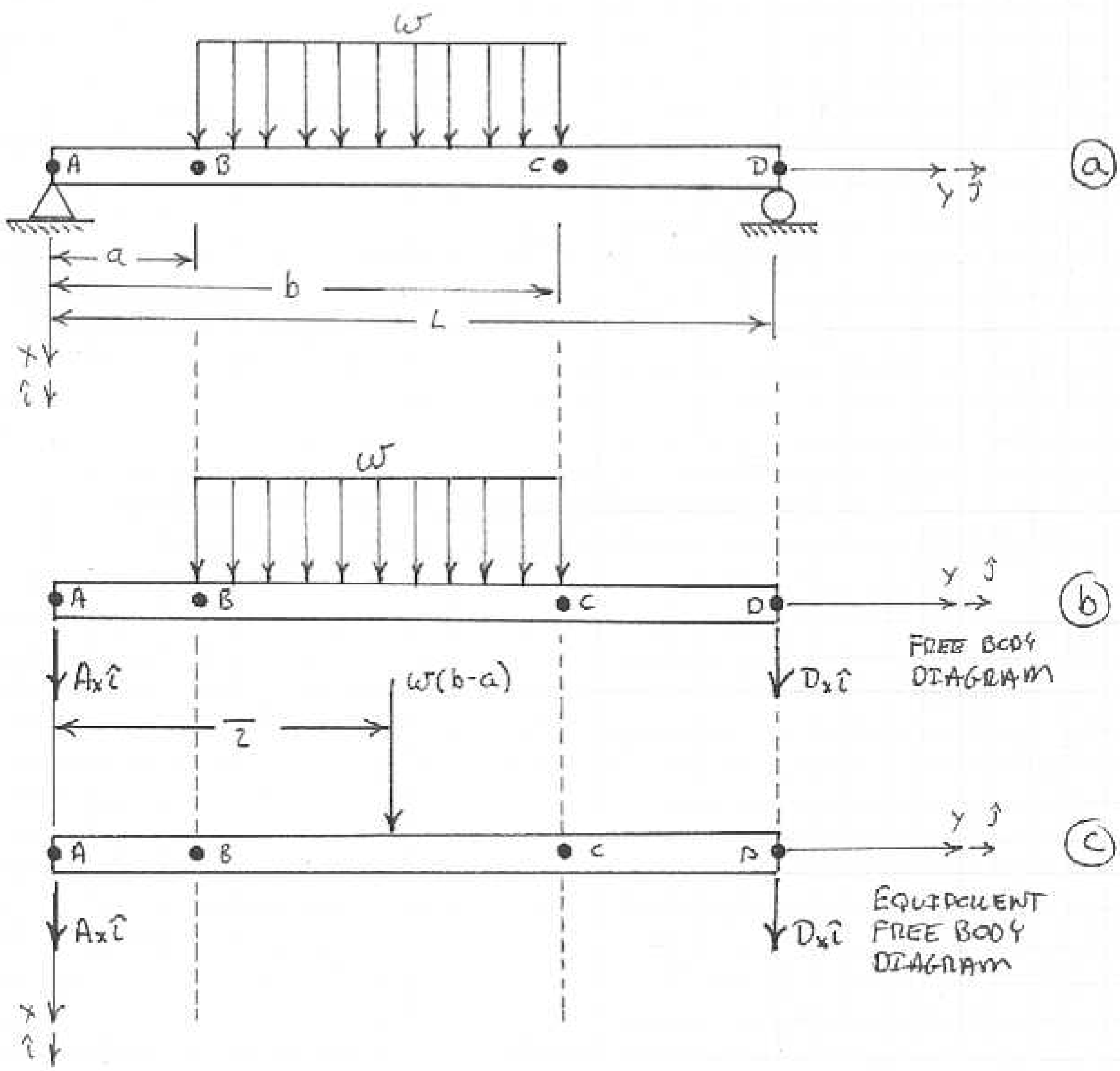
PROBLEM 3.24 DETERMINE THE DEFLECTION EQUATIONS FOR THE BEAM SHOWN USING (a) SUPERPOSITION AND (b) SINGULARITY FUNCTIONS.

GIVEN:
1. BEAM OF LENGTH L
2. BOTH ENDS OF THE BEAM ARE SIMPLY SUPPORTED
3. DISTRIBUTED LOAD APPLIED TO THE BEAM FROM A DISTANCE "a" FROM THE END TO "b".

ASSUMPTIONS:
1. Small DEFLECTIONS
2. RESPONSE IS LINEARLY ELASTIC

FIND:
1. DEFLECTION USING SUPERPOSITION.
2. DEFLECTIONS USING SINGULARITY FUNCTIONS.

FIGURE:



Solution:

Solving for the reactions at A and D using (c)

$$\Sigma F_x = 0 = A_x + D_x + \omega(b-a) \Rightarrow \underline{\underline{A_x + D_x = -\omega(b-a)}} \qquad (1)$$

$$\Sigma M_{z@A} = 0 = -\left(\frac{a+b}{2}\right)(\omega)(b-a) - D_x \cdot L$$

$$\Rightarrow D_x = -\frac{\omega}{2L}\cdot(b+a)(b-a) = -\frac{\omega}{2L}(b^2-a^2) \qquad (2)$$

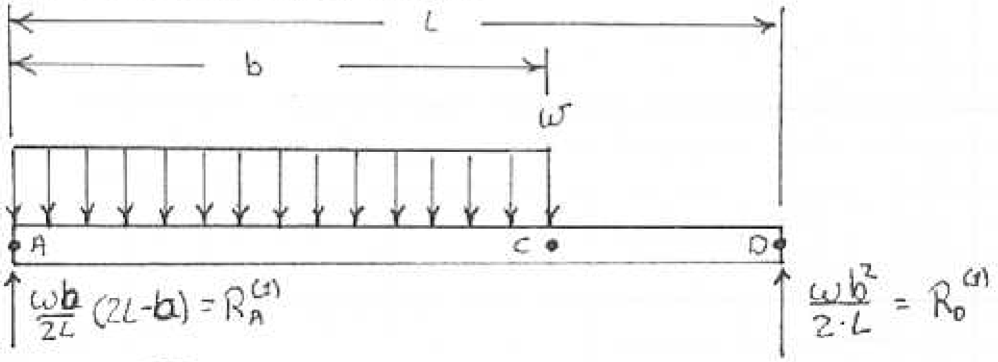
From (1)

$$A_x = -\omega(b-a) - D_x = -\omega(b-a) + \frac{\omega}{2L}(a+b)(b-a)$$

$$= -\frac{2\cdot L\cdot \omega}{2L}(b-a) + \frac{\omega}{2L}(a+b)(b-a) = \frac{\omega}{2L}(b-a)\cdot(a+b-2L)$$

$$= \frac{\omega}{2L}(a\cdot b - b^2 - 2Lb - a^2 - ab + 2La) = \frac{\omega}{2L}(b^2 - 2\cdot L\cdot b - a^2 + 2La) \qquad (3)$$

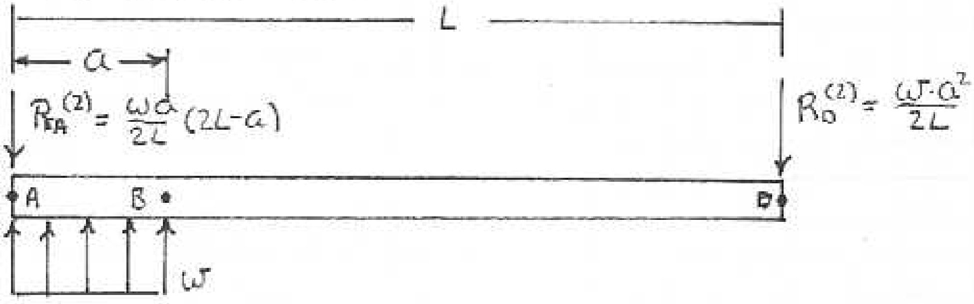
The superposition solution uses the beam in Appendix C.10. The use of C.10 is illustrated below




(d)

$$u_{AC}^{(1)} = \frac{\omega\cdot y}{24EI\cdot L}\left[2by^2(2L-b) - L\cdot y^3 - b^2(2L-b)^2\right]$$

$$u_{CD}^{(1)} = (X_C)_{AC} - \frac{\omega}{24EI}(y-b)^4$$




(e)

$$u_{AB}^{(2)} = -\frac{\omega\cdot y}{24EI\,L}\left[2ay^2(2L-a) - Ly^3 - a^2(2L-a)^2\right]$$

$$u_{BD}^{(2)} = (X_c)_{AB} + \frac{\omega}{24\cdot E\cdot I}(y-a)^4$$

(e) IS SUBTRACTED FROM (d). STARTING WITH THE REACTIONS AT A

$$R_A = R_A^{(1)} - R_A^{(2)} = \frac{w\cdot a}{2\cdot L}(2L-a) - \frac{wb}{2L}\cdot(2L-b)$$

$$= \frac{w}{2L}\left[2\cdot a\cdot L - a^2 - 2\cdot b\cdot L + b^2\right] \qquad (7)$$

$$R_D = R_D^{(2)} - R_D^{(1)} = \frac{w\cdot a^2}{2L} - \frac{w\cdot b^2}{2L} = \frac{w}{2L}(a^2-b^2) = -\frac{w}{2L}(b^2-a^2) \qquad (8)$$

EQUATIONS (7) AND (8) DETERMINED THROUGH SUPERPOSITION MATCH EQUATIONS (3) AND (2) THAT WERE ARRIVED AT THROUGH THE EQUATIONS OF EQUILIBRIUM.

THE DEFLECTION IN THE THREE REGIONS OF THE BEAM (A-B, B-C, C-D) CAN ALSO BE FOUND THROUGH THE SUPERPOSITION OF THE BEAM SOLUTIONS FOUND IN (d) & (e)

$$u_{AB} = u_{AC}^{(1)} + u_{AB}^{(2)} = \frac{w\cdot y}{24EIL}\left[2by^2(2L-b) - Ly^3 - b^2(2L-b)^2\right]$$
$$- \frac{w\cdot y}{24EIL}\left[2ay^2(2L-a) - Ly^3 - a^2(2L-a)^2\right]$$

$$= \frac{w\cdot y}{24\cdot E\cdot I\cdot L}\left[2\cdot y^2(2bL-b^2) - Ly^3 - b^2(2L-b)^2 - 2y^2(2aL-a^2) + Ly^3 + a^2(2L-a)^2\right]$$

$$= \frac{w\cdot y}{24EIL}\left[2y^2(2L(b-a)-b^2+a^2) - b^2(2L-b)^2 + a^2(2L-a)^2\right] \qquad (9)$$

$$u_{BC} = u_{AC}^{(1)} + u_{BD}^{(2)}$$
$$= \frac{wy}{24EIL}\left[2by^2(2L-b) - Ly^3 - b^2(2L-b)^2\right] + (\theta_c)_{AB} + \frac{w}{24EI}(y-a)^4$$

$$(\theta_c)_{AB} = -\frac{w\cdot a}{24EIL}\cdot\left[2a^3(2L-a) - La^3 - a^2(2L-a)^2\right]$$

$$= -\frac{wa}{24EIL}\cdot\left[4a^3L - 2a^4 - La^3 - a^2(4L^2 - 4La + a^2)\right]$$

$$= -\frac{wa}{24EIL}\left[4a^3L - 2a^4 - La^3 - 4a^2L^2 + 4a^3L - a^4\right]$$

$$= -\frac{wa}{24EI}\left[-3a^4 + 7a^3L - 4a^2L^2\right]$$

$$u_{BC} = \frac{w \cdot y}{24EIL}\left[2 \cdot b \cdot y^2(2L-b) - Ly^3 - b^2(2L-b)^2\right] + -\frac{wa}{24EIL}\left[-3a^4 + 7a^3L - 4a^2L^2\right] + \frac{w}{24EI}(y-a)^4$$

$$= \frac{w}{24EIL}\left[2by^3(2L-b) - Ly^4 - b^2y(2L-b)^2 + 3a^5 - 7a^4L + 4a^3L^2 + L(y-a)^4\right] \quad (10)$$

$$u_{CD} = u_{CD}^{(1)} + u_{BD}^{(2)} = (x_c)_{AC} - \frac{w}{24EI}(y-b)^4 + (x_c)_{AB} + \frac{w}{24EI}(y-a)^4$$

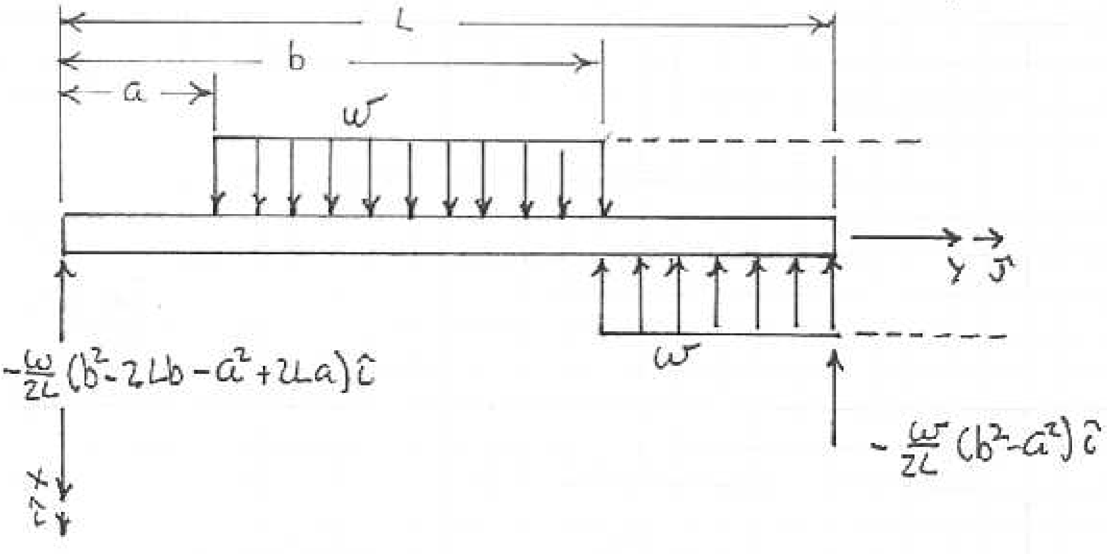
$$(x_c)_{AC} = \frac{w \cdot b}{24EIL}\left[2b^4(2L-b) - Lb^3 - b^2(2L-b)^2\right]$$

$$(x_c)_{AB} = -\frac{wy}{24EIL}\left[2a^4(2L-a) - La^3 - a^2(2L-a)^2\right]$$

$$u_{CD} = \frac{w \cdot b}{24EIL}\left[2b^4(2L-b) - Lb^3 - b^2(2L-b)^2\right] - \frac{w}{24EI}(y-b)^4 - \frac{wy}{24EIL}\left[2a^4(2L-a) - La^3 - a^2(2L-a)^2\right] + \frac{w}{24EI}(y-a)^4$$

$$= \frac{w}{24EIL}\left[2b^4(2L-b) - Lb^3 - b^2(2L-b)^2 - L \cdot (y-b)^4 - 2a^4(2L-a) + La^3 + a^2(2L-a)^2 + L \cdot (y-a)^4\right] \quad (11)$$

NOW SOLVING FOR THE DEFLECTION USING SINGULARITY FUNCTIONS



$$q = -\frac{w}{2L}(b^2-2Lb-a^2+2La)\langle y-0\rangle_{-1} + w\langle y-a\rangle^0 - w\langle y-b\rangle^0 - \frac{w}{2L}(b^2-a^2)\langle y-L\rangle_{-1}$$

$$V = -\int q\,dy$$
$$= +\frac{w}{2L}(b^2-2Lb-a^2+2La)\langle y-0\rangle^0 - w\langle y-a\rangle^1 + w\langle y-b\rangle^1 + \frac{w}{2L}(b^2-a^2)\langle y-L\rangle^0$$

$$M = \int V\cdot dy$$
$$= \frac{w}{2L}(b^2-2L\cdot b-a^2+2L\cdot a)\langle y-0\rangle^1 - \frac{w}{2}\langle y-a\rangle^2 + \frac{w}{2}\langle y-b\rangle^2 + \frac{w}{2L}(b^2-a^2)\langle y-L\rangle^1$$

$$\Theta = -\frac{1}{EI}\int M\cdot dy$$
$$= -\frac{w}{4EIL}(b^2-2Lb-a^2+2La)\langle y-0\rangle^2 + \frac{w}{6EI}\langle y-a\rangle^3 - \frac{w}{6EI}\langle y-b\rangle^3 - \frac{w}{4EIL}(b^2-a^2)\langle y-L\rangle^2 + C_1$$

$$u = \int\Theta\,dy$$
$$= -\frac{w}{12EIL}(b^2-2Lb-a^2+2La)\langle y-0\rangle^3 + \frac{w}{24EI}\langle y-a\rangle^4 - \frac{w}{24EI}\langle y-b\rangle^4 - \frac{w}{12EIL}(b^2-a^2)\langle y-L\rangle^3 + C_1y + C_2 \quad (12)$$

THE CONSTANTS IN (12) ARE DETERMINED FROM THE BOUNDARY CONDITIONS $u(0)=0$ AND $u(L)=0$

$$u(0)=0=C_2$$
$$u(L)=0=-\frac{w}{12EI}(b^2-2Lb-a^2+2La)L^3+\frac{w}{24EI}(L-a)^4-\frac{w}{24EI}(L-b)^4+C_1L$$
$$\Rightarrow C_1=\frac{w}{24EIL}\left[(b^2-2Lb-a^2+2La)L^3-(L-a)^4+(L-b)^4\right]$$
$$u = -\frac{w}{12EIL}(b^2-2Lb-a^2-2La)\langle y-0\rangle^3+\frac{w}{24EI}\langle y-a\rangle^4-\frac{w}{24EI}\langle y-b\rangle^4 - \frac{w}{12EIL}(b^2-a^2)\langle y-L\rangle^3+\frac{wy}{24EIL}\left[(b^2-2Lb-a^2+2La)L^3-(L-a)^4+(L-b)^4\right]$$