As a student at Union College, I am part of a community that values intellectual effort, curiosity and discovery. I understand that in order to truly claim my educational and academic achievements, I am obligated to act with academic integrity. Therefore, I affirm that I carried out the work on this exam with full academic honesty, and I rely on my fellow students to do the same.

For this exam I understand that:

1. I **must** work alone in writing out the answers to this exam.
2. I **cannot** copy solutions to these problems from any person or any other resource.
3. I **can** only use my calculator and the programs that I developed in **this course** to solve the problems in this exam I **cannot** use any other electronic resources to assist me in the solution to the questions on this exam.
4. I **cannot** discuss any part of this exam or discuss what was covered on this exam with anyone else, post it in any electronic form, or communicate in any way that would provide assistance to anyone as to what is being covered on the exam.

Signature: ______________________

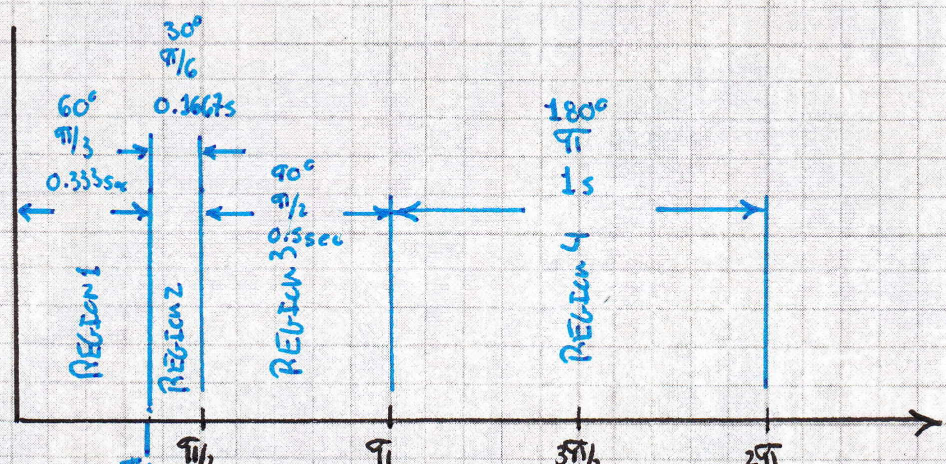
Print Name: SOLUTION

Exam Date: ______________________

PROBLEM 1 (50 pts): Design a double-dwell cam to move a follower 0 to 2in in 60°, Dwell for 30°, Fall 2in in 90°, and dwell for the remainder of the cycle. The total cycle must take 2 seconds.

1a). Illustrate below the segments of motion for the described follower, the length of the segments, duration of the segments, and appropriate segment boundary conditions to insure two piecewise continuous derivatives of the displacement functions.

$$\omega = \frac{2\pi \text{ rad}}{2 \text{ sec}} = \underline{\underline{\pi \text{ rad/s}}}$$



θ[RAD]	0	π/3	π/2	π	3π/2	2π
θ[°]	0	60°	90°	180°		360
t[s]	0	0.333	0.5	1		2
S[in]	0	2	2	0		0
S'[in/rad]	0	0	0	0		0
S''[in/rad²]	0	0	0	0		0
S'''[in/rad³]	–	–	–	–		–

1b. Determine the parameters of CYCLOIDAL functions that will cause the follower to RISE to the DWELL and then FALL to the return DWELL.

RISE: $0 \le \theta \le 60°$, $0 \le \theta_1 \le 60°$, $0 \le \theta_1 \le \pi/3$, $\beta_1 = \pi/3$, $h = 2_{in}$ (SEGMENT 1)

$\theta_1 = 0$ $\theta_1 = \beta_1$

$\theta_1/\beta_1 = 0$ $\theta_1/\beta_1 = 1$

$$s = h\left[\frac{\theta_1}{\beta_1} - \frac{1}{2\pi}\cdot \sin\left(\frac{\theta_1}{\beta_1}\cdot 2\pi\right)\right]$$

$$s' = v = \frac{h}{\beta_1}\left[1 - \cos\left(2\pi\cdot\frac{\theta_1}{\beta_1}\right)\right]$$

$$s'' = a = 2\pi\frac{h}{\beta_1^2}\cdot \sin\left(2\pi\frac{\theta_1}{\beta_1}\right)$$

$$s''' = j = 4\pi^2\cdot\frac{h}{\beta_1^3}\cos\left(2\pi\frac{\theta_1}{\beta_1}\right)$$

DWELL: $60° \le \theta \le 90°$, $0 \le \theta_2 \le \pi/6$, $0 \le \theta_2 \le 30°$, $\beta_2 = 30° = \pi/6$, $h = 2_{in}$ (SEGMENT 2)

$$s = h$$
$$s' = v = 0$$
$$s'' = a = 0$$
$$s''' = j = 0$$

Fall: $90° \le \theta \le 180°$, $0 \le \theta_3 \le \pi/2$, $0 \le \theta_3 \le 90°$, $\beta_3 = 90° = \pi/2$, $h = 2in$ (SEGMENT 3)

$$s = h - h\left[\frac{\theta_3}{\beta_3} - \frac{1}{2\pi}\cdot \sin\left(2\pi\cdot\frac{\theta_3}{\beta_3}\right)\right]$$

$$s' = v = \frac{h}{\beta_3}\left[\cos\left(2\pi\cdot\frac{\theta_3}{\beta_3}\right) - 1\right]$$

$$s'' = a = -\frac{2\pi\cdot h}{\beta_3^2}\cdot \sin\left(2\pi\cdot\frac{\theta_3}{\beta_3}\right)$$

$$s''' = j = -\frac{4\cdot\pi^2\cdot h}{\beta_3^3}\cdot\cos\left(2\pi\frac{\theta_3}{\beta_3}\right)$$

DWELL: $180° \le \theta \le 360°$, $0 \le \theta_4 \le \pi$, $0 \le \theta_4 \le 180°$, $\beta_4 = 180° = \pi$, $h = 0$ (SEGMENT 4)

$$s = 0$$
$$s' = v = 0$$
$$s'' = a = 0$$
$$s''' = j = 0$$

1c. Determine the parameters of POLYNOMIAL functions that will cause the follower to RISE to the DWELL and then FALL to the return DWELL.

<u>SEGMENT 1 – RISE:</u> $0 \leq \theta \leq 60^\circ,\ 0 \leq \theta_1 \leq 60^\circ,\ 0 \leq \theta_1 \leq \pi/3,\ \beta_1 = \pi/3 = 60^\circ,\ h = 0\,in - 2\,in$

$\theta_1 = 0,\ \theta_1/\beta_1 = 0:$ $S_1 = 0\,in$ (1), $S_1' = 0\,in/rad$ (2), $S_1'' = 0\,in/rad^2$ (3)

$\theta_1 = \beta_1 = \pi/3,\ \theta_1/\beta_1 = 1:$ $S_1 = 2\,in$ (4), $S_1' = 0\,in/rad$ (5), $S_1'' = 0\,in/rad^3$ (6)

$$S_1 = \cancel{C_0}^{\,0\,(1)} + \cancel{C_1}^{\,0\,(2)}\left(\frac{\theta_1}{\beta_1}\right) + \cancel{C_2}^{\,0\,(3)}\left(\frac{\theta_1}{\beta_1}\right)^2 + C_3\left(\frac{\theta_1}{\beta_1}\right)^3 + C_4\left(\frac{\theta_1}{\beta_1}\right)^4 + C_5\left(\frac{\theta_1}{\beta_1}\right)^5 \quad (7)$$

$$S_1' = \frac{\cancel{C_1}^{\,0}}{\beta_1} + \frac{2\cdot\cancel{C_2}^{\,0}}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right) + 3\frac{C_3}{\beta_1}\cdot\left(\frac{\theta_1}{\beta_1}\right)^2 + 4\cdot\frac{C_4}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^3 + 5\cdot\frac{C_5}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^4 \quad (8)$$

$$S_1'' = 2\frac{\cancel{C_2}^{\,0}}{\beta_1^2} + 6\frac{C_3}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right) + 12\frac{C_4}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^2 + 20\frac{C_5}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^3 \quad (9)$$

$$S_1''' = 6\frac{C_3}{\beta_1^3} + 24\frac{C_4}{\beta_1^3}\cdot\left(\frac{\theta_1}{\beta_1}\right) + 60\frac{C_5}{\beta_1^3}\left(\frac{\theta_1}{\beta_1}\right)^2 \quad (10)$$

(4) → (7): $S_1\left(\frac{\theta_1}{\beta_1}=1\right) = 2\,in = \underline{C_3 + C_4 + C_5 = 2\,in}$ (11)

$S_1'\left(\frac{\theta_1}{\beta_1}=1\right) = 0 = \frac{3\cdot C_3}{\beta_1} + \frac{4\cdot C_4}{\beta_1} + \frac{5C_5}{\beta_1} \Rightarrow \underline{3\cdot C_3 + 4\cdot C_4 + 5\cdot C_5 = 0}$ (12)

$S_1''\left(\frac{\theta_1}{\beta_1}=1\right) = 0 = 6\frac{C_3}{\beta_1^2} + 12\frac{C_4}{\beta_1^2} + 20\frac{C_5}{\beta_1^2} \Rightarrow \underline{6\cdot C_3 + 12\cdot C_4 + 20\,C_5 = 0}$ (13)

(11) – (13) CAN BE WRITTEN IN MATRIX FORM AND SOLVED SIMULTANEOUSLY.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{bmatrix} \cdot \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} 2\,in \\ 0\,in \\ 0\,in \end{Bmatrix} \Rightarrow \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} 20\,in \\ -30\,in \\ 12\,in \end{Bmatrix}$$

THEREFORE (7) – (10) CAN BE WRITTEN

$$S_1 = 20\,in\left(\frac{\theta_1}{\beta_1}\right)^3 - 30\left(\frac{\theta_1}{\beta_1}\right)^4 + 12\left(\frac{\theta_1}{\beta_1}\right)^5$$

$$S_1' = \frac{60\,in}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^2 - \frac{120\,in}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^3 + \frac{60\,in}{\beta_1}\left(\frac{\theta_1}{\beta_1}\right)^4$$

$$S_1'' = \frac{120\,in}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right) - \frac{360\,in}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^2 + \frac{240\,in}{\beta_1^2}\left(\frac{\theta_1}{\beta_1}\right)^3$$

$$S_1''' = \frac{120\,in}{\beta_1^3} - \frac{720\,in}{\beta_1^3}\left(\frac{\theta_1}{\beta_1}\right) + \frac{720\,in}{\beta_1^3}\left(\frac{\theta_1}{\beta_1}\right)^2$$

<u>SEGMENT 2 – Dwell</u> $60 \leq \theta \leq 90,\ 0 \leq \theta_2 \leq 30^\circ,\ 0 \leq \theta_2 \leq \pi/6,\ \beta_2 = \pi/6 = 30^\circ,\ h = 2\,in$

$S_2 = 2\,in$
$S_2' = 0\,in/rad$
$S_2'' = 0\,in/rad^2$
$S_2''' = 0\,in/rad^3$

Segment 3 - Fall: $90 \le \theta \le 180°$, $0 \le \theta_3 \le 90°$, $0 \le \theta_3 \le \pi/2$, $\beta_3 = \pi/2 = 90°$, $h = 2in - 0in$

$\theta_3 = 0,\ \theta_3/\beta_3 = 0:$ $S_3 = 2in$ (14) $\quad S_3' = 0\,in/rad$ (15) $\quad S_3'' = 0\,in/rad^2$ (16)

$\theta_3 = \beta_3,\ \theta_3/\beta_3 = 1:$ $S_3 = 0\,in$ (17) $\quad S_3' = 0\,in/rad$ (18) $\quad S_3'' = 0\,in/rad^2$ (19)

(14) (15) (16)

$$S_3 = \cancelto{2in}{C_0} + \cancelto{0}{C_1}\left(\frac{\theta_3}{\beta_3}\right) + \cancelto{0}{C_2}\left(\frac{\theta_3}{\beta_3}\right)^2 + C_3\left(\frac{\theta_3}{\beta_3}\right)^3 + C_4\left(\frac{\theta_3}{\beta_3}\right)^4 + C_5\left(\frac{\theta_3}{\beta_3}\right)^5 \quad (20)$$

$$S_3' = \cancelto{0}{\frac{C_1}{\beta_3}} + 2\cancelto{0}{\frac{C_2}{\beta_3}}\cdot\left(\frac{\theta_3}{\beta_3}\right) + 3\frac{C_3}{\beta_3}\left(\frac{\theta_3}{\beta_3}\right)^2 + 4\frac{C_4}{\beta_3}\left(\frac{\theta_3}{\beta_3}\right)^3 + 5\frac{C_5}{\beta_3}\left(\frac{\theta_3}{\beta_3}\right)^4 \quad (21)$$

$$S_3'' = 2\cdot\cancelto{0}{\frac{C_2}{\beta_3^2}} + 6\cdot\frac{C_3}{\beta_3^2}\cdot\left(\frac{\theta_3}{\beta_3}\right) + 12\frac{C_4}{\beta_3^2}\left(\frac{\theta_3}{\beta_3}\right)^2 + 20\frac{C_5}{\beta_3^2}\left(\frac{\theta_3}{\beta_3}\right)^3 \quad (22)$$

$$S_3''' = 6\frac{C_3}{\beta_3^3} + 24\frac{C_4}{\beta_3^3}\left(\frac{\theta_3}{\beta_3}\right) + 60\frac{C_5}{\beta_3^3}\left(\frac{\theta_3}{\beta_3}\right)^2 \quad (23)$$

(17)→(20): $S_3\left(\frac{\theta_3}{\beta_3}=1\right) = 0in = 2in + C_3 + C_4 + C_5 \Rightarrow C_3 + C_4 + C_5 = -2in$ (24)

(18)→(21): $S_3'\left(\frac{\theta_3}{\beta_3}=1\right) = 0\,in/rad = 3\frac{C_3}{\beta_3} + 4\frac{C_4}{\beta_3} + 5\frac{C_5}{\beta_3} \Rightarrow 3\cdot C_3 + 4\cdot C_4 + 5\cdot C_5 = 0in$ (25)

(19)→(22): $S_3''\left(\frac{\theta_3}{\beta_3}=1\right) = 0\,in/rad^2 = 6\frac{C_3}{\beta_3^2} + 12\frac{C_4}{\beta_3^2} + 20\frac{C_5}{\beta_3^2} \Rightarrow 6\cdot C_3 + 12\cdot C_4 + 20\cdot C_5 = 0in$ (26)

(24)-(26) CAN BE PLACED IN MATRIX FORM AND SOLVED SIMULTANEOUSLY

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{bmatrix} \cdot \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} -2in \\ 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} -20in \\ 30in \\ -12in \end{Bmatrix}$$

NOW (20)-(23) CAN BE WRITTEN

$$S_3 = 2in - 20in\left(\frac{\theta_3}{\beta_3}\right)^3 + 30in\left(\frac{\theta_3}{\beta_3}\right)^4 - 12in\left(\frac{\theta_3}{\beta_3}\right)^5$$

$$S_3' = -\frac{60in}{\beta_3}\left(\frac{\theta_3}{\beta_3}\right)^2 + \frac{120in}{\beta_3}\left(\frac{\theta_3}{\beta_3}\right)^3 - \frac{60in}{\beta_3}\left(\frac{\theta_3}{\beta_3}\right)^4$$

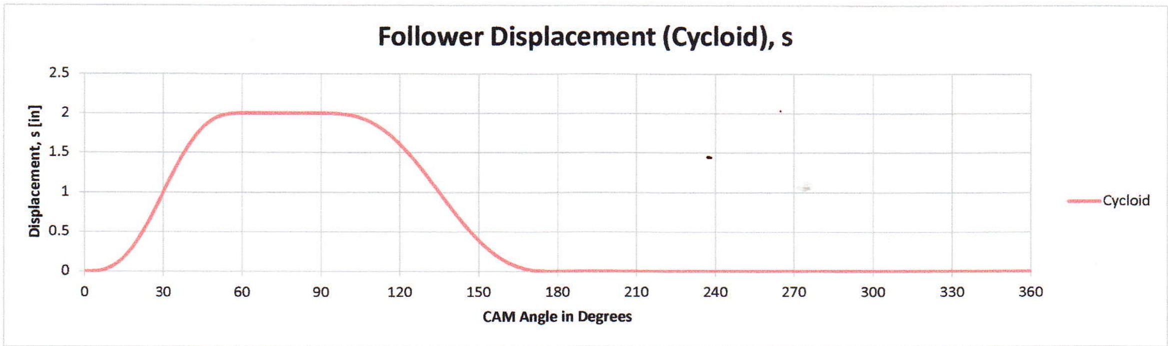
$$S_3'' = -\frac{120in}{\beta_3^2}\left(\frac{\theta_3}{\beta_3}\right) + \frac{360in}{\beta_3^2}\left(\frac{\theta_3}{\beta_3}\right)^2 - \frac{240in}{\beta_3^2}\left(\frac{\theta_3}{\beta_3}\right)^3$$

$$S_3''' = -\frac{120in}{\beta_3^3} + \frac{720in}{\beta_3^3}\left(\frac{\theta_3}{\beta_3}\right) - \frac{720in}{\beta_3^3}\left(\frac{\theta_3}{\beta_3}\right)^2$$

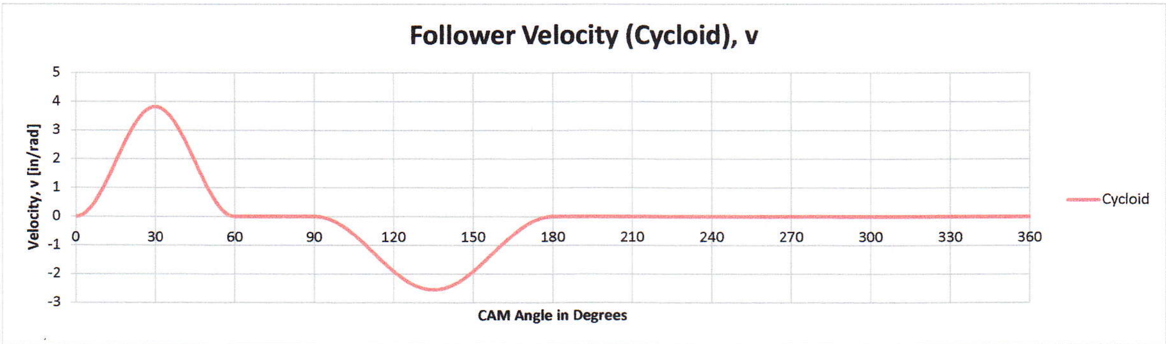
Segment 4: Dwell

$S_4 = 0in$
$S_4' = 0\,in/rad$
$S_4'' = 0\,in/rad^2$
$S_4''' = 0\,in/rad^3$

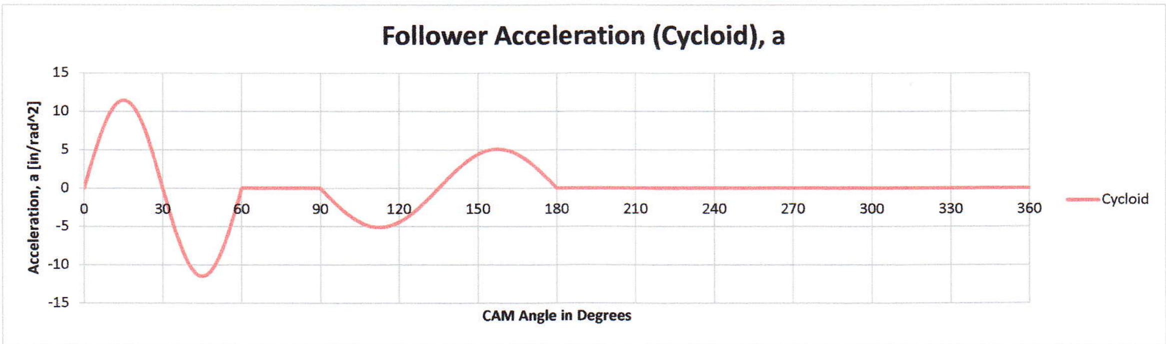
1d: Using the computer program that you developed in class plot the CYCLOIDAL and POLYNOMIAL s-s'-s''-s''' (s-v-a-j) diagrams for the entire 360° of cam rotation. Attach the diagrams directly after this page. (This last instruction is part of the exam. Failing to comply with this request will result in a 2pt deduction on the exam grade.)



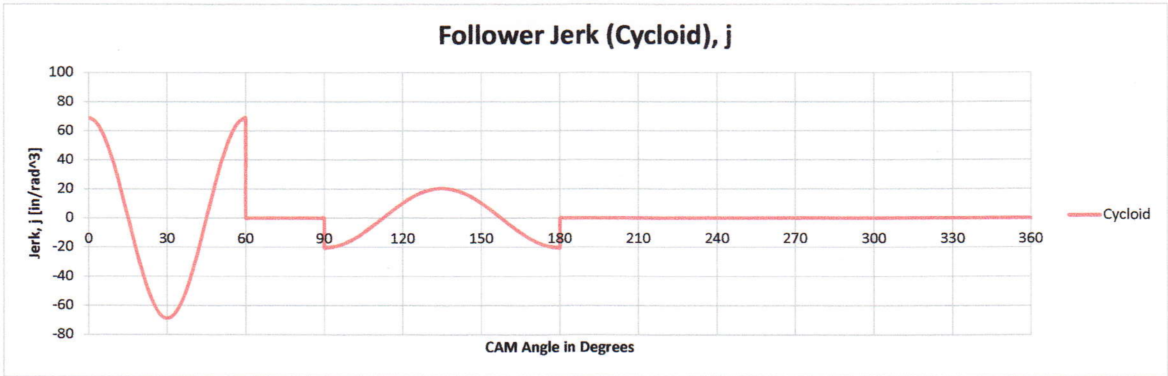
Follower Displacement (Cycloid), s
Displacement, s [in]
2.5
2
1.5
1
0.5
0
0
30
60
90
120
150
180
210
240
270
300
330
360
CAM Angle in Degrees
Cycloid



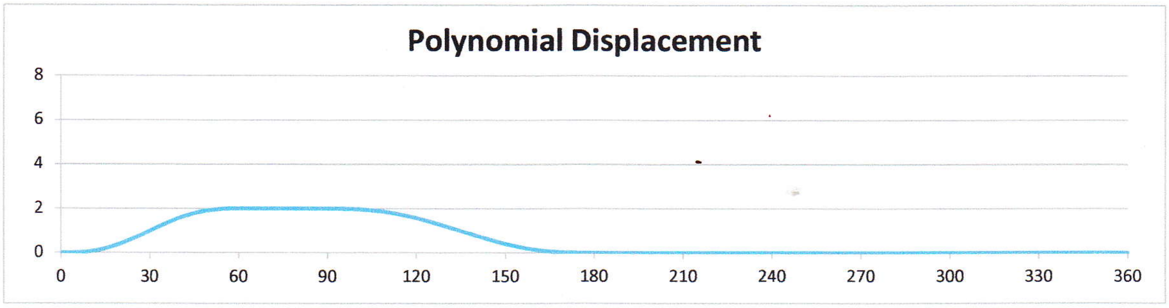
Follower Velocity (Cycloid), v
Velocity, v [in/rad]
5
4
3
2
1
0
-1
-2
-3
0
30
60
90
120
150
180
210
240
270
300
330
360
CAM Angle in Degrees
Cycloid

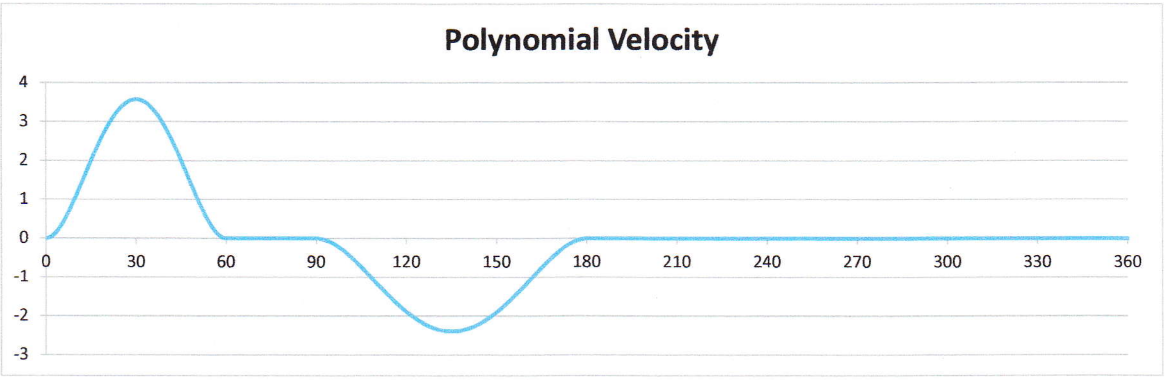


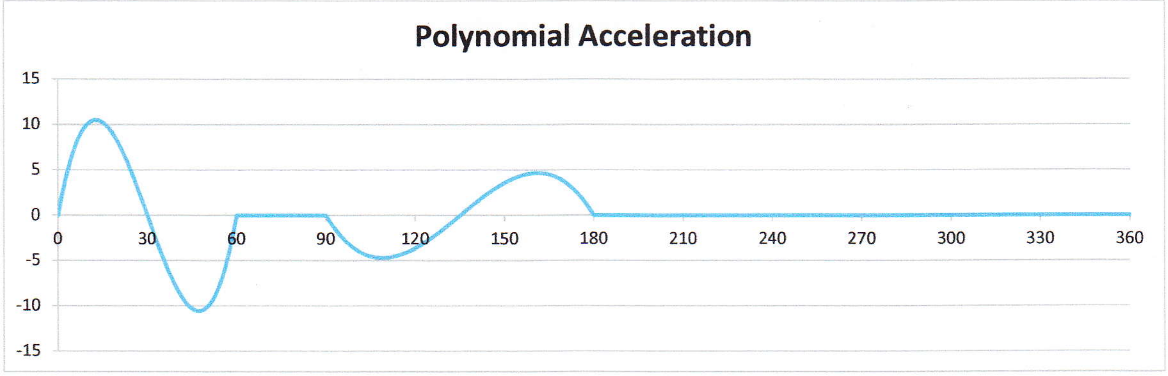
Follower Acceleration (Cycloid), a
Acceleration, a [in/rad^2]
15
10
5
0
-5
-10
-15
0
30
60
90
120
150
180
210
240
270
300
330
360
CAM Angle in Degrees
Cycloid

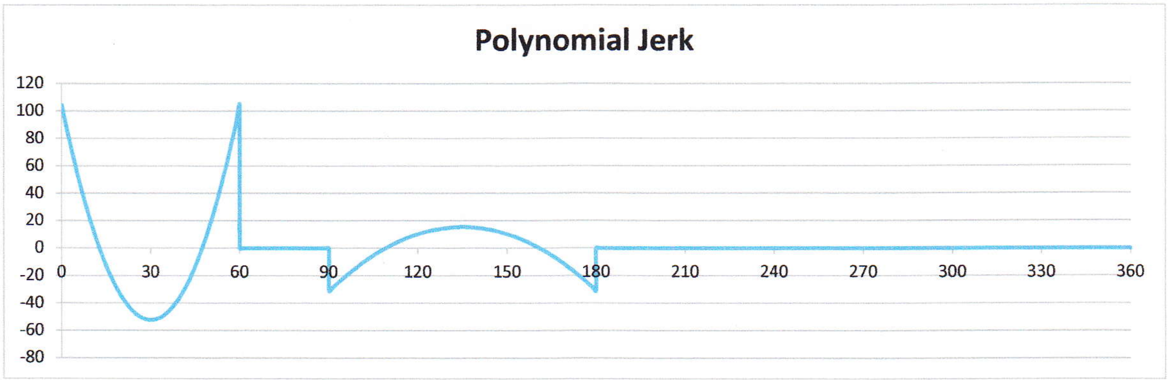


Follower Jerk (Cycloid), j
Jerk, j [in/rad^3]
100
80
60
40
20
0
-20
-40
-60
-80
0
30
60
90
120
150
180
210
240
270
300
330
360
CAM Angle in Degrees
Cycloid


Polynomial Displacement
8
6
4
2
0
0 30 60 90 120 150 180 210 240 270 300 330 360


Polynomial Velocity
4
3
2
1
0
-1
-2
-3
0 30 60 90 120 150 180 210 240 270 300 330 360


Polynomial Acceleration
15
10
5
0
-5
-10
-15
0 30 60 90 120 150 180 210 240 270 300 330 360


Polynomial Jerk
120
100
80
60
40
20
0
-20
-40
-60
-80
0 30 60 90 120 150 180 210 240 270 300 330 360

PROBLEM 2 (~~20~~ 50 pts): Using the computer program you developed in class, design a four-bar linkage to carry the object in the figure on the next page through the three positions shown in their numbered order without regard for fixed pivots. Use points A and B for your attachment points.

2a. Write out your GIVENS, FREE CHOICES, and what you need to find below. Input the appropriate parameters into your computer model. Print out your model and insert it directly after the page that follows. (This last instruction is part of the exam. Failing to comply with this request will result in a 2pt deduction on the exam grade.)

GIVEN:

$$p_{21} = \sqrt{(421\text{mm}-0\text{mm})^2 + (963\text{mm}-0\text{mm})^2} = 1051\text{mm} \qquad \delta_2 = \tan^{-1}\left(\frac{963\text{mm}}{421\text{mm}}\right) = 66.386°$$

$$p_{31} = \sqrt{(184\text{mm}-0\text{mm})^2 + (1400\text{mm}-0\text{mm})^2} = 1412\text{mm} \qquad \delta_3 = \tan^{-1}\left(\frac{1400\text{mm}}{184\text{mm}}\right) = 82.513°$$

$$\alpha_2 = 27° - 0° = 27°$$

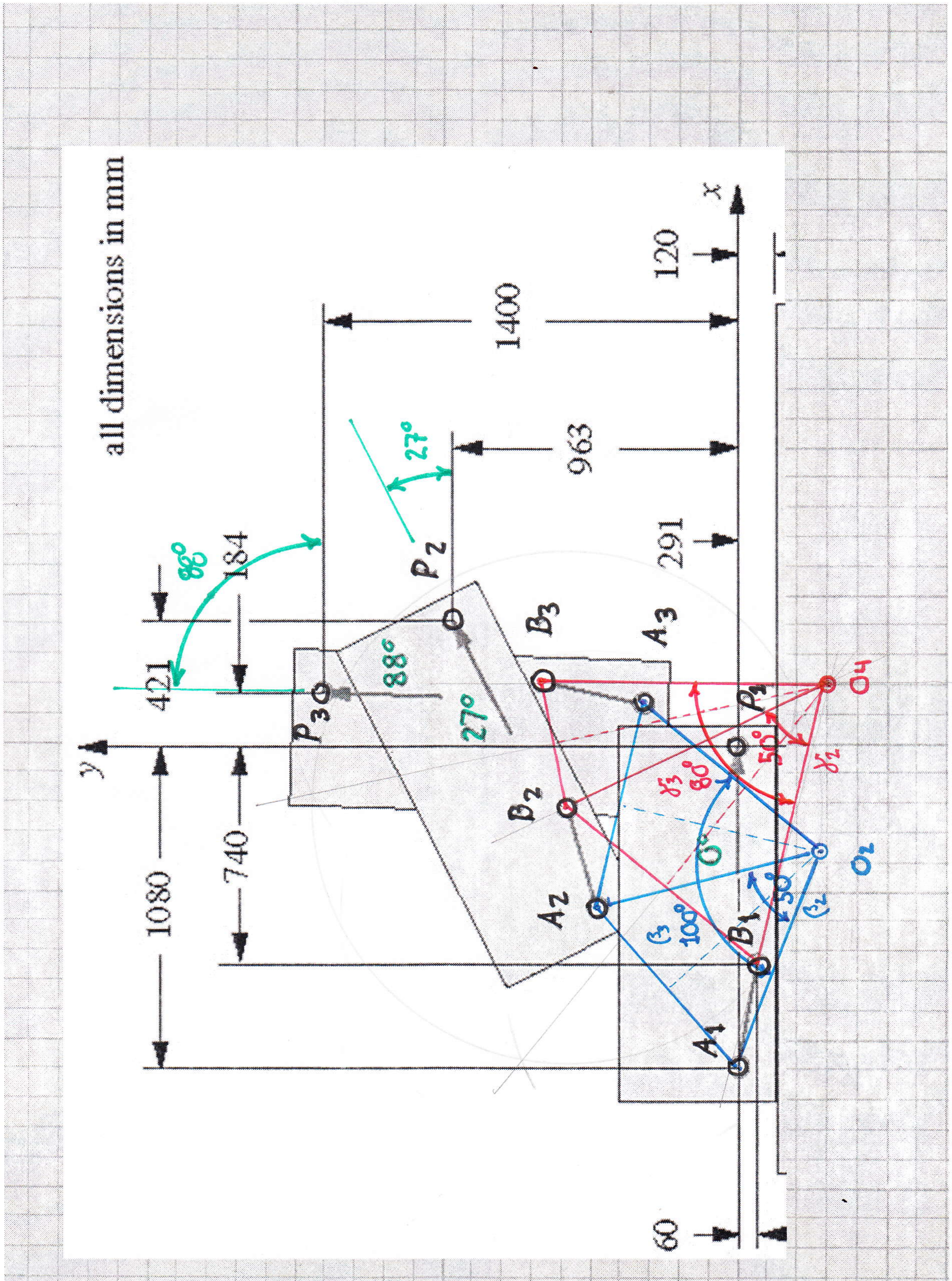
$$\alpha_3 = 88° - 0° = 88°$$

FREE CHOICES: BEST DETERMINED BY PEFORMING A GRAPHICAL SOLUTION

$\beta_2 = -50°$ $\qquad\qquad$ $\gamma_2 = -50°$

$\beta_3 = -100°$ $\qquad\qquad$ $\gamma_3 = -80°$

THE NEGATIVE SIGNS INDICATE CLOCKWISE ROTATION.


all dimensions in mm
x
y
1400
963
291
120
60
1080
740
421
184
88°
27°
P_1
P_2
P_3
A_1
A_2
A_3
B_1
B_2
B_3
O_2
O_4
50°
80°
100°

THREE POSITION ANALYTICAL MOTION SYNTHESIS

$$\vec{W}_2+\vec{Z}_2=\vec{W}_1+\vec{Z}_1+\vec{P}_{21};\quad \vec{W}_3+\vec{Z}_3=\vec{W}_1+\vec{Z}_1+\vec{P}_{31}$$

$$|\vec{W}_1|=|\vec{W}_2|=|\vec{W}_3|=w;\quad |\vec{Z}_1|=|\vec{Z}_2|=|\vec{Z}_3|=z$$

$$\vec{W}_1=w\cdot\left[\cos(\theta)\hat{i}+\sin(\theta)\hat{j}\right]$$

$$\vec{W}_2=w\cdot\left[\cos(\theta+\beta_2)\hat{i}+\sin(\theta+\beta_2)\hat{j}\right]$$

$$\vec{W}_3=w\cdot\left[\cos(\theta+\beta_3)\hat{i}+\sin(\theta+\beta_3)\hat{j}\right]$$

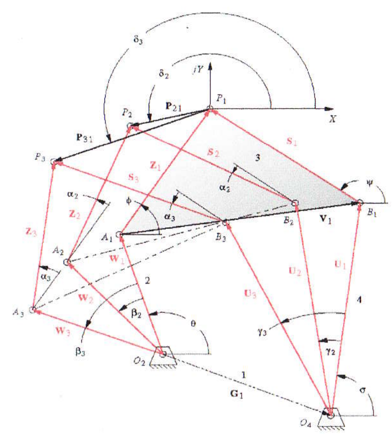
$$\vec{Z}_1=z\cdot\left[\cos(\phi)\hat{i}+\sin(\phi)\hat{j}\right]$$

$$\vec{Z}_2=z\cdot\left[\cos(\phi+\alpha_2)\hat{i}+\sin(\phi+\alpha_2)\hat{j}\right]$$

$$\vec{Z}_3=z\cdot\left[\cos(\phi+\alpha_3)\hat{i}+\sin(\phi+\alpha_3)\hat{j}\right]$$

$$\vec{P}_{21}=p_{21}\cdot\left[\cos(\delta_2)\hat{i}+\sin(\delta_2)\hat{j}\right]$$

$$\vec{P}_{31}=p_{31}\cdot\left[\cos(\delta_3)\hat{i}+\sin(\delta_3)\hat{j}\right]$$



$$\vec{U}_2+\vec{S}_2=\vec{U}_1+\vec{S}_1+\vec{P}_{21};\quad \vec{U}_3+\vec{S}_3=\vec{U}_1+\vec{S}_1+\vec{P}_{31}$$

$$|\vec{U}_1|=|\vec{U}_2|=|\vec{U}_3|=u;\quad |\vec{S}_1|=|\vec{S}_2|=|\vec{S}_3|=s$$

$$\vec{U}_1=u\cdot\left[\cos(\sigma)\hat{i}+\sin(\sigma)\hat{j}\right]$$

$$\vec{U}_2=u\cdot\left[\cos(\sigma+\gamma_2)\hat{i}+\sin(\sigma+\gamma_2)\hat{j}\right]$$

$$\vec{U}_3=u\cdot\left[\cos(\sigma+\gamma_3)\hat{i}+\sin(\sigma+\gamma_3)\hat{j}\right]$$

$$\vec{S}_1=s\cdot\left[\cos(\psi)\hat{i}+\sin(\psi)\hat{j}\right]$$

$$\vec{S}_2=s\cdot\left[\cos(\psi+\alpha_2)\hat{i}+\sin(\psi+\alpha_2)\hat{j}\right]$$

$$\vec{S}_3=s\cdot\left[\cos(\psi+\alpha_3)\hat{i}+\sin(\psi+\alpha_3)\hat{j}\right]$$

$$\vec{P}_{21}=p_{21}\cdot\left[\cos(\delta_2)\hat{i}+\sin(\delta_2)\hat{j}\right]$$

$$\vec{P}_{31}=p_{31}\cdot\left[\cos(\delta_3)\hat{i}+\sin(\delta_3)\hat{j}\right]$$

FIRST DYAD

GIVEN:		CHOSEN:		FIND:	
P12	1051.00	β2	-50.00	w	864.431
P13	1412.00	β3	-100.00	θ	155.183
δ2	66.39			z	1093.078
δ3	82.51			φ	2.096
α2	27.00			W1x	-784.603
α3	88.00			W1y	362.822
				Z1x	1092.347
				Z1y	39.972

	x-coord	y-coord.
O2	-307.744	-402.794
A1	-1092.347	-39.972
A2	-534.139	431.464
A3	185.810	306.885
P1	0.000	0.000
P2	421.002	962.994
P3	183.985	1399.962

$$\begin{bmatrix} -0.3572 & 0.7660 & -0.1090 & -0.4540 \\ -0.7660 & -0.3572 & 0.4540 & -0.1090 \\ -1.1736 & 0.9848 & -0.9651 & -0.9994 \\ -0.9848 & -1.1736 & 0.9994 & -0.9651 \end{bmatrix} \begin{Bmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{Bmatrix} = \begin{Bmatrix} 421.0021 \\ 962.9944 \\ 183.9853 \\ 1399.9619 \end{Bmatrix}$$

$$\begin{bmatrix} \cos\beta_2-1 & -\sin\beta_2 & \cos\alpha_2-1 & -\sin\alpha_2 \\ \sin\beta_2 & \cos\beta_2-1 & \sin\alpha_2 & \cos\alpha_2-1 \\ \cos\beta_3-1 & -\sin\beta_3 & \cos\alpha_3-1 & -\sin\alpha_3 \\ \sin\beta_3 & \cos\beta_3-1 & \sin\alpha_3 & \cos\alpha_3-1 \end{bmatrix} \cdot \begin{Bmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{Bmatrix} = \begin{Bmatrix} p_{21}\cdot\cos\delta_2 \\ p_{21}\cdot\sin\delta_2 \\ p_{31}\cdot\cos\delta_3 \\ p_{31}\cdot\sin\delta_3 \end{Bmatrix}$$

SECOND DYAD

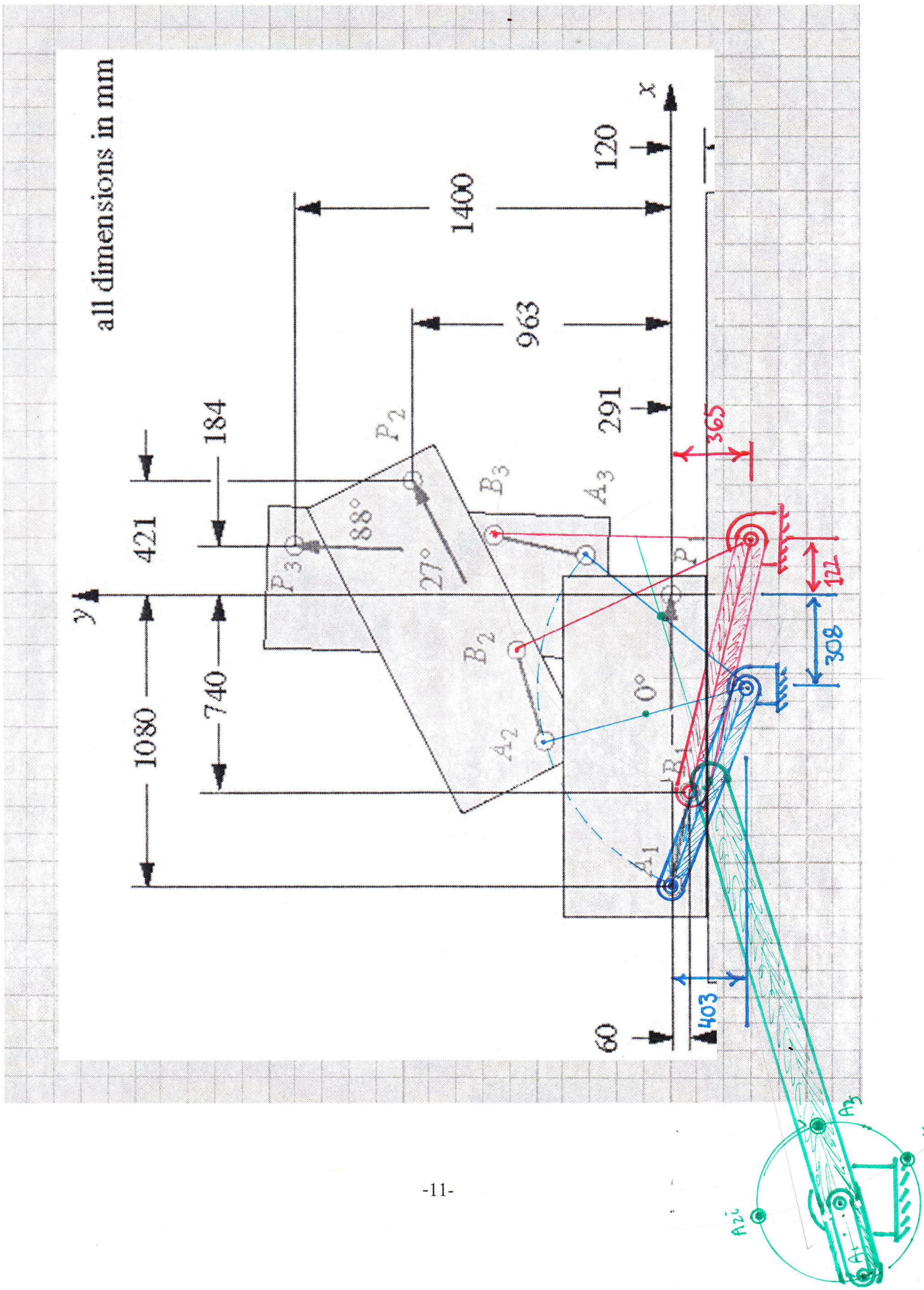
GIVEN:		CHOSEN:		FIND:	
P12	1051.00	γ2	-50.00	u	966.523
P13	1412.00	γ3	-80.00	σ	163.051
δ2	66.39			s	806.988
δ3	82.51			ψ	5.891
α2	27.00			U1x	-924.542
α3	88.00			U1y	281.759
				S1x	802.726
				S1y	82.830

	x-coord	y-coord.
O4	121.816	-364.589
B1	-802.726	-82.830
B2	-256.628	524.763
B3	238.750	594.834
P1	0.000	0.000
P2	421.002	962.994
P3	183.985	1,399.962

$$\begin{bmatrix} -0.3572 & 0.7660 & -0.1090 & -0.4540 \\ -0.7660 & -0.3572 & 0.4540 & -0.1090 \\ -0.8264 & 0.9848 & -0.9651 & -0.9994 \\ -0.9848 & -0.8264 & 0.9994 & -0.9651 \end{bmatrix} \begin{Bmatrix} U1x \\ U1y \\ S1x \\ S1y \end{Bmatrix} = \begin{Bmatrix} 421.0021 \\ 962.9944 \\ 183.9853 \\ 1399.9619 \end{Bmatrix}$$

$$\begin{bmatrix} \cos\gamma_2-1 & -\sin\gamma_2 & \cos\alpha_2-1 & -\sin\alpha_2 \\ \sin\gamma_2 & \cos\gamma_2-1 & \sin\alpha_2 & \cos\alpha_2-1 \\ \cos\gamma_3-1 & -\sin\gamma_3 & \cos\alpha_3-1 & -\sin\alpha_3 \\ \sin\gamma_3 & \cos\gamma_3-1 & \sin\alpha_3 & \cos\alpha_3-1 \end{bmatrix} \cdot \begin{Bmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{Bmatrix} = \begin{Bmatrix} p_{21}\cdot\cos\delta_2 \\ p_{21}\cdot\sin\delta_2 \\ p_{31}\cdot\cos\delta_3 \\ p_{31}\cdot\sin\delta_3 \end{Bmatrix}$$

1b. Draw the mechanism that you synthesized with your computer model to scale on the figure below. Show the mechanism in position 1.



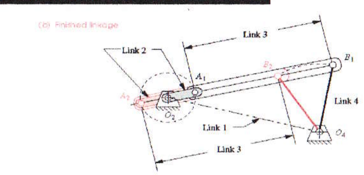
2c: Using the computer model you developed in class, synthesize a drive dyad for the mechanism that you synthesized in Part 2a. Print out your model and insert it directly after the page that follows. (This last instruction is part of the exam. Failing to comply with this request will result in a 2pt deduction on the exam grade.) VERIFY THAT THE MECHANISM IS A GRASHOF MECHANISM.

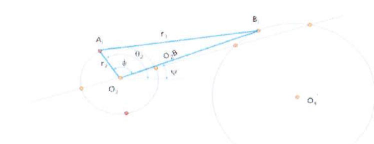
2d. On the figure in 2b, draw the synthesized drive dyad in the configuration that is appropriate for position 1 of the mechanism that you synthesized in 2a. Make sure that you show the location of all of the positons of Point A on the drive dyad.

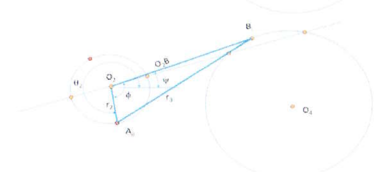
NON-QUICK-RETURN (From Three Position Results)

	X-pos	Y-pos	mag	angle	i	j
3P-O2 => O4	**-307.744**	**-402.794**	**506.902**	**-127.4**	**-0.6071**	**-0.7946**
3P-A1	**-1092.347**	**-39.972**	**1093.078**	**-177.9**	**-0.9993**	**-0.0366**
3P-A2	**-534.139**	**431.464**	**686.634**	**141.1**	**-0.7779**	**0.6284**
3P-A3	**185.810**	**306.885**	**358.754**	**58.8**	**0.5179**	**0.8554**

Factors		
P	0.5	**% dist up Link 4**
K	-2.5	**Length of Link 3+Link 2 wrt B1B2**
Link 1	**2005.911**	
Link 2	**331.096**	
Link 3	**1986.578**	
Link 4	**432.216**	**Grashof**







$\phi=$	84.2
$\psi=$	19.6
$\theta_{2i}=$	103.8
$\theta_{2ii}=$	295.4

$$r_3^2 = r_2^2 + (O_2B)^2 - 2\cdot r_2\cdot(O_2B)\cdot\cos\phi$$

$$\phi = \cos^{-1}\frac{r_2^2 + (O_2B)^2 - r_3^2}{2\cdot r_2\cdot(O_2B)}$$

$$\psi = \tan^{-1}\frac{B_{iy} - O_{2y}}{B_{ix} - O_{2x}}$$

$$\theta_{2i} = \phi + \psi$$

$$\theta_{2ii} = 360 + \psi - \phi$$

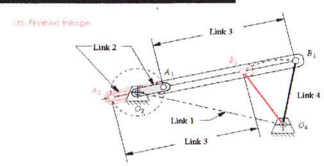
	x comp	y comp	mag	angle	Normal (r) i	Normal (r) j	Perpendicular (θ) i	Perpendicular (θ) j
rO4	**-307.744**	**-402.794**	**506.902**	**-127.4**	**-0.6071**	**-0.7946**	**0.7946**	**-0.6071**
rP3O2-A1	-784.603	362.822	864.431	155.2	-0.9077	0.4197	-0.4197	-0.9077
rP3O2-A2	-226.395	834.258	864.431	105.2	-0.2619	0.9651	-0.9651	-0.2619
rP3O2-A3	493.555	709.679	864.431	55.2	0.5710	0.8210	-0.8210	0.5710
rB1	**-700.046**	**-221.383**	**734.217**	**-162.5**	**-0.9535**	**-0.3015**	**0.3015**	**-0.9535**
rO4B1	-392.301	181.411	432.216	155.2	-0.9077	0.4197	-0.4197	-0.9077
rB2	**-60.967**	**-47.954**	**77.567**	**-141.8**	**-0.7860**	**-0.6182**	**0.6182**	**-0.7860**
rO4B2	246.777	354.840	432.216	55.2	0.5710	0.8210	-0.8210	0.5710
rBi	**-420.942**	**14.335**	**421.186**	**178.0**	**-0.9994**	**0.0340**	**-0.0340**	**-0.9994**
rO4Bi	-113.197	417.129	432.216	105.2	-0.2619	0.9651	-0.9651	-0.2619
rB1B2	639.079	173.429	662.193	15.2	0.9651	0.2619	-0.2619	0.9651
rO2	**-2297.743**	**-654.954**	**2389.265**	**-164.1**	**-0.9617**	**-0.2741**	**0.2741**	**-0.9617**
rB1O2	-1597.697	-433.571	1655.482	-164.8	-0.9651	-0.2619	0.2619	-0.9651
rBiO2	-1876.801	-669.289	1992.568	-160.4	-0.9419	-0.3359	0.3359	-0.9419
rB2O2	-2236.776	-607.000	2317.674	-164.8	-0.9651	-0.2619	0.2619	-0.9651
rA1	**-2617.282**	**-741.669**	**2720.338**	**-164.2**	**-0.9621**	**-0.2726**	**0.2726**	**-0.9621**
rO2A1	-319.539	-86.714	331.096	-164.8	-0.9651	-0.2619	0.2619	-0.9651
rA2	**-1978.203**	**-568.240**	**2058.199**	**-164.0**	**-0.9611**	**-0.2761**	**0.2761**	**-0.9611**
rO2A2	319.539	86.714	331.096	15.2	0.9651	0.2619	-0.2619	0.9651
rAi	-2376.841	-333.445	2400.116	-172.0	-0.9903	-0.1389	0.1389	-0.9903
rO2Ai	-79.098	321.509	331.096	103.8	-0.2389	0.9710	-0.9710	-0.2389
rAii	-2155.556	-953.966	2357.217	-156.1	-0.9144	-0.4047	0.4047	-0.9144
rO2Aii	142.187	-299.011	331.096	-64.6	0.4294	-0.9031	0.9031	0.4294
rB1A1	-1917.236	-520.286	1986.578	-164.8	-0.9651	-0.2619	0.2619	-0.9651
rBiAi	-1955.899	-347.780	1986.578	-169.9	-0.9846	-0.1751	0.1751	-0.9846
rBiAii	-1734.614	-968.301	1986.578	-150.8	-0.8732	-0.4874	0.4874	-0.8732
rB2A2	-1917.236	-520.286	1986.578	-164.8	-0.9651	-0.2619	0.2619	-0.9651
rO4O2	-1989.998	-252.160	2005.911	-172.8	-0.9921	-0.1257	0.1257	-0.9921

K IS NEGATIVE IN THIS SECTION TO KEEP IT BELOW THE MECHANISM.

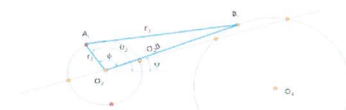
NON-QUICK-RETURN (From Three Position Results)

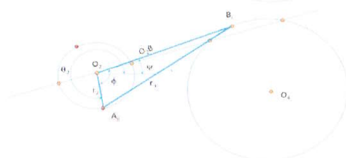
	X-pos	Y-pos	mag	angle	i	j
3P-O2 => O4	**-307.744**	**-402.794**	**506.902**	**-127.4**	**-0.6071**	**-0.7946**
3P-A1	**-1092.347**	**-39.972**	**1093.078**	**-177.9**	**-0.9993**	**-0.0366**
3P-A2	**-534.139**	**431.464**	**686.634**	**141.1**	**-0.7779**	**0.6284**
3P-A3	**185.810**	**306.885**	**358.754**	**58.8**	**0.5179**	**0.8554**

Factors		
P	0.5	**% dist up Link 4**
K	2.5	**Length of Link 3+Link 2 wrt B1B2**
Link 1	**1353.212**	
Link 2	**331.096**	
Link 3	**1324.385**	
Link 4	**432.216**	**Grashof**



ϕ=	81.3
ψ=	-171.5
θ2i=	-90.2
θ2ii=	107.2





$$r_3^2 = r_2^2 + (O_2B)^2 - 2 \cdot r_2 \cdot (O_2B) \cdot \cos\phi$$

$$\phi = \cos^{-1}\frac{r_2^2 + (O_2B)^2 - r_3^2}{2 \cdot r_2 \cdot (O_2B)}$$

$$\psi = \tan^{-1}\frac{B_{iy} - O_{2y}}{B_{ix} - O_{2x}}$$

$$\theta_{2i} = \phi + \psi$$

$$\theta_{2ii} = 360 + \psi - \phi$$

					Normal (r)		Perpendicular (θ)	
	x comp	y comp	mag	angle	i	j	i	j
rO4	**-307.744**	**-402.794**	**506.902**	**-127.4**	**-0.6071**	**-0.7946**	**0.7946**	**-0.6071**
rP3O2-A1	-784.603	362.822	864.431	155.2	-0.9077	0.4197	-0.4197	-0.9077
rP3O2-A2	-226.395	834.258	864.431	105.2	-0.2619	0.9651	-0.9651	-0.2619
rP3O2-A3	493.555	709.679	864.431	55.2	0.5710	0.8210	-0.8210	0.5710
rB1	**-700.046**	**-221.383**	**734.217**	**-162.5**	**-0.9535**	**-0.3015**	**0.3015**	**-0.9535**
rO4B1	-392.301	181.411	432.216	155.2	-0.9077	0.4197	-0.4197	-0.9077
rB2	**-60.967**	**-47.954**	**77.567**	**-141.8**	**-0.7860**	**-0.6182**	**0.6182**	**-0.7860**
rO4B2	246.777	354.840	432.216	55.2	0.5710	0.8210	-0.8210	0.5710
rBi	**-420.942**	**14.335**	**421.186**	**178.0**	**-0.9994**	**0.0340**	**-0.0340**	**-0.9994**
rO4Bi	-113.197	417.129	432.216	105.2	-0.2619	0.9651	-0.9651	-0.2619
rB1B2	639.079	173.429	662.193	15.2	0.9651	0.2619	-0.2619	0.9651
rO2	**897.651**	**212.189**	**922.389**	**13.3**	**0.9732**	**0.2300**	**-0.2300**	**0.9732**
rB1O2	1597.697	433.571	1655.482	15.2	0.9651	0.2619	-0.2619	0.9651
rBiO2	1318.593	197.853	1333.354	8.5	0.9889	0.1484	-0.1484	0.9889
rB2O2	958.618	260.143	993.289	15.2	0.9651	0.2619	-0.2619	0.9651
rA1	**578.112**	**125.474**	**591.572**	**12.2**	**0.9772**	**0.2121**	**-0.2121**	**0.9772**
rO2A1	-319.539	-86.714	331.096	-164.8	-0.9651	-0.2619	0.2619	-0.9651
rA2	**1217.190**	**298.903**	**1253.354**	**13.8**	**0.9711**	**0.2385**	**-0.2385**	**0.9711**
rO2A2	319.539	86.714	331.096	15.2	0.9651	0.2619	-0.2619	0.9651
rAi	896.724	-118.907	904.573	-7.6	0.9913	-0.1315	0.1315	0.9913
rO2Ai	-0.927	-331.095	331.096	-90.2	-0.0028	-1.0000	1.0000	-0.0028
rAii	799.592	528.431	958.429	33.5	0.8343	0.5514	-0.5514	0.8343
rO2Aii	-98.059	316.242	331.096	107.2	-0.2962	0.9551	-0.9551	-0.2962
rB1A1	1278.157	346.857	1324.385	15.2	0.9651	0.2619	-0.2619	0.9651
rBiAi	1317.666	-133.242	1324.385	-5.8	0.9949	-0.1006	0.1006	0.9949
rBiAii	1220.534	514.096	1324.385	22.8	0.9216	0.3882	-0.3882	0.9216
rB2A2	1278.157	346.857	1324.385	15.2	0.9651	0.2619	-0.2619	0.9651
rO4O2	1205.396	614.983	1353.212	27.0	0.8908	0.4545	-0.4545	0.8908

SOLUTION FOR A POSITIVE VALUE OF K

As a student at Union College, I am part of a community that values intellectual effort, curiosity and discovery. I understand that in order to truly claim my educational and academic achievements, I am obligated to act with academic integrity. Therefore, I affirm that I carried out the work on this exam with full academic honesty, and I rely on my fellow students to do the same.

For this exam I understand that:

1. I **must** work alone in writing out the answers to this exam.
2. I **cannot** copy solutions to these problems from any person or any other resource.
3. I **can** only use my calculator and the programs that I developed in **this course** to solve the problems in this exam I **cannot** use any other electronic resources to assist me in the solution to the questions on this exam.
4. I **cannot** discuss any part of this exam or discuss what was covered on this exam with anyone else, post it in any electronic form, or communicate in any way that would provide assistance to anyone as to what is being covered on the exam.

Signature: ______________________________

Print Name: ______________________________

Exam Date: ______________________________

PROBLEM 1 (50 pts): Design a double-dwell cam to move a follower 0 to 2in in 60°, Dwell for 30°, Fall 2in in 90°, and dwell for the remainder of the cycle. The total cycle must take 2 seconds.

1a). Illustrate below the segments of motion for the described follower, the length of the segments, duration of the segments, and appropriate segment boundary conditions to insure two piecewise continuous derivatives of the displacement functions.

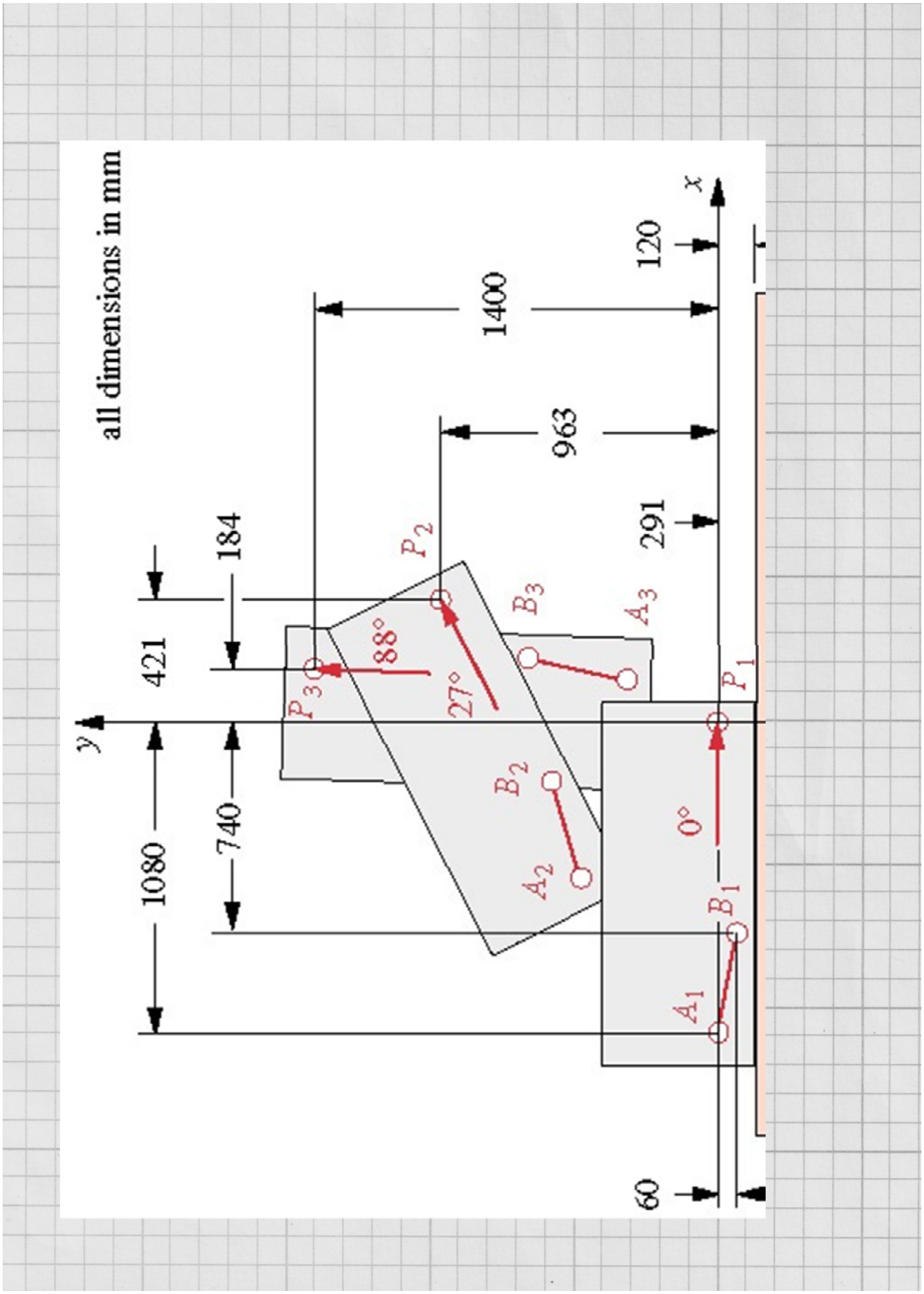
1b. Determine the parameters of CYCLOIDAL functions that will cause the follower to RISE to the DWELL and then FALL to the return DWELL.

1c. Determine the parameters of POLYNOMIAL functions that will cause the follower to RISE to the DWELL and then FALL to the return DWELL.

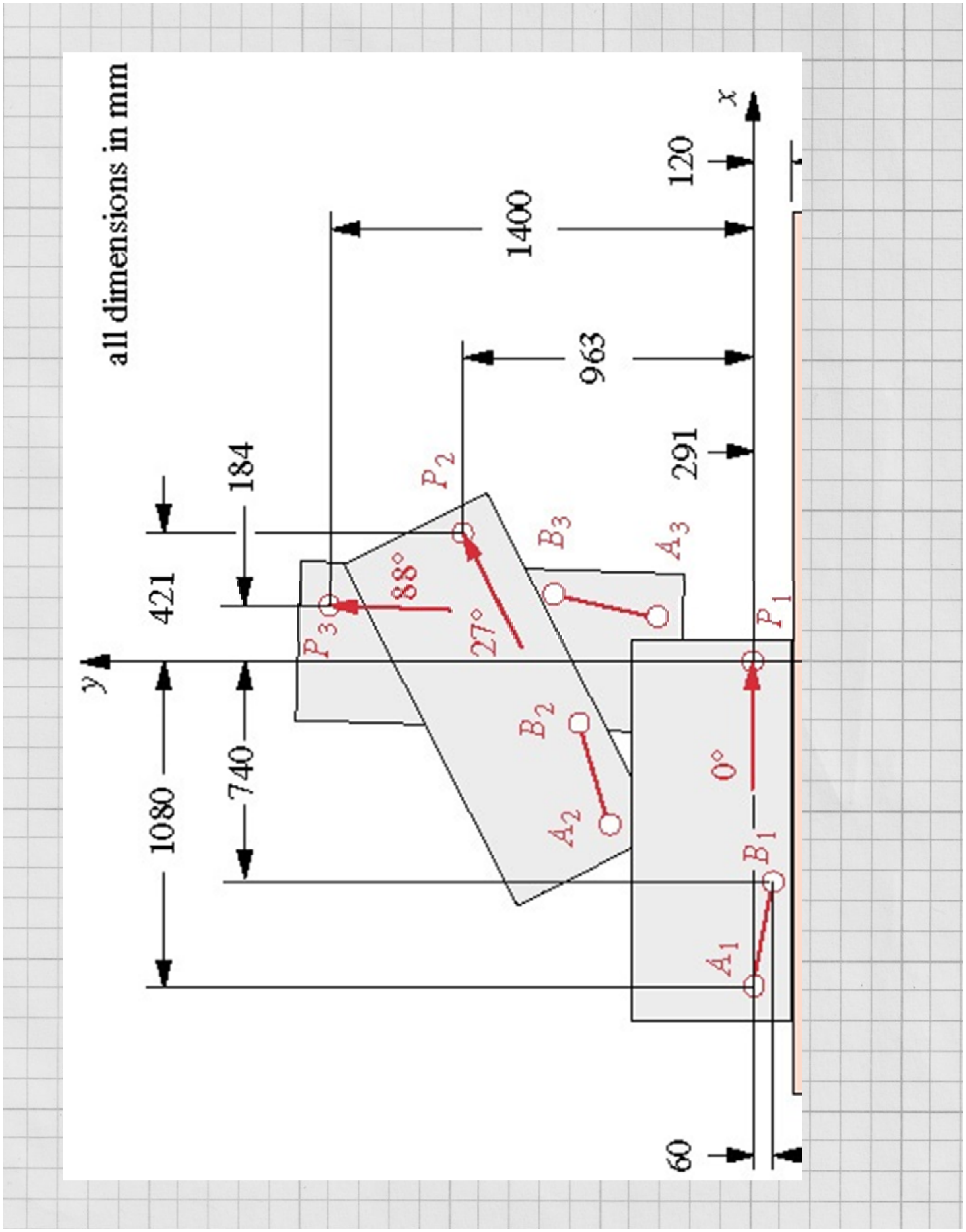
1d: Using the computer program that you developed in class plot the CYCLOIDAL and POLYNOMIAL s-s'-s''-s''' (s-v-a-j) diagrams for the entire 360° of cam rotation. Attach the diagrams directly after this page. (This last instruction is part of the exam. Failing to comply with this request will result in a 2pt deduction on the exam grade.)

PROBLEM 2 (50 pts): Using the computer program you developed in class, design a four-bar linkage to carry the object in the figure on the next page through the three positions shown in their numbered order without regard for fixed pivots. Use points A and B for your attachment points.

2a. Write out your GIVENS, FREE CHOICES, and what you need to find below. Input the appropriate parameters into your computer model. Print out your model and insert it directly after the page that follows. (This last instruction is part of the exam. Failing to comply with this request will result in a 2pt deduction on the exam grade.)


all dimensions in mm
x
y
120
1400
963
291
184
421
1080
740
60
88°
27°
0°
P_1
P_2
P_3
A_1
B_1
A_2
B_2
A_3
B_3

1b. Draw the mechanism that you synthesized with your computer model to scale on the figure below. Show the mechanism in position 1.



2c: Using the computer model you developed in class, synthesize a drive dyad for the mechanism that you synthesized in Part 2a. Print out your model and insert it directly after the page that follows. (This last instruction is part of the exam. Failing to comply with this request will result in a 2pt deduction on the exam grade.) VERIFY THAT THE MECHANISM IS A GRASHOF MECHANISM.

2d. On the figure in 2b, draw the synthesized drive dyad in the configuration that is appropriate for position 1 of the mechanism that you synthesized in 2a. Make sure that you show the location of all of the positons of Point A on the drive dyad.