PROBLEM 4 A SIMPLE BEAM AB WITH AN OVERHANG BC IS LOADED BY TWO FORCES P AND A COUPLE Pa THROUGH THE ARRANGEMENT SHOWN IN THE FIGURE. DRAW THE SHEAR AND BENDING MOMENT DIAGRAM FOR BEAM ABC.

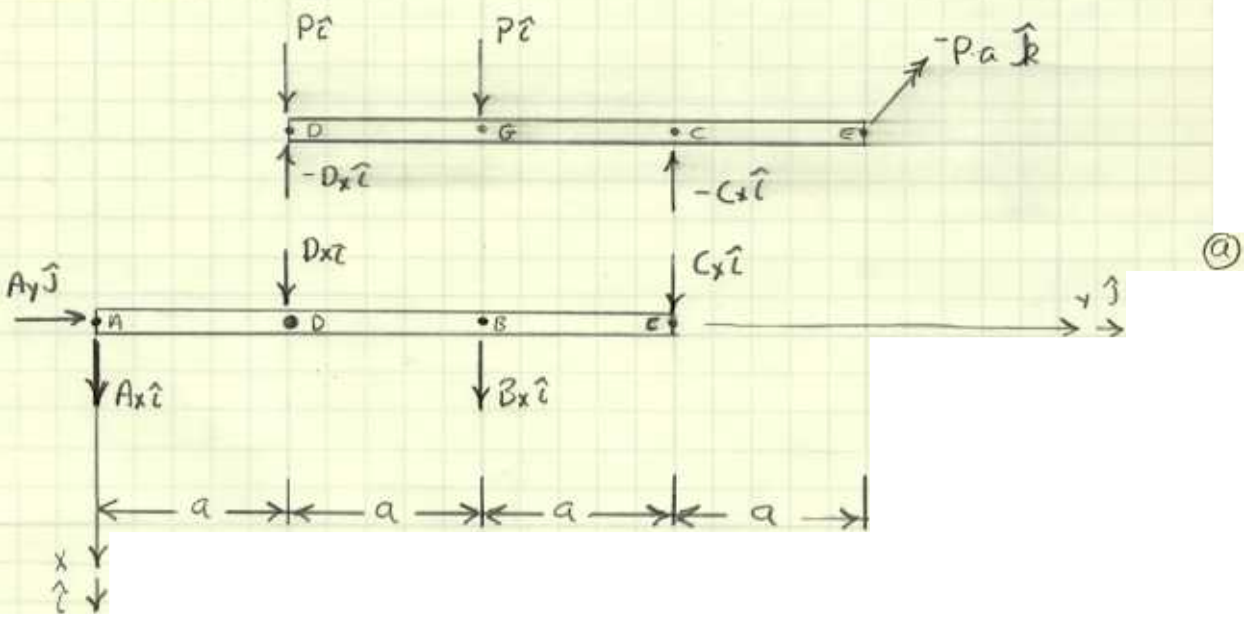
GIVEN:

CONSTRAINTS
1) TWO BEAMS CONNECTED BY PIN JOINTS
2) BEAM LOADED WITH TWO POINT LOADS AND A COUPLE

ASSUMPTIONS
1) MATERIAL RESPONDS IN A LINEAR ELASTIC MANNER
2) DEFLECTIONS ARE SMALL

FIND:
1) SHEAR AND BENDING MOMENT DIAGRAM FOR ABC

FREE BODY DIAGRAM:



MECHANICS:

THE SOLUTION STARTS BY CONSIDERING THE EQUILIBRIUM OF DGCE

$$\Sigma F_x = 0 = 2P - D_x - C_x \Rightarrow \underline{D_x + C_x = 2P} \quad (1)$$

$$\Sigma M_{z/at\,D} = 0 = -Pa - 2aC_x - Pa \Rightarrow \underline{C_x = P} \quad (2)$$

$$(2)\rightarrow(1) \Rightarrow \underline{D_x = P} \quad (3)$$

THESE RESULTS ARE NOW USED TO DETERMINE THE REACTIONS IN ADBC.

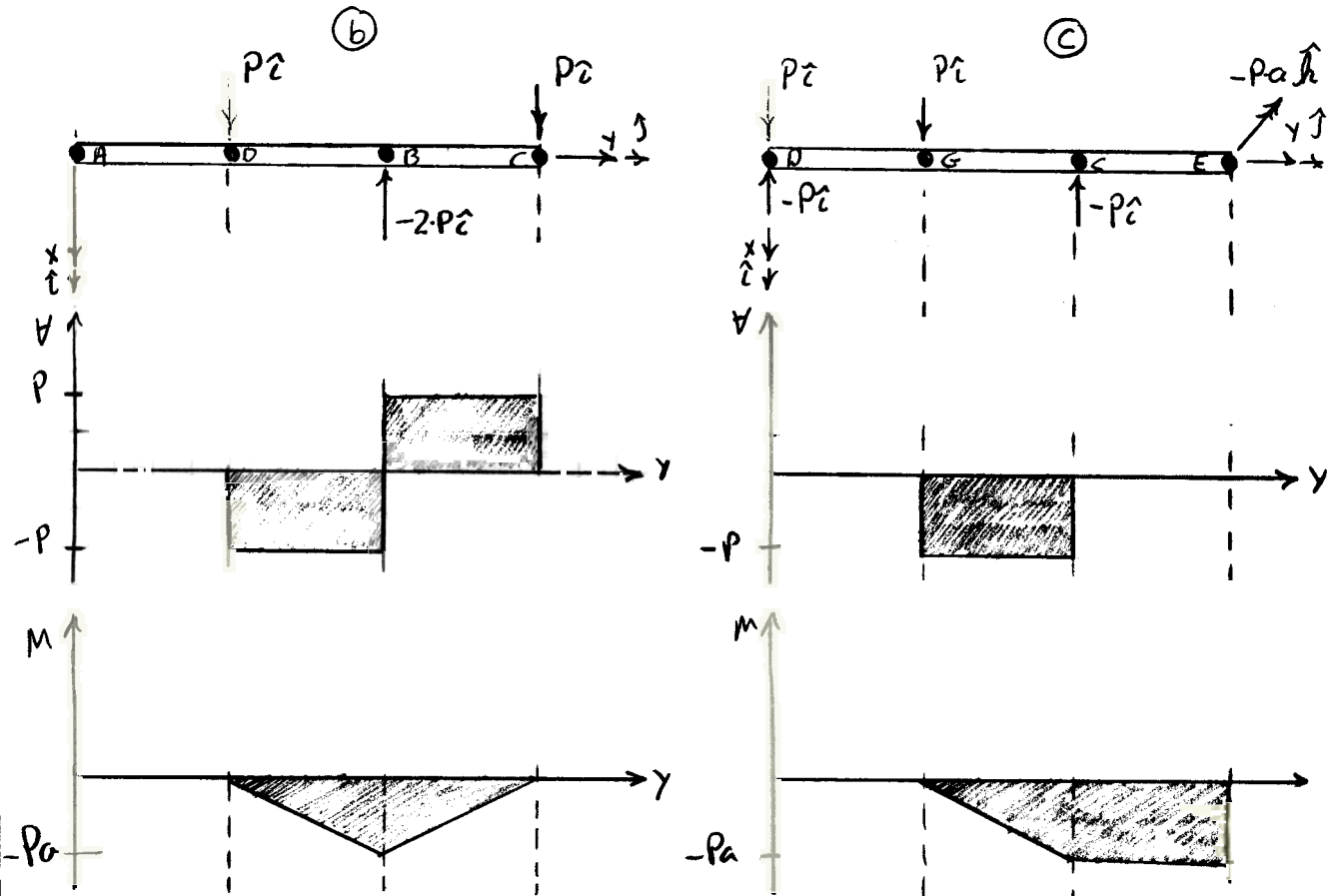
$$\Sigma F_y = 0 = \underline{A_y}$$

$$\Sigma F_x = 0 = A_x + P + B_x + P = 0 \Rightarrow \underline{A_x + B_x = -2P} \quad (4)$$

$$\Sigma M_{z/at\,A} = 0 = -P\cdot a - 2\cdot a\cdot B_x - 3\cdot a\cdot P \Rightarrow \underline{B_x = -2P} \quad (5)$$

$$(5)\rightarrow(4) \Rightarrow \underline{A_x = 0} \quad (6)$$

NOW THE SHEAR AND BENDING MOMENT DIAGRAMS CAN BE DRAWN



Summary: THE SOLUTION IS QUICKLY REALIZED ONCE THE PROBLEM IS DECOMPOSED INTO TWO BEAMS. INTERNAL REACTIONS ARE EQUAL BUT OPPOSITE.

BEAM ADBC

$0 < y < a$ ⓓ

$\Sigma F_x = 0$ ↑ ⇒ $V = 0$

$\circlearrowleft M_{z/D} = 0$ ↺ ⇒ $M = 0$

$a < y < 2a$ USING ⓔ

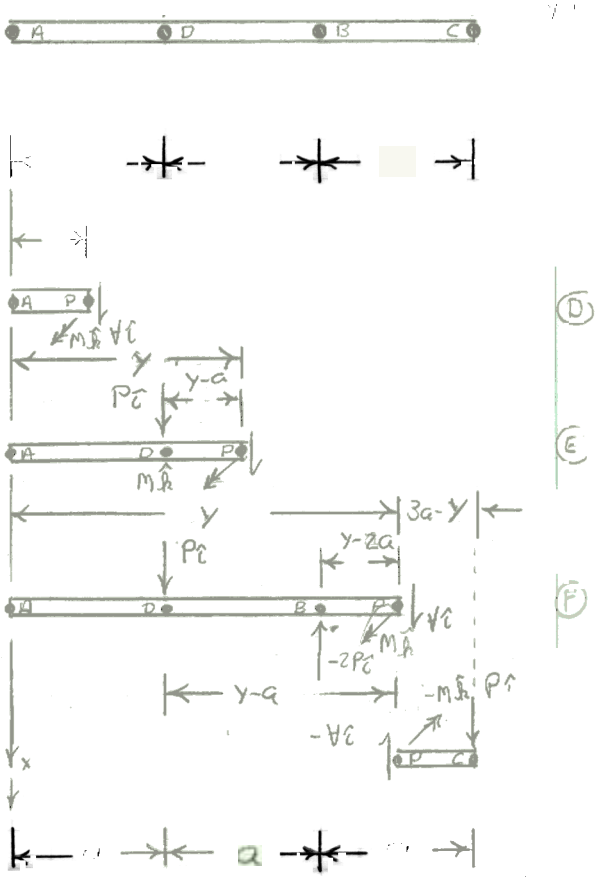
$\Sigma F_x = 0$: P ↓ ⇒ $V = -P$

$\Sigma M_{z/p} = 0$ ↺ ⇒ $M = -P \cdot y + a \cdot P$

$2a < y < 3a$ USI ⓕ

$\Sigma F_x = 0$: ↑ ⇒ $V = P$

$\Sigma M_{z/p} = 0$ (3a y) ⇒ $M = Py - 3a \cdot P$



FOR BEAM DGCE

$0 < y < a$ USING ⓗ

ΣF_y P ↓ ⇒ $V = 0$

$\Sigma M_{z/p} = 0$ ↺ ⇒ $M = 0$

$a < y$ USING ⓘ

$\Sigma F_y = 0$ P ↓ ⇒ $V = -P$

$\Sigma M_{z/p}$ $-P_y + P_y$ $P(y-a)$ ⇒ $M = -P_y + Pa$

$2a < y < 3a$ ⓚ

$\Sigma F_x = 0$ ↑ ⇒ $V = 0$

$\Sigma M_{z/p} = 0$ Pa ↺ ⇒ $M = -Pa$