MER-40: DESIGN I　　　　EXAM 1　　　　FEBRUARY 2, 1994

NAME: SOLUTION

A 4ft long beam is cantilevered from a wall as shown in the figure on Page 2. The beam's cross-section is in the shape of an I-beam with a height of 8 in., and upper flange length of 5.5 in., a lower flange length of 3.5 in., and a 0.5 in wall thickness throughout. On the top of the beam a 480 lb/in load is distributed over the entire length of the beam.

a) On the figure provided (page 2) draw the free-body diagram and determine the reactions at the wall.

$$\sum F_x = 0 = A_x + 23\,040\text{lb} \Rightarrow \boxed{A = {}^{-}23\,040\text{ lb}}$$

$$\sum M_z|_{\text{at }A} = 0 = M_{Az} - (2\text{ft})\left(\frac{12\text{in}}{\text{ft}}\right)(23\,040\text{lb}) \Rightarrow \boxed{M_{Az} = 553\,000\text{ in}\cdot\text{lb}}$$

b) Draw the shear and bending moment diagram for this beam on the figures provided.

For the general section

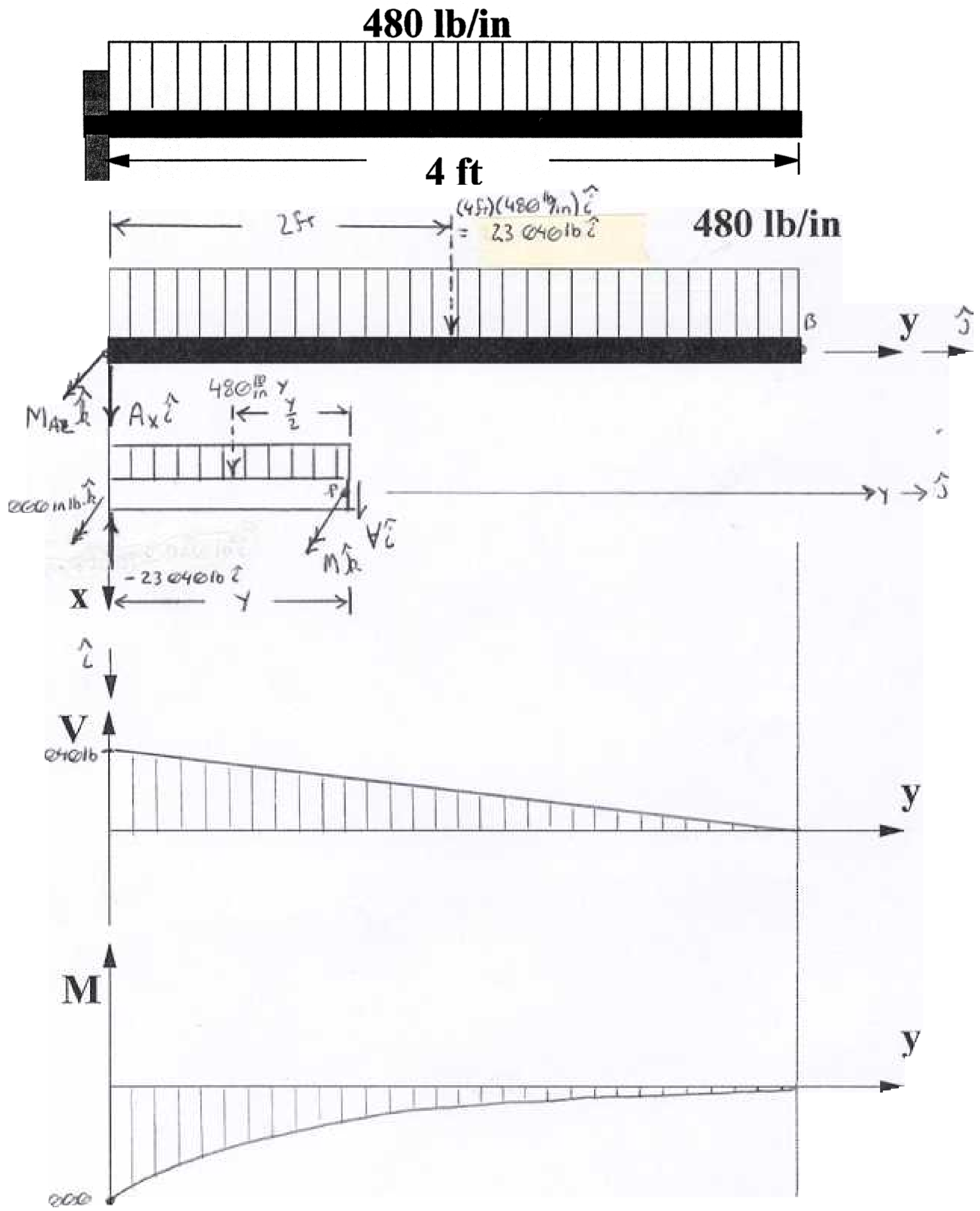
$$\sum F_x = 0 = V + 480\frac{\text{lb}}{\text{in}}\cdot y - 23\,040\text{ lb} \Rightarrow \boxed{V = {}^{-}480\frac{\text{lb}}{\text{in}}\cdot y + 23\,040\text{ lb}}$$

$$V(0) = 23\,040\text{ lb}$$

$$\sum M_z|_{\text{at }p} = 0 = M + 553\,000\text{in}\cdot\text{lb} + \left(480\frac{\text{lb}}{\text{in}}\cdot y\right)\frac{y}{2} - 23\,040\text{lb}\cdot y$$

$$\boxed{M = {}^{-}240\frac{\text{lb}}{\text{in}}\cdot y^2 + 23\,040\text{ lb}\cdot y - 553\,000\text{ in}\cdot\text{lb}}$$

$$M(0) = -553\,000\text{ in}\cdot\text{lb}$$


480 lb/in
4 ft
2 ft
(4 ft)(480 lb/in) î
= 23 040 lb î
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c) Locate the centroid for the cross-section of this beam and plot the coordinate axes on the figure (page 4) provided such that the axes intercept at the centroid.

Since the beam is symmetric about the x-axis no calculation is required; however, the beam is not symmetric about the z-axis. Using the bottom of the lower flange as our reference

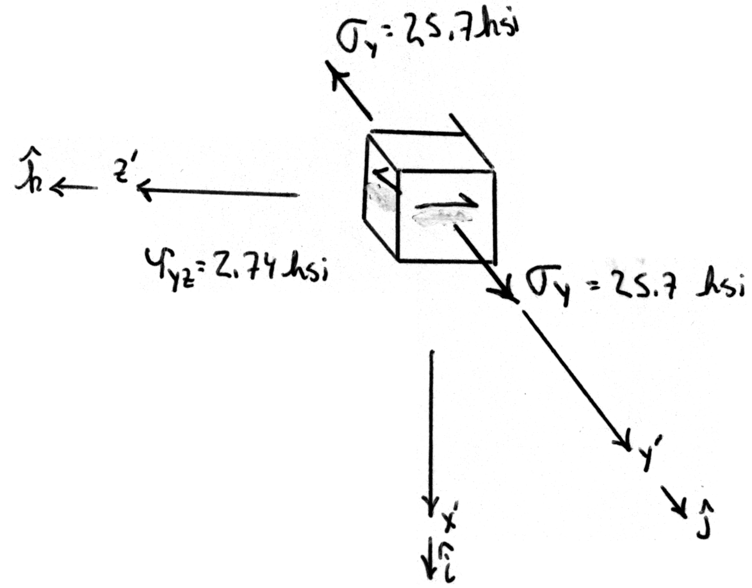
$$\bar{x} = \frac{\sum \tilde{x}_i A_i}{\sum A_i} = \frac{(7.75in)(0.5in)(5.5in) + (4in)(0.5in)(7in) + (0.25in)(0.5in)(3.5in)}{(0.5in)(5.5in) + (0.5in)(7in) + (0.5in)(3.5in)}$$

$$\boxed{\bar{X} = 4.47 \text{ in}}$$

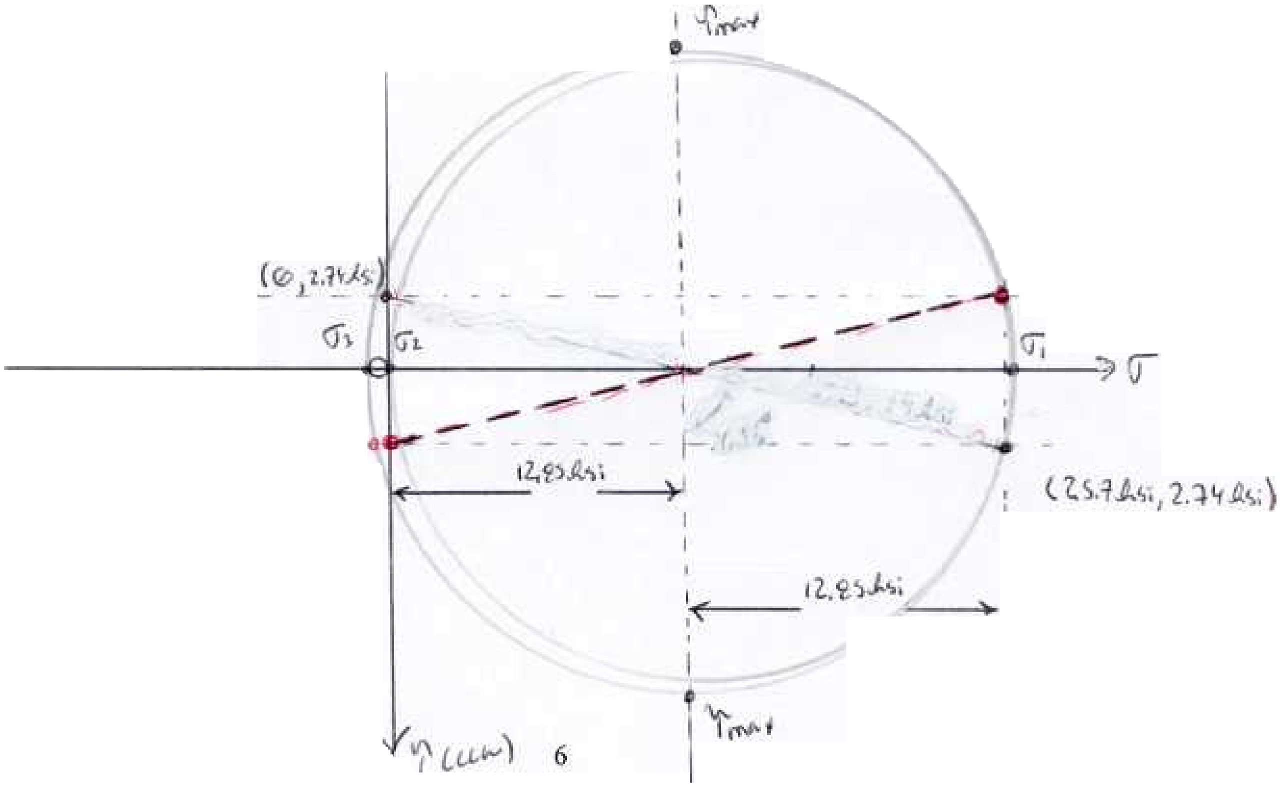
d) Where along the length of the beam do you expect the state of stress to be most severe? Justify your answer.

The highest states of stress will occur near the wall. The reason that the state of stress at the wall is the highest is because this is ~~where~~ where bending moment and transverse shear force are greatest.

f) Draw the stress element at the position in the center of the top flange (point a) just to the left of center.



g) Draw a 3-D Mohr's circle representation for this element.



h) What are the principal stresses and maximum shear stress?

$12.85\,ksi + 13.14\,ksi = 25.99\,ksi = \underline{26.0\,ksi}$

$= 12.85\,ksi - 13.14\,ksi = \underline{\underline{-0.290\,ksi}}$

$\sigma_2 = \underline{\underline{0}}$

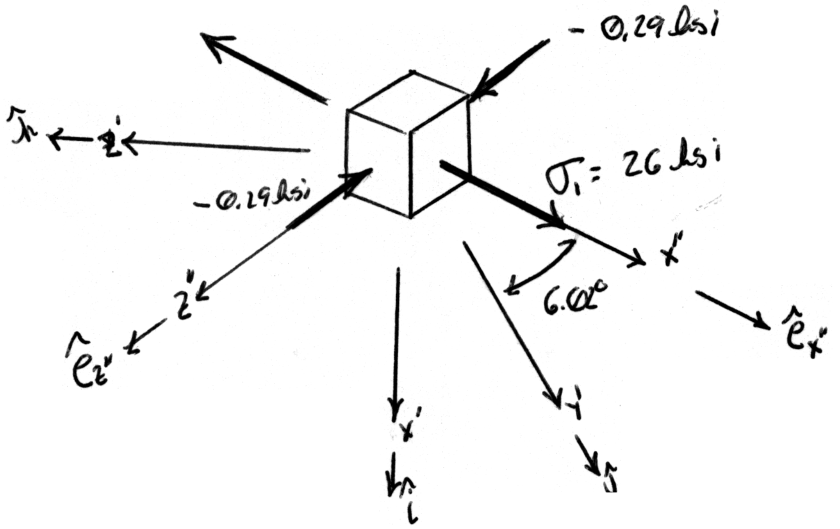
$\tau_{max} = \underline{\underline{13.14\,ksi}}$

i) How much does the element in part "f" have to rotate in order for the stresses on each of the faces to be the principal stresses.

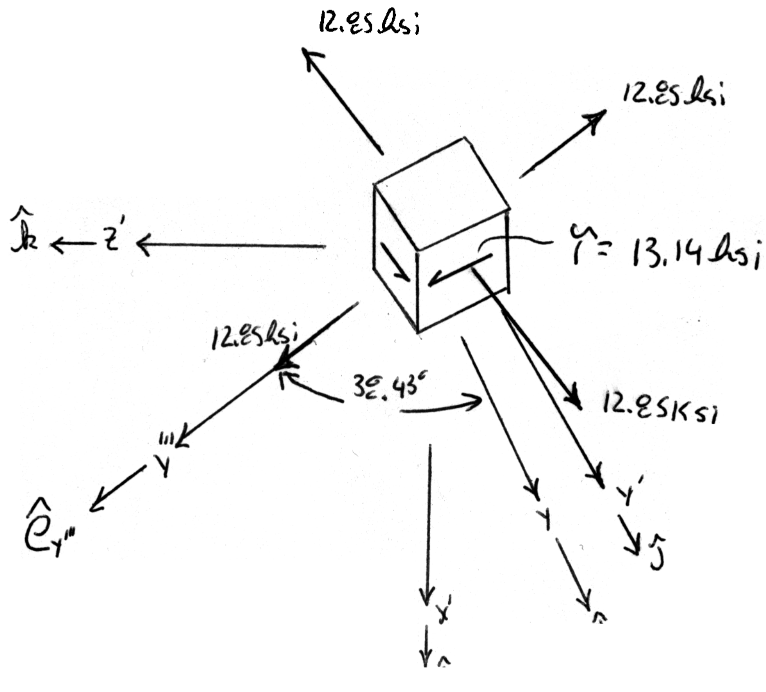
$\tan 2\theta = \dfrac{2.74\,ksi}{12.85\,ksi} = 0.213 \Rightarrow 2\theta = 12.04$

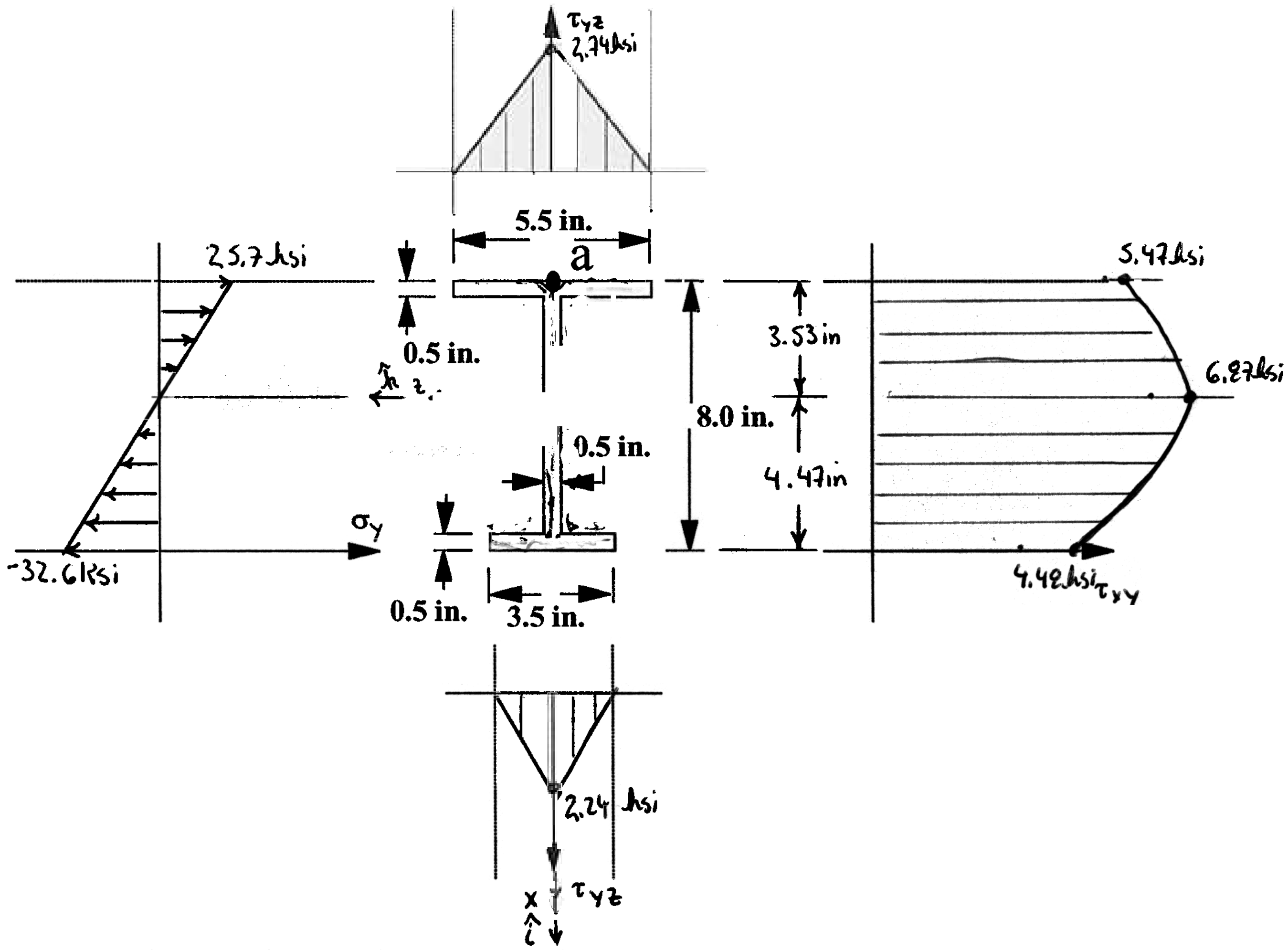
$\theta = 6.02^\circ$ ccw

j) Draw the stress element rotated such that only the principal stresses are on the cube. Be sure to use the original coordinate system to reference the rotation.



k) Draw the stress element such that it contains the maximum shearing stress. Again use the original coordinate system to reference the rotation.





e) Draw the normal stress, shear stress, and shear flow diagrams for the point that you identified in part "d" on the figure provided. Be sure to calculate and label the maximum and minimum points on these figures. On the figures identify which stress component the figure is referring to (i.e., σ_x,σ_y, σ_z, τ_{xy},τ_{xz},τ_{yz}).

normal stress will be given by $\sigma = \frac{M_z y}{I_{zz}}$

We must find I

$$\frac{1}{12}(5.5\,in)(0.5\,in)^3 + (0.5\,in)(5.5\,in)(3.28\,in)^2$$
$$+ \frac{1}{12}(3.5\,in)(0.5\,in)^3 + (0.5\,in)(3.5\,in)(4.22\,in)^2$$
$$+ \frac{1}{12}(0.5\,in)(7\,in)^3 + (0.5\,in)(7.\,in)(0.47\,in)^2$$

$$\boxed{I_{zz} = 75.91\,in^4}$$

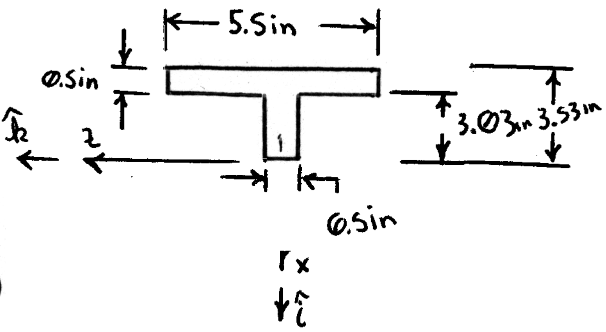
$$\sigma_y(4.47\text{in}) = \frac{(^-553\ 000\ \text{in}\cdot\text{lb})(4.47\text{in})}{75.91\ \text{in}^4} = {}^-32{,}564\ \text{psi}$$

$\underline{\underline{32.6\ \text{Ksi}}}$ (compression)

$$\sigma_y(^-3.53\text{in}) = \frac{(^-553\ 000\ \text{in}\cdot\text{lb})(-3.53\text{in})}{75.91\ \text{in}^4} = 25{,}716\ \text{ps}$$

$\underline{25.7\ \text{ksi}}$ (tension)

The shear stress is calculated using



$\underline{\text{at } x = 0}$

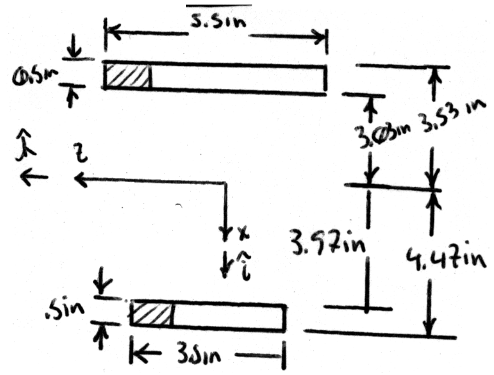
$$\tau_{xy} = \frac{VQ}{It} \quad \text{for } \tau_{xy} \text{ at the centroid } x = 0$$

$$Q_{max} = \sum \bar{x}_i A = (^+3.28\text{in})(0.5\text{in})(5.5\text{in}) + (^+1.52\text{in})(3.03\text{in})(0.5\text{in})$$

$$Q_{max} = {}^+11.32\ \text{in}^3$$

$$\tau_{xy}(0) = \frac{(23\ 040\ \text{lb})(^+11.32\text{in}^3)}{(75.91\ \text{in}^4)(0.5\text{in})} = {}^+6.87(10^3)\ \text{psi} = 6.87\ \text{ksi}$$

$\underline{\text{at } x = {}^-3.03}$



$$Q = (^+3.28\text{in})(0.5\text{in})(5.5\text{in}) = {}^+9.02\ \text{in}^3$$

$$\tau_{xy}(-3.03) = \frac{(23\ 040\ \text{lb})(^+9.02\ \text{in}^3)}{(75.91\ \text{in}^4)(0.5\text{in})} = {}^+\underline{\underline{5.47\ \text{ksi}}}$$

$\underline{\text{at } x = 3.97}$

$$Q = (4.22\text{in})(.5\text{in})(3.5\text{in}) = 7.38\ \text{in}^3$$

$$\tau_{xy}(3.97\text{in}) = \frac{(23\ 040\text{lb})(7.38\text{in}^3)}{(75.91\ \text{in}^4)(0.5\text{in})} \quad 4.48\ \text{ksi}$$

$\underline{\underline{\tau_{yz}}}$

$\underline{\text{Top Flange max}}$

$$Q = (^+3.28\text{in})(0.5\text{in})(2.75\text{in}) = 4.51\ \text{in}^3$$

$$\tau_{yz}(\text{Top}) = \frac{(23\ 040\text{lb})\ (4.51\ \text{in}^3)}{(75.91\ \text{in}^4)\ (0.5\text{in})} = \underline{2.74\ \text{ksi}}$$

$\underline{\text{Bottom Flange max}}$

$$Q = (4.22\text{in})(.5\text{in})(1.75\text{in}) = 3.69\ \text{in}^3$$

$$\tau_{yz}(\text{Bottom}) = \frac{(23\ 040\ \text{lb})\ (3.69\ \text{in}^3)}{(75.91\ \text{in}^4)\ (0.5\text{in})} = \underline{2.24\ \text{ksi}}$$