PROBLEM 6.8-4 3|| A cross section in the shape of an unbalanced I-beam is shown. Derive the following formula for the distance e from the centerline of the web to the shear center S.

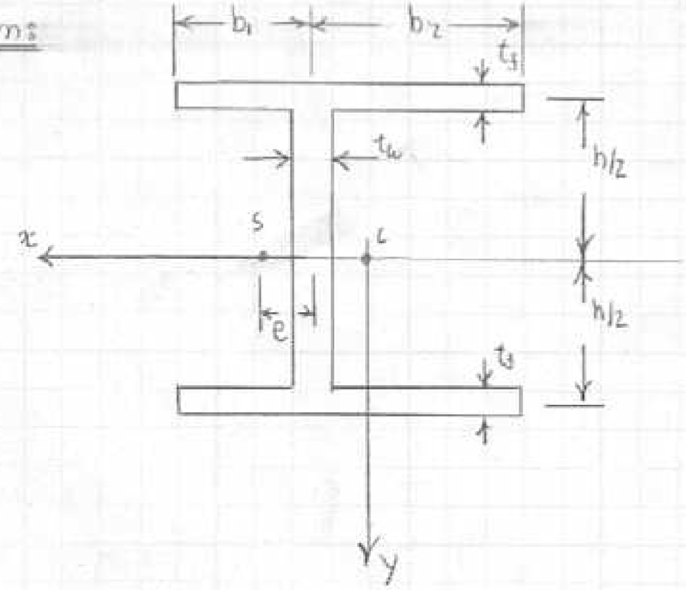
$$e = \frac{3 \cdot t_f \cdot (b_2^2 - b_1^2)}{h \cdot t_w + 6 \cdot t_f \cdot (b_1 + b_2)}$$

Also, check the formula for the special cases in which $b_1 = 0$ and $b_2 = b$ (channel section) and $b_1 = b_2 = b/2$ (doubly symmetric I beam).

GIVEN:
1) Constraint
   - unbalanced I-beam
2) Assumptions
   - linear elastic material response
   - small deflections

FIND:
1) Develop a formula for the shear center
2) check formula for the case of a channel section
3) Check formula for the case of an I-beam

DIAGRAM:



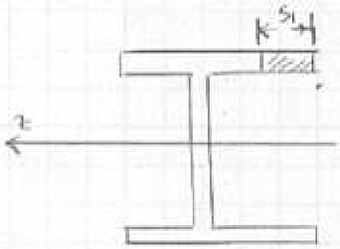
Mechanics:

We start the solution of this problem by determining the moment of inertia about the z axis

$$I_{zz} = \underbrace{\frac{1}{12}(b_1+b_2)\cdot(t_f)^3\cdot 2}_{\text{This term is considered small compared to the rest}} + 2(b_1+b_2)(t_f)\left(\frac{h}{2}\right)^2 + \frac{1}{12}\cdot t_w\cdot h^3$$

$$= \frac{1}{12}\cdot t_w\cdot h^3 + 2\cdot(b_1+b_2)\cdot t_f\cdot\left(\frac{h}{2}\right)^2 = \underline{\underline{\frac{h^2}{12}\left[h\cdot t_w + 6\cdot t_f\cdot(b_1+b_2)\right]}} \quad (1)$$

Now we need to determine the shear stress in each leg



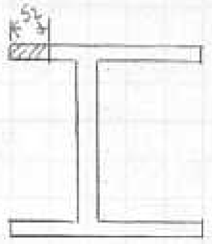
$$\tau = \frac{VQ}{It}$$

$$Q_1 = \frac{h}{2}\cdot s_1\cdot t_f$$

$$\tau_1 = \frac{V\cdot\frac{h}{2}\cdot t_f}{I_{zz}\,t_f}\cdot s_1 = \underline{\underline{\frac{V\cdot h}{2\cdot I_{zz}}\cdot s_1}}$$

$$F_1 = \int_0^{b_2}\tau_1\cdot t_f\cdot ds_1 = \int_0^{b_2}\frac{V\cdot h\cdot t_f}{2\cdot I_{zz}}\cdot s_1\, ds_1 = \underline{\underline{\frac{V\cdot h\cdot t_f}{4\, I_{zz}}\, b_2^2}} \quad (2)$$

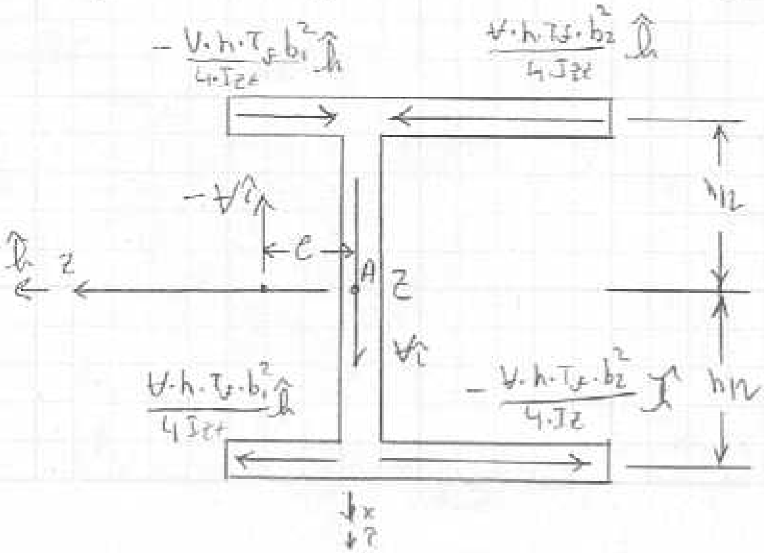
Now for the other part of the flange



$$Q_2 = \frac{h}{2}\cdot s_2\cdot t_f$$

$$\tau_2 = \frac{V\cdot\frac{h}{2}\cdot \cancel{t_f}}{I_{zz}\cdot\cancel{t_f}}\cdot s_2 = \underline{\underline{\frac{V\cdot\frac{h}{2}}{I_{zz}}\cdot s_2}}$$

$$F_2 = \int_0^{b_1}\tau_2\cdot t_f\cdot ds_2 = \int_0^{b_1}\frac{V\cdot h}{2\, I_{zz}}\cdot s_2\cdot t_f\, ds_2 = \underline{\underline{\frac{V\cdot h\cdot t_f\, b_1^2}{4\, I_{zz}}}}$$



Summing moments about point A

$$\sum M_y|_{\text{pt }A} = 0 = -V \cdot e - \frac{V \cdot h^2 \cdot t_f \cdot b_1^2}{4 \cdot I_{zz}} + \frac{V \cdot h^2 t_f \cdot b_2^2}{4 I_{zz}}$$

$$e = \frac{h^2 t_f}{4 I_{zz}}\left(b_2^2 - b_1^2\right) = \frac{h^2 \cdot t_f \cdot (b_2^2 - b_1^2)}{4 \cdot \frac{h^2}{12}\left[h \cdot t_w + 6 \cdot t_f \cdot (b_1 + b_2)\right]}$$

$$= \boxed{\frac{3 \cdot t_f (b_2^2 - b_1^2)}{h \cdot t_w + 6 \cdot t_f \cdot (b_1 + b_2)}}$$

Now lets consider the special cases $b_1 = 0$, $b_2 = b$

$$\boxed{e = \frac{3 \cdot t_f \cdot b^2}{h \cdot t_w + 6 \cdot t_f \cdot b}}$$

and when $b_1 = b_2$

$$\boxed{e = 0}$$

Summary

The solution was facilitated by assuming that $t_w^3 \approx 0$. After this point we simply impose equilibrium.