MER 42: Design III
Exam I

Union College
October 26, 1994

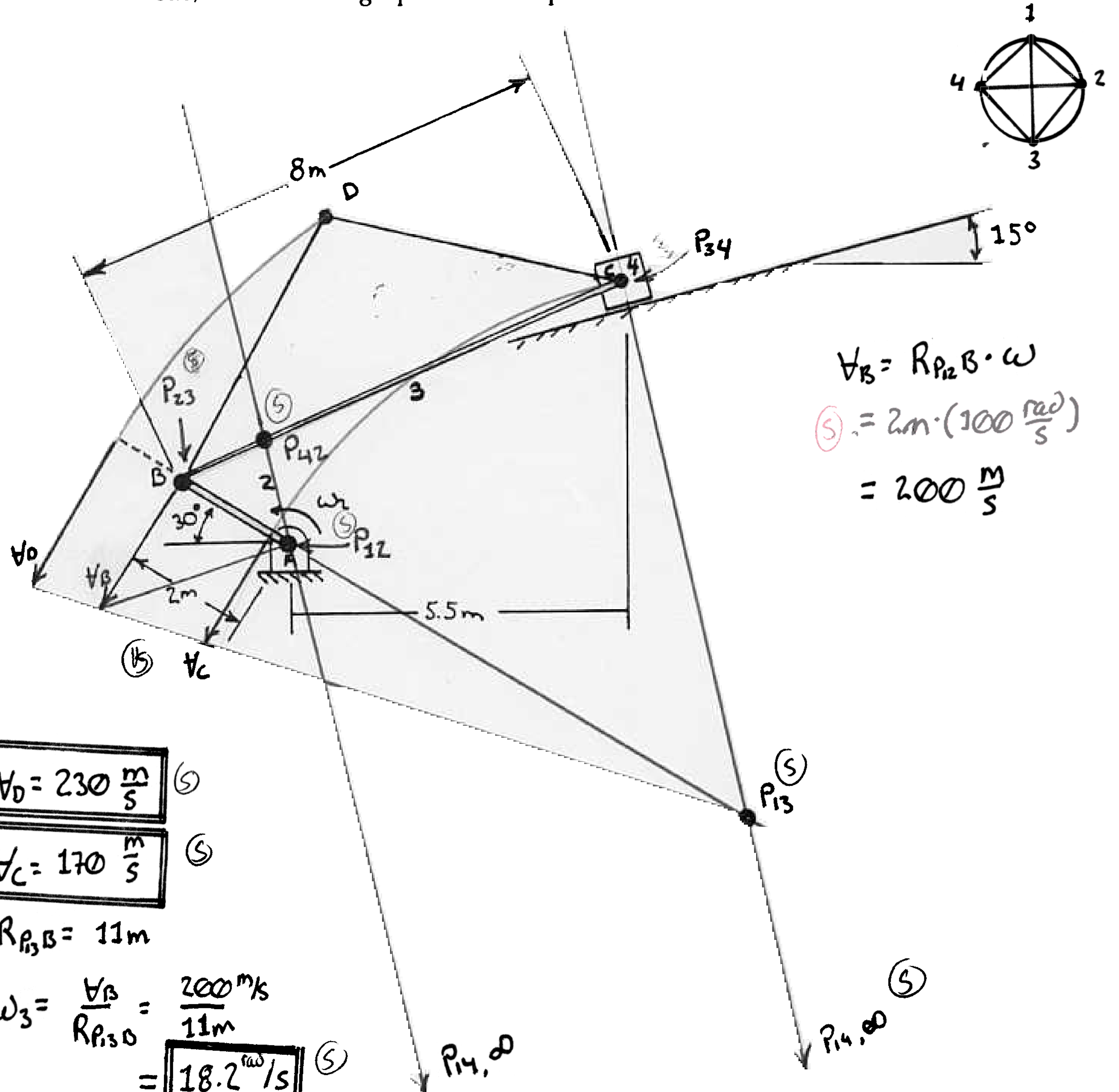
NAME: Solution

PROBLEM 1:

Using the linkage A-B-C illustrated (to scale) below,

a) locate all the instant centers for this configuration and draw them below,

b) determine the absolute velocities of points C and D (attached to link 3) and the angular velocity of link 3 knowing the angular velocity of link 2 is 100 rad/sec. Use the graphical technique.



$V_B = R_{P_{12}B} \cdot \omega$

$= 2m \cdot (100 \frac{rad}{s})$

$= 200 \frac{m}{s}$

$\boxed{V_D = 230 \frac{m}{s}}$

$\boxed{V_C = 170 \frac{m}{s}}$

$R_{P_{13}B} = 11m$

$\omega_3 = \frac{V_B}{R_{P_{13}B}} = \frac{200 \, m/s}{11m}$

$= \boxed{18.2 \, rad/s}$

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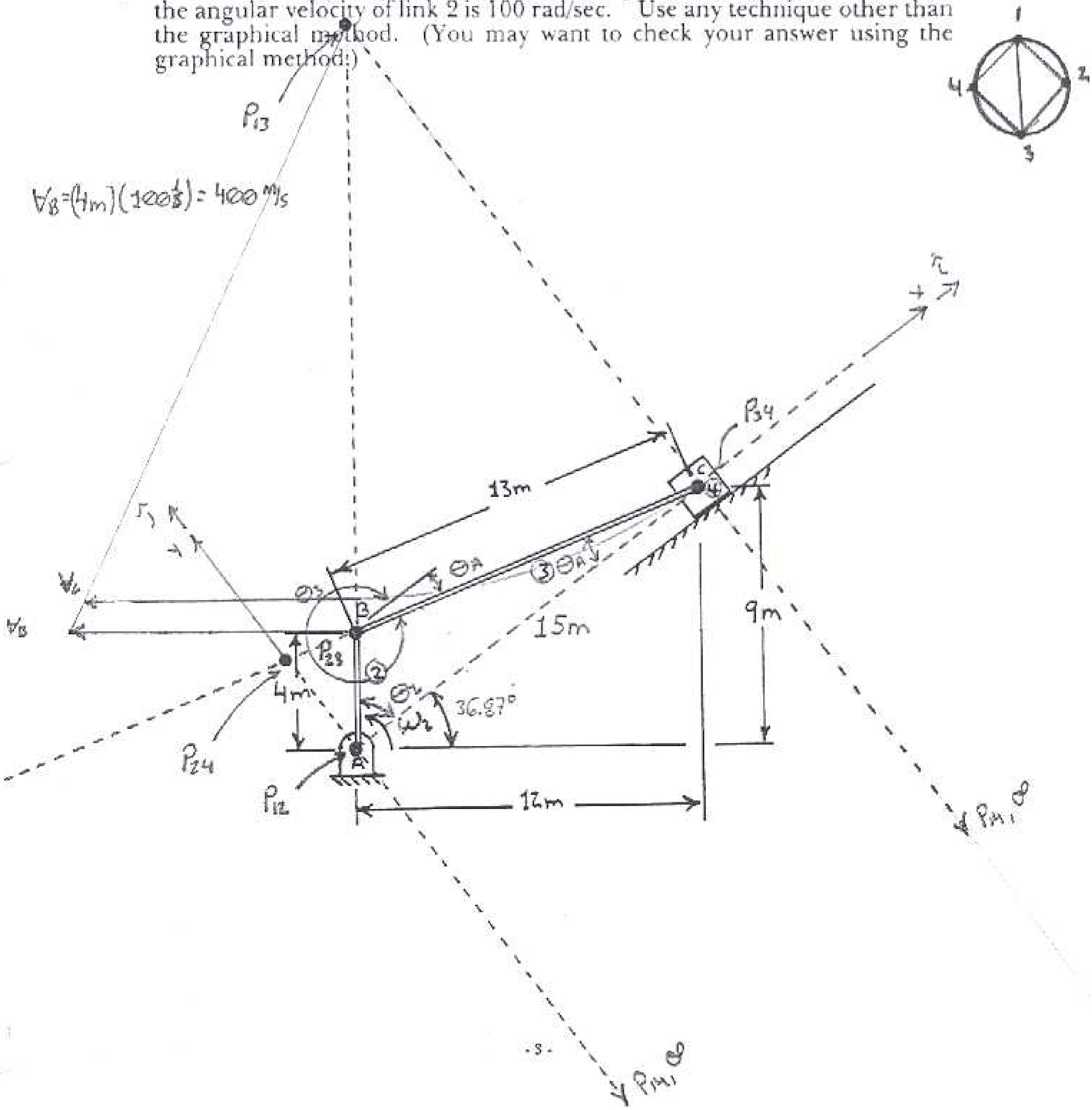
PROBLEM 2:

Using the linkage A-B-C shown (to scale) below,

a) write the loop-closure equation for this configuration,

b) determine the velocity of point C and the angular velocity of link 3 knowing the angular velocity of link 2 is 100 rad/sec. Use any technique other than the graphical method. (You may want to check your answer using the graphical method.)



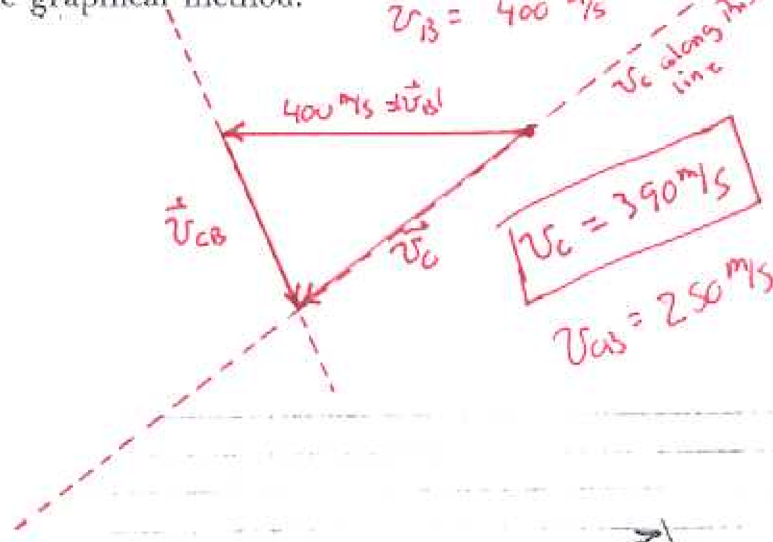
PROBLEM 2:

Using the linkage A-B-C shown (to scale) below,

a) write the loop-closure equation for this configuration,

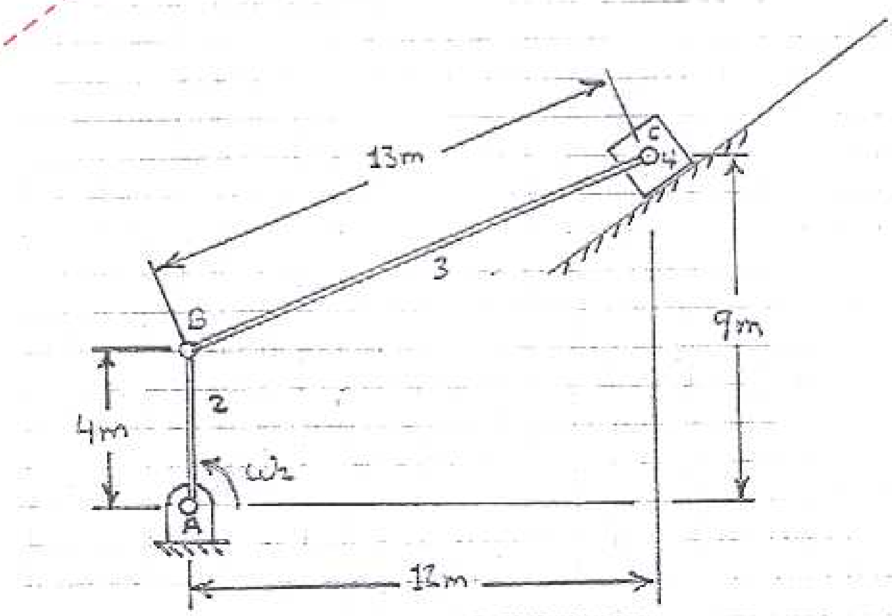
b) determine the velocity of point C and the angular velocity of link 3 knowing the angular velocity of link 2 is 100 rad/sec. Use the graphical method.

$v_B = 400\ m/s$



$\boxed{v_C = 390\ m/s}$

$v_{CB} = 250\ m/s = \omega_3 \cdot r \Rightarrow \boxed{\omega_3 = 19.2\ 1/s}$



a)
$$(13m)^2 = (4m)^2 + (15m)^2 - 2(4m)(15m)\cos\theta_2$$
$$\theta_2 = \cos^{-1}\left[-\frac{13m^2 - (4m)^2 - (15m)^2}{2(4m)(15m)}\right]$$
$$\underline{\underline{\theta_2 = 53.13^\circ}}$$
$$(4m)^2 = (13m)^2 + (15m)^2 - 2(13m)(15m)\cos\theta_A$$
$$\theta_A = \cos^{-1}\left[-\frac{(4m)^2 - (13m)^2 - (15m)^2}{2(13m)(15m)}\right] = \underline{\underline{14.25^\circ}}$$
$$\theta_3 = 360^\circ - \theta_A = \underline{\underline{345.75^\circ}}$$

The loop closure equation can now be written

$$\vec{R}_{AC} = \vec{R}_{AB} + \vec{R}_{BC}$$
$$\vec{R}_{AC} = R_{AC}\hat{e}_{AC} = 15m\,\hat{i}$$
$$\vec{R}_{AB} = R_{AB}\hat{e}_{AB} = 4m(\cos 53.13\,\hat{i} + \sin 53.13\,\hat{j}) = 2.4m\,\hat{i} + 3.20m\,\hat{j}$$
$$\vec{R}_{BC} = R_{BC}\hat{e}_{BC} = 13m(\cos 345.75\,\hat{i} + 345.75\,\hat{j}) = +12.56\,\hat{i} - 3.20m\,\hat{j}$$

b) To determine velocity we must differientate the loop closure equation with respect to time

$$\dot{R}_{AC}\hat{e}_{AC} + R_{AC}\underbrace{\dot{\hat{e}}_{AC}}_{\emptyset} = \underbrace{\dot{R}_{AB}}_{\emptyset}\hat{e}_{AB} + R_{AB}\dot{\hat{e}}_{AB} + \underbrace{\dot{R}_{BC}}_{\emptyset}\hat{e}_{BC} + R_{BC}\dot{\hat{e}}_{BC}$$

$$\dot{R}_{AC}\hat{e}_{AC} = R_{AB}\dot{\hat{e}}_{AB} + R_{BC}\dot{\hat{e}}_{BC} = R_{AB}\cdot\omega_2(\hat{k}\times\hat{e}_{AB}) + R_{BC}\,\omega_3(\hat{k}\times\hat{e}_{BC})$$

To calculate the velocity of point C, $V_c = \dot{R}_{AC}$, we dot this equation with $\hat{k}\times(\hat{k}\times\hat{e}_{BC})$

$$\dot{R}_{AC}\hat{e}_{AC}\cdot[\hat{k}\times(\hat{k}\times\hat{e}_{BC})] = R_{AB}\,\omega_2(\hat{k}\times\hat{e}_{AB})\cdot[\hat{k}\times(\hat{k}\times\hat{e}_{BC})] + R_{BC}\,\omega_3\underbrace{(\hat{k}\times\hat{e}_{BC})\cdot[\hat{k}\times(\hat{k}\times\hat{e}_{BC})]}_{\emptyset}$$

Knowing

$\hat{e}_{AC} = \hat{i}$

$\hat{e}_{AB} = 0.6\hat{i} + 0.8\hat{j}$

$\hat{e}_{BC} = 0.969\hat{i} - 0.246\hat{j}$

$$(\hat{k} \times \hat{e}_{AB}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ .6 & .8 & 0 \end{vmatrix} = -.8\hat{i} + .6\hat{j}$$

$$(\hat{k} \times \hat{e}_{BC}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ .969 & -.246 & 0 \end{vmatrix} = .246\hat{i} + .969\hat{j}$$

$$\hat{k} \times (\hat{k} \times \hat{e}_{BC}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ .246 & .969 & 0 \end{vmatrix} = -.969\hat{i} + .246\hat{j}$$

we can write

$$\dot{R}_{AC} = R_{AB} \cdot \omega_2 \; \frac{(\hat{k} \times \hat{e}_{AB}) \cdot [\hat{k} \times (\hat{k} \times \hat{e}_{BC})]}{\hat{e}_{AC} \cdot [\hat{k} \times (\hat{k} \times \hat{e}_{BC})]} =$$

$$= (4m)(100\tfrac{1}{s}) \cdot \frac{(-.8\hat{i} + .6\hat{j}) \cdot [-.969\hat{i} + .246\hat{j}]}{\hat{i} \cdot (-.969\hat{i} + .246\hat{j})} = \boxed{-380 \ m/s = V_C}$$

To find the angular velocity of link 3 we dot with $(\hat{k} \times \hat{e}_{AC})$

$$\dot{R}_{AC} \underbrace{\hat{e}_{AC} \cdot (\hat{k} \times \hat{e}_{AC})}_{0} = R_{AB} \cdot \omega_2 (\hat{k} \times \hat{e}_{AB}) \cdot (\hat{k} \times \hat{e}_{AC}) + R_{BC} \cdot \omega_3 (\hat{k} \times e_{BC}) \cdot (\hat{k} \times \hat{e}_{AC})$$

$$\omega_3 = -\omega_2 \frac{R_{AB}}{R_{BC}} \frac{(\hat{k} \times \hat{e}_{AB}) \cdot (\hat{k} \times \hat{e}_{AC})}{(\hat{k} \times \hat{e}_{BC}) \cdot (\hat{k} \times e_{AC})}$$

$\hat{k} \times \hat{e}_{AC} = \hat{k} \times \hat{i} = -\hat{j}$

$$\omega_3 = -(100\tfrac{1}{s}) \cdot \left(\frac{4m}{13m}\right) \cdot \left[\frac{(-.8\hat{i} + .6\hat{j}) \cdot (-\hat{j})}{(.246\hat{i} + .969\hat{j}) \cdot (-\hat{j})}\right] = \boxed{-19.1 \tfrac{1}{s} = \omega_3}$$

we can also write the loop closure equation in terms of complex variables

$$\vec{R}_{AC} = \vec{R}_{AB} = \vec{R}_{BC}$$

$$R_{AC}e^{j\theta_1} = R_{AB}e^{j\theta_2} + R_{BC}e^{j\theta_3}$$

Now differentiating with respect to time

$$\dot{R}_{AC}\, e^{j\theta} + R_{AC} j \underbrace{\cancel{\dot{\theta}_1}}_{0} e^{j\theta} = \underbrace{\cancel{\dot{R}_{AB}}}_{0} e^{j\theta_2} + R_{AB}\, j\, \dot{\theta}_2 e^{j\theta_2} + \underbrace{\cancel{\dot{R}_{BC}}}_{0} e^{j\theta_3} + R_{BC}\, j\, \dot{\theta}_3 e^{j\theta_3}$$

$$\dot{R}_{AC} e^{j\theta} = R_{AB}\, j\, \dot{\theta}_2 e^{j\theta_2} + R_{BC}\, j\, \dot{\theta}_3\, e^{j\theta_3}$$

Using Euler's identity, $\pm e^{j\theta} = \cos\theta \pm j\sin\theta$

$$\dot{R}_{AC}(\cos\theta_1 + j\sin\theta_1) = R_{AB}\cdot j\,\dot{\theta}_2\,(\cos\theta_2 + j\sin\theta_2) + R_{BC}\cdot j\cdot\dot{\theta}_3(\cos\theta_3 + j\sin\theta_3)$$

Since $\theta_1 = 0$, ~~and~~ we can now seperate the real and imaginary components of the above equation

$$\dot{R}_{AC} = -R_{AB}\dot{\theta}_2 \sin\theta_2 + R_{BC}\dot{\theta}_3 \sin\theta_3$$

$$0 = R_{AB}\dot{\theta}_2\cos\theta_2 + R_{BC}\dot{\theta}_3\cos\theta_3$$

from the second equation

$$\dot{\theta}_3 = -\frac{R_{AB}\dot{\theta}_2\cos\theta_2}{R_{BC}\cos\theta_3} = -(100\tfrac{1}{s})\left(\frac{4m}{13m}\right)\left(\frac{.6}{.9692}\right) = \boxed{-19.1\tfrac{1}{s}}$$

from the first equation

$$\dot{R}_{AC} = -(4m)(100\tfrac{1}{s})(.8) - (13m)(-19.\ \tfrac{1}{s})(-.2461) = \boxed{-381\tfrac{m}{s}}$$