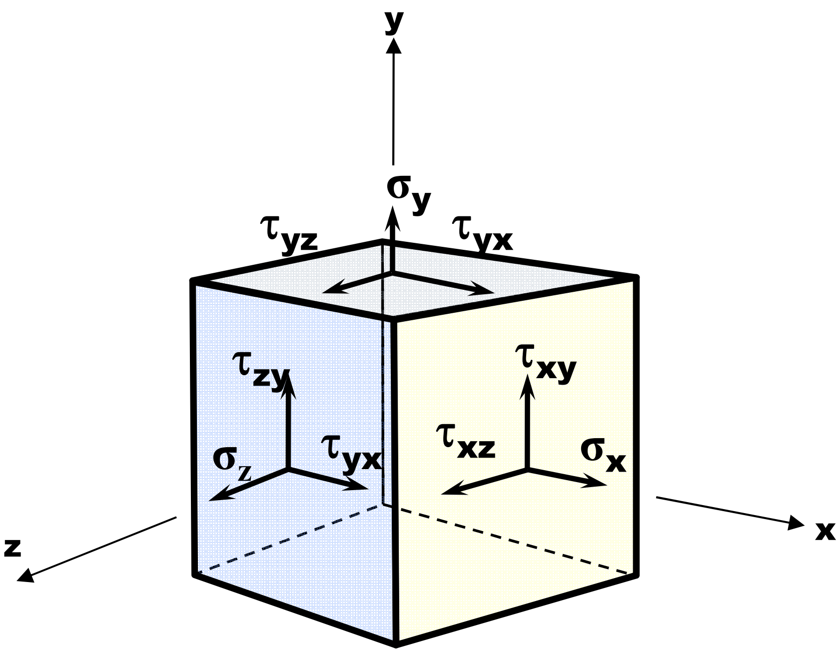
MER311: Advanced Strength of Materials

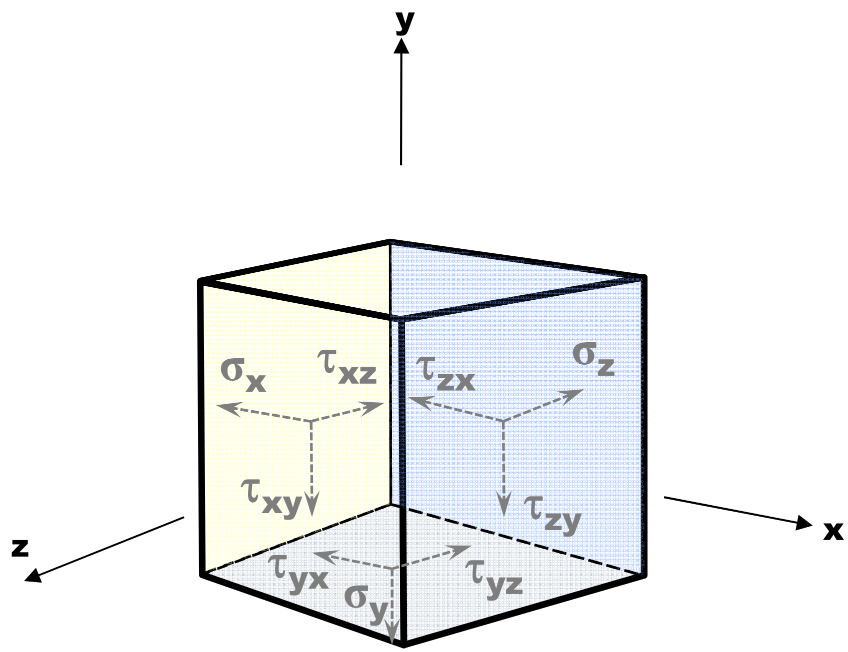
LECTURE OUTLINE

- Principal Stress (σ_1, σ_2, σ_3)
- Eigenvalues and Eigenvectors of Stress Tensor
- Mohr's Circle

Stress at a Point
Shown in the Tensile (+) Direction



Surfaces with a Positive Directed Area Normal



Surfaces with a Negative Directed Area Normal

Principal Stresses are found by Solving Quadratic Equation

$$\begin{vmatrix} \sigma_x - \sigma_p & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_p & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_p \end{vmatrix} = 0$$

$$\begin{aligned} &\sigma_p^3 - (\sigma_x + \sigma_y + \sigma_z) \cdot \sigma_p^2 \\ &\quad + (\sigma_x \cdot \sigma_y + \sigma_y \cdot \sigma_z + \sigma_x \cdot \sigma_z - \tau_{yz}^2 - \tau_{zx}^2 - \tau_{xy}^2) \cdot \sigma_p \\ &\quad - (\sigma_x \cdot \sigma_y \cdot \sigma_z + 2 \cdot \tau_{yz} \cdot \tau_{xz} \cdot \tau_{xy} - \sigma_x \cdot \tau_{yz}^2 - \sigma_y \cdot \tau_{zx}^2 - \sigma_z \cdot \tau_{xy}^2) = 0 \end{aligned}$$

$$\sigma_p^3 - I_1 \cdot \sigma_p^2 + I_2 \cdot \sigma_p - I_3 = 0$$

I_1, I_2, I_3 Stress Invariants

EXAMPLE: Eigenvalues and Functions

Determine the principal stresses and their directions for the tensor shown.

$$[\sigma] = \begin{bmatrix} 50 & 10 & 0 \\ 10 & 20 & 40 \\ 0 & 40 & 30 \end{bmatrix} MPa$$

SOLUTION: In MatLab Eigenvalues and Functions

```
>> S=[50 10 0 ; 10 20 40; 0 40 30]

S =

   50   10    0
   10   20   40
    0   40   30
```

```
>> [DCS,PS]=eig(S)

DCS =

  -0.1135    0.9263    0.3592
   0.7509   -0.1568    0.6415
  -0.6506   -0.3425    0.6778


PS =

  -16.1676         0         0
         0   48.3076         0
         0         0   67.8600
```

SOLUTION: Eigenvalues and Functions

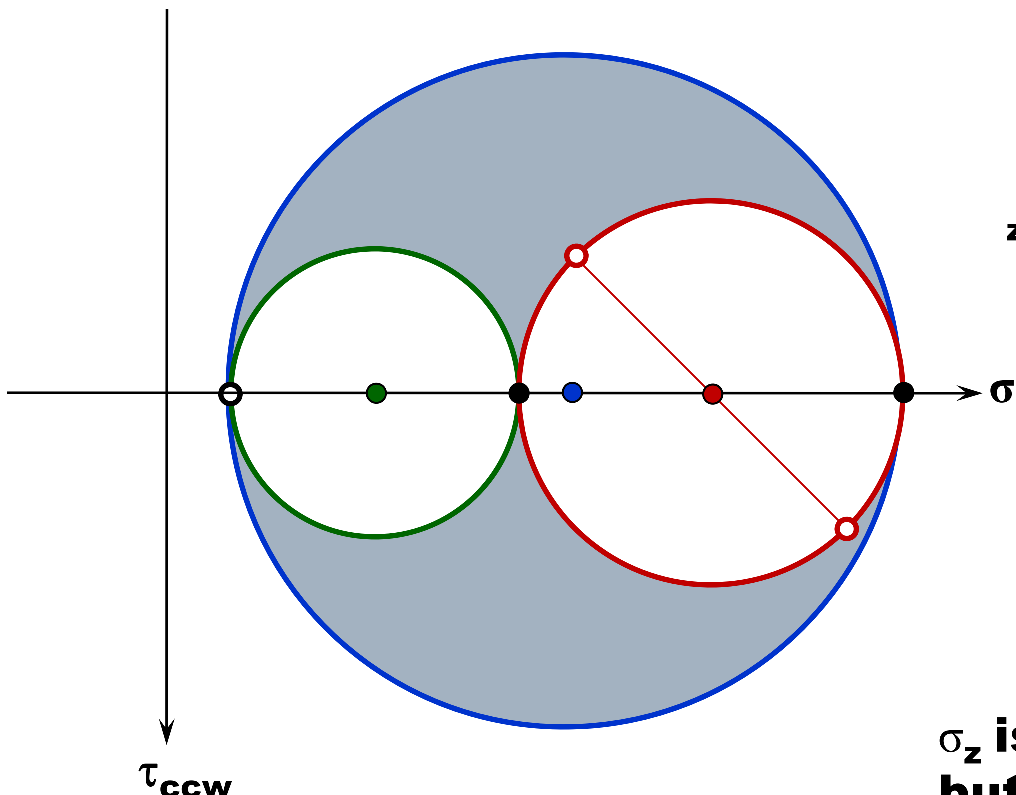
- **Principal Stress**

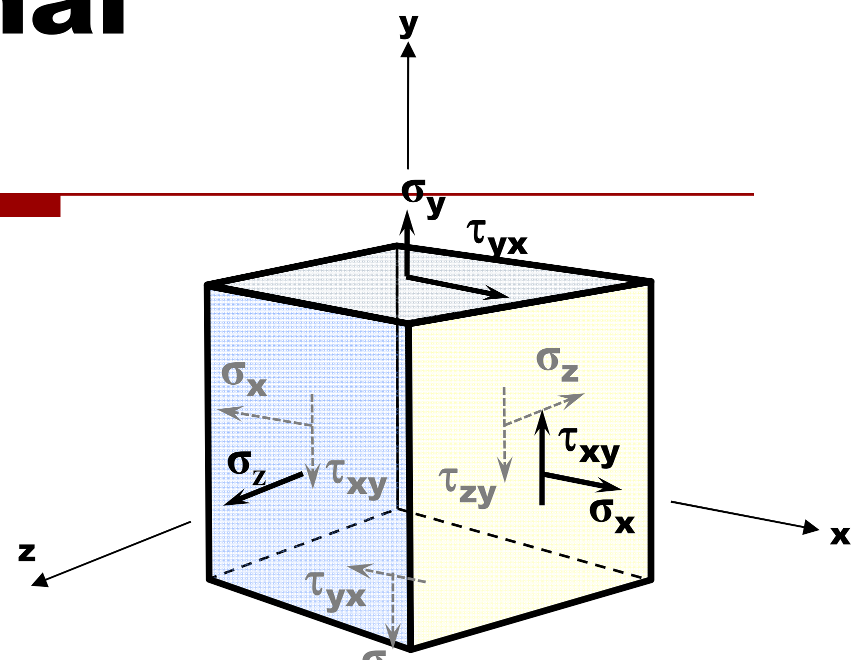
$$\left[\sigma_p\right] = \begin{bmatrix} -16.17 & 0 & 0 \\ 0 & 48.3 & 0 \\ 0 & 0 & 67.9 \end{bmatrix} MPa$$

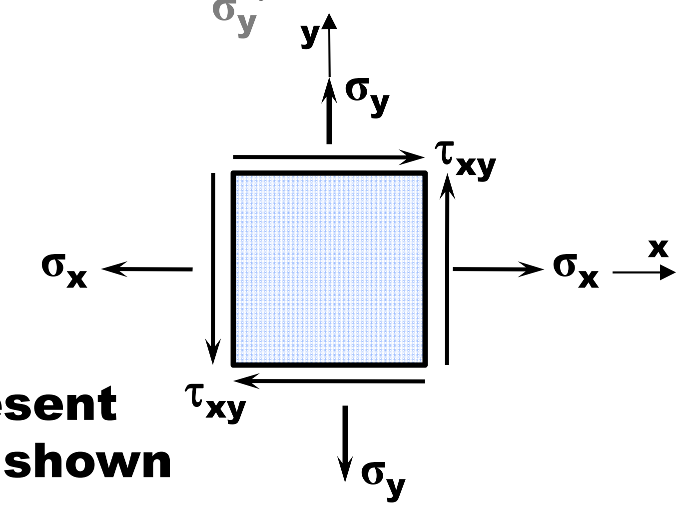
- **Direction Cosines**

$$[T] = \begin{bmatrix} -0.1135 & 0.7509 & -0.6506 \\ 0.9263 & -0.1568 & -0.3425 \\ 0.3592 & 0.6415 & 0.6778 \end{bmatrix}$$

Three Dimensional Mohr's Circle







σ_z is present but not shown

EXAMPLE:
3D Mohr's Circle

A structural member is found to have an axial stress of 150MPa and a transverse stress of 100MPa. The stress orthogonal to these stresses is zero. Calculate the maximum shear stress in this member at this point.

EXAMPLE:
3D Mohr's Circle

Determine the principal stresses and the maximum shear stress for the following state of stress.

$$[\sigma] = \begin{bmatrix} 12 & 4 & 0 \\ 4 & -8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$