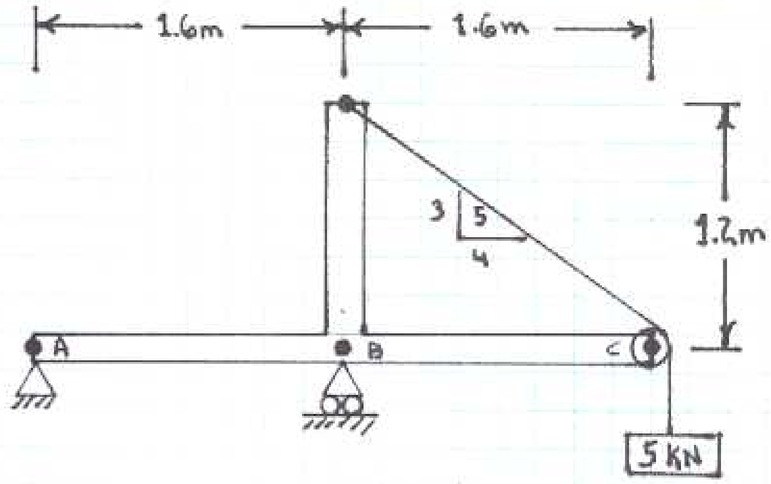
PROBLEM 11 | DETERMINE THE DEFLECTION AND CURVATURE OF POINT C USING THE MOMENT AREA METHOD. COMPARE THESE RESULTS TO YOUR PREVIOUS SOLUTIONS.




GIVEN:

CONSTRAINTS

1. 3.2m LONG BEAM WITH 1.2m VERTICAL EXTENSION AT MID-SPAN
2. SIMPLY SUPPORTED AT ONE END AND AT CENTER SPAN
3. CABLE ATTACHED TO THE TOP OF THE VERTICAL EXTENSION, TRAVELS OVER A FRICTIONLESS ROLLER, AND HOLDS A 5kN MASS.

ASSUMPTIONS

1. GRAVITY ACTS IN THE VERTICAL DIRECTION
2. MATERIAL IS LINEARLY ELASTIC
3. DEFLECTIONS AND STRAINS ARE SMALL

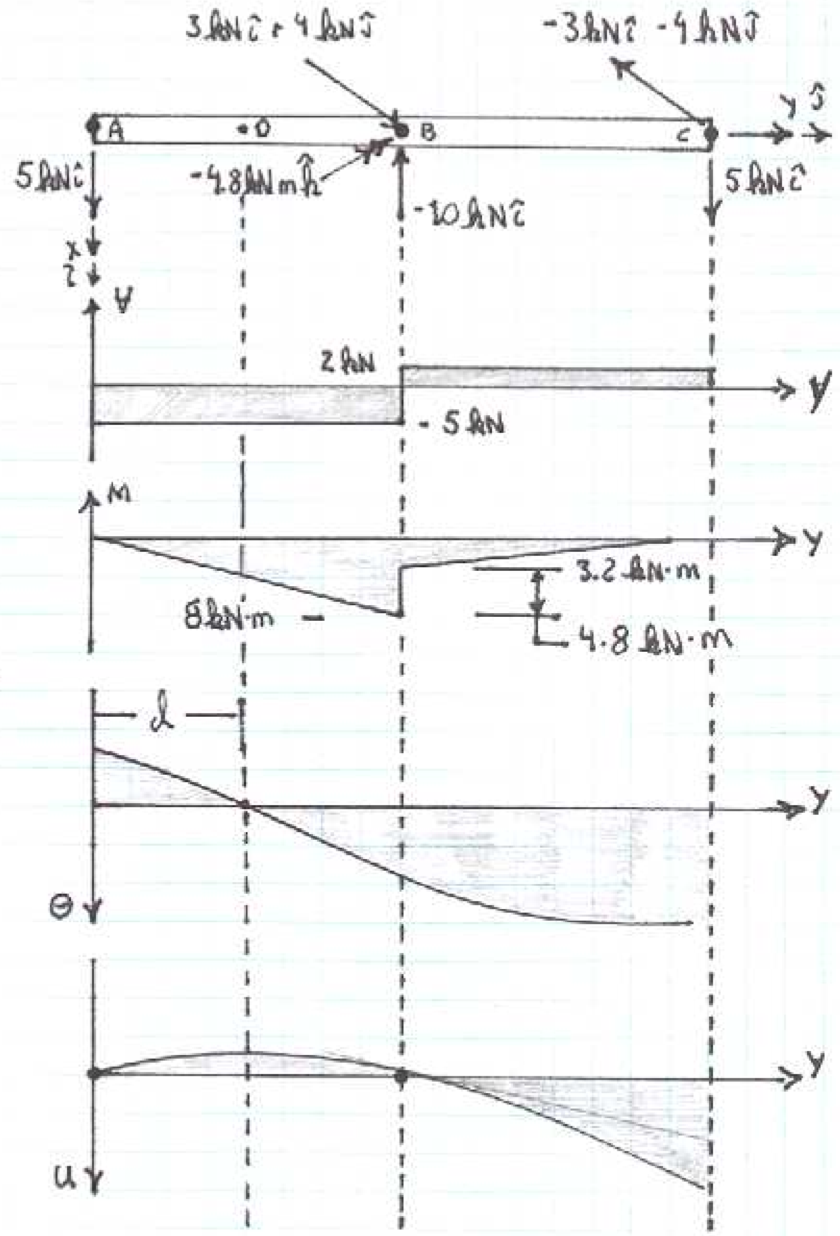
FIND:

1. THE DEFLECTION OF POINT C USING THE MOMENT AREA METHOD

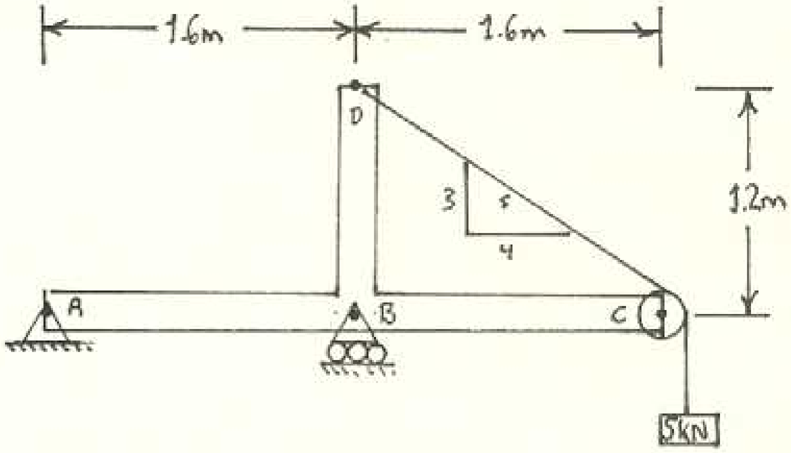
SOLUTION:

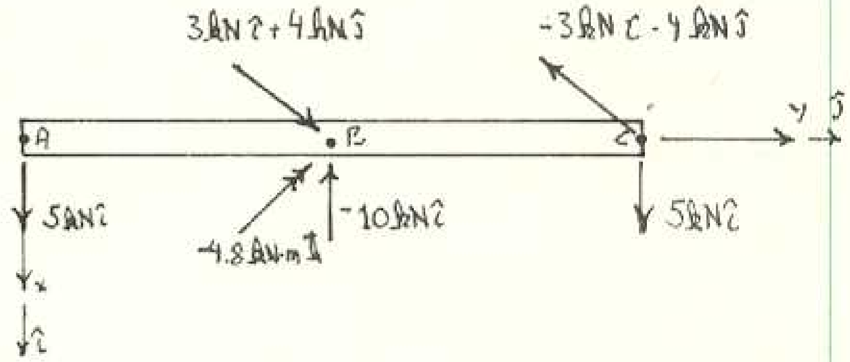
THE FIGURES TO THE RIGHT SHOW THE ORIGINAL BEAM, A FREE BODY DIAGRAM OF THE BEAM, THE SHEAR FORCE, BENDING MOMENT, CURVATURE, AND DEFLECTION DIAGRAMS.

THE NORMAL FORCE DIAGRAM HAS NO BEARING ON THE SOLUTION TO THE PROBLEM ASKED AND THEREFORE IS NOT SHOWN.



SOLVING THE PROBLEM USING SINGULARITY FUNCTIONS




$$q(y) = 5\,kN\langle y-0\rangle_{-1} \underbrace{- 10\,kN\langle y-1.6m\rangle_{-1} + 3\,kN\langle y-1.6m\rangle_{-1}}_{-7\,kN\langle y-1.6m\rangle_{-1}} - 4.8\,kN\cdot m\langle y-1.6m\rangle_{-2}$$
$$\underbrace{+5\,kN\langle y-3.2m\rangle_{-1} - 3\,kN\langle y-3.2m\rangle_{-1}}_{2\,kN\langle y-3.2m\rangle_{-1}}$$

$$= 5\,kN\langle y-0\rangle_{-1} - 7\,kN\langle y-1.6m\rangle_{-1} - 4.8\,kN\cdot m\langle y-1.6m\rangle_{-2} + 2\,kN\langle y-3.2m\rangle_{-1} \quad (1)$$

$$V = -\int q(y)\,dy$$
$$= \int\left[-5\,kN\langle y-0\rangle_{-1} + 7\,kN\langle y-1.6m\rangle_{-1} + 4.8\,kN\cdot m\langle y-1.6m\rangle_{-2} - 2\,kN\langle y-3.2m\rangle_{-1}\right]dy$$
$$= -5\,kN\langle y-0\rangle^{0} + 7\,kN\langle y-1.6m\rangle^{0} + 4.8\,kN\cdot m\langle y-1.6m\rangle_{-1} - 2\,kN\langle y-3.2m\rangle^{0} \quad (2)$$

$$M = \int V(y)\,dy$$
$$= \int\left[-5\,kN\langle y-0\rangle^{0} + 7\,kN\langle y-1.6m\rangle^{0} + 4.8\,kN\cdot m\langle y-1.6m\rangle_{-1} - 2\,kN\langle y-3.2m\rangle^{0}\right]dy$$
$$= -5\,kN\langle y-0\rangle^{1} + 7\,kN\langle y-1.6m\rangle^{1} + 4.8\,kN\cdot m\langle y-1.6m\rangle^{0} - 2\,kN\langle y-3.2m\rangle^{1} \quad (3)$$

$$\theta(y) = \frac{-1}{EI}\int M\,dy$$

$$= \frac{1}{EI}\int[5\,kN\langle y-0\rangle^1 - 7\,kN\langle y-1.6m\rangle^1 - 4.8\,kN\cdot m\langle y-1.6m\rangle^0 + 2\,kN\langle y-3.2m\rangle^1]\,dy$$

$$= \frac{5\,kN}{2EI}\langle y-0\rangle^2 - \frac{7\,kN}{2EI}\langle y-1.6m\rangle^2 - \frac{4.8\,kN\cdot m}{EI}\langle y-1.6m\rangle^1 + \frac{2\,kN}{2EI}\langle y-3.2m\rangle^2 + C_1$$

$$= \frac{5\,kN}{2\cdot EI}\langle y-0\rangle^2 - \frac{7\,kN}{2EI}\langle y-1.6m\rangle^2 - \frac{4.8\,kN\cdot m}{EI}\langle y-1.6m\rangle^1 + \frac{1\,kN}{EI}\langle y-3.2m\rangle^2 + C_1$$

$$= \frac{2.5\,kN}{EI}\langle y-0\rangle^2 - \frac{3.5\,kN}{EI}\langle y-1.6m\rangle^2 - \frac{4.8\,kN\cdot m}{EI}\langle y-1.6m\rangle^1 + \frac{1\,kN}{EI}\langle y-3.2m\rangle^2 + C_1 \quad (4)$$

$$u(y) = \int\theta(y)\,dy$$

$$= \int\left[\frac{2.5\,kN}{EI}\langle y-0\rangle^2 - \frac{3.5\,kN}{EI}\langle y-1.6m\rangle^2 - \frac{4.8\,kN\cdot m}{EI}\langle y-1.6m\rangle^1 + \frac{1\,kN}{EI}\langle y-3.2m\rangle^2 + C_1\right]dy$$

$$= \frac{2.5\,kN}{3EI}\langle y-0\rangle^3 - \frac{3.5\,kN}{3EI}\langle y-1.6m\rangle^3 - \frac{4.8\,kN\cdot m}{2EI}\langle y-1.6m\rangle^2 + \frac{1}{3}\frac{kN}{EI}\langle y-3.2m\rangle^3 + C_1\cdot y + C_2 \quad (5)$$

THE TWO BOUNDARY CONDITIONS FOR THIS PROBLEM ARE

$$u(0) = 0 \quad (6)$$

$$u(1.6m) = 0 \quad (7)$$

USING (5) AND APPLYING THE FIRST BOUNDARY CONDITION, (6)

$$u(0) = 0 = \frac{2.5\,kN}{3EI}(0)^3 + C_1(0) + C_2 \Rightarrow \underline{C_2 = 0} \quad (8)$$

SUBSTITUTING (8) INTO (5) AND APPLYING THE SECOND BOUNDARY CONDITION (7)

$$u(1.6m) = 0 = \frac{2.5\,kN}{3EI}(1.6m)^3 + C_1\cdot(1.6m)$$

$$\Rightarrow C_1 = -\frac{2.5\,kN}{3EI}\cdot\frac{(1.6m)^3}{(1.6m)} = -\frac{2.5\,kN}{3EI}(1.6m)^2 = \underline{\frac{-2.133\,kN\cdot m^2}{EI}} \quad (9)$$

THEREFORE

$$u(y) = \frac{2.5\,kN}{3EI}\langle y-0\rangle^3 - \frac{3.5\,kN}{3EI}\langle y-1.6m\rangle^3 - \frac{4.8\,kN\cdot m}{2\cdot EI}\langle y-1.6m\rangle^2 + \frac{1}{3}\frac{kN}{EI}\langle y-3.2m\rangle^3 - \frac{2.133\,kN\cdot m^2}{EI}\cdot y$$

$$\theta(y) = \frac{2.5\,kN}{EI}\langle y-0\rangle^2 - \frac{3.5\,kN}{EI}\langle y-1.6m\rangle^2 - \frac{4.8\,kN\cdot m}{EI}\langle y-1.6m\rangle^1 + \frac{1\,kN}{EI}\langle y-3.2m\rangle^2 - \frac{2.133\,kN\cdot m^2}{EI}$$

$$M(y) = -5\,kN\langle y-0\rangle^1 + 7\,kN\langle y-1.6m\rangle^1 + 4.8\,kN\cdot m\langle y-1.6m\rangle^0 - 2\,kN\langle y-3.2m\rangle^1$$

$$V(y) = -5\,kN\langle y-0\rangle^0 + 7\,kN\langle y-1.6m\rangle^0 + 4.8\,kN\cdot m\langle y-1.6m\rangle_{-1} - 2\,kN\langle y-3.2m\rangle^0$$