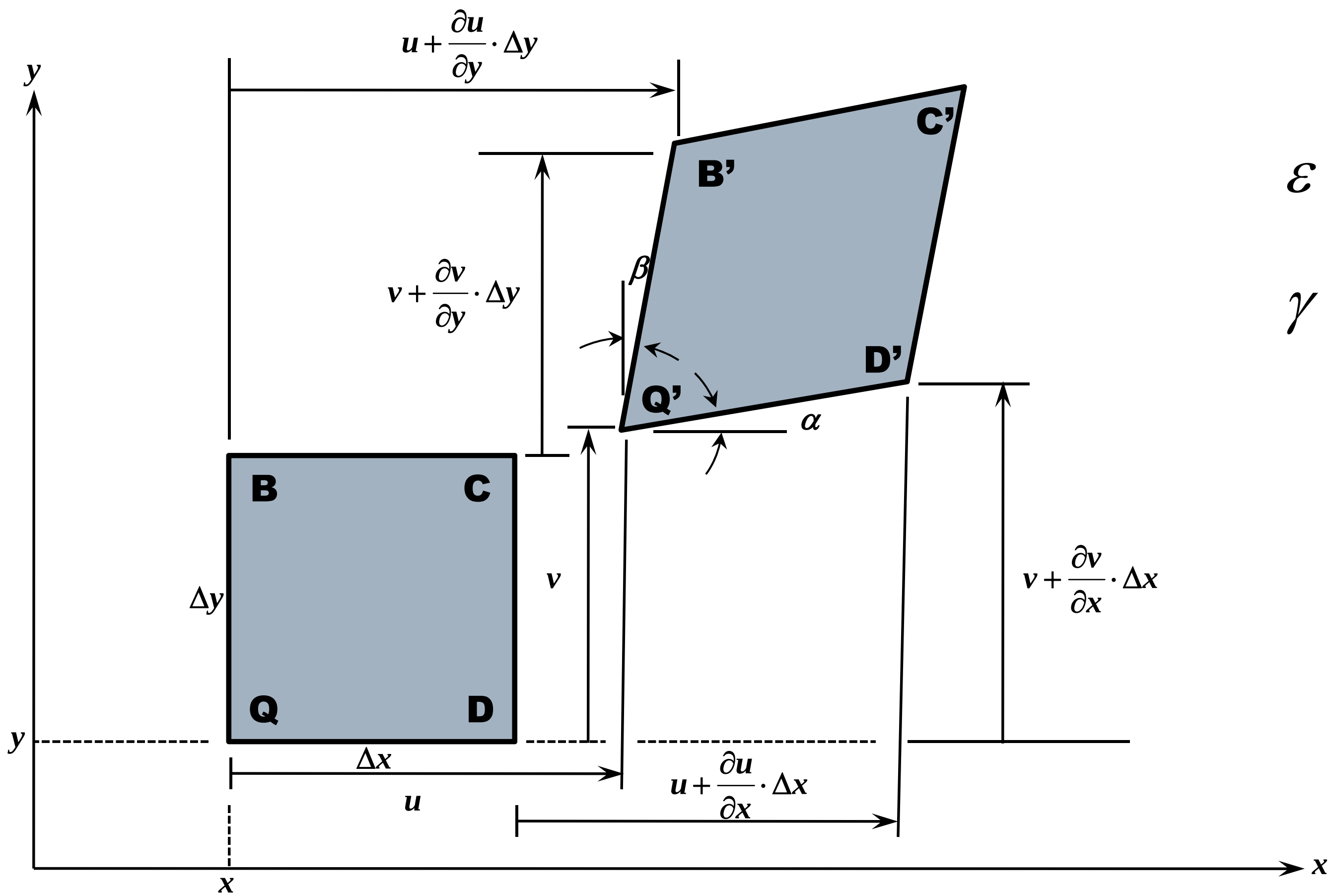
MER311: Advanced Strength of Materials

LECTURE OUTLINE

- Strain – Displacement Relations
- Compatibility
- Mohr's Circle for Strain

Strain-Displacement Relationships, ε and γ



$$\varepsilon = \frac{\Delta l}{l}$$

$$\gamma = \theta \approx \operatorname{Tan}\theta$$

Normal Strain - Displacements

$$\varepsilon_x = \partial u \big/ \partial x$$

$$\varepsilon_y = \partial v \big/ \partial y$$

$$\varepsilon_x = \partial w \big/ \partial z$$

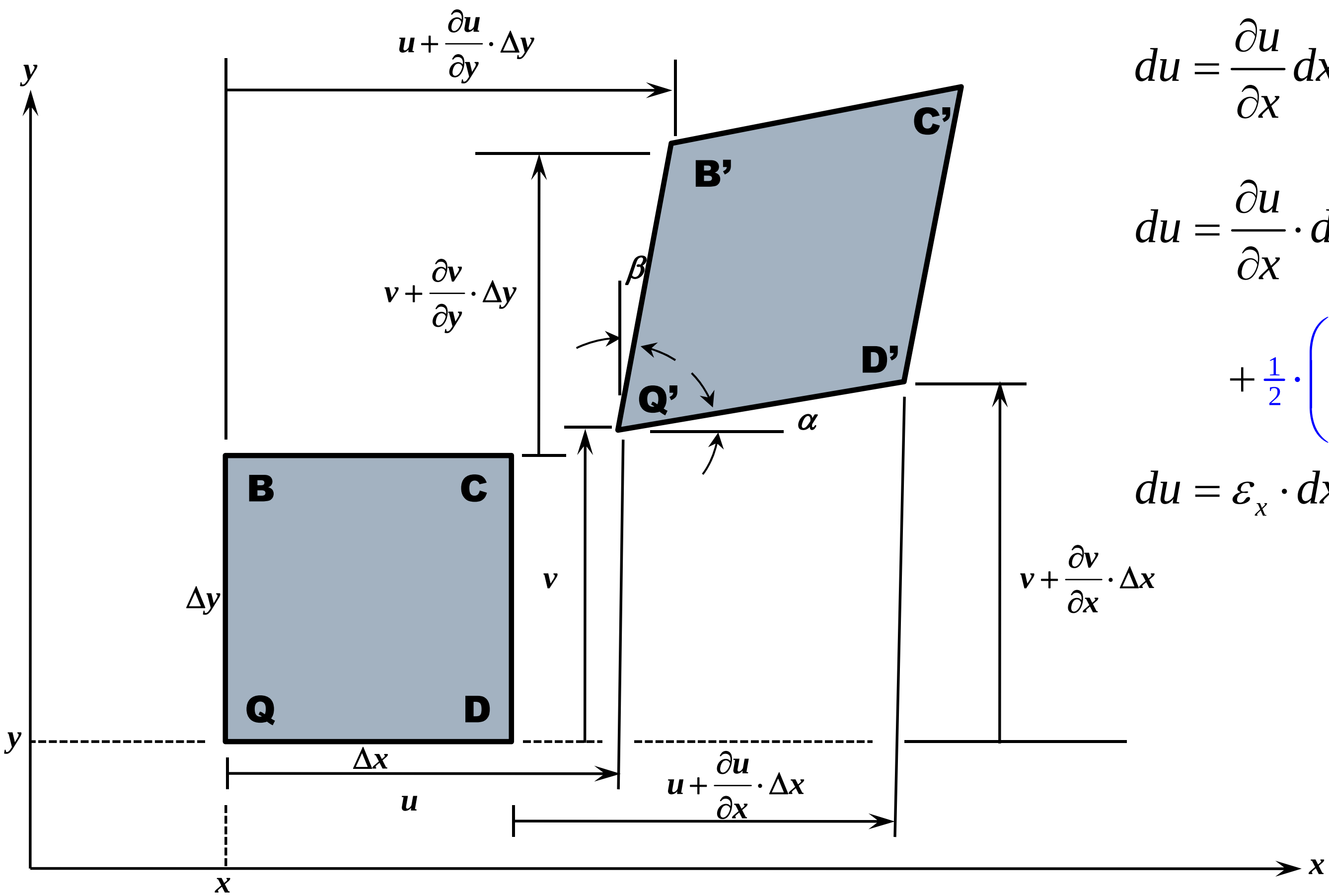
Shear Strain - Displacements

$$\gamma_{xy} = \partial v / \partial x + \partial u / \partial y$$

$$\gamma_{xz} = \partial w / \partial x + \partial u / \partial z$$

$$\gamma_{zy} = \partial v / \partial z + \partial w / \partial y$$

Strain-Displacement Relationships, Θ



$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$$

$$du = \frac{\partial u}{\partial x}\cdot dx + \tfrac{1}{2}\cdot\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\cdot dy$$

$$+\tfrac{1}{2}\cdot\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)\cdot dy$$

$$du = \varepsilon_x \cdot dx + \tfrac{1}{2}\cdot\gamma_{xy}\cdot dy + \Theta_z \cdot dy$$

Curvature - Displacements

$$\Theta_{xy} = \frac{1}{2}\left(\partial v \big/ \partial x - \partial u \big/ \partial y \right)$$

$$\Theta_{xz} = \frac{1}{2}\left(\partial u \big/ \partial z - \partial w \big/ \partial x \right)$$

$$\Theta_{zy} = \frac{1}{2}\left(\partial w \big/ \partial y - \partial v \big/ \partial z \right)$$

Example

The following displacement field is applied to a certain body where $k=10^{-4}$.

$u=k(2x+y^2)$, $v=k(x^2 -3y^2)$, $w=0$

(a) Show the distorted configuration of a two-dimensional element with sides dx and dy and its lower left corner (point A) initially at the point (2,1,0), i.e., determine the new length and angular position of each side.

Compatibility: 6 Strain Equations in 3 Displacements

$$\varepsilon_x = \frac{\partial u}{\partial x} \xrightarrow[\text{With Respect to y}]{\text{Differentating Twice}} \frac{\partial^2 \varepsilon_x}{\partial^2 y} = \frac{\partial^3 u}{\partial^2 y \cdot \partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \xrightarrow[\text{With Respect to x}]{\text{Differentating Twice}} \frac{\partial^2 \varepsilon_y}{\partial^2 x} = \frac{\partial^3 v}{\partial^2 x \cdot \partial y}$$

$$\xrightarrow[\text{These}]{\text{Adding}} \frac{\partial^2 \varepsilon_x}{\partial^2 y} + \frac{\partial^2 \varepsilon_y}{\partial^2 x} = \frac{\partial^3 u}{\partial^2 y \cdot \partial x} + \frac{\partial^3 v}{\partial^2 x \cdot \partial y}$$

$$\gamma_{xy} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \xrightarrow[\text{Respect to x and y}]{\text{Differentating With}} \frac{\partial^2 \gamma_{xy}}{\partial x \cdot \partial y} = \left(\frac{\partial^3 v}{\partial^2 x \cdot \partial y} + \frac{\partial^3 u}{\partial^2 y \cdot \partial x} \right)$$

$$\boxed{\frac{\partial^2 \gamma_{xy}}{\partial x \cdot \partial y} = \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2}}$$

Compatibility: Repeating For Other Strain Combinations

$$\frac{\partial^2 \gamma_{xy}}{\partial x \cdot \partial y} = \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2}$$

$$\frac{\partial^2 \gamma_{yz}}{\partial y \cdot \partial z} = \frac{\partial^2 \varepsilon_y}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial y^2}$$

$$\frac{\partial^2 \gamma_{xz}}{\partial x \cdot \partial z} = \frac{\partial^2 \varepsilon_x}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial x^2}$$

Compatibility Continued

$$2 \cdot \frac{\partial^2 \varepsilon_x}{\partial y \cdot \partial z} = \frac{\partial}{\partial x} \cdot \left(-\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} \right)$$

$$2 \cdot \frac{\partial^2 \varepsilon_y}{\partial z \cdot \partial x} = \frac{\partial}{\partial y} \cdot \left(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} \right)$$

$$2 \cdot \frac{\partial^2 \varepsilon_z}{\partial x \cdot \partial y} = \frac{\partial}{\partial z} \cdot \left(\frac{\partial \gamma_{yx}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} \right)$$

Strain Tensor

$$[\varepsilon]=\begin{bmatrix}\varepsilon_x & \dfrac{\gamma_{xy}}{2} & \dfrac{\gamma_{xz}}{2}\\ \dfrac{\gamma_{yx}}{2} & \varepsilon_y & \dfrac{\gamma_{yz}}{2}\\ \dfrac{\gamma_{zx}}{2} & \dfrac{\gamma_{zy}}{2} & \varepsilon_z\end{bmatrix}$$

$$=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz}\\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz}\end{bmatrix}=\varepsilon_{ij}$$

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \dfrac{\gamma_{xz}}{2}\\ \dfrac{\gamma_{yz}}{2}\\ \dfrac{\gamma_{xy}}{2}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \varepsilon_{xz}\\ \varepsilon_{yz}\\ \varepsilon_{xy}\end{Bmatrix}$$

Strain Transformations

$$T = \begin{bmatrix} n_{x',x} & n_{x',y} & n_{x',z} \\ n_{y',x} & n_{y',y} & n_{y',z} \\ n_{z',x} & n_{z',y} & n_{z',z} \end{bmatrix}$$

$$[\varepsilon]_{x'y'z'} = [T]\cdot[\varepsilon]_{xyz}\cdot[T]^T$$