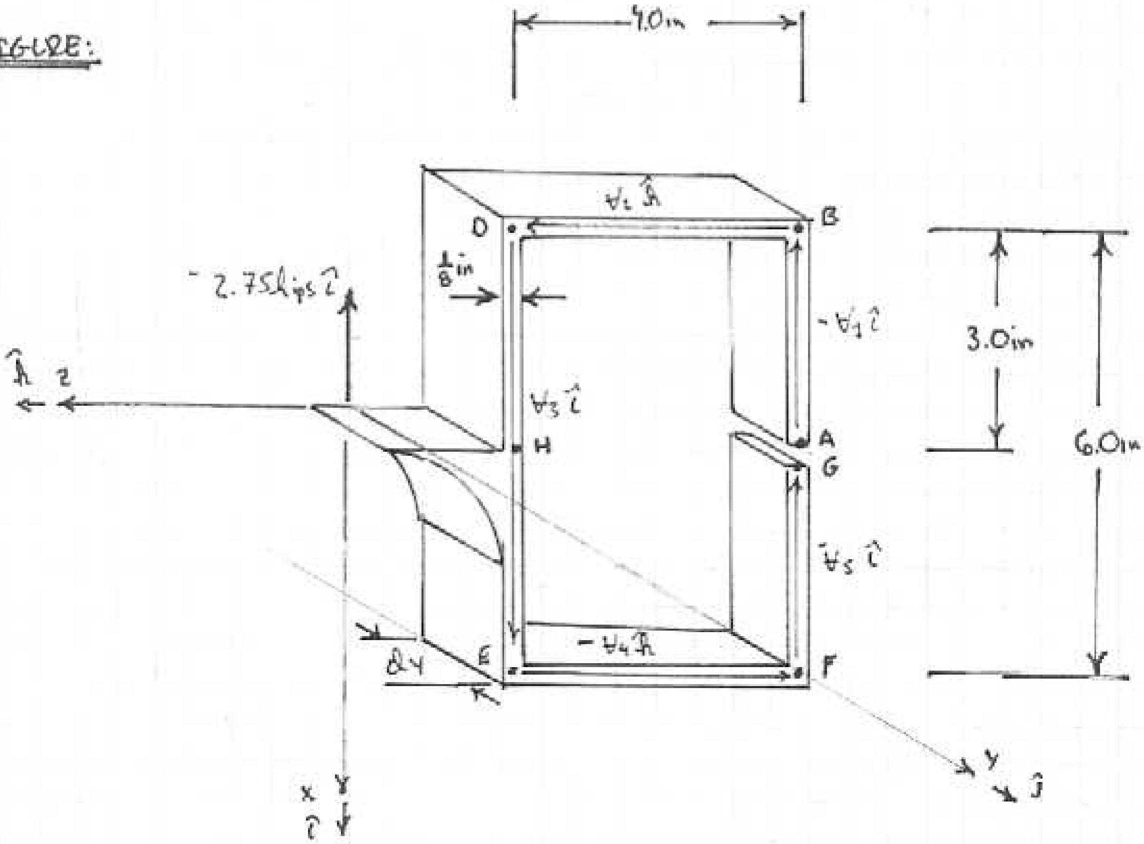
PROBLEM 6.66 | AN EXTRUDED BEAM HAS THE CROSS SECTION SHOWN. DETERMINE (a) THE LOCATION OF THE SHEAR CENTER O, (b) THE DISTRIBUTION OF THE SHEARING STRESS CAUSED BY THE 2.75 KIP VERTICAL SHEARING FORCE APPLIED AT O.

GIVEN:
1. 4 in BY 6 in BOX SECTION WITH ONE SIDE WALL CUT ALONG THE LENGTH
2. WALL THICKNESS OF 1/8 in.
3. VERTICAL SHEARING FORCE OF 2.75 kips.

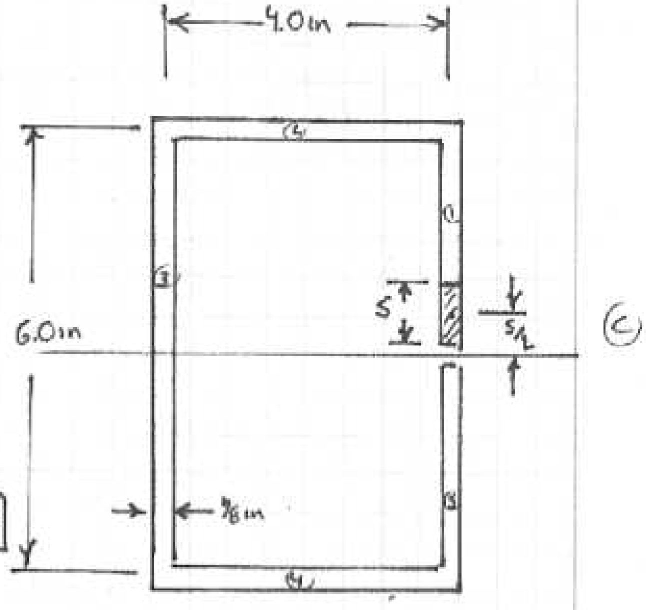
ASSUMPTIONS:
1. LINEAR ELASTIC MATERIAL
2. SMALL DEFORMATIONS

FIND:
1. THE LOCATION OF THE SHEAR CENTER
2. THE DISTRIBUTION OF THE SHEARING STRESS

FIGURE:



(a)



SOLUTION:

CALCULATING THE EXPRESSION FOR THE SHEARING STRESS IN THE FIRST SECTION OF THE BEAM.

$$\tau = \frac{V \cdot Q}{I \cdot t} \quad (1)$$

$$I = 2\cdot\left[\tfrac{1}{12}(\tfrac{1}{8}\text{in})(3\text{in})^3 + (\tfrac{1}{8}\text{in})(3\text{in})(1.5\text{in})^2\right] + \tfrac{1}{12}(\tfrac{1}{8}\text{in})(6\text{in})^3 + 2\left[\tfrac{1}{12}(4.0\text{in})(\tfrac{1}{8}\text{in})^3 + (4.0\text{in})(\tfrac{1}{8})(3.0\text{in})^2\right]$$

$$= 13.50\ \text{in}^4 \quad (2)$$

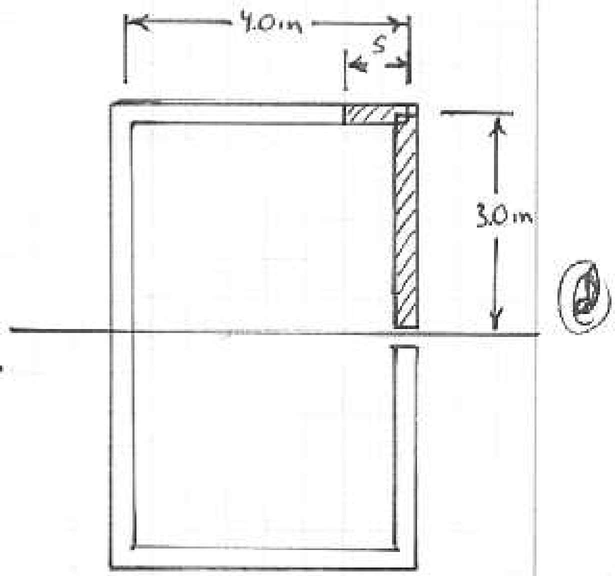
$$Q_1 = \bar{x}A = \left(\tfrac{s}{2}\right)\cdot\left(\tfrac{1}{8}\text{in}\right)\cdot s = \tfrac{1}{16}\text{in}\cdot s^2 \quad (3)$$

$$\tau_1 = \frac{V\cdot Q_1}{I\, t_1} = \frac{2.75(10^3)\ \text{lb}\cdot\frac{1}{16}\text{in}\cdot s^2}{13.50\text{in}^4\cdot \frac{1}{8}\text{in}} = \underline{101.8\ \tfrac{\text{lb}}{\text{in}^4}\cdot s^2} \quad (4)$$

AT THE END POINTS OF THIS SECTION

$$\tau_1(0) = \underline{0} \quad (5)$$

$$\tau_1(3\text{in}) = 101.8\ \tfrac{\text{lb}}{\text{in}^4}\cdot(3\text{in})^2 = \underline{916.7\ \tfrac{\text{lb}}{\text{in}^2}} \quad (6)$$



IN THE SECOND SECTION OF THE BEAM

$$Q_2 = \sum \bar{x}A = (1.5\text{in})(3.0\text{in})(\tfrac{1}{8}\text{in}) + (3.0\text{in})(\tfrac{1}{8}\text{in})\cdot s$$

$$= 0.5625\text{in}^3 + 0.3750\text{in}^2\cdot s \quad (7)$$

$$\tau_2 = \frac{2.75(10^3)\ \text{lb}\cdot(0.5625\text{in}^3 + 0.3750\text{in}^2\cdot s)}{13.50\ \text{in}^4\cdot \frac{1}{8}\text{in}}$$

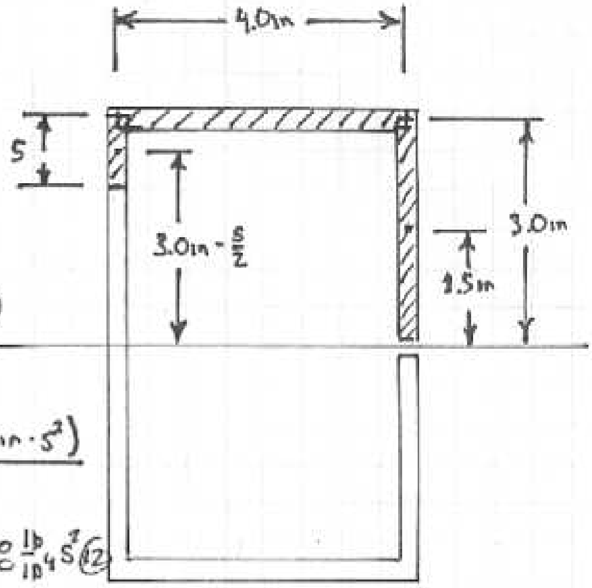
$$= 916.7\ \tfrac{\text{lb}}{\text{in}^2} + 611.1\ \tfrac{\text{lb}}{\text{in}^3}\cdot s \quad (8)$$

AT THE ENDS OF THIS SECTION

$$\tau_2(0) = \underline{916.7\ \text{lb/in}^2} \quad (9)$$

$$\tau_2(4.0\text{in}) = 916.7\ \tfrac{\text{lb}}{\text{in}^2} + 611.1\ \tfrac{\text{lb}}{\text{in}^3}\cdot(4.0\text{in}) = \underline{3\ 361\ \text{lb/in}^2} \quad (10)$$

IN THE THIRD SECTION OF THE BEAM



(b)

$$Q_3 = \sum \bar{x}_i A_i = (1.5\text{in})(3.0\text{in})(\tfrac{1}{8}\text{in}) + (3.0\text{in})(4.0\text{in})(\tfrac{1}{8}\text{in}) + (3\text{in} - \tfrac{s}{2})(s)(\tfrac{1}{8}\text{in})$$

$$= 2.062\,\text{in}^3 + 0.375\,\text{in}^2 \cdot s - \tfrac{1}{16}\text{in} \cdot s^2 \quad (11)$$

$$\tau_3 = \frac{2.75(10^3)\,\text{lb} \cdot (2.062\,\text{in}^3 + 0.375\,\text{in}^2 \cdot s - \frac{1}{16}\text{in} \cdot s^2)}{13.50\,\text{in}^4 \cdot 1/8\,\text{in}}$$

$$= 3\,361\,\tfrac{\text{lb}}{\text{in}^2} + 611.1\,\tfrac{\text{lb}}{\text{in}^3} \cdot s - 101.8\,\tfrac{\text{lb}}{\text{in}^4} s^2 \quad (12)$$

AT THE ENDS OF THIS SECTION

$$\tau_3(0) = \underline{3\,361\,\text{lb/in}^2} \quad (13)$$

$$\tau_3(3\text{in}) = 3\,361\,\tfrac{\text{lb}}{\text{in}^2} + 611.1\,\tfrac{\text{lb}}{\text{in}^3} \cdot (3\text{in}) - 101.8\,\tfrac{\text{lb}}{\text{in}^4}(3\text{in})^2$$

$$= \underline{4277\,\tfrac{\text{lb}}{\text{in}^2}} \quad (14)$$

THE SHEAR STRESS CAN NOW BE INTEGRATED ALONG THE DIFFERENT SECTIONS OF THIS CROSS SECTION IN ORDER TO DETERMINE THE MAGNITUDE OF THE SHEAR FORCES IN EACH SECTION.

$$V_1 = V_5 = \int \tau_1 \cdot dA = \int \tau_1 \cdot t \cdot ds$$

$$= \int_0^{3\text{in}} 101.8\,\tfrac{\text{lb}}{\text{in}^4} \cdot (\tfrac{1}{8}\text{in}) \cdot s^2 \cdot ds = 12.72\,\tfrac{\text{lb}}{\text{in}^3} \cdot \frac{s^3}{3}\Big|_0^{3\text{in}}$$

$$= 4.242\,\tfrac{\text{lb}}{\text{in}^3}\left[(3\text{in})^3 - (0)^3\right] = \underline{114.5\,\text{lb}} \quad (15)$$

$$= 0.1145\ \text{kips}$$

$$V_2 = V_4 = \int \tau_2 \cdot dA = \int \tau_2 \cdot t \cdot ds$$

$$= \int_0^{4\text{in}} (916.7\,\tfrac{\text{lb}}{\text{in}^2} + 611.1\,\tfrac{\text{lb}}{\text{in}^3} \cdot s)(\tfrac{1}{8}\text{in})\,ds$$

$$= \int_0^{4\text{in}} (114.5\,\tfrac{\text{lb}}{\text{in}} + 76.39\,\tfrac{\text{lb}}{\text{in}^2} \cdot s)\,ds$$

$$= \left[114.5\,\tfrac{\text{lb}}{\text{in}} \cdot s + 76.39\,\tfrac{\text{lb}}{\text{in}^2} \cdot \frac{s^2}{2}\right]_0^{4\text{in}} = \left[114.5\,\tfrac{\text{lb}}{\text{in}} \cdot s + 38.20\,\tfrac{\text{lb}}{\text{in}^2} \cdot s^2\right]_0^{4\text{in}}$$

$$= 114.5\,\tfrac{\text{lb}}{\text{in}} \cdot (4\text{in}) + 38.20\,\tfrac{\text{lb}}{\text{in}^2} \cdot (4\text{in})^2 = \underline{1069\,\text{lb}} \quad (16)$$

$$= 1.069\ \text{kips}$$

$$V_3 = 2\cdot\int \tau_3\, dA = 2\int \tau_3 \cdot t \cdot ds$$

$$= 2\cdot\int_0^{3\,in}\left[3361\tfrac{lb}{in^2} + 611.1\tfrac{lb}{in^3}\cdot s - 101.8\tfrac{lb}{in^4}\cdot s^2\right]\left(\tfrac{1}{8}\,in\right)\cdot ds$$

$$= 2\cdot\int_0^{3\,in}\left[420.1\tfrac{lb}{in^2} + 76.39\tfrac{lb}{in^3}\cdot s - 12.73\tfrac{lb}{in^3}\cdot s^2\right] ds$$

$$= 2\cdot\left[420.1\tfrac{lb}{in^2}\cdot s + \tfrac{76.39}{2}\tfrac{lb}{in^3}\cdot s^2 - \tfrac{12.73}{3}\tfrac{lb}{in^4}\, s^3\right]_0^{3\,in}$$

$$= 2\cdot\left[420.1\tfrac{lb}{in^2}\cdot(3\,in) + 38.19\tfrac{lb}{in^3}\cdot(3\,in)^2 - 4.242\tfrac{lb}{in^4}\cdot(3\,in)^3\right]$$

$$= \underline{\underline{2979\ lb}}$$

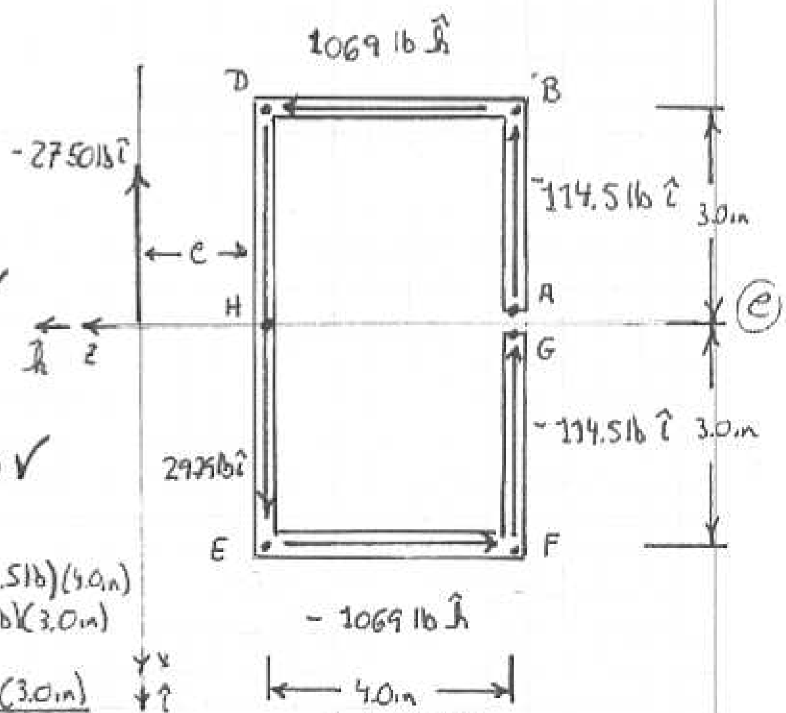


FIGURE (e) SUMMARIZES THE FORCES THAT RESULT FROM THE SHEAR STRESS DISTRIBUTION AND THE EXTERNAL LOAD. NOW LETS CONSIDER EQUILIBRIUM

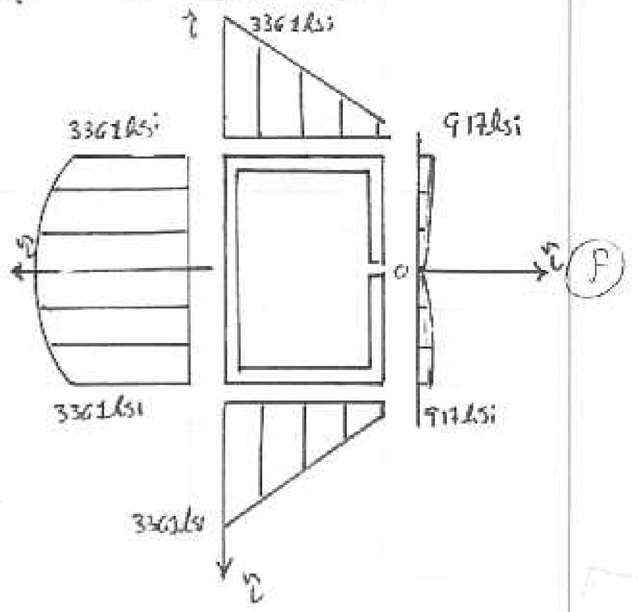
$$\Sigma F_z = 0 = 1069\,lb - 1069\,lb = 0 \checkmark$$

$$\Sigma F_x = 0 = -114.5\,lb - 114.5\,lb + 2979\,lb - 2750\,lb = 0 \checkmark$$

$$\Sigma M_{z@H} = 0 = -(2750\,lb)\cdot e + 2\cdot(114.5\,lb)(4.0\,in) + (1069\,lb)(3.0\,in) + (1069\,lb)(3.0\,in)$$

$$e = \frac{2(114.5\,lb)(4.0\,in) + 2(1069\,lb)(3.0\,in)}{2750\,lb}$$

$$= \boxed{2.665\,in}$$



SUMMARY:

FIGURE (F) SUMMARIZES THE DISTRIBUTION OF SHEAR IN THIS CROSS-SECTION. THE ONLY REASON WHY THE SHEAR STRESS IS THE SAME AT EACH OF THE REGION INTERSECTION POINTS IS BECAUSE THE WALL THICKNESS IS CONSTANT THROUGHOUT THE SECTION.