Kinematics Fundamentals

- Vector Overview
- Velocity Analysis
- Four Bar Linkage
- Slider Crank
- Inverted Slider Crank

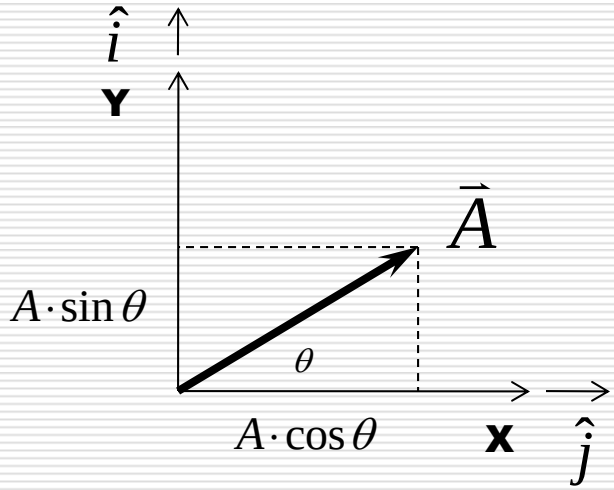
Representing Vectors

Vector

$$\hat{A} = A \cdot \cos\theta \cdot \hat{i} + A \cdot \sin\theta \cdot \hat{j}$$

$$= A \cdot \left(\cos\theta \cdot \hat{i} + A \cdot \sin\theta \cdot \hat{j} \right)$$

$$= A \cdot \hat{e}_A$$



Complex

$$\vec{A} = A \cdot \cos\theta + j \cdot A \cdot \sin\theta$$

$$= A \cdot e^{j \cdot \theta}$$

Differentiating Vectors

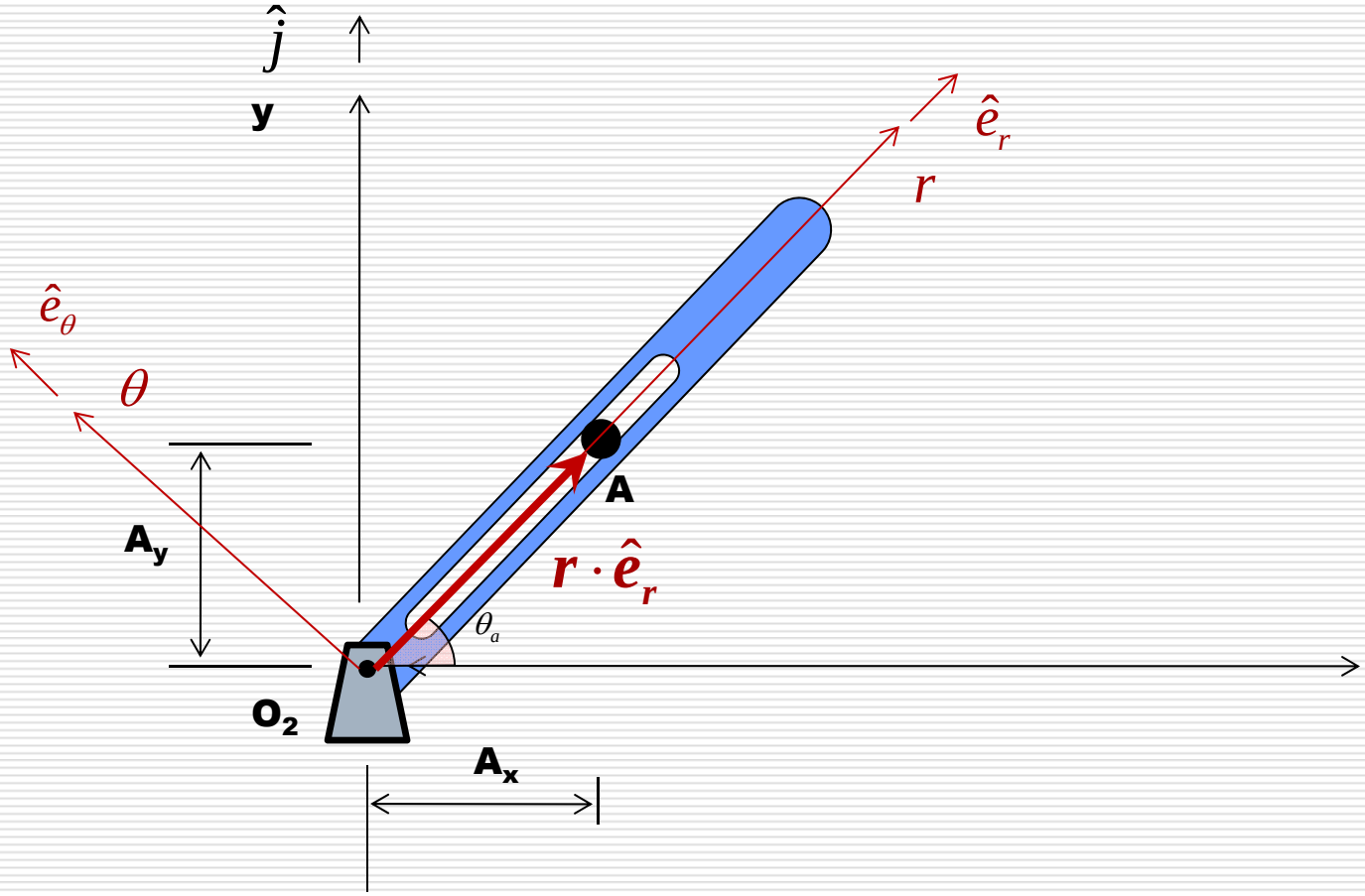
Chain Rule

$$\frac{d}{dt}\left(A(t)\cdot B(t)\right)=\frac{dA(t)}{dt}\cdot B(t)+A(t)\cdot\frac{dB(t)}{dt}$$

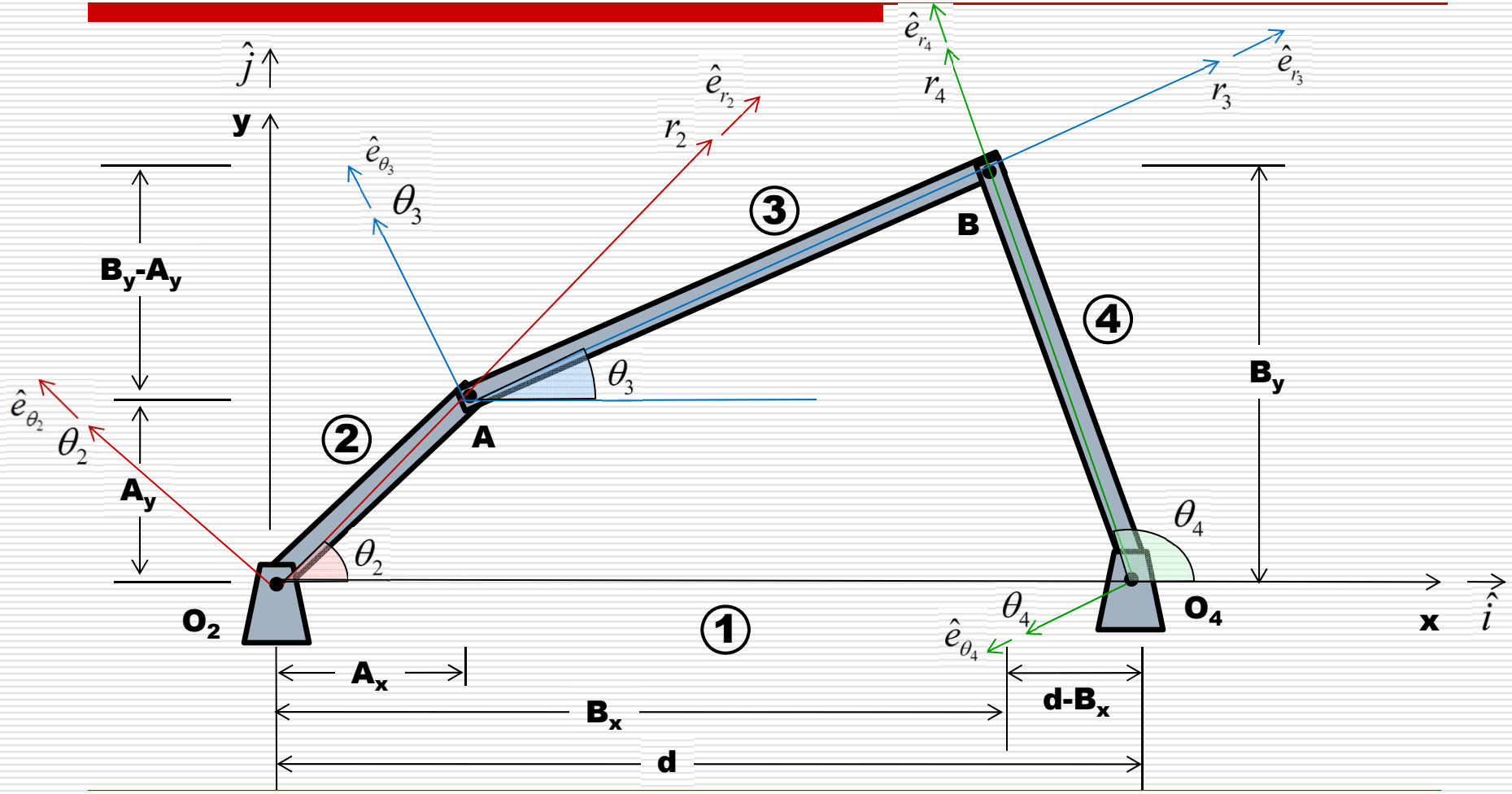
Omega Theorem

$$\frac{d\hat{e}}{dt}=\dot{\hat{\theta}}\otimes\hat{e}$$

Velocity Fundamentals



TYPE I: (RRRR)
Four Bar Linkage



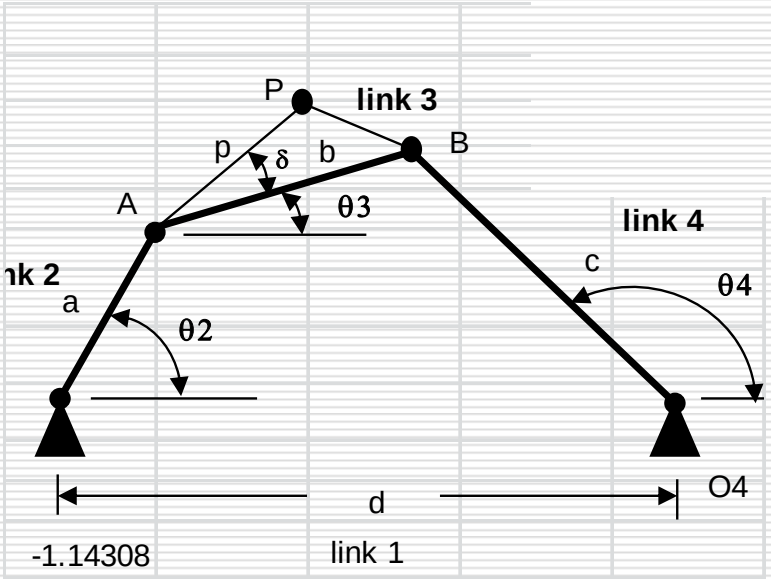
4-Bar Algorithm Inputs

a=	5	Link 2
b=	12	Link 3
c=	10	Link 4
d=	15	Link 1
$\theta_2 =$	60	1.047197551
$\dot{\theta}_2 =$	-25	$1/s$

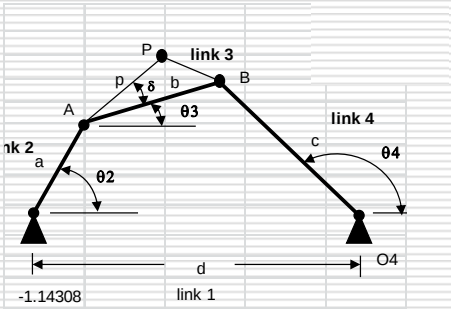
K1=	9.7600E+00
K2=	3.4641E-01
K3=	3.2414E+00
K4=	-6.4770E+01

p=	5
δ=	331

By=	9.83	-6.59
Bx=	13.17	7.48
$\theta_3 =$	27.3	-65.5
$\theta_4 =$	100.6	-138.8
$\dot{\theta}_3 =$	7.0737E+00	-3.5022E+00
$\dot{\theta}_4 =$	-7.0545E+00	1.0626E+01

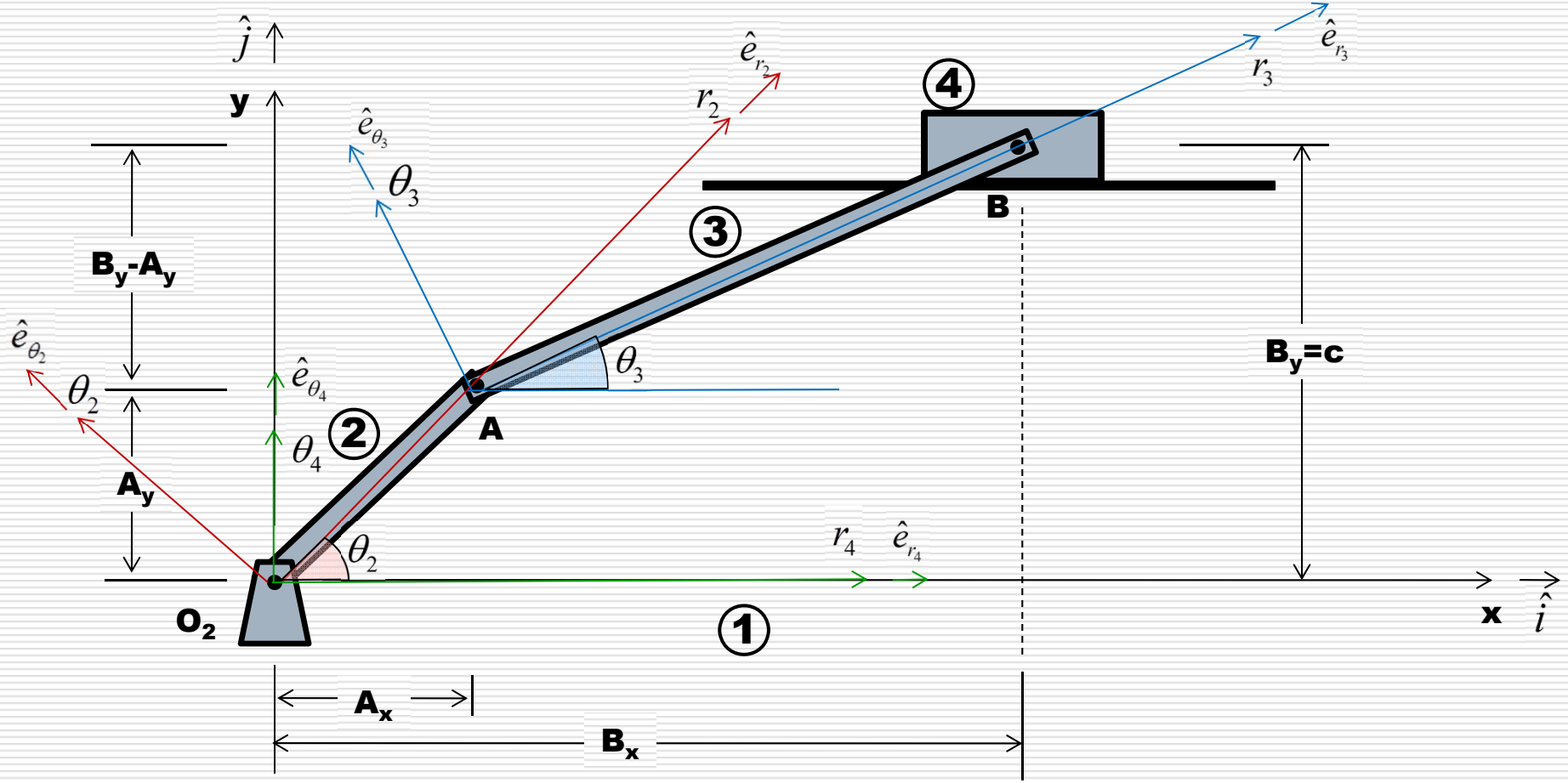


4-Bar Algorithm Velocity Results



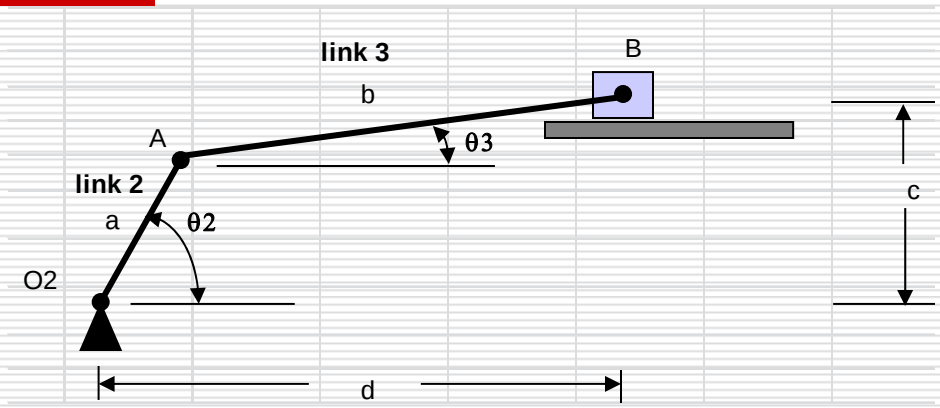
					e_r		e_θ	
	x comp	y comp	mag	angle	i	j	i	j
r04=	**15.00**	**0.00**	**15.000**	**0.0**	**1.000**	**0.000**	**0.000**	**1.000**
rA=	**2.50**	**4.33**	**5.000**	**60.0**	**0.500**	**0.866**	**-0.866**	**0.500**
rBA=	10.67	5.50	12.000	27.3	0.889	0.458	-0.458	0.889
rBO4=	-1.83	9.83	10.000	100.6	-0.183	0.983	-0.983	-0.183
rB=	**13.17**	**9.83**	**16.430**	**36.7**	**0.801**	**0.598**	**-0.598**	**0.801**
rPA=	5.00	-0.15	5.000	-1.7	1.000	-0.030	0.030	1.000
rP=	**7.50**	**4.18**	**8.584**	**29.1**	**0.873**	**0.487**	**-0.487**	**0.873**
vA=	**108.25**	**-62.50**	**125.000**	**-30.0**	**0.866**	**-0.500**	**0.500**	**0.866**
vBA=	-38.91	75.44	84.884	117.3	-0.458	0.889	-0.889	-0.458
vB=	**69.35**	**12.94**	**70.545**	**10.6**	**0.983**	**0.183**	**-0.183**	**0.983**
vPA=	1.06	35.35	35.368	88.3	0.030	1.000	-1.000	0.030
vP=	**109.31**	**-27.15**	**112.635**	**-13.9**	**0.971**	**-0.241**	**0.241**	**0.971**
ALT	**x comp**	**y comp**	**mag**	**angle**	**i**	**j**	i	j
rO4=	**15.00**	**0.00**	**15.000**	**0.0**	**1.000**	**0.000**	**0.000**	**1.000**
rA=	**2.50**	**4.33**	**5.000**	**60.0**	**0.500**	**0.866**	**-0.866**	**0.500**
rBA=	4.98	-10.92	12.000	-65.5	0.415	-0.910	0.910	0.415
rBO4=	-7.52	-6.59	10.000	-138.8	-0.752	-0.659	0.659	-0.752
rB=	**7.48**	**-6.59**	**9.966**	**-41.4**	**0.750**	**-0.661**	**0.661**	**0.750**
rPA=	-0.39	-4.98	5.000	-94.5	-0.078	-0.997	0.997	-0.078
rP=	**2.11**	**-0.65**	**2.208**	**-17.2**	**0.955**	**-0.296**	**0.296**	**0.955**
vA=	**108.25**	**-62.50**	**125.000**	**-30.0**	**0.866**	**-0.500**	**0.500**	**0.866**
vBA=	-38.24	-17.43	42.027	-155.5	-0.910	-0.415	0.415	-0.910
vB=	**70.01**	**-79.93**	**106.259**	**-48.8**	**0.659**	**-0.752**	**0.752**	**0.659**
vPA=	-17.46	1.37	17.511	175.5	-0.997	0.078	-0.078	-0.997
vP=	**90.80**	**-61.13**	**109.456**	**-34.0**	**0.830**	**-0.558**	**0.558**	**0.830**

TYPE II: (RRRP)
Slider Crank Linkage



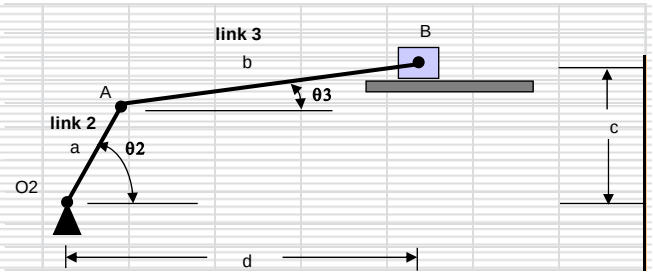
Slider Crank Algorithm Inputs

a=	5.5	Link 2
b=	21	Link 3
c=	2	Offset
$\theta_2 =$	55	0.959931089
$\dot{\theta}_2 =$	-20	$1/s$



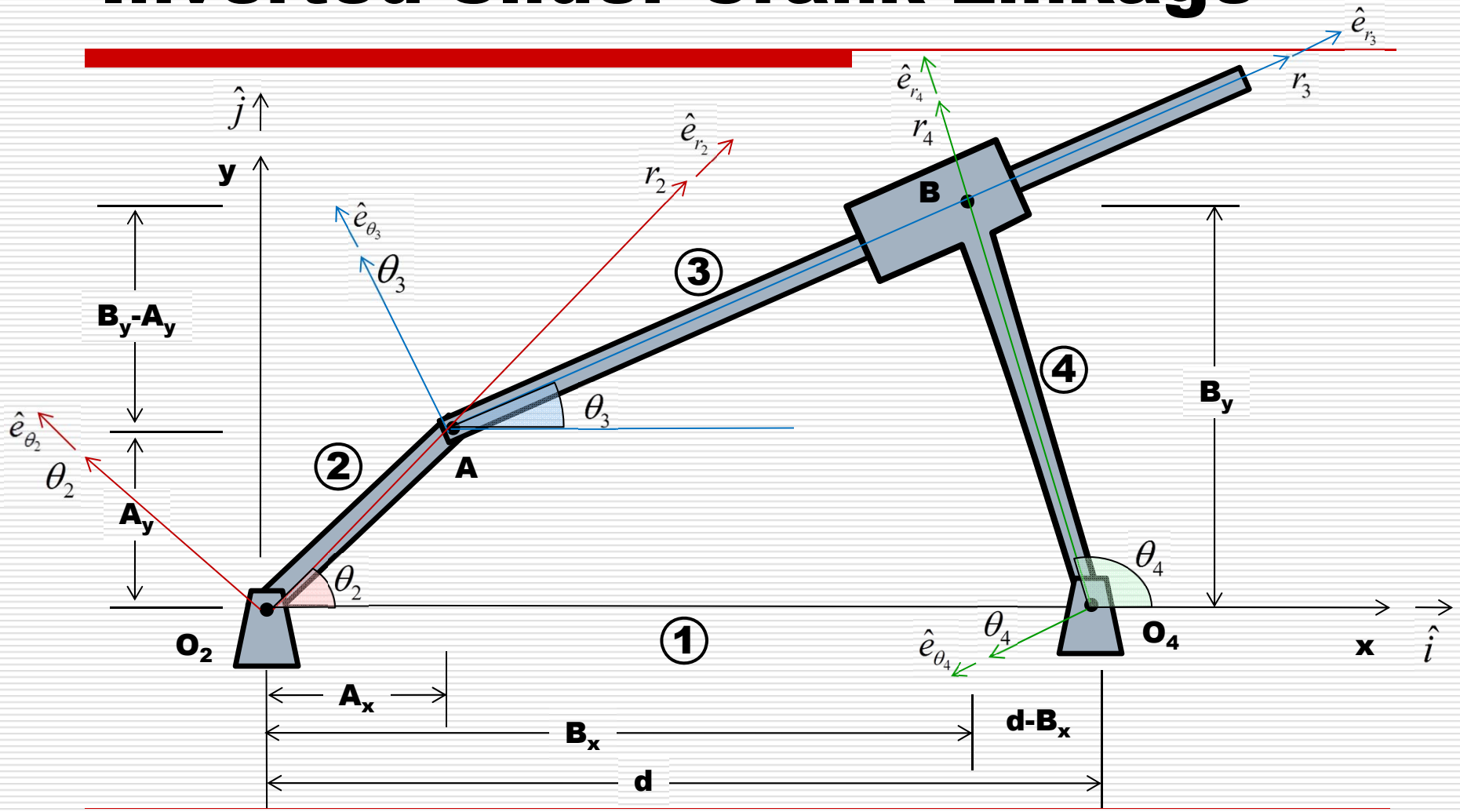
By=	2.00	2.00
Bx=	24.00	-17.70
$\theta_3 =$	-6.9	-173.1
$\dot{\theta}_3 =$	3.03	-3.03
$\ddot{\theta}_3 =$		

Slider Crank Algorithm Velocity Results



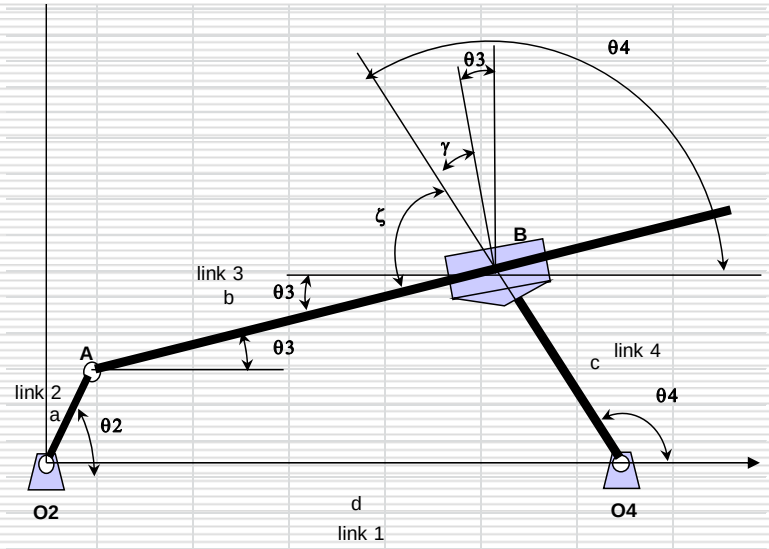
					e_r		e_θ	
	x comp	y comp	mag	angle	i	j	i	j
rB=	**24.00**	**2.00**	**24.09**	**4.8**	**0.997**	**0.083**	-0.083	0.997
rA=	**3.15**	**4.51**	**5.50**	**55.0**	**0.574**	**0.819**	-0.819	0.574
rBA=	20.85	-2.51	21.00	-6.9	0.993	-0.119	0.119	0.993
vB	**97.69**	**0.00**	**97.69**	**0.0**	**1.000**	**0.000**	0.000	1.000
vA	**90.11**	**-63.09**	**110.00**	**-35.0**	**0.819**	**-0.574**	0.574	0.819
vBA	7.58	63.09	63.55	83.1	0.119	0.993	-0.993	0.119
alt	**x comp**	**y comp**	**mag**	**angle**	**i**	**j**	**i**	**j**
rB=	**-17.70**	**2.00**	**17.81**	**173.6**	**-0.994**	**0.112**	-0.112	-0.994
rA=	**3.15**	**4.51**	**5.50**	**55.0**	**0.574**	**0.819**	-0.819	0.574
rBA=	-20.85	-2.51	21.00	-173.1	-0.993	-0.119	0.119	-0.993
vB	**82.53**	**0.00**	**82.53**	**0.0**	**1.000**	**0.000**	0.000	1.000
vA	**90.11**	**-63.09**	**110.00**	**-35.0**	**0.819**	**-0.574**	0.574	0.819
vBA	-7.58	63.09	63.55	96.9	-0.119	0.993	-0.993	-0.119

TYPE II: (RRRP)
Inverted Slider Crank Linkage



Slider Crank Algorithm
Inputs and Initial Results

a=	10	Link 2	
c=	6	Link 4	
d=	3	Link 1	
$\theta_2 =$	45		
γ=	45		
$\dot{\theta}_2 =$	100	$1/s$	
$\ddot{\theta}_2 =$	-65	$1/s^2$	
b=	2.73		-11.21
$\theta_4 =$	46.40		163.74
$\theta_3 =$	-88.60		28.74
$\dot{\theta}_4 =$	98.95		137.74
$\dot{\theta}_3 =$	98.95		137.74
b-dot=	304.37		-304.37

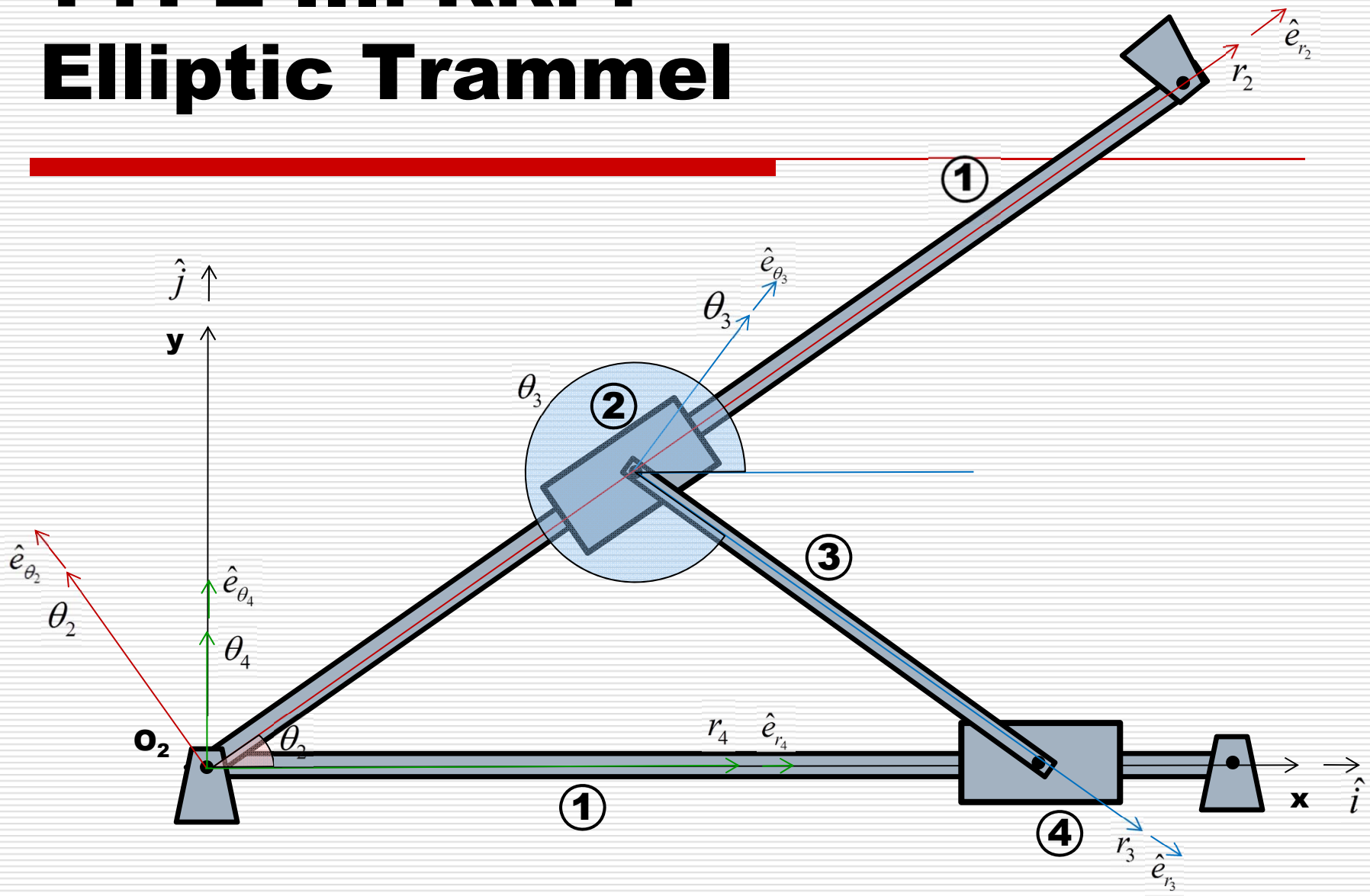


Slider Crank Algorithm
Velocity Vector Analysis

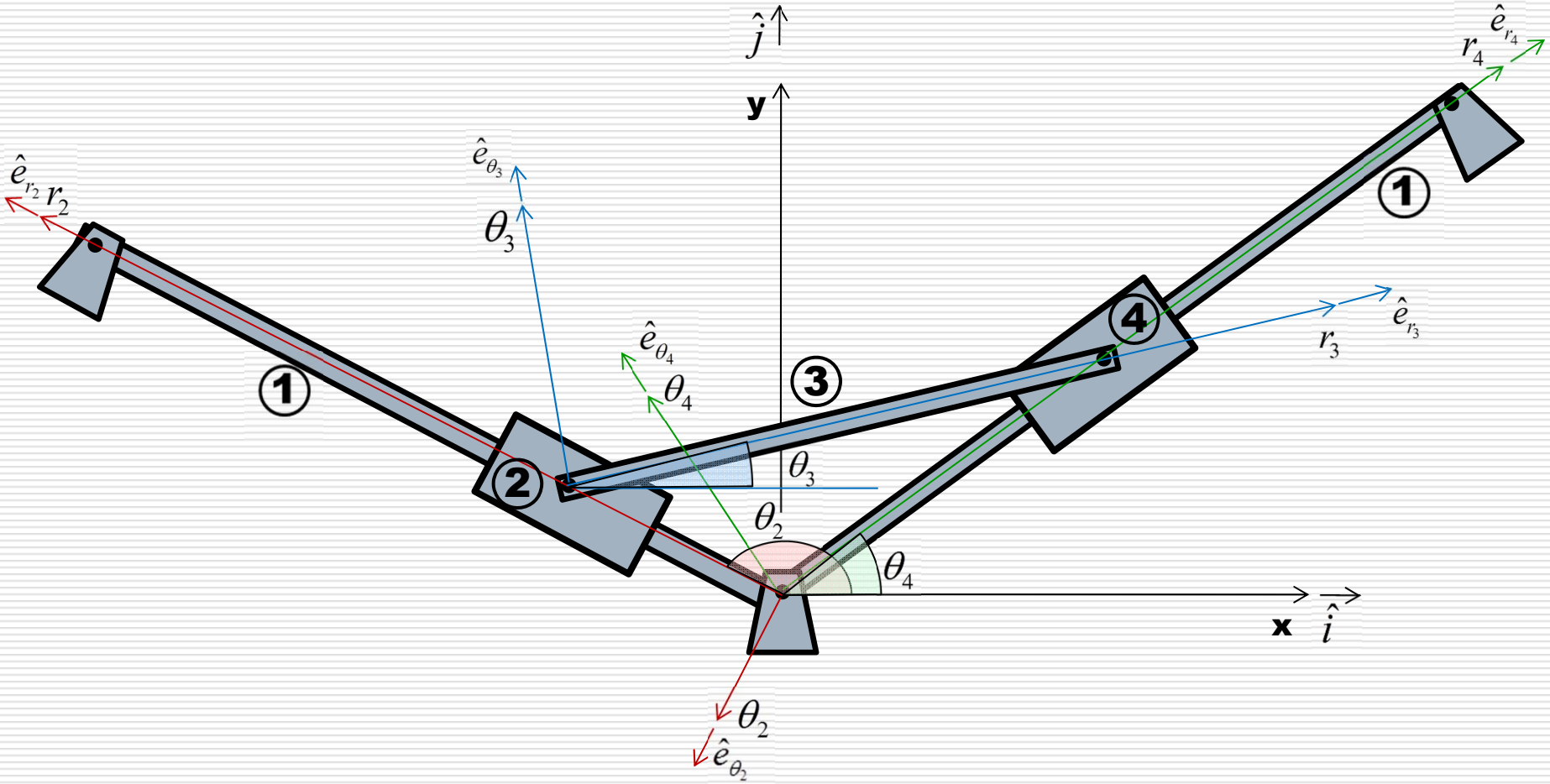
					e_r		e_θ	
	x comp	**y comp**	**mag**	**angle**	**i**	**j**	**i**	**j**
rO4=	**3.00**	**0.00**	**3.00**	**0.0**	**1.000**	**0.000**	0.000	1.000
rA=	**7.07**	**7.07**	**10.00**	**45.0**	**0.707**	**0.707**	-0.707	0.707
rBA=	0.07	-2.73	2.73	-88.6	0.024	-1.000	1.000	0.024
rBO4=	4.14	4.35	6.00	46.4	0.690	0.724	-0.724	0.690
rB=	**7.14**	**4.35**	**8.36**	**31.3**	**0.854**	**0.520**	-0.520	0.854
vA=	**-707.11**	**707.11**	**1000.00**	**135.0**	**-0.707**	**0.707**	-0.707	-0.707
vBA=	277.17	-297.69	406.75	-47.0	0.681	-0.732	0.732	0.681
vB=	**-429.93**	**409.42**	**593.69**	**136.4**	**-0.724**	**0.690**	-0.690	-0.724

alt	**x comp**	**y comp**	**mag**	**angle**	**i**	**j**	**i**	**j**
rO4=	**3.00**	**0.00**	**3.00**	**0.0**	**1.000**	**0.000**	0.000	1.000
rA=	**7.07**	**7.07**	**10.00**	**45.0**	**0.707**	**0.707**	-0.707	0.707
rBA=	-9.83	-5.39	11.21	-151.3	-0.877	-0.481	0.481	-0.877
rB04=	-5.76	1.68	6.00	163.7	-0.960	0.280	-0.280	-0.960
rB=	**-2.76**	**1.68**	3.23	148.7	-0.854	0.520	-0.520	-0.854
vA=	**-707.11**	**707.11**	**1000.00**	**135.0**	**-0.707**	**0.707**	-0.707	-0.707
vBA=	475.68	-1500.50	1574.09	-72.4	0.302	-0.953	0.953	0.302
vB=	**-231.42**	**-793.39**	**826.46**	**-106.3**	**-0.280**	**-0.960**	0.960	-0.280

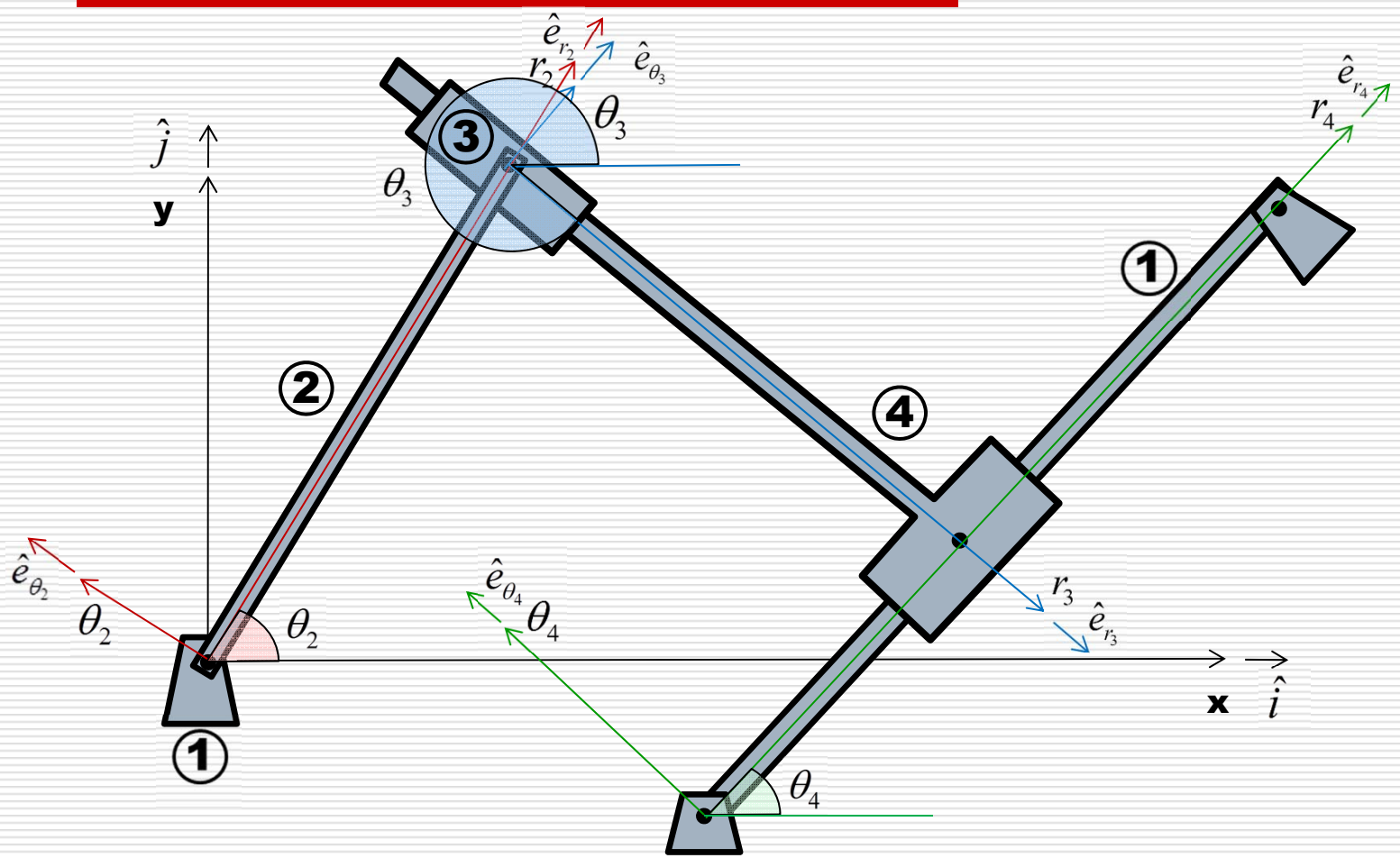
TYPE III: RRPP
Elliptic Trammel



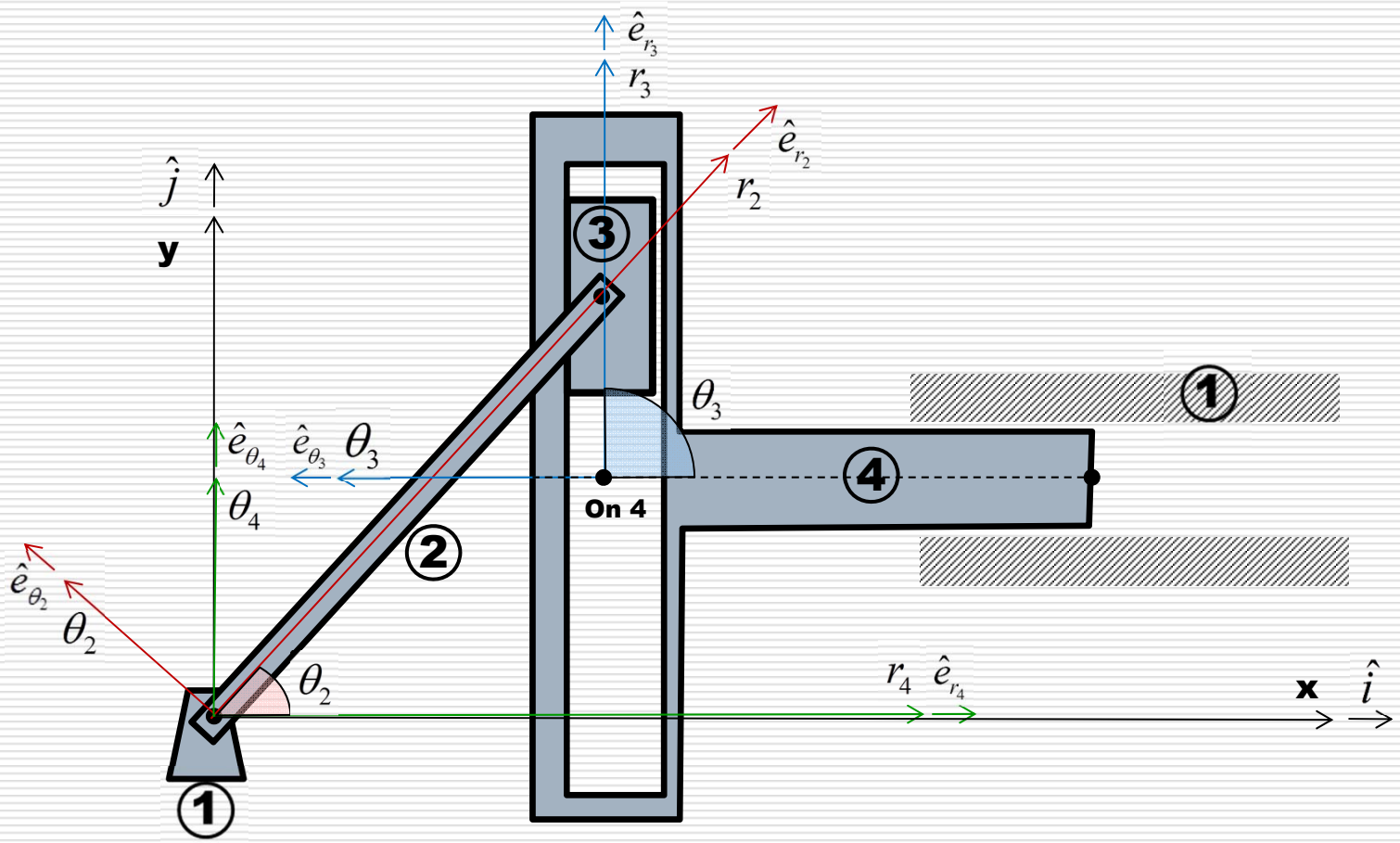
Type III: RRPP
Elliptic Trammel



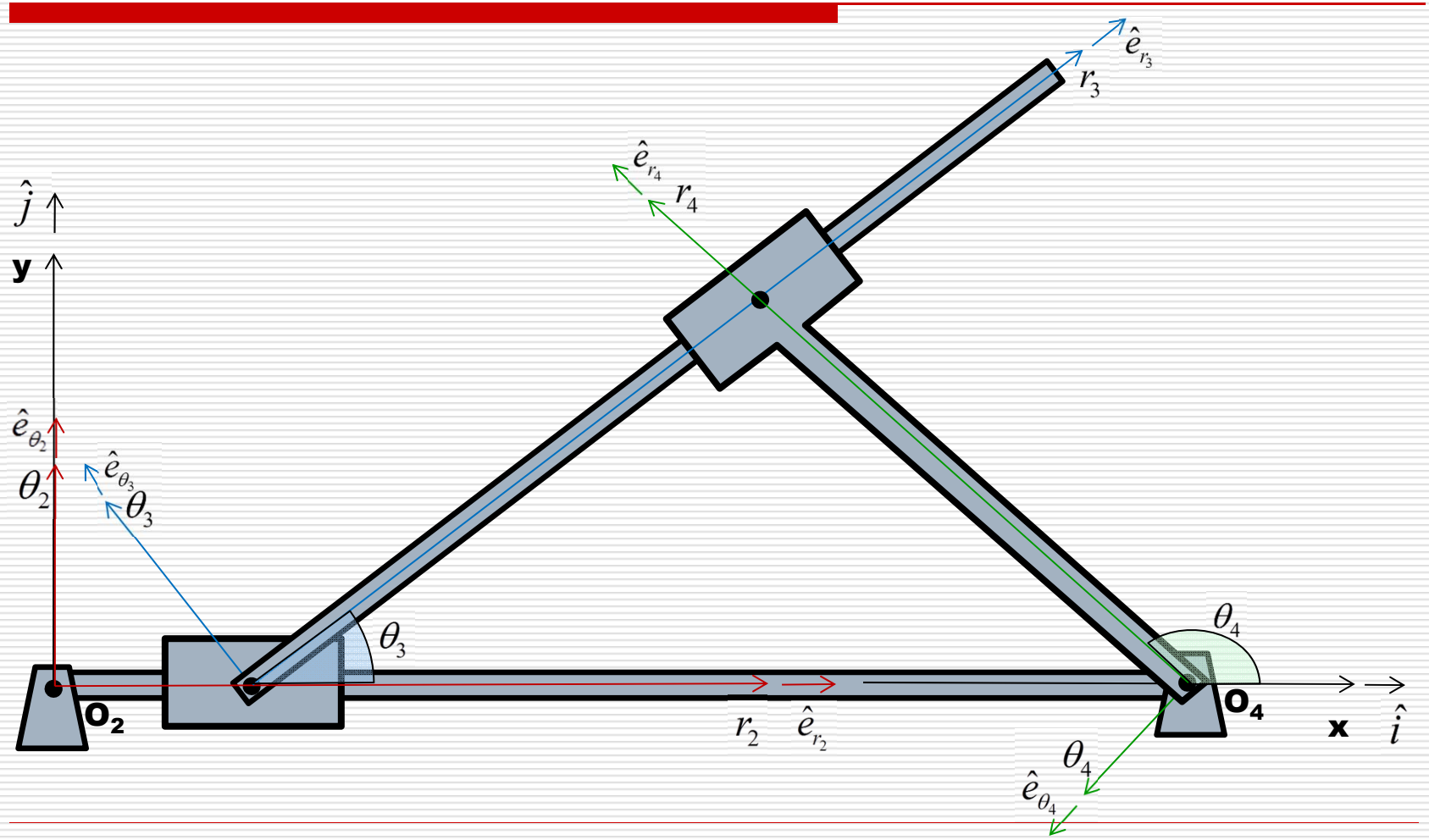
Type III: RRPP
Elliptic Trammel



Type III: RRPP
Elliptic Trammel: Scotch Yoke



Type IV: RPRP
Rapson Slide Linkage



Example 1

Rods "*R*" and "*L*" are pinned at "O" and "O'" to a frame. Rod "*L*" is also pinned to the slotted body "*B*" at "B". ?The upper end of "*R*" is pinned at P to a roller that moves freely in the slot of "*B*". The angular velocities of the rod "*R*" and link "*L*" are constants:

ω_R=-0.2 rad/s

ω_L=-0.4 rad/s

Determine the velocity of "P" and the angular velocity of "B" at the given instant.

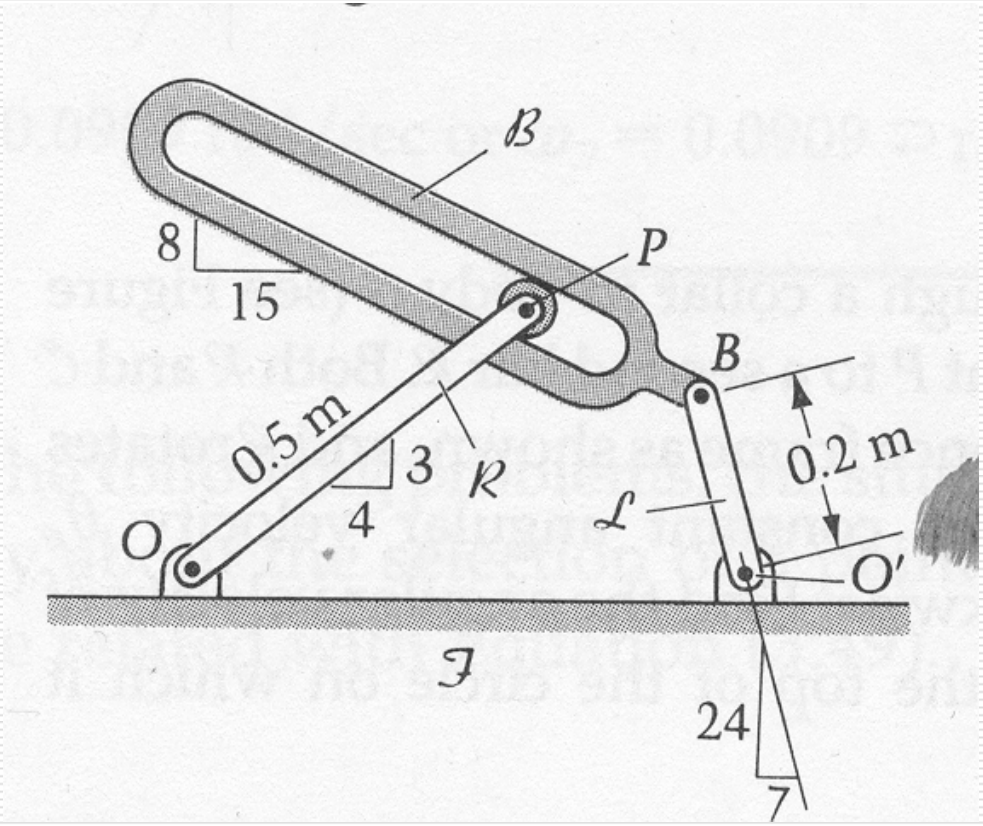
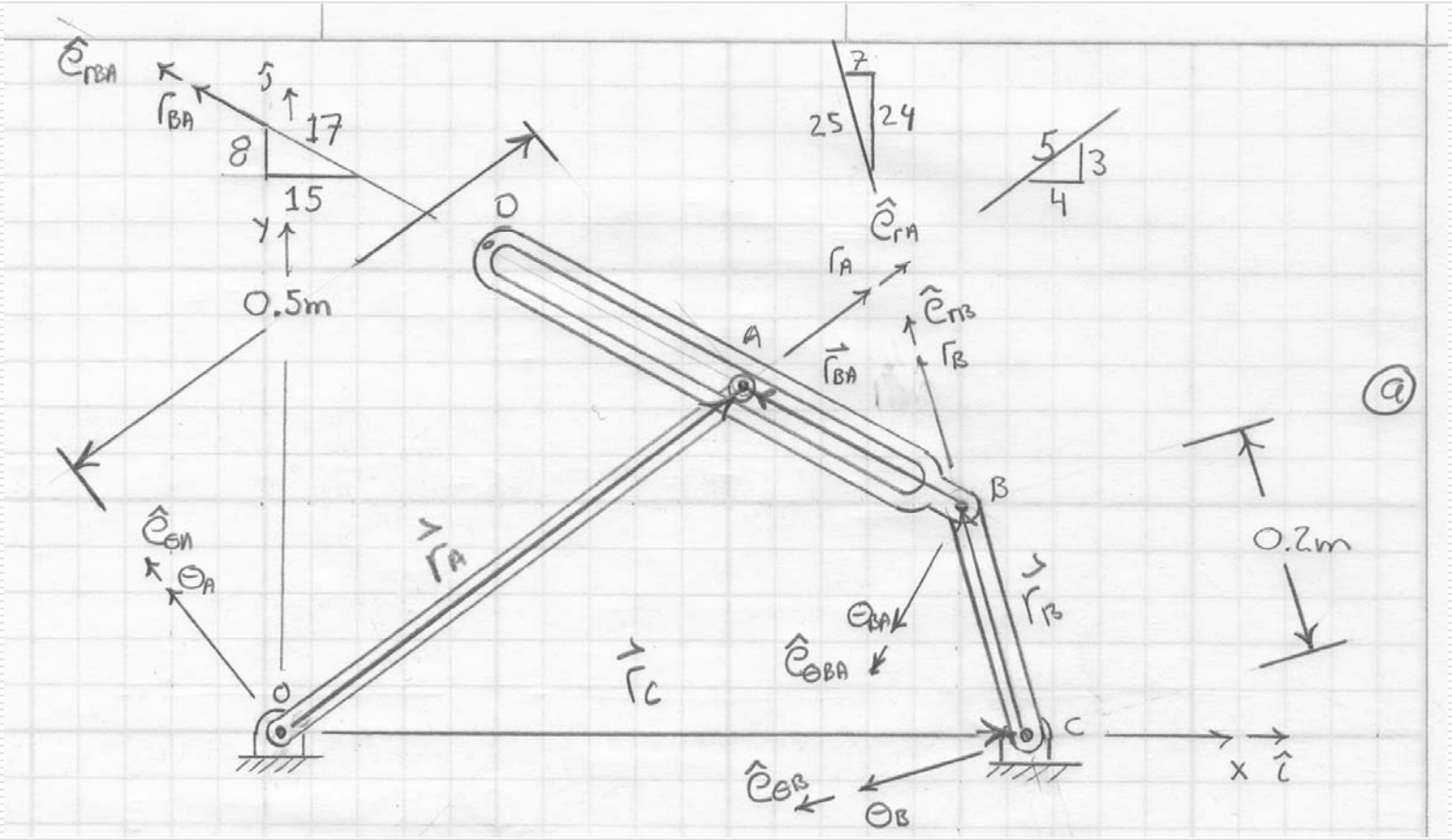


Diagram of Problem



Problem Geometry