ESC043
Advanced Mechanics

EXAM1
Union College

Winter 2002
4 February

NAME: Solution

PROBLEM 1: The L-bracket shown is 12mm in diameter and is built into the wall at A. At point C a 10 kN load is applied as shown.

1a. Using the diagram below, draw the free-body diagram and determine the reactions at A. Using the diagram provided, illustrate the resultant bending moment, torque, normal force, and shear force at the wall.

(2) $\Sigma F_x = 0 = A_x + 8.66\,kN \Rightarrow \underline{\underline{A_x = -8.66\,kN}}$

(2) $\Sigma F_y = 0 = A_y - 5.0\,kN \Rightarrow \underline{\underline{A_y = 5.0\,kN}}$

(2) $\Sigma F_z = \underline{\underline{0 = A_z}}$

$$\Sigma \vec{M}_A = \vec{0} = \vec{r}_{AC} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.5m & 0 & 0.5m \\ 8.66kN & 5kN & 0 \end{vmatrix} + \vec{M}_A$$

$$= [-(0.5m)(-5kN)]\hat{i} - [-(0.5m)\cdot(8.66kN)]\hat{j} + [(1.5m)(-5kN)]\hat{k} + M_{Ax}\hat{i} + M_{Ay}\hat{j} + M_{Az}\hat{k}$$

Dotting with $\hat{i}$

(2) $0 = 2.5\,kN{\cdot}m + M_{Ax} \Rightarrow \underline{\underline{M_{Ax} = -2.5\,kN{\cdot}m}}$

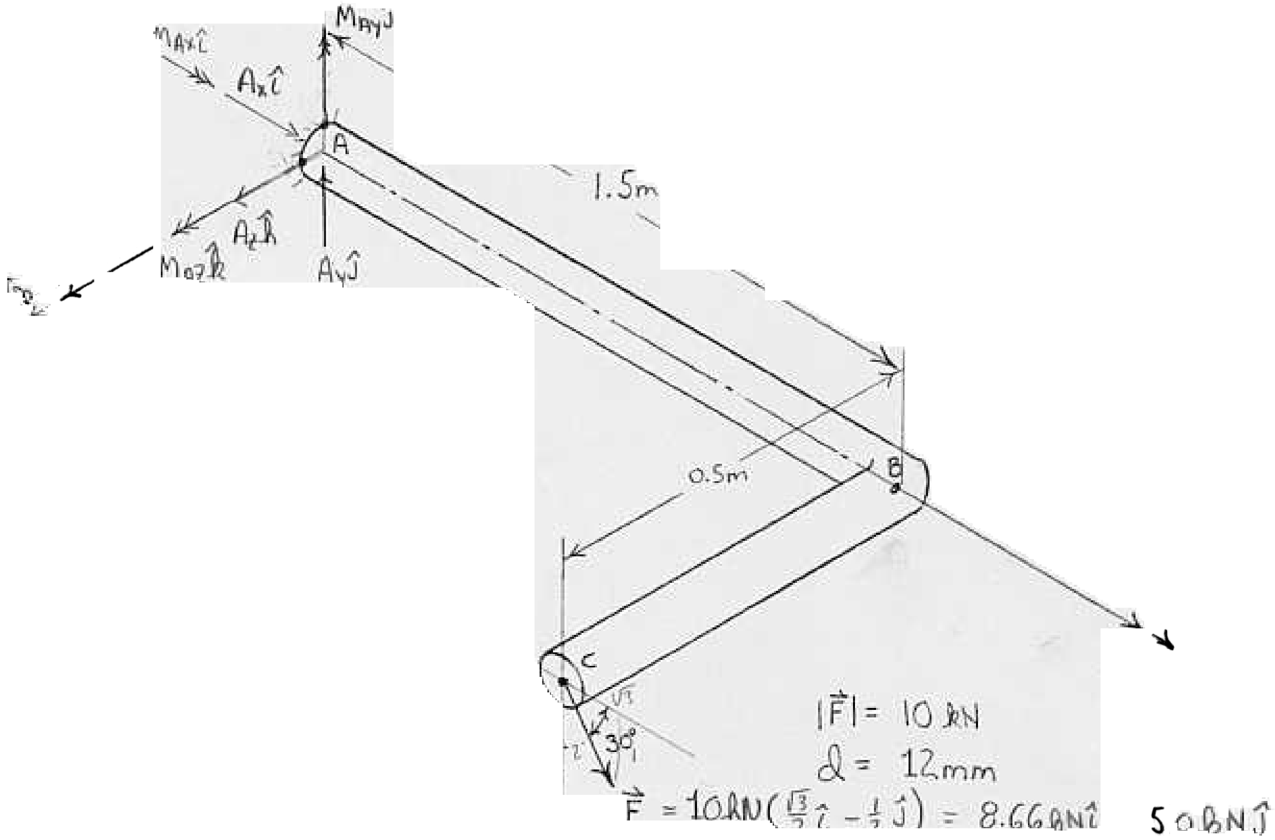
Dotting with $\hat{j}$

(2) $0 = 4.33\,kN{\cdot}m + M_{Ay} \Rightarrow \underline{\underline{M_{Ay} = -4.33\,kN{\cdot}m}}$

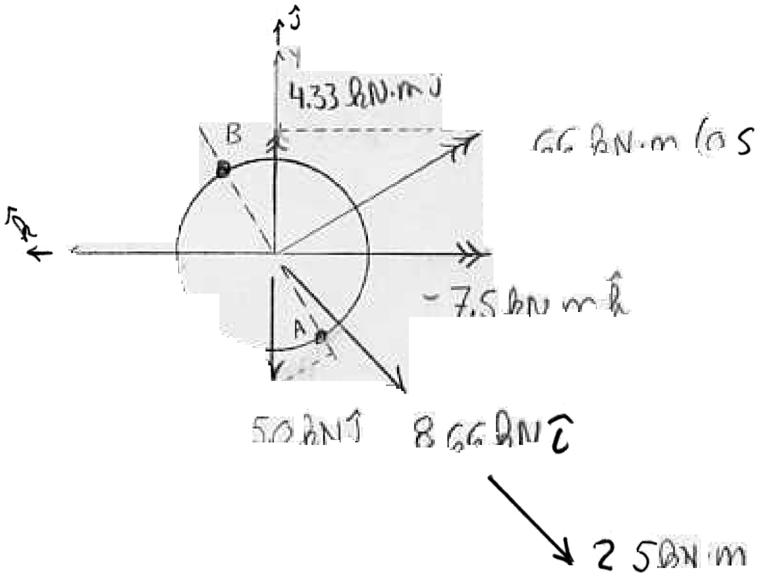
Dotting with $\hat{k}$

$0 = -7.5\,kN{\cdot}m + M_{Az} \Rightarrow \underline{\underline{M_{Az} = 7.5\,kN{\cdot}m}}$

4)



$M_{Ax}\hat{i}$
$M_{Ay}\hat{j}$
$A_x\hat{i}$
A
1.5m
$A_z\hat{k}$
$M_{Az}\hat{k}$
$A_y\hat{j}$
B
0.5m
C
30°
$|\vec{F}| = 10\ kN$
$d = 12mm$
$\vec{F} = 10kN(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}) = 8.66kN\hat{i}$ 5.0kN$\hat{j}$



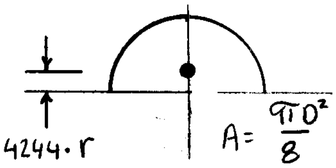
$\hat{j}$
y
4.33 kN·m $\hat{j}$
B
66 kN·m (0.5 866$\hat{a}$
$\hat{k}$
-7.5 kN·m $\hat{k}$
A
5.0kN$\hat{j}$ 8.66kN$\hat{i}$
2.5kN·m

1b. At the point of the maximum bending stress in the beam at the wall, determine the complete stress state, designate the location on the previous illustration, and draw the resulting stress cube.

MAXIMUM TENSION AT A : 76.56(10⁶)   <1.05(10⁹)

$$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{8.66(10^3)\,N}{\pi\,[12(10^{-3})m/2]^2} + \frac{8.66(10^3)\,N\cdot m \cdot [12(10^{-3})\,m/2]}{\dfrac{\pi\,[12(10^{-3})m]^4}{64}}$$

$$51.07(10^9)\,\frac{N}{m^2} = \boxed{51.12\ GPa}$$



SHEAR STRESS DUE TO TORQUE

(5) $$\tau_T = \frac{Tr}{J} = \frac{2.5\,kN\cdot m \cdot 6(10^{-3})\,m}{\dfrac{\pi\cdot[12(10^{-3})m]^4}{32}} = 7.368(10^9)\,\frac{N}{m^2}$$

$$\boxed{7.368\,GPa}$$

SHEAR STRESS DUE TO SHEAR FORCE

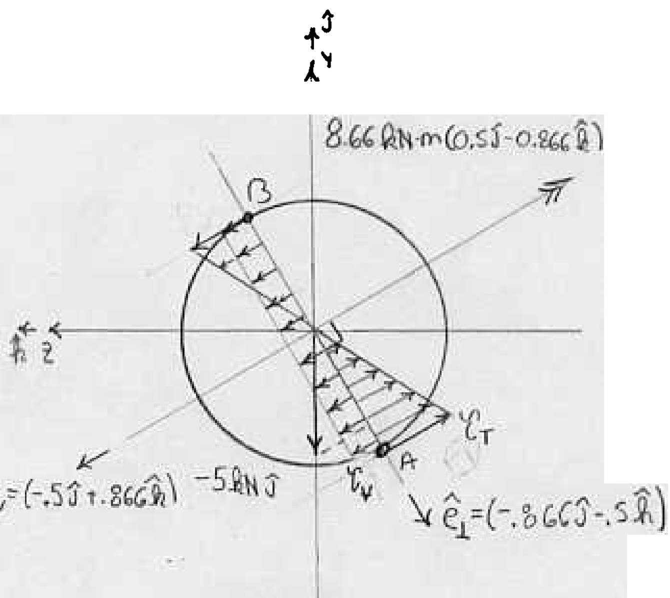
SHEAR FORCE RESULTANT PARALLEL TO MOMENT

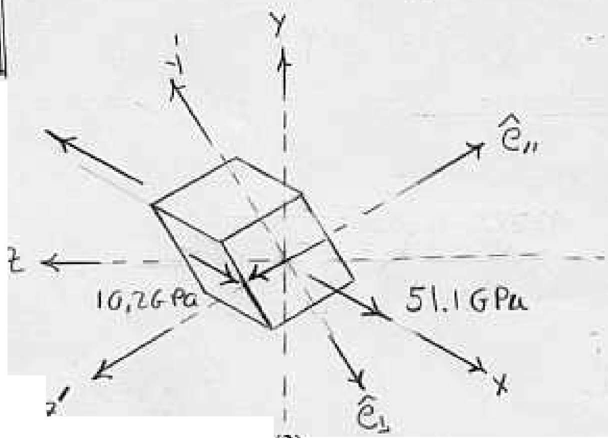
(5) $5.8\,kN\,\hat{j} \circ (-.5\hat{j} + .866\hat{k}) = 2.9\,kN$

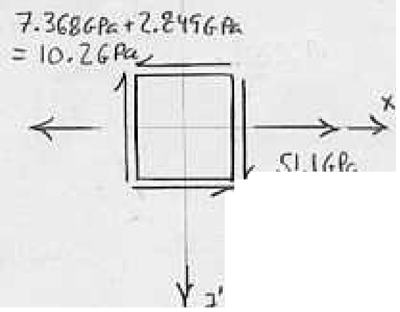
(5) $$\tau_V = \frac{VQ}{It} = \frac{2.9\,kN \cdot 0.4244 \cdot \left(\dfrac{12(10^{-3})m}{2}\right) \cdot \dfrac{\pi\cdot(12\cdot(10^{-3})m)}{8}}{\dfrac{\pi\cdot[12(10^{-3})m]^4}{64} \cdot 12(10^{-3})\,m}$$

$$2.849(10^9)\,\frac{N}{m^2}$$

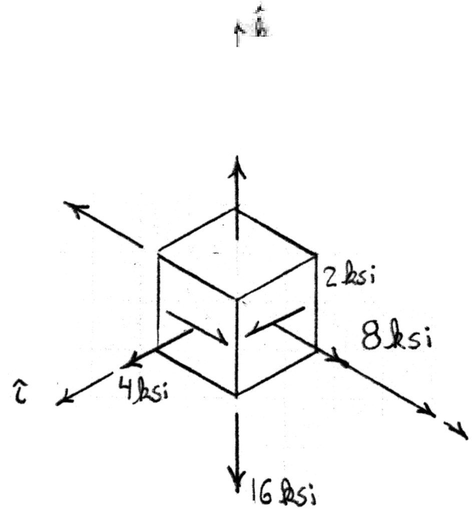
$$\boxed{2.849\ GPa}$$







PROBLEM 2: For the stress cube shown below, draw Mohr's circles and determine the principle stresses, maximum shear stress, and explain the orientations of these states of stress with respect to the original stress cube.



$\sqrt{(2ksi)^2+(2ksi)^2}$ 2.828 ksi

9.586 ksi 3.172 ksi 6.414 ksi 3)

r_3 ksi 8.828 ksi 72 ksi /3

σ_1 6 ksi 2.828 ksi 72 ksi 3)

σ_2 6 ksi 2.828 ksi 8.828 ksi

σ_3 16 ksi

τ_{max1} 2.828 ksi

τ_{max2} 6. ksi

τ_{max3} 172 ~~ksi~~ 2.586 5)

θ_{p1} 22.5 θ_{s1} 22.5° 3)

$\theta_{p2} = 0$ $\theta_{s2} = 45°$ 3)

$\theta_{p3} = 0$ $\theta_{s3} = 45°$ 3

5.